MERRIMAN

Horace M Sturges

AMERICAN
CIVIL ENGINEERS' HANDBOOK

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MANSFIELD MERRIMAN

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The Publishers and the Editor-in-Chief will be grateful to readers of this volume who will kindly call attention to any errors of omission or of commission therein. It is intended to make our publications standard works of study and reference, and, to that end, the greatest accuracy is sought. It rarely happens that the early editions of books are free from errors; but it is the endeavor of the Publishers to have them removed, and it is therefore desired that the Editor-in-Chief may be aided in his task of revision, from time to time, by the kindly criticism of readers.

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PREFACE TO FIRST EDITION

In November of 1908 the undersigned laid out a rough plan for the contents of a new Civil Engineers' Pocket Book and invited several highly qualified men to act as Associate Editors. In January of 1909 preliminary outlines of each of the sections were arranged by correspondence. In April of 1909 these outlines were perfected, and a preliminary table of contents of the entire book was sent to each editor together with instructions relating to the plan of treatment, the details of arrangement and typography, and the preparation of manuscripts and drawings. In September of 1909 the first copy was sent to the engraver and printer. Now, after more than two years of arduous work in corresponding, compiling, writing, editing, proof reading, and indexing, the plates are about to be put on the press.

The Editor-in-Chief is responsible for the division of the book into sections and, in most cases, for the arrangement of chapters and articles. To every sheet of manuscript, every drawing, every galley proof, and every page of type and plate proof he has given careful attention. He and the printer are responsible for the typography of the volume.

Each Associate Editor is responsible for the subject-matter of his section and has read and corrected the page proofs of the same. Thanks are due to each and all for hearty cooperation and cheerful compliance with the instructions and wishes of the Editor-in-Chief.

From the instructions issued to Associate Editors in April of 1909 the following sentences are cited as explanatory of the plan of this volume:

> Pocket books are consulted by engineers because facts, formulas, tables, and methods can be found more quickly than by referring to text books or treatises. A pocket book must be prepared from this point of view. It should cover the ground with great conciseness and clearness, and be thoroughly up to date.
>
> The work is to be on a higher plane than former American Pocket Books. While Taschenbuch Hütte may generally be taken as a model regarding the presentation of mathematical matter, it is certain that this new Pocket Book should be very much better in respect to practical subjects. The reader is supposed to have a good knowledge of elementary mathematics, so that it is unnecessary to express formulas in words.
>
> Demonstrations are not required, but methods which cannot be expressed by formulas should be clearly explained and usually be illustrated by numerical examples. Logical order is not so important as in text-books, and there should be no hesitation in deviating from it when deemed desirable. This is a reference book only.
>
> The fundamental principles to be kept in mind in preparing the copy are these: (1) Select those topics to which civil engineers desire to refer. (2) Condense the matter so that the greatest amount may be put in the assigned space and at the same time be clearly presented. (3) This Civil Engineers' Pocket Book must be better and fuller than any heretofore published in the English language.

The thirteen sections of the book contain 75 chapters, 620 articles, 495 tables, and 944 numbered figures which are equivalent to about 1200 ordinary cuts since in many cases several similar figures are grouped together. The

number of tables is so large that it is impracticable to mention them in the table of contents, for the mere list of their titles would occupy over eight pages of fine type, but references to all will be found in the index. Sections 2 to 11 inclusive deal with civil engineering proper, while Section 1 gives tables for approximate mathematical computations and Sections 12 and 13 treat of mathematics, mechanics, physics, meteorology, and weights and measures. These thirteen sections fill 1314 pages, and they are followed by a detailed alphabetic index which occupies 66 pages.

Regarding typography it has been the aim to render this Pocket Book more legible and artistic than any heretofore published. The page is longer and wider than usual, all type is leaded, there are no tables reading lengthwise of the page, and there are no short pages except at the beginning and end of a section. The effort has been made to economize space at every step where this did not conflict with the rules of good typography.

It is not claimed that this first edition is perfect. There are some duplications in minor topics and a few have been overlooked, while full unity of treatment has not been secured. Completeness, unity, and perfect accuracy are difficult of attainment in a first edition, and it can only here be affirmed that earnest efforts have been made to secure these desirable qualities. It is believed, moreover, that all whose names appear on the title-page have a pride in their connection with the volume and feel that it is a contribution to engineering literature which will promote sound engineering practise.

In conclusion it seems advisable to call the attention of readers of this book to the words of caution written by John B. Henck in 1854: "It may be remarked that it was no part of the purpose of this volume to furnish a collection of mere rules, professing to require only an ability to read for their successful application. Rules can seldom be safely applied without a clear understanding of the principles on which they rest."

MANSFIELD MERRIMAN

NEW YORK, December, 1910

NOTE TO FOURTH EDITION

This edition has been thoroughly revised and so much enlarged that it is now properly called Handbook instead of Pocket-book. To Section 1 is added an Appendix on colored paper which gives the excellent mathematical tables of Searles for the use of computors. Section 3A is entirely devoted to Electric Railroads. The new Section 17 treats of Irrigation and Drainage. All other Sections have been revised and brought up to date, being in some cases rewritten and much enlarged. The third edition had 1580 pages; this edition has 1955 pages.

The editorial staff has been changed by the withdrawal of Walter L. Webb, Rudolph P. Miller, and Frank P. McKibben, whose places have been filled by Fred. A. Barnes, Herbert F. Moore, and Frank H. Constant. The new Sections 3A and 17 were written by William A. Del Mar and Horace W. King. It is believed that the result of the work of the eighteen associate editors renders this Handbook, more than ever before, a valuable and authoritative book of reference to all engaged in civil engineering.

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n	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957
	0	1	2	3	4	5	6	7	8	9

of Numbers from 000 to 999

n	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957
	0	1	2	3	4	5	6	7	8	9

2. Logarithms of Trigonometric Functions

Angle	Log Arc	Log Sin	Log Tan	Log Sec	Log Csc	Log Cot	Log Cos		
1°	$\bar{2}.2419$	$\bar{2}.2419$	$\bar{2}.2419$	0.0001	1.7581	1.7581	$\bar{1}.9999$	0.1913	89
2	$\bar{2}.5429$	$\bar{2}.5428$	$\bar{2}.5431$	0.0003	1.4572	1.4569	$\bar{1}.9997$	0.1864	88
3	$\bar{2}.7190$	$\bar{2}.7188$	$\bar{2}.7194$	0.0006	1.2812	1.2806	$\bar{1}.9994$	0.1814	87
4	$\bar{2}.8439$	$\bar{2}.8436$	$\bar{2}.8446$	0.0011	1.1564	1.1554	$\bar{1}.9989$	0.1764	86
5	$\bar{2}.9408$	$\bar{2}.9403$	$\bar{2}.9420$	0.0017	1.0597	1.0580	$\bar{1}.9983$	0.1713	85°
6°	$\bar{1}.0200$	$\bar{1}.0192$	$\bar{1}.0216$	0.0024	0.9808	0.9784	$\bar{1}.9976$	0.1662	84
7	$\bar{1}.0870$	$\bar{1}.0859$	$\bar{1}.0891$	0.0032	0.9141	0.9109	$\bar{1}.9968$	0.1610	83
8	$\bar{1}.1450$	$\bar{1}.1436$	$\bar{1}.1478$	0.0042	0.8564	0.8522	$\bar{1}.9958$	0.1557	82
9	$\bar{1}.1961$	$\bar{1}.1943$	$\bar{1}.1997$	0.0054	0.8057	0.8003	$\bar{1}.9946$	0.1504	81
10	$\bar{1}.2419$	$\bar{1}.2397$	$\bar{1}.2463$	0.0066	0.7603	0.7537	$\bar{1}.9934$	0.1450	80°
11°	$\bar{1}.2833$	$\bar{1}.2806$	$\bar{1}.2887$	0.0081	0.7194	0.7113	$\bar{1}.9919$	0.1395	79
12	$\bar{1}.3211$	$\bar{1}.3179$	$\bar{1}.3275$	0.0096	0.6821	0.6725	$\bar{1}.9904$	0.1340	78
13	$\bar{1}.3558$	$\bar{1}.3521$	$\bar{1}.3634$	0.0113	0.6479	0.6366	$\bar{1}.9887$	0.1284	77
14	$\bar{1}.3880$	$\bar{1}.3837$	$\bar{1}.3968$	0.0131	0.6163	0.6032	$\bar{1}.9869$	0.1227	76
15	$\bar{1}.4180$	$\bar{1}.4130$	$\bar{1}.4281$	0.0151	0.5870	0.5719	$\bar{1}.9849$	0.1169	75°
16°	$\bar{1}.4460$	$\bar{1}.4403$	$\bar{1}.4575$	0.0172	0.5597	0.5425	$\bar{1}.9828$	0.1111	74
17	$\bar{1}.4723$	$\bar{1}.4659$	$\bar{1}.4853$	0.0194	0.5341	0.5147	$\bar{1}.9806$	0.1052	73
18	$\bar{1}.4971$	$\bar{1}.4900$	$\bar{1}.5118$	0.0218	0.5100	0.4882	$\bar{1}.9782$	0.0992	72
19	$\bar{1}.5206$	$\bar{1}.5126$	$\bar{1}.5370$	0.0243	0.4874	0.4630	$\bar{1}.9757$	0.0931	71
20	$\bar{1}.5429$	$\bar{1}.5341$	$\bar{1}.5611$	0.0270	0.4659	0.4389	$\bar{1}.9730$	0.0870	70°
21°	$\bar{1}.5641$	$\bar{1}.5543$	$\bar{1}.5842$	0.0298	0.4457	0.4158	$\bar{1}.9702$	0.0807	69
22	$\bar{1}.5843$	$\bar{1}.5736$	$\bar{1}.6064$	0.0328	0.4264	0.3936	$\bar{1}.9672$	0.0744	68
23	$\bar{1}.6036$	$\bar{1}.5919$	$\bar{1}.6279$	0.0360	0.4081	0.3721	$\bar{1}.9640$	0.0680	67
24	$\bar{1}.6221$	$\bar{1}.6093$	$\bar{1}.6486$	0.0393	0.3907	0.3514	$\bar{1}.9607$	0.0614	66
25	$\bar{1}.6398$	$\bar{1}.6259$	$\bar{1}.6687$	0.0427	0.3741	0.3313	$\bar{1}.9573$	0.0548	65°
26°	$\bar{1}.6569$	$\bar{1}.6418$	$\bar{1}.6882$	0.0463	0.3582	0.3118	$\bar{1}.9537$	0.0481	64
27	$\bar{1}.6732$	$\bar{1}.6570$	$\bar{1}.7072$	0.0501	0.3430	0.2928	$\bar{1}.9499$	0.0412	63
28	$\bar{1}.6890$	$\bar{1}.6716$	$\bar{1}.7257$	0.0541	0.3284	0.2743	$\bar{1}.9459$	0.0343	62
29	$\bar{1}.7042$	$\bar{1}.6856$	$\bar{1}.7438$	0.0582	0.3144	0.2562	$\bar{1}.9418$	0.0272	61
30	$\bar{1}.7190$	$\bar{1}.6990$	$\bar{1}.7614$	0.0625	0.3010	0.2386	$\bar{1}.9375$	0.0200	60°
31°	$\bar{1}.7332$	$\bar{1}.7118$	$\bar{1}.7788$	0.0669	0.2882	0.2212	$\bar{1}.9331$	0.0127	59
32	$\bar{1}.7470$	$\bar{1}.7242$	$\bar{1}.7958$	0.0716	0.2758	0.2042	$\bar{1}.9284$	0.0053	58
33	$\bar{1}.7604$	$\bar{1}.7361$	$\bar{1}.8125$	0.0764	0.2639	0.1875	$\bar{1}.9236$	$\bar{1}.9978$	57
34	$\bar{1}.7734$	$\bar{1}.7476$	$\bar{1}.8290$	0.0814	0.2524	0.1710	$\bar{1}.9186$	$\bar{1}.9901$	56
35	$\bar{1}.7859$	$\bar{1}.7586$	$\bar{1}.8452$	0.0866	0.2414	0.1548	$\bar{1}.9134$	$\bar{1}.9822$	55°
36°	$\bar{1}.7982$	$\bar{1}.7692$	$\bar{1}.8613$	0.0920	0.2308	0.1387	$\bar{1}.9080$	$\bar{1}.9743$	54
37	$\bar{1}.8101$	$\bar{1}.7795$	$\bar{1}.8771$	0.0977	0.2205	0.1229	$\bar{1}.9023$	$\bar{1}.9662$	53
38	$\bar{1}.8217$	$\bar{1}.7893$	$\bar{1}.8928$	0.1035	0.2107	0.1072	$\bar{1}.8965$	$\bar{1}.9579$	52
39	$\bar{1}.8329$	$\bar{1}.7989$	$\bar{1}.9084$	0.1095	0.2011	0.0916	$\bar{1}.8905$	$\bar{1}.9494$	51
40	$\bar{1}.8439$	$\bar{1}.8081$	$\bar{1}.9238$	0.1157	0.1919	0.0762	$\bar{1}.8843$	$\bar{1}.9408$	50°
41°	$\bar{1}.8547$	$\bar{1}.8169$	$\bar{1}.9392$	0.1222	0.1831	0.0608	$\bar{1}.8778$	$\bar{1}.9321$	49
42	$\bar{1}.8651$	$\bar{1}.8255$	$\bar{1}.9544$	0.1289	0.1745	0.0456	$\bar{1}.8711$	$\bar{1}.9231$	48
43	$\bar{1}.8753$	$\bar{1}.8338$	$\bar{1}.9697$	0.1359	0.1662	0.0303	$\bar{1}.8641$	$\bar{1}.9140$	47
44	$\bar{1}.8853$	$\bar{1}.8418$	$\bar{1}.9848$	0.1431	0.1582	0.0152	$\bar{1}.8569$	$\bar{1}.9046$	46
45	$\bar{1}.8951$	$\bar{1}.8495$	0.0000	0.1505	0.1505	0.0000	$\bar{1}.8495$	$\bar{1}.8951$	45°
		Log Cos	Log Cot	Log Csc	Log Sec	Log Tan	Log Sin	Log Arc	Angle

Explanation on p. 38

3. Natural Trigonometric Functions

Angle	Arc	Sin	Tan	Sec	Cosec	Cot	Cos		
1°	0.0175	0.0175	0.0175	1.0002	57.299	57.290	0.9998	1.5533	89
2	0.0349	0.0349	0.0349	1.0006	28.654	28.636	0.9994	1.5359	88
3	0.0524	0.0523	0.0524	1.0014	19.107	19.081	0.9986	1.5184	87
4	0.0698	0.0698	0.0699	1.0024	14.336	14.301	0.9976	1.5010	86
5	0.0873	0.0872	0.0875	1.0038	11.474	11.430	0.9962	1.4835	85°
6°	0.1047	0.1045	0.1051	1.0055	9.5668	9.5144	0.9945	1.4661	84
7	0.1222	0.1219	0.1228	1.0075	8.2055	8.1443	0.9925	1.4486	83
8	0.1396	0.1392	0.1405	1.0098	7.1853	7.1154	0.9903	1.4312	82
9	0.1571	0.1564	0.1584	1.0125	6.3925	6.3138	0.9877	1.4137	81
10	0.1745	0.1736	0.1763	1.0154	5.7588	5.6713	0.9848	1.3963	80°
11°	0.1920	0.1908	0.1944	1.0187	5.2408	5.1446	0.9816	1.3788	79
12	0.2094	0.2079	0.2126	1.0223	4.8097	4.7046	0.9781	1.3614	78
13	0.2269	0.2250	0.2309	1.0263	4.4454	4.3315	0.9744	1.3439	77
14	0.2443	0.2419	0.2493	1.0306	4.1336	4.0108	0.9703	1.3265	76
15	0.2618	0.2588	0.2679	1.0353	3.8637	3.7321	0.9659	1.3090	75°
16°	0.2793	0.2756	0.2867	1.0403	3.6280	3.4874	0.9613	1.2915	74
17	0.2967	0.2924	0.3057	1.0457	3.4203	3.2709	0.9563	1.2741	73
18	0.3142	0.3090	0.3249	1.0515	3.2361	3.0777	0.9511	1.2566	72
19	0.3316	0.3256	0.3443	1.0576	3.0716	2.9042	0.9455	1.2392	71
20	0.3491	0.3420	0.3640	1.0642	2.9238	2.7475	0.9397	1.2217	70°
21°	0.3665	0.3584	0.3839	1.0711	2.7904	2.6051	0.9336	1.2043	69
22	0.3840	0.3746	0.4040	1.0785	2.6695	2.4751	0.9272	1.1868	68
23	0.4014	0.3907	0.4245	1.0864	2.5593	2.3559	0.9205	1.1694	67
24	0.4189	0.4067	0.4452	1.0946	2.4586	2.2460	0.9135	1.1519	66
25	0.4363	0.4226	0.4663	1.1034	2.3662	2.1445	0.9063	1.1345	65°
26°	0.4538	0.4384	0.4877	1.1126	2.2812	2.0503	0.8988	1.1170	64
27	0.4712	0.4540	0.5095	1.1223	2.2027	1.9626	0.8910	1.0996	63
28	0.4887	0.4695	0.5317	1.1326	2.1301	1.8807	0.8829	1.0821	62
29	0.5061	0.4848	0.5543	1.1434	2.0627	1.8040	0.8746	1.0647	61
30	0.5236	0.5000	0.5774	1.1547	2.0000	1.7321	0.8660	1.0472	60°
31°	0.5411	0.5150	0.6009	1.1666	1.9416	1.6643	0.8572	1.0297	59
32	0.5585	0.5299	0.6249	1.1792	1.8871	1.6003	0.8480	1.0123	58
33	0.5760	0.5446	0.6494	1.1924	1.8361	1.5399	0.8387	0.9948	57
34	0.5934	0.5592	0.6745	1.2062	1.7883	1.4826	0.8290	0.9774	56
35	0.6109	0.5736	0.7002	1.2208	1.7434	1.4281	0.8192	0.9599	55°
36°	0.6283	0.5878	0.7265	1.2361	1.7013	1.3764	0.8090	0.9425	54
37	0.6458	0.6018	0.7536	1.2521	1.6616	1.3270	0.7986	0.9250	53
38	0.6632	0.6157	0.7813	1.2690	1.6243	1.2799	0.7880	0.9076	52
39	0.6807	0.6293	0.8098	1.2868	1.5890	1.2349	0.7771	0.8901	51
40	0.6981	0.6428	0.8391	1.3054	1.5557	1.1918	0.7660	0.8727	50°
41°	0.7156	0.6561	0.8693	1.3250	1.5243	1.1504	0.7547	0.8552	49
42	0.7330	0.6691	0.9004	1.3456	1.4945	1.1106	0.7431	0.8378	48
43	0.7505	0.6820	0.9325	1.3673	1.4663	1.0724	0.7314	0.8203	47
44	0.7679	0.6947	0.9657	1.3902	1.4396	1.0355	0.7193	0.8029	46
45	0.7854	0.7071	1.0000	1.4142	1.4142	1.0000	0.7071	0.7854	45°
		Cos	Cot	Cosec	Sec	Tan	Sin	Arc	Angle

Explanation on p. 38

4. Reciprocals

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
0.10	10.00	9.901	9.804	9.709	9.615	9.524	9.434	9.346	9.259	9.174
0.11	9.091	9.009	8.929	8.850	8.772	8.696	8.621	8.547	8.475	8.403
0.12	8.333	8.264	8.197	8.130	8.065	8.000	7.937	7.874	7.813	7.752
0.13	7.692	7.634	7.576	7.519	7.463	7.407	7.353	7.299	7.246	7.194
0.14	7.143	7.092	7.042	6.993	6.944	6.897	6.849	6.803	6.757	6.711
0.15	6.667	6.623	6.579	6.536	6.494	6.452	6.410	6.369	6.329	6.289
0.16	6.250	6.211	6.173	6.135	6.098	6.061	6.024	5.988	5.952	5.917
0.17	5.882	5.848	5.814	5.780	5.747	5.714	5.682	5.650	5.618	5.587
0.18	5.556	5.525	5.495	5.464	5.435	5.405	5.376	5.348	5.319	5.291
0.19	5.263	5.236	5.208	5.181	5.155	5.128	5.102	5.076	5.051	5.025
0.20	5.000	4.975	4.950	4.926	4.902	4.878	4.854	4.831	4.808	4.785
0.21	4.762	4.739	4.717	4.695	4.673	4.651	4.630	4.608	4.587	4.566
0.22	4.545	4.525	4.505	4.484	4.464	4.444	4.425	4.405	4.386	4.367
0.23	4.348	4.329	4.310	4.292	4.274	4.255	4.237	4.219	4.202	4.184
0.24	4.167	4.149	4.132	4.115	4.098	4.082	4.065	4.049	4.032	4.016
0.25	4.000	3.984	3.968	3.953	3.937	3.922	3.906	3.891	3.876	3.861
0.26	3.846	3.831	3.817	3.802	3.788	3.774	3.759	3.745	3.731	3.717
0.27	3.704	3.690	3.676	3.663	3.650	3.636	3.623	3.610	3.597	3.584
0.28	3.571	3.559	3.546	3.534	3.521	3.509	3.497	3.484	3.472	3.460
0.29	3.448	3.436	3.425	3.413	3.401	3.390	3.378	3.367	3.356	3.344
0.30	3.333	3.322	3.311	3.300	3.289	3.279	3.268	3.257	3.247	3.236
0.31	3.226	3.215	3.205	3.195	3.185	3.175	3.165	3.155	3.145	3.135
0.32	3.125	3.115	3.106	3.096	3.086	3.077	3.067	3.058	3.049	3.040
0.33	3.030	3.021	3.012	3.003	2.994	2.985	2.976	2.967	2.959	2.950
0.34	2.941	2.933	2.924	2.915	2.907	2.899	[illegible].890	2.882	2.874	2.865
0.35	2.857	2.849	2.841	2.833	2.825	2.817	2.809	2.801	2.793	2.786
0.36	2.778	2.770	2.762	2.755	2.747	2.740	2.732	2.725	2.717	2.710
0.37	2.703	2.695	2.688	2.681	2.674	2.667	2.660	2.653	2.646	2.639
0.38	2.632	2.625	2.618	2.611	2.604	2.597	2.591	2.584	2.577	2.571
0.39	2.564	2.558	2.551	2.545	2.538	2.532	2.525	2.519	2.513	2.506
0.40	2.500	2.494	2.488	2.481	2.475	2.469	2.463	2.457	2.451	2.445
0.41	2.439	2.433	2.427	2.421	2.415	2.410	2.404	2.398	2.392	2.387
0.42	2.381	2.375	2.370	2.364	2.358	2.353	2.347	2.342	2.336	2.331
0.43	2.326	2.320	2.315	2.309	2.304	2.299	2.294	2.288	2.283	2.278
0.44	2.273	2.268	2.262	2.257	2.252	2.247	2.242	2.237	2.232	2.227
0.45	2.222	2.217	2.212	2.208	2.203	2.198	2.193	2.188	2.183	2.179
0.46	2.174	2.169	2.165	2.160	2.155	2.151	2.146	2.141	2.137	2.132
0.47	2.128	2.123	2.119	2.114	2.110	2.105	2.101	2.096	2.092	2.088
0.48	2.083	2.079	2.075	2.070	2.066	2.062	2.058	2.053	2.049	2.045
0.49	2.041	2.037	2.033	2.028	2.024	2.020	2.016	2.012	2.008	2.004
0.50	2.000	1.996	1.992	1.988	1.984	1.980	1.976	1.972	1.969	1.965
0.51	1.961	1.957	1.953	1.949	1.946	1.942	1.938	1.934	1.931	1.927
0.52	1.923	1.919	1.916	1.912	1.908	1.905	1.901	1.898	1.894	1.890
0.53	1.887	1.883	1.880	1.876	1.873	1.869	1.866	1.862	1.859	1.855
0.54	1.852	1.848	1.845	1.842	1.838	1.835	1.832	1.828	1.825	1.821
n	0	1	2	3	4	5	6	7	8	9

f Numbers

n	0	1	2	3	4	5	6	7	8	9
0.55	1.818	1.815	1.812	1.808	1.805	1.802	1.799	1.795	1.792	1.789
0.56	1.786	1.783	1.779	1.776	1.773	1.770	1.767	1.764	1.761	1.757
0.57	1.754	1.751	1.748	1.745	1.742	1.739	1.736	1.733	1.730	1.727
0.58	1.724	1.721	1.718	1.715	1.712	1.709	1.706	1.704	1.701	1.698
0.59	1.695	1.692	1.689	1.686	1.684	1.681	1.678	1.675	1.672	1.669
0.60	1.667	1.664	1.661	1.658	1.656	1.653	1.650	1.647	1.645	1.642
0.61	1.639	1.637	1.634	1.631	1.629	1.626	1.623	1.621	1.618	1.616
0.62	1.613	1.610	1.608	1.605	1.603	1.600	1.597	1.595	1.592	1.590
0.63	1.587	1.585	1.582	1.580	1.577	1.575	1.572	1.570	1.567	1.565
0.64	1.562	1.560	1.558	1.555	1.553	1.550	1.548	1.546	1.543	1.541
0.65	1.538	1.536	1.534	1.531	1.529	1.527	1.524	1.522	1.520	1.517
0.66	1.515	1.513	1.511	1.508	1.506	1.504	1.502	1.499	1.497	1.495
0.67	1.493	1.490	1.488	1.486	1.484	1.481	1.479	1.477	1.475	1.473
0.68	1.471	1.468	1.466	1.464	1.462	1.460	1.458	1.456	1.453	1.451
0.69	1.449	1.447	1.445	1.443	1.441	1.439	1.437	1.435	1.433	1.431
0.70	1.429	1.427	1.425	1.422	1.420	1.418	1.416	1.414	1.412	1.410
0.71	1.408	1.406	1.404	1.403	1.401	1.399	1.397	1.395	1.393	1.391
0.72	1.389	1.387	1.385	1.383	1.381	1.379	1.377	1.376	1.374	1.372
0.73	1.370	1.368	1.366	1.364	1.362	1.361	1.359	1.357	1.355	1.353
0.74	1.351	1.350	1.348	1.346	1.344	1.342	1.340	1.339	1.337	1.335
0.75	1.333	1.332	1.330	1.328	1.326	1.325	1.323	1.321	1.319	1.318
0.76	1.316	1.314	1.312	1.311	1.309	1.307	1.305	1.304	1.302	1.300
0.77	1.299	1.297	1.295	1.294	1.292	1.290	1.289	1.287	1.285	1.284
0.78	1.282	1.280	1.279	1.277	1.276	1.274	1.272	1.271	1.269	1.267
0.79	1.266	1.264	1.263	1.261	1.259	1.258	1.256	1.255	1.253	1.252
0.80	1.250	1.248	1.247	1.245	1.244	1.242	1.241	1.239	1.238	1.236
0.81	1.235	1.233	1.232	1.230	1.229	1.227	1.225	1.224	1.222	1.221
0.82	1.220	1.218	1.217	1.215	1.214	1.212	1.211	1.209	1.208	1.206
0.83	1.205	1.203	1.202	1.200	1.199	1.198	1.196	1.195	1.193	1.192
0.84	1.190	1.189	1.188	1.186	1.185	1.183	1.182	1.181	1.179	1.178
0.85	1.176	1.175	1.174	1.172	1.171	1.170	1.168	1.167	1.166	1.164
0.86	1.163	1.161	1.160	1.159	1.157	1.156	1.155	1.153	1.152	1.151
0.87	1.149	1.148	1.147	1.145	1.144	1.143	1.142	1.140	1.139	1.138
0.88	1.136	1.135	1.134	1.133	1.131	1.130	1.129	1.127	1.126	1.125
0.89	1.124	1.122	1.121	1.120	1.119	1.117	1.116	1.115	1.114	1.112
0.90	1.111	1.110	1.109	1.107	1.106	1.105	1.104	1.103	1.101	1.100
0.91	1.099	1.098	1.096	1.095	1.094	1.093	1.092	1.091	1.089	1.088
0.92	1.087	1.086	1.085	1.083	1.082	1.081	1.080	1.079	1.078	1.076
0.93	1.075	1.074	1.073	1.072	1.071	1.070	1.068	1.067	1.066	1.065
0.94	1.064	1.063	1.062	1.060	1.059	1.058	1.057	1.056	1.055	1.054
0.95	1.053	1.052	1.050	1.049	1.048	1.047	1.046	1.045	1.044	1.043
0.96	1.042	1.041	1.040	1.038	1.037	1.036	1.035	1.034	1.033	1.032
0.97	1.031	1.030	1.029	1.028	1.027	1.026	1.025	1.024	1.022	1.021
0.98	1.020	1.019	1.018	1.017	1.016	1.015	1.014	1.013	1.012	1.011
0.99	1.010	1.009	1.008	1.007	1.006	1.005	1.004	1.003	1.002	1.001
n	0	1	2	3	4	5	6	7	8	9

5. Squares of Num-

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188
1.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416
1.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664
1.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850	1.877	1.904	1.932
1.4	1.960	1.988	2.016	2.045	2.074	2.103	2.132	2.161	2.190	2.220
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465	2.496	2.528
1.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856
1.7	2.890	2.924	2.958	2.993	3.028	3.063	3.098	3.133	3.168	3.204
1.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572
1.9	3.610	3.648	3.686	3.725	3.764	3.803	3.842	3.881	3.920	3.960
2.0	4.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285	4.326	4.368
2.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796
2.2	4.840	4.884	4.928	4.973	5.018	5.063	5.108	5.153	5.198	5.244
2.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712
2.4	5.760	5.808	5.856	5.905	5.954	6.003	6.052	6.101	6.150	6.200
2.5	6.250	6.300	6.350	6.401	6.452	6.503	6.554	6.605	6.656	6.708
2.6	6.760	6.812	6.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236
2.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784
2.8	7.840	7.896	7.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352
2.9	8.410	8.468	8.526	8.585	8.644	8.703	8.762	8.821	8.880	8.940
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.5
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.4
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.2
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.1
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.0
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.0
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.9
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.9
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.9
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.9
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.9
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.9
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.0
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.1
n	0	1	2	3	4	5	6	7	8	9

bers from 1.00 to 9.99

n	0	1	2	3	4	5	6	7	8	9
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80
n	0	1	2	3	4	5	6	7	8	9

6. Square Roots of

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343
n	0	1	2	3	4	5	6	7	8	9

Numbers from 1.00 to 99.9

Continued on p. 20

n	0	1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161
n	0	1	2	3	4	5	6	7	8	9

Square Roots of

Continued from p. 19

n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302
11	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450
12	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592
13	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987
16	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111
17	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231
18	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347
19	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572
21	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680
22	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785
23	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889
24	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089
26	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187
27	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282
28	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736
33	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992
36	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075
37	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156
38	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237
39	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395
41	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473
42	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550
43	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626
44	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848
47	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134
51	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204
52	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273
53	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342
54	7.348	7.355	7.362	7.369	7.376	7.3[illegible]	7.389	7.396	7.403	7.409
n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9

Numbers from 1.00 to 99.9

n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477
56	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
57	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609
58	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675
59	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804
61	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868
62	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931
63	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994
64	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118
66	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179
67	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240
68	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301
69	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361
70	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420
71	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479
72	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538
73	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597
74	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712
76	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769
77	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826
78	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883
79	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939
80	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994
81	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050
82	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105
83	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160
84	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268
86	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
87	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375
88	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429
89	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
91	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586
92	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638
93	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690
94	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793
96	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844
97	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894
98	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945
99	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995
n	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9

7. Cubes of Num-

Explanation on p. 38

n	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50
2.2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44
2.5	15.62	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47
2.7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50
3.1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70
4.6	97.34	97.97	98.61	99.25	99.90	100.5	101.2	101.8	102.5	103.2
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5
n	0	1	2	3	4	5	6	7	8	9

bers from 1.00 to 9.99

n	0	1	2	3	4	5	6	7	8	9
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9
6.1	227.0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2
6.2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9
6.3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9
6.4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2
6.6	287.5	288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4
6.7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0
6.8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1
6.9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4
7.1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7
7.2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4
7.3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6
7.4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2
7.6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8
7.7	456.5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7
7.8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2
7.9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5
8.1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4
8.2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7
8.3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6
8.4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8
8.6	636.1	638.3	640.5	642.7	645.0	647.2	649.5	651.7	654.0	656.2
8.7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2
8.8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6
8.9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1
9.1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2
9.2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8
9.3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9
9.4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0
9.6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9
9.7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3
9.8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4
9.9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0
n	0	1	2	3	4	5	6	7	8	9

8. Cube Roots of Numbers

n	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	n	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
10	2.1544	4.6416	10.000	55	3.8030	8.1932	17.652
11	2.2240	4.7914	10.323	56	3.8259	8.2426	17.758
12	2.2894	4.9324	10.627	57	3.8485	8.2913	17.863
13	2.3513	5.0658	10.914	58	3.8709	8.3396	17.967
14	2.4101	5.1925	11.187	59	3.8930	8.3872	18.070
15	2.4662	5.3133	11.447	60	3.9149	8.4343	18.171
16	2.5198	5.4288	11.696	61	3.9365	8.4809	18.272
17	2.5713	5.5397	11.935	62	3.9579	8.5270	18.371
18	2.6207	5.6462	12.164	63	3.9791	8.5726	18.469
19	2.6684	5.7489	12.386	64	4.0000	8.6177	18.566
20	2.7144	5.8480	12.599	65	4.0207	8.6624	18.663
21	2.7589	5.9439	12.806	66	4.0412	6.7066	18.758
22	2.8020	6.0368	13.006	67	4.0615	8.7503	18.852
23	2.8439	6.1269	13.200	68	4.0817	8.7937	18.945
24	2.8845	6.2145	13.389	69	4.1016	8.8366	19.038
25	2.9240	6.2996	13.572	70	4.1213	8.8790	19.129
26	2.9625	6.3825	13.751	71	4.1408	8.9211	19.220
27	3.0000	6.4633	13.925	72	4.1602	8.9628	19.310
28	3.0366	6.5421	14.095	73	4.1793	9.0041	19.399
29	3.0723	6.6191	14.260	74	4.1983	9.0450	19.487
30	3.1072	6.6943	14.422	75	4.2172	9.0856	19.574
31	3.1414	6.7679	14.581	76	4.2358	9.1258	19.661
32	3.1748	6.8399	14.736	77	4.2543	9.1657	19.747
33	3.2075	6.9104	14.888	78	4.2727	9.2052	19.832
34	3.2396	6.9795	15.037	79	4.2908	9.2443	19.916
35	3.2711	7.0473	15.183	80	4.3089	9.2832	20.000
36	3.3019	7.1138	15.326	81	4.3267	9.3217	20.083
37	3.3322	7.1791	15.467	82	4.3445	9.3599	20.165
38	3.3620	7.2432	15.605	83	4.3621	9.3978	20.247
39	3.3912	7.3061	15.741	84	4.3795	9.4354	20.328
40	3.4200	7.3681	15.874	85	4.3968	9.4727	20.408
41	3.4482	7.4290	16.005	86	4.4140	9.5097	20.488
42	3.4760	7.4889	16.134	87	4.4310	9.5464	20.567
43	3.5034	7.5478	16.261	88	4.4480	9.5828	20.646
44	3.5303	7.6059	16.386	89	4.4647	9.6190	20.724
45	3.5569	7.6631	16.510	90	4.4814	9.6549	20.801
46	3.5830	7.7194	16.631	91	4.4979	9.6905	20.878
47	3.6088	7.7750	16.751	92	4.5144	9.7259	20.954
48	3.6342	7.8297	16.869	93	4.5307	9.7610	21.029
49	3.6593	7.8837	16.985	94	4.5468	9.7959	21.105
50	3.6840	7.9370	17.100	95	4.5629	9.8305	21.179
51	3.7084	7.9896	17.213	96	4.5789	9.8648	21.253
52	3.7325	8.0415	17.325	97	4.5947	9.8990	21.327
53	3.7563	8.0927	17.435	98	4.6104	9.9329	21.400
54	3.7798	8.1433	17.544	99	4.6261	9.9666	21.472

Explanation on page 38

9. Three-Halves Powers of Numbers

n	0	1	2	3	4	5	6	7	8	9
0.0	0.000	0.001	0.003	0.005	0.008	0.011	0.015	0.019	0.023	0.027
0.1	0.032	0.036	0.042	0.047	0.052	0.058	0.064	0.070	0.076	0.083
0.2	0.089	0.096	0.103	0.110	0.118	0.125	0.133	0.140	0.148	0.156
0.3	0.164	0.173	0.181	0.190	0.198	0.207	0.216	0.225	0.234	0.244
0.4	0.253	0.263	0.272	0.282	0.292	0.302	0.312	0.322	0.333	0.343
0.5	0.354	0.364	0.375	0.386	0.397	0.408	0.419	0.430	0.442	0.453
0.6	0.465	0.476	0.488	0.500	0.512	0.524	0.536	0.548	0.561	0.573
0.7	0.586	0.598	0.611	0.624	0.637	0.650	0.663	0.676	0.689	0.702
0.8	0.716	0.729	0.743	0.756	0.770	0.784	0.798	0.811	0.826	0.840
0.9	0.854	0.868	0.882	0.897	0.911	0.926	0.941	0.955	0.970	0.985
1.0	1.000	1.015	1.030	1.045	1.061	1.076	1.091	1.107	1.122	1.138
1.1	1.154	1.170	1.185	1.201	1.217	1.233	1.249	1.266	1.282	1.298
1.2	1.315	1.331	1.348	1.364	1.381	1.398	1.414	1.431	1.448	1.465
1.3	1.482	1.499	1.517	1.534	1.551	1.569	1.586	1.604	1.621	1.639
1.4	1.657	1.674	1.692	1.710	1.728	1.746	1.764	1.782	1.800	1.819
1.5	1.837	1.856	1.874	1.893	1.911	1.930	1.948	1.967	1.986	2.005
1.6	2.024	2.043	2.062	2.081	2.100	2.119	2.139	2.158	2.178	2.197
1.7	2.217	2.236	2.256	2.275	2.295	2.315	2.335	2.355	2.375	2.395
1.8	2.415	2.435	2.455	2.476	2.496	2.516	2.537	2.557	2.578	2.598
1.9	2.619	2.640	2.660	2.681	2.702	2.723	2.744	2.765	2.786	2.807
2.0	2.828	2.850	2.871	2.892	2.914	2.935	2.957	2.978	3.000	3.021
2.1	3.043	3.065	3.087	3.109	3.131	3.153	3.175	3.197	3.219	3.241
2.2	3.263	3.285	3.308	3.330	3.353	3.375	3.398	3.420	3.443	3.465
2.3	3.488	3.511	3.534	3.557	3.580	3.602	3.626	3.649	3.672	3.695
2.4	3.718	3.741	3.765	3.788	3.811	3.835	3.858	3.882	3.906	3.929
2.5	3.953	3.977	4.000	4.024	4.048	4.072	4.096	4.120	4.144	4.168
2.6	4.192	4.217	4.241	4.265	4.289	4.314	4.338	4.363	4.387	4.412
2.7	4.437	4.461	4.486	4.511	4.536	4.560	4.585	4.610	4.635	4.660
2.8	4.685	4.710	4.736	4.761	4.786	4.811	4.837	4.862	4.888	4.913
2.9	4.939	4.964	4.990	5.015	5.041	5.067	5.093	5.118	5.144	5.170
3.0	5.196	5.222	5.248	5.274	5.300	5.327	5.353	5.379	5.405	5.432
3.1	5.458	5.485	5.511	5.538	5.564	5.591	5.617	5.644	5.671	5.698
3.2	5.724	5.751	5.778	5.805	5.832	5.859	5.886	5.913	5.940	5.968
3.3	5.995	6.022	6.049	6.077	6.104	6.132	6.159	6.186	6.214	6.242
3.4	6.269	6.297	6.325	6.352	6.380	6.408	6.436	6.464	6.492	6.520
3.5	6.548	6.576	6.604	6.632	6.660	6.689	6.717	6.745	6.774	6.802
3.6	6.831	6.859	6.888	6.916	6.945	6.973	7.002	7.031	7.059	7.088
3.7	7.117	7.146	7.175	7.204	7.233	7.262	7.291	7.320	7.349	7.378
3.8	7.408	7.437	7.466	7.495	7.525	7.554	7.584	7.613	7.643	7.672
3.9	7.702	7.732	7.761	7.791	7.821	7.850	7.880	7.910	7.940	7.970
4.0	8.000	8.030	8.060	8.090	8.120	8.150	8.181	8.211	8.241	8.272
4.1	8.302	8.332	8.363	8.393	8.424	8.454	8.485	8.515	8.546	8.577
4.2	8.607	8.638	8.669	8.700	8.731	8.762	8.793	8.824	8.855	8.886
4.3	8.917	8.948	8.979	9.010	9.041	9.073	9.104	9.135	9.167	9.198
4.4	9.230	9.261	9.293	9.324	9.356	9.387	9.419	9.451	9.482	9.514
n	0	1	2	3	4	5	6	7	8	9

Explanation on page 38

10. Fifth Powers and Roots; Five-Halves Powers and Roots

n	n^5	$n^{\frac{1}{5}}$	$n^{\frac{5}{2}}$	$n^{\frac{2}{5}}$	n	n^5	$n^{\frac{1}{5}}$	$n^{\frac{5}{2}}$	$n^{\frac{2}{5}}$
0.1	0.0000	0.6310	0.0032	0.3981	4.6	2059.6	1.3569	45.383	1.8412
0.2	0.0003	0.7248	0.0179	0.5253	4.7	2293.5	1.3628	47.890	1.8571
0.3	0.0024	0.7860	0.0493	0.6178	4.8	2548.0	1.3685	50.478	1.8728
0.4	0.0102	0.8326	0.1012	0.6931	4.9	2824.8	1.3742	53.148	1.8883
0.5	0.0312	0.8706	0.1768	0.7579	5.0	3125.0	1.3797	55.902	1.9037
0.6	0.0778	0.9029	0.2789	0.8152	5.1	3450.3	1.3852	58.739	1.9188
0.7	0.1681	0.9311	0.4100	0.8670	5.2	3802.0	1.3906	61.661	1.9338
0.8	0.3277	0.9564	0.5724	0.9146	5.3	4182.0	1.3959	64.668	1.9485
0.9	0.5905	0.9791	0.7684	0.9587	5.4	4591.7	1.4011	67.762	1.9632
1.0	1.0000	1.0000	1.0000	1.0000	5.5	5032.8	1.4063	70.943	1.9776
1.1	1.6105	1.0192	1.2691	1.0389	5.6	5507.3	1.4114	74.211	1.9919
1.2	2.4883	1.0371	1.5774	1.0757	5.7	6016.9	1.4164	77.569	2.0061
1.3	3.7129	1.0539	1.9269	1.1107	5.8	6563.6	1.4213	81.016	2.0201
1.4	5.3782	1.0696	2.3191	1.1441	5.9	7149.2	1.4262	84.553	2.0340
1.5	7.5938	1.0845	2.7557	1.1761	6.0	7776.0	1.4310	88.182	2.0477
1.6	10.486	1.0986	3.2382	1.2068	6.1	8446.0	1.4357	91.902	2.0613
1.7	14.199	1.1120	3.7681	1.2365	6.2	9161.3	1.4404	95.715	2.0747
1.8	18.896	1.1247	4.3469	1.2651	6.3	9924.4	1.4450	99.621	2.0880
1.9	24.761	1.1370	4.9760	1.2927	6.4	10737.	1.4496	103.62	2.1012
2.0	32.000	1.1487	5.6569	1.3195	6.5	11603.	1.4541	107.72	2.1143
2.1	40.841	1.1600	6.3907	1.3455	6.6	12523.	1.4585	111.91	2.1272
2.2	51.536	1.1708	7.1789	1.3708	6.7	13501.	1.4629	116.19	2.1401
2.3	64.363	1.1813	8.0227	1.3954	6.8	14539.	1.4672	120.58	2.1528
2.4	79.626	1.1914	8.9234	1.4193	6.9	15640.	1.4715	125.06	2.1654
2.5	97.656	1.2011	9.8821	1.4427	7.0	16807.	1.4758	129.64	2.1779
2.6	118.81	1.2106	10.900	1.4655	7.1	18042.	1 4800	134.32	2.1903
2.7	143.49	1.2198	11.979	1.4878	7.2	19349.	1.4841	139.10	2.2026
2.8	172.10	1.2287	13.119	1.5096	7.3	20731.	1.4882	143.98	2.2148
2.9	205.11	1.2373	14.322	1.5309	7.4	22190.	1.4923	148.96	2.2269
3.0	243.00	1.2457	15.588	1.5518	7.5	23730.	1.4963	154.05	2.2388
3.1	286.29	1.2539	16.920	1.5723	7.6	25355.	1.5002	159.23	2.2507
3.2	335.54	1.2619	18.318	1.5924	7.7	27068.	1.5042	164.52	2.2625
3.3	391.35	1.2697	19.783	1.6122	7.8	28872.	1.5081	169.92	2.2742
3.4	454.35	1.2773	21.316	1.6315	7.9	30771.	1.5119	175.42	2.2859
3.5	525.22	1.2847	22.918	1.6505	8.0	32768.	1.5157	181.02	2.2974
3.6	604.66	1.2920	24.590	1.6692	8.2	37074.	1.5232	192.55	2.3202
3.7	693.44	1.2991	26.333	1.6876	8.4	41821.	1.5306	204.50	2.3427
3.8	792.35	1.3060	28.149	1.7057	8.6	47043.	1.5738	216.89	2.3648
3.9	902.24	1.3128	30.037	1.7236	8.8	52773.	1.5449	229.72	2.3867
4.0	1024.0	1.3195	32.000	1.7411	9.0	59049.	1.5518	243.00	2.4082
4.1	1158.6	1.3260	34.038	1.7584	9.2	65908.	1.5587	256.73	2.4295
4.2	1306.9	1.3324	36.151	1.7754	9.4	73390.	1.5654	270.91	2.4505
4.3	1470.1	1.3387	38.342	1.7922	9.6	81537.	1.5720	285.55	2.4712
4.4	1649.2	1.3449	40.610	1.8088	9.8	90392.	1.5785	300.65	2.4917
4.5	1845.3	1.3510	42.957	1.8251	10	100000	1.5849	316.23	2.5119

Explanation on page 39

11. Circumferences of Circles

Diameters in Units and Tenths

d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.314	0.628	0.942	1.257	1.571	1.885	2.199	2.513	2.827
1	3.142	3.456	3.770	4.084	4.398	4.712	5.027	5.341	5.655	5.969
2	6.283	6.597	6.912	7.226	7.540	7.854	8.168	8.482	8.796	9.111
3	9.425	9.739	10.05	10.37	10.68	11.00	11.31	11.62	11.94	12.25
4	12.57	12.88	13.19	13.51	13.82	14.14	14.45	14.77	15.08	15.39
5	15.71	16.02	16.34	16.65	16.96	17.28	17.59	17.91	18.22	18.54
6	18.85	19.16	19.48	19.79	20.11	20.42	20.73	21.05	21.36	21.68
7	21.99	22.31	22.62	22.93	23.25	23.56	23.88	24.19	24.50	24.82
8	25.13	25.45	25.76	26.08	26.39	26.70	27.02	27.33	27.65	27.96
9	28.27	28.59	28.90	29.22	29.53	29.85	30.16	30.47	30.79	31.10
10	31.42	31.73	32.04	32.36	32.67	32.99	33.30	33.62	33.93	34.24
11	34.56	34.87	35.19	35.50	35.81	36.13	36.44	36.76	37.07	37.38
12	37.70	38.01	38.33	38.64	38.96	39.27	39.58	39.90	40.21	40.53
13	40.84	41.15	41.47	41.78	42.10	42.41	42.73	43.04	43.35	43.67
14	43.98	44.30	44.61	44.92	45.24	45.55	45.87	46.18	46.50	46.81
15	47.12	47.44	47.75	48.07	48.38	48.69	49.01	49.32	49.64	49.95
16	50.27	50.58	50.89	51.21	51.52	51.84	52.15	52.46	52.78	53.09
17	53.41	53.72	54.04	54.35	54.66	54.98	55.29	55.61	55.92	56.23
18	56.55	56.86	57.18	57.49	57.81	58.12	58.43	58.75	59.06	59.38
19	59.69	60.00	60.32	60.63	60.95	61.26	61.58	61.89	62.20	62.52

Explanation on p. 39

12. Circumferences of Circles

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.3927	0.7854	1.1781	1.5708	1.9635	2.3562	2.7489
1	3.1416	3.5343	3.9270	4.3197	4.7124	5.1051	5.4978	5.8905
2	6.2832	6.6759	7.0686	7.4613	7.8540	8.2467	8.6394	9.0321
3	9.4248	9.8175	10.210	10.603	10.996	11.388	11.781	12.174
4	12.566	12.959	13.352	13.744	14.137	14.530	14.923	15.315
5	15.708	16.101	16.493	16.886	17.279	17.671	18.064	18.457
6	18.850	19.242	19.635	20.028	20.420	20.813	21.206	21.598
7	21.991	22.384	22.777	23.169	23.562	23.955	24.347	24.740
8	25.133	25.525	25.918	26.311	26.704	27.096	27.489	27.882
9	28.274	28.667	29.060	29.452	29.845	30.238	30.631	31.023
10	31.416	31.809	32.201	32.594	32.987	33.379	33.772	34.165
11	34.558	34.950	35.343	35.736	36.128	36.521	36.914	37.306
12	37.699	38.092	38.485	38.877	39.270	39.663	40.055	40.448
13	40.841	41.233	41.626	42.019	42.412	42.804	43.197	43.590
14	43.982	44.375	44.768	45.160	45.553	45.946	46.338	46.731
15	47.124	47.517	47.909	48.302	48.695	49.087	49.480	49.873
16	50.265	50.658	51.051	51.444	51.836	52.229	52.622	53.014
17	53.407	53.800	54.192	54.585	54.978	55.371	55.763	56.156
18	56.549	56.941	57.334	57.727	58.119	58.512	58.905	59.298
19	59.690	60.083	60.476	60.868	61.261	61.654	62.046	62.439

Explanation on p. 39

13. Circular

Explanation on p. 39

Central Angle Degrees	Length of Chord	Rise of Arc	Area of Segment	Central Angle Degrees	Length of Chord	Rise of Arc	Area of Segment
1	0.0175	0.0000	0.00000	46	0.7815	0.0795	0.04176
2	0.0349	0.0002	0.00000	47	0.7975	0.0829	0.04448
3	0.0524	0.0003	0.00001	48	0.8135	0.0865	0.04731
4	0.0698	0.0006	0.00003	49	0.8294	0.0900	0.05025
5	0.0872	0.0010	0.00006	50	0.8452	0.0937	0.05331
6	0.1047	0.0014	0.00010	51	0.8610	0.0974	0.05649
7	0.1221	0.0019	0.00015	52	0.8767	0.1012	0.05978
8	0.1395	0.0024	0.00023	53	0.8924	0.1051	0.06319
9	0.1569	0.0031	0.00032	54	0.9080	0.1090	0.06673
10	0.1743	0.0038	0.00044	55	0.9235	0.1130	0.07039
11	0.1917	0.0046	0.00059	56	0.9389	0.1171	0.07417
12	0.2091	0.0055	0.00076	57	0.9543	0.1212	0.07808
13	0.2264	0.0064	0.00097	58	0.9696	0.1254	0.08212
14	0.2437	0.0075	0.00121	59	0.9848	0.1296	0.08629
15	0.2611	0.0086	0.00149	60	1.0000	0.1340	0.09059
16	0.2783	0.0097	0.00181	61	1.0151	0.1384	0.09502
17	0.2956	0.0110	0.00217	62	1.0301	0.1428	0.09958
18	0.3129	0.0123	0.00257	63	1.0450	0.1474	0.10428
19	0.3301	0.0137	0.00302	64	1.0598	0.1520	0.10911
20	0.3473	0.0152	0.00352	65	1.0746	0.1566	0.11408
21	0.3645	0.0167	0.00408	66	1.0893	0.1613	0.11919
22	0.3816	0.0184	0.00468	67	1.1039	0.1661	0.12443
23	0.3987	0.0201	0.00535	68	1.1184	0.1710	0.12982
24	0.4158	0.0219	0.00607	69	1.1328	0.1759	0.13535
25	0.4329	0.0237	0.00686	70	1.1472	0.1808	0.14102
26	0.4499	0.0256	0.00771	71	1.1614	0.1859	0.14683
27	0.4669	0.0276	0.00862	72	1.1756	0.1910	0.15279
28	0.4838	0.0297	0.00961	73	1.1896	0.1961	0.15889
29	0.5008	0.0319	0.01067	74	1.2036	0.2014	0.16514
30	0.5176	0.0341	0.01180	75	1.2175	0.2066	0.17154
31	0.5345	0.0364	0.01301	76	1.2313	0.2120	0.17808
32	0.5512	0.0387	0.01429	77	1.2450	0.2174	0.18477
33	0.5680	0.0412	0.01566	78	1.2586	0.2229	0.19160
34	0.5847	0.0437	0.01711	79	1.2722	0.2284	0.19859
35	0.6014	0.0463	0.01864	80	1.2856	0.2340	0.20573
36	0.6180	0.0489	0.02027	81	1.2989	0.2396	0.21301
37	0.6346	0.0517	0.02198	82	1.3121	0.2453	0.22045
38	0.6511	0.0545	0.02378	83	1.3252	0.2510	0.22804
39	0.6676	0.0574	0.02568	84	1.3383	0.2569	0.23578
40	0.6840	0.0603	0.02767	85	1.3512	0.2627	0.24367
41	0.7004	0.0633	0.02976	86	1.3640	0.2686	0.25171
42	0.7167	0.0664	0.03195	87	1.3767	0.2746	0.25990
43	0.7330	0.0696	0.03425	88	1.3893	0.2807	0.26825
44	0.7492	0.0728	0.03664	89	1.4018	0.2867	0.27675
45	0.7654	0.0761	0.03915	90	1.4142	0.2929	0.28540

Segments

Central Angle Degrees	Length of Chord	Rise of Arc	Area of Segment
91	1.4265	0.2991	0.29420
92	1.4387	0.3053	0.30316
93	1.4507	0.3116	0.31226
94	1.4627	0.3180	0.32152
95	1.4746	0.3244	0.33093
96	1.4863	0.3309	0.34050
97	1.4979	0.3374	0.35021
98	1.5094	0.3439	0.36008
99	1.5208	0.3506	0.37009
100	1.5321	0.3572	0.38026
101	1.5432	0.3639	0.39058
102	1.5543	0.3707	0.40104
103	1.5652	0.3775	0.41166
104	1.5760	0.3843	0.42242
105	1.5867	0.3912	0.43333
106	1.5973	0.3982	0.44439
107	1.6077	0.4052	0.45560
108	1.6180	0.4122	0.46695
109	1.6282	0.4193	0.47844
110	1.6383	0.4264	0.49008
111	1.6483	0.4336	0.50187
112	1.6581	0.4408	0.51379
113	1.6678	0.4481	0.52586
114	1.6773	0.4554	0.53807
115	1.6868	0.4627	0.55041
116	1.6961	0.4701	0.56289
117	1.7053	0.4775	0.57551
118	1.7143	0.4850	0.58827
119	1.7233	0.4925	0.60116
120	1.7321	0.5000	0.61418
121	1.7407	0.5076	0.62734
122	1.7492	0.5152	0.64063
123	1.7576	0.5228	0.65404
124	1.7659	0.5305	0.66759
125	1.7740	0.5383	0.68125
126	1.7820	0.5460	0.69505
127	1.7899	0.5538	0.70897
128	1.7976	0.5616	0.72301
129	1.8052	0.5695	0.73716
130	1.8126	0.5774	0.75144
131	1.8199	0.5853	0.76584
132	1.8271	0.5933	0.78034
133	1.8341	0.6013	0.79497
134	1.8410	0.6093	0.80970
135	1.8478	0.6173	0.82454

Central Angle Degrees	Length of Chord	Rise of Arc	Area of Segment
136	1.8544	0.6254	0.83949
137	1.8608	0.6335	0.85455
138	1.8672	0.6416	0.86971
139	1.8733	0.6498	0.88497
140	1.8794	0.6580	0.90034
141	1.8853	0.6662	0.91580
142	1.8910	0.6744	0.93135
143	1.8966	0.6827	0.94700
144	1.9021	0.6910	0.96274
145	1.9074	0.6993	0.97858
146	1.9126	0.7076	0.99449
147	1.9176	0.7160	1.01050
148	1.9225	0.7244	1.02658
149	1.9273	0.7328	1.04275
150	1.9319	0.7412	1.05900
151	1.9363	0.7496	1.07532
152	1.9406	0.7581	1.09171
153	1.9447	0.7666	1.10818
154	1.9487	0.7750	1.12472
155	1.9526	0.7836	1.14132
156	1.9563	0.7921	1.15799
157	1.9598	0.8006	1.17472
158	1.9633	0.8092	1.19151
159	1.9665	0.8178	1.20835
160	1.9696	0.8264	1.22525
161	1.9726	0.8350	1.24221
162	1.9754	0.8436	1.25921
163	1.9780	0.8522	1.27626
164	1.9805	0.8608	1.29335
165	1.9829	0.8695	1.31049
166	1.9851	0.8781	1.32766
167	1.9871	0.8868	1.34487
168	1.9890	0.8955	1.36212
169	1.9908	0.9042	1.37940
170	1.9924	0.9128	1.39671
171	1.9938	0.9215	1.41404
172	1.9951	0.9302	1.43140
173	1.9963	0.9390	1.44878
174	1.9973	0.9477	1.46617
175	1.9981	0.9564	1.48359
176	1.9988	0.9651	1.50101
177	1.9993	0.9738	1.51845
178	1.9997	0.9825	1.53589
179	1.9999	0.9913	1.55334
180	2.0000	[illegible].0000	1.57080

14. Areas of Circles for Diam-

Explanation on p. 39

d	0	1	2	3	4	5	6	7	8	9
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933
1.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.012	4.047	4.083	4.119
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.01	10.07	10.12
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35
5.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16
5.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98
5.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82
5.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67
d	0	1	2	3	4	5	6	7	8	9

eters in Units and Hundredths

d	0	1	2	3	4	5	6	7	8	9
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54
5.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43
5.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33
5.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25
5.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13
6.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09
6.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07
6.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07
6.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11
6.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15
6.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21
6.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28
6.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48
7.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60
7.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74
7.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89
7.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25
7.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45
7.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66
7.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89
7.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40
8.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68
8.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98
8.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29
8.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95
8.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31
8.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68
8.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07
8.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90
9.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33
9.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78
9.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25
9.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23
9.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75
9.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28
9.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82
9.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38
d	0	1	2	3	4	5	6	7	8	9

15. Areas of Circles

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.0123	0.0491	0.1104	0.1963	0.3068	0.4418	0.6013
1	0.7854	0.9940	1.2272	1.4849	1.7671	2.0739	2.4053	2.7612
2	3.1416	3.5466	3.9761	4.4301	4.9087	5.4119	5.9396	6.4918
3	7.0686	7.6699	8.2958	8.9462	9.6211	10.321	11.045	11.793
4	12.566	13.364	14.186	15.033	15.904	16.800	17.721	18.665
5	19.635	20.629	21.648	22.691	23.758	24.850	25.967	27.109
6	2[illegible].[illegible]74	29.465	30.680	31.91[illegible]	33.183	34.472	35.785	37.122
7	38.485	39.871	41.282	42.718	44.179	45.664	47.173	48.707
8	50.265	51.849	53 456	55.088	56.745	58.426	60.132	61.862
9	63.617	65.397	67.201	69.029	70.882	72.760	74.662	76.589
10	78.540	80.516	82.516	84.541	86.590	88.664	90.763	92.886
11	95.033	97.205	99.402	101.62	103.87	106.14	108.43	110.75
12	113.10	115.47	117.86	120.28	122.72	125.19	127.68	130.19
13	132.73	135.30	137.89	140.50	143 14	145.80	148.49	151.20
14	153.94	156.70	159.48	162.30	165.13	167.99	170.87	173.78
15	176.71	179.67	182.65	185.66	188.69	191.75	194.83	197.93
16	201.06	204.22	207.39	210.60	213.82	217.08	220.35	223.65
17	226.98	230.33	233.71	237.10	240.53	243.98	247.45	250.95
18	254.47	258.02	261.59	265.18	268.80	272.45	276.12	279.81
19	283.53	287.27	291.04	294.83	298.65	302.49	306.35	310.24
20	314.16	318.10	322.06	326.05	330.06	334.10	338.16	342.25
21	346.36	350.50	354.66	358.84	363.05	367.28	371.54	375.83
22	380.13	384.46	388.82	393.20	397.61	402.04	406.49	410.97
23	415.48	420.00	424.56	429.13	433.74	438.36	443.01	447.69
24	452.39	457.11	461.86	466.64	471.44	476.26	481.11	485.98
25	490.87	495.79	500.74	505.71	510.71	515.72	520.77	525.84
26	530.93	536.05	541.19	546.35	551.55	556.76	562.00	567.27
27	572.56	577.87	583.21	588.57	593.96	599.37	604.81	610.27
28	615.75	621.26	626.80	632.36	637.94	643.55	649.18	654.84
29	660.52	666.23	671.96	677.71	683.49	689.30	695.13	700.98
30	706.86	712.76	718.69	724.64	730.62	736.62	742.64	748.69
31	754.77	760.87	766.99	773.14	779.31	785.51	791.73	797.98
32	804.25	810.54	816.86	823.21	829.58	835.97	842.39	848.83
33	855.30	861.79	868.31	874.85	881.41	888.00	894.62	901.26
34	907.92	914.61	921.32	928.06	934.82	941.61	948.42	955.25
35	962.11	969.00	975.91	982.84	989.80	996.78	1003.8	1010.8
36	1017.9	1025.0	1032.1	1039.2	1046.3	1053.5	1060.7	1068.0
37	1075.2	1082.5	1089.8	1097.1	1104.5	1111.8	1119.2	1126.7
38	1134.1	1141.6	1149.1	1156.6	1164.2	1171.7	1179.3	1186.9
39	1194.6	1202.3	1210.0	1217.7	1225.4	1233.2	1241.0	1248.8
40	1256.6	1264.5	1272.4	1280.3	1288.2	1296.2	1304.2	1312.2
41	1320.3	1328.3	1336.4	1344.5	1352.7	1360.8	1369.0	1377.2
42	1385.4	1393.7	1402.0	1410.3	1418.6	1427.0	1435.4	1443.8
43	1452.2	1460.7	1469.1	1477.6	1486.2	1494.7	1503.3	1511.9
44	1520.5	1529.2	1537.9	1546.6	1555.3	1564.0	1572.8	1581.6

d 0 1/8 1/4 3/8 1/2 5/8 3/4 7/8

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16. Volumes of Spheres

Diameters in Units and Tenths

d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.001	0.004	0.014	0.034	0.065	0.113	0.180	0.268	0.382
1	0.524	0.697	0.905	1.150	1.437	1.767	2.145	2.572	3.054	3.591
2	4.189	4.849	5.575	6.371	7.238	8.181	9.203	10.31	11.49	12.77
3	14.14	15.60	17.16	18.82	20.58	22.45	24.43	26.52	28.73	31.06
4	33.51	36.09	38.79	41.63	44.60	47.71	50.97	54.36	57.91	61.60
5	65.45	69.46	73.62	77.95	82.45	87.11	91.95	96.97	102.2	107.5
6	113.1	118.8	124.8	130.9	137.3	143.8	150.5	157.5	164.6	172.0
7	179.6	187.4	195.4	203.7	212.2	220.9	229.8	239.0	248.5	258.2
8	268.1	278.3	288.7	299.4	310.3	321.6	333.0	344.8	356.8	369.1
9	381.7	394.6	407.7	421.2	434.9	448.9	463.2	477.9	492.8	508.0
10	523.6	539.5	555.6	572.2	589.0	606.1	623.6	641.4	659.6	678.1
11	696.9	716.1	735.6	755.5	775.7	796.3	817.3	838.6	860.3	882.3
12	904.8	927.6	950.8	974.3	998.3	1023	1047	1073	1098	1124
13	1150	1177	1204	1232	1260	1288	1317	1346	1376	1406
14	1437	1468	1499	1531	1563	1596	1630	1663	1697	1732
15	1767	1803	1839	1875	1912	1950	1988	2026	2065	2105
16	2145	2185	2226	2268	2310	2352	2395	2439	2483	2527
17	2572	2618	2664	2711	2758	2806	2855	2903	2953	3003
18	3054	3105	3157	3209	3262	3315	3369	3424	3479	3535
19	3591	3648	3706	3764	3823	3882	3942	4003	4064	4126

Explanation on p. 39

17. Volumes of Spheres

Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.0010	0.0082	0.0276	0.0654	0.1278	0.2209	0.3508
1	0.5236	0.7455	1.0227	1.3612	1.7671	2.2468	2.8062	3.4515
2	4.1888	5.0243	5.9641	7.0144	8.1812	9.4708	10.889	12.443
3	14.137	15.979	17.974	20.129	22.449	24.942	27.612	30.466
4	33.510	36.751	40.194	43.846	47.713	51.800	56.115	60.663
5	65.450	70.482	75.766	81.308	87.114	93.189	99.541	106.17
6	113.10	120.31	127.83	135.66	143.79	152.25	161.03	170.14
7	179.59	189.39	199.53	210.03	220.89	232.12	243.73	255.71
8	268.08	280.85	294.01	307.58	321.56	335.95	350.77	366.02
9	381.70	397.83	414.40	431.43	448.92	466.88	485.30	504.21
10	523.60	543.48	563.86	584.74	606.13	628.04	650.47	673.42
11	696.91	720.94	745.51	770.64	796.33	822.58	849.40	876.80
12	904.78	933.35	962.51	992.28	1022.7	1053.6	1085.2	1117.5
13	1150.3	1183.8	1218.0	1252.8	1288.2	1324.4	1361.2	1398.6
14	1436.8	1475.6	1515.1	1555.3	1596.3	1637.9	1680.3	1723.3
15	1767.1	1811.7	1857.0	1903.0	1949.8	1997.4	2045.7	2094.8
16	2144.7	2195.3	2246.8	2299.0	2352.1	2405.9	2460.6	2516.1
17	2572.4	2629.6	2687.6	2746.5	2806.2	2866.7	2928.2	2990.5
18	3053.6	3117.7	3182.6	3248.5	3315.2	3382.9	3451.5	3520.9
19	3591.4	3662.7	3735.0	3808.2	3882.4	3957.6	4033.7	4110.7

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18. Natural Hyperbolic Functions

u	Sinh *u*	Cosh *u*	Tanh *u*	*u*	Sinh *u*	Cosh *u*	Tanh *u*
0.00	0.0000	1.0000	0.0000	2.25	4.6912	4.7966	0.9780
0.05	0.0500	1.0013	0.0500	2.30	4.9370	5.0372	0.9801
0.10	0.1002	1.0050	0.0997	2.35	5.1951	5.2905	0.9820
0.15	0.1506	1.0113	0.1489	2.40	5.4662	5.5569	0.9837
0.20	0.2013	1.0201	0.1974	2.45	5.7510	5.8373	0.9853
0.25	0.2526	1.0314	0.2449	2.50	6.0502	6.1323	0.9866
0.30	0.3045	1.0453	0.2913	2.55	6.3645	6.4426	0.9879
0.35	0.3572	1.0619	0.3364	2.60	6.6947	6.7690	0.9890
0.40	0.4108	1.0811	0.3800	2.65	7.0417	7.1123	0.9900
0.45	0.4653	1.1030	0.4219	2.70	7.4063	7.4735	0.9910
0.50	0.5211	1.1276	0.4621	2.75	7.7894	7.8533	0.9918
0.55	0.5782	1.1551	0.5005	2.80	8.1919	8.2527	0.9926
0.60	0.6367	1.1855	0.5370	2.85	8.6150	8.6728	0.9933
0.65	0.6967	1.2188	0.5717	2.90	9.0596	9.1146	0.9940
0.70	0.7586	1.2552	0.6044	2.95	9.5268	9.5791	0.9945
0.75	0.8223	1 2947	0.6352	3.00	10.018	10.068	0.9950
0.80	0.8881	1.3374	0.6640	3.05	10.534	10.581	0.9955
0.85	0.9561	1.3835	0.6911	3.10	11.076	11.122	0.9959
0.90	1.0265	1.4331	0.7163	3.15	11.647	11.690	0.9963
0.95	1.0995	1.4862	0.7398	3.20	12.246	12.287	0.9967
1.00	1.1752	1.5431	0.7616	3.25	12.876	12.915	0.9970
1.05	1.2539	1.6038	0.7818	3.30	13.538	13.575	0.9973
1.10	1.3356	1.6685	0.8005	3.35	14.234	14.269	0.9976
1.15	1.4208	1.7374	0.8178	3.40	14.965	14.999	0.9978
1.20	1.5095	1.8107	0.8337	3.45	15.734	15.766	0.9980
1.25	1.6019	1.8884	0.8483	3.50	16.543	16.573	0.9982
1.30	1.6984	1.9709	0.8617	3.55	17.392	17.421	0.9984
1.35	1.7991	2.0583	0.8741	3.60	18.285	18.313	0.9985
1.40	1.9043	2.1509	0.8854	3.65	19.224	19.250	0.9987
1.45	2.0143	2.2488	0.8957	3.70	20.211	20.236	0.9988
1.50	2.1293	2.3524	0.9052	3.75	21.249	21.272	0.9989
1.55	2.2496	2.4619	0.9138	3.80	22.339	22.362	0.9990
1.60	2.3756	2.5775	0.9217	3.85	23.486	23.507	0.9991
1.65	2.5075	2.6995	0.9289	3.90	24.691	24.711	0.9992
1.70	2.6456	2.8283	0.9354	3.95	25.958	25.977	0.9993
1.75	2.7904	2.9642	0.9414	4.0	27.290	27.308	0.9993
1.80	2.9422	3.1075	0.9468	4.1	30.162	30.178	0.9995
1.85	3.1013	3.2585	0.9518	4.2	33.336	33.351	0.9996
1.90	3.2682	3.4177	0.9562	4.3	36.843	36.857	0.9996
1.95	3.4432	3.5855	0.9603	4.4	40.719	40.732	0.9997
2.00	3.6269	3.7622	0.9640	4.5	45.003	45.014	0.9998
2.05	3.8196	3.9483	0.9674	4.6	49.737	49.747	0.9998
2.10	4.0219	4.1443	0.9705	4.7	54.969	54.978	0.9999
2.15	4.2342	4.3507	0.9732	4.8	60.751	60.759	0.9999
2.20	4.4571	4.5679	0.9757	4.9	67.141	67.149	0.9999

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19. Naperian Logarithms of Numbers from 1 to 119

n	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	−∞	0.0000	0.6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
1	2.3026	2.3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2.9444
2	2.9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
3	3.4012	3.4340	3.4657	3.4965	3.5264	3.5553	3.5835	3.6109	3.6376	3.6636
4	3.6889	3.7136	3.7377	3.7612	3.7842	3.8067	3.8286	3.8501	3.8712	3.8918
5	3.9120	3.9318	3.9512	3.9703	3.9890	4.0073	4.0254	4.0430	4.0604	4.0775
6	4.0943	4.1109	4.1271	4.1431	4.1589	4.1744	4.1897	4.2047	4.2195	4.2341
7	4.2485	4.2627	4.2767	4.2905	4.3041	4.3175	4.3307	4.3438	4.3567	4.3694
8	4.3820	4.3944	4.4067	4.4188	4.4308	4.4427	4.4543	4.4659	4.4773	4.4886
9	4.4998	4.5109	4.5218	4.5326	4.5433	4.5539	4.5643	4.5747	4.5850	4.5951
10	4.6052	4.6151	4.6250	4.6347	4.6444	4.6540	4.6634	4.6728	4.6821	4.6913
11	4.7005	4.7095	4.7185	4.7274	4.7362	4.7449	4.7536	4.7622	4.7707	4.7791

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20. Multipliers for Transferring Logarithms

Common to Naperian					Naperian to Common			
1	2.302585093	Example.			1	0.434294482	Example.	
2	4.605170186	Find Nap log of 105			2	0.868588964	Find number corresponding to Nap log $\bar{1}.6078$	
3	6.907755279				3	1.302883446		
		Com log 105 = 2.02119						
4	9.210340372	2	4.605170		4	1.737177928	.6	0.26058
5	11.512925465	.02	46052		5	2.171472410	.07	304
6	13.815510558	1	2303		6	2.605766891	8	35
		1	230					+0.26397
7	16.118095651	9	207		7	3.040061373	$\bar{1}$	−0.43429
8	18.420680744				8	3.474355855	Com log =	$\bar{1}.8297$
9	20.723265837	Nap log 105 = 4.65396			9	3.908650337	Number = 0.6756	

21. Multipliers for Finding Lengths of Circular Arcs

	Degrees	Minutes	Seconds		
1	0.017453293	0.000290888	0.000004848	Example.	
2	0.034906585	0.000581776	0.000009696	Find length of arc for a central angle of 48° 45′ in circle of 12 ft. radius.	
3	0.052359878	0.000872665	0.000014544		
4	0.069813170	0.001163553	0.000019393	40°	0.698132
5	0.087266463	0.001454441	0.000024241	8°	.139626
6	0.104719755	0.001745329	0.000029089	40′	.011636
				5′	.001454
					0.85085
7	0.122173048	0.002036217	0.000033937		12
8	0.139626340	0.002327106	0.000038785		
9	0.157079633	0.002617994	0.000043633	Length = 10.210 ft	

22. Mathematical Constants

Symbol	Number	Logarithm
π	3.1415927	0.4971499
2π	6.2831853	0.7981799
3π	9.4247780	0.9742711
4π	12.5663706	1.0992099
5π	15.7079633	1.1961200
6π	18.8495559	1.2753011
7π	21.9911486	1.3422479
8π	25.1327412	1.4002399
9π	28.2743339	1.4513924
$4\pi/3$	4.1887902	0.6220886
$\pi/2$	1.5707963	0.1961199
$\pi/4$	0.7853982	$\bar{1}.8950899$
$\pi/6$	0.5235988	$\bar{1}.7189986$
$\pi/30$	0.1047198	$\bar{1}.0200286$
$\pi/180$	0.0174533	$\bar{2}.2418774$
$1/\pi$	0.3183099	$\bar{1}.5028501$
$2/\pi$	0.6366198	$\bar{1}.8038801$
$180/\pi$	57.2957795	1.7581226
$10800/\pi$	3437.74677	3.5362739
$648000/\pi$	206264.806	5.3144251
π^2	9.8696044	0.9942997
$1/\pi^2$	0.1013212	$\bar{1}.0057003$
π^3	31.0062767	1.4914496
$1/\pi^3$	0.0322516	$\bar{2}.5085504$

Symbol	Number	Logarithm
$\sqrt{\pi}$	1.7724539	0.2485749
$1/\sqrt{\pi}$	0.5641896	$\bar{1}.7514251$
$\pi\sqrt{2}$	4.4428829	0.6476649
$\sqrt{2\pi}$	2.5066282	0.3990899
$\sqrt{\pi/2}$	1.2533141	0.0980599
$\sqrt{2/\pi}$	0.7978844	$\bar{1}.9019401$
ε	2.7182818	0.4342945
ε^2	7.3890568	0.8685890
$1/\varepsilon$	0.3678794	$\bar{1}.5657055$
$1/\varepsilon^2$	0.1353353	$\bar{1}.1314110$
μ	0.4342945	$\bar{1}.6377843$
$1/\mu$	2.3025851	0.3622157
$\sin 1°$	0.0174524	$\bar{2}.2418553$
$\sin 1'$	0.0002909	$\bar{4}.4637261$
$\sin 1''$	0.0000048	$\bar{6}.6855749$
2	2.	0.3010300
$\sqrt{2}$	1.4142136	0.1505150
$\sqrt{1/2}$	0.7071068	$\bar{1}.8494850$
3	3.	0.4771213
$\sqrt{3}$	1.7320508	0.2385606
$\sqrt{1/3}$	0.5773503	$\bar{1}.7614394$

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23. Decimal Equivalents of Common Fractions

Fract.	Decimal	Logarithm
1/2	0.5	$\bar{1}.69897$
1/3	0.33333	$\bar{1}.52288$
2/3	0.66667	$\bar{1}.82391$
1/4	0.25	$\bar{1}.39794$
3/4	0.75	$\bar{1}.87506$
1/5	0.2	$\bar{1}.30103$
2/5	0.4	$\bar{1}.60206$
3/5	0.6	$\bar{1}.77815$
4/5	0.8	$\bar{1}.90309$
1/6	0.16667	$\bar{1}.22185$
5/6	0.83333	$\bar{1}.92082$
1/7	0.14286	$\bar{1}.15490$
2/7	0.28571	$\bar{1}.45593$
3/7	0.42857	$\bar{1}.63202$
4/7	0.57143	$\bar{1}.75696$
5/7	0.71429	$\bar{1}.85387$
6/7	0.85714	$\bar{1}.93305$

Fract.	Decimal	Logarithm
1/8	0.125	$\bar{1}.09691$
3/8	0.375	$\bar{1}.57403$
5/8	0.625	$\bar{1}.79588$
7/8	0.875	$\bar{1}.94201$
1/12	0.08333	$\bar{2}.92082$
5/12	0.41667	$\bar{1}.61979$
7/12	0.58333	$\bar{1}.76592$
11/12	0.91667	$\bar{1}.96211$
1/16	0.0625	$\bar{2}.79588$
3/16	0.1875	$\bar{1}.27300$
5/16	0.3125	$\bar{1}.49485$
7/16	0.4375	$\bar{1}.64098$
9/16	0.5625	$\bar{1}.75012$
11/16	0.6875	$\bar{1}.83727$
13/16	0.8125	$\bar{1}.90982$
15/16	0.9375	$\bar{1}.97197$

Fract.	Decimal	Logarithm
1/32	0.03125	$\bar{2}.49485$
3/32	0.09375	$\bar{2}.97197$
5/32	0.15625	$\bar{1}.19382$
7/32	0.21875	$\bar{1}.33995$
9/32	0.28125	$\bar{1}.44909$
11/32	0.34375	$\bar{1}.53624$
13/32	0.40625	$\bar{1}.60879$
15/32	0.46875	$\bar{1}.67094$
17/32	0.53125	$\bar{1}.72530$
19/32	0.59375	$\bar{1}.77360$
21/32	0.65625	$\bar{1}.81707$
23/32	0.71875	$\bar{1}.85658$
25/32	0.78125	$\bar{1}.89279$
27/32	0.84375	$\bar{1}.92621$
29/32	0.90625	$\bar{1}.95725$
31/32	0.96875	$\bar{1}.98621$

24. Explanation and Use of the Mathematical Tables

Arguments and Functions are the two kinds of numbers that appear in a table, the former being the numbers for which the values of the functions have been computed; thus, in $\sin\theta$, values of θ are arguments and those of $\sin\theta$ are functions. In this book arguments are generally in bold-face type and functions in common type. An argument is at the side of the table and sometimes part of it is at the side and part at the top or foot; thus, when the reciprocal of 0.192 is sought, 0.19 is found at left-hand side and .002 at top, then 5.208 is found at the intersection of the horizontal row and the vertical column. Interpolation is the process of finding the function for an argument that falls between two tabular arguments. This is generally done by regarding the function as varying uniformly between the two adjacent tabular values. Hence if the given argument a is half-way between two tabular arguments a_1 and a_2, the required function f is the mean of the two corresponding tabular functions f_1 and f_2. In general when $a_2 > a_1$ and $f_2 > f_1$, the value of f for an argument a which lies between a_1 and a_2 is

$$f = f_1 + \frac{a - a_1}{a_2 - a_1}(f_2 - f_1) \quad \text{or} \quad f = f_2 - \frac{a_2 - a}{a_2 - a_1}(f_2 - f_1)$$

the first formula being preferable when $a < \frac{1}{2}(a_1 + a_2)$, altho both are applicable for any a. Thus by Table 14 the area of a circle for diameter 2.533 is found to be 5.039. When the functions decrease as the arguments increase, $f_2 - f_1$ is negativ; thus log cot 42° 17′ is found from Table 2 to be 0.0413.

When a function is given and it is required to find the argument, the same process applies if a_1, a, a_2 be regarded as the functions and f_1, f, f_2 as the arguments. For example, when a cotangent is given as 1.8501 and the angle is required, the function is seen from Table 3 to fall between 1.8807 and 1.8040 which correspond to the arguments 28° and 29°; here the difference of the tabular functions is .0767 for a difference of 60′ in the arguments, while the difference 1.8807 − 1.8501 is 0.0306; then 306/767 = .4 and .4 × 60 = 24, whence the required angle is 28° 24′, closely.

An error of one or two units usually occurs in the last figure of a function obtained by interpolation, since the tabular values do not really vary in the manner supposed. An interpolated argument is liable to a similar error, and hence care should be taken not to extend the figures too far; in the last numerical example, however, it happens that the last figures should be 231.

Logarithms of Numbers (Tables 1 and 26). The word logarithm and its abbreviation log, when used without qualification, refer to a common logarithm which is defined by the equation $10^{\log n} = n$. Table 1 gives the decimal part, or mantissa of a logarithm, while the integral part or characteristic is to be supplied according to the following rules. When the number is greater than 1, the characteristic of its log is positiv and is one less than the number of figures preceding the decimal point; thus,

$\log 6.54 = 0.81558 \quad \log 65.4 = 1.81558 \quad \log 654 = 2.81558 \quad \log 6540 = 3.81558$

When the number is less than 1, the characteristic of its log is negativ and is numerically one greater than the number of ciphers immediately following the decimal point, thus the four-place log of 6 is 0.7782, and

$\log 0.6 = \bar{1}.7782 \quad \log 0.06 = \bar{2}.7782 \quad \log 0.006 = \bar{3}.7782 \quad \log 0.0006 = \bar{4}.7782$

Here the mantissa is positiv, so that $\bar{2}.7782$ is the same as $-2 + 0.7782$. When the given number is an integral power of 10, the mantissa is zero, so that log 1000 = 3, log 100 = 2, log 10 = 1, log 1 = 0, log 0.1 = −1, log 0.01 = −2, log 0.001 = −3, or in general $\log 10^m = m$.

Multiplication and Division of numbers may be performed by the help of logarithms and the use of the following rules:

To multiply a by b, $\log a + \log b = \log ab$
To divide a by b, $\log a - \log b = \log a/b$

Here $\log a$ and $\log b$ are obtained from Table 1 or 26 and the above rules for the characteristic; then the numbers corresponding to $\log ab$ and $\log a/b$ are found from the Table. For example, to multiply 68.31 by 0.2754, the sum of the logs is 1.27444 and its corresponding number is 18.812, the last decimal being one unit in error.

Powers and Roots of numbers are most conveniently computed by logarithms and the use of the following rules:

To raise a to the nth power, $n \log a = \log a^n$

To extract the nth root of a, $\frac{1}{n} \log a = \log a^{\frac{1}{n}}$

For example to raise 0.6831 to the 1.53 power: $1.53 \times \bar{1}.83448 = -1.53 + 1.27675 = \bar{2}.47 + 1.27675 = \bar{1}.74675$ which is log of 0.55815. To find the fifth root of 0.6831: one-fifth of $\bar{1}.83448$ is $^1/_5\,(-5 + 4.83448) = \bar{1}.96690$ which is log of 0.9262; or it is perhaps better to multiply by 0.2 instead of dividing by 5, thus $0.2\,(\bar{1}.83448) = 0.2\,(-1 + 0.83448) = -0.2 + 0.16690 = -1 + 0.8 + 0.16690 = \bar{1}.96690$.

Table 2 contains logarithms of trigonometric functions to four decimal places at intervals of 1°, the characteristics being given. When the angle is less than 45° look for it on the left side and for the name of the function at the top; when greater than 45° look for it on the right side and for the function at the foot. In many books these functions are called logarithmic sines, logarithmic tangents, etc., while the characteristics are written 8 and 9 instead of $\bar{2}$ and $\bar{1}$, thus requiring some multiple of 10 to be subtracted later. Here the final logarithm of a computation is correct without such subtraction; thus to compute $(\cos 18°)^3$ write $\log (\cos 18°)^3 = 3 \log \cos 18° = \bar{1}.9346$, which is log of 0.8601. If any one wishes to use the old but illogical method, he can, in taking a log from a table, write 9. instead of $\bar{1}$. and 8. instead of $\bar{2}$. as in Table 27.

Table 2 gives four-place logs of trigonometric functions. The columns for log arc refer to the angles exprest in radians or to the arc of the circle exprest in terms of radius unity. Thus for 10°, the arc is 0.1745 and log arc to 10° is $\bar{1}.2419$; the arc of 57°.3 is 1.0000 and log arc 57°.3 is 0.0000. The above remarks regarding characteristics should be noted.

Natural Trigonometric Functions (Arts. 3 and 28–29). Table 3 gives four-place functions for every degree and Tables 28–29 give five-place for functions for intervals of 1′ in angle. For explanation of the term arc, see preceding paragraph. For rules regarding the signs of functions greater than 90° see Sect. 12. The secant of an angle is the reciprocal of its cosine and cosecant is the reciprocal of sine. The logarithms of the functions in Table 3 are given in Table 2, and those of the functions in Tables 28 and 29 are in Table 27. Log sec = − log cos, and log cosec = − log sin.

Reciprocals, Powers, and Roots (Arts. 4–10, pp. 14–26). The arrangement of most of these is like that of a logarithmic table, the last figure of the argument being given at the top and foot. By properly moving the decimal point in both argument and function, Tables 4–8 may be used for other numbers than those given. For example, reciprocals of 0.0504, 0.504, 5.04, 50.4 are 19.84, 1.984, 0.1984 0.01984 to four significant figures; square roots of 3.04 and 304 are 1.744 and 17.44, while those of 30.4 and 3040 are 5.514 and 55.14. When the decimal point is moved one place in a number it is moved two places in the square and three places in the cube.

The Three-halves Powers in Table 9 have been taken from Horton's Weir Experiments, Coefficients and Formulas (Water Supply and Irrigation Paper No. 200 of U. S. Geological Survey, 1907) and are here given to four decimal places only. All those which end in 5 in Horton's table have been recomputed.

Table 10 has been computed for this Handbook, the work being done in duplicate. To find $(10\,n)^5$, $(10\,n)^{1/5}$, $(10\,n)^{5/2}$, and $(10\,n)^{2/5}$ multiply the tabular values by 100000, 1.5849, 316.23, and 2.5119. To find $(0.1\,n)^5$, $(0.1\,n)^{1/5}$, $(0.1\,n)^{5/2}$, and $(0.1\,n)^{2/5}$ multiply the tabular values by 0.00001, 0.63096, 0.0031623, and 0.39811.

Circles and Spheres (Arts. 11–17, pp. 27–33). Tables 11, 14, 16 give circumferences of circles, areas of circles, and volumes of spheres for diameters in units and tenths. When the decimal point is moved one place in a diameter, it is to be moved one place in a circumference, two places in an area, and three places in a volume. Thus, for diameters 1.53 and 15.3 ft, the areas of the circles are 1.839 and 183.9 sq ft, and the volumes of the spheres are 1.875 and 1875 cu ft. When the diameters are given in units and eighths Tables 12, 15, 17 are to be used.

Table 13, which gives properties of circular segments, has been taken from Taschenbuch Verein Hütte, 1905. Lengths of the arc of the segment are given in Table 3 up to 90° and for larger angles may be found from Table 21. Example: for central angles of 35° and 105° and a radius r, the arcs are 0.6109 r and 1.8326 r, the chords are 0.6014 r and 1.5867 r, the rises are 0.0463 r and 0.3912 r, and the areas are 0.01864 r^2 and 0.4 3333r^2.

Areas of Spheres may be found by multiplying the circular areas in Tables 14 and 15 by 4, since the area of a sphere is equal to that of four great circles. Thus, area of a sphere for diameter 1.53 is 7.356, the last figure being one unit in error.

Miscellaneous Tables (Arts. 18–23, pp. 34–36). Table 18 gives hyperbolic sines, cosines and tangents for arguments up to 4.9. The hyperbolic cotangent, coth u, may be found by taking the reciprocal of tanh u. Powers of the Naperian base, $\epsilon = 2.71828$, may be obtained by taking the sum of cosh u and sinh u when u is positiv and their difference when u is negativ; thus, $\epsilon^{2.5} = 12.1825$ and $\epsilon^{-2.5} = 0.0821$.

Naperian Logarithms, often called hyperbolic logs or natural logs, are given in Table 19 for numbers from 1 to 119. The abbreviation Nap log designates these, and they are defined by the equation $\epsilon^{\text{Nap log}\, n} = n$, where ϵ is the Naperian base 2.71828. Extended tables of Naperian logs are unnecessary since they may be easily computed from common logs as shown by the example in Table 20.

Table 21 gives multiples for computing lengths of circular arcs for any given central angle. In Table 22, the symbol π represents the ratio of the circumference of a circle to its diameter, ϵ the base of the Naperian system of logarithms, and μ the modulus of the common system of logarithms; $1/\mu$ is the Nap log of 10.

Circumferences and areas of circles may be computed with accuracy to six or seven significant figures by the help of the multiples of π given in Table 22 using circumference $= \pi d$ and area $= \pi r^2$ where d is the diameter and r the radius.

25. Precision of Results Computed from Tables

Numbers in Tables are not generally precise in the last figure, and hence a lack of precision occurs in the final result of a computation. It is important that a computor should use the tables so as to obtain the most precise result and also that he should not attribute to the result a precision that does not exist. In general no more than four reliable significant figures can be obtained from a four-place table; indeed the fourth significant figure is liable to an error of one-quarter of one unit. Hence the Tables 28–29 cannot be used for cases where six or more significant figures are required to be accurately determined, but their use is limited to cases where only four or

five are needed. The greater part of the computations in engineering require only three or four significant figures to be determined with accuracy.

The values given in tables may have a maximum error of one-half a unit in the last figure. Thus in Table 3 the value of sin 18° is given as 0.3090, this meaning that the true value lies between 0.30895 and 0.30905; in Table 28 it is given as 0.30902, this meaning that the true value lies between 0.309015 and 0.309025. When a quantity like 576.48 sin 18° is to be computed by the four-place natural sine, the multiplication gives 178.13232, but the last four figures have no precision, since all that can be concluded by the use of the number 0.3090 is that the result lies between 178.103498 and 178.161142. Hence the result of this multiplication should be written 178.1, and it should be recognized that the last figure may have an error of one-half a unit. When a five-place sine is used the numerical value of a sin θ is liable to an error of half a unit in the fifth significant figure when sin θ is taken from Table 28 without interpolation.

An interpolated value may have double the error of a tabular value, since it is obtained by using the difference of two tabular values one of which may have a positiv error while the other may have a negativ error. Thus sin 8° 18′.5 is found from Table 28 to be 0.14450, this meaning that the true value lies between 0.14449 and 0.14451. Hence the quantity a sin θ when obtained by the multiplication of a by a sine interpolated from a five-place table may have an error of one unit in the fifth significant figure. All figures beyond the fifth have no precision whatever and are entirely misleading.

The Probable Error of a number taken from a table is that error which is as likely as not to be exceeded. When the number is directly taken from the table its probable error is one-fourth of a unit in the last figure; when it is obtained by interpolation its probable error is one-half of a unit in the last place. An interpolated value has really a probable error slightly greater than that just stated, since the function does not vary uniformly between two tabular values as the method of interpolation implies.

When the product of two quantities f_1 and f_2 is to be obtained by using tabular values, the product f_1f_2 may have a maximum error $(f_1+f_2)r$ and its probable error is $\frac{1}{2}(f_1+f_2)r$ where r is half a unit in the last place when the numbers are directly taken from the table and one unit when interpolation is made. Thus, the four-place product of 4.12^2 and 7.43^2, when obtained by multiplying the four-place squares given in Table 5, is 936.7, the probable error of which is 0.18; the last figure is here really three units in error but an error so large will occur only about one time in a hundred. For cases of this kind logarithms give more precise results; thus by four-place logs the product of 4.12^2 and 7.43^2 is 937.2 where the last figure is only one unit in error.

The quotient f_1/f_2 also has the probable error $\frac{1}{2}(f_1+f_2)r$. When several quantities taken from tables are combined either by direct multiplication or division, the probable error of the result is $\frac{1}{2}(f_1+f_2+f_3+\ .\ .\ .)r$ and its maximum error may be twice as great. When logarithms are used, the probable and maximum errors are much smaller.

Inexperienced computors sometimes, in making interpolations, use all the figures obtained in the multiplication of differences, and thus carry the work several places further than the tabular values warrant. This procedure not only entails additional work and gives extra figures which are wholly inaccurate, but it leads the computor to suppose that his results have a far higher degree of precision than is actually the case, hence vitiating his judgment and perhaps leading to the deceit of others as well as of himself. The above indicates that no more significant figures should appear in intermediate work or final results than are given in the tables which are used.

In this discussion the data of the computation are regarded as free from error. When the data are obtained by measurement, these are affected by the errors and uncertainties of the measurements and hence the probable errors of the computed results are greater than those noted above. See Sect. 12, Art. 14.

SECTION 2

SURVEYING, GEODESY, RAILROAD LOCATION

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INSTRUMENTS AND MAPS

LAND AND TOWN SURVEYING

TOPOGRAPHIC SURVEYING

ASTRONOMICAL OBSERVATIONS*

GEODETIC SURVEYING*

RAILROAD LOCATION

EARTHWORK COMPUTATIONS

* By George L. Hosmer, Associate Professor of Topographical Engineering in Massachusetts Institute of Technology.

INSTRUMENTS AND MAPS

1. Chains and Tapes

The Engineer's Chain has 100 links, each one foot long. Gunter's chain, formerly much used in land surveys, is 66 ft in length and has 100 links, each 7.92 inches long. The metric chain is 20 meters in length and has 100 links, each 20 centimeters long.

Each end link is provided with a handle, the outside edge of which is the zero point. Every tenth link counting from either end is marked by a brass tag having one, two, three or four points on the tag corresponding to the number of tens of links it marks; the middle of the chain is marked by a round tag. The long links are connected by two short links for flexibility. This introduces about 600 wearing surfaces which causes the chain to lengthen with use. All chains should therefore be frequently compared with a Standard; the length is adjusted by means of a screw and nut in one handle which allows the length of the end link to be changed, thus putting all of the error in length into the end link. Chains are gradually being displaced by the heavy steel tapes which will stand rough usage and are not subject to change of length from wearing.

Cloth, Metallic, and Steel Tapes are in common use. Cloth tapes stretch so readily that they are useless for surveying. Metallic tapes are cloth tapes with fine brass wires woven into them to prevent stretching. They are usually graduated into feet, tenths, and half-tenths and are made in lengths of from 25 ft to 100 ft. It is not uncommon for metallic tapes when subjected to rough usage and to alternate wetting and drying to become 0.2 to 0.5 ft in error in a length of 50 ft. Metallic tapes should not be used except for measurement of short lines where great precision is not required.

Steel Tapes may be obtained in lengths up to 500 ft but the most common lengths are 50 ft, 100 ft and 300 ft. The shorter tapes are usually made of thin ribbons of steel, while the longer tapes are made heavier in cross-section so that they will withstand rougher usage and will not become kinked so readily. The light tapes are graduated thruout their length into feet, tenths, and hundredths, while the heavier tapes are generally marked only at every foot, the last foot on each end being divided into tenths. In most tapes the zero point is at the outer edge of the ring attached to the end of the ribbon of steel, but this is not always the case; some tapes are graduated so that the zero point comes on the tape 0.2 or 0.3 ft from the end ring. Light tapes are not conveniently handled in rough work; they become easily kinked and broken, but can be mended by riveting to the back of the tape a piece of an old tape of the same width. A new tape is usually of standard length at 62° F. with a pull of 12 pounds and supported thruout its length. While the steel tape varies in length slightly with variations in temperature and pull, still in much of the work of surveying it is accurate enough to use it without regard to its change in length, and for surveys which require great accuracy the change in length can be determined and proper corrections applied to measured distances.

Special steel tapes fitted with a thermometer and with a spring-balance handle for registering the amount of pull can be obtained from the manufacturers, also tapes graduated in feet and inches for use in building construction. Pocket Steel Tapes from 3 ft up are very useful in the field.

Steel tapes are broken usually by jerking them when they are looped or kinked or by stepping on them or allowing a vehicle to pass over them in soft ground. A tape properly cared for to prevent rusting and kept intact will last until the graduations are worn off.

To change measurements recorded in tenths and hundredths of a foot into inches and fractions, the following equivalents may be used:

Decimal of Foot	= .01	.08	.17	.25	.50	.75
Inches	= 1/8 −	1 −	2 +	3	6	9

If these values are memorized, decimals of a foot may be quickly transposed into inches as follows: 0.44 ft = 0.50 ft − 0.06 ft = 6 in − ¾ in = 5¼ in.

The Odometer is an instrument attached to a wheel of a vehicle which records the number of revolutions the wheel makes in traversing between two given points. If the circumference of the wheel is known the approximate distance can be computed.

The Pedometer is an instrument which records the number of steps of a walker carrying it. A modern form records the distance traversed by a person carrying the pedometer, it being adjusted for the length of pace of the walker.

2. Verniers

The Vernier is a device for determining the subdivision of the smallest division of a scale more accurately than can be done by simply estimating the fractional part. Its accuracy depends upon the fact that one can judge more accurately when two lines coincide than he can estimate a fractional part of a space. The simplest form of vernier (Fig. 1) is used on some kinds of leveling rods. It is constructed by making the entire length of the vernier equal to the length of 9 divisions on the scale, and subdividing this vernier length into 10 equal parts. In this way one division on the vernier equals 9/10 of a division on the scale, so that ab is 1/10 and cd is 2/10 of a scale division. If the vernier is raised until a comes opposite b then the reading will be 701; if raised until c comes opposite d the reading is 702. The number of the line on the vernier that coincides with some line on the scale is the number of tenths of the smallest division on the scale that the index point lies above the scale division just below it.

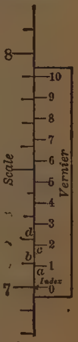

Fig. 1

Transit Verniers are usually made double, one on each side of each index, so that angles can be read in either direction. For transits reading to minutes the vernier scale is made by dividing a space equal to 29 half-degrees of arc into 30 parts, so that the difference in length of one division of the circle and one division of the vernier is equal to one minute of arc. If the vernier is placed so that its zero point (or the index) coincides with the 0° mark on the circle, then the first lines on either side of the zero of the vernier will fail to coincide with the corresponding lines on the circle by just one minute, the second lines on each side of the zero of the vernier will fail to coincide with their corresponding marks on the circle by just 2 minutes, and so on. If the vernier is moved along the circle so that the first line next to the zero of the vernier exactly coincides with its corresponding line on the circle the reading will be 0° 01′; and if the vernier is moved one minute more the second lines will coincide and the reading will be 0° 02′. Therefore in

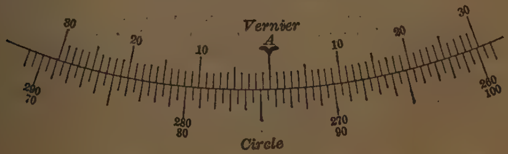

Fig. 2. Transit Circle with Double Vernier

reading an angle on a transit, read the position of the vernier index on the circle, reading the angle in the same direction that the telescope has moved and estimate roughly the number of minutes the index has passed over from

the graduation next back of it on the circle, then follow along the vernier in the same direction and find the vernier line which coincides with some line on the circle and add the number of this line to the circle reading. Thus, Fig. 2 represents a one-minute transit circle with double vernier which reads, if the telescope (which always moves with the vernier) has moved clockwise, $274^\circ\ 23'$. If it has moved anticlockwise it reads $85^\circ\ 37'$. For a one-minute instrument the circle is divided into 30′ spaces and there are 30 divisions on the vernier; a 30-second instrument has the circle divided into 20′ spaces with 40 divisions on the vernier; a 20-second instrument has the circle divided usually into 20′ spaces with 60 divisions on the vernier; and a 10-second instrument has the circle divided into 10′ spaces with 60 divisions on the vernier.

Single Verniers which are to be read in either direction must be numbered from both ends, and the index at either one end or the other is set at 0° on the circle depending upon which direction the angle is to be measured. These single verniers are used on transits reading to 10 seconds or 20 seconds where a double vernier would be so long as to interfere with the standards. Another type of single vernier called the FOLDED vernier is sometimes used on the vertical circle of transits and on alidades of plane tables. It is like any single vernier except that the index is the middle division of the vernier. In reading it, if a coincidence is not reached by passing along the vernier in the proper direction, it is necessary to pass to the other end of the vernier and continue in the same direction, toward the center, until the coincidence is found.

A type of vernier which reads in hundredths of a degree is used to a limited extent. It is claimed by some that this is particularly useful in laying out railroad curves, but its advantage is so slight that it is outweighed by the disadvantage of breaking away from the American practice of recording angles in degrees and minutes and the inconvenience in the use of the ordinary tables.

Retrograde Vernier. The verniers described above are all DIRECT verniers in which the smallest division of the vernier is smaller than the smallest division of the circle. The RETROGRADE vernier is one in which the smallest division of the vernier is larger than the smallest division of the circle. It is seldom used on surveying instruments.

3. The Compass

The Surveyor's Compass is an instrument for determining the difference in direction between any horizontal line and a magnetic needle. The needle is balanced on a pivot in the center of a compass-box so that it can swing freely in a horizontal plane, the pivot being the center of a circle which is graduated usually to half-degrees and numbered from 0° to 90° in both directions from the N and S end of the compass-box. The pivot and the circle are secured to a brass frame on which are two vertical sights placed on the line passing thru the pivot and the N and S points of the compass-box. When not in use the needle should be kept raised from the pivot by the screw and lever provided for that purpose so as not to dull the pivot-point. The compass is connected to the tripod by a ball-and-socket joint which is clamped in position after the instrument has been leveled by means of two spirit levels attached to the frame at right angles to each other. The frame has a spindle which fits into the ball-and-socket joint so that after the instrument has been leveled it can be swung around in a horizontal plane. Since in the northern hemisphere the N end of the needle if not counterbalanced would dip downward, a little counterweight is attached to the S end of the needle.

To Use the Compass it is first set up over the proper point and leveled, and then sighted along the line whose direction is desired. Since the needle stands still and the box turns under it, the letters E and W on the box have been reversed from their natural position so that the reading of the needle will give the proper quadrant. To obtain the BEARING of the line in the direction

it is sighted it is important to follow this rule: When the N point of the compass-box is toward the station whose bearing is desired, read the N end of the needle (the end to which the counterbalance is not attached in northern hemisphere). When the S point of the box is toward the station, read the south end of the needle. The term "station" as here used means any point on the survey line. Bearings are usually read to the nearest quarter of a degree. The bearing looking in the opposite direction along the line is called the REVERSE BEARING. To avoid a parallax error the needle should be read by looking along the needle, not across it.

Precautions. Before the bearing is read the glass should be tapped lightly over the end of the needle to be sure that it is not touching the under side of the glass. If the needle appears to cling to the glass it probably indicates that the glass has become electrified, and this difficulty can be removed by placing the moistened finger on the glass.

In the use of the compass great care should be taken that no iron or steel is near the instrument to attract the needle from its true position. The chain, pins, ax, pocket knife, iron wire in a stiff hat are fruitful sources of errors. Electric currents affect the needle so seriously that it is of little use in cities for obtaining even an approximate magnetic meridian. It is customary to take the forward and reverse bearing of lines so that any local attraction of the needle may be detected. If the bearing of AB taken from Station A and the bearing of BA from Station B do not agree it indicates that at either A or B there is local attraction. To determine at which station it exists, take the bearing of BC with the compass at B, and then with the compass at C determine the bearing of CB. If these agree it indicates that there is no local attraction at B.

Adjustments of the Compass. (1) The bubbles are adjusted like those on the horizontal plate of a transit (Art. 4). (2) Straightening needle and centering pivot-point: Bent needle may be detected by reading both N and S ends when N end is held opposite a given graduation, and then by reading both ends again when S end is held opposite same graduation; take needle out and bend it with pliers at the middle until the end reads the same in either position. Pivot may be centered by finding (by trial) position of maximum difference between end readings on different parts of the circle, and then bending pivot so that end readings agree. (3) Remagnetizing needle; rub with a bar magnet from center toward ends, using N end of magnet for S end of needle, and vice versa.

The Pocket Compass is a hand instrument for obtaining roughly the bearing of a line. There are two kinds, the PLAIN and the PRISMATIC. The former is much like the surveyor's compass, except that it has no sights. In the prismatic compass the graduations, instead of being on the compass-box, are on a card which is fastened to the needle (like a mariner's compass) and which moves with it. This compass is generally provided with sights. The bearing can be read, by means of a prism, at the same instant that the compass is sighted along the line.

4. The Transit and its Adjustments

The Engineer's Transit, a view of which is shown in Fig. 3, has two spindles, one inside the other, to each of which is attached a horizontal circular plate, the outer spindle being attached to the lower plate and the inner one to the upper plate. Attached to this upper plate are two standards supporting the horizontal axis of the telescope. The motion of this horizontal axis and of the two spindles is controlled by clamps N and M and slow-motion (tangent) screws n and m. On the upper plate are two spirit-levels, which are used in leveling the instrument. Most transits have four leveling screws; but those made for triangulation work usually have only three. When the instrument is provided with a telescope bubble and a vertical arc it is called an ENGINEER'S (or SURVEYOR'S) TRANSIT; if it does not have these attachments it is called a PLAIN TRANSIT.

The Telescope has two lenses cemented together for its objective, and an eyepiece composed of four lenses if the instrument is an erecting transit, two lenses if inverting. An inverting instrument gives a much more illuminated field. Between the objective and the eyepiece is the cross-hair ring.

The line of sight, or line of COLLIMATION, is the straight line drawn thru the optical center of the objective and the point of intersection of the cross-hairs. The adjustment of the eyepiece and of the objective so that the cross-hairs and image of the object can be clearly seen at the same time is called focusing. The MAGNIFYING POWER of a telescope is the amount by which an object is increased in apparent size. It is equal to tan $\frac{1}{2} A/\tan \frac{1}{2} a$ (or nearly A/a), A being the angle subtended by the object as seen thru the telescope and a the angle as seen by the unaided eye. It may be found approximately as follows: Hold a rod a short distance from the instrument and view it thru the telescope with one eye; at the same time look at it directly with the other eye. It will be noticed that one space as viewed thru the telescope will appear to cover several spaces as seen with the naked eye. This number is approximately the magnifying power of the telescope.

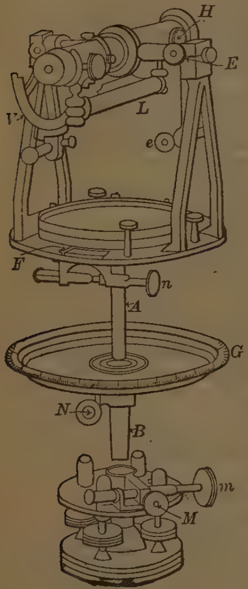

Fig. 3

In Fig. 3 the Engineer's Transit is shown separated into three parts. A is the inner spindle; B, outer spindle; N, upper clamp; n, tangent screw for upper clamp; M, lower clamp; m, tangent screw for lower clamp; E, telescope vertical motion clamp; e, tangent screw for vertical motion; F, vernier; G, graduated circle; H, focusing screw for objective; V, vertical arc.

Common Special Attachments are: (1) Diagonal or prismatic eyepiece for sighting high altitudes. (2) A reflector attached to the telescope for illuminating the cross-hairs when working in the dark. (3) A gradienter screw for measuring rates of grades directly. (4) Stadia hairs, which are two extra horizontal cross-hairs used in measuring distances by stadia. (5) Solar attachment, for determining a meridian by observation on the sun.

By Observing a Few Important Precautions much of the wear of instruments and accidents to them can be avoided. Neither the tripod legs nor the shoes should be allowed to become loose. In taking the instrument out of its box always lift it by placing the hands beneath the leveling base. When about to move the transit from one point to another be sure that the four leveling screws are properly bearing, that the needle is lifted, and that the clamps are set just firmly enough so that any slight shock will allow motion. A waterproof bag should be carried at all times to cover the instrument if it rains or when the instrument is exposed to dust. When the transit must stand out in the rain the cap should be put over the object glass; if water should get into the telescope, take out the eyepiece, cover the open end of telescope tube with cloth, and allow it to dry out. Any parts of the instrument that have been wet should be wiped dry, especially the vertical arc, but be careful not to touch the edges of the arcs. One should always avoid placing the hands on exposed graduations, as it will tarnish the metal. In cleaning the lenses use a fine camel-hair brush, and for screw threads use a stiff tooth brush and apply a very little watch oil after cleaning, but do not apply oil to exposed screws.

Important Adjustments of the Transit are: (1) to make the plane of the plate bubbles perpendicular to the vertical axis of the instrument, (2) to

nake the line of sight perpendicular to the horizontal axis, (3) to make the orizontal axis perpendicular to the vertical axis of the instrument. These hree adjustments are made to depend on the principle of reversion, the effect f an error being doubled by a reversal of the instrument. Each adjustment hould be repeated to test its accuracy.

(1) **Adjustment of the Bubble Tubes.** Bring plate bubbles in center f respective tubes by means of leveling screws. Turn 180° in azimuth; half ne apparent error in bubbles is the real error; so bring bubble half-way back y screws on the bubble tubes. Adjust each bubble independently.

(2) **Adjustment of the Line of Sight.** First adjust the vertical cross-air so it will lie in a plane perpendicular to the horizontal axis. Set up and evel the instrument. Leveling the instrument is not necessary for this part f the adjustment, but as it is an essential part of the second portion of this djustment it is well to level at this time. Sight the vertical hair on a well-efined point, clamp both plates, rotate telescope about horizontal axis. If oint does not appear to travel along the vertical cross-hair, loosen screws olding cross-hair ring, and by tapping lightly on one screw, rotate ring until bove condition is fulfilled. Then tighten screws and proceed with the econd part of the adjustment as follows. Sight telescope at point *A* (200 to oo ft away), and clamp vertical axis; revolve telescope on horizontal axis nd set a point *B* in line of sight and same distance approximately from nstrument as *A* but in the opposite direction. Points *A* and *B* should be at bout the same elevation. Loosen clamp, turn in azimuth and sight *A* again, lamp, revolve telescope on horizontal axis and set point *C* in line of sight eside point *B*. Mark or note a point *D* one-fourth the distance between *C* and *B* measured from *C*. To adjust, move the cross-hair ring until *D* is ighted by loosening the screw on one side of the telescope and tightening that pposite. If the transit has an erecting eyepiece move the ring in the direction *D* to *C*; if an inverting eyepiece, move ring in the same direction *C* to *D*.

(3) **Adjustment of the Standards.** Set up and level the transit. Sight n some high point *A* and clamp vertical axis. Lower telescope and set oint *B* in line of sight about level with telescope. Reverse telescope and urn in azimuth 180° and sight on *B* and clamp. Raise telescope until point *A* is visible and note point *C* in line of sight and at same height at *A*. If *C* loes not fall on *A*, loosen pivot cap screws at adjustable end of horizontal axis and raise or lower the end of the axis by means of capstan-headed screw under axis. The adjusting screw should be brought into position by a right-hand turn, otherwise the block on which the horizontal axis rests may stick and not follow the screw. The cap screws should then be tightened just enough to avoid looseness of the bearing.

If the instrument is badly out of adjustment it is better to bring it as a whole gradually into adjustment rather than to attempt to completely adjust one part at a time. In this way the adjustment of one part will not disturb the preceding adjustments, the parts are not subjected to strains, and the instrument will remain in adjustment longer.

To Adjust the Vernier of the Vertical Circle so it will read 0° when the telescope bubble is in the center, loosen the screws holding the vernier, and tap lightly until the zeros coincide. This adjustment eliminates index correction in measuring vertical angles, provided the line of sight is parallel to the bubble tube.

To Adjust the Objective Slide so it will move parallel to the line of sight. Adjust the line of sight as described under (2) above, but use very distant objects, then repeat this same adjustment using points very near by, and if in this last test there is an apparent error, it indicates that the objective slide does not move parallel to the line of sight. The adjustment is made by moving the adjusting screws of the objective slide so as to apparently increase the error, one-quarter of the apparent error. Then test adjustment of line

of sight again by using distant points. The adjustments of the objective slide, centerin the eyepiece tube, centering the circles are usually done by the instrument makers.

To Eliminate Effects of Errors in Adjustment and still obtain accu rate results the instrument must be used as follows. To avoid errors in th plate bubble, level up, turn 180° in azimuth and bring bubbles half-way bac by means of leveling screws. This makes the vertical axis truly vertical, an the bubbles should remain in the same parts of their respective tubes as th instrument is turned about vertical axis. Errors in line of sight and horizonta axis are avoided by using the instrument with telescope in its direct and the in its reversed position and taking the mean of the results, whether the worl is running lines or measuring angles. Errors of eccentricity are eliminate by taking the mean of the readings of the two opposite verniers, and errors o graduation of the circle are nearly eliminated by reading the angle in differen parts of the circle or by measuring the angle by repetition. Where only on vernier is read in determining an angle always read the one that was set.

5. The Solar Attachment

The Solar Attachment is a small instrument sometimes attached to transit and with which a true meridian can be established by observation o the sun. The commonest form of solar attachment (SAEGMULLER'S) consist of a small axis called the polar axis attached at right angles to the telescop and to its horizontal axis, and on which is mounted a small telescope. Anothe form of solar attachment (BURT'S) has the small telescope replaced by a len and a screen on which the sun's image can be thrown. This instrument i provided with a special arc for setting off the sun's declination.

Astronomic Terms. In Fig. 4, which represents half of a celestial spher the circle $NWSE$ is the observer's horizon, $SZPN$ is the observer's meridia

Fig. 4. Celestial Sphere

(a vertical circle thru the pole). The circle EQW is the celestial equator, an AMB, parallel to the equator, is a parallel of declination, or the path of th sun on a certain day. The sun's DECLINATION is its angular distance nort or south of the equator, or arc OT; it is + when north and − when sout The POLAR DISTANCE of the sun is the complement of the declination, th

arc OP; when the declination is minus (in winter) the polar distance is more than 90°. If the polar axis points toward the pole, the small telescope can be made to follow the sun's daily path by giving it an inclination to the polar axis equal to the sun's polar distance and then revolving it about the polar axis.

To Find the True Meridian by an observation on the sun with a Saegmuller solar: (1) Make the angle between the polar axis and the solar telescope equal to the sun's polar distance at the time of the observation. This is done by turning the solar telescope into the same plane as the main telescope by sighting both on some distant object, and then making the angle between the two telescopes equal to the sun's declination. Incline the main telescope until the reading of the vertical circle equals the declination, and clamp; then level the solar telescope by means of the attached level. The angle between the polar axis and the solar telescope is then 90° plus or minus the reading of the vertical circle.

(2) By means of the vertical circle of the transit incline the polar axis to the vertical by an angle equal to the colatitude of the place, which is 90° minus the latitude. The polar axis will then have the same angle of elevation as the celestial pole.

(3) If the observation is in the forenoon, place the solar telescope on the left of the main telescope (on the right if in the afternoon); then, by moving the whole instrument about the vertical axis and the solar telescope about the polar axis, point the solar telescope at the sun. The sun's image is brought to the center of the square which is formed by four cross-hairs in the solar telescope. The final setting is made by the tangent screw controlling the horizontal motion of the transit and the one controlling the motion of the solar about the polar axis. Only one position can be found where the solar telescope will point to the sun. In this position the vertical axis points to the zenith, the polar axis to the pole, and the solar telescope to the sun. Since the solar telescope is pointing to the sun the main telescope must be in the plane of the meridian. If all of the work has been correctly done it will be observed that the sun's image will remain between the cross-hairs set parallel to the equator, and therefore the sun can be followed in its path by a motion of the solar telescope alone revolving about the polar axis. If it is necessary to move the instrument about the vertical axis to point the solar telescope again at the sun, this shows that the main telescope was not truly in the meridian. For good results observations should not be made near noon, or near sunrise or sunset when the altitude is less than 10°.

The Sun's Polar Distance may be obtained from the "American Ephemeris and Nautical Almanac," published by the Government. The polar distance is not given directly, but its complement, the sun's apparent declination, is given for each day and for the instant of Greenwich Mean Noon, as well as the rate of change of the declination, or the difference for 1 hour. In order to use this for any given locality, it is first necessary to find the local or the standard time corresponding to mean noon of Greenwich. In the United States, where STANDARD TIME is used, the relation to Greenwich time is very simple. In the Eastern time belt the time is exactly 5 hours earlier than at Greenwich; in the Central, 6 hours earlier; in the Mountain, 7 hours earlier; in the Pacific, 8 hours earlier. If a certain declination corresponds to Greenwich mean noon, then the same declination corresponds to 7 A.M. in the Eastern belt or 6 A.M. in the Central belt, etc. The declination for any subsequent hour of the day may be found by adding (algebraically) the difference for 1 hour multiplied by the number of hours elapsed. Declinations marked North must be regarded as positiv and those marked South as negativ. An examination of the values of the declination for successive days will show which way the correction is to be applied. It will be useful also to remember that the declination is 0° about March 21, and increases until about June 22, when it is approximately $23^\circ 27'$ North; it then decreases, passing the 0° point about September 22, until about December 21, when it is approximately $23^\circ 27'$ South; it then goes North and is 0° on March 21.

Atmospheric Refraction. After the correct declination is found it has still to be corrected for refraction of the atmosphere. The effect of refraction is to make the sun appear higher up in the sky than it actually is. In the northern hemisphere, when the declination is North this correction must be added, when South, subtracted; or algebraically it is always added. The amount of refraction can readily be found by COMSTOCK'S METHOD as follows. Set the vertical hair on one edge (or limb) of the sun and note the instant by a watch. Set the vernier of plate 10′ ahead and note the time when the same edge meets the cross-hair. If n is the number of seconds of time between the observations and h is the altitude in degrees, then the refraction in minutes equals $2000/hn$ approximately.

The Colatitude which must be set off on the vertical circle may be obtained from a map, or may be determined by an observation as follows. Set off the sun's declination for noon, as for any other observation, the two telescopes being in the same vertical plane, and point the small telescope at the sun. By varying the angle of elevation of the main telescope, keep the solar telescope pointing at the sun until the maximum altitude is reached. The angle read on the vertical circle is the colatitude. This observation necessarily comes near noon, but in order to be sure of the maximum altitude it is necessary to begin the observation some time before noon, for the maximum does not occur at exactly apparent noon because the declination is continually changing, the interval between apparent and Standard noon may not be known, and the watch may not be exactly right.

6. The Level and its Adjustments

A Level Instrument is a telescope with a delicate spirit-level attached to it so that when the bubble is in the center the line of sight is horizontal, or tangent to a level surface, which is a curve every point of which is perpendicular to the direction of gravity. The two common types are the WYE and the DUMPY levels. In both of these instruments the telescope is mounted on a vertical axis about which the telescope can swing, and is leveled by means of four leveling screws.

In the Wye Level the spirit-level is attached to the telescope which rests in two Y-shaped supports, which in turn are fastened to a horizontal bar to which the vertical axis is attached. The telescope can be taken out of the Y's, turned end for end and replaced, when testing the bubble for adjustment.

The Dumpy Level has its vertical axis, the horizontal bar and the supports of the telescope all in one piece, to which the spirit-level is attached. The dumpy level will stand much rougher usage than the wye level, has fewer wearing parts and allows of fully as precise work. Practically the only advantage the wye level has over the dumpy is that the adjustment of the line of sight can be a little more conveniently tested. The precautions in use of the level and its care are similar to those described for the transit in Art. 4.

The Locke Hand Level is simply a metal tube with plain glass covers at its ends and with a small spirit-level on top. When looking thru the tube the bubble is seen on one side of the tube in a mirror thru a lens; on the other side the landscape is viewed. When the bubble is in the center of the tube the observer can note where the horizontal line which appears in the center of the bubble tube cuts a rod and in this way do approximate leveling (see Art. 28).

Principal Adjustments of the Wye Level are: (1) to make the line of sight coincide with the axis of the pivots, or parallel to it, (2) to make the line of sight and axis of bubble tube parallel, (3) to make the axis of the bubble tube perpendicular to the vertical axis. As in the transit instrument, most of the adjustments depend upon the principle of reversion.

(1) **Adjustment of Line of Sight.** First make the horizontal cross-hair truly horizontal when the instrument is level. This adjustment is tested by sighting, after leveling, on a point and noting if the cross-hair appears to remain on the point as the telescope is revolved about its vertical axis. If an adjustment is necessary it is done by rotating the cross-hair ring as described in the case of the second adjustment of the transit. After this adjustment is

made loosen the clips which hold the telescope in the wyes. Sight the intersection of the cross-hairs at a point and clamp. Rotate the telescope 180° in wyes. If the cross-hairs do not remain on the point they must be moved halfway back to the point by means of screws on the cross-hair ring, each cross-hair being adjusted separately by means of its proper ring screws.

(2) **Adjustment of Bubble by Indirect Method.** Bring the bubble in the center of its tube and clamp in that position. Rotate telescope in wyes a few degrees around its horizontal axis; if the bubble moves, correct the entire error by means of the horizontal capstan screws at one end of the bubble tube. Then clamp the telescope over a pair of leveling screws and bring the bubble into the center of the tube, lift the telescope from the wyes, turn it end for end, and replace in the wyes without jarring the instrument. Correct half the apparent error by the vertical adjusting screw of the bubble tube.

(3) **Adjustment of the Wyes.** Level the instrument, then bring the bubble exactly to the middle over a pair of leveling screws. Then turn the telescope 180° about its vertical axis and correct half the apparent error by means of the adjusting screw of the wye support. Since the bubble is brought to the center of the tube at each rod-reading this last adjustment in no way affects the accuracy of the leveling work but is a convenience.

The Adjustments of the Dumpy Level are the same in purpose as for the wye level, but are, owing to the construction of the instrument, done in a different order and by a different procedure in some cases. They are: (1) to make the horizontal cross-hair truly horizontal when the instrument is level, (2) to make the axis of the bubble tube perpendicular to the vertical axis, (3) to make the line of sight parallel to the axis of the bubble.

(1) **The Adjustment of the Cross-Hairs** is done as described under the first adjustment of the wye level.

(2) **Adjustment of Bubble Tube.** Level the instrument and carefully center the bubble over a pair of leveling screws. Turn the telescope 180° in azimuth and correct half the apparent error in the bubble by means of the adjusting screws of the level tube.

(3) **Adjustment of Line of Sight by Direct or "Peg" Method.** Select points A and B 200 ft or more apart. Set up the level beside A so that when a rod is held on A the eyepiece will swing and just clear the rod. Look through the telescope wrong end to at the rod and find the reading opposite the center of the field. Turn the telescope toward B and take a rod-reading in the usual manner, being sure that the bubble is in the middle of the tube. Then set up the level at B and repeat the above operation. These two sets of observations give two independent determinations of the difference in elevation between the two points. The true difference in elevation is the mean of these two results. Leaving the instrument at B, set the rod at A so that it will read the height the instrument is above B plus or minus the true difference in elevation between A and B. Then if the level is sighted on the target of the rod it will define a level line. While the bubble is in the center of its tube the line of sight should be brought to coincide with the target by moving the cross-hair ring by means of its adjusting screws. The following example illustrates the method:

Instrument at Sta. A		Instrument at Sta. B.	
Rod-reading on Sta. A	= 3.971	Rod-reading on Sta. B	= 5.064
Rod-reading on Sta. B	= 4.937	Rod-reading on Sta. A	= 4.036
Diff. in elev. of A and B	= 0.966	Diff. in elev. of B and A	= 1.028

Mean of two dif. in elev. = ½ (0.966 + 1.028) = 0.997, true diff. in elev.
Instrument is now 5.064 above Sta. B.
Rod-reading on Sta. A should be 5.064 − 0.997 = 4.067 to give a level sight.

This "peg" adjustment may be used for adjusting the line of sight of the wye level except that in the wye level after the target at *A* has been set at the correct elevation to define a level line the line of sight is made to coincide with the target by means of the leveling screws and then the bubble is brought to its mid position by means of its adjusting screws. This "peg" adjustment is also used for the hand level. In using the "peg" method for transits the adjustment may be made either by moving the cross-hair ring or by moving one end of the level tube. If the cross-hair is moved the adjustment of the line of collimation must be tested; if the bubble tube is moved the vertical arc must be adjusted.

To eliminate the effect of errors in adjustment of the line of sight, of the bubble tube or the wyes the observer must be sure that the bubble is in the center of the tube at the instant that the rod is read, and the length of the backsights and foresights should be made equal.

7. Leveling Rods

Two General Types of Rods, TARGET and SELF-READING, are in common use. The target rods are read only by the rodman, while the self-reading rods are read directly by the levelman. The commonest forms of target rod are the Boston, the New York and the Philadelphia rods; the latter may be used also as a self-reading rod.

The Boston Rod is an extension target rod made of two strips one of which slides in a groove in the other, and provided with clamps to hold the two parts in any desired position. There is a scale on each side of the rod graduated to hundredths of a foot, each scale being provided with a vernier for reading to thousandths of a foot. The target is fastened to one of the strips, the other one is held on the ground and the target strip raised to the proper reading, the highest reading being 5.8 ft; these are called "short rod-readings." For "long rod-readings" the rod is turned end for end and the target strip is raised, its highest reading being 11.4 ft. A serious objection to this kind of rod is that in reversing it any error in the position of the target with reference to the zero of the scale is doubled.

The New York Rod has two strips arranged similarly to those of the Boston rod, but the circular target is movable and for short rod the target is moved up and down on the rod, the scale graduated to hundredths being in the face of the rod; it is read by means of a vernier on the target. The scale for long rod is on the side of one of the strips and the vernier is on the other strip. When used as a long rod the target is clamped to the face of the rod at the reading corresponding to the lowest reading on the side scale; then the target is raised just as in the Boston rod.

The Philadelphia Rod is marked on its face thruout its entire length (extended) with red numbers to designate the feet and black numbers for the tenths so that it can be used as a self-reading rod. The red figures are 0.1 ft high and the black figures 0.06 ft. When it is used as a target rod, it is operated just as the New York rod; the only difference being that the short scales at the center of the metal target and on the back of the rod (for long rod-readings) are not verniers, but these scales are graduated to 0.005 ft and are used in estimating the readings to thousandths of a foot.

Special Designs of self-reading rods are in use, the figures on the face being as a rule made of some definite height (0.06 or 0.08 ft) and of a thickness of 0.01 or 0.02 ft to aid in estimating the readings. Some of these rods are in three parts, giving an extended length of 16 ft. In nearly every respect self-reading rods are preferable to target rods.

The Tape-rod is a wooden rod made in one piece with a metal roller near each end around which is a continuous movable steel band 20 ft long and about 0.1 ft wide, on the outside of which the scale is painted so that it can be used as a self reading rod; the band has a clamp to fasten it to the rod at any desired position. Unlike the ordinary rod the scale reads down instead of up. In using this rod the band is set so that the level sights the reading of the elevation of the bench-mark. For example, if the B.M. elevation is

142.36 the band is moved and clamped so that the level sights 12.36 on the scale. When the rod is held at any other point the rod-reading plus 130 gives the elevation of the point. Such a rod is of great convenience in cross-section work.

Self-reading Rods for Precise Leveling are used by the U. S. Coast and Geodetic Survey and are made of single pieces of wood soaked in paraffin to prevent changes in length due to moisture; metal plugs are inserted at equal distances to detect any changes in length. The rod is divided into centimeters painted alternately black and white. The U. S. Geological Survey and other government surveys use similar rods for precise leveling (Art. 25). For plumbing the rod in precise work special devices are used such as spirit-levels attached to brass angles or watch levels held on the corner of the rod.

8. Drafting Instruments and their Uses

It is assumed that the reader is familiar with the ordinary drafting instruments; only a few of the more uncommon will be described.

The Pantograph is a jointed framework of several pieces (or arms) of wood or metal so joined as to form a parallelogram; and used for enlarging or reducing maps. It rests upon three points, one of which, A, is fixt and the other two, B and C, are movable. There are other bearing points but they simply support the instrument and are not essential to its principle. The two movable points B and C are in such positions that they will trace exactly similar figures, so that the instrument is used for copying plans either to the same or different scales, which latter is accomplished by varying the positions of the points B and C on the arms. Thus two points B and C can be attached to their respective arms at any desired position, but the essential condition is that A, B, and C shall lie in a straight line and each of the three points must be attached to one of three different sides (or sides produced) of the parallelogram. Any one of the three points can be the fixt point. These instruments are usually provided with scales on the arms indicating the proper settings for various reductions or enlargements. Because of lost motion in the joints very accurate results cannot as a rule be reached, but the best metal pantographs are sufficiently accurate for most topographical maps.

The Planimeter is an instrument for determining the area of a figure by moving the tracing point of the instrument around the perimeter of the plotted area however irregular its shape. The most common form is the AMSLER POLAR PLANIMETER, which has two arms, one fixt in length, at the end of which is the anchor point which has a needle point to attach the instrument to the paper. The length of the other arm, the tracing arm, can be changed to give results in different units. At the end of the tracing arm is a point which is moved along the outline of the area to be measured. This arm passes thru a collar to which the fixt arm is attached by a pivot. Connected with this collar is a graduated wheel which, together with a little disk which records the revolution of the wheel, gives the area in units depending upon the length of the movable arm. The planimeter rests on three points, the anchor, the tracing point and the periphery of the wheel. To measure an area, press the anchor point into the paper at a position outside of the perimeter of the figure, if it is not too large, and start the tracer from a definite point on the periphery of the area, preferably such as will bring the two arms approximately at right angles to each other. Read the disk, wheel and its vernier, giving four figures. The tracing point is then moved carefully around the outline of the area until the starting point is again reached, when the disk and wheel are again read. The difference of the two readings gives the area in the unit depending upon the length of the movable arm. The setting of the scale marked on the movable arm for different units is given by the maker. If the area is so large that the anchor point cannot be set outside its limits it can be divided into smaller parcels and the area of each deter-

mined separately; or the anchor can be placed inside the area provided the area of a CORRECTION CIRCLE is added to the result, which value is also given by the maker. Results correct to within one percent may be easily obtained with this instrument.

The Rolling Planimeter is not anchored to the drawing. It has a tracing point at the end of an adjustable pivoted arm which is fastened to a frame supported on two rollers. The whole instrument is rolled forward and backward in a straight line while the tracing point traverses the outline of the area. Results correct to a tenth of one percent are easily reached with this instrument. The rolling planimeter is much more expensive than the polar planimeter.

The Three-armed Protractor is used to solve the "three-point problem" graphically. It is similar to the ordinary protractor except that it has three arms, the middle one fixed at 0° of the circle and the other two movable. On either side of the 0° point angles can be laid off. If a point is located by two angles taken between signals *A* and *B* and between *B* and *C* this point sought can be located by laying off the two angles one on either side of the zero point of the protractor and then moving the instrument about on the plan until the plotted points *A*, *B*, and *C* lie on the beveled edges of their respective arms. When this position is found the center of the protractor locates the point sought.

9. Drawing Papers and Blueprinting

Drawing Papers for Working Plans are of all grades from manila detail paper, which costs about 10 cents per yard, to well-seasoned mounted paper, which will not change greatly with changes in moisture, and which costs from $0.75 to $2.00 per yd, depending upon the width and quality. Mounted paper comes in 10, 20 and 30-yd rolls and in widths of 36, 42, 58, 62 and 72 inches. The best grades of drawing papers can also be obtained in sheets, either plain or mounted; the common sizes are Demy, 15 by 20; Medium, 17 by 22; Royal, 19 by 24; Super Royal, 19 by 27; Imperial, 22 by 30; Double Elephant, 27 by 40; Antiquarian, 31 by 53 inches.

Transparent Paper, similar to bond paper, is used largely for studies and for temporary copies of plans. But for more permanent copies tracing cloth is used. Tracing paper comes in sheets of the same standard sizes as mounted paper; it also comes in 10 and 20 yd rolls and in widths 21, 24, 27, 30, 36, 40, 42, 48 and 54 inches, which cost from 5 to 25 cents per yard.

Tracing Cloth is a very uniform quality of cotton coated with a preparation to make it transparent. This material is manufactured in 24-yd rolls in widths 30, 36, 38, 42, 48 and 54 in, and costs from 35 to 85 cents per yd. Most tracing cloth has to be rubbed with powdered chalk before it will take ink. It has a smooth and a rough side; most draftsmen prefer to work on the rough side because it will take pencil lines and will not show erasures as much as the smooth side when process prints are made from them. In making a tracing of another tracing place white paper under the under tracing. From one tracing any number of process prints can be made.

Cross-section and Profile Papers can be procured with colored lines, both on heavy paper and on transparent paper or cloth. Cross-section paper is printed in orange, green, red, blue and black in sheets 16 by 20 and 17 by 22 in, and also on smaller sheets called "coordinate paper." Cross-section papers can be procured in 20 and 50 yd rolls with the engraving 20 and 30 in wide, and also in metric units 50 cm and 75 cm wide. Profile papers are printed in green, orange and red in three scales, called Plates A, B, and C. Plate A has 4 spaces horizontally to the inch and 20 vertically; Plate B has 4 horizontally and 30 vertically; and Plate C has 5 horizontally and 25 vertically. This material is mostly manufactured in 20 and 50 yd rolls with the engraving 9, 10 and 20 in wide and costs from 12 to 25 cents per yd on paper and 50 to 75 cents a yd on tracing cloth.

Blueprint Paper is the most common of the process papers. It is a white paper coated on one side with a solution which is sensitive to light. When fresh it has a yellowish green color. The following is a formula for this solution; solution (a) citrate of iron and ammonia 1 part (by weight) to 5 parts of water; solution (b) red prussiate of potash (recrystallized), 1 part (by weight) to 5 parts of water. These two solutions should be mixt in equal parts in the dark or in subdued light. The mixt solution is applied to the paper by means of a flat brush or sponge in a dark room or in subdued light, care being taken to coat the paper uniformly. Then the paper is hung in a dark room to dry and is stored there until used. The above coating will require an exposure of about 5 minutes in bright sunlight; for quick printing paper use a larger proportion of citrate of iron and ammonia. Prepared blueprint paper and cloth can be procured in 10 and 50 yd rolls in widths 24, 30, 36, 42 and 54 inches. The paper costs from 15 to 25 cents and the cloth 30 to 60 cents per yd. In making the print expose the tracing in a printing frame (with the blueprint paper under it) to the sunlight a proper length of time depending upon the sensitiveness of the paper and the brilliancy of the light. Then remove the paper and wash thoroughly in water. Great care should be exercised so that the tracing may not become wet, for it is impossible to eradicate such spots on tracing cloth.

Vandyke Paper is a sensitized paper which is printed in the same manner as blueprint paper, except that the tracing is put into the frame with the ink lines adjacent to the sensitive side of the paper. After an exposure of about 5 minutes in bright sunlight the print is washt for about 5 minutes in clear water and then put into a solution of half an ounce of hyposulphite of soda to a quart of water. It is left in this solution about 5 minutes and then put into clear water again for 20 minutes. This process gives a negativ of the drawing, a dark brown paper with white transparent portions where the ink lines of the tracing covered the paper in printing. This Vandyke print can then be used like a tracing to make other positiv prints on Vandyke paper, giving brown lines on white background, or to make blueprints with blue lines and white background. This paper can be obtained in 10 and 50 yd rolls and in widths 30, 36 and 42 inches; it costs from 20 to 30 cents a yard. BLACKPRINT PAPERS are used to some extent; they require a chemical developing bath, and give a black-line copy of the original.

Blueprinting Machines, equipped with a brilliant artificial light, so that the prints may be made at any time of day or night, are now somewhat common. In one of the best types several horizontal rollers are provided, with the light so arranged that as the tracing and blueprint paper pass from one roller to another the exposure is made. The speed of the machine is controllable and the length of the tracing that can be printed is limited only by the length of the roll of blueprint paper; thus long plans or profiles can be printed without the necessity of frequent splicing which is required with other types of printing frame; furthermore the print is of uniform color thruout. Some machines are provided with an apparatus for washing and drying the prints as fast as they come out.

10. Methods of Plotting Traverses

The Precision of Plotting depends upon the purpose of the survey. If it has been made simply to produce a map the angles may be laid off by means of a protractor and the distances scaled, but it is advisable to plot the triangulation system or the traverses on which the entire survey is based by a more exact method, leaving the details only to be plotted with protractor and scale. A traverse that is closed in the field should close on the plan; any error of closure in the plot indicates an error in scaling some distance or in laying off an angle, one or both. The bearing of each line of the traverse can always be computed from the bearing of some one line, assumed to be correct, and then any line can be extended till it meets the plotted meridian and its direction checked independently. Such checks are absolutely necessary if the traverse is not closed; and also the scaling of each line of the traverse

should be checked. But in the case of a closed traverse, if it closes on the plan, it is a good indication that it has been accurately plotted.

Plotting by Rectangular Coordinates is a common method for accurate results. It consists in referring all the angle points in the traverse to a pair of coordinate axes; for convenience these axes are the same ones used in calculating the area if the latter is required. The advantages of this method are that each point is plotted independently of the others, that all plotting is done by means of the scale only and that the plotting can be readily checked. First, compute the ordinate and abscissa, called TOTAL LATITUDE and TOTAL DEPARTURE, of each angle point of the traverse (Art. 16). If a meridian thru the most westerly point and a perpendicular thru the most southerly point are chosen for axes there will be no negativ ordinates or abscissas, for east departures and north latitudes are always regarded plus values. The total latitude of any point is the algebraic sum of all the latitudes of the intervening courses beginning with the most southerly point, and similarly for the total departure but beginning with the most westerly point. Construct a rectangle whose height equals the difference in latitude of the most northerly and southerly points and whose width equals the difference in departure of the most westerly and easterly points. This rectangle should be plotted very accurately by straight-edge and reliable triangles or by use of a beam compass, and checked by scaling the diagonals. To plot any point, lay off its total latitude on both the easterly and westerly sides of the rectangle from the southerly side. The point will lie on a line connecting these two points. Along this line scale off the total departure of the point, thus plotting the position of the point. Similarly all of the other points are plotted. To check a plot of this character scale the lengths of the traverse lines. This method is particularly useful in plotting closed traverses where the traverse encloses a field whose area is desired, for in this case the latitude and departure of the courses have to be computed in determining the area; but in the case of traverses that do not close the Tangent Method or the Chord Method is generally preferable.

The Tangent Method consists in laying off the angles by constructing at each vertex a right triangle whose base is 10 inches, or any other unit, and whose other leg is the natural tangent of the angle laid off in the same unit as the base. The traverse should first be

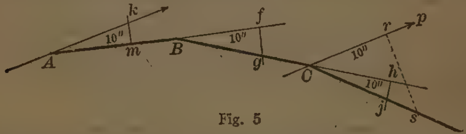

Fig. 5

roughly plotted with protractor and scale so as to determine its extent and shape and so that the first line of the plot can be laid in its proper place on the paper. This first line, AB (Fig 5), is then drawn to the proper scale. The forward end is extended, say 10 in, at which point f a perpendicular is erected by using triangles. On this perpendicular the natural tangent fg of the deflection angle is scaled off and line Bg is drawn. This gives the direction of the second line of the traverse on which the proper length BC is laid off, which is in turn extended 10 in further to form the base of a new right triangle in which the acute angle hCj will be the deflection angle measured at that station. To check such a traverse, calculate the bearing of all the lines referred to any line as a meridian and draw such a meridian line accurately by plotting the bearing angle kAm from the first line by the tangent method. Then draw a line Cp parallel to this meridian line so that it will intersect the course Cj whose direction is to be checked, and measure the angle between this course and the meridian by laying off a 10-inch base and erecting a perpendicular rs and scaling rs to determine the tangent of the angle pCj.

When the angle much exceeds 45° it is more accurate to plot the complement of the deflection angle rather than the angle itself, in which case a right angle is erected at the station point and this is used as the base of the right triangle; in such a case and thruout this method it is important that the triangles used shall be true.

In the Chord Method of plotting a traverse the angles are laid off by scaling off a chord across an arc so that it will subtend the required angle at the center of the arc. In detail, plot the first line AB to scale (Fig. 6). With B as a center swing an arc with radius of say 10 in. Extend AB till it cuts the arc at f, from which point scale off the chord fg along the arc in the proper direction. Chords for different angles have been published, but any natural sine table is practically as convenient, since the chord distance equals twice the

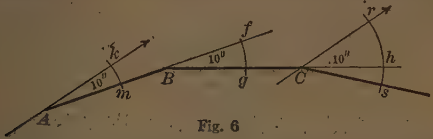

Fig. 6

sine distance of half the angle. The necessity for multiplying by 20 can be avoided by plotting the natural sine of half the angle directly with a scale of 10 ft to an inch, while the radius is scaled with a 20-ft scale. The direction of any course Cs may be checked by calculating the bearings of all the lines and measuring the angle between a meridian line rC and the course in question Cs by drawing an arc rs as in plotting the angle, measuring the chord rs and finding the angle rCs corresponding.

For Maps of Large Areas, such as a state or portion of a country, it is not sufficiently accurate to draw the meridians and parallels of latitude as rectangles. The most common form of projection used is the POLYCONIC, in which the surface of the sphere representing the earth is developed on a series of cones. In the U. S. Coast Survey Report for 1884, Appendix No. 6, p. 135, is a table giving coordinates of curvature for plotting the parallels of latitude and meridian lines for this projection.

11. Finishing the Plan

The Style of a Drawing and the data which it should give depend upon the use to which it is to be put. On every plan, however, there should be a complete title which should be a brief description of the drawing, the owner's name, the location of property, the scale, the date of the survey, the surveyor's name and address. Besides these, if the plan is a land plan, a meridian line should be drawn and some designation as to whether it is the true or magnetic meridian, the limits of abutting property and the names of their present owners. Notes are frequently added to give such further information as is necessary for a correct interpretation of the plan. All essential dimensions are lettered in their proper places. In the case of a land plan the area is usually expressed in square feet or acres and lettered in the middle of the parcel; the lengths of the sides are lettered in the middle of each line, and sometimes the bearings of each side or the angles between them are given, also any stone bounds, iron pipes or other physical boundaries which may exist are represented by abbreviations such as S.B. for stone bound, I.P. for iron pipe, sp. for spike, and stk. for stake. It is the practice of most surveyors to omit from the plan the calculated bearings of the lines or the angles. The bearings or angles are frequently of no use to the owner, but they are of great value to any other surveyor who may have occasion to rerun the lines of the property, and to fail to give them on the plan is in some instances at least withholding data for which the owner of the property has paid and to which he is rightfully entitled. Oftentimes the fact that the angles have been omitted from a drawing necessitates a resurvey which would have been unnecessary had the first surveyor given these data on his plan.

On Working Drawings and sometimes on finished plans the traverse line is drawn, usually as a colored full line, the angle points being shown either by very small circles the center of which marks the exact point or else by very short lines drawn through the angle points so as to bisect the angles. Triangulation stations are represented by small equilateral triangles with the point in the center, stadia stations by small squares, and other auxiliary stations by circles. The location of bench-marks is frequently represented by a small cross and figures thus, B.M. × 427.62.

The boundaries of the property and the physical features, such as streets or buildings, are usually drawn as full black ink lines. Shore lines are represented by black or by Prussian blue lines, and they should as a rule be the heaviest lines on the plan unless they are a very unimportant part of the plan. Whenever colors are used better results can always be obtained with water colors than with bottled inks. The following, burnt sienna, raw sienna, yellow ocher, scarlet vermilion, carmine, sepia, cadmium yellow, gamboge, Hooker's green, Prussian blue and indigo, are the colors commonly used by surveyors. All of these except gamboge may be used on tracing linen without danger of running, but if sun process prints are to be made from the tracing it will be found that Prussian blue and indigo will not print well. Some colors which do not give good prints may be made to do so by adding a little of some color to give it body (such as Chinese white or cadmium yellow) but not in quantity enough to change the original color.

Lettering should be plain but neat. Roman letters give the best appearance, but Gothic and Italic letters are often used on detail plans for construction. Lettering should be arranged to be read from the bottom or from the right-hand side of the sheet. Lines of titles should be centered and properly spaced. In large drafting rooms titles are often set up in type and printed on the map.

Sizes of Type. The thickness of a line of printer's type is measured by "points," one point being 1/72 of an inch, so that a line of six-point type is 6/72 or 1/12 of an inch thick. The larger print in this book is seven-point type, and the smaller is six-point type. Following shows the sizes up to twelve-point with names frequently used for them.

3 1-2 point or Brilliant	9 point or Bourgeois
4 1-2 point or Diamond	10 point or Long Primer
5 point or Pearl	11 point or Small Pica
5 1-2 point or Agate	12 point or Pica
6 point or Nonpareil	
7 point or Minion	
8 point or Brevier	

Notes which may be required are usually lettered with a plain letter like the Reinhardt style. Border lines are not as a rule used on construction drawings, but on finished land plans a single line drawn 3/4 in to 2 in from the edge of the drawing gives a good appearance if the line is not too heavy. A fine line just inside of a heavier one makes a good border line; it is the tendency of most draftsmen to make the border lines too heavy.

The meridian should be represented on all land plans as a full arrow if it is a true meridian and as a half arrow if it represents the magnetic meridian, the half arrow head being on the side toward the declination. It should be of simple design, not too conspicuous in size or weight of lines. It is good practice to show both the true and the magnetic meridian and to letter on the magnetic the declination at the place and time the survey was made.

On Mounted Plans accurate scaling may be difficult on account of shrinkage of the paper. It is hence well to draw lines parallel to the borders, forming sides of 100 or 1000 ft, so that allowance may be made in scaling after the paper shrinks.

To Clean the Drawing use a soft pencil eraser, a sponge eraser or dry bread crumbs. For tracings, gasoline or benzine will remove pencil marks without affecting the ink lines.

12. Topographic Signs

Conventional Signs, which have come to be used so generally that they are practically accepted as standard, are used on topographic maps to show certain physical features. A few of the most common are shown in Fig. 7.

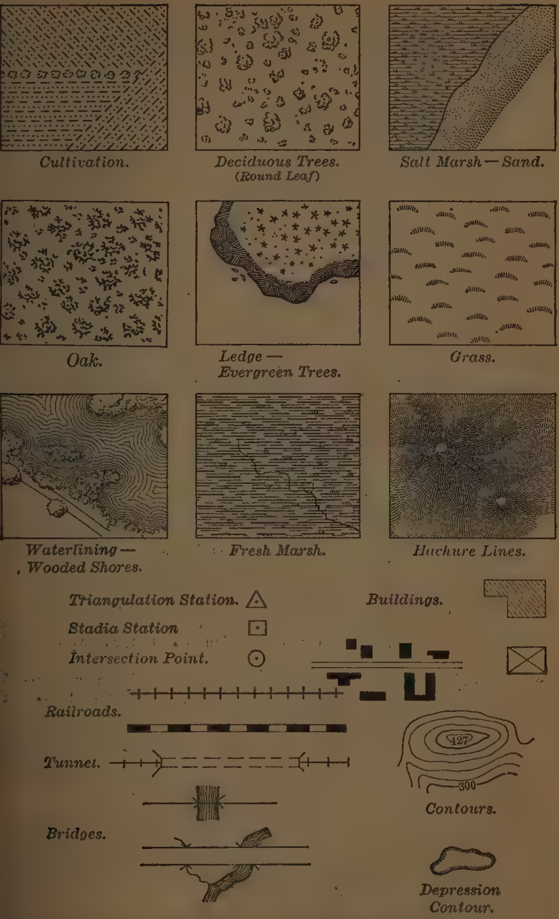

Fig. 7. Conventional Topographic Signs for Maps

They are executed with a free-hand pen except that the horizontal lines on the "salt marsh" signs and the straight lines in "cultivated land" are ruled, also fences, railroads and buildings. In all cases the symbol for grass and marsh land should be parallel to the bottom of the map. In the grass symbol the little lines which represent a tuft of grass should all appear to radiate from some point below the base of the group. It is usually composed of five or seven short lines, the tops of the lines forming a curve. They are made beginning at the left-hand side with a short dash and increasing in length up to the middle line, which is vertical, and then diminishing and ending with a short dash or dot at the right-hand side. In executing "water-lining" the first line outside the shore line should be a light full line drawn just as close to the shore line as possible, and should follow very carefully every irregularity. The next line should be drawn parallel to the first but with a little more space between them than was left between the shore line and the first line. Then the third line should be spaced a little farther out and so on; five to ten lines are sufficient. The aim should be to change the spacing of the lines so gradually that no one place can be observed where the spacing suddenly increases. Water-lining, fresh marsh and salt marsh symbols are often represented in Prussian blue. A distinction is generally made between evergreen and deciduous trees by using different symbols, and a further distinction is made by using a special symbol for oak trees. The symbol for an evergreen tree is made up of five or six short free-hand radial lines of uniform weight, giving a star-like appearance to each individual symbol. They are made of different weight of lines, some much lighter than others. To avoid the tendency of making them in rows it is well to make irregular groups of the symbols in different parts of the area to be covered and to fill in between these groups with smaller or lighter symbols. In the symbol for deciduous trees it is attempted to represent the plan of a tree in foliage, with a slight shading toward the lower right-hand side. These little symbols are placed regularly over the area if they represent cultivated trees and are given a smoother outline than those representing wild growth, which are scattered irregularly over the area. On some topographic maps, such as the U. S. Geological Survey maps, most of the topographic signs are represented in colors, while the U. S. Coast and Geodetic maps are all produced in black. When colors are used the trees should be green; the grass, a light green tint (Hooker's green No. 2); water, a light Prussian blue tint; cultivated land, yellow ocher.

Contour Lines are almost always drawn in burnt sienna water-color. Every fifth or tenth contour is usually represented by a line slightly heavier and a little darker in color. These may be drawn either with a Gillott's No. 303 pen or with a contour pen. In numbering the contours just enough numbers should be marked so that the elevation of any contour can be found without difficulty. The numbers on the contours should be small figures in burnt sienna. A common mistake is to make the contours too heavy so that they subordinate some of the more important features on the map.

Hachure Lines instead of contours are sometimes used to represent the shape of the surface of the ground (Fig. 7). Contour lines are first sketched lightly in pencil as guides in drawing the hachure lines, which are drawn normal to the contours from the summit downward in rows, each row touching the next preceding; the steeper the slope the heavier and shorter the line. All of the lines are equally spaced.

Sub-aqueous Contours are usually represented on hydrographic maps by dot-and-dash black lines, the shallowest contour having one dot between the dashes, the next contour in depth having two dots between the dashes and so on. In some cases contours representing fathoms of water are shown as single dots for the first fathom, two dots and then a pace for the second fathom, dots in threes for the next and so on.

LAND AND TOWN SURVEYING

13. Measurements of Length

Variations in Tape Measurements are due to erroneous length of tape, improper pull on the tape, careless plumbing, incorrect alignment, wind, changes in temperature, and sag of tape. The erroneous length of a tape can be discovered by comparing it with some standard; most cities have a standard which has been established with more or less care. The government Bureau of Standards at Washington will, for a nominal charge, standardize tapes. The tape should be tested at intermediate points as well as for its total length; its temperature and pull should be noted, and whether it is suspended or supported. If the tape is too long the measurements made with it are recorded too short, and the proper corrections should be added.

The Amount of Pull on a tape will have a very appreciable effect upon its length; ordinary light 100-ft tapes will stretch 0.01 to 0.02 ft with an increase of 10 pounds pull over the ordinary pull. This increase will be different for different tapes, so it is well to investigate it by fastening the ring end of a tape to a nail in the floor, and with the tape supported on the floor to exert different amounts of tension on the tape and measure them with a spring balance. In this manner the variations in length due to different tensions can readily be determined for any tape.

Plumbing is the operation of transferring any point on a horizontal tape to the ground under the tape by means of a plumb-bob. Inaccuracies due to this process can only be avoided by using great care. It should be borne in mind, however, that if a line must be measured by plumbing, more accurate results can be obtained by measuring downhill than uphill. Even when great care is taken in plumbing, so much error is introduced that for accurate results it is better to measure the inclined distance from the horizontal axis of the instrument to the station mark ahead, measure with the vertical arc the angle of inclination of the tape, and compute the horizontal distances by means of the versine of the angle of inclination; horizontal distance = inclined distance minus (inclined distance × versine angle). This computation can usually be done with sufficient accuracy by means of a slide rule. This method requires a set up of the transit at every other tape-length. Another way to measure these inclined distances is to tape directly from stake to stake in one tape-length; set the instrument up at every other stake and measure the vertical angle to a point above the two station points on either side of the instrument equal to the distance the horizontal axis is above the stake under the transit. This will give the inclination of the tape. Still another method is to measure the inclined distance from stake to stake and obtain the difference in elevation by leveling. When much of this inclined work is to be done it is more expeditious to use a 200-ft or 300-ft tape.

Alignment Errors are not likely to be large; lining in the tape by eye is for most measurements exact enough. If in measuring 100-ft tape-lengths a point is 1 ft out of line it will introduce an error of only 0.005 ft in that tape-length. The error due to poor alignment may be computed by the formula $c - a = h^2/2c$, where c is the tape-length or fractional tape-length, h is the offset from the correct line, and a is the correct length. Thus, if one end of the tape is on line and the other 0.8 ft off line the error in that one tape-length is $0.8^2/200 = 0.0032$ ft. Of course there will be a similar error in the next tape-length, making a total error of 0.0064 ft. The shorter the tape-length the greater is the error due to poor alignment.

Errors due to Wind can be avoided only by making measurements in calm weather, because it is impossible to determine accurately the amount of these errors under such variable conditions as exist when the wind is blowing.

Temperature Changes of the ordinary 100-ft steel tapes are about 0.01 ft per 16° F. Tapes are usually made to be of standard lengths at 62° F. The coefficient of expansion of steel is about 0.0000063 for 1° F. Special tapes are made of alloys whose coefficient of expansion is very small; "Invar" tapes of nickel steel have been used by the U. S. Coast Survey, the coefficient of expansion being about 1/25th that of ordinary steel tapes. By their use the temperature correction is eliminated except for very exact measurements. When ordinary steel tapes are used the temperature of the tape must be obtained and the temperature correction applied if exact results are desired. Small tape-thermometers are made for this purpose, but unless the thermometer is in contact with the tape and protected from the direct sunlight it will not register the tape temperature. Such an attachment is at best awkward. If the tape can be compared with a standard in sunlight and also in shade and the air temperature taken in the shade at both tests, then the correction to be made in measurements on account of temperature change can be readily determined if the air temperature in the shade is observed when the measurements are taken, for we have an empirical relation between the air temperatures and tape corrections. This is on the assumption, however, that the ratio between the tape temperature when lying in the sunlight and the air temperature in the shade is a constant.

Sag. Unless the tape is supported thruout its length, which is often impracticable, a correction must be applied for the sag of the tape due to its own weight, or, what is more commonly done, the pull is increased so as to stretch the tape an amount equal to the shortening due to sag. If supported at both ends it will hang in a curve of the form of a catenary, and the ends of the tape will therefore be less than its length apart, the amount of error depending upon the weight of the tape, the distance apart of the points of suspension, and the pull. With a 12-lb pull on a 100-ft ribbon steel tape supported at its ends the effect of sag will be about 0.01 ft. This may be found for any particular tape by the formula, Shortening due to Sag = $(L/24)\ (wl/t)^2$, where w = weight of tape in pounds per foot of length, t = tension in pounds, l = length of tape in feet between supports, and L = total length of tape in feet. The result will be in feet.

If a tape can be compared in a suspended condition with a standard the shortening due to different amounts of sag may be determined, and then the proper approximate corrections applied to any measurements by judging the amount of sag.

With a steel tape, if ordinary care is taken in plumbing and in alignment and with rough corrections made for temperature, an accuracy of 1 in 5000 can easily be obtained. For accuracy greater than 1 in 10 000 it is necessary to apply corrections for pull, temperature, alignment, and sag.

In all measurements it is of utmost importance to distinguish between errors which tend to balance and those which continually accumulate, the latter being far more important. For those which tend to balance, the number of errors which will probably remain uncompensated, according to the Method of Least Squares, will be the square root of the total number of opportunities for error. For example, if a 100-ft tape is 0.02 ft too long an accumulative error of about 1 ft will be made in measuring a line a mile long. If, on the other hand, the tape-lengths are not marked closer than 0.05 ft, the total error made by this compensating error of 0.05 will only be $\sqrt{52} \times 0.05 = 0.36$ ft.

14. Magnetic Variations

Surveys with Compass and Chain cannot be relied upon to be closer than about 1 part in 500, since the bearings are read only to the nearest quarter of a degree. This method is hence adapted only to rough surveys of woodlands

where the land is cheap and where there is little danger from local attraction of the needle.

For Transit and Tape Surveys it is desirable in some instances to determine the direction of the true meridian by observations on the sun or on Polaris (the "north star"), as described in Art. 44. In many cases exact bearings of the lines are not needed and it is sufficiently accurate to determine the magnetic bearings by reading the needle. The approximate true bearings may then be determined by applying the DECLINATION of the needle to the observed bearings; but in most transit and tape surveys in cities the magnetic bearings are so unreliable that when taken at all they are used merely for the purpose of plotting a needle on the map to show approximately the "points of the compass."

The Declination of the needle is the angle which it makes with the true meridian. The needle rarely points in the true meridian; if it points east of the true meridian it is called an east declination. The declination of the needle at places east of Ohio and the Carolinas is West (in 1915), being a maximum of 22½° in Maine; in the western part of the country it is East, being a maximum of 24° in the state of Washington. The average declination in Alaska is 34° E, Porto Rico 2½° W, Canal Zone 4½° E, Hawaiian Islands 10½° E, Philippine Islands ¾° E. The declination continually changes; these changes are called VARIATIONS.

The Secular Variation is a long slow swing, periodic in its character, and covering many years. In the United States all east declinations are now (1915) gradually decreasing and all west declinations are increasing at the rate of about 3 min per year.

The Daily Variation consists of a swing from the extreme easterly position at about 8 A.M. to its most westerly position at about 1.30 P.M. It is in its mean position at about 10 A.M. and 5 P.M. This daily variation is from 5 to 15 min of arc.

The Annual Variation (about one minute per year) is so small that it need not be considered in surveying work.

Irregular Variations, due to so-called magnetic storms, are uncertain in character and cannot be predicted. These variations are sometimes large.

Isogonic Lines are lines drawn on a map so as to connect all places where the declination of the needle is the same at a given time. The U. S. Coast Survey has constructed isogonic charts of the United States and these can be obtained from Washington. These charts do not give results with great precision, but are useful in finding approximate values of the declination. They are prepared by plotting upon the map the observed declination at magnetic stations located thruout the country and interpolating results at intermediate places. Observed declinations at these interpolated places frequently show results quite different from those given on these isogonic charts, so that it is necessary whenever the declination of the needle is desired with precision to make observations for finding the true meridian.

Changes in Declination make it necessary in rerunning old lines to modify the given bearings an amount equal to the change in declination which has taken place since the lines were first run. To determine the declination for some past date, records of the U. S. Coast Survey are of assistance provided they had a magnetic station in the vicinity of the place in question. A better way of finding the difference in declination is to take the magnetic bearing of any well-defined line of the old survey, such as between two identified stone heaps, and compare the present bearing of this line with its original bearing.

An Isogonic Chart for Jan. 1, 1915. copied from one issued by the U. S. Coast and Geodetic Survey, will be found on the next page. The full lines are the isogonic lines. or lines of equal magnetic declination, the heavy one marked 0° passing through the places in the U. S. where the north end of the magnetic needle points to the true north. East of the 0° line the north end of the magnetic needle points west of true north, west of that line it points east

Isogonic Chart of United States for 1915

of true north. Thus, at Boston, Mass., the declination in 1915 was about 14° W, and at Helena, Mont., it was about 21° E.

These isogonic lines are constantly shifting; the 0° line is moving westward at a rate between 1′ and 2′ per year. On the chart two parallel lines are seen marked 'Annual Change 0'; at all places on that double line there was no change in declination in 1915, at all places east of it the west declination was increasing, at all places west of it the east declination was decreasing. Thus, near Denver, Colo. the east declination decreased about 3′ in 1915.

In 1915 the U. S. Coast and Geodetic Survey published 'Distribution of the Magnetic Declination in the U. S.' which gives 12 tables showing the magnetic declination as observed for many places since 1750. The following is one of these tables which gives the secular change of the magnetic declination for eight places:

Secular Changes of Magnetic Declination

Region.	New York, east.	New York, middle.	New York, west.	North Carolina, east.	North Carolina, middle.	North Carolina, west.	North Dakota, east.	North Dakota, middle.
Place...	Albany	Elmira	Buffalo	Newbern	Greensboro	Asheville	Jamestown	Bismarck
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
1750	7 41 W	4 40 W	......	0 18 W	1 14 E	......	......	......
1760	6 59	3 57	......	0 18 E	1 50	......	......	......
1770	6 23	3 18	......	0 50	2 24	......	......	......
1780	5 56	2 46	1 52 W	1 27	2 53	......	......	......
1790	5 40	2 24	1 24	1 35	3 14	......	......	......
1800	5 34	2 13	1 08	1 44	3 26	4 06 E	......	......
1810	5 40	2 13	1 01	1 44	3 29	4 12	......	......
1820	5 56	2 24	1 08	1 35	3 23	4 09	......	......
1830	6 23	2 46	1 24	1 16	3 07	3 57	14 02 E	......
1840	6 59	3 18	1 52	0 50	2 43	3 35	14 12	......
1850	7 45	3 57	2 26	0 17 E	2 12	3 07	14 12	16 23 E
1860	8 31	4 46	3 10	0 19 W	1 38	2 35	14 02	16 18
1870	9 10	5 23	3 49	1 00	1 00	1 57	13 44	16 04
1880	9 57	6 16	4 40	1 40	0 20 E	1 17	13 15	15 38
1890	10 18	6 57	5 22	2 16	0 17 W	0 41	12 32	14 58
1900	10 56	7 32	5 57	2 52	0 51	0 09 E	12 09	14 40
1905	11 11	7 50	6 11	3 08	1 04	0 02 W	12 17	14 50
1910	11 37	8 12	6 32	3 25	1 19	0 13	12 25	15 00
1915	12 05 W	8 37 W	6 53 W	3 42 W	1 34 W	0 23 W	12 32 E	15 09 E

These figures show that the variation in different decades is not the same and that the values of the declination are represented by a curved line. The curve of sines approximately represents these values for a given place; thus for Bethlehem, Pa., the west declination is closely given by $D = 5°.27 + 3°.05 \sin(1°.46n + 38°.2)$ where n is the number of years since Jan. 1, 1900; for that place the formula indicates that the west declination had a minimum value of 2°.2 about 1812 and will have a maximum value of 8°.3 about 1936.

15. Traversing with Transit and Tape

The Transit and Tape Method is commonly used for running traverses. The instrument is set up at every angle point in the traverse and the angle measured with the transit. The distance between the angle points is measured with the tape. The distances are sometimes measured and recorded as

separate distances from angle point to angle point, the angle points being designated by letters or by numbers. Another method is to measure the entire traverse from the first point, which is called "Station 0," continuously thruout the traverse, each 100-ft point being called a station. If the second transit point is 672.43 ft from Station 0 it is called "Sta. 6 + 72.43," if the next point is 350 ft farther it would be recorded "10 + 22.43" and so on, so that the station of any point is its distance from the point of beginning of the traverse measured along the traverse.

The Precision with which the measurements should be taken will depend upon the object of the survey. If it is a city survey of valuable property the distances would be measured to the nearest hundredth of a foot, and the angles to 15 seconds; while in a farm survey the angles to the nearest minute and the distances to a tenth of a foot or in some cases even to a foot (by the stadia method) is exact enough. Transits used for general surveying work read to minutes or to half-minutes. It is therefore necessary in accurate surveys to measure the angles by repetition, as explained at the end of this article, to obtain results consistent with the accuracy of the tape measurements. A proper appreciation of the relation of distances and angles may be had if it is borne in mind that 0.03 ft a hundred feet away subtends one minute.

The Angles Measured may be the deflection angle (the angle between the last course produced and the next course), the interior angle, or the azimuth angle. In measuring a deflection angle the telescope has to be inverted, and any error in the line of collimation will therefore be introduced into the deflection angle. This error may be eliminated if the angle is measured first with the telescope direct and then, without changing the circle reading, invert the telescope and "double" the angle; half the final angle is the correct angle. To avoid this necessity of reversing the telescope and repeating the angles some surveyors prefer to measure the interior angles directly.

For Azimuth Angles the instrument is first set up at *A* with the circle set at 0°, and the lower clamp loose. The telescope is turned so as to point toward magnetic south or true south (whichever is desired for a reference direction) and lower motion clamped. Then the upper plate is unclamped, *B* sighted, and the angle read in a clockwise direction. This gives the azimuth of the line *AB*, an azimuth angle being the total angle read in a clockwise direction from the south around to 360°. The instrument is then taken to *B* so as to obtain the azimuth of *BC*. Invert the telescope and backsight on *A*, the vernier remaining at the same reading it had when azimuth *AB* was read, clamp the lower motion, turn the telescope to its direct position, loosen the upper clamp and sight *C*. This will give the azimuth of *BC* referred to the same meridian as *AB*. Evidently this method does not eliminate any error in the line of collimation adjustment. By the azimuth method, when the survey is about to be closed the azimuth of the first line *AB* can be obtained by setting up again on *A* and using the last course as a backsight. The difference between the first determination of the azimuth of *AB* and its final determination gives directly the total error in the angular work.

As a check against large errors in the angles the magnetic bearing of each line should be read when practicable and compared with the calculated bearing. The calculated bearing of the first line is assumed to be its observed bearing; the calculated bearing of any line is obtained from the calculated bearing of the line next preceding combined with the measured angle between the two lines.

Deflection Angles should be recorded as R (right) or L (left). The algebraic sum of the deflection angles of a closed traverse should equal 360°. If the interior angles are read, the sum of all the interior angles of a closed traverse should be $(n - 2) \times 180°$, where n is the number of lines.

The Error of Closure of a traverse in which the angles are measured to the nearest minute and the distances to a tenth of a foot should not exceed 1/5000. But where the angles are measured to 15 seconds and the distances to hundredths of a foot, results with an error of not more than 1 in 20 000 to 40 000 may be obtained.

Checks on Traverses. In traverses which enclose an area there is a mathematical check on the distances provided the angles are correct, as shown in Art. 16. But in traverses which do not close there is no check on the distances other than by remeasurement of the lines (preferably in the opposite direction from the first measurement), except by cut-off lines which form closed traverses of portions of the survey. The angles can be checked by determining the true bearing of the first line of the traverse by solar or stellar observation, and whenever it is desired to check the angles the true bearing of any course can be determined by another meridian observation. The meridian can be determined in this way to the nearest minute.

To Measure an Angle by Repetition, set up the transit at A, set the verniers at $0°$, sight B, and clamp lower clamp. Loosen upper clamp and sight C, read and record the angle. Leaving the two plates clamped together, unclamp the lower clamp and sight B, unclamp upper clamp and sight C, and read and record the angle. Half this angle should check the first angle read, and the result obtained is more exact than the first angle. It is evident that by repeating this process with a one-minute instrument for six times and dividing by 6 an angle to ten seconds can be obtained. This method of repetition might be carried even further, but ordinary transits are not good for results much closer than ten seconds.

This method of repetition can be readily applied to laying off an angle. The angle is first laid off and a point set, and then the angle which has been laid off is measured by repetition as described in the previous paragraph. If it is found to be in error, say 20 sec., the point on the stake is moved in the proper direction a distance equal to the distance from the point set to the instrument × tan 20 sec. = 0.0001' = 0.01' per 100'

16. Areas of Fields

Corrections of Field Notes are made before computation for area is begun. Errors in tape measurements due to erroneous length of tape are corrected. Errors in angles, provided they are not large enough to indicate mistakes, are eliminated by "balancing the angles," which means altering the value of those angles which were taken from short sights or those angles where the error is most likely to lie. The calculated bearing of each course is computed starting from one course whose bearing is either known or assumed.

The Double Meridian Distance Method of computing an area is as follows: The data are usually tabulated as shown in the computation below. The LATITUDE of a course equals the distance times the cosine of the bearing; the DEPARTURE is the distance times the sine of the bearing. North latitudes and East departures are regarded as positiv values and South latitudes and West departures are negativ. After these have been computed the error in latitude and in departure is found, which is the difference between the plus and the minus latitudes and the plus and minus departures. If these errors are not large enough to indicate a mistake in the measurements or computations then they are distributed among the latitudes and departures according to the following rule if it is a transit and tape survey. The correction applied to the $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ as the $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ of that course is to the sum of all the $\left\{\begin{matrix}\text{latitudes}\\\text{departures}\end{matrix}\right\}$ (without regard to algebraic signs). Any knowledge of difficulties met in the field which would lead the surveyor to suspect that the error lay in certain lines should take precedence over this rule. Furthermore it is more probable that on account of sag of the tape and small obstacles on the line the recorded distances are too long rather than too short, and therefore it is not good practice

to apply the above rule when it lengthens any of the distances. In the example here given, however, the rule is rigidly followed in balancing this survey simply to show its application. The algebraic sum of balanced latitudes and the departures should be zero.

The next column to compute is the double meridian distance (D.M.D.) of each course. The D.M.D. of the first course equals the departure of the first course. The D.M.D. of any other course equals the D.M.D. of the course preceding plus the departure of the preceding course plus the departure of the course itself. The D.M.D. of the last course should be numerically equal to the departure of the last course but should have the opposite algebraic sign. The algebraic signs must be carefully observed not only in computing the D.M.D's. but in all of this computation for area. If the D.M.D's. are computed beginning at the most westerly point in the traverse all the D.M.D's. will be plus. The last column of positiv and negativ double areas is found by multiplying each D.M.D. by its corresponding latitude. Half the algebraic sum of the double areas gives the area of the field.

Computation for Area by Double Meridian Distance Method

Bearings.	Dist. Feet.	Lats.	Deps.	Balanced Lats.	Balanced Deps.	D.M.D.	Double Areas.
N 56° 25′ E	540.91	+ 299.20	+ 450.62	+299.19	+450.66	+ 450.66	+134 833
S 64° 27′½ E	198.44	− 85.56	+ 179.05	− 85.56	+179.06	+1080.38	− 92 437
S 67° 50′ E	212.46	− 80.16	+ 196.76	− 80.16	+196.78	+1456.22	−116 731
S 68° 34′ E	98.13	− 35.86	+ 91.34	− 35.86	+ 91.35	+1744.35	− 62 553
S 29° 17′ E	186.75	− 162.89	+ 91.35	−162.90	+ 91.36	+1927.06	−313 918
S 61° 26′½ E	651.70	− 311.55	+ 572.41	−311.57	+572.45	+2590.87	−807 234
S 66° 50′ W	910.92	− 358.37	− 837.46	−358.39	−837.40	+2325.92	−833 583
N 45° 21′ W	1046.25	+ 735.28	− 744.32	+735.25	−744.26	+ 744.26	+547 217
	3845.56	+1034.48	+1581.53				2)1 544 406
		−1034.39	−1581.78				772 203
		error +0.09	−0.25				sq. ft.

This result should be roughly checked by determining the area by use of the planimeter (Art. 8) or by dividing it into triangles, scaling the bases and altitudes and computing the area. The agreement between the sum of the + and − latitudes and + and − departures is a check on the multiplication of the distance by the cosine and sine of the bearing. The D.M.D's. are checked as shown above. But there is no check on the double areas. A good exact check on the area is to compute from the latitudes the double parallel distances (D.P.D.) just as the D.M.D's. were computed from the departures, and then obtain double areas by multiplying each D.P.D. by its departure. If l is the error in latitudes and d the error in departures, the error of closure of the survey equals $\sqrt{l^2 + d^2}$. Thus, the error of closure of the survey shown above is 0.266 ft, and

$$\frac{\sqrt{0.09^2 + 0.25^2}}{3845.56} = \frac{0.266}{3846} = \frac{1}{14\,500} = \text{relative error of closure}$$

Sometimes it is not possible to follow the perimeter of the field with the traverse, in which case the fractional areas to be added to or subtracted from the area of the traverse are computed by dividing them into triangles, rectangles or trapezoids.

Large mistakes made in the fieldwork will be detected in the computation by the D.M.D. method. If the latitudes and departures do not balance within reasonable limits, the error in departure divided by the error in latitude equals the tangent of the bearing of the line which would represent the error of closure, and if only one mistake has been made in measuring the distances it probably lies in a line having this angle for its bearing.

Compass Surveys should be balanced by the following rule, because the errors are more likely to be due to the rough results obtained in the angles than in the distances. The correction to be applied to the $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ as the length of the course is to the perimeter of the field.

Area of a Field by Rectangular Coordinates. Where the coordinates of the corners of a property are known the coordinate method of computing its area is more direct than the D.M.D. method. Let x_a, x_b, x_c, etc., and y_a, y_b, y_c, etc., represent the abscissas and ordinates of points A, B, C, etc., of the same area computed above by the D.M.D. method. By the rectangular coordinate method the area is found as follows: Every abscissa is multiplied by the difference between the following and preceding ordinates, always subtracting the following from the preceding. The algebraic sum of these products divided by 2 gives the area. Exprest as a formula, this rule is

$$\text{Area} = \tfrac{1}{2}\left\{x_a(y_b - y_h) + x_b(y_c - y_a) + x_c(y_d - y_b) + x_d(y_e - y_c) + \text{etc.}\right\}$$

Below is the computation by coordinates of the same area as that above. The origin of coordinates is 100 ft W and 2000 ft S of the initial point.

Computation for Area by Coordinate Method

Sta.	x	y	Diff. bet. Adjacent y's.	Double Areas. +	Double Areas. −
A	100.00	2000.00	−1034.44	....	103 444
B	550.66	2299.19	− 213.63	....	117 637
C	729.72	2213.63	+ 165.72	120 929	....
D	926.50	2133.47	+ 116.02	107 493	....
E	1017.85	2097.61	+ 198.76	202 308	....
F	1109.21	1934.71	+ 474.47	526 287	....
G	1681.66	1623.14	+ 669.96	1 126 645	....
H	844.26	1264.75	− 376.86	....	318 168
				2 083 662	539 249

Area = ½ (2 083 662 − 539 249) = 772 206 sq. ft.

17. Crooked Boundaries

Curved Boundaries, such as brooks, are often located by measuring the perpendicular offsets from the traverse line which has been run alongside of the brook. The curvature of the boundary may be so flat and uniform that it will be feasible to take the offsets at regular intervals, while in cases where the boundary changes in direction abruptly it is necessary to take the offsets only at the points where these changes occur. In either case there will be the areas of several trapezoids to compute to find the area between the traverse line and the boundary, but if the offsets are taken at equal intervals then the computation may be made by one of the following methods.

By the Trapezoidal Rule Area $= \tfrac{1}{2}\, d\,(h_e + 2\,\Sigma h + h_e')$, where d = common interval between offsets, h_e and h_e' = end offsets of the series of trapezoids, and Σh = sum of intermediate offsets. This rule assumes that the boundary is a straight line between adjacent offsets.

Simpson's One-third Rule assumes that between adjacent offsets the boundary is a curve (a parabola). By this rule Area $= d/3\,(h_e + 2\,\Sigma h_{\text{odd}} + 4\,\Sigma h_{\text{even}} + h_e')$, where d = common interval between offsets, h_e and h_e' = end offsets of the series, $2\,\Sigma h_{\text{odd}}$ = twice the sum of the odd offsets except the first and last (the 3rd, 5th, 7th, etc.); $4\,\Sigma h_{\text{even}}$ = four times the sum of the even offsets (the 2nd, 4th, 6th, etc.). For this rule to apply there must be an odd number of offsets; if there is an even number, compute the area of one end trapezoid separately.

To Straighten a Crooked Boundary is to run a straight line cutting the crooked boundary so as to make the lots on either side of the straight line have the same areas as they have with the crooked boundary for the division line. To do this, first run a trial line AB (Fig. 8), then measure proper offsets to the crooked boundary and compute the areas X, Y and Z on each side of AB between it and the crooked boundary. The sum of X and Z should equal Y, for $X + Z$ is the amount taken from Lot M, and Y is the amount taken from Lot N. If $X + Z$ does not equal Y the difference is the amount the trial line has taken from one lot more than it has taken from the other, so this whole difference must be returned. For example, suppose the original area of Lot M = 50 000 and Lot N = 40 000. When AB is run it is found that the area of X = 300, Y = 1000, and Z = 500, that is, the trial line AB has made Lot M = 50 000 − 800 + 1000 = 50 200, and Lot M = 40 000 − 1000 + 800 = 39 800. An area 1000 − (300 + 500) = 200 must therefore be taken from Lot M and added to Lot N by running the final line AD so as to make the area ADB equal to 200.

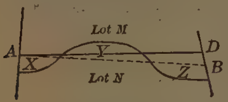

Fig. 8

18. Old Lines

In Rerunning Old Property Lines the surveyor must first determine where the original boundaries of the property lie and then survey those boundary lines. He should not attempt to correct the original lines even tho he may be sure that errors exist in them. He must first of all find the physical evidence of the location of the boundaries, and failing in this he should base his judgment as to their location on such evidence as occupancy or dimensions given in deeds or the word of competent witnesses. It must not be assumed that a boundary is missing because it is not at once visible. Stone bounds are often buried two or three feet deep; the top of a stake soon rots off, but evidences of the existence of the stake are often found many years after the top has disappeared, and the supposed location should be carefully dug over to find traces of the old stake.

In Interpreting a Deed it is assumed that it was intended to convey property the boundaries of which will form a closed traverse. Therefore, if it is found that the omission of a whole chain-length or the reversing of the direction of a bearing will make a deed description close, this change in dimensions may be made, for it is assumed that the description is of a closed field. Where the record of the original survey does not close, the deeds of adjoining property may be of assistance. Where artificial features are mentioned as boundaries, these always take precedence over the recorded measurements or angles, but these marks must be mentioned in the deed in order to have the force or authority of monuments. When the area does not agree with the boundaries as described in the deed the boundaries control. All distances unless otherwise specified are to be taken as straight lines; but distances given as so many feet along a wall or highway are supposed to follow these lines even if they are not straight. When a deed refers to a plan the dimensions on this plan become a part of the description of the property.

Legal Boundaries. Where property is bounded by a highway the abutters own to the center line, but where it is an accepted street each abutter yields his portion of the street for public use; if, however, the street is abandoned the land reverts to the original owners. If a street has been opened and used for a long period, bounded by walls or fences, and there has been no

protest regarding them, these lines usually hold as legal boundaries. In the case of a line between private owners acquiescence in the location of the boundary will, in general, make it the legal line; but if there is a mistake in its location and it has not been brought to the attention of the interested parties or the question of its position raised, then occupancy for many years does not make it a legal line.

Where property is bounded by a non-navigable stream it extends to the thread of the stream. If the property is described as running to the bank of the river it is interpreted to mean to the low-water mark unless otherwise stated. Where original ownership ran to the shore line of a navigable river and the water has subsequently receded, the proper subdivision is one that gives to each owner along the shore his proportional share of the channel of the river; these lines will therefore run, in general, perpendicular to the channel of the stream from the original intersection of division lines and shore lines.

A more complete statement of the principles mentioned above, particularly with reference to the U. S. Public Land Surveys, will be found in an address on "The Judicial Functions of Surveyors," by Chief-Justice Cooley of the Michigan Supreme Court. See Proceedings Mich. Association of Engineers and Surveyors, 1882, pp. 112–122.

Should all evidence of artificial boundaries of the property be missing the surveyor will have to use the deed description as the best evidence. Where the directions of the lines are given as magnetic bearings it is necessary to first determine the declination of the needle at the date of the survey. The declination should be stated in the deed or on the original plan, but it seldom does appear in either. If the date can be established the declination for that year and place may be obtained from the records of local surveyors or from past U. S. Coast Survey records. If one line can be identified as a boundary the difference between its present magnetic bearing and its original bearing gives the difference in declination directly, and all the rest of the deed lines can be run out by correcting all the bearings by this amount. The chain used in the original survey may have been of different length from the one now used; this can be readily determined by measuring the length of any of the well-defined lines of the property.

19. Obstacles and Inaccessible Distances

A Random Line is sometimes required on account of obstacles on line. If it be required to run the line AB (Fig. 9), neither point being visible from the other, a random line AX can be run. Measure the perpendicular distance BC and also AC. The length of AB may then be computed and any desired point may be set on the line AB as follows. Suppose a point D is to be set on AB and that a perpendicular from D meets AC at E, then $AE = AD \times AC/AB$, and $ED = BC \times AD/AB$. By this method, then, it may not be necessary to run out and actually measure the line AB. The angle CAB can be found from CB and AC.

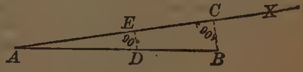

Fig. 9

If a straight line such as AX cannot be run, a traverse composed of several straight lines and angles may be made to connect A and B, and using AB as the closing side of a closed traverse, its length and direction may be computed (Art. 16), and also the coordinates of any point in the traverse may be found referred to AB so that distances may be laid off from the traverse which will define any desired points on AB. This traverse method, since it involves the measurement of angles, will not lend itself as readily to accurate results as the case first described where the random line is a straight line. In running a straight line where it is necessary to produce the line by reversing the telescope it should always be done by taking the mean result of a double reversal, the telescope being erect in taking the first backsight and inverted when taking the second backsight, so as to eliminate any error in the line of collimation.

An Equilateral Triangular Traverse about an obstacle is a special case of the above method. Let the obstacle be on the line AB (Fig. 10); the instrument is set up at C, an

angle of 120° is laid off, and a stake set a sufficient distance away at D, and CD is measured. The transit is then set up at D and an interior angle of 60° laid off so as to run a line that will cut AB; this line is made equal to CD and stake E is set. With the instrument at E an angle DEB is laid off equal to 120° which should be sighting along AB if the work has been done accurately. Evidently $CD = ED = CE$. This method is weak in accuracy owing to its dependence upon angle measurements. A slight error in any angle will introduce an appreciable error in CE. If CD is short the error in CE is small, but the error in producing EB by an angle laid off from the short side ED is likely to be large.

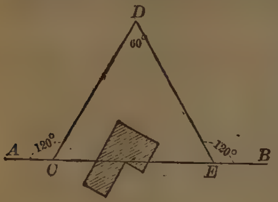

Fig. 10

Running Parallel Line Past Obstacle. One of the most exact ways of producing a straight line past an obstacle of limited size, such as a house, is to run an offset parallel transit line past the obstacle as follows. The instrument is set up at C (Fig. 11) and a right angle ACC' laid off with the transit. CC' is made any convenient distance which will bring the auxiliary line beyond the obstacle. Similarly point A' is set opposite point A, and sometimes a second point B' opposite B. These points A' and B' need not be set by means of a transit set up at A and B if AA' is short. The instrument

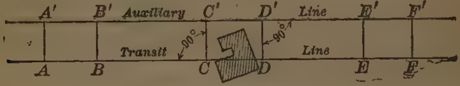

Fig. 11. Running Parallel Line past Obstacle

is then set up at C' and backsighted on A', the sight is checked on B', the telescope inverted, and points D', E' and F' set on line. Leaving the telescope inverted, another backsight is taken on A', and the process repeated to eliminate any error in the line of collimation. Then the transit is moved to point D', and a right angle turned off, and point D set, the distance $D'D$ being made equal to $C'C$. Then by setting up at D and sighting ahead on F ($FF' = DD'$), and checking on point E ($EE' = DD'$), the transit line is again run forward in its original location. The distance $C'D'$ is carefully measured, which gives the distance CD, and thus it appears why it is so necessary that the lines CC' and $D'D$ shall be laid off at right angles by means of the transit. The other offsets AA', BB', EE' and FF' are not in any way connected with the measurement along the line; they simply define the direction of the line, so that if convenient it is often only necessary to show these distances as swing offsets for the transitman to sight on. The swing offset is given by swinging a tape in a horizontal arc about A as a pivot. The zero point of the tape is held at A, and at the distance AA' out on the tape a pencil is held vertically on which the transitman sights when the pencil appears to be swung out farthest from A so that his line of sight will be tangent to the arc. The offsets BB' and EE' are not absolutely necessary, but they serve as desirable checks on the work, and in first-class surveying they should not be omitted. For obvious reasons the offsets AA' and FF' should be taken as far back from the obstacle as is practicable.

Should the house be in a hollow so that it is possible to see over it with the instrument at A, the point F, or a foresight of some sort, should be set on line beyond the house to be

used as a foresight when the transit is set up again on the original line. The distance may be obtained by an offset line around the house. Sometimes it is possible to place exactly on line on the ridgepole of the house a nail, which gives an excellent backsight when extending the line on the other side of the building.

If the building has a flat roof and it is possible to set a point on the roof exactly on line, move the instrument to this point on the roof, and prolong the line in this way. Under these conditions the transitman will have to be extremely careful in the use of his instrument on account of its insecure foundation. If he walks around the transit he will find that it affects the level bubbles and the position of the line of sight; it is therefore well for him if possible to stand in the same tracks while he backsights and foresights. Sometimes two men, one in front and one behind the transit, can carry on the work under these conditions more accurately and conveniently. This method insures an accurate prolongation of the line, but the distance through the building must be found by an offset method, by plumbing from the edge of the flat roof, or by inclined measurements.

To Measure an Inaccessible Distance where the line is visible, as across a pond, several methods may be employed:

(1) Lay off from the transit line AC (Fig. 12) a line AB which passes by the end of the pond so that it can be taped and set stake B; then set up the instrument at B and lay off $ABC = 90°$, point C being obtained by intersecting the main transit line. Measure angle CAB and side AB, from which $AC = AB/\cos CAB$, or better, $AC = AB + AB$ exsec CAB. The measurement of the angle at C and of CB will serve as checks.

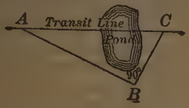

Fig. 12

Fig. 13

(2) With the instrument at A (Fig. 13) and a swing offset of 100 ft from C (some point on the main traverse line on the other side of the pond) measure the angle between CA and a tangent AB to the swing offset. $AC = 100/\sin CAB$.

(3) Any line AB (Fig. 14) may be laid off along the shore of a river, and a point C set on the main traverse line across the river, measure angles A and B, and C as a check, and from AB as a base compute AC by trigonometry. If AB is run perpendicular to CA and made some number of hundred feet long the computation is greatly simplified.

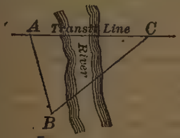

Fig. 14

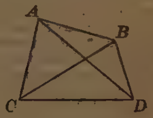

Fig. 15

To Obtain the Distance between Two Inaccessible Points A and B (Fig. 15) by observations from two accessible points C and D, measure DC and angles ADC, ADB, ACB and BCD. Compute CB in triangle CBD, AC in triangle ACD, and then in triangle ACB compute AB.

To Obtain the Inaccessible Distance AB (Fig. 15) between two accessible points by observation on two inaccessible points C and D when distance CD is known. Measure angles CAD, DAB, CBD and ABC. Assume $AB = 1$. Then compute CD by same process as described in the above paragraph. This gives a ratio between AB and CD, and since CD is known, the actual length of AB may be computed.

20. City Surveying

Staking Out Line and Grade for streets, curbs, sewers, and pavements is constantly done by a city engineering department. This class of work as a rule calls for lines and grades to the nearest hundredth of a foot. For this reason it is customary to use transits with the horizontal arcs graduated so as to read 30″, 20″ or even 10″, and to use steel tapes graduated to hundredths of a foot. In some cases the spring balance and the thermometer should be used so as to make the proper tension and temperature corrections (Art. 13).

A Standard of Length is established as a rule by carefully transferring the length of some other standard by means of different tapes and under different weather conditions, or by means of tapes which have been standardized by the U. S. Bureau of Standards. This standard should be placed where it will not be exposed to the direct rays of the sun. When the tape is tested it should be stretched out at full length beside the standard and left there until it acquires the same temperature as the standard before the comparison is made.

Street Lines are usually marked by monuments, but the best practise requires that besides these monuments, which may become misplaced, accurate offsets to the underpinning of buildings along the line shall be measured and recorded so that if the monuments become disturbed, due to building operations or paving of the sidewalks, they can readily be replaced in their exact location. These monuments are usually ordinary stone bounds 3 or 4 ft long and 4 to 8 in square on top. The bound should be long enough so as to rest on ground below frost. A drill-hole in the center of the top marks the point; a more exact method of marking is to fill this drill-hole with lead and drive a very small copper tack in the lead at the exact point. A copper bolt, with a punch hole marking the exact point, is sometimes inserted in the drill-hole.

To Set a Stone Bound in the place of a stake marking a corner, first drive four temporary stakes around the corner stake several feet from it and in such a way that a line stretched from two opposite stakes will pass over the tack in the head of the corner stake. Then tacks are carefully set in the tops of these temporary stakes in such positions that a stretching line running from the tack on one stake to the tack on the opposite stake will pass exactly over the tack in the corner stake. Then the corner stake is removed and the hole dug for the stone bound. Care should be taken not to dig the hole any deeper than is necessary so that the bound may be set on firm earth. When the hole for the bound has been dug to the proper depth the monument is dropped into the hole in such a manner that it will be plumb. The fill around the bound should be placed with considerable care, the material being properly rammed as the filling proceeds and the bound kept in such a position that the drill-hole in the top of it shall be exactly under the intersection of the strings. It is sometimes desirable to put in a foundation of concrete and to fill with concrete around the monument to within a foot of the surface where a very substantial bound is required, or where the ground is so soft as to furnish an insecure foundation.

Curved Street Lines are as a rule composed of simple or compound circular curves and are laid out by the method of deflection angles as explained in Art. 56. But the lengths of these curves are the actual length of arcs; this calls for a slight modification in the computations and in measuring the lengths from that used in railroad practise. The length of the curve in city practise equals the radius × the circular measure of the central angle. Since the tape must be stretched along chords in making the measurements the station points to be set by the deflection angle method will be located by measuring the chord subtending the arc of say 50 or 100 ft depending upon how frequently the points are to be located on the curve. A convenient formula for determin-

ing the difference in length between the arc and its chord is $L - C = L^3/24\,R^2$, where L is the arc, C is the chord, and R the radius, all in the same units. This formula is correct to the nearest hundredths of a foot for chords of 50 ft or less where the radius is 150 ft or greater, or for chords of 25 ft or less and radii 25 ft or greater. It is sufficiently exact to compute the results by such a formula with the ordinary slide rule. The deflection angle to any point on the curve equals half the total central angle multiplied by the ratio of its distance from the instrument to the total length of curve (see Art. 56).

In Staking out Street Grades for setting curbs, for pavement or sewer construction, self-reading rods, such as the Philadelphia rod, are the most practicable, altho target rods are used to some extent. The target rod is particularly useful in taking an individual exact reading at any considerable distance from the instrument, which is required in city surveying more than in topographic surveying.

A target rod is also useful in "shooting in grades," for curbs or sewers as follows. If it is a straight grade from Sta. 0 to 5 and grade stakes have been set at 0 and 5 the instrument is set up just to one side of Sta. 0, the rod held on the grade stake at Sta. 0, and target clamped at the height of the telescope, then the rod is held on the grade stake at Sta. 5 and the telescope inclined so as to sight the target and clamped. The line of sight is then parallel to the grade line, and any intermediate stake may be set by driving it until the target coincides with the horizontal hair of the telescope when the rod is held on the stake.

It often happens that the pavement is so hard that stakes cannot be driven or that it is not advisable to use stakes on account of the danger to pedestrians, in which case the grades are marked on fences and buildings, and frequently a convenient place is not found at the grade height. In such cases the grade is marked one or two feet higher or lower than the grade height and marked on the building thus $\underline{\diagdown^{2'}\diagup}$ meaning that the correct grade is 2 ft lower than this mark, or $\overline{\diagup_{1'}\diagdown}$ meaning that the grade is 1 ft above this mark; the arrow points in the direction and the number gives the distance to measure to reach the proper elevation.

When the grades are marked in hard pavements it is customary to drive stout spikes flush with the ground and take the elevation of the tops of these spikes and make a record of the amount the several spikes are above or below grade to be given to the foreman.

Both curb and sewer grades are laid out as a rule as above described. The lines given for such construction are usually parallel lines, 3 to 6 ft to one side of the center line, and marked by stakes or spikes.

Parabolic Vertical Curves are used to connect grades of different slope, the same system of computing elevations for points on the curve being used as explained in Art. 60.

A Rectangular coordinate system based upon plane surveying, disregarding the effect of curvature of the earth, is used in the survey of some of the large cities, such as Baltimore and Boston. Two arbitrary lines at right angles to each other are chosen as axes of a coordinate system, and all important points, such as street corners, are located with reference to these coordinates. One of the chief advantages of this system is that any number of lost points can be easily replaced because their relation to the remaining located points in the city is known.

In laying out a system of this sort a base-line is measured and a triangulation scheme computed as a basis, secondary triangles being formed from the larger ones, and then traverses are run from these triangulation points and closed on other triangulation points. All of the triangles are considered to be plane triangles in the computation. The origin of the coordinate axes is either taken so as to lie outside of the city limits or else, if it be inside, it is given a value such as 20 000, 20 000, so that negativ values for coordinates are avoided.

21. Special Problems

Setting Batter-Boards for a Building. One of the most common tasks of the surveyor is to set the batter-boards for the excavation and construction of the cellar of a new building. For a brick or stone building the lines to be defined are the outside lines of the building, and the elevation

desired is usually the top of the first floor. In the case of a wooden building the line usually given is the outside line of the brick or stone underpinning, and the elevation given is the top of this underpinning on which the sill of the house is to rest. Sometimes the outside line of the sill is desired instead of the outside line of the underpinning. There should be a definite understanding in regard to these points before the work of staking out is begun.

The first work is to stake out the location of the building by accurately setting temporary stakes at all of the corners of the building, in Fig. 16, at A, B, C, D, E and F. A stake should be set at G also so that the entire work can be checked by measuring the diagonals AC and BG, and FD and EG. These checks should always be applied where possible. Then the posts for the batter-boards are driven into the ground 3 or 4 ft outside the line of the cellar so that they will not be disturbed when the walls are being constructed. On these posts, which are usually of 2 by 4-inch scantling, 1-in boards are nailed. These boards are set by the surveyor so that their top edges are level with the grade of the top of the underpinning or for whatever other part of the building the grades are required. After the batter-boards are all in place they should be checked roughly by sighting across them; they should all appear at the same level. Sometimes, however, on account of the slope of the ground some of them have to be set a definite number of feet above or below grade.

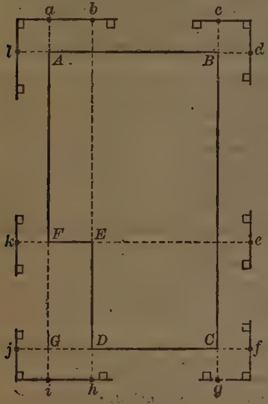

Fig. 16

Then the lines are to be marked by nails driven in the top of these batter-boards. The transit is set up on one of the corner stakes of the house at B (Fig. 16), for example, and a sight is taken on C. This line is then marked on the batter-board beyond (at g) and on the one near the transit (at c). Then a sight is taken along BA and this line is produced both ways and nails set on the batter-boards at l and d. In a similar manner all of the lines are marked on the batters. These points should be marked with nails driven in the top-edges of the batter-boards, and there should be some lettering on the boards to make clear which lines have been given. It is well for the surveyor also to show these marks to the builder or inspector and have it clearly understood just what parts of the structure these lines and grades govern.

Supplying Closing Side of a Traverse. If the latitudes and departures of the several courses of a traverse which does not close are computed the algebraic sum of the latitudes gives the latitude of the closing side and the algebraic sum of the departures gives the departure of the closing side (Art. 16). The square root of the sum of the squares of these two elements gives the length of the closing side, and the tangent of its bearing is its departure divided by its latitude.

To Cut off an Area by a straight line from a point on a side: Plot the field and the known point; draw a trial line from this point to an angle in the other side of the field so as to lay off approximately the required area. Then the sides and bearings of all the lines except the trial line across the field are known; and its length and bearing can be computed as explained in the previous article, and the area of the portion cut off computed by the D.M.D. method (Art. 16). The difference between this area and the area required is a triangle whose base is the trial line and whose altitude can be readily computed, from which the distance along the side of the field from the end of the trial line to the correct point can be computed. Then the closing side can again be calculated and the area cut off computed by the D.M.D. method, which should this time give the desired result.

To find area cut off by line in given direction from given point, let the cut-off line be AC. This problem may be readily solved by drawing a line from A in the traverse to the corner B which lies nearest the other extremity C of the cut-off line. The area of the portion of the traverse cut off by AB is then computed (Art. 16), and to this area is added or from it is subtracted the area of the triangle ABC.

Athletic Grounds. In the layout of athletic grounds the matter of orientation, so that the sun will not shine in the eyes of the players or in the eyes of the spectators, is of great importance. A baseball, football or tennis ground should therefore be laid out with its length running in a north and south direction, and with the grand stand on the west side because most games are played in the afternoon.

Running Tracks should be as a rule one-eighth or one-quarter mile in length, because on tracks of these lengths a full number of laps (or half laps) are required for the ordinary dashes of 220 yards, 440 yards and 880 yards, thus bringing both the start and the finish of the race in front of the grand stand. A good running track should be 15 ft wide on all portions except in front of the grand stand, where it should be made 20 ft wide for use in 100-yd. dashes, in which race the contestants are usually "bunched." For these short dashes the straightaway portion of the track is usually lengthened beyond the curve so as to allow at least 40 ft beyond the finish line. The curves at the ends of an oval field-track should not be sharper than 100-ft radius if it is possible to avoid it. The line for measurement of the length of a running track is 18 inches from the pole, and in a horse race-track it is 3 ft from the pole. A good running track can be made in three layers, 8 inches rock, 2 of cinders, and 2 inches of 3 parts screened cinders to 2 parts screened clay loam. A quarter-mile track with curves of 100-ft radius at its ends will allow of placing a football gridiron inside the oval with 20 ft of turf between the side lines of the football field and the running track, which should be about the minimum distance. One of the best examples of this arrangement is the quarter-mile track at the Ohio State University, Columbus, Ohio, shown in Fig. 17. The Harvard Stadium is another example of this arrangement, but there the side lines of the football field are too near the running track.

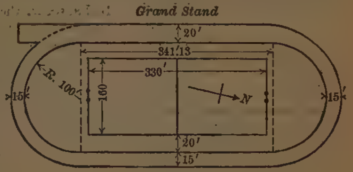

Fig. 17

Fig. 18

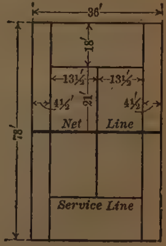

Fig. 19

Football Grounds should be 330 ft × 160 ft, and on level ground. Lines 5 yards apart and parallel to the goal lines are laid out to aid in estimating distances made in the plays. The goals are placed in the center of the ends of the field; the goal poles are 18½ ft apart, and the horizontal cross-bar 10 ft above the ground. It is of importance, if a good football field is to be made, to screen the surface material thru a half-inch screen.

A Baseball diamond is laid out as shown in Fig. 18. The size of the field itself is limited only by the space available, except that the shortest distance from home plate to any fence between the foul lines should be 235 ft. The ground should be as near level as possible; there must not be more than 15 inches fall from the pitcher's box to the base lines or to home plate, and the base lines should be level. Where possible to obtain it, there should be 90 ft behind the home plate, with a minimum of 50 ft.

A Tennis Court is laid out as shown in Fig. 19. An area of 120 ft × 50 ft is about the minimum for a good tennis court. It should have surface drainage by grading the court on a slight slope (about 6 inches per 100 ft) from the center toward the sides. The net posts are placed 3 ft outside the side lines, are 3.5 ft high, and the net should be 3 ft high.

Field Hockey grounds are laid out as shown in Fig. 20. The length is always 300 ft, but the width must be not less than 150 ft nor more than 180 ft.

In Lacrosse, the field is rectangular in shape, 375 ft long and of any desired width. The goals are in the center of the two ends; the goal posts are 6 ft apart and 6 ft high. Each pair of goal posts is in the center of a rectangle which measures 18 ft crosswise of the field and 12 ft lengthwise of the field.

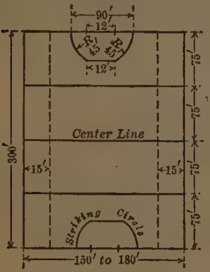

Fig. 20

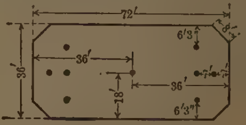

Fig. 21

For Basket-Ball the rectangle is 50 × 75 ft, with the goals in the centers of the ends.

A **Bowling Green** is 120 ft square, with each rink not less than 19 ft nor more than 21 ft. It should be as nearly level as it is possible to make it.

Croquet Grounds are laid out as shown in Fig. 21; the located points within the border show the position of the center of each wicket.

22. U. S. Public Lands

The System. The United States System of Surveying the Public Lands, which was inaugurated in 1784, and modified since by various acts of Congress, requires that the public lands "shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles so as to form townships six miles square," and that the corners of the townships thus surveyed "must be marked with progressive numbers from the beginning." Also, that the townships shall be subdivided into thirty-six sections, each of which shall contain six hundred and forty acres, as nearly as may be, by a system of two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of a mile. Since the meridians converge it is evident that the requirement that the lines shall conform to true meridians and also that townships shall be six miles square, is mathematically impossible.

The Subdivision Work is carried on as follows. FIRST, the establishment of an INITIAL POINT, a PRINCIPAL MERIDIAN, by astronomical observations,

which is a true meridian thru the initial point, and a BASE-LINE which is a true parallel of latitude thru the initial point. These operations are performed in different localities as a basis for the surveys in that portion of the country. The principal meridian is a straight line and the base-line a curve, being at every point at right angles to the meridian thru that point. The base-line is laid out by first running a straight l ne and measuring from it offsets to locate points on the parallel of latitude. Two methods are used for this, called the Secant and the Tangent Methods. The "Manual of Surveying Instructions for the Survey of the Public Lands of the United States," issued by the General Land Office, contains complete explanations of these methods and tables, giving the offsets as well as general instructions.

SECOND, the division of the area into tracts approximately 24 miles square by the establishment of STANDARD PARALLELS, which are true parallels of latitude extending east and west thru 24-mile points on the principal meridian, and GUIDE MERIDIANS, which are true meridians thru 24-mile points on the base-line and on the standard parallels and extending north to the intersection of the next standard parallel or base-line. Since these guide meridians converge these tracts will be 24 miles on their southern and less on their northern boundaries.

THIRD, the division of each 24-mile tract into TOWNSHIPS, each approximately 6 miles square, by establishment of MERIDIONAL LINES, or RANGE LINES, which are true meridians thru standard township corners established at intervals of 6 miles on the base-line and on the standard parallels and extending north to an intersection with the next standard parallel or base-line; and the establishment of LATITUDINAL LINES, or TOWNSHIP LINES, joining the township corners previously established at intervals of 6 miles on the principal meridian, guide meridians, and range lines. Neglecting discrepancies in fieldwork the east and west boundaries of townships will be 6 miles in length, but the north and south will vary in length, being a maximum of 6 miles at the standard parallel or base line forming the southern limit of a 24-mile tract and a minimum at that forming its northern limit.

FOURTH, the division of each township into SECTIONS, each approximately 1 mile square (640 acres), by establishing SECTION LINES, both meridional and latitudinal, parallel to and at intervals of 1 mile from the eastern and southern boundaries of the township.

The subdivisions of sections into "quarters," "eighties" and "forties" constitutes the principal work of local surveyors. In establishing a point in the center of a section to divide it into quarters it is placed at the intersection of lines joining the quarter corners on the opposite sides of the section.

Townships are designated by numbering them in order both north and south from the base-line and east and west from the principal meridian. Any series of contiguous townships or sections situated north and south of each other constitute a RANGE, and such a series in an east and west direction constitutes a TIER. Thus "Township 8 south, Range 18 east of the Sixth Principal Meridian" locates its position; this is usually abbreviated to "T. 8 S., R. 18 E., 6th P.M."

The Sections in a township are numbered beginning with No. 1 at the northeast corner of the township and proceeding west to No. 6, then the next tier runs east to No. 12, the next tier west to No. 18, and so on, No. 36 being in the southeast corner.

23. Leveling for Profile

In Leveling to Determine a Profile the line is first stationed, every 100-ft point, or such other interval as is desired, being distinctly marked. The level is set up and a rod-reading called a BACKSIGHT (B.S., or +S) taken on a point called a BENCH-MARK (B.M.) whose elevation is known. When

this B.S. is added to the elevation of the B.M. it gives the HEIGHT OF THE INSTRUMENT (H.I.). Rod-readings called FORESIGHTS (F.S., or −S) are then read on as many station points on the line as can conveniently be seen from the instrument, and the elevation of the point on which the rod rests when a F.S. is taken is found by subtracting the F.S. from the H.I. Rod-readings are taken at all distinct changes in slope which occur on the line which is being profiled, whether these changes come at the full station points or not, and the intermediate stations are determined by tape measurements (sometimes by pacing) and are recorded as plus stations as shown in the sample notes below.

Field Notes in Running a Profile

Profile of Hudson St. Curb (West side). June 17, 1908. Brown, level. Jones, rod.

Sta.	+S	H.I.	−S	Elev.	B.M. & T.P. Elev.	Descriptions.
B.M$_1$.	7.218	104.912	.....	......	97.694	N.E. cor. top granite post S.W. cor. Town Hall.
0	.....	....	4.24	100.67	....	
1	.....	....	4.28	100.63	....	
2	.....	....	4.33	100.58	....	
3	.....	....	4.38	100.53	....	
T.P.	6.473	106.623	4.762	......	100.150	Rim M.H. opp. house #73.
4	.....	....	6.12	100.50	....	
4+60	.....	....	6.11	100.51	....	
5	.....	....	6.13	100.49	....	
6	.....	....	6.15	100.47	....	
6+85	.....	....	6.17	100.45	....	
7	.....	....	6.14	100.48	....	
8	.....	....	6.10	100.52	....	
B.M$_2$.	6.071	107.977	4.717	......	101.906	Top S.B. cor. Burrill St. Elev. 101.912
9	.....	....	7.43	100.55	....	

When it is necessary to move the level to a new position to proceed along the line to be profiled a TURNING POINT (T.P.) is selected and its elevation determined to use at the next set-up of the level in finding the new H.I. When this new H.I. is found, F.S. readings are taken at the proper stations on the line as far as is consistent with the precision required and then a new T.P. is established, and so on. The readings on the B.M's. and T.P's. should be taken to one more decimal place than those taken for the profile. In the notes shown, for example, the readings were taken on the curb to the nearest 0.01 ft, while those on the T.P's. were read to 0.001 ft. The B.M's. are all carefully described in the notes; as a rule T.P's. are not described, but where they are taken on points easily identified it is well to describe them also. The distances from the level to the rod when held on the T.P. or B.M. should be about equal for a backsight and its corresponding foresight, so that any errors in the adjustment of the line of sight or wyes may be eliminated.

If possible the levels should form a circuit, returning to the original B.M. so as to check the intermediate work. A common method is to check the work by readings on other B.M's. which are passed in progressing along the line. When this is done those B.M's. should be used as T.P's.

The calculation of the level notes may be checked by finding the difference between the sum of the B.S's. and F.S's. which were taken on the first B.M., on all the intermediate T.P's., and the final B.M. This gives the difference in elevation.

In taking levels for a profile it is necessary to start from some datum to which to refer the levels, and a B.M. whose elevation above some datum plane (such as mean sea level) is used if one is near the locality of the work. Or a B.M. of assumed elevation may be used, in which case all of the levels will refer to an assumed datum. At a later period, if necessary, these levels may be connected to a sea-level datum by running an accurate line of levels between the assumed B.M. and some other B.M. whose elevation referred to sea level is known.

The Proper Length of Sights will depend upon the distance at which the rod appears distinct and upon the precision required. Under ordinary conditions sights should not exceed 300 ft where elevations are required to the nearest 0.01 ft, and even at a much shorter distance the boiling of the air may prevent a precision of this degree.

Curvature and Refraction Correction. Since a level line is a curved line which at every point is perpendicular to the direction of gravity and the line of sight of a level is along a tangent to this curve it is necessary to take this into account in the more precise leveling work. This correction is usually combined with that due to the refraction of the atmosphere; and the combined correction, for sights of 300 ft, is about 0.002 ft; for 500 ft, 0.005 ft; for 1000 ft, 0.020 ft. These corrections are to be applied to any single rod-reading by subtracting from the reading; but if the rod is equally distant from the instrument on the foresight and backsight the effect of curvature and refraction is eliminated from the result.

To Establish a Datum from tidal observations, set up a vertical staff, graduated to feet and tenths, in such a manner that the high and low water can be read. Read the positions of high and low water for each day for as long a period as practicable. The mean value obtained from an equal number of high and low water observations will give the approximate value of mean sea level. If the observations extend over one lunar month the result will be fairly good; to determine this accurately will require observations extending over about eighteen years.

Double Rodded Lines are frequently used when it will be impracticable to run a circuit or when a profile is to be made thru a new country where no intermediate B.M's. have been previously established. Instead of taking a foresight on a single T.P., foresights are taken on two different T.P's. near together and varying in elevation by a foot at least. When the level is set up again a backsight is taken on both T.P's. and two H.I's. computed, which should agree within whatever limits of error are allowable for the class of work at hand. This method of carrying on a check on the leveling is particularly applicable to such cases as running preliminary surveys for a railroad.

24. Leveling for Cross-Sections

Cross-Sections for Borrow-Pits. When it is desired to determine the shape of the ground with a nicety such as is required for landscape architect studies or for computing the material taken from a borrow-pit the area may be divided into squares (or rectangles) and elevations taken at the corners of these squares and at as many intermediate points as are necessary to determine the shape of the ground. These surface elevations are usually taken to the nearest tenth of a foot, and the squares, which are anywhere from 10 ft to 100 ft on a side, are laid out with transit and tape. The corners may be designated by a system of letters for the lines running in one direction and numbers for the system perpendicular to the lettered system; for example, point D, 7 lies at the intersection of lines D and 7, and intermediate point $E + 8, 6 + 4$ lies 8 ft from line E toward F and 4 ft from line 6 toward line 7. The notes are kept like profile notes (Art. 23) except that the stations are designated as just described. The tape-rod, described in Art. 7, is of particular use in work of this sort.

When cross-sections are taken to determine the amount of material taken out of a borrow-pit, readings are taken as described above before the excavation is started, and then after the excavation is completed the same system of cross-section lines is again run out

and the new elevations at corners, and intermediate points if necessary, are determined. The quantity of material removed is computed as explained in Art. 66.

Road Cross-Sections are taken as a basis for estimating the quantity of earthwork in railroad or highway construction, but by an entirely different method from the borrow-pit method. From the plan of the proposed road its alignment is staked out and a profile is taken along the center line, which is subsequently plotted. On this profile the grade line is drawn, which corresponds to the finished surface of the road. Roads are usually first finished to sub-grade, which is below the completed surface by an amount equal to the thickness of the road covering, the pavement of a highway or the ballast in the case of a railroad. The width of the base of the road and the inclination of the side slopes are known. For ordinary gravel the slope is usually 1½ ft horizontal to 1 ft vertical, called "a slope of 1½ to 1."

For construction work the engineer sets grade stakes at every full station or oftener on the center line and at both sides where the finished slope intersects the surface of the ground. All three of these stakes are marked, giving the amount of "cut" or "fill" to be made at these points. The cut or fill marked on the stakes where the slopes meet the original surface is the vertical distance from the base of the road to the surface of the ground at these points. The process of setting these slope stakes is described in Art. 62.

Cross-sections for dams, for canals and other engineering structures are common problems for the surveyor. This work consists of making a short profile at right angles to the line of the structure, at intervals of from 10 to 100 ft apart depending upon various conditions, and extending far enough on either side so as to include the possible location of the structure. These cross-section lines representing the surface are plotted for each station, and on them are also plotted the proposed cross-sections of the structure at the respective stations. This gives a complete record of the conditions before the work is started, and as it progresses, by the use of different colored lines, a continual graphical record of the work may be kept from month to month, which is the customary interval for making estimates for partial payments to contractors.

25. Precise Leveling

Precise Spirit-leveling is like ordinary leveling except that certain refinements are introduced into the field methods and in the construction and manipulation of the instrument. The characteristic features of a precise level are an inverting telescope of high power and good definition provided with a vertical and three horizontal hairs, and a sensitive spirit-level of uniform curvature having a mirror or set of prisms so adjusted that the observer can see the bubble while he is looking at the rod. The instrument rests on three leveling screws. There are two spirit-levels set at right angles to each other for approximate leveling and a micrometer screw by which one end of the telescope may be raised or lowered slightly when centering the main bubble. Some of these instruments are of the wye and some of the dumpy patterns; the latest (1900) U. S. Coast Survey level is a dumpy. This Coast Survey instrument differs from most of the previously constructed precise levels; its telescope is of iron-nickel alloy, which has a very low coefficient of expansion; the bubble is placed as near the line of sight as possible so that the parallelism of the two will not be disturbed by local temperature changes. A second tube is set on the left-hand side of the telescope, and carries a set of prisms so arranged that the observer sees the bubble thru this tube with his left eye at the same time that he views the rod thru the telescope with his right eye. The great advantage of this instrument is in the rapidity with which the readings can be taken. (For a complete description see Coast Survey reports for 1900, p. 521, and 1903, p. 200.)

The Rods used in recent precise leveling work are of the non-extensible self-reading pattern. They are usually in sections in the form of a + or a

T, and since they are always used with an inverting telescope the figures on the rod are made upside down. Rods graduated in the metric scale, the smallest division being a centimeter, are used by the U. S. Coast Survey, while the U. S. Geological Survey use a rod graduated in yards, the smallest division being one-hundredth of a yard. The foot of this rod carries a metal shoe rounded so as to fit the cup-shaped cavity of the foot-piece which is driven into the ground.

While being carried from one station to another the level should be protected from the sun's rays, the micrometer should be unscrewed so that the telescope will not bear on the point of the screw, the nuts holding the tripod legs should be loosened, the central clamp should be tight if the instrument is carried on the tripod. When the level is set up the tripod nuts are tightened and the tension on the central clamp screw released so that the instrument is held on the tripod by its own weight. In using the instrument the bubbles of the small levels are brought to a central position by means of the leveling screws, the telescope is turned toward the rod and the large bubble centered by the micrometer screw.

The observations are made in such a manner as to eliminate the recognized common errors in leveling work, which are (1) settling of level on soft ground; (2) unequal expansion and contraction of different parts of instrument due to temperature changes; (3) irregular refraction of air near ground; (4) unequal length of backsights and foresights; (5) selecting poor turning points; (6) rod not held plumb; (7) bubble not in center of tube at instant reading is taken.

Errors due to settling of tripod are eliminated by taking two backsights and two foresights at each set-up in the following order: first the backsight is read, then the foresight, and then the foresight is again read and lastly the backsight again. The mean gives the correct elevation of the T.P. provided the level has settled the same amount in the time which has elapsed between each set of readings. Errors due to temperature changes may be partly avoided by shortening the time elapsing between the reading of the backsight and foresight. Rapidity is also of great advantage in eliminating errors due to settling of tripod. In all cases the instrument should be shielded from sun's rays and from wind by an umbrella or special shield. Errors due to refraction of the air may be partly avoided by using long-legged tripods so as to set the level high above the ground and by making the observations in the middle of the day rather than in early morning or late afternoon. The distances to backsights and foresights are kept nearly equal by reading these distances with the stadia hairs; the notes should be kept so that they will show at a glance whether the sums of the foresight and backsight distances are equal or not. Foot-plates or pins are driven into the ground to use for T.P's. when a satisfactory T.P. cannot be found at the proper distance from the instrument. To aid in holding the rod plumb a spirit-level is used permanently attached to it or held against its edge. The temperature of the rod is also taken occasionally during the day and proper corrections applied to readings.

On the U. S. Coast Survey work two rods are used, each rod being held alternately for a foresight and a backsight, the same rod is held on a T.P. for both the backsight and the foresight, all three cross-hairs are read, the interval between the upper and lower giving the stadia distance. At odd-numbered instrument stations the backsight is taken before the foresight and at even-numbered stations the foresight is taken first. All levels are run in circuits of a mile or two and, if possible, the return trip is made under different atmospheric conditions. The maximum length of sight allowable is 150 meters, and the difference of backsight and foresight distances must not become greater than 20 meters. The instrument adjustments are tested daily.

The Allowable Errors in precise leveling work of various surveys is as follows: U. S. Coast Survey, $4^{mm}\sqrt{\text{kilometers}}$; U. S. Lake Survey, $10^{mm}\sqrt{\text{kilometers}}$; Mississippi River Survey, $5^{mm}\sqrt{\text{kilometers}}$; U. S. Geological Survey, $0.017^{ft}\sqrt{\text{miles}}$. The results actually reached fall well within these limits. A high grade of leveling has been done on the Barge Canal Survey of New York with an ordinary wye level of good construction; in this work the allowable error of closure was only $0.02^{ft}\sqrt{\text{miles}}$. An accuracy equal to that required by the use of a precise leveling instrument can be reached with the ordinary wye or dumpy level but probably not as economically.

TOPOGRAPHIC SURVEYING

26. Barometric Leveling.

The Barometer may be used as a means of measuring differences in elevation, since one inch in the height of the mercury column is equivalent to about 900 ft in altitude. The atmospheric pressure varies also with changes of temperature and humidity, so that it is necessary in measuring differences in altitudes with the barometer to determine the amounts of these variations and to make proper allowance for them. The ordinary cistern mercurial barometer and the aneroid are used in surveying. The latter on account of its compactness and sensitiveness is more generally employed, but it is liable to derangements if subjected to great ranges of pressure or if roughly handled; it is also subject to errors due to temperature changes.

Barometric Leveling is useful in making a reconnoissance or in determining the contours for a map of very small scale. It is approximate at best, but with a mercurial barometer results as close as 5 ft may be obtained by careful handling of the instrument, and with an aneroid in good adjustment results correct to 5 or 10 ft may be obtained but not without repeating the observations and taking great care in the use of the aneroid. The error in height determined by a barometer is approximately a constant, so that the percentage error is much smaller for large differences in elevation than for small differences. The barometer does not give actual heights, but the difference in its readings will be a function of the difference in elevation of the two points provided the atmospheric conditions have not changed so as to affect the readings.

To Read a Mercurial Barometer the screw at the bottom of the barometer is turned until the ivory point is just in contact with the mercury The vernier is then raised or lowered until its lower edge is just at the height of the mercury column. More than one reading should be taken, and the ivory point should be set in contact with the mercury each time. The temperature of the air and of the mercury are both read when the mercury height is recorded. In carrying the mercurial barometer from place to place the bottom screw should be turned up until the mercury just fills the tube, then the barometer should be inverted and carried in its case upside down.

On the Aneroid Barometer are two scales, the inner one corresponding to inches of mercury and the outer one to feet of altitude, the zero of the altitude scale being in most instruments at 31 inches on the mercury scale. The outer scale should not be movable with reference to the inner scale, for the number of feet of altitude corresponding to one inch of mercury is different in different parts of the scale. Aneroids marked "compensated" are supposed to be adjusted so that changes in temperature of the instrument will not affect the readings. The instrument should be handled carefully in order to avoid disturbing its delicate mechanism. When it is to be read, tap the case lightly to be sure the instrument has adjusted itself to the changed pressure. It should stand a few minutes before it is read so as to allow it to come to the true reading; it should not be heated by the sun's rays or the body; and it may be held in either a vertical or horizontal position when being read, but as the readings in these two positions are different it should be held in the same position at all stations. As accurate results may be obtained from small as from large aneroids.

The Best Field Method requires the use of two barometers, one to remain at some fixt station whose elevation is known, and a continuous record of this instrument will give the atmospheric changes. It is assumed

since the other aneroid will be in use in the same general locality, that the atmospheric changes will be the same at both instruments. It is well to use a mercurial barometer at the fixt station. The second instrument (an aneroid) is read at the start at the fixt station. The difference between the readings of the two barometers at this station is an index correction to be applied to all readings of the moving barometer. The aneroid is then carried to the next station whose elevation is required and read, and so on to as many points as are desired, making the time of travel between stations as short as possible. At each point the pressure, time and air temperature are read and recorded. Upon the return trip with the aneroid it will be well, as a check, to take readings again at the stations as they are past, and upon arriving at the fixt station the aneroid is read again. At intervals during the day, every half hour, or oftener if the atmospheric conditions are rapidly changing, the barometer at the fixt station is read and the time, air temperature, and mercury temperature (if a mercurial) are recorded. The interpolated readings of the fixt barometer are entered in the notes of the moving barometer and the index correction applied, thereby eliminating all changes in reading except those due to changes in altitude. When only one barometer is used a reading is taken at the first station when starting on the trip and again at the end of the trip. The difference between these two readings represents the total change due to weather conditions, and interpolated readings for the times when the barometer was read at the other stations must be made for the first station, as illustrated by the example below.

Calculating the Altitude. Differences in elevation are calculated from the differences in pressure either by formula or by table, or it may be found with an aneroid by reading directly the altitude scale which is based on one of the formulas. LAPLACE'S FORMULA is one of the most accurate; it is $D = 60\,158.6\,(\log h - \log H)\left\{1 + (t_a' + t_a - 64^\circ)/900\right\}$, where D = difference in elevation in feet, h = height of mercury at lower station in inches, H = height of mercury at upper station in inches; t_a' and t_a are the observed air temperatures in Fahrenheit degrees. The correction due to air temperature is often a large amount. In case a mercurial barometer is used H is the height of mercury at the upper station reduced to the temperature of the mercury at the lower station by $H = h'\left\{1 + 0.00008967\,(t_m - t_m')\right\}$, where t_m = the temperature of mercury at lower station and t_m' = the temperature of mercury at upper station. The following table condensed from Guyot is based on this formula.

EXAMPLE using one aneroid barometer: find difference in elevation between Overlook and Bald Mt.

Station	Time	Barom. Inches	Air Temp.
Overlook	9.25 A.M.	29.42	47° F.
Bald Mt.	10.30 A.M.	27.15	30°
Overlook	10.57 A.M.	29.36	43°

Here 10.30 − 9.25 = 1.05 = 65 min, time going up, and 10 57 − 9.25 = 1.32 = 92 min, total time. Also 29.42 − 29.36 = 0.06 in, diff. in barometer height at Overlook, whence 29.42 − (0.06 × 65/92) = 29.38, probable reading at Overlook at 10.30 A.M.

From Barometric Table,	29.38 =	28 157
	27.15 =	26 095
		2 062
Temp. Corr. = $2\,062\left[\left\{\tfrac{1}{2}(47 + 43) + 30\right\} - 64\right]/900$	=	+ 25
Difference in elevation	=	2 087 feet.

For Determining Difference in Elevation by Barometer

Barom. Inches	Feet	Barom. Inches	Feet	Barom. Inches	Feet	Barom. Inches	Feet	Barom. Inches	Feet
16.00	12 280	20.00	18 110	22.75	21 466	25.50	24 457	28.25	27 133
16.10	12 442	20.05	18 175	22.80	21 533	25.55	24 508	28.30	27 179
16.20	12 604	20.10	18 240	22.85	21 590	25.60	24 559	28.35	27 225
16.30	12 765	20.15	18 305	22.90	21 647	25.65	24 610	28.40	27 271
16.40	12 925	20.20	18 370	22.95	21 704	25.70	24 661	28.45	27 317
16.50	13 084	20.25	18 434	23.00	21 761	25.75	24 712	28.50	27 362
16.60	13 242	20.30	18 499	23.05	21 818	25.80	24 762	28.55	27 409
16.70	13 398	20.35	18 563	23.10	21 874	25.85	24 813	28.60	27 454
16.80	13 554	20.40	18 627	23.15	21 931	25.90	24 864	28.65	27 500
16.90	13 709	20.45	18 691	23.20	21 987	25.95	24 914	28.70	27 545
17.00	13 864	20.50	18 755	23.25	22 044	26.00	24 964	28.75	27 591
17.10	14 017	20.55	18 818	23.30	22 100	26.05	25 014	28.80	27 637
17.20	14 169	20.60	18 882	23.35	22 156	26.10	25 065	28.85	27 682
17.30	14 321	20.65	18 945	23.40	22 212	26.15	25 115	28.90	27 727
17.40	14 471	20.70	19 008	23.45	22 267	26.20	25 164	28.95	27 772
17.50	14 621	20.75	19 071	23.50	22 323	26.25	25 214	29.00	27 817
17.60	14 770	20.80	19 134	23.55	22 378	26.30	25 264	29.05	27 862
17.70	14 918	20.85	19 197	23.60	22 434	26.35	25 314	29.10	27 907
17.80	15 065	20.90	19 260	23.65	22 489	26.40	25 363	29.15	27 952
17.90	15 211	20.95	19 322	23.70	22 544	26.45	25 412	29.20	27 997
18.00	15 357	21.00	19 384	23.75	22 599	26.50	25 462	29.25	28 041
18.10	15 502	21.05	19 446	23.80	22 654	26.55	25 511	29.30	28 086
18.20	15 646	21.10	19 508	23.85	22 709	26.60	25 560	29.35	28 131
18.30	15 789	21.15	19 570	23.90	22 764	26.65	25 609	29.40	28 175
18.40	15 931	21.20	19 632	23.95	22 818	26.70	25 658	29.45	28 220
18.50	16 073	21.25	19 694	24.00	22 873	26.75	25 707	29.50	28 264
18.55	16 143	21.30	19 755	24.05	22 927	26.80	25 755	29.55	28 308
18.60	16 214	21.35	19 816	24.10	22 982	26.85	25 805	29.60	28 352
18.65	16 284	21.40	19 877	24.15	23 036	26.90	25 853	29.65	28 396
18.70	16 354	21.45	19 938	24.20	23 090	26.95	25 902	29.70	28 440
18.75	16 423	21.50	19 999	24.25	23 144	27.00	25 950	29.75	28 484
18.80	16 493	21.55	20 060	24.30	23 198	27.05	25 999	29.80	28 528
18.85	16 562	21.60	20 120	24.35	23 251	27.10	26 047	29.85	28 572
18.90	16 632	21.65	20 181	24.40	23 305	27.15	26 095	29.90	28 616
18.95	16 701	21.70	20 241	24.45	23 358	27.20	26 143	29.95	28 659
19.00	16 769	21.75	20 301	24.50	23 412	27.25	26 191	30.00	28 703
19.05	16 838	21.80	20 361	24.55	23 465	27.30	26 239	30.05	28 746
19.10	16 907	21.85	20 421	24.60	23 518	27.35	26 287	30.10	28 790
19.15	16 975	21.90	20 481	24.65	23 571	27.40	26 334	30.15	28 833
19.20	17 043	21.95	20 540	24.70	23 624	27.45	26 382	30.20	28 877
19.25	17 111	22.00	20 600	24.75	23 677	27.50	26 430	30.25	28 920
19.30	17 179	22.05	20 659	24.80	23 730	27.55	26 477	30.30	28 963
19.35	17 246	22.10	20 718	24.85	23 782	27.60	26 524	30.35	29 006
19.40	17 314	22.15	20 777	24.90	23 835	27.65	26 572	30.40	29 049
19.45	17 381	22.20	20 836	24.95	23 887	27.70	26 619	30.45	29 092
19.50	17 448	22.25	20 894	25.00	23 940	27.75	26 666	30.50	29 135
19.55	17 516	22.30	20 954	25.05	23 992	27.80	26 713	30.55	29 178
19.60	17 582	22.35	21 012	25.10	24 044	27.85	26 760	30.60	29 220
19.65	17 648	22.40	21 071	25.15	24 096	27.90	26 807	30.65	29 263
19.70	17 715	22.45	21 129	25.20	24 148	27.95	26 854	30.70	29 306
19.75	17 781	22.50	21 187	25.25	24 199	28.00	26 900	30.75	29 348
19.80	17 847	22.55	21 245	25.30	24 251	28.05	26 947	30.80	29 391
19.85	17 913	22.60	21 303	25.35	24 303	28.10	26 994	30.85	29 433
19.90	17 979	22.65	21 360	25.40	24 354	28.15	27 040	30.90	29 475
19.95	18 044	22.70	21 418	25.45	24 406	28.20	27 086	30.95	29 518

27. Trigonometric Leveling

Finding the Difference in Elevation of two points by means of the horizontal distance between them and the vertical angle is called TRIGONOMETRIC LEVELING. It is used chiefly in determining the elevation of triangulation stations and in obtaining the elevation of a plane-table station from any visible triangulation point of known elevation. In triangulation work the vertical angles are usually measured at the same time the horizontal angles are measured, so as to obtain the elevations of triangulation points as well as their horizontal positions. The vertical angle is measured to some definite point on the signal whose height above the center mark of the station was determined when the signal was erected, and the height of the instrument above its station should be measured and recorded. In the most exact work the angles are measured with a special vertical circle instrument. In less precise work an ordinary theodolite whose vertical arc reads by verniers to 30″ or to 20″ may be used, but with such instruments only single readings can be made. The best results with such an instrument are obtained by taking the average of several independent readings half of which are taken with the telescope direct and the other half with the telescope inverted. In every case the index correction, or reading of the vertical arc when the telescope is level, must be recorded.

Simultaneous Observations. The chief difficulty in obtaining accurate results by trigonometric leveling is due to the uncertainty of the angle of refraction, that is, the angular deviation of the line of sight on account of the refraction of the air. This varies with the locality, the temperature and the atmospheric pressure, so that the only way its effect can be practically eliminated is by taking simultaneous observations between two stations, in which case

$$h_1 - h_2 = K \tan \tfrac{1}{2}(\alpha_2 - \alpha_1) \left\{1 + (h_1 + h_2)/2R + K^2/12R^2\right\}$$

where $h_1 - h_2$ = difference in elevation of stations in feet; K = arc of the earth subtended between plumb lines thru the two station points, approximately equal to the distance between the two stations in feet; α_1 and α_2 are the simultaneously observed vertical angles; R = radius of earth at latitude of the stations; for most work it is close enough to use log R (in feet) = 7.32068.

Observation at One Station. If the observation is made at only one station the REFRACTION COEFFICIENT must be known and applied to the vertical angle. The refraction coefficient is $m = r/c$, whence $r = mc$, where r = the angle of refraction and c = the angle at the center of the earth between the two stations. For a single observation

$$h_2 - h_1 = K \tan\left[\alpha_1 + (\tfrac{1}{2} - m)(K/R \sin 1'')\right]\left[1 + (h_1 + h_2)/2R + K^2/12R\right]$$

In this formula the letters have the same significance as in the formula for simultaneous observations above. In solving for $h_2 - h_1$, approximate values are first obtained by omitting the last factor; the approximate values are then substituted in the last factor and corrected values computed.

From a large number of observations the U. S. Coast Survey has determined values of m as follows:

For lines crossing the sea	0.078
Between primary points (high elevation)	0.071
For interior of the country, about	0.065

Clarke gives in his "Geodesy"

For rays crossing the sea	0.0809
For rays not crossing the sea	0.0750

A Rough Determination of the difference in height may be found by multiplying the horizontal distance by the tangent of the vertical angle and applying a single correction for curvature and refraction by the formula $h = K^2/1.7426$, where K is the distance in miles between the stations and h is the correction in feet. If K is exprest in units of 1000 ft then $h = 0.02 \times K^2$ (nearly). This correction is applied so as to increase the difference in elevation if the vertical angle is plus, and it should decrease the difference in elevation if the vertical angle is minus. This method is used for the stadia and plane table except that for ordinary sights or for rough work the curvature and refraction correction is omitted.

28. Contours

A Contour Line is the intersection of a level surface with the surface of the ground. For example, a shore line of a pond is a contour line, and if the pond level were raised a foot its new shore line would be the contour line of 1 ft greater elevation. Contours are used to represent the shape of the ground, and give a means of reading elevations directly from the map. It is customary to locate them a whole number of feet above the datum, such as the 50-ft contour, and at regular intervals, at, say, every 2 ft or every 5 ft, the number of the contour is its elevation above the datum. Since the contours are equidistant vertically their horizontal distance apart gives a measure of the steepness of slopes.

Characteristics of Contours are: (1) All points on any one contour have the same elevation. (2) Every contour closes on itself, either within or beyond the limits of the map. (3) A contour which closes within the limits of the map encloses either a summit or a depression. In depressions there will usually be found a pond or a lake; but where there is no water the contours are usually marked in some way to indicate a depression. (4) Contours can never cross each other except where there is an overhanging cliff, in which case there must be two intersections. (5) On a uniform slope contours are spaced equally. (6) On a plane surface they are straight and parallel. (7) In crossing a valley the contours run up the valley on one side and, turning at the stream, run back on the other side. Since the contours are always at right angles to the lines of steepest slope they are at right angles to the thread of the stream at the point of crossing. (8) Contours cross the ridge lines (divides) at right angles.

Contours and Profiles. If a line is drawn across a contour map the profile of the surface along that line may be constructed, since the points where the contours are cut by the line are points of known elevation and the horizontal distances between these points can be scaled or projected from the map. The profile shown in Fig. 22 is constructed by first drawing, as a basis for the profile, equidistant lines, corresponding to the contour interval, and parallel to XY. From the points where XY cuts the contours, lines are projected to the corresponding line on the profile. Conversely, if the profiles of a sufficient number of lines on the map are given it is possible to plot these lines on the map, mark the elevations, and then to sketch the contours.

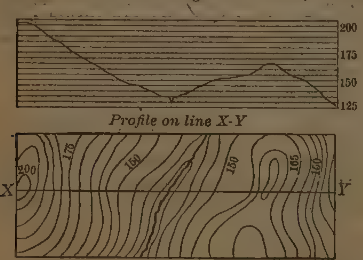

Fig. 22

Locating Contours. Since the topography of much of the country is due to stream erosion the position and fall of streams gives much information regarding the location of contours. These, together with a few ridge elevations, give information from which contours may be readily sketched. It should be borne in mind in sketching contours that they cross the stream at right angles and curve around from the hill on either side so as to represent the valley; they are usually farther apart at the top and bottom of a natural eroded slope (where no wave action is present) than in the middle; a stream is usually steeper near its source than in its lower portion.

Where the contour interval is small and much detail is desired the ground may be cross-sectioned (Art. 24) and from the elevations determined the contours may be sketched. Or the contours may be actually traced out and located by stadia. In this case the rodman moves up or down the slope until a level rod-reading shows that the foot of his rod is on a contour. The position of the rod is then located by distance and azimuth.

The hand level is used to a considerable extent on railroad preliminary surveys to locate contours, and is satisfactory for such work because the contours are located only a short distance from the transit line so that the inaccuracies of hand-level work cannot become very great. In using a hand level for locating contours, first measure the distance from the ground to the levelman's eye, which may be, say, 5.2 ft. Then from a reading of the rod held on a point of known elevation the elevation of the eye of the levelman is determined. The leveler then directs the rodman uphill or downhill until the rod-reading is such as will correspond to the foot of the rod being on a contour. When this point is found it is located by tape or by pacing. Then, if the contours are being located on a side-hill, the levelman moves uphill past the located point on which the rodman stands, until the levelman reads the contour interval plus 5.2 on the rod. When this point is reached the levelman is standing on a contour which is then located. The levelman stands on this point and the rodman travels uphill until the levelman reads 5.2 minus the contour interval on the rod, when the rod is resting on the next higher contour. In going downhill the rodman holds his rod on a contour and the levelman will back down the hill until he reads 5.2 minus the contour interval on the rod, at which point the levelman will be standing on the contour next below the rodman; then the rodman passes the levelman and backs down the hill until the rod-reading is 5.2 plus the interval, which determines the next lower contour. Sometimes profiles are run and the elevations plotted on the plan give a basis for sketching the contours. Profiles of valleys and ridges as a rule give the best data.

29. The Stadia

The Stadia Method is one in which distances are measured by observing thru the telescope of a transit the space, on a graduated rod, included between two horizontal hairs called STADIA HAIRS. If the rod is held at different distances from the instrument different intervals on the rod are included between the stadia hairs, the spaces on the rod being proportional to the distances from the instrument to the rod, so that the intercepted space is a measure of the distance to the rod. This method furnishes a rapid means of measuring distances in filling in details of topographic and hydrographic surveys. It is used either with a transit or a plane table which is provided with the two additional horizontal cross-hairs. It has the great advantage that the intervening country does not have to be traveled, provides a means of measuring inaccessible distances such as across water surfaces, the errors of measurements are compensating rather than cumulative, and it affords an accuracy sufficient for many kinds of work, being applicable even to surveying for area of such lands as wood lots or farms, for an accuracy of one part in 500 may be attained with the stadia. The ROD used may be of any desired pattern of self-reading rod, the Philadelphia rod being a good type for short distances. It is well to use a rod with the graduations represented by some diagram which can be seen distinctly a long distance, 1000 to 2000 ft away. Portions of two of these rods are shown in Fig. 23, with the reading of the three cross-hairs marked in the figure.

The Fundamental Principle of the stadia is the simple geometric proposition that in two similar triangles homologous sides are proportional. The vertex of these triangles is not the center of the transit but is at a point in front of the objective a distance equal to F, the focal length of the objective, so that the distance from the center of the instrument to the rod $= (F/i)\ s + (F + c)$, where i is the distance between the upper and lower stadia hairs, s is the intercepted space on the rod and c is the distance from the center of the instrument to the objective. Evidently $(F + c)$ is practically a constant for any given instrument; for transits it varies from 0.75 to about 1.35 depending upon the style of instrument. For all ordinary purposes it may be taken as 1 ft for transits and 2 ft for plane-table alidades. The other function $(F/i)\ s$ is a variable, but as it is almost always customary to have the stadia hairs spaced so that $F/i = 100$ this reduces the equation to Distance $= 100\,s + 1$. Every hundredth of a foot on the rod is then a measure of one foot of distance.

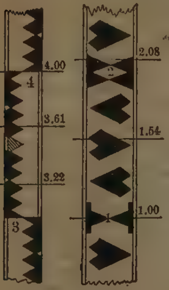

Fig. 23

Inclined Sights. Since it is the horizontal distance which is required it is necessary to reduce all inclined readings to the horizontal, and when the difference in elevation between the instrument station and the rod station is desired the vertical distance also must be computed from the inclined sight. In practise it is customary to hold the rod plumb rather than perpendicular to the line of sight, so that it is evident that if the line of sight is inclined it will subtend an interval on the rod which is too great depending upon the angle α of inclination and the distance the rod is from the instrument. By trigonometric proof, with one slight assumption, it may be shown that the

$$\text{Vertical Distance} = (F/i)\ s \cdot \tfrac{1}{2} \sin 2\alpha + (F + c) \sin \alpha$$
$$\text{Horizontal Distance} = (F/i)\ s \cdot \cos^2 \alpha + (F + c) \cos \alpha$$

For the ordinary conditions (when $F/i = 100$ and $F + c = 1$) and making the approximations that $\sin \alpha = \frac{1}{2} \sin 2\alpha$ and $\cos \alpha = \cos^2 \alpha$ in the last terms of the above formulas, which are nearly correct for small angles,

$$\text{Vertical Distance} = (100 \times \text{rod interval} + 1)\ \tfrac{1}{2} \sin 2\alpha$$
$$\text{Horizontal Distance} = (100 \times \text{rod interval} + 1) \cos^2 \alpha$$

These are in the form used for ordinary transit work. The reduction of the field-notes can be best accomplished, however, by the use of tables, diagrams or stadia slide rules, all of which are based upon these formulas.

The rod intervals are usually read to the nearest hundredth of a foot, giving the distance to the nearest foot only, so that in most topographic work, where the distances are not required to within a foot, the constant $(F + c)$ may be neglected, and in such work there will be no need of applying the horizontal correction for vertical angles under 3 degrees.

30. Stadia Fieldwork

Points are Located by (1) the azimuth, (2) the distance, and (3) the angle of elevation or depression. If elevations are not required the vertical angle is read only to the nearest 10 minutes, for that is close enough for determining the horizontal distance, but if elevations are required the vertical angle should be read to the nearest minute and sometimes to the half-minute

and the index correction also read for each vertical angle. The distance and azimuth angle are read with whatever refinement is necessary for the work in hand; as a rule the distances are recorded to the nearest foot and the azimuth to the nearest minute, altho for single "shots" which are to be plotted by use of the ordinary protractor the azimuth to the nearest 5 minutes is close enough. Traverse lines which control the survey are measured by tape or by stadia as the accuracy of the survey may demand. For large-scale maps it may be necessary to tape the distances of the control, but for small-scale maps the stadia method is sufficiently accurate. In the latter case the distances should be read both on the foresight and backsight.

Azimuths. In starting the survey, if the true azimuth of any line is known all of the azimuths of the survey may refer to that meridian, but if no meridian has been established, as is frequently the case, the azimuths may be referred to the magnetic meridian. Azimuth angles are read from the south right-handed for 360°. The horizontal circle is set on 0°, the telescope turned so that it points toward magnetic S and the lower clamp tightened. Then if the upper clamp is loosened and the telescope sighted toward the next station the horizontal arc, if read right-handed, will give the azimuth of the first line referred to the magnetic meridian. The azimuth of any other line may be obtained by simply resetting the upper clamp as each sight is taken. When it is necessary to move to the next instrument station the telescope is oriented, by the method explained in Art. 15, so that it will read 0° when pointing to the south. The bearings of the traverse lines will give a rough check on the azimuths of those lines.

Distances are read by setting one of the stadia hairs on a whole foot-mark, by means of the vertical arc clamp and tangent screw, and counting the feet, tenths and hundredths of a foot between the stadia hairs. Great care should be taken not to mistake the middle horizontal cross-hair for a stadia hair. This can be avoided by always making a mental estimate of the distance. Occasionally a half-interval may be read when something obstructs the view of the whole interval.

The Vertical Angles are taken by sighting the middle cross-hair on a point on the rod whose distance above the foot of the rod is equal to the distance from the horizontal axis of the telescope to the station beneath the transit. The distance is known as the HEIGHT OF INSTRUMENT (H.I.); it is not the same as the H.I. in ordinary leveling, which is the distance of the telescope above the datum. When the middle hair is sighted at the H.I. point on the rod the line of sight is parallel to a line drawn from the station under the transit to the foot of the rod, and the vertical angle read is the inclination of this line, so that the difference in elevation of the two stations can be directly computed.

It is sometimes impossible to set on the H.I. point on the rod owing to obstacles in the line of sight. In such a case the angle is taken when sighting at some convenient foot-mark on the rod, and this reading of the middle hair on the rod is recorded in the notes with the vertical angle. The H.I. point is sometimes marked by a red cloth band; its position being changed at each new set-up of the instrument.

The Order of Fieldwork is as follows: First read the distance and record it, then set the middle hair on the H.I. point of the rod and the vertical hair on the rod. Signal the rodman to go to the next point, and while he is moving to the next point read the horizontal circle, the vertical arc and its index correction if necessary. Unless elevations are required the H.I. is not used, and the vertical arc will not be read except for angles of 3° or more.

Stadia Notes require a large amount of sketching and description of details in order to convey sufficient information to enable a draftsman who may not be familiar with the locality to plot the notes. Where the notes are merely for flat topography, with no elevations, the difficulties are not great. The side shots are numbered, and these numbers should be put on the sketch in such a manner as not to be mistaken for any measured distance which may also be recorded on the sketch. Where the survey is for a contour map the accuracy of the map depends in a large measure upon the completeness of the descriptions given in the notes regarding the slope of the ground between located points. The elevation of points on ridges, valleys, knolls, and depressions will be determined in the field, but without a full description of the slopes of the surface the correct interpretation is often next to impossible. One method is to describe the shape of the ground in the notes; another method is to sketch contours in the notebook, showing approximately the form of the surface.

31. Stadia Computations

Reduction of Stadia Field Notes is usually done by means of tables, diagrams or stadia slide rule rather than by direct use of the formulas. The stadia slide rule is the most convenient when the reductions are required in the field as in plane-table surveys. The difference in elevation can be most rapidly obtained by means of a diagram or stadia slide rule, while the horizontal distances can be rapidly computed by use of a table of HORIZONTAL CORRECTIONS which gives the distances to be subtracted from the inclined readings to obtain the horizontal distance, to which must be added the instrument constant $(F + c)$. The following stadia reduction tables give, for vertical angles up to 20°, the difference in elevation for an inclined distance of 100 ft. If the inclined reading is 612, the instrument constant 1 ft, and the vertical angle 4° 42′, the difference in elevation = 6.13 × 8.17 = 50.1 ft. Since the rod intervals are read as a rule to only three significant figures this multiplication can be accomplished with sufficient accuracy by means of the ordinary 10-inch slide rule. The 50.1 ft is recorded in the notes and this difference in elevation is applied to the elevation of the station over which the transit is set to give the elevation of the point desired. It will be seen from an inspection of the tables that a half minute greater vertical angle would have increased the difference in elevation by a little less than 0.1 ft, which shows the importance of reading the vertical angle carefully and of applying its index correction.

Even without this table vertical heights may be computed if a table of horizontal corrections is available. A condensed table of horizontal corrections can be made which can be pasted on a leaf in the back of the notebook. After the horizontal distance has been found by the use of this table the vertical height can be computed by multiplying the horizontal distance by the tangent of the vertical angle, which can be conveniently accomplished by use of the tangent scale on an ordinary slide rule.

Where the vertical angle is not taken to the H.I. mark on the rod this must be taken into account in the working up of the column of differences in elevation as follows. Suppose at a point the distance read was 135, vertical angle − 8° 50′ on 2.5, H.I. 4.5; then $-(1.36 \times 15.17) + 2 = -18.6$ ft. Here the 2 ft is + because the vertical angle was taken to a point 2 ft lower than the H.I. point. Had the vertical angle been a + angle the 2 ft would still have been applied so as to increase the difference in elevation.

Beaman's Stadia Arc. Another method of simplifying the calculations of elevations in the field consists in using only those vertical angles for which the differences in elevation are simple multiples of the rod interval. This principle is used in the attachment devised by Beaman for the vertical arc of the transit or plane table. On this attachment are marked the numbers giving multiples of the rod interval, the zero graduation being marked

Reductions of Stadia Observations

VERTICAL HEIGHTS FOR 100 FEET INCLINED DISTANCE

Minutes	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45
2	0.06	1.80	3.55	5.28	7.02	8.74	10.45	12.15	13.84	15.51
4	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56
6	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62
8	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67
10	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73
12	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78
14	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84
16	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89
18	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95
20	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00
22	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06
24	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11
26	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17
28	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16.22
30	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28
32	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33
34	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39
36	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44
38	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50
40	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55
42	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61
44	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66
46	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72
48	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77
50	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83
52	1.51	3.[illegible]6	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88
54	1.57	3.31	5.05	6.79	8.51	10.22	11.93	13.61	15.28	16.94
56	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99
58	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05
60	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10

HORIZONTAL CORRECTIONS

Dist.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
100	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5
200	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9
300	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4
400	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8
500	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3
600	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7
700	0.0	0.2	0.8	1.9	3.4	5.3	7.6	1[illegible].4	13.6	17.2
800	0.0	0.2	1.0	2.2	3.9	6.1	8.7	[illegible]1.9	15.5	19.6
900	0.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1
1000	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5

Reductions of Stadia Observations—Continued

VERTICAL HEIGHTS FOR 100 FEET INCLINED DISTANCE

Minutes	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
0	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.1[illegible]
16	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.1[illegible]
18	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.1[illegible]
20	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.2[illegible]
22	17.70	19.32	20.92	22.49	24.04	25.55	27.04	28.49	29.90	31.2[illegible]
24	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.3[illegible]
26	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.3[illegible]
28	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.4[illegible]
30	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.4[illegible]
32	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.5[illegible]
34	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.5[illegible]
36	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.[illegible]
38	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.[illegible]
40	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.[illegible]
42	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.[illegible]
44	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.[illegible]
46	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.[illegible]
48	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.[illegible]
50	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.[illegible]
52	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.[illegible]
54	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.[illegible]
56	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.[illegible]
58	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32[illegible]
60	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78	32[illegible]

HORIZONTAL CORRECTIONS

Dist.	10°	11°	12°	13°	14°	15°	16°	17°	18°
100	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9.5
200	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1
300	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6
400	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2
500	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7
600	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3
700	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8
800	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4
900	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9
1000	30.2	36.4	43.2	50.6	58.5	67.0	76.0	85.5	95.5

50 instead of 0 so as to make it easier to determine which are plus and which minus angles. In using a transit with this attachment the observer reads the distance, turns the tangent screw of the telescope until the index line is opposite some line on the attached arc, and then notes the rod reading of the middle cross-hair. The difference in elevation between the telescope and the reading on the rod is (50 minus the reading of the arc) times the rod interval. From this result must be subtracted the rod reading to give the difference in elevation between the telescope and the foot of the rod. Evidently no stadia tables or slide rule are needed when this attachment is used.

32. The Plane Table and its Use

The Plane Table is an instrument by means of which points are located in the field by graphical methods directly on the map, which is fastened to a table top supported on a tripod. The accuracy of the map is usually limited by the accuracy of plotting rather than by the field measurements, for it is used mostly for small-scale maps. In the field the map must be protected from becoming distorted by moisture, so as to preserve the accuracy of the plot. The plane table is the only surveying instrument admitting of a rapid solution of the Three-point Problem in the field; this makes it practicable to locate stations independently of each other, so that errors cannot accumulate as they do in traversing. The most important advantage of the plane-table method over other topographic methods, however, is that all of the sketching is done in the field, where the topographer can see the form of the ground that he is mapping. He can sketch details at once in their proper position, without burdening his memory and without making elaborate notes. For this reason the details may be accurately sketched from a much smaller number of located points than would be required, for instance, by the transit and stadia method.

The plane-table method has the disadvantage of requiring more time for the fieldwork than other methods, and it is also more dependent upon favorable weather than a method where the map is not exposed. But taking into account both the field and office work the plane table will prove to be much more economical than the transit and stadia for work in open country, and at the same time the results obtained will be sufficiently accurate for most topographic work.

The Plane Table itself consists of a board, usually about 24 by 30 in mounted on a tripod with a device for leveling and clamping the board, the Johnson ball-and-socket leveling device and clamp being the most commonly employed. The ALIDADE consists of a telescope mounted on a horizontal axis resting in wye supports, which are connected to a metal column at the base of which is attached a flat piece of metal about 18 in long having both edges straight and parallel to the telescope. On this base are two spirit-levels for leveling the table. The telescope has a vertical motion only and a vertical arc. The entire instrument is moved about in azimuth on the board and sighted as desired.

Locating Points by Intersection. In order to begin a survey with a plane table it will in general be necessary to have on the map two plotted points corresponding to two points on the ground the distance between which is known and at least one of which can be occupied with the table. The simplest method of locating points by means of the plane table without measuring any distance is as follows. The base-line *ab* is plotted on the plane-table sheet, representing, to some scale, the measured base *AB*. The table is set so that *a* on the map is vertically above *A* on the ground and the table is leveled; then one edge of the alidade is placed along the base-line *ab* drawn on the map, the table is then turned in azimuth until the telescope sights the signal *B*, and the horizontal motion clamped. The line *ab* is now parallel to *AB* and the table is said to be oriented. The alidade is then placed so that the straight-

edge passes through a, the telescope is sighted to some signal C, and an indefinite line drawn toward C. The point c on the map (representing the signal C) is somewhere on this line. If the table is now moved to B (b being set vertically over B) and the process of orienting the table and sighting toward C is repeated, the point c is located on an indefinite line through b; hence it lies at the intersection of these two lines ac and bc. The triangle abc is similar to the triangle ABC, and each line on the map is parallel to the corresponding line on the ground. In a similar manner any number of points may be located.

Locating Points by Direction and Distance. The simplest way of locating points by the plane table is by obtaining the direction with the alidade and measuring the distance by stadia. This is the method most commonly used for filling in the details of a plane-table survey after the table has been oriented.

Locating Points by Resection. It sometimes happens that it is desired to locate a plane-table station from a base only one end of which can be occupied with the table. In such a case we may proceed as follows. Let A and B represent the points on the ground at the ends of the base-line; C is the signal which is to be located, and ab represents the base-line plotted on the plane-table sheet. Set up at A, the end of the base which is accessible; and orient the table by sighting B with the alidade along ab. Then, centering the alidade on a, draw an indefinite line toward C. This line should be drawn the full length of the alidade. The table is then taken to C and oriented by means of the indefinite line just drawn. Since the position of c on the indefinite line is not known it is necessary to estimate its position on the map and to use this point in placing the table over the point C. If the alidade is now centered on b and sighted toward B, a resection line may be drawn, and this line will cut the first indefinite line, thus locating the point c desired. Thus, without going to station B, the same line has been drawn on the map that would have been obtained by an intersection from B. It is evident that this resection method may be advantageous even if B could be occupied, for the point C has been located without taking the time required to go to station B. The position of c found by this method should be checked if possible by resection lines from other points whose positions are known to be correct.

The Three-Point Problem. One of the great advantages of the plane table is that it may be set up at any place where three triangulation points (plotted on the sheet) can be seen and the position of this plane-table station located on the sheet simply by observations from this point. The position of the plane table is found by means of the so-called Three-Point Problem, which, in this case, is an application of the principle of resection. The two graphical solutions of this problem chiefly used in plane table surveying are known as (1) LEHMANN'S method, or the TRIANGLE-OF-ERROR method; and (2) BESSEL'S method, or the INSCRIBED QUADRILATERAL. The former is a trial method, but it is the more rapid of the two for ordinary work and is used in practise far more than the latter method. Bessel's method gives a direct solution and consequently requires less experience than the former. It has the disadvantage, however, that in certain positions of the signals a part of the required geometric construction falls outside the limits of the plane-table sheet, in which case the solution is not practicable.

Lehmann's Method. If three signals A, B, and C have their plotted positions at a, b, and c, and if the table be set up at D and oriented correctly, the resection lines drawn from a, b, and c will all pass thru d, the plotted position of D. Since there is no means of accurately orienting the table, the position of d being unknown at the start, the table must be oriented approximately by estimation. If the plane table is not oriented exactly the three resection lines will not ordinarily pass thru a common point but will form a triangle known as the triangle of error (Fig. 24). From this triangle of error the true posi-

ion of d may be estimated, and by a second trial a new triangle of error may be obtained which is smaller than the former. By successive trials this triangle may be made so small hat it is almost a point. In practise very few trials are necessary, the triangle often being educed to a point in the second trial, so that the method is in reality a rapid one.

If the table is on the circumference of the circle thru the three signals its position is ndeterminate. When point D is inside the triangle ABC it is in a favorable position for n accurate location. If the table is outside this riangle there are certain positions of the signals which are not favorable, especially when the ngles subtended by the sides of the triangle ormed by the signals are small and the middle ignal is farthest from D, but if the middle signal s near D the location of d is strong.

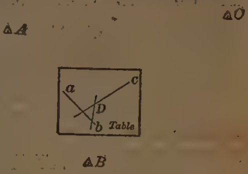

Fig. 24

If D lies inside the triangle ABC then d will ie inside the triangle of error and vice versa. If circle is passed thru a, b and the intersection f the resection lines from a and b it will pass hru the true position of d; similarly a circle thru , c and resection lines from b and c may be ketched and in this manner a close estimate of the position of d made for the second trial. The distance of d from any resection line is proportional to the distance of the table from he signal from which that line was drawn.

33. Hydrographic Surveying

Hydrographic Surveying is the term applied to the processes used in urveying any body of water. The determination of the topography of the ottom of a lake, harbor, or other body of water is one of the common probems. In connection with such surveys the character of the material comosing the bottom is often desired. The Fieldwork is usually done by irst establishing certain points on shore, by triangulation or traverse, to which the hydrographic survey may be referred, and then measuring (usually rom a boat) the depth of the water at various points and determining the osition of these points. Besides the ordinary transit and tape outfit, a ounding-pole or lead-line and a boat with the necessary equipment will be equired. A tide gage should be set up in tidal waters and in lakes where he water level changes rapidly. The sounding-poles are made similar to elf-reading rods graduated to tenths of a foot. A shoe is sometimes attached o the bottom, provided with a cup-shaped cavity which, if smeared with allow or soap, enables samples of material to be collected. The lead-line onsists of a long chain or a hemp or cotton line at the end of which is ttached a lead weight. A brass sash-chain gives very satisfactory results, with cloth tags of various colors for foot-marks. Where there is not much current a 6 to 10 pound weight will suffice for depths up to 40 ft.

Methods of Locating Soundings. There are six general methods of ocating soundings, as follows: (1) The boat is rowed on a range at a uniform rate of speed and the soundings are located by time intervals. (2) The oat is rowed on a range line and the positions of the soundings are "cut in" by a transit angle taken on shore or by an angle taken with a sextant from the boat. (3) The boat may or may not be rowed on any definite range, and its position is located by angles taken simultaneously by two transits on shore or by angles taken simultaneously to shore points from the boat by means of two sextants. (4) The positions of the soundings are located by the stadia method. (5) The positions of the soundings are defined by the ntersection of fixed ranges. (6) A wire or line is stretched across a stream from shore to shore and soundings are taken at different points along this wire and located by measured distances from one end of it.

Locating a Sounding by a Range and an Angle. In still water where there is no difficulty in keeping the boat in any desired position soundings may be conveniently located by keeping the boat on a range line of known position marked by two objects on shore, such as range poles, and then "cutting in" the position of the leadsman by means of a transit angle taken from shore at the instant the sounding is made. The ranges may be fan-shaped if the pivot-point of the system of ranges is some steeple or other object located far enough back from the shore so that the lines will not diverge too rapidly. The recorder writes in his notebook the depths as they are called off by the leadsman and also the times when the soundings are made. He also notifies the leadsman by calling out "Sound" about 5 seconds before each sounding is desired. The leadsman takes the soundings as quickly as possible, usually within two or three seconds of the desired time. In the hydrographic work of the Corp of Engineers, U. S. A., where nearly all of the soundings are located by two angles taken with transits on shore, soundings are usually taken at 15, 20, or 30 second intervals, and a location made each minute by the instrument men at the instant the signal is given by the signalman in the boat. The chief of party usually acts as signalman and directs the work in the boat, and sees that the boat is kept on the ranges (if any are used) and that the boat is so propelled as to properly cover the area to be sounded. The signal is given by holding up a flag for about 10 seconds and dropping it suddenly the instant the sounding is taken, at which moment the transitmen on shore take angles to the leadsman or to his hand if visible. In the work of the U. S. Engineers, white, red, and sometimes black flags are used for signaling. Both the recorder's and the instrument men's notes should show the colors of the signal as well as the time for each located sounding, thus giving two means of identifying the angles and the corresponding soundings. This double check is of particular value where the lines of soundings are long. In tidal waters the time record is required also as a means of reducing the soundings to Mean Low Water or to whatever other datum is used. The tide gage readings and times are recorded throughout the day.

Where the soundings are taken with a view to obtaining every slight change in the slope of the bottom, instead of taking the soundings at a given interval of time, it is desirable to take them as frequently as the leadsman can conveniently handle the pole. In this method the boat is usually rowed on a range and the instrument men on shore "cut in" only those soundings that are designated by the signalman in the boat. The time in this case is recorded to the nearest second. As a rule every sixth or eighth sounding is located.

In plotting notes like these where the soundings are quite close together the points which were "cut in" are located on the plan and the intermediate readings are interpolated between them; the soundings are assumed to be equally spaced between the located ones.

Locating a Sounding by Two Angles from a Boat. A common method of locating soundings when ranges are not used is by taking two angles simultaneously from the boat to three signals, or any previously determined points, A, B, and C, on shore. This is an application of the Three-Point Problem which is frequently used in plane-table work. It is essential that the signals or points should be fixt stations; such points as buoys or floats will not give satisfactory locations.

In measuring angles from a boat it is necessary to employ some instrument which does not require a steady support like the transit. For this reason the angles are usually taken with two sextants These two angles are sufficient to locate the position of the soundings, except in the one case where the boat happens to be on the circumference of the circle passing thru the three signals between which the angles are measured.

This method is less frequently employed than the "range and angle" method or the "two angles from shore" method, because it often happens that the two angles taken with

the sextant do not give good intersections; this is especially true when the soundings extend far from shore. Furthermore, if the signal happens to be high above the shore, and consequently not at the level of the boat, the angle measured will be enough different from the horizontal angle to introduce serious error into some parts of the work.

The Sextant is an instrument adapted to measuring angles in any plane. Owing to the fact that it can be used by an observer who is on a moving object, such as a boat, it is especially valuable for hydrographic work. It is employed not only for taking angles from a boat in locating soundings but is also in common use for making astronomical observations which are necessary in determining the latitude, longitude, and time at sea. The FRAME *ABI* (Fig. 25) is usually of brass, on the under side of which is attached the

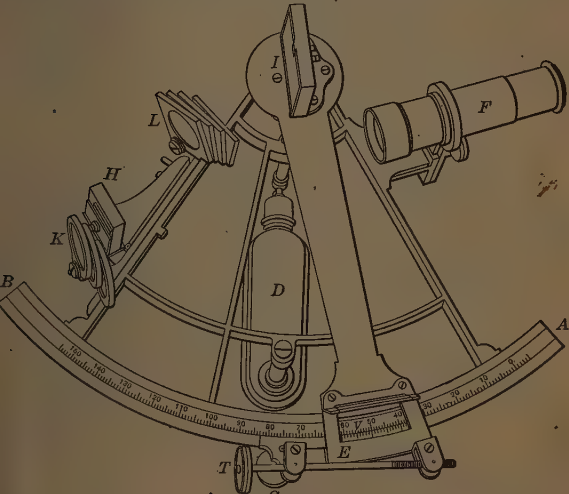

Fig. 25. The Sextant

wooden handle *D*. The index arm *IE* is pivoted at *I*, the center of the arc *AB*, and this arm can be swung around *I* as a center so that the vernier *V* can pass from *A* to *B* on the arc (or limb) and can be set at any position on the arc by means of the clamp *C* and tangent screw *T*. At *I* is a plane glass mirror called the index glass; it is attached rigidly to the index arm and perpendicular to the plane of the sextant, its reflecting surface being over the pivot about which the index arm revolves. Rigidly attached and perpendicular to the frame of the sextant is the plane horizon glass *H*, the upper half of which is transparent, while the lower half is a mirror; this glass is set so that its plane is parallel to the index glass when the vernier is set at o°. The telescope is at *F*. *K* and *L* are colored glasses which are hinged so that they can be swung around the pivot into the path of the rays of light to protect the eye of the observer in making observations on the sun.

The limb AB is graduated into spaces which are really half-degrees, but on account of the construction of the instrument each of these is marked as a whole degree, so that the scale has an extent of 120 degrees. The graduations are so subdivided that the angles can be read in most instruments to 10 seconds; in some of the smaller instruments the vernier reads only to half-minutes. In the ordinary sextant the arm is from 5 to 8 in in length. A pocket sextant having an arm about 2 in long is very convenient for reconnoissance surveys and for filling in the details of more accurate surveys.

To Measure an Angle with the sextant, hold the instrument by its handle in the right hand and turn it so that the plane of the sextant coincides with the plane thru the two objects to be observed, with the telescope on the upper side of the sextant if the angle is horizontal, or on the left-hand side in the case of a vertical angle. Without changing the plane of the sextant twist it in the hand so as to turn the telescope toward the left-hand object and observe it through the upper (transparent) portion of the horizon glass. Then, holding the instrument as steady as possible, turn the index arm with the left hand until the other object appears in the silvered portion of the horizon glass opposite the first point. Bring the second point exactly opposite the first one by means of the clamp and tangent screw of the index arm. The coincidence of the images should be tested by twisting the instrument a little so as to make the reflected image move back and forth across the direct image. Read the vernier and apply the index correction. The telescope is not used for rapid work such as locating soundings; the sight is simply made thru the ring in which the telescope fits.

34. Photographic Surveying

Surveying by Photography is one of the most rapid methods of locating topographic details, and consequently it is one of the cheapest methods of making a map. It is best adapted to maps of very small scale, say 2 or 3 miles to an inch. The accuracy to be expected from this method is not as great as that given by the plane table. To locate a point on a map by means of photographs it is necessary that the point should appear on two photographs taken from two different camera stations, and in addition the following data must be known: the positions of the two camera stations, the direction in which the camera was pointed in each case, and the focal length of the lens. If a photographic print is held vertically in front of the eye at a distance equal to the focal length and in the same direction as when the exposure was made, then every point on the print will appear to cover its corresponding point on the ground as shown in Fig. 26. Hence the print when held at this distance gives a measure of the angles between points shown in the view.

Fig. 26

The Camera is provided with spirit-levels for making the plate vertical, with notches near the edges of the plate showing the position of the true horizon, and a vertical line thru the center of the plate called the principal line. The horizontal direction of any point in the picture is given by the angle it makes with the principal line. This angle is determined by the relation

tan horizontal angle = dist. from principal line/focal length.

Plotting. When the position of the station and the focal length are known, a circle may be drawn with the station as center and the focal length as radius, which is the locus of the centers of all pictures taken from this station. When the direction of the view is known the position of the picture may be put on the plan. Distances may then be scaled off the print or negativ from the different points to the principal line and laid off on the line on the plan representing the picture. If radial lines are drawn from the station to these various points then the plotted positions of the points will be somewhere on the corre-

sponding line. Another set of these radial lines from another station will give locations for all points shown in both views.

Points may also be located vertically by means of angles determined by the picture. The distance of a point above the horizon line (on print) divided by the horizontal distance from the station to the point on the print is the tangent of the vertical angle. Since the actual distance to the point may now be scaled from the map the difference in elevation may be computed by proportion, or found graphically by construction. The details of the map are sketched from the picture after the prominent points have been plotted.

35. Mine Surveying

In Mine Surveys, all accurate measurements are made with the transit and steel tape. Transits used in this work are provided with an auxiliary telescope attached to one side of or above the main telescope, so that it is possible to sight down a vertical shaft. The instrument known as the "eccentric bearing" transit has one telescope which can be used either in its center bearings as in an ordinary transit or in bearings built out over the limb of the instrument so as to take vertical sights.

Transferring a Meridian into the mine is one of the important steps. This is usually done by setting up the transit at the mouth of the shaft and after taking a backsight on a surface station to take a foresight down the shaft, the line of sight being made as much inclined as possible, and the inclined distance measured. The transit is then set up at the bottom of the shaft, a backsight taken on the top station and the traverse carried into the mine. The utmost care must be taken to eliminate all errors of adjustment of the transit, for the accuracy of all the underground survey depends upon this process. Another and probably more accurate method of transferring a meridian into a mine is by means of two heavy plumb-bobs hung from the staging above the shaft. The transit is set up both above and below on this line and thus an azimuth line is established between the surface and the workings. In order to have as long a base as possible the plumbs should be hung near the shaft casing.

Underground Traverses are run thru the passages in the mine. It is often necessary to introduce very short lines into the traverse, and since the azimuth is transferred to distant parts of the mine thru these short lines great care must be taken to eliminate all instrumental errors by reversing the telescope and using the mean of the two results. The positions of walls and workings are located by measurements from the traverse line. The station points are often placed in the roof; a nail in a wooden plug is a good station mark.

Notes of Mine Surveys are kept as a rule in the form of sketches, especially the details, such as the location and extent of the stopes. The different station points of the survey are numbered consecutively. For convenience, stations on the first level are numbered 101, 102, etc., on the second level 201, 202, and so on. A column for vertical angles is required, for most of the traverse lines are inclined.

To Plot the Survey the three sets of coordinates must be computed, which give all the data necessary to plot the mine in plan, longitudinal section, and transverse section. The horizontal and vertical distances equal the inclined distance multiplied by the cosine and sine respectively of the vertical angle, and the azimuth or bearings together with the horizontal distances give a means of computing the latitudes and departures of the courses.

Surveying for Patent. By patent proceedings are meant the proceedings necessary to obtain from the government a fee-simple deed of a mining claim. Title to metalliferous land, as granted by the United States, conveys the right to all minerals included in the downward prolongation of the portion of vein cut off by the vertical end boundaries of the claim. The Federal law allows a claim to cover 1500 ft located along the direction of a vein and 300 ft of surface ground on either side of it. These are maximum dimensions and may be reduced by local laws. For further information see the "Manual of Instructions for the Survey of the Mineral Land of the United States," issued by the General Land Office, Washington, D. C.

ASTRONOMIC OBSERVATIONS

36. Definitions

Practical Astronomy treats of the theory and use of astronomical instruments and the methods of computing the results obtained by observation. In this branch of astronomy only the relative directions of heavenly bodies are considered, not the actual distances, hence it is convenient to regard all such bodies as situated on the surface of a sphere, called the CELESTIAL SPHERE, Fig. 27, whose radius is infinite and whose center is the center of the earth. The VERTICAL at any point on the earth's surface is the direction of the force of gravity at that point. This direction is shown by the plumb-line. The vertical produced upward pierces the celestial sphere in a point called the ZENITH. The point where this line pierces the sphere on the opposite side is the NADIR. A plane thru the center of the earth perpendicular to the vertical cuts the sphere in a great circle called the HORIZON. It is sometimes called the true horizon to distinguish it from the sea horizon. The latter is a small circle, below the true horizon, and whose plane is parallel to the plane of the true horizon.

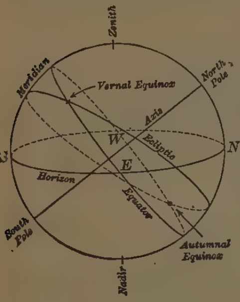

Fig. 27

The Earth's Rotation upon its axis causes all celestial bodies to appear to revolve in the opposite direction at the same rate. To an observer looking along the axis from north to south all bodies appear to revolve in a right-handed, or clockwise, direction. The AXIS of the celestial sphere is the earth's axis produced. The POLES are the points where this axis pierces the sphere. The CELESTIAL EQUATOR is a great circle of the celestial sphere midway between the poles; its plane coincides with the plane of the earth's equator. The MERIDIAN of an observer is that great circle which passes thru his zenith and the poles; its plane contains the observer's vertical. The intersection of the plane of the meridian and the plane of the horizon is the MERIDIAN LINE. This line cuts the horizon in the north and south points. The PRIME VERTICAL is a great circle thru the zenith whose plane is perpendicular to the meridian. It cuts the horizon in the east and west points.

The ECLIPTIC is a great circle of the celestial sphere which the sun appears to describe during the year. It is inclined to the equator at an angle of nearly 23° 27′. The points of intersection of the equator and the ecliptic are the EQUINOXES, and the points on the equator midway between the equinoxes are the SOLSTICES.

37. Astronomic Coordinates

Two Spherical Coordinates may be used to designate the position of any point on the sphere. The circles of reference in any system are first, a great circle called the primary, and second, a system of great circles perpendicular to the primary, called secondaries. The following systems are commonly used.

In the Horizon System the circles of reference are the horizon and great circles perpendicular to the horizon, called VERTICAL CIRCLES (Fig. 28).

The coordinates are called ALTITUDE and AZIMUTH. The altitude of a point is its angular distance above the horizon measured on a vertical circle through the point. The complement of the altitude is called the ZENITH DISTANCE. The azimuth of a body is the arc of the horizon included between the meridian and the vertical circle through the body. It is commonly measured from the south point westward from 0° to 360°; it is sometimes measured from the north point.

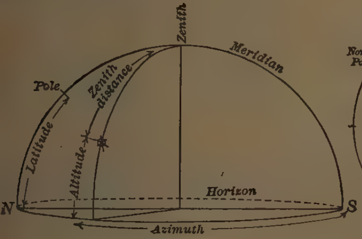

Fig. 28

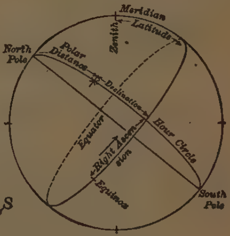

Fig. 29

In the Equator System (Fig. 29) the circles of reference are the equator and great circles at right angles to the equator, called HOUR CIRCLES. The coordinates are called DECLINATION and RIGHT ASCENSION. The declination of a point is its angular distance north or south of the equator. Declinations are considered positiv when north and negativ when south. The complement of the declination is called the POLAR DISTANCE. The right ascension of a point is the arc of the equator between the vernal equinox and the hour circle through the point. It is reckoned eastward from the equinox from 0° to 360° or from 0^h to 24^h. The position of a point may also be designated by its DECLINATION and HOUR ANGLE. The hour angle is the arc of the equator between the meridian and the hour circle thru the body. It is reckoned from the meridian westward from 0° to 360° or from 0^h to 24^h. It will be seen that declination and right ascension are independent of the observer's position, while the coordinates of the horizon system are dependent upon the observer's position.

Coordinates of the Observer. The observer's position is defined by his latitude and longitude. The LATITUDE is the angular distance of the observer's zenith north or south of the equator. The LONGITUDE is the arc of the equator between the meridian of Greenwich (or other primary meridian) and the meridian of the observer.

The American Ephemeris and Nautical Almanac contains the right ascension and declination of the sun, moon, principal planets and many of the stars, given in most cases for the instant of Greenwich Noon each day. For any other instant these coordinates must be obtained by interpolation, the rate of change per hour being given for that purpose. The Almanac also contains other data required in reducing astronomical observations.

Transformation of Coordinates. For a BODY ON THE MERIDIAN let Fig. 30 represent the plane of the meridian, and EZ the distance of the zenith north of the equator, or the latitude. PN is the altitude of the pole. The arc PN = arc EZ. Hence the altitude of the pole equals the latitude of the observer. If m be a point on the meridian the relation between the altitude h, the declination D, and the latitude L, is shown by the equation $90° - L = h - D$. If m is south of the equator the equation still holds true if D is considered

to be negativ. For a point between the zenith and the pole the relation is exprest by $L = h - p$, where p is the polar distance of the point. For a point below the pole the equation becomes $L = h + p$. For a BODY OFF THE MERIDIAN Fig. 31 shows the relation between the altitude, declination, azimuth and hour angle of point S, and the latitude of Z, found by

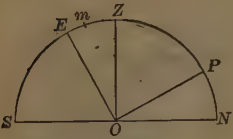

Fig. 30

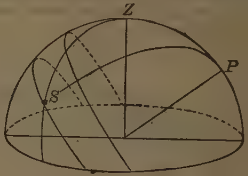

Fig. 31

solving the spherical triangle PZS. The sides of the triangle are the colatitude PZ, the zenith distance ZS, and the polar distance PS. The angles are P the hour angle, Z the azimuth, and S the parallactic angle. The solution may be effected by means of the ordinary formulas of spherical trigonometry, but it is usually more convenient to apply special formulas derived from the fundamental relations; these will be given in the following paragraphs when required.

38. Measurement of Time

Measurement of Time. Every celestial body passes the meridian twice each day in describing its apparent path around the earth, once on the upper side, that containing the zenith, and once on the lower side. These are called the upper and lower transits or culminations. A SIDEREAL DAY is the interval of time between two successive upper transits of the vernal equinox over the same meridian. A SOLAR DAY is the interval of time between two successive upper transits of the sun's center over the same meridian. The SIDEREAL TIME at any instant is the hour angle of the vernal equinox. The relation between sidereal time S, hour angle t, and right ascension R is given by the equation $S = R + t$. If the body is on the meridian, $t = 0$, and for this instant $S = R$. The SOLAR TIME at any instant is the hour angle of the sun's center.

Mean and Apparent Time. The apparent angular motion of the sun is not uniform, hence for ordinary purposes it is more convenient to use the time kept by a fictitious sun, or MEAN SUN, which moves along the equator uniformly at the average speed of the true sun. Time kept by this mean sun is MEAN SOLAR TIME. Time kept by the true sun is APPARENT SOLAR TIME. The difference between the two is the EQUATION OF TIME, which is given in the Nautical Almanac for Greenwich Noon each day.

Example. Given the local apparent time $8^h\ 51^m\ 52^s$ A.M. on Jan. 9, 1909, in longitude $4^h\ 44^m\ 18^s$ west. At Greenwich Mean Noon the equation was $7^m\ 08^s$, to be added to the apparent time. The increase was $1^s.0$ per hour. The Greenwich time was known to be 2^h (nearly). The corrected equation of time is therefore $7^m\ 10^s$ and the mean local time was $8^h\ 59^m\ 02^s$ A.M.

Astronomical and Civil Time. In astronomical reckoning the day begins at noon, or when the sun's center is on the meridian. It is divided into 24^h. The civil day begins at midnight and is divided into two parts of 12^h each. From midnight to noon is called A.M.; from noon to midnight is called P.M. The civil day begins 12^h earlier than the astronomical day of

the same date. Examples: Astr. Time May 10, 15^h = Civ. Time, May 11, 3 A.M; Astr. Time Jan. 3, 7^h = Civ. Time Jan. 3, 7^h P.M.

Longitude and Time. The hour angle of the sun at any meridian is the local solar time at that meridian. The hour angle of the sun at Greenwich at that instant is the Greenwich time. The difference is the longitude of the place exprest in hours, minutes and seconds. Similarly, for the vernal equinox the result is the sidereal time at the two places and the difference of time is the longitude.

An angle exprest in hours, minutes and seconds of either solar or sidereal time may be exprest in degrees, minutes and seconds by multiplying by 15. Every circle may be divided into 24 hours, or into 360 degrees; hence $15° = 1^h$ for either solar or sidereal time; also $15' = 1^m$ and $15'' = 1^s$. In changing from degrees to hours the following relations are also convenient: $1° = 4^m$, and $1' = 4^s$.

Examples. To convert a west longitude of 166° 20′ 05″ into time: $166° 20' 05'' = 165° + 1° + 15' + 05' + 05'' = 11^h + 04^m + 01^m + 20^s + 0^s.33 = 11^h\ 05^m\ 20^s.33$. To convert $4^h\ 44^m\ 18^s.0$ west longitude into degrees: $4^h\ 44^m\ 18^s.0 = 60° + 11° + 04'\ 30'' = 71°\ 04'\ 30''$.

Solar and Sidereal Intervals of Time. On account of the earth's motion in its orbit the sun appears to move eastward among the stars nearly 1° per day. This causes the sun to reach the meridian later each day as compared with the time of transit of the equinox. For this reason the mean solar day is nearly 4^m longer than the sidereal day. All intervals of mean solar time are correspondingly longer than those of sidereal time. This retardation is just sufficient to bring the sun back to its starting point at the end of a year, so that the number of sidereal days in the year is just one greater than the number of solar days. The year contains 365.2422 solar days and 366.2422 sidereal days. Therefore 1 sidereal day = 0.99726957 solar day and 1 solar day = 1.00273791 sidereal days. Solar time might be converted into sidereal time and vice versa by means of these two equations, but a more convenient way is to calculate the correction to be added in the one case and subtracted in the other. This correction is nearly 10^s per hour. Values of these corrections have been worked out for all hours and minutes from 0^h to 24^h and will be found in the Nautical Almanac.

Example. Convert $9^h\ 26^m\ 32^s.120$ of solar time into sidereal time. From Table III, Appendix to Nautical Almanac,

For $9^h\ 26^m$, reduction is	$1^m\ 32^s.979$
For $32^s.120$, reduction is	$.088$
	$1^m\ 33^s.067$
	$9^h\ 26^m\ 32^s.120$
Corresponding sidereal interval =	$9^h\ 28^m\ 05^s.187$

To convert $9^h\ 28^m\ 05^s.187$ into mean solar time. Table II, Nautical Almanac, gives:

For $9^h\ 28^m$	$1^m\ 33^s.053$
For $05^s.187$	$.014$
	$1^m\ 33^s.067$
	$9^h\ 28^m\ 05^s.187$
Corresponding mean solar interval =	$9^h\ 26^m\ 32^s.120$

These results may be obtained with sufficient accuracy for many purposes by the following approximate rule. Express the given interval in hours and decimals, multiply by 10 and call the result seconds. Then subtract from this result 1^s for every 6^h in the interval. The result is the desired correction, correct within a few tenths of a second in ordinary cases. In the above example $9^h\ 26^m\ 32^s = 9^h.44$, which gives $94^s.4$. The amount to be subtracted from this is $1^s.6$, leaving $1^m\ 32^s.8$ as the correction to reduce the given interval to sidereal units.

Sidereal and Mean Time at any Instant. The relation between sidereal time S, the right ascension of the mean sun, R_s, and the hour angle of

the mean sun, t_s, is shown by the equation $S = R_s + t_s$. From the Nautical Almanac may be obtained the value of R_s at Greenwich Mean Noon, and by increasing this by the change for the number of hours in the west longitude it is reduced to the right ascension at local mean noon. The above equation may be used more conveniently in the form $S = R_s + t_s + C$, where C is the correction to reduce t_s to a sidereal interval and R_s is the right ascension for the instant of local noon. If it is desired to find the mean time the equation is Mean Time $= S - R_s - C'$, where C' is the correction to reduce $S - R_s$ to a solar interval. It will be seen that C and C' represent the increase in R_s since local noon.

Example. To find the local sidereal time when the local mean time is $9^h\ 22^m\ 18^s.6$, Jan. 7, 1907, in longitude $4^h\ 44^m\ 18^s.0$ West.

$$
\begin{aligned}
R_s \text{ at Greenwich Mean Noon} &= 19^h\ 03^m\ 36^s.38 \\
\text{Increase in } 4^h\ 44^m\ 18^s.0 &= 46\ .70 \\
\hline
R_s \text{ at local mean noon} &= 19^h\ 04^m\ 23^s.08 \\
t_s &= 9\ \ 22\ \ 18\ .60 \\
C &= \ 1\ \ 32\ .37 \\
\hline
S &= 28^h\ 28^m\ 14^s.05 = 4^h\ 28^m\ 14^s.05
\end{aligned}
$$

Whenever the resulting sidereal time exceeds 24^h the actual sidereal time is found by subtracting 24^h. When the sun's right ascension is larger than the sidereal time and is to be subtracted from it, 24^h may be added to the sidereal time in order to make the subtraction possible.

Example. To reduce $4^h\ 28^m\ 14^s.05$ sidereal time to the corresponding local mean time in longitude $4^h\ 44^m\ 18^s.0$ West. Take $S = 28^h\ 28^m\ 14^s.05$; then $S - R_s = 9^h\ 23^m\ 50^s.97$, and $S - R_s - C' = 9^h\ 22^m\ 18^s.60$, mean local time.

Standard Time. In the United States the territory is divided into time belts, all places in the same belt using the local mean time at one of the meridians named below. Eastern Time is that of the 75th meridian, Central Time that of the 90th, Mountain Time the 105th and Pacific Time the 120th meridian. By this arrangement the minutes and seconds of time are the same in all parts of the country and also at Greenwich, the clocks in the various belts differing by whole hours. For example, when it is mean noon at Greenwich it is 7^h A.M. in the Eastern belt, 6^h A.M. in the Central belt, 5^h A.M. in the Mountain belt and 4^h A.M. in the Pacific belt.

To convert local mean time into Standard Time, take the difference in longitude between the meridian of the place and the Standard Meridian, convert this into hours, minutes and seconds, and add it to the local time if the place is west of the Standard Meridian, subtract if east.

Example 1. Local Mean Time $9^h\ 26^m\ 14^s$ A.M. Longitude, $71°\ 04'$ West of Greenwich. Find the Eastern Time. $75°\ 00' - 71°\ 04' = 3°\ 56' = 15^m\ 44^s$. $9^h\ 26^m\ 14^s - 15^m\ 44^s = 9^h\ 10^m\ 30^s$, Eastern Time.

Example 2. Central Time, 8^h P.M. Longitude, $91°$. Find the local mean time. $91° - 90° = 1° = 4^m$. Then $8^h - 04^m = 7^h\ 56^m$, local mean time.

39. Altitudes

Correcting Measured Altitudes. When the altitude of a heavenly body is measured with a transit or a sextant it is necessary to apply certain corrections to the observed altitude in order to obtain the true altitude. REFRACTION is the bending downward of rays of light from a celestial body upon entering the atmosphere. This causes the object to appear too high above the horizon. To reduce an observed altitude to the true altitude it is necessary to subtract a correction for this refraction, the amount of the correction depending upon the altitude, upon the temperature of the air and upon the barometric pressure. For accurate work the correction must be taken from refraction tables. For rough computations it will be close enough

to take the refraction correction in minutes equal to the natural cotangent of the altitude, provided the altitude is not less than about 10°. For low altitudes the refraction correction is uncertain.

Parallax. An observer on the earth's surface sees an object at a lower altitude than he would if he were at the center of the earth. This is called the error of parallax. The displacement of the object is greatest when it is at the horizon and is zero when the body is in the zenith. The parallax for a body on the horizon is called its horizontal parallax. The correction for any altitude equals the horizontal parallax times the cosine of the altitude. The sun's horizontal parallax is about 9″; that of the moon is nearly a degree.

Dip. If the altitude is measured at sea it must be taken with a sextant and measured from the sea horizon. In this case the observed altitude must be referred to the true horizon by subtracting the angle of dip. The amount of the correction varies with the height of the eye above the surface of the sea. For ordinary observations the dip in minutes may be taken as the square root of the height of the eye in feet above sea level.

Semidiameter. If the object observed is the sun, the moon or a planet, the edge (or limb) of the disk may be observed and the altitude reduced to the center by adding or subtracting the semidiameter. The semidiameters are given in the Nautical Almanac for every day at Greenwich Noon. For approximate work the sun's semidiameter may be taken as 16′ in March and September, 15′ 45″ in June and 16′ 15″ in December.

40. Constellations near the Pole

Polaris, the Pole-Star, is a second-magnitude star situated about 1° 08′ from the pole. Its right ascension is about $1^h\ 30^m$; consequently at the instant when the sidereal time is $1^h\ 30^m$ Polaris is vertically above the pole, on the meridian. Directly above Polaris at this time ($1^h\ 30^m$) is the constellation Cassiopeia, shaped like an inverted letter W. Below Polaris is the great dipper, Ursa Major. A line through the two stars in the outer side of the dipper bowl points nearly to Polaris, and these stars are called the "pointers." As this constellation is a conspicuous one the pointers are easily recognized and may be used to identify the pole-star. The positions of these constellations are shown in Fig. 32. When the star is vertically above or below the pole it is said to be at culmination (upper or lower); when the star is at its extreme eastern or western position it is said to be at its greatest elongation (east or west). The figure shows the constellations as they appear when Polaris is at its upper culmination; by looking at it inverted they are seen as they appear at its lower culmination. By looking at the figure from the left and right margins of the page, the positions are seen for the western and eastern elongations of Polaris.

Fig. 32

41. Observations for Latitude

By the Pole-Star. If the altitude of the pole-star (Polaris) be observed when it is above the pole (upper culmination) and its maximum altitude

found by trial, then this will be the meridian altitude. The latitude is then found by the equation $L = h - p$. The measured value of the altitude h must be corrected for refraction as well as for any index error of the instrument. The polar distance p is found by looking up the declination of Polaris in the Nautical Almanac and subtracting it from 90°. If the star is at the lower culmination the polar distance must be added.

Example. Observed altitude = 43° 37′ 00″, index error = + 30″, and refraction correction = − 1′ 01″; then true altitude = 43° 36′ 29″. Since declination of star was 88° 44′ 35″, polar distance = 1° 15′ 25″, and hence latitude = 42° 21′ 04″.

By the Sun at Noon. If the maximum altitude of the sun be found at noon this may be taken as the meridian altitude. This altitude is found with the transit by setting the horizontal cross-hair on the lower edge of the sun and following it as long as it continues to rise. When the sun drops below the cross-hair the altitude is read from the vertical arc. This altitude should be corrected for index error, refraction, semidiameter and parallax. The latitude is then found from the equation $L = 90° - (h - D)$. This method applies also to a star when crossing the meridian.

Example. In longitude $4^h.74$ West, the observed altitude of the sun's lower limb was 25° 06′.0, index error = + 01′.0, refraction correction = − 02′.0, and sun's semidiameter = 16′.3; hence true altitude = 25° 21′.3. On this day sun's declination at Greenwich apparent noon was − 22° 19′ 33″, change in one hour = + 19″.59; hence increase in declination = 1′ 33″, and true declination = − 22° 18′.0. The formula then gives latitude = 42° 20′.7.

42. Observations for Time

By Transit of Star. If the line of sight of a transit instrument be placed in the plane of the meridian the time can be accurately determined by noting the instant by a watch or a chronometer when some known star passes the vertical cross-hair. The sidereal time at the instant is the same as the star's right ascension. If a sidereal chronometer is used the difference between the chronometer reading and the right ascension is the error of the chronometer on local sidereal time. If a watch is used, then this sidereal time must be converted into mean solar time of the meridian for which the watch is regulated. This is done by subtracting from the right ascension of the star the right ascension of the mean sun corrected to local mean noon. This result reduced to solar interval is the local mean time. This is reduced to standard time by taking the difference between the longitude of the place and that of the standard meridian, converting this into time, and adding it if the place is west of the standard meridian, subtracting if east.

Example. Transit of Star across Meridian. Longitude, $5^h\ 20^m\ 10^s.0$ West. Date April 5, 1902. Observed time of Transit of α Hydræ = $8^h\ 48^m\ 58^s.5$. Right Ascension of Mean Sun at G.M.N. = $0^h\ 51^m\ 24^s.6$. Increase in $5^h\ 20^m\ 10^s = 52^s.6$. Corrected Right Ascension = $0^h\ 52^m\ 17^s.2$.

Right Ascension α Hydræ	= $9^h\ 22^m\ 48^s.4$
Right Ascension Mean Sun	= 0 52 17 .2
	$8^h\ 30^m\ 31^s.2$
Reduction to Solar Time = C′	= 1 23 .6
	$8^h\ 29^m\ 07^s.6$
Reduction to Eastern Time	= 20 10 .0
Eastern Time of Transit	= $8^h\ 49^m\ 17^s.6$
Observed Time	= 8 48 58 .5
Watch slow	$19^s.1$

This method is the one used in the most precise observations for time except that corrections are introduced to allow for the non-adjustment of the

instrument, and the observations are recorded with great precision on an automatic register called a chronograph.

By Transit of the Sun. A similar observation may be made on the sun. The watch times of transit of the west and east edges of the sun across the meridian should be noted and the mean taken as the time for the center. This instant is $12^h\ 00^m\ 00^s$ local apparent time. This must be reduced to mean time by applying the equation of time and the result converted into standard time as already explained. The difference between this and the watch reading is the error of the watch on standard time.

Example. Transit of the Sun across the Meridian. Longitude, $71°\ 04\frac{1}{2}'$. Observed time West and East edges of sun, $11^h\ 30^m\ 08^s$ and $11^h\ 32^m\ 26^s$.

Local Apparent Time of transit of center	=	$12^h\ 00^m\ 00^s$
Equation of Time	=	$-12\ \ 07$
Mean Local Time	=	$11^h\ 47^m\ 53^s$
Reduction to Eastern Time		$-15\ \ 42$
Eastern Standard Time	=	$11^h\ 32^m\ 11^s$
Mean of Observed Times	=	$11\ \ 31\ \ 17$
Watch slow		54^s

Time by Altitude of Sun. The time may be determined by measuring the altitude of the sun's lower edge and noting the watch reading at the same instant. This altitude must be corrected for refraction, parallax, and semidiameter. The hour angle of the sun is then found by solving the *PZS* triangle for the angle at the pole. A convenient formula for this is

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\cos s \sin (s - h)}{\cos L \sin p}}$$

where t is the hour angle and $s = \frac{1}{2}(L + h + p)$. The latitude L must be known in order to solve the triangle; the polar distance p is found from the Nautical Almanac by subtracting the declination from 90°; h is the corrected altitude. If it is in the forenoon the resulting hour angle (converted into time) must be subtracted from 12^h in order to obtain the apparent time. This apparent time must be corrected for the equation of time and then reduced to standard time as before.

Example. Altitude of Sun's Lower Limb for Time. Latitude, $42°\ 21'.0$ N. Longitude, $4^h\ 44^m\ 18^s.0$ W. Date, Jan. 9, 1909. Mean of watch readings $8^h\ 45^m\ 01^s$ A.M. Mean of altitudes $12°\ 19'.6$.

Observed Altitude	=	$12°\ 19'.6$
Refraction	=	4.4
		$12°\ 15'.2$
Semidiameter	=	16.3
		$12°\ 31'.5$
Parallax	=	$.1$
True Altitude	=	$12°\ 31'.6$
Declination at G.M.N.	=	$22°\ 09'\ 23''$
$20'' \times 1.75$	=	35
Declination at $8^h\ 45^m$	=	$22°\ 08'\ 48''$
p	=	$112°\ 08'.8$
Equa. of time at G.M.N.	=	$+\ 7^m\ 08^s$
$1^s.04 \times 1.75$	=	2
Equa. of time at $8^h\ 45^m$	=	$+\ 7^m\ 10^s$

$L =$	$42°\ 21'.0$	sec. =	0.13133
$h =$	$12\ \ 31\ .6$		
$p =$	$112\ \ 08\ .8$	csc =	0.03329
$2s =$	$166°\ 61'.4$		
$s =$	$83°\ 30'.7$	cos =	9.05308
$s - h =$	$70°\ 59'.1$	sin =	9.97563
			$2)9.19333$
		$\sin \frac{1}{2} t$ =	9.59666
		$\frac{1}{2} t$ =	$23°\ 16'.17$
		t =	$46°\ 32'.3$
		=	$3^h\ 06^m\ 09^s$
		App. time =	$8^h\ 53^m\ 51^s$
		Equa. of time =	$7\ \ 10$
		Mean time =	$9^h\ 01^m\ 01^s$
			$15\ \ 42$
		Eastern time =	$8^h\ 45^m\ 19^s$
		Watch time =	$8\ \ 45\ \ 01$
		Watch slow	18^s

By Altitude of a Star. If a star is observed in this manner the calculation is the same except that the resulting hour angle must be added to the star's right ascension to obtain the sidereal time. The sidereal time may then be converted into standard time as already explained. The sun or star should not be near the meridian, nor within 10° of the horizon, if an accurate result is sought.

43. Determination of Longitude

If the local time is found at two places and these times compared, the difference is the difference in longitude exprest in hours. The most precise method of making this comparison is by means of the telegraph, signals being recorded simultaneously on the two chronographs which are used to record the observations for local time at the two places. Longitude may also be found by carrying a chronometer or a watch from one place to the other, determining its error on the local time of each place and the rate at which the timepiece gains or loses in order to allow for the variation between the two observations. If M = the difference in longitude, e the error of the watch at the first station, e' the error at the second (+ when slow, − when fast), r the gain or loss per day (+ when losing, − when gaining), and d the number of days between the observations, then $M = e + dr - e'$.

Example. At a place $4^h\,44^m\,18^s$ west of Greenwich a watch was $15^m\,42^s$ slow on local mean time. At a place farther west the same watch was $14^m\,10^s$ slow. The watch was gaining $6^s.5$ daily. The second observation was made 26 hours after the first. The difference in longitude is then $15^m\,42^s - 26/24 \times 6^s.5 - 14^m\,10^s = 1^m\,25^s$ and the longitude of the second place is $4^h\,45^m\,43^s$.

44. Observations for Azimuth

By a Circumpolar Star. The simplest method of determining a meridian is by observing the direction of a circumpolar star when it is at eastern or western elongation. A common transit instrument being thoroly adjusted, it is set over a well-defined point several minutes prior to the time of elongation, its cross-hairs being illuminated from a lantern held by an assistant. As the star moves toward elongation keep it covered by the vertical hair by means of the tangent screw on the vernier plate until the star remains on the stationary hair for some time and is then about to leave it in a direction contrary to its former motion. At this instant depress the telescope and set a point in the line of sight. The next day lay off to the east or west, as the case may require, the azimuth of the star at elongation which is computed as explained below, or which, in the case of Polaris, may be taken from the table on the next page with a probable error of a little less than half a minute. The line thus determined is the true meridian. The azimuth of any other line is then found by measuring with the transit the horizontal angle between the line and the true meridian.

The Azimuth at Elongation may be computed by the equation $\sin Z = \cos D / \cos L$, where D is the declination of the star as found from the Nautical almanac, L the latitude of the place, and Z the required azimuth. For example, let declination of a star be 88° 48′ 13″ and the latitude of the place be 42° 21′, then azimuth of the star at elongation is 1° 37′ 08″. For an eastern elongation lay off this angle to the left and for a western elongation lay it off to the right in order to determine the true meridian.

By Elongation of Polaris. The circumpolar star generally used is Polaris (Art. 40). Its azimuths at elongation are given in the following table with sufficient precision for the purposes of common surveying, while the table on page 112 gives the times when the elongations occur. These tables are reprinted from "Principal Facts of the Earth's Magnetism," issued by the U. S. Coast and Geodetic Survey.

Azimuths of Polaris at Elongation

Latitude	1918	1919	1920	1921	1922	1923	1924	1925
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
10	1 09.0	1 08.7	1 08.4	1 08.1	1 07.8	1 07.4	1 07.2	1 06.8
11	09.2	08.9	08.6	08.3	08.0	07.7	07.4	07.0
12	09.5	09.2	08.9	08.6	08.2	07.9	07.6	07.3
13	09.8	09.4	09.1	08.8	08.5	08.2	07.8	07.6
14	10.0	09.7	09.4	09.1	08.8	08.5	08.2	07.8
15	1 10.4	1 10.0	1 09.7	09.4	09.1	08.8	08.5	08.1
16	10.7	10.4	10.1	09.8	09.4	09.1	08.8	08.5
17	11.1	10.8	10.4	10.1	09.8	09.5	09.2	08.8
18	11.5	11.1	10.7	10.5	10.2	09.8	09.5	09.2
19	11.9	11.6	11.2	10.9	10.6	10.2	09.9	09.6
20	1 12.3	1 12.0	1 11.7	1 11.4	1 11.0	1 10.7	1 10.4	1 10.0
21	12.8	12.5	12.2	11.8	11.5	11.2	10.8	10.5
22	13.3	13.0	12.6	12.3	12.0	11.6	11.3	11.0
23	13.8	13.5	13.2	12.8	12.5	12.2	11.8	11.5
24	14.4	14.1	13.7	13.4	13.0	12.7	12.4	12.0
25	1 15.0	1 14.7	1 14.3	1 14.0	1 13.6	1 13.3	1 13.0	1 12.6
26	15.6	15.3	14.9	14.7	14.2	13.9	13.6	13.2
27	16.3	15.9	15.6	15.2	14.9	14.6	14.2	13.9
28	17.0	16.6	16.3	15.9	15.6	15.2	14.9	14.6
29	17.7	17.4	17.0	16.6	16.3	16.0	15.6	15.2
30	1 18.5	1 18.1	1 17.8	17.4	17.0	16.7	16.4	16.0
31	19.3	18.9	18.6	18.2	17.9	17.5	17.2	16.8
32	20.1	19.8	19.4	19.1	18.7	18.3	18.0	17.6
33	21.0	20.7	20.3	19.9	19.6	19.2	18.8	18.5
34	22.0	21.6	21.2	20.9	20.5	20.1	19.8	19.4
35	1 23.0	1 22.6	1 22.2	1 21.8	1 21.5	1 21.1	1 20.7	1 20.4
36	24.0	23.6	23.3	22.9	22.5	22.1	21.7	21.4
37	25.1	24.7	24.3	24.0	23.6	23.2	22.8	22.4
38	26.2	25.9	25.5	25.1	24.7	24.3	23.9	23.5
39	27.5	27.1	26.7	26.3	25.8	25.5	25.1	24.7
40	1 28.7	1 28.3	1 27.9	1 27.5	1 27.1	1 26.7	1 26.3	1 25.9
41	30.0	29.6	29.1	28.8	28.4	28.0	27.6	27.2
42	31.5	31.0	30.6	30.2	29.8	29.4	29.0	28.6
43	32.9	32.5	32.1	31.8	31.2	30.8	30.4	30.0
44	34.5	34.1	33.6	33.2	32.8	32.4	31.9	31.5
45	1 36.1	1 35.7	1 35.3	34.8	34.4	34.0	33.5	33.1
46	37.8	37.4	37.0	36.5	36.1	35.6	35.2	34.8
47	39.7	39.2	38.8	38.3	37.9	37.4	37.0	36.5
48	41.6	41.1	40.7	40.2	39.8	39.3	38.8	38.4
49	43.6	43.1	42.7	42.2	41.7	41.3	40.8	40.3
50	1 45.7	1 45.3	1 44.8	1 44.3	1 43.8	1 43.4	1 42.9	1 42.4

The above table was computed with the mean declination of Polaris for each year. More accurate results will be had by applying to the tabular values the corrections taken from the supplementary table at the top of the next page. For example, the azimuth of Polaris at elongation on July 1, 1918, in latitude 41°, is $1° 30'.0 + 0'.2 = 1° 30'.2$; again the azimuth of Polaris at elongation on Dec. 1, 1918, in latitude 41° is $1° 29'.8 - 0'.7 = 1° 29'.1$. For latitude 42° 15′ a double interpolation gives the azimuth of Polaris at elongation on Feb. 15, 1919, as 1° 36′.2. The azimuth as thus deduced from these two tables will, in general, be correct within 0′.3.

Corrections to Table on page 111

For middle of	Correction	For middle of	Correction
January	−0′.5	July	+0.′2
February	−0 .4	August	+0 .1
March	−0 .3	September	−0 .1
April	0 .0	October	−0 .4
May	+0 .1	November	−0 .6
June	+0 .2	December	−0 .8

Local Mean (Astronomic) Times of Elongations and Culminations of Polaris in the year 1915

Computed for latitude 40° north and longitude 90° west of Greenwich

Astronomic Day	East Elongation	Upper Culmination	West Elongation	Lower Culmination
1915	h m	h m	h m	h m
January 1	0 51.7	6 46.9	12 42.1	18 44.9
January 15	23 52.5	5 51.6	11 46.8	17 49.6
February 1	22 45.3	4 44.5	10 39.7	16 42.5
February 15	21 50.1	3 49.2	9 44.4	15 47.2
March 1	20 54.8	2 54.0	8 49.2	14 52.0
March 15	19 59.6	1 58.8	7 54.0	13 56.8
April 1	18 52.7	0 51.9	6 47.1	12 49.9
April 15	17 57.7	23 52.9	5 52.0	11 54.8
May 1	16 54.8	22 50.0	4 49.2	10 52.0
May 15	15 59.9	21 55.1	3 54.2	9 57.0
June 1	14 53.3	20 48.5	2 47.6	8 50.4
June 15	13 58.5	19 53.7	1 52.8	7 55.6
July 1	12 55.9	18 51.1	0 50.2	6 53.0
July 15	12 01.1	17 56.3	23 51.5	5 58.2
August 1	10 54.5	16 49.7	22 44.9	4 51.7
August 15	9 59.8	15 55.0	21 50.2	3 56.9
September 1	8 53.2	14 48.4	20 43.6	2 50.3
September 15	7 58.3	13 53.5	19 48.7	1 55.4
October 1	6 55.5	12 50.7	18 45.9	0.52 7
October 15	6 00.6	11 55.8	17 51.0	23.53 8
November 1	4 53.7	10 48.9	16 44.1	22 46.9
November 15	3 58.6	9 53.8	15 49.0	21 51.8
December 1	2 55.6	8 50.8	14 46.0	20 48.8
December 15	2 00.4	7 55.6	13 50.8	19 53.6

To refer the above quantities to years other than 1915:

For 1917	subtract 0.7 minutes	For 1921	add 1.6 minutes
1918	add 0.9	1922	add 3.1
1919	add 2.5	1923	add 4.5
1920	add 4.0 before March 1 add 0.1 after Feb. 29	1924	add 5.9 before March 1 add 2.0 after Feb. 29

The time in this table is that of the astronomic day, which begins at noon on the civil day of the same date and is reckoned from 0^h to 24^h. Hence an astronomic time less than 12^h refers to the civil day of the same date, but an astronomic time greater than 12^h refers to the morning of the next civil day. Thus, the east elongation given in the table for May 1 occurs at 4^h 54.8^m A.M. local time in the civil day, May 2.

For any day of the month other than those given in the table add to the next following tabular value 3.93 minutes for every day from it to the given date. For example, the east elongation on Nov. 12, 1915, occurs at 4^h 10.4^m P.M. local time.

For any place south or north of latitude 40°, add to the time of west elongation 0.10^m

for each degree south of 40° and subtract 0.16m for each degree north of 40°. Reverse these operations for east elongation. Thus, for a place in longitude 90° and latitude 43° the east elongation on Nov. 12, 1915, occurs at 4^h 09.9^m P.M. local time.

For any place east or west of longitude 90° add 0.16^m for each 15° east and subtract 0.16^m per each 15° west. Thus, for a place in latitude 40° and longitude 69° west of Greenwich the east elongation on Nov. 12, 1915, occurs at 4^h 10.6^m P.M. local time.

To refer all the above local times to standard time add 4^m for each degree the place is west of the standard meridian and subtract 4^m for each degree east. Thus, for longitude 69° and latitude 40°, the east elongation on Nov. 12, 1915, occurs at 2^h 46.6^m P.M. Central Standard Time.

As an example involving all these corrections the Eastern Standard time when the lower culmination of Polaris occurs in latitude 36° and longitude 77° on the astronomic day, May 24, 1919, will be found. For June 1, 1915, the table gives 8^h 50.4^m and for June 1, 1919, it is 8^h 52.9^m local mean astronomic time, or 8^h 52.9^m P.M. local mean civil time. Then for May 24 it is 9^h 24.3^m P.M. for latitude 40° and longitude 90°. The latitude correction is -0.64^m and the longitude correction is $+0.14^m$, so that the time is 9^h 24.4^m P.M. in latitude 36° and longitude 77°. To this is to be added 8^m for the difference in time between longitude 75° and 77°, thus giving 9^h 32.4^m P.M. for the Eastern Standard time at which the lower culmination occurs on the civil day May 24, 1919, in latitude 36° and longitude 77°. The uncertainty of this result is about 0.2^m.

By Culmination of Polaris. The time of a culmination of Polaris having been found by help of the above table, a rough determination of the meridian may be made by pointing the telescope upon Polaris at that exact instant. Such a determination is liable to be in error 1′ or 2′, since the star moves most rapidly in azimuth when at its culmination. It is always preferable to observe Polaris at elongation as explained above.

By the Altitude of the Sun. The observation on the sun is made by observing first the altitude of the lower edge of the sun and the horizontal angle to the left edge, this angle being measured from some reference mark; second, the altitude of the upper edge is taken together with the horizontal angle to the right edge. The time should be noted at each pointing. The mean of the two altitudes corrected for refraction and parallax is taken as the altitude of the sun's center corresponding to the mean of the two horizontal angles and also to the mean of the two watch readings. The azimuth of the sun's center is then computed by the formula

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\cos s \cos (s - p)}{\cos L \cos h}}$$

in which $s = \tfrac{1}{2}(L + h + p)$, and L, h and p stand for the latitude, altitude and polar distance respectively.

Example. Corrected mean of observed altitudes = 15° 22′.5, mean of horizontal angles = 74° 07′ 5, mean of watch readings = 8^h 43^m 47^s A.M. Eastern Time. Latitude = 42° 21′.0. Sun's declination at Greenwich Noon (or 7^h A.M.) = −21° 14′ 54″. Difference for 1 hour = −26″.81. Corrected declination = −21° 15′ 40″. Polar distance = 111° 15′ 40″.

$$\begin{aligned}
L &= 42^\circ\ 21' & \log \sec &= 0.13133\\
h &= 15\ .22\ .5 & \log \sec &= 0.01583\\
p &= \underline{111\ \ 15\ .7} & &\\
2s &= 168^\circ\ 59'.2 & &\\
s &= 84^\circ\ 29'.6 & \log \cos &= 8.98212\\
s - p &= -26^\circ\ 46'.1 & \log \cos &= \underline{9.95077}\\
& & 2)&\underline{9.08005}\\
\cos \tfrac{1}{2} Z &= 9.54002 & \tfrac{1}{2} Z &= 69^\circ\ 42'.7\\
& & Z &= 139^\circ\ 25\ .4\\
& & \text{Horizontal Angle} &= \underline{74\ 07\ .5}\\
& & \text{Azimuth of Mark} &= \text{N } 65^\circ\ 17'.9 \text{ E}
\end{aligned}$$

The azimuth of a star may be found in exactly the same way, except that the star is bisected with both cross-hairs and it is not necessary to note the time. The object observed should not be near the meridian, nor within 10° of the horizon, if an accurate result is desired.

GEODETIC SURVEYING

45. Definitions

Geodetic Surveying may be defined as that branch of surveying in which, on account of the extent of the survey and the required precision of the results, it is necessary to consider the spheroidal form of the earth's surface. The object of an extensive geodetic survey is usually twofold: first, to locate certain points with great accuracy in order to connect different topographic or hydrographic maps and furnish an accurate control of the whole survey; second, to furnish data for perfecting our knowledge of the form and dimensions of the earth.

Triangulation. The most economical way of locating points with the accuracy required for these purposes is to establish a system of triangulation extending over the desired area. A triangulation system consists of a series of triangles formed by lines connecting observing stations built on prominent mountains or hills, or other points which it is desirable to locate. In this system of triangles the length of one side of some triangle must be known; then if all of the angles in each triangle are measured the lengths of all other lines in the system may be calculated by trigonometry. There are three types of systems which may be recognized: first, a series of approximately equilateral triangles; second, a series of central polygons, for example a row of hexagons each with an interior station; third, a series of quadrilaterals with both diagonals drawn. The first system is the cheapest when it is desired to extend the survey along a narrow belt, as in surveying a river. It has the disadvantage of having but few "checks," that is, few geometric conditions which must be satisfied by the measurements. The second system is adapted to surveying an area of greater width than the first. The third is the most accurate because it has the greatest number of checks.

Triangulation is usually divided into three classes, according to the lengths of lines and the accuracy with which the angles are to be measured. The largest triangles are called PRIMARY and are used to locate controlling points with great accuracy. The smallest triangles are called TERTIARY and are used for locating details on topographic or hydrographic maps. The SECONDARY or intermediate class is necessary in locating the tertiary points from those of the primary system on account of the great difference in the lengths of the lines. The classification is relative only and differs greatly in different localities. Primary lines may be anywhere from 10 miles to 150 miles; secondary from 5 miles to 25 miles; tertiary from ½ mile to 5 miles. The angular measurements on primary work are made with the greatest precision, while tertiary angles are taken with only sufficient accuracy for the detail work.

Subdivision of Fieldwork. The process of carrying out a triangulation scheme may be divided as follows: (a) Reconnaissance and Preliminary Work. (b) Base-line Measurement. (c) The Measurement of the Angles. (d) The Astronomical Observations.

46. Reconnaissance

The Reconnaissance includes: (1) Making a rough sketch map of the region and studying the general scheme with regard to the shape of the triangles and the general distribution of stations. Very acute angles should

not be chosen if this can be avoided. (2) Testing the lines to see that in each triangle the points are visible, each from the others, and that sights will not be seriously interfered with by excessive atmospheric refraction or by smoke of near-by cities. (3) Making notes in regard to condition of roads, approaches to station, transportation, the lumber available, and any other details that will be useful when the work of measuring the angles is being carried out.

Direction of Station. In clearing out lines thru woods for the purpose of sighting at distant stations it is sometimes necessary to compute the direction of a station which is invisible. Suppose that at points A and B (Fig. 33) the angles CAD, DAB, ABC and CBD have been measured and it is desired to know the direction of C from D. In the triangle ABC the side AB may be taken as the base and assumed equal to unity; the side AC may then be calculated. In the triangle ABD the side AD may be calculated. From AC, AD and the included angle the angle ADC may be found. Then if either B or A can be seen from D the direction of C may be determined since the angles at D are known.

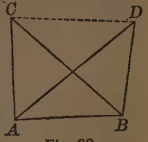

Fig. 33

Elevation of Signals. In ascertaining whether one station is visible from another it is always preferable to test the line by sighting directly over it. In some cases this is not practicable and it becomes necessary to obtain the result by calculation. Suppose for example that station A (Fig. 34) has an elevation of 900 feet and it is desired to sight to station B which is 60 miles

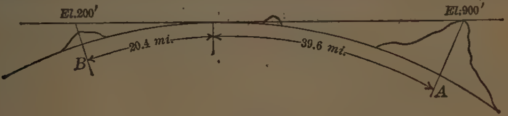

Fig. 34

distant and has an elevation of 200 ft. In this case the sight cannot be taken on account of the curvature of the earth's surface, and it becomes necessary to calculate the height of a tower which when built on B will be visible from A. The distance from a tangent line to the curve may be calculated from the given horizontal distance and the dimensions of the earth. The distance so calculated must be decreased (by about one-seventh part) on account of the refraction of the atmosphere. The combined curvature and refraction is given with sufficient accuracy by the formula $h = k^2/1.743$, where h is the offset from the tangent in feet and k is the horizontal distance in statute miles. In the above example the distance which an observer at A could see over a surface at sea level is given by $\sqrt{h \times 1.743} = 39.6$ miles. The remaining distance, 20.4 miles, corresponds to $(20.4)^2/1.743 = 238.7$ ft; that is, the lowest point which an observer at A can see has an elevation of 238.7 ft above sea level or 38.7 ft above B. Hence a tower 38.7 ft high must be erected at B to make the sight possible. In practice it would be necessary to raise the line at least 10 ft higher in order to avoid the errors due to high refraction of the air near the surface of the ground.

In the above solution it has been assumed that the intervening ground is at sea level, but if there are summits between the two stations it will be necessary to determine how much they rise above the sight line. Suppose that there is a hill 30 miles from A (on

line AB) whose elevation is 75 ft. The hill is 9.6 miles from the tangent point and therefore the curvature correction is 53.0 ft. The top of this hill is $75 - 53.0 = 22.0$ ft above the computed sight line, and the line must be raised 22.0 ft at this point either by increasing the elevation of both stations by 22.0 ft or by building the tower at B 44.0 ft higher.

47. Base-Line

Base-Line. To compute the lengths of the triangle sides it is necessary to choose some base-line, usually not a side of a primary triangle, and to measure its length with great accuracy, and then to connect this base-line with a line of the primary system by means of a special system of triangles called the BASE-NET or EXPANSION. The position of the base should be chosen both with regard to its accurate connection with the main system and to convenience in the measurement of the length itself; but the accuracy of the connection should not be sacrificed to convenience in the measurement.

Measurement of the Base-Line may best be made by the invar tape apparatus. This tape is made of an alloy of nickel and steel which has a very low coefficient of expansion (from $^1/_{25}$ to $^1/_{30}$ that of steel tapes) and is in consequence much less affected by errors in the determination of the temperature of the tape during the measurements. Invar is softer than steel and is easily bent, but if it is wound on a reel of not less than 16 in diameter there is practically no error caused by bending the tape. The tape used in this work is usually 50 meters in length, and is suspended at the ends and at the middle point on stakes or tripods. A definite tension is given by means of a spring balance mounted on a special tension apparatus. The temperature of the tape is determined by means of thermometers clamped on to the tape. The points of support are carefully lined in and brought to a uniform grade by means of a telescope. The rate of grade of each tape-length is determined by direct leveling or else by measuring the vertical angles. The position of the end points of the tape on the intermediate stakes is marked by means of lines scratched on copper or zinc strips tacked to the tops of the stakes. The ends of the base-line are marked by means of copper bolts set in heavy stones or concrete blocks sunk in the ground. The transfer of the end marks to the tape, or vice versa, is made by means of a transit or by an apparatus called a cut-off cylinder.

In making the measurement the tape is hung on the first set of supports and carefully lined in; the tension is then applied and the zero mark is set over the end bolt and the position of the 100-meter end of the tape is marked on the metal strip. The temperature is noted at the same time. Additional measurements are made as a check. The tape is then carried forward to the second set of supports, the zero end being set on the mark previously made, and the operation is repeated. This process is continued until the final mark is reached. The base is measured at least twice to verify the work and to give a value of the probable error of the measurements. The accuracy of the base-line should not be less than $^1/_{500\,000}$ for primary triangulation.

Corrections. In order to reduce the field measurements to the true length of base it is necessary to apply the following corrections: (1) Grade; (2) alinement; (3) variation of tension from normal; (4) sag (if number of supports is changed); (5) temperature; (6) error in absolute length of tape at standard temperature and tension; (7) reduction to sea level.

Correction for Grade may be made by the formula

$$C_g = -\tfrac{1}{2} L(a^2 + \tfrac{1}{4} a^4), \quad \text{where} \quad a = h/L$$

in which L = the length of the tape or any section of it and h = the fall in the distance L. The second term $\frac{1}{4}a^4$ is negligible for ordinary grades, say less than 5%. The correction for errors of alinement may be made by means of the same formula.

The Normal Tension usually adopted is that at which the shortening due to sag (hanging in a curve between supports) and the lengthening due to tension will just equal each other. If the tension applied at any one measurement is greater or less than the normal tension t, allowance may be made for the resulting error by the formula

$$\Delta L = \frac{L}{S} \cdot \frac{\Delta t}{E} + \frac{1}{12}\left(\frac{w}{t}\right)^2 n \cdot l^3 \cdot \frac{\Delta t}{t}$$

where ΔL = the increase in length, L = the length, S = cross-section of tape, E = modulus of elasticity (in the units used for w and L), w = weight of unit length of tape, Δt = increase in tension, l = length of a single span, n = number of spans. w and t must be in the same units; L, l and S must be in the same units.

The Temperature Correction is made by adding to the length of the tape the correction $C_t = 100^m \times k \times (t_1^\circ - t_0^\circ)$, where k is the coefficient of expansion for the tape used, t_0° standard temperature for which the length of the tape has been determined, and t_1° the actual observed temperature.

Absolute Length. The error in the length may be found by comparing the tape with the U. S. Standard and then computing the correction for the difference in tension and the manner of supporting; or better, some base-line of known length may be measured with the tape apparatus, the tape being supported and stretched in exactly the same manner as it will be in the subsequent base measurements. This method eliminates the uncertainty in the computed corrections for sag and tension. The error thus found will be the error at some definite temperature.

Reduction to Sea Level. In order that the whole triangulation system shall be referred to the spheroid selected to represent the figure of the earth it is necessary to reduce the length of the base-line to what it would be if originally measured at sea level. If the elevation h of the base above sea level be determined by leveling, then the reduction is approximately equal to $-Bh/R$, where B is the length of base and R the earth's radius at the point in question; h and R must be in the same units. (Average value of log R (in meters) = 6.80470; (in feet) = 7.32068.)

Marking the Stations. The position of the triangulation station should be marked by a copper bolt set in a drill-hole or else by a stone monument set in the ground. In addition to the center mark several witness marks should be set near by, and their azimuths and distances from the center accurately determined so that the center mark could be replaced in case it is disturbed. When a stone monument is used as a mark the precaution is sometimes taken to set another mark several feet below the surface to be used in case the surface marks are destroyed.

48. Angle Measurements

Signals. Before the angular measurements are begun signals of some sort are placed over the selected stations to be used for sighting when the angles are measured. In some cases these consist of a mast made of 4 by 4 in lumber supported by a tripod and marked by black and white stripes, sometimes also by flags (Fig. 35). The foot of the mast is set about 7 or 8 ft above the ground so that the instrument may be set beneath the signal when measuring angles. Where it is necessary to build a high signal on account of woods this may be done by raising a tall mast made of two or three poles spliced together and bracing it by wire guys. Such signals are suitable for short lines, say up to 15 or 20 miles. For longer lines heliotropes are used. A heliotrope is an apparatus for reflecting sunlight to a distant station, and consists of a movable mirror and a device for pointing it with sufficient accuracy so that the light will reach the station. In cases where it is necessary for the triangulation instrument to be raised in order to see over intervening obstacles, towers are

built over the station, consisting of an inner tripod for the instrument and an outer structure for the observer, the two being disconnected so that the observer will not disturb the instrument when moving about on the tower.

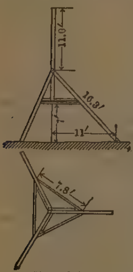

Fig. 35

Measuring the Angles. Instruments designed for triangulation differ from the ordinary transit chiefly in their size, in the fineness of the graduation and in the power of the telescope. The diameter of the circle is sometimes as great as 30 in; it is found, however, that there is little or no advantage in having a circle larger than about 12 in. in diameter. The instrument is usually designed with three leveling screws, and rests either on a pier or on a tripod, according to its size. There are two types of instrument used in this work, the REPEATING instrument and the DIRECTION instrument, the latter being used for the most refined geodetic work.

The Repeating Instrument. The repeating instrument is constructed like the ordinary transit except that it has three leveling screws. The telescope is of high magnifying power and the cross-hairs are usually of the X pattern instead of vertical and horizontal. The verniers read either to 10″ or to 5″. In measuring angles in primary triangulation the instrument is mounted on a pier or other firm support, and is protected from sun and wind by a tent or a temporary building. For work of a less precise character the instrument may be used on its tripod and sheltered by an umbrella. A common method of observation is to measure the angle six times by repetition, the left-hand signal being sighted first and then the right-hand signal, each time, the telescope being in the direct position; when six repetitions have been made, invert the telescope and measure the angle again six times, beginning with the right-hand object. It is not necessary to reset the vernier on zero for the second half of the set of angles, but it may be left at the reading it had at the end of the first six repetitions, then at the end of the second half of the set it should read approximately zero again. Both verniers should be read at the beginning and at the end of each half-set. The twelve repetitions made in this manner constitute a "set." For primary triangulation 5 or 6 sets should be measured; for secondary work, 2 to 4 sets; for tertiary, one or more sets according to the size of the instrument. In order to eliminate errors of graduation the first set may be taken starting with vernier A at 0°, the second set starting at $360°/mn$ and so on, m being the number of sets proposed to be taken and n the number of verniers on the instrument.

Record for a Repeating Instrument

Sta. Blue Hill. Date, May 21, 19-9. Observer, A. L. Instr. No. 196[illegible].

Station	Time	Tel.	Rep	Ver. A.	B.	Mean	Angle	Final Angle
Prospect	4h 00m	D	0	0° 00′ 00″	00″	00″		
to	P.M.		1	123 28 10	20	15		
Shaw			6	380 49 40	40	40	123° 28′ 16″.7	
								123° 28′ 15″.8
		R	0	20 49 40	40	40		
			6	0 00 10	10	10	123 28 15 .0	

The Direction Instrument. In the direction instrument the horizontal circle and the upper portion of the instrument can be moved independently. The method of repetition is not used, but the directions of signals are read in

order of azimuth around the circle, the angles being derived from the differences in these directions. The degrees and 5-minute spaces are read directly from the circle by means of an index microscope. For obtaining the minutes and seconds several micrometer microscopes (usually three) are attached to the upper part of the instrument and placed at equal intervals around the circle. The angular space between the zero point of any micrometer and the last preceding graduation of the circle passed over by that micrometer is measured by means of the micrometer screw. The precision is increased by reading all the microscopes and taking the mean. To read a direction the circle is clamped in any desired position, the telescope is turned so that the cross-hairs point at the signal; the degrees and 5-minute spaces are then read on the index microscope and the minutes, seconds and fractions of seconds are read on all of the other microscopes. The mean of all the micrometer readings added to the reading of the index is the direction of the signal referred to the zero graduation of the circle.

Use of Direction Instrument. At each pointing the index microscope is read for the degrees and 5-minute spaces and all the micrometer microscopes are read for the seconds and fractions. After the directions of all signals have been read the telescope is reversed in its bearings and the series is repeated in the reverse order. This completes one "set." Sixteen of these sets are taken on primary work; on secondary triangulation 5 to 10 sets are taken, and on tertiary, one set. In order to distribute the readings over all parts of the 360° the circle is shifted each time by an amount equal to $360°/mn$ for an instrument having 2 or 4 microscopes, or $180°/mn$ if the instrument has 3 microscopes.

49. Computations

Reducing the Notes. The reduction of the notes and calculation of results include the following steps: (1) Station adjustment. (2) Correction of angles for spherical excess. (3) Distribution of error of closure of triangle. (4) Computation of lengths of sides of triangles. (5) Reduction to center when angles are measured at eccentric station, and a repetition of steps 2, 3 and 4. (6) Computation of the geodetic positions of the stations. (7) Figure adjustment. (8) Final recomputation of 4 and 5.

The Station Adjustment is made by applying the Method of Least Squares. The following illustrates the method when the angles are measured by a repeating instrument. Let four lines OA, OB, OC, OD radiate from the station O, thus giving 12 angles that can be measured. Of these only three are independent, and when the probable values of these are known, those of the others are found at once. First, for each of the measured angles an observation equation is written; second, from these are derived three normal equations; third, the solution of the normal equations gives the most probable values of the three independent angles; fourth, the most probable values of all other angles are then found by simple addition or subtraction.

Example: Let the measured angles and their weights be as follows, the weights being numbers proportional to the numbers of repetitions:

$$
\begin{aligned}
AOB &= 55° \ 57' \ 58''.68 \text{ with weight } 3\\
BOC &= 48 \ \ 49 \ \ 13 \ .64 \text{ with weight } 19\\
AOC &= 104 \ \ 47 \ \ 12 \ .66 \text{ with weight } 17\\
COD &= 54 \ \ 38 \ \ 15 \ .53 \text{ with weight } 13\\
BOD &= 103 \ \ 27 \ \ 28 \ .99 \text{ with weight } 6
\end{aligned}
$$

Let O_1, O_2, O_3 designate the observed values of AOB, BOC, COD, and let z_1, z_2, z_3 be corrections to be applied to O_1, O_2, O_3, in order to give the most probable values of those angles. Then result the five simpler observation equations:

$$
\begin{aligned}
&z_1 = 0. &&\text{with weight } 3 \qquad &&z_3 = 0 &&\text{with weight } 13\\
&z_2 = 0. &&\text{with weight } 19 &&z_2 + z_3 = -0''.18 &&\text{with weight } 6\\
&z_1 + z_2 = +0''.34 &&\text{with weight } 17
\end{aligned}
$$

Next multiply each observation equation by the coefficient of z_1 in that equation and also by its weight, and add the results, thus finding the normal equation for z_1; similarly form normal equations for z_2 and z_3. The three normal equations are

$$
\begin{aligned}
20 z_1 + 17 z_2 \qquad\quad &= +5''.78 \\
17 z_1 + 42 z_2 + 6 z_3 &= +4.70 \\
6 z_2 + 19 z_3 &= -1.08
\end{aligned}
$$

and the solution of these gives $z_1 = +0''.28$, $z_2 = +0''.01$, $z_3 = -0''.06$ as the most probable corrections. The following are hence the adjusted values of the five observed angles:

$$
\begin{aligned}
O_1 + z_1 &= 55^\circ\ 57'\ 58''.96 \\
O_2 + z_2 &= 48\ \ 49\ \ 13.65 \\
O_1 + O_2 + z_1 + z_2 &= 104\ \ 47\ \ 12.61 \\
O_3 + z_3 &= 54\ \ 38\ \ 15.47 \\
O_2 + O_3 + z_2 + z_4 &= 103\ \ 27\ \ 29.12
\end{aligned}
$$

which satisfy all the geometric requirements and from which the other 7 angles are readily found. By Art. 14½ of Sect. 12 it may be computed that the probable error of an observation of weight unity, that is, of one repetition, is $0''.31$.

Spherical Excess. The spherical excess is the amount by which the sum of the three angles of a spherical triangle exceeds 180°. The amount of this correction depends upon the area of the triangle and upon the latitudes of the points. It may be computed from the formula $e'' = bc \sin A/2R^2 \sin 1''$, in which b and c are the sides and A the included angle of the triangle, and R the radius of the earth at the point; the mean value of log R (in meters) = 6.80470. For a more precise value use $e'' = bc \sin A/2RN \sin 1''$, in which R is the radius of curvature of the meridian and N the normal for the given latitude. One-third of the spherical excess of the triangle is to be subtracted from each angle, which reduces it to the corresponding plane angle.

Error of Closure. The sum of the three plane angles will not be 180°, however, unless the measurements were perfectly made. A test of the accuracy of the work is found in the error of the sum of these angles. In the triangulation of the U. S. Coast Survey the average error of closure for primary triangles is about 1″, for secondary 2″ to 3″, and for tertiary 4″ to 5″. After the spherical excess has been calculated the remaining error is due to errors of measurement and is distributed equally among the three angles of the triangle.

Computation of Triangles. After the triangle has been adjusted for error of closure the two unknown sides of the triangle are computed from the formula $a/b = \sin A/\sin B$. The plane angles are used for this calculation because for any triangle which occurs in practise it is sufficiently exact to calculate it as a plane triangle and it is simpler than to compute the spherical triangle.

Computation of Triangle Sides

Stations	Obs. Angles	Cor.	Sph. Ang.	Sph. Exc.	Plane Angles and Distances	Logarithms
P to B					22 723ᵐ.08	4.356 4673
O	61° 47′ 18″.8	0″.4	18″.4	0″.3	61° 47′ 18″.1	0.054 9218
P	82 27 27 .9	0 .4	27 .5	0 .3	82 27 27 .2	9.996 2261
B	35 45 15 .4	0 .4	15 .0	0 .3	35 45 14 .7	9.766 6415
O to B					25 563.20	4.407 6152
O to P					15 067.13	4.178 0306

Reduction to Center. If owing to obstructions it is impossible to place the instrument over the center mark when observing the angles, it may be placed at some other point, called the ECCENTRIC STATION, and the angle observed with the same accuracy as for an angle at the center. These angles may then be reduced to the values they would have at the center mark provided the distance and direction to the center from the instrument be accurately determined. This reduction is made by expressing the direction of each line

as an azimuth reckoned from the center mark as the zero degree point, and then calculating the difference in azimuth of each distant signal as seen from the instrument and from the center mark. This is done by solving the triangle (Fig. 36) formed by joining the instrument, the center mark, and any distant signal, the result being approximately $s'' = d \sin \alpha / D \sin 1''$, where s'' is the azimuth correction in seconds, d is the eccentric distance (measured with a tape), α is the azimuth of the distant station, D is the approximate calculated distance to the distant station. The azimuth α may have any value from 0° to 360°; careful attention should be given to the algebraic signs of $\sin \alpha$. After the corrections (s'') have been calculated they are added algebraically to the corresponding azimuths previously found. The differences of the corrected azimuths then give the true angles at the center.

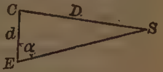

Fig. 36

Example of Reduction to Center

Stations	A	B	C	D	E
Azimuth	42° 14′ 20″.0	46° 01′ 59″.1	104° 47′ 30″.1	161° 10′ 05″.2	205° 10′ 03″.6
Log sin	9.8275	9.8572	9.9853	9.5090	9.6286 *n*
Log $\frac{1}{D}$	6.1052	6.1025	6.0640	6.2672	6.0909
Log $\frac{1.342^m}{\sin 1''}$	5.4422	5.4422	5.4422	5.4422	5.4422
Log S	1.3749	1.4019	1.4915	1.2184	1.1617
S	+23″.7	+25″.2	+31″.0	+16″.5	−14″.5
Azimuth	42° 14′ 43″.7	46° 02′ 24″.3	104° 48′ 01″.1	161° 10′ 22″.7	205° 09′ 49″.1

50. Dimensions of the Earth

Elements of the Spheroid deduced by Clarke in 1866, and used since 1880 in geodetic work in the United States, are as follows:

Semi-major axis = 6 378 278 meters = 20 926 062 feet
Semi-minor axis = 6 356 654 meters = 20 855 121 feet
Meridian quadrant = 10 001 997 meters = 32 814 885 feet
Eccentricity = 0.082 271 Ellipticity = 1/294.98

The radius of a sphere which is equal in volume to this spheroid is 6 371 062 meters or 20 902 416 feet or 3959 statute miles. The radius 3957 statute miles corresponds to a sphere whose quadrant equals the elliptical quadrant.

Hayford's discussion of 1909 gives 6 356 909 meters for the semi-minor axis and 1/297.0 for the ellipticity of the spheroid which best fits the North American continent.

Lengths of One Degree, in Meters, for the Clarke Spheroid

Lat.	On Meridian	On Parallel	Lat.	On Meridian	On Parallel
0°	110 568	111 322	45°	111 132	78 850
5°	110 577	110 901	50	111 231	71 699
10	110 602	109 642	55	111 326	63 997
15	110 644	107 553	60	111 416	55 803
20	110 706	104 650	65	111 497	47 178
25	110 769	100 953	70	111 567	38 189
30	110 850	96 489	75	111 624	28 904
35	110 939	91 291	80	111 666	19 395
40	111 034	85 397	85	111 692	9 735
45	111 132	78 850	90	111 701	0

RAILROAD LOCATION

51. Reconnaissance Survey

Three Types of Survey are employed in laying out a new railroad: first, the RECONNAISSANCE, or an examination of the country thru which the railroad is to run for the purpose of choosing the general route (or routes) which it seems worth while to further investigate; second, a topographic or PRELIMINARY SURVEY of a belt of country chosen by the reconnaissance; and third, the laying out or LOCATION of the proposed railroad within the belt of the preliminary. If the reconnaissance is carelessly made the subsequent work will not rectify this defect. It is in the reconnaissance that most of the bad errors in railroad location have occurred. While financial considerations frequently determine that a road must pass thru certain towns between its termini, the location of the road depends largely upon the topography, and the choice of route upon the locating engineer.

For a Reconnaissance the first step is to obtain the best available maps of the country in question. The U. S. Geological Survey maps are particularly valuable for such purposes, for their contour lines show elevations. On such maps routes may be sketched and a very good idea of the maximum grades of the road obtained. Routes which are obviously unsatisfactory are eliminated from further consideration and only those which look favorable need be thoroly examined by traveling over them. The map will save much time and expense and minimize the amount of actual field examination. Unfortunately large portions of the country have never been surveyed and it is not uncommon for the engineer to find himself without any map of the country he is to examine.

The engineer will travel over the most favorable routes on foot, horseback, or by vehicle. Distances are obtained by pacing or by odometer. Elevations are obtained by the use of the aneroid barometer and a hand level, and such data incorporated into a rough map of the proposed road. The engineer must choose not only a route which can be built at a reasonable cost but one that can be maintained and operated economically. He must therefore take into consideration the grades, curvature, length of line, earthwork and character of the soil, bridging, tunneling and general drainage. To give proper weight to these matters he must be familiar with the economic principles of location, the relative economic value of additional length of line, of grade and curvature. The traffic to be accommodated by the proposed road may be heavy in one direction and light in the other, in which case a grade of 1% may not be as serious in one direction as a grade of 0.5% will be in the opposite direction. A large traffic also justifies an expensive line in order to save operating expense.

A ridge location is advantageous because of the small amount of drainage and bridging to be provided. But it more often happens that the termini and the intermediate towns to be tapped are located on rivers, so that a valley location is required. Valley locations are very common; they frequently allow low grades but often require many structures to bridge the lateral streams and are more subject to washouts. Where it is necessary to cross from one bank of a river to another especial care in the selection of the bridge site must be exercised. In valley locations the grade is practically determined by the rate of fall of the river, but in mountainous countries the rivers may be so steep as to be prohibitive for a railroad grade, in which case the road is purposely lengthened by passing up some lateral valley so as to keep the ruling grade down to the desired amount. In routes which cross the drainage of a country it is of the utmost importance to discover the lowest pass and to use this point as one of the governing points on the location. In very mountainous country it may be feasible in getting over the pass to introduce a short stretch of much steeper grade than the working maximum grade and to operate this grade by the use of one or more helper engines.

While in early railroading in this country crossings of highways at grade were not avoided, practically every state now requires that new railroads shall be constructed with no highway grade crossings, a requirement which introduces another important consideration into the choice of location.

In making the reconnaissance the engineer should not only choose routes from the standpoint of cost of construction, maintenance and operation, but he should also gather data of the industries and natural resources of the country thru which the road is to pass.

52. Preliminary Survey

The Preliminary Survey is a transit and tape traverse run as near as practicable to the probable location of the railroad from one governing point to another. A governing point may be a town, a river crossing, a pass or other fixed point on the route. Between these points every effort will be made to maintain the lowest practicable and economical rate of grade. The ruling grade of each engine division should be adjusted with reference to those of adjoining divisions, and to conditions of local traffic, so as to avoid breaking and making up of trains. All surveys should be made with regard to future permanent construction and every effort used to reduce the amount of temporary construction to the lowest limits. The preliminary should be run with considerable accuracy, for the location survey is to be checked against the preliminary. The distances are usually measured to tenths of a foot, and the deflection angles to the nearest minute.

The Purpose of the Preliminary is to serve as a basis for a topographic survey of a belt of country, varying in width from 300 ft to 1000 ft depending upon the character of the country, a belt in which the located railroad will probably lie. The organization of the preliminary survey party is usually as follows: a chief of party, transitman, two chainmen, two flagmen, levelman and rodman, topographer and assistants, and axmen. A stake is driven at every 100-ft station and marked with the station number. Each transit point is carefully marked by a nail in the head of the stake, but intermediate points are set to the nearest tenth only and are marked merely by stakes. A preliminary party in fairly open country will make 5 to 8 miles a day. The level party follows immediately behind the transit party, taking readings for a profile of the preliminary line.

Topographic Details are sketched to scale on cross-section paper, using the elevations obtained by the level party as a basis. As a rule a 5-ft contour interval is used and the contours are located by means of the hand level and metallic tape, as described in Art. 28. The stadia is employed in some cases to obtain topographic details, and a small plane table is well adapted for this work in open country. The plot of the preliminary map and topographic details together with the profile of the preliminary line should be kept up to date so as to give the locating engineer information as regards grade and curvature to aid him in judging whether to push ahead with his line or to "back up" and run the line over somewhat different ground. A common scale for preliminary maps is 200 or 400 ft to an inch.

The preliminary survey practically fixes the general alinement and grades, and where more than one preliminary has been run the best one can be chosen from a study of the plans and profiles and from other data accumulated when the preliminary is made. The kinds of materials in the excavations, the bridge and tunnel sites, and an estimate of the cost are data which the preliminary should include. The map and profile of the best preliminary line form a basis for the location.

53. Location Survey

The Located Railroad is composed of straight track, called TANGENTS, and circular curves. On most roads a spiral is introduced between the tangents and curves, but this refinement need not enter into the work until the location line is actually staked out. First a trial location line is plotted on the preliminary map and its profile constructed from the data given by the contours on the preliminary. This trial line will pass thru the governing

points, such as bridge sites or passes, and will be laid out so as to make the total quantity of excavations and embankment a minimum, due regard being given to the grades and the amount of curvature in the alinement. On all curves the rate of grade should be flattened about 0.04% per degree of curve to neutralize the additional train resistance; this is called CURVE COMPENSATION. The drainage of the road is of utmost importance, so that it is good practice to lay out the line across flat country a little higher than the surrounding land. The line thru cuts which are more than 500 ft long should be on a grade for proper drainage, which requires about 0.2 ft per 100 ft. It should be borne in mind that on account of the necessity for side ditches the cuts are wider than the fills and that this should be taken into account when judging from the profile of the relative amounts of cut and fill.

Several shifts of the location line will probably be made and profiles constructed for each position until the best line is determined. This final line will probably cross and recross the preliminary traverse line many times if the preliminary has been skilfully run. Ties from the location line to the preliminary line are then scaled from the map and are used in the field in laying out the located line as determined by the office study. This method is called PAPER LOCATION. Sometimes, in a canyon for example where there is no choice of location, the preliminary survey is omitted and the located line is run out in the field. This is called FIELD LOCATION.

After the location line has been actually run it may be found advisable in some instances not to follow exactly the line laid out on the plan. Minor changes and modifications may be made, for example, in fixing with care the location of stations, water tanks, or coaling plants and in adjusting the grades near such points so as to reduce the cost and disadvantage of train stops to the minimum. When train stops at or near the foot of a grade cannot be avoided the rate of grade should be flattened to facilitate the starting of trains. It is not infrequently found that stream diversions, even when of considerable magnitude, prove cheaper than bridging, both in first cost and in maintenance, particularly when the excavated material can be used in embankments. The locating engineer will give special attention to the determination of the necessary length of bridges and size of culverts, character and area of waterway. A cross-section of streams showing along the center line of the railroad, the river bottom, flood height and surface indications of rock should be plotted for each crossing. Thorough drainage is a maxim to be impressed on the mind, and the engineer must not be misled in so-called "rainless districts." In ravines carrying mountain torrents the openings must be left much larger than the appearance of the banks would seem to make necessary.

In Staking Out the center line of location the direction of the tangents is first defined and run to an intersection with adjacent tangents which locate the vertices of the curves. The curves are then staked out as explained in Art. 56. For location work a one-minute transit and 100-ft steel tape are usually employed, the measurements being made to tenths of a foot. A stake is driven at every full station point and a "hub" with a nail is set at all transit stations together with a proper witness stake. The stakes at the beginning and end of each curve should be marked "P.C." and "P.T." respectively and should be carefully referenced by stakes set far enough on each side of the location to lie outside of the construction or filling. The true bearings of tangents should be accurately determined occasionally by solar or stellar observation so as to check the fieldwork and for the purpose of describing the location line. Connections should be made in the location survey to all property lines crossed and to all government land lines and corners.

Immediately behind the transit party comes the level party, which runs a profile of the center line, reading the surface elevations to 0.1 ft and T.P's. and B.M's. to 0.01 ft. A substantial B.M. should be established every 2000 ft and plainly marked. When the final alinement has been staked out the level party, using the grade line on the location profile as a basis, take the cross-sections (see Art. 62) at each full station or oftener to determine the amount of earthwork and at the same time to give stakes properly marked as guides to the contractor in excavating and filling. These grades refer to the sub-grade, on top of which are to be placed the ballast and track.

54. Simple Circular Curves

A Simple Curve is a circular curve connecting two tangents. The points where the curve is tangent to the tangents are called the POINT OF CURVATURE P.C. and the POINT OF TANGENCY P.T., these points being at the beginning and at the end respectively of the curve. The tangents meet at the VERTEX V. The deflection angle at V between the two tangents is the INTERSECTION ANGLE I, which is equal to the CENTRAL ANGLE between radii drawn to the P.C. and P.T. In railroad practice the LENGTH OF CURVE is the distance from P.C. to P.T. measured by 100-ft chords (a sub-chord may occur at one or both ends of the measurement). The DEGREE OF CURVE D is the angle at the center of the curve subtended by a chord of 100 ft. A chord of less than 100 ft is called a SUB-CHORD and its central angle is a SUB-ANGLE. The relation between D and R is expressed by the formula $\sin \frac{1}{2} D = 50/R$.

Radius and Degree of Curve

Degree D	Radius R Feet	Log R
0° 0′	∞	∞
10	34377.5	4.536274
20	17188.8	4.235244
30	11459.2	4.059154
40	8594.42	3.934216
50	6875.55	3.837308
1° 0′	5729.65	3.758128
10	4911.15	3.691183
20	4297.28	3.633194
30	3819.83	3.582044
40	3437.87	3.536289
50	3125.36	3.494900
2° 0′	2864.93	3.457115
10	2644.58	3.422356
20	2455.70	3.390176
30	2292.01	3.360217
40	2148.79	3.332193
50	2022.41	3.305869
3° 0′	1910.08	3.281051
10	1809.57	3.257576
20	1719.12	3.235305
30	1637.28	3.214122
40	1562.88	3.193925
50	1494.95	3.174627
4° 0′	1432.69	3.156151
10	1375.40	3.138430
20	1322.53	3.121404
30	1273.57	3.105022
40	1228.11	3.089236
50	1185.78	3.074005
5° 0′	1146.28	3.059290
10	1109.33	3.045059
20	1074.68	3.031281
30	1042.14	3.017927
40	1011.51	3.004972
50	982.64	2.992393
6° 0′	955.37	2.980170
10	929.57	2.968282

Degree D	Radius R Feet	Log R
6° 20′	905.13	2.956711
30	881.95	2.945442
40	859.92	2.934459
50	838.97	2.923747
7° 0′	819.02	2.913295
10	800.00	2.903089
20	781.84	2.893118
30	764.49	2.883371
40	747.89	2.873840
50	732.01	2.864514
8° 0′	716.78	2.855385
10	702.18	2.846446
20	688.16	2.837687
30	674.69	2.829102
40	661.74	2.820685
50	649.27	2.812428
9° 0′	637.27	2.804327
10	625.71	2.796374
20	614.56	2.788566
30	603.80	2.780897
40	593.42	2.773361
50	583.38	2.765955
10° 0′	573.69	2.758674
10	564.31	2.751514
20	555.23	2.744471
30	546.44	2.737541
40	537.92	2.730721
50	529.67	2.724008
11° 0′	521.67	2.717397
10	513.91	2.710887
20	506.38	2.704473
30	499.06	2.698154
40	491.96	2.691926
50	485.05	2.685788
12° 0′	478.34	2.679735
30	459.28	2.662074
13° 0′	441.68	2.645111
30	425.40	2.628794

Degree D	Radius R Feet	Log R
14° 0′	410.28	2.613076
30	396.20	2.597914
15° 0′	383.06	2.583272
30	370.78	2.569116
16° 0′	359.26	2.555415
30	348.45	2.542140
17° 0′	338.27	2.529268
30	328.68	2.516774
18° 0′	319.62	2.504638
30	311.06	2.492839
19° 0′	302.94	2.481361
30	295.25	2.470186
20° 0′	287.94	2.459300
30	280.99	2.448688
21° 0′	274.37	2.438337
30	268.06	2.428235
22° 0′	262.04	2.418371
30	256.29	2.408734
23° 0′	250.79	2.399315
30	245.53	2.390103
24° 0′	240.49	2.381091
30	235.65	2.372270
25° 0′	231.01	2.363633
30	226.55	2.355173
26° 0′	222.27	2.346882
30	218.15	2.338755
27° 0′	214.18	2.330785
30	210.36	2.322967
28° 0′	206.68	2.315295
30	203.13	2.307764
29° 0′	199.70	2.300370
30	196.38	2.293108
30° 0′	193.19	2.285974
30	190.09	2.278963

In the Metric System the degree of curve is usually defined as the angle at the center subtended by a chord of 20 meters. The first and second columns of the following table give degree of curve and radius according to this definition, while the third column gives the equivalent degree of curve corresponding to the definition stated on the preceding page.

Degree of Curve and Radius for Metric System

Degree of curve is angle at center subtended by a chord of 20 meters.

Degree.	Radius, Meters.	Equivalent U. S. Degree.	Degree.	Radius, Meters.	Equivalent U. S. Degree.	Degree.	Radius, Meters.	Equivalent U. S. Degree.
0° 0′	∞	0° 0′	6° 0′	191.97	9° 09′	12° 0′	95.67	18° 20′
20	3437.8	0 30	20	181.03	9 40	20	93.09	18 51
40	1718.9	1 01	40	171.98	10 10	40	90.65	19 21
1° 0′	1145.9	1° 31′	7° 0′	163.80	10° 41′	13° 0′	88.34	19° 52′
20	859.46	2 02	20	156.37	11 11	20	86.14	20 23
40	687.57	2 32	40	149.58	11 42	40	84.05	20 54
2° 0′	572.99	3° 03′	8° 0′	143.36	12° 12′	14° 0′	82.06	21° 24′
20	491.14	3 33	20	137.63	12 43	20	80.16	21 55
40	429.76	4 04	40	132.35	13 13	40	78.34	22 26
3° 0′	382.02	4° 34′	9° 0′	127.45	13° 44′	15° 0′	76.61	22° 57′
20	343.82	5 05	20	122.91	14 15	20	74.96	23 28
40	312.58	5 35	40	118.68	14 45	40	73.37	23 59
4° 0′	286.54	6° 06	10° 0′	114.74	15° 16′	16° 0′	71.85	24° 29′
20	264.51	6 36	20	111.05	15 47	20	70.40	25 0
40	245.62	7 07	40	107.58	16 17	40	69.00	25 31
5° 0′	229.26	7° 37′	11° 0′	104.33	16° 48′	17° 0′	67.66	26° 02′
20	214.94	8 08	20	101.28	17 19	18 0	63.92	27 35
40	202.30	8 38	40	98.39	17 49	19 0	60.59	29 08

55. Elements of a Simple Curve

In Fig. 37 let I be the central angle between radii drawn to the P.C. and P.T. of the simple curve; this is equal to the intersection angle between the tangents. Also let

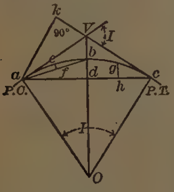

Fig. 37

Oc = Radius = R,
ac = Chord = C,
Vc = Tangent distance = T,
bd = Middle ordinate = M,
Vb = External distance = E,
ka = Offset from tangent = t.

The following relations between these elements are true when I is less than 180°. The trigonometric function exsec $\frac{1}{2}I$, used in the formulas, is called the external secant, and vers $\frac{1}{2}I$ is called the versed sine (see Sect. 12, Art. 4).

Central Angle $I = D \times L/100$ (railroad practice only)
$= \text{arc } abc/R$ (in circular measure) $= (\text{arc } abc/2\pi R) \times 360°$

$\sin \frac{1}{2} I = \frac{1}{2} C/R = (E + M)/T$ $\qquad$ $\tan \frac{1}{2} I = T/R$
$\tan \frac{1}{4} I = M/\frac{1}{2} C = E/T$ $\qquad$ $\text{vers} \frac{1}{2} I = M/R$
$\cos \frac{1}{2} I = \frac{1}{2} C/T = M/E$ $\qquad$ $\text{exsec} \frac{1}{2} I = E/R$

Radius $R = 50/\sin \frac{1}{2} D = 5730/D$ (approx.) $= \frac{1}{2} C/\sin \frac{1}{2} I = T \cot \frac{1}{2} I = E/\text{exsec} \frac{1}{2} I = M/\text{vers} \frac{1}{2} I = (\text{arc } abc/I)(180°/\pi)$.

Degree of Curve D = twice the angle whose sine is $50/R$.

$$D_{x^\circ} = \frac{5730}{R_{x^\circ}} \text{ (approx.)} = \frac{T_{1^\circ}}{T_{x^\circ}} \text{ (approx.)} = \frac{E_{1^\circ}}{E_{x^\circ}} \text{ (approx.)}$$

Chord $C = 2\,R \sin \tfrac{1}{2} I = 2\,T \cos \tfrac{1}{2} I = 2\,M \cot \tfrac{1}{4} I = 2\sqrt{M\,(2\,R - M)}$
$= 2\,E \sin \tfrac{1}{2} I/\text{exsec}\,\tfrac{1}{2} I = \text{arc}\,abc - (\text{arc}\,\overline{abc}^3/24\,R^2)$ (approx.).

Arc $abc = \pi RI/180^\circ = C + (C^3/24\,R^2)$ approximately
$= R \times$ (Length of arc for radius 1). See Tables 5 and 21, Sect. 1.

Length of Curve $L = 100\,I/D$ (railroad practice only).

Tangent Distance $T = R \tan \tfrac{1}{2} I = \tfrac{1}{2} C/\cos \tfrac{1}{2} I = E \cot \tfrac{1}{4} I = \sqrt{E(2\,R + E)}$
$= T_{1^\circ}/D$ approximately.

External Distance $E = R\,\text{exsec}\,\tfrac{1}{2} I = T \tan \tfrac{1}{4} I = \sqrt{T^2 + R^2} - R$
$= \tfrac{1}{2} C\,\text{exsec}\,\tfrac{1}{2} I/\sin \tfrac{1}{2} I = M/\cos \tfrac{1}{2} I = RM/(R - M) = E_{1^\circ}/D$ (approx.).

Middle Ordinate $M = R \text{ vers } \tfrac{1}{2} I = R - \sqrt{(R + \tfrac{1}{2} C)(R - \tfrac{1}{2} C)}$
$= E \cos \tfrac{1}{2} I = \tfrac{1}{2} C \tan \tfrac{1}{4} I = T \cot \tfrac{1}{2} I \times \text{vers } \tfrac{1}{2} I$
$= C^2/8\,R$ (approx.) $= 4\,ef$ (approx.) $= \tfrac{1}{4}\,t$ (approx.).

The chord for M must be the same as the chord for t (see Fig. 37).

Middle Ordinate $ef = \tfrac{1}{4} M$ (approx.).

Ordinate $gh = \sqrt{(R^2 - \overline{dh}^2)} - \sqrt{R^2 - \tfrac{1}{4} C^2}$
$= \sqrt{(R + dh)(R - dh)} - \sqrt{(R + \tfrac{1}{2} C)(R - \tfrac{1}{2} C)}$
$= M - (\overline{dh}^2/2\,R)$ (approx.) $= (ah \times hc)/2\,R$ (approx.).

Offset from Tangent $t = C^2/2\,R = C \sin \tfrac{1}{2} I = 4\,M$ (approx.). The chord for t must be the same as the chord for M (see Fig. 37).

56. Laying Out Curves

Four Methods of laying out circular curves are: (1) by deflection angles, (2) by offsets from the tangent, (3) by chord deflections, (4) by middle ordinates. It is assumed that the two tangents which are to be connected by a curve have been run out to an intersection and the vertex V set and I measured. The degree of curve being known, T is computed and the P.C. and P.T. stakes are set. The station of P.C. is thus established and the station of P.T. = Sta. P.C. + L. Approximately $L = 100\,I/D$.

The Deflection Angle Method is by far the most common and for nearly all railroad work it is sufficiently exact. To lay out a curve by this method set up the transit at the P.C., vernier reading 0°, and sight on V (Fig. 38). Lay off deflection angle VAa and measure chord Aa, thus locating point a. Angle $VAa = \tfrac{1}{2}\,\alpha_a$, and $\alpha_a = D \times Aa/100$. Leaving the lower motion of transit clamped loosen the upper clamp and lay off $VAb = \tfrac{1}{2}\,\alpha_b$ and measure chord ab, and so on, setting each stake and checking at the end of the curve on the P.T. stake.

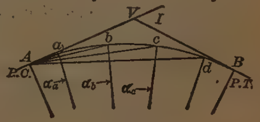

Fig. 38

The total deflection angle to any point on the curve is evidently half the central angle from the P.C. to that point; and where the stations are 100 ft. apart the deflections increase by $\tfrac{1}{2} D$ for each successive station.

Deflection angle for any sub-chord $= cD/200$ (in degrees)
$= c \times 0.3 \times D$ (result in minutes when D is in degrees)

Example. Find deflection angles of a 6° curve connecting P.T. 12 + 73.4 and P.C. 8 + 41.7, the curve being to the right.

$$58.3 \times 0.3 \times 6 = 105' = 1^\circ 45' \text{ to Sta. } 9.$$
$$1^\circ 45' + 3^\circ = 4^\circ 45' \text{ to Sta. } 10.$$
$$4^\circ 45' + 3^\circ = 7^\circ 45' \text{ to Sta. } 11.$$
$$7^\circ 45' + 3^\circ = 10^\circ 45' \text{ to Sta. } 12.$$
$$73.4 \times 0.3 \times 6 = 132' = \underline{2^\circ 12'} \text{ to Sta. } 12 + 73.4.$$
$$12^\circ 57'$$
$$\text{P.T. } 12 + 73.4 - \text{P.C. } 8 + 41.7 = 431.7 = L$$
$$4.317 \times 6^\circ = 25^\circ.902 = 25^\circ 54' = I \text{ and } 12^\circ 57' = \tfrac{1}{2} I.$$

This checks the deflection angle from P.C. to P.T. As a rule the deflection angles should be computed to the nearest half-minute.

It is better practise to lay the curves out backward, starting the chord measurements from the P.C., for example, while the instrument is at the P.T. In this case the vernier is set at 0°, a sight taken on the P.C., and the same deflections used for each station as would be used in laying out the curve if the instrument were at the P.C. If the instrument were set up at the P.C. the vernier should be set at ½ I, a sight taken on P.T., and chords measured beginning with the P.T., using the same deflection angles as before.

Intermediate Set-ups on Curve are necessary when the curve cannot be laid out for its entire length from either the P.C. or P.T. Set the transit up on the intermediate point, then apply one of these two methods: 1. Set the vernier at 0°, reverse the telescope and sight P.C., using lower clamp. Turn telescope direct and lay off the same deflection angle to the next station on the curve that was computed for it when transit was at P.C. 2. Set the vernier beyond 0° so as to read the same angle that was used in setting the intermediate station, reverse telescope, sight P.C. and clamp lower clamp. Loosen upper clamp and turn vernier to read roughly 0° and see if it appears to be sighting along a tangent to the curve thru the intermediate point. (If the deflection angle has been laid off on the correct side of the vernier the telescope will be sighting along this auxiliary tangent when the vernier reads 0°.). Turn telescope direct and lay off the proper deflection angle for the next point ahead on the curve, which will be ½ D if the next station is a full station from the intermediate set-up. The curve beyond the intermediate set-up point is therefore laid out like a new curve, starting at the intermediate point. This second method is the one to be used in laying out compound and reversed curves with the instrument set up at the P.C.C. or P.R.C. (Arts. 58 and 59.)

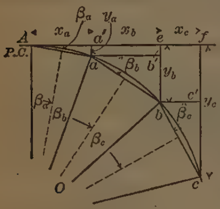

Fig. 39

Offsets-from-Tangent Method. Here no angles are used, every point on the curve being set by measured distances. The first step is to compute the rectangular coordinates of each point, the origin of coordinates being the P.C. and the axes being the tangent and the radius thru the P.C. (Fig. 39). In the triangles $Aa'a$, $ab'b$, $bc'c$, etc., the acute angles are equal to the central angle from the P.C. to the middle of the chord forming the hypotenuse of the respective triangles.

$$x_a = Aa \cos \beta_a \qquad y_a = Aa \sin \beta_a$$
$$x_b = x_a + ab \cos \beta_b \qquad y_b = y_a + ab \sin \beta_b$$
$$x_c = x_b + bc \cos \beta_c \qquad y_c = y_b + bc \sin \beta_c$$

As a check, $x_c = R \sin AOc \qquad y_c = R \text{ vers } AOc$

In the fieldwork the transit is set up at P.C., sighted along the tangent, and points a', e, f, etc., set and marked by stake and nail. Point a is located by measuring with one tape y_a and with another tape Aa; the intersection of these two measurements locates a. Point b is set by intersecting the measurement y_b with ab, etc. This method allows greater precision than the deflection angle method. It is not often used except in setting points where there are obstacles on the curve which prevent sighting across it as is required in the deflection angle method.

Deflection Distance Method. (Fig. 40.) This method is employed to lay ut a curve by the use only of tapes, plumb-lines, and lining-poles. From the hord or sub-chord Aa, the distances ea and Ae are first computed by

$$ea = \overline{Aa}^2/2\,R \quad \text{and} \quad Ae = Aa - (\overline{ae}^2/2\,Aa).$$

ight from A to V and set point e. Set a by measuring ea and Aa. If Aa a sub-chord, set point d by making $Ad = ea$ and $ad = Ae$. Then daf is a angent to the curve thru a. Compute $bf = \overline{ab}^2/2\,R$, and $af = ab - (\overline{bf}^2/2$ $b)$. Then sight along da and set the point f. Set point b by the ties fb nd ab. If $ab = bc$, produce ab to g, making $g = bc$. Set point c by the ties bc and gc. 'he chord deflection $gc = 2 \times fb$. The chord eflections can be used, then, whenever the hords on both sides of the last station are qual. When they are not equal an auxiliary angent must be erected as at a. If chord Aa is ery short, point b should also be set by a tanent offset from the main tangent so that the urve will not be produced from too short a base. The chords ab, bc, etc., re often taken less than 100 ft. This method is particularly useful in reetting a stake which has been knocked out of place.

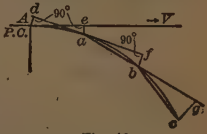

Fig. 40

Middle Ordinate Method. This is employed only for short sharp urves which are to be laid out without the use of a transit. In Fig. 41, the '.C. and P.T. stakes being in place, chord AB is measured and stake e set y lining it in by eye in the middle of AB. Middle ordinate eb is computed $M = C^2/8\,R$) and laid off perpendicular to AB by eye. Similarly points a nd c are located, af and $cg = be/4$.

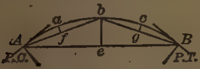

Fig. 41

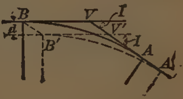

Fig. 42

57. Curve Problems

Parallel Tangent Problems. Fig. 42, both curves of same degree.

$$AA' = VV' = BB' = d/\sin I$$

Fig. 43, both curves have the same P.C.

$$R - R' = d/\text{vers } I$$

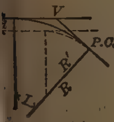

Fig. 43

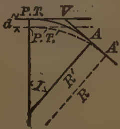

Fig. 44

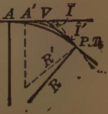

Fig. 45

Fig. 44, both curves have P.T's. on same radial line.

$$R - R' = d/\text{exsec } I \text{ and } AA' = (R - R')\tan I$$

Miscellaneous Problems. Fig. 45. The direction of the forward tanent to be changed at the P.T.

$$R' = R \text{ vers } I/\text{vers } I' \text{ and } AA' = R\sin I - R'\sin I'$$

Fig. 46. To find the station of B on a simple curve from which a tangent will pass thru C; the radius is known. Measure β and AC. In triangle ACO, compute OC and AOC. In right triangle OBC, compute BOC. Then $AOB = AOC - BOC$, and Sta. B = Sta. A + 100 AOB/D.

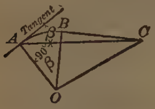

Fig. 46

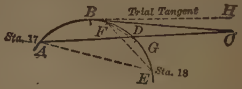

Fig. 47

Fig. 47. To find by approximate field-method the station of B from which a tangent line will pass thru C. First find by approximate method the station of D, where line AC cuts the curve. Station A is the nearer full station to B. (If E were nearer than A then CE should be produced until it cuts the curve near A.) To find where CA cuts the curve, set point F in the middle of arc ABE by method of ordinates (Art. 56). If the arc FE is practically a straight line find the intersection of CA and FE. If the arc and chord do not practically coincide then set point G in the middle of arc FE by the method of ordinates, and then find the intersection of AC and FG. The station D being known, assume point B so as to make AB slightly larger than BD and locate B carefully for a transit point. Set up the transit at B and lay off a tangent to the curve at that point. If this tangent does not pass thru C, measure the angle between the tangent and the line BC, angle HBC in this case. Then move point B, forward or backward on the curve as the case requires, a distance = 100 × $HBC/$degree of curve. A tangent from this new point B should pass almost exactly thru C.

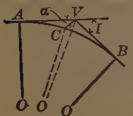

Fig. 48

Fig. 48. To find R of curve which shall join tangents AV and BV and pass thru a given point C. Measure angle α and VC. Then $\sin VCO = \cos(\frac{1}{2}I + \alpha)/\cos\frac{1}{2}I$. In triangle VOC, $CVO = 90° - \frac{1}{2}I - \alpha$, VCO and VC are known; solve for radius OC. If angle VOC is very small find radius by $VC \sin\alpha/\text{vers}\,AOC$.

Fig. 48. To find distance VC, given R, I and α. Solve triangle VOC for VC; $OC = R$, $OV = R/\cos\frac{1}{2}I$, $OVC = 90° - \frac{1}{2}I - \alpha$.

Obstacles on Curves. When obstacles occur, as frequently happens, so that the entire curve cannot be laid out by deflection angles from one position of the instrument, the difficulty can often be overcome by running out the curve as far as possible and then setting up the transit at the last located point on the curve and running the curve ahead from this intermediate set-up (Art. 56). This method is applicable when the obstacle is not large or when it does not lie directly on the curve. If a large obstacle lies on the curve proper, the curve may be run out until it meets the obstacle, both from the P.C. and P.T., or such obstacles may be passed by use of the Offset-from-Tangent Method, the offset being measured from the main tangent or from an auxiliary tangent thrown off from some intermediate point on the curve. Sometimes an obstacle on a curve can be passed by laying off the deflection angle for a point beyond the obstacle, computing and measuring the long chord to that point, thus locating a station without making a new set-up of the transit, and regular chord measurements may start again from this station.

If the P.C. is inaccessible lay off the curve from the P.T. as far as possible toward the P.C. and check the last point by the Method of Offset from Tangent. In applying this latter method, compute the distance along the tangent from the P.C. and subtract this distance from T, which gives the distance from V to the point on the tangent from which the tangent offset is laid off perpendicular to the tangent.

The vertex V is frequently inaccessible. The central angle I, however, can be found y connecting the tangents thru the P.C. and P.T. by a traverse (ABC, Fig. 49). Comute AC, then in triangle AVC compute AV and VC, also determine the value of I. Then the P.C. and P.T. stakes are set by measuring $(T - AV)$ and $(T - VC)$ respecively from points A and C.

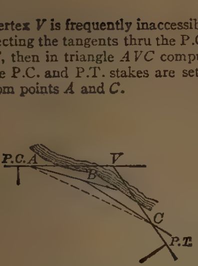

Fig. 49

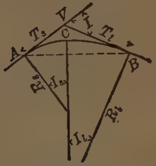

Fig. 50

58. Compound Curves

Compound Curves are formed by two simple circular curves running in he same general direction, and which lie upon the same side of their common angent at their point of junction. The point where these curves join is called he "point of compound curvature" (P.C.C.). Where more than two curves re in succession the curve is called a multiple compound curve. In laying ut compound curves in the field, each simple curve portion is laid out separately. From the P.C. the simple curve will be laid out to the P.C.C., and the ommon tangent is used as a basis for laying out the simple curve to the next .C.C. and so on, the methods being the same as those employed in laying out imple curves (Art. 56).

Elements of Compound Curve. The following apply only to compound urves composed of two simple curves. Where there are more than two simple urves the formulas become quite long. In Fig. 50 it will be observed that all he elements marked with the subscript s refer to the curve of shorter radius, nd those marked l refer to the curve of longer radius. Any compound curve omposed of two simple curves may be defined by any four of the seven elements marked on Fig. 50. These four elements having been chosen the emainder may be computed by using the following formulas.

Given R_s, R_l, I_s, I_l; required I, T_s, T_l. Here $I = I_l + I_s$ and

$$T_s = R_s \tan \tfrac{1}{2} I_s + \frac{(R_s \tan \frac{1}{2} I_s + R_l \tan \frac{1}{2} I_l) \sin I_l}{\sin I}$$

$$T_l = R_l \tan \tfrac{1}{2} I_l + \frac{(R_s \tan \frac{1}{2} I_s + R_l \tan \frac{1}{2} I_l) \sin I_s}{\sin I}$$

Given T_s, R_s, R_l, I; required T_l, I_l, I_s.

$$\text{vers } I_l = \frac{T_s \sin I - R_s \text{ vers } I}{R_l - R_s} \qquad I_s = I - I_l$$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I.$$

Given T_l, R_s, R_l, I; required T_s, I_l, I_s.

$$\text{vers } I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l - R_s} \qquad I_l = I - I_s$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I.$$

Given T_s, R_s, I_s, I; required T_l, R_l, I_l. Here $I_l = I - I_s$ and

$$R_l = R_s + \frac{T_s \sin I - R_s \operatorname{vers} I}{\operatorname{vers} I_l}$$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I$$

Given T_l, R_l, I_l, I; required T_s, R_s, I_s. Here $I_s = I - I_l$ and

$$R_s = R_l - \frac{R_l \operatorname{vers} I - T_l \sin I}{\operatorname{vers} I_s}$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I$$

Given T_l, T_s, R_s, I; required R_l, I_l, I_s.

$$\tan \tfrac{1}{2} I_l = \frac{T_s \sin I - R_s \operatorname{vers} I}{T_l + T_s \cos I - R_s \sin I} \qquad I_s = I - I_l$$

$$R_l = R_s + \frac{T_l + T_s \cos I - R_s \sin I}{\sin I_l}$$

Given T_l, T_s, R_l, I; required R_s, I_l, I_s.

$$\tan \tfrac{1}{2} I_s = \frac{R_l \operatorname{vers} I - T_l \sin I}{R_l \sin I - T_l \cos I - T_s} \qquad I_l = I - I_s$$

$$R_s = R_l - \frac{R_l \sin I - T_l \cos I - T_s}{\sin I_s}$$

Given AB, VAB, VBA, either R_s or R_l; required either R_l, or R_s, I_l, I_s, I. Solve the triangle AVB for AV and VB, which gives T_l and T_s . $I = VAB + VBA$. Then the four elements T_l, T_s, I and either R_s or R_l are given. The remaining elements may be computed by formulas given above.

Compound Curve Problems. Given a simple curve between two tangents. Curve is to be compounded so as to end in a parallel tangent (Fig. 51). In case the new tangent is inside the old tangent, as in Fig. 51, the radius of the original simple curve becomes R_l, and $\operatorname{vers} I_s = d/(R_l - R_s)$. If the new tangent falls outside the old tangent, then the original simple curve becomes R_s, and $\operatorname{vers} I_l' = d/(R_l - R_s)$.

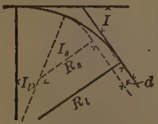

Fig. 51

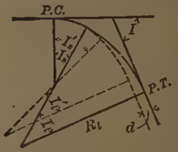

Fig. 52

Given a compound curve between two tangents. Required to retain the same radii but to change the P.C.C. so as to end in a parallel tangent.

(a) When the new tangent lies inside the old tangent and the curve of larger radius is at the P.T. end (Fig. 52).

$$\operatorname{vers} I_l' = \operatorname{vers} I_l - \frac{d}{R_l - R_s}$$

(b) When the new tangent lies outside the old tangent and the curve of larger radius is at the P.T. end.

$$\operatorname{vers} I_l' = \operatorname{vers} I_l + \frac{d}{R_l - R_s}$$

(c) When the new tangent lies inside the old tangent and the curve of shorter radius is at the P.T. end.

$$\text{vers } I_s' = \text{vers } I_s + \frac{d}{R_l - R_s}$$

(d) When the new tangent lies outside the old tangent and the curve of shorter radius is at the P.T. end.

$$\text{vers } I_s' = \text{vers } I_s - \frac{d}{R_l - R_s}$$

In the above formulas the angles with the prime mark are the new angles.

59. Reversed Curves

Reversed Curves are formed by two simple circular curves running in the same general direction but which lie on opposite sides of a common tangent at their point of junction. Where these curves meet is called the "point of reversed curvature" (P.R.C.).

Connecting Parallel Tangents (Fig. 53). In this case the central angles are equal, the P.R.C. lies on the straight line AB, and $BAE = \frac{1}{2} I$.

$$\text{vers } I = \frac{d}{R_1 + R_2} \text{ and } AB = \sqrt{2\, d\,(R_1 + R_2)}$$

If both curves have the same radius, then

$$\text{vers } I = \frac{d}{2R} \text{ and } AB = 2\sqrt{dR}$$

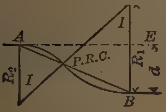

Fig. 53

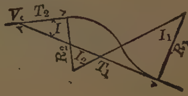

Fig. 54

Connecting Nonparallel Tangents (Fig. 54). As in the compound curve, the choice of four of its elements defines the reversed curve. Given T_1, I, R_1, R_2; required I_1, I_2, T_2.

$$\text{vers } I_2 = \frac{R_1 \text{ vers } I + T_1 \sin I}{R_1 + R_2} \qquad I_1 = I_2 - I$$

$$T_2 = T_1 \cos I + R_1 \sin I - (R_1 + R_2) \sin I_2$$

Given T_2, I, R_1, R_2; required I_1, I_2, T_1.

$$\text{vers } I_1 = \frac{R_2 \text{ vers } I + T_2 \sin I}{R_1 + R_2} \qquad I_2 = I_1 + I$$

$$T_1 = T_2 \cos I + R_2 \sin I + (R_1 + R_2) \sin I_1$$

The two tangents thru the P.C. and P.T. are frequently so nearly parallel that the intersection V cannot be readily found, in which case a line V_1V_2 (Fig. 55) may be chosen as the common tangent and angles I_1 and I_2 measured, and the line V_1V_2. To connect these tangents by curves having equal radii:

$$R = \frac{V_1V_2}{\tan \frac{1}{2}I_1 + \tan \frac{1}{2}I_2}$$

Another way to define a reversed curve is, having chosen the P.C. and P.T. (Fig. 56), to measure the distance AB between them and also angles α and β. To connect these tangents by curves having equal radii:

$$\sin C = \tfrac{1}{2}(\cos\alpha + \cos\beta) \qquad R = \frac{AB}{\sin\alpha + \sin\beta + 2\cos C}$$

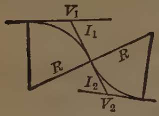

Fig. 55

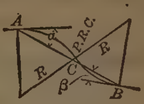

Fig. 56

In reversed curves with nonparallel tangents at the P.C. and P.T. the line AB (Fig. 56) does not pass thru the P.R.C.

60. Easement Curves

The Outer Rail on a curve should be elevated to overcome the effect of centrifugal force, but on straight track the rails should be level across. Hence at the P.C. and P.T. conflicting conditions are present; the outer rail cannot be elevated and at the same time have the rails level across. The old practise was to begin to elevate the outer rail for a hundred feet or more along the tangent from the P.C. and P.T. and to reach its proper elevation either at the P.C. or a short distance beyond on the curve. This method was a makeshift, and considerable shock was given to the rolling stock at each end of the curve. To obviate this difficulty an EASEMENT CURVE is introduced between the tangent and the circular curve. This is a curve of varying radius which leaves the tangent as a very flat curve and grows sharper and sharper until it has the same radius as the circular curve. While the easement curve is developing from a very flat curve to one as sharp as the circular curve the outer rail is gradually elevated so that at any point from the beginning to the end of the easement curve the outer rail is elevated to the proper height.

Cubic Spiral Easement Curve (Fig. 57). AV is a tangent, ABC a spiral connecting the tangent and the circular curve CF, CE is the circular curve produced backward to E, where it is parallel to tangent AV. Let D_c = degree of circular curve, and R_c = its radius; l_c = total length of spiral AC; l = length of spiral from P.S. to any point on spiral; s_c = spiral angle, angle between radius at P.S. and at P.C.C. = angle EOC; s = angle between radius at P.S. and radius at any point on spiral; $q = AD$; $p = DE$; i = deflection angle (instrument at P.S.) to any point on spiral; i_c = deflection angle to P.C.C.; x = offset from tangent to any point on the spiral; x_c = offset GC; y = distance from P.C. to any point on the spiral measured along the tangent; $y_c = AG$. Point B is on the easement curve opposite D.

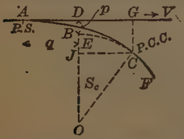

Fig. 57

The equation of the Cubic Parabola is $x = y^3/6\,R_c l_c$.

The equation of the Cubic Spiral is $x = l^3/6\,R_c l_c$. Since y practically equals l for the flat spirals used in railroad practise there is only slight difference between these two curves. The properties of the Cubic Spiral are:

(1) Degree of curve varies directly with its length from P.S.

(2) Deflection angle varies as square of length from P.S.

(3) Offset from tangent varies as cube of length from P.S.
(4) $x_c = l_c^3/6\,R_c l_c$ (the equation of Cubic Spiral).
$y_c = l_c - l_c^3/40\,R_c^2$ (this latter term is in most cases negligible).
$x_c = GC = 4/3\,EJ = 4/3\,R_c$ vers $s_c = 4\,p$. $EJ = 3\,DE = 6\,DB = 3\,p$.
$q = y_c - R_c \sin s_c$ and $p = x_c - R_c$ vers s_c.
(5) $s_c = l_c D_c/200$.
(6) Deflection angle to any point $= i = s/3$, $\therefore i_c = s_c/3$.
(7) The back deflection is equal to twice the forward deflection.

Example. Given $D_c = 4°$ and $l_c = 300$ ft. (a) Find data to lay out spiral by fsets from tangent, every 50-ft point to be set. (b) Find data to lay out spiral by deection angles, every 50-ft point to be set.

(a) From (5), $s_c = 300 \times 4/200 = 6°$. From (4), $x_c = 4/3\,R_c$ vers $6° = 10.47$ nd $y_c = 300 - 300^3/40\,R_c^2 = 300 - 0.3 = 299.7$. From (3), offset to 50-ft point $50^3/300^3 \times 10.47 = 0.05$, offset to 100-ft point $= 10.47/27 = 0.39$, to 150-ft point = .47/8 = 1.31, to 200-ft point $= 8 \times 10.47/27 = 3.11$, to 250-ft point $125 \times 10.4/216 =$ .06. The y distances to measure along the tangent may be computed by the formula $= l - l^3/40\,R^2$, in which R for any point is found from (1). But this computation is ldom necessary; it is sufficiently accurate to compute q from (4), or $q = 299.7 - R_c$ n $s_c = 149.9$, and to call the y for the 50-ft point $= 50.0$. Also for the 100-ft point = 100.0, for the 150-ft point $y = q = 149.9$, for the 200-ft point $y = 199.9$, for the 250- point $y = 249.8$.

(b) From (5) $s_c = 300 \times 4/200 = 6°$. From (6) $i_c = 6°/3 = 2°$. From (2) deflecon to 50-ft point $= 1/36 \times 2° = 0°\ 03'\ 20''$, to 100-ft point $= 1/9 \times 2° = 0°\ 13'\ 20''$, 150-ft point $= 1/4 \times 2° = 0°\ 30'$, to 200-ft point $= 4/9 \times 2° = 0°\ 53'\ 20''$, to 250-ft oint $= 25/36 \times 2° = 1°\ 23'\ 20''$. Evidently the amount of computation necessary for e deflection angle method is very small.

On many railroads it is the practise to use an easement on all curves sharper than 30′, and the sharper the curve and greater the speed of trains the longer the spiral hould be. See Report of Committee on Track of Am. Ry. Eng. & M. W. Assoc., Vol. , Part 1, 1909.

Example. Given $D_c = 4°$ and $p = 4$ ft. (a) Find data to lay out spiral by offsets, tting every quarter point on spiral. (b) Find data to lay out spiral by deflection ngles, setting every quarter point on spiral.

(a) From (4) $x_c = 16.00$; for $1/4\,l_c$, $x = 1/64 \times 16 = 0.25$; for $1/2\,l_c$, $x = 1/8 \times 16 = 2.00$; or $3/4\,l_c$, $x = 27/64 \times 16 = 6.75$. Then vers $s_c = 3\,p/R_c = 12/R_c$, and $s_c = 7°\ 25'.3$.
From (5) $l_c = (s/D) \times 200 = 371.1$,
From (4) $y_c = 371.1 - (371.1^3/40\,R_c^2) = 370.5$,
$q = 370.5 - R_c \sin 7°\ 25'.3 = 185.6$.
The other values of y are interpolated between these values of y_c and q.
(b) Find s as in above $= 7°\ 25'.3$. Then $i_c = 7°\ 25'.3/3 = 2°\ 28'.4$, and
for 1/4 point, $i = 1/16 \times 2°\ 28'.4 = 0°\ 09'.3$,
for 1/2 point, $i = 1/4 \times 2°\ 28'.4 = 0°\ 37'.1$,
for 3/4 point, $i = 9/16 \times 2°\ 28'.4 = 1°\ 23'$.

Fieldwork. In new locations the tangent is usually run as far as point D Fig. 58), then an offset $p = DC$ is measured, point C set and the circular urve run out from C to E. $DV = (R + p) \tan 1/2\,I$. ater on, when the track is to be laid the P.S. is ocated by measuring the distance q back from D. Then with the transit at P.S. the spiral is laid out y deflection angles to the P.C.C., where a new et-up of the transit is made. The back deflecion (twice the forward deflection) is laid off to stablish the auxiliary tangent at P.C.C. from which the circular curve is run to the second P.C.C. The central angle of the circular curve

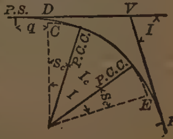

Fig. 58

between the P.C.C.'s $= I -$ sum of spiral angles on both ends. If both spirals have the same length, which is the usual case, then $I_c = I - 2\,s = I - l_c D_c/100$.

Then the transit is taken to the second P.S. and the spiral at that end is run back and checked on the second P.C.C.

Compound Curves. Easement curves are required between the two circular curves forming a compound curve for the same reason that calls for their use between a tangent and a simple curve. In case of a compound curve the sharper curve D_s must be at an offset p inside the flatter curve D_l at the point where these two curves are parallel. First determine points A and B (Fig. 59) where the curves will be parallel. $AB = p$, which defines l_c and q. Locate the ends of the spiral C and F by measuring $AC = q$, and $BF = l_c - q$. Set up the transit at C, lay off an angle (clockwise) which equals the deflection angle $XCA = \frac{1}{2} D_l \times q$, and sight on A. If the lower motion is left clamped and the circle turned to 0° the telescope will be sighting along the auxiliary tangent CX. Then lay out the spiral by deflection angles as usual.

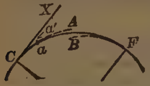

Fig. 59

The deflection angle $XCa = a'Ca + XCa'$. The angle $a'Ca$ is the same as the deflection angle to the first point on a spiral that connects a tangent and a circular curve of degrees $D_s - D_l = D_x$. Find the deflection angles for a spiral of the given length which will join a tangent and a D_x curve. Then the deflection angle for any point on the spiral to use in laying out the spiral between the compound curves = the deflecting angle found for that point for the spiral to connect D_x + the deflection from the auxiliary tangent to a point on the circular curve CA opposite the required spiral point.

In Revising Old Simple Curve Alinement to introduce spirals, the many existing conditions will as a rule limit the choice of spiral and introduce special problems, one of the most common of which is the following. In Fig. 60, half of a circular curve AB is shown. One of the methods of introducing a spiral into this alinement is to move this simple curve 2 to 4 ft toward its center and then introduce a spiral on each end. But this requires more track shifting than is allowable under many conditions and another expedient is resorted to as shown in Fig. 60; the circular curve is thrown slightly outward in its middle portion and sharpened so that it will fall inside the tangent, making $ED = p$. Calling the distance $BC = k$, R_1 the original curve and R_2 the new curve, then

$$R_1 - R_2 + k = (k + p)/\text{vers } \tfrac{1}{2} I \text{ and}$$
$$AF = q - (R_1 - R_2 + k) \sin \tfrac{1}{2} I$$

If the requirement is that the track shall not be thrown at any point more than 1 ft, then since the greatest shift comes along the circular curve between the P.C.C's. assume $k = 1$; and then assume p about 2 k. From the first equation above R_2 may be found, and then AF, which defines the P.S., while D_2 and p determine l_c.

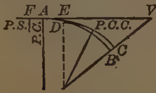

Fig. 60

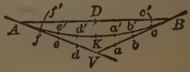

Fig. 61

A Vertical Parabolic Curve is used at the vertex of a grade to avoid the sudden change of direction in passing from one grade to another. If the change of grade is very slight a vertical curve may not be needed, but it is customary to introduce one wherever the change in rate of grade amounts to about 0.2 per cent. In Fig. 61, AV and BV represent the straight grade lines, AKB is the parabolic curve, and Elev. $D = \frac{1}{2}$ (Elev. A + Elev. B). In a parabola, point K is always midway between D and V, so that Elev. $K = \frac{1}{2}$ [$\frac{1}{2}$ (Elev. A + Elev. B) + Elev. V]. Since the offsets from the tangent vary as the squares of the distances out along the tangent, $cc' = ff' = \frac{1}{16} KV$, also $bb' = ee' = \frac{1}{4} KV$ and $aa' = dd' = \frac{9}{16} KV$. To find the elevations of points f', e', d', a', b' and c', compute on the straight grade lines the elevation of f, e, d, a, b and c, and then apply the distances ff', ee', and dd'.

EARTHWORK COMPUTATIONS

61. Profiles and Sections

A Profile on which the grade line has been drawn will show the excavated portions above and the embankment portions below the grade line. In a cut, on account of the necessity of providing for drainage, the base of the roadbed is made wider than in fill, so that the area of cut as shown on profile represents more earthwork than an area of the same shape and size in the fill portion of the profile. While it is true that the depth of the cut or fill and the shape of the ground on either side of the center line have much to do with the amount of earthwork, still it is found satisfactory, before the cross-sections have been taken, to make rough comparisons between the relative amounts of cut and fill by comparing the area of the cut and of the fill as represented on the profile; these may be determined by use of the planimeter.

Cross-sections for the computation of earthwork are determined at each full station, and oftener where the shape of the ground demands it. The usual form of these cross-sections (Fig. 62) is level across the base *ab*, with a straight slope extending from the ends of the base until it intersects the surface of the ground *e* and *f*, while its fourth side is formed by the ground surface *ef*. In determining cross-sections, enough dimensions must be taken to define the shape of the surface *ef*. Every slight elevation or hollow in the ground surface is not required; dimensions are taken so as to represent the quantity in the section and its general shape rather than its exact shape, so that there is an opportunity for the exercise of good judgment.

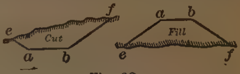

Fig. 62

Ordinary Cross-sections, shown in Fig. 63, are: (1) Level section, in which the ground surface is parallel to the base. (2) Three-level section, in which the ground surface is estimated to be straight from the center stake to where each side slope intersects the surface. (3) Five-level section, in which the surface is estimated to be straight from the center stake to points over the ends of the base, and from these two points to run straight to where the side slopes intersect the surface. (4) Irregular section, where the surface is very broken, requiring it to be divided into several straight slopes, the levels being taken where these slopes intersect and also where the two side slopes meet the surface. (5) Cut-and-fill section, where part of the section is in cut and part in fill. These will be either in the shape of triangles or of shapes similar to a portion of one of the above-named sections. (6) Compound sections are formed when materials of different classification occur in the same section, such as rock covered by earth. The ordinary classifications are earth, loose rock and solid rock.

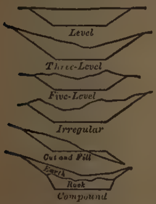

Fig. 63

62. Fieldwork

Cross-sections are taken not only at every full station, but wherever there is a distinct change in slope along the center line and also where the surface on either side of the center line demands an intermediate section to properly represent the volume included between the cross-sections. Cross-

sections are taken perpendicular to the center line of the road, and radially on curves. Before this work is begun the following data should be at hand: (1) The center line, marked with alinement stakes at every full station and properly numbered. (2) Notes of alinement and profile. (3) Record of B.M's. established by the preliminary or location survey party. (4) Width of base for cut and for fill and side slopes to use for each class of material. A slope of 1½ to 1 means 1½ ft horizontal to 1 vertical.

To Find the Cut or Fill at the Center, set up the level and find the H.I. to the nearest hundredth of a foot. From the profile obtain the grade elevation at the given station. The H.I. minus the grade elevation gives the ROD-READING FOR GRADE, which is computed to the nearest tenth of a foot. This grade is usually the sub-grade on top of which the ballast is to be placed. Take a rod-reading on the ground at the center stake; the difference between the rod-reading for grade and the surface rod-reading gives the cut or fill at that point. It is customary to record cuts as + and fills −. The center cut or fill could be found by determining the elevation of the ground and subtracting it from the H.I., but by using the rod-reading for grade these computations can be readily made mentally. The surface elevation is obtained by adding the cut to the grade elevation, or by subtracting in the case of fill. A grade stake is driven on the center line and marked with the cut or fill as follows, "C 3.2" or "F 6.7."

Slope Stakes are set at the points where the side slopes meet the ground. These stakes are also marked, giving the amount the ground is above or below the base of the section; it is called the cut or fill at the side slope, but strictly speaking there is no cut or fill at the slope stakes. The position of a slope stake is found by trial as follows. In the case of a cut, estimate from the center cut and slope of the surface what the probable side cut will be. The distance from the center stake to a point on the side slope having this cut equals (½ base + cut × slope). Make this computation roughly and measure out this distance from the center stake and take a rod-reading at that point. The rod-reading for grade (distance from H.I. to base of section) minus this surface rod-reading gives the cut at the trial point. Compute the distance out from the center stake to a point on the side slope having this cut. If this computed distance equals the measured distance from the center to the rod the trial point was the correct point; if not, then a second trial must be made by holding the rod on another point and repeating the operation. The difference between the measured and calculated distances is an aid in judging where the rod should be held for the second trial. After a little practise it will be possible in most cases to set the slope stake at the second trial. When the correct point is found the stake is marked "C," followed by the feet and tenths this surface point is above the base. The cut and the distance from the center to the slope stakes are entered in the notes. The same process is then repeated for the slope stake on the other side of the center. Rod-readings are taken at intermediate points if they are needed to define the shape of the surface; their positions are located by measuring the distances from the center stake, and the cut at these points is the difference between the rod-reading for grade and the surface rod-reading.

A slope-board is sometimes used in setting slope stakes and in obtaining intermediate elevations. It consists of a long straight wooden board with a spirit-level mounted in its upper edge. After the center cut has been obtained by means of the level instrument, the leveling for the side stakes is done by means of the level-board and a rod. It is particularly useful where there is considerable difference in elevation between the center and side stakes because it obviates the necessity of making a new set-up of the level to obtain the side heights.

In Passing from Cut to Fill it is customary to take three intermediate sections: (1) at the point where the cut runs out (is zero) on the down-hill side line, (2) where the cut is zero on the center line, and (3) where the fill begins on the up-hill side line, as shown by sections B, C and D in Fig. 64.

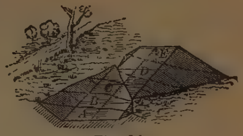

Fig. 64

63. Methods of Computation

To Compute the Volume of earthwork between two parallel cross-sections, let A_1 and A_2 be the end areas, A_m the area of a section midway between the ends, c_1 and c_2 the center heights at the end sections, D_1 and D_2 the sums of the side distances d_r and d_l (Fig. 65), l the length of the solid and V its volume.

By Average End Areas, $V = \frac{1}{2} l (A_1 + A_2)$,
By Prismoidal Formula, $V = \frac{1}{6} l (A_1 + 4 A_m + A_2)$,
By Average End Areas with prismoidal correction,

$$V = \tfrac{1}{2} l (A_1 + A_2) - \tfrac{1}{12} l (c_1 - c_2)(D_1 - D_2).$$

When the given quantities are in feet V is in cubic feet and must be divided by 27 to obtain cubic yards.

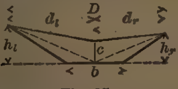

Fig. 65

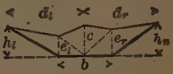

Fig. 66

The Areas of Sections must be found in order to use the above formula. Let b = width of base, c = center height, s = slope. Let h_r and h_l be side heights, d_r and d_l be side distances, and $D = d_r + d_l$. Then

For a Level Section, Fig. 63, $A = c\,(b + sc)$.
For a Three-level Section, Fig. 65, $A = \frac{1}{2}\left\{\frac{1}{2} b\,(h_r + h_l) + cD\right\}$
For a Five-level Section, Fig. 66, $A = \frac{1}{2}\,(cb + e_r d_r + e_l d_l)$

Irregular Sections may be divided into triangles by drawing diagonals from where the verticals meet the base (or base produced) to the surface end of the next vertical out, beginning at the center vertical, as shown in Fig. 67. If this rule is followed it will be found that the dimensions of the pairs of triangles chosen will be readily taken from the cross-section notes.


Fig. 67

It is the practice of some engineers to plot irregular sections on cross-section paper and to obtain their areas by use of the planimeter. This is not so rapid nor so accurate a method as to compute the area as shown. Where the cross-sections are plotted, however, it gives an excellent opportunity to record on the cross-sections by different colored ink lines the progress of the work from month to month, this is particularly useful where the cutting is deep or for cross-sections of dams or canals. The dimensions of the middle-section area, used in the three-term prismoidal formula, are found by proportion from the dimensions of the two bases.

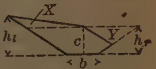

Fig. 68

The prismoidal formula gives an exact volume for a prismoid, which may be defined as "a solid having for its two ends any dissimilar parallel figures of the same number of sides, and all the sides of the solids plane figures." It applies to prisms, wedges and pyramids bounded by planes and to similar solids bounded by warped surfaces. Any solid met with in earthwork computation may be divided into these solids, so that it applies to all ordinary cases of earthwork. It consumes considerable time to compute earthwork by the prismoidal formula method, which is for many cases more exact than is re-

quired, consequently the method of average end areas has become the most common. The end area method, however, gives results almost always too large. To obtain an exact result by the most rapid means, compute by the average end area method and then apply the prismoidal correction, which gives the same value as the prismoidal formula.

Correction for Curvature. In making the field measurements the cross-sections on curves are taken along radial lines AB, CD and EF (Fig. 69). But when the earthwork is computed each solid is assumed to have a length equal to the chord (GH or HJ) and to have end sections whose planes are perpendicular to the chord. For example, solids $KGL - MHN$ and $OHP - QJR$ are computed. It will be seen that the solid PMH has been included twice and that NOH has been omitted. If the cross-section taken at H is symmetrical about the center line, then these discrepancies balance, but if at H there is an unsymmetrical cross-section, then a correction must be applied. In Fig. 70 DF has been drawn so as to form a symmetrical figure with DE. For the cross-section $EDFBA$ there is no correction, but for the portion FDC a correction must be applied. This correction may be found by an application of Pappus' Theorem, "If a plane area lying wholly on the same side of a straight line in its own plane revolves about that line and thereby generates a solid of revolution, the volume of that solid thus generated is equal to the product of the revolving area and the path described by the center of gravity of the plane area during the revolution." For the case of Fig. 70 there applies Correction $= (\frac{1}{2} b + sc)(h_r - h_l)(d_r + d_l) \times 0.00291\,\beta$, in which β = sum of half the central angle under the chords GH and HJ, so that, if GH and HJ are each 100 ft then β = Degree of curve. In above formula β should be taken in degrees. If the greater area is on the outside of the curve the correction should be added, and subtracted when the greater area is on the inside of the curve.

Fig. 69

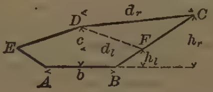

Fig. 70

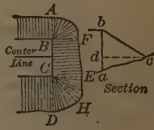

Fig. 71

When Openings are left in Embankments for bridge sites the mass $ABCDEF$ (Fig. 71) must be computed separately from the rest of the earthwork. It is composed of the wedge $BCEF$ and of the quarter cones ABF and DCE, while abc represents a section of the toe slope along the lines BA, BF, CE or CD. The volume by end-area method of the wedge $BCEF = \frac{1}{2}$ (Area CE + Area BF) $\times BC$. The volumes of the cones are found by the Pappus Theorem, or Volume of cone ABF = area $abc \times \frac{1}{3}\, dc \times \frac{1}{2}\pi$.

The prismoidal correction should be added to the volume obtained by the average end-area method. If the prismoidal correction comes out minus, the numerical value is to be subtracted. This formula for prismoidal correction applies to complete sections of earthwork whose bases are of equal width and bounded at their ends by 3 level sections, triangles or lines. The prismoidal correction for a pyramid or cone is one-third the volume by end-area method and is always subtracted. In applying the prismoidal correction to a solid bounded by irregular sections, however, it is customary to find the dimensions of level sections having the same areas respectively as the two irregular sections and determine the prismoidal correction by using the dimensions of these two level sections. This is an approximation which is consistent with the accuracy of the original measurements taken in the field to define the irregular sections.

64. Earthwork Tables

Level-Section Tables are given below for bases from 12 to 42 feet and for slopes of 1 to 1 and 1½ to 1. They are especially useful in making preliminary estimates, but may also be used for other sections by finding the heights of level sections which give equivalent areas.

The Prismoidal Correction Tables below give quantities to be subtracted from volumes obtained by the method of average end areas. Here $c_1 - c_2$

and $D_1 - D_2$ must be in feet, and then the quantity found will be in cubic yards. See pages 146 and 147.

Triangular Prism Tables for prisms with triangular bases and 100 feet in length are given on pages 148–151. To use this table each end section is divided into triangles, and the base a and altitude b of each triangle determined in feet. Then $\frac{1}{2} ab$ is the area of the triangle and $\frac{100}{54} ab$ is the volume in cubic yards of the triangular prism 100 feet long. The tables here given have been compiled from the triangular prism tables computed by C. Frank Allen. His tables, however, were computed for prisms 50 feet in length, and for some methods of earthwork computation they are more serviceable than tables for triangular prisms 100 feet long.

65. Haul and Mass Diagram

Haul. It is the practise with many engineers to pay the contractor for hauling earthwork as well as for excavation and embankment. This item of haul is usually computed as so many cubic yards hauled 100 ft. It is obviously impracticable to trace each cubic yard of excavation to its place of deposit in embankment, so the average haul is computed. The AVERAGE LENGTH OF HAUL is the distance between the center of gravity of a mass of earth in cut and the center of gravity of the same mass after it has been deposited in fill. This distance (in feet) times the quantity hauled gives the haul in units of cubic yards hauled one foot. The best way to treat this computation is to consider it in two parts: (1) compute the haul of the cut portion to a vertical plane where the cut ends and the fill begins, (2) compute the haul of the fill portion beginning at the same plane.

To Find the Center of Gravity of a large mass of earthwork composed of several individual volumes of different shape and size it is necessary either to find the position of the center of gravity of each solid and treat the computation of haul as separate loads comprising the volumes contained in a length of 100 ft (or shorter), or else to assemble these 100-ft solids in some manner so as to obtain the position of the center of gravity of a series of solids. The latter method may be applied when the individual solids are of equal lengths. Where the lengths are not alike, such as for example where sections have been taken at plus stations, the haul of the material in these volumes of odd length must be computed as a separate item as follows.

If the solid had end sections of equal area its center of gravity would be midway between the ends, but where the end sections are unequal the center of gravity of the solid will lie nearer the larger than the smaller end. An expression for the distance from the mid-section of a volume of earthwork to its center of gravity is

$$x_l = l^2 (A_1 - A_2)/12\, V_f \text{ for solid of length } l$$

$$\text{or} \quad x_{100} = 100\, (S_1 - S_2)/6\, V_y \text{ for solid 100 ft long,}$$

where x and x_{100} = distance in feet from mid-section to the center of gravity of solids of length l and 100 ft respectively, A_1 and A_2 = area of end sections in square feet, l = length of solid in feet, S_1 and S_2 = volume in cu yds of prisms each 50 ft long whose end sections are A_1 and A_2 respectively, V_f = volume in cubic feet in a solid of length l and with A_1 and A_2 for end sections, and V_y = same volume in cubic yards of a 100-ft solid with A_1 and A_2 for end sections. If the solid is less than 100 ft in length then $x_l = x_{100} \times l/100$. The distance of the center of gravity from the end of the volume being known, its distance from any station is easily found, and the haul to that station will equal this distance times the cubic yards in the solid.

The Mass Diagram gives the best means of measuring haul and of making studies of the comparative economy of different schemes of haul and of excavation and filling. Fig. 72 represents a portion of a railroad profile and its mass diagram which is plotted as follows. At Sta. 0 the ordinate is zero, at

Level Sections, Slopes 1 to 1. Cubic Yards for 100 ft in Length

Depth in Feet	Base in Feet							
	12	14	16	18	20	22	24	26
1	48	56	63	70	78	85	93	100
2	104	119	133	148	163	178	193	207
3	167	189	211	233	256	278	300	322
4	237	267	296	326	356	385	415	444
5	315	352	389	426	463	500	537	574
6	400	444	489	533	578	622	667	711
7	493	544	596	648	700	752	804	856
8	593	652	711	770	830	889	948	1007
9	700	767	833	900	967	1033	1100	1167
10	815	889	963	1037	1111	1185	1259	1333
11	937	1019	1100	1181	1263	1344	1426	1507
12	1067	1156	1244	1333	1422	1511	1600	1689
13	1204	1300	1396	1493	1589	1685	1781	1878
14	1348	1452	1556	1659	1763	1867	1970	2074
15	1500	1611	1722	1833	1944	2056	2167	2278
16	1659	1778	1896	2015	2133	2252	2370	2489
17	1826	1952	2078	2204	2330	2456	2581	2707
18	2000	2133	2267	2400	2533	2667	2800	2933
19	2181	2322	2463	2604	2744	2885	3026	3167
20	2370	2519	2667	2815	2963	3111	3259	3407
21	2567	2722	2878	3033	3189	3344	3500	3656
22	2770	2933	3096	3259	3422	3585	3748	3911
23	2981	3152	3322	3493	3663	3833	4004	4174
24	3200	3378	3556	3733	3911	4089	4267	4444
25	3426	3611	3796	3981	4167	4352	4537	4722
26	3659	3852	4044	4237	4430	4622	4815	5007
27	3900	4100	4300	4500	4700	4900	5100	5300
28	4148	4356	4563	4770	4978	5185	5393	5600
29	4404	4619	4833	5048	5263	5478	5693	5907
30	4667	4889	5111	5333	5556	5778	6000	6222
31	4937	5167	5396	5626	5856	6085	6315	6544
32	5215	5452	5689	5926	6163	6400	6637	6874
33	5500	5744	5989	6233	6478	6722	6967	7211
34	5793	6044	6296	6548	6800	7052	7304	7556
35	6093	6352	6611	6870	7130	7389	7648	7907
36	6400	6667	6933	7200	7467	7733	8000	8267
37	6715	6989	7263	7537	7811	8085	8359	8633
38	7037	7319	7600	7881	8163	8444	8726	9007
39	7367	7656	7944	8233	8522	8811	9100	9389
40	7704	8000	8296	8593	8889	9185	9481	9778
41	8048	8352	8656	8959	9263	9567	9870	10174
42	8400	8711	9022	9333	9644	9956	10267	10578
43	8759	9078	9396	9715	10033	10352	10670	10989
44	9126	9452	9778	10104	10430	10756	11081	11407
45	9500	9833	10167	10500	10833	11167	11500	11833
46	9881	10222	10563	10904	11244	11585	11926	12267
47	10270	10619	10967	11315	11663	12011	12359	12707
48	10667	11022	11378	11733	12089	12444	12800	13156
49	11070	11433	11796	12159	12522	12885	13248	13611
50	11481	11852	12222	12593	12963	13333	13704	14074
51	11900	12278	12656	13033	13411	13789	14167	14544
52	12326	12711	13096	13481	13867	14252	14637	15022
53	12759	13152	13544	13937	14330	14722	15115	15507

Level Sections, Slopes 1 to 1. Cubic Yards for 100 ft in Length

Depth in Feet	Base in Feet							
	28	30	32	34	36	38	40	42
1	107	115	122	130	137	144	152	159
2	222	237	252	267	281	296	311	326
3	344	367	389	411	433	456	478	500
4	474	504	533	563	593	622	652	681
5	611	648	685	722	759	796	833	870
6	756	800	844	889	933	978	1022	1067
7	907	959	1011	1063	1115	1167	1219	1270
8	1067	1126	1185	1244	1304	1363	1422	1481
9	1233	1300	1367	1433	1500	1567	1633	1700
10	1407	1481	1556	1630	1704	1778	1852	1926
11	1589	1670	1752	1833	1915	1996	2078	2159
12	1778	1867	1956	2044	2133	2222	2311	2400
13	1974	2070	2167	2263	2359	2456	2552	2648
14	2178	2281	2385	2489	2593	2696	2800	2904
15	2389	2500	2611	2722	2833	2944	3056	3167
16	2607	2726	2844	2963	3081	3200	3319	3437
17	2833	2959	3085	3211	3337	3463	3589	3715
18	3067	3200	3333	3467	3600	3733	3867	4000
19	3307	3448	3589	3730	3870	4011	4152	4293
20	3556	3704	3852	4000	4148	4296	4444	4593
21	3811	3967	4122	4278	4433	4589	4744	4900
22	4074	4237	4400	4563	4726	4889	5052	5215
23	4344	4515	4685	4856	5026	5196	5367	5537
24	4622	4800	4978	5156	5333	5511	5689	5867
25	4907	5093	5278	5463	5648	5833	6019	6204
26	5200	5393	5585	5778	5970	6163	6356	6548
27	5500	5700	5900	6100	6300	6500	6700	6900
28	5807	6015	6222	6430	6637	6844	7052	7259
29	6122	6337	6552	6767	6981	7196	7411	7626
30	6444	6667	6889	7111	7333	7556	7778	8000
31	6774	7004	7233	7463	7693	7922	8152	8381
32	7111	7348	7585	7822	8059	8296	8533	8770
33	7456	7700	7944	8189	8433	8678	8922	9167
34	7807	8059	8311	8563	8815	9067	9319	9570
35	8167	8426	8685	8944	9204	9463	9722	9981
36	8533	8800	9067	9333	9600	9867	10133	10400
37	8907	9181	9456	9730	10004	10278	10552	10826
38	9289	9570	9852	10133	10415	10696	10978	11259
39	9678	9967	10256	10544	10833	11122	11411	11700
40	10074	10370	10667	10963	11259	11556	11852	12148
41	10478	10781	11085	11389	11693	11996	12300	12604
42	10889	11200	11511	11822	12133	12444	12756	13067
43	11307	11626	11944	12263	12581	12900	13219	13537
44	11733	12059	12385	12711	13037	13363	13689	14015
45	12167	12500	12833	13167	13500	13833	14167	14500
46	12607	12948	13289	13630	13970	14311	14652	14993
47	13056	13404	13752	14100	14448	14796	15144	15493
48	13511	13867	14222	14578	14933	15289	15644	16000
49	13974	14337	14700	15063	15426	15789	16152	16515
50	14444	14815	15185	15556	15926	16296	16667	17037
51	14922	15300	15678	16056	16433	16811	17189	17567
52	15407	15793	16178	16563	16948	17333	17719	18104
53	15900	16293	16685	17078	17470	17863	18256	18648

Level Sections, Slopes 1½ to 1. Cubic Yards for 100 ft in Length

Depth in Feet	Base in Feet							
	12	14	16	18	20	22	24	26
1	50	57	65	72	80	87	94	102
2	111	126	141	156	170	185	200	215
3	183	206	228	250	272	294	317	339
4	267	296	326	356	385	415	444	474
5	361	398	435	472	509	546	583	620
6	467	511	556	600	644	689	733	778
7	583	635	687	739	791	843	894	946
8	711	770	830	889	948	1007	1067	1126
9	850	917	983	1050	1116	1183	1250	1317
10	1000	1074	1148	1222	1296	1370	1444	1519
11	1161	1243	1324	1406	1487	1569	1650	1731
12	1333	1422	1511	1600	1689	1778	1867	1956
13	1517	1613	1709	1806	1902	1998	2094	2191
14	1711	1815	1919	2022	2126	2230	2333	2437
15	1917	2028	2139	2250	2361	2472	2583	2694
16	2133	2252	2370	2489	2607	2726	2844	2963
17	2361	2487	2613	2739	2865	2991	3117	3243
18	2600	2733	2867	3000	3133	3267	3400	3533
19	2850	2991	3131	3272	3413	3554	3694	3835
20	3111	3259	3407	3556	3704	3852	4000	4148
21	3383	3539	3694	3850	4005	4161	4317	4472
22	3667	3830	3993	4156	4318	4481	4644	4807
23	3961	4131	4302	4472	4642	4813	4983	5154
24	4267	4444	4622	4800	4978	5156	5333	5511
25	4583	4769	4954	5139	5324	5509	5694	5880
26	4911	5104	5296	5489	5681	5874	6067	6259
27	5250	5450	5650	5850	6050	6250	6450	6650
28	5600	5807	6015	6222	6430	6637	6844	7052
29	5961	6176	6391	6606	6820	7035	7250	7465
30	6333	6556	6778	7000	7222	7444	7667	7889
31	6717	6946	7176	7406	7635	7865	8094	8324
32	7111	7348	7585	7822	8059	8296	8533	8770
33	7517	7761	8006	8250	8494	8739	8983	9228
34	7933	8185	8437	8689	8941	9193	9444	9696
35	8361	8620	8880	9139	9398	9657	9917	10176
36	8800	9067	9333	9600	9867	10133	10400	10667
37	9250	9524	9798	10072	10346	10620	10894	11169
38	9711	9993	10274	10556	10837	11119	11400	11681
39	10183	10472	10761	11050	11339	11628	11917	12206
40	10667	10963	11259	11556	11852	12148	12444	12741
41	11161	11465	11769	12072	12376	12680	12983	13287
42	11667	11978	12289	12600	12911	13222	13533	13844
43	12183	12502	12820	13139	13457	13776	14094	14413
44	12711	13037	13363	13689	14015	14341	14667	14993
45	13250	13583	13917	14250	14583	14917	15250	15583
46	13800	14141	14481	14822	15163	15504	15844	16185
47	14361	14709	15057	15406	15754	16102	16450	16798
48	14933	15289	15644	16000	16356	16711	17067	17422
49	15517	15880	16243	16606	16968	17331	17694	18057
50	16111	16481	16852	17222	17592	17963	18333	18704
51	16717	17094	17472	17850	18228	18606	18983	19361
52	17333	17719	18104	18489	18874	19259	19644	20030
53	17961	18354	18746	19139	19531	19924	20317	20709

Level Sections, Slopes 1½ to 1. Cubic Yards for 100 ft in Length

Depth in Feet	Base in Feet							
	28	30	32	34	36	38	40	42
1	109	117	124	131	139	146	154	161
2	230	244	259	274	289	304	319	333
3	361	383	406	428	450	472	494	517
4	504	533	563	593	622	652	681	711
5	657	694	731	769	806	843	880	917
6	822	867	911	956	1000	1044	1089	1133
7	998	1050	1102	1154	1206	1257	1309	1361
8	1185	1244	1304	1363	1422	1481	1541	1600
9	1383	1450	1517	1583	1650	1717	1783	1850
10	1593	1667	1741	1815	1889	1963	2037	2111
11	1813	1894	1976	2057	2139	2220	2302	2383
12	2044	2133	2222	2311	2400	2489	2578	2667
13	2287	2383	2480	2576	2672	2769	2865	2961
14	2541	2644	2748	2852	2956	3059	3163	3267
15	2806	2917	3028	3139	3250	3361	3472	3583
16	3081	3200	3319	3437	3556	3674	3793	3911
17	3369	3494	3620	3746	3872	3998	4124	4250
18	3667	3800	3933	4067	4200	4333	4467	4600
19	3976	4117	4257	4398	4539	4680	4820	4961
20	4296	4444	4592	4741	4889	5037	5185	5333
21	4628	4783	4939	5094	5250	5406	5561	5717
22	4970	5133	5296	5459	5622	5785	5948	6111
23	5324	5494	5665	5835	6006	6176	6346	6517
24	5689	5867	6044	6222	6400	6578	6756	6933
25	6065	6250	6435	6620	6806	6991	7176	7361
26	6452	6644	6837	7030	7222	7415	7607	7800
27	6850	7050	7250	7450	7650	7850	8050	8250
28	7259	7467	7674	7881	8089	8296	8504	8711
29	7680	7894	8109	8324	8539	8754	8969	9183
30	8111	8333	8555	8778	9000	9222	9444	9667
31	8554	8783	9013	9243	9472	9702	9931	10161
32	9007	9244	9482	9719	9956	10193	10430	10667
33	9472	9717	9962	10206	10450	10694	10939	11183
34	9948	10200	10452	10704	10956	11207	11459	11711
35	10435	10694	10954	11213	11472	11731	11991	12250
36	10933	11200	11467	11733	12000	12267	12533	12800
37	11443	11717	11991	12265	12539	12813	13087	13361
38	11963	12244	12526	12807	13089	13370	13652	13933
39	12494	12783	13072	13361	13650	13939	14228	14517
40	13037	13333	13630	13926	14222	14519	14815	15111
41	13591	13894	14198	14502	14806	15109	15413	15717
42	14156	14467	14778	15089	15400	15711	16022	16333
43	14731	15050	15369	15687	16006	16324	16643	16961
44	15319	15644	15970	16296	16622	16948	17274	17600
45	15917	16250	16583	16917	17250	17583	17917	18250
46	16526	16867	17207	17548	17889	18230	18570	18911
47	17146	17494	17843	18191	18539	18887	19235	19583
48	17778	18133	18489	18844	19200	19556	19911	20267
49	18420	18783	19146	19509	19872	20235	20598	20961
50	19074	19444	19815	20185	20556	20926	21296	21667
51	19739	20117	20494	20872	21250	21628	22006	22383
52	20415	20800	21185	21570	21956	22341	22726	23111
53	21102	21494	21887	22280	22672	23065	23457	23850

Prismoidal Corrections in Cubic

$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9
$D_1 - D_2$									
0.1	0.03	0.06	0.09	0.12	0.15	0.19	0.22	0.25	0.28
0.2	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.49	0.56
0.3	0.09	0.19	0.28	0.37	0.46	0.56	0.65	0.74	0.83
0.4	0.12	0.25	0.37	0.49	0.62	0.74	0.86	0.99	1.11
0.5	0.15	0.31	0.46	0.62	0.77	0.93	1.08	1.23	1.39
0.6	0.19	0.37	0.56	0.74	0.93	1.11	1.30	1.48	1.67
0.7	0.22	0.43	0.65	0.86	1.08	1.30	1.51	1.73	1.94
0.8	0.25	0.49	0.74	0.99	1.23	1.48	1.73	1.98	2.22
0.9	0.28	0.56	0.83	1.11	1.39	1.67	1.94	2.22	2.50
1.0	0.31	0.62	0.93	1.23	1.54	1.85	2.16	2.47	2.78
1.1	0.34	0.68	1.02	1.36	1.70	2.04	2.38	2.72	3.06
1.2	0.37	0.74	1.11	1.48	1.85	2.22	2.59	2.96	3.33
1.3	0.40	0.80	1.20	1.60	2.01	2.41	2.81	3.21	3.61
1.4	0.43	0.86	1.30	1.73	2.16	2.59	3.02	3.46	3.89
1.5	0.46	0.93	1.39	1.85	2.31	2.78	3.24	3.70	4.17
1.6	0.49	0.99	1.48	1.98	2.47	2.96	3.46	3.95	4.44
1.7	0.52	1.05	1.57	2.10	2.62	3.15	3.67	4.20	4.72
1.8	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00
1.9	0.59	1.17	1.76	2.35	2.93	3.52	4.10	4.69	5.28
2.0	0.62	1.23	1.85	2.47	3.09	3.70	4.32	4.94	5.56
2.1	0.65	1.30	1.94	2.59	3.24	3.89	4.54	5.19	5.83
2.2	0.68	1.36	2.04	2.72	3.40	4.07	4.75	5.43	6.11
2.3	0.71	1.42	2.13	2.84	3.55	4.26	4.97	5.68	6.39
2.4	0.74	1.48	2.22	2.96	3.70	4.44	5.19	5.93	6.67
2.5	0.77	1.54	2.31	3.09	3.86	4.63	5.40	6.17	6.94
2.6	0.80	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22
2.7	0.83	1.67	2.50	3.33	4.17	5.00	5.83	6.67	7.50
2.8	0.86	1.73	2.59	3.46	4.32	5.19	6.05	6.91	7.78
2.9	0.90	1.79	2.69	3.58	4.48	5.37	6.27	7.16	8.06
3.0	0.93	1.85	2.78	3.70	4.63	5.56	6.48	7.41	8.33
3.1	0.96	1.91	2.87	3.83	4.78	5.74	6.70	7.65	8.61
3.2	0.99	1.98	2.96	3.95	4.94	5.93	6.91	7.90	8.89
3.3	1.02	2.04	3.06	4.07	5.09	6.11	7.13	8.15	9.17
3.4	1.05	2.10	3.15	4.20	5.25	6.30	7.35	8.40	9.44
3.5	1.08	2.16	3.24	4.32	5.40	6.48	7.56	8.64	9.72
3.6	1.11	2.22	3.33	4.44	5.56	6.67	7.78	8.89	10.00
3.7	1.14	2.28	3.43	4.57	5.71	6.85	7.99	9.14	10.28
3.8	1.17	2.35	3.52	4.69	5.86	7.04	8.21	9.38	10.56
3.9	1.20	2.41	3.61	4.81	6.02	7.22	8.43	9.63	10.83
4.0	1.23	2.47	3.70	4.94	6.17	7.41	8.64	9.88	11.11
4.1	1.27	2.53	3.80	5.06	6.33	7.59	8.86	10.12	11.39
4.2	1.30	2.59	3.89	5.19	6.48	7.78	9.07	10.37	11.67
4.3	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.94
4.4	1.36	2.72	4.07	5.43	6.79	8.15	9.51	10.86	12.22
4.5	1.39	2.78	4.17	5.56	6.94	8.33	9.72	11.11	12.50
4.6	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78
4.7	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.06
4.8	1.48	2.96	4.44	5.93	7.41	8.89	10.37	11.85	13.33
4.9	1.51	3.02	4.54	6.05	7.56	9.07	10.59	12.10	13.61
5.0	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.35	13.89
$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9

Yards for a Solidity 100 feet long

$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9
$D_1 - D_2$									
5.1	1.57	3.15	4.72	6.30	7.87	9.44	11.02	12.59	14.17
5.2	1.60	3.21	4.81	6.42	8.02	9.63	11.23	12.84	14.44
5.3	1.64	3.27	4.91	6.54	8.18	9.81	11.45	13.09	14.72
5.4	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00
5.5	1.70	3.40	5.09	6.79	8.49	10.19	11.88	13.58	15.28
5.6	1.73	3.46	5.19	6.91	8.64	10.37	12.10	13.83	15.56
5.7	1.76	3.52	5.28	7.04	8.80	10.56	12.31	14.07	15.83
5.8	1.79	3.58	5.37	7.16	8.95	10.74	12.53	14.32	16.11
5.9	1.82	3.64	5.46	7.28	9.10	10.93	12.75	14.57	16.39
6.0	1.85	3.70	5.56	7.41	9.26	11.11	12.96	14.81	16.67
6.1	1.88	3.77	5.65	7.53	9.41	11.30	13.18	15.06	16.94
6.2	1.91	3.83	5.74	7.65	9.57	11.48	13.40	15.31	17.22
6.3	1.94	3.89	5.83	7.78	9.72	11.67	13.61	15.56	17.50
6.4	1.98	3.95	5.93	7.90	9.88	11.85	13.83	15.80	17.78
6.5	2.01	4.01	6.02	8.02	10.03	12.04	14.04	16.05	18.06
6.6	2.04	4.07	6.11	8.15	10.19	12.22	14.26	16.30	18.33
6.7	2.07	4.14	6.20	8.27	10.34	12.41	14.48	16.54	18.61
6.8	2.10	4.20	6.30	8.40	10.49	12.59	14.69	16.79	18.89
6.9	2.13	4.26	6.39	8.52	10.65	12.78	14.91	17.04	19.17
7.0	2.16	4.32	6.48	8.64	10.80	12.96	15.12	17.28	19.44
7.1	2.19	4.38	6.57	8.77	10.96	13.15	15.34	17.53	19.72
7.2	2.22	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00
7.3	2.25	4.51	6.76	9.01	11.27	13.52	15.77	18.02	20.28
7.4	2.28	4.57	6.85	9.14	11.42	13.70	15.99	18.27	20.56
7.5	2.31	4.63	6.94	9.26	11.57	13.89	16.20	18.52	20.83
7.6	2.35	4.69	7.04	9.38	11.73	14.07	16.42	18.77	21.11
7.7	2.38	4.75	7.13	9.51	11.88	14.26	16.64	19.01	21.39
7.8	2.41	4.81	7.22	9.63	12.04	14.44	16.85	19.26	21.67
7.9	2.44	4.88	7.31	9.75	12.19	14.63	17.07	19.51	21.94
8.0	2.47	4.94	7.41	9.88	12.35	14.81	17.28	19.75	22.22
8.1	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50
8.2	2.53	5.06	7.59	10.12	12.65	15.19	17.72	20.25	22.78
8.3	2.56	5.12	7.69	10.25	12.81	15.37	17.93	20.49	23.06
8.4	2.59	5.19	7.78	10.37	12.96	15.56	18.15	20.74	23.33
8.5	2.62	5.25	7.87	10.49	13.12	15.74	18.36	20.99	23.61
8.6	2.65	5.31	7.96	10.62	13.27	15.93	18.58	21.23	23.89
8.7	2.69	5.37	8.06	10.74	13.43	16.11	18.80	21.48	24.17
8.8	2.72	5.43	8.15	10.86	13.58	16.30	19.01	21.73	24.44
8.9	2.75	5.49	8.24	10.99	13.73	16.48	19.23	21.97	24.72
9.0	2.78	5.56	8.33	11.11	13.89	16.67	19.44	22.22	25.00
9.1	2.81	5.62	8.43	11.23	14.04	16.85	19.66	22.47	25.28
9.2	2.84	5.68	8.52	11.36	14.20	17.04	19.88	22.72	25.56
9.3	2.87	5.74	8.61	11.48	14.35	17.22	20.09	22.96	25.83
9.4	2.90	5.80	8.70	11.60	14.51	17.41	20.31	23.21	26.11
9.5	2.93	5.86	8.80	11.73	14.66	17.59	20.52	23.46	26.39
9.6	[illegible].96	5.93	8.89	11.85	14.81	17.78	20.74	23.70	26.67
9.7	2.99	5.99	8.98	11.98	14.97	17.96	20.96	23.95	26.94
9.8	3.02	6.05	9.07	12.10	15.12	18.15	21.17	24.20	27.22
9.9	3.06	6.11	9.17	12.22	15.28	18.33	21.39	24.44	27.50
10.0	3.09	6.17	9.26	12.35	15.43	18.52	21.60	24.69	27.78
$c_1 - c_2 =$	1	2	3	4	5	6	7	8	9

Triangular Prisms. Cubic Yards for 100 ft in Length

Height in Feet	Width in Feet 1	2	3	4	5	6	7	8	9
0.2	0.4	0.7	1.1	1.5	1.9	2.2	2.6	3.0	3.3
0.4	0.7	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.7
0.6	1.1	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.0
0.8	1.5	3.0	4.4	5.9	7.4	8.9	10.4	11.9	13.3
1.0	1.9	3.7	5.6	7.4	9.3	11.1	13.0	14.8	16.7
1.2	2.2	4.4	6.7	8.9	11.1	13.3	15.6	17.8	20.0
1.4	2.6	5.2	7.8	10.4	13.0	15.6	18.2	20.7	23.3
1.6	3.0	5.9	8.9	11.9	14.8	17.8	20.7	23.7	26.7
1.8	3.3	6.7	10.0	13.3	16.7	20.0	23.3	26.7	30.0
2.0	3.7	7.4	11.1	14.8	18.5	22.2	25.9	29.6	33.3
2.2	4.1	8.1	12.2	16.3	20.4	24.4	28.5	32.6	36.7
2.4	4.4	8.9	13.3	17.8	22.2	26.7	31.1	35.6	40.0
2.6	4.8	9.6	14.4	19.3	24.1	28.9	33.7	38.5	43.3
2.8	5.2	10.4	15.6	20.7	25.9	31.1	36.3	41.5	46.7
3.0	5.6	11.1	16.7	22.2	27.8	33.3	38.9	44.4	50.0
3.2	5.9	11.9	17.8	23.7	29.6	35.6	41.5	47.4	53.3
3.4	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7
3.6	6.7	13.3	20.0	26.7	33.3	40.0	46.7	53.3	60.0
3.8	7.0	14.1	21.1	28.1	35.2	42.2	49.3	56.3	63.3
4.0	7.4	14.8	22.2	29.6	37.0	44.4	51.9	59.3	66.7
4.2	7.8	15.6	23.3	31.1	38.9	46.7	54.4	62.2	70.0
4.4	8.1	16.3	24.4	32.6	40.7	48.9	57.0	65.2	73.3
4.6	8.5	17.0	25.6	34.1	42.6	51.1	59.6	68.1	76.7
4.8	8.9	17.8	26.7	35.6	44.4	53.3	62.2	71.1	80.0
5.0	9.3	18.5	27.8	37.0	46.3	55.6	64.8	74.1	83.3
5.2	9.6	19.3	28.9	38.5	48.1	57.8	67.4	77.0	86.7
5.4	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
5.6	10.4	20.7	31.1	41.5	51.9	62.2	72.6	83.0	93.3
5.8	10.7	21.5	32.2	43.0	53.7	64.4	75.2	85.9	96.7
6.0	11.1	22.2	33.3	44.4	55.6	66.7	77.8	88.9	100.0
6.2	11.5	23.0	34.4	45.9	57.4	68.9	80.4	91.9	103.3
6.4	11.9	23.7	35.6	47.4	59.3	71.1	83.0	94.8	106.7
6.6	12.2	24.4	36.7	48.9	61.1	73.3	85.6	97.8	110.0
6.8	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.7	113.3
7.0	13.0	25.9	38.9	51.9	64.8	77.8	90.7	103.7	116.7
7.2	13.3	26.7	40.0	53.3	66.7	80.0	93.3	106.7	120.0
7.4	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
7.6	14.1	28.1	42.2	56.3	70.4	84.4	98.5	112.6	126.7
7.8	14.4	28.9	43.3	57.8	72.2	86.7	101.1	115.6	130.0
8.0	14.8	29.6	44.4	59.3	74.1	88.9	103.7	118.5	133.3
8.2	15.2	30.4	45.6	60.7	75.9	91.1	106.3	121.5	136.7
8.4	15.6	31.1	46.7	62.2	77.8	93.3	108.9	124.4	140.0
8.6	15.9	31.9	47.8	63.7	79.6	95.6	111.5	127.4	143.3
8.8	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
9.0	16.7	33.3	50.0	66.7	83.3	100.0	116.7	133.3	150.0
9.2	17.0	34.1	51.1	68.1	85.2	102.2	119.3	136.3	153.3
9.4	17.4	34.8	52.2	69.6	87.0	104.4	121.9	139.3	156.7
9.6	17.8	35.6	53.3	71.1	88.9	106.7	124.4	142.2	160.0
9.8	18.1	36.3	54.4	72.6	90.7	108.9	127.0	145.2	163.3

For explanation see page 141

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Triangular Prisms. Cubic Yards for 100 ft in Length

Height in Feet	Width in Feet								
	1	2	3	4	[illegible]	6	7	8	9
10.0	18.5	37.0	55.6	74.1	92.6	111.1	129.6	148.2	166.7
10.2	18.9	37.8	56.7	75.6	94.4	113.3	132.2	151.1	170.0
10.4	19.3	38.5	57.8	77.0	96.3	115.6	134.8	154.1	173.3
10.6	19.6	39.3	58.9	78.5	98.2	117.8	137.4	157.0	176.7
10.8	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
11.0	20.4	40.7	61.1	81.5	101.9	122.2	142.6	163.0	183.3
11.2	20.7	41.5	62.2	83.0	103.7	124.4	145.2	165.9	186.7
11.4	21.1	42.2	63.3	84.4	105.6	126.7	147.8	168.9	190.0
11.6	21.5	43.0	64.4	85.9	107.4	128.9	150.4	171.9	193.3
11.8	21.9	43.7	65.6	87.4	109.3	131.1	153.0	174.8	196.7
12.0	22.2	44.4	66.7	88.9	111.1	133.3	155.6	177.8	200.0
12.2	22.6	45 2	67.8	90.4	113.0	135.6	158.2	180.7	203.3
12.4	23.0	45.9	68.9	91.9	114.8	137.8	160.7	183.7	206.7
12.6	23.3	46.7	70.0	93.3	116.7	140.0	163.3	186.7	210.0
12.8	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
13.0	24.1	48.1	72.2	96.3	120.4	144.4	168.5	192.6	216.7
13.2	24.4	48.9	73.3	97.8	122.2	146.7	171.1	195.6	220.0
13.4	24.8	49.6	74.4	99.3	124.1	148.9	173.7	198.5	223.3
13.6	25.2	50.4	75.6	100.7	125.9	151.1	176.3	201.5	226.7
13.8	25.6	51.1	76.7	102.2	127.8	153.3	178.9	204.4	230.0
14.0	25.9	51.9	77.8	103.7	129.6	155.6	181.5	207.4	233.3
14.2	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
14.4	26.7	53.3	80.0	106.7	133.3	160.0	186.7	213.3	240.0
14.6	27.0	54.1	81.1	108.2	135.2	162.2	189.3	216.3	243.3
14.8	27.4	54.8	82.2	109.6	137.0	164.4	191.9	219.3	246.7
15.0	27.8	55.6	83.3	111.1	138.9	166.7	194.4	222.2	250.0
15.2	28.1	56.3	84.4	112.6	140.7	168.9	197.0	225.2	253.3
15.4	28.5	57.0	85.6	114.1	142.6	171.1	199.6	228.2	256.7
15.6	28.9	57.8	86.7	115.6	144.4	173.3	202.2	231.1	260.0
15.8	29.3	58.5	87.8	117.0	146.3	175.6	204.8	234.1	263.3
16.0	29.6	59.3	88.9	118.5	148.2	177.8	207.4	237.0	266.7
16.2	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
16.4	30.4	60.7	91.1	121.5	151.9	182.2	212.6	243.0	273.3
16.6	30.7	61.5	92.2	123.0	153.7	184.4	215.2	245.9	276.7
16.8	31.1	62.2	93.3	124.4	155.6	186.7	217.8	248.9	280.0
17.0	31.5	63.0	94.4	125.9	157.4	188.9	220.4	251.9	283.3
17.2	31.9	63.7	95.6	127.4	159.3	191.1	223.0	254.8	286.7
17.4	32.2	64.4	96.7	128.9	161.1	193.3	225.6	257.8	290.0
17.6	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.7	293.3
17.8	33.0	65.9	98.9	131.9	164.8	197.8	230.7	263.7	296.7
18.0	33.3	66.7	100.0	133.3	166.7	200.0	233.3	266.7	300.0
18.2	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
18.4	34.1	68.1	102.2	136.3	170.4	204.4	238.5	272.6	306.7
18.6	34.4	68.9	103.3	137.8	172.2	206.7	241.1	275.6	310.0
18.8	34.8	69.6	104.4	139.3	174.1	208.9	243.7	278.5	313.3
19.0	35.2	70.4	105.6	140.7	175.9	211.1	246.3	281.5	316.7
19.2	35.6	71.1	106.7	142.2	177.8	213.3	248.9	284.4	320.0
19.4	35.9	71.9	107.8	143.7	179.6	215.6	251.5	287.4	323.3
19.6	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
19.8	36.7	73.3	110.0	146.7	183.3	220.0	256.7	293.3	330.0

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Triangular Prisms. Cubic Yards for 100 ft in Length

Height in feet	Width in Feet								
	1	2	3	4	5	6	7	8	9
20.0	37.0	74.1	111.1	148.2	185.2	222.2	259.3	296.3	333.3
20.2	37.4	74.8	112.2	149.6	187.0	224.4	261.9	299.3	336.7
20.4	37.8	75.6	113.3	151.1	188.9	226.7	264.4	302.2	340.0
20.6	38.1	76.3	114.4	152.6	190.7	228.9	267.0	305.2	343.3
20.8	38.5	77.0	115.6	154.1	192.6	231.1	269.6	308.2	346.7
21.0	38.9	77.8	116.7	155.6	194.4	233.3	272.2	311.1	350.0
21.2	39.3	78.5	117.8	157.0	196.3	235.6	274.8	314.1	353.3
21.4	39.6	79.3	118.9	158.5	198.2	237.8	277.4	317.0	356.7
21.6	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
21.8	40.4	80.7	121.1	161.5	201.9	242.2	282.6	323.0	363.3
22.0	40.7	81.5	122.2	163.0	203.7	244.4	285.2	325.9	366.7
22.2	41.1	82.2	123.3	164.4	205.6	246.7	287.8	328.9	370.0
22.4	41.5	83.0	124.4	165.9	207.4	248.9	290.4	331.9	373.3
22.6	41.9	83.7	125.6	167.4	209.3	251.1	293.0	334.8	376.7
22.8	42.2	84.4	126.7	168.9	211.1	253.3	295.6	337.8	380.0
23.0	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.7	383.3
23.2	43.0	85.9	128.9	171.9	214.8	257.8	300.7	343.7	386.7
23.4	43.3	86.7	130.0	173.3	216.7	260.0	303.3	346.7	390.0
23.6	43.7	87.4	131.1	174.8	218.5	262.2	305.9	349.6	393.3
23.8	44.1	88.2	132.2	176.3	220.4	264.4	308.5	352.6	396.7
24.0	44.4	88.9	133.3	177.8	222.2	266.7	311.1	355.6	400.0
24.2	44.8	89.6	134.4	179.3	224.1	268.9	313.7	358.5	403.3
24.4	45.2	90.4	135.6	180.7	225.9	271.1	316.3	361.5	406.7
24.6	45.6	91.1	136.7	182.2	227.8	273.3	318.9	364.4	410.0
24.8	45.9	91.9	137.8	183.7	229.6	275.6	321.5	367.4	413.3
25.0	46.3	92.6	138.9	185.2	231.5	277.8	324.1	370.4	416.7
25.2	46.7	93.3	140.0	186.7	233.3	280.0	326.7	373.3	420.0
25.4	47.0	94.1	141.1	188.2	235.2	282.2	329.3	376.3	423.3
25.6	47.4	94.8	142.2	189.6	237.0	284.4	331.9	379.3	426.7
25.8	47.8	95.6	143.3	191.1	238.9	286.7	334.4	382.2	430.0
26.0	48.2	96.3	144.4	192.6	240.7	288.9	337.0	385.2	433.3
26.2	48.5	97.0	145.6	194.1	242.6	291.1	339.6	388.1	436.7
26.4	48.9	97.8	146.7	195.6	244.4	293.3	342.2	391.1	440.0
26.6	49.3	98.5	147.8	197.0	246.3	295.6	344.8	394.1	443.3
26.8	49.6	99.3	148.9	198.5	248.1	297.8	347.4	397.0	446.7
27.0	50.0	100.0	150.0	200.0	250.0	300.0	350.0	400.0	450.0
27.2	50.4	100.7	151.1	201.5	251.9	302.2	352.6	403.0	453.3
27.4	50.7	101.5	152.2	203.0	253.7	304.4	355.2	405.9	456.7
27.6	51.1	102.2	153.3	204.4	255.6	306.7	357.8	408.9	460.0
27.8	51.5	103.0	154.4	205.9	257.4	308.9	360.4	411.9	463.3
28.0	51.9	103.7	155.6	207.4	259.3	311.1	363.0	414.8	466.7
28.2	52.2	104.4	156.7	208.9	261.1	313.3	365.6	417.8	470.0
28.4	52.6	105.2	157.8	210.4	263.0	315.6	368.1	420.7	473.3
28.6	53.0	105.9	158.9	211.9	264.8	317.8	370.7	423.7	476.7
28.8	53.3	106.7	160.0	213.3	266.7	320.0	373.3	426.7	480.0
29.0	53.7	107.4	161.1	214.8	268.5	322.2	375.9	429.6	483.3
29.2	54.1	108.2	162.2	216.3	270.4	324.4	378.5	432.6	486.7
29.4	54.4	108.9	163.3	217.8	272.2	326.7	381.1	435.6	490.0
29.6	54.8	109.6	164.4	219.3	274.1	328.9	383.7	438.5	493.3
29.8	55.2	110.4	165.6	220.7	275.9	331.1	386.3	441.5	496.7

Continued on next page

Triangular Prisms, Cubic Yards for 100 ft in Length

Height in feet	Width in Feet								
	1	2	3	4	5	6	7	8	9
30.0	55.6	111.1	166.7	222.2	277.8	333.3	388.9	444.4	500.0
30.2	55.9	111.9	167.8	223.7	279.6	335.6	391.5	447.4	503.3
30.4	56.3	112.6	168.9	225.2	281.5	337.8	394.1	450.4	506.7
30.6	56.7	113.3	170.0	226.7	283.3	340.0	396.7	453.3	510.0
30.8	57.0	114.1	171.1	228.2	285.2	342.2	399.3	456.3	513.3
31.0	57.4	114.8	172.2	229.6	287.0	344.4	401.9	459.3	516.7
31.2	57.8	115.6	173.3	231.1	288.9	346.7	404.4	462.2	520.0
31.4	58.2	116.3	174.4	232.6	290.7	348.9	407.0	465.2	523.3
31.6	58.5	117.0	175.6	234.1	292.6	351.1	409.6	468.1	526.7
31.8	58.9	117.8	176.7	235.6	294.4	353.3	412.2	471.1	530.0
32.0	59.3	118.5	177.8	237.0	296.3	355.6	414.8	474.1	533.3
32.2	59.6	119.3	178.9	238.5	298.1	357.8	417.4	477.0	536.7
32.4	60.0	120.0	180.0	240.0	300.0	360.0	420.0	480.0	540.0
32.6	60.4	120.7	181.1	241.5	301.9	362.2	422.6	483.0	543.3
32.8	60.7	121.5	182.2	243.0	303.7	364.4	425.2	486.0	546.7
33.0	61.1	122.2	183.3	244.4	305.6	366.7	427.8	488.9	550.0
33.2	61.5	123.0	184.4	245.9	307.4	368.9	430.4	491.9	553.3
33.4	61.9	123.7	185.6	247.4	309.3	371.1	433.0	494.8	556.7
33.6	62.2	124.4	186.7	248.9	311.1	373.3	435.6	497.8	560.0
33.8	62.6	125.2	187.8	250.4	313.0	375.6	438.1	500.7	563.3
34.0	63.0	125.9	188.9	251.9	314.8	377.8	440.7	503.7	566.7
34.2	63.3	126.7	190.0	253.3	316.7	380.0	443.3	506.7	570.0
34.4	63.7	127.4	191.1	254.8	318.5	382.2	445.9	509.6	573.3
34.6	64.1	128.2	192.2	256.3	320.4	384.4	448.5	512.6	576.7
34.8	64.4	128.9	193.3	257.8	322.2	386.7	451.1	515.6	580.0
35.0	64.8	129.6	194.4	259.3	324.1	388.9	453.7	518.5	583.3
35.2	65.2	130.4	195.6	260.7	325.9	391.1	456.3	521.5	586.7
35.4	65.6	131.1	196.7	262.2	327.8	393.3	458.9	524.4	590.0
35.6	65.9	131.9	197.8	263.7	329.6	395.6	461.5	527.4	593.3
35.8	66.3	132.6	198.9	265.2	331.5	397.8	464.1	530.4	596.7
36.0	66.7	133.3	200.0	266.7	333.3	400.0	466.7	533.3	600.0
36.2	67.0	134.1	201.1	268.2	335.2	402.2	469.3	536.3	603.3
36.4	67.4	134.8	202.2	269.6	337.0	404.4	471.9	539.3	606.6
36.6	67.8	135.6	203.3	271.1	338.9	406.7	474.4	542.2	610.0
36.8	68.2	136.3	204.4	272.6	340.7	408.9	477.0	545.2	613.3
37.0	68.5	137.0	205.6	274.1	342.6	411.1	479.6	548.1	616.7
37.2	68.9	137.8	206.7	275.6	344.4	413.3	482.2	551.1	620.0
37.4	69.3	138.5	207.8	277.0	346.3	415.6	484.8	554.1	623.3
37.6	69.6	139.3	208.9	278.5	348.1	417.8	487.4	557.0	626.7
37.8	70.0	140.0	210.0	280.0	350.0	420.0	490.0	560.0	630.0
38.0	70.4	140.7	211.1	281.5	351.9	422.2	492.6	563.0	633.3
38.2	70.7	141.5	212.2	283.0	353.7	424.4	495.2	565.9	636.7
38.4	71.1	142.2	213.3	284.4	355.6	426.7	497.8	568.9	640.0
38.6	71.5	143.0	214.4	285.9	357.4	428.9	500.4	571.9	643.3
38.8	71.9	143.7	215.6	287.4	359.3	431.1	503.0	574.8	646.7
39.0	72.2	144.4	216.7	288.9	361.1	433.3	505.6	577.8	650.0
39.2	72.6	145.2	217.8	290.4	363.0	435.6	508.1	580.7	653.3
39.4	73.0	145.9	218.9	291.9	364.8	437.8	510.7	583.7	656.7
39.6	73.3	146.7	220.0	293.3	366.7	440.0	513.3	586.7	660.0
39.8	73.7	147.4	221.1	294.8	368.5	442.2	515.9	589.6	663.3

For explanation see page 141

Sta. 1 the ordinate represents the quantity of earth between Sta. 0 and Sta. 1, at Sta. 2 it represents the quantity from Sta. 0 to Sta. 2. So that at any station the ordinate represents the algebraic sum of all the quantities from Sta. 0 up to that station, excavation being considered positiv and embankment negativ. This diagram (Fig. 72) shows that from Sta. 0 to 28 the cut and fill balance and that from Sta. 0 to 40 there is a little excess of cut.

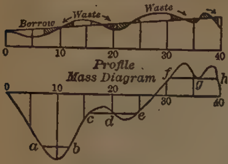

Fig. 72

The Properties of a Mass Diagram are: (1) High and low points in the mass diagram curve occur at points of no cut and fill on the profile. (2) In the mass diagram descending lines denote embankment and ascending lines excavation. (3) The difference in length (algebraically) between any two ordinates is a measure of the total quantity of material between the stations at which the ordinates are drawn. (4) Excavation equals embankment between any two points where a horizontal line intersects the mass diagram. (5) The area between a horizontal line and the curve of mass diagram is a measure of the haul between the two stations where the horizontal line cuts the diagram.

Referring to Fig. 72, the horizontal lines *ab*, *ce*, and *fh* are drawn in a manner explained below. Vertical lines projected from *a*, *b*, *c*, *d*, etc., to the profile will show between which stations the material must be hauled, borrowed or wasted. From Sta. 7 to Sta. 12, for example, the material is to be hauled. The earth in excavation just beyond Sta. 10 is thrown over into fill, each load being carried a short distance at first, the distance the material is hauled increasing until material at Sta. 12 is hauled to Sta. 7, which is called the limit of economical haul. If the cost of earthwork in cut and in fill and the cost of haul are known the length of haul beyond which it is more economical to waste excavated material and to borrow embankment may be readily calculated. This assumes, of course, that the material in cut can be wasted and that filling may be taken from a near-by borrow-pit, both without any appreciable haul or cost for material. Assuming that the economical limit of haul is 1000 ft, then the line *ab* should be drawn 1000 ft long. Line *fgh* is drawn tangent to the curve at *g*, for had it been drawn lower than *g*, *fg* would be greater than 1000 ft, and had it been drawn a little higher than its present position then point *f* would fall further to the right and point *h* further to the left, with the result that more material would be wasted between Sta. 24 and 31 and more borrowed beyond Sta. 39 than is here shown, and this extra material would be wasted and borrowed while it was more economical to haul it. The line *fgh* as drawn, then, represents the best economy. Similarly the line *cde* is in the most economical position. It is drawn so as to make the sum of the areas above *cd* and below *de* a minimum, and these are a minimum when $cd = de$. This is the most economical condition as regards the cost of earthwork because, had *cd* been drawn any lower, it would have decreased the waste at Sta. 16 and increased the waste at Sta. 25 exactly the same amount, but it would have made the sum of the two areas cut off by the horizontal line larger than at present, and this would have meant an increase in the amount of haul which is measured by these areas. Evidently the amount borrowed from Sta. 0 to 7 is the ordinate at *a*, the waste from 12 to 16 is the difference between the ordinates at *b* and *c*, and the waste from Sta. 25 to 31 is the sum (algebraic difference) of the ordinates at *e* and *f*. The beginning of a borrow is shown at Sta. 40.

Free Haul. It is sometimes provided that no payment shall be made for earth hauled less than a specified distance. The haul of any material carried beyond this free-haul limit is called OVERHAUL. The limits of free haul and the method of computing overhaul have been defined by the Amer. Ry. Engrs. and M. of W. Assoc. in Vol. 7, Report of 1906, as follows:

"The limits of free haul shall be determined by fixing on the profile two points, one on each side of the neutral grade point, one in excavation and the other in embankment, such that the distance between them equals the specified free-haul limit and the included quantities of excavation and embankment balance. All haul on material beyond this

e-haul limit will be estimated and paid for on the basis of the following method of mputation:

"All material within this limit of free haul will be eliminated from further considera n. The distance between the center of gravity of the remaining mass of excavation d center of gravity of the resulting embankment, less the limit of free haul as above scribed, shall be the length of overhaul, and the compensation to be rendered therefor ll be determined by multiplying the yardage in the remaining mass as above described, the length of the overhaul."

66. Borrow-Pits

In railroad construction it is often more economical when the length of ul becomes great to borrow material from near-by borrow-pits to form ıbankments. To calculate the amount of earth taken from a borrow-pit is customary to divide the surface of the ground where the pit is to be exca- ted into squares and to take levels at each corner of these squares, and at termediate points when the slope of the surface is not straight from corner corner. Then when the excavation is completed the same system of uare cross-sections is again staked out and elevations are taken at the rners and at the necessary intermediate points. Care should be taken ıen the first system is staked out that it extends far enough to include the tire area which may possibly be covered by the borrow-pit. The field- ork of laying out these cross-sections is done as explained in Art. 24. The fference between the original and the final elevations at the corners gives the ıgths of the vertical edges of a series of vertical truncated rectangular prisms. ward the edges of the borrow-pits, where the slopes occur, there probably ll be several triangular and trapezoidal truncated prisms. Let A = area of ht section of truncated prism, h_1, h_2, h_3, etc. = the lengths of the vertical ges of the prism, and V = volume of prism; then

For Truncated Triangular Prism, $V = A \times (h_1 + h_2 + h_3)/3$.

For Truncated Rectangular Prism, $V = A \times (h_1 + h_2 + h_3 + h_4)/4$.

When Additional Heights have been taken at intermediate points the computation made as follows. (1) When an intermediate level is taken in the center of the square rectangle, the volume of the rectangular prism is computed by the formula given above, d to it is added algebraically the volume of a pyramid having the rectangle for its base d the center height minus the average of the four corner heights for its altitude. The lume of this entire solid would then be ex- essed

$$V = A(h_1 + h_2 + h_3 + h_4)/4 + A\left\{h_c - (h_1 + h_2 + h_3 + h_4)/4\right\}/3$$

which h_c = altitude of prism along its center rtical line.

(2) When an intermediate level is taken at e middle of one side of a rectangle, the volume the rectangular prism is computed as usual, d to it is added the volume of a pyramid ving the rectangle for its base and the differ- ce between the additional height and the erage of the heights at the two corners which at the extremity of the side on which the ditional height has been taken If the addi- nal height is h_b and it is taken on the side ose end heights are h_3 and h_4 respectively, en an expression for the entire volume would

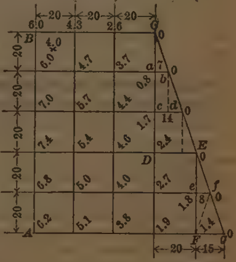

Fig. 73

$$V = A(h_1 + h_2 + h_3 + h_4)/4 + A\left\{h_b - \tfrac{1}{2}(h_3 + h_4)\right\}/3$$

(3) When intermediate heights are taken at any other point in the rectangle except the center of the rectangle or in the middle of one of its sides, then the rectangular

prism should be divided into four triangular prisms; the additional height forms the vertical edge at one of the corners of each of these four triangular prisms.

In arranging the data for computation it is customary to lay out the borrow-pit in plan on cross-section paper and to mark at each corner the depth of excavation, and to sketch any lines representing the proper subdivision of the rectangles or trapezoids as given in the field notes. A plan of a portion of a borrow-pit is shown in Fig. 73, and by the above method the volume of excavation will be found to be 1057 cubic yards.

Where there are several rectangular prisms having the same area of cross-section A the computation of the quantity of earthwork may be simplified as follows: Volume of Assembly of Truncated Rectangular Prisms $= A\,(p_1 + 2\,p_2 + 3\,p_3 + 4\,p_4)/4$ in which p_1 = sum of heights common to one prism, p_2 = sum of heights common to two prisms, p_3 = those common to three, p_4 = those common to four prisms.

Measurements of Dredged Material are taken in two ways, (1) measurement in place, and (2) scow measurement. For the MEASUREMENT-IN-PLACE METHOD soundings are taken before and again after the dredging work is done, and the volume of the material which has been removed is computed either by the Average End Area Method or by the Borrow-Pit Method. If the dredging covers a large area, contours may be drawn showing the shape of the bottom before and after the dredging has been done, and these may be used as bases of vertical solids. Where a channel has been dredged, sometimes it is more convenient to compute the volume as a series of horizontal solids with vertical end sections similar to a railroad excavation. In channel dredging the contractor is usually required to form the bottom at a given grade and the side slopes at a given slope, in which case it is not necessary to take cross-sections after the work is done, but soundings are taken as the dredging progresses to insure that the excavation is of proper depth.

When Scow Measurements are used to determine the quantity dredged each pocket of the scow is carefully measured and its capacity computed. When the scows have been loaded with the dredged material the surveyor or inspector makes a note of the number of full pockets; if they are not full he measures the distance down from the top of the coaming of the scow to where he estimates that the material in a pocket would come if it were leveled off. The volume of this small rectangular prism is calculated and deducted from the capacity of the corresponding pocket. For each scow in use tables giving the quantity in each pocket at various distances below the top of coaming are usually prepared for convenience. These scow measurements should be taken just before the tow starts for the dumping ground. When scows remain moored for a day or so before being towed to the dumping ground some of the material in the pockets leak out thru the bottom doors if they are not tightly closed, and much material may find its way back again into the dredged portion of the channel. In the case of a deck scow where the material is piled on the deck any practical and convenient method may be used to determine the volume, the measurements depending upon the shape of the pile. For ROCK the amount taken out can be calculated by obtaining its weight, and this is ascertained by determining the displacement of the scow before and after loading.

Earthwork Computation from Contours is used in many landscape problems for determining an approximate value of the quantity of earth to be moved in a proposed grading project or in connection with a grading problem to form the shape of the final surface of a certain locality after a definite quantity of material has been removed from it. The use of contours in earthwork computations has been confined almost exclusively to preliminary estimates, but the principles involved in such computations are sufficiently sound so that as accurate determinations may be made in this way as by any of the other more common forms of earthwork computation, provided the contours are determined in the field with sufficient accuracy. It is frequently necessary to use a map, however, in which the contours have been determined by means of the stadia method or by some method of sketching their positions, using as a basis several characteristic points which have been located by the transit and whose elevation has been determined by leveling. It is not customary or economical to define the

ntours accurately enough to admit of computing earthwork for payment of ntracts because the borrow-pit method (Art. 66) is more readily adapted this problem. For preliminary studies of all grading problems, however, contour map is invaluable; its use can be extended to determining approx-ate quantities of earthwork without the necessity of any additional field-rk, so that it is adapted to preliminary estimates.

There are three general methods of computing earthwork from data given on a ntour map: (1) by computing directly the amount of cut or fill between ccessive contours; (2) by assuming a horizontal plane below the lowest part any of the earthwork and first computing the volume of earth between that ane and the original surface, and afterward computing the volume between e same plane and the proposed surface; the difference between these two lumes will be the amount of excess cut or fill; (3) by drawing on the plan first ine of no cut or fill, a second line representing, say, 2 ft cut or fill, a third line 4 ft cut or fill, and so on, and finally computing the volume between these ccessive 2-ft layers. In all three cases the most practical way of determining e areas of the end sections is by means of a planimeter. The volumes are eferably computed by the end area method.

Some surveyors use the prismoidal formula in problems of this nature by either inter-lating by the eye the "middle area" or by treating every other area at a contour plane a "middle area." The error in sketching the contours, in scaling the map, in the inkage of the paper on which the map is drawn, and the uncertainty in the amount of inkage in the earthwork are large enough to offset any advantage which the pris-idal formula may have over the end area method in point of accuracy. The end area thod therefore is in practically all instances not only as exact as is the map upon which computations are based, but is also sufficiently exact for preliminary estimates.

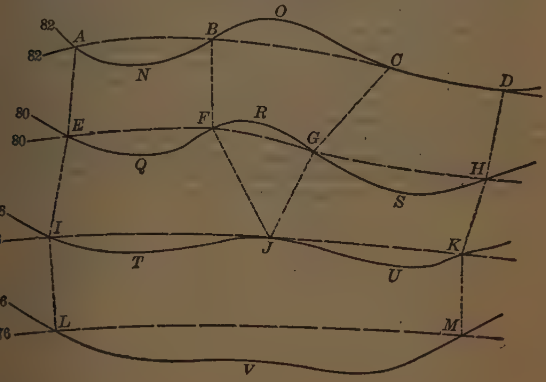

Fig. 74

Method (1). The simplest application of this problem is illustrated by Fig. 74, which the full lines are contours representing the shape of the existing surface, ile the dash-line contours represent the proposed shape of the ground; the tted lines are construction lines marking the limits of the solids to be com-

puted. Such solids as *ANB-FQE*, *EQF-JTI*, *ITJUK-MVL* represent excavation, and solids like *BOC-GRF* and *FRG-J* are embankment. The solid *ANB-FQE* is substantially an inclined prismoid having two parallel horizontal bases whose areas are *ANB* and *FQE*, the perpendicular distance between these being the contour interval, in this case 2 ft. To compute the volume of solid *ANB-FQE*, first determine the area of the parallel bases *ABN* and *EFQ* by planimeter. The perpendicular distance between these bases is the contour interval, 2 ft. The volume is therefore computed by averaging the end areas and multiplying by the contour interval. All the other six excavation solids here shown should be computed in a similar manner; solid *CD-HSG* is a wedge. The solid *FRG-J* is a pyramid whose base is *FRG* and whose altitude is 2 ft.

Method (2). Where the problem is so complicated that the quantities of earthwork cannot be readily separated into prisms, wedges, and pyramids, as was done in Fig. 74, it can be solved by computing the quantity of earth by method (2). Such a treatment is particularly applicable to the problem of determining the excess of cut or fill in a given grading project as represented by a set of proposed contours. It can be applied to determining the actual amount of cut and of fill, but for this purpose it is not so convenient of application as method (1).

Method (3). This method is particularly applicable when the original ground is very irregular and the proposed surface is to have an entirely different shape, so that the proposed contours do not cut many of the original contours of the same elevation. The first step in this process is to mark at every intersection of a new with an old contour the cut (or fill) and then to connect by means of a smooth curve the successive points of equal cut. These new curves will enclose areas which are the horizontal projections of irregular surfaces which are parallel to the final surface of the land and which are (if the contour interval is 2 ft) at the line of no cut or fill, at 2 ft, 4 ft, etc., above the final surface. The solids included between these 2-ft irregular surfaces are layers of earth each 2 ft thick, and their volumes are computed by determining the areas of their horizontal projections and applying the usual end area method for volumes. To find the volume of the solid included between the curves of 2-ft cut and the curves of 4-ft cut, for example, take for the upper surface the area within the dotted 2-ft line as a plane surface; for the lower surface, the area within the dotted 4-ft line; then multiply the mean of these two areas by 2 ft in this case (the difference between the 2-ft and 4-ft lines). The solids at the top or bottom of these layers may be treated as wedges or pyramids.

SECTION 3

STEAM RAILROADS

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TRACTION, ROADWAY, DRAINAGE

1. Train Resistances

1. **Locomotive Resistances.** The Amer. Rwy. Eng. Assoc. gives (Manual, 1915 Ed.) the following formulas for locomotive resistances: (*a*) Cylinder to rim of drivers: resistance = 18.7 (tons weight on drivers) + 80 (number of driving axles); (*b*) Engine truck and tender trucks: resistance = 2.6 (tons weight on trucks) + 20 (number of truck axles); (*c*) Atmospheric resistance = 0.25 V^2, in which V = velocity in miles per hour. Total resistance equals the sum. For example, a consolidation engine weighs, engine and tender, 330 000 lbs, with 170 000 lbs (= 85 short tons) on the four driving axles. The resistance between the cylinders and the rims of the drivers = 18.7 × 85 + 80 × 4 = 1910 lbs; the truck resistance = 2.6 × (165 − 85) + 20 × 5 = 308; atmospheric resistance at say 20 m p h = 0.25 × 400 = 100; the total resistance is therefore 2318. At the assumed velocity of 20 m p h the H P consumed would be 2318 × 29.33/550 = 123, which must be subtracted from the total indicated horse-power to determine the effective power at the draw bar. The above does not include the effect of grade. On a 1% grade there is an additional tax on the engine, merely on account of the weight of the engine and tender of 3300 × 29.33/550 = 176 H P, which must also be deducted. It has been found that the resistance between cylinders and rims of drivers is practically constant, regardless of speed, after the bearings have become warmed up.

2. **Atmospheric Resistance** depends on end area and length of train and is independent of tonnage. The end-area pressure is approximately one-half of the unit pressure times the area of the car end. According to elaborate experiments by Goss, the resistance of trains of locomotives and cars of ordinary dimensions may be exprest by the following equations, in which P = total atmospheric resistance in pounds, n = the number of cars, L = length of the whole train, l = combined length of the cars composing the train, and V = velocity in miles per hour.

Locomotive, tender, and freight cars	$P = (0.13 + 0.01n)V$
Locomotive, tender, and passenger cars............	$P = (0.13 + 0.02n)V$
Freight cars following locomotive	*$P = (0.016 + 0.01n)V$
Passenger cars following locomotive...............	*$P = (0.016 + 0.02n)V$
Locomotive and any kind of train..................	$P = 0.0003(L + 347)V$
Any train of cars (freight or passenger) following locomotive	*$P = 0.0003(l + 53)V$

3. **Oscillation and Concussion.** These resistances are usually considered to vary as the square of the velocity. Their determination, independent of other forms of resistance, is impracticable and even useless, since they vary with the particular condition of the track and roadbed at the time.

4. **Rolling Friction.** The resistance to the wheel rolling on the rail, independent of other forms of resistance, has not yet been definitely determined. It evidently depends on the stiffness of the rail and on the rigidity of the support given to the rail by the ties.

5. **Journal Friction** per ton of load is (*a*) less for higher pressures or heavy wheel loads; (*b*) greater for very low velocities; (*c*) a minimum for speed corresponding to train velocity of about 10 miles per hour; (*d*) greater for higher velocities; (*e*) less for higher temperatures; and (*f*) very dependent on perfection of lubrication.

* Excludes atmospheric resistance of locomotive and tender.

Total Train Resistance per Short Ton on Grades

Grade		When tractive resistance *on a level* in pounds per ton is					Grade		When tractive resistance *on a level* in pounds per ton is				
Rate per cent	Feet per mile	5	6	8	10	12	Rate per cent	Feet per mile	5	6	8	10	12
0.00	0.00	5	6	8	10	12	2.00	105.60	45	46	48	50	52
.05	2.64	6	7	9	11	13	.05	108.24	46	47	49	51	53
.10	5.28	7	8	10	12	14	.10	110.88	47	48	50	52	54
.15	7.92	8	9	11	13	15	.15	113.52	48	49	51	53	55
.20	10.56	9	10	12	14	16	.20	116.16	49	50	52	54	56
0.25	13.20	10	11	13	15	17	2.25	118.80	50	51	53	55	57
.30	15.84	11	12	14	16	18	.30	121.44	51	52	54	56	58
.35	18.48	12	13	15	17	19	.35	124.08	52	53	55	57	59
.40	21.12	13	14	16	18	20	.40	126.72	53	54	56	58	60
.45	23.76	14	15	17	19	21	.45	129.36	54	55	57	59	61
0.50	26.40	15	16	18	20	22	2.50	132.00	55	56	58	60	62
.55	29.04	16	17	19	21	23	.55	134.64	56	57	59	61	63
.60	31.68	17	18	20	22	24	.60	137.28	57	58	60	62	64
.65	34.32	18	19	21	23	25	.65	139.92	58	59	61	63	65
.70	36.96	19	20	22	24	26	.70	142.56	59	60	62	64	66
0.75	39.60	20	21	23	25	27	2.75	145.20	60	61	63	65	67
.80	42.24	21	22	24	26	28	.80	147.84	61	62	64	66	68
.85	44.88	22	23	25	27	29	.85	150.48	62	63	65	67	69
.90	47.52	23	24	26	28	30	.90	153.12	63	64	66	68	70
.95	50.16	24	25	27	29	31	.95	155.76	64	65	67	69	71
1.00	52.80	25	26	28	30	32	3.00	158.40	65	66	68	70	72
.05	55.44	26	27	29	31	33	.05	161.04	66	67	69	71	73
.10	58.08	27	28	30	32	34	.10	163.68	67	68	70	72	74
.15	60.72	28	29	31	33	35	.15	166.32	68	69	71	73	75
.20	63.36	29	30	32	34	36	.20	168.96	69	70	72	74	76
1.25	66.00	30	31	33	35	37	3.25	171.60	70	71	73	75	77
.30	68.64	31	32	34	36	38	.30	174.24	71	72	74	76	78
.35	71.28	32	33	35	37	39	.35	176.88	72	73	75	77	79
.40	73.92	33	34	36	38	40	.40	179.52	73	74	76	78	80
.45	76.56	34	35	37	39	41	.50	184.80	75	76	78	80	82
1.50	79.20	35	36	38	40	42	4.00	211.20	85	86	88	90	92
.55	81.84	36	37	39	41	43	4.50	237.60	95	96	98	100	102
.60	84.48	37	38	40	42	44	5.00	264.00	105	106	108	110	112
.65	87.12	38	39	41	43	45	5.50	290.40	115	116	118	120	122
.70	89.76	39	40	42	44	46	6.00	316.80	125	126	128	130	132
1.75	92.40	40	41	43	45	47	6.50	343.20	135	136	138	140	142
.80	95.04	41	42	44	46	48	7.00	369.60	145	146	148	150	152
.85	97.68	42	43	45	47	49	8.00	422.40	165	166	168	170	172
.90	100.32	43	44	46	48	50	9.00	475.20	185	186	188	190	192
1.95	102.96	44	45	47	49	51	10.00	528.00	205	206	208	210	212

The above table is mathematically correct except as noted in § 6.

6. Grade Resistance is 20 lbs per short ton (2000 lbs) for each percent of grade. This rule is not mathematically precise but the error is less than 0.5% for a 10% grade, 0.08% for a 4% grade, and is inappreciable for ordinary railroad grades. The GRADE OF REPOSE is the grade on which the effect of gravity just equals the tractive resistance. If a wagon or car is started down such a grade, it would continue to move indefinitely at a low velocity as long as the conditions are constant. The total resistance up such a grade is precisely twice that on a level.

7. Curve Resistance is considered the equivalent of a 0.04% grade per degree of curve, or 0.8 lb per short ton per degree of curve. See also Art. 2.

8. Inertia Resistance is measured by the energy required to impart velocity to the train. It is a tax on the locomotive, but it does not waste work, since every foot-pound of energy spent in this way is stored up as kinetic energy, which is later utilized in overcoming track resistances, or is transformed into potential energy by surmounting a grade. Of course it may be wasted by the

Accelerativ (or Retarding) Force in Pounds per Short Ton
(Original)

Distances s, Feet	Velocity in miles per hour, V									
	15	20	25	30	35	40	45	50	55	60
100	158	281	439	632	860	.....	.....	.....	.....	.....
200	79	140	219	316	430	562	711	878	.....	.....
300	53	94	146	211	287	375	474	585	708	843
400	40	70	110	158	215	281	355	439	531	632
500	32	56	88	126	172	225	284	351	425	506
600	26	47	73	105	143	187	237	293	354	421
700	23	40	62	90	123	161	203	251	303	361
800	20	35	55	79	108	141	178	218	266	316
900	18	31	48	70	96	124	158	195	236	281
1000	16	28	44	63	86	112	142	176	212	253
1100	14	25	40	57	78	102	129	160	193	230
1200	13	23	37	53	72	94	117	146	177	211
1300	12	22	34	49	66	86	109	135	163	194
1400	11	20	31	45	61	80	102	125	152	181
1500	10	19	29	42	57	75	95	117	142	168
1600	9	18	27	40	54	70	89	109	133	158
1700	9	17	25	37	51	66	84	103	125	149
1800	8	16	24	35	48	62	79	97	118	140
1900	8	15	23	33	45	59	75	92	112	133
2000	8	14	22	32	43	56	71	88	106	126
2100	7	13	21	30	41	54	68	84	101	119
2200	7	13	20	29	39	51	65	80	97	115
2300	7	12	19	27	37	49	62	76	92	110
2400	7	12	18	26	36	47	59	73	88	106
2500	6	11	18	25	34	45	57	70	85	102
2600	6	11	17	24	33	43	55	67	82	97
2640	6	11	17	24	33	43	54	66	80	96

Formula: $P = 70.22\ V^2/s$, in which P = force in pounds, s = distance in feet, V = velocity in miles per hour, 5% being allowed for rotative energy of wheels.

use of brakes. The greater part of this energy is the kinetic energy of translation, but about 5% is the kinetic energy of rotation of the various wheels and axles of the train. Let P represent the required force in pounds per short ton, s be the distance in feet between two points at which the train velocities are V_1 and V_2 respectively. Then $P = 70.22(V_2^2 - V_1^2)/s$, if V_1 and V_2 are in miles per hour. V_1 is the lower velocity; when the train is starting from rest, V_1 is zero.

9. **Brake Resistance** wastes the kinetic energy of the train by transforming it into heat and, unless such energy has been acquired by running down a grade, constitutes a source of double loss since the application of the brakes also requires power. For ordinary stops it may be considered as equivalent to a 7.5% grade, or 150 lbs per short ton; while for emergency stops it may reach double that amount. The corresponding distances may be obtained from the table on page 160.

Effect of Temperature. Decrease of temperature increases journal friction, but on the other hand, a frozen roadbed if smooth decreases the oscillatory resistances. At very low temperatures the increase in journal friction (combined with decreased power of the locomotive) requires a material reduction in the permissible train loading. Some railroads have a percentage scale for the reduction of the rating of locomotives according to the temperature. The increased tractive resistance for winter traffic over summer traffic is about two pounds per ton, with two pounds per ton additional for temperatures below zero.

Extra Resistance of Starting. The resistance of journals and axles is very much greater at the instant of starting and until they have become warmed up. Cars which have been stationary overnight, especially in cold weather, will become "frozen up" and will require a much greater force to start them, but the added resistance is only momentary and consumes but little energy, measured in foot-pounds. The frequent practise among locomotive engineers of backing the locomotive for a few feet and then immediately reversing and starting ahead has three effects, all of which help to start the train: (1) the journals are loosened from the somewhat rigid condition they will assume even during a short stop; (2) the springs in many of the couplers are more or less compress during the backward movement, and their expansion materially assists in starting the cars; (3) if the train is very long, the total slack in the couplers is very considerable, and the locomotive will have moved forward several feet and will have a considerable velocity before the last car starts; the cars are therefore started one by one. Since so much depends on the method of handling the locomotive, there is a corresponding variation in the results of tests which have been made to determine the value of this resistance. Thirty-five tests on the Rock Island system, with trains of 34 to 45 cars, gave results varying from 10.6 to 18.2 lbs per ton. The weighted mean of the values was 14.1 lbs per ton. The same tests quoted 30 lbs per ton when a train had stood overnight and was "frozen up"; also a resistance of only 6 lbs when the stop was merely instantaneous. Other tests have shown an average of 14 lbs per ton.

Effect of Weight of Cars. The resistance per ton of horse cars, light trolley cars and contractors' dump cars is as high as 20 lbs per ton and even more when the rails are light or the track in poor condition. The heavier the wheel loads, the less the resistance per ton. The resistance of an empty standard gage car, weighing say 17 tons, on good track, is about 8 lbs per ton, and for a loaded car, weighing say 60 to 70 tons, about 4 lbs per ton or even less on a good track.

Rating of Locomotives. Since grade resistance varies with the total weight, while tractive resistance depends on the concentration of weight on the

wheels, the gross weight of cars which may be attached to a locomotive depends not only on the grade, but also on whether the cars are loaded or empty. The greater the number of cars for a given weight of train, the greater the necessary drawbar pull, and therefore, for a given drawbar pull, the gross weight of a trainload of empties must be less than that of a train of loaded cars. For example, assume that an engine, weighing with the tender 336 500 lbs, has a tractive effort at the rim of the drivers of 32 075 lbs. On the basis of the resistance formula for " A " rating given in the next paragraph, the rating for a 0.6% grade is 2091 tons, with an allowance or " adjustment " of 8.6 tons for each car.* Then the permissible weight of the train which could be attached to that engine when the ruling grade is 0.6% may be exprest by the formula $W = A - 8.6n$, in which W = the weight of cars, A = the rating, and n = the number of cars.

For 30 loaded cars	2091 − 258 = 1833 tons;	average 61 tons per car;
For 50 partly loaded cars	2091 − 430 = 1661 tons;	average 33 tons per car;
For 90 empties	2091 − 774 = 1317 tons;	average 15 tons per car.

But a ruling grade of 0.6% is unusually low. As another example, with a ruling grade of 1.6%, the adjustment allowance is 3.6 tons per car, while the rating A is reduced to 770 tons. The possible train loads would be:

For 10 loaded cars	770 − 36 = 734 tons;	average 73 tons per car;
For 25 partly loaded cars	770 − 90 = 680 tons;	average 27 tons per car;
For 40 empties	770 − 144 = 626 tons;	average 16 tons per car.

On the 1.6% grade the weight of the train of empties is 85% that of the loaded train; on the 0.6% grade it is only 72%; which shows the relatively greater importance of heavy loads with low ruling grades. Formulas for tractive force of locomotives are given in Art. 21.

Total Train Resistance. No one simple formula can be devised which is equally applicable to all conditions of wheel loading, character of cars, condition of track, weight of rail, etc. Dennis' tests, corroborated by those of Shurtleff, indicated a practically uniform resistance for freight trains at all velocities between 7 and 35 miles per hour. Later experiments by Prof. Schmidt at Univ. of Illinois indicate some increase in resistance with increase in velocity for both freight and passenger equipment and the resistance of high-speed passenger trains unquestionably increases with the velocity, and some say, with the square of the velocity. Formulas based on tests made many years ago, when the rails were light, track conditions relatively poor, wheel loads light, and running gear not as perfect as present standards, are certainly not applicable to present standard conditions; and results obtained by tests of passenger trains are not applicable to freight trains, or vice versa.

A very few of the multitudinous formulas which have been proposed are here quoted.

Engineering News, 1892	$R = 2 + \frac{1}{4}V$
Baldwin Locomotive Works, prior to 1892	$R = 3 + \frac{1}{6}V$
Shurtleff, 1906	$R = 0.5 + (90/w)$

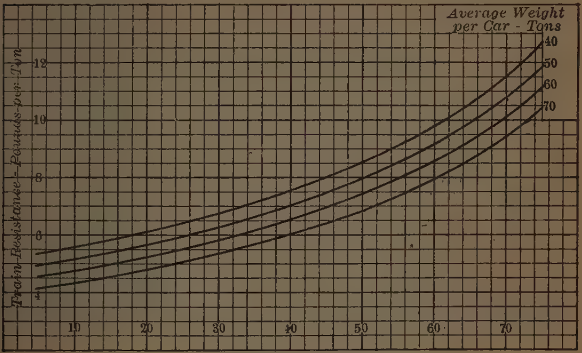
Amer. Rwy. Eng. Assoc., Manual, 1915 Ed.

A Rating, Temp. above 35° (Fahr.)	$T = 2.2t + 122n$
B Rating, Temp. 20° to 35°	$T = 3.0t + 137n$
C Rating, Temp. 0° to 20°	$T = 4.0t + 153n$
D Rating, Temp. below 0°	$T = 5.4t + 171n$

* This assumes that the engine and tender form part of the train which is approximately correct for A and B ratings on account of track and atmospheric resistances, not otherwise considered.

in which R = total resistance in pounds per short ton, V = velocity in miles per hour, T = the resistance in pounds per train, n = number of cars in train, t = weight of train in short tons, w = average weight in short tons of cars in train.

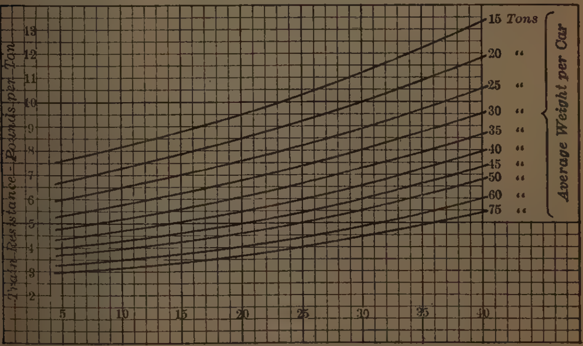
The first two are based mainly on tests with the passenger equipment of 1890–92 and may be compared with the results obtained by Prof. Schmidt in 1916 as shown below.



Speed - Miles per Hour

Fig. 1. Passenger Train Resistance

The second two formulas are for heavy freight service and assume that the resistance per ton is constant at the ordinary freight speeds of 7 to 35 miles per hour. They may



Speed - Miles per Hour

Fig. 2. Freight Train Resistance

in turn be compared with the results obtained by Prof. Schmidt with freight equipment in 1908–09 given in Fig. 2.

2. Compensation of Grade on Curves

The Principle involved is that the grade should be reduced by such an amount that the saving in grade resistance will compensate for the additional resistance caused by the curvature. The proper rate of compensation evidently depends somewhat on the speed of the train; it also depends largely on the resistance to the rotation of the car trucks about their king-bolts, being very greatly reduced when ball bearings are used under the center and side plates; it is apparently far less per degree of curve on very sharp curvature than on easy curvature. The rate of compensation should be made greater on a curve occurring immediately above a stopping place for trains. Since the added resistance of curvature virtually increases the grade, it is unimportant whether there is any compensation on curves which are on minor grades, provided the total resistance does not become greater than that of the ruling grade. The rules recommended by the Amer. Rwy. Eng. Assoc. are as follows: (1) Compensate 0.03 ft per degree when the length of curve is less than ½ the length of the longest train, when the curve occurs within the first 20 ft of rise of a grade or when curvature is in no sense limiting. (2) Compensate 0.035 ft per degree when the curve is between ½ and ¾ as long as the longest train and when the curve occurs between 20 ft and 40 ft of rise from the bottom of the grade. (3) Compensate 0.04 ft per degree when the curve is habitually operated at low speed, when the length of curve is more than ¾ that of the longest train, when the elevation is excessive for freight trains or at all places where curvature is likely to be limiting. (4) Compensate 0.05 ft per degree wherever the loss of elevation can be spared.

3. Cross-Sections of Railroads

For Railroad Roadbeds the specifications adopted by the Amer. Rwy. Eng. Assoc. call for flat sub-grades having minimum widths of 20 ft, 16 ft, and 14 ft for Classes A, B, and C respectively, see Art. 8. These are for depths of ballast shown in Figs. 14 and 15, Art. 10, and must be increased for deeper ballast in order to keep the same berm. They apply to both cut and fill, but in cuts the total width at sub-grade must be increased by the width of the two ditches.

Side Slopes of Excavations should ordinarily be 1.5 to 1. Firm earth can sometimes retain its slope permanently at 1 : 1. Loose rock will stand at 0.5 to 1, and solid rock at 0.25 to 1. Very soft earth, such as quicksand, may require a slope as flat as 4 to 1. Such earth as is proper to use for embankments will stand permanently when made with 1.5 to 1 slope. Loose-rock embankments will stand with 1 to 1 slopes.

Sodding the side slopes of embankments and excavations was formerly considered as extravagant landscape gardening, but it is now being more and more realized that the protection which it affords to the slopes from rain-wash is frequently more than worth its cost. The shoulders and toes of embankments should be rounded off to circles of about 4 ft radius, rather than make sharp edges which are not easily maintained.

4. Right-of-way

The Usual Width for single-track roads is four rods or 66 ft. The usual width taken for western railroads is 100 ft. A Class A roadbed (width 20 ft) on a 15-ft embankment, with slopes of 1.5 to 1, will require a width of 65 ft. A double-track roadbed, 33 ft wide at sub-grade in a level cut 20 ft deep, the slopes being 1.5 to 1, will require 93 ft of width beside the berm, in which to locate

Area of Right-of-Way for Varying Widths

Width in Feet	Acres per Mile	Acres per 100 Ft.	Width in Feet	Acres per Mile	Acres per 100 Ft.	Width in Feet	Acres per Mile	Acres per 100 Ft.
16.5 / 1 rod	2.00	0.038	27	3.27	0.062	40	4.85	0.092
17	2.06	.039	28	3.39	.064	41	4.97	.094
18	2.18	.041	29	3.52	.067	41.25 / 2.5 rods	5.00	.095
19	2.30	.044	30	3.64	.069	42	5.09	.096
20	2.42	.046	31	3.76	.071	43	5.21	.099
21	2.55	.048	32	3.88	.073	44	5.33	.101
22	2.67	.051	33 / 2 rods	4.00	.076	45	5.45	.103
23	2.79	.053	34	4.12	.078	46	5.58	.106
24	2.91	.055	35	4.24	.080	47	5.70	.108
24.75 / 1.5 rods	3.00	.057	36	4.36	.083	48	5.82	.110
25	3.03	.057	37	4.48	.085	49	5.94	.112
26	3.15	.060	38	4.61	.087	49.5 / 3 rods	6.00	.114
			39	4.73	.090			

For greater widths use sums or multiples; thus, for 87 ft width take sums of the acres for 3 and 44 ft.

he drainage ditches at the top of the bank which will keep surface water out f the cut and the additional width required for side ditches. The area required er mile of road is 2 acres for each rod of width.

5. Fences

Wire Fences with wood or concrete posts are recommended as standard ight-of-way fences by the Amer. Rwy. Eng. Assoc. The height is generally bout 4 ft 6 in but it and other features must conform to the statutory require-ents, if any. Four classes are provided, with the longitudinal wires spaced s follows:

Class A: *5 in above ground, 4, 4½, 5, 5½, 6, 7, 8 and 9 in.
Class B: 7 in above ground, 6½, 7, 7½, 8, 8½ and 9 in.
Class C: 9 in above ground, 7½, 8, 8½ and 9 in.
Class D: 10 in above ground, 10, 10, 12 and 12 in.

* This is reduced to 3 in and a barbed wire stretched in the middle of the space to ake fence "hog-tight."

Smooth, round wire of No. 9 gage is specified for Classes A, B and C, except hat for the top and bottom strands of a Class A fence No. 7 wire is required. ibbon, smooth, round or barbed wire may be used for Class D fences; the mooth wire being preferred, tho barbed wire is still quite generally used. heavy smooth wire or plank at the top is recommended for use with barbed vire. Vertical stay wires of No. 9 gage are specified for classes A, B and C; paced 12, 18 and 24 in, respectively. These must be attached to the longi-udinal wires with a mechanical lock or fastening which will prevent slipping ither longitudinally or vertically, or they may be electrically welded. Voven wire fencing giving the same effect or any combination of spacing, both ongitudinal and vertical, and material is increasingly used. It is preferably abricated in the shop if the ground is fairly level but may be woven in the ield to better fit rough ground if desired. In any case, all longitudinal wires

should be provided with tension curves or "spring coiled" to take up expansion and contraction. Combinations of narrow woven wire fencing with barbed wire strands above are also used in some cases. Wires should be placed on the side of the post away from the track. Staples should be of No. 9 wire, 1 in long for hardwood and 1½ in for softwood. They should be set diagonally with the grain of the wood and driven home tight. The top wire should be double stapled.

Wire for fencing and staples should be steel; galvanized with an even coating of zinc and the fence should be so fabricated as not to remove the galvanizing or impair the tensile strength of the wire. If electrically welded the galvanizing should be applied after fabrication. The galvanizing must stand one-minute immersion tests in a solution of commercial sulphate of copper crystals and water, the specific gravity of which is 1.185 and temperature between 60 and 70 degrees F. The sample should be washed with water and wiped dry after each immersion, and if the zinc is removed or a copper colored deposit formed after the fourth immersion, the lot from which the sample was taken is rejected.

Wooden Posts should be straight and free from splits, rot or other defects. They may be made of cedar, locust, chestnut, Bois d'Arc, white oak, catalpa, or other durable wood native to the locality or of treated timber. The dimensions of sawed or split posts should at least equal those given below for round ones. End, corner, anchor and gate posts should be at least 8 ft long, 8 in in diameter at small end, and set 3 ft 4 in in the ground; intermediate posts should be at least 7 ft long, not less than 4 in in diameter at small end, and set 2 ft 4 in in the ground. The posts should be set with the large end down. The spacing of posts should be 16½ ft. Holes should be dug to full depth, even if blasting is necessary. To avoid blasting holes in solid rock, intermediate posts may be set on 6 by 6 in sills, 4 ft long, braced on both sides by 2 by 6 in braces, 3 ft long. Not more than two such posts should be placed consecutively. When the posts and wire are set, the tops of the posts should be sawed off on a one-fourth pitch, the high side being that on which the wire is fastened and 2 in above it. Intermediate posts in hollows or sags should be anchored down by gaining and spiking two cleats near the bottom. End and corner posts should be anchored by cleats set one near the bottom and the other just below the ground surface, and also braced by braces to adjacent posts. The cleats should be made of 2 by 6 in common lumber, 3 ft long. The braces should be 4 by 4 in common lumber. Wire braces made of a double strand of No. 9 wire should be used where bracing by tension is needed.

Concrete Posts are recommended as practical and a suitable substitute for wood where economy would result therefrom. The cross-section may be a square, rectangle, isosceles triangle, giving V section with rounding of acute angle, a T or a circle; the last having the most extended use while square or nearly square sections are slightly more efficient in resisting the forces that ordinarily cause failure. This may be offset by greater resistance to deterioration and better methods of manufacture. Square corners should be rounded off to a radius of not less than 1 in and posts should taper from base to top. End, corner, anchor and gate posts should be at least 8 ft long, 8 in at base and 6 in at top, set 3 ft 4 in in the ground and reinforced with four ⅜ in rods. Intermediate posts should be at least 7 ft long, 5½ in at base and 4 in at top, set 2 ft 4 in in the ground and reinforced with three or four ¼ in rods, depending on design; which should be such that post, supported at the ground line and acting as a cantilever, will withstand a force of 180 lbs applied 60 in above the ground line. Braces should be 4 by 4 in, reinforced with four ¼ in rods. Concrete should be made of one part Portland cement to not more than four

parts of clean, hard aggregate so proportioned as to produce a dense concrete, using screen analysis as a guide. The minimum size of gravel or crusht stone used should be not less than ¼ in nor more than ½ in. Concrete should be thoroly mixed in a batch mixer and of such consistency that water may be brought to the surface by tamping. Reinforcement of stiff round or square rods, preferably moderately deformed, made from steel with high elastic limit, should be placed as near the surface of the post as practicable, say ½ in from it, and should be positively held in its proper place thruout its length. Molds should be soaped or oiled to prevent sticking and jogged or vibrated to compact concrete. Posts should be sprinkled with water for eight to ten days after being made and cured for 90 days before being set or transported. They should not be made out of doors in freezing weather or exposed to the sun. They should be carefully handled and packed in straw, sawdust or other suitable material for shipment. The fencing may be attached to the posts by means of wires wound around them, by staples, loops or lugs cast into the concrete or by casting holes in the posts thru which wires may be passed, tho the last method weakens the post somewhat.

Steel Posts are used to some extent, the design depending on the manufacturer. A comparison of costs is given below. Special types of fences from picket to solid board, sometimes surmounted by one or two strands of barbed wire, in wooden construction and from ornamental iron to woven wire on steel posts, also with barbed wire at the top, in metal construction, are used around yards and shops.

Farm Gates should be of all metal construction and from 12 ft to 16 ft wide depending on the width of agricultural implements used in the vicinity or legal requirements. The height should be at least 4 ft 6 in. They should swing away from the track and, if hinged as is desirable, should shut by gravity and overlap the post so as not to swing toward track.

The First Cost of fencing depends on prices of material and labor and efficiency of the latter and should be estimated in each case from quotations on material and estimated output of labor. *The Annual Cost* involves also the estimated life and interest rate and should be used in making comparisons. The following data give a rough idea of costs and information from which detailed estimates may be prepared. Data collected in 1915 by a Committee of the Amer. Rwy. Eng. Assoc. showed the average cost and life of wood posts to be as follows:

Kind of Wood	Life, Years	First Cost, Cents	Handling, Cents	Setting, Cents	Total, Cents
Bois D'Arc	26	14	1.72	8.52	24¼
Catalpa	12	17	1.72	8.52	27¼
Cedar	15	16	1.72	8.52	26¼
Chestnut	11	14	1.72	8.52	24¼
Cypress	11	20	1.72	8.52	30¼
Locust	17	20½	1.72	8.52	30¾
Oak	9	14½	1.72	8.52	24¾
Pine	8	11	1.72	8.52	21¼

Information received in 1917 by the same Committee indicated an average first cost and cost of handling and setting of 27 and 7 cents, respectively, for steel posts and 25 and 20 cents for the same items for concrete posts. With an estimated average life of 20 years for steel and 50 years for concrete and interest

at 6%, the total annual costs are 2.7, 2.96 and 2.85 cents for cedar, steel and concrete, respectively. At present prices, the average annual cost of the steel post would be over 4 cents, that of the cedar, nearly 3 cents and of the concrete over 3 cents. These are averages and an exact comparison should always be made in a given case.

Materials Required for Board Fences
Per Mile of Fence One Board High

Spacing of Posts	Number of Posts	Nails in Pounds		Lumber in Feet B M				
		8d	10d	1×4 in	1×6 in	1×8 in	1×10 in	1×12 in
8 ft ...	660	20.7	31.5	1760	2640	3520	4400	5280
10 ft....	528	22.0	33.5	1760	2640	3520	4400	5280
12 ft ...	440	18.3	28.0	1760	2640	3520	4400	5280
14 ft ...	378	15.7	24.0	1760	2640	3520	4400	5280
16 ft. ..	330	13.7	21.0	1760	2640	3520	4400	5280

The quantity of nails is figured on the basis of two nails to a board to each post and an allowance of 5% is made for loss. Where posts are 8 ft apart it is figured that 16 ft boards will be used.

For Wire Fences the 16½ ft spacing, if used, will require 320 posts per mile. Barbed wire varies in weight with the type but may be estimated roughly at 1 lb per rod, or 320 lbs per mile, while No. 7 and No. 9 wire weigh 439 and 306 lbs per mile, respectively.

Materials, Except Posts, Required for Wire Fences. Amer. Rwy. Eng. Assoc. Standards
Per Mile of Fence

Class	Wire				Staples		Locks
	Longitudinals		Verticals No. 9	Total	1-in	1½-in	
	No. 7	No. 9					
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	No.
A........	878	2142	1300	4320	31.1	46.4	47 500
B........	....	2142	825	2967	24.9	37.1	24 640
C........	....	1530	450	1980	18.7	27.8	13 200

One inch extra was allowed at each end of vertical or stay wires and 5% was allowed for loss of staples. Nothing was allowed for loss of locks or ties, as, if fabricated in the field, some of the stay wires would be omitted at posts.

Staples are put up in 100-lb kegs, the number per keg if of No. 9 wire being approximately as follows: 1-in, 10 800; 1¼-in, 8700; 1½-in, 7200 and 2-in, 5800. The number of pounds per strand per mile of fence with posts 16½ ft apart may be found by dividing the quantities in the above table for a Class A fence by ten. Locks are of various types, the weight may be estimated at 25 lbs per 1000.

Prices of both materials and labor are high (1918) and subject to wide variations and fluctuations. Barbed wire costs from 5 to 7 cents per lb and smooth from 8 to 10. Staples cost about the same per lb as the wire and locks 50 to 75% more.

The Output of Labor is affected by topography of site, presence of trees or brush, and kind of soil as well as experience and efficiency of workmen. Assuming that one man will dig the holes and carry and set 35 posts per day, Camp estimates the labor per mile for various types of fences as follows:

Four board fence, 16 ft boards, posts 8 ft apart, without battens	29½ days
Same, with battens	36 days
Barbed wire fence, 4 strands, posts 16 ft apart	13 days
Same, with posts 12 ft apart	16 days
Same, with top board and three wires, posts 12 ft apart	17 days
Same, with top board and four wires, posts 12 ft apart	18 days

He allows about one day's labor, more or less, for each strand. Other data indicate that inexperienced men, particularly on short jobs, will take from 25 to 50% more time than stated above and cost of foreman, whose time may be estimated at one-sixth that of men, and of clearing brush and distributing material must also be considered. Holes may be dug with an auger or digger at about half the cost with bar and shovel or 100 holes may be dug by one man in a day instead of about fifty.

Highway Grade Crossings. Three inch planks, spiked to blocks on the ties, on either side of each rail, the outer ones tight against the heads of the rails and the inner allowing a flangeway of 1¾* in are commonly used for country roads and farm crossings. The space between the inside planks should be filled in flush with ballast. An old rail laid on its side with its head against the web of the track rail, or standing with separators between it and the track rail, is often used for the flangeway. The ends of such rails should be bent inward to give an opening of at least 4 in. On important crossings the planking should entirely fill the space between the rails and the ends of the planking should be beveled both between and outside the rails to a thickness of 1 in, commencing at a point 10 in from the end. On improved roads and paved streets the macadam or pavement is continued between the rails leaving flangeways as above. The width of the crossing should be not less than 16 ft. The width of a farm or private road crossing should not be less than 12 ft.

Snow Fences. A snow fence is a structure erected for the purpose of accumulating drifting snow. Snow is carried by the wind close to the surface of the ground and is deposited in railway cuts on account of the eddies and diminution of velocities which these cuttings cause in the wind. A snow fence is so placed that artificial eddies will form on the windward side of a cut at a sufficient distance to cause the snow to be deposited between the snow fence and the cut. A tight fence of sufficient height will cause snow to accumulate on the windward side of a fence. An open fence causes snow to accumulate principally on the leeward side. Frequently the best type of fence and its proper location may only be determined by experiment, therefore portable snow fences are used. When practicable a permanent snow fence located on the right-of-way line is the most economical. Where this line is 50 ft or less from the center of the track, the boards should be close. For greater distances space may be left between the boards. When the distance is 100 ft, 50% of the fence area should be open space. The height of the fence should not exceed 10 ft. Hedges of California privet, Armour Barberry, evergreens, spruce, and locust have been planted in recent years for this purpose. Their economi-

* For curved track increase 1/16 in for each 2 degrees over a 2 degree curve.

cal efficiency is still doubtful. Stone walls have been used effectually where field stone are plentiful.

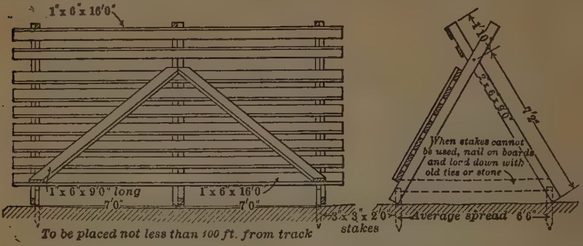


Fig. 3. Portable Snow Fence

A specification, quoted from Orrock's "Railroad Structures and Estimates," calls for cedar posts 8 in diameter, 12 ft long set 3 ft 6 in in the ground and spaced 8 ft on centers. The boards are ⅞ in × 6 in × 16 ft, and are nailed on with 6 in clear spaces. This requires eight boards, four breaking joints at each post. A *portable fence* is made in panels 16 ft long. Six boards 1 in × 6 in × 16 ft, spaced 6 in clear, are fastened onto three pieces 2 in × 6 in × 9 ft, spaced 7 ft apart. Two pieces 1 in × 6 in × 10 ft, act as stiffeners. Three other pieces, also 2 in × 6 in × 9 ft, are bolted, using ½ in × 4¾ in carriage bolts, with plate washers, scissorwise to the other three similar pieces. Two more boards are fastened to the tops of the second set of pieces. These sections may be folded up flat for storage. When in use, the scissors are opened until the lower ends are spread about 6 ft 6 in apart, and are fastened to stakes, 3 in × 3 in × 2 ft, driven into the ground. On hard, rocky ground a tie-board, 2 in × 6 in × 7 ft, may be nailed to the lower ends of the scissors and weighted down with old ties, stones, etc. This construction is estimated to cost $6 per panel of 16 ft.

Fig. 4. Slide Fall, Northern Pacific Ry.

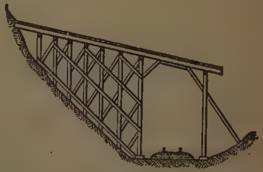


Fig. 5. Slide Fall, Northern Pacific Ry.

Snow Sheds are expensive to build and to maintain. Their probable necessity and the possibility of avoiding them should have its influence in deciding on the location of the road. There are two types of construction: (1) those to protect the track against slides, the track being on a side hill or at the base of a hill, and (2) those to prevent the filling of cuts or to protect the track from an excessive level fall. Altho the detail design varies somewhat with nearly every location, typical designs for timber sheds are shown in Figs. 4 to 7.

Sheds cut off the view considerably and are disagreeable to operate; hence a "summer track" is sometimes laid outside. To partially open up level fall sheds and to localize fires, movable sections 100 ft long are provided which are mounted on wheels and telescope into adjacent sections enlarged for that

purpose. The cost of maintaining the extremely heavy avalanche sheds, particularly those for double track, and the great fire risk have led some roads

Fig. 6. Level Fall, Southern Pacific Ry.

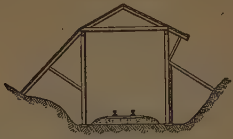


Fig. 7. Level Fall, Canadian Pacific Ry.

to build combination concrete and timber sheds and reinforced concrete ones. See *Eng. News*, Dec. 15, 1910.

6. Drainage and Waterway for Culverts

Longitudinal Drains are of three kinds: (*a*) intercepting ditches which protect the slopes of cuts, and intercepting ditches or tile drains which similarly protect embankments on saturated soil; (*b*) side ditches in cuts; (*c*) subsurface drains, which are usually made of 6-in tile pipes and are located about 2 ft underneath those of class (*b*). The SLOPE must ordinarily conform to the natural slope of the ground for class (*a*) and of the roadbed for class (*b*). Since tile pipes will carry water on a flatter slope than mere earth ditches, drains of type (*c*) are particularly necessary when the grade of the road is level or very

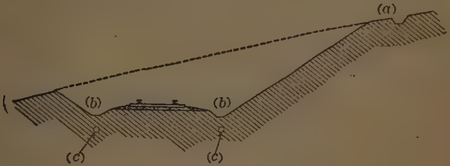


Fig. 8. Longitudinal Drains.

flat, and especially if the soil is retentive and impermeable. Care should be taken that all such drains have a free and unobstructed outfall.

Cross Drains are used only when it is necessary, on account of the lack of a proper outfall, to conduct the water thru the roadbed. When lumber is cheap, they may be made of boards so as to give an interior cross-section of 6 by 8 in. 6-in cast-iron pipes are far better.

The Area of Waterway, or size of opening, of a culvert should be sufficient to care for the flow resulting from all ordinary floods but, unless the probability of loss of life or damages from interruption of traffic be very great, it is not considered necessary to provide for those phenomenal floods which occur at very long intervals; it being more economical to repair such damages once in fifty to seventy-five years than to tie up additional capital in construction for such long periods. It should be noted, however, that cost does not increase in proportion to capacity, this fact together with the impossibility of accurately determining required sizes and the advantages of standards in estimating and

construction, has led most roads to adopt a few standard sizes for small openings rather than try to fit each case accurately.

The Maximum Rate of Flow, or Runoff, at a given point depends upon the following factors:

(*a*) Rate of rainfall, not necessarily the maximum which is likely to be of such short duration as not to give time for the water to reach the culvert from any considerable portion of the area unless the watershed is very small.

(*b*) Area of the watershed.

(*c*) Permeability of surface and vegetation. Saturation by long continued rains makes most soils more or less impervious, also freezing. Vegetation retards flow.

(*d*) Shape and slope of the watershed, which affect the time required for the water to reach the culvert from different points. See (*a*).

The Form of Entrance, Slope and Smoothness of a culvert affect somewhat its carrying capacity tho these factors are not usually considered in determining sizes but are simply made as favorable as local conditions and type of structure permit.

A Theoretical Determination of the proper area of waterway being impossible on account of the unknown influence of the factors affecting the runoff, recourse is had to observation and the use of empirical formulas. Consensus of opinion seems to favor "careful field observation and the use of intelligent judgment," particularly in settled country where other culverts and bridges exist over the same stream, thus giving opportunity for a study of their sufficiency at times of high water. In new country, high water marks, preferably at points where the stream has a well-defined channel, will furnish valuable evidence. Where there are no such guides the safest plan is to build a temporary trestle, which will not only provide an ample waterway for all floods during its life, but which will permit the construction of a culvert of proper dimensions under the trestle. The life of such a trestle should be sufficient to determine the area with all-sufficient accuracy.

Empirical Formulas are useful as a guide where only general information is available but their use in other cases is warranted only to the extent that the formulas and the constants used therein are known from long usage and observation of results to fit local conditions. All of the above conditions should be considered in estimating the value of the constant to be used in a particular case in applying any empirical formula, of which some are quoted below. Let a = area of waterway in square feet and A = drainage area in acres.

E. T. D. Myer's formula, $a = C\sqrt{A}$, in which C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground.

A. N. Talbot's formula, $a = C\sqrt[4]{A^3}$. "For steep and rocky ground C varies from 2/3 to 1. For rolling agricultural country, subject to floods at times of melting snow, and with the length of the valley three of four times its width, C is about 1/3; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, 1/5 or 1/6 or even less may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert."

J. T. Fanning's formula, $a = 0.23\sqrt[6]{A^5}$, in which no allowance is made for variations in conditions which affect the flow.

R. McMath's formula, $a = 0.5908\sqrt[5]{A^4}$, which was designed primarily for application to sewers. Like Fanning's, it makes no allowance for variations in conditions.

C. B. & Q. formula, $a = 0.46875\ A/(3 + 0.079\sqrt{A})$.

Fanning's formula and the C. B. & Q. formula agree very closely, and also agree with Talbot's for a mean value of C. But Talbot's has the advantage

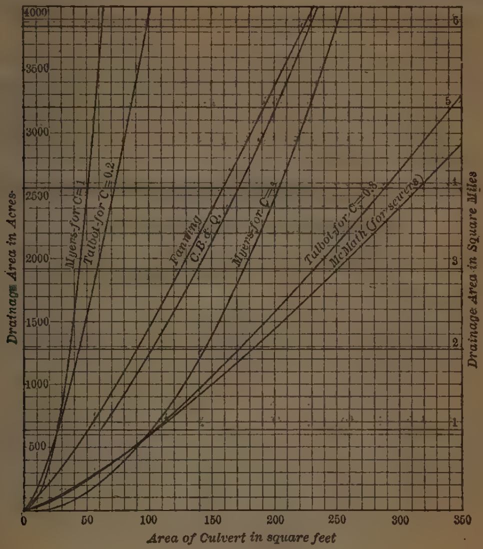


Fig. 9. Required Culvert Areas by Different Formulas

of flexibility. Curves for these formulas are plotted in Fig. 9, from which values of a may be readily found.

Areas of Full Semicircular Arches with Vertical Abutment Walls

Height Abut Walls, Feet	Span of arch, feet											
	5	6	7	8	9	10	11	12	13	14	15	16
1	14.8	20.1	26.2	33.1	40.8	49.3	58.5	68.5	.....	.....	.....	.....
2	19.8	26.1	33.2	41.1	49.8	59.3	69.5	80.5	92.4	.....	.....	.....
3	24.8	32.1	40.2	49.1	58.8	69.3	80.5	92.5	105.4	119.0	.....	.....
4	29.8	38.1	47.2	57.1	67.8	79.3	91.5	104.5	118.4	133.0	148.4	.....
5	34.8	44.1	54.2	65.1	76.8	89.3	102.5	116.5	131.4	147.0	163.4	180.5
6	....	50.1	61.2	73.1	85.8	99.3	113.5	128.5	144.4	161.0	178.4	196.5
7	....	...	68.2	81.1	94.8	109.3	124.5	140.5	157.4	175.0	193.4	212.5
8	....	....	....	89.1	103.8	119.3	135.5	152.5	170.4	189.0	208.4	228.5
9	....	....	....	....	112.8	129.3	146.5	164.5	183.4	203.0	223.4	244.5
10	....	....	....	....	.....	139.3	157.5	176.5	196.4	217.0	238.4	260.5

As a check on the waterway area found by the above methods, the rules for the proper size of a sewer to carry the storm flow of a watershed may be used. See Sect. 10, Art. 26, p. 1253.

The table on p. 173 will be useful in selecting a culvert having a required waterway area.

Pipes of standard sizes are extensively used for small openings and the following table, based on Talbot's formula, probably more used than any other given above, will be found convenient. For example, if the drainage area is 14 acres, a 16-in pipe culvert might answer under the most favorable conditions, but the size should be increased up to 36 inches, according to the coefficient deemed proper for the locality.

Drainage Area in Acres Drained by Standard Culvert Pipes, for Various Coefficients in Talbot's Formula

Formula: Drainage area = (culvert area)$^{\frac{4}{3}}/C^{\frac{4}{3}}$

Diam. of Pipe, Inches	Area, sq ft	Talbot's formula: C =					
		0.2	0.3	0.4	0.5	0.75	1.0
8	0.35	2	1	*	*	*	*
10	0.55	4	2	2	1	*	*
12	0.79	6	4	3	2	1	*
16	1.40	13	8	5	4	2	2
18	1.77	18	11	7	5	3	2
20	2.18	24	14	10	7	4	3
24	3.14	39	23	16	12	7	5
30	4.91	71	42	28	21	12	8
36	7.07	116	57	46	34	20	14
48	12.57	250	146	99	74	43	29

* Indicates less than one acre.

Circular Pipes for Culvert Openings

Diameter	12 in	16 in	18 in	20 in	24 in	30 in	3 ft	4 ft	5 ft	6 ft
Area, sq ft......	0.79	1.40	1.77	2.18	3.14	4.9	7.1	12.6	19.6	28.3
Weight of 12-ft sections of cast-iron pipe......	899	1322	1510	1798	2458	3325	4862	8038	12500	18500
Thickness of cast-iron pipe, inches	9/16	5/8	5/8	11/16	3/4	13/16	1 1/16	1 1/4	1 5/8	2

Cast-iron pipe of other thicknesses and weights can also be had in the market. See Sect. 10, Art. 20, p. 1235. Sewer, corrugated iron and concrete, both plain and reinforced, pipes are also used.

7. Designs for Culverts

The Length of the barrel of a culvert between the insides of the head walls is $L = 2\ sd + b$, in which s = slope ratio of the embankment (horizontal to vertical), d = depth to top of culvert, b = width of roadbed. (Fig. 10*a*.)

End or Wing Walls. These act as retaining walls for the lower part of the embankment and are of three types (see Fig. 10*b*):

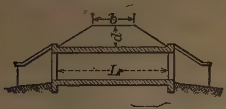


Fig. 10*a*

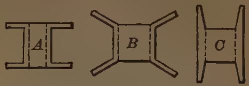


Fig. 10*b*. Types of End Walls

Type A is the simplest form, but the volume of masonry is usually greater than that required by the other types except in narrow gorges. It is generally used for pipe and other low and small culverts.

Type B requires less masonry and is more commonly used for large culverts.

Type C is liable to the objection that the water may work its way behind the wing walls, scour them out and endanger the embankment and culvert. When this can be unquestionably prevented, the type has the advantage of simplicity and economy of construction.

The Length of an end wall of type A should be a little more than sufficient to keep the earth of the embankment spilling around its ends from reaching the opening or more than twice the slope ratio times the height under coping plus the width of opening minus twice the thickness of the wall. Type B is tapered down to a height of from 2 to 4 ft and type C, to practically nothing. The types are often combined for culverts on a skew in order to better guide the stream into the culvert.

Standard Designs. Many railroads have prepared sets of standard plans for culverts and other structures, track work, signs, etc. These usually give quantities and are convenient for both estimating and construction as the men become accustomed to their use. Some examples are given below.

Pipe Culverts should be well bedded in firm earth or have plank, concrete or even piling foundation in soft soils. If slope is light they should be laid with a small camber and ends should usually be protected with end walls. Sewer pipe should have a cover of at least 3 ft of earth and no stones should be allowed in contact with it. Joints should be at least partially filled to smooth surface by centering spigots in hubs and prevent scour.

Stone Box Culverts. The thickness of the roof slabs for stone culverts usually varies from 10 in for a 2-ft span to 15 in for a 4-ft span. Fig. 11 repre-

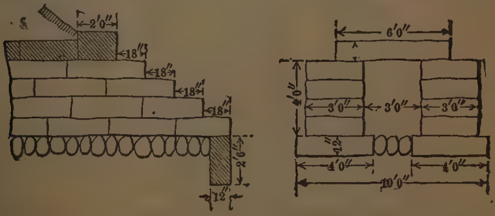


Fig. 11. Typical Stone Box Culvert

sents what may be considered as a typical design. Such a culvert requires 7.0 cu yds of masonry for each of the end walls and 1.41 cu yds of masonry for

each foot of length between the inside faces of the coping stones. This estimate does not include the volume of the paving.

Wooden Box Culverts, similar in design to the stone box, are used to save time and expense during construction, where timber is cheap. They are made large enough to allow a pipe culvert to be put thru them before the timber gives out. The sides are of 12 by 12 in timbers, laid up solid and the top varies in thickness from 8 in for 3-ft span to 12 in for 5 ft. The bottom should be planked and the ends protected from undercutting.

Masonry Arch Culverts. These usually are built as full semicircular arches unless the span is very great and the rise would be objectionably high. Designs are various, with resulting variations in quantities. Those for stone masonry and plain concrete are similar and data for Erie Railroad standards for the latter are given. The subscripts to the dimensions (A_1, A_2, etc.) indicate that all of the A dimensions are of the same general character. The plans indicate a culvert of type B for the up-stream end and of type C for the down-stream end. If there are two or more tracks, let Z = distance between track centers of the outer tracks and other notation as in Fig. 12; then

For single track, $L = 2(s_1 y + T + T_1 + T_2)$.

For two or more tracks, $L = Z + 2(s_1 y + T + T_1 + T_2)$.

The neat line is supposed to be U inches below the surface of the ground. Carry foundations as deep as conditions may require. Minimum depth U' inches except on rock bottom.

The volume of one of these culverts may be easily computed by the use of the quantities found in the accompanying table. These tables can also be used to compute the volumes of stone culverts if the quality of masonry is fairly good and joints thin. For rough rubble the thickness of walls and arch should be increased.

Volume of Standard Plain Concrete Arch Culverts, Erie Railroad

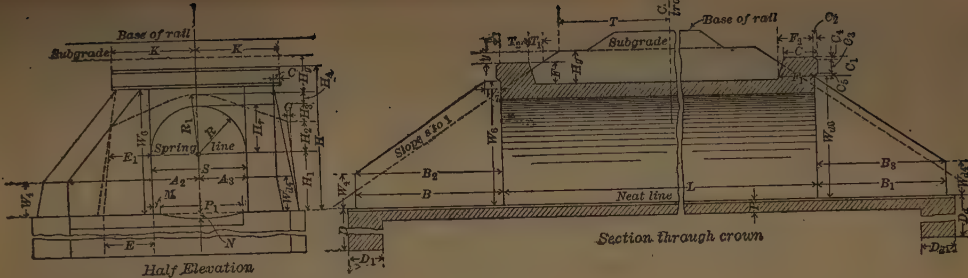
Condensed from several tables in Bulletin 105, Am. Ry. Eng. & Main. Way Assoc.

Span	Per lin ft		Paving between wing walls	Curtain walls 1 ft. deep	Two portals, wing walls and parapet
	Barrel	Paving under barrel			
3	0.873	0.055	0.60	0.30	11.0
4	1.034	0.083	1.10	0.44	13.2
5	1.117	0.111	1.31	0.59	13.8
6	1.979	0.185	7.50	1.48	28.7
8	2.784	0.260	12.7	2.0	48.2
10	3.703	0.333	20.38	2.63	74.5
12	4.792	0.408	29.23	3.04	105.2
14	5.998	0.482	54.68	5.48	158.3
16	6.703	0.556	64.11	6.00	186.7
18	7.598	0.630	76.29	6.7	212.0
20	8.087	0.704	88.86	7.3	242.6

For example, if a fill for a single-track road is 30 ft deep and an 8-ft culvert is to be used, since $H = 12'\ 1''$, $v = 17'\ 11'' = 17.92$,

$$L = 2\ (1.5 \times 17'\ 11'' + 9'\ 6'' + 10\tfrac{1}{2}'' + 2'\ 6'') = 79'\ 6'',$$

$$\text{Vol.} = 79.5\ (2.784 + 0.260) + 12.7 + 2.0 + 48.2 = 304.9 \text{ cu yd.}$$



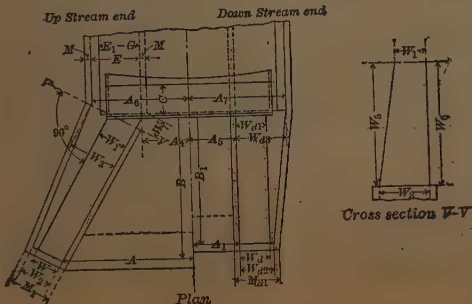


Fig. 12. Standard Plain Concrete Arch Culverts, Erie Railroad

From Bull. 105, Am. Ry. Eng. & M. W. Assoc.

In these plans A_4 and A_5 always equal the half-span, C is constant at 2′ 10,″ C_1 = 1′ 6″, C_2 = 0′ 4″, C_3 = 1′ 2″, $C_4 = C_5$ = 0′ 2″, D and Dd are variable, $D_1 = Dd_1$ = 2′ 0″, F = 2′ 0″, Hg = 3′ 0″, M = 0′ 6″, P_1 is one foot less than the span, S is the slope ratio 1.5 : 1, T = 9′ 6″, T_1 = 0′ 9″ for 3, 4 and 5 ft spans and 10½″ for longer spans when wings are at 30° with the axis. U = 1′ 0″. Wd and Wd_1 are the same as W, also W_6 and Wd_6 are the same as W_5.

Dimensions of Standard Plain Concrete Arch Culverts, Erie Railroad

Condensed from Bulletin 105 (1908) of Am. Ry. Eng. & Main. Way Assoc.

Dimensions	Span in feet										
	3	4	5	6	8	10	12	14	16	18	20
	ft in	ft in	ft in	ft in	ft in	ft in	ft in	ft in	ft in	ft in	ft in
A A_2	1 6	2 0	2 6	8 6	11 3	14 1	16 10	19 9	21 8	23 7	25 6
A_1 A_3	1 6	2 0	2 6	3 0	4 0	5 0	6 0	19 9	21 8	23 7	25 6
A_6 A_7	4 0	4 9	5 6	6 10	8 3	9 8	11 2	12 7	13 11	15 4	16 8
B B_2	4 11	5 6	5 5	9 6	12 7	15 9	18 10	22 1	23 9	25 3	26 11
B_1 B_3	4 11	5 6	5 5	8 3	11 4	14 6	17 7	22 1	23 9	25 3	26 11
E	2 6	2 9	3 0	3 10	4 3	4 8	5 2	5 7	5 11	6 4	6 8
E_1	2 2	2 3	2 6	3 6	3 10	4 2	4 7	4 11	5 3	5 8	6 0
F_1	2 0	2 0	2 0	← 2′ 10½″ →							
F_2	2 0	2 0	2 0	← 3′ 0″ →							
G				0 2	0 3	0 3	0 4	0 4	0 5	0 5	0 6
H	7 5	7 10	7 9	10 0	12 1	14 2	16 3	18 5	19 6	20 6	21 7
H_1	4 0	4 0	3 6	4 0	5 0	6 0	7 0	← 8′ 0″ →			
H_2	[illegible]	0	0	2 0	2 6	3 1	3 8	4 3	4 9	5 3	5 10
H_3	0 8	1 0	1 4	1 0	1 6	1 11	2 4	2 9	3 3	3 9	4 2
H_4	[illegible] 9	0 10	0 [illegible]	1 0	1 1	1 2	1 3	1 5	1 6	1 6	1 7
H_7	[illegible] 8	1 0	1 4	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0
K	4 0	4 9	5 6	5 10½	6 10½	7 10½	8 10½	9 [illegible]½	10 10½	11 10½	12 10½
M_1	3 6	3 9	4 0	3 10	4 0	4 2	4 3	4 3	4 4	4 4	4 4
N	[illegible]	0	0	← 0′ 6″ →							
P	0 9	0 9	0 9	← 1′ 0″ →							
R	2 0	2 6	3 0	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0
R_1	2 9	3 4	3 11	4 0	5 1	6 2	7 3	8 5	9 6	10 6	11 7
T_2	2 0	2 0	2 0	← 2′ 6″ →							
W W_1	2 0	2 0	2 0	2 6	2 6	2 6	2 10	2 10	2 10	2 10	2 10
W_2 W_{d2}	2 6	2 9	3 0	2 10	3 0	3 2	3 3	3 3	3 4	3 4	3 4
W_3 W_{d3}	2 6	2 9	3 0	3 4	4 3	5 2	6 0	7 0	7 5	7 10	8 4
W_4	6 1	6 6	6 5	← 3′ 0″ →							
W_{d4}	2 10	2 10	2 10	← 3′ 0″ →							
W_5 W_{d5}	6 1	6 6	6 5	8 6	10 7	12 8	14 9	16 11	18 0	19 0	20 1
W_7				← 1′ 5½″ →							
W_{d7}	0	0	0	0	0	0	← 1′ 5½″ →				
Area	13.3	18.7	22.1	39.8	67.5	102.3	144.2	193.3	233.5	276.9	323.4

The last line gives area of waterway in square feet.

Reinforced Concrete Culverts. The ability to resist stress, which permits the use of flat slabs, both in top and bottom, makes this method of construction of especial value. In general, any soil which is firm enough to support an embankment will support a reinforced concrete box culvert, with a slab bottom, with almost any span, without resorting to piling. Spread footings, unless very narrow, are less economical and less effective than the slab bottom. The design of culverts for railroad work permits the adoption of standard designs, using standard forms which may be removed and used several times with great economy. The forms may be standardized regardless of the use of a slab bottom, which in any case must be laid first. Excavation in the gully or stream bed will frequently expose a rocky bottom into which trenches may be dug which will not only give a sufficient footing but will also furnish sufficient resistance against any lateral thrust of the side walls. In each case it becomes a question of judgment whether to use a slab bottom or perhaps to dig a little deeper and make the side walls a little higher in order to set the side walls in firm soil. In either case the upper corners should be filled out and suitably reinforced

in order to resist any unbalanced thrust perpendicular to the barrel of the culvert. When a bottom slab is used the thrust at the bottom of the side walls may be taken up by shallow trenches in the top of the slab, immediately under the side walls, but it is cheaper and sufficiently effective to place greased plugs in the slab at the proper places for the vertical bars of the side walls. These plugs are afterward removed and the bars grouted in. Short pieces of scrap

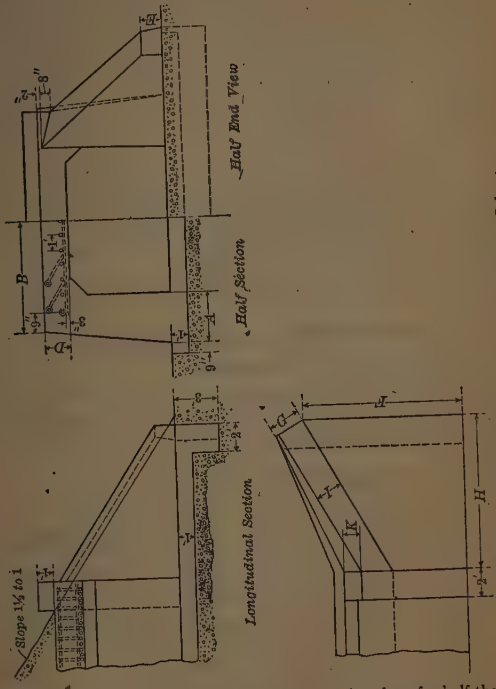


Fig. 13. Standard Reinforced Concrete Culvert

reinforcement may also be used as dowels by inserting them for half their length in the slab while still soft. Typical plans and data for the same are here given. Some roads use longer spans but reinforced concrete girders are commonly used for spans from 15 to 30 ft and arches for greater spans.

The longitudinal reinforcement is of ½-in bars the lower spaced 12-in centers for all spans, the upper used on 8- and 10-ft spans, only, and spaced 10 and 12 in, respectively. The transverse reinforcement varies with span from 9/16 to 19/16 in and is spaced 3-in centers for the center section and 8-in for the end sections. One-third the bars are straight, one-third bent up near the side walls and one-third nearer the center as shown on the half section. Other designs carry reinforcing in the side walls and bottom as shown in Sect. 5, which see also for retaining walls and other structures.

Data for Standard Reinforced Concrete Culverts, Nashville, Chattanooga and St. Louis Railroad

Condensed from Vol. 10, Proc. Amer. Rwy. Eng. & Main. of Way Assoc. (1909)

Letter	Span by Height													
	4×3		4×5		6×4		6×6		8×6		8×8		10×10	
	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
A	1	9	1	9	2	6	2	6	3	0	3	0	3	6
B	3	9	3	9	5	0	5	0	6	0	6	0	7	6
D	0	10	0	10	0	11	0	11	1	2	1	2	1	4
E	1	0	1	0	1	6	1	6	1	6	1	6	1	6
F	2	0	2	0	5	9	7	6	8	8	10	4	13	3
G	1	9	1	9	2	3¾	2	2½	2	3	2	2¼	2	1¾
H	5	0	9	0	4	10	7	10	8	3	11	3	14	6
I	1	9	1	9	2	0	2	0	2	0	2	0	2	0
K	0	0	0	0	0	11	0	11	1	2	1	2	1	2
Area	12		20		24		36		48		64		99	
Barrel (*a*)	0.963		1.222		1.497		1.829		2.258		2.628		3.790	
Barrel (*b*)	21		21		66		66		100		100		190	
Portals	8.11		16.0		16.4		28.4		31.5		47.1		71.9	

Barrel (*a*) gives cubic yards of concrete and (*b*) pounds of steel per lineal foot of barrel.

STEAM RAILROAD TRACK

8. Classification

The following classification of railways, based on tonnage and on maximum speed of passenger trains, is that used by the American Railway Engineering Association as the basis for recommended practise in the construction of roadbed, dimensions and quality of ballast, cross-sections, etc.

Class "A" shall include all districts of a railway having more than one main track, or those districts of a railway having a single main track with a traffic that equals or exceeds the following: Freight car mileage passing over district per year per mile, 150 000; or, Passenger car mileage per year per mile of district, 10 000; with maximum speed of 50 miles per hour.

Class "B" shall include all districts of a railway having a single main track with a traffic that is less than the minimum prescribed for Class "A," and that equals or exceeds the following: Freight car mileage passing over district per year per mile, 50 000; or, Passenger car mileage per year per mile of district, 5000; with maximum speed of 40 miles per hour.

Class "C" shall include all districts of a railway not meeting the traffic requirements of Classes "A" or "B."

9. Ballasting

The following definitions and specifications have been officially adopted by the Amer. Rwy. Eng. Assoc., and are therefore quoted as desirable standards for the classification and selection of ballast.

Ballast: Selected material placed on the roadbed for the purpose of holding the track in line and surface. BROKEN OR CRUSHT STONE: Stone broken by artificial means into small fragments of specified sizes. CHATS: Tailings from mills in which zinc, lead, silver and other ores are separated from the rocks

in which they occur. GRAVEL: Worn fragments of rock occurring in natural deposits, that will pass thru a 2½-in ring and be retained on a No. 10 screen. SAND: Any hard, granular, comminuted rock which will pass thru a No. 10 screen and be retained upon a No. 50 screen. CHERT: An impure flint or hornstone, occurring in natural deposits. CINDERS: The residue from the coal used in locomotives and other furnaces. SLAG: The waste product, in a more or less vitrified form, of furnaces for the reduction of ore; usually the product of a blast furnace. BURNT CLAY: A clay or gumbo which has been burned into material for ballast. GUMBO: A term commonly used for a peculiarly tenacious clay, containing no sand. DISINTEGRATED GRANITE: A natural deposit of granite formation, which, on removal from its bed by blasting or otherwise, breaks into particles of size suitable for ballast.

Stone Ballast. Stone shall be durable enough to resist the disintegrating influences of the climate where it is used; it shall be hard enough to prevent pulverizing under the treatment to which it is subjected; it shall break in angular pieces when crusht. The maximum size of ballast shall not exceed pieces which will pass in any position thru a 2½-in ring; the minimum size shall not pass thru a ¾-in ring. It should be free from dirt, dust or rubbish.

Gravel Ballast. At least 25% of sand is essential in gravel ballast to prevent the stones from shifting under their load. For Class A roads bank gravel containing more than 2% of dust (material finer than sand) or 40% of sand must be washed or screened and the product should contain between 25 and 35% of sand. For Class B roads bank gravel containing more than 3% of dust or 60% of sand should be washed or screened and the product should contain from 25 to 50% of sand. For Class C roads any material which makes better track than the natural soil may be used.

Cinders. The use of cinders as ballast is recommended for the following situations: On branch lines with a light traffic; on sidings and yard tracks near point of production; as sub-ballast in wet, spongy places; as sub-ballast on new work where dumps are settling, and at places where the track heaves from frost. It is recommended that provision be made for wetting down cinders immediately after being drawn.

Burnt Clay. The material should be black gumbo or other suitable clay free from sand or silt. The suitability of the material should be determined by thoro testing in a small test kiln before establishing a ballast kiln. The material should be burned hard and thoroly. The fuel used should be fresh and clean enough to burn with a clean fire. It is important that a sufficient supply be kept on hand to prevent interruption of the process of burning. Burning should be done under the supervision of an experienced and competent burner. Ballast should be allowed to cool before it is loaded out of the pit. Absorption of water should not exceed 15% by weight.

10. Standard Cross-Sections

The Cross-Sections in Figs. 14 and 15 are those officially adopted and recommended as good practise by the Amer. Rwy. Eng. Assoc., and on account of the care used to obtain the consensus of opinion of the best officials of the country, they may be considered as the most authoritative designs obtainable.

These sections are intended to show minimum depth under the ties and are recommended for use only on the firmest, most substantial and well-drained subgrade. For the deeper ballast commonly used on heavy traffic lines, a similar shape and same slopes are recommended; resulting, for a section adopted by the association in 1918 for 12-in sub-ballast and 12-in top-ballast (total under

tie 24 in), in a distance of 5 ft 7½ in from end of the tie (8 ft) to toe of ballast. With 1 ft 6 in berms, toe of ballast to shoulder, this requires 22 ft 3 in for width of roadbed on embankments and same plus ditches for cuts.

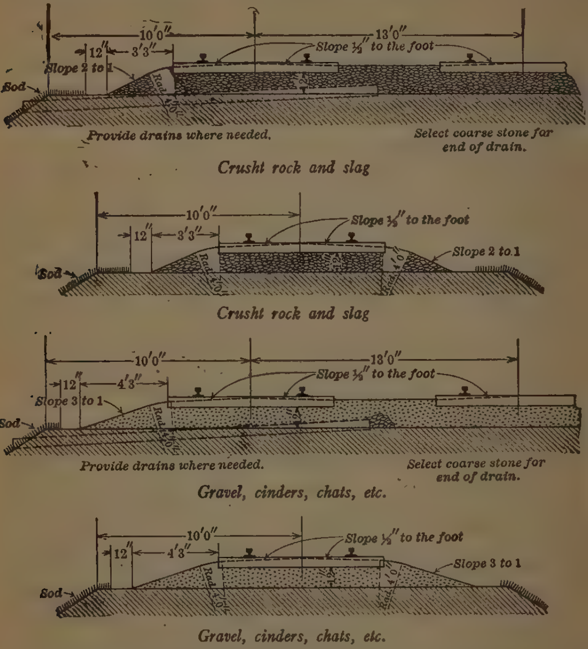


Fig. 14. Standard Railroad Cross-Sections, Class A Track

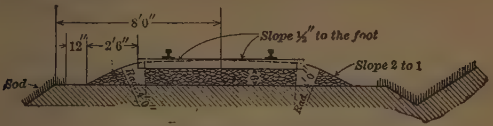
NOTE.—Slag is broken and similar in character to crusht rock. For granulated slag the same section as for gravel, cinders, etc., should be used.

The Quantity of Ballast required in cubic yards per mile is as follows:

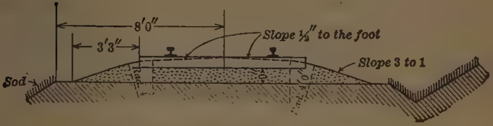
Kind of Ballast	Class A Double track	Class A Single track	Class B	Class C
Crusht rock and slag	6277	2974	2195	....
Gravel, cinders, chats, etc	6459	3156	2284	1695
Cementing gravel and chert	....	....	1882	1309

These computed volumes are based on the standard cross-sections as given in Figs. 14 and 15, and on the use of 6 in by 8 in by 8-ft cross ties, spaced 24 in on centers, for all classifications of track. The use of wider ties (same depth) would slightly decrease the required volume of ballast; the use of deeper ties

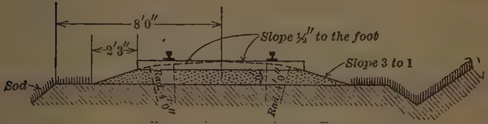
(same width) would increase the volume; wider spacing would increase the volume; the use of longer ties would decrease the volume.



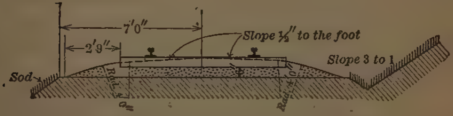
Crusht rock and slag, Class B Track



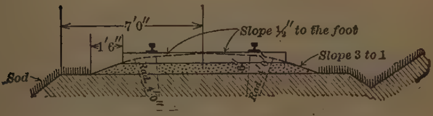
Gravel, cinders, chats, etc., Class B Track



Cementing gravel and chert, Class B Track



Gravels, cinders, chats, etc., Class C Track



Cementing gravel and chert, Class C Track

Fig. 15. Standard Railroad Cross-Sections

Cost. The total cost is made up of three items: the cost of the material on the cars, the cost of hauling to the place of distribution, and the cost of placing and tamping under the track. The cost of LOADING GRAVEL at a gravel pit is quoted at 7 cents per cu yd. Another quotation, which included "washing" the gravel, was 18 cents; still other quotations ranged from 5.62 cents to 13.9 cents. The cost of crusht limestone at the crusher is quoted

at 53.5 cents; other quotations for "crusht stone" were 48 cents, 59 cents, and 61.5 cents. The cost of HAULING 50 miles is quoted as 5.5 cents; on another road, hauling 50 miles and unloading, 10.7 cents; on another road the cost of hauling combined with unloading and placing in track was given as 40 cents. The cost of PLACING IN TRACK is about 12 to 15 cents for cinders, a little more for gravel, and up to 30 cents for broken stone; the cost of placing stone ballast is given in one case as 31.9 cents, and the cost of placing gravel under the same conditions was 11.8 cents. The cost of digging out old worn-out gravel ballast was given as 15 cents per cu yd. Tie renewals in stone ballast cost more than in gravel ballast; the relative cost (to quote one case) was 16.8 cents per tie for stone and 10.3 cents per tie for gravel.

These costs were all obtained before the present era of high and fluctuating prices and should be increased from 50 to 100% for the same methods. The increasing use of labor-saving machinery on many operations serves to partially neutralize these increases, but conditions are so variable that it is impossible to cover them.

11. Cross Ties

Wooden Ties. The minimum thickness should be 6 in; the minimum width (of a sawed tie) 8 in. Pole ties should have a face at least 6 in wide and a cross-section not less than 48 sq in. The minimum length should be 8 ft. For higher class roads the thickness is sometimes increased to 7 in, the width to 9 and even 10 in, and the length to 8 ft 6 in and even 9 ft.

Choice of Wood. About 44% of all ties used are made of the various species of oaks. The southern pines come next with about 18%. Douglas fir, cedar, chestnut, cypress, western pine, tamarack, hemlock, redwood, lodge pole pine and white pine are used in the order named. The average cost varies from 51 cents for oak to 28 cents for hemlock. The best of oak ties sometimes command 80 cents.

Durability. This depends on the weight and amount of the traffic, the character and drainage of subsoil and ballast, the use of tie plates, the climate, the time of year of cutting the timber, the age of the timber, and the amount of its seasoning before being placed in the track, and chiefly on the kind of wood. Therefore any quoted figures are necessarily subject to wide variations. A quoted consensus of opinion as to the life of untreated ties, which may be considered very reliable, is as follows: white oak, 7 to 12 years; pine, 5 to 8 years; chestnut, 8½ years; cypress, 7 to 9 years; cedar, 15 years; tamarack, 5 to 6 years; hemlock, 5 years. From 75 to 98% of white oak ties fail from decay, the remainder failing from rail cutting and spike cutting. For pine, chestnut, cedar, cypress, gum and similar timber, the corresponding figures for failures from decay are from 25 to 80%. The softer the wood, the greater is its liability to fail from rail cutting or spike cutting. Failures from these causes are practically eliminated by the use of tie plates. The durability of chemically treated ties depends on the kind of chemical treatment and on the protection of the ties against mechanical injury, such as rail cutting and spike cutting. It is possible to treat ties chemically so that they will resist decay for 30 years, but it is useless to do so unless they are adequately protected against mechanical destruction by the use of screw spikes and tie plates.

Spacing. The usual standard spacing is 2 ft on centers. Wider, heavier ties usually justify wider spacing. The number of ties per mile for various spacings is as follows:

Number per 33 ft rail	22	21	20	19	18	17	16.5	16	15	14	13
Aver. spac. in inches	18.0	18.9	19.8	20.9	22.0	23.3	24.0	24.75	26.4	28.3	30.[illegible]
Number per mile	3520	3360	3200	3040	2880	2720	2640	2560	2400	2240	2080

The Effect of Chemical Treatment on the strength of cross-tie woods has been particularly investigated by the U. S. Government, and the final conclusions are: (*a*) A high degree of steaming is injurious; (*b*) the presence of zinc chlorid does not decrease the static strength, but it renders the wood brittle and therefore more liable to fracture under impact; (*c*) creosote is not injurious.

Cost of Treatment. Neglecting the variable items of royalties on patents, profit, interest or depreciation, the cost of the three most common types of treatment will average about as follows, for a tie containing three cubic feet: zinc chlorid, 16 cents; zinc chlorid and creosote, 27 cents; creosote, 10 lbs to the cu ft, 55 cents.

Economics of Treated Ties. The durability of ties treated by various methods and with various amounts of chemicals is approximately proportional to the expense incurred in treating them. It is possible to waste money in extra creosote so that the tie will be worn out mechanically long before any appreciable decay has set in. The method of treatment should correspond to the other elements of deterioration. The true relative value of two ties, one of which is long-lived but costly, and the other is cheaper but less durable, may be readily determined from the table below when the cost

Annual Charge against a Tie, Based on Original Cost and Assumed Life of Tie; Interest Compounded at 5%

From Webb's Railroad Construction

Life of tie in years	Original cost of tie in cents									
	.20	30	40	50	60	70	80	90	100	Each 5 cents
3	7.34	11.02	14.69	18.36	22.03	25.70	29.38	33.05	36.72	1.836
4	5.64	8.46	11.28	14.10	16.92	19.74	22.56	25.38	28.20	1.410
5	4.62	6.93	9.24	11.55	13.86	16.17	18.48	20.79	23.10	1.155
6	3.94	5.91	7.88	9.85	11.82	13.79	15.76	17.73	19.70	0.985
7	3.46	5.18	6.91	8.64	10.37	12.10	13.83	15.55	17.28	0.864
8	3.09	4.64	6.19	7.74	9.28	10.83	12.38	13.92	15.47	0.774
9	2.81	4.22	5.63	7.03	8.44	9.85	11.25	12.66	14.07	0.703
10	2.59	3.89	5.18	6.48	7.77	9.07	10.36	11.66	12.95	0.648
11	2.41	3.61	4.81	6.02	7.22	8.43	9.63	10.84	12.04	0.602
12	2.26	3.38	4.51	5.64	6.77	7.90	9.03	10.15	11.28	0.564
13	2.13	3.19	4.26	5.32	6.39	7.45	8.52	9.58	10.65	0.532
14	2.02	3.03	4.04	5.05	6.06	7.07	8.08	9.09	10.10	0.505
15	1.93	2.89	3.85	4.82	5.78	6.74	7.71	8.67	9.63	0.482
16	1.85	2.77	3.79	4.61	5.54	6.46	7.38	8.30	9.23	0.461
17	1.77	2.66	3.55	4.43	5.32	6.21	7.10	7.98	8.87	0.443
18	1.71	2.57	3.42	4.28	5.13	5.99	6.84	7.70	8.55	0.428
19	1.65	2.48	3.31	4.14	4.96	5.79	6.62	7.45	8.27	0.414
20	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22	8.02	0.401

of each in the track and the expected life of each in years is known. For example, assume that a cheap untreated hemlock tie costs 35 cents, that the cost of placing it in the track is 20 cents, and that the tie will last 5 years. Assume that a yellow-pine tie is bought for 35 cents, is treated thoroly with creosote at a cost of 60 cents, and that, as before, the cost of placing it in the track is 20 cents. The cheap tie costs 55 cents in the track and will last 5 years. By the table, the annual charge against it is $11.55 + 1.15 = 12.70$ cents. The other tie costs \$1.15, and is assumed to last 20 years, hence from the table the annual charge against it is $(8.02 + 3 \times 0.401) = 9.223$ cents. Even if the treated tie lasted only 15 years, the annual charge would be only 11.076 cents. Reducing the rate of interest from 5% to say 4% of course reduces the annual charge against any tie; it also makes the longer-lived ties relatively more valuable, or with a less relative annual charge.

12. Rails

Standard Length. The standard length is 33 ft. Shorter lengths, varying by single feet down to 25 ft, are permissible up to 10% of the entire order.

Expansion. The allowance for expansion should be 0.000 006 5 of the length per degree Fahrenheit. For a 33-ft rail and a change of temperature of 25°, the expansion would be 0.005 36 ft = 0.0643 in, or 1/16 in very nearly. The following allowances are therefore recommended for rail laying:

Temp. (F.°) =	−20° to 0°,	0° to 25°,	25° to 50°,	50° to 75°,	75° to 100°
Rail opening =	5/16 in,	1/4 in,	3/16 in,	1/8 in,	1/16 in.

Weight of Rail. Baldwin Locomotive Works rule: "Each ten pounds weight per yard of ordinary steel rail, properly supported by cross ties (not less than 14 per 30-ft rail), is capable of sustaining a safe load per wheel of 3000 lbs." A consolidation locomotive with 170 000 lbs on the eight drivers has a load of 21 250 lbs per wheel and should therefore have at least 71-lb rails (say 75-lb), to run on.

Of the rails rolled in 1917, 10% weighed less than 50 lbs per yard; 30%, 50 to 85; 34%, 85 to 100 and 26%, over 100. The maximum is about 140 lbs per yard, but very few of this weight have been used.

The reported axle loads vary from 30 000 lbs to 68 000 lbs, the most common maximum being from 45 000 to 55 000 lbs. It may be noted that an axle load of 42 500 lbs means a wheel load of 21 250 lbs, which, by the Baldwin rule, would call for a rail 71 lbs per yd.

The New York State Law requires that on narrow-gage roads the weight shall not be less than 25 lbs per yard; on standard-gage roads it must not be less than 56 lbs per yard on grades of 110 ft to the mile or under, and must be at least 70 lbs per yard on steeper grades. These are minimum requirements regardless of weight of rolling stock.

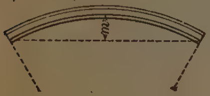


Fig. 16

The gross tons per mile of rails of any weight is readily found by multiplying the weight per yard by 11/7. For example, $70 \times 11/7 = 110$, the gross tons of 70-lb rail per mile of track. The rule is theoretically exact, but there should practically be an allowance of about 2% for rail cutting.

Curving. An approximate but practical rule for the middle ordinate m (Fig. 16) is that it is equal to the square of the length divided by 8 times the radius of the curve. Applying the approximate rule that radius of curve $= 5730/D$, where D is the degree of the curve, the rule becomes, for a standard

33-ft rail, m (in inches) $=0.285\ D$; or for any length rail, m (in inches) $=0.00026\ L^2D$, where $L=$ length in feet.

For ordinary curvature and lengths up to 33 ft there is no practical error in this rule, the maximum error below for a 30° curve being ⅛ in and that for even a 50° curve being only ½ in. The following table was computed by its use. For greater lengths the error varies practically as the square of the distance.

Middle Ordinates for Curving Rails, in Inches

D	Length of Rail in Feet													D
	10	15	20	24	25	26	27	28	29	30	31	32	33	
2	..	⅛	¼	¼	⅜	⅜	⅜	⅜	½	½	½	½	⅝	2
3	⅛	⅛	⅜	½	½	½	⅝	⅝	⅝	¾	¾	¾	⅞	3
4	⅛	¼	⅜	⅝	⅝	¾	¾	⅞	⅞	1	1	1⅛	1⅛	4
5	⅛	¼	½	¾	⅞	⅞	1	1	1⅛	1⅛	1¼	1⅜	1⅜	5
6	⅛	⅜	⅝	⅞	1	1	1⅛	1¼	1⅜	1⅜	1½	1⅝	1¾	6
7	⅛	⅜	¾	1	1⅛	1¼	1⅜	1⅜	1½	1⅝	1¾	1⅞	2	7
8	¼	½	⅞	1¼	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	8
9	¼	½	1	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2⅝	9
10	¼	⅝	1	1½	1⅝	1¾	1⅞	2	2¼	2⅜	2½	2⅝	2⅞	10
12	⅜	¾	1¼	1¾	1¾	2⅛	2¼	2½	2⅝	2⅞	3	3¼	3⅜	12
14	⅜	⅞	1½	2	2¼	2½	2⅝	2⅞	3⅛	3¼	3½	3¾	4	14
16	⅜	1	1⅝	2⅜	2⅝	2⅞	3	3¼	3½	3¾	4	4¼	4½	16
18	½	1	1⅞	2¾	2⅞	3⅛	3⅜	3¾	4	4¼	4½	4⅞	5⅛	18
20	½	1⅛	2	3	3¼	3½	3⅞	4⅛	4⅜	4¾	5	5⅜	5¾	20
25	⅝	1½	2⅝	3¾	4⅛	4⅜	4¾	5⅛	5½	5⅞	6¼	6¾	7⅛	25
30	¾	1¾	3⅛	4½	4⅞	5¼	5¾	6⅛	6⅝	7⅛	7½	8	8½	30
35	⅞	2	3⅝	5¼	5¾	6¼	6⅝	7⅛	7¾	8¼	8¾	9⅜	10	35
40	1	2⅜	4⅛	6	6½	7⅛	7⅝	8¼	8¾	9⅜	10	10¾	11⅜	40
45	1⅛	2⅝	4¾	6¾	7⅜	8	8⅝	9¼	9⅞	10⅝	11⅝	12⅛	12⅞	45
50	1¼	3	5¼	7½	8⅛	8⅞	9½	10¼	11	11¾	12⅜	13⅜	14¼	50

Failures are classified by the Amer. Rwy. Eng. Assoc. under the following heads:

1. Broken Rail, rail broken thru or having crack which might result in break.
2. Flow of Metal, rolling out of top without indication of breaking down of head.
3. Crusht Head.
4. Split Head, split at or near center sometimes accompanied by seam or hollow head.
5. Split Web, longitudinal split generally starting thru bolt holes.
6. Broken Base, any break in base, give sketch.
7. Damaged, broken or injured by wrecks, etc. Rails failing from causes 2 to 6 inc., are further classified as defective. Complete reports are required of all failures classified as above, and giving also complete data regarding characteristics, manufacture, location and condition of rail, ties and ballast, alinement, weather, etc. These reports are collected and summarized for study on various bases such as length of service, kind of steel, mills etc.

The total average failures per 100 track miles (32 000 rails, 33 ft) classified on basis of length of service up to five years are given in the table on p. 188.

The steady improvement indicated by these statistics is due principally to the improvement in the metal used for rails, the adoption of heavier rails better adapted to the increased loads and the gradual replacement of Bessemer by open-hearth rails. It should be noted that less than two rails per thousand fail each year on the average for the first five years and that the failures per year increase with the life of the rail, which is as it should be and just the reverse

Rail Failures per 100 Track Miles

From Bulletin 203, Jan., 1918, Amer. Rwy. Eng. Assoc.

Year Rolled	Years of Service				
	1	2	3	4	5
1908					698.1
1909				224.1	277.8
1910			124.0	152.7	198.5
1911		77.0	104.4	133.3	176.3
1912	28.9	32.1	49.3	78.9	
1913	12.5	24.8	44.8		
1914	8.2	19.8			
1915	8.9				

of common experience some fifteen to twenty years ago when the inferior quality of rails resulted in very heavy replacements from defects during the first two or three years of use.

Life. Rails not replaced on account of failure wear out in service unless replaced by heavier sections and the life depends on the rate of wear and the total wear permitted before renewal, each road fixing its limit of wear to suit its own conditions. Wear is usually considered proportional to tonnage, tho heavy axle loads and high speeds undoubtedly increase the destructive effect of traffic. Grade also increases wear 50 to 100% due to the slipping of drivers and use of sand. The excess wear on curves may be taken at 1/6 that on tangents per degree of curve, or, the wear on a 6° curve will be double that on tangents. The normal wear of rails may be estimated at 1/16 in per 25 000 000 to 35 000 000 tons of traffic. Assuming the allowable depth of wear for main track at 3/8 in gives, say, 200 000 000 tons as its total tonnage life in main track. It may then be used on branch lines or sidings, with or without rerolling. Since the quality of the metal inside the rail is inferior to that on the surface some roads remove rail from main track early and reroll it, thus again securing a better wearing surface but, of course, a reduced section. The tonnage treated in this way and the practical displacement of Bessemer steel rails by open-hearth steel ones are shown by the table on p. 189.

Sections. The section in common use originated in the so-called "flat bottom" pattern designed by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad, in 1830. It was reinvented in England in 1836 by Charles Vignoles and is used as the "Vignoles" rail in Europe and to some extent in England, tho the standard in the latter country is the double headed or "bull head" rail which requires chairs for its support. The early rails had pear shaped heads, being designed for the use of iron, on account of the danger of the side of the head breaking down with that material. After the introduction of steel in 1865, many modifications were made by the various roads using rails until in 1891, nearly 300 different patterns were in use with 27 different weights per yard practically all 80 lbs and less. This caused needless expense and higher prices due to the large stock of rolls which the eleven Bessemer steel rail mills had to carry on hand and in 1893 the Amer. Soc. of Civil Engrs. having studied the subject exhaustively thru various committees since 1873, adopted standard sections whose use was recommended. These sections were extensively adopted as at one time about 75% of the output conformed to those standards, but when the heavier rails came into common use much difficulty was experienced. Tho much of this was due to poor quality

Production of Rails by Processes, in Gross Tons

Year	Open Hearth	Bessemer	Rerolled	Electric	Iron	Total
1902	6 029	2 935 392	......	......	6 512	2 947 933
1903	45 054	2 946 756	......	......	667	2 992 477
1904	145 883	2 137 957	......	......	871	2 284 711
1905	183 264	3 192 347	......	......	318	3 375 929
1906	186 413	3 791 459	......	......	15	3 977 887
1907	252 704	3 380 025	......	......	925	3 633 654
1908	571 791	1 349 153	......	......	71	1 921 015
1909	1 256 674	1 767 171	......	*	....	3 023 845
1910	1 751 359	1 884 442	......	*	230	3 636 031
1911	1 676 923	1 053 420	91 751	462	234	2 822 790
1912	2 105 144	1 099 926	119 390	3 455	....	3 327 915
1913	2 527 710	817 951	155 043	2 436	....	3 502 780
1914	1 525 851	323 897	95 169	178	....	1 945 095
1915	1 775 168	326 952	102 083	......	....	2 204 203
1916	2 269 606	440 092	144 826	......	....	2 854 518
1917	2 292 197	533 325	118 639	......	....	2 944 161

* Small tonnages rolled but included with Bessemer and open-hearth. Rerolled rails were also included 1902 to 1910. There were also rerolled from new second quality and defective rails 22 010 tons in 1911; 42 586, 1912; 43 793, 1913; 26 772, 1914; 9 129, 1915; 3 860. 1916 and 9 007, 1917. The totals show that the practise of rerolling has decreased markedly since 1913, but total production has also decreased.

of metal and poor methods of manufacture a change in section was also necessary and the Amer. Rwy. Assoc. appointed a committee to consider the matter in 1905. This committee made its final report in 1909, recommending the adoption of two types of sections, A and B, the first with shallower head and greater height than the second. The association concurred and referred the sections to the Amer. Rwy. Eng. Assoc. whose Committee on Rail reported in 1915 in favor of the "A" section for a single type for the 90-lb rail and in favor of sections of their own design for 100-, 110- and 120-lb rails, thus leaving the A. S. C. E. sections standard for weights under 90 lbs per yard. These recommendations were adopted. Considerable tonnage of both sections A and B has been rolled, however, and they are likely to continue in use to some extent as the A section satisfies those who believe that the head should be thin and the moment of inertia as great as possible and the B those who believe that the head should be narrow and deep and that the moment of inertia is comparatively unimportant. The A section is used on the comparatively straight prairie roads of the Central West while B section has been adopted by some of the crooked, heavy-traffic, coal roads thru the mountains in the East.

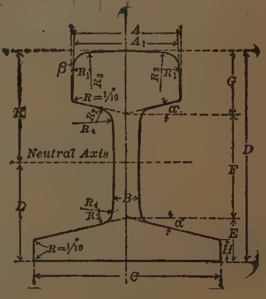


Fig. 16

In Fig. 16 is given a diagram showing the dimensions which are common to all designs and indicating by letters the dimensions which vary. The

Angles and Dimensions of Standard Designs for Rails (see Fig. 16)

Standard	Radii, inches				Angles		Wt. of rail, lbs per yard	Dimensions, inches									
	Upper corner of head R_1	Fillet corners R_2	Top of head R_3	Side of web R_4	Bot. of head and top of flange α	Side of head β		A	A_1	B	C	D	E	F	G	H	K
A.S.C.E.	5/16	1/4	12	12	13°	Vert.	60	2 3/8		21/64	4 1/4	4 1/4	49/64	2 17/64	1 7/32		
							70	2 7/16		33/64	4 5/8	4 5/8	13/16	2 15/32	1 11/32		
							80	2 1/2		35/64	5	5	7/8	2 5/8	1 1/2		
							90	2 5/8		9/16	5 3/8	5 3/8	59/64	2 55/64	1 19/32		2 5/8
							100	2 3/4		9/16	5 3/4	5 3/4	31/32	3 5/64	1 45/64		3 1/32
Am. Rwy. Assoc. A	3/8	3/8	14	14	4:1 14° 02′	1/16:1 3° 35′	60	2 1/4		15/32	4	4 1/2	13/16	2 29/64	1 15/64	5/16	2.37
							70	2 3/8		1/2	4 1/4	4 3/4	29/32	2 1/2	1 11/32	3/8	2.55
							80	2 1/2		33/64	4 5/8	5 1/8	31/32	2 23/32	1 7/16	3/8	2.81
							90	2 9/16		9/16	5 1/8	5 5/8	1	3 5/32	1 15/32	3/8	3.08
							100	2 3/4		9/16	5 1/2	6	1 1/16	3 3/8	1 9/16	3/8	3.25
Am. Rwy. Assoc. B	3/8	5/16	12	12	13°	3°	60	2 1/8	2 1/16	31/64	3 11/16	4 3/16	7/8	2 7/16	1 1/4	29/64	2 15/64
							70	2 3/8	2 5/16	33/64	4 3/64	4 35/64	59/64	2 17/64	1 23/64	29/64	2 23/64
							80	2 7/16	2 11/32	35/64	4 7/16	4 15/16	1	2 15/32	1 15/32	31/64	2 85/128
							90	2 9/16	2 15/32	9/16	4 49/64	5 17/64	1 1/32	2 5/8	1 39/64	31/64	2 105/128
							100	2 21/32	2 9/16	9/16	5 9/64	5 41/64	1 5/64	2 55/64	1 45/64	31/64	3 1/64
Am. Rwy. Eng. Assoc.	3/8	3/8	14	14	4:1 14° 02′	1/16:1 3° 35′	90	2 9/16		9/16	5 1/8	5 5/8	1	3 5/32	1 15/32		3.08
							100	2 11/16		9/16	5 3/8	6	1 1/16	3 9/32	1 21/32		3.25
							110	2 25/32		19/32	5 1/2	6 1/4	1 1/8	3 13/32	1 23/32		3.42
							120	2 7/8		5/8	5 3/4	6 1/2	1 3/16	3 17/32	1 25/32		3.51
Penn. R.R.	1/16	1/4	10	8	13°	4°	85	2 3/16		17/32	5	5	7/8	2 3/8	1 3/4		2 9/16
							100	2 13/16		5/8	5 1/2	5 1/2	15/16	2 11/16	1 7/8		2 27/32

Areas, Proportions and Properties of Rails

Standard	Weight Lbs/Yd	Area Sq In	Head		Web		Base		Moment of Inertia
			Area Sq In	Per cent	Area Sq In	Per cent	Area Sq In	Per cent	
Amer. Soc. C E.	75	7.40	3.11	42	1.55	21	2.74	37	22.848
	80	7.89	3.31	42	1.66	21	2.92	37	27.25
	85	8.34	3.50	42	1.75	21	3.09	37	30.00
	90	8.80	3.68	42	1.84	21	3.28	37	34.19
	100	9.82	4.13	42	2.06	21	3.63	37	43.80
Amer. Rwy. Eng. Assoc.	90	8.82	3.20	36.2	2.12	24.0	3.50	39.8	38.7
	100	9.95	3.80	38.2	2.25	22.6	3.90	39.2	49.0
	110	10.82	4.04	37.4	2.49	23.0	4.29	39.6	57.0
	120	11.85	4.40	37.1	2.69	22.7	4.76	40.2	67.6

A. S. C. E. designs including thirteen different weights of rails. The later designs recommended a reduction in the number of standards to five designs varying from 60 to 100 lbs per yard. One dimension only is common to all standards and all sizes of rails, the radii of the upper and lower corners of the flanges and the lower corners of the head being invariably $^1/_{16}$ in. Some other dimensions and angles are constant for a set of standards as shown in the table, p. 190.

Specifications. The following are the standard specifications of the Amer. Rwy. Eng. Assoc. for carbon steel rails:

1. **Access to Works.**—Inspectors representing the purchaser shall have free entry to the works of the manufacturer at all times while the contract is being executed, and shall have all reasonable facilities afforded them by the manufacturer to satisfy them that the rails have been made and loaded in accordance with the terms of the specifications.

2. **Place for Tests.**—All tests and inspections shall be made at the place of manufacture, prior to shipment, and shall be so conducted as not to interfere unnecessarily with the operation of the mill.

3. **Material.**—The material shall be steel made by the Bessemer or Open-Hearth process as provided by the contract.

4. **Chemical Composition.**—The chemical composition of each heat of the steel from which the rails are rolled, determined as prescribed in paragr. 6, shall be within the following limits:

Elements	Percent for Bessemer Process		Percent for Open-Hearth Process	
	70 lbs and over but under 85 lbs	85-100 lbs inclusive	70 lbs and over but under 85 lbs	85-100 lbs inclusive
Carbon	0.40 to 0.50	0.45 to 0.55	0.53 to 0.66	0.62 to 0.75
Phosphorus, not to exceed	0.10	0.10	0.04	0.04
Manganese	0.80 to 1.10	0.80 to 1.10	0.60 to 0.90	0.60 to 0.90
Silicon, not less than . .	0.10	0.10	0.10	0.10

When other acceptable deoxidizing agents are used, the minimum limit for silicon will be omitted.

5. **Average Carbon.**—It is desired that the percentage of carbon in an entire order of rails shall average as high as the mean percentage between the upper and lower limits specified.

6. **Analyses.**—In order to ascertain whether the chemical composition is in accordance with the requirements, analyses shall be furnished as follows:

(a) For Bessemer process the manufacturer shall furnish to the inspector, daily, carbon determinations for each heat before the rails are shipped, and two chemical analyses, every twenty-four hours representing the average of the elements, carbon, manganese, silicon, phosphorus and sulphur contained in the steel, one for each day and night turn, respectively. These analyses shall be made on drillings taken from the ladle test ingot not less than ⅛ in beneath the surface.

(b) For Open-Hearth process, the makers shall furnish the inspectors with a chemical analysis of carbon, manganese, silicon, phosphorus and sulphur, for each heat.

(c) On request of the inspector, the manufacturer shall furnish a portion of the test ingot for check analyses.

7. **Physical Qualities.**—Tests shall be made to determine:

(a) Ductility or toughness as opposed to brittleness. (b) Soundness.

8. **Method of Testing.**—The physical qualities shall be determined by the Drop Test.

9. **Drop Testing Machine.**—The drop testing machine used shall be the standard of the American Railway Engineering Association.

(a) The tup shall weigh 2000 lbs and have a striking face with a radius of five inches.

(b) The anvil block shall weigh 20 000 lbs, and be supported on springs.

(c) The supports for the test pieces shall be spaced three feet between centers and shall be a part of, and firmly secured to, the anvil. The bearing surfaces of the supports shall have a radius of five inches.

10. **Pieces for Drop Test.**—Drop tests shall be made on pieces of rail not less than four feet and not more than six feet long. These test pieces shall be cut from the top end of the top rail of the ingot, and marked on the base or head with gage marks one inch apart for three inches each side of the center of the test piece, for measuring the ductility of the metal.

11. **Temperature of Test Pieces.**—The temperature of the test pieces shall be between 60 and 100 degrees Fahrenheit.

12. **Height of Drop.**—The test piece shall preferably be placed base upwards on the supports, and be subjected to impact of the tup falling free from the following heights:

For 70-lb rail	16 feet
For 80-, 85- and 90-lb rail	17 feet
For 100-lb rail	18 feet

13. **Elongation or Ductility.**—(a) Under these impacts the rail under one or more blows shall show at least 6 per cent elongation for one inch, or 5 per cent each for two consecutive inches of the six-inch scale, marked as described in paragr. 10.

(b) A sufficient number of blows shall be given to determine the complete elongation of the test piece of at least every fifth heat of Bessemer steel, and of one out of every three test pieces of a heat of Open-Hearth steel.

14. **Permanent Set.**—It is desired that the permanent set after one blow under the drop test shall not exceed that in the following table, and a record shall be made of this information.

Rail			Permanent Set, measured by Middle Ordinate in Inches in a Length of 3 Feet	
Section	Weight per Yard	Moment of Inertia	Bessemer Process	O.-H. Process
A.R.A.-A	100	48.94	1.65	1.45
A.R.A.-B	100	41.30	2.05	1.80
A.R.A.-A	90	38.70	1.90	1.65
A.R.A.-B	90	32.30	2.20	2.00
A.R.A.-A	80	28.80	2.85	2.45
A.R.A.-B	80	25.00	3.15	2.85
A.R.A.-A	70	21.05	3.50	3.10
A.R.A.-B	70	18.60	3.85	3.50

15. **Test to Destruction.**—The test pieces which do not break under the first or subsequent blows shall be nicked and broken, to determine whether the interior metal is sound. The words "interior defect," used below, shall be interpreted to mean seams, laminations, cavities, or interposed foreign matter made visible by the destruction tests, the saws, or the drills.

16. **Bessemer Process Drop Tests.**—One piece shall be tested from each heat of Bessemer steel.

(*a*) If the test piece does not break at the first blow and shows the required elongation (paragr. 13), all of the rails of the heat shall be accepted, provided that the test piece when broken does not show interior defect.

(*b*) If the test piece breaks at the first blow, or does not show the required elongation (paragr. 13), or if the test piece does not break and shows the required elongation, but when broken shows interior defect, all of the top rails from that heat shall be rejected.

(c) A second test shall then be made of a test piece selected by the inspector from e top end of any second rail of the same heat, preferably of the same ingot. If the test ece does not break at the first blow, and shows the required elongation (paragr. 13), of the remainder of the rails of the heat shall be accepted, provided that the test piece ıen broken does not show interior defect.

(d) If the test piece breaks at the first blow, or does not show the required elongation aragr. 13), or if the test piece does not break and shows the required elongation, but ıen broken shows interior defect, all of the second rails from that heat shall be rejected

(e) A third test shall then be made of a test piece selected by the inspector from e top end of any third rail of the same heat, preferably of the same ingot. If the test ece does not break at the first blow and shows the required elongation (paragr. 13), all the remainder of the rails of the heat shall be accepted, provided that the test piece ıen broken does not show interior defect.

(f) If the test piece breaks at the first blow, or does not show the required elonga-on (paragr. 13), or if the test piece does not break and shows the required elongation, t when broken shows interior defect, all of the remainder of the rails from that heat all be rejected.

17. **Open-Hearth Process Drop Tests.**—Test pieces shall be selected from the cond, middle and last full ingot of each Open-Hearth heat.

(a) If two of these test pieces do not break at the first blow, and if both show the quired elongation (paragr. 13), all of the rails of the heat shall be accepted, provided at none of the three test pieces when broken show interior defect.

(b) If two of the test pieces break at the first blow, or do not show the required ongation (paragr. 13), or if any of the three test pieces when broken show interior fect, all of the top rails from that heat shall be rejected.

(c) Second tests shall then be made from three test pieces selected by the inspector om the top end of any second rails of the same heat, preferably of the same ingots. If o of these test pieces do not break at the first blow and if both show the required elonga-on (paragr. 13), all of the remainder of the rails of the heat shall be accepted, provided at none of the three test pieces when broken shows interior defect.

(d) If two of these test pieces break at the first blow, or do not show the required ongation (paragr. 13), or if any of the three test pieces when broken show interior fect, all of the second rails of the heat shall be rejected.

(e) Third tests shall then be made from three test pieces selected by the inspector om the top end of any third rails of the same heat, preferably of the same ingots. If o of these test pieces do not break at the first blow, and if both show the required elonga-on (paragr. 13), all of the remainder of the rails of the heat shall be accepted, provided at none of the three test pieces when broken shows interior defect.

(f) If two of these test pieces break at the first blow, or do not show the required ongation (paragr. 13), or if any of the three test pieces when broken show interior defect, of the remainder of the rails from that heat shall be rejected.

18. **No. 1 Rails.**—No. 1 classification rails shall be free from injurious defects and ws of all kinds.

19. **No. 2 Rails** (a) Rails which, by reason of surface imperfections, or for causes entioned in paragr. 29 hereof, are not classed as No. 1 rails, will be accepted as No. 2 ils, but No. 2 rails which contain imperfections in such number or of such character will, in the judgment of the inspector, render them unfit for recognized No. 2 uses, ll not be accepted for shipment.

(b) No. 2 rails to the extent of 5% of the whole order will be received. All rails cepted as No. 2 rails shall have the ends painted white and shall have two prick nch marks on the side of the web near the heat number near the end of the rail, so aced as not to be covered by the splice bars.

20. **Quality of Manufacture.**—The entire process of manufacture shall be in accord-ce with the best current state of the art.

21. **Bled Ingots.**—Bled ingots shall not be used.

22. **Discard.**—There shall be sheared from the end of the bloom, formed from the p of the ingot, sufficient metal to secure sound rails.

23. **Lengths.**—The standard length of rails shall be 33 feet, at a temperature of ° Fahr.; 10% of the entire order will be accepted in shorter lengths varying by 1 ft

from 32 ft to 25 ft. A variation of 1/4 in from the specified lengths will be allowe excepting that for 15% of the order a variation of 3/8 in from the specified lengths w be allowed. No. 1 rails less than 33 ft long shall be painted green on both ends.

24. **Shrinkage.**—The number of passes and speed of train shall be so regulated th on leaving the rolls at the final pass, the temperature of the rail will not exceed th which requires a shrinkage allowance at the hot saws, for a rail 33 ft in length and 100-lb section, of 6 3/4 ins and 1/8 in less for each 10 lbs decrease in section.

25. **Cooling.**—The bars shall not be held for the purpose of reducing their temperatur nor shall any artificial means of cooling them be used after they leave the finishing pa Rails, while on the cooling beds, shall be protected from snow and water.

26. **Section.**—The section of rails shall conform as accurately as possible to the ter plate furnished by the Railroad Company. A variation in height of 1/64-inch less or 1/ in greater than the specified height, and 1/16 in in width of flange, will be permitted; b no variation shall be allowed in the dimensions affecting the fit of the splice bars.

27. **Weight.**—The weight of the rails specified in the order shall be maintained nearly as possible, after complying with the preceding paragraph. A variation of 1/2 1% from the calculated weight of section, as applied to an entire order, will be allowe

28. **Payment.**—Rails accepted will be paid for according to actual weights.

29. **Straightening.** (a) The hot straightening shall be carefully done, so th gagging under the cold presses will be reduced to a minimum. Any rail coming to t straightening presses showing sharp kinks or greater camber than that indicated by middle ordinate of 4 ins in 33 ft, for A. R. A. type of sections, or 5 ins for A. S. C. type of sections, will be at once classed as a No. 2 rail. The distance between the su ports of rails in the straightening process shall not be less than 42 ins. The suppor shall have flat surfaces and be out of wind.

(b) Rails heard to snap or check while being straightened shall be at once rejecte

30. **Drilling.**—Circular holes for joint bolts shall be drilled to conform to the dra ing and dimensions furnished by the Railroad Company.

31. **Finishing.** (a) All rails shall be smooth on the heads, straight in line a surface, and without any twists, waves or kinks. They shall be sawed square at t ends, a variation of not more than 1/32 in being allowed; and burrs shall be careful removed.

(b) Rails improperly drilled or straightened, or from which the burrs have not be removed, shall be rejected, but may be accepted after being properly finished.

(c) When any finished rail shows interior defects at either end or in a drilled ho the entire rail shall be rejected.

32. **Branding.**—Rails shall be branded for identification in the following manner:

(a) The name of the manufacturer, the month and year of manufacture, and t weight and type of section of rail shall be rolled in raised letters and figures on the si of the web. The type shall be marked by letters which signify the name by which it known, as for example:

Sections of the Amer. Soc. of Civil Engrs. .... A.S.C.
Sections of the Amer. Rwy. Assoc. .... R. A.-A, R.A.-
Sections of the Amer. Rwy. Eng. Assoc. .... R.

(b) The number of the heat and letter indicating the portion of the ingot fro which the rail was made shall be plainly stamped on the web of each rail where it w not be covered by the joint bars. The top rail shall be lettered "A" and the succeedi ones "B", "C", "D", etc., consecutively; but in case of a top discard of 20 to 35 the letter "A" will be omitted, the top rail becoming "B". If the top discard be great than 35% the letter "B" shall be omitted, the top rail becoming "C".

(c) Open-Hearth rails shall be branded or stamped "O-H" in addition to the oth marks.

(d) All markings of rails shall be done so effectively that the marks may be read long as the rails are in service.

33. **Separate Classes.**—All classes of rails shall be kept separate from each other.

34. **Loading.**—Rails shall be carefully handled and loaded in such manner as not injure them.

13. Elevation of Outer Rail on Curves

(See also Sect. 2, Art. 60.) The theoretic formula is $e = dV^2/gR$, where d= stance between centers of rails, V= speed of train, g= acceleration of gravity, ıd R= radius of curve, or, very closely for standard gage, $e = 0.00069\ DV^2$, ıere D is degree of curve, V is speed in miles per hour, and e is elevation of ıter rail in inches. The elevation to be chosen evidently depends on the speed, ıich should be that of the fastest passenger trains. The Lehigh Valley R. R. s successfully used a superelevation of 10 inches; 8 inches is generally con- lered the limit, while some roads limit it to 6 inches. Whatever the limit, e speed of trains on sharp curves should be reduced, if necessary, to that rresponding to the limiting elevation. A common rule for level grade is one ch for each degree of curve, which agrees with the above formula for a speed 38 miles per hour. An extension of this rule is that if the grade is over two rcent, the elevation is ¾ inch for each degree, which is the equivalent of ying that on the 2% grade the speed up the grade would not be greater than m p h, and that the speed down the grade should be limited to that.

Superelevation of Outer Rail in Inches, Standard Gage

(Elevation of center of outer rail above center of inner rail)

Deg of curve	Velocity in miles per hour												Deg of curve
	15	20	25	30	35	40	45	50	55	60	65	70	
1	⅛	¼	⅜	⅝	⅞	1⅛	1⅜	1¾	2⅛	2½	2⅞	3⅜	1
2	¼	½	⅞	1¼	1¾	2¼	2¾	3½	4⅛	5	5⅞	6¾	2
3	½	⅞	1¼	1⅞	2½	3¼	4¼	5⅛	6¼	7½	8¾	10⅛	3
4	⅝	1⅛	1¾	2½	3⅜	4⅜	5⅝	6⅞	8⅜	10			4
5	¾	1⅜	2⅛	3⅛	4¼	5½	7	8⅝					5
6	⅞	1⅝	2⅝	3¾	5⅛	6⅝	8⅜						6
7	1⅛	1⅞	3	4⅜	5⅞	7¾							7
8	1¼	2¼	3½	5	6¾	8⅞							8
9	1⅜	2½	3⅞	5⅝	7⅝								9
10	1½	2¾	4¼	6¼	8½								10
11	1¾	3	4¾	6⅞									11
12	1⅞	3¼	5⅛	7½									12
13	2	3⅝	5⅝	8⅛									13
14	2⅛	3⅞	6	8¾									14
15	2⅜	4⅛	6½	9¼									15

Standard Gage of Track is 4 ft 8½ in on tangents and on curves of 8° and s. For sharper curves gage is widened ⅛ inch for each 2 degrees of curve up a maximum of 4 ft 9¼ in, for tracks of standard gage. Gage should not be wi- ned unless the locomotive wheels bind in going around the curve. The gage a track is the distance between the heads of the rails, measured at right angles ereto at a point ⅝ in below the top of the rail. Gage, including widening due wear, should never exceed 4 ft 9½ in. Gage of track at a frog should be ndard even when on a curve, tho the flangeway may be widened to com- nsate for the increase in gage in this case.

14. Rail Fastenings

General Requirements. The ideal rail joint should have the same strength, ffness and elasticity, no more and no less, both laterally and vertically, as e rails which it joins. Such a joint therefore would not be deformed or take permanent set under any load which the rail can support without yielding. e joint must also permit longitudinal slipping of the rail to allow for expansion

and contraction. It is impossible that any form of scarfed joint should have at the same time, the same strength, stiffness and elasticity as the original rail. Only a perfect butt welding would accomplish this, but welding would not permit expansion and contraction. Therefore the above conflicting requirements must be filled as perfectly as possible by a compromise design whose cost is not prohibitive.

Designs. The old-fashioned fish-plate design (see Fig. 17*a*) proved so inefficient, especially in lateral strength, that its use has been abandoned except in connection with the very light rails used in industrial railways or the light temporary track used by contractors. A typical modern design for the ordinary angle plate is given in Fig. 17*b*. Among the multitudinous designs which have been brought out the following have come into more common use: the Continuous, Fig. 17*c*; the Weber, Fig. 17*d*; and the Wolhaupter, Fig. 17*e*. The last three designs furnish a plate which runs under the rail between the two joint ties and which keeps the ends of the rails absolutely in line vertically as well

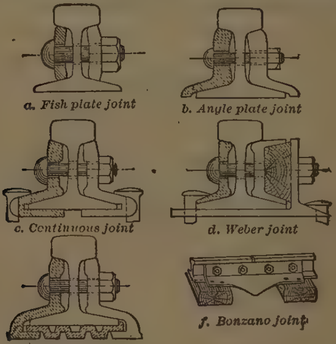


Fig. 17. Various Types of Rail Joints

as laterally. The Bonzano rail joint, illustrated in Fig. 17*f*, is a development of the angle-plate joint with an extended flange which is bent down between the two joint ties thus giving a stiffer joint. This can be used only in suspended joints or those in which the rail gap is midway between two ties and the three base joints shown above are also intended for such use. The supported joint, with the rail gap directly over a tie, is very little used and then generally in the so-called three-tie joint. The Bonzano bars are usually heavier and therefore more expensive than the common angle bars and the base joints are not only subject to this disadvantage but also require adzing of the joint ties or setting them on an angle, thus being more expensive to install. The consensus of opinion seems to be that their advantages over the angle bar are not commensurate with the extra cost and many roads are returning to the use of angle bars, sometimes heavily ribbed to furnish additional stiffness, but more often of high-carbon or quenched carbon or alloy steel as specified below.

Specifications. Bessemer steel is no longer specified for joint bars, high carbon steel or quenched carbon steel (both open-hearth) and quenched alloy

eel being the materials covered by the present (1918) specifications of the mer. Rwy. Eng. Assoc. The principal requirements are as follows:

	High-Carbon	Quenched Carbon	Alloy
arbon, percent	....	0.42 to 0.55	*
hosphorus, percent, maximum	0.04	0.04	0.04
ensile strength, lbs per sq in	85 000	100 000	110 000
lastic limit, lbs. per sq in	....	70 000	85 000
longation percent in 2 in length	....	$\frac{1\ 600\ 000}{\text{Tens Str}}$	
Minimum, percent	16	12	
eduction in area, percent	....	$\frac{3\ 500\ 000}{\text{Tens Str}}$	
Minimum, percent	....	25	
ending	Cold bending without sign of fracture on outside of bent portion thru arc of 90° with diameter 3 times the thickness of the test specimen.		

* Nickel and chromium to the extent of 1.0% and 0.35%, respectively, are considered quivalent to 0.07% carbon.

All test specimens must be cut from finished bars and all bars must be punched, slotted nd shaped at a temperature of not less than 800° C (1470° F.)

"Quenched" bars must be quenched in oil, or water if so specified, from a temperaure of about 810° C (1490° F) and kept in the bath until cold enough to handle. laterial requiring quenching in water but otherwise meeting specifications may be ccepted at the option of the purchaser. All bars must be finished smooth and true ith no variation in fishing angle and height, which factors affect the fit of the bar to

Splice Bars, Bolts and Spikes for Various Weights of Rails

Rail, pounds per yard	Length of bar. Inches	Pounds per foot	Pounds per pair	Proper size of track bolt. Inches	Proper size of spikes. Inches
30	21	4.49	15.1	2½ × ⅝	4 × ½
35	21	4.7	15.9	2¾ × ⅝	4½ × ½
40	21	5.54	18.8	3 × ⅝	5 × ½
45	21	6.3	21.5	3 × ⅝	5½ × 9/16
50	21	6.97	23.4	3½ × ¾	5½ × 9/16
55	23	7.5	28.0	3¾ × ¾	5½ × 9/16
60	23	8.4	31.4	3¾ × ¾	5½ × 9/16
65	23	9.2	34.4	4 × ¾	5½ × 9/16
	33	9.6	51.5	4¼ × ⅞	5½ × 9/16
70	23	9.0	33.6	4 × ¾	5½ × 9/16
	33	10.0	53.3	4 × ¾	5½ × 9/16
75	23	10.68	39.9	4¼ × ¾	5½ × 9/16
	33	12.9	63.7	4 × ¾	5½ × 9/16
80	23	10.61	39.7	4¼ × ⅞	5½ × 9/16
	33	14.65	78.5	4½ × ⅞	5½ × 9/16
85	33	12.4	66.4	4½ × ⅞	5½ × 9/16 or ⅝
90	33	13.5	72.3	4¾ × ⅞	5½ × 9/16 or ⅝
95	33	14.7	78.7	4¾ × ⅞	5½ × 9/16 or ⅝
100	33	15.78	85.0	4¾ × ⅞	5½ × 9/16 or ⅝

Three hundred and twenty pairs of splice bars will be required per mile of single track, using 33-ft rails. The maximum number permitted by the rail specification governing short-length rails is 330. Allowing individually for switches and sidings, the number per mile should not exceed 325.

the rail. Branding indicating manufacturer, year, design, material and treatment if any, is required and the number of the melt must be stenciled on the bars.

Bolt Holes. The standard spacing for bolt holes recommended by the Amer. Rwy. Eng. Assoc. is 5½ in, giving lengths of about 24 in and 32 in respectively, for the 4-bolt and 6-bolt splices. The hole in the rail should be $^3/_{16}$ in larger than the bolt used and the centers of the end holes should be 2 $^{21}/_{3}$ in from the end of the rail to allow for expansion and contraction. This combination of measurements permits a total relative motion of the rail ends o ⅜ in. The holes in the bars are made oval, fitting the bolt and preventing it from turning.

Dimensions of Angle Bars. Altho the dimensions and weight per linear foot of angle bars, even for the same weight of rail, vary with different manufacturers, the table at bottom of p. 197 gives average figures. The net weight of the bars is from 2.5% to 4% less than their weight unpunched.

Spikes. It has been demonstrated that of all the various forms of driven spikes which have been proposed, the plain spike of uniform section has the greatest holding power. Making the spike jagged, twisting the spike, or even swelling the spike at about the middle of its length, actually reduces its holding power. The adhesion, however, is increased by having cutting edges at the lower end which shall cut rather than crush the fibers, and by having beveled wedges whose length is about twice the width of the spike rather than a short blunt point. The sizes of spikes recommended with different rails are given in the table.

Railroad Spikes

Size measured under head. Inches	Average number per keg of 200 pounds	Ties 24 in between centers, 4 spikes per tie, quantity per mile.		Suitable rail. Pounds per yard
		Pounds	Kegs	
5½ × ⅝	275	7680	38.40	85 to 100
5½ × 9/16	375	5632	28.16	45 to 100
5 × 9/16	400	5280	26.40	40 to 56
5 × ½	450	4692	23.46	40
4½ × ½	530	3984	19.92	35
4 × ½	600	3520	17.60	30
4½ × 7/16	680	3104	15.52	25 to 30

The specifications of the Amer. Rwy. Eng. Assoc. require the steel to be made by the open-hearth or other approved process and permit of heat treatment if necessary to secure the desired properties, which are:

Ultimate strength, not less than 55 000 lbs per sq in.
Elastic limit, not less than 50% of ultimate strength.
Elongation, not less than 20% in 2 in.
Reduction of area, not less than 40%.
Bending, the finished spike when bent back upon itself 180° shall show no signs of fracture. When the head of the spike is bent backward cold it shall show no signs of fracture.
Twisting, when the body of the spike is twisted cold 1½ turns it shall show no signs of fracture.
Other requirements cover workmanship, finish, inspection etc.

The Driving of Spikes should be by blows which are in line with the spike. Eccentric or glancing blows tend to enlarge the spike hole and reduce the holding power of the spike. The spikes should not be driven directly opposite each other, but should be staggered. The staggering should be reversed for

he two rails on one tie; that is, the spikes on the inner side of the two rails hould be nearer the same side of the tie.

Screw Spikes. The advantages of screw spikes are: (1) The increase in the life of he tie by the avoidance of spike killing; (2) The decreased cost of maintenance (altho his is denied), and (3) their greater holding power. It is said, as an objection to their se, that when they are rusted r the head broken off by ccident, it is difficult or impossible to extract the stump. They were adopted as standard by the D. L. & W. R. R. Fig. 18), after extensive trials nd are reported as continuing to give satisfactory service n that and other roads. The Pennsylvania Railroad, on the other hand, has recently 1918) rejected them after nine years of trial and careful observation in two sections of rack carrying very heavy traffic. The figures in this case show them to be more costly, oth in first cost and maintenance, than drive spikes and the report states that they were unsatisfactory and less efficient than the ordinary spike. One strong objection was the entire loss of holding power when the wood fiber between the threads became worn as it did under the action of the extremely heavy traffic. The screws cost considerably more than common spikes, and require more work to put them in place. The screwing must be done with a track wrench. The auger hole should have the same diameter as that of the screw at the base of the thread, or perhaps 1/16 in larger. The A. R. E. A. specifications require them to be made of open-hearth steel, having not more than .05 per cent of either phosphorus or sulphur. The minimum allowable figures for the finished spike are ultimate strength 60 000 lb, elastic limit 50 per cent of ultimate, elongation 22 per cent in 2 in, reduction of area 40 per cent. The material must be capable of being bent 180° and hammered down flat and the finished spike bent 90° without sign of fracture.

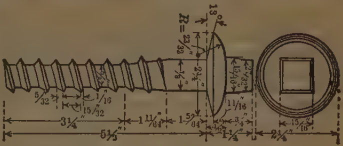


Fig. 18. Screw Spike

Track Bolts. The length of track bolts necessarily depends on the distance between the outer faces of the angle plate, and to this must be added the thickness of the nut lock and the thickness of the nut. Sometimes there is a slight

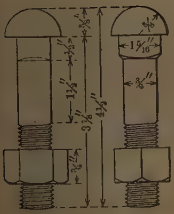


Fig. 19. Track Bolt

Track Bolts

Average number in a keg of 200 pounds

Size of bolt. Inches	Square nut	Hexagonal nut	Suitable rail. Lbs per yd
2½×⅝	390	425	30
2¾×⅝	379	410	35
3 ×⅝	366	395	40
3 ×¾	250	270	
3¼×¾	243	261	
3½×¾	236	253	50
3¾×¾	229	244	55 to 60
4 ×¾	222	236	65 to 70
4¼×¾	215	228	75
3½×⅞	170	180	
3¾×⅞	165	175	
4 ×⅞	161	170	
4¼×⅞	157	165	80
4½×⅞	153	160	85
4¾×⅞	149	156	90 to 100

margin beyond this, but any unnecessary length is objectionable. The heads of track bolts are usually hemispherical. Directly under the head the bolt has an oval form which fits loosely in a corresponding oval hole in the angle plate. This prevents the bolt from turning. The table gives proper size of track bolt for various weights of rail and their corresponding angle bars. A typical design for a track bolt is shown in Fig. 19.

On the basis of 325 joints per mile, the number of bolts per mile of single track is 1300 for 4-bolt splices and 1950 per mile for 6-bolt splices. Divide 1300 (or 1950) by the number per keg to obtain the number of kegs per mile.

The specifications of the Amer. Rwy. Eng. Assoc. call for steel made by the Open Hearth or other approved process and permit of heat treatment if necessary to secur the desired properties which are as follows:

Material	Elastic Limit* Minimum Lbs per Sq In	Elongation in 2 in Minimum Percent	Reduction in Area Percent
Carbon steel	35 000	25	50
Nickel or other alloy steel, treated	45 000	15	40
Nickel or other alloy steel, untreated	45 000	20	40

* The elastic limit shall in no case be less than 50% of the ultimate strength

The material must bend cold thru 180° and flatten on itself without fracture. Othe requirements cover workmanship, finish, inspection, etc.

Nut Locks are a practical necessity to prevent the nut from working loose The most common kind is essentially an open ring made of spring steel, th ends of the ring being bent outward. Screwing u the nut compresses the spring and makes the shar edges bite into the nut and thereby prevent it turning backward. Another form combines a nu and nut lock, the nut being slightly open on or side. The hole is drilled thru the nut and th thread is cut slightly smaller than the bolt. Whe the nut is screwed up, it is forced slightly open, which makes such a pressur on the screw threads that vibration cannot jar it loose.

Verona *National*

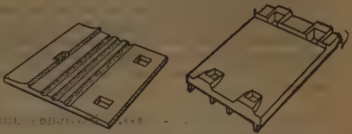
Fig. 20. Nut Locks

The specifications of the Amer. Rwy. Eng. Assoc. for spiral spring nutlocks requi steel made by the open-hearth or other approved process, containing not more than 0.05 of either phosphorus or sulfur. The physical tests specified are: After the finish nutlock has been subjected for one hour to a pressure sufficient to compress it flat ar has been released, its reaction shall not be less than two-thirds its height or thickne of section provided its thickness is less than width of section. If section is square, t reaction must be not less than one-half its thickness. If height or thickness of sectio is more than width, the reaction shall be not less than the width of the section. (The requirements are for nutlocks of internal diameters from $^{13}/_{16}$ to $1\,^{5}/_{16}$ in.) With o end of the finished nutlock secured in a vise, and the opposite end twisted to 45°, the must be no sign of fracture. When further twisted until broken, the fracture mu show a good quality of steel.

Tie Plates reduce the unit pressure of the rail on the tie, and prevent t rail from cutting the tie. They relieve the lateral pressure of the rail again a spike, since the tie plate distributes the lateral pressure to both spikes. T pressure against the spikes is still further relieved by the lugs or corrugatio which are found on the under side of many forms of plates. The wear on spik

called "necking" which is caused by the vertical vibration of the rail against the spike is very much reduced.

Design of Tie Plates. The following statement of principles to be followed in the design of tie plates was ordered incorporated in the Manual of the Amer. Rwy. Eng. Assoc. during their Convention in 1913.



Flat bottom Tie Plate *Economy Tie Plate*
Fig. 21. Tie Plates

The plates shall not be less than 6 in in width, and as much wider as consistent with the class of ties to be used.

The length of the plates shall not be less than the safe bearing area of the ties divided by the width of the plate, and, when made for screw spikes, shall be so shaped as to provide proper support for the screw spikes.

The thickness of the plate shall be properly proportioned to the length.

The plates shall have a shoulder at least ½ in high. The distance from the edge of rail base to the end of the tie-plate on the outer side must be uniform, and in excess of the projection inside of the rail base.

Where treated ties are used or where plates are used with screw spikes, a flat bottom plate is preferable. Where ribs of any kind are used on base of plate, these shall be few in number and not to exceed ¼ in in depth.

The punching must correspond to the slotting in the splice bars and, where advisable, may be so arranged that the plates may be used for joints. Spike holes may be punched for varying widths of rail base where the slotting will permit such punching without the holes interfering with each other and where the plate is of such design that the additional holes will not impair the strength of the plate.

In the above statements the width is the distance parallel to the rail and the length is the distance perpendicular to it. The length of a tie plate should be about 4 in greater than the width of the rail base. Joint tie plates must be longer still on account of the greater width of the rail joint as assembled. The tie plates used underneath frogs (Fig. 27) are extra lengths.

The specifications of the Am. Ry. Eng. Assoc. provide for steel, both Bessemer and open-hearth, and wrought and malleable iron, the latter being used on account of its comparative immunity from rust. Steel plates are required to have an ultimate strength of not less than 55 000 lbs per sq in with elastic limit at least 50% of ultimate; elongation, not less than 20% in 2 in and reduction of area, not less than 40%. They must also bend cold for 90° without sign of fracture. Wrought-iron plates must have an ultimate strength of at least 45 000 lbs per sq in and bend cold across the grain 90° without sign of fracture. Malleable plates must be cast with a lug for test purposes which, when broken off must not break easily, as cast-iron, but must bend and show signs of toughness. The fracture must show a narrow band of white metal on the surface, center portion being dark and fiberless.

An Order for Tie Plates should include the following items of information; (a) width of rail base; (b) size of spike under head; (c) number of spike holes, two, three or four; (d) spacing, lengthwise of rail, of slots in opposite angle bars on one tie; (e) net width, measured perpendicular to rail, between such slots. The proportion of joint plates to intermediates will depend on the number of ties per rail, on whether the joints are supported, suspended, or are three-tie joints. In the latter case there will be no slots in the angle bars over the middle joint tie, and the spacing of the holes must be a little wider than the extreme width, out to out, of the angle plates as assembled.

Setting Tie Plates. The efficiency depends largely on proper setting. Plates may be set on soft-wood ties by merely raising the rail, setting the plate squarely in place and driving the spikes. The first train passing over will force any corrugations or claws

into the tie. The spikes should at once be driven home. Plates must be set in hardwood ties by direct driving A follower plate of steel about ¾ in to ⅞ in thick and about the size of the tie plate, will protect the tie plate from the direct blows of the swages used in driving. A still better method is to use a maul or beetle to strike a straddler which just straddles the rail and rests on the ends of the plate. A hand car carrying a hammer similar to a small-scale pile driver is far better and more effective than a hand maul. They are most economically and efficiently set in large numbers by means of a tie-plating machine.

15. Switches and Frogs

Mechanical Features. A switch usually implies at least two breaks in one of the main rails. The flanges on the wheels make it necessary either to lift every pair of wheels so that the flanges can pass over the rail, or else to make breaks in the rail so that the flanges may pass thru the head of the rail. The two most common methods are those sketched in Figs. 22 and 23. Both methods employ a frog at *F*. The STUB SWITCH cuts both of the main rails at *H* and *K*. Both rails from *S′* to *H* and from *S* to *K* are

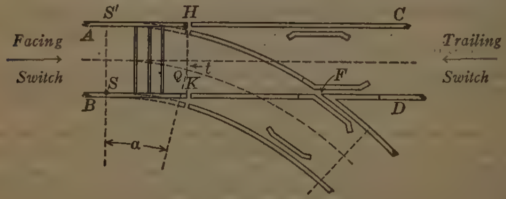


Fig. 22. Stub Switch

not fastened to the ties, but are mutually tied to each other by four or more tie-rods. The gap at *H* and *K* is always sufficient to cause a considerable jar to the rolling stock. When operated as a trailing switch, a derailment is inevitable if the switch is misplaced. Their use is now confined to sidings running from switch tracks; they should never be used in main tracks. A POINT SWITCH leaves one main rail unbroken. The point rail *S″H* varies in length from 10 to 30 ft and is planed and bent to give a straight gage line with a thickness of ¼ in at the point. The point is afterward ground to a thickness of ⅛ in beginning at a point 2 ft from the end. The angle of the switch point varies from about 0° 45′ to about 2° 30′ according to the dimensions and frog angle of the switch. There is an abrupt bend in the main rail at the point *S′*, equal to the angle of the point rail. The point rails, *S″H* and *SK*, are tied together with tie-rods. They are held in place by a very stiff spring (see Fig. 24) which will yield sufficiently to permit the wheels to remain on the rails if a train trails thru with the switch misplaced. The SWITCH POINTS should fit the stock rail thruout the planing. The planing must be such that the point is ⅛

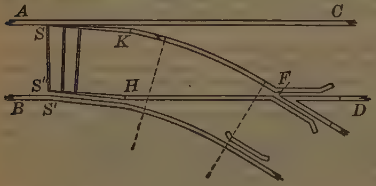


Fig. 23. Point Switch

in below the top of stock-rail at the end, rising to ¼ in above same in about 40% of length of point in order to care for hollow treads on car wheels. The

Fig. 24. Section of Main and Switch Rails about 2 ft. back of Point

rails of the frog are always made straight, therefore the lead rails between the switch point and the frog must be curved to a circular arc which is tangent both to the switch rail and the wing rail.

Frogs. A diagrammatic design of a frog is illustrated in Fig. 25. The frogs are always built up of four pieces of rail, which are usually of the cross-

Fig. 25. Diagrammatic Design of Frogs

section and weight used on the main track. The actual point of the frog is rounded off as shown. Fixed frogs are made either by riveting the four

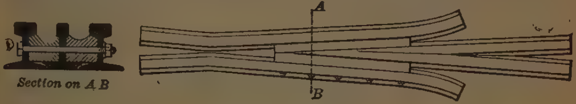


Fig. 26. Stiff Frog

rails to a base plate, or else by placing cast-iron fillers between the rails and bolting or clamping the rails and fillers together or by a combination of riveting

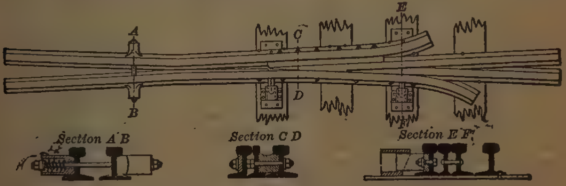


Fig. 27. Spring Rail Frog

and bolting. Spring rail frogs usually have one wing rail (the one connecting with the main rail) movable (both may be made so) and yet normally pressing against the frog point so that there is no gap to be past over by the wheels running on the main track. When wheels are running thru the frog onto the switch, the guard rail opposite the frog forces the opposite wheel to run in its

proper line, and this causes the inside of the flange of the wheel running thru the frog to press against the wing rail and force it back so as to leave a sufficient opening between the wing rail and the frog point for the wheel flange to pass thru. Movable point frogs operated with and in the same manner as switches controlling routes thru them are used at slip switches in yards to avoid breaks in rail for high speed traffic. The frog number (n) may be determined by dividing CH by AB (Fig. 25), or SH by ($AB+DE$). When no tape or scale is available, use any unit of measure about 4 in long, such as a stick or piece of paper, and find by trial two points on the frog where the distance between the gage lines (as at AB and DE) just equals the assumed unit of length. Then step off that unit of length between the points H and S and note the number of times the unit is contained in the distance HS. The result equals twice the frog number.

Unbroken Main Rails. A switch which permits the use of unbroken main rails necessarily lifts the train a vertical height somewhat greater than the depth of the wheel flange, and as this must be accomplished in a distance of a very few feet, it is impossible to operate such switches at high speed. Altho there are undoubted advantages in having an unbroken main rail, these devices have not come into common use.

The Dimensions of Stub Switches are usually computed on the basis that the lead rails are circular from the switch point to the frog point. For

Trigonometrical Functions of the Frog Angles

Frog no.	Frog angle F	Nat sin F	Nat cos F	Log sin F	Log cos F	Log cot F	Log vers F	Frog no.
4	14°15′00″	.24615	.96923	9.39121	9.98643	10.59522	8.48812	4
4.5	12°40 49	.21951	.97561	.34145	.98927	.64782	.38721	4.5
5	11°25 16	.19802	.98020	.29671	.99131	.69461	.29670	5
5.5	10°23 20	.18033	.98360	.25606	.99282	.73676	.21467	5.5
6	9°31 38	.16552	.98621	.21884	.99397	.77513	.13966	6
6.5	8°47 51	.15294	.98823	.18453	.99486	.81033	.07059	6.5
7	8°10 16	.14213	.98985	.15269	.99557	.84288	8.00655	7
7.5	7°37 41	.13274	.99115	.12301	.99614	.87313	7.94691	7.5
8	7°09 10	.12452	.99222	.09522	.99661	.90138	.89111	8
8.5	6°43 59	.11724	.99310	.06909	.99699	.92791	.83865	8.5
9	6°21 35	.11077	.99385	.04442	.99732	.95290	.78915	9
9.5	6°01 32	.10497	.99448	9.02107	.99759	.97652	.74232	9.5
10	5°43 29	.09975	.99501	8.99891	.99783	10.99892	.69787	10.0
10.5	5°27 09	.09502	.99548	.97782	.99803	11.02021	.65559	10.5
11	5°12 18	.09072	.99588	.95770	.99821	.04051	.61527	11
11.5	4°58 45	.08679	.99623	.93849	.99836	.05987	.57676	11.5
12	4°46 19	.08319	.99653	.92007	.99849	.07842	.53986	12
15	3°49 06	06659	.99778	.82343	.99904	.17561	.34631	15
16	3°34 47	.06244	.99804	.79544	.99915	.20371	.29028	16
18	3°10 56	.05551	.99846	.74438	.99933	.25494	.18807	18
20	2°51 51	.04997	.99875	.69869	.99946	.30076	7.09663	20
24	2°23′13″	.04165	.99913	8.61959	9.99962	11.38003	6.93835	24

blunt frog angles such a theory may be practically applied, and the computations are very simple. Let l = the lead or the distance SF in Fig. 22, r = mean radius of the switch rails, n = number of the frog, g = gage of track. Then $l=2gn$, and $r=nl=2gn^2$. The length of switch rails ($S'H=SK$ in Fig. 22) must be such that the offset to the curve at the point Q shall equal the required switch-throw. If t = the switch-throw, then t/r = vers of the angle (α) subtended by the switch rails; then length of switch rails $= r \sin \alpha$. The above

heory ignores the fact that the wing rail between the frog point and its junction ith the switch rails is straight rather than curved.

A Point Switch also has straight point rails, Fig. 28. The effect of these wo details is to shorten the lead and also decrease the radius of the switch rails.

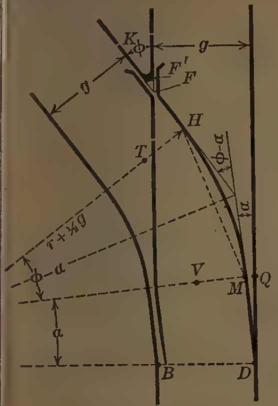


Fig. 28. Point Switch

Let α = angle of point rails
F = angle of frog = ϕ in Fig. 28
g = gage of track
n = frog number
w = length of wing rail = FH
c = chord of switch rail arc = HM
t' = thickness of switch point
s = length of switch rail = MD
l = length of lead = BF'
r = radius of center line of lead curve = OT = OV
h = heel distance = MQ
f = distance FF'
t = thickness of frog point

Then

$$c = \frac{g - w \sin F - s \sin \alpha - t'}{\sin \frac{1}{2}(F + \alpha)}$$

$$r = \frac{c}{2 \sin \frac{1}{2}(F - \alpha)} - \frac{1}{2}g$$

or

$$r = \frac{g - w \sin F - h}{\cos \alpha - \cos F} - \frac{1}{2}g$$

$$l = (g - w \sin F - h) \cot \tfrac{1}{2}(F + \alpha) + w \cos F + s \cos \alpha + f$$

r

$$l = (r + \tfrac{1}{2}g)(\sin F - \sin \alpha) + w \cos F + s \cos \alpha + f$$

r

$$l = (s - w)\frac{\sin \frac{1}{2}(F - \alpha)}{\sin \frac{1}{2}(F + \alpha)} + (g - t') \cot \tfrac{1}{2}(F + \alpha) + f$$

$$\alpha = \sin^{-1}\left(\frac{h - t'}{s}\right)$$

The dimensions of a point switch for a given frog number, as computed from the bove equations, depend on the values of w, s, t' and h, all of which may be chosen at leasure within certain narrow limitations. The length of the wing rail ($w = FH$), as vell as other frog dimensions, depends (within narrow limitations) on the switch manuacturer. The Am. Rwy. Eng. Assoc. recommends as standard practice for standard age (g = 4 ft 8½ in), 6¼ in. for h and ¼ in for t' (for computation purposes, actual hickness is ⅛ in as already given) and values of s and w for the various frog numbers s given in the table on page 206.

The length (L) is measured from the point of the switch to the actual blunt point f the frog (F'). This distance is greater than the distance to the theoretical point of he frog by the amount of the "frog bluntness (f)." On the basis that the actual width f the blunt point is ½ in as recommended the frog bluntness equals the frog number imes ½ in or times 0.0417 ft, and is given in the second column in the table.

The closure is the distance from the heel of the switch rail, or "point," to the toe of the rog and equals, for the straight rail, $L - (s + w + f)$ and, for the curved rail, $2\pi(R + \frac{1}{2}g) \times (F - \alpha)/360$, or, approximately $(F - \alpha)/D$.

The lengths and closures given in the table are the theoretical ones for the dimensions given, those for other values of s, t', h and w are easily computed

Switch and Frog Dimensions

(Condensed from Manual of Amer. Rwy. Eng. Assoc., 1915 Ed.)

Frog number	Frog bluntness	Frog				Switch rail		Switch dimensions					Frog number
		Wing rail		Total length		Length	Angle	Radius	Degree of lead curve	Length	Closure		
											Straight rail	Curved rail	
n	f	w		HK		s	α	r	D	$L=BF'$			n
	ft	ft	in	ft	in	ft	° ′ ″	ft	° ′ ″	ft	ft	ft	
4	0.17	3	2	8	6	11	2 36 19	112.26	52 53 56	37.22	22.88	23.29	4
5	0.21	3	7	10	0	11	2 36 19	183.22	31 40 24	42.98	28.19	28.55	5
6	0.25	4	0	11	0	11	2 36 19	273.95	21 01 58	48.36	33.11	33.38	6
7	0.29	4	5	12	6	16½	1 44 11	364.88	15 47 19	62.23	41.02	41.24	7
8	0.33	4	9	13	6	16½	1 44 11	488.71	11 44 40	67.80	46.22	46.42	8
9	0.37	6	0	16	0	16½	1 44 11	516.27	9 18 27	72.61	49.74	49.92	9
9½	0.40	6	0	16	0	16½	1 44 11	699.97	8 11 33	75.30	52.40	52.58	9½
10	0.42	6	0	16	6	16½	1 44 11	790.25	7 15 18	77.93	55.01	55.17	10
11	0.46	6	0	17	6	22	1 18 08	940.21	6 05 48	92.52	64.06	64.20	11
12	0.50	6	5	18	6	22	1 18 08	1136.34	5 02 38	97.75	68.83	68.96	12
15	0.62	7	8	22	6	33	0 52 05	1744.38	3 17 06	131.12	89.83	89.94	15
16	0.67	8	0	24	0	33	0 52 05	2005.98	2 51 24	136.62	94.95	95.05	16
18	0.75	8	10	26	6	33	0 52 05	2587.66	2 12 52	147.13	104.54	104.61	18
20	0.83	9	8	29	0	33	0 52 05	3262.98	1 45 22	157.18	113.68	113.76	20
24	1.00	11	4	34	6	33	0 52 05	4932.77	1 09 42	176.09	130.66	130.77	24

by means of the formulas. The theoretical closures are usualy modified in practice to avoid if possible cutting more than one rail per turnout, giving "practical" closures and "practical" leads (or lengths) as given below.

Practical Leads and Closures and Ordinates for Curving Lead Rails

For Standard Turnouts of Amer. Rwy. Eng. Assoc.

Frog Number	Lead	Closures				Ordinates			
		Straight Rail		Curved Rail		Center		Quarter Points	
	ft					ft	in	ft	in
4	37.94	1-23.60		1-24		0.59	7⅛	0.44	5⅜
5	42.47	1-27.68		1-28		0.55	6⅝	0.41	5
6	47.98	1-32.73		1-33		0.50	6	0.38	4½
7	62.10	1-13.89	1-27	1-14.11	1-27	0.58	7	0.43	5¼
8	67.98	1-16.40	1-30	1-16.60	1-30	0.55	6⅝	0.41	5
9	72.28	1-16.41	1-33	1-16.59	1-33	0.50	6	0.38	4½
9½	75.71	1-25.82	1-27	1-26	1-27	0.49	5⅞	0.37	4⅜
10	77.93	1-27	1-28	1-27.17	1-28	0.48	5¾	0.36	4⅜
11	94.31	1-32.85	1-33	2-33		0.55	6⅝	0.41	4⅞
12	100.80	1-23.88	2-24	3-24		0.52	6¼	0.39	4¾
15	131.19	2-30	1-29.89	3-30		0.58	7	0.43	5¼
16	137.57	1-29.90	2-33	1-30	2-33	0.56	6¾	0.42	5
18	146.51	1-25.93	3-26	4-26		0.53	6⅜	0.40	4¾
20	157.42	1-26.92	2-27 1-33	3-27	1-33	0.50	6	0.37	4½
24	177.22	1-32.89	3-33	4-33		0.44	5⅜	0.33	4

Rails shorter than 12 or 13 ft should not be used. The maximum modifi-
tion from the theoretical leads is slightly over 3% while modifications up to
to 8% in either direction may be made without seriously affecting the riding
alities of the turnout.

Turnouts from Curved Tracks. (a) *Stub Switch, lead rails considered to be
rved with uniform curvature from switch point to frog point.*

1. Dimensions for turnout from the inner side of a curved track; see Fig. 29.

$$\tan \tfrac{1}{2}\theta = gn/R \qquad (r + \tfrac{1}{2}g) = (R - \tfrac{1}{2}g)\frac{\sin \theta}{\sin (F + \theta)}$$

$$\text{Lead} = BF = 2(R - \tfrac{1}{2}g)\sin \tfrac{1}{2}\theta$$

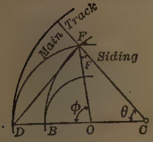


Fig. 29

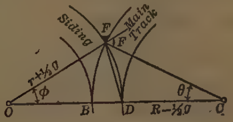


Fig. 30

2. Dimensions for turnout from the outer side of a curved track; see Fig. 30.

$$\tan \tfrac{1}{2}\theta = gn/R \qquad (r + \tfrac{1}{2}g) = (R + \tfrac{1}{2}g)\frac{\sin \theta}{\sin (F - \theta)}$$

the curvature of the main track is very sharp, θ may be greater than F and
en the center of curvature C will be on the concave side of the curved main
ack. Fig. 29 may then be used for this case by merely transposing O and
ϕ and θ, and " main track " and " siding." The equation for r then becomes

$$(r - \tfrac{1}{2}g) = (R + \tfrac{1}{2}g)\ \frac{\sin \theta}{\sin (\theta - F)} \qquad \text{Lead} = BF = 2\,(R + \tfrac{1}{2}g)\sin \tfrac{1}{2}\theta.$$

The lead is practically the same as for straight track and is most easily found
om the formula, Lead $= BF = 2gn$. Then $\theta = 2gnD$, where D is degree of main
ack curve. The degree of the turnout curve is also approximately that of
e main track curve plus or minus that of the turnout curve from straight track
r the same frog number according to whether the turnout is from the inner
outer side of the main track curve.

(b) *Stub Switch, with straight frog.* Frogs are usually made straight, with
e result on straight track of shortening each tangent to the turnout curve,
d therefore the lead, by the length of the leg of the frog. The radius and
gree of the new turnout curve are easily computed for the new tangent
ngth. For curved track the effect on lead may be assumed the same as on
raight track and the new degree of curve used as above.

(c) *Point Switch.* The gage lines of the switch rails have the curvature
the track so that the turnout at the heel of the switch rail makes the angle
with the main track the same as for straight track. The precise mathematical
mputations are quite complicated and it will be sufficient to assume the same
fference in lead between stub and point switches as obtains on straight track,
at is, use the table for leads, and to find the degree of the turnout curve in
e manner described above for the stub switch.

Double Turnout from Straight Track. The computations are quit simple when it is assumed that all of the frog rails as well as the switch rail are circular thruout. As may be seen from Fig. 31, the dimensions would b considerably changed by assuming the frog rails to be straight, and mucl depends on the length of the straight rails. In any case, the sections of curve rails would be very short. Only the equations for curved lead rails will her be given.

$$\text{vers } \tfrac{1}{2} F_m = \frac{g}{2(r + \tfrac{1}{2} g)} \qquad MF_m = \frac{r}{2 n_m}$$

in which n_m is the frog number corresponding to the middle frog F_m. The abov equations also depend on the assumption that F_l and F_r are equal. Thei values are obtained from the relation that vers $\tfrac{1}{2} F_m = \tfrac{1}{2}$ vers $F_l = \tfrac{1}{2}$ vers F_r Altho approximate, there is no sensible error in the relation $n_m = 0.707$ between the frog numbers, in which n_m is the frog number of F_m, and n i the frog number of F_l and F_r.

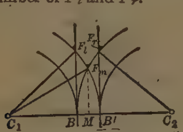


Fig. 31

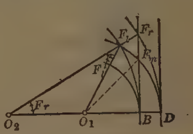


Fig. 32

If both turnouts run to the same side as indicated in Fig. 32, the relation of the frog will be practically identical with those in the previous paragraph, since the inner switc may be regarded merely as a curved main track, and since, as previously shown, th dimensions of switches are but little altered by a moderate curvature of the main track.

Connecting Curves from Turnouts. The following solutions for con necting curves and crossovers between straight parallel tracks are based o the use of straight frog rails beyond the frog point. Let d = distance betwee track centers, g = gage, F = frog angle, w' = length of wing rail back o theoretical frog point = $(HK - w)$ of the Switch Table = DF in the figures,

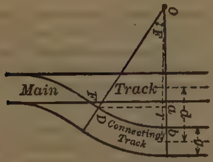


Fig. 33

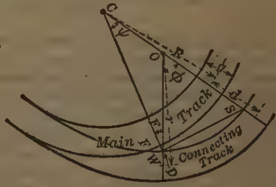


Fig. 34

R = radius of main curved track, r = radius of connecting curved track n = frog number corresponding to frog angle F, ψ = central angle, measure at center C of main curved track, required for connecting curve, ϕ = cen tral angle, required for connecting curve, measured at center O of connectin curve.

For a connecting curve from a straight track, see Fig. 33,

$$(r - \tfrac{1}{2} g) = \frac{d - g - w' \sin F}{\text{vers } F}$$

Altho the distance from F to the "fouling point" (which is on the line ab) may be shortened somewhat by using a sharper curve and using a short tangent at each end, the method indicated in Fig. 33 is preferable. Under such conditions, using the standard dimensions given in the Switch Table, the length of the connecting curve for a straight track, measured along the main track, is a definite quantity for any frog number, and is as given in the accompanying table for $d = 13$; $g = 4.708$.

Connecting Curve from Turnout, Straight Main Track

Frog no.	Dist.	Radius	Deg. of curve
4	61.31	229.20	25°12′02″
5	76.50	356.92	16 06 20
6	90.55	519.49	11 02 47
7	108.02	706.05	8 07 18
8	123.91	928.77	6 10 19
9	139.25	1169.71	4 54 00
9½	147.54	1313.15	4 22 08
10	155.35	1455.03	3 56 19
11	170.93	1760.31	3 15 18
12	187.29	2108.98	2 43 02
15	234.42	3300.35	1 44 10
16	250.52	3777.65	1 31 00
18	280.90	4743.75	1 12 28
20	312.31	5866.22	0 58 36
24	374.86	8447.05	0°40′42″

For a connecting curve on the outside of a main curved track, see Fig. 34,

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g - w' \sin F)}{2R + d + w' \sin F}$$

$$(r - \tfrac{1}{2}g) = (R + \tfrac{1}{2}g + w' \sin F)\frac{\sin \psi}{\sin (F + \psi)}$$

$$DS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F + \psi)$$

For a connecting curve on the inside of a main curved track, see Fig 35,

$$\tan \tfrac{1}{2}\phi = \frac{2n(d - g - w' \sin F)}{2R - d - w' \sin F}$$

$$(r + \tfrac{1}{2}g) = (R - \tfrac{1}{2}g - w' \sin F)\frac{\sin \phi}{\sin (\phi - F)}$$

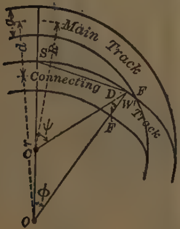


Fig. 35

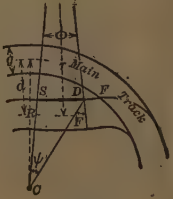


Fig. 36

In case the frog angle F is larger than ϕ, the center of curvature of the connecting curve rails will be on the other side of the main track as shown in Fig. 36. Then

$$(r - \tfrac{1}{2}g) = (R - \tfrac{1}{2}g - w' \sin F)\frac{\sin \phi}{\sin (F - \phi)}$$

In case ϕ should exactly equal F, the connecting rails would be straight and r = infinity. This is true when

$$2R - d - w' \sin F = 4n^2(d - g - w' \sin F)$$

The solution of this equation will show the value of R which makes this case possible for any given values of n, d, g and w'.

Crossover between Straight Parallel Tracks. With straight connecting track, the length between the frog wing rails

$$VZ = \frac{d - g}{\sin F} - 2\,w' - g \cot F.$$

The distance between frog points measured along either track

$$F_2Y = (d - g) \cot F - \frac{g}{\sin F}$$

The distance (measured along either track) from the switch point on one track to the switch point on the other track equals F_2Y plus twice the theoretical lead, or twice the distance from either switch point to the theoretical frog point.

Distance between Switch Points, Crossover between Straight Parallel Tracks, with 13 feet between Track Centers

Frog no.	F_2Y	Total distance	Frog no.	F_2Y	Total distance	Frog no.	F_2Y	Total distance
4	13.52	87.62	9	31.89	176.37	15	53.54	319.58
5	17.29	102.83	9½	33.71	183.51	16	57.13	329.03
6	20.96	117.18	10	35.51	190.53	18	64.32	357.08
7	24.62	148.50	11	39.13	223.25	20	71.51	384.21
8	28.27	163.21	12	42.73	237.23	24	85.87	436.05

With reversed curve connecting track, but with $F_1 = F_2 = F$, $w_1 = w_2 = w'$, and $r_1 = r_2 = r$, an indefinite number of combinations of r and θ may be selected by choosing some reasonable value of r in the following equation and solving

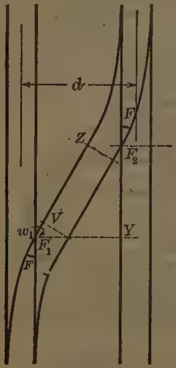


Fig. 37

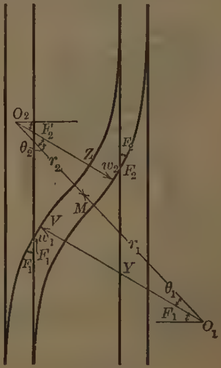


Fig. 38

the equation for θ, which is the only remaining unknown quantity, provided the frog angles are equal. (Similar to Fig. 38.) In this case the point M is midway between the tracks and between the frogs. The use of reversed curves saves distance measured along the main track, as shown in the illustration.

$$\tfrac{1}{2}(d - g) = w' \sin F + \tfrac{1}{2} g \cos F + r\,[\text{vers}\,(F + \theta) - \text{vers}\,F]$$

and $$F_2Y = 2\,[w' \cos F + \tfrac{1}{2} g \sin F + r\,[\sin (F + \theta) - \sin F]]$$

In case it is desired to make the crossing with two unequal frogs it may be done within limitations as shown in Fig. 38. The following equation may be derived:

$$d - g = w_1 \sin F_1 + \tfrac{1}{2} g\ (\cos F_1 + \cos F_2) + r_1\ [\text{vers}\ (F_2 + \theta_1) - \text{vers}\ F_1] + r_2\ [\text{vers}\ (F_2 + \theta_2) - \text{vers}\ F_2] + w_2 \sin F_2.$$

But since $(F_1 + \theta_1)$ must always equal $(F_2 + \theta_2)$ above equation becomes

$$\text{vers}\ (F_1 + \theta_1) = \frac{d - g - w_1 \sin F_1 - w_2 \sin F_2 - \tfrac{1}{2} g\ (\cos F_1 + \cos F_2) + r_1\ \text{vers}\ F_1 + r_2\ \text{vers}\ F_2}{r_1 + r_2}$$

All of the quantities in the right-hand side of the above equation are known except r_1 and r_2. Their values may be selected at pleasure (within limitations) and may be equal or unequal. The distance along the track

$$F_2Y = w_1 \cos F_1 - \tfrac{1}{2} g \sin F_1 + r_1\ [\sin\ (F_1 + \theta_1) - \sin F_1] + w_2 \cos F_2 - \tfrac{1}{2} g \sin F_2 + r_2\ [\sin\ (F_2 + \theta_2) - \sin F_2]$$

Cross Ties for Turnouts and Crossovers. The following table is based on the dimensions of the turnouts and crossovers given in previous tables. The switch block rests on two ties which are 4 ft longer than the standard. The tie under the point of the frog is 5 ft longer than the standard. The longest tie for a turnout is twice the standard length. The ties between the frog points (for the distance F_2Y, Fig. 37) are standard length plus 13 ft.

Number and Lengths of Ties in Turnouts and Crossovers
Standard tie, 8 ft 6 in long; 13 ft between track centers

Length	Frog number														
	4	5	6	7	8	9	9½	10	11	12	15	16	18	20	24
9 ft 0 in	5	5	6	8	9	9	10	10	12	13	15	15	16	18	20
9 6	3	4	4	5	5	6	6	6	7	8	12	12	12	13	15
10 0	3	3	3	4	4	5	5	5	6	6	9	9	9	10	11
10 6	2	3	3	4	4	4	4	5	5	6	8	8	9	9	10
11 0	2	2	3	3	4	4	4	4	5	5	7	7	8	9	10
11 6	2	2	2	3	3	3	3	4	4	4	6	7	7	8	9
12 0	1	2	2	3	3	3	3	3	4	4	6	7	7	7	7
12 6	3	3	3	4	5	5	5	5	6	6	7	7	8	8	9
13 0	1	1	2	2	2	3	3	3	3	3	5	5	6	6	7
13 6	1	1	1	1	1	2	2	2	2	2	2	2	2	2	4
14 0	1	1	1	1	2	2	2	2	2	2	3	3	3	4	5
14 6	1	1	1	1	2	2	2	2	3	3	3	4	4	4	5
15 0	1	1	1	2	2	2	2	2	3	3	4	4	5	5	6
15 6	1	1	2	2	2	2	2	2	3	3	4	4	5	5	6
16 0	1	1	2	2	2	2	2	3	3	3	4	4	5	6	6
16 6	1	2	2	2	2	2	3	3	3	3	4	4	5	6	7
17 0	1	2	2	2	2	2	3	3	3	4	4	5	6	6	7
21 ft 6 in	8	10	12	13	15	16	18	19	22	23	28	30	35	38	46

For a turnout, use the number of ties called for from 9 ft 0 in to 17 ft 0 in inclusive; for a crossover use twice the number from 9 ft 0 in to 13 ft 0 in inclusive, plus the number of 21 ft 6 in ties. The average spacing of these ties is 22 inches.

The standards of the Amer. Rwy. Eng. Assoc. for Nos. 8, 10, 11 and 16 turnouts and crossovers show an average spacing of 20 to 21 in, the detailed spacing being arranged to give increased support to switch points and frogs and to give suspended joints with the arrangement of lead rails recommended. The two ties for the switch block are 15 ft long and the maximum length is 6 in less than double the standard. For cross-overs the long ties are used for the entire distance between the toes of the frogs.

Bill of Timber for Standard Turnouts and Crossovers. Amer. Rwy. Eng. Assoc.
Standard tie, 8 ft 6 in long; 13 ft between track centers.

Length		Turnouts				Crossovers			
		No. 8	No. 10	No. 11	No. 16	No. 8	No. 10	No. 11	No. 16
Ft	In								
9	0	9	9	12	20	16	18	24	40
9	6	7	7	10	14	14	14	20	28
10	0	5	5	8	10	10	10	16	20
10	6	4	5	5	9	8	10	10	18
11	0	3	4	5	6	6	8	10	12
11	6	3	4	5	6	6	8	10	12
12	0	3	3	3	5	6	6	6	10
12	6	3	4	3	5	6	6	8	10
13	0	3	4	4	5				
13	6	2	3	4	5				
14	0	3	3	3	5				
14	6	2	3	3	4				
15	0	5	5	4	7	4	4	4	4
15	6	2	3	3	5				
16	0	3	3	3	4				
16	6	2	4	3	5				
21	6	-				24	31	32	49
Total BM		3651	4407	4814	7111	6925	8156	9534	13878
Distance		25.77	36.29	32.60	49.68	7.97	10.00	10.98	15.98

Total BM is in feet, board measure, on the basis of 7×9 in ties and the distance given in the last line is for turnouts from the actual frog point to the end of the ties provided for in the table. Beyond that point standard ties are used. For crossovers the distance is the change in length between frog points due to change of 1 ft in distance between track centers.

Crossover between Curved Parallel Tracks. (a) *Using a Straight Connecting Track:* There are two cases as illustrated in Figs. 39 and 40, depending on whether F_2 is greater or less than θ_2. The following equations apply to both figures. One frog (say F_1) may be chosen at pleasure within narrow limitations, then the relation between F_1 and F_2 is given by

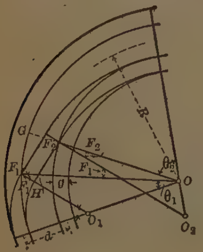


Fig. 39

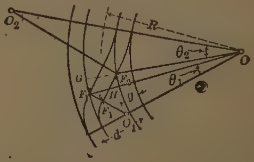


Fig. 40

$$\cos F_2 = \cos F_1 \frac{R + \frac{1}{2}d - \frac{1}{2}g - g \sec F_1}{R - \frac{1}{2}d + \frac{1}{2}g}$$

The distance between the frogs measured along the inner rail of the outer track is

$$GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2)$$

F_2 will not in general equal the angle of any standard frog, even though the frog F_1 be standard.

(b) Using a Reverse Curve for the Connecting Curve: In Fig. 41 the lead rails are assumed to be circular thruout, and the lead rail curves are assumed to have been continued until they meet at the point of reverse curve. Within narrow limitations F_1 and F_2 may be selected at pleasure, and will of course be made of standard sizes. The problem should be solved on this basis, which will determine the dimensions of the connecting curve, and then the switch rails may be altered if desired.

$$\text{vers}\,\phi = \frac{d\,(r_1 + r_2 - \frac{1}{2}d)}{(R - \frac{1}{2}d + r_2)\,(R + \frac{1}{2}d - r_1)};\ \sin OO_2O_1 = \sin\phi\,\frac{R + \frac{1}{2}d - r_1}{r_1 + r_2}$$

$$O_2O_1D = \phi + O_1O_2O;\ NF_2 = 2\,(R - \frac{1}{2}d + \frac{1}{2}g)\sin\frac{1}{2}(\phi - \theta_1 - \theta_2).$$

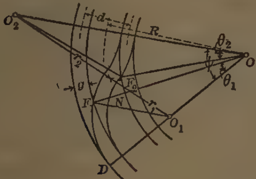


Fig. 41

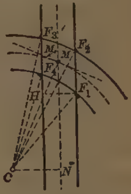


Fig. 42

Crossings. FOR TWO STRAIGHT TRACKS the frog angles are the angle (and its supplement) made by the two alinements. For ONE STRAIGHT AND ONE CURVED TRACK, Fig. 42 applies with these equations:

$$\cos F_1 = \frac{R\cos M + \frac{1}{2}g}{R - \frac{1}{2}g},\quad \cos F_2 = \frac{R\cos M + \frac{1}{2}g}{R + \frac{1}{2}g}$$

$$\cos F_3 = \frac{R\cos M - \frac{1}{2}g}{R + \frac{1}{2}g},\quad \cos F_4 = \frac{R\cos M - \frac{1}{2}g}{R - \frac{1}{2}g}$$

$$F_3F_4 = (R + \frac{1}{2}g)\sin F_3 - (R - \frac{1}{2}g)\sin F_4$$

$$HF_4 = (R - \frac{1}{2}g)(\sin F_4 - \sin F_1)$$

$$F_1F_2 = (R + \frac{1}{2}g)\sin F_2 - (R - \frac{1}{2}g)\sin F_1$$

For TWO CURVED TRACKS all four frogs are unequal, see Fig. 43. The radii of the two tracks are of course known and also the angle of intersection of their tangents (M). Then r_1, r_2, r_3 and r_4 become known by adding (or subtracting) $\frac{1}{2}g$ to the known track center radii. In the triangle C_1MC_2,

$$\tfrac{1}{2}(C_1 + C_2) = 90° - \tfrac{1}{2}M$$

$$\tan\tfrac{1}{2}(C_1 - C_2) = \cot\tfrac{1}{2}M\,\frac{R_2 - R_1}{R_2 + R_1}$$

From these equations C_1 and C_2 become known. Then

$$c = C_1C_2 = R_2\,\frac{\sin M}{\sin C_1}$$

For abbreviation $s_1 = \frac{1}{2}(c + r_1 + r_4)$; $s_2 = \frac{1}{2}(c + r_2 + r_4)$; $s_3 = \frac{1}{2}(c + r_1 + r_3)$ and $s_4 = \frac{1}{2}(c + r_2 + r_3)$.

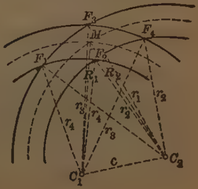


Fig. 43

Then $\text{vers}\,F_1 = \dfrac{2\,(s_1 - r_1)\,(s_1 - r_4)}{r_1 r_4}$ $\text{vers}\,F_2 = \dfrac{2\,(s_2 - r_2)\,(s_2 - r_4)}{r_2 r_4}$

$\text{vers}\,F_3 = \dfrac{2\,(s_3 - r_1)\,(s_3 - r_3)}{r_1 r_3}$ $\text{vers}\,F_4 = \dfrac{2\,(s_4 - r_2)\,(s_4 - r_3)}{r_2 r_3}$

$\sin C_1C_2F_4 = \sin F_4 \dfrac{r_3}{c}$; $\sin C_1C_2F_2 = \sin F_2 \dfrac{r_4}{c}$; $F_2C_2F_4 = C_1C_2F_4 - C_1C_2F_2$

$\sin F_1C_1C_2 = \sin F_1 \dfrac{r_1}{c}$; $\sin F_2C_1C_2 = \sin F_2 \dfrac{r_2}{c}$; $F_1C_1F_2 = F_1C_1C_2 - F_2C_1C_2$

From these equations the chords F_1F_2 and F_2F_4 are readily computed.

16. Stock Guards

Essential Features. A stock guard consists essentially of wing fences extending from the right-of-way fences to APRONS, which are short sections of fence, usually flaring, set parallel with the track at the track end of the wing fences; also a pit or a rough surface, on which stock will not walk, which extends the entire width of the track between the outer ends of the ties. A pit is about three feet deep, five feet long (parallel with track) and as wide as necessary; it has vertical walls of masonry or wood; the rails are supported on two wooden stringers spanning the pit. A surface guard covers the ties for a distance of about 8 feet with some one of a variety of wooden or metal slats or tiles.

Recommended Practise. The Amer. Rwy. Eng. Assoc. recommends surface guards in preference to pit guards, chiefly on account of the disastrous consequences due to a derailment at the guard or a failure of the pit structure. A surface guard should be so designed that it will not catch dragging brake chains or other rigging; it should not endanger employees who must necessarily walk over it; it should not rattle during the passage of trains; it should be reasonable in first cost, durable and easily applied and removed for track repairs; finally, it should be effective to deter all kinds of live stock from attempting to cross and yet should not catch and hold such as should make the attempt.

Surface Guards. Home-made wooden guards consist of slats about 8 ft long, 2½ in wide, 4 in high, spaced 4½ in c.c. using 2-in fillers and tied together with three ⅝-in round rods running thru slats and fillers. The slats are chamfered on the upper corners. Metal guards are usually patented and consist of rods (replacing the slats) or some one of a variety of designs in stamped sheet metal which, by bending out lugs, will make a rough surface. A surface may also be made by a combination of interlocking tiles about 4 in wide, 15 in long, and whose upper surface forms ridges. This type is free from rust or decay but is more liable to breakage.

17. Yards and Terminals

(Condensed from Definitions and Recommended Practise of the Am. Rwy. Eng. Assoc., as published in its "Manual.")

A Yard is a system of tracks arranged in series within defined limits for separating and making up trains, storing cars and other purposes. A large yard will have some or all of the following features. An incoming train leaves the main track and enters a receiving yard where the train may wait temporarily until the separation of the cars begins. The train runs from the receiving yard to a separating yard, in which the cars are separated according to district, commodity or other required order. Some cars will be run to a clas-

sification yard in which cars are classified or grouped in accordance with requirements preliminary to forwarding in trains. Some cars may be sent to a storage yard, awaiting further disposition. Others will be sent to special tracks referred to later. The cars which are to be sent out of the yard are then sent to the departure or forwarding yard, where they are assembled into trains. The distribution of separate cars on to various tracks is often accomplished by pushing them over a summit, beyond which they run by gravity. Such a yard is called a summit or hump yard. A yard in which the classification is accomplished by the use of a pole operated by an engine on an adjacent parallel track is called a poling yard. Tracks in a group or set of parallel tracks all used for some one purpose are called body tracks. They should be spaced from 13 to 14 ft c.c. A group of such tracks all lead into a ladder track which should be not less than 15 ft c.c. from any parallel track. No. 8 is the minimum frog number which should be used on such tracks. The track connecting either end of the yard with the main line is called a lead track, which should be interlocked with the main line. Running tracks and open tracks are provided so as to permit the free movement of cars and switching engines from any portion of the yard to any other. Caboose tracks should be located between the receiving and forwarding yards, and are preferably so constructed that a caboose may be readily pushed thereon from a receiving track and then dropt by gravity to the train departing in the direction from which the caboose has arrived. Scale tracks should be located between the receiving and separating yards. Coaling, ash-pit, sand and engine tracks should be located on the route to and from the engine house; they should be so arranged that water, coal and sand may be taken and ashes disposed of in convenient rotation and also provide that switching engines may clean fires, take coal, water and sand and pass around the waiting engines. Bad-order tracks should be convenient to the classification yard, so that bad-order cars may be set off and easily run to the repair tracks. The repair tracks should have a maximum capacity of about 15 cars each and be spaced alternately 16 and 24 ft apart c.c. Fifty linear feet of track should be estimated in rating the capacity of freight-car repair tracks, in order to provide working room about each car. Part of the yards should be provided with air and water pipes with outlets 50 ft apart for testing cars. A material supply track should be placed in the space between each pair of tracks. Heavy freight car repairs should be under cover and provided with overhead traveling cranes to facilitate heavy lifting. Icing tracks should be located between the receiving and separating yards so that the cars to be iced may readily be moved from the receiving to the icing tracks and thence to the separating yard. A coach-cleaning yard should be located for ready and quick access to and from the station. The tracks should be long enough to accommodate trains without cutting, and should preferably be stub-ended, with a car cleaners' repair supply building located at right angles at their ends. Yard track capacity is estimated by allowing 42 linear feet of track to each car. Team delivery yards should be located convenient to the freight house so that the receipt and shipment of freight may be easily under the control of the freight agent's force. The tracks should be stub tracks in parallel pairs, the tracks of each pair 12 ft between centers and the pairs 52 ft between centers. For convenience of shifting, the tracks should have a capacity of about 20 cars each. A crane for handling heavy freight should be provided. If possible, ingress and egress for teams should be provided for each end of each teamway. Wagon scales should be provided near the team entrance of the yard, and track scales should be provided and located for convenient switching.

A Hump Yard should have receiving, classification and departure tracks. Trains may be handled thru it faster and at less cost than thru any other form

of yard. The receiving tracks should be of sufficient length to hold a maximum train, and should be sufficient in number to hold as many trains, arriving in quick succession, as the character of the road renders probable. If possible the grades of the receiving tracks should be such that one engine can push the maximum train over the hump. The length and number of the classification tracks must depend on local conditions. The departure tracks should be of full train length and sufficient in number to provide ample standing room for trains while being tested for air and while waiting for engines. An air hose and an air brake testing plant should be provided conveniently near the departure tracks. The grades after leaving the summit should be sufficient to run the cars to their proper destination in the yard. The following grades are recommended: 60 feet of not over 2% and 50 feet of 4% with vertical curves 60 ft long between ascending grade and 2% (at hump), 40 ft long between 2% and 4% and about 155 ft from 4% to head of ladder tracks; thence on 1% down thru ladder tracks and turnouts and 0.5% thru classification tracks. The length of 4% grade may be reduced to a minimum of 30 ft and the grade thru ladder track and turnouts to a minimum of 0.8% as proportion of loaded cars to be handled increases. Where the traffic and climatic conditions require it, the grades should be made steeper in winter and restored again in the spring. When required, scales should be located at such a distance from the summit (75 ft is recommended) that when the car to be weighed reaches the scales it will be properly spaced from the following cars and running slowly enough to render correct weighing easy. For average conditions it is recommended that a No. 8 frog be the sharpest used for classification yards.

Freight Houses. For an inbound-freight house 50 ft is recommended as good average width. Platforms should be provided beside the tracks and coontinuous doors so as to obviate the necessity for spotting cars. For an outbound freight house the recommended width is 30 ft. Not advisable to load thru more than four to six cars. For a very great number of cars there should be stub tracks in pairs with covered platforms between, the pairs, the platforms leading to the freight house at the ends of the tracks. For a roadway between freight house on one side and wall on the other, minimum width 30 ft; with team track or another freight house on the other side, minimum clear width 40 ft.

Stock Yards for receiving cattle vary from a small pen at a way station to a series of large pens at a terminal. They include arrangements for feeding and watering, as well as loading and unloading.

STEAM RAILROAD EQUIPMENT

18. Water Stations

Chemical Treatment is necessary when the water contains objectionable amounts of incrusting or corrosive matter or of alkali salts. When water is boiled at a pressure of over 60 lbs the carbonates of lime and magnesia will precipitate and form mud or soft scale whose presence in the boiler is objectionable. It may be blown off but this wastes water and more or less heat. The sulfates of lime and magnesia when boiled form a hard scale which adheres to the tubes and is only removed with difficulty. Treatment with lime in a tank will precipitate the carbonates, which are thus easily removed. The cost of this treatment is very moderate. The use of sodium carbonate (or soda ash) in a water containing sulfate of lime will produce carbonate of lime, which precipitates, forming soft scale and also sulfate of soda which is objectionable because of "foaming." FOAMING is produced by the alkali salts, sulfate, carbonate or chloride of sodium, and is objectionable because it is difficult to maintain the proper amount of water in the boiler and keep wet steam out of

the cylinders. If the sulfate hardness is very great the water is practically worthless, since too much alkali salts would be developed by treatment. Even when the sulfate hardness is comparatively low, the simultaneous presence of considerable amounts of alkali salts renders the water useless, since the sulfate cannot be treated without producing a foaming water. Water may be classified as follows, the figures representing parts per 100 000:

With respect to	Very good	Fair	Very bad
Incrustation.....	No sulfates; sodium carbonates and carbonate hardness less than 35	Sulfate hardness, 5 to 10; total hardness less than 30	Sulfate hardness over 15; total hardness over 50
Foaming.........	Alkali salts less than 7	Alkali salts between 15 and 25	Alkali salts over 40

The Quantity of pure reagents required to remove 1 lb of various scaling and corroding substances is given in the following table. These should be increased to give equivalent quantities of pure reagents in case commercial products are used. The pounds of matter per 1000 gal may be found by dividing parts per 100 000 by 12 or grains per gal by 7. The total amount of lime must be sufficient to take care of the free carbonic acid as well as the solids.

Reagents Required for Water Softening. Per Pound of Substance

Amer. Rwy. Eng. Assoc.

Substance	Reagent and Amount	Foaming Matter Increased
Sulfuric acid.................	0.57 lb lime and 1.08 lbs soda ash......	1.45 lbs
Free carbonic acid	1.27 lbs lime..........................	None
Calcium carbonate............	0.56 lb lime	None
Calcium sulfate...............	0.78 lb soda ash......................	1.04 lbs
Calcium chloride.............	0.96 lb soda ash......................	1.05 lbs
Calcium nitrate..............	0.65 lb soda ash......................	1.04 lbs
Magnesium carbonate.........	1.33 lbs lime	None
Magnesium sulfate...........	0.47 lb lime and 0.88 lb soda ash.......	1.18 lbs
Magnesium chloride..........	0.59 lb lime and 1.11 lbs soda ash......	1.22 lbs
Magnesium nitrate...........	0.38 lb lime and 0.72 lb soda ash.......	1.15 lbs
Calcium carbonate...........	3.15 lbs barium hydrate...............	None
Magnesium carbonate........	3.76 lbs barium hydrate	None
Magnesium sulfate...........	2.62 lbs barium hydrate...............	None
*Calcium sulfate.....	2.32 lbs barium hydrate...............	None

* In precipitating the calcium sulfate, there would also be precipitated 0.74 lb of calcium carbonate or 0.31 lb of magnesium carbonate, the 2.32 lbs of barium hydrate performing the work of 0.41 lb of lime and 0.78 lb of soda ash, or for reacting on either magnesium or calcium sulfate, 1 lb of barium hydrate performs the work of 0.18 lb of lime and .034 lb of soda ash, and the lime treatment can be correspondingly reduced.

These data will enable one to determine the quantities of reagents necessary to treat a given water and local quotations will supply data for a cost estimate. The cost of operation (aside from chemicals) will depend largely on the method used and local conditions and may be very little in addition to the cost of operating a water plant.

Concentration of foaming solids reaches the critical point between 100 and 200 grains per gal, depending on the character of the solids and the amount of suspended matter in the water. The percent of wastage by blowing off for keeping below the limit of 100 grains per gal equals the number of grains of foaming matter per gallon in the water as supplied to locomotives, and varies inversely as the limit for other values.

Another method of treatment is to pass the water thru tanks between steel plates. A 110-volt current sent thru the plates passes thru the water. Hydrate of iron is liberated which precipitates the incrusting solids. The cost depends on the quality of the water and the amount of current used. At one station the cost was 47.9 cents per 1000 gallons. This is more expensive than the chemical treatment, but it permitted the treatment and use of a badly foaming water.

As an illustration of the importance of the subject, the El Paso and Southern Railway found that even after chemical treatment of the hard-water supply on a division 128 miles long, the engine tonnage was reduced 25%, while the cost of locomotive maintenance was increased $1000 per year per engine over the normal amount. To avoid this a waterworks system from a supply of pure mountain water 130 miles distant was constructed at a cost of $1 300 000. Even this expenditure was proven to be amply justified.

Tests have proven that for a scale thickness of say 1/8 in the loss of heat transmission may amount to 10 or 12%. A porous scale increases the heat loss even more than a solid scale. The chemical composition has no practical effect except as it may produce a porous scale. The heat loss increases as the thickness of the scale.

Supply. The Amer. Rwy. Eng. Assoc. recommends the purchase of water where it can be obtained in sufficient quantity and of suitable quality at a reasonable price. Otherwise the source may be springs, lakes, ponds, creeks, rivers or wells and it may be possible to supply the tanks by gravity, tho usually pumping will be necessary. The quantity and quality of the water should be investigated for a sufficient time to give accurate results, future increased demands and possible necessity for treatment being taken into account. Ordinarily the quantity should be sufficient, if economically possible, to give a 24-hour supply in 7 hours at terminal stations and 4 hours at intermediate stations, except for large stations when one may figure on 10-hour or even 20-hour pumping service.

Pumping. Steam and gasoline engines are most used in this service; the former especially in regions where slack coal is obtainable and the latter on account of the small amount of attention required. The sharp advances in

Costs of Fuel for Various Types of Pumps and Engines

Type		Fuel		B H P Hour		Eff H P	
Pump	Engine	Kind	Price	Fuel Used	Cost	No.	Cost 10 Hr
Recip	Steam, Sl Val	Bit Coal	$2.00 per ton	14 lbs	$0.0126	40	$3.15
Recip	Int Combust	Gasoline	0.16 per gal	1/8 gal	0.0200	50	4.00
Recip	Int Combust	Ill Gas	0.75 M cu ft	12 cu ft	0.0090	50	1.90
Recip	Int Combust	Nat gas	0.25 M cu ft	8 cu ft	0.0020	50	0.40
Recip	Int Combust	Fuel oil	0.06 per gal	1/8 gal	0.0075	50	1.50
Recip	Electric Mot	Elec	0.03 KWH	0.746 KW	0.0224	50	4.48
Centr	Electric Mot	Elec	0.03 KWH	0.746 KW	0.0224	50	4.48
Centr	Int Combust	Gasoline	0.16 per gal	1/8 gal	0.0200	50	4.00
Centr	Int Combust	Fuel oil	0.06 per gal	1/8 gal	0.0075	50	1.50

Note.—The last column covers the work required to elevate 400 gals per min 100 ft, this being equivalent to the delivery of 240 000 gals per day of ten hours, which is an average requirement of a railroad water station.

the price of gasoline have led to the introduction of other forms of power such as oil, producer gas, etc. Electric motors are practically automatic and may be used to advantage where current is cheap. The Amer. Rwy. Eng. Assoc. recommends steam for plants up to 5 E. H. P. (E. H. P. =gallons per minute times sum of static and friction heads divided by 3960) where 100 lbs of coal delivered at the pump house is cheaper than 1 gal of oil delivered at the oil storage tank, especially where a steam plant is maintained for other purposes and the interest on its cost is less than on an oil plant and unless the quality of the water is such as to occasion heavy boiler repairs. The table on p. 218 of costs of fuel presented by the Comm. on Water Service of the Amer. Rwy. Eng. Assoc. in 1915 will be useful in comparing methods:

Tanks of ordinary size are commonly built of wood, cedar or cypress. Steel is also used especially for the larger sizes and permanent installations and a few reinforced concrete tanks have been constructed with varying results. No doubt these will be more used in the future as they should be permanent and satisfactory if properly constructed. The construction of wooden and steel tanks is now a specialized business which may be left to the builders under general specifications. See Manual of the Amer. Rwy. Eng. Assoc. Their capacity usually varies from 10 000 to 80 000 gals. Two or even three smaller tanks are preferable to one excessively large tank.

Capacity in U. S. Gallons of Tanks of Various Inside Dimensions

Height Feet	Diameter Feet	Gallons	Height Feet	Diameter Feet	Gallons	Height Feet	Diameter Feet	Gallons
10	12	8 460	14	16	21 057	18	22	51 185
	13	9 929		18	26 650		24	60 914
	14	11 515		20	32 901		26	71 489
	15	13 219		22	39 810		28	82 910
12			16			20		
	14	13 817		18	30 457		24	67 682
	15	15 863		20	37 601		26	79 432
	16	18 049		22	45 498		28	92 123
	18	22 843		24	54 146		30	105 752

The total cost of wooden tanks per 1000 gals capacity may be roughly estimated at from $30 to $35 for 100 000 gal tanks to $45 to $50 for 30 000 gal tanks to $70 to $75 for 15 000 gal tanks to $130 to $140 for 5000 gal tanks. Steel tanks will cost from 50% to 75% more.

Track Tanks are used to obtain a supply of water while the train is in motion—even 60 miles per hour. A scoop, lowered from under the tender, is attached to a pipe leading to the tender tank. The rapid motion forces the water up the pipe from the long shallow tanks between the rails. The height of the top of the scoop above the track tank is usually about 9 ft, and this means that water will not flow into the tank unless the speed of the engine is more than 16 miles per hour. Even at 20 miles an hour more water is wasted by slopping over the sides than reaches the tender. The minimum wastage is about ⅛ and occurs at a speed of 45 to 50 miles per hour.

The track tank is made of $^3/_{16}$-in plate, is 19 in wide, 6 in deep, bottom rounded to 1½ in radius, plates are 15 ft long, riveted with $^7/_{16}$-in rivets, 20 rivets per joint; on each side is riveted an angle 1⅛ by 2 by ¼ in; the edges are stiffened by a molding of 1⅛ by ½ in bar iron. The ties are dapped 1½ in deep so as to lower the tank a little. At the center of its length it is rigidly attached to the ties. Ordinary track spikes are driven beside the angles. thus holding the tank in place laterally and vertically, but permitting longitudinal expan-

sion with temperature. There are inclined planes at each end, both inside and outside the tank, which will automatically raise the scoop in case the fireman neglects to raise it or lowers it too soon. About 1200 ft is an ordinary length, tho double this is sometimes used where trains are frequent, and the track must be absolutely level, which frequently requires reconstruction of the roadbed. They are usually placed on tangents but are successfully used on flat curves when necessary. To prevent freezing in winter, steam pipes having ⅛-in nozzles are inserted every 40 to 50 ft thruout the length of the tank, and jets of live steam are forced thru them from the boiler in the pump house in the "direct" system. In the "circulatory" the water is kept in circulation and steam is fed in with it at the inlets. The size of the required boilers depends on the amount of pumping required and on the severity of the climate, as it affects the demand for steam to prevent freezing. Altho 25 H.P. would generally suffice for warm-weather pumping, 150 H.P. might be required to prevent freezing. The cost of installation may amount to $10 000 and the cost of maintenance may be $125 to $150 per month.

19. Miscellaneous Structures

Coaling Stations. Located at division terminals and in general near engine houses. HAND SHOVELING of coal from a gondola car directly to engine tender involves delay of car until coal is required. A trestle costing $250 to $500, or a convenient natural embankment which lifts the car about 5 ft above the locomotive track, facilitates the work of shoveling. The shoveling has been done for about 17 cents per ton. To save delaying the cars the coal is sometimes shoveled onto a platform, or bin with low sides, and again shoveled into the tender. This doubles the cost of shoveling and with bituminous coal produces more slack. As an improvement a JIB CRANE is set on a platform, which lifts buckets of about one ton capacity and stores them on the platform from which they are again lifted and dumped in the tender when needed. Two quoted costs for this method are 23.2 and 32.0 cents per ton, but in these cases the plants handled only 12 and 13 tons per day. The use of small dump cars rather than buckets at a plant handling 235 tons a day reduced the cost to 17.8 cents per ton. The next step in reduction of operating costs by increase in cost of plant is to construct a TRESTLE with coal-car track 30 to 40 ft above the engine track, the grade of approach not exceeding 5%. Self-dumping cars, dumping the coal directly into bins, eliminate the cost of shoveling and reduce breakage of the coal. The cost of coal handling on 26 such plants varied from 9 to 12 cents per ton. When space for a 5% approach is not available, the grade may be increased to 20%, and the cars hauled up the grade by the use of cable and hoisting engine. Another type is a LOCOMOTIVE CRANE, which is essentially a jib crane mounted on a self-moving car. Such a crane, costing about $7500, can load 70 engines per day. It loads directly from the cars to the locomotive tender and can handle cinders and ashes as well as coal. It is also particularly useful when self-dumping cars are not regularly obtainable, especially since flat-bottom cars are almost useless for many types of coal-handling plants. The average cost of operating seven locomotive crane plants, each handling 106 to 230 tons of coal per day, was 7.3 cents per ton. Another crane handling only 45 tons per day cost 14.0 cents per ton. An added advantage in it is that such a crane may be transported and can be utilized for temporary emergencies, especially where permanent construction is for any reason inadvisable. The bucket conveyor type has the advantage of minimum ground space and indefinite flexibility to suit local conditions, provided the amount of coal to be handled is very large. The advantage and economy are in these particulars. The cost of handling coal in nine plants varied from 9.4 cents to 14.3 cents per ton. Storage bins should be constructed with hopper bottoms so as to facilitate the movement of the coal and also to prevent the accumulation of slack coal which sometimes

ignites spontaneously. Coal is sometimes weighed by passing it thru auxiliary bins holding 5 to 10 tons and mounted on scales. Sometimes the scales are omitted, and the measuring is done by volume. When sand and cinders are handled in the same plant, the handling machinery should be separate, since sand and cinders produce excessive wear on moving parts. The growing scarcity of lumber is constantly increasing the relative ultimate economy of plants of reinforced concrete and fireproof construction is desirable.

Ash Pits. The simplest form of ash pit is made by laying the rails on 12-in by 14-in wooden stringers, which rest on cross ties leaving a net space 4 ft wide. The stringers and ties are covered with old boiler plate to protect them from the hot ashes. Water service with hose and suitable drainage are almost essential to quench burning cinders. Altho such a pit is justifiable for a road doing small business, it requires prompt removal of the ashes by shoveling. The next step is to construct a pit with concrete walls and bottom and with a clear depth of about 3 ft. Light all-metal cars may be provided in the bottom of such a pit for the prompt removal of the ashes. Another method makes the pit wider (perhaps 10 ft), runs one rail on one side wall and the other rail on a series of cast-iron columns. This affords greater freedom in the removal of the ashes. A variety of designs agree in dumping the ashes from the locomotive into a hopper or car directly under the locomotive. The hopper or car is then drawn out sidewise, or, after the locomotive has past on, the car is lifted vertically by some form of mechanical hoist and dumped into a gondola car on a storage track. Ashes are also handled by the belt type of conveyor, the same as coal.

Sand Houses. A sand house consists essentially of a wet sand storage bin and a sand drier and screen. The dried sand may be stored in buckets from which it may be dumped by hand into the sand boxes on the engine. A more elaborate and economical plan uses compress air to pump the dry sand into an elevated storage tank from which it falls by gravity through a pipe to the engine.

Oil Houses. The essential features are a fireproof place of economical construction for the storage of oil, combined with conveniences for its distribution. The oil should be stored in tanks located in the basement, the basement having masonry walls and the floor above being made preferably of reinforced concrete. A trap door of fireproof materials should be the only opening into the vault beside the pipes. No illumination except electric lights should be permitted. The vault is surmounted by a house which may be of wood, and which is used for pumping the oil from the tanks into small cans and to distribute the oil to the employees. Since the oil is put in and removed entirely by pumping, the vault need not be opened except for occasional inspection and repairs.

Track Scales for freight-car service have a capacity of from 100 to 150 tons. The platform is generally from 36 to 46 ft long, but the length is sometimes increased to 60 ft, which permits the weighing of cars without stopping them as they cross the scales on a gravity track. Scales 100 ft long have been built, but their use is diminishing on account of greater unreliability. To economize yard room and yet not subject the scale mechanism to needless wear, a pair of thru rails is run about 10 to 12 in from the weighing rails. One of these thru rails rests on the wall of the scale pit and the other rests on iron posts which extend thru the platform and are separate from it. The weighing rails switch into the thru rails by means of point rails at about one rail length each side of the scales. The cost varies from about $25 to $35 per ton capacity, according to capacity and style, the lower prices applying to the larger-capacity scales.

Turntables. Length for modern locomotives must be from 60 up to 100 ft, the most common length being 70 ft. The pivot pier foundation must be designed for a load of perhaps 300 tons. The depth of the pit depends on whether a deck table or one made of two pony girders is used. The circular walls of the pit should be made of masonry, but are sometimes made of wood. They must support the circular rail on which the end of the table runs when unbalanced. The pit floor should be paved. A drain pipe with suitable outfall should be provided. The cost of a 70-ft steel turntable will vary from $4500 to $8000 according to the details of depth of foundation, lining of pit, etc. Temporary structures which will serve the purpose for light-weight engines may be built for $700 to $800 where timber is cheap.

20. Block Signaling

(The following has been condensed by permission from the Manual of the Am. Ry. Eng. and Main. of Way Assoc. It is largely a statement of "recommended practise.")

Signals, if practicable, should be placed either over (on a signal bridge) or upon the right of and adjoining the track to which they refer. Semaphore arms which govern should be displayed to the right of the signal mast, as seen from an approaching train. A mast may have a crosspiece on which two uprights (no more) may be mounted on which to place signals. One upright may be a stub to indicate that the corresponding track has no governing signals. Not more than one track should intervene between a bracket signal mast and the track for which its left upright carries a signal arm. There should be a definite place for flags and hand lanterns when used for signals; they should be fixt by a flag socket and lantern hook on the side of the signal station toward the direction of an approaching train and convenient for the operator to reach from one of the windows. The recommended semaphore for either train order, interlocking or automatic block signal is as shown in Fig. 44. The arm should have a sweep of 90°. The design may be used for either a two-position or a three-position signal. When the lamp is on the side (as in figure) spectacle A is always blank. Spectacle B has red glass. Spectacle C is red for a two-position signal and yellow or green (whichever color is used for "caution") in a three-position signal. Spectacle D is white or green, whichever color is used for "clear." Sometimes for economy on a single-track road the post is cut off so that the lantern is placed on top of the post directly back of (A) in the figure and another arm and spectacle casting is swung on the other side of the pole. In such a case spectacle A is red; B is red for two-position and yellow or green for three-position signal (like spectacle C in other design); spectacle C is white or green and spectacle D is blank. It is recommended that the "electric slot," an appliance for automatically disengaging the signal arm connection from its actuating lever and

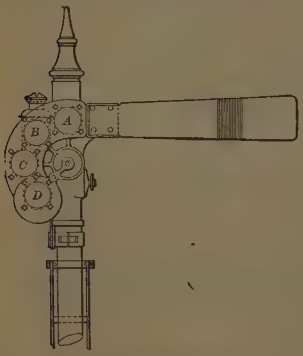


Fig. 44. Semaphore

returning signal arm to "stop," as a train passes, should be utilized. High-speed movements should be governed by high signals and low-speed movements by low signals. Not more than two high-speed signals should be displayed on one mast, the top arm to govern unrestricted speed and the lower arm to govern all other high speeds. All low-speed movements should be governed by one-arm low signals of dwarf construction. A distant signal should be provided for each high-speed route. "Red" should be the "color" stop indication, and the "horizontal" position of the arm should be the "position" stop indication for all home signals. A mark of distinction should be made between automatic block signals and all other home signals, whether interlocking, train-order or manually operated block signals. Home block signals should be provided at all interlocking plants used as block stations. All mechanically operated high-speed signals should be pipe-connected. Low-speed signals may be wire-connected. One distant signal only should be provided for a high-speed route, and when "clear" it should mean that all high-speed home signals along that route thru the interlocking plant, including the home block signal, are "clear." Every movement within the limits of an interlocking plant should be governed by an interlocked signal. In view of the recent trend of the development of the art, the following recommendations were made as desirable improvements on present practise: (a) that a red light shall be the night indication for "stop," a yellow light for "caution" and green for "clear"; (b) that day indications shall be given by semaphore signals in the upper right-hand quadrant; (c) that the semaphore arm horizontal shall indicate "stop," inclined upward 45° "caution," and inclined upward 90° "proceed."

Compensators. One should be provided for each pipe line over 50 ft in length and under 800 ft in length and the crank arms should be 11 in by 13 in centers. When the length is between 800 and 1200 ft, the crank arms

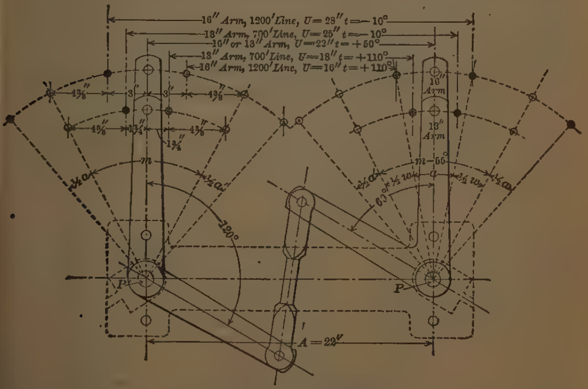


Fig. 45. Compensator for Signal Pipe Lines

shall be 11 in by 16 in centers. Pipe lines over 1200 ft long shall have two compensators. The compensators shall have one 60° and one 120° angle-

cranks and connecting link mounted in cast-iron base, having top of center pins supported. The distance between center of pin-holes shall be 22 in. The standard design is as shown in Fig. 45, where P and P' are the bearing pins, A = distance between them, t = temperature Fahr., a = temperature angle, w = stroke angle, and $m = a + w$. The distance U between the ends of the arms should be adjusted for the temperature at the time of adjustment as per following table.

Values of Spacing for U in Compensator (Fig. 45)

Temp. Fahr.	Length of lines compensated in feet											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
110°	$21\frac{1}{2}$	$21\frac{1}{16}$	$20\frac{9}{16}$	$20\frac{1}{8}$	$19\frac{5}{8}$	$19\frac{1}{8}$	$18\frac{5}{8}$	$18\frac{3}{16}$	$17\frac{11}{16}$	$17\frac{3}{16}$	$16\frac{3}{4}$	$16\frac{1}{4}$
90	$21\frac{11}{16}$	$21\frac{3}{8}$	$21\frac{1}{16}$	$20\frac{3}{4}$	$20\frac{3}{8}$	$20\frac{1}{8}$	$19\frac{3}{4}$	$19\frac{7}{16}$	$19\frac{1}{8}$	$18\frac{13}{16}$	$18\frac{1}{2}$	$18\frac{3}{16}$
70	$21\frac{13}{16}$	$21\frac{11}{16}$	$21\frac{1}{2}$	$21\frac{3}{8}$	$21\frac{3}{16}$	$21\frac{1}{16}$	$20\frac{7}{8}$	$20\frac{3}{4}$	$20\frac{9}{16}$	$20\frac{3}{8}$	$20\frac{1}{4}$	$20\frac{1}{8}$
50			Mean Temperature				$U = A = 22$					
30	$22\frac{3}{16}$	$22\frac{5}{16}$	$22\frac{1}{2}$	$22\frac{5}{8}$	$22\frac{13}{16}$	$22\frac{15}{16}$	$23\frac{1}{8}$	$23\frac{1}{4}$	$23\frac{7}{16}$	$23\frac{5}{8}$	$23\frac{3}{4}$	$23\frac{15}{16}$
10	$22\frac{5}{16}$	$22\frac{5}{8}$	$22\frac{15}{16}$	$23\frac{1}{4}$	$23\frac{5}{8}$	$23\frac{15}{16}$	$23\frac{1}{4}$	$24\frac{9}{16}$	$24\frac{7}{8}$	$25\frac{3}{16}$	$25\frac{1}{2}$	$25\frac{3}{8}$
0	$22\frac{3}{8}$	$22\frac{13}{16}$	$23\frac{3}{16}$	$23\frac{5}{8}$	24	$24\frac{3}{8}$	$24\frac{13}{16}$	$25\frac{3}{16}$	$25\frac{5}{8}$	26	$26\frac{3}{8}$	$26\frac{13}{16}$
−10	$22\frac{1}{2}$	$22\frac{15}{16}$	$23\frac{7}{16}$	$23\frac{15}{16}$	$24\frac{3}{8}$	$24\frac{7}{8}$	$25\frac{3}{8}$	$25\frac{13}{16}$	$26\frac{5}{16}$	$26\frac{13}{16}$	$27\frac{1}{4}$	$27\frac{3}{4}$

Values of U are based on 0.08 of an inch as coefficient of expansion for an increase of 10° Fahr. for each 100 ft of line, and the nearest 1/16 in is given. Since the mean temperature varies, it must be taken for the latitude where the work is done.

21. Locomotives.

Classification is indicated by three numbers, of which the first is the number of pilot-truck wheels, the second is the number of drivers and the third is the number of trailing wheels. All three numbers are given even tho the number is zero. For example, the common American type having four pilot wheels, four drivers and no trailing wheels is indicated by 4–4–0; the six-wheel switching engine, having neither pilot nor trailing wheels, by 0–6–0.

Classification of Domestic Locomotives Ordered in the U. S., 1911 to 1917

Type	Class	1911	1912	1913	1914	1915	1916	1917
Mikado	2–8–2	590	1309	796	333	562	754	834
Switching	0–8–0	...	...	...	...	...	...	110
Switching	0–6–0	443	821	638	201	221	730	282
Switching	0–4–0	...	...	...	...	...	...	47
Consolidation	2–8–0	577	858	823	166	194	63	60
Mallet	A–B–B–C*	112	168	72	59	120	218	175
Pacific	4–6–2	496	594	566	174	102	278	342
Santa Fe	2–10–2	...	...	...	63	75	325	370
Ten-Wheel	4–6–0	238	364	255	48	39	40	28
Mogul	2–6–0	127	61	42	24	12	28	13
Mountain	4–8–2	2	...	24	12	9	182	55
Atlantic	4–4–2	9	5	46	34	1	2	...
American	4–4–0	27	8	8	19	1	1	...
Electric	Various	133	75	94	59	69	32	43
Other	Various	406	252	103	73	168	238	188
Totals	.....	2850	4515	3467	1265	1573	891	2704

* A and C are truck wheels, usually 2 or 0 and B drivers, usually 6 or 8. There may be three sets of drivers, as 2-8-8-8-2 for the large Mallet mentioned below, the third set being under the tender in this case.

The tendencies as to types up to the present year (1918) is indicated by the able at bottom of p. 224, showing locomotives ordered.

As each of these types has been made of various weights, the number of ifferent kinds and sizes of locomotives in service is very great. The present Jnited States Railroad Administration intends to reduce this number and has dopted twelve standards, only, tho just what leeway if any, will be given he roads in deviating from these is at present unknown.

Standard Locomotives U. S. Railroad Administration, 1918

Specification No.	Class	Weight		Cylinders		Drivers	Boiler Pressure	Tractive Force	Axle Load
		Total lbs	On Drivers lbs	Kind	Dia × Str in	in	lbs/sq in	lbs	lbs
1-A	2 8-2	290 000	220 000	Simple	26×30	63	200	54 600	55 000
2-A	2-8-2	325 000	240 000	Simple	27×32	63	190	60 000	60 000
7	2-10-2	360 000	275 000	Simple	27×32	57	200	69 400	55 000
8	2-10-2	390 000	300 000	Simple	30×32	63	190	74 000	60 000
11	2-6-6-2	440 000	360 000	Comp	23&35×32	57	225	80 300	60 000
12	2-8-8-2	540 000	480 000	Comp	25&39×32	57	240	106 000	60 000
5-A	4-6-2	270 000	165 000	Simple	25×28	73	200	40 700	55 000
6-A	4-6-2	300 000	180 000	Simple	27×28	79	200	43 800	60 000
3-A	4-8-2	320 000	220 000	Simple	27×30	69	200	53 900	55 000
4-A	4-8-2	350 000	240 000	Simple	28×30	69	200	58 000	60 000
9	0 6-0	165 000	165 000	Simple	21×28	51	190	39 100	55 000
10	0-8-0	220 000	220 000	Simple	25×28	51	175	51 200	55 000

The first group of six are for freight service, the first four of the second group are for assenger and the remaining two for switching service. All are designed for the use of ituminous coal and all are equipped with superheaters and brick arches. Probably he Santa Fes and Mallets, Nos. 7, 8, 11 and 12, will have mechanical stokers. The over ll height is 15 ft except for the heavy Santa Fe, No. 8, and the 2-8-8-2 Mallet, No. 12, where it is 15 ft 9 in. The width over cylinders is 10 ft 4 in for all designs xcept small Mallet, No. 11, which has 10 ft 6 in and the heavy Santa Fe, No. 8, and arge Mallet, No. 12, which have 10 ft 9 in. The width over cab body and cab eaves, ncluding the cab handles, is the same for all designs, being 10 ft in the first case and 10 t 2 in in the second.

Weights. These vary from about 25 tons for a 4-4-0 type of narrow-gage ocomotive to that of a Mallet standard-gage locomotive weighing 853 050 lbs vith 63 466 lbs on each of twelve driving axles. Between these limits there are ocomotives of any total weight and any reasonable ratio of driving weight to otal weight tho the limits in any given type would of course be smaller. In 916 there were 64 073 locomotives in the U. S., or 252 per 1000 miles of line. This emphasizes the practical difficulties of reducing to the twelve standard lesigns while clearances and strength of structures will prohibit their use in many ases without extremely expensive reconstruction. For instance, none of them ould pass thru the Hoosic Tunnel.

The **tenders** for the first three standard freight locomotives carry 10 000 als of water and weigh 172 000 lb while the other three carry 12 000 gal and veigh 206 000 lb. Those for the Pacific and switching locomotives carry 8000 al and weigh 144 000 lb and the other two carry 10 000 gal and weigh 172 000 b. All carry 16 tons of coal and are mounted on two four-wheel trucks.

The Maximum Weight and clearance dimensions of locomotives are required by an engineer when designing structures and these vary on different lines. They will usually not exceed the standards given above tho a few roads are using locomotives of greater total weight and with slightly greater axle loads.

Wheel Bases, Heating Surfaces and Grate Areas for U. S. Standard Locomotives 1918

Number	1–A		2–A		7		8		11		12	
Wheel Base:	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
Driving	16	9	16	9	21	0	22	4	31	2	42	1
Rigid									10	4	15	6
Total	36	1	36	1	40	4	42	2	50	2	57	4
Eng and Ten	71	5½	71	9½	76	0½	82	10½	88	10	93	3
	Square Feet											
Grate Area	66.7		70.8		76.3		88.2		76.3		96.2	
Heating Surface:												
Total*	3783		4297		4666		5156		5456		6217	
Superheater	882		993		1078		1230		1260		1475	
Equivalent †	4706		5787		6283		7001		7346		8420	

Number	5–A		6–A		3–A		4–A		9		10	
Wheel Base:	ft	in	ft	in	ft	in	ft	in	ft	in	ft	in
Driving	13	0	14	0	18	3	18	3	11	0	15	0
Rigid	......		......		......		......		......		......	
Total	34	9	36	2	40	0	40	0	11	0	15	0
Eng and Ten	68	7½	70	8½	75	8½	75	8½	48	10½	52	10½
	Square Feet											
Grate Area	66.7		70.8		70.8		76.3		33		46.6	
Heating Surface:												
Total*	3333		3808		4130		4666		1894		2781	
Superheater	794		882		957		1078		475		637	
Equivalent †	4524		5133		5566		6283		2607		3737	

* Total is for tubes, flues, firebox and arch tubes.
† Equivalent is total evaporating surface plus 1.5 times superheating surface.

The Tractive Force of a locomotive may be limited by any one of three factors:

(a) Frictional force or "adhesion" between drivers and rail, a function of the weight on the drivers.

(b) Capacity of the boiler to produce steam, depending on design of boiler, fuel, water and stoking.

(c) Capacity of the mechanism to convert the energy of steam into motion, depending on size of cylinders and drivers.

At low speeds, adhesion is usually the limiting factor as most locomotives are "over-cylindered" and the rate of steam consumption is low; hence they are able to "slip their drivers" and the tractive force using sand may be as much as one-third the weight on the drivers, tho it is not usual to count on more than one-fourth as a maximum. As the speed increases a point is reached where the boiler is no longer able to furnish steam at full cut-off and maintain its normal pressure. So the cut-off is reduced and the "Mean Effective Pres-

sure" becomes smaller and smaller as the speed increases. Hence boiler capacity limits tractive force at speeds beyond the point mentioned above as could be expected, since weight on drivers and size of cylinders can be increased easily while the size of the boiler is limited.

Determination of Tractive Force. The tractive force of a locomotive may be determined by actual measurement with a dynamometer car or at a locomotive testing plant or by computation. With a dynamometer car, speeds, alinement and grades must be accurately known in order to make the proper allowances. The Amer. Rwy. Eng. Assoc. recommends that speeds be determined to the nearest 0.1 mile per hour. Cylinder power may be determined by means of indicator cards and used to find tractive force. By computation: for low speeds (or maximum tractive force) one may simply take one-fifth to one-fourth the weight on the drivers, assuming reasonably good design. For higher speeds, recourse must be had to the theoretical formula with empirical coefficients or to empirical methods. Let P = the boiler pressure in pounds per sq in; S, the length of stroke in inches; d, the diameter of piston for simple engines; d_h and d_l, the diameters of high-pressure and low-pressure pistons respectively for compound engines; R, the ratio of the area of the low-pressure to the high-pressure cylinders; D, the diameter of drivers; and T.F. the tractive force at the circumference of the drivers. In a simple engine the work done by both cylinders during a complete revolution of the drivers = piston area $\times$ effective average cylinder pressure $\times$ stroke $\times 2 \times 2$. But the work also equals the tractive adhesion developed at the circumference of the drivers $\times$ the distance traveled by the drivers during one revolution, which of course equals the circumference of the drivers. Therefore, for simple engines,

$$\left.\begin{array}{c}\text{Theoretical}\\ \text{tractive force}\end{array}\right\} = \frac{\text{Piston area} \times \text{effective cylinder pressure} \times \text{stroke} \times 2 \times 2}{\text{circumference of drivers}}$$

The effective area of the piston is reduced about 1.5% on account of the area of the piston rod. The effective energy at the wheel rim is reduced on account of friction of the piston, piston rod, crosshead and the various bearings. The effective steam pressure in the cylinder is always considerably less than that in the boiler, even at low speed and full cut-off. These reductions may be allowed for by figuring the steam pressure (effective at the drivers) to be 80 to 85% of the boiler pressure. Therefore, dividing both numerator and denominator by π(3.1416) we have

$$\text{T.F.} = 0.8Pd^2S/D \quad \text{(for simple engines)}$$

To prevent racking the engine frame, two-cylinder compounds or four-cylinder compounds of the Baldwin type are designed with R at such a ratio that the work done by the high- and low-pressure cylinders is approximately equal. Altho there is not such necessity for balancing with a tandem compound, the same ratio is used, which averages 2.81. The formula is

$$\text{T.F.} = \frac{PS}{D}(.67d_h^2 + .25d_2^2) \qquad \left(\begin{array}{c}\text{for Baldwin compounds and}\\ \text{tandem compounds}\end{array}\right)$$

Even greater power may be temporarily obtained when starting a train, or, in an emergency, on a grade (if not limited by poor adhesion) by exhausting from the high-pressure cylinder directly to the atmosphere and by admitting high pressure steam directly into the low-pressure cylinders. Of course all economy due to compounding is lost while this is done.

The Tractive Force of a Two-Cylinder Compound, when the work is equal in both cylinders, and when working compound, is

$$\text{T.F.} = \frac{.8Pd_l^2S}{(R+1)D} \qquad \text{(for two-cylinder compounds)}$$

As before, the tractive force may be increased when starting by exhausting the high-pressure cylinder directly into the atmosphere and by admitting boiler steam to the low-pressure cylinder thru a special valve which proportionately reduces its pressure. Here the T.F. is the same as that of a simple engine with both cylinders of the same diameter as the high-pressure cylinder.

The Equivalent Diameter of Simple Cylinders which, with the same boiler pressure, diameter of drivers, and stroke, will have the same tractive force, may be exprest as follows, when all cylinders perform the same work:

$$d = \sqrt{\frac{d_l^2 d_h^2}{d_l^2 + d_h^2}} \quad \text{(for two-cylinder compounds)}$$

$$d = 1.41\sqrt{\frac{d_l^2 d_h^2}{d_l^2 + d_h^2}} \quad \text{(for four-cylinder compounds)}$$

The Tractive Force as a Function of the Velocity is exprest by the following formula by Isaacs and Adams (Bulletin 112, Am. Ry. Eng. and Main. Way Assoc.), the notation being somewhat revised to correspond with that given above, and in which V = train speed in miles per hour,

$$\text{T.F.} = d^2 P \frac{S}{D}\left(0.95 - \frac{392\,S}{11\,000\,D} V\right)$$

The reduction of tractive force with increase in velocity is shown in the table (computed on the basis of the above formula) in which is given the ratio of tractive force at speeds between 10 and 30 miles per hour to the tractive force at 10 miles per hour for engines with five combinations of ratio of stroke to diameter of drivers.

Ratio of Tractive Force at Various Speeds to Tractive Force at 10 Miles per Hour

Stroke = Drivers = Ratio S/D =	24 in 56 in 0.429	28 in 62 in 0.453	24 in 50 in 0.480	28 in 56 in 0.500	30 in 56 in 0.536
Velocity, miles per hr					
10	1.000	1.000	1.000	1.000	1.000
11	0.981	0.980	0.978	0.977	0.975
12	.962	.959	.956	.954	.950
13	.942	.939	.934	.931	.925
14	.923	.918	.912	.908	.899
15	.904	.898	.890	.885	.874
16	.885	.877	.868	.862	.849
17	.866	.857	.846	.838	.824
18	.847	.837	.824	.815	.799
19	.827	.816	.802	.792	.774
20	.808	.796	.780	.769	.748
21	.789	.775	.758	.746	.723
22	.770	.755	.736	.723	.698
23	.751	.734	.714	.700	.673
24	.731	.714	.692	.677	.648
25	.712	.694	.671	.654	.623
26	.693	.673	.649	.631	.597
27	.674	.653	.627	.608	.572
28	.655	.632	.605	.584	.547
29	.636	.612	.583	.561	.522
30	.616	.592	.561	.538	.497

Average Evaporation in Locomotive Boilers in Pounds of Steam per Pound of Coal

Thermal Value of Coal B. T. U.	Pounds of Coal Fired per Hour per Sq Ft of Heating Surface											
	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
10 000	5.24	4.87	4.55	4.25	3.98	3.74	3.51	3.31	3.13	2.96	2.80	2.66
12 000	6.29	5.85	5.46	5.10	4.78	4.49	4.22	3.98	3.75	3.55	3.37	3.19
14 000	7.34	6.82	6.37	5.95	5.57	5.24	4.92	4.64	4.38	4.14	3.93	3.73

Feed water at 60° F, boiler pressure 200 lbs per sq in. In bad water districts deduct 10% for each 1/16 in accumulated scale and 1% for each grain per gal of foaming salts. Heating surface does not include superheater if used but is water evaporating surface only.

Weight of Steam Used in One Foot of Stroke in Locomotive Cylnders

For Locomotives Using Saturated Steam

Diameter of Cylinder	Gage Pressures						
	160 lb	170 lb	180 lb	190 lb	200 lb	210 lb	220 lb
Inches	lb	lb	lb	lb	lb	lb	lb
12	0.304	0.321	0.337	0.354	0.370	0.389	0.405
13	0.357	0.376	0.396	0.415	0.435	0.456	0.475
14	0.414	0.436	0.459	0.482	0.504	0.529	0.551
15	0.476	0.501	0.527	0.553	0.579	0.607	0.633
15½	0.508	0.535	0.562	0.590	0.618	0.649	0.675
16	0.541	0.570	0.599	0.629	0.658	0.691	0.720
17	0.611	0.643	0.676	0.710	0.744	0.780	0.812
18	0.685	0.722	0.759	0.796	0.834	0.875	0.911
18½	0.724	0.762	0.801	0.841	0.881	0.924	0.962
19	0.763	0.804	0.845	0.887	0.928	0.975	1.015
19½	0.804	0.847	0.890	0.934	0.978	1.027	1.069
20	0.846	0.891	0.936	0.983	1.029	1.080	1.125
20½	0.888	0.936	0.984	1.032	1.081	1.134	1.181
21	0.932	0.982	1.032	1.083	1.134	1.191	1.240
22	1.023	1.078	1.133	1.189	1.245	1.307	1.361
23	1.118	1.178	1.238	1.300	1.361	1.428	1.487
28	1.657	1.745	1.835	1.926	2.017	2.117	2.204

For Simple Locomotives Using Superheated Steam

Diameter of Cylinder	160 lb	170 lb	180 lb	190 lb	200 lb	210 lb
18	0.415	0.443	0.470	0.498	0.524	0.551
19	0.465	0.496	0.526	0.557	0.587	0.618
20	0.515	0.549	0.582	0.617	0.650	0.684
21	0.565	0.605	0.641	0.679	0.715	0.752
22	0.623	0.665	0.705	0.747	0.787	0.827
23	0.682	0.728	0.772	0.818	0.861	0.905
24	0.741	0.791	0.838	0.889	0.931	0.984
25	0.804	0.859	0.910	0.965	1.016	1.065
26	0.868	0.927	0.983	1.041	1.097	1.150
27	0.937	1.000	1.057	1.123	1.183	1.241
28	1.008	1.078	1.143	1.209	1.275	1.340
29	1.083	1.156	1.225	1.199	1.368	1.438
30	1.157	1.234	1.308	1.387	1.460	1.533

Cylinder diameter is for high-pressure cylinders in compounds. Superheat of 200° F and drop of five pounds per sq in between boilers and cylinders are assumed.

The Amer. Rwy. Eng. Assoc. recommends the following method of determining tractive force. Assuming that the maximum amount of coal that can be fired is 4000 lb per hour for hand-fired locomotives and 6000 lb per hour for stoker-fired locomotives with grates of less than 70 sq ft and 8000 lb per hour for stoker-fired locomotives with grates of more than 70 sq ft, find the rate of steam production from the first table on p. 229.

The steam used per revolution at full cut-off equals the quantity given in the second table on p. 229 multiplied by four times the length of the stroke in feet for simple and four-cylinder compounds and by twice the length of stroke in feet for two-cylinder compounds. The weight of steam produced by the boiler per minute divided by the weight used per revolution will give the maximum number of revolutions per minute at which full cut-off can be maintained. The corresponding train speed is found by multiplying by the diameter of the driver in inches and dividing by 336.13 and this speed is called M.

The pounds of steam used per I. H. P. hour at speed M are:

	For Locomotives Using Saturated Steam						
Gage pressure	160	170	180	190	200	210	220
Simple	39.45	39.10	38.80	38.53	38.30	38.11	37.99
Compound	26.57	26.34	26.14	25.95	25.80	25.67	25.59
	For Locomotives Using Superheated Steam						
Simple	24.72	24.50	24.31	24.14	24.00	23.88	23.81

Percent of Cylinder Tractive Force for Various Multiples of *M*

For Locomotives Using Saturated Steam

Speed	Compound Percent	Simple Percent	Speed	Compound Percent	Simple Percent	Speed	Compound Percent	Simple Percent
M			*M*			*M*		
Start	135.00*	106.00	3.6	32.40	44.75	6.4	.....	23.59
0.5	103.00	103.00	3.7	31.25	43.56	6.5	.....	23.18
1.0	100.00	100.00	3.8	30.10	42.39	6.6	.....	22.79
1.1	96.28	95.57	3.9	29.14	41.24	6.7	.....	22.42
1.2	92.55	91.53	4.0	28.24	40.10	6.8	.....	22.06
1.3	88.83	87.83	4.1	27.38	39.00	6.9	.....	21.71
1.4	85.12	84.46	4.2	26.56	37.96	7.0	.....	21.38
1.5	81.40	81.37	4.3	25.77	36.97	7.1	.....	21.06
1.6	77.68	78.55	4.4	25.03	36.03	7.2	.....	20.75
1.7	73.96	75.97	4.5	24.34	35.13	7.3	.....	20.45
1.8	70.25	73.60	4.6	23.69	34.26	7.4	.....	20.16
1.9	66.54	71.41	4.7	23.07	33.41	7.5	.....	19.88
2.0	63.21	69.37	4.8	22.48	32.59	7.6	.....	19.61
2.1	60.20	67.47	4.9	21.92	31.82	7.7	.....	19.34
2.2	57.48	65.67	5.0	21.38	31.11	7.8	.....	19.08
2.3	54.97	63.94	5.1	20.87	30.42	7.9	.....	18.82
2.4	52.68	62.22	5.2	20.37	29.75	8.0	.....	18.57
2.5	50.42	60.55	5.3	19.89	29.10	8.1	.....	18.33
2.6	48.16	58.92	5.4	19.43	28.48	8.2	.....	18.09
2.7	46.08	57.33	5.5	18.99	27.87	8.3	.....	17.86
2.8	44.10	55.78	5.6	.....	27.33	8.4	.....	17.64
2.9	42.29	54.26	5.7	.....	26.81	8.5	.....	17.43
3.0	40.57	52.78	5.8	.....	26.30	8.6	.....	17.22
3.1	38.95	51.33	5.9	.....	25.81	8.7	.....	17.01
3.2	37.42	49.91	6.0	.....	25.34	8.8	.....	16.82
3.3	35.98	48.55	6.1	.....	24.88	8.9	.....	16.63
3.4	34.66	47.24	6.2	.....	24.44	9.0	.....	16.45
3.5	33.53	45.97	6.3	.....	24.01			

* Operated as simple engine.

For Simple Locomotives Using Superheated Steam

Speed	Percent	Speed	Percent	Speed	Percent	Speed	Percent
M		*M*		*M*		*M*	
Start	106.00	2.7	47.12	4.5	31.19	6.3	22.90
0.5	103.00	2.8	45.82	4.6	30.61	6.4	22.56
1.0	100.00	2.9	44.61	4.7	30.05	6.5	22.21
1.1	92.42	3.0	43.49	4.8	29.52	6.6	21.89
1.2	86.55	3.1	42.30	4.9	29.00	6.7	21.57
1.3	81.20	3.2	41.21	5.0	28.48	6.8	21.24
1.4	76.95	3.3	40.17	5.1	27.96	6.9	20.92
1.5	73.00	3.4	39.22	5.2	27.47	7.0	20.62
1.6	69.55	3.5	38.30	5.3	27.00	7.1	20.32
1.7	66.60	3.6	37.42	5.4	26.53	7.2	20.07
1.8	63.66	3.7	36.61	5.5	26.10	7.3	19.78
1.9	61.27	3.8	35.89	5.6	25.69	7.4	19.52
2.0	58.96	3.9	35.11	5.7	25.26	7.5	19.26
2.1	56.94	4.0	34.39	5.8	24.86	7.6	19.01
2.2	55.12	4.1	33.72	5.9	24.46	7.7	18.76
2.3	53.26	4.2	33.06	6.0	24.04	7.8	18.52
2.4	51.53	4.3	32.40	6.1	23.66	7.9	18.28
2.5	48.50	4.4	31.79	6.2	23.28	8.0	18.06
2.6	48.50						

The steam produced per hour, divided by the amount used per I. H. P. hour will give the I. H. P. at speed *M* and the corresponding tractive force = 375 times I. H. P. divided by *M* (in miles per hour). The tractive force at other speeds may then be found by use of the preceding tables.

The cylinder tractive force must be reduced by the engine resistances (see Art. 1) to give draw-bar pull and the tractive force between drivers and rail found by the theoretical method should be reduced by items "b" and "c" of those resistances, tho the uncertainty in the value of the coefficients makes it useless to attempt refinements.

The Horse-Power Developed by a Locomotive equals the actual tractive force developed, multiplied by the speed in miles per hour, and divided by 375; if the speed is given in feet per second, divide by 550 instead of 375. An actual test of the draw-bar pull of a locomotive measured with a dynamometer showed that at very low velocities the pull was about 30 000 lbs, which corresponded to an adhesion ratio of about 22%. At a velocity of 10 miles per hour the pull had decreased to 26 500 lbs, which indicated a development of 706 h.p. At 20 miles per hour the tractive force had dropt to 17 000 lbs, in spite of the fact that owing to the greater velocity the horse-power developed had increased to 906. At 30 miles per hour the tractive force had dropt to 10 500 lbs, which indicated 840 h.p. Since the above figures represented draw-bar pulls at the rear of the tender, the actual power developed at the drivers was greater, and the actual horse-power developed by the cylinders was still greater in each case. At low velocities the horse-power increases almost directly as the velocity. At velocities above 13 miles per hour, the horse-power increases very slowly, and at high speeds it decreases as the velocity increases.

Cost of Locomotives varies with details of the construction and with the particular makes of the attachments which are used, as well as with general fluctuations in business. Present prices are abnormal and fluctuating but the government will fix prices on standard locomotives. A common pre-war figure (1915) for rough estimates was $300 per short ton.

Fuel Consumption. Wood is used only where it is comparatively cheap and coal is comparatively expensive. Green wood contains about 50% of

moisture and has far less calorific value than dry wood. About 2.5 lbs of dry wood are required to produce the same heat as one lb of average soft coal. One cord of dry hickory will weigh about 4500 lbs and in calorific value equals about 1800 lbs of average soft coal. One cord of average pine will weigh only 2000 lbs and is the equivalent of 800 lbs of coal. OIL is used for locomotive fuel where crude petroleum is cheap. One pound has a heat value of about 21 000 B.T.U., or 1.5 times that of the best coal. Less weight to be carried in the tender, ease of handling and firing, more uniform heat, less repairs to firebox and no expenses for ash handling are additional advantages to be considered in comparing relative economy. In 1911, 3627 locomotives out of a total of 61 327, or nearly 6% burned oil for fuel. Its use is constantly increasing. ANTHRACITE COAL is used on a few railroads passing thru the anthracite regions. The calorific value is no higher than the best grades of bituminous and it costs considerably more, but it makes less smoke and soot and for passenger traffic this has its advantage. Wood, oil and anthracite may be considered the exceptions. BITUMINOUS COAL is the standard locomotive fuel. The average cost of fuel per train mile in the U. S. increased from 8.73 cents in 1897 to 16.8 cents in 1907. This is chiefly due to the large ncrease in weight of locomotives and in the weight of trains hauled.

Economical Speed. When the velocity of trains is very low (say 5 miles per hour) the coal burned per mile is greater and train wages are greater both on train under consideration and others due to greater interference with traffic, while any saving is small. For high velocities the cost per train mile is likewise comparatively high. The most economical speed is about 17 miles per hour for single track and nearly 20 miles per hour for double track according to results of computations by Isaacs and Adams, in Bul. 115 of the Amer. Rwy. Eng. and M. of W. Assoc. These were, of course, based on certain assumptions but there is no doubt that it is often not most economical to haul maximum train loads or full " ratings " on account of low speeds.

Rating of Locomotives. The meaning of this term, with two illustrations, is given in Art. 1. The following derivation of the formula with tables for its use is condensed from a report of the Economics Committee to the Amer. Rwy. Eng. Assoc. in 1910. Let p =" pulling power of locomotive " or the tractive force as measured at the rim of the drivers; e the weight of the engine and tender; w the weight of the train, exclusive of locomotive; r the rate of grade; k a constant, depending on the weight of the train (w); n the number of cars; c a constant, depending on the number of cars; and A =the " rating." Then

$$p = (e+w)(r+k) + nc$$

Transforming, $\dfrac{p}{r+k} - e = w + n\dfrac{c}{r+k}$; but $w + n\dfrac{c}{r+k} = A$

therefore $A = \dfrac{p}{r+k} - e$

The last formula for tractive resistance on a level given in Art. 1 is—$T = 2.2t + 122n$ in which t corresponds to $(e+w)$. But since the formula applies only to level track, $r = 0$. Therefore the constant k, which depends on the weight of the train, =2.2 lbs per ton or .0011 lb per pound, and the constant c, which depends on the number (n) of cars, =122 lbs. The rating A for any grade is therefore some definite weight (depending on the grade), from which must be subtracted a constant for that grade times the number (n) of cars to obtain the actual weight (w) which may be hauled. The constant for each grade, $c/(r+k)$, is readily computed. For example, for a 0.6% grade, it equals $122/(.006+.0011) = 17\ 183$ lbs or 8.6 tons per car. Therefore on a 0.6% grade,

$A = w + 8.6n$, or $w = A - 8.6n$. Similarly, the constant with which to multiply may be determined for any grade, and, as this is independent of the type of locomotive, it is given in the following table:

Values of $c/(r+k)$ for Various Grades

Grade	Tons per car	Grade	Tons per car	Grade	Tons per car	Grade	Tons per car	Grade	Tons per car
Level	55	0.5%	10.	1.0%	5.5	1.5%	3.8	2.0%	2.88
0.1%	29	0.6	8.6	1.1	5.0	1.6	3.6	2.1	2.75
0.2	20	0.7	7.5	1.2	4.7	1.7	3.4	2.2	2.63
0.3	15	0.8	6.7	1.3	4.3	1.8	3.2	2.3	2.52
0.4	12	0.9	6.0	1.4	4.0	1.9	3.0	2.4	2.42

The rating of some particular engine for any and all grades may be determined by noting its hauling capacity on some one grade. For example, it hauls 50 cars, weighing 3033 tons, up a 0.3% grade at ordinary freight-train velocity and without acceleration. The adjustment for that grade (taken from the table) is 15 tons per car. Therefore the rating is $3033 + (15 \times 50) = 3783$ tons or 7 566 000 lb. The engine weighs 336 500 lb. Then $A + e = 7\,566\,000 + 336\,500 = 7\,902\,500 = p/(r+k)$. But $(r+k) = .003 + .0011 = .0041$. Therefore $p = 32\,400$ lb, the pulling power of the locomotive. Then, knowing p, the rating A for any grade may be determined by substituting in the formula various values for r. For example, the ruling grade of the road is 1.2%; it is desired to know the power of that locomotive on the ruling grade.

$$A = p/(r+k) - e = 32\,400/(.012 + .0011) - 336\,500 = 2\,473\,000$$
$$- 336\,500 = 2\,136\,500 \text{ lb} = 1068 \text{ tons.}$$

The actual tonnage of cars which may be hauled is less than this, since on this grade 4.7 tons must be subtracted for each car of the train. If there are 20 cars in the train, their aggregate weight must not exceed $1068 - (20 \times 4.7) = 974$ tons, or an average of 48.7 tons per car.

22. Cars

The Total Number of Cars in Service on the roads of the United States in 1916 was 2 478 159, of which 2.2% were in passenger service; 3.9% were in "the company's service," and the remaining 93.9% were in freight service. There were 8.7 freight cars per mile of line, but only 0.205 passenger cars per mile of line, or one passenger car for 4.9 miles of line. The percentages of the various classes of passenger cars were as follows: In 1915, Coaches, 54.0; passenger combination, 10.5; other combination, 5.3; emigrant, 0.1; dining, 2.5; parlor, 1.2; sleeping, 1.3; baggage and express, 17.8; postal, 2.8; and all others, 4.5. The above figures for the number of cars do not include 19 861 cars "in fast-freight-line service." A very large proportion of the sleeping cars in use are "leased." The cars in "the company's service" include chiefly cabooses and gravel cars, together with derrick cars, officers' and pay cars, and other road cars such as maintenance of way cars. Practically all cars are fitted with train brakes and automatic couplers, the number of those not so fitted being less than one percent.

The average capacity of all cars in 1916 was 40.5 tons. In 1911 it was 37 tons and in 1904 it was only 30 tons, and this illustrates the growth in average capacity from about 15 tons, which was once the standard.

The present U. S. Railroad Administration standards for freight cars provide for a 40-ton, steel underframe, double sheathed box car; 40- and 50-ton, steel frame single sheathed box cars; a 50-ton steel high side gondola car; a 50-ton composite high side gondola car; a 70-ton low side steel gondola car with drop ends; a 50-ton hopper car and a 70-ton hopper car. The length over striking plate is 42 ft 1½ in for the box cars, 42 ft 10½ in for the 50-ton gondolas, 48 ft 7 in for the 70-ton gondola and 31 ft 11 in and 40 ft 5 in for the 55- and 70-ton hoppers, respectively. Of the freight cars ordered in 1917, 40½% were all-steel, 20% steel frame, 26½% steel underframe, 5% steel center sill and only 8% all wood.

Usual Dimensions of Passenger Cars are: length, 50 to 80 ft; width 10 ft; height above the rail, 14 ft. The width and height above the rail are approximately constant, the variations being chiefly in the length. The weight varies from 25 to 70 tons. The all-steel passenger car weighs about 62 tons. The nominal capacity of an ordinary passenger coach varies from 50 to 80 passengers. At 125 lbs per passenger, the total live load will vary from 6250 lbs, to 10 000 lbs, which is only 8 or 10% of the dead load. On the other hand, a freight car with 100 000 lbs capacity will weigh about 34 000 lbs, and the live load is therefore nearly three times the dead load. The proportion of paying load to dead load is therefore many times greater in freight service than in passenger service which is one reason for the disparity in rates. Of the 2000 passenger cars ordered in 1917, 1874 were all-steel, 93 were steel underframe and only 33 wood. All-steel cars weigh from 2 to 4 tons more than wooden cars of the same dimensions but their life is greater and they are superior from the standpoints of safety and sanitation.

Car Maintenance. The relative economy of steel and wooden cars is indicated from a report made on the question of retiring from service 4600 wooden coal cars with a capacity of from 40 000 to 60 000 lbs and with ages varying from 9 to 23 years. It was shown that the annual cost of repairs per car was $95.98 or 37.8% of their present value, while 3000 steel cars would have 20% greater capacity than 4600 wooden cars. The amount actually spent on the maintenance of the wooden cars would pay 6% on the cost of the steel cars and leave an annual surplus of $215 000. Experience on the Harriman lines has shown that the average cost of repairs on steel and wooden cars was in the ratio of 100 to 161, respectively. The average annual cost of repairs on the Southern Pacific Railroad between the years 1902 and 1907 was $3165 per locomotive, $759 per passenger car, and $70 per freight car.

23. Stations.

Essential Requirements of a railroad station, given approximately in the order of their importance, are (1) platform; (2) shelter, developing from shed to waiting room; (3) station agent's office; (4) toilet facilities, developing from a mere privy to modern toilet room; (5) separate waiting room for ladies; (6) baggage and express room. In addition there is the legal requirement in many states that separate waiting rooms shall be provided for colored people. The policy of adding a freight room and agent's living quarters to the station building is debatable. For small stations there are advantages in the method.

The Cost of stations may, when necessary or advisable, be reduced to a very few hundred dollars, which will build a "shelter." As facilities and size are added the cost increases indefinitely. The reader is referred to Orrock's "Railroad Structures and Estimates," Berg's "Buildings and Structures of American Railroads," and similar works for detailed plans, accompanied by estimates of cost, of various types of structures. For use in preliminary

estimates the following approximate costs of wooden frame structures, compiled from Orrock's "Railroad Structures," will be found useful:

(1) Shelter: platform. 50 × 6 ft; house 22 × 12 ft.................. $125 to $200
(2) Station: platform, 250 × 8 ft; waiting room, 10 × 20 ft; office, 10 × 10 ft; baggage and express room, 10 ft × 10 ft 6 in...... $1000 to $1500
(3) Station: same as (2) but with four rooms in second story for agent's dwelling.................................... $1500 to $2000
(4) Station: same as (2) but with freight room 16 × 20 ft............ $1400 to $1800
(5) Station: waiting room, 16 × 16 ft; ladies' waiting room, 10 × 20 ft; office. 12 × 10 ft; baggage and express room, 16 × 16 ft; corridor between ladies' room and waiting room and two lavatories; platform 8 × 300 ft broadened and surrounding the station except at rear.................................... $2000 to $[illegible]
(6) Station: similar to (5) but with four rooms in second story for agent's dwelling.................................... $2500 to $3500

The following unit costs per sq ft of ground covered are taken from Gillette's "Hand Book of Cost Data" as being applicable to station buildings in the Mississippi Valley region. The unit costs are somewhat lower than Orrock's estimates. These figures do not include the cost of platform.

	Per sq ft
Station, frame, with living rooms, on piles	$1.30
Station, frame, with living rooms, stone foundation	1.50
Station, passenger and freight, frame, on piles	1.15
Station, passenger and freight, brick	1.80
Station, modern passenger, brick and stone, slate roof, hardwood finish	3.50

Another method is to allow from 5 to 7 cents per cu ft for the cost of a frame structure with shingle roof, and from 7 to 9 cents per cu ft for a brick building. Add ¼ cent per cu ft for slate or metal roof rather than shingle. The higher figures apply to smaller buildings and the figures only apply to plain, unpretentious buildings.

24. Engine Houses and Shops

(Condensed from recommendations in Manual of Am. Rwy. Eng. Assoc.)

The Circular Form is preferable tho a rectangular house may be desirable when not more than three or four engines are to be housed, or when it is more economical to provide a Y than a turntable, or where engines need not be turned, or at shops where a transfer table is available which may serve engine house also.

A Round House implies a turntable located at its center. The turntable, preferably of the deck type, should be long enough to balance the engine when tender is empty and should be operated by power, preferably electric except where only a few light engines are turned. The turntable pit should be well drained and paved, with side walls of brick or concrete. Pivot masonry of concrete with stone cap. Ties under the circular rail should be supported on concrete walls. The distance from center of turntable to inner line of roundhouse is determined by number of stalls required in a full circle. The angle between consecutive stall tracks should be an even divisor of 180° so that tracks at opposite ends of the turntable will simultaneously line up with it. The clear stall length should not be less than 15 ft greater than the over-all length of the locomotive. The INTERIOR ARRANGEMENT is designed on the basis that the locomotive is always run in with the tender toward the turntable. The entrance doorways should be 13 ft by 16 ft clear. The doors should be made of non-corrosive material, fit snugly, operate easily and admit of the use of small doors. The walls and roof should be made of non-corrosive material unless protected against corrosion. Security against interruption to traffic from fire warrants the serious consideration of fire-proof construction. Engine pits should be not less than 60 ft long, with convex floor and drainage toward the turntable. Walls and floors may be of concrete. Supports for jacking

timbers should be provided. Smoke jacks, not less than 7 sq. ft in area should be fixt, should have dampers and large hoods and be made of non-corrosive and non-combustible material. The bottom of the jack should be as low as the height of the, locomotive stacks will permit, at least 42 in wide and long enough to receive smoke from stack at its limiting positions, due to adjustments of the driving wheels to bring the side rods at proper position for repairs. The floor should be permanent and be crowned between pits. Drop pits should be provided for truck wheels, driving wheels and tender wheels. When hot-air heating is used, the air should be led thru ducts under the floor to the pits under the engine portion of the locomotive. Ducts should be provided with dampers so that air can be turned off when workmen are in the pit. A general temperature of 50° to 60° is recommended. Air should be heated by exhaust steam in so far as possible. The supply should be taken from the external air and no recirculation allowed. Light should be obtained from the exterior as far as possible and from windows rather than skylights. General illumination, avoiding shadows, should be obtained by using a number of lights between stalls and a plug for incandescent lights should be placed in each alternate space between stalls. Provision should be made for a few necessary machine tools, preferably electrically driven. Piping for air, steam and water supply should be provided.

STEAM RAILROAD OPERATION

25. Statistics of U. S. Railroads for years ending June 30, 1906 and 1916

	1906	1916
Number of miles of road in operation	230 761	266 031
Average construction for previous 10 years, miles	4 408	3 527
Number of miles of line per 100 sq mi of territory	7.55	8.55
Number of miles of line per 10 000 inhabitants	26.78	25.03
Average gross earnings per mile of line	$11900	$14214
Average gross earnings per inhabitant	$30.17	$36.04
Number of passengers reported as carried, (millions)	798	1 005
Gross pass. mileage (number carried one mile) "	25 167	34 214
Number of tons reported as carried "	1 631	2 226
Total ton mileage "	215 877	343 100
Average number of passengers in train	49	55
Average journey per passenger, miles	31.54	34.02
Average number of tons in train	344	535
Average revenue per passenger per mile, cents	2.003	2.006
Average revenue per ton of freight per mile, cent	.748	.716
Average revenue per passenger train-mile	$1.20	$1.39
Average revenue per freight train-mile	$2.61	$3.83
Average revenue per train-mile, all trains	$2.08	$2.80
Average cost of running a train one mile, all trains.	$1.37	$1.83

Average Cost for U. S. of Operating a Train One Mile

Year	Cents	Year	Cents	Year	Cents	Year	Cents
1890	96.006	1897	92.918	1904	131.375	1911	154.338
1891	95.707	1898	95.635	1905	132.140	1912	159.077
1892	96.580	1899	98.390	1906	137.060	1913	170.375
1893	97.272	1900	107.288	1907	146.993	1914	176.917
1894	93.478	1901	112.282	1908	147 340	1915	177.641
1895	91.829	1902	117.960	1909	143.370	1916	183.279
1896	93.838	1903	126.604	1910	148.865		

26. Cost of Operation

The Average Cost of a Train Mile for the whole United States in the year 1890 was 96.0 cents. It decreased to 91.8 cents in 1895, and since that time there has been an almost steady increase up to $1.83 in 1916. Altho the variations from these figures are large in many cases for individual roads, the average for individual trunk lines is seldom very different from the general average. While the averages for the individual short lines with light traffic will vary considerably from the general average, the average for any group of short lines is usually but little different from the general average, which means that there is no general tendency for the average figure for a short light-traffic line to be either less or greater than the general average.

For the purpose of analyzing costs of operation, etc., the Interstate Commerce Commission has divided the railroads into three classes on the basis of the gross annual operating revenue: Class I, over $1 000 000; Class II, $1 000-000 to $100 000; Class III, less than $100 000. Nearly 90% of the mileage comes under Class I and detailed data for operating expenses of that Class are shown in the following table, which gives the classification of accounts required by the Interstate Commerce Commission together with the total amount of each account, its percentage of the total operating expenses and the cost per mile of line and per train mile, respectively, for the year ending June 30, 1915.

Operating Expenses of Class I Railroads, 228 433.59 Miles. Year Ending June 30, 1915

Account No.	Account Name	Total Dollars	Average per Mile Dollars	Per cent of Total Op. Exp.	Cost per Train Mile Cents
	I. Maintenance of Way and Structures:				
201	Superintendence	21 066 188	92	1.042	1.85
202	Roadway Maintenance	36 171 765	158	1.790	3.18
203	Roadway Depreciation	345 331	2	0.017	0.03
204	Underground Power Tubes	....	...	.....	....
205	Underground Power Tubes—Dep	....	...	.....	....
206	Tunnels and Subways	1 022 320	5	0.051	0.09
207	Tunnels and Subways—Dep	44 273	(1)	0.002	(3)
208	Bridges, Trestles and Culverts	28 181 309	123	1.394	2.48
209	Bridges, Trestles and Culverts—Dep	244 817	1	0.012	0.02
210	Elevated Structures	36 323	(1)	0.002	(3)
211	Elevated Structures—Dep	....	...	.....	....
212	Ties	65 166 274	285	3.222	5.62
213	Ties—Dep	1 083 882	5	0.054	0.09
214	Rails	16 183 523	71	0.801	1.40
215	Rails—Dep	700 930	3	0.035	0.06
216	Other Track Material	16 155 789	71	0.799	1.39
217	Other Track Material—Dep	362 257	2	0.018	0.03
218	Ballast	6 278 528	28	0.311	0.54
219	Ballast—Dep	179 992	1	0.009	0.02
220	Track Laying and Surfacing	93 454 569	409	4.624	8.06
221	Right-of-Way Fences	3 049 668	13	0.151	0.26
222	Right-of-Way Fences—Dep	1 096	(1)	(2)	(3)
223	Snow and Sand Fences and Snowsheds	311 643	1	0.015	0.03

Operating Expenses of Class I Railroads—*Continued*

Account No.	Account Name	Total Dollars	Average per Mile Dollars	Per-cent of Total Op. Exp.	Cost per Train Mile Cents
224	Snow and Sand Fences and Snowsheds—Dep	50	(1)	(2)	(3)
225	Crossings and Signs	4 998 961	22	0.247	0.43
226	Crossings and Signs—Dep	1 239	(1)	(2)	(3)
227	Station and Office Buildings	13 755 826	61	0.681	1.18
228	Station and Office Buildings—Dep	64 876	(1)	0.003	0.01
229	Roadway Buildings	1 409 170	6	0.070	0.12
230	Roadway Buildings—Dep	11 252	(1)	0.001	(3)
231	Water Stations	4 523 730	20	0.224	0.39
232	Water Stations—Dep	27 603	(1)	0.001	(3)
233	Fuel Stations	1 844 994	8	0.091	0.16
234	Fuel Stations—Dep	5 760	(1)	(2)	(3)
235	Shops and Enginehouses	7 947 506	35	0.393	0.68
236	Shops and Enginehouses—Dep	63 928	(1)	0.003	0.01
237	Grain Elevators	201 063	1	0.010	0.02
238	Grain Elevators—Dep	2 000	(1)	(2)	(3)
239	Storage Warehouses	40 850	(1)	0.002	(3)
240	Storage Warehouses—Dep	....	...	.....	....
241	Wharves and Docks	1 979 871	9	0.098	0.17
242	Wharves and Docks—Dep	19 224	(1)	0.001	(3)
243	Coal and Ore Wharves	1 348 419	6	0.067	0.12
244	Coal and Ore Wharves—Dep	456 356	2	0.023	0.04
245	Gas Producing Plants	28 659	(1)	0.001	(3)
246	Gas Producing Plants—Dep	....	....	.....	....
247	Telegraph and Telephone Lines	3 963 875	17	0.196	0.34
248	Telegraph and Telephone Lines—Dep	852	(1)	(2)	(3)
249	Signals and Interlockers	9 906 160	43	0.490	0.85
250	Signals and Interlockers—Dep	11 334	(1)	(2)	(3)
251	Power Plant Dams, Canals and Pipe Lines	3 312	(1)	(2)	(3)
252	Power Plant Dams, Canals and Pipe Lines—Dep	....	....	....	....
253	Power Plant Buildings	149 584	1	0.007	0.01
254	Power Plant Buildings—Dep	36 166	(1)	0.002	(3)
255	Power Substation Buildings	8 190	(1)	(2)	(3)
256	Power Substation Buildings—Dep	19 871	(1)	(2)	(3)
257	Power Transmission Systems	33 844	(1)	0.002	(3)
258	Power Transmission Systems—Dep	20 206	(1)	0.001	(3)
259	Power Distribution Systems	719 357	3	0.036	0.06
260	Power Distribution Systems—Dep	96 994	(1)	0.005	0.01
261	Power Line Poles and Fixtures	68 615	(1)	0.003	0.01
262	Power Line Poles and Fixtures—Dep	5 514	(1)	(2)	(3)
263	Underground Conduits	6 544	(1)	(2)	(3)
264	Underground Conduits—Dep	4 825	(1)	(2)	(3)
265	Miscellaneous Structures	280 643	1	0.014	0.02
266	Miscellaneous Structures—Dep	8 239	(1)	(2)	(3)
267	Paving	207 505	1	0.010	0.02
268	Paving—Depreciation	....	...	.....	....
269	Roadway Machines	1 340 554	6	0.066	0.12
270	Roadway Machines—Dep	769	(1)	(2)	(3)
271	Small Tools and Supplies	3 376 160	15	.167	0.29

Operating Expenses of Class I Railroads—*Continued*

Account No.	Account Name	Total Dollars	Average per Mile Dollars	Per cent of Total Op. Exp.	Cost per Train Mile Cents
272	Removing Snow, Ice and Sand	4 388 787	19	0.217	0.38
273	Assessments for Public Improvements	422 014	2	0.021	0.04
274	Injuries to Persons	2 741 076	12	0.136	0.24
275	Insurance	2 424 292	11	0.120	0.21
276	Stationery and Printing	617 069	3	0.031	0.05
277	Other Expenses	214 986	1	0.011	0.02
	Sum of Foregoing Items	359 799 151	1 575	17.802	31.01
278	Maintaining Joint Tracks, Yards, and Other Facilities—Dr	14 552 809	63	0.720	1.25
279	Maintaining Joint Tracks, Yards, and Other Facitities—Cr	10 347 782	45	0.512	0.89
	Total Maintenance of Way and Structures	364 004 178	1 593	18.010	31.37
	II. Maintenance of Equipment:				
301	Superintendence	15 737 689	69	0.779	1.36
302	Shop Machinery	8 916 891	39	0.441	0.77
303	Shop Machinery—Dep	28 476	(1)	0.001	(3)
304	Power Plant Machinery	1 155 671	5	0.057	0.10
305	Power Plant Machinery—Dep	218 729	1	0.011	0.02
306	Power Substation Apparatus	30 500	(1)	0.002	(3)
307	Power Substation Apparatus—Dep	72 594	(1)	0.004	1.00
308	Steam Locomotives—Repairs	158 343 388	693	7.834	13.65
309	Steam Locomotives—Dep	21 515 447	94	1.064	1.85
310	Steam Locomotives—Retirements	3 432 239	15	0.170	0.30
311	Other Locomotives—Repairs	649 996	3	0.032	0.06
312	Other Locomotives—Dep	246 736	1	0.012	0.02
313	Other Locomotives—Retirements	21 512	(1)	0.001	(3)
314	Freight Train Cars—Repairs	162 841 807	713	8.057	14.4
315	Freight Train Cars—Dep	46 974 928	206	2.324	4.04
316	Freight Train Cars—Retirements	10 140 629	45	0.502	0.87
317	Passenger Train Cars—Repairs	32 497 659	142	1.608	2.80
318	Passenger Train Cars—Dep	8 476 875	37	0.419	0.73
319	Passenger Train Cars—Retirements	819 027	4	0.041	0.07
320	Motor Equipment of Cars—Repairs	637 953	3	0.032	0.06
321	Motor Equipment of Cars—Dep	231 651	1	0.011	0.02
322	Motor Equipment of Cars—Retirements	3 350	(1)	(2)	(3)
323	Floating Equipment—Repairs	4 831 485	21	0.239	0.42
324	Floating Equipment—Dep	1 835 587	8	0.091	0.16
325	Floating Equipment—Retirements	93 693	(1)	0.005	0.01
326	Work Equipment—Repairs	6 001 000	26	0.297	0.52
327	Work Equipment—Dep	2 029 358	9	0.100	0.18
328	Work Equipment—Retirements	1 067 414	5	0.053	0.09
329	Miscellaneous Equipment—Repairs	31 848	(1)	0.002	(3)
330	Miscellaneous Equipment—Dep	3 996	(1)	(2)	(3)
331	Miscellaneous Equipment—Retirements	4 483	(1)	(2)	(3)
332	Injuries to Persons	2 157 328	10	0.107	0.19
333	Insurance	3 563 660	16	0.176	0.31
334	Stationery and Printing	954 048	4	0.047	0.08

Operating Expenses of Class I Railroads—*Continued*

Account No.	Account Name	Total Dollars	Average per Mile Dollars	Per cent of Total Op. Exp.	Cost per Train Mile Cents
335	Other Expenses	288 784	1	0.014	0.02
	Sum of Foregoing Items	495 856 431	2 171	24.533	42.75
336	Maintaining Joint Equipment at Terminals—Dr	1 614 427	7	0.080	0.14
337	Maintaining Joint Equipment at Terminals—Cr	731 297	3	0.036	0.06
	Total Maintenance of Equipment	496 739 561	2 175	24.577	42.82
	III. Traffic:				
351	Superintendence	16 359 473	72	0.810	1.41
352	Outside Agencies	23 757 340	104	1.176	2.06
353	Advertising	7 141 908	31	0.353	0.62
354	Traffic Associations	1 377 560	6	0.068	0.12
355	Fast Freight Lines	2 606 607	12	0.129	0.22
356	Industrial and Immigration Bureaus	1 399 376	6	0.069	0.12
357	Insurance	23 695	(1)	0.001	(3)
358	Stationery and Printing	6 649 850	29	0.329	0.57
359	Other Expenses	87 610	(1)	0.004	0.01
	Total Traffic Expenses	59 403 419	260	2.939	5.12
	IV. Transportation—Rail Line:				
371	Superintendence	26 073 727	114	1.290	2.25
372	Despatching Trains	17 155 505	75	0.849	1.48
373	Station Employees	144 222 128	631	7.136	12.43
374	Weighing, Inspection and Demurrage Bureaus	2 556 258	11	0.126	0.22
375	Coal and Ore Wharves	4 504 664	20	0.223	0.39
376	Station Supplies and Expenses	12 708 104	56	0.629	1.10
377	Yardmasters and Yard Clerks	17 818 063	78	0.881	1.54
378	Yard Conductors and Brakemen	54 076 853	237	2.676	4.66
379	Yard Switch and Signal Tenders	4 640 978	20	0.230	0.40
380	Yard Enginemen	31 436 725	138	1.555	2.71
381	Yard Motormen	246 903	1	0.012	0.02
382	Fuel for Yard Locomotives	29 864 063	131	1.477	2.57
383	Yard—Switching Power Produced	116 037	1	0.006	0.01
384	Yard—Switching Power Purchased	5 899	(1)	(2)	(3)
385	Water for Yard Locomotives	2 183 381	10	0.108	0.19
386	Lubricants for Yard Locomotives	551 312	2	0.027	0.05
387	Other Supplies for Yard Locomotives	638 383	3	0.032	0.06
388	Enginehouse Expenses—Yard	10 560 811	46	0.523	0.91
389	Yard Supplies and Expenses	1 696 888	7	0.084	0.15
392	Train Enginemen	115 089 314	504	5.694	9.92
393	Train Motormen	1 196 546	5	0.059	0.10
394	Fuel for Train Locomotives	179 104 928	784	8.861	15.42
395	Train Power Produced	1 207 687	5	0.060	0.10
396	Train Power Purchased	1 187 834	5	0.059	0.10
397	Water for Train Locomotives	12 202 039	53	0.064	1.05

Operating Expenses of Class I Railroads—*Continued*

Account No.	Account Name	Total Dollars	Average per Mile Dollars	Per cent of Total Op. Exp.	Cost per Train Mile Cents
398	Lubricants for Train Locomotives	3 329 859	15	0.165	0.29
399	Other Supplies for Train Locomotives	3 280 789	14	0.162	0.28
400	Enginehouse Expenses—Train	34 457 002	151	1.705	2.97
401	Trainmen	127 347 403	558	6.301	10.98
402	Train Supplies and Expenses	37 625 766	165	1.862	3.24
403	Operating Sleeping Cars	631 176	3	0.031	0.05
404	Signal and Interlocker Operation	11 185 653	49	0.553	0.96
405	Crossing Protection	7 872 632	35	0.390	0.68
406	Drawbridge Protection	1 026 900	4	0.051	0.09
407	Telegraph and Telephone Operation	5 178 593	23	0.256	0.45
408	Operating Floating Equipment	10 936 534	48	0.541	0.94
409	Express Service	884	(1)	(2)	(3)
410	Stationery and Printing	7 080 487	35	0.395	0.69
411	Other Expenses	2 366 927	10	0.117	0.20
414	Insurance	3 644 056	16	0.180	0.31
415	Clearing Wrecks	3 905 776	17	0.193	0.34
416	Damage to Property	3 939 109	17	0.195	0.34
417	Damage to Live Stock on Right-of-Way	3 848 596	17	0.190	0.33
418	Loss and Damage—Freight	29 528 016	129	1.461	2.54
419	Loss and Damage—Baggage	216 935	1	0.011	0.02
420	Injuries to Persons	21 527 480	94	1.065	1 85
	Sum of the Foregoing Items	990 875 603	4338	49.025	85.40
390	Operating Joint Yards and Terminals—Dr.	24 703 671	108	1.222	2.13
391	Operating Joint Yards and Terminals—Cr.	13 596 249	60	0.673	1.15
412	Operating Joint Tracks and Facilities—Dr.	6 056 190	27	0 300	0.52
413	Operating Joint Track and Facilities—Cr.	5 298 481	23	0.262	0.46
	Total Transportation—Rail Line	1 002 740 734	4390	49.612	86.42
	V. Transportation—Water Line:				
431	Operation of Vessels	4 814 121	21	0.238	0.42
432	Operation of Terminals	3 025 449	13	0.149	0.26
433	Incidentals	334 423	2	0.017	0.03
	Total Transportation—Water Line	8 173 993	36	0.404	0.71
	VI. Miscellaneous Operations:				
441	Dining and Buffet Service	14 820 918	65	0.733	1.28
442	Hotels and Restaurants	4 540 933	20	0.225	0.39
443	Grain Elevators	644 472	3	0.032	0 06
444	Stock Yards	723 597	3	0.036	0.06
445	Producing Power Sold	926 901	4	0.046	0 08
446	Other Miscellaneous Operations	1 214 964	5	0.060	0.10
	Total Miscellaneous Operations	22 871 785	100	1.132	1.97
	VII. General:				
451	Salaries and Expenses of General Officers	10 192 445	44	0.504	0.87
452	Salaries of Clerks and Attendants	33 666 372	147	1.666	2.90

Operating Expenses of Class I Railroads—*Concluded*

No.	Account Name	Total Dollars	Average per Mile Dollars	Per cent of Total Op. Exp.	Cost per Train Mile Cents
453	General Office Supplies and Expenses	3 437 457	15	0.170	0.30
454	Law Expenses	12 017 288	52	0.594	1.04
455	Insurance	117 916	1	0.006	0.01
456	Relief Department Expenses	880 597	4	0.044	0.08
457	Pensions	4 272 033	19	0.211	0.37
458	Stationery and Printing	2 907 504	13	0.144	0.25
459	Valuation Expenses	2 933 949	13	0.245	0.25
460	Other Expenses	3 126 557	14	0.155	0.27
	Sum of the Foregoing Items	73 552 118	322	3.639	6.34
461	General Joint Facilities—Dr.	889 359	4	0.044	0.08
462	General Joint Facilities—Cr.	269 370	1	0.013	0.02
	Total General Expenses	74 172 107	325	3.670	6.39
	VIII. Transportation for Investment—Cr.	6 945 163	31	0.344	0.60
	Grand Total—Operating Expenses	2 021 160 614	8848	100.00	174.20

(1), (2) and (3) = less than $1, 0.001% and 0.01c., respectively.

Comparison of Classes I, II, and III Railroads. Year ending June 30, 1915

	Class I	Class II	Class III	Total
Miles of Road*	228 433.59	19 312.05	8 467.97	256 213.61
Percent of Total	89.2	7.5	3.3	100
Miles of Track	357 519.66	23 748.16	9 873 69	391 141.51
Percent of Total	91.4	6.1	2.5	100
Capital	$16 024 548 404	$738 801 963	$173 673 196	$16 937 023 563
Capital per mile	$70 150	$38 250	$20 500	$66 150
Gross Income	$2 977 540 809	$72 206 348	$14 762 952	$3 064 510 109
Gross Income per Mile	$13 035	$3 739	$1 743	$11 961
Operating Expenses	$2 021 160 614	$54 420 217	$13 102 125	$2 088 682 956
Operating Expenses per mile	$8.848	$2.818	$1.545	$8 152
Train Miles	1 160 237 137	38 483 667	9 532 142	1 208 252 941
Operating Expenses per Train Mile	$1.742	$1.412	$1.370	$1.729
Operating Ratio	70.385%	77.841%	89.020%	70.655%
Principal Items of Operating Expenses				
M. of W. & S. per Mile	$1 593	$702	$470	$1 489
Percent of Total Operating Expenses	18.01	24.90	30.40	18.27
M. of E.	$2 175	$572	$240	$1 990
Percent of Total Operating Expenses	24.58	20.30	15.55	24 41
Transportation Rail Line	$4 390	$1 258	$640	$4 030
Percent of Total Operating Expenses	49.61	44.62	41.42	49.43
General Expenses per Mile	$325	$189	$144	$309
Percent of Total Operating Expenses	3.67	6.70	9.35	3.78

* Submitting reports, practically entire mileage.
[illegible] expenses to operating income.

The operating expenses of Class II and Class III roads are successively simplified, there being only nine and seven accounts, respectively, under M. of W. & S. instead of seventy-nine as shown for Class I, and so on.

Maintenance of Way. Altho the basis of the classification of operating expenses has been so changed that an exact comparison on items for 1904 and 1914 is not easy, it is possible to make approximate comparisons which are instructive. From 1904 to 1914 the cost of maintenance of way per train mile increased about 15%. This is due very largely to the great increase in the wages paid to track labor, the increase being shown very clearly in the following figures:

Average Daily Wages of Trackmen, from 1904 to 1914

	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
Section foremen	1.78	1.79	1.80	1.90	1.95	1.96	1.99	2.07	2.09	2.14	2.20
Other trackmen	1.33	1.32	1.36	1.46	1.45	1.38	1.47	1.50	1.50	1.58	1.59
No. trackmen per 100 m.	136	143	155	162	130	136	157	147	138	145	128

The average number of section foremen per 100 miles of line has remained almost constant at 17.

Data for later years are not comparable on account of change in classification of employees from 18 classes to 68 classes, but data submitted to the Railroad Wage Commission indicated that the average pay of Section Foremen in 1917 was $73.89 per month and of Sectionmen, $50.31. These are increased by 41% and 42.35%, respectively, under the recommendations of the Railroad Wage Commission.

Maintenance of Equipment. The three great items constituting 90% of the cost of maintenance of equipment are the repairs, depreciation and retirements of locomotives, passenger cars and freight cars. The cost of repairs and renewals of locomotives increased from 10.2 c. per train mile in 1904 to 17.4 c. in 1914. This is largely compensated by the increased train loads which can be hauled by the heavier locomotives now in use. The average tractive force of locomotives was about 27 949 lbs in 1911 and increased to 32 100 in 1916, an increase of about 15% in five years. Altho the greater tractive force gives greater earning power, it increases the cost of engine repairs, depreciation and retirements per train mile. The cost of passenger car repairs and renewals remained practically constant at 5.8 c. per passenger train mile. The cost of repairs and renewals of freight cars increased from 19.4 c. per train freight mile in 1904 to 32.1 c. in 1914. This is partly compensated by an increase in average capacity, which increased from 30 tons in 1904 to 39.7 tons in 1914.

Conducting Transportation. The cost per train mile increased from 75.1 c. in 1904 to 86.5 c. in 1914. Again this increase was largely due to increase in wages, the increase being indicated as follows:

Average Daily Wages of Enginemen, Conduc ors, Stationmen, etc., 1904–14

	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
Enginemen	4.10	4.12	4.12	4.30	4.45	4.44	4.55	4.79	5.00	5.20	5.24
Firemen	2.35	2.38	2.42	2.54	2.64	2.67	2.74	2.94	3.02	3.13	3.32
Conductors	3.50	3.50	3.51	3.69	3.81	3.81	3.91	4.16	4.29	4.39	4.47
Other trainmen	2.27	2.31	2.35	2.54	2.60	2.59	2.69	2.88	2.96	3.04	3.09
Station agents	1.93	1.93	1.94	2.05	2.09	2.08	2.12	2.17	2.20	2.28	2.33
Other stationmen	1.69	1.71	1.69	1.78	1.82	1.82	1.84	1.89	1.89	1.96	1.98
Switch tenders, etc.	1.77	1.79	1.80	1.87	1.78	1.73	1.69	1.74	1.70	1.70	1.71

The following table gives the average monthly wage for 1917 of certain classes of employees as indicated by data submitted to the Railroad Wage Commission and the increase under the recommendations of that body:

Monthly Wages of Various Employees, 1917

Class	Dollars	Increase, Percent	New Wage, Dollars
Clerks. $900 per annum and upwards	102.08	30.39	134 30
Clerks, below $900 per annum	56.77	41.00	80 37
Assistant Engineers and Draftsmen	95.22	33.70	128.35
Machinists	116.35	24.96	146.20
Structural Ironworker	84.38	40.00	119 00
Mechanics Helpers and Apprentices	68.58	41 00	97 29
Train Despatchers and Directors	149.76	16.17	174.25
Road Freight Engineers and Motormen	175.64	11.56	196 35
Road Freight Conductors	154.56	15.16	178.50
Road Passenger Engineers and Motormen	185.93	10.14	204 85
Road Passenger Conductors	163.75	13.51	186.15
Road Freight Brakemen and Flagmen	100.17	31.29	132.60
Road Passenger Brakemen and Flagmen	91.10	35.82	124.95

Similar increases to all employees receiving less than $250 per month indicate an estimated increase in wages alone of nearly $600 000 000, or 37%, over the operating expenses for the year ending June 30, 1915, given above or nearly $300 000 000, or 15%, over those for the calendar year 1917, the other 15% having been already obtained from the railroads.

The cost of fuel and other engine supplies increased from 17.3 c. per train mile in 1904 to 19.9 c. per train mile in 1914. The item of train supplies and expenses increased from 1904 to 1914, not only in percentage (1.555% to 1.862%) but also in cents per train mile (2.06 c. to 3.24 c.).

27. Taxes on Steam Railroads per Mile of Line

Condensed from Interstate Commerce Commission Reports for 1907, 1911, and 1915

The table on p. 245 gives the total taxes paid per mile of line in the various States and Territories during the fiscal years ending June 30, 1907, 1911 and 1915. The figures are given for the three years, so as to indicate the increase or decrease and the rate. The table is useful to give some idea of the probable taxes on some existing road, or a new project, for a few years in the near future. Altho all of the increase in some States is undoubtedly due to a proportionate increase, thru betterments, in the real value of the roads, some of the increase is undoubtedly due to a revision in the method of assessment.

No figures are given to show the relation between the taxes per mile in any State and the capitalization (or any other measure of value) of the roads in that State. The total taxes paid during 1915 amounted to $134 895 161, which is 0.67% of the total capitalization, $20 162 122 323.

28. Earnings of Steam Railroads

The Interstate Commerce Commission have abandoned their former method of dividing the railroads of the country into ten territorial groups, and have

State	Taxes per mile			State	Taxes per mile		
	1907	1911	1915		1907	1911	1915
Alabama	218	317	342	Nevada	265	627	373
Arkansas	224	306	465	New Mexico	139	192	369
Arizona	142	209	532	New Hampshire	358	572	602
California	390	524	633	New Jersey	2047	2335	3110
Colorado	287	326	389	New York	686	898	1435
Connecticut	1339	1407	783	North Carolina	177	225	341
Delaware	391	388	467	North Dakota	265	317	371
Dist. Columbia	1480	1521	1816	Ohio	569	680	972
Florida	176	212	276	Oklahoma	114	460	464
Georgia	166	218	267	Oregon	228	360	538
Idaho	233	337	549	Pennsylvania	510	689	785
Illinois	472	501	691	Rhode Island	1100	1336	1888
Indiana	481	562	717	South Carolina	176	248	293
Iowa	243	269	364	South Dakota	101	213	312
Kansas	296	316	421	Tennessee	267	334	441
Kentucky	366	396	513	Texas	153	199	275
Louisiana	218	265	412	Utah	320	394	529
Maine	292	345	510	Vermont	172	272	519
Maryland	620	754	797	Virginia	376	488	609
Massachusetts	1525	1631	1260	Washington	415	568	847
Michigan	398	490	477	West Virginia	413	461	589
Minnesota	429	425	550	Wisconsin	414	456	671
Mississippi	214	252	465	Wyoming	141	395	405
Missouri	206	232	289				
Montana	271	385	468	United States	367	444	569
Nebraska	429	360	426				

substituted three new divisions, the Eastern, the Southern and the Western, whose limitations are shown in Fig. 46.

Fig. 46.—U. S. Divisions of Interstate Commerce Commission.

The operating revenues in these three districts for the year ending June 30, 1915, were as follows:

Source of revenue	Eastern District		Southern District		Western District		Whole U. S.	
	per mile of line	%	per mile of line	%	per mile of line	%	per mile of line	%
Freight revenue......	$14 949	69.04	$7 193	72.08	$6 329	68.57	$8 709	69.28
Passenger revenue....	4 819	22.27	2 068	20.73	2 163	23.45	2 832	22.54
Mail revenue	352	1.63	178	1.78	225	2.44	249	1.98
Express revenue......	525	2.43	245	2.45	217	2.35	302	2.40
Milk revenue........	182	0.84	11	0.11	22	0.24	61	0.48
Switching revenue....	244	1.13	89	0.89	90	0.97	130	1.03
Special Ser. Revenue..	10	0.05	6	0.06	10	0.10	9	0.08
Other revenue*.......	567	2.61	190	1.90	174	1.88	278	2.21
Totals............	21 648	100.00	9 980	100.00	9 231	100.00	12 570	100.00

* Includes water transfers—vehicle, live stock and other dining, hotel, and restaurant service, privileges, storage, demurrage, rentals and miscellaneous.

The large territory covered by each of the "Districts" tends to make the relative percentage for freight and passenger revenue nearly constant. Smaller areas would show marked differences, as, for instance, in the New England States the passenger revenue was about 35% and the freight revenue only 56% while in Ohio, Indiana and Michigan, the variation was the other way. Combining these in the "Eastern District" covers up the local variations.

Analysis of Earnings of Steam Railroads for Year Ending June 30, 1915

	Eastern District		Southern District		Western District		Av. U. S.
	Class I	Class II	Class I	Class II	Class I	Class II	
Av. rev. per pass. per mile......	$1 848	$1 736	$2 142	$2 694	$2 081	$2 770	$1 985
Av. rev. per pass. carried	0 484	0 222	0 792	0 464	1 042	0 478	0 658
Rev. per pass. train mile........	1 432	0 709	1 130	0 486	1 320	0 726	1 309
Pass. train earn. per mile of road.	5 844	1 304	2 501	625	2 621	769	3 229
Av. rev. per ton per mile.........	0 646	1 410	0 639	1 535	0 878	1 890	0 752
Av. rev. per ton carried.........	0 874	0 287	1 304	0 607	1 778	0 513	1 125
Rev. per freight train mile	3 736	3 183	2 867	2 142	3 533	2 771	3 470
Freight train earn. per mile of road	14 884	3 816	7 184	2 048	6 268	2 230	8 182
Op. rev. per train mile..........	2 708	1 824	2 167	1 486	2 531	1 978	2 519
Op. exp. per train mile	.962	1 454	1 584	1 154	1 698	1 519	1 776
Oo. rev. per mile of road.........	21 633	5 325	9 980	2 762	9 162	3 155	1 827
Op. exp. per mile of road.........	15 667	4 249	7 295	2 146	6 147	2 423	8 341
Op. ratio—Exp. to rev., %	72.42	79.78	73.09	77.68	67.09	76.79	70.52

The above table is especially instructive in illustrating the effect of magnitude of business (Class I or II) and of section of country, East, West or South.

The average revenue per passenger per mile and per ton per mile is always

higher on the weaker roads. They are compelled (and permitted) to make the higher charges in order to pay expenses. The unit charges are lower in the East, on account of the greater business, while the West and South are about equal.

The average per passenger carried and per ton carried is least in the East on account of the enormous volume of short distance travel and haulage, and more than double in the West, on account of the longer average distances between important traffic centers. The average journey per passenger is 26 miles in the East, 37 miles in the South, and 50 miles in the West. The weaker roads in each section obtain less per passenger in spite of the higher charges per mile.

The passenger train earnings and the freight train earnings per mile of road

Summary of U. S. Freight Traffic Movement for Year Ending June 30, 1916

Items	Tonnage (thousands)	Percentage of total
Grain	108 511	4.88
Fruits and vegetables	41 393	1.86
Flour; other mill prod	44 245	1.99
Hay	13 025	0.59
Cotton	8 795	0.40
Other products of agriculture	17 795	0.80
Total products of agriculture	233 764	10.52
Live stock	23 875	1.07
Dressed meats	6 230	0.29
Hides and leather	3 190	0.14
Other products of animals	17 835	0.80
Total products of animals	51 130	2.30
Anthracite coal	128 881	5.80
Bituminous coal	615 748	27.68
Other mine products	457 755	20.60
Total products of mines	1 202 384	54.08
Lumber	153 537	6.90
Other products of forests	48 207	2.16
Total products of forests	201 744	9.06
Iron, pig and bloom, and rails	45 803	2.06
Petroleum; other oils	33 603	1.51
Cement, brick and lime	77 461	3.48
Castings and machinery	32 426	1.46
Bar and sheet metal	43 396	1.95
Other manufactures	133 568	6.01
Total manufactures	366 257	16.48
Merchandise	76 820	3.45
Miscellaneous	91 182	4.11
Grand total *	2 223 281	100.00

* Excludes 2 662 206 tons not classified

are, of course, roughly proportional to the figures determining the classification. The South and West are about equal in the corresponding classes, while the East earns more than double. It is significant that, in spite of the lower charges per passenger mile and per ton mile on the Class I eastern roads, they are able to obtain a much greater revenue per train mile than the southern roads and somewhat more than the western roads Altho the operating expenses per train mile show a tendency to keep roughly proportional to the revenue, yet the weaker roads in each district have a higher ratio of operating expenses to revenue. In 1910 ten large roads showed operating expenses of $1.50 per train mile while ten small roads had $1.54, but the ratio of operating expenses to gross earnings was 0.672 for the large roads and 0.746 for the small ones. In the same year the roads of the whole United States gave $1.49 and 0.663 for these figures. Both the large and the small roads were higher than the average in ratio of expenses to earnings.

29. Accidents

During the year ending June 30, 1916, 9364 persons were killed, about two-thirds of whom were neither passengers nor employees but "others," mainly trespassers. Only 283 of the killed were passengers while 2 687 were employees, of whom 1 960 were killed in train service accidents, 145 in collisions, 133 in derailments and 35 in miscellaneous accidents. As 1 005 000 000 passengers were carried during that year, only one in about three and one-half million was killed; similarly, the total passenger mileage was 34 214 000 000, which means one death per 120 million passenger miles. Of course the chances of injury are much greater as indicated by the following statistics:

Railroad Accidents in the United States

Year	Passengers		Employees		Others		Total	
	Killed	Injured	Killed	Injured	Killed	Injured	Killed	Injured
1906	359	10 764	3 929	76 701	6 330	10 241	10 618	97 706
1907	610	13 041	4 534	87 644	6 695	10 331	11 839	111 016
1908	381	11 556	3 405	82 487	6 402	10 187	10 188	104 230
1909	253	10 311	2 610	75 006	5 859	10 309	8 722	95 626
1910	324	12 451	3 382	95 671	5 976	11 385	9 682	119 507
1911	356	13 433	3 602	126 039	6 438	10 687	10 396	150 159
1912	318	16 386	3 635	142 442	6 632	10 710	10 585	169 538
1913	403	16 539	3 175	171 417	6 846	12 352	10 964	200 308
1914	265	15 121	3 259	165 212	6 778	12 329	10 302	192 662
1915	222	12 110	2 152	138 092	6 247	11 838	8 621	162 040
1916	283	8 379	2 687	160 663	6 394	11 333	9 364	180 375

Accidents are more or less fortuitous and hence the data are somewhat erratic, but those for passengers and employees give some indication of results from the "safety first" movement, especially in view of the increase of over 20% in passenger train mileage and about 35% in freight car mileage during the decade covered by the above table.

30. Cost of Railroads

The following items of cost are inserted as a catalog of the expenses which are usually met with and also to give an approximate idea of average costs per mile of track, which

may be utilized in approximate preliminary calculations. Except where definite unit costs are given, the estimates are frequently subject to wide variations.

Right-of-way. Land for railroad purposes is valued at a higher rate than for farm purposes on account of consequential damages and injury to adjoining property. Even State Commissions for appraising railroads make an allowance. The Minnesota Commission generally allowed three times the ordinary farm value for farm lands. In cities from 1.25 to 1.75 times the ordinary market value was allowed. In Michigan from 2 to 2.25 times the ordinary market value is allowed.

Clearing. Usually $25 to $100 per acre.

Grubbing. Usually $100 to $200 per acre where required.

Earthwork. When the grading is light, fills are frequently made from borrow pits and the material in cuts is wasted, and then the price per cu yd applies to the sum of the yardage of cut and fill plus shrinkage. This price is usually 20 to 35 cents per cu yd for earth; about 60 to 80 cents for loose rock and 90 to $1.50 for solid rock. When earthwork is heavy, care is taken to make the fills from the material in the cuts. The disposition of material is directed by the railroad engineer, and the contractor is paid according to the amount excavated. Borrowing is reduced to a minimum, and the contractor is often allowed extra for overhaul, usually one cent per cu. yd per 100 ft, for material hauled in excess of some limit, say 500 to 1000 feet. When only the yardage excavated is paid for and the contractor must make the embankments without extra pay, the price per yard for earth is nearly double the price given above unless machine methods are used when the cost may be about the same as above or even less. Much depends on haul and local conditions. The prices for loose rock and solid rock will be but little more than the values given above, since the extra work is chiefly due to the loosening. Work on and near cities and on lines under traffic will usually cost more. The grading on all the railroads of Wisconsin was estimated to average $5098 per mile of main line; those in Michigan, $2778 per mile; those in Minnesota, $7372. The variations in this item are evidently very large.

Tunnels. This item is so exceedingly variable that general figures are worthless except those of unit cost. The average of a large number of cases is shown in the following tabular form, condensed from Drinker's Tunneling:

Material	Cost per cubic yard				Cost per lineal foot	
	Excavation		Masonry			
	Single	Double	Single	Double	Single	Double
Hard rock.....	$5.89	$5.45	$12.00	$8.25	$69.76	$142.82
Loose rock.....	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground ...	3.62	4.64	15.06	10.50	135.31	174.42

Bridges, Trestles and Culverts. The price per mile of road is evidently very variable. The average for the railroads of Minnesota was computed at $2576 per mile; those of Michigan, $1027 per mile. Trestle timber in place is worth approximately $55 per M feet; trestle piling in place, 60 cents per lineal foot of pile; pile trestling from $9.00 to $15.00 per foot of trestle; vitrified culvert pipe in place, $1.50 per lin ft for 18 in, $3.00 per lin ft for 24 in; cast-iron pipe culverts in place about 5 cents per lb.

Rails. Tons (2240 lbs) per mile of track equals $^{11}/_{7}$ weight of rail per yard in lbs. Allow 2% for cutting and waste. The freight from mill to delivery point is frequently a considerable gross item. Normal prices for rails are $55 and $57 per ton for Bessemer and open hearth, respectively. Prices of RAIL FASTENINGS at Pittsburgh have been fixed by the government as follows: steel splice bars 3. 25 cents per lb; spikes 3.9 cents per lb; track bolts 4.9 cents per lb.

Cross Ties. 35 cents to $1.10 per tie; common average 70 cents.

Frogs, Switches and Crossings. Switches in place will cost from $200 to $450 per switch. The cost of a crossing of two railroads will depend on the angle of intersection, and will cost from $150 up.

Ballast. Quantities and unit costs are given under the heading of BALLASTING under TRACK. The cost may run from $1000 to $5000 per mile for the best of broken stone. An ordinary average is $1250 per mile of track.

Track Laying and Surfacing. This really includes the three items of track laying, surfacing and the train service of hauling track materials from the point of delivery to the places where used. The train service is frequently furnished by the railroad company, but its actual cost is from $75 to $150 per mile. The contract price for track laying alone is frequently $350 per mile, but there are many records of track laying by means of special track-laying cars for as little as $150 per mile. Sometimes ballasting is deferred until the road is in operation, at least for construction trains. The surfacing may then be a separate contract at about $300 per mile. Allowing an average figure of $125 per mile for the train service, the total cost to the company for this item will average about $775 per mile.

Fencing will average about $175 per mile of fence, or about $350 per mile of road when both sides are completely fenced. Often only a small fraction of the total length is fenced.

Buildings and Miscellaneous Structures. Station buildings and fixtures in Wisconsin averaged $476 per mile; those in Minnesota, $771; those in Michigan, $526. Unit allowance in valuation of a Texas railroad for small frame passenger stations, $1 per square foot; for platforms, 16 cents per square foot. The analyzed cost of six section and tool houses averaged 30.7 cents per sq ft of area. The cost of water stations of ordinary capacity will vary from 2 to 3 cents per gallon of capacity. Sign boards, $7 to $9 each. Whistle posts, mile posts and rail rests, about $1 each. Road crossings require about 260 ft B.M., which at an average price in place of $40 per M will cost $10.40 each. A pair of stock guards is usually required for each crossing, which may cost about $50 per pair, including the short fences from the right-of-way lines to the ends of the ties. A Y can take the place of a turntable providing that land is available at no special cost. Using a radius of 300 ft, with 100 ft of track for the engine at the tail of the Y, about 1050 ft of track will be required together with three switches. The tail of the Y will require land for 400 ft from the main track.

Turntables may cost anywhere from $1000 to $2500. The turntable itself is made of cast iron or of structural steel, or sometimes of a combination of structural steel trusses with a cast-iron center.

Coaling Stations may vary from a mere platform or bunker from which the coal is shoveled into the tender (altho at a considerable cost per ton) to the very elaborate and costly coal pockets in which coal is deposited by coal conveyors or dumped from cars drawn up an incline, and from which the coal slides thru chutes to the tender. The cost must evidently be computed for

individual cases. Ash Pits likewise vary from a mere pit between the rails to an elaborate adjunct of a coaling station by which the ashes are immediately transported on a "conveyor" to an ash car.

Terminal Grounds. Altho the area of terminal grounds may be not more than 3 or 4% of the total property area, its gross cost, or its cost per mile of road, may be actually more than the cost of all the remaining right-of-way. In Minnesota, the valuation of terminal grounds was 71% of the total valuation of all the line right-of-way, gravel pits, station grounds and terminal grounds.

Miscellaneous: Shops, Roundhouses and Shop Tools and Machinery will average about 2% of the total cost of a road. Snow fences, Bridge Ticklers, Track Scales, Mail Cranes and Bumping Posts must be allowed for on all large roads, but their relative cost is insignificant.

Grain Elevators, Warehouses, Docks and Wharves are mentioned as possible items of expense, but do not apply to many roads. For Signals the cost is so variable, depending on the degree of elaboration of the system, that average unit prices per mile of road would only be misleading.

Telegraph Line. Safe estimate $250 per mile of road for a single-wire line. If poles are very cheap even this cost may be materially cut.

Freight on Construction Material. This applies chiefly to track material such as cross ties, rails, etc. While very variable, it may amount to nearly 1% of the cost of the road. Frequently the item is ignored, the freight being added to the cost of each item.

Contingencies. Usually estimated at 5 to 10% of the above items.

Engineering, Superintendence and Legal. Frequently figured at 5% of all the construction work. The legal work will cost about 1%.

Equipment. The approximate cost of locomotives and cars is about $5000 per mile of road. Marine equipment must occasionally be allowed for.

Item	Cost	Per-cent	Item	Cost	Per-cent
Engineering	$1 319	2.97	Roundhouse and shops	$ 328	0.74
Right-of-way	4 056	9.13	Fuel and water stations	258	0.58
Real estate	230	0.52	Docks, wharves, inclines	44	0.10
Clearing and grubbing	1 098	2.47	Columbia river incline	122	0.27
Grading	11 343	25.54	Other buildings and structures	25	0.06
Tunnels	5 624	12.66	Fences	22	0.05
Masonry	942	2.12	Telegraph	47	0.11
Cribbing and bulkheading	714	1.61	Shop tools and machinery	96	0.22
Bridges and culverts	4 318	9.72	Protection against snow and ice	158	0.36
Cattle guards, road crossings and signs	234	0.53	Locomotive and car service	86	0.19
Ties	1 198	2.70	General expense	108	0.24
Rails	5 932	13.35	Transportation, men and materials	92	0.21
Rail fastenings	774	1.74	Insurance	1	0.00
Frogs, switches, etc.	169	0.38	Operating constr. trains	514	1.16
Track laying, surfacing	532	1.20	Interest on advances	501	1.13
Ballasting	1 088	2.45	Bond expenses	74	0.17
Surfacing, filling and lining track	61	0.14	Bond int. during constr.	1 569	3.53
Trans. Dept. bldgs	615	1.38	Wagon roads	32	0.07
Road Dept. bldgs	88	0.20	Total average, per mile of line	$44 412	100.00

Organization Expenses will amount to 1 to 1.5% of the cost and frequently even more.

Interest on Cost during Construction. Theoretically this should mean interest on money from the time that bills are payable until the road begins operation. This should average about half the time required to build the road, and therefore about 3% is frequently allowed. Financiers sometimes require interest on the full amount from the time of signing the contract, and this will more than double this item.

The original cost per mile of main line of 488 miles of the Great Northern Rwy. in Washington is given in the table on p. 251. The total length of track was about 8% greater. The cost of grading, tunnels and bridges and culverts was unusually high and indicates the mountainous character of the country. One tunnel, 13 813 ft long, costing $2 524 212, or $183.40 per linear ft accounted for 92% of the whole cost of the tunneling.

Appraisal commissions appointed in Wisconsin, Michigan and Minnesota have appraised all of the railroads of each state, the appraisal value being made both from the standpoint of the cost of reproduction and also the present value as affected by depreciation. The systems of analyzing the cost are not identical, which makes comparison difficult and even slightly inaccurate as given below, but the average prices are very instructive. The cost per mile of main line and branches and the percentage of the item to the total cost are given separately for each state.

Miles of U. S. Railroads in Operation on Dec. 31 of Each Year

Year	Miles	Year	Miles	Year	Miles	Year	Miles
1830	23	1852	12 908	1874	72 385	1895	181 115
1	95	3	15 360	1875	74 096	6	182 769
2	229	4	16 720	6	76 808	7	184 591
3	380	1855	18 374	7	79 082	8	186 810
4	623	6	22 016	8	81 747	9	190 818
1835	1 098	7	24 503	9	86 556	1900	194 262
6	1 273	8	26 968	1880	93 262	1	198 743
7	1 497	9	28 789	1	103 108	2	202 938
8	1 913	1860	30 626	2	114 677	3	207 335
9	2 302	1	31 286	3	121 422	4	212 394
1840	2 818	2	32 120	4	125 345	1905	217 341
1	3 535	3	33 170	1885	128 320	6	222 766
2	4 026	4	33 908	6	136 338	7	228 128
3	4 185	1865	35 085	7	149 214	8	232 046
4	4 377	6	36 801	8	156 114	9	238 356
1845	4 633	7	39 050	9	161 276	1910	242 107
6	4 930	8	42 226	1890	166 703	1*	254 732
7	5 598	9	46 844	1	170 729	2	258 033
8	5 996	1870	52 922	2	175 170	3	261 036
9	7 365	1	60 301	3	177 516	4	263 547
1850	9 021	2	66 171	4	179 415	1915	264 378
1	10 982	3	70 268			6	266 031

* 1911 and later years are for June 30.

For the World at close of 1913 the railroad mileage was 687 123, of which 354 248 miles were in North and South America, 215 140 in Europe, 68 198 in Asia, 27 531 in Africa, and 22 006 in Australia. Canada had 29 298, Mexico had 15 840, Argentina had 20 639, and Brazil had 15 525 miles.

State Appraisals of Steam Railroads. Average Cost per Mile

No.	Items	Minnesota (1907) Cost	Minnesota (1907) Per-cent	Michigan (1900) Cost	Michigan (1900) Per-cent	Wisconsin (1903) Cost	Wisconsin (1903) Per-cent
1	Land; right-of-way, yards and terminals	$9 637	17.78	$3 665	14.00	$3 719	12.03
2	Grading, clearing and grubbing	7 372	13.60				
3	Protection work, riprap, retaining walls	318	0.59	2 778	10.63	5 098	16.50
4	Tunnels	33	0.06	147	0.56	122	0.39
5	Cross ties and switch ties	2 303	4.25	1 426	5.46	1 529	4.95
6	Ballast	1 239	2.29	476	1.83	788	2.55
7	Rails	4 348	8.02	3 674	14.05	3 773	12.21
8	Track fastenings	782	1.44	492	1.88	617	2.00
9	Switches, frogs and railroad crossings	183	0.34	188	0.72	151	0.48
10	Track laying and surfacing	703	1.30	839	3.21	447	1.45
11	Bridges, trestles and culverts	2 576	4.75	1 027	3.93	2 372	7.67
12	Track and bridge tools	27	0.05	......	......	19	0.06
13	Fences, cattle guards and signs	364	0.67	431	1.66	277	0.90
14	Stockyards and appurtenances	74	0.14	......	......	......	......
15	Water stations	211	0.39	93	0.36	161	0.52
16	Fuel stations	95	0.18	39	0.15	54	0.17
17	Station buildings and fixtures	771	1.42	526	2.01	476	1.54
18	Miscellaneous buildings	572	1.05	158	0.60	353	1.14
19	General repair shops	543	1.00	276	1.05	428	1.38
20	Engine houses, turntables, cinder pits	373	0.69				
21	Shop machinery and tools	241	0.44	142	0.54	182	0.59
22	Track scales	24	0.04	......	......	......	......
23	Docks and wharves	799	1.47	707	2.71	260	0.84
24	Interlocking plants	53	0.10	64	0.25	52	0.17
25	Signal apparatus	20	0.04				
26	Telegraph lines and appurtenances	173	0.32	33	0.13	19	0.06
27	Telephone lines and appurtenances	12	0.02				
28	Grain elevators and warehouses	......	......	204	0.78	163	0.52
29	Adaptation and solidification of roadbed	1 546	2.85	......	......	......	......
30	Engineering, superintendence and legal	1 598	2.95	776	2.97	940	3.04
31	Locomotives	2 249	4.15	1 155	4.42	1 342	4.35
32	Passenger equipment	871	1.61	408	1.57	627	2.03
33	Freight car equipment	6 176	11.40	2 527	9.67	3 630	11.75
34	Miscellaneous equipment	175	0.32	89	0.34	70	0.22
35	Marine equipment	6	0.01	220	0.84	0	0.00
36	Steam-, gas- and electric-power plants	105	0.19	13	0.05	9	0.03
37	Freight on track material	478	0.88	......	......	193	0.62
38	Organization expenses	......	......	340	1.30	275	0.89
39	Contingencies	2 353	4.34	2 360	9.02	1 512	4.96
40	Stores and supplies	686	1.27	188	0.72	427	1.38
41	Interest during construction	4 115	7.59	677	2.59	825	2.67
	Total average cost per mile	54 184	100.00	26 138	100.00	30 910	100.00

Item 30 assumed to be 4.5% of items 1–28 incl. for Mich. and Wisc. and of items 1–29 incl. and also item 36 for Minnesota.

Item 38 assumed to be 1.5% for Mich. and 1% for Wisc. of items 1–37 incl.

Item 39 assumed to be 10% for Mich., 5.5% for Wisc. and 5% for Minn. of items 1–37 incl. Item 41 assumed to be 3% for Mich. and Wisc. of items 1–37 incl. Basis for Minn. estimate not stated, but corresponding figure would be 8.75%.

Capital Stock of U. S. railroads was $2 708 673 373 at end of 1880, $4 590 471 560 at end of 1890, $5 804 346 250 at end of 1900, $8 380 819 190 at end of 1910, and $9 058-982 773 on June 30, 1916. Per mile of road, the capital stock for each of these dates was $29 395, $28 101, $30 205, $34 922, and $34 050.

Funded Debt of U. S. railroads was $2 530 874 943 at end of 1880, $5 055 225 025 at end of 1890, $5 758 592 750 at end of 1900, $9 600 634 906 at end of 1910, and $12 033-389 512 on June 30, 1916. Per mile of road, the funded debt for each of these dates was $27 466, $30 945, $29 967, $40 004 and $45 233.

Total Indebtedness, per mile of road, including capital stock, bonded debt, and equipment obligations, was $58 624 at end of 1880, $61 343 at end of 1890, $61 884 at end of 1900, $78 714 at end of 1910 and $79 283 on June 30, 1916.

Total Track in United States in 1916, including first, second, third, and fourth tracks, but excluding sidings and yards, was 293 075.03 miles, in which invested capital per mile was $71 935.

31. Economics of Location

Economic Location is the determination of the proper relation between construction and operating expenses and the discovery, in any individual case, of that location which will afford the most economical combination of cost and operating expenses. The PRINCIPLES must be practically applied by comparing proposed plans for new construction or by changing existing lines and abandoning existing facilities. For new work the method must include for each proposed route these features: (1) the estimated cost of the line; (2) the annual interest charge on capital at a rate which the circum-

Distance. Percentage of Operating Expenses Affected by Length of Line According to Three Estimates

Division of total cost of railroad operation	Wellington, 1887			Webb, 1906			Harriman Line,* 1909		
	Total percentage	Am't affected	Per cent to total	Total percentage	Am't‡ affected	Per cent to total	Total percentage	Am't affected	Per cent to total
Maintenance of way	†23.0	§11.6 19.0	50.4 82.7	21.0	§15.7 19.0	75.0 90.3	24.97	23.45	93.9
Maintenance of equipment	15.6	5.7 8.2	36.6 52.6	18.0	6.7 7.6	37.2 42.1	20.53	0	0
Conducting transportation	31.4	7.5 24.2	23.9 77.1	56.6	8.9 25.0	15.8 44.0	50.22	15.02	29.8
General expenses	30.0	0 0	0 0	4.3	0 0	0 0	4.28	0	0
Totals.........	100.0	24.8 51.4	....	100.0	31.3 51.6	....	100.00	38.47	......

* Auditors' estimate for one of the Harriman lines.

† Interpretation of first line of figures: the maintenance of way expenses are estimated at 23.0% of total cost of operation; 11.6% of the total are affected by changes of length of line; the 11.6% is 50.4% of 23.0%. Other groups interpreted similarly.

‡ Including several sub-items really belonging under maintenance of way or transportation which should decrease materially the percentages for "conducting transportation."

§ The upper figures show the percentage on the basis of changes so small that they are measured only in feet, which will not appreciably affect several items of expense; the lower figures are for changes measured in miles. The Harriman figures make no such distinction.

stances of the project make proper; (3) the estimated annual traffic, not only the gross tonnage, but also the number of trains required to handle it; (4) the effect, if any, of change of location on the traffic obtainable; (5) the estimated annual cost of handling such traffic; (6) the effect, as far as it is possible or proper to predict it, of future changes in traffic or in operating conditions. Having determined the gross revenue, the interest on first cost and the operating expenses, the line giving the largest net revenue is evidently the most economical line. When revenue is not affected (generally true for small changes) the line having a minimum sum of operating expenses and interest on first cost is the most economical. When studying changes of an existing line, in addition to the features above-mentioned, the possible injury to existing facilities must be considered. The proposed plan should, in such a case, so increase the net income that the excess will at least pay a proper return on the entire cost of the change.

The general method of analysis consists in separating the different items of cost of operation and determining the proportion of each item which is affected by each physical element in the proposed change of line. The physical elements are Distance, Curvature, Minor Grades and Ruling Grades. The table gives the aggregate results for each of the main groups of operating expenses and illustrates the method. Both the grouping and the percentages are now changed, see detail of operating expenses on p 237.

The lack of agreement of these figures is partly due to some differences in the basis of computation, partly to the different times at which they were computed and partly to the fact that such figures can never be constant for all times and conditions, but must be modified to suit local conditions. It is, however, remarkable that the mean values for the large and small changes give 38.1%, 41.4% and 38.5% respectively as the percentage of total operating expenses which are affected by changes of length of line.

J. B. Berry's estimate divides changes of distance into three classes: Class (A), distance so short as not to affect wages of engine or train men, 32.10%; Class (B), distance affecting train wages but not requiring additional side tracks, 45.94%; Class (C), distances so great as to require additional side tracks and sta ons, 59.5%.

W. G. Raymond's estimate is 35.3% + a flat amount of $350 per year per mile for expenses which are independent of the number of trains. This flat amount is a little less than $1 per day. With ten trains per day each way or twenty train-miles over each mile, it would add a little less than five cents and make the total approximately 40%.

Reduction of Distance may also reduce the receipts as well as operating expenses and thus reduce the advantage gained by shorter distance, provided the change of distance is so great that it is measured in miles. The EFFECT ON RECEIPTS is not computable except in the most approximate way; it depends on the following elements: (1) Since the cost of operating additional distance is approximately 40% of the average, while passenger receipts and some other kinds are based more or less strictly on actual mileage, there is a real advantage and profit in added distance for such kinds of traffic. (2) For competitive business, where receipts are absolutely independent of distance, any added distance is a pure loss, without any compensation. (3) For thru business, where the division of receipts between roads is more or less strictly according to the relative mileage of hauls on the two or more roads, there is profit or loss according to the percentage of length of haul on the home road to the total haul, the larger the percentage the larger the possible profit, which also depends on whether the rate is competitive or non-competitive. (4) Since a road always has varying proportions of these several kinds of traffic, the net advantage or disadvantage depends on the combined effect. As a final conclusion, there is always some addition to receipts to partially

offset the added cost of operating added distance; for a road whose business is very largely non-competitive the increase in receipts due to added distance might be greater than the increase in operating expenses; for a road whose business is chiefly competitive the net loss will be great; in any case there is some reduction on the advantage computed above by decreasing the distance.

Curvature. The basis of the analysis of cost is the determination, first, of the degree of curve on which the power requirements are precisely double the power used on a straight level track, and then the determination of the additional cost of the doubled resistance. It is then assumed that train resistance and added cost of curvature are proportional to the number of degrees of central angle and independent of the radius of curvature. The degree of curve on which the resistance is precisely double that on a level tangent has been variously estimated from 10° to 12° 30′. A mile of such continuous curvature will have from 528° to 660° of curvature. Doubling the power requirements on the engine will increase certain items of cost of a train-mile by a computable percentage of such cost. A continuous 10° (or 12° 30′) curve will increase items of maintenance of way and equipment by percentages which may be closely estimated. Many of the items of cost of a train-mile are unaffected. In this way the total increase in the cost of one mile of such curved track is computed. Dividing the total extra cost by the number of degrees (528 to 660) gives the value of 1°. The chief items affected by curvature are rails, of which the extra wear may be from 200 to 300%; repairs to locomotives, 100 to 200%, and repairs of cars which are increased from 50 to 100%. Some other items of maintenance of way are increased 25 to 50%, while fuel and water would be increased about 50%. The extra cost per degree has been variously computed as follows:

Author	Date	Deg. per mile	Percentage	Extra cost per degree
Wellington	1887	600°	35.6%	0.0593%
Berry	1904	660	{35.35* 29.88†	{0.0536 0.0498
Webb	1908	528	34.6	0.0655

* Uncompensated curvature. † Compensated curvature.

The percentage of extra cost per degree chosen should be multiplied by the estimated cost per train-mile and this value multiplied by the number of trains per year, which gives the yearly cost of 1° of curvature. Capitalizing this at the proper rate of interest gives the amount which may justifiably be spent to avoid 1°. For calculations of this kind, estimates must be made on the following items which will vary according to the local conditions: (1) the local cost per train-mile; (2) the percentage by which the various items of cost per train-mile are affected by curvature; (3) the number of trains which will use the tracks. For new construction even the last item is an uncertain quantity.

Minor Grades. The basis of computing the cost is to determine, first, the rate of grade which will exactly double the resistance on a level tangent; second, to compute the effect on operating expenses of a mile of such a grade; and third, on the assumption that the cost of lifting a train thru so many vertical feet is proportional to the height, the extra cost of one foot of elevation equals the added cost of operating a mile of such grade divided by the number of vertical feet in a mile of that grade. It is assumed that the grade which doubles tractive resistance on a level tangent does not limit the length or weight of trains it is designed to haul with one engine, or that any limiting effect is a separate matter, and that the only effect of the grade is to increase the operating expenses over those required for operation over a level track.

Minor grades are necessarily divided into three classes on account of a distinction in their effects on the operation of trains. Class (A) includes those grades which, on account of the rate of grade or the length, do not require any change in the handling of the train, no brakes and no change in the throttle or cut-off. Class (B) includes those grades which are steep enough or long enough to require shutting off steam (but no brakes) in order to avoid objectionable velocity while descending and to require more steam or longer cut-off when ascending. Class (C) includes the grades which require that steam shall be shut off and brakes applied on down grades and full steam power, with perhaps sand, to assist traction on the up grades. Since a fast passenger train may safely attain very high speed in a sag and since the effect of a hump is a relatively small reduction of a previously high velocity, a grade which should properly be classified as an (A) grade for fast passenger trains should be classified as a (B) or even a (C) grade for slow freight trains. The average tractive resistance of a train at ordinary velocity is about 10 lbs per short ton. For slow freight trains the figure is too high; for fast passenger trains it is too low. Ten lbs per ton is the equivalent of the grade resistance on a 0.5% grade which has a rise of 26.4 feet per mile. SAGS AND HUMPS: This section applies only to the possible elimination of sags and humps, since a long uniform grade connecting two predetermined points is practically unchangeable. The problem therefore becomes the determination of the value of removing a hump or filling a sag whose height is the vertical distance from the vertex of the sag or hump from the uniform grade by which it would be replaced. VALUE OF 1 FOOT OF RISE-AND-FALL: Class (A) grades have no effect on the power required of the locomotive, and no effect on operation except a harmless temporary increase or decrease in velocity; they have also no appreciable effect on operating expenses and can therefore be ignored. The effect of Class (B) and Class (C) grades has been variously computed as follows:

Author	Date	Percentage Affected		Annual value*	
		Class B	Class C	Class B	Class C
Wellington	1887	3.03%	9.67%	84%	276%
Webb	1908	6.08	7.92	168	219
Berry	1904	3.38	7.13	93	197

* Annual value in percent of average cost per train-mile for each round trip daily train per foot of rise-and-fall.

Ruling Grades are those which limit the length or weight of the trains which may be handled by one locomotive. The classification does not include steep grades which will always be operated by two or more engines and are called pusher grades, nor does it include very short steep grades which may always be operated by momentum. The maximum train load which may be hauled on a given ruling grade by an engine of given weight, dimensions and boiler power, may be determined by dividing the tractive power by the total resistance per ton. Calculating the gross tonnage to be handled upon any division, the number of train loads required for a given type of engine may be determined. Dividing the gross tonnage by one less number of engines gives the tonnage for one less number of trains; dividing the tractive power by the increased train tonnage plus weight of engine gives the tractive power per ton; and the table in Art. 1 gives the corresponding grade for the increased tractive power. The effect of a change of grade is shown more clearly in the table on the following page.

The cost per train-mile of each additional train for hauling the same tonnage of cars up a somewhat steeper grade has been computed by Webb as about 53% of the average cost of a train-mile. J. B. Berry (Bul. 49, Am. Ry. Eng. & Main.

Table Showing Percent of Trains Necessary to Handle a Given Traffic with Single Engine, if Gradients are Changed from Those in First Column to Any Other Given Gradient

Gradient to be Reduced, Percent	Proposed Gradient, Percent, with Percent of Trains to handle same Traffic as on Original Gradient															
	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
2.0	26	30	34	38	42	46	51	55	60	65	69	74	79	84	89	95
1.9	27	31	36	40	44	49	54	58	63	68	73	78	84	89	94	100
1.8	29	33	38	42	47	52	57	62	67	72	78	83	89	94	100	
1.7	31	35	40	45	50	55	60	66	71	77	82	88	94	100		
1.6	33	37	43	48	53	59	64	70	76	82	87	94	100			
1.5	35	40	46	51	57	63	68	75	81	87	93	100				
1.4	37	43	49	55	61	67	73	80	87	93	100					
1.3	40	46	53	59	66	72	79	86	93	100						
1.2	43	50	57	64	71	78	85	92	100							
1.1	47	54	61	69	76	84	92	100								
1.0	51	59	67	75	83	91	100									
0.9	55	64	73	82	91	100										
0.8	61	71	80	90	100											
0.7	68	79	89	100												
0.6	76	88	100													
0.5	87	100														
0.4	100															

This table is taken from Bulletin 49 of Am. Ry. Eng. & M. W. Assoc., by J. B. Berry; calculated for engine and tender weighing 150.3 tons in working order, tractive power 35 268 lb, weight of caboose 17 tons, velocity and frictional resistance 6 lb per ton. As percentages, the figures apply approximately to almost any freight engine.

Way Assoc.) computes the cost similarly as 43.29%. Isaacs and Adams (Bul. 112 of same Assoc.) computed that 30.64% of the total operating expenses are "affected by locomotive mileage." They use the figure by multiplying the additional number of train-miles required by the higher grade by 30.6% of the average cost per train-mile and consider this as the additional cost of the higher grade.

Pusher Grades. A road may be designed for pusher grades by determining a pair of grades such that two engines of a certain type can haul up the steeper grade the same train which will just tax the capacity of one such engine on the lower grade. This pair of grades must be selected according to the character of the country and must be such that all the grades of the line may be reduced, without unreasonable cost, to the limit of the grade for one engine, except such grades as it is intended to operate with pusher engines. Little or nothing is saved by reducing these pusher grades below the limit computed for two engines. In exceptional cases a combination of three grades may be computed, of which the two lower grades are to be operated as above, and the highest grade is such that three engines are required to haul the same train over it. The following table is computed for one type of engine. The precise balance between any thru grade and the corresponding grades for one and for two pushers depends on the type of engine, the available ratio of adhesion and the tractive resistance; but it may be demonstrated that reasonable variations in the type of engine or tractive resistance will only alter the required grades by a few hundredths of one percent. The corresponding pusher grades may therefore be accepted as approximately true for all cases. The accuracy of the table is theoretically impaired by the fact that

the rating tonnage on the pusher-engine service is somewhat different from that on the thru-engine service. The variation will depend on the ratio of rating-ton resistance to tare-ton resistance. As an illustration, a 1.9% one-pusher grade corresponds to a 0.92% thru grade according to the table. Working out the corresponding grade, using the values proposed by Dennis of 2.6 lbs as the resistance of a rating ton and 9 lbs as the resistance of a tare ton, the corresponding thru grade is computed to be 0.99%. Some increase should be expected since the pusher locomotive has a higher resistance per ton than average train resistance. This shows that, altho no table can be compiled which will suit all conditions, the differences will not probably exceed a few hundredths of one percent. The variations are of less importance with the higher pusher grades. The cost of a pusher engine mile as a percentage of the average cost of a train-mile was estimated by Wellington (1887) as 38.3%; by Berry (1904) 34.4%; by Webb (1908) 38.8%.

Balanced Grades for One, Two, and Three Engines

Thru grade, percent	Track resistance, 6 lbs			Track resistance, 8 lbs		
	Net load for one engine in tons (2000 lbs)	Corresponding pusher grade for same net load, percent		Net load for one engine in tons (2000 lbs)	Corresponding pusher grade for same net load, percent	
		One Pusher	Two Pushers		One Pusher	Two Pushers
Level	3868	0.28	0.55	2874	0.37	0.72
0.10	2874	0.47	0.82	2278	0.56	0.98
0.20	2278	0.66	1.08	1880	0.74	1.23
0.30	1880	0.84	1.33	1596	0.92	1.47
0.40	1596	1.02	1.57	1384	1.09	1.70
0.50	1384	1.19	1.80	1218	1.27	1.92
0.60	1218	1.37	2.02	1085	1.44	2.14
0.70	1085	1.54	2.24	977	1.60	2.36
0.80	977	1.70	2.46	887	1.77	2.56
0.90	887	1.87	2.66	810	1.93	2.76
1.00	810	2.03	2.86	745	2.09	2.96
1.10	745	2.19	3.06	688	2.24	3.15
1.20	688	2.34	3.25	638	2.40	3.33
1.30	638	2.50	3.43	594	2.55	3.51
1.40	594	2.65	3.61	555	2.70	3.68
1.50	555	2.80	3.78	521	2.85	3.85
1.60	521	2.95	3.95	489	2.99	4.02
1.70	489	3.09	4.12	461	3.13	4.17
1.80	461	3.23	4.27	435	3.27	4.33
1.90	435	3.37	4.43	411	3.42	4.49
2.00	411	3.52	4.59	390	3.55	4.63
2.10	390	3.65	4.73	370	3.68	4.78
2.20	370	3.78	4.88	352	3.81	4.92
2.30	352	3.91	5.02	335	3.94	5.05
2.40	335	4.04	5.15	319	4.07	5.19
2.50	319	4.17	5.29	304	4.20	5.32

Basis: Thru and pusher engines alike; consolidation type; total weight, 107 tons; weight on drivers, 53 tons; adhesion, 9/40, giving a tractive force for each engine of 23 850 lbs; normal track resistance 6 (also 8) lbs per ton. It is assumed that the engine is so designed and that the velocity and steam pressure are such that the engine can always develop a tractive force of 23 850 lbs.

The mileage must be computed as twice the total distance between th
sidings at top and bottom of the grade, this distance being always somewha
in excess (and sometimes very much so) of the length of the pusher grade o
the profile.

Momentum Grades are grades which, altho somewhat steeper than th
ruling grade, can invariably be approached at such a velocity that the kineti
energy of the train will assist the engine sufficiently to carry the train past th
summit with a minimum velocity of say 10 miles per hr. If the resistance o
trains were independent of velocity, the mechanics of the problem would b
much simplified, but the extensive dynamometer tests of Dennis, confirme
by those of Shurtleff, have shown that the resistance per ton of freight trains a

Velocity Head of Trains Moving at Various Velocities

Vel., miles per hr	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10	3.51	3.58	3.65	3.72	3.79	3.87	3.95	4.02	4.10	4.17
11	4.25	4.33	4.41	4.49	4.57	4.65	4.73	4.81	4.89	4.97
12	5.06	5.15	5.23	5.32	5.41	5.50	5.58	5.67	5.75	5.84
13	5.93	6.02	6.12	6.21	6.31	6.40	6.50	6.59	6.69	6.78
14	6.88	6.98	7.08	7.19	7.29	7.39	7.49	7.60	7.70	7.80
15	7.90	8.00	8.11	8.22	8.33	8.44	8.55	8.66	8.77	8.88
16	8.99	9.10	9.21	9.32	9.43	9.55	9.67	9.79	9.91	10.03
17	10.15	10.27	10.39	10.51	10.63	10.75	10.87	10.99	11.12	11.25
18	11.38	11.50	11.63	11.76	11.89	12.02	12.15	12.28	12.41	12.55
19	12.68	12.81	12.95	13.08	13.22	13.35	13.49	13.63	13.77	13.91
20	14.05	14.19	14.33	14.47	14.61	14.75	14.89	15.04	15.19	15.34
21	15.49	15.64	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84
22	17.00	17.15	17.30	17.46	17.62	17.78	17.94	18.10	18.26	18.42
23	18.58	18.74	18.90	19.06	19.22	19.38	19.55	19.72	19.89	20.06
24	20.23	20.40	20.57	20.74	20.91	21.08	21.25	21.42	21.59	21.77
25	21.95	22.12	22.30	22.48	22.66	22.84	23.02	23.20	23.38	23.56
26	23.74	23.92	24.10	24.28	24.46	24.65	24.84	25.03	25.22	25.41
27	25.60	25.79	25.98	26.17	26.36	26.55	26.74	26.93	27.13	27.33
28	27.53	27.73	27.93	28.13	28.33	28.53	28.73	28.93	29.13	29.33
29	29.53	29.73	29.93	30.13	30.34	30.55	30.76	30.97	31.18	31.39
30	31.60	31.81	32.02	32.23	32.44	32.65	32.86	33.08	33.30	33.52
31	33.74	33.96	34.18	34.40	34.62	34.84	35.06	35.28	35.50	35.72
32	35.95	36.17	36.39	36.62	36.85	37.08	37.31	37.54	37.77	38.00
33	38.23	38.46	38.69	38.92	39.15	39.38	39.62	39.86	40.10	40.34
34	40.58	40.82	41.06	41.30	41.54	41.78	42.02	42.26	42.51	42.76
35	43.01	43.26	43.51	43.76	44.01	44.26	44.51	44.76	45.01	45.26
36	45.51	45.76	46.01	46.26	46.52	46.78	47.04	47.30	47.56	47.82
37	48.08	48.34	48.60	48.86	49.12	49.38	49.64	49.91	50.18	50.45
38	50.72	50.99	51.26	51.53	51.80	52.07	52.34	52.61	52.88	53.15
39	53.42	53.69	53.96	54.23	54.51	54.79	55.07	55.35	55.63	55.91
40	56.19	56.47	56.75	57.03	57.31	57.59	57.87	58.16	58.45	58.74
41	59.03	59.32	59.61	59.90	60.19	60.48	60.77	61.06	61.35	61.64
42	61.94	62.23	62.52	62.82	63.12	63.42	63.72	64.02	64.32	64.62
43	64.92	65.22	65.52	65.82	66.12	66.43	66.74	67.05	67.36	67.67
44	67.98	68.29	68.60	68.91	69.22	69.53	69.84	70.15	70.46	70.78
45	71.10	71.42	71.74	72.06	72.38	72.70	73.02	73.34	73.66	73.98
46	74.30	74.62	74.94	75.26	75.59	75.92	76.25	76.58	76.91	77.24
47	77.57	77.90	78.23	78.56	78.89	79.22	79.55	79.89	80.23	80.57
48	80.91	81.25	81.59	81.93	82.27	82.61	82.95	83.29	83.63	83.97
49	84.32	84.66	85.00	85.34	85.69	86.04	86.39	86.74	87.09	87.44
50	87.79	88.14	88.49	88.85	89.20	89.55	89.91	90.26	90.61	90.97

elocities between 10 and 35 miles per hr is so nearly uniform that the follow-
ng method gives a close approximation to accuracy. It is also fortunately true
at a very considerable error in velocity height at high velocities is compara-
vely unimportant. If it is assumed that a train were running without track
esistance, its kinetic energy at a velocity v in feet per sec would lift it up a
rade to a total vertical height $h = v^2/2g$, in which g is the acceleration of
ravity. If V=vel in miles per hr, $h=0.03344\ V^2$; adding 5% for the rotative
inetic energy of the wheels, we have $h=0.0351\ V^2$. If an engine is develop-
ng just sufficient energy at the foot of a grade to overcome all tractive resist-
nces, it can (theoretically) climb a grade to a total vertical height of h ft.
ractically, since resistance increases at very low velocities, the train would
ot reach the total height, but the method practically holds as long as the
elocity does not fall below 10 miles per hr. The values of h (or "velocity
eights") are in the table opposite. For example, if an engine were so loaded
hat it could only overcome its tractive resistances and were to approach a
.3% grade 3000 ft long at a speed of 35 miles per hour, it would have at the
ottom of the grade a "velocity head" (as per the table) of 43.01 feet. The
ertical rise on this grade will be 39 ft and at the top of the grade the train
ould still have a velocity head of 4.01 ft, which corresponds to a velocity of
0.7 miles per hour, which would be sufficient for it to pass over the summit.

But it will not do to assume that the work done by the engine is only sufficient
overcome the tractive resistances. Even tho an engine is often throttled
own to this amount of work, no engine would be so loaded that it could do no
ore than haul a train at freight-train speed on a level. An engine properly
aded will haul its train up the ruling grade and the test of the momentum
ade or hump is the possibility of hauling the train up the excess grade with
e help of momentum. For example, a train, with the engine loaded for a 0.7%
ade, approaches a 1.2% grade 7850 ft long with a velocity of 35 miles per hour.
he excess grade is 0.5%, and in a distance of 7850 feet the excess rise is 39.25
. The velocity head corresponding to 35 miles per hr is 43.01 ft; subtracting
.25, we have left a velocity head of 3.76, which corresponds to a velocity of
.3 miles per hr, which is a safe speed for going over the summit. The secondary
uestion, whether it is safe to assume that no fully loaded freight train will
er need to stop, or even slacken speed, along this grade of 7850 feet, is a
atter of practical operation rather than of engineering.

The accuracy of this method depends on the assumptions that the resistance
uniform within the range of the velocities, and also that the work done by
e engine is uniform. Altho the resistance may be considered substantially
niform, the tractive power of the engine decreases as the velocity increases.
hen, as in the above illustration, the train approaches the grade at a velocity
35 miles per hr, the engine is not producing as much tractive power as it can
roduce at a lower velocity. But if the rating of the engine (see Art. 21) is
ased on a fairly high velocity on the ruling grade, the excess tractive power
lower velocities may so balance the deficiency at the higher velocities that the
ethod is applicable without material error. Experience has shown this to be
ue.

Sags in Grade Line. Economies are often possible provided that sags having
rades exceeding the ruling grade on one or both sides may be harmlessly intro-
uced. Their harmlessness may be tested by the velocity-head table. It is
f course assumed that running through the sag without a stop may be depended
n. Deciding first the maximum-permissible speed for freight trains in the
ottom of the sag, the ability of the train to mount the grade in either direction
nd have sufficient speed at the top of the grade may be tested. It must also be

determined what speed of approach is necessary at the other end of the sag in order to attain that maximum speed at the bottom, and whether it is practicabl to approach the sag at that speed.

As an illustration: the ruling grade is 1.2%; economical construction suggest a down grade of 2.0% for a distance of 4500 feet, followed by an up grade o 2.0% for 7000 ft. Assume a maximum-permissible freight-train velocity of 4 miles per hr, the velocity head being 71.10 ft. The excess grade is 0.8%, whic in 7000 ft has a rise of 56 ft. At the top of the grade the remaining velocity hea is 15.10 ft, which corresponds to 20.7 miles per hour, which shows that a spee of 45 miles per hr through the sag would be unnecessary. The drop on th other slope of the sag is 90 ft, which would give to the train a velocity of ove 50 miles per hour, even without using steam. There is therefore no questio in this case of the ability of the train to acquire that speed at the bottom o the sag. It will also be still easier to run the train through the sag in th opposite direction.

Classification of Grades. The velocity-head table may be utilized to predic the behavior of trains, and therefore to classify the grades into the A, B, an C classes discussed under Minor Grades. The above case should be classifie as (C) grade, since steam must be shut off and brakes used to prevent objec-tionably high speed in the sag, for either direction of movement.

SECTION 3A

ELECTRIC RAILROADS

ASSOCIATE EDITOR

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GENERAL

POWER STATIONS AND SUBSTATIONS

TRANSMISSION AND DISTRIBUTION

CARS AND LOCOMOTIVES

CAR BARNS AND OTHER BUILDINGS

SIGNALS AND TRACK

* By Walter Loring Webb, who was Associate Editor for previous editions.

GENERAL

1. Classes of Traffic Suitable for Electric Traction

General. The essential characteristic of electric traction which distinguishes it from other forms, is the separation of the source of motive power from the rolling stock. This gives it a virtual monopoly for street and interurban systems because it permits the generation of motive power to be concentrated in a single economical plant instead of being distributed over a number of comparatively uneconomical ones, as in the case of steam traction.

Conditions for Electric Traction on Railroads. On electrified railroad trunk lines the generating and distributing systems must have large capacity in order to handle the heavy trains, but are not kept fully employed due to the comparative infrequency of traffic. This results in the investment of a great deal of capital which becomes idle, the fixed charges on which generally exceed the operating economies. Exceptions to this generalization occur under the following conditions:

(*a*) When the cost of coal is very high, the operating economies may be sufficient to counterbalance the fixed charges, as electric traction requires only half the amount of coal needed with steam traction.

(*b*) When cheap water power is available, a similar condition may occur.

(*c*) When it is desired to increase the weight of trains to be hauled on steep grades, the elimination of the power generating system from the locomotive, in electric traction, permits an increased concentration of motors on the locomotive, thereby obtaining the desired increase of hauling power, at a lower cost than by the elimination of grades.

(*d*) When it is desired to increase the capacity of a mountain division and this can be accomplished by increasing the weight of trains as described above, rather than by increasing the number of tracks so as to permit more trains to be hauled.

(*e*) When it is necessary to eliminate smoke and gases in tunnels or city streets, the element of economy being necessarily subordinated to safety or convenience.

(*f*) When the traffic density is unusually great, as in certain suburban zones.

(*g*) When by the elimination of smoke and gases, trains thru cities can be run underground and the ground surface utilized for lucrative building purposes.

2. Systems of Electric Traction

Economic Differences Between Systems. The stationary equipment of an electric traction system consists of three parts—the generating, transmission, and distribution systems. These must have sufficient capacity to carry the peak loads and are therefore likely to be comparatively idle during a large part of the time. The activity of these parts depends upon the number of cars in operation.

The moving equipment of an electric railway system consists of the motors, controllers and other parts of the rolling stock necessary for electric operation. The activity of these parts does not depend upon the number of cars in operation, but only upon the activity of each car.

There are several systems of electric traction which differ in the distribution of the investment between stationary and moving equipment. Stationary equipment costs less than moving equipment of the same capacity, because it is concentrated in larger units and its design is not restricted by the limitations of car construction. It is, therefore, obvious that for railways operating a large number of cars, it pays to have a comparatively large investment in stationary equipment; whereas, railways with few cars must economize in stationary equipment and concentrate their investment as far as practicable on their limited rolling stock.

Technological Differences between Systems. The various systems of electric traction now in use may be classified as follows according to the above principle:

Low-voltage, direct current
High-voltage, direct current
Three-phase
Single-phase.

In every practical electric railway system current is generated for transmission at a high voltage and utilized in the motors at low voltage. In the direct-current systems, the transformation from high to low voltage is accomplished at stationary substations, which also change the alternating current from the transmission lines, into direct current, which is fed to the trolley or third rail, and thence to the motors. In the alternating-current systems, the high-voltage current is delivered to the trolley lines and transformed down to a voltage suitable for the motors by means of transformers on the cars or locomotives. As the direct-current systems carry a greater proportion of investment in their stationary equipment, they are especially adapted to urban and important interurban lines. The alternating-current systems, which carry a proportionately greater investment in moving equipment, are especially suitable for trunk lines having infrequent service.

The high-voltage, direct-current system is intermediate, in the above respect, between the low-voltage, direct-current and the alternating-current systems.

The choice of systems does not depend merely upon the investment, but also upon the operating costs. It has been found, however, that, with the present high rate of interest, these are seldom the governing factor.

3. Costs and Economics

General. Even more than in the case of steam railroads, electric urban and interurban railways labor under the difficulty that their rates are fixed by law, while their operating costs are constantly rising. This, added to the uncertainties of limited franchises and hostile regulation, makes it difficult to obtain capital for electric railways except at excessive rates of interest. The consequence is that the utmost economy must be exercised in construction and operation.

Revenues and Expenses of Street and Interurban Railways. The table on the next page shows the average annual revenues and expenses of street and interurban railways, 1916 being a normal year and 1917, peculiar by reason of high operating costs, due to the war.

Cost per Mile for Street and Interurban Railways. On page 267 is given the prescribed classification of capital charges used by the New York State Public Service Commission (2d District). It is useful in preparing estimates. An average total capital cost, including all items shown in the following classification, is $72 000 per mile of road. This figure covers all electric railways in New York State except those in the City of New York, and two lines of a very special character, namely, the New York, Westchester and Boston R.R. and the Niagara R.R. The former cost $1 269 219 and the latter $308 602 per mile of road owned.

Cost of Steam Railroad Electrification. The total cost of steam railroad electrification depends so much upon the amount of traffic and the character of the line, that no general average figures are of use. The distribution of those

features of annual expense which are purely electrical is shown for a typical electrified steam railroad, in a table near the end of this Article. Valuable data on this subject were published by J. B. Cox for the Butte, Anaconda & Pacific Railway, Trans., A.I.E.E., 1914, Vol. 33, p. 1369, and by W. S. Murray for the New York, New Haven & Hartford R.R., Trans. A.I.E.E., 1915, Vol. 34, p. 85; the data in both cases being too voluminous to abstract.

Unit Costs of Electrical Construction. Prices of electrical construction have been so radically affected by the Great War that it is difficult to gage the value of cost data. The unit costs on page 267 are based upon pre-war conditions.

Annual Revenues and Expenses of Electric Railways

(Street and Interurban)

(From Monthly Reports to American Electric Railway Association)

	Eastern District		Southern District		Western District		Entire United States	
	1916	1917	1916	1917	1916	1917	1916	1917
Revenues and Expenses, Dollars:								
Operating revenues	19 909	20 979	15 565	16 342	28 303	30 249	21 296	22 525
Operating expenses	12 435	14 143	8 899	9 535	18 090	20 236	13 312	15 013
Net earnings	7 474	6 836	6 666	6 807	10 213	10 013	7 984	7 512
Operating ratio. percent	62.46	67.42	57.17	58.35	63.91	66.90	62.51	66.65
Average miles of line represented	5 793	5 843	785	793	1 786	1 801	8 364	8437
Components of Operating Expenses. Dollars:								
Way and structures	1 405	1 452	1 067	1 096	2 079	2 026	1 517	1 540
Equipment	1 005	1 161	958	985	1 863	2 122	1 191	1 355
Total maintanance and renewal	3 280	3 552	2 025	2 081	4 908	5 144	3 503	3 742
Power	1 847	2 412	1 120	1 376	2 540	2 820	1 921	2 388
Conducting transportation	4 973	5 467	4 227	4 576	7 449	8 299	5 440	5 995
Traffic	40	23	61	57	96	106	55	45
General and miscellaneous	1 654	1 890	1 466	1 454	2 774	2 948	1 882	2 076
Transportation for Investment (credit)	......	......	......	......	33	31	7	7
Average miles of line represented	4 702	4 752	786	793	1 569	1 582	7 057	7 127
Per Revenue Car Mile:								
Operating revenues, cents	......	32.99	......	24.91	......	28.65	......	30.87
Operating expenses, cents	......	21.45	......	14.90	......	18.90	......	20.06
Net earnings	......	11.54	......	10.01	......	9.75	......	10.81
Revenue car miles, billions	......	272	......	36	......	161	......	470
Average miles of line represented	......	4 447	......	603	......	1 618	......	6 668

(Aera, 1918, Vol. 6, pp.794–797.)

Third rails cost from $5000 to $8000 per mile, depending largely upon whether the railroad is operating during construction.

Ordinary trolley systems cost from $1500 to $2500 per mile of single-track road or from $3500 to $5000 per mile of double track. The cost of catenary construction of the most substantial type is shown in Table on page 268.

Substations cost from $20 to $40 per kilowatt capacity, the former figure being for large substations, cost of real estate and building being omitted in each case.

Power stations cost from $80 to $100 per kilowatt capacity.

Underground conduits for railways cost from 25 cents to 75 cents per duct-foot, the latter figure being appropriate where the conduit lines are built along the narrow right-of-way of an operating railroad.

Transmission pole lines cost from $2000 per mile for wooden poles, to $4000 per mile for light steel poles.

Classification of Capital Expenditures

(New York State Public Service Commission, Second District)

Intangible Street Railway Capital

1. Organization.
2. Franchises.
3. Patent rights.
4. Other intangible street railway capital.

Direct Expenditures for Tangible Street Railway Capital

5. Right of way.
6. Other street railway land.
7. Grading.
8. Ballast.
9. Ties.
10. Rails, rail fastenings and joints.
11. Special work.
12. Underground construction.
13. Track laying and surfacing.
14. Paving.
15. Roadway tools.
16. Tunnels.
17. Elevated structures and foundations.
18. Bridges, trestles and culverts.
19. Crossings, fences and signs.
20. Interlocking and other signal apparatus.
21. Telephone and telegraph lines.
22. Poles and fixtures.
23. Underground conduits.
24. Transmission system.
25. Distribution system.
26. Dams, canals and pipe lines.
27. Power plant buildings.
28. Substation buildings.
29. General office buildings and equipment.
30. Shops and car houses.
31. Stations, waiting rooms and miscellaneous buildings.
32. Docks and wharves.
33. Park and resort properties.
34. Furnaces, boilers and accessories.
35. Steam engines.
36. Turbines and water wheels.
37. Gas power equipment.

38. Power plant electric equipment.
39. Miscellaneous power plant equipment.
40. Substation equipment.
41. Cable power equipment.
42. Shop equipment.
43. Locomotives.
44. Revenue cars.
45. Electric equipment of cars.
46. Other rail equipment.
47. Miscellaneous equipment.

General Expenditures for Street Railway Fixed Capital

48. Engineering and superintendence.
49. Law expenditures during construction.
50. Injuries during construction.
51. Taxes during construction.
52. Miscellaneous construction expenditures.
53. Interest during construction.
54. Total classified by prescribed accounts.
57. Total not classified by prescribed accounts.
58. Total fixed capital.

Distribution of Annual Expenses for Typical Electrified Railroad

	Per Cent
Fixed Charges	59
Operating charges:	
Power station	12
Transmission and distribution	5
Substations	3
Signals and communications	5
Trains	16
	100

Notes. The above table excludes all items not peculiar to electric operation. It includes multiple-unit cars but not standard cars drawn by electric locomotives. The cost of train crews, except for electric locomotives, is omitted. Taxes on real estate are also omitted.

Cost of Catenary Construction, Dollars, per Mile of Road

(W. S. Murray, Trans. A.I.E.E., 1915, Vol. 34, p. 85)

	Compound Catenary			Simple Catenary	
	6 Tracks	4 Tracks	2 Tracks	4 Tracks	2 Tracks
Steel	14 390	9 350	6 900	8 800	6 680
Concrete	4 920	3 110	3 580	2 930	3 000
Catenary material	16 650	11 050	5 580	7 035	3 530
Catenary labor	2 800	1 980	1 130	1 238	510
Total	38 760	25 490	17 190	20 003	13 720

Life Expectancy Table of Electrical Equipment

(W. S. Gorsuch)

Equipment	Total Life in Years
Steam Power Plants:	
Buildings	75
Boilers	30
Stokers and grates	20
Conveyors, elevators and hoists	20
Turbines, complete	15
Engines and condensers	15
Piping	15
Heaters	15
Pumps	15
Alternators	15
Switchboard apparatus and instruments	15
Synchronous converters	30
Transformers	20
Exciters	25
Motors	25
Storage batteries	10
Tools and sundries	15
Substations:	
Synchronous converters	30
Transformers	20
Motor generator sets	25
Switchboard apparatus and instruments	20
Storage	10
Tools and sundries	15
Transmission and distribution:	
Conduit (vitrified clay)	50
Feeders (direct-current)	20
Feeders (alternating-current)	25

Note.—In estimating the time in years during which equipment of a generating station may be reasonably expected to perform its functions, three elements must be clearly kept in mind; namely, obsolescence, inadequacy and inefficiency. How long apparatus or equipment will remain in service before it becomes obsolete, inadequate or ineffective, is purely a speculative matter, and can only be predicted in advance by past experience and a knowledge of the art, and with careful judgment.

4. Power Consumption in Relation to Schedule

Forces Acting on a Train. The forces tending to accelerate a train are the tractive effort developed by the motors and the component of the weight along the track on down-grades. The forces which retard the motion of the train are the various frictional forces and the component of the weight along the track on up-grades; also in braking, the frictional force due to the brakes. All the various frictional forces, except the braking resistance, such as track friction, journal friction, air friction, etc., which oppose the motion of a train on a straight track, are usually considered together and are referred to as the "train resistance." The extra friction due to track curvature is usually considered as an equivalent up-grade.

Train Resistance for Electric Trains

N = number of cars (including electric locomotive, if any);
w = average weight of car loaded, in tons (= total weight of train divided by N);
r = train resistance in pounds per ton;
v = speed in miles per hour;
a = cross-section of car in square feet;

A and B are constants in the formula; K taken as 0.0030 thruout.

$$r = A + Bv + \frac{Ka(0.9 + 0.1N)v^2}{Nw} \quad (1)$$

The constant A depends chiefly upon the average total weight of car and load. Thus

for w =	15	20	25–30	35	40–45	50	70
A =	6.0	5.5	5.0	4.5	4.0	3.5	3.0

The constant B depends primarily upon the nature of the track and roadbed and also to some extent upon the weight and type of the car. Burch gives the following values:

Passenger cars on excellent track	0.06–0.11
Passenger cars on ordinary track	0.10–0.15
Freight cars on ordinary track	0.05–0.06

The heavier the car the higher the value of this coefficient.

Grades and Curvatures. An actual up-grade of $G\%$ produces a retarding force of $20\,G$ lb per ton and a down-grade of $G\%$ produces an accelerating force of $20\,G$ lb per ton. A curve always gives rise to a retarding force, which ranges from 0.5 to 1 lb per ton per degree of curvature. Using the higher figure, each degree of curvature may be taken equivalent to an up-grade of 0.05%. Note that for angles of curvature up to 12° the angle in degrees may be taken equal to $5730 \div R$, where R is the radius of curvature in feet.

Average Acceleration Rates

Service	Miles per Hour per Second
Steam locomotive, freight service	0.1 to 0.2
Steam locomotive, passenger service	0.2 to 0.5
Electric locomotive, passenger service	0.3 to 0.6
Electric motor cars, interurban service	0.8 to 1.3
Electric motor cars, city service	1.5 to 2.0
Electric motor cars, rapid transit service	1.5 to 2.0
Highest practical rate	2.0 to 2.5

Acceleration Constant. The tractive effort required to give to 1 ton (2000 lb) a linear acceleration of 1 mphps is 91.2 lb. To accelerate a train of W tons requires a tractive effort of $91.2\,aW$ lb to produce a linear acceleration of a mphps; but on account of the accompanying angular acceleration of the rotating parts, an additional force is required.

The acceleration constant is raised by the flywheel effect by about 5% (i.e., $Wr/W = 0.05$) for heavy cars and locomotives, and between 5% and 10% for light, low-speed cars, 8% being an average figure. However, C is usually taken as 100, corresponding to an increase in effective weight of about 10%. A given linear acceleration of a mphps. then requires an accelerating force of $100\,a$ lb per ton.

Tractive Effort and Adhesion Coefficient. Let

F = tractive effort, in pounds per ton, exerted by motors;
G = per cent actual grade (+ for up-grade);
g = degrees of curvature;
r = train resistance, in pounds per ton;
a = acceleration in mphps (− for retardation).

Then the tractive effort required per ton of total train weight is

$$F = 100\,a + r + 20\,G + g. \qquad (2)$$

The adhesion or "tractive" coefficient is the quotient (expressed usually as per cent) of the tractive effort in pounds which will slip the drivers, divided by the weight in pounds on the drivers. Burch gives the values in Table below. The maximum possible tractive effort is the product of the adhesion coefficient (as a decimal fraction) by the weight (in pounds) on the drivers.

Adhesion Coefficients

Condition of Track	Without Sand	With Sand
Most favorable condition	35	40
Clean, dry rail	28	30
Thoroly wet rail	18	24
Greasy moist rail	15	25
Sleet-covered rail	15	20
Dry snow-covered rail	11	15

The Weight of Locomotive required to accelerate a train weighing W tons at the rate of a miles per hour per second up a grade of $G\%$ on a g degree curve against a frictional resistance of r lb per ton, when the $q\%$ of the weight is on the drivers and the coefficient of adhesion, is $p\%$, is given by the following formula:

$$\text{Weight of locomotive} = \frac{5\,W}{pq}(100\,a + r + 20\,G + g).$$

Example. What weight of locomotive is required to accelerate a 400-ton train at the rate of 0.05 mile per hour per second up a 0.1% grade against a frictional resistance of 8 lb per ton, when 80% of the weight is on the drivers and the coefficient of adhesion is 20%?

$$\text{Weight of locomotive} = \frac{5 \times 400}{20 \times 80}(50 + 8 + 2) = 75 \text{ tons.}$$

Maximum Over-all Efficiency of Motors and Gears at Rated Voltage

Horse-power, 1-hour Rating	Kind of Motor	Max. Eff., Percent
30–100	D-C. geared	83–88
100–250	D-C. geared	88 89
250–500	D-C. gearless	91–93
50–200	A-C. series geared	70–80 *
200–500	3-phase induction geared	85–89

* Including step-down transformers.

Power Required at Given Speed. Let

r = train resistance in pounds per ton of total weight of train;
G = percent grade;
g = degree of curvature;
a = acceleration in mphps;
v = speed in mph;
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99\,v\,(r + 20\,G + g + 100\,a)$ watts per ton, or $2.67 \times 10^{-3}\,vW\,(r + 20\,G + g + 100\,a)$ horse power, total,

The power input, p_i, to the car or locomotive is equal to the power at the rims of the drivers divided by the over-all efficiency ϵ of the controller, motors and gears, i.e.,

$$p_i = \frac{1.99\, Wv(r + 20\,G + g + 100\,a)}{1000\epsilon} \text{ kilowatts} \qquad (5)$$

Approximate Method of Calculating Energy Consumption. The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the more accurate step-by-step method.

When the method is applied to checking purposes, the column of the table below headed "Actual Energy Output" should be used and the input calculated from the known efficiencies. When applied to rough calculations, the column headed "Approximate Electrical Energy Input" should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required. The total energy in watt-hours per ton-mile will be the sum of the amounts required for acceleration and overcoming frictional train resistance grades and curves. Let

V = average running speed in miles per hour;
V_m = maximum speed in miles per hour;
L = length of run in miles;
L_p = distance traveled, with power on, in miles;
$n = l/L$ = number of stops per mile including one terminus;
r = average train resistance, in pounds per ton (say that corresponding to a speed from 10 to 20% greater than the average speed);
G = average equivalent grade, in per cent;
g = average curvature in degrees;
$K = \frac{V_m}{V}$ = ratio of maximum to average speed; see Table on page 273;
$Q = \frac{L}{L_p}$ = ratio of length of run to distance traveled with power on. See Table on page 273.

Output at Wheel Rim and Input to Cars in Watt-hours per Ton-mile

Energy For	Actual Energy Output at Wheel Rims of Cars	Approximate Electric Energy Input to Cars
Acceleration	$\frac{V^2_m}{36.2L}$	$\frac{K^2nV^2}{25}$
Frictional train resistance	$\frac{1.99\,r\,Lp}{L}$	$\frac{2.9\,r}{Q}$
Grades	$\frac{39.8\,GLp}{L}$	$\frac{57\,G}{Q}$
Curves	$\frac{1.99\,gLp}{L}$	$\frac{2.9\,g}{Q}$
Total	Sum	Sum

NOTE.—$25 = 36.2\,\epsilon$, $57.8 = 39.8 \div \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of 1 lb per ton, which is probably high.

Values of K and Q

Stops per Mile, n	K		Q
	Locomotive Passenger Trains	Single Cars Multiple Unit Trains and Freight Trains	All Trains
0	1.0	1.0	1.0
0.1	1.2	1.1	1.1
0.2	1.35	1.2	1.25
0.3	1.5	1.25	1.4
0.4	1.6	1.3	1.5
0.5	1.7	1.35	1.7
0.6	1.75	1.4	1.8
0.7	1.8	1.45	1.9
0.8	1.85	1.45	2.0
0.9	1.9	1.5	2.1
1.0	1.95	1.5	2.15
1.2	1.95	1.6	2.2
1.4	1.95	1.6	2.3
1.6	1.95	1.6	2.4
1.8	1.95	1.65	2.5
2.0	1.95	1.7	2.6
2.5	1.95	1.75	2.7
3.0	2.0	1.8	2.8
3.5	2.0	1.85	2.9
4.0	2 0	1.9	2.9
4.5	2.0	1.95	2.95
5.0	2.0	2.0	3.0

More exact methods involve a knowledge of the motor characteristics, gears, brakes, and control.

Power Required for Car Heating and Lighting. In addition to the energy required for propelling the cars, a very appreciable amount is also required in the winter, for heating them, and a small amount at night for lighting. See table below. In making up a load diagram this energy should be included. See Art. 17 for power required for heating.

Lighting of Cars

Length of Car, Feet	Average * Kw for Lighting
14–20	0.25
20–28	0.35
28.34	0.55
34.40	0.70

* During the hours lights are on, using tungsten lamps.

POWER STATIONS AND SUBSTATIONS

5. Power Stations

General. The power plant of an electric railroad should be located with reference to the load center, the water and coal supply, and the cost of real estate. The load center is usually the least important consideration as the

cost of transmission lines and the losses therein do not rank in importance with considerations of plant economy.

Water is required for boilers and condensers. The coal supply should be brought by water where practicable, with auxiliary rail connection.

Buildings. Buildings are usually of brick with steel frames and concrete floors. Floors are of concrete with cement or tile finish for engine rooms and concrete with cement finish or bricks laid on edge in cement mortar for boiler rooms. Foundations for machinery are usually made of concrete and separate from the wall foundations in order to reduce vibration. Boiler-room equipment is carried on the steel structure.

Tidal limits and flood-water stages fix the levels of both condenser intake and outlet tunnels, and sometimes of engine- and boiler-room floors.

Standard Design. Present practise for large stations provides a boiler room and generating room side by side and separated by a thick wall. The generating room is divided in two parts, either with or without a separating wall, the part nearer the boiler room containing the generating equipment and provided with a crane; the other part, without a crane, having the electrical control apparatus.

Equipment. The table on p. 275 shows the principal items in a typical electric railway power station, and their relation to the general processes of the plant. At the head of the upper part of the table are listed the raw materials which enter the plant, followed by the machinery and apparatus thru which they pass to the boiler. At the head of the lower part of the table are listed the products of the boilers, followed by the machinery and apparatus thru which they are dissipated.

6. Substations

General. A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system. Most commonly, substations transform high-voltage alternating current to low-voltage direct current, the former affording the most economical transmission, and the latter the most economical conversion of electrical into mechanical energy. Substations are usually housed in substantial brick and concrete buildings, altho where the current has to be changed only in voltage and not in kind, as on single-phase railway systems, the building is often unnecessary, as no rotating machinery is required, and the transformers and switching apparatus can be built for out-door operation.

Economical Location. Substations are located at intervals of 3 to 20 miles along the railroad, the spacing being limited by considerations of line voltage regulation at the cars and economy, both of first cost and operation. Theoretically, substations should be located so as to give the least annual expense, taking into account both the fixed charges on the investment in substations and feeders, the operating charges and the cost of energy losses in the system. An investment in copper feeders, especially in the case of bare or weatherproof aerial cables, causes little annual expense beyond the interest and taxes on the investment; whereas, a substation involves, in addition, heavy depreciation charges and labor charges for operation. In recent years automatic substations, not requiring constant attendance, have been successfully used on interurban lines. In such a substation, the machines automatically start into operation when a car is about to enter the section of line fed by it, and they stop when the car has passed beyond. Such systems have

Relation of Machinery in Typical Power-house

COAL	AIR	FEEDWATER	CONDENSING WATER
Coal barge	(Blower)	Meter	Screen
Clam-shell bucket			
Car			
Coal pile			
Gantry crané		Feed water pump	
Conveyors	(Air ducts)		
Bunkers			
Downspout		Feed water heater	
Apron			
Stoker			
Boiler furnace	Boiler furnace	Boiler drum	Intake flume

ASHES — GASES — STEAM

Individual hoppers
Ash cars
Collective hoppers
Bucket conveyor
Storage bin
Spout
CAR

SUPERHEATER

STACKS — PIPING — Centrifugal Pump

Turbine *

Condenser
(Surface type assumed)

Hot well pump — OUTLET

Hot well
(Condensed steam returns
to feed water pump.)

* Turbine drives electric generators, which supply current to the switching and protective apparatus and thence to the outgoing feeders.

not found favor on important urban lines due to the complex controlling devices not having proven sufficiently reliable under the severe conditions occurring on those lines.

The permissible distance between substations depends on the voltage of distribution, being greater the higher the voltage. On an ideal line, doubling the operating voltage, quarters the distance between substations, but on actual lines no such ratio exists, as substations have to be located with reference to junctions, yards and terminals.

Where a length of track receives its power from only one direction, the distance that can be fed, with a given voltage drop, is ¼ the distance allowable between substations.

Apparatus. In the usual type of substation for the conversion of high pressure alternating current into low-pressure direct current, the high-pressure current first passes thru oil-switches to the high-pressure bus, then thru oil-switches to the primary windings of the transformers. From the secondary

windings of the transformers, the low-pressure alternating current enters the slip-rings of the converters, and emerges from the commutators as direct current of a slightly higher pressure. From the commutators it passes thru circuit breakers to the direct-current bus bars and thence thru circuit breakers to the outgoing feeders. In the design of substations, it is good practise to arrange all the apparatus so as to make the route of the electric current as direct as possible.

All substations are not equipped with synchronous converters, motor-generators having been used for pressures in excess of 750 volts. Such a motor-generator usually consists of an induction motor driving a direct-current generator. The present practise, however, is to use interpole synchronous converters supplying direct-current at 1200 volts, two machines in series being used for 2400 volts.

Portable Substations. Portable substations, i.e., substations of small size installed in a box car, are sometimes used where unexpected or occasional loads are liable to occur. They are especially useful to carry holiday loads for which it would not pay to install a permanent substation.

Power Capacity. The power capacity required in a substation depends upon the size, number and speed of cars; on the length of line fed and upon the number of stops per mile.

Railway substation machines and transformers are usually rated by the kilovolt-ampere output which having produced a constant temperature, may be increased 50% for 2 hours without producing temperatures or temperature rises exceeding by more than 5° C. the standard limiting values established by the American Institute of Electrical Engineers. This is known as the nominal rating.

Buildings. Substations having all apparatus on one level require about 0.2 sq ft per kilowatt nominal rating. Buildings are usually of brick with concrete foundations for walls and machines, except where the machines are small, in which case the machines are often carried by the floor beams. It is usual to provide a crane which, in important substations, should be equipped for both manual and electrical operation.

Provision must be made for ventilation and good illumination, but roofs must be constructed so that there will be no danger of leaks over the machines and other live apparatus. Lavatory, toilet and telephone booth should be conveniently near the operators' desks.

7. Transformer Stations

General. A transformer station, as used on alternating current railway systems, is an aggregation of apparatus for converting high-voltage alternating current from transmission lines, into lower-voltage alternating current for use in the trains, the apparatus being usually out-of-doors and protected only by a fence.

Special designs of oil-cooled transformers, switching apparatus, and lightning arresters must be used for this purpose.

All apparatus is set on concrete foundations, and steel structures are used to support overhead wires and switching apparatus.

Auto-transformer System. The New York, New Haven & Hartford Railroad system transmits at 22 000 volts between trolley and feeder, but the locomotives receive current at 11 000 volts between trolley and track rail. This is accomplished by transformer stations about 2 miles apart, where the

trolley and feeder are each connected to the track rails through a separate autotransformer. This arrangement not only effects economy in transmission, but also greatly reduces the current in the track rails and earth, with consequent reduction of disturbances to telephone lines.

TRANSMISSION AND DISTRIBUTION

8. Duct Lines and Underground Conductors

General. Underground cables in the United States are almost invariably installed in conduits made either of glazed tile, wood, or paper fiber. The conduits are laid so as to form a series of continuous ducts of a length of not over 400 ft, which are terminated in brick or concrete chambers from which the cables are pulled into the ducts.

Conduit lines are used for the distribution of electrical energy wherever the unsightliness, danger or instability of pole lines prohibit the use of the latter. They are used for both the transmission and distribution cables of many railways, and for telephone and telegraph lines.

Types of Conduits. Figs. 1 and 2 show the conduits used by the New York Central Railroad Company, and are representative of the best modern practise. The single-duct conduit weighs 16½ lb per length of 18 in, and the four-duct conduit weighs 100 lb per length of 3 ft.

Fiber conduit consists of tubes made by rolling paper saturated with bituminous compound around a mandrel. Like iron pipe, which it resembles in

Sizes and Weights of Electric Conduits

Inside Diameter, Inches	Thickness of Walls, Inches	Approx. Average Weight, per Foot, Pounds	Length of Section, Inches	Inside Diameter, Inches	Thickness of Walls, Inches	Approx. Average Weight, per Foot, Pounds	Length of Section, Inches
SLEEVE JOINT				SLEEVE JOINT			
1	¼	0.45	30	1	¼	0.45	30
1½	¼	0.75	60	1½	¼	0.75	60
2	¼	0.90	60	2	¼	0.90	60
2½	¼	1.05	60	2½	¼	1.05	60
3	¼	1.30	60	3	¼	1.30	60
3½	¼	1.60	60	3½	7/16	2.50	60
4	¼	1.85	60	4	½	3.20	60
SCREW JOINT				DRIVE JOINT			
1½	5/16	0.85	60	2	¼	0.90	60
2	3/8	1.35	60	2½	¼	1.05	60
2½	3/8	1.70	60	3	¼	1.30	60
3	7/16	2.20	60	3½	¼	1.60	60
3½	7/16	2.50	60	4	¼	1.85	60
4	1/2	3.20	60				

appearance, lengths of fiber conduit are usually joined by screw and coupling, altho it is also made for socket, sleeve and drive joints. The usual dimensions of fiber conduits are given in the table on page 277.

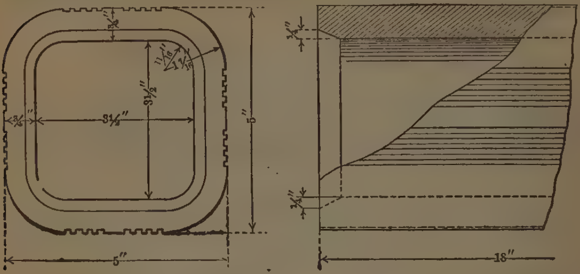


Fig. 1. Typical Single-Duct Conduit.

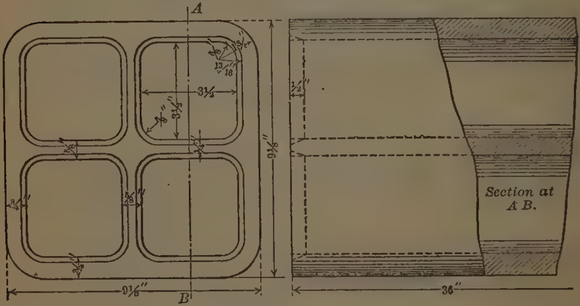


Fig. 2. Typical Four-Duct Conduit

Wrought-iron pipe is used in city streets where the conduit line has to twist about sub-surface obstructions. It is more expensive than tiled conduit,

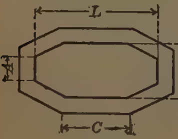


Fig. 3.

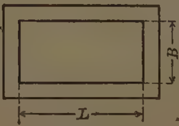


Fig. 4.

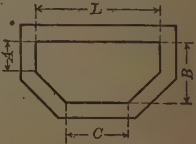


Fig. 5.

Splicing Chambers.

and does not last as long on account of rusting. The usual sizes are 3 in and 3½ in pipe, 20 ft long, provided with threaded ends and couplings.

Splicing Chambers. Splicing chambers for straight runs are usually built in the shapes shown in Figs. 3, 4, 5, and 6. The height of large splicing chambers is usually determined by the height in which a man can stand upright, and is seldom less than 6½ ft. The width is similarly influenced by the space required for working, which is at least 4 ft. The length depends upon the length of splice and the space required to curve the cable from the ducts to the supporting shelves or racks, considerations which make a length of 8 ft a practical minimum where there are large cables. The usual dimensions are as follows:

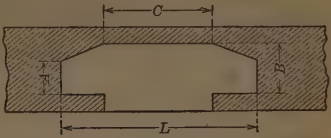


Fig. 6. Splicing Chamber

Type	*A*	*B*	*C*	*L*
Fig. 3	2 ft 2 in	5 ft 0 in	3 ft 6 in	9 ft 0 in
Fig. 4	....	4 6	....	8 0
Fig. 5	2 0	4 6	3 8	10 0
Fig. 6	1 6	3 0	4 0	10 0

Cables in splicing chambers are usually supported on iron brackets attached to the chamber walls. This construction is very satisfactory, especially if the power cables are wrapped with asbestos; but some engineers consider a shelf preferable, as it affords insulation and provides more protection between cable splices.

Testing of Duct Lines. Ducts are usually tested by rodding with a mandrel in order to ascertain whether they are continuous and unobstructed. The rods used for this purpose are of hickory 1 in in diameter and 3 or 4 ft long, and are fitted at the ends with steel couplings such as that shown in Fig. 7. The first rod is attached to a mandrel and pushed into the duct. Another rod is coupled to the first, and the pair pushed further into the duct. By successfully coupling other rods and pushing them into the duct, the mandrel is made to travel from one chamber to another.


Fig. 7. Testing Mandrel

As soon as the mandrel emerges into the receiving chamber, the rods are pulled thru and uncoupled. If an obstruction stops the mandrel, an attempt is made to force it thru by repeated blows, failing which it becomes necessary to break into the conduit line from the side. It is usual to attach a No. 10 or No. 12 galvanized steel wire to the last rod, and leave the wire in the duct after the removal of the rods. This wire is subsequently used for drawing a heavy rope through the duct, by means of which the cable is pulled in place.

Types of Cables Used. Cables for underground conduit lines are almost invariably insulated with impregnated paper and covered with a lead sheath. Three conductor cables are used for high-pressure, three-phase transmission, and either single conductor or two conductor concentric cables for low-pressure distribution.

9. Pole Lines

General. The most usual construction for street and interurban railways consists of wooden poles spaced about 100 ft apart, carrying wooden cross-arms, porcelain insulators, and strung with bare or weatherproof copper wire. Where the transmission line parallels the railway, the same poles are used for supporting both trolley system and transmission lines. Steel towers are used on electrified steam railways.

Tubular steel poles are largely used for urban railways and reinforced concrete poles are gaining favor.

Dimensions of Wooden Poles. Poles are specified by their total length and either diameter or circumference at the top. Standard lengths are multiples of 5 ft, and vary from 30 to 60 ft. Standard diameters are even multiples of 1 in and vary from 7 to 8 in, except on the Pacific Coast, where 8 to 10 in prevail. The taper is exprest as the difference in inches between two circumferences 10 ft apart and is usually as follows:

Michigan white cedar	5.2
Maryland chestnut	3.8–4.0
* California yellow pine	4.0
Montana lodgepole pine	3.0
* Texas loblolly pine	2.4
Washington cedar	3.5

The minimum dimensions specified by the American Electric Railway Association are given in the table on page 282.

Tubular Steel Poles. Tubular steel poles are made of three pieces, each of a different size, of steel tubing. The three pieces are joined by shrinking the larger onto the smaller, until an overlap of 18 in is obtained. Such poles are made in lengths of 28 to 35 ft, and vary in weight from 384 to 1688 lb. Comprehensive tables of data for such poles are given in A.E.R.A. Engineering Manual.

Reinforced Concrete Poles. Reinforced concrete poles are made in lengths of 28 to 35 ft, and with butt sections, from 5 to 15 in square, the corners, however, being eliminated. They are reinforced with four steel rods. Hexagonal poles are also used, containing six steel rods. Comprehensive tables of data for such poles are given in A.E.R.A. Engineering Manual.

Location of Poles. Side poles should have a minimum clear distance of 7 ft 6 in from the center line of track at level of top of rail; center poles should have a minimum clear distance of 7 ft from the center of track, these clearances to be suitably increased at curves. Poles are usually spaced between 90 and 110 ft apart.

Depth of Setting. The usual depth of pole setting for tangent construction is shown in the table at end of this Article. On curves, and other special localities, an extra depth may be necessary.

Wires and Cables for Pole Lines. Three classes of wire are used for aerial conductors; Bare, weatherproof and insulated. Bare and weatherproof wires are usually made of hard-drawn copper, the stranding being as shown in the table below. Bare wires are used for operating pressures over 2500 volts, but where the pressure is not above this amount, weatherproof wire is preferred by many engineers, because it affords some protection from grounds due to the contact of foreign bodies, especially tree twigs.

Insulated cables, except telephone cables, are seldom used on pole lines on account of their expense, but where used are made of soft annealed copper

Stranding of Concentric Lay-Cables for Aerial Use

Size (See note 1)	Number of Wires (See note 2). Bare, insulated or weatherproof cables for aerial use	Size (See note 1)	Number of Wires (See note 2). Bare, insulated or weatherproof cables for aerial use
2.0 Cir. Inches	91	0000 A.W.G.	19 or 7 (See note 3)
1.5 "	61	00 "	7
1.0 "	61	2 "	7
0.6 "	37	7 and smaller	1
0.5 "	37		
0.4 "	19		

1. For intermediate sizes, use stranding for next larger size.
2. Conductors of 0000 A.W.G. and smaller are often made solid and this table of stranding should not be interpreted as excluding this practice.
3. Cables of sizes 0000 and 000 A.W.G., are usually made of 7 strands when bare and 19 strands when insulated or weatherproof.

either insulated with paper and sheathed in lead, or insulated with varnished cambric or rubber and covered with impregnated cotton braid. Such cables must be supported from a steel messenger cable by means of clips spaced from 12 to 18 in apart. Where rubber insulation is used on aerial lines, a high-grade compound is essential if it is to withstand the effects of the weather.

Guy Wires. Galvanized iron cables are used for guying poles. They are usually made of 7 strands of No. 12 or 14 B.W.G. wire, having an ultimate breaking strength of 2300 or 5000 lb per cable, respectively.

Preservation of Poles. It is estimated that it takes 190 years to grow a 30-ft cedar pole which, when set in the ground, will not last over 15 years. When butt-treated, the life is increased to about 20 years. The preservatives most generally used are creosote, zinc chloride and bichloride of mercury. (See U. S. Forest Service Circulars 84 and 147.) Decay occurs at the butt of the pole, which must be particularly well protected.

Cross Arms. Cross arms are rated by the number of pins they are made to carry, the number being from 2 to 10. The usual cross-sections are 3¼ or 3¾ by 4¼ or 4¾, the length depending upon the number of wires to be carried and their voltage. Cross arms, 3¼ by 4¼ in in cross-section are attached to wooden poles by a thru-bolt driven from the back of pole toward and thru the arm, having a washer at each end, with hole at back of pole counterbored to secure a good seat for the washer. Cross arms should be steadied by strap braces secured to the pole by a lag screw and to side of arm away from pole by carriage bolts on the center line of arm, with nuts next to the braces. Arms up to and including 48 in in length should have braces 24 in long fastened to arm 16 in from center; arms over 48 in long should have braces 28 in long fastened to arms 19 in from center. Cross arms larger than 3¼ by 4¼ in should be steadied by angle braces.

10. Trolley Systems

General. The trolley wire is usually of hard-drawn copper but sometimes of steel which is suspended from insulators some 16 to 30 ft above the ground, and presents a continuous contact surface to a trolley wheel or bow attached to the rolling stock. There are two classes of trolley construction, the span wire

Minimum Circumferences of Poles in Inches
(A.E.R.A. Engineering Manual)

A. For span construction where a 35-ft span is required, or for heavy feeder lines carrying from one to six cross arms.

B. For span or bracket construction where spans are not more than 35 ft. or bracket line or construction carrying two transmission circuits, one feeder arm and two telephone and signal arms.

C. For telephone, signal and other light auxiliary lines where no side strain is required.

CHESTNUT POLES

Total Length, Ft	Class A		Class B		Class C	
	Top	6 Ft from Butt	Top	6 Ft from Butt	Top	6 Ft from Butt
25	24	36	21	31	20	30
30	24	40	22	36	20	33
35	24	43	22	40	20	36
40	24	45	22	43	20	40
45	24	48	22	47	20	43
50	24	51	22	50	20	46
55	22	54	22	53	20	49
60	22	57	22	56	....	....
65	22	60	22	59	....	....
70	22	63	22	62	....	....
75	22	66	22	65	....	....

EASTERN WHITE CEDAR POLES

Total Length, Ft	Class A	Class B	Class C
	Top 24 In	Top 22 In	Top 18¾ In
	6 Ft from Butt	6 Ft from Butt	6 Ft from Butt
30	40	36	33
35	43	38	36
40	47	43	40
45	50	47	43
50	53	50	46
55	56	53	49
60	59	56	....

WESTERN WHITE CEDAR POLES

Total Length, Ft	Class A	Class B	Class C
	Top 28 In	Top 25 In	Top 22 In
	6 Ft from Butt	6 Ft from Butt	6 Ft from Butt
30	37	34	30
35	40	36	32
40	43	38	34
45	45	40	36
50	47	42	38
55	49	44	40
60	52	46	41
65	54	48	43

Depth of Setting Poles in Ground

Total Length of Pole, Ft	Trolley Construction		Transmission Line Construction
	Rock or Concrete	Earth	Earth
30	5 ft 0 in	6 ft 0 in	5 ft 0 in
35	5 6	6 0	5 6
40	5 6	6 6	6 0
45	6 0	6 6	6 5
50	6 6	7 0	6 5
55	6 6	7 6	7 0
60	7 0	8 0	7 0
65	7 0	8 6	7 6
70	7 0	9 0	7 6
75	....	....	8 0
80	....	....	8 0

and the side bracket; each may have either simple or catenary suspension. (See Art. 9 for details of poles.)

Span-wire and Side-bracket Construction. In the simple span-wire construction the trolley wire is supported by wires stretched across the tracks between poles or building walls. The side-bracket construction resembles the span-wire except that instead of the supporting wire being stretched between two poles, it is stretched between two supports on the same pole. In both of these types of construction, the trolley wire is supported at intervals of 100 ft or more and sags between supports, making it necessary for the trolley to be in vertical vibration as the cars move.

Catenary Construction. The speed attained upon modern electric roads makes it difficult to obtain satisfactory service with a trolley wire which dips between supports and sways with every impulse. The catenary construction was devised to meet this condition. In general, it consists of a grooved copper trolley wire suspended horizontally from a sagging messenger cable, which is suitably insulated and firmly held in place. The supporting structure employed for interurban single- or double-track roads usually is of the side-bracket type, but for some conditions cross-span construction becomes necessary. The latter method of support differs only in the substitution of a catenary cross-span for the bracket arm and doubling the number of poles required for single track.

Applications of Various Types of Construction. The overhead trolley system is used on urban railways, wherever the unsightliness or danger of its exposed construction is not considered objectionable. It is used on interurban, suburban and trunk lines, wherever, by reason of low load or high voltage, the current taken by the trains is not too great to be economically carried on copper wires.

Center-pole construction is approved practise for double-track railways, side-pole construction being generally used for single-track lines. Span-wire construction is used where, for any reason, it is impracticable to have the poles near the tracks or where, as is commonly the case in Europe, the span wires are supported from the walls of buildings. Prejudice against overhead lines is often due to the excessive loading of poles, which is both unsightly and dangerous.

Parts of Overhead Trolley Construction. The trolley wire is secured to an " ear " by soldering, clamps or other means and the ear is bolted to a " sus-

pension " which may or may not be provided with an insulating portion. These suspensions are carried by span wires which in turn are fastened to the poles or brackets, or they may be fastened directly to the bracket. If the " suspension " is not insulated, " strain " insulators are inserted in the span wire between the ear and its point of attachment to the poles or brackets. A slightly different form of suspension, called a " pull-off " is used on curves. At turnouts " trolley frogs " must be used to guide the trolley wheel. These various parts are illustrated in the accompanying cuts.

Trolley Ears. Figs. 8 to 12 show ears for round wire. Fig. 8 shows an ear with flaps, which are bent around the wire to hold it in place; this type is

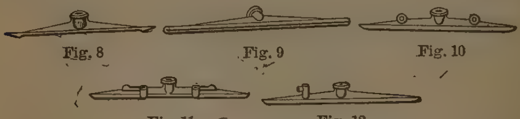


Fig. 8 Fig. 9 Fig. 10 Fig. 11 Fig. 12

Trolley Ears for Round Wire

seldom used on account of arcing at the flaps. The ear shown in Fig. 9 which has a deep groove into which the wire is soldered, is the type in common use. These ears may be provided with rings as in Fig. 10 to which guy wires are attached to relieve the strain at curves and for steadying the line at intervals. Fig. 11 shows the type of ear used at points where the trolley is spliced, which should always be at an ear. Fig. 12 shows an ear with a terminal for a feeder connection.

Figs. 13 to 15 are designed for grooved or " figure 8 " trolley wire. Special grooved (Fig. 16) or " figure 8 " wire affords a smoother running surface for the

Fig. 13 Fig. 14 Fig. 15

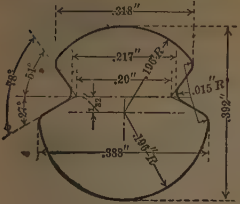
Trolley Ears for "figure 8" Wires

trolley wheel or bow as the ear only grips the upper part, leaving the lower part absolutely even. This construction is practically essential for bow trolleys.

Suspensions for Straight Line Work. Fig. 17 shows an uninsulated suspension with ear attached for straight line work. Fig. 18 shows a suspension with strain insulators at each end; this is also used on curves for double-track work. Figs. 19 and 20 show solid insulated suspensions for span wire and side-bracket construction, respectively. These types are frequently used, and are quite satisfactory if the insulating material used in their construction is properly made. Fig. 21 shows a section of an assembled cap and cone suspension. Fig. 22 shows a cap and cone suspension with ear attached. The type in which cap and cone are made separate is preferred by some engineers on account of the possibility of replacing injured bolts and insulation without removing the whole suspension. This advantage is partly offset by the greater liability to trouble due to multiplicity of parts.

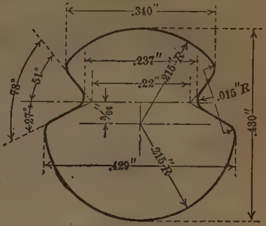
Strain Insulators. Fig. 23 shows a " globe " strain insulator, the type most commonly used. Fig. 24 shows a " Brooklyn " strain insulator. This type is used on wooden pole and light iron pole construction to draw span

wires taut. It is required for heavy iron pole construction if spans are long and temperature variations great. Bolts may be provided at both ends if an extra



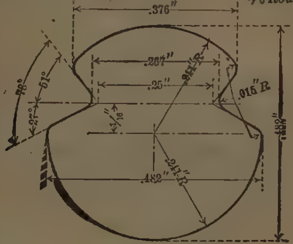
$\frac{2}{0}$ *Wire*

Area 0.1083 *sq. in.*
$^2/_0$ *Round wire* 0.1045 *sq. in.*



$\frac{3}{0}$ *Wire*

Area 0.1314 *sq. in.*
$^3/_0$ *Round wire* 0.1318 *sq. in.*



$\frac{4}{0}$ *Wire*

Area 0.1665 *sq. in.*
$^4/_0$ *Round wire* 0.1662 *sq. in.*

Fig. 16. Standard Grooved Wire Sections

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Types of Suspensions

amount of adjustment is required. A globe strain and a Brooklyn strain may be used in series where extra insulation is required.

Pull-offs. Fig. 25 shows a cap and cone pull-off for single-curve construction and Fig. 26, a cap and cone pull-off for double-track curve construc

tion. Pull-offs of the types corresponding to the uninsulated and solid insulated suspension shown in Fig. 17 and 18, are also used.

Fig. 23 Fig. 24

Strain Insulators

Fig. 25 Fig. 26

Pull-offs

Trolley Frogs. A trolley frog is a malleable iron casting used at switches or crossovers where trolley wires from different tracks unite. Its function is to hold the diverging wires together and afford a smooth running path to the trolley wheel when a car passes or enters a switch. A common type is illustrated in Fig. 27. Frogs are made for various angles of divergence, and both right- and left-handed. The usual angles are 8, 15 and 20 degrees.

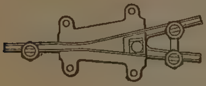


Fig. 27. Trolley Frog

Sag and Tension in Overhead System. Particular attention must be paid to designing the overhead structure in such a manner that it will safely stand the extra tension due to the contraction of the wires at low temperatures and the extra loads due to wind and sleet, and a sufficient allowance should be made in the height of the trolley wire to take care of the extra sag which it experiences at high temperatures.

Height of Trolley Wire above Rail. The height of wire varies between a minimum of 16 ft and a maximum of 22 ft, the usual height being about 18 ft. It is usual to raise the wire at railroad crossings to a height of 22 ft or more.

Rake of Poles. Bracket-arm poles on tangent construction should have a rake backwards not exceeding 3 in, and span-wire poles in hard ground a rake of from 4 to 5 in. In soft ground a rake of 12 in is not uncommon. Center poles should be set vertically except at curves, where they should bend away from the curve along the perpendicular to the tangent at that point of the curve.

Anchorage. At both ends of every grade and curve, there should be a permanent anchorage. If there are not many grades and curves, anchorages should be provided at intervals of from ¼ to ¾ of a mile. An anchorage is made by means of a steel cable running from the trolley wire to one or more anchor poles through an anchor ear and strain insulators.

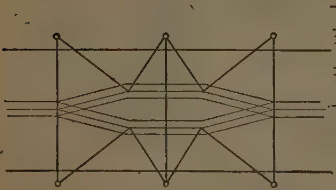


Fig. 28

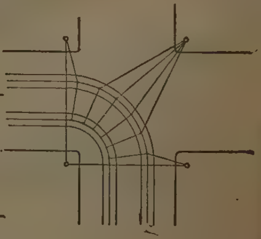


Fig. 29

Simple Pull-off Arrangement

Curves. At curves in the track the trolley wire should be made to follow the curve by means of pull-offs or wires pulling the trolley wire outward as shown in Fig. 28 and 29.

Wherever possible the pull-off wires should be radial to the trolley wire. This, however, requires a large number of poles, and is therefore impracticable in

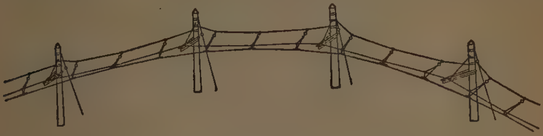


Fig. 30. Bridle Construction at Curve

cities. In such cases, the bridle, bow-string or backbone construction shown in Fig. 30, 31 and 32 is resorted to. Here the pull-off wires, instead of being

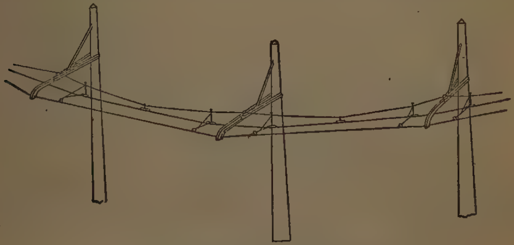


Fig. 31. Bow String Construction at Curve

anchored to individual poles, are fastened to a wire which is stretched between poles. While this construction is almost universally used in cities, it is more

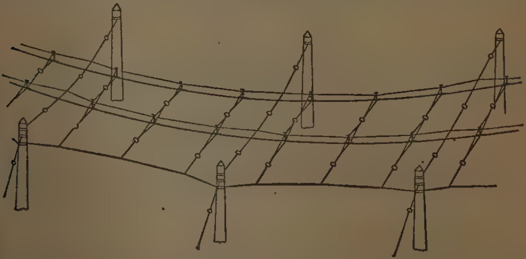


Fig. 32. Backbone Construction at Curve

expensive to maintain than single pull-offs, and is therefore less favored for inter-urban lines.

A combination of the two types of construction is shown in Fig. 33 (Fig. 28 to 33 are from General Electric Co. publications).

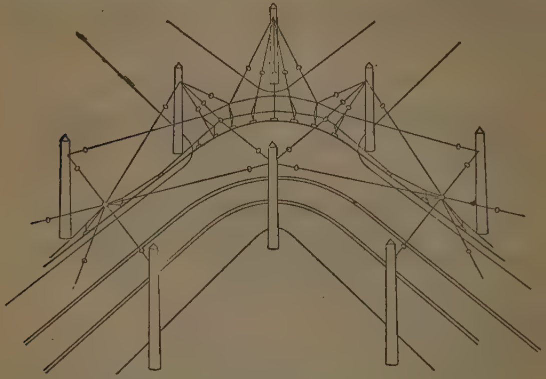


Fig. 33. Combination Construction at Curve

Spacing of Pull-offs at Curves. The number of pull-offs should be sufficient to keep the wire within about 2½ in from the theoretical curve. This may be accomplished by spacing the successive ears in accordance with the following relations:

Let L = distance between pull-offs in feet;
R = radius of curve in feet;
a = offset of wire in inches from theoretical curve, midway between pull-offs, the ears being assumed to lie on the theoretical curve.

Then
$$L = \sqrt{\frac{2\,aR}{3} - \left(\frac{a}{6}\right)^2},$$

Or
$$L = 0.815\,\sqrt{aR},$$

with an error less than ¼% for all radii greater than 40 ft.

If a is to be 2½ in, then

$$L = 1.29\,\sqrt{R} \text{ approximately.}$$

If the ears are set exactly on the theoretical curve, the wire will depart 2½ in from the correct position half way between the two ears; if the ears are set 1¼ in from the curve, the mid-point of the wire will be only 1¼ in when L is taken equal to $1.29\,\sqrt{R}$.

Offset of Trolley Wire at Curves. It is usual, at curves, to offset the trolley wire from the center of the track because the trolley wheel is tilted inward due to the elevation of the outer rail, and because in order to keep the wheel on the wire, the latter must be so placed that the projection of the pole on the plane of the track is always tangential to the wire.

(*a*) The former offset, measured horizontally toward the inside of the curve, uals $h \tan \theta$ where h is the normal height of the wire above the top of rail, d θ is the angle of elevation between the plan of the track and the horizontal. or standard gage and wire 18 ft above the top of rail this offset is about 4 in r every inch of elevation.

(*b*) The latter offset, also measured horizontally toward the center of the curve, calculated as follows, assuming the curve to be longer than the car itself; r shorter curves the offset will be less. Let

R = radius of curve;
L = horizontal distance of center of trolley wheel to center of trolley base usually about 11 ft;
G = distance from center of car to center of truck.

the case where the trolley base is located over the truck center, as on cars ith two trolleys, the offset is

$$R - \sqrt{R^2 - L^2}.$$

the trolley base is located over the car center as on cars with one trolley, e offset equals

$$R - \sqrt{R^2 - G^2 - L^2}.$$

he total offset is the sum of the offsets (*a*) and (*b*).

Curves of Small Curvature. If the curvature* is less than 10°, the poles e frequently spaced closer together and no pull-offs used. The table may be nsidered typical, the divergence of the wire from the theoretical curve being pt within 3½ in.

Spacing of Poles on Curves, Interurban Road

Degree of Curvature of Track	Pole Spacing, Ft	Divergence of Trolley Wire from Track Center, Inches	Degree of Curvature of Track	Pole Spacing, Ft	Divergence of Trolley Wire from Track Center, Inches
Tangent	120	0.	6	60	2.87
1	120	1.87	7	50	2.34
2	110	3.37	8	50	2.64
3	90	3.06	9	50	2.94
4	80	3.37	10	50	3.24
5	70	3.06			

Turnouts. The location of the trolley frog at turnouts may be determined follows: Referring to Fig. 34, from switch point A draw a line to center oint D of track frog distance BC and om switch point B draw a line to center oint E of arc AEC. The intersection of ese two lines at F will be the proper cation of the frog. While certain variles, such as superelevation of the outer il on the curve, length of wheel base d projection of trolley pole rearward om center of car, may necessitate slight variation of setting, this location

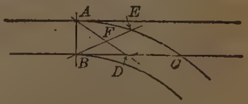


Fig. 34. Location of Frog

* The degree of curvature of a track is the angle which subtends a chord 100 ft long, tween points on the center line of the track.

will be found so nearly correct that a very small alteration which must be determined by experiment, will compensate for the variable conditions.

The following table gives the range of distance from track switch point to track frog with which each set of trolley frogs may be most satisfactorily used:

Track-frog Distance	Divergence Angle of Trolley Frog
Up to 22 ft.....	20°
From 20 to 30 ft	15°
Above 28 ft.....	8°

The minimum frog distance given in the table with which the 15° frogs may be used to best advantage corresponds to a turnout radius of 40 ft, but when suburban cars, using high-speed trolley wheels, run over city tracks it is advisable to use 15° frogs rather than 20° frogs throughout the city construction even where the minimum frog distance is less than 20 ft.

Simple and Compound Catenary Construction. Catenary construction may be divided into simple and compound. In the former the trolley wire or wires are carried by one or two messenger cables which are supported only at the poles, bents or span wires; in the latter, the horizontal wire or wires are carried by a messenger cable, which is itself suspended from another messenger cable which is supported at the poles, bents or span wires. The advantages of the compound catenary are greater flexibility, reduced stresses in the supporting wires, shorter hangers, better lightning protection and superior curve construction.

To obtain a line which will not require frequent readjustment, the messenger cable must be installed with practically uniform tension throughout its length making it necessary to have less sag in the shorter spans. For this reason certain definite pole spacings and corresponding hanger lengths have been standardized.

Number of Suspensions. The number of suspensions depends upon the speed at which the cars are to be run, and upon whether a bow or wheel trolley is used. The three-point suspension in which, with 150 ft spacing, the hangers are 50 ft apart has been found ample for wheel collectors. With the sliding pantagraph or bow trolley an eleven-point suspension has been found sufficient with 150 ft pole spacing. (See following Table.)

Hangers. Where only one horizontal wire is suspended from the messenger cable, the hangers should hang loosely from the cable and should be screwed fast to the trolley wire. This permits the trolley wire to rise slightly as the trolley passes under it, thereby making the wire equally flexible along its entire length. Where a steel contact wire is clipped to the horizontal conductor, the hangers are usually rigidly attached at both ends, as the duplication of horizontal wires assures uniform flexibility, regardless of the hangers. (See following Table.)

Wear of Trolley Wheel, Bow and Trolley Wire. The wear of the trolley wires is not serious with either wheel or bow collectors. On the lines of the Indianapolis & Cincinnati Traction Company, a copper trolley wire lost less than 1% in weight after it had experienced 39,000 car movements, each car taking an average of about 40 amperes by an aluminum slider.

The vertical wear of the steel contact wire on the N. Y., N. H. & H R.R. was 0.028 in in 30 months, which is practically 4.5% per year of the half diameter

Tangent Construction

Number of Hangars per Span. Pantagraph or Bow Trolleys

Length Pole Spacing, Feet	Number of Points of Suspension.	Length of Hangers, Inches										
		6	6¾	8½	11	12	13½	14¾	16	17½	19¼	20½
150	11	1	2	2	2	....	....	2	....	....	2	....
125	9	....	....	....	1	2	2	....	2	....	2	....
110	8	....	....	....	....	....	2	2	....	2	....	2
95	7	....	....	....	....	....	....	....	[illegible]	2	....	2
80	6	....	....	....	....	....	....	....	....	2	2	2
70	5	....	....	....	....	....	....	....	....	....	3	2
55	4	....	....	....	....	....	....	....	....	....	....	4

Number of Hangers per Span. Wheel Trolleys

Length Pole Spacing, Feet	Number of Points of Suspension	Length of Hangers, Inches							
		6	11	13½	14¾	16	17½	19¼	20½
150	3	1	......	......	2	......	......	......	......
125	3	......	1	......	......	......	2	......	......
110	3	......	......	1	......	......	2	......	......
9[illegible]	3	......	......	......	......	1	......	2	......
80	3	......	......	......	......	......	1	2	......
70	2	......	......	......	......	......	......	......	2
55	2	......	......	......	......	......	......	......	2

of the wire (one-half taken to permit wire to be held in clips) which, even on this vertical diameter basis, indicates a life of over 20 years; but as a matter of fact it will be much more than this, for the reason that as the vertical diameter lessens the breadth of contact increases thruout, thus diminishing the rate of vertical wear. Of further interest is the fact that there is practically no corrosion of the wire as the wire is covered with a film of grease deposited by the pantograph shoe (W. S. Murray).

Change in Length of Trolley Wire. The change in length of copper due to changes in temperature is one of the greatest difficulties in the maintenance of overhead work. A drop of 100° F. in temperature will cause a copper bar to contract approximately 1 in for every 100 ft of length. If it be restrained at the ends this will cause an additional stress of 2500 lb in the case of a No. 0000 trolley.

European catenary lines are usually maintained at constant tension by means of weights pulling on the free ends of the trolley wire at the end of every section.

Prevention of Formation of Sleet. Sleet may be prevented from forming on the wires by greasing the latter with petroleum jelly and if it does form it may be easily removed by any of the numerous commercial forms of sleet cutters.

11. Third Rail Systems

General. Third rails are used for low voltage railways where the power requirements are in excess of the current carrying capacity of a trolley wire, and where there is private right-of-way.

Collector Shoes. Current is collected from the third rail by projecting collector shoes on the cars. These shoes are placed on each side of each truck, and are all alive, when any one is alive.

Rails are usually made of special high conductivity steel, it being not unusual to have a conductivity $^1/_6$ or $^1/_7$ that of copper. (See Article 12 for resistance of rails of various sections.)

Types of Construction. Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

(*a*) **Interborough Top-contact Type.** One of the most commonly used types is illustrated in Fig. 35. It is often called the "Interborough Type" on account of its use in the subways of the Interborough Rapid Transit Co., of New York. The rail is a standard T-section and rests on reconstructed granite insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

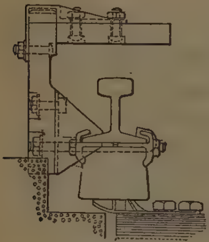


Fig. 35. Interborough Top-Contact Type

(*b*) **Pennsylvania Top-contact Type.** (Fig. 36.) Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail, when repairs are being made, but it cannot be relied upon as well as the Interborough type to keep the rail and protection in perfect alignment.

(*c*) **Under-contact Types.** (Fig. 37.) While the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed locations as they cannot be wholly protected from snow, ice and sleet. The lower

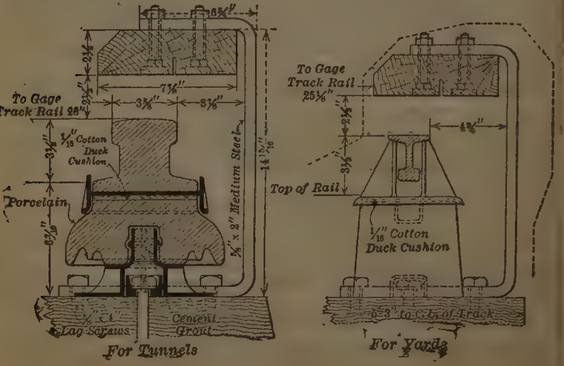


Fig. 36. Pennsylvania Top-Contact Type

part is only a few inches from the ties, while holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow, and from flooding. The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped in insulators by hook-bolts hung from brackets, with the top and sides of the rail completely sheathed in a flexible insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow and spray. With this type of rail the protection is of such character that there is no packing of snow between the sheathing and the contact rail, as in some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but hanging down clear of the edge of the rail, are easily broken off by the passing shoe.

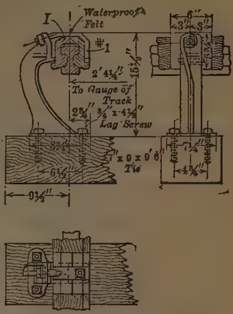


Fig. 37. N. Y. C. R. R. Under-Contact Type

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type.

Location and Weight of Third Rails. There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right-of-way and in the maximum equipment lines of various railroads. The following standard has been recommended (1911 and 1912) by the American Electric Railway Engineering Association:

The gage line of the third rail to be located not less than 26 in and not more than 27 in from the gage line of the track and the contact surface of the third rail to be not less than 2¾ in or more than 3½ in above the plane of the top of the track rail.

This standard has since been abandoned and three standard clearance lines adopted instead. The maximum equipment line for rolling equipment is given in Article 13; those for third rail structures and permanent way, respectively, are given below:

Third-rail Structures Must be Contained within the Line Indicated by the Following Ordinates and Abscissas

Height Above Top of Rail, Inches	Distance Out from Gage Line, Inches
Top of Tie	19¾
⅝	19¾
3¹¹⁄₁₆	25
8	25
9⅞	26⅞
9⅞	33¾
6⅛	36¾
¼	36¾
—2¼	35¼
Top of Tie	35¼

(On curves of less radii than 800 ft and at switches, this line must be moved out 2½ in.)

Permanent Way Structures Must not Encroach upon the Line Indicated by the Following Ordinates and Abscissas

Height Above Top of Rail, Inches	Distance Out from Gage Line, Inches
Top of Tie	36¼
—2½	36¼
0	37¾
$7^5/_{16}$	37¾
10⅞	34¼

(On curves of less radii then 800 ft and at switches, this line must be moved out 2½ in.)

Location and Weight of Third Rail

Name of Railroad	Center of Third Rail to Near Gage Line, Inches	Contact Face above Top of Running Rail, Inches	Weight, Pounds per Yard
Albany & Hudson	27	6	....
Aurora, Elgin & Chicago	20⅛	$6^5/_{16}$	100
Baker St. & Waterloo Ry	....	....	90
Baltimore & Ohio	30	3⅛	....
Berlin Elevated and Subway	13.25	7	....
Boston Elevated and Subway	20⅝	6	85
Brooklyn Rapid Transit	21¾	6	70
Camden & Atlantic City R.R	26	3½	100
Central London Ry	Center	1½	....
Columbus, London & Springfield	27	6	....
Columbus & Newark	27	6	....
Fayet-Chamounix	23	9	....
Grand Rapids, Gd. Haven & Muskegon	20⅝	5¾	....
Great Northern Ry., England	19¼	....	80
Hudson Tunnels, New York	26	4	75
Interborough Rapid Transit	26	4	75
Kings County El., Brooklyn	19½	5¼	....
Lackawanna & Wyoming	20⅝	3	75
Lake St. El., Chicago	20⅛	6½	....
Lancashire & Yorkshire Ry	19¼	3	70
Liverpool Elevated	Center	1½	....
Long Island R.R	27	3½	100
Manhattan Ry., New York	20¾	7½	100
Mersey Ry	22	4½	....
Metropolitan & District, London	16	3	100
Metropolitan Elevated, Chicago	20⅛	6¼	48
*New York Central R.R	28¼	2¾	70
Northeastern Ry., England	19¼	3¼	80
Northwestern Elevated, Chicago	20⅛	6½	48
North Shore R.R. Cal	27	6	50-60
Paris Orleans Ry	25⅝	7⅞	....
Paris Versailles Ry	25⅝	7⅞	....
Pennsylvania R.R	27	3½	150
*Philadelphia & Western	27	3⅝	....
*Philadelphia Rapid Transit	27	6	70
Seattle & Tacoma R.R	20	7½	100
South Side Elevated, Chicago	20⅛	6¾	....
Wamseebahn (Berlin)	33½	12⅝	....
Waterloo & City Ry	28¼	0	....
West Jersey & Seashore	26	3½	100
*West Shore R.R	32	2¾	70
Wilkesbarre & Hazelton	28	5	80

* Bottom-contact surface. All others have top-contact surface.

Inclines. Where it is necessary to break the third rail at cross-overs, grade crossings or electrical section breaks, END INCLINES are required to catch the contact shoes and bring them to the normal level of the third rail.

The exact location of the inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the car equalizer bar and journal box project outward. If there is a train-bus line connecting all the cars, the end inclines may be situated many feet back from the switch point of frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.

12. Rail Bonds

General. Rail bonds are electrical conductors for bridging the joints of rails. They consist either of a series of thin strips of annealed copper, or of one or more cables of copper wire, the ends of which are usually prest or cast into solid terminals. Ribbon is more compact, but stranded wire is more flexible; the latter should always be used if space permits.

Types of Bonds. Bonds may be classified, according to the method of

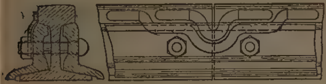


Fig. 38.

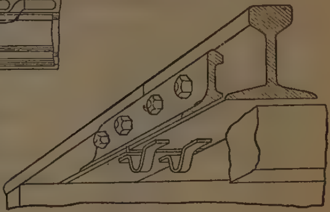


Fig. 39.

fastening them to the rail, as soldered bonds, brazed bonds and bonds applied by mechanical pressure. Soldered bonds and compress terminal head bonds find their best application in third-rail work, where good electrical contact is of greater importance than mechanical strength. Expanded terminal web bonds, especially of the concealed type with two stranded conductors, are regarded as the best for heavy track work, where mechanical strength and cost of installation are of the utmost importance.

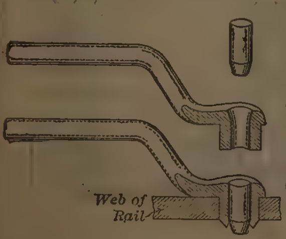


Fig. 40

Soldered Bonds usually consist of a series of thin strips of annealed copper with tinned terminals as shown in Figs. 38 and 39. They are soldered direct to the head, foot or web of the rail. One or more bonds per joint may be used.

Brazed Bonds resemble soldered bonds except that the terminals are enveloped in brass. They are brazed or welded to the rail by heat generated electrically in a carbon electrode which constitutes one jaw of a clamp holding the bond against the rail.

Expanded and Comprest Terminal Bonds. Bonds fastened to the rail by mechanical pressure may be divided into two general classes, expanded terminal and comprest terminal bonds.

Pin-expanded Terminal Bonds (Fig. 40) have their heads drilled with an axial hole, through which a tapered steel pin *d* is driven, forcing the copper outward and against the steel. This type of bond is fastened to the web of the rail.

Comprest Terminal Bonds. (Fig. 41 and 42.) There are two kinds of comprest terminal bonds, in one of which direct pressure is applied at both ends of the head,

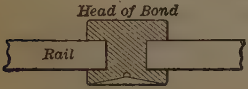


Fig. 41

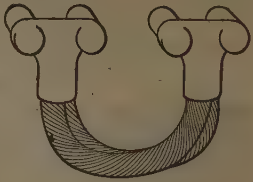


Fig. 42

and in the other, at one end only. The first type of bond is usually applied to the web of the rail by means of a heavy screw or hydraulic press (Fig. 43)

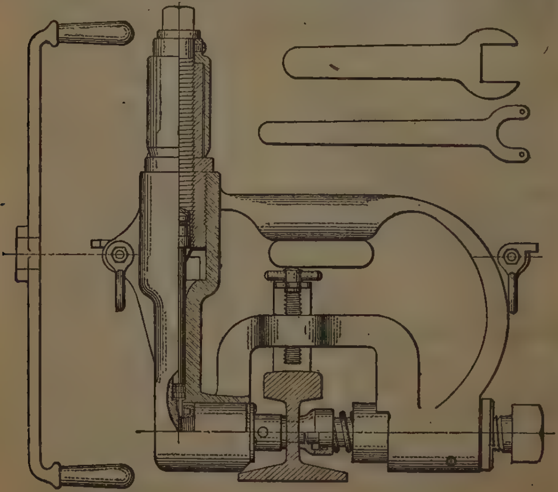


Fig. 43

which engages the bond head and causes it to compress longitudinally and expand laterally as the pressure is applied, bringing the copper into firm contact with the steel and spreading the projecting end of the terminal into a button-

shaped rivet-head, as shown in Fig. 41. The second type of bond (Fig. 42) is applied only to the head of the rail, the terminal lugs being set in holes therein and expanded into contact by means of hammer blows.

Exposed Versus Concealed Bonds. Whether soldered, brazed, expanded or comprest, bonds may be either exposed or concealed (Fig. 44) under the fish plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made. Where the bonds are exposed to theft, as, for example, on track rails unprotected by paving, concealed bonds are almost a necessity.

While concealed bonds are necessarily applied to the web of the rail, exposed bonds may be applied to the foot or head. Head bonds have the advantage of greater contact surface at the terminal studs, while foot bonds are less exposed to mechanical violence. Web bonds, unless concealed, have to be excessively long in order to span the fish-plates.

Substitutes for Bonding. Several efficient substitutes for bonding are now in use, such as electrical welding, thermit welding and the "Romapac" continuous rail system. (See Article 21.)

Installation of Bonds. The foremost consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Soldered Bonds. The rail surface is brightened by means of a carborundum or emery wheel, and tinned, using an acid flux. The bond is then clamped in place and the rail and bond heated by means of a blow-torch, to a temperature at which the solder will melt and cause the bond to adhere firmly to the rail.

Fig. 44

Brazed Bonds. The preliminary processes are the same as for soldered bonds except that a special clamp is used, the terminals of which are the electrodes of an electrical circuit, one being of copper and the other of carbon. The surface of the rail being previously ground bright at the point where the braze is to be made, the brass-enveloped bond terminal is prest against the rail by the carbon electrode, the copper electrode being in contact with the opposite side of the rail. The current on passing from one electrode to the other, traverses the bond terminal and rail, the carbon becoming incandescent. The incandescent carbon (pressing the copper against the rail) quickly transmits sufficient heat at exactly the point where it is required to produce the weld.

Welding Outfit. It is claimed by the manufacturers that an average of over 100 bonds per day are readily installed by a car operating with four men, a bonder and three helpers. Such a car carries a rotary converter and transformer, with accessory apparatus. To weld an average-sized rail bond to the rail, an alternating current of about 2000 amp at 5 volts is employed. On direct-current railways this is obtained by converting and transforming current taken from the trolley wire.

Pin-expanded Bonds. The rail is drilled, usually through the web, with or without lubricant, using some form of drill especially adapted to this service. Drilling without lubricant has the advantage of giving a perfectly clean hole, but is believed by some to cause excessive wear of the drills. It is doubtful, however, whether the small amount of oil which would be used could be kept

constantly at the cutting edges, the only places where it would be useful. Dry drilling has been found successful on many railroads. If oil is used, it should be wiped out with a clean cloth saturated with gasoline, in which case the joint resistance will be increased less than 3%. Lubricants containing water are likely to cause rust, especially if the drilling gang precedes the bond installers by any considerable time. The hole having been drilled, the bond head is inserted into it and a long taper punch lubricated with grease is driven entirely through the terminal. Then a short drift pin is driven home, as shown in Fig. 40.

This type of bond requires a smaller equipment in tools and materials than most other types and does not necessitate the use of any apparatus which obstructs the track and would thereby endanger traffic.

Compress-Terminal Web and Foot Bonds. The drilling having been performed as for a pin-expanded bond, and the bond heads inserted into their respective holes, a screw or hydraulic compressor, as shown in Fig. 43, is applied at both ends of the bond head, the conical point of the press fitting into the conical depression of the bond. Pressure is applied either until a collar on the ram touches the rail, or until the head of the bond acquires the proper shape. Where no collar is used the point of the press (if of the screw type) sometimes cuts into the bond head; this may be avoided by placing a small amount of flake graphite mixed with oil in the depression of the bond head.

Compress-Terminal Head Bonds. A four-spindle drill is used to drill four holes simultaneously in the rail heads. It is important to avoid drilling the holes too deep lest the copper should not touch bottom and therefore be unable to expand laterally. If, on the other hand, the hole is too shallow, expansion will occur too soon.

Tests of Bonds after Insulation. Every rail in service should be periodically tested and a complete record of the tests kept. The frequency of the tests will depend upon local conditions, once in 9 months being an average for 14 urban railways, and once in 12 months an average of 11 interurban railways (See Proc. Am. El. Ry. Assn., 1911, p. 751.)

Resistance Test. The usual method of testing is to measure the drop of potential across the bonded joint and find simultaneously the length of continuous rail in which the same drop occurs, i.e., the "equivalent" length of the bonded joint. Several ingenious instruments have been devised for making this comparison with ease and accuracy.

Resistance of T-rails, A.S.C.E. Standard Section (20° C.)
(Full Cross-section)

Weight, Pounds per Yard	Cross-section, Square Inch	Area, Millions of Circular Mils	Spec. Res. 12.5 times that of Copper		Spec. res. 8 Times that of Copper*	
			Ohms per 1000 ft	Ohms per Mile	Ohms per 1000 ft	Ohms per Mile
70	6.9	8.77	0.0148	0.0779	0.00944	0.0499
75	7.4	9.45	0.0138	0.0729	0.00884	0.0467
80	7.8	9.9	0.0131	0.0689	0.00835	0.0441
85	8.3	10.5	0.0122	0.0645	0.00781	0.0413
90	8.8	11.2	0.0115	0.0609	0.00738	0.0390
95	9.3	11.8	0.0109	0.0570	0.00701	0.0370
100	9.8	12.5	0.0104	0.0547	0.00664	0.0350

* To find the resistance of rails of any specific resistance x referred to copper as unity multiply these resistances by x and divide by 8.

Mechanical Strength. The mechanical adhesion of soldered bonds may be tested by means of a lever as shown in Fig. 45. It may be used as soon as the terminals are cool. The operation of testing consists simply in submitting each bond terminal to a predetermined pull. A properly soldered bond should stand a shearing force of 1200 lb per sq in of contact. Calling S this shearing force per square inch, A the square inch of contact, and P the pull, as registered on the balance, then

$$P = \frac{ASl}{L}.$$

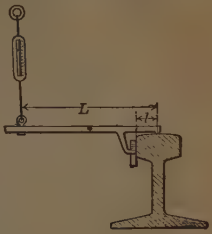


Fig. 45

Rebonding. The resistance at which a joint should be rebonded depends upon how much potential drop is permissible in the tracks, and upon the relative cost of the energy loss and the cost of rebonding. In the Proc. Am. Elec. Ry. Assn., 1911, this resistance is given by 22 railways, in terms of the equivalent length of continuous rail. This length ranged from 2 to 12 ft, with an average of 6.6. ft.

CARS AND LOCOMOTIVES

13. Cars

City cars vary in length from 42 to 50 ft, the average of twenty typical city cars being 45 ft 6 in over the bumpers. The width varies between 8 ft and 8 ft 10 in, the average of twenty typical cars being 8 ft 4 in. Truck centers vary from 19 to 24 ft, averaging 21 ft 10 in. The number of seats varies from 32 to 56, the average of twenty typical cars being 47.

The standard height of couplers for city cars, measured from top of rail to center of coupler is 1 ft 8 in. The standard height of bumpers is 2 ft 7 in from top of rail to top of bumper. The standard height of platforms is 2 ft 7 in from top of rails.

Recent trend has been toward the drop platform type with rear entrance, and front exit, as it is one of the safest types and seems ideal for quiet loading and unloading of passengers. The center door type has also found favor. These types lend themselves readily to the pay-as-you-enter system of fare collection.

Interurban cars vary in dimensions from those of city cars to those of standard steam passenger cars, depending on the class of service, speed and length of run. A typical recent car is 61 ft long 8 ft 1 in wide, 38 ft 4 in truck centers, and weighs 85 600 lb complete. The standard height of bumpers for interurban cars is 4 ft 3 in from top of rail to top of bumper. The usual height of platform is 4 ft 3 in.

Suspension of Motors. Motors, on double-truck cars, are hung between an axle and the transom. A lug cast on the side of the motor frame away from the axle rests on the transom either with or without intervening springs. About 60% of the weight is carried directly on the axles.

Maximum Equipment Line. The maximum equipment line for cars on third rail systems has been standardized as follows by the American Electric Railway Engineering Association and other interested bodies.

Points Determining Maximum Equipment Line

Height Above Top of Rail, Inches	Distance Out from Gage Line, Inches
6	0
6	2½
20	2½
24	4 11/16
24	9
25⅞	10⅛
31	10⅞
34¼	24

Trucks. Cars not more than 20 ft long can be satisfactorily carried on single trucks having a wheel base of about 7 ft. Longer cars should have two trucks of the swivel type, having a wheel base of from 4 ft to 6 ft 6 in. the shorter wheel base being used where there are sharp curves. Maximum traction trucks have unequal size pairs of wheels the bolster being near the larger pair, to the axle of which the motor is geared. This has the effect of throwing from 70 to 85% of the total weight on the driving wheels, although only two motors are used on the car, an especial advantage where high acceleration is desired.

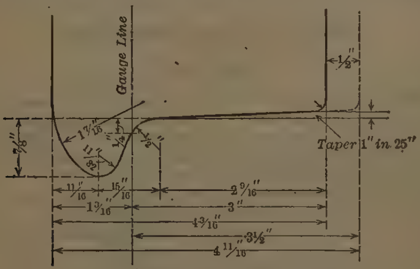


Fig. 46. Standard Sections of Wheel Flanges

Wheels. Electric railways use the standard gage of 4 ft 8 in. The standard form of wheel flange is shown in Fig. 46.

14. Locomotives

General. Electric locomotives, unlike steam locomotives, can be made of practically any desired power capacity without excessive complication and without increase in the number of operators. This arises from the absence of boilers and the possibility of controlling a number of motors or even a number of complete locomotive units as if they were a single unit.

Rating. The American Institute of Electrical Engineers has standardized the rating of electric locomotives in terms of the weight on drivers, the nominal 1-hour tractive effort, the continuous tractive effort and corresponding speeds.

(*a*) Weight on Drivers. The weight on drivers, exprest in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

(*b*) Nominal Tractive Effort. The nominal effort, exprest in pounds, shall be that exerted at the rims of the drivers, when the motors are operating at their nominal (1 hour) rating. (See Art. 15.)

(*c*) Continuous Tractive Effort. The continuous tractive effort, exprest in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full voltage continuous rating. (See Art. 15.)

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for ½ or ¾ voltage, but in such cases the voltage shall be clearly specified.

(*d*) Speed. The rated speed, exprest in miles per hour, shall be that at which the continuous tractive effort is exerted.

Classification. Locomotives may be classified as follows, with reference to trucks:

1. Rigid wheel base.
   - (*a*) Without leading and trailing trucks.
   - (*b*) With leading and trailing trucks.

2. Separated bogie truck (the trucks being connected through the upper frames as in a car).
   - (*a*) Symmetrical
   - (*b*) Unsymmetrical.

3. Articulated (wherein the two trucks are hinged back to back).

All of these types of locomotives are practically steady at speeds under 40 mph but above this speed marked differences appear: the steadiest riding machines, according to G. Gibbs, having high centers of gravity and long unsymmetrical wheel bases. Considerable difference of opinion prevails in regard to proper mechanical design for high speeds.

Transmissions. Motors are connected to locomotive drivers by the following forms of transmission:

(*a*) Gearless, the motor armatures being mounted direct on the axles.

(*b*) Gearless quill, each motor armature being mounted on a hollow shaft or quill which surrounds the axle, but is free with respect to it. At each end of the quill is a disc having projecting arms which fit into spaces between the spokes of the driving wheels. The arms are connected to the driving wheels by springs which tend to keep the arms centered between spokes but allow sufficient play to relieve the excessive starting stresses in the armature.

(*c*) Direct gears, the motors being mounted either between or over the axles, a motor having a pinion engaging a gear wheel on the locomotive axle.

(*d*) Quilled gears, the gear wheel being on a hollow shaft or quill which engages the drivers through springs, thereby relieving both the armature and gears of excessive starting stresses.

(*e*) Cranks and Scotch yoke, in which two motors are connected together and have crank shafts which operate three sets of drivers.

(*f*) Crank and countershaft, in which the motors are located above the tracks and are connected through a countershaft to side rods joining the drivers.

Control. Practically all modern electric locomotives have multiple unit control, so that two or more locomotives may be coupled together under the control of one operator. Indeed some locomotives consist of two sections, each of which is virtually a separate locomotive, coupled together permanently. Such a locomotive weighing 157 tons, with all the weight on the drivers, is in use on the Pennsylvania Railroad.

Weight. The weight per driver axle for high speed locomotives should not exceed 40 000 lb with ordinary track, and 50 000 lb with very good track, and in

slow-speed service more usual weights are 15 000 to 20 000 lb per driver axle. European practise indicates a maximum of 35 000 to 40 000 lb per axle.

The weight in pounds per foot of total wheel base usually varies from 4500 to 7500 lb but is occasionally as high as 11 000 lb.

The following weight analysis by E. P. Burch (Electric Traction for Railway Trains) is based upon data for about twenty representative American and European locomotives.

Weight Distribution, Electric Locomotives

PER CENT

Distribution of Weight	Direct Current		Three Phase		Single Phase		Motor Generator
		Ave.		Ave.		Ave.	Ave.
Mechanical parts......	50–72	68	48–56	51	46 59	58	43
Motor..............	20–27	24	26–40	30	26–36	27	30
Transformer..........	0	0	0–10	10	8	7	6
Other electrical parts ..	5–10	8	7–10	9	7–11	8	21
HP 1 hr per ton, about.	....	16	....	18	....	14	8

The mechanical parts of direct-current locomotives are high in percentage because the total weights are low. Three-phase motor weights appear high because European designers use light frames.

15. Motors and Controllers

Selection of Motor. The following information relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(*a*) Weight of total number of cars in train (in tons of 2000 lb) exclusive of electrical equipment and load.
(*b*) Average weight of load and durations of same, and maximum weight of load and durations of same.
(*c*) Number of motor cars or locomotives in train, and number of trailer cars in train.
(*d*) Diameter of driving wheels.
(*e*) Weight on driving wheels, exclusive of electrical equipment.
(*f*) Number of motors per motor car.
(*g*) Voltage at train with power on the motors—average, maximum and minimum.
(*h*) Rate of acceleration in miles per hour per second.
(*i*) Rate of braking (retardation in miles per hour per second).
(*j*) Speed limitations, if any (including slowdowns).
(*k*) Distances between stations.
(*l*) Duration of station stops.
(*m*) Schedule speed including station stops in mph.
(*n*) Train resistance in pounds per ton of 2000 lb at stated speeds.
(*o*) Moment of inertia of revolving parts, exclusive of electrical equipment.
(*p*) Profile and alignment of track.
(*q*) Distance coasted as a percent of the distance between station stops.
(*r*) Time of layover at end of run, if any.

Rating of Motors. The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator, and 75° C. at any other normally accessible part after one-hour's continuous run at its rated voltage (and frequency in the case of an alternating-current motor) on a stand with the motor covers arranged

to secure maximum ventilation without external blower. The rise in temperature as measured by resistance, shall not exceed 100° C. (The statement of the nominal rating shall also include the corresponding voltage and armature speed.)

The continuous ratings of a railway motor shall be the inputs in amperes at which it may be operated continuously at ½, ¾ and full voltage, respectively, without exceeding the specified temperature rises (see table below) when operated on stand test with motor covers and cooling system, if any, arranged as in service. Inasmuch as the same motor may be operated under different conditions as regards ventilation, it will be necessary in each case to define the system of ventilation which is used. In case motors are cooled by external blowers, the flow of air on which the rating is based shall be given.

Maximum Permissible Temperatures and Temperature Rises

Class of Insulation	Maximum Observable Temperature of Windings when in Continuous Service		Stand-test Temperature Rise of Windings	
	By Thermometer	By Resistance	By Thermometer	By Resistance
A. Cotton, silk, paper and similar materials when so treated or impregnated as to increase the thermal limit	85° C.	110° C.	65° C.	85° C.
B. Mica, asbestos and other materials capable of resisting high temperatures in which any class A material or binder is used for structural purposes only, and may be destroyed without impairing* the insulating or mechanical qualities of the insulation	100° C.	130° C.	80° C.	105° C.

* The word impairing is here used in the sense of causing any change which would disqualify the insulation for continuous service. For infrequent occasions, due to extreme ambient temperatures, it is permissible to operate at 15° C. higher temperature.

Voltages. Direct current motors are usually made for line voltages of 500, 550 or 600 volts and sometimes for 1200 volts or somewhat more. On 2400 volt lines, it is usual to have the motors in pairs connected permanently in series.

Alternating current, single-phase motors, are usually made for 400 to 500 volts, the required voltage being obtained by transforming down the line voltage by means of a transformer or auto-transformer on the train.

Weight. Motors vary in weight from 30 lb per horse-power (40 lb per kilowatt) for the largest sizes (200 hp) to 70 lb per horse-power (93 lb per kilowatt) for the smaller sizes (35 hp), the ratings being the nominal.

Controllers. The speed of direct current motors is controlled by connecting resistances in series with the motors, by connecting the motors at first in series and then in parallel, and sometimes by varying the strength of the fields. Alternating current motors are similarly controlled, except that auto-transformers are used instead of resistances.

The direction of rotation, in both kinds of motors, is changed by reversing the current in *either* the fields or armature, this being usually accomplished by rotating an auxiliary handle of the controller.

The type K controller is largely used for light weight cars, and consists of an operating handle which moves a cylindrical drum with projecting contact pieces which come in contact with stationary fingers. The first three points correspond to accelerating steps, by means of which the resistance in series with the motors is gradually cut out. The fourth step, full series, gives about half speed and is a continuous running point. The following steps restore the resistance and put the motors in parallel. The last step leaves the motors in parallel without any resistance.

Multiple-unit control is used on large cars, especially where the combined motor capacity exceeds 300 hp. The equipment consists of a small master controller which enables a comparatively weak current to operate contactors large enough to make the necessary changes in the circuit, as in the case of type K controllers. The master controller on any car will operate the contactors on all cars, if a jumper cable be run from car to car connecting together all the control circuits.

The weight of control equipment of type K varies from 1200 to 2250 lb and of multiple-unit equipment, from 2800 to 3200 lb.

16. Brakes

General Principles. Practically all existing brakes make use of the frictional adhesion between the wheels and track, and between the brake shoes and wheels. The frictional adhesion between wheels and track varies from 15 to 30% of the weight on the wheels. An adhesion of 15% applies to normal track and the higher values to sanded track. This refers to rolling friction; if the wheels begin to slide, the coefficient of friction drops. For this reason braking must be such as not to allow the wheels to slip. The frictional adhesion between brake shoes and wheels is given by the formula,

$$h = \frac{1 + 0.000\,472\,l}{1 + 0.00\,2390\,l}F,$$

where h = coefficient of friction after elapsed distance l;
l = distance wheel has traveled in frictional contact with the brake shoe;
F = coefficient of brake shoe friction at speed and pressure at which brake shoe was applied at the beginning of the distance l
$= \dfrac{0.382}{1 \times 0.02933\,S}$ where S = speed, mph.

The braking retardation depends upon the various factors involved, approximately as indicated by the following formula:

$$a = 0.01098\,k\,(Ph + fW);$$

where a = retardation, mph per second;
k = ratio of linear inertia to total inertia of train;
P = total braking pressure applied normal to wheel treads by brake shoes, pounds;
h = cœfficient of brake friction;
f = train resistance, pounds per ton weight of train;
W = weight of train, tons.

If we neglect rotational energy and train resistance

$$a = 0.01098 \frac{R}{W}$$

where R = total retarding force, lb.

The usual rate of retardation for multiple unit electric trains is from 1.5 to 2.0 miles per hour per second, and on street cars from 2 to 2½ miles per hour per second.

Construction. Any brake which depends upon the friction between the wheels and brake shoes, is composed of four parts: the shoes, the truck rigging, the foundation rigging and the source of braking force.

Brake Shoes. In order to avoid excessive costs for the renewal of brake shoes, the weight should be limited so that no individual shoe should weigh more than 24 lb. To avoid an excessive loss of weight in scrap, the minimum weight should be 20 lb. The cast iron should be closely granular, of uniform texture and with the combined and graphitic carbon evenly balanced. The graphitic carbon should be in the form of nodules rather than flakes.

Wear averages 3.75 to 6.5 lb per 1000 wheel miles and the minimum scrap weight should be 6¼ lb per shoe; at 5½ lb there is danger of cutting the head.

Ordinarily, brake shoes are placed to bear on the inside of the truck frame; i. e., between wheels. With this arrangement, it is possible, by varying the angularity of the hanger link, to introduce a force which will equalize, to any desired degree, the transfer of weight from the rear to the forward axle.

Truck Rigging. The braking force must be distributed equally between the brake shoes, or there will be danger of sliding the wheels. Separate levers are used for each shoe. On double-truck cars, which must travel around sharp curves, the bar connecting the brake levers is made of circular form, and is then known as a "radius bar"; and the brake pull rod is connected to it thru a roller working in a clevis, which allows the brakes to act in spite of the swivelling of the truck.

Foundation Brake Rigging. The pull rods which pull on the radius bar rods are each attached to one end of the cylinder levers, the centers of these levers (which are normally about parallel) being joined by a rod. The other ends of the cylinder levers are connected to the piston and the slack adjuster respectively.

The purpose of the slack adjuster is to pull back the piston without loosening the brakes in case it travels so far as to unduly reduce the cylinder pressure. This is usually accomplished by causing the excess piston motion to operate a device which shifts the fulcrum of one of the cylinder levers. The normal piston travel is 8 in on standard equipment.

Hand Brakes. It is usual to provide all cars with hand brakes, even if also equipped with air brakes. Hand brakes communicate their motion to the foundation rigging by means of a chain which winds on the brake staff at one end and is attached, thru a multiplying lever, to the pull rods. In high-ratio hand brakes, the bottom of the brake staff carries a gear, the chain connection being made through the meshing gear.

Air Brakes. Air brakes have the advantage of being capable of quicker application than hand brakes, and they save power both by permitting more coasting and by making sure that the brake shoes do not rub when the car is running, a condition which frequently occurs with hand brakes due to the desire of the motorman to be able to apply the brakes as quickly as possible.

The maximum pressure of the brake shoes on the wheel may be definitely limited by proper design and wheel skidding prevented. Air brakes also relieve the motorman of severe manual strain.

The New York Public Service Commission requires cars weighing more than 25 100 lb to be equipped with air brakes.

There are two kinds of air brakes: the "straight" and the "automatic." In the former, the pressure of a main reservoir acts directly on the foundation rigging; in the latter, the pressure comes from an auxiliary reservoir, the main air pressure being used to control the admission of air to and from the auxiliary reservoir and to release the air from the brake cylinder to the outside atmosphere. In "straight" air brakes, the control consists essentially of a valve which can connect the cylinder to the main reservoir, can disconnect the cylinder yet allow it to retain the air, or can release the air from it. This system is used for single car operation.

In long trains, the air has so far to flow from the reservoir on the forward car or locomotive, that the brakes on the rear cars are applied later than on the front cars. This may impose severe stresses in the draw bars and may cause the train to break in two. This trouble does not exist with the "automatic" air brake. In this system, under normal running conditions, the auxiliary reservoirs on each car are fully charged to the pressure of the train pipe and the brake cylinders are open to the atmosphere. To apply the brakes, the train pipe pressure is reduced, an air operated "triple valve" automatically disconnects the auxiliary reservoir from the train pipe and connects it with the brake cylinder, meanwhile closing the cylinder exhaust.

Inspection of Air Brakes. The American Electric Railway Association gives the following instructions for inspection: Start the air pump to its maximum capacity; see that brake-valve handle is in release position and where automatic air is used see that gages indicate 20 lb difference between train line and auxiliary. If they do not, the governors need to be reset. Apply brake to show reduction of 40 lb. Place brake-valve handle to lap position; see that air gage operates properly and that no leaks are in or around the brake-valves or pipes leading thereto; examine all pipes, reservoirs, triple valves, cylinders, etc., while brake is set and see that none are leaking and that brake does not release while the brake-valve handle is in lap position. If the cylinder piston has a travel of more than 5 in an adjustment of brakes is necessary. Where slack adjuster is used, see that it is placed to its minimum of travel before any adjustment of brakes.

Inspect all shoes and see that they are in alignment with the wheel so that none are broken and renew those that will not give sufficient wear until the next inspection.

In renewing brake shoes put shoes of same thickness on opposite wheels, be they either old or new.

Examine all keys, bolts, pins, beams, turn-buckles, rods, etc., and lubricate where necessary.

Compressors. Independent motor-driven air compressors are most commonly used for air brakes, and are usually directly connected. Compressors should be of a size such as will not be required to operate more than one-third the time.

17. Car Heating

General. Street cars should be heated to a temperature of 55° to 60° Fahr. which is found to be comfortable for passengers in street attire. Suburban and interurban cars, which have longer runs, should be heated to between 65° and

70° Fahr. in order that the passengers may be comfortable without their wraps.

Systems. Two systems of heating are in use on modern railways: The hot-water furnace and the electric heater. The former is used for locomotive trains and sometimes for interurban cars making long runs. The latter is used for urban cars and generally for interurban cars.

In the case of locomotive trains, the water is heated by an oil-burning flash boiler on the locomotive; whereas, for interurban cars there is a small boiler on the car.

Electric Heating. The electric system of heating, altho more expensive to maintain than the others, finds the greatest favor, and has the advantage of good distribution of heat, cleanliness, ease of regulation, low fire hazard and no attendance.

Electric heaters are all equally efficient in regard to the amount of heat developed but may differ in respect to durability and cost of maintenance. They are usually made of resistance wire wound on porcelain forms, but sometimes the wire is imbedded in insulating material. The latter type has greater heat storage capacity than the former.

The power required for car heating is as follows:

Power for Car Heating

Length of Car, Feet	Average Kilowatts for Heating	
	Average Conditions.	Severe Conditions.
14 to 20	3.5	4.
20 to 28	4.5	5.5
28 to 34	5.5	7.5
34 to 40	7.5	10.5

CAR BARNS AND OTHER BUILDINGS

18. Car Houses and Inspection Sheds

General. Car houses are used for the storage and inspection of street and interurban cars and on small railways are combined with the repair shop. On electrified railroads, car houses are for inspection only, as out-of-door storage is the usual practise.

When about to design a car house, the engineer should first acquaint himself with municipal and underwriters' regulations and conforming with these, should provide room for car storage and inspection, administration offices, line department, road department, car employees' lobby, car sign storage, sand drying and storage, salt storage, oil storage, wash-room and toilets.

Concrete foundations are used for walls and columns, and the walls are usually made of brick; but where cheapness is essential, a mill frame with 2-in cement curtain wall may often be used. Heavy wooden columns and roof trusses are generally used, although cast-iron columns are favored by some designers. Wooden roofs covered with felt, pitch and gravel are usual, and they are provided with copper flashings and counter-flashings of either copper or lead.

Floors are made of concrete with cement finish, and are sloped to provide

ample drainage. It is especially important to keep the inspection pits clear of water.

Inspection Pits. Inspection pits are placed between the rails and are usually 4 ft 6 in deep below the top of rails. The track rails are carried on wooden stringers (usually 10 by 12) on the masonry side walls of the pit. More modern practise in large car houses, is to have a basement as deep as an ordinary pit but extending under all the tracks. In this case the track rails are supported on stringers or T beams resting on posts.

Tracks. Car tracks should be spaced not less than 11 ft between centers where there are no posts between tracks, and not less than 13 ft where there are posts. Ordinary tee rails are used. The dead ends of the car-house tracks should be provided with bumpers. The trolley wire should be at least 16 ft 6 in above the top of rail in order to avoid injuring the trolley springs by keeping them in undue compression. It is usual to support the trolley wire on a flat board to prevent the trolley rising and striking the building structure if it should leave the wire.

Tracks at Entrance. The tracks at the entrance to the car house should be designed to facilitate the rapid entrance and departure of cars with the least interruption to traffic on the main tracks. This is sometimes accomplished by having an extra track in front of the car house and running all storage tracks into it; the extra track being connected at either or both ends, to the main tracks.

Heating. Car houses are best heated by means of a blower system where the air is blown over steam coils and through the building. The heating plant should be either in a separate building or enclosed in fireproof walls.

Doors. Where there is sufficient clearance, swinging and sliding wooden doors are preferred; but where space is restricted, rolling steel doors are used for large openings.

Lighting may be either by incandescent lamps or mercury vapor lamps, the former being most usual. Lamps should be fed, wherever practicable, from a regular lighting system, as the voltage fluctuations of the trolley or third rail circuit are usually excessive. Where, however, the lights must be fed from the traction circuits, the requisite number of lamps must be grouped together in series. The pits or basements should be well lighted.

Painting. Car-house walls are usually covered with cold water paint except for distances of several feet from the ground, where oil paint is to be preferred.

Fire Risks. Due to the great value of cars and the loss of revenue which would result from their destruction, it is important to make the fire risk as low as possible by the adoption of the following precautions:

(1) Use of automatic sprinklers.
(2) Provision of two sources of water supply such as mains and tank.
(3) Division of building by fire-proof walls. The underwriters stipulate that no section shall contain cars to the value of over $200 000.
(4) Avoiding proximity to inflammable buildings.
(5) Having ample provision of water pails, sand pails, chemical extinguishers and short hoses, the last being usually 2½ in in diameter and provided with 1⅛ in nozzles.
(6) Locating car house near a fire station.
(7) Building tracks on a grade so that cars may be easily pushed out by hand.
(8) Having numerous auxiliary fire alarms.
(9) Consulting underwriters when planning.

The Operating Force should be located at the front of the house and starters should be in a position to see all incoming and outgoing cars.

19. Repair Shops

General. Car repair shops are used for the repair and renovation of cars and are generally similar in construction to car houses, being often combined therewith. The details of building construction, heating and lighting given under CAR HOUSES apply equally well to car shops, except that only about half the floor area need be equipped with tracks, the remainder being devoted to departments where unassembled parts are handled. Where tracks are installed, the usual spacing is from 15 to 16 ft between centers.

Heating. While large repair shops are usually heated by the blower system (see Car Houses), an exception must be made in the case of the paint shop where direct steam radiation is preferred. Small shops are usually heated directly by steam or hot-water radiators.

Space Distribution. The table below gives the principal subdivisions of a typical repair shop with the average space distribution, the actual space required being from 140 to 240, averaging about 200 sq ft, of shop floor per car owned, exclusive of yard space or transfer tables. Other departments, not included in the table either because they are affected by the amount the shop depends upon outside aid or because of their smallness, are as follows: Dry kiln, lumber store, boiler room, brass foundry, pattern store, indoor transfer table, locker room and toilets.

Space Distribution in Car Repair Ships

No.	Department	Work of Department	Percent of Floor Area Occupied: Departments with Car Tracks	Percent of Floor Area Occupied: Departments without Car Tracks
1	Repair shop	General repair work on cars:		
		(a) Main repair shop	15	
		(b) Truck repair shop		5
2	Machine shop	Metal work on parts, including slotting commutators		10
3	Blacksmith shop	Forgings, welds, etc., on parts		4
4	Carpenter or erecting shop.	Heavy repairs on car bodies	17	
5	Store-room and Offices.	Stock storage exclusive of heavy material		15
6	Electrical shop	Electrical repairs to armatures, etc., except as cited under No. 2		5
7	Wood mill	Machine work on wood		8
8	Paint shop and wash-rooms.	Painting of cars and storage of painting materials, etc.:		
		(a) General paint shop	18	
		(b) Cabinet room, varnish room		3
	Total		50	50

Means for Moving Cars. Cars are lifted by any of the following means: Traveling cranes, jacks, hydraulic lifts, screw hoists and chain hoists. Traveling cranes are generally used except in the smallest shops.

Transfer tables are preferred to ladder tracks for moving cars between departments.

Proportion of Cars in Shops. The percentage of cars in the shops varies from 8 to 12 depending upon the quality of the equipment, and its usage.

Storage Space around Shops. Shop buildings should be surrounded with sufficient ground to serve for storage of trucks, lumber, scrap material, etc. The total area of land required for shops and storage varies from 1 to 2 acres per 100 cars owned, depending upon whether all departments are concentrated in one building or scattered among several.

SIGNALS AND TRACKS

20. Signals

General. The purpose of signals are to promote safety and to increase line capacity.

In addition to audible signals such as bells, whistles, and torpedoes, movable visible signals, such as lanterns, flags and fuses, single-aspect fixed signals such as slow, stop, whistle and drawbridge signs, electric railways use two-aspect or three-aspect fixed signals, such as switch targets, train order signals, interlocking signals and block signals.

Signal Indications. The two-or three-aspect fixed signals are either semaphores or lamps, or more commonly, a combination of both. Where semaphore signals are used, they are arranged to indicate three positions in the upper left-hand quadrant. These positions are, horizontal for "stop," 45° for "proceed with caution" and vertical for "proceed."

Where semaphores have pointed ends, the horizontal position indicates "stop and proceed" and 45° indicates "proceed, next signal at stop."

Where lights are used, the indications are as follows:

Red Red Red	Stop and stay
Red Red Red Yellow	Stop and proceed
Yellow Yellow Yellow	Proceed, next signal at stop
Green Green Green	Proceed

A "stop" signal combined with a yellow flag indicates "proceed under control, prepared to stop short of any obstructions."

Location of Poles. The standard location for a signal pole is 7 ft 9 in from the center of the track to the center of the pole, with semaphore 15 ft from top of rail to bottom of arm. The wooden blade of the semaphore is 3 ft 6 in by

7 in and from top to vertical projection of center of rotation when horizontal is 3 ft 11 ½ in, the center of rotation being 16 ft 7 in from top of rail.

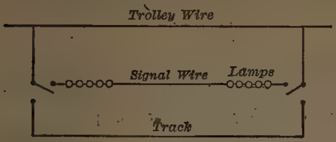


Fig. 47. Manually Operated Signal System.

Lenses. (*a*) For high-speed interurban service, a lens of not less than 8⅜ in in diameter should be used on all light signals operated by continuous track circuits.

(*b*) For moderate speed roads, a lens of not less than 5⅜ in in diameter should be used where light-signals are operated by trolley contact or other end-set devices.

Manually Operated Signals. Signals employing a wire circuit are used on the shorter interurban roads and on some city roads. A typical system is shown in Fig. 47 in which a double-throw electric switch is installed on a pole at each end of the block, so as to be accessible to the motormen. When a car enters the block at either end, the conductor or motorman throws the switch to the opposite position, thereby lighting the lamps at both ends. (If there is already a car in the block, the lamps will not light, as the motorman coming the other way will have thrown up his switch.) When he leaves the block he throws up the switch, thereby extinguishing lamps at both ends. The man on the next car puts down the switch and if it lights he proceeds. A light always means safety, whereas no light is unsafe.

A development of this is a semaphore in place of lamps and automatic throwing of the switches by mechanical trip or magnet on the car.

Manual Block System. This is similar to the system used on steam roads, wherein block signals are operated manually upon information received by telephone or telegraph.

Controlled Manual Block System. This is similar to the simple manual block system except that the signals at the end of each block are interconnected electrically, so that a clear signal cannot be obtained without the cooperation of the signalmen at each end of the block.

Token System. This is a manual block system applied to single track lines. In this system no train is allowed to occupy a block section without obtaining possession of a tablet, staff or other characteristic token. These tokens are taken from signals at the ends of blocks and form part of an interconnected electric system. such that but one token can be removed at a time from a pair of signals, and until the token has been replaced in one signal, no token can be taken from the other. The possession of a token is authority to proceed.

Where it is desired to admit two trains to a block, divided tokens are used, each train taking one, and no further tokens can be withdrawn until all of the pieces of the first are replaced.

Automatic Block System. This is a system in which signals controlled by trains govern the entrance to each block, thereby affording both head and rear protection.

Protection is obtained either by a sectionalized trolley wire, sectionalized track rails or short insulated sections of track rail at the ends of blocks.

Due to the use of the track rails as part of the traction current circuit, some means must be adopted to keep the signal and traction currents separate. On direct-current railways, this may be accomplished either by bridging the insulated rail joints by inductance bonds, which permit the traction current to flow

and choke back the alternating signal current, or by confining the traction current to one rail, and the signal current to the other, the latter system being favored where the track work is complicated, as in yards, junctions, etc. On alternating-current railroads, it is usual to reserve one track rail for signal purposes.

Insulating Rail Joints. Insulating joints at the ends of blocks are made by inserting a piece of insulating fiber between the rail ends, insulating bushings around the bolts and a plate of insulating fiber between the joint plate and rails, the former being prepared especially for the purpose.

Effect of Preservatives in Ties. Zinc-treated ties when new tend to short-circuit the rails, due to the conductivity of the zinc salts. Circuits of 2000 ft length may be operated successfully with 50% of ties so treated, but 5000 ft circuits with all new zinc-treated ties will not operate.

Crossing Protection. Automatic highway crossing protection is usually accomplished by means of a gong (200 cycles per min) operated by a "setting switch" placed in the trolley wire at the approach to the crossing and a "restoring switch" at the crossing.

Dispatcher's Signal Systems. These are for the operation of special signals to control train movement from a dispatcher's office.

One system has a pendulum of a different length at each signal, and a duplicate of each of these in the dispatcher's office. When the dispatcher starts one of his pendulums, it makes and breaks an electric circuit, thereby periodically energizing electromagnets near all the line pendulums. Only that pendulum which is synchronized with it starts into motion and it trips a release which both sets the signal and closes a sounder circuit in the dispatcher's office, thus indicating that the signal has been set.

Standard Practice. The reports of the Joint Committee on Block Signals for Electric Railways, American Electric Railway (Engineering) Association, give full details of actual and recommended practise. See Proceedings American Electric Railway Engineering Association and A.E.R.E.A. Engineering Manual. The standards cited in this article are those of the above association.

21. Track

(By Walter Loring Webb)

General. Track for urban electric roads is identical with that for steam roads (Sect. 3) except in the following items: high girder rails, to comply with paving and street traffic requirements, and rail bonding, to facilitate the return of the current to the power house with minimum power loss and electrolytic effect.

Rails. Some of the forms of high rails together with suitable rail joints are shown in Figs 48 and 49. All such rails, being imbedded in pavement and prevented from lateral displacement, are laid with close joints, no allowance being made for temperature changes. Being buried in the pavement the range of temperature is less than if fully exposed to the atmosphere, and temperature changes merely produce a harmless tension or compression in the metal. This permits the use of WELDED JOINTS, which have the advantage of greater permanency, less cost for maintenance and better conductivity. THERMIT PROCESS: A mixture of powdered aluminum and oxide of iron is placed in a crucible and ignited. The chemical reaction develops a heat of about 5000° Fahr. The oxygen combines with the aluminum, and the iron becomes a molten low-carbon steel. The molten iron sinks to the bottom of the crucible. An iron pin, covered and protected by an asbestos washer,

forms a plug for a hole in the bottom of the crucible. When the iron is thoroly melted, the pin is struck up so that the molten metal escapes from the bottom and pours into a mold which has been made surrounding the joint. The intense heat welds the ends of the rails together and to the molten iron surrounding them, producing a solid welded joint. The cost is approximately \$5.00 per joint. CAST-IRON welded joints (the Falk method) are made by melting cast iron in a portable cupola which is transported over the line. The joint is about 14 in long and will require 70 to 140 lb of cast iron, according to the size of the rail. No splice plates are used. Cast-iron molds are placed around the joint. The cast iron is heated to a somewhat higher temperature than in ordinary cast-iron molding. The molten metal adjoining the mold chills, hardens and contracts first, forcing the molten iron into the finest interstices in the surface of the steel rail, which is itself heated to a white heat by the molten iron. It has been found that if the rails are high in manganese and low in phosphorus, they will be self-hardening, but a low-carbon rail will become annealed by such a process and will wear rapidly at the joint. The conductivity of such a joint is designed to be equal to that of the rail, but tests have shown that there is frequently a considerable percentage of discount. The cost of applying cast-iron welded joints (details of method not given) is quoted in one case at about \$3.75 per joint, including removal and replacement of pavement. The ELECTRICALLY-WELDED joint as made by the Lorain Steel Co. welds two plates 18 in long by 3 in wide and 1 in thick to the webs of the rails. Each plate has three raised bosses on the surface, the bosses being the only portions of the plate actually welded to the rail web. The plates are prest against the web with a pressure as high as 35 tons, while a current which may amount to 25,000 amperes runs thru the joint. The mechanism for making such joints is carried on a car and the current taken directly from the trolley wire. The cost is approximately from \$5.00 to \$6.00 per joint.

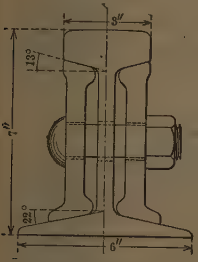


Fig. 48. Shanghai Rail

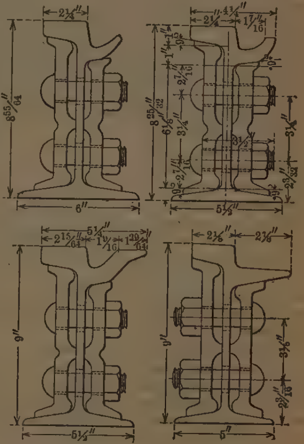


Fig. 49. Girder Rails

Track Gage. The convenience of operating standard-gage steam railroad equipment on electric roads has resulted in the almost universal adoption

of standard gage (4 ft 8½ in) for electric roads. Philadelphia (5 ft 2¼ in) and St. Louis (4 ft 10 in) are the most prominent exceptions to this rule. The flanges of wheels have conoidal surfaces and there are therefore no definite points on their surfaces between which the gage of the wheels may be measured. The gage is therefore arbitrarily measured between points on the flanges ¼ in from (or below) the treads, and this gage-width for the wheels is made ¼ in less than the gage-width between rails.

Track Clearances. The spacing between track centers for double track or turnouts depends on the width over all of the cars in use, and also indirectly on the speed, which would affect the clearance. The extreme width of cars out to out varies from 8 ft to 9 ft 1 in. This width plus the clearance, which usually varies from 5 to 13 in, equals the distance between track centers, which therefore varies from 8 ft 6½ in in Philadelphia to 9 ft 5 in in Chicago. When streets are very wide, the clear distance between cars may be increased to 2 ft. For center-pole construction even greater space must be used and the distance between track centers may be increased to 15 ft. The large cars now commonly used overhang the track so much when going around sharp curves that the standard clearance between the tracks on tangents is too narrow to permit the passing of two cars on sharp curves. It is frequently impossible to increase the track clearance so that the largest cars may pass on the curves, and it is then necessary to require one car to wait for another and not permit them to run on the curve simultaneously. The amount of added clearance depends on whether the car is mounted on single or double trucks, its length (including the possible fender at either end) and the spacing between the center pins of the trucks. The simplest rule for determining the extra width between track centers is to draw at a proper scale the curves at the desired radius, and draw a plan view of a car together with its wheel base, somewhat as shown in Fig. 50. The excess over the width of the car of the sum of the distances (a and b in Fig. 50) from the track center to the extreme projecting points of the car on both sides is the required excess in track clearance. When no excess clearance is provided, both tracks have a common center of curvature. By using curves with different centers of curvature and with suitable radii, the desired extra clearance may be obtained. Since the projection of the car over the rails varies with each change of curvature, the necessary combination of curves to provide sufficient clearance for a given type of rolling stock must usually be determined by trial.

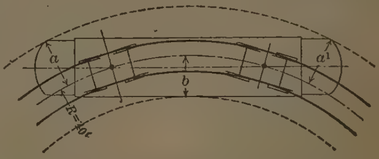


Fig. 50. Clearance for Car on Curve

Transition Curves or Spirals should be used at the beginning and ending of all curves. The spirals for easy curves should be those already developed for steam railway work. The curves around street corners are usually so sharp that spirals having more rapid changes of curvature must be used. The manufacturers of "special work" now use spirals not only in constructing approaches to simple curves but also for branch-offs, Y's, etc. The design of such special work may be left to them by furnishing them with the following items: (a) total central angle of curve, (b) total required clearance at center of curve between curb and nearest rail or distance from vertex

to inner rail, (c) distance from rails to curb in each street. The following description of spirals which may be used with plain curves is compiled from the catalog of Wm. Wharton Jr. & Co., Inc. Four spirals have been developed, all of which have one common base represented in spiral No. 1. The other spirals are derived from it by multiplying the functions of this spiral by the figures indicated for the number of the spirals; that is, by 1½ for spiral No. 1½, by 2 for spiral No. 2 and by 3 for spiral No. 3. It is recommended to use the spiral No. 1 for curves with a central radius not exceeding 62 ft 6 in; No. 1½, 112 ft 6 in; No. 2, 200 ft, and No. 3, 500 ft. The system has also been designed so that switches can be set into the plain ends of curves without changing the lines. The curves whose dimensions are given by the table are those of the gage line of the inner rail and not the center line of the track as in steam railroad work. The spirals are of the multiform-compound-curve class. The central angles for each chord length from

Chord No.	Central angle	
	Partial	Total (y)
1	0° 34′ 23″	0° 34′ 23″
2	1 08 45	1 43 08
3	1 40 16	3 23 24
4	2 06 39	5 30 03
5	3 27 27	8 57 30
6	3 00 56	11 58 26
7	3 49 11	15 47 37
8	4 38 44	20 26 21

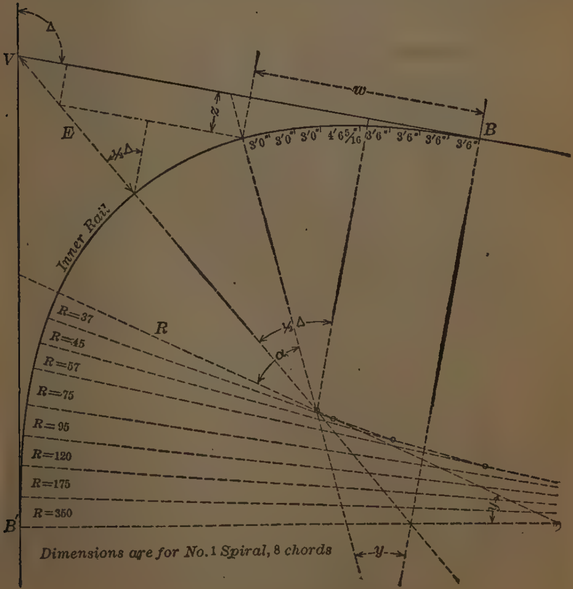


Fig. 51. Spiral for Street Railway Curve

the tangent are identical for each spiral (as far as they go) and are as given in the tabular form.

To Select a Curve with Spirals. Given the central angle Δ and the desired external distance E from the intersection of the tangents to the desired middle point of the inner rail curve. As a trial $R' = E/\text{exsec}\,\frac{1}{2}\Delta$, which gives the radius of a simple curve thru that point and connecting the tangents. The radius R for the central part of the curve must be somewhat less than R' and also somewhat less than the radius of the last chord of the spiral which it is proposed to use. For example, with $\Delta = 100°\ 20'$ and $E = 21'$, $R' = 37.424$. (Note that R, E and R' here refer to the gage line of the inner rail and not to the track center line.) All the chords of a No. 1 spiral will be needed and the angle $y = 20°\ 26'\ 21''$.

$$R = \frac{E \cos \frac{1}{2}\Delta - z}{\cos y - \cos \frac{1}{2}\Delta} = \frac{E \cos \frac{1}{2}\Delta - z}{2 \sin (y + \frac{1}{4}\alpha) \sin \frac{1}{4}\alpha}$$

In this equation, $Z = 3$ ft $3\frac{3}{8}$ in $= 3.281$. Then $R = 34.304$. The central part of the curve with radius R has a central angle of $\alpha = \Delta - 2y = 59°\ 27'\ 18''$; $\frac{1}{4}\alpha = 14°\ 51'\ 50''$; $(y + \frac{1}{4}\alpha) = 35°\ 18'\ 11''$; w for this case $= 27$ ft $2\frac{1}{4}$ in $= 27.187$ ft. The ordinates and abscissas of the various points of the spiral are as given in the table below. The distance from the vertex V to the point of tangency $B = z \tan \frac{1}{2}\Delta + R\,(\sin \frac{1}{2}\alpha / \cos \frac{1}{2}\Delta) + w$.

Dimensions of Spirals for Street Railroad Tracks
(Wm. Wharton Jr. & Co., Inc.)

Spiral No.	Chord No.	Radius inner rail	Length of chord	Coordinates of farther end of chord	
				Ordinate	Abscissa
1	1	35[illegible]′ [illegible]″	3′ 6″	3′ 6″	0′ 0 3/16″
	2	175 0	3 6	7 0	0 1/16
	3	120 0	3 6	10 5 5/16	0 2 15/16
	4	95 0	3 6	13 11 3/16	0 6 3/16
	5	75 0	4 6 5/1[illegible]	18 5 11/16	1 [illegible]
	6	57 0	3 0	21 5 1/16	1 7 3/16
	7	45 0	3 0	24 4	2 4 3/16
	8	37 0	3 0	27 2 1/4	3 3 3/8
1½	[illegible]	525 0	5 3	5 3	0 0 5/16
	[illegible]	262 6	5 3	10 6	0 1 9/16
	[illegible]	180 0	5 3	15 8 15/16	0 4 3/8
	4	142 6	5 3	20 11 3/4	0 9 1/4
	5	112 6	6 9 7/16	27 8 9/16	1 7 1/2
	[illegible]	85 6	4 6	32 1 5/8	2 5 5/16
	7	67 6	4 6	36 6	3 0 9/16
2	1	700 0	7 0	7 0	0 0 7/16
	2	350 0	7 0	14 0	0 2 1/8
	3	240 0	7 0	20 11 7/8	0 5 13/16
	4	190 0	7 0	27 11 5/8	1 0 3/8
	5	150 0	9 0 5/8	36 11 3/8	2 2
3	1	1050 0	10 6	10 6	0 0 5/8
	2	525 0	10 6	21 0	0 3 1/8
	3	360 0	10 6	31 5 7/8	0 8 3/4
	4	285 0	10 6	41 11 7/16	1 6 9/16

As an example assume a curve in a track following a highway where the central angle Δ is $30° 20'$ and the maximum permissible distance E is 13.5 ft. A simple curve thru the desired middle point and joining the tangents will have a radius $R' = E/\text{exsec}\ \frac{1}{2}\Delta = 11.5/\text{exsec}\ 15° 10' = 318.67$. R must be somewhat less than this. We must use the first three chords of a No. 3 spiral and $z = 0$ ft $8\frac{3}{4}$ in $= 0.73$ ft; $y =$ the total central angle for three chords $= 3° 23' 24''$; $a = \Delta - 2y = 30° 20' - 6° 46' 48'' = 23° 33' 12''$, $(y + \frac{1}{4}a) = 3° 23' 24'' + 5° 53' 18'' = 9° 16' 42''$. Solving for R we have 313.51 ft. To determine the tangent distance we must use 31 ft $5\frac{7}{8}$ in $= 31.490$ ft as the value of w for this case. The tangent distance in this case equals 46.618 ft.

Spacing of Turnouts; Single-Track Road. Economical and efficient operation absolutely requires that turnouts shall be located as nearly as possible at the points where the cars will naturally meet (according to the system adopted)

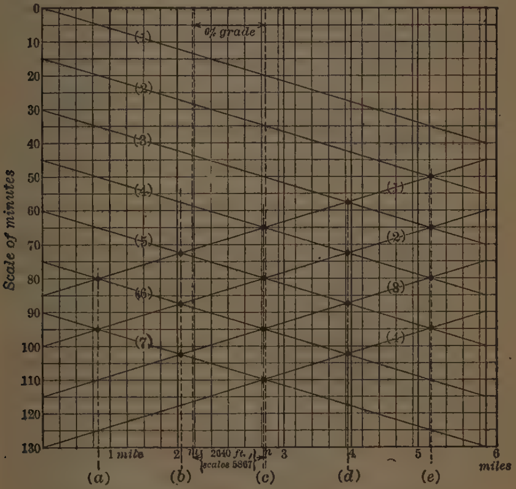


Fig. 52. Chart to Determine Location of Turnouts or Passing Points

without requiring one car to wait for another at any meeting point. Before these meeting points can be determined, the maximum number of cars to be operated on one section and their effective schedule time must be known. When there are unusually steep grades on the line which will cause certain sections to be run more slowly than others, allowance must be made by a corresponding shortening of the sections having the steep grades. The necessity for slow speed thru village streets and the possibility of comparatively high speed thru the open country or on private right-of-way, and even excessive curvature, will have substantially the same effect in modifying the distance between adjacent turnouts. The principles of spacing turnouts may be best stated by an example. Assume a road 6 miles long, operating six cars; assume maximum velocity of 20 miles per hour with stops averaging 800 ft apart. Assume that the

grades are equivalent to an average of 2%, but that there is a half-mile stretch of 6%. The energy which will move the car at 20 miles per hr on a 2% grade will move it only 9 miles per hr on the 6% grade. The half mile will require 200 sec, which at 20 miles per hr would run the car $1^1/_9$ miles or 5867 ft. Therefore that half-mile stretch must be increased to 5867 ft in Fig. 52, and the virtual length of the line must be considered as increased by 5867 − 2640 = 3227 ft = 0.61 miles, or that the line is virtually 6.61 miles long. The velocity will be 10.2 miles per hour and the entire 6.61 miles will be run in 38.9 min. With an allowance of say 6 min for terminal wait, the run must be practically on 45 min schedule. Lay off on profile or cross-section paper, at suitable scales, times as vertical ordinates and distances as horizontal spaces. The 2640 ft of 6% grade is expanded to the 5867 ft of virtual distance requiring the same time interval. Two inclined lines represent the course of the car during the round trip. The six cars run at 15 min interval. The location of the meeting places is graphically determined. The location of turnout (*c*) must be determined by scaling $^{2640}/_{5867}$ of the space from *m* (or *n*) from the line (*c*). From the above example the following general principles may be determined:

(*a*) The number of cars is one greater than the number of turnouts.

(*b*) The running time between consecutive turnouts is one-half the running-time interval between cars.

(*c*) The time interval between consecutive turnouts is necessarily uniform thruout.

(*d*) When, as illustrated above, the velocity for any section is necessarily reduced, the spacing must be reduced accordingly.

(*e*) If variations in traffic density seem to require a temporary or periodical reduction in number of cars, it must be done by cutting out one or more cars from the regular schedule, or perhaps by cutting out every alternate car, and operating the others precisely as before. This merely means that some turnouts are not always utilized.

(*f*) Since in the example given the lay-over is 6 min and the time interval between successive car passings at (*a*) is 15 min the time interval from the terminus to (*a*) is 9 min and the distance is 0.6 the normal distance between turnouts. The distance between (*e*) and the other terminal is the same. There are therefore 4 + 0.6 + 0.6 = 5.2 intervals in the virtual distance of 6.61 miles. The interval is therefore 6.61/5.2 = 1.271 miles, = 6711 feet, which is the distance (*a*) to (*b*) and (*d*) to (*e*). If (*b*) to *m* is 920 ft, then *m* to (*c*) is virtually 6711 − 920 = 5791 ft, which multiplied by $^{2640}/_{5867}$ = 2606 ft, the actual distance. The spacing from (*b*) to (*c*) is therefore 920 + 2606 = 3526 ft. 5867 − 5791 = 76, which multiplied by $^{2640}/_{5867}$ = 34 ft, the true distance from (*c*) to *n*. As a check, 2606 + 34 = 2640. (*a*) and (*e*) are each distant from their termini by $^6/_{10}$ of 1.271, or 0.74 mile.

SECTION 4

MATERIALS OF CONSTRUCTION

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MECHANICS OF SIMPLE STRESSES

1. Tension, Compression, Shear

Stress is the internal force which, when a body is subjected to extern forces, tends to hold the molecules in their original relation and to preser the integrity of the body. Stresses are measured by the same units as force namely, in pounds, tons, kilograms.

Unit Stress is the measure of intensity of a stress. It is the quotient ol tained by dividing a total uniform stress by the number of units of area ov which the stress is distributed. Unit stresses are exprest in pounds per squa inch, tons per square foot, kilograms per square centimeter, and the like.

Ultimate Stress. If the external forces acting on a body are increased such an extent that the internal force or stress is no longer able to preserve tl integrity of the body, rupture occurs. Ultimate stress is the greatest stre which can be produced in a body before rupture occurs. Ultimate un stress is the ultimate stress on one unit of area. Ultimate stress and ultima strength are interchangeable terms.

Tension is the name for the stress which tends to keep two adjoining plan of a body from being pulled apart under the influence of two forces actir away from each other. If a bar of uniform cross-section is hung vertical from a rigid support, and a load of 2000 pounds suspended at the lower en a tensile stress is developed in every section of the bar. In order that equilil rium may obtain, the internal stress or tension in the bar due to the loa must equal the external force or load, or 2000 pounds.

Compression is the name for the stress which tends to keep two adjoinin planes of a body from being pushed together under the influence of tw forces acting toward each other. A brick pier of uniform cross-sectio supporting a column carrying a load of 40 short tons will be subjected to compressive stress due to the load. When equilibrium obtains this stre must equal 40 short tons. If the area of the pier be two feet square, th unit stress in the pier due to the load will be 10 short tons per square foot.

Shear is the name for the stress which tends to keep two adjoining plan of a body from sliding one on the other under the influence of two equal an parallel forces acting in opposite directions. The forces which induc shearing stresses in a body are termed SHEARING FORCES, and usually a but slightly separated, so that their action is similar to that of a pair of shear whence the name Shear. When two flat plates, secured together by mear of rivets, are subjected to forces which tend to pull the plates apart by slidin one on the other, as in the case of the plates of a boiler under internal stea pressure, shearing stresses are induced in the rivets which hold the plat together. Unless otherwise noted, shearing stress is assumed to be distribute uniformly over the section upon which it acts.

Axial Forces and Axial Stresses. When forces producing tension compression act on a body of symmetrical form in such a way that the resultant coincides with the axis of the body, they are termed AXIAL FORCE and the stresses induced thereby, AXIAL STRESSES. In simple axial tensio or compression alone, the stress is distributed uniformly over every sectio of the body normal to the direction of the force.

Axial tension and compression are the two commonest forms in which stress is m with in engineering structures. An example of simple axial tension is to be found in vertical eyebar in a bridge truss, where the only force acting on the bar is along its axi A horizontal eyebar or one inclined to the vertical, while acted upon by axial forc transmitted thru the end pins, is also subjected to a force acting normal to or at an ang

with the axis, due to the weight of the bar, which produces stresses not axial, so that the resultant stress in the bar is not uniformly distributed over each section, and the case is not one of simple axial tension.

A short vertical prism, whose length does not exceed about six times the least side or diameter, is subjected to simple axial compression under a load the resultant of which passes thru the centers of gravity of the end sections. If the length of the prism is more than eight or ten times its least side or diameter it becomes a COLUMN, and altho the resultant force is still axial as regards the end sections, some bending is assumed to have taken place in the shaft of the column, resulting in an unequal distribution of stress over a given section, and the case is not considered one of simple axial compression.

Formulas for Simple Stresses. Let P be any force producing tension, compression or shear, A the area over which the induced stress is uniformly distributed, and S the unit stress; then

$$P = SA \qquad S = P/A \qquad A = P/S \qquad (1)$$

which are expressions applying to all cases of simple axial tension, compression, or shear. Two quantities being given the other one can be found.

Factor of Safety, Working Stress. In order that the safety of a structure shall be assured, there must be no danger of rupture in any of its members. To secure this assurance of safety the stress induced in any member by any load which the member will be called upon to carry must never approach the ultimate strength of the material. The WORKING STRESS for any material is the unit stress which, by experiment, has been found safe to allow in that material and still give a proper degree of security against rupture, and is the unit stress to be used in determining the sizes of structural members of that material. The FACTOR OF SAFETY is the number by which the ultimate stress must be divided to give the working stress. If in formula (1) S be taken as the ultimate stress, n a factor of safety, and S_w the safe allowable stress or working stress, then $S_w = S/n$.

The purpose of the factor of safety and the working stress is twofold; first, to guard against undiscoverable defects in the structural material employed, which defects reduce the ultimate strength of the material; and second, to provide against the possibility of an increase in the load to be carried due to unforeseen circumstances. The selection of the working stress and the factor of safety therefore depends, first, upon the degree of definiteness with which the ultimate stress for the particular material is known. Thus, the factor of safety for steel or iron is smaller than for timber, since the ultimate strength of each of the various grades of steel or iron is very nearly a constant quantity, while there is a wide variation in the values for the ultimate strength of timber obtained from different specimens of the same variety. Second, upon the definiteness with which the loads to be carried can be computed. Thus, when "dead" load capable of exact calculation forms a large proportion of the total load to be carried, as in the truss members of long and heavy bridges, the factor of safety is smaller than when live or moving loads form a large proportion of the total load, as in the case of girders supporting traveling cranes or heavy moving machinery.

Riveted Joints, such as the longitudinal joints in boilers, are examples of simple stress and may be designed or investigated by formula (1).

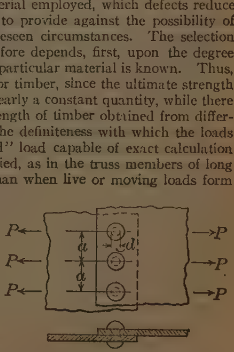


Fig. 1. Single Lap Joint

CASE I. Lap joint with single riveting. (Fig. 1.) Here the shear in each rivet comes in one cross-section only and is termed "single shear." Let a = the pitch or rivet spacing, d = the diameter of the rivets, t = the thickness of the plates, P = the load to be transmitted from one plate to the other in the distance a, S_t = the unit tensile stress produced in the plates by the load P, S_c = the unit com-

pressive stress produced in the plates where the rivets bear against them (commonly called "bearing"), S_s = the unit shear produced in the rivets; then

$$S_t = P/t(a-d) \qquad S_c = P/td \qquad S_s = P/\tfrac{1}{4}\pi d^2$$

Case II. Lap joint with double riveting. (Fig. 2.) As in Case I, the rivets are in single shear. Using the same symbols as before,

$$S_t = P/t(a-d) \qquad S_c = P/2\,td \qquad S_s = P/2 \cdot \tfrac{1}{4}\pi d^2$$

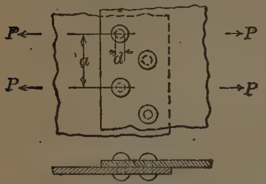


Fig. 2. Double Lap Joint

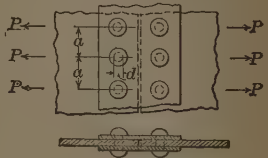


Fig. 3. Butt Joint

Case III. Butt joint with single riveting. (Fig. 3.) In this case the shear in the rivets comes in two cross-sections and is termed "double shear." Using the same symbols as before,

$$S_t = P/t(a-d) \qquad S_c = P/td \qquad S_s = P/2 \cdot \tfrac{1}{4}\pi d^2$$

2. Deformation under Stress

Deformation is the amount of the change in the shape of a body caused by the action of an external force. Deformations are measured by the same unit as the linear dimensions of a body, namely, inches and millimeters. If a weight is suspended at the end of a steel bar, the effect of the weight is to increase slightly the length of the bar. This increase is the deformation.

Hooke's Law. Whenever a body is subjected to an external force a stress and an accompanying deformation result. From experiment it has been found that when the unit stress does not exceed a certain limit, which limit varies with each particular material, the stress bears a constant ratio to the accompanying deformation. Thus, if a weight W, suspended from a steel bar produces an elongation of 1/100 inch, a weight of twice W will produce an elongation of 2/100 inch, and a weight of three times W will produce an elongation of 3/100 inch.

Stress and Strain. The word STRAIN has been frequently used for the internal force in a body, or stress. It is present practise, however, to use the word strain as a synonym for deformation only, or the effect of a stress. Thus the expression "stress and strain" is equivalent to the expression "stress and deformation." Owing to the conflicting meanings of the word Strain it has been suggested that it be avoided, the word DEFORMATION being used in its place.

Kinds of Deformation. Deformation may be of three kinds: elongation, or increase in length, due to tension; shortening, or decrease in length, due to compression; detrusion, or the slipping of one plane on another, due to shear.

Elasticity and Set. It has been found by experiment that up to a certain limit, in addition to the constant ratio existing between stress and the accompanying deformation, a body deformed under stress will return to its original shape when the stress is removed. This ability to return to its original form after deformation is termed ELASTICITY. If the unit stresses be increased, however, beyond such limit, it is found that not only do the deformations

increase more rapidly than the corresponding stresses, but that the material undergoes a SET, or certain amount of permanent change, and is no longer capable of returning to its original shape when the stress is removed.

Elastic Limit. The elastic limit of a material is the highest unit stress to which that material may be subjected and still return to its original shape when the stress is removed.

Proportional Limit. The proportional limit of a material is the highest unit stress for which the deformation is proportional to the stress.

With actual materials it is probable that some very slight inelastic action occurs under any stress whatever, and that Hooke's law is a very close approximation rather than a rigid statement of fact. The 1916 " Standards " of the American Society for Testing Materials specify that in making laboratory determinations of elastic limit and of proportional limit deformations shall be measured to the nearest 0.0001 inch. Within this degree of precision the elastic limit and the proportional limit are practically coincident, and the terms are, for practical purposes, interchangeable.

Various methods of determining the proportional limit or other practical " limits " closely related to it are in use. The " Useful Limit Point " proposed by the U. S. Bureau of Standards and " Johnson's Elastic Limit " are discussed on p. 377.

Modulus of Elasticity. The modulus of elasticity of a material is the constant which, within the proportional limit, expresses the ratio between unit stress and unit deformation. Thus, let E = the modulus of elasticity, P = an axial force, A = the cross-sectional area of a bar, S = the unit stress produced by the force P, or P/A, d = the deformation produced by the force P in a bar of length l, and $d/l=e$; then

$$E=(P/A)/(d/l) \quad \text{or} \quad E=S/e$$

Since l and d are linear dimensions, e is an abstract number, and E is exprest in the same units as S, such as pounds per square inch, tons per square foot, or kilograms per square centimeter.

COEFFICIENT OF ELASTICITY, and YOUNG'S MODULUS are expressions sometimes used for the Modulus of Elasticity. They are not now in general use in the United States, Modulus of Elasticity being the generally accepted expression for the quantity E.

Deformation within Elastic Limit. The moduli of elasticity of materials measure their relative ability to resist deformation under unit stresses within their proportional limits. The above formula may be written in the form $d=Sl/E$. For the same unit stress S, d decreases when E increases. It will thus be seen from a comparison of the various moduli of elasticity (Art. 3) that for the same unit stress timber in tension will be deformed ten times as much as cast iron, and cast iron about twice as much as steel. This formula may be employed to determine the total deformation of a bar of a length l when S and E are known.

From the values of the proportional limit and the moduli of elasticity in tension (Art. 3) the unit elongation at the proportional limit may be computed for each of the common structural materials, as follows:

For timber $\frac{1}{500}=0.0020$ For wrought iron $\frac{1}{900}=0.0011$

For medium steel $\frac{1}{857}=0.0012$

The ultimate elongation, or the elongation at rupture, cannot be computed, since beyond the proportional limit the ratio of the unit stress to the unit deformation is a constantly varying quantity.

3. Phenomena of Stress

Stress-Deformation Diagrams. The action of a bar under stress may best be represented by a curve the ordinates of which represent unit stresses and the abscissas the corresponding unit deformations. For all unit stresses below the proportional limit the stress-deformation curve will have the form of a straight line, since the formula $S = Ee$ is an equation which represents a straight line thru the origin of coordinates, the slope of which is represented by E. Beyond the proportional limit the curve will take various forms and radii of curvature depending upon the material. The end of the curve will represent the point of rupture, and the greatest ordinate will represent the ultimate strength of the material. The stress-deformation curve constructed from information derived from actual experiment, a sufficient number of points of the curve being plotted to determine accurately its curvature, affords in most cases the most satisfactory method available for determining the proportional limit of the material, the proportional limit being represented by the ordinate of the point of tangency between the straight line and the rest of the curve. Fig. 4 is a typical stress-deformation diagram, being that for medium steel in tension. The curves for cast iron and timber are shown for comparison.

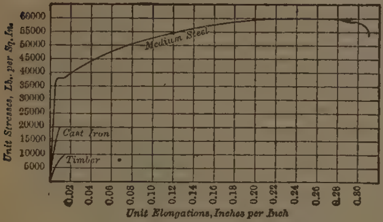


Fig. 4. Stress-elongation Diagram.

The stress-deformation curve has various forms depending upon the material in question and whether the stress is tension or compression. In some cases the curve is of such form as to make it difficult to locate the proportional limit with accuracy. In the case of medium steel in tension (Fig. 4) the point of tangency between the straight line and the rest of the curve is fairly well defined, but in the corresponding curve for medium steel in compression the transition from the straight line to the curve is more gradual, and the point of tangency is more difficult to locate. Thus again, timber and cast iron have curves entirely different from medium steel or wrought iron, in that they are curved thruout their length and, for cast iron especially, the proportional limit cannot be at all definitely fixt.

Yield Point. In tests for the determination of the elastic properties and ultimate strength in tension of such metals as structural steel and wrought iron it is noted that after the elastic and proportional limits have been exceeded a point is reached where the unit elongations increase rapidly without any, or at most very slight, increase in the unit stresses. The unit stress at this point is termed the yield point.

The yield point is often confused with the elastic limit and the former is given for the latter in many reports of tests, owing to the facility with which the yield point may be determined with some testing machines. It is that point where the scale beam of the testing machine drops and remains down until the operating screws have stretched the metal a certain amount. After an interval the bar again rises and the stress in the specimen increases. The yield point is beyond the elastic limit—for structural steel from 3000 to 6000 lb per sq in beyond it. Yield points are observed only in ductile metals, such as wrought iron and steel.

Elasticity and Set. The portion of the stress-deformation curve between the origin of coordinates and the elastic limit represents the range of unit stress and unit deformation within which the material will return to its original shape upon the removal of the stress. Beyond the elastic limit a body under stress only partly recovers its original shape upon the removal of the stress, the body being more or less permanently deformed. The amount of this permanent deformation is termed the PERMANENT SET. Thus a bar of wrought iron subjected to a tensile stress of 30 000 lb per sq in shows a unit elongation of 0.003 in per in. Upon the removal of the stress the bar will show a recovery of, say, 0.001 in per in, having a permanent set of 0.002 in per in.

Ultimate Strength and Stress at Rupture. The ultimate strength of a material is represented by the greatest ordinate in the stress-deformation diagram and is the greatest unit stress to which the material may be subjected before rupture. The ultimate strength of a material as indicated by the stress-deformation diagram is not necessarily at the point of rupture. The stress-deformation curve for such materials as steel and wrought iron shows a decided drop after the ultimate strength has been reached, and the end of the curve corresponds to a unit stress in some cases considerably below the ultimate strength.

Average Properties of Structural Materials

Material	Weight per cubic foot. Pounds	Modulus of elasticity		Elastic limit and proportional limit		Ultimate strength		
		Tension compression	Shear	Tension compression	Shear	Tension	Compression	Shear
		Pounds per square inch						
Cast iron......	450	15 000 000	6 000 000	*	*	20 000	80 000	†
Wrought iron..	480	27 000 000	10 000 000	30 000	18 000	50 000	31 000‡	40 000
Medium steel..	490	30 000 000	12 000 000	35 000	21 000	60 000	40 000‡	48 000
Nickel steel (3.5% nickel)	490	30 000 000	12 000 000	42 000	25 000	85 000	48 000‡	63 000
Timber........	35	1 500 000	300 000 along grain	3 000	......	8 000	8 000	500 along grain
Stone.........	165	5 000 000	2 700 000	*	*	......	6 000	1 500
Brickwork.....	125	....	....	*	*	......	1 200	......
Terra cotta masonry.......	120	....	....	*	*	......	3 000	......
Portland cement concrete (1 : 2 : 4)...	150	2 500 000	....	*	*	150	2 000	1 300
Gypsum.......	80	1 000 000	....	*	*	......	1 400	......

* No well-defined elastic limit or proportional limit.

† Strength in shear greater than strength in tension, in shear tests a tension failure on an oblique plane takes place.

‡ No well-defined ultimate in compression. The yield point, which is slightly higher than the proportional limit, is the practical ultimate.

To determine the unit stress in direct tension or compression divide the applied load by the area of cross-section of the original bar before the loads were applied. This is not strictly correct, since when the bar is approaching rupture the area of cross-section is considerably reduced. If after the ultimate strength has been reached the actual load, as indicated by the scale beam of the testing machine, could be divided by the actual area at the moment that particular load is acting, it would probably be found that the curve, if properly constructed, would curve upward beyond the point of ultimate strength.

Ultimate Deformation. Since there can be no general expression for the relation between stress and deformation beyond the elastic limit, it is not possible to determine the ultimate deformation, or deformation at the point of rupture, save by actual experiment. Ultimate deformations are rarely determined save in the case of tension tests, where the ultimate deformation becomes the ultimate elongation. It is usually exprest in percent. and is found by determining the amount that a measured length of bar has elongated after rupture and dividing the amount of this elongation by the original measured length. Thus, if a measured length of eight inches of a bar before testing increases to ten and one half inches at rupture, there would be 31.2 percent ultimate elongation. The elongation after fracture is an index of the ductility of the material.

4. Methods of Fracture

Brittleness. A material which cannot be deformed to any extent without rupture is termed brittle. Brittleness is relative, no material being perfectly brittle, that is, capable of no deformation before rupture. Many materials are brittle to a greater or less degree, glass being one of the most brittle of materials. Brittle materials have relatively short stress-deformation curves, the deformations of which they are capable before rupture being relatively very small. Of the common structural materials cast iron, brick, and stone are to be considered brittle in comparison with steel. Brittle materials rupture without appreciable reduction of area, and show a clean, sharp fracture, no flow of material taking place before rupture occurs.

Plasticity is the ability to change shape without fracture. A perfectly plastic material is one in which an applied load however small produces a permanent deformation. Such a material has no elastic limit and has no point of rupture. The material flows continually under the applied loads, a prism being ultimately reduced to a thin sheet without actual fracture. Lead is the best example of a plastic metal, having little or no elasticity. Structural steel near its ultimate strength becomes plastic, and the deformation or flow of material under the stress becomes very marked.

Brittleness and plasticity are opposite terms. Materials which have a high degree of plasticity have no brittleness, and rupture with considerable reduction of area. The reduction of area at rupture may be considered the measure of the plasticity or brittleness of a material, a large reduction of area indicating a high degree of plasticity and little or no reduction of area indicating a high degree of brittleness.

Ductility is the ability to undergo great stretch without fracture.

Toughness is the ability to withstand high stress together with great deformation. Toughness is an index of the ability of a material to withstand impact without complete fracture. Toughness is measured by the area under the stress-deformation diagram for a material, or, less exactly, by the product of ultimate strength and elongation after fracture.

Fracture Under Tension. Figs. 5, 6, 7 represent typical fractures under tension. Fig. 5 shows the fracture of a brittle material like hard steel, with

a very small reduction of area. Figs. 6 and 7 show the fracture of more plastic materials, such as wrought iron and steel, with a typical cup fracture as for structural steel (Fig. 6), and a fibrous fracture as for wrought iron (Fig. 7), the reduction of area in both cases being marked.

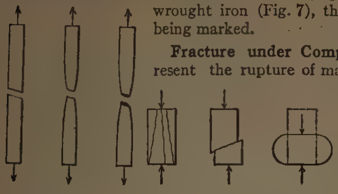


Fig. 5 Fig. 6 Fig. 7 Fig. 8 Fig. 9 Fig. 10

Fracture under Compression. Figs. 8, 9, 10 represent the rupture of materials under compression. For very brittle materials, such as brick (Fig. 8), fracture occurs by shearing along planes nearly parallel to the direction of the applied load with little or no increase in area. For more plastic materials, such as timber (Fig. 9), fracture occurs along planes inclined to the direction of the load, the greater the degree of plasticity the greater the inclination of the planes of fracture. A perfectly plastic material (Fig. 10) under its ultimate stress has a large increase in area without any shearing planes of fracture.

Relation of Shear to Tension and Compression. A body subjected to a force producing direct tension or compression has induced in it shearing stresses along certain planes of that body. Thus if an axial force P be applied to a bar (Fig. 11) along any plane $a-a$ inclined to the direction of the force, P may be resolved into two components, P_1 and P_2, acting respectively normal and parallel to the plane $a-a$. The component P_2 parallel to $a-a$ produces a shearing stress over the plane $a-a$. For all planes making angles 0° or 90° with the direction of the force P the shearing stress will be zero. It will be of maximum intensity along planes inclined 45° to the direction of P, and its value will be one-half of the direct tensile or compressive stress. Thus, if S be the unit tensile or compressive stress, the maximum unit shearing stress S_s along planes inclined 45° to the direction of P will be $S_s = \frac{1}{2}S$.

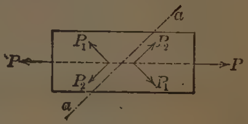


Fig. 11

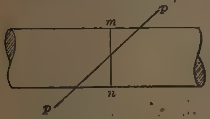


Fig. 12

In a body subjected to shearing stress, such as a shaft in torsion, there are set up tensile or compressive stresses on oblique planes. If in the body shown in Fig. 12 there is set up shearing stress along the plane mn there is set up shearing stress of equal intensity along planes at right angles to mn and tensile or compressive stress, in addition to shearing stress, along an inclined plane pp. If pp makes an angle of 45° with mn the shearing stress on pp becomes zero and the tensile or compressive stress is a maximum and its intensity is equal to that of the shearing stress along mn. The strength of a body is sometimes determined by stresses along oblique planes. Many brittle materials have great strength in compression, less strength in shear, and still less strength in tension. Under torsion such materials fail by tension along inclined planes; under compression they fail by shearing along inclined planes.

5. Work and Resilience

External Work. When a load P is applied to a bar and a deformation d results, a force has acted over a certain path in producing this deformation

and therefore a certain amount of work has been expended. When the forc P is applied in small increments or is increased in amount gradually fror zero up to P, the elastic limit of the material not being exceeded, the mea force which has acted to produce the deformation is $\frac{1}{2}P$, and the path ov which the force has acted is the deformation d. If K be the amount of wor expended, $K = \frac{1}{2}Pd$. Or, if A is the cross-section of the bar, l its length, the unit stress produced, and e the unit elongation under the unit stress S, the this formula becomes $K = \frac{1}{2}SeAl$, which is an expression for the extern work required to produce deformation within the elastic limit. The facto $\frac{1}{2}Se$ is the external work per unit of volume, the volume of the bar being Al.

Resilience is the amount of work which may be stored up in a body unde stress within the elastic limit, in the form of stress energy, and which ma be recovered when the force producing the stress is removed. When th force has been applied gradually so that no energy has been converted int heat, from the law of the conservation of energy the resilience must equ the external work. The resilience of a bar may be exprest, therefore, b the last formula, which, after making the necessary substitutions, may b written $K = \frac{1}{2}(S^2/E)Al$. When S is the elastic limit of the material, th factor $\frac{1}{2}S^2/E$ is termed the Modulus of Resilience.

The above formulas apply to any stress, whether tension, compression, or shear. I the case of tension and compression the deformation or path of the force is normal to th planes of the body over which the stress is distributed, and parallel to the length l; whil in the case of shear the deformation is parallel to the planes over which the stress is dis tributed and normal to the length l. It should be further noted that the formulas appl only to elastic resilience, that is, to the resilience within the elastic limit of the material

Work Required for Rupture. Since beyond the elastic limit the defor mations are not proportional to the stresses, $\frac{1}{2}P$ does not express the mea value of the force acting. The formula $K = \frac{1}{2}(S^2/E)Al$ therefore does no express the work required for deformations after the elastic limit of the materia has been past, and cannot express the work required for rupture. The worl per unit volume required to produce deformations beyond the elastic limi or for rupture may, however, be determined from the stress-deformation dia gram, it being measured by the area included between the axis of abscissas and the stress-deformation curve up to the deformation in question.

6. Cylinders and Rollers

Thin Cylinders. Under the internal pressure of water or steam, a pip or a cylindrical boiler tends to rupture longitudinally along an element. Th internal force or pressure acts normally to the inne surface and with equal intensity at all points. Th tendency to rupture is resisted by the tensile strengt of the material. When the thickness of the material i very small compared with the diameter, the stress ma be considered uniformly distributed over the thicknes without appreciable error and the case is considered on of a thin cylinder. Ordinary pipes and boilers are considered thin cylinders.

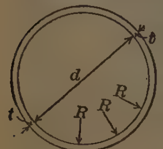


Fig. 13

Let R be the unit pressure (Fig. 13), d the diamete of the pipe or boiler, l its length, t the thickness of the shell, and S the unit stress in the material. From a principle of hydrostatics the force which tends to produce rupture is ldR. The total resisting stress is

$2ltS$. In order that equilibrium may obtain, the resisting stress must equal the pressure; hence

$$2tS = dR \quad \text{or} \quad S/R = d/2t \tag{1}$$

This is the formula commonly used in investigating pipes, or the cylindrical shells of boilers, under internal pressure.

The Head of a Cylindrical Boiler under pressure tends to tear away from the cylindrical shell by transverse rupture under a force of $\frac{1}{4}\pi d^2R$. This tendency is resisted by the tensile stress in the material distributed over an area equal to πdt. The stress S being considered uniformly distributed over the thickness t,

$$\pi dtS = \tfrac{1}{4}\pi d^2R \quad \text{or} \quad S/R = d/4t \tag{2}$$

A comparison of formulas (1) and (2) shows that in a thin cylinder the resistance to transverse rupture is twice the resistance to longitudinal rupture.

Cylinders under External Pressure, such as fire tubes in a boiler, fail by collapsing, the pressure tending to distort the cross-section of the cylinder from a true circle to an ellipse. While the formula $2tS = Rd$ applies to cylinders under external pressure as long as the cross-section remains a true circle, actual experience shows that irregularities of manufacture result in distortion, which the continued pressure tends to increase. No rational method being available for the investigation of cylinders under external pressure, recourse has been made to empirical methods. For tubes having a ratio of length to diameter greater than 6 and a ratio of thickness to diameter greater than 0.03, Carman and Carr found that the collapsing pressures are given by the following empirical formula for lap-welded steel pipe:

$$R = 83\,370\, t/d - 1025 \tag{3}$$

in which R is the external pressure in pounds per square inch, d is the external diameter in inches, and t is the thickness of the tube in inches. Univ. of Ill. Eng. Expt. Sta. Bulletins 5 and 99.

Thick Cylinders. When the thickness of metal in a pipe or cylinder is such that the difference between the internal and the external radius is large compared with the mean radius, the stresses due to the internal pressure cannot be considered as uniformly distributed over the sectional area of the annulus, and formula (1) therefore does not apply. The most widely used formula for the design and investigation of thick cylinders is Lamé's formula as modified by Clavarino. Let r_1 and r_2 (Fig. 14) be respectively the internal and external radii, R_1 the pressure per square inch on the inside of the cylinder, R_2 the pressure per square inch on the outside of the cylinder, S the tangential unit stress at a distance x from the axis of the cylinder. Then assuming the factor of the lateral contraction, or Poisson's ratio, to be $\frac{1}{3}$,

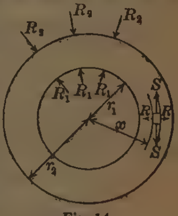


Fig. 14

$$S = \left[r_1^2R_1 - r_2^2R_2 + \frac{4r_1^2r_2^2}{x^2}(R_1 - R_2) \right] \Big/ 3(r_2^2 - r_1^2) \tag{4}$$

It will be noted that S decreases as x increases, wherefore S will be a maximum when $x = r_1$, or at the inner surface, and will be a minimum when $x = r_2$, or at the outer surface. When S is positiv the stress is tension; when negativ, compression. Under ordinary conditions R_2 may be neglected in comparison with R_1, being usually only the atmospheric pressure of 15 lb per sq in. Usually

only the maximum pressure, or that on the inside of the cylinder, is required. Therefore, making $R_2 = 0$ and $x = r_1$, formula (4) becomes

$$S = \tfrac{1}{3}R_1(r_1^2 + 4r_2^2)/(r_2^2 - r_1^2) \qquad (5)$$

which is generally employed for common cases of investigation or design.

In a thick cylinder, in addition to the tangential stress S, which may be either tension or compression depending on the relative values of r_1, r_2, R_1 and R_2, there is a radial compressive stress R which will have its maximum value at the inner surface, where it will equal R_1, and its minimum value at the outer surface, where it will equal R_2.

Cylindrical Rollers are commonly used in providing expansion bearings at the ends of long girders and trusses, to provide for the difference in length due to temperature changes. These rollers are designed to travel between steel plates and to transmit the load from the upper to the lower plate, each roller taking its proper proportion of the total load (Fig. 15).

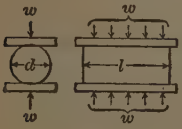


Fig. 15

Let W be the load carried by one roller of a length l and a diameter d, and let S be the maximum compressive stress in the roller and E the modulus of elasticity for the material; then

$$S = [(9W^2E)/(8l^2d^2)]^{\frac{1}{3}}$$

or

$$dl \doteq (3W/2S)(E/2S)^{\frac{1}{2}}$$

The first formula is generally used for the investigation of cylindrical rollers, the load W, the length l, and the diameter d being given; while the second is the proper formula for the design of rollers with a given load, working stress and modulus of elasticity. If w be the load per unit of length of roller, or $w = W/l$, then this formula becomes $w = \tfrac{2}{3}dS(2S/E)^{\frac{1}{2}}$. Substituting $S = 25\,000$ and $E = 30\,000\,000$ there results $w = 680\,d$ for steel. The rule for bridge rollers given in most recent specifications is $w = 600\,d$.

From tests by McDaniel at the University of Illinois the safe load in pounds per inch of length for hard wood rollers rolling on hard wood planks would seem to be about 25 times the diameter in inches.

These formulas are deduced under the assumption that the plates are not deformed. Experiments seem to indicate, however, that the plates are deformed as well as the rollers. The formulas err on the side of safety.

Ball Bearings. The permissible load on a hardened steel ball as determined by Stribeck from experimental data is given by the equation

$W = 565\,d^2$ for balls bearing against flat, hardened steel surfaces, and
$W = 1400\,d^2$ for balls supported in a hardened steel race in the shape of a groove with a radius $= \tfrac{2}{3}d$

In the above W is the load in pounds, and d the diameter in inches.

7. Repetitive and Impact Stresses

Fatigue of Metals. It is a well-known fact based on experiment and general experience that stresses which can be applied to a body a few times without causing apparent structural damage may, if applied a great many times, cause failure. If a polished surface on a heavily stressed part of a body is examined under a microscope as stress is repeatedly applied, minute flaws will be

seen to develop and spread, finally developing cracks. The phenomena of failure under repeated stress is known as Fatigue.

A view formerly common was that under repeated stress metal "crystallized." This view became common because it seemed to explain the fact that under repeated stress even ductile materials fail by snapping sharply off, and the fracture has a crystalline appearance. The gradual spread of minute flaws would cause the same result, the effect of the flaw being to reduce the available area resisting stress as if a sharp notch were cut in the body at the plane where flaws form. The crystallization theory of failure under repeated stress has given way to the micro-flaw theory.

Wöhler's Laws of Fatigue. All formulas for strength under repeated stress are based on test results. The earliest and the most noted repeated stress tests were made by Wöhler, 1859 to 1870, and the following statements summarize the results of his tests:

(1) "The rupture of a bar may be caused by repeated application of a unit stress less than the ultimate strength of the material.

(2) The greater the range of stress the less is the unit stress required to produce rupture after an enormous number of applications.

(3) When the unit stress in a bar varies from zero up to the elastic limit, an enormous number of applications are required to cause rupture.

(4) A range of stress from tension into compression and back again produces rupture with a less number of applications than the same range in stress of one kind only.

(5) When the range of stress in tension is equal to that in compression the unit stress that produces rupture after an enormous number of applications is a little greater than one-half the elastic limit."

The relation of the ultimate strength of materials under fatigue to their ultimate strength under gradually applied loads may be exprest by the formulas of Launhardt and Weyrauch (Art. 32), or by Merriman's formula

$$S = S_e + \tfrac{1}{2}(S_u - S_a)P'/P + \tfrac{1}{2}(S_u + S_a - 2S_e)(P'/P)^2$$

in which S is the ultimate unit stress under fatigue, S_u the ultimate unit stress under loads gradually applied, S_e the unit stress at the elastic limit, S_a the unit stress which causes rupture when the stress alternates from a certain value in tension to the same value in compression, usually taken as $\tfrac{1}{2}S_e$, and P and P' two values between which the total stress or the load alternates an enormous number of times. The word enormous as used above is intended to mean about 40 million.

Exponential Formula for Repeated Stress. If within the elastic limit materials of construction were absolutely elastic, below the elastic limit the material would be capable of withstanding an infinite number of repetitions of stress without failure. As a matter of experience, however, material will fail under a large number of repetitions of stress smaller than the elastic limit. From a study of available test data, Basquin in 1910 proposed the following formula for failure under repeated stress

$$S = \frac{A}{\sqrt[q]{N}} \qquad \text{or} \qquad \log S = \log A - 1/q \log N$$

in which S is the unit stress which will cause failure after N repetitions, and A and q are constants depending on range of stress, manner of loading, and nature of the material. In 1915 Moore and Seely proposed the following modification of the exponential formula

$$S = \frac{B}{(1 - P'/P)\sqrt[8]{N}} \qquad \text{or} \qquad \log S = \log B - \log(1 - P'/P) - \tfrac{1}{8}\log N$$

in which S and N have the same significance as above, B is an experimental determined constant for the material, and P'/P is the ratio of minimum maximum stress. The accompanying table gives values of B as determin from test data for a number of materials:

Tentative Values of B

Material	B	Log B
Structural steel and soft machinery steel..	250 000	5.39794
Wrought iron........................	250 000	5.39794
Steel, 0.45 per cent carbon..............	350 000	5.54407
Cold-rolled steel shafting................	400 000	5.60206
Tempered spring steel..................	400 000 to 800 000	5.60206 / 5.90309
Hard-steel wire........................	600 000	5.77815
Gray cast iron........................	100 000	5.00000
Cast aluminum........................	80 000	4.90309
Hard-drawn copper....................	140 000	5.14613

For low stresses, corresponding to numbers of N above 10 000 000, test data are ve few, but such as are available give values of S higher than are given by the exponent formula, which seems to err on the side of safety. This may be explained on the su position that under low stresses the over-stressed crystals of material are scattered a damage spreads slowly. Improved agreement with test data is obtained if the valu of S given by the exponential formula are multiplied by a "probability factor." Valu of such a probability factor are given in the accompanying table:

N	Probability factor
1 000 000	1.000
10 000 000	1.135
20 000 000	1.368
50 000 000	1.670
100 000 000	1.818
500 000 000	1.960
Infinity	2.000

A comparison of Merriman's formula and the exponential formula is given by the following example:

The girders supporting an elevated railway track are of structural steel, and are subjected to a stress varying from a minimum to a maximum of opposite sign and twice as great as the minimum, $P'/P=-0.5$. $S_u=60\ 000$, $S_e=35\ 000$, $S_a=\frac{1}{2}S_e$, B in the exponential formula $=250\ 000$, $\log B=5.398$, and N is taken as 40 000 000.

By Merriman's formula,

$$S=35\ 000+\tfrac{1}{2}(60\ 000-17\ 500)(-0.5)$$
$$+\tfrac{1}{2}(60\ 000+17\ 500-70\ 000)(0.25)$$
$$=25\ 300 \text{ lb per sq in,}$$

by the exponential formula,

$$\log S=\log 250\ 000-\log(1-0.5)-\tfrac{1}{8}\log 40\ 000\ 000$$
$$=5.398-0.1761-\frac{7.6021}{8}=4.2716,$$
$$S=18\ 700$$

From the above table the "probability factor" for $N=40\ 000\ 000$ is, by interpolatio 1.57. Using this factor S becomes $18\ 700\times 1.57=29\ 400$ lb per sq in.

Allowable stresses under repeated loading as computed by Merriman's formula do n differ greatly from the stresses computed by the exponential formula for $N=$ abo 10 000 000. Beyond 10 000 000 repetitions of stress there is very little experiment data available. Many machine members and some structural members, e g., the girde carrying an electric elevated railway—have to withstand more than 10 000 000 repetitio of stress in a normal "lifetime." The exponential formula indicates a very rapid increa of endurance for a slight reduction of stress, which is in accord with observed facts, a it gives a finite endurance for any repeated stress however small. In parts of structur

or machines whose failure would endanger human life it is recommended that in using the exponential formula the "probability factor" be omitted.

In designing parts to resist repeated stress it must be borne in mind that the static strength is a criterion independent of the strength under repeated stress. The safe static stress must never be exceeded in any member, although for low values of the number of repetitions the exponential formula may give results higher than the safe static stress for the material. Static strength and strength under repeated stress are two independent properties of a material.

Static and Sudden Loads. A load at rest, producing no change in the unit stress S, or a load which is increased gradually by increments from o up to P, is termed a Static Load, and unless otherwise noted is the load usually understood in the discussion of structural members. Many structural members, however, are subject to load applied in such a manner that the full intensity of the load is acting during much of the time that it is producing deformation. Such loads are termed Dynamic or Sudden Loads. It is obvious that the effect of a load suddenly applied is much greater than the effect of the same load applied in small increments.

Stresses Due to Sudden Loads. In a bar acted on by a load gradually applied within the elastic limit the load increases gradually from O to P, and the load-deformation diagram for the bar is a triangle. The energy stored up in the bar equals the area under the load-deformation triangle, or $\frac{1}{2}Pd$, where d is the deformation. If the load is applied very suddenly the load-deformation diagram is no longer a triangle, the area under it increases, and, as the limiting case, the load may be considered to be instantly applied, and its value equal to P for the whole period of deformation. In this limiting case the load-deformation diagram is a rectangle, the area under it equals Pd. Instead of there being equilibrium at the deformation d, as in the case of gradually applied load, the deformation and the stress increase until the additional energy under instantaneous load is expended. As this energy is twice as great as in the case of gradually applied load and, as the stress-deformation diagram is still a triangle the stress produced by an instantaneously applied load is twice as great as is the stress produced by a gradually applied load. Actually no load is instantaneously applied, but the instantaneously applied load may be regarded as the limiting case of rapidly applied loading.

Impact is a word used to denote the effect of a moving load. The blow of a hammer is a good example of impact, the velocity with which the weight of the hammer or the load is moving when it strikes being an important factor in the effect produced. If P be a load in motion with a velocity V at the moment of striking a horizontal bar, the energy due to the velocity, or the kinetic energy, could be exprest by $PV^2/2g$, in which g is the acceleration due to gravity. If h be the height thru which a load must fall in order to acquire the velocity V, then $V^2/2g=h$, and the kinetic energy of the moving load may be exprest by Ph. The load P striking the bar produces a stress which increases from o up to Q, with a corresponding deformation increasing from o up to d_1. The energy stored in the bar is evidently exprest by $\frac{1}{2}Qd_1$, which must be equal to the external work, provided no energy has been expended as heat or in giving velocity to the bar, or $\frac{1}{2}Qd_1=Ph$. If d be the deformation produced by a static load P, then $d_1/d=Q/P$, whence by combining and solving for Q and d_1 there results.

$$Q=P(2h/d)^{\frac{1}{2}} \quad . \quad d_1=d(2h/d)^{\frac{1}{2}}$$

from which it appears that Q and d_1 increase with h or in other words with the velocity with which the load is moving when impact occurs. If the bar

be vertical instead of horizontal the external work expended is exprest by $P(h+d_1)$. Substituting this value for the external work above.

$$Q=P+P(1+2\,h/d)^{1/2} \qquad d_1=d+d(1+2\,h/d)^{1/2}$$

It is obvious that these formulas are valid only when the stresses do not exceed the elastic limit of the material.

Compared with actual experiments the above formulas give values somewhat too large. This is due to the fact that some of the external work is not effective in producing stress, it being expended in giving motion to the bar and in producing heat, the heat being caused by the friction between the displaced molecules. For light bars, however, the values given by the formulas give results approximately correct.

Rupture from Impact. Since the stresses caused by moving loads increase with the velocity of the load, it is obvious that rupture may be caused by impact provided the load has the requisite velocity. The above formulas, however, do not apply, since they are valid only for stresses within the elastic limit. There being no rational formulas for rupture due to impact, the only information available has been obtained thru experiment. The relation between the work required for rupture from impact and the work required under static loads has been determined by Hatt. From nearly 200 experiments he determined that the work required for rupture from impact was about 30 percent greater than that required by static loads. He also found that the ultimate elongation was about 20 percent greater than for static loads.

Live Load Stresses, Coefficient of Impact. In computing stresses in structural members two classes of loads are usually considered; the dead load, or the weight of the various parts of the structure, and the live loads or the superimposed loads which the structure has been designed to carry. The effects of these two loads are usually computed separately. The stresses due to the dead loads are computed from static loads in the ordinary manner, the actual stresses increasing gradually as the structure is built. Stresses due to the live loads, however, may often be considerably greater than those due to corresponding static loads, since under some circumstances the loads are applied suddenly, as when a heavy, rapidly moving train runs onto the floor system of a bridge. It is evident, therefore, that in computing stresses due to various live loads proper allowance should be made for the suddenness with which the stresses may be induced. The Coefficient of Impact is the factor by which the corresponding static stress must be multiplied in order to give the amount that the live-load stress exceeds the corresponding static stress. Thus let S be the corresponding static unit stress due to a live load W, and i the coefficient of impact. Then the amount that the live-load stress exceeds the corresponding static stress would be iS and the total stress $S+iS$. Values of the coefficient of impact i have been determined by empirical methods, the values varying with existing conditions. For loads suddenly applied, such as in the case of the sheave beams of elevators or hoists, i is taken as unity. In other cases where the application of the load is more gradual, as in the case of a moving train on a bridge, i is taken as a fraction less than unity.

It should be noted that the meaning of the word impact as here used differs somewhat from its strict theoretical meaning. The use of the terms "impact" and "coefficient of impact" in connection with live-load stresses is, however, very general.

8. Combined Stresses

Combinations of Axial Forces. Assume a bar of a cross-section A acted upon at the same time by a number of axial forces some of which produce

tension and some compression. Assuming those producing tension to be positiv and those producing compression negativ, five forces may be represented by P_1, P_2, P_3 and $-P_1$, $-P_2$. Let the unit stress in the bar be S. It is obvious that the unit stress in the bar is equivalent to the stress produced by a load represented by the algebraic sum of the several loads acting, or $S = (P_1 + P_2 + P_3 - P_1 - P_2)/A$. If the forces represented by P_1 and $-P_1$ and P_2 and $-P_2$ have the same numerical values, then $S = P_3/A$. P_3 represents the algebraic sum of the forces acting, and being positiv indicates tension.

Change in Cross-section. A body under either tension or compression is subject also to shearing stresses along planes inclined to the direction of the applied force. The deformation resulting from these shearing stresses causes a decrease in the cross-section of the body in the case of tension and an increase in cross-section in the case of compression. When the elastic limit of the material is not exceeded, experiments show that the lateral unit deformation, or change in diameter or other lateral dimensions, bears a constant ratio to the linear unit deformation. Let this ratio be exprest by p. Let a bar of a diameter d and a length l under a unit stress S within the elastic limit have a linear unit deformation e, then the total linear deformation under the unit stress S will be represented by el, and the total lateral deformation, or change in diameter, will be exprest by ped. If d' be the diameter and l' the length of the bar while under the stress S, then

$$\text{For tension, } l' = (1+e)l \quad \text{and} \quad d' = (1-pe)d$$

$$\text{For compression, } l' = (1-e)l \quad \text{and} \quad d' = (1+pe)d$$

The quantity p is sometimes called the "factor of lateral contraction" but more commonly Poisson's Ratio.

Average values for Poisson's Ratio for common materials are given by the accompanying table:

Material	Poisson's Ratio
Steel	0.333
Wrought iron	0.333
Cast iron	0.250
Brass	0.333
Concrete	0.100

The True Internal Stresses due to three forces acting on a body so that each force has a direction prependicular to the plane of the other two, as would be the case of three forces acting normal to the faces of the parallelopiped (Fig. 16), each force being assumed to be opposed by an equal and opposite force on the opposite face, may be investigated by means of the factor of lateral contraction or Poisson's Ratio. Assuming the forces to be all tension, and the tensile unit stress in the direction parallel with the edges $O1$, $O2$, $O3$, to be respectively S_1, S_2, and S_3, then the forces producing the unit elongations e_1, e_2, and e_3 in the directions of S_1, S_2, and S_3, respectively, will, if E is the modulus of elasticity, be exprest by

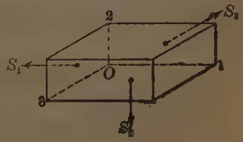


Fig. 16

$$Ee_1 = S_1 - pS_2 - pS_3 \qquad Ee_2 = S_2 - pS_1 - pS_3 \qquad Ee_3 = S_3 - pS_1 - pS_2$$

Representing by T_1, T_2, and T_3 the unit stresses Ee_1, Ee_2, and Ee_3, each bein respectively the unit stress which would produce the corresponding un deformation were the other two lateral forces not acting,

$$T_1 = S_1 - pS_2 - pS_3 \qquad T_2 = S_2 - pS_3 - pS_1 \qquad T_3 = S_3 - pS_1 - pS_2$$

Here T_1, T_2, and T_3 are called the true unit stresses in the three directior although they are not stresses as defined on p. 320 but rather quantities measure in pounds per square inch, obtained by multiplying the unit deformation by th modulus of elasticity of the material. The stresses in this formula are assume to be all tension and are therefore taken as positiv. Taking compression stress as negativ, the formula may be modified for any combination of tensile and com pressive stresses by changing the proper signs.

Apparent Stresses are those in the determination of which no account taken of the lateral deformation due to a force itself or to other forces actin in directions normal to that force. TRUE STRESSES are those determined fror the actual existing deformations, all forces acting on the body being considerec

Assume a bar 2 in × 2 in in cross-section to be subjected to an axial load of 24 oc lb with no other forces acting. Using symbols as above, $S_1 = 6000$ lb per sq in, an from formula $T_1 = S_1 = 6000$ lb per sq in, or the true stress is equal to the apparent stres assuming $p = 0.333$, $T_2 = 0.333 \times -6000 = -2000$. Whence it is seen that a true stre of 2000 lb per sq in exists at right angles to the direction of the load. Again assum a bar 3 in × 3 in in cross-section and 10 in long to carry a load of 54 000 lb compressio with a second load of 90 000 lb compression applied normal to one side. Then from th formula, assuming $p = 0.333$ as before, the true stresses are

$$T_1 = -6000 + 0.333 \times 3000 = -5000 \text{ lb per sq in}$$

$$T_2 = -3000 + 0.333 \times 6000 = -1000 \text{ lb per sq in}$$

Shear under Tension or Compression. A body under tensile or com pressive forces is subjected to tensile or compressive stresses in planes norm to the direction of the forces, and to shearing stresses between planes incline to the directions of the forces, the intensity of the shear varying with th inclination of the planes. Assume two forces P_1 and P_2 acting on a bar a right angles to each other, producing apparent normal unit stresses of S and S_2. Let S' represent the maximum shearing stress. Then from the rela tion existing between the maximum shear and tension or compression,

$$S' = \tfrac{1}{2}(S_1 - S_2')$$

The planes in which the maximum shearing stress occur are found to be thos which make angles of 45° with the directions of the two forces.

A bar of cast iron 1 sq in in cross-section under a compressive load of 2400 lb subjected to unit stresses $S_1 = 2400$ and $S_2 = 0$. The maximum shearing stress $S' = 2400/2 = 1200$.

Failure under Combined Stress. Several theories have been advance to explain the phenomena of failure under combined stress. Three of these ar of practical importance.

The Maximum Strain is the criterion of safety or danger for materi according to one of these theories. This is equivalent to using as a criterio the maximum " true " internal stress as defined on p. 335. Using this theor as a basis for calculation it is necessary to take account of the lateral deformatio accompanying axial stress, in other words, Poisson's Ratio must be con sidered. An example of the use of this theory is given under the paragrap on True Internal Stress above.

The Maximum Stress, or rather the maximum apparent stress as defined on p. 336 is the criterion of safety for a material according to the second of these heories. Using this theory as a basis for computation no account is taken of the lateral deformation accompanying axial stress. This is equivalent to aking Poisson's Ratio equal to o. This theory is frequently called the Rankine theory or the common theory, and is the one commonly used by engi-neers. While the weight of experimental evidence seems to show that it is not strictly true, yet in most cases it gives results not widely divergent from the maximum strain theory. An example of the use of this theory is given under he paragraph on Apparent Stresses on p. 336.

The Maximum Shearing Stress developed under load is the criterion of afety for materials according to the third of these theories. This theory in-volves the relation of strength of a material in shear to its strength in tension or compression. If the maximum shear theory is true for all cases it must be rue for a bar under simple axial tension. For such a bar the maximum shear-ng stress is on a plane making $45°$ with the axis of the bar, and is equal to one-half the tensile stress. For brittle materials the strength in shear is distinctly greater than one-half the strength in tension, for ductile metals the strength in hear varies from about one-half to 0.6 of that in tension. An illustration of he use of this theory is given in the paragraph Shear under Tension or Com-pression, p. 336.

Experimental Study of Failure under Combined Stress has been made by Guest, by Scoble, by Mason, by Hancock, by Becker, and by Matsumura and Hamabe. Recent investigations are those of Becker on steel (Univ. of Ill., Eng. Expt. Sta., Bulletin 85) and of Matsumura and Hamabe on cast iron (Memoirs of the Coll. of Eng., Kyoto Imperial Univ., Feb., 1915). These investigations indicate that for cast iron and robably for other brittle materials the maximum strain theory holds, and that for ductile naterials it is necessary to compute both the maximum "true" stress, and the maximum hearing stress, and to know the ratio of shearing strength to tensile or compressive trength for the material. For steel this ratio varies from about 0.5 to 0.6.

9. Miscellaneous Cases

Eccentric Loads in a Rectangular Bar. An eccentric load is one whose ine of action does not coincide with the axis of the bar upon which it acts. In concentric or axial loading the load, or if there are several loads all acting at he same time, their resultant, coincides with the axis of the bar. The result of an eccentric load is an uneven distribution of stress over the area of cross-section, the unit stress in one portion of the cross-section being considerably greater than in another portion, the actual variation depending upon he position of the resultant load with refer-ence to the axis.

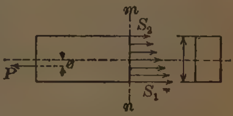


Fig. 17

Let P be a load acting on a rectangular bar (whose area is A) at a distance e from the axis of the bar, e being measured in the di-rection of the width d (Fig. 17). Were the load P axial the unit stress S would be the same all over the area of cross-section and would be $S = P/A$. The load P being eccentric, P/A expresses only the average unit stress, the maximum unit stress and the minimum unit stress varying from the average an amount dependent upon the position of the load. Let $m-n$ be any section and S_1 be unit stress along edge nearer to P and S_2 unit stress along opposite edge. It has been found that the intermediate stresses between S_1 and S_2 will vary

uniformly or as a straight line, so that the plane figure (Fig. 17), of which S_1 and S_2 are two parallel sides, is a trapezoid. In order that equilibrium may obtain, the resultant stress must equal P, and its line of action must be in the same line as P. From these two conditions of equilibrium it results that

$$S_1 = (P/A)(1 + 6\,e/d) \qquad S_2 = (P/A)(1 - 6\,e/d)$$

and these formulas are applicable alike to cases of tension or compression. The same result might have been obtained by considering the bar subject to an axial load P and a bending moment Pe.

Making $e = o$ in the formulas, $S_1 = P/A$, and $S_2 = P/A$, which is a case of axial loading. When $e \doteq d/6$, $S_1 = 2P/A$ and $S_2 = 0$, which indicates that when the resultant force is at the edge of the middle-third the maximum stress is twice the average stress and the minimum stress is zero. Making e greater than $d/6$, the sign of the stress S_2 changes, indicating a change from tension to compression or vice versa. A brick pier 5 ft $\times$ 4 ft in cross-section, loaded at a point 1 ft from the center of the top in the direction of the width, with a load of 16 000 lb, would have a maximum unit compression of $S_1 = (16\,000/20)(1 + 6 \times \frac{1}{4}) = 2000$ and a stress on the opposite edge of the pier of $S_2 = (16\,000/20)(1 - 6 \times \frac{1}{4}) = -400$. The minus sign indicates a change in stress, or tension

Centrifugal Stress in a Revolving Bar. If a weight is secured to an axis of revolution by means of a cord or a bar, and is made to revolve about that axis at a certain radius, a tensile stress is generated in the cord or bar. Let P be a weight revolving about an axis B (Fig. 18) at a radius r from the axis to the center of gravity of P and with a velocity V. Let Q be the centrifugal tension generated. Then from mechanics $Q = PV^2/gr$, in which g is the acceleration due to gravity, the mean value of which is usually taken as 32.16 feet per second per second. Let n be the number of revolutions per second; then $V = 2\pi nr$ and $Q = (4\,P\pi^2n^2r)/g$, which gives the centrifugal stress in the member securing the weight to the axis when that member has no appreciable weight, as would be the case were it a cord. If the connecting member is a bar of a length l and weight W, then

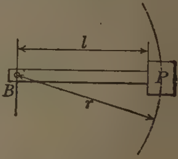


Fig. 18

$$Q = (4\,P\pi^2n^2r)/g + (2\,W\pi^2n^2l)/g \quad \text{or} \quad Q = 4\,(Pr + \tfrac{1}{2}\,Wl)\,\pi^2n^2/g$$

In this formula a portion of the stress Q is due to the weight P revolving with its center of gravity at a distance r from the axis, and a portion is due to the weight of the bar revolving with its center of gravity at a distance $\frac{1}{2}\,l$ from the axis. Q in this case is the stress in the bar at the axis. The stress in a simple bar revolving about an axis at one end varies from Q at the axis to o at the outer end. If w be the weight of the bar per unit of volume and A its cross-sectional area, the stress Q' at any point distant x from the axis is

$$Q' = 2\,wA\,(l^2 - x^2)\pi^2n^2/g$$

Giving Q' its proper value and making $x = 0$, the number of revolutions required to rupture a bar of any given size, length, and material may be found. Thus the number of revolutions required to rupture a steel bar one square inch in cross-section and 4 feet long will be found by substituting in the formula $x = 0$, $l = 4$ ft, $A = 1$, $w = 3.4$ lbs, $Q' = 60\,000$ lbs per sq in, $g = 32.16$; then n equals about 42 revolutions per second.

Revolving Thin Hoop. A thin circular hoop having a thickness t and a radius r revolving about its center generates a tension in the hoop due to the centrifugal force acting radially in a manner similar to the action of an interral pressure on a section of a thin cylinder. If S be the tensile unit stress in the hoop, W = weight per foot of the hoop, and the other symbols as above, then

$$S = 4\,\pi^2 w n^2 r^2 / g \quad \text{or} \quad S = 4\,(\pi n r)^2 w / g$$

which applies only when t is very small compared with the radius r.

Revolving Thick Hoop or Solid Wheel. Let a thick hoop have an inside radius of r_1 and an outside radius of r_2, and let R be the radial unit stress and S the tangential unit stress, at a distance x from the axis, due to the angular velocity a or $2\,\pi n$. Other symbols remaining as above, then

$$R = (3/2\,g)\,(w\pi^2 n^2)\,(r_1^2 + r_2^2 - r_1^2 r_2^2/x^2 - x^2)$$
$$S = (3/2\,g)\,(w\pi^2 n^2)\,(r_1^2 + r_2^2 + r_1^2 r_2^2/x^2 - \tfrac{1}{3}\,x^2)$$

For a solid wheel, such as a millstone or a grindstone, $r_1 = 0$ and $r_2 = r$. Whence, when $x = 0$, the unit stresses at the center are $R = S = 3/2\,(\pi n r)^2 w/g$. When $x = r$, then $R = 0$, and $S = (\pi n r)^2 w/g$ is the tangential stress at the circumference. The above formulas are deduced by supposing that the revolving body does not change its form, but they give the correct actual stresses at the circumferences. For the center of a solid wheel, the formula for S does not give the correct stress by making $r_1 = 0$, but the true radial and tangential stresses corresponding to the actual deformation are $\tfrac{3}{2}\,(1 - p)\,(\pi n r)^2 w/g$. Here p is Poisson's Ratio, or the factor of lateral contraction, the mean value of which is $\tfrac{1}{3}$ for steel, $\tfrac{1}{4}$ for cast iron, and $\tfrac{1}{5}$ or less for stone. For a solid steel wheel, the true unit stress at the axis is $(\pi n r)^2 w/g$, but if there be a very small hole at the axis the true unit stress is two times as great. All these formulas apply only when the elastic limit of the material is not exceeded.

Temperature Stresses. All structural materials undergo changes in length due to changes in temperature. THE COEFFICIENT OF EXPANSION for any material is the factor which expresses the change per unit of length for each degree of temperature. A bar or other structural member of a length l under a change in temperature of t degrees will, if free to move, undergo a change in length of tln, n being the coefficient of expansion. If the bar or member is fixt so that the change in length cannot occur it is evident that there is generated in the bar or member a stress equal in amount to that required to produce a deformation of tln or a unit deformation tn. If E be the modulus of elasticity and S the unit stress produced, then

$$S/E = tn \quad \text{or} \quad S = tnE$$

from which it is seen that S is independent of the length of the member.

Average values for the coefficient of expansion based on one degree (Fahrenheit) are as follows: $n = 0.000\,006\,2$ for cast iron, $0.000\,006\,5$ for steel, $0.000\,006\,7$ for wrought iron, $0.000\,005\,0$ for brick and stone, and $0.000\,005\,5$ for concrete.

Shrinkage of Hoops. A hoop surrounding a cylinder such as a reinforcing band on a gun or a tire on a wagon wheel or locomotive driver, is usually held in place by turning the hoop to an inside diameter slightly smaller than that of the cylinder it is intended to surround, and then expanding it by heat until large enough to fit, the shrinkage in cooling holding it securely in place. In such a hoop a tangential stress is produced in the hoop and a radial pressure in the cylinder which it encloses. When the thickness of the hoop is small compared with its diameter, all the deformation produced may be considered as confined to the hoop. The tension in the hoop will be proportional to its change in diameter. Let d = the diameter of the cylinder to be enclosed, which is assumed to be the same after the hoop is in place, d_1 = the diameter to which the hoop has been turned, S = the tangential unit stress, E = the modulus of elasticity, R = the radial unit stress acting on the inside of the hoop, and t = the thickness of the hoop. Then

$$S = E\,(d - d_1)\,/d_1 \quad \text{and} \quad R = 2\,tS/d_1$$

The values found for S from this formula are somewhat too large since some chan is made in the diameter d due to the radial pressure. For thick hoops such as the ban on heavy guns more exact formulas are usually employed

Internal Friction. In 1893 the discovery was made by Hartmann th lines of stress became visible on polished metal specimens these lines remai ing after the removal of the load if the elastic limit had been exceeded. O a cylindrical specimen these lines are two sets of helices; on a flat specime they are two sets of straight lines. In tension they make angles with the ax greater than 45°, in compression the angles are less than 45°. These lines ind cate the direction of planes along which sliding or shearing is occurrin Internal friction occurs along these planes from the theory of which Merrima deduced the ratio between the shearing and compressive strength of britt materials (Mechanics of Materials, 1910, p. 380) finding 0.13 for anthraci coal, 0.18 for sandstone, 0.23 for hard brick, 0.29 for concrete, and 0.35 fo cast iron.

BEAMS, COLUMNS, SHAFTS

10. Moments and Shears

Flexure, or Bending, is the phenomenon which occurs when a straigh bar is subjected to a force or a combination of forces so applied that the axi of the bar is caused to assume the form of a curve. The phenomenon c flexure is a combination of the three simple stresses of tension, compressio and shear. Thus a horizontal bar simply supported at the ends under th influence of its own weight assumes the form of a curve, concave upward, an is undergoing flexure. The fibers on the convex side of the bar are elon gated and therefore are in tension, while the fibers on the concave side ar shortened and are therefore in compression. Shear is taking place betwee each vertical plane of the bar and the one adjoining, between the middle o the bar and each support. The structural members which are ordinaril subject to flexure are called BEAMS and are usually horizontal members, carry ing loads acting vertically. Flexure, however, is not confined entirely t beams, since it may occur in any member of a structure under the influenc of loads other than axial loads. Even in the case of a strut or column whic is under compression, flexure may occur when the acting forces are eccentri with respect to the axis.

A Simple Beam is a horizontal member simply supported at the ends so that all part have free movement in a vertical plane under the influence of vertical loads. Simple beam are the commonest structural members. Under ordinary loading the upper fibers are i compression and the lower fibers in tension. A CANTILEVER BEAM is a member wit one end projecting beyond the point of support, free to move in a vertical plane under th influence of vertical loads placed between the free end and the support. The fibers i the upper side of such a beam are in tension and those in the lower side in compression A beam with one end rigidly fixt in a brick wall and the other end free is an exampl of a cantilever beam. Constrained beams are those rigidly fixt at one or both point of support. A beam with one or both ends rigidly built into brickwork is an example A CONTINUOUS BEAM is one having more than two points of support. More than tw points of support cause a distribution of stress similar to that in a constrained beam.

Neutral Surface and Neutral Axis. Any beam under flexure takes th form of a curve. The fibers of the beam on the concave surface are sub jected to compression, while those on the convex surface are subjected t tension. It is obvious that these stresses must decrease toward the middle o the depth of the beam; therefore at some point in the depth of the beam ther is a surface where the fibers are neither in tension nor in compression an

where no deformation is taking place. This surface is termed the NEUTRAL SURFACE. The trace of this surface or plane on any cross-section of the beam is termed the NEUTRAL AXIS of that section.

From experiment it has been shown that where the elastic limit of the material has not been exceeded the deformation in any fiber and in consequence the stress in that fiber is proportional to its distance from the neutral surface. It may also be demonstrated analytically that the neutral surface passes thru the center of gravity of the cross-section.

End Reactions. In order that equilibrium may obtain in any vertical system of forces acting in one plane, as in the case of the loads and reactions of beams, it is known from analytical mechanics that, first, the algebraic sum of all vertical forces must equal zero, and second, the algebraic sum of all moments must equal zero. From the first of these laws it is apparent that the sum of the reactions must equal the sum of the loads. In a simple beam when the loads are systematically placed with reference to the supports, as in the case of loads uniformly distributed, such as the weight of the beam itself, or equal concentrated loads placed at equal distances from the supports or center of beam, each reaction will equal ½ the sum of the loads. When the loads are not systematically placed the reaction at each support may be ascertained from the second of the laws stated above. Thus consider the system of loads, Fig. 19.

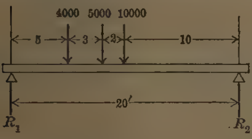


Fig. 19

Taking moments about the left support $R_2 \times 20 - 4000 \times 5 - 5000 \times 8 - 10\,000 \times 10 = 0$, or $R_2 = 8000$ lb. Similarly moments taken at the right support give $R_1 = 11\,000$. It will be noted that the sum of the reactions equals 19 000 the sum of the loads, thus fulfilling the first law of equilibrium. When some of the loads are uniformly distributed over a portion of or the whole of the beam the same method may be applied to find the reactions by considering the uniform loads as concentrated at their centers of gravity.

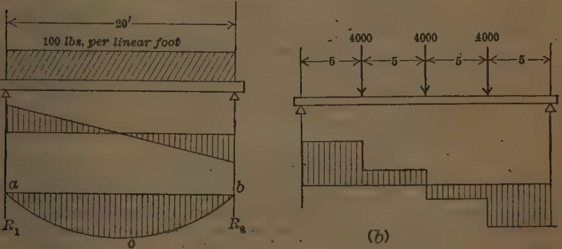


Fig. 20

Vertical Shear. At every section in a beam two equal and opposite forces, the one on the right and the other on the left of the section, tend to shear the beam at that point. This shearing tendency is greatest at the reactions where the shearing forces are each equal in amount to the reaction. The vertical shear at any section is the measure of the shearing tendency at that section and is equal to the algebraic sum of all the forces to the left of that section, upward forces or reactions being taken as positiv and the downward forces

or loads being taken as negativ. The vertical shear may be either negativ or positiv, depending upon the relative values of the loads and reactions to the left of the section.

Figs. 20a and 20b illustrate diagrammatically the vertical shear. Fig. 20a shows a beam with a uniform load of 100 lb per ft, the vertical shear at each support being equal to the reaction of 1000 lb at that support and gradually decreasing to zero at the center. Fig. 20b shows diagrammatically the vertical shear under concentrated loads, the vertical shear between the left reaction and first load being 6000 pounds, between the first and second load 2000 pounds, at the center zero, between the second and third loads 2000, and between the third load and the right reaction 6000 pounds.

The Bending Moment at any section is the algebraic sum of the moments of all forces on the left of that section, moments tending to cause rotation in the same direction as the hands of a clock being taken as positiv, and those tending to cause rotation in the opposite direction being taken as negativ. The bending moment at any section is the measure of the flexural stress at that section.

In Fig. 21a let the beam have a length l and be loaded with a uniform load of w lbs per lin ft. The left reaction will then be $wl/2$, and if M be the bending moment at the distance x from the left support, then $M = \frac{1}{2}wlx - \frac{1}{2}wx^2$, whence it is seen that $M = 0$ when $x = 0$, and $M = \frac{1}{8}wl^2$ when $x = \frac{1}{2}l$. In Fig. 21b let the beam be loaded with the

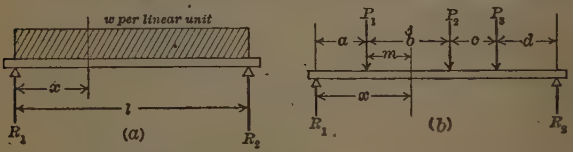


Fig. 21

loads P_1, P_2, P_3. The reactions R_1 and R_2 may be found by the principle of moments. The bending moment at any point between the first and second loads will be $M = R_1x - P_1m$.

Maximum Bending Moment. In a cantilever beam the maximum bending moment is at the support irrespective of the position of the loads. In a simple beam the point of maximum bending moment, or the dangerous section in a beam, is at the point where the shear passes thru zero, that is, where the shear changes from positiv to negativ, or vice versa. This point may be located in any system of loading by beginning at one reaction and subtracting the loads in order from the reaction until a point is reached where the sum of the loads equals the reaction.

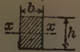
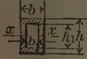
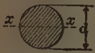
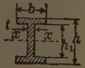
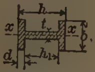
In Fig. 20a the bending moment under a uniformly distributed load is illustrated diagrammatically. From the expression for the bending moment at any point, $M = \frac{1}{2}wlx - \frac{1}{2}wx^2$, it will be noted that the bending moment at any section may be represented by the corresponding ordinate of the plane figure (Fig. 20a) formed by the straight line ab and the parabola acb. It will be noted that the maximum ordinate is at the center, coinciding with the axis of the parabola, and that at the supports the ordinates are zero.

11. Formulas for Flexure

Conditions of Equilibrium. The investigation of beams is governed by the following three conditions of equilibrium, which hold for any section:

Sum of all tensile stresses = Sum of all compressive stresses.
Resisting Shear = Vertical Shear.
Resisting Moment = Bending Moment.

Properties of Common Sections of Beams

Sections of beams, Fig. 22	Distance from neutral axis to extreme fiber of section	Moment of inertia I	Section modulus I/c	Radius of Gyration r $\sqrt{I/A}$
	$\frac{1}{2}h$	$\frac{1}{12}bh^3$	$\frac{1}{6}bh^2$	$\frac{h}{\sqrt{12}}=0.289\,h$
	$\frac{2}{3}h$	$\frac{1}{36}bh^3$	$\text{Min}=\frac{bh^2}{24}$	$\frac{h}{\sqrt{18}}=0.236\,h$
	$\frac{1}{2}h$	$\frac{1}{12}(bh^3-b_1h_1^3)$	$\frac{bh^3-b_1h_1^3}{6h}$	$\sqrt{\frac{bh^3-b_1h_1^3}{12(bh-b_1h_1)}}$
	$\frac{1}{2}d$	$\frac{\pi d^4}{64}=.0491\,d^4$	$\frac{\pi d^3}{32}=.0982\,d^3$	$\frac{d}{4}$
	$\frac{1}{2}d$	$\frac{\pi(d^4-d_1^4)}{64}=.0491(d^4-d_1^4)$	$\frac{\pi(d^4-d_1^4)}{32\,d}=.0982\frac{(d^4-d_1^4)}{d}$	$\frac{\sqrt{d^2+d_1^2}}{4}$
	$\frac{1}{2}h$	$\frac{bh^3-h_1^3(b-t)}{12}$	$\frac{bh^3-h_1^3(b-t)}{6\,h}$	$\sqrt{\frac{bh^3-h_1^3(b-t)}{12\,[bh-h_1(b-t)]}}$
	$\frac{1}{2}b$	$\frac{2\,db^3+h_1t^3}{12}$	$\frac{2\,db^3+h_1t^3}{6\,b}$	$\sqrt{\frac{2\,db^3+h_1t^3}{12\,[bh-h_1(b-t)]}}$
	$\frac{1}{2}h$	$\frac{bh^3-h_1^3(b-t)}{12}$	$\frac{bh^3-h_1^3(b-t)}{6\,h}$	$\sqrt{\frac{bh^3-h_1^3(b-t)}{12\,[bh-h_1(b-t)]}}$
	c	$\frac{h_1t}{12}\left[t^2+12\left(b-c-\frac{t}{2}\right)^2\right]+\frac{db}{6}\left[b^2+3(2c-b)^2\right]$	$\frac{I}{c}$	$\sqrt{\frac{I}{A}}$
	c	$\frac{bd}{12}\left[d^2+12\left(h-c-\frac{d}{2}\right)^2\right]+\frac{th_1}{12}\left[h_1^2+12\left(c-\frac{h_1}{2}\right)^2\right]$	$\frac{I}{c}$	$\sqrt{\frac{I}{A}}$

Resisting Shear. At any section in a beam the internal forces must equal the external forces in order that equilibrium may obtain. At any section, therefore, some internal force must oppose and be equal to the vertical shear at that section. This force is the shearing stress in the material and is called the RESISTING SHEAR. Let V be the vertical shear at any section, S_s the maximum unit shearing stress at that section and A the area of the section. The average shearing stress at the section is

$$S'_s=V/A$$

The shearing stress is not, however, uniformly distributed over the cross-section and the maximum shearing stress on the section is

$$S_s=kV/A\,, \qquad (1)$$

in which k is a constant depending on the shape of the section. For a rectangle k is 1.50, for a triangle k is 1.33, and for a circle k is 1.33. Values of factors for determining maximum shearing stress in I beams and channel beams are given on pp. 441, 458.

In a beam the shearing stress is greatest at the neutral axis and is 0 at the extreme fibers. At any point in a beam the horizontal shearing stress is equal in intensity to the vertical shearing stress at that point. Wooden beams which are weak in shear along the grain are in special danger of failure by horizontal shear.

Resisting Moment. The bending moment at any section tends to cause rotation about that section. The tendency to rotate is resisted by the moment of tensile and compressive stresses in the material at that section, which act as an internal couple. This internal couple is called the resisting moment. Let S be the unit stress at any extreme fiber on the surface of the beam due to the bending moment and c the distance from that fiber to the neutral surface, M the resisting moment or its equal, the bending moment; then

$$M = \frac{SI}{c}, \quad \text{or} \quad S = \frac{Mc}{I} \qquad (2)$$

in which I is the moment of inertia of the section about a gravity axis.

The Moment of Inertia of an area with reference to any axis may be defined as the sum of the products obtained by multiplying each elementary area of cross-section, da, by the square of the distance of that particular elementary area from the axis. The moment of inertia is represented by I, and

$$I = \Sigma z^2 da \qquad (3)$$

in which z is the distance of the elementary area da from the axis. The MOMENT OF INERTIA is a factor depending on the shape of the cross-section. From the above expression it will be found that the values of I will be greatest for sections having the largest area at the greatest distance from the axis of reference. Unless otherwise noted, the axis of reference is always the neutral axis of the section.

Section Modulus. This is the term I/c in the formula for the resisting moment. It is the measure of the resisting moment or the strength of a beam of given cross-section and is largely used as a basis of computation in the design and investigation of beams. In the table on page 343 are given the values of the moment of inertia I and of the section modulus I/c for the various sections most commonly used in structural design.

Investigation and Design. Formulas (1) and (2) are the formulas used in the design and investigation of beams. Except in cases of short spans and heavy loads the question of resisting shear is not usually the controlling factor. Having obtained the bending moment from the conditions of loading in any particular case of design, and having assumed a proper working value for S, formula (2) is solved for I/c and the proper beam selected. In the investigation of beams, M is determined from the conditions of loading and from the kind of beam employed. The proper substitutions are then made in formula (2) and the equation solved for S, which may be either tension or compression. A comparison of the value of S with the allowable working stress for the material will determine the degree of stability. See also the tabulated statement on p. 360 of the various ways in which beams may fail.

Effect of Combined Shearing and Bending Stress. In a beam of I section or channel section both the shearing stress and the bending stress at the junction of web and flange may be high, and the combination of the two stresses

sometimes causes stress on an inclined plane which is greater than the direct stress in the extreme fibers of the flange. This effect of combined stresses is rarely of importance except for deep beams with short spans. If S_t is the maximum stress on an inclined plane at the point under consideration, S' the direct bending stress at the junction of web and flange, and S'_s the shearing stress at the same point, which is slightly less than the shearing stress for the same cross-section at the neutral axis, then

$$S_t = \tfrac{1}{2}S' + \sqrt{(S_s')^2 + (\tfrac{1}{2}S')^2}$$

See also paragraph on Flexure and Tension, p. 370.

Elastic Curve. In a horizontal beam under a system of vertical loads the fibers in the upper surface are shortened, while those in the lower surface are elongated. This causes the beam to deflect downward and the neutral surface to assume the form of a curve whose radius of curvature at any section is dependent upon the bending moment at that section and the moment of inertia of the section.

Thus in Fig. 23 let mn represent the elementary section dl of a beam whose length is l and let $b'b$ and $c'c$ represent two sections separated by the distance dl. These sections, before bending, are assumed to be parallel; after bending, the sections produced intersect at some point o, the distance om being the radius of curvature R. Let $c_1'c_1$ be drawn parallel to $b'b$ thru n. The distance $c_1'c'$ represents the deformation of the upper fibers in the length dl, and the distance cc_1 represents the corresponding deformation in the lower fibers. Let $c'c_1'$ be represented by e; then $e = (S/E)dl$, in which S is the unit stress producing the deformation e, and E is the modulus of elasticity. Let c represent the distance from the neutral surface to the extreme fiber; it then follows that $R/dl = c/e$, whence $R = EI/M$, which expresses the value of the radius of curvature R at any section in terms of the modulus of elasticity, the moment of inertia and the bending moment at the section. When $M = o$, $R =$ infinity, or the curve at that point is a straight line; when M is a maximum, R is a minimum, or the curve is the sharpest. If the curve be referred to a system of coordinate axes in which x represents abscissas or horizontal distances and y ordinates or vertical distances, giving R its value as determined for very flat curves by the differential calculus in terms of x, y and l; then

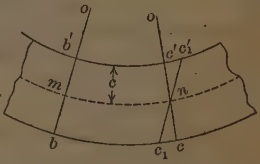


Fig. 23

$$\frac{EI}{R} = EI\frac{d^2y}{dx^2} = M \qquad (4)$$

which is the general differential equation of the elastic curve of any beam under any system of loading exprest in terms of the modulus of elasticity, the moment of inertia and the bending moment.

Slope of Beams. For a beam the integration

$$\int EI\frac{d^2y}{dx^2} = EI\frac{dy}{dx} + C$$

gives an expression involving $\frac{dy}{dx}$, the change of slope, or inclination of the elastic curve for any section of the beam. C is a constant of integration depending upon loading and upon end conditions. A beam so rigidly held that under bending action the slope remains horizontal at a support is said to be fixt at that support, and at the support $\frac{dy}{dx} = o$.

Deflection. In formula (4), assuming the origin of coordinates at one support, where the deflection of a beam is zero, y becomes the deflection at any point at a distance x from the support or origin of coordinates. Substituting proper values for E, I, and M, integrating twice, and giving proper values to the constants of integration, the value of y, or the deflection, may be determined for any point in the beam.

12. Beams of Uniform Cross-section

Values for shear, V, and for moment, M, at any section; for maximum moment, M_{max}, for slope, dy/dx, and for deflection, y, at any section, and for maximum deflection, y_{max} are given in the accompanying table for a number of common kinds of beams of uniform cross-section.

It is possible to combine values given in the table for different loadings to give values for loadings not given in the table. For example, by combining the values for a beam loaded with two symmetrical loads with the values for a beam loaded at one point in its span values for a beam with three loads may be obtained.

A beam with ends and loads overhanging the supports may be treated as a combination of a cantilever beam and an end-supported beam.

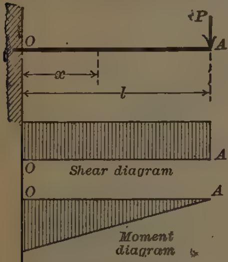


Fig. 24. Cantilever Beam, Concentrated Load at Outer End.

Shear at any section,

$$V = P \text{ (constant)};$$

Moment at any section,

$$M = -P(l - x);$$

$$M_{max} = -Pl \text{ at } O;$$

Equation of slope,

$$dy/dx = (-P/EI)(lx - x^2/2);$$

Equation of elastic curve,

$$y = (-P/EI)\left(\frac{lx^2}{2} - \frac{x^3}{6}\right);$$

Maximum deflection,

$$y_{max} = -Pl^3/3EI \text{ at } A.$$

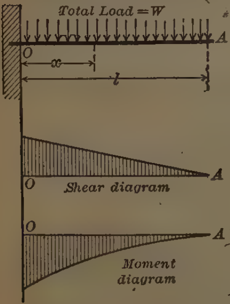


Fig. 25. Cantilever Beam, Uniform Load.

Shear at any section,

$$V = W - \frac{Wx}{l};$$

$$V_{max} = W \text{ at } O;$$

Moment at any section,

$$M = -W\left(l/2 - x + \frac{x^2}{2l}\right);$$

$$M_{max} = -Wl/2 \text{ at } O;$$

Equation of slope,

$$dy/dx = -(W/EI)\left(\frac{lx}{2} - \frac{x^2}{2} + \frac{x^3}{6}\right);$$

Equation of elastic curve,

$$y = -(W/EI)\left(\frac{lx^2}{4} - \frac{x^3}{6} + \frac{x^4}{24l}\right);$$

Maximum deflection,

$$y_{max} = -Wl^3/8\,EI \text{ at } A.$$

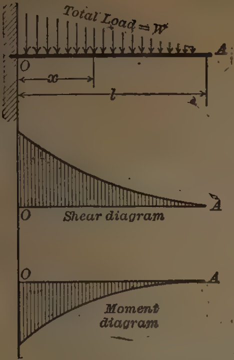


Fig. 26. Cantilever Beam, Load Increasing Uniformly from Free End to Support.

Shear at any section,

$$V = W\left(1 - 2\frac{x}{l} + \frac{x^2}{l^2}\right);$$

$$V_{\max} = W \text{ at } O;$$

Moment at any section,

$$M = W\left(\frac{x^3}{3l^2} - \frac{x^2}{l} + x - \frac{l}{3}\right);$$

$$M_{\max} = -Wl/3 \text{ at } O;$$

Equation of slope,

$$dy/dx = \frac{W}{EI}\left(\frac{x^4}{12l^2} - \frac{x^3}{3l} + \frac{x^2}{2} - \frac{lx}{3}\right);$$

Equation of elastic curve,

$$y = \frac{W}{EI}\left(\frac{x^3}{6} - \frac{x^4}{12l} + \frac{x^5}{60l^2} - \frac{lx^2}{6}\right);$$

Maximum deflection,

$$y_{\max} = -Wl^3/15EI \text{ at } A.$$

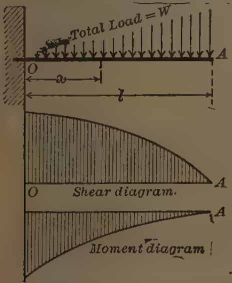


Fig. 27. Cantilever Beam, Load Increasing Uniformly from Support to Free End.

Shear at any section,

$$V = W\left(1 - \frac{x^2}{l^2}\right);$$

$$V_{\max} = W \text{ at } O;$$

Moment at any section,

$$M = W\left(x - \frac{x^3}{3l^2} - \frac{2}{3}l\right);$$

$$M_{\max} = -\tfrac{2}{3}Wl \text{ at } O;$$

Equation of slope,

$$dy/dx = \frac{W}{EI}\left(\frac{x^2}{2} - \frac{x^4}{12l^2} - \frac{2}{3}lx\right);$$

Equation of elastic curve,

$$y = \frac{W}{EI}\left(\frac{x^3}{6} - \frac{x^5}{60l^2} - \frac{1}{3}lx^2\right);$$

Maximum deflection,

$$y_{\max} = -\frac{11}{60}\frac{Wl^3}{EI} \text{ at } A.$$

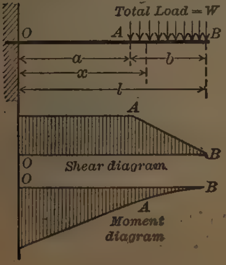


Fig. 28. Cantilever Beam, Uniform Load Over Part of Beam.

Shear at any section,

$$V = W \text{ for } OA;$$

$$V = W - \frac{W}{b}(x - a) \text{ for } AB;$$

Moment for any section,

$$M = -W\left(l - x - \frac{b}{2}\right) \text{ for } OA;$$

$$M = -\frac{W}{2b}(l - x)^2 \text{ for } AB.$$

$$M_{\max} = -W\left(l - \frac{b}{2}\right) \text{ at } O.$$

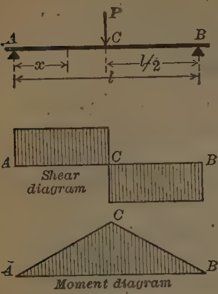


Fig. 29. Beam Supported at Ends, Concentrated Load at Middle of Span.

End reactions, $R_A = R_B = P/2$.

Shear at any section, $V = \pm P/2$;

Moment at any section,

$$M = Px/2 \text{ for } AC;$$

$$M_{max} = Pl/4 \text{ at } C;$$

Equation of slope,

$$\frac{dy}{dx} = \frac{P}{EI}\left(\frac{x^2}{4} - \frac{l^2}{16}\right);$$

Equation of elastic curve,

$$y = \frac{P}{EI}\left(\frac{x^3}{12} - \frac{l^2x}{16}\right);$$

Maximum deflection,

$$y_{max} = -\frac{Pl^3}{48EI} \text{ at mid span.}$$

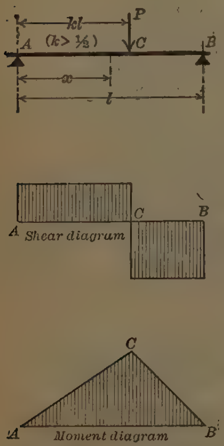


Fig. 30. Beam Supported at Ends, Concentrated Load at any Point in Span.

End reactions,

$$R_A = P(1-k);$$

$$R_B = Pk;$$

Shear at any section,

$$V = P(1-k) \text{ for } AC;$$

$$V = -Pk \text{ for } CB;$$

Moment at any section,

$$M = P(1-k)x \text{ for } AC;$$

$$M = Pk(l-x) \text{ for } CB;$$

$$M_{max} = Pkl(1-k) \text{ at } C;$$

Equation of slope,

$$dy/dx = \frac{P}{2EI}\left[\left(1-k\right)x^2 - \left(\frac{2}{3}k - k^2 + \frac{k^3}{3}\right)l^2\right] \text{ for } AC;$$

$$dy/dx = \frac{P}{EI}\left(lx - \frac{x^2}{2} - \frac{k^2l^2}{6} - \frac{l^2}{3}\right)k \text{ for } CB;$$

Equation of elastic curve,

$$y = \frac{P}{6EI}[(1-k)x^3 - (2k - 3k^2 + k^3)l^2x] \text{ for } AC;$$

$$y = \frac{P}{6EI}[-x^3 + 3lx^2 - k^2l^2x - 2l^2x + k^2l^3] \text{ for } CB$$

Maximum deflection,

$$y_{max} = -\frac{Pl^3}{3EI}(1-k)\left(\frac{2}{3}k - \frac{1}{3}k^2\right)^{3/2} \text{ at } x = l\sqrt{\frac{2}{3}k - \frac{k^2}{3}}.$$

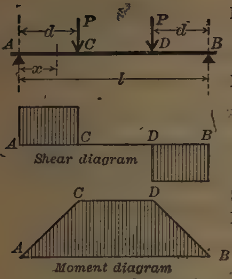


Fig. 31. Beam Supported at Ends with Two Equal, Symmetrical, Concentrated Loads.

End reactions,

$$R_A = R_B = P;$$

Shear,

$$V = P \text{ for } AC,\ O \text{ for } CD,\ -P \text{ for } DB;$$

Moment,

$$M = Px \text{ for } AC,\ Pd \text{ for } CD,\ P(l-x) \text{ for } DE;$$

$$M_{max} = Pd \text{ at any section between } C \text{ and } D;$$

Equation of slope,

$$dy/dx = \frac{P}{2EI}(x^2 + d^2 - dl) \text{ for } AC;$$

$$dy/dx = \frac{Pd}{EI}\left(x - \frac{l}{2}\right) \text{ for } CD;$$

Slopes for DB symmetrical with those for AC.

Equation of elastic curve,

$$y = (P/EI)\left(\frac{x^3}{6} + \frac{d^2x}{2} - \frac{dlx}{2}\right) \text{ for } AC;$$

$$y = (Pd/EI)\left(\frac{x^2}{2} - \frac{lx}{2} + \frac{d^2}{6}\right) \text{ for } CD;$$

Elastic curve for DB symmetrical with that for AC.

Maximum deflection,

$$y_{max} = \frac{Pd}{24EI}(4d^2 - 3l^2) \text{ at mid span.}$$

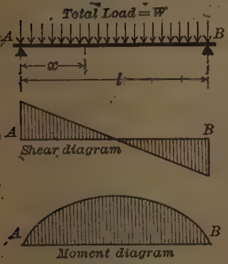


Fig. 32. Beam Supported at Ends, Uniform Load.

End reactions,

$$R_A = R_B = W/2;$$

Shear at any section,

$$V = W/2 - \frac{Wx}{l};$$

$$V_{max} = \pm\frac{W}{2} \text{ at supports;}$$

Moment at any section,

$$M = \frac{Wx}{2} - \frac{Wx^2}{2l};$$

$$M_{max} \doteq \frac{Wl}{8} \text{ at mid span;}$$

Equation of slope,

$$\frac{dy}{dx} = \frac{W}{24EI}\left(6x^2 - \frac{4x^3}{l} - l^2\right);$$

Equation of elastic curve,

$$y = \frac{W}{24EI}\left(2x^3 - \frac{x^4}{l} - l^2x\right);$$

Maximum deflection,

$$y_{max} = -\frac{5}{384}\frac{Wl^3}{EI} \text{ at mid span.}$$

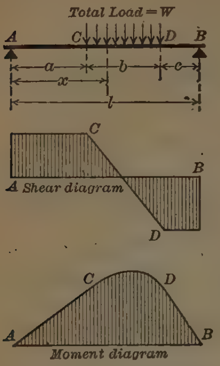


Fig. 33. Beam Supported at Ends, Uniform Load over Part of Span.

End reactions,

$$R_A = \frac{W(2c+b)}{2l};$$

$$R_B = \frac{W(2a+b)}{2l};$$

Shear at any section,

$$V = R_A \text{ for } AC;$$

$$V = R_A - \frac{W}{b}(x-a) \text{ for } CD;$$

$$V = -R_B \text{ for } DB;$$

Moment for any section,

$$M = R_A x \text{ for } AC;$$

$$M = R_A x - \frac{W}{2b}(x-a)^2 \text{ for } CD;$$

$$M = R_B(l-x) \text{ for } DB;$$

$$M_{max} = R_A\left(\frac{a+R_A b}{2W}\right) \text{ at } x = a + \frac{R_A b}{W}.$$

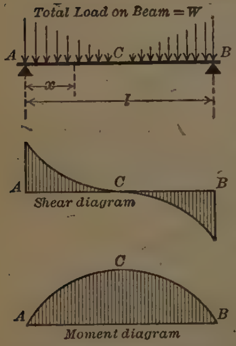


Fig. 34. Beam Supported at Ends, Load Increasing Uniformly from Center to Ends.

End reactions,

$$R_A = R_B = W/2;$$

Shear at any section,

$V = -W\left(\frac{2x}{l} - \frac{2x^2}{l^2} - \frac{1}{2}\right)$ for AC; shear diagram symmetrical about mid span;

$$V_{max} = \pm W/2 \text{ at supports};$$

Moment at any section,

$M = Wx\left(\frac{1}{2} - \frac{x}{l} + \frac{2}{3}\frac{x^2}{l^2}\right)$ for AC; moment diagram symmetrical about mid span.

$$M_{max} = Wl/12 \text{ at } C;$$

Equation of slope,

$$\frac{dy}{dx} = \frac{W}{EI}\left(\frac{x^2}{4} - \frac{x^3}{3l} + \frac{x^4}{6l^2} - \frac{l^2}{32}\right);$$

Equation of elastic curve,

$$y = -\frac{Wx}{EI}\left(\frac{l^2}{32} - \frac{x^2}{12} + \frac{x^3}{12l} - \frac{x^4}{30l^2}\right);$$

Maximum deflection,

$$y_{max} = -\frac{3}{320}\frac{Wl^3}{EI} \text{ at } C.$$

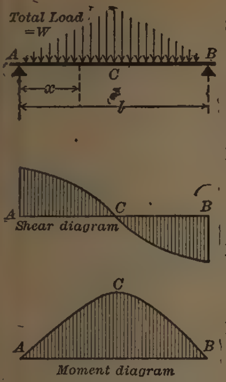


Fig. 35. Beam Supported at Ends, Load Increasing Uniformly from Ends to Center.

End reactions,

$$R_A = R_B = W/2.$$

Shear at any section,

$$V = -W\left(2\frac{x^2}{l^2} - \frac{1}{2}\right) \text{ for } AC;$$

Shear diagram symmetrical about mid span.

$$V_{\max} = \pm W/2 \text{ at supports.}$$

Moment of any section,

$$M = Wx\left(\frac{1}{2} - \frac{2}{3}\frac{x^2}{l^2}\right);$$

$$M_{\max} = Wl/6 \text{ at } C.$$

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^4}{6l^2} - \frac{x^2}{4} + \frac{5}{96}l^2\right);$$

Equation of elastic curve,

$$y = -\frac{Wx}{EI}\left(\frac{x^4}{30l^2} - \frac{x^2}{12} + \frac{5}{96}l^2\right);$$

Maximum deflection,

$$y_{\max} = \frac{Wl^3}{60EI} \text{ at mid span.}$$

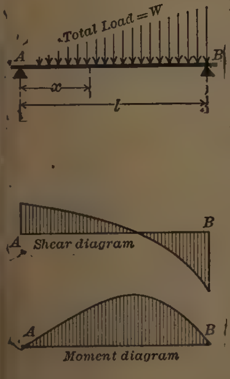


Fig. 36. Beam Supported at Ends, Load Varying Uniformly from One End to the Other.

End reactions,

$$R_A = W/3;\ R_B = ⅔\,W;$$

Shear at any section,

$$V = -W(x^2/l^2 - ⅓);$$

$$V_{\max} = -⅔\,W \text{ at } B;$$

Moment at any section,

$$M = -\frac{Wx}{3}\left(\frac{x^2}{l^2} - 1\right)$$

$$M_{\max} = 0.128\,Wl \text{ at } x = 0.577l;$$

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^4}{12l^2} - \frac{x^2}{6} + \frac{7}{180}l^2\right);$$

Equation of elastic curve,

$$y = -\frac{Wx}{EI}\left(\frac{x^4}{60l^2} - \frac{x^2}{18} + \frac{7}{180}l^2\right);$$

Maximum deflection,

$$y_{\max} = 0.0131\frac{Wl^3}{EI} \text{ at } x = 0.52l.$$

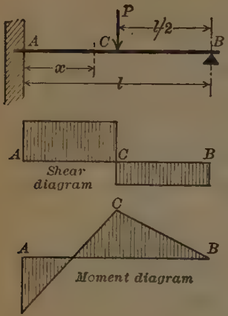


Fig. 37. Beam Fixt at One End, Supported at the Other, Concentrated Load at Mid Span.

End reaction at B,

$$R_A = \tfrac{11}{16}P.$$

Shear at any section,

$$V = +\tfrac{11}{16} P \text{ for } AC;$$
$$V = -\tfrac{5}{16} P \text{ for } CB;$$

Moment at any section,

$$M = P(\tfrac{11}{16} x - \tfrac{3}{16} l) \text{ for } AC;$$
$$M = \tfrac{5}{16} P(l - x) \text{ for } CB;$$

Moment at mid span,

$$M_C = \tfrac{5}{32} Pl;$$

Moment at fixt end,

$$M_A = -\tfrac{3}{16} Pl;$$

Inflection point (M changes sign) at $x = \frac{3}{11} l$;

Equation of slope,

$$\frac{dy}{dx} = \frac{P}{EI}(\tfrac{11}{32} x^2 - \tfrac{3}{16} lx) \text{ for } AC;$$
$$\frac{dy}{dx} = \frac{P}{EI}\left(\tfrac{5}{16} lx - \tfrac{5}{32} x^2 - \frac{l^2}{8}\right) \text{ for } CB;$$

Equation of elastic curve,

$$y = \frac{Px^2}{EI}(\tfrac{11}{96} x - \tfrac{3}{32} l) \text{ for } AC;$$
$$y = \frac{P}{EI}\left(\tfrac{5}{32} lx^2 - \tfrac{5}{96} x^3 - \frac{l^2 x}{8} + \frac{l^3}{48}\right) \text{ for } CB;$$

Maximum deflection,

$$y_{max} = -0.0093 \frac{Pl^3}{EI} \text{ at } x = 0.553 l.$$

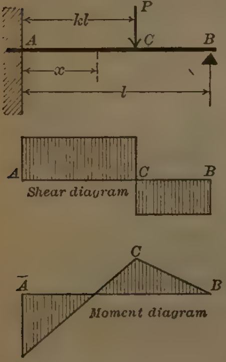


Fig. 38. Beam Fixt at One End, Supported at the Other, Concentrated Load at any Point.

Shear for AC,

$$V = (P/2)(2 - 3k^2 + k^3);$$

Reaction at supported end. Shear for CB,

$$R_B = V = \frac{P}{2}(3k^2 - k^3);$$

Moment for any section,

$$M = \frac{P}{2}[x(2 - 3k^2 + k^3) - l(2k - 3k^2 + k^3)] \text{ for } AC;$$
$$M = -\frac{P}{2}[x(3k^2 - k^3) + l(k^3 - 3k^2)] \text{ for } CB;$$

Moment at fixt end,

$$M_A = -\frac{Pl}{2}(2k - 3k^2 + k^3);$$

Moment under load,

$$M_C = \frac{Pl}{2}(3k^2 - 4k^3 + k^4).$$

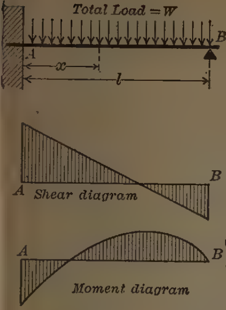


Fig. 39. Beam Fixt at One End, Supported at the Other; Uniform Load.

End reaction at B,

$$R_B = \tfrac{3}{8}W;$$

Shear at any section,

$$V = -W\left(\frac{x}{l} - \tfrac{5}{8}\right);$$

$$V_{\max} = \tfrac{5}{8}W \text{ at fixt end;}$$

Moment at any section,

$$M = -W\left(\frac{x^2}{2l} - \frac{5}{8}x + \frac{l}{8}\right);$$

Moment at fixt end,

$$M_A = -Wl/8;$$

Maximum moment in span,

$$M_{\max} = \tfrac{9}{128}\,Wl \text{ at } x = \tfrac{5}{8}l;$$

Inflection point (M changes sign) at $x = l/4$;

Equation of slope,

$$\frac{dy}{dx} = -\frac{W}{EI}\left(\frac{x^3}{6l} - \tfrac{5}{16}x^2 + \frac{lx}{8}\right);$$

Equation of elastic curve,

$$y = -\frac{Wx^2}{EI}\left(\frac{x^2}{24l} - \frac{5}{48}x + \frac{l}{16}\right);$$

Maximum deflection,

$$y_{\max} = -0.00543\frac{Wl^3}{EI} \text{ at } x = 0.578l.$$

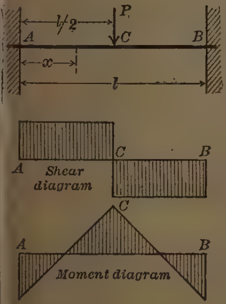


Fig. 40. Beam Fixt at Both Ends, Concentrated Load at Mid Span.

Shear at any section,

$$V = \pm\frac{P}{2};$$

Moment at any section,

$$M = P\left(\frac{x}{2} - \frac{l}{8}\right) \text{ for } AC;$$

Moment diagram symmetrical about mid span;

Moment at fixt ends,

$$M_A = M_B = -Pl/8;$$

Moment at mid span,

$$M_C = +Pl/8;$$

Inflection points (M changes sign) at $x = l/4$ and $x = \tfrac{3}{4}l$;

Equation of slope,

$$\frac{dy}{dx} = \frac{Px}{EI}\left(\frac{x}{4} - \frac{l}{8}\right) \text{ for } AC;$$

Equation of elastic curve,

$$y = \frac{Px^2}{EI}\left(\frac{x}{12} - \frac{l}{16}\right) \text{ for } AC;$$

Slope curve and elastic curve symmetrical about mid span;

Maximum deflection,

$$y_{\max} = \frac{Pl^3}{192EI} \text{ at mid span.}$$

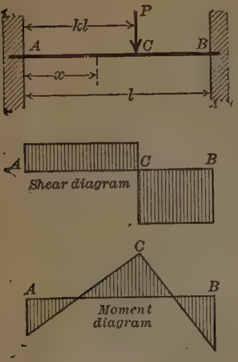


Fig. 41. Beam Fixt at Both Ends, Concentrated Load at any Point in Span.

Shear,

$$V = P(1 - 3k^2 + 2k^3) \text{ for } AC;$$

$$V = -Pk^2(3 - 2k) \text{ for } CB;$$

Moment,

$$M = P[(1 - 3k^2 + 2k^3)x - l(k - 2k^2 + k^3)] \text{ for } AC;$$

$$M = P[(2k^3 - 3k^2)x + (2k^2 - k^3)l] \text{ for } CB;$$

Moment at fixt ends,

$$M_A = -Plk(1 - 2k + k^2);$$

$$M_B = -Plk^2(1 - k);$$

Moment under load,

$$M_C = Plk^2(2 - 4k + 2k^2).$$

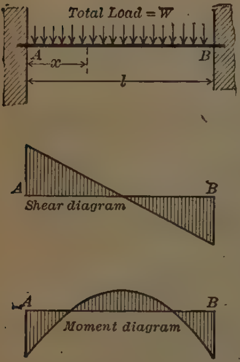


Fig. 42. Beam Fixt at Both Ends, Uniform Load.

Shear at any section,

$$V = -W\left(\frac{x}{l} - \frac{1}{2}\right);$$

$$V_{max} = \pm W/2 \text{ at ends};$$

Moment at any section,

$$M = -W\left(\frac{x^2}{2l} - \frac{x}{2} + \frac{l}{12}\right);$$

Moment at fixt ends,

$$M_A = M_B = -\frac{Wl}{12};$$

Moment at mid span,

$$M_C = \frac{Wl}{24};$$

Inflection points (M changes sign) at $x = 0.211l$ and $x = 0.789l$;

Equation of slope,

$$\frac{dy}{dx} = -\frac{Wx}{EI}\left(\frac{x^2}{6l} - \frac{x}{4} + \frac{l}{12}\right);$$

Equation of elastic curve,

$$y = -\frac{Wx^2}{EI}\left(\frac{x^2}{24l} - \frac{x}{12} + \frac{l}{24}\right);$$

Maximum deflection,

$$y_{max} = -\frac{Wl^3}{384EI} \text{ at mid span.}$$

13. Beams of Uniform Strength

General Principles. The general flexure formula for the investigation and design of beams, $M = SI/c$, indicates that the stress S varies with the bending moment. In beams of uniform strength, S must be constant. Therefore from the flexure formula, since the bending moment varies, the quantity I/c, or section modulus, must be made to vary with the bending moment, in order to provide uniformity of strength thruout all sections under various conditions of loading. Only beams which are rectangular in cross-section will be considered. From the table in Art. 11, $I/c = \frac{1}{6}bh^2$. Substituting in the flexure formula there results

$$M = \tfrac{1}{6}Sbh^2 \qquad (1)$$

from which beams of uniform strength may be designed, giving proper values to the bending moment M, determined from the conditions of loading.

Deflection of Beams of Uniform Strength. The accompanying table gives values for dimensions, stresses, and deflections for the common kinds of uniform strength beams. (Figs. 43, 43*a*, 43*b*, 43*c*.)

Dimensions, Stresses, and Deflections for Uniform Strength Beams with Rectangular Cross-section.

At the ends of the beams where shear governs the design, the dimensions of the cross-section must be increased.

I. Cantilever beam, constant depth, load at outer end. Fig. 43*a*.

Depth $= h$; width, $b = b_1 \dfrac{x}{l}$.

Fiber stress due to flexure, $S = \dfrac{6Pl}{b_1h^2}$.

Maximum deflection, $y_{max} = \dfrac{6Pl^3}{Eb_1h^3}$.

Note. A flat spring made up of a number of leaves held together by a band acts approximately like a beam of varying width $b =$ the sum of the widths of the leaves piled together at any section.

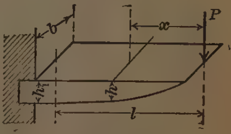


Fig. 43.

II. Cantilever beam, constant width, load at outer end. Fig 43.

Depth, $h = h_1\sqrt{\dfrac{x}{l}}$; width $= b$.

Fiber stress due to flexure, $S = \dfrac{6Pl}{bh_1^2}$.

Maximum deflection, $y_{max} = \dfrac{8Pl^3}{Ebh_1^3}$.

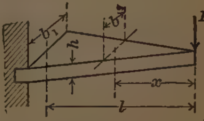


Fig. 43*a*.

III. Beam supported at ends, constant depth, load at mid span. Fig. 43*b*.

Depth $= h$; width, $b = \dfrac{2b_1x}{l}$.

Fiber stress due to flexure, $S = \dfrac{3}{2}\dfrac{Pl}{b_1h^2}$.

Maximum deflection, $y_{max} = \dfrac{3}{8}\dfrac{Pl^3}{Eb_1h^3}$.

See note under I. above.

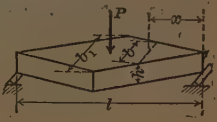


Fig. 43*b*.

IV. Beam supported at ends, constant width, load at mid span. Fig. 43c.

Depth, $h = h_1\sqrt{\frac{2x}{l}}$; width $= b$;

Fiber stress due to flexure, $S = \frac{3}{2}\frac{Pl}{bh_1^2}$,

Maximum deflection, $y_{max} = \frac{Pl^3}{2Ebh_1^3}$.

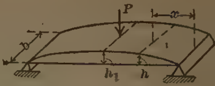


Fig. 43c.

In the table of beams of uniform strength, the bending moment has been assumed to be the controlling factor. The shearing strength of the material, however, would modify a number of these forms near the ends, and this fact should be taken into account in actual practise. While the discussion of beams of uniform strength is of considerable interest, theoretically, it is of little practical value, except in the case of plate girders, which are designed in a manner somewhat different from that discussed above, and in the case of flat springs.

14. Continuous Beams

A Continuous Beam is one for which there are more than two supports. In the discussion here given it will be assumed that the supports are all on one level and that the beam is of constant cross-section. The expressions for the resisting shear, for the resisting moment SI/c, and the differential equation for the elastic curve $M = EI(d^2y/dx^2)$ apply to continuous beams with the same force as they do to simple, cantilever, or constrained beams. The elements which must be determined in order to apply these expressions are the vertical shears and the bending moments under various conditions of loading. Since the number of reactions will always be greater than two, it is obvious that they cannot be determined by the usual method of moments. It will be noted, moreover, that the several reactions of a continuous beam will depend for their values upon the elastic behavior of the beam under load. Therefore, instead of determining the bending moments from the reactions in the case of continuous beams, the process must be reversed and the bending moments must be determined from the elastic behavior of the beam under load and the reactions and vertical shears determined from the bending moments.

General Formulas. As in the case of simple beams, the vertical shear at any section is the algebraic sum of the reactions and the loads on the left of that section. Let V be the vertical shear at any section distant x to the right of any support. Let V' be the vertical shear at a section to the right of but infinitely close to the support. Let ΣP_1 denote the sum of the concentrated loads on the distance x, and let w be the uniform load per linear unit; then

$$V = V' - wx - \Sigma P_1 \qquad (1)$$

Assume any two supports of a continuous beam. The bending moment M at any section distant x from the left support being the algebraic sum of the moments of all forces to the left of that section, rotation in the direction of the hands of a clock being considered positiv, and rotation in the opposite direction negativ, assuming M' the moment at the left support, then

$$M = M' + V'x - \tfrac{1}{2}wx^2 - \Sigma P_1(x - kl) \qquad (2)$$

in which k is a fraction less than unity and l the distance between supports. If M'' be the moment at the right support, there results the relation

$$V'l = M'' - M' + \tfrac{1}{2}wl^2 + \Sigma P_1(l - kl) \qquad (3)$$

From formulas (1), (2), and (3) it appears that V and M can always be determined, when M' and M'', or the bending moments at the supports, are known.

Theorem of Three Moments for Uniformly Distributed Load. For the purpose of determining the bending moments at the supports, which from the preceding paragraph it appears are necessary for the determination of the bending moment and the vertical shear at any section, the relation existing between the moment at any support in a continuous beam and the moments at the supports on either side is utilized. This relation between the moments at any three consecutive supports constitutes the theorem of three moments. In Fig. 44 let M', M'', and M''' be the moments at any

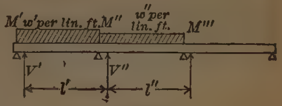


Fig. 44

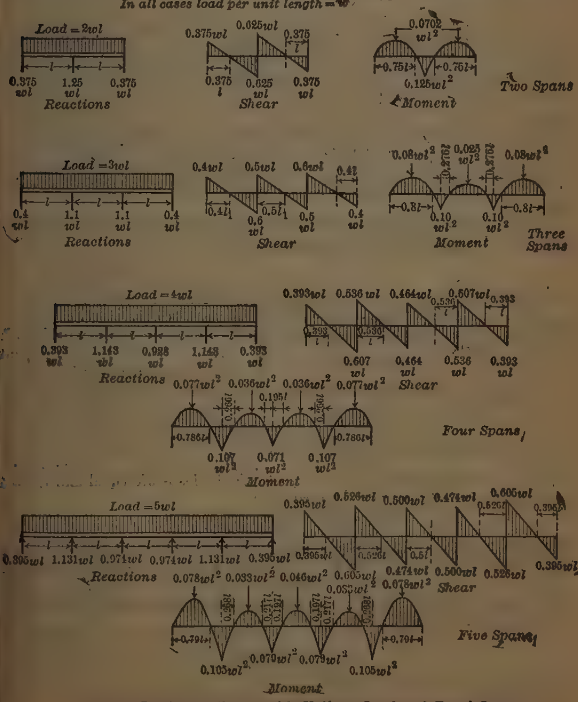


Fig. 45. Continuous Beams with Uniform Load and Equal Spans

three consecutive supports in a continuous beam, and let V' and V'' be the vertical shears on the right of the supports where the moments are respectively M' and M''. Let the span between the first and second supports be l' and between the second and third supports l'', and let the uniform load on the first span be w' per linear unit, and the uniform load on the second span w'' per linear unit. From the properties of the elastic curves, and by substituting for V' and V'' their values in terms of M', and M'', and M''' obtained from the use of formula (3), there follows the relation

$$M'l' + 2M''(l' + l'') + M'''l'' = -\tfrac{1}{4}w'l'^3 - \tfrac{1}{4}w''l''^3 \qquad (4)$$

Formula (4) is the general expression for the theorem of three moments under uniform loads. When $w' = w''$ and when $l' = l''$, or, in other words, when the load is uniform thruout and equal to w per linear unit and the spans equal, formula (4) becomes

$$M' + 4M'' + M''' = -\tfrac{1}{2}wl^2 \qquad (5)$$

Formulas (4) and (5) are used as follows: In any continuous beam of n spans there will be $n+1$ supports, and assuming the beam not to overhang the end supports, there will be $n-1$ unknown moments, the moments at the end supports being each zero. An equation in the form of (4) or (5) is written for each of the unknown moments at the supports, and the solution of these equations will give the unknown quantities M', M'', M''', and the like. Substituting M' and M'' in formula (3), the vertical shear V' at a support may be determined, from which the bending moment at any point may be found by means of formula (2). The accompanying Fig. 45 gives reactions, shears, and bending moments for continuous beams with uniform load over from two to five equal spans.

Theorem of Three Moments for Concentrated Loads. If M', M'', and M''' are the moments at three successive supports of a continuous beam shown in Fig. 46, and P' and P'' are concentrated loads in the two spans considered, P' being distant an amount a from the support where the moment is M' and P'' being distant an amount b from the support, where the moment is M''; then the theorem of three moments becomes

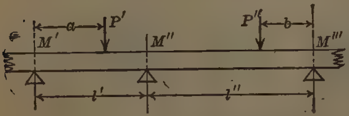


Fig. 46. Continuous Beam Notation

$$M'l' + 2M''(l' + l'') + M'''l'' = -\frac{P'a(l'^2 - a^2)}{l'} - \frac{P''b(l''^2 - b^2)}{l''} \qquad (6)$$

If all the spans are the same length l, and all the loads are equal and located at the mid-points of the spans this reduces to

$$M' + 4M'' + M''' = -\tfrac{3}{4}Pl$$

For a combination of uniformly distributed load and concentrated load or for spans containing more than one concentrated load the moments and shears may be computed separately for uniformly distributed loading and for single concentrated loads in each span, and the resulting moments and shears superposed for the combined loading.

15. Tests of Beams

Phenomena. In the discussion of beams, the general flexure formula $M = SI/c$ and the formulas for deflection are based upon the assumption that

the values of E for stress and deformation below the neutral axis are the same as for stress and deformation above the neutral axis. While in most cases there is a slight difference between the value of E determined from compression tests and its value determined from tension tests, for the materials of which beams are usually constructed, steel, wrought iron, or timber, the difference is so slight that they may be assumed to be the same, as is usually done, without appreciable error. The formulas for the resisting moment and deflection are obviously valid only when the stresses produced are within the elastic limit. Beams under test loads, when the loads are small compared to the ultimate capacity and the deflections slight, exhibit phenomena which tend to confirm the assumption made regarding the distribution of stresses and the deflection. In tests on beams the measured deflections agree closely with the computed deflections and are proportional to the applied loads, as is indicated in the deflection formulas, up to a certain point, beyond which the deflections are found to increase much more rapidly than the loads, indicating the elastic limit of the beam.

Modulus of Rupture. Since the expression $M = SI/c$ is no longer valid after the elastic limit of the material has been exceeded, it cannot be applied to determine actual conditions after that point has been reached. For purposes of comparison of brittle materials, however, it has been found convenient to determine the value of S from this formula when M equals the bending moment causing rupture. The value of S thus obtained for any material is termed the modulus of rupture and is found to be intermediate between the ultimate tensile and the ultimate compressive strength. It is to be noted that the modulus of rupture does not express the actual stress in the extreme fiber of a beam, but is a quantity useful only as a basis of comparison.

Average values for the moduli of rupture of common structural materials are as follows, in pounds per square inch:

Brick	800	Timber	9000
Stone	2000	Cast iron	40 000

It has been noted from tests that the shape of cross-section and the ratio of diameter to span affect the value of the modulus of rupture to some extent.

The manner of failure of beams under test depends upon the material of which they are constructed and the relative shape. Beams of brittle materials, such as plain concrete, stone or cast iron, fail by the fibers giving way in tension. In other cases where the material is less brittle and more plastic, such as timber, the beam fails by the crushing of the compression fibers. Beams of ductile materials, such as structural steel, bend when the yield point is reached and no value for the modulus of rupture can be obtained. Beams that are very deep in comparison to their length sometimes fail by the material shearing off near the supports. This manner of failure is, however, rare, except in the case of plain concrete or stone beams, where failure sometimes takes place along a diagonal line near a support.

16. Methods of Failure of Beams

The table on page 360 gives a number of kinds of typical failures of beams, with the general precaution to be observed in order to prevent such failures.

17. Resilience and Impact on Beams

Resilience of a beam is the measure of its elastic resistance to external work. If in a simple beam a load P is gradually applied at the center until the beam comes to rest with a deflection of f, the load P has acted over the path f with a mean intensity of $\frac{1}{2}P$, so that the external work K producing the deflection f is $K = \frac{1}{2}Pf$. This formula will also express the resilience of the beam pro-

Typical Failures of Beams and General Methods of Prevention

Kind of beam	Danger of failure by	Precaution against failure
All beams............	Rupture or excessive deformation of extreme fibers.	Sufficiently large section modulus I/c.
Long-span I beams and channel beams; long-span timber beams.	Sidewise buckling of compression fibers.	Additional material on compression side; sidewise bracing; low working stress in flexure formula, $S=Mc/I$.
Short-span beams.	Buckling of web of beam.	Sufficient thickness of web to resist diagonal column action; stiffening angles riveted to web
Short-span beams with thin webs.	Excessive shearing stress in web.	Sufficient thickness of web.
Wooden beams.......	Horizontal shear along neutral axis.	Sufficient breadth of beam to allow low working stress in shear.
Short beams, beams with short end-bearings, beams carrying concentrated loads.	Crushing of web adjacent to load or to end-bearing.	Sufficient thickness of web to carry compression under load or over bearing block; stiffening angles riveted to web and fitted against flange of beam under load or over bearing block.

vided the load P has been applied so that no energy has been lost in producing heat. For any beam of length l and under any conditions of loading, the work K may be exprest in terms of the volume of the beam by substituting for its value obtained from the general flexural formula for the particular condition of loading, and substituting for f its value obtained from the proper deflection formula, thus:

$$K=\frac{a^2}{b}\left(\frac{r}{c}\right)^2 \tfrac{1}{2}\frac{S^2}{E}Al$$

in which a is a coefficient depending upon the kind of beam and the manner of loading being 1 for a cantilever loaded at the end, 2 for a cantilever with uniform load, and 4 for a simple beam loaded at the middle; b is a coefficient obtained from the formula for deflection, being 3 for a cantilever loaded at the end and 48 for a simple beam loaded at the middle and r is the radius of gyration of the section with respect to the neutral axis and is equal to $\sqrt{I/A}$, where A is the sectional area. The quantity Al is the volume of the beam.

Stresses Due to Impact. The effect of a weight falling upon a beam is to produce a deflection considerably greater than would be produced by the same weight being applied to the beam gradually or in small increments. If P be a load which when falling from a height h on a horizontal beam produces a maximum deflection d_1. The work performed in producing this deflection is therefore $P(h+d_1)$. Let S_1 be the unit stress in the extreme fiber produced by the load P falling from the height h and S the unit stress produced by static load P, and let d be the deflection which would be produced by the static load P. The elastic limit not being exceeded, the deflections are pr

portional to the unit stresses or $d_1/d = S_1/S$, or if Q be the static load which would produce the deflection d_1, then $d_1/d = Q/P$, or $Q/P = S_1/S$. If none of the energy is lost in producing heat or giving movement to the beam, the external work and the internal energy stored in the beam at the moment the deflection reaches d_1 may be exprest by $\frac{1}{2}Qd_1$, whence $\frac{1}{2}Qd_1 = P(h + d_1)$, whence by combination with the above

$$S_1 = S + S\left(1 + \frac{2\,h}{d}\right)^{\frac{1}{2}} \quad \text{and} \quad d_1 = d + d\left(1 + \frac{2\,h}{d}\right)^{\frac{1}{2}}$$

These formulas give the maximum unit fiber stress and the maximum deflection due to the dynamic load P. In place of h in the above discussion its value $V^2/2\,g$ may be substituted, in which V is the velocity at the moment of impact and g the acceleration due to gravity, usually taken at 32.16 feet per second per second. S may be obtained from the general formula for flexure, $M = SI/c$, and d from the proper formula for deflection under a static load.

If a force P moving horizontally with a velocity V strikes a beam the ends of which are secured against horizontal movement, producing a lateral deflection d_1, the work expended in producing this deflection may be exprest by Pd_1, whence

$$S_1 = S(2h/d)^{\frac{1}{2}} \quad \text{and} \quad d_1 = d(2h/d)^{\frac{1}{2}}$$

give values for the maximum unit stress in the extreme fiber under impact from a horizontal load moving with a velocity V, in which $h = V^2/2g$.

18. Columns

A Column is a structural member in which the length is such compared with its least lateral dimension that failure under compression is most likely to occur by the rupture of the material in the extreme fiber owing to flexural stresses accompanying lateral deflection. Any compression member in which the length exceeds 8 or 10 times its least lateral dimension is usually termed a column, strut, or post.

Columns differ from other compression members in that in a column it is assumed that an unequal distribution of stress is liable to take place, resulting in flexure and lateral deflection. While theoretically an axial load in a perfectly straight symmetrical member can produce only uniformly distributed stresses with consequently no flexure, actual experience teaches that when the length exceeds 8 or 10 times the least lateral dimension some flexure will result owing to the fact that it is practically impossible to construct an absolutely straight and symmetrical member and to load it absolutely axially. It is evident, moreover, that a lateral deflection once present, due to whatever cause, tends to increase with a corresponding increase in the flexural stress.

The Length of a Column is usually taken as the distance between the points at which it is rigidly secured against lateral deflection. A long compression member with a number of points of support dividing it into several sections may consist of several columns as in the case of the columns in a high steel building. In determining the unsupported length of each section, however, only such points of support are to be considered as will prevent deflection of the column in any direction.

Radius of Gyration. Since the strength of a column is so largely dependent upon its ability to resist flexural stress, the moment of inertia of its cross-section is an important factor in the determination of its carrying capacity. In place of the moment of inertia, however, for the purpose of comparison it has been found more convenient to use the RADIUS OF GYRATION, which is a linear dimension. If I be the moment of inertia of a section with reference

to an axis, A the area of the section, and r the radius of gyration with reference to that axis, then the value of the radius of gyration in terms of the area and the moment of inertia may be found from the relation $r^2 = I/A$. Unless otherwise noted the radius of gyration is always taken with respect to the axis thru the center of gravity of the cross-section.

The axis of a column is a line drawn thru the centers of gravity of the cross-sections. If the column be perfectly symmetrical the moment of inertia and the radius of gyration will be the same for every axis thru the center of gravity of every section. In many columns used in structural design the cross-sections are not perfectly symmetrical, and there are therefore several values for the radius of gyration, depending upon the position of the axis thru the center of gravity. In the design of columns, therefore, it is necessary to know the value for the least radius of gyration in each particular case.

The radius of gyration for a circle is $d/4$, where d is the diameter. For a rectangle of height d and width b the radius of gyration for an axis parallel to d is $0.289\,b$ and that for an axis parallel to b is $0.289d$. For sections of rolled beams and shapes the values of the radius of gyration are given in Arts. 47–48.

The Ratio of Slenderness of a column is the number obtained by dividing the length by the radius of gyration. The length and the radius of gyration both being linear dimensions, the ratio of slenderness is an abstract number The radius of gyration and the length must be exprest in the same linear unit, and since it is current practise to give the radius of gyration in inches, it is necessary also to reduce the length of the column to inches before the ratio of slenderness can be obtained.

Condition of Ends. The strength of a column being dependent largely upon its ability to resist lateral deflection, the condition of the ends has a marked effect upon the carrying capacity. The various conditions which may exist at the ends of columns are usually divided into four classes. Columns with ROUND ENDS are such that at the bearing at either end there is perfect freedom of motion, as would be the case were there a ball-and-socket joint at each end. Columns with HINGED ENDS are such as have perfect freedom of motion at the ends in one plane, as would be the case of compression members in bridge trusses where the loads are transmitted thru end pins. Columns with FLAT ENDS have the bearing surface normal to the axis of the column and of sufficient area to give at least partial fixity to the ends of the columns against lateral deflection. Columns with FIXT ENDS have the ends rigidly secured so that under any load the tangent to the elastic curve at the ends will be parallel to the axis in its original position.

Experiments prove that columns with fixt ends are stronger than columns with either flat, hinged, or round ends, and that columns with round ends are weaker than any of the other types. Columns with hinged ends are equivalent to those with round ends in the plane in which they have free movement, while columns with flat ends have a value intermediate between columns with fixt ends and columns with round ends. It often happens that columns have one end fixt and one end hinged or have various other combinations. Their relative values may be taken as intermediate between those represented by the condition at either end.

Euler's Formula. Let A be the cross-sectional area of a column whose length is l and let P be an axial load under which the column is considered to assume a small lateral deflection f. Were the column absolutely straight and loaded absolutely axially and the material homogeneous it is evident that the load P could produce no lateral deflection. The deflection f, however, is assumed to be present, due to whatever cause, the load P being just sufficient to maintain the deflection f without the column returning to its original position or deflecting further. Assuming the origin of coordinates at one end with abscissas measured in the direction of the length and ordinates representing the deflections, the elastic curve assumed by the column would have the

differential equation $EId^2y/dx^2 = -Pf$. From the integration of this equation and the determination of the constants of integration there results

$$P = n^2\pi^2EI/l^2 \qquad (1)$$

in which E has its usual value and n is a number dependent upon the inclination of the tangent of the elastic curve at the ends. Giving to n proper values, formulas may be written for the several conditions of ends. For round ends n is taken as 1 and formula (1) becomes $P = \pi^2EI/l^2$, or, since $I = Ar^2$, in which r is the radius of gyration,

$$P/A = \pi^2E(r/l)^2 \qquad (2)$$

For a column with fixt ends n is taken as 2, and formula (1) becomes

$$P = 4\pi^2\frac{EI}{l^2} \quad \text{or} \quad \frac{P}{A} = 4\pi^2E\left(\frac{r}{l}\right)^2 \qquad (3)$$

Formula (1), of which formulas (2) and (3) are special cases, is the general form of Euler's formula for long columns. It is to be noted that the quantity f, or the deflection, does not appear. The load P in Euler's formula, therefore, is the load which would be required to maintain in the column any deflection which it might happen to have. A decrease in the load P would cause the column to return to its original position, while an increase in the load P would cause the deflection to increase indefinitely. The value of P in formula (1), therefore, may be taken as the ultimate strength of the column. For very long columns, where small loads produce marked lateral deflections, experimental results agree very closely with values computed from Euler's formula. It will be noted that Euler's formula does not involve the crushing strength of the material. For short columns failure occurs by crushing, not by flexure. For columns with values of l/r from 50 to 150 failure occurs by a combination of crushing and flexure. For short columns Euler's formula does not apply at all, the criterion being the ultimate in compression for brittle materials, or the yield point in compression for ductile materials. For columns with a value of l/r less than about 150 Euler's formula gives results distinctly higher than those observed in tests. Euler's formula is rarely used in practise.

Rankine's Formula is widely used for the design and investigation of columns employed in engineering practise. It is a modification of the formula which for many years was in current use under the name of "Gordon's formula." Rankine's formula is based on the assumption that columns in engineering practise are intermediate between long columns and short prisms failing by a combination of direct compression and flexural stresses accompanying lateral deflection. While under an axial load a perfectly straight column will be strest uniformly over its entire cross-section, any variation in structure will produce uneven distribution of stresses, resulting in lateral deflection, which must be taken into account in safe design. Thus in a column in which P is the direct load, A the cross-sectional area, and S_1 the compressive stress due to bending, the maximum unit stress in the column will be exprest by $P/A + S_1$. Determining the values of S_1 from the ordinary phenomena of flexure assuming that flexure action in columns is analagous to that in beams and taking into consideration the condition of ends of the column, Rankine's formula may be written

$$\frac{P}{A} = \frac{S}{1 + \phi(l/r)^2}$$

in which S is the maximum stress in the column, P the axial load, A the area of cross-section, l the unsupported length of the column, r the radius of gyration of the cross-section, and ϕ a number depending upon the condition of the ends of the column and the material of which it is constructed. The values of ϕ for various conditions are determined experimentally. Wide variations in the values are found in columns of different shapes and materials. Rankine's formula in the form above stated is applicable to the investigation

of columns in which the factors P, A, l and r are known and ϕ is assumed f the given conditions. The value of S then determined when compared wi the ultimate strength and yield point of the material will determine the fact of safety and the degree of stability of the column.

In Rankine's formula the following average values for ϕ are to be used: see also Sectio 8, pp. 823, 844, and 912.

Material	Columns both ends fixt	Columns one end fixt, one end round	Columns both ends round
Timber........	1/3000	1.95/3000	4/3000
Cast iron......	1/5000	1.95/5000	4/5000
Wrought iron..	1/36 000	1.95/36 000	4/36 000
Steel..........	1/25 000	1.95/25 000	4/25 000

The value of S to be us in the design of colum should be the allowable co pressive unit stress, while f computing in cases of ruptu it should be the ultimate co pressive unit stress for britt materials or the yield poi in compression for ducti materials. Rankine's formu should be used between t limits of 20 and 150 for l/r.

Straight-Line Formula. The plotted results of actual tests on colum show that the relation between ultimate load and l/r is fairly well represente by a straight line, for columns which fail by flexure of the whole column, and n by local collapse. For a value of $l/r = 0$ the average unit stress on the sectio P/A, is equal to the ultimate in compression for brittle materials and to t yield point in compression for ductile materials. From the point thus dete mined the straight line representing the average results of tests is drawn tange to the curve of Euler's formula for the material. The equation of the straight li

$$P/A = S - Cl/r$$

is in which S is the unit stress at the ultimate in compression for a brittle materi or at the yield point in compression for a ductile material, and C is a consta determined by experiment.

The following table gives average values of S and C for structural ste cast iron, and wood. The loads given by the straight line formula using t constants in this table are ultimate loads:

Ultimate Strength Constants for the Straight-Line Column Formula

$$P/A = S - Cl/r$$

Material and end condition	S	C	Limit of l/r
Structural steel:			
Round ends..........................	35 000	150	160
Fixt ends...........................	35 000	75	320
One end round, one fixt.............	35 000	100	240
Cast iron:			
Round ends..........................	34 000*	175	90
Fixt ends...........................	34 000*	88	160
One end round, one fixt.............	34 000*	116	115
Wood:			
Round ends..........................	5 000*	75	75
Fixt ends...........................	5 000*	37	150
One end round, one fixt.............	5 000*	50	112

* This is less than the ultimate in compression for small specimens of cast iron or woo but from tests of full-size columns seems to be the value to be used for full-size castin or timbers which may contain defects.

For purposes of design the straight line formula is usually put in a form which gives working loads directly. Both the constant S and the constant C are divided by a factor of safety. Examples of such straight-line formulas for working loads on columns are given on p. 844, Section 8 for structural steel columns, on p. 843, Section 8 for cast-iron columns, and on p. 762, Section 7, for wooden columns. In a general way, in the straight-line formula P/A represents the average unit stress on the cross-section of the column at the middle of its length, and Cl/r represents the unit stress at the extreme fiber on the concave side due to flexure. S represents the stress at the extreme fiber on the concave side due to both direct compression and flexure.

Eccentric Loads. In many column designs the loading will not act along the axis of the column but at some distance from the axis; such loads are called eccentric loads, and the distance from the axis is called the eccentricity. It is evident that the greater the eccentricity the greater the flexural stresses, and that the unit compressive stress S on the most comprest side of the column will be greater than for axial loading. Let P be the load at distance z from axis of column; then $Pz = M$, the moment at axis. If the amount of unit stress along the side nearest the load is called S, then Rankine's formula may be written in the following form:

$$S = \frac{P}{A}\left(1 + \phi\left(\frac{l}{r}\right)^2 + \frac{cz}{r^2}\right)$$

where c = distance from axis of column to side of greatest compression. The straight-line formula may be notified for eccentrically loaded columns as follows:

$$(P/A)(1 + cz/r^2) = S - Cl/r$$

Shear in Columns. For columns built up of two or more members held together by lattice bars the shear is of importance because the stress in the lattice bars is due mainly to shear in the column. The distribution of flexural stress in actual structural columns is too irregular to permit a satisfactory analysis for shear. Tests at the University of Illinois and general successful practise in the design of large columns indicate that the total shear on any cross-section of a column may be taken as not greater than 2.5 percent of the axial load, and that the lattice bars may be designed to take the stress corresponding to this shear.

If P is the axial load on the column, then the shear at any cross-section may be taken as $0.025P$. If there are lattice bars on each side of the column and θ is the angle which the lattice bars make with the axis of the column then the load carried by each bar is $0.025\ P/2 \cos\theta$ for single latticing and $0.025P/4 \cos\theta$ for double latticing. The average stress in the lattice bars should be very low as they themselves are often long, slender and eccentrically loaded columns. The average stress for a lattice bar should not be over 5000-6000 lb per sq in. Lattice bar design is further discussed in Section 8, p. 917.

19. The Behavior of Columns under Load

Steel Columns may be either one-piece columns, such as H-section columns, or built-up columns, such as latticed channel columns. Fig. 47 shows various forms of cross-section for structural steel columns, and others are shown in section 8, page 911. When loaded to failure well-designed steel columns fail by buckling of the whole column, showing points of inflection if the test column is fixt-ended. Columns made up of very thin parts may fail by local "wrinkling" of metal. The Manual of the Am. Ry. Eng. Assn. specifies that the minimum thickness of web for a built-up steel column shall be 1/30 the width between the lines of rivets connecting it to the flanges, and that the minimum thickness of cover plate shall be 1/40 the distance between nearest rivet lines.

Three important series of tests of steel columns were made in 1917 by the U. S. Bureau of Standards (see Engineering News-Record for June 28, 1917 and Feb. 7, 1918, and U. S. Bureau Standards Tech. Paper 101 for summaries of results). These

Fig. 47. Sections of Steel Columns

tests included a variety of forms of steel columns and emphasize the dependence of strength of column on the yield point in compression of the material. The "useful limit point," see p. 377, was used as a criterion of compressive elastic strength. The strength of structural steel in compression was found in some cases to be distinctly less than the strength in tension. No marked superiority was found for any particular type of column, but columns made up of channels connected by batten plates were found to be weaker than columns in which the flange members were connected by latticing.

Cast-Iron Columns. The majority of designers at the present time have turned from the use of cast-iron columns to those of steel, but yet there seems to exist in the mind of some designers an impression that cast iron fulfills the necessary qualification as a material for columns. Tests of full-size cast-iron columns have been made at the Watertown Arsenal and at Phoenixville, Pa., and both series of tests showed that for short columns the strength in compression is much less than the strength of small test specimens of cast iron. This is probably due to the imperfections always present in iron castings. The cross-section of cast-iron columns is usually a hollow circle. For short columns failure of test columns takes place by shearing on an inclined plane; for long columns failure takes place by rupture on the convex side. Failure of cast-iron columns is always a sudden, shattering failure. Where the load is concentric and the stresses can be accurately determined, cast iron is used, by a good many designers, in certain places where steel would be more expensive. It is important that a very careful inspection be made of the cast iron and all pieces with flaws and blowholes be rejected.

Wood Columns are usually square in cross-section. Tests of full-size wood columns show lower strength for short columns than is shown by compression tests of small specimens of wood, the cause being, doubtless, the presence of defects and the non-homogeniety of the material. Wood columns fail less suddenly than cast-iron columns, but more suddenly and completely than steel columns. Wood columns are especially sensitive to effect of length on strength.

Brick and Terra Cotta Columns usually have low values of l/r so that length does not play much part in determining their strength. The care used in laying up the brick or the terra cotta tile, and the strength of the mortar used are important strength factors. If loaded to the danger point brick and terra cotta columns are liable to fail very suddenly and to collapse completely. See p. 421.

Concrete Columns are discussed in Section 5.

20. Torsion of Shafts

If a straight bar of material such as a round shaft be thoroly gript at one end and a twisting force applied at the other end, a state of stress is set up in the bar called TORSION. The angle thru which a fiber of unit's length is turned is called the ANGLE OF TORSION. The twisting of the bar is produced by two couples, one consisting of the applied force with its lever arm, and one at the grip, with its lever arm. The conditions of equilibrium are that the moments of these couples must be equal.

Resisting Moment is the couple acting at the grip multiplied by its lever arm. If S be the unit shearing stress acting at a distance c from the axis of the bar, then S/c = the unit shearing stress at a unit's distance from axis of bar, and Sz/c would be the unit stress at any distance z from the axis. If this be considered to act over an elementary area da, then the moment of this stress, at distance z from axis, about axis would be $da.Sz^2/c$, and the summation of this between proper limits would be the resisting moment. The summation of da multiplied by z^2 is the sum of the products of each elementary area multiplied by its distance from the axis of bar. This expression is called the polar moment of inertia and will be represented by the letter J.

The Polar Moment of Inertia of an area is the moment of inertia about an axis which is perpendicular to the plane of the area. It is equal to the sum of the moments of inertia of the area about two axes which are in the plane of the area, which intersect at the point where the axis of the polar moment of inertia pierces that plane, and which are at right angles to each other. For torsion the polar moment of inertia about an axis thru the center of gravity of the area is of most importance. The polar moment of inertia of a circle is $\frac{\pi}{32}d^4$, twice its moment of inertia about a diameter.

Torsion Formula for Round Shafts. Considering a force P acting at one end of a shaft at a distance e from the axis, the moment of this force about the axis is equal to Pe. The resisting moment at the gript end would be SJ/c, and from the conditions of equilibrium $SJ/c = Pe$, or $S = Pec/J$, which is the torsion formula for round shafts.

The twist of a circular shaft of length l subjected to a twisting moment Pe is $\theta = Pel/JF$ in which F is the modulus of elasticity of the material in shear, and θ the angle of twist in radians. Values of F for various materials are approximately:

Steel (all grades)	12 000 000
Wrought iron	10 000 000
Cast iron	6 000 000

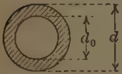
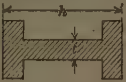
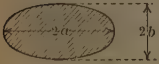
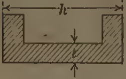
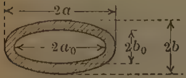
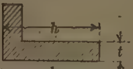
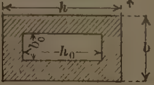
Solid and Hollow Shafts. In the design of a solid round shaft $J = \pi d^4/32$ and $c = \frac{1}{2}d$. These values substituted in the preceding formula give $Pe = {}^1/_{16}\pi d^3 S$. If the shaft is to be designed for the transmission of power, $d = 68.5 (H/nS)^{1/3}$, where H is the horse-power transmitted, n the number of revolutions per minute, S the allowable unit stress in pounds per square inch. On account of the greater strength of hollow round shafts over solid shafts of the same section area the former are often used for large sizes or where light weight is essential. In the design of hollow round shafts the value of J must be exprest in terms of the inner and outer diameters. Letting d_1 be the outer diameter, d_2 the inner diameter, $J = {}^1/_{32}\pi(d_1^4 - d_2^4)$, and $c = \frac{1}{2}d_1$, the torsion formula then becomes $S(d_1^4 - d_2^4)/d_1 = 321\,000\, H/n$. In hollow shafts as in hollow beams the material is removed from the axis where it is but little strest and placed where it can resist the twisting moment more efficiently. It is obvious that the formulas for solid and for hollow shafts become the same when d^3 equals $(d_1^4 - d_2^4)/d_1$. When the section areas are equal then d^2 must equal $(d_1^2 - d_2^2)$. From these two equations the ratio of strength of a hollow shaft to a solid shaft of the same section area is $(2\,d_1^2 - d^2)/dd_1$.

Shafts with Non-Circular Cross-Section. The distribution of shearing stress in such shafts, while somewhat similar to the round shafts, is more complex. As an example, it has been found from experiments that no shearing stress exists at the corners of a rectangular shaft, while at the middle of

the wide side the shearing unit stress is greater than at the middle of the long side. For any section with re-entrant angles, such as an I-section, the stress at the apex of the angle is, theoretically, infinite, and practically there is a minute area of very high stress which may not be important under static loading, but forms a nucleus for cracks under repeated loading. The accompanying table gives values for the approximate stress set up under torsion in shafts of various cross-section. The values are those determined by Bach, and later modified by Kommers as the result of experiments on specimens of brittle material.

Shearing Stress under Torsion in Shafts of Various Cross-sections

S, maximum unit shearing stress, in lb per sq in
Pe, twisting moment in inch pounds

Cross-section Fig. 48a	S	Cross-section Fig. 48b	S
	$\frac{16}{\pi}\frac{Pe}{d^3}$		$\frac{9}{2}\frac{Pe}{b^2h}$
	$\frac{16}{\pi}\frac{Ped}{d^4-d_0^4}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{2}{\pi}\frac{Pe}{ab^2}$ If $2a > 2b$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{2}{\pi}\frac{Peb}{ab^3-a_0b_0^3}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{1.09\,Pe}{b^3}$		$\frac{9}{2}\frac{Pe}{ht^2}$
	$\frac{20\,Pe}{b^3}$		$\frac{9}{2}\frac{Peb}{b^3h-b_0^3h_0}$

For a shaft of any cross-section not having re-entrant angles St. Venant gives an approximate expression for the angle of twist measured in radians,

$$\theta=\frac{4\pi^2PeJ}{FA^4}$$

in which A is the area of cross-section and the other symbols have the same significance as in the preceding paragraphs.

Computed Twisting Strength. In the twisting of a round bar to the point of rupture, failure takes place first on the circumference and then on the interior until complete shearing of the shaft is effected. The value of S

computed from the case of failure does not agree with the ultimate shearing strength of the material. Average ultimate computed twisting strengths of the different materials are as follows in pounds per square inch: cast iron 30 000, wrought iron 55 000, structural steel 65 000.

By the use of these average values the probable twisting moment that will cause rupture of round shafts, or the size of shaft that will fail under a certain twisting moment, may be approximately ascertained. The above values are larger than the ultimate shearing strength of the same materials, and this may be said to be due to the fact that these values have been computed from a formula which does not correctly represent the distribution of the internal stresses, while the ultimate strength of a material is a constant. When the angle of twist in a test due to the moment Pe reaches a point where it does not increase uniformly, the shearing elastic limit is reached, and from the formula $S=Pec/J$ may be computed the elastic limit for shear. Therefore the constants given above should be used with a proper factor of safety.

Shaft Couplings. When two lengths of shafting are to be joined together the connection is made with a coupling. The bolts used in such coupling transmit the torsion from one length of shafting to the other and therefore are subjected to shearing stress. This shearing stress is similar to that in the body of the shaft and is the greatest upon the side of the bolt most remote from the axis of shaft. If J_1 represents the polar moment of inertia of one bolt with respect to the axis of shaft, being equal to the polar moment of inertia of its cross-section about its axis plus the section area of the bolt into the square of the distance between the axis of shaft and axis of bolt, then in order that the strength of bolts and shaft be the same $J/c=J_1n/c_1$, c_1 being distance of the most remote fiber of bolt from axis of shaft, and n the number of bolts. Calling D the diameter of shaft, d the diameter of each bolt, and h the distance between axis of bolt and axis of shaft, and substituting the values of J and J_1 in the equation $J/c=J_1n/c_1$, an expression for finding the proper diameter of the bolts would be formed. As this equation with proper substitutions is an awkward expression for finding the value of the diameter, it is general practise to assume the shearing stress uniformly distributed over the area of the bolts. On this assumption $c_1=h$, $J_1=\frac{1}{4}\pi d^2h^2$, and $J/c=\frac{1}{16}\pi D^3$. Equating the equation $J/c=J_1n/c_1$, with the proper substitutions it reduces to $D^3=4\,nhd^2$ and $d=\frac{1}{2}(D^3/nh)^{\frac{1}{2}}$, from which if n is known the diameter of bolt necessary can be found. The value found from the approximate formula is about 10 percent larger than the value found from the accurate formula.

21. Combined Stresses

Flexure and Compression. The ordinary manner of determining the amount of compression existing in a bar under an axial force is to consider the axial compression and the compression produced by flexure. Thus, calling S the maximum unit stress due to the axial force and the flexure, S_a the axial compressive stress, S_f the unit compressive stress due to flexure, then $S=S_f+S_a$, where $S_f=Mc/I$ and $S_a=P/A$, I being the moment of inertia, c the distance of the extreme fiber, and A the area of cross-section. If the member is slender, a more exact method must be used in finding the total compression. Calling M the bending moment due to the transverse loads, P the axial load, e the deflection, then the total bending moment is $M+Pe$ and $S_f=(M+Pe)c/I$. The value of e may be assumed to equal $\alpha l^2S_f/\beta Ec$ from the action of beams under transverse loads. Substituting this value of e, the maximum unit stress on the concave side of the column is

$$S=\frac{P}{A}+\frac{Mc/I}{1-\dfrac{\alpha Pl^2}{\beta EI}}$$

in which E is the modulus of elasticity, l the span, and α and β numbers depending upon the arrangement of ends and the kind of loading of the beam.

	α	β
Cantilever beam loaded at end	1	3
Cantilever beam uniformly loaded	2	8
Simple beam loaded at middle	4	48
Simple beam uniformly loaded	8	384/5
Beam fixt at one end, supported at other, uniform load	8	186
Beam fixt at both ends, loaded at middle	8	192
Beam fixt at both ends, uniformly loaded	12	384

In Flexure and Tension the method of finding the total amount of tension is similar to that for flexure and compression. Calling S the maximum unit stress due to the axial force and flexure, S_a the axial tensile unit stress S_f the tensile unit stress due to flexure, then $S = S_a + S_f$, where $S_f = Mc/I$ and $S_a = P/A$. For a more exact formula, let M_1 equal the bending moment that produces the stress S, M the bending moment due to flexure, P the axial load or force, e the decreased deflection; then $M_1 = M - Pe$, and using the same reasoning as in the more exact method of flexure and compression, the maximum tensile unit stress is $S = \frac{P}{A} + \frac{Mc}{I} \bigg/ \left(1 + \frac{\alpha P l^2}{\beta E I}\right)$, the value of α and β being the same as given in the previous paragraph.

Flexure and Torsion result when a shaft is supported by hangers and loaded with pulleys for the transmission of the horse-power H. Find the flexural unit stress from $S = 32\ M/\pi d^3$ and the torsional unit stress from $S_s = 321\ 000\ H/nd^3$. Then the maximum apparent tensile and compressive unit stresses S_t and S_c, and the maximum shearing unit stress S_{sh}, are given by

$$S_t = S_c = \tfrac{1}{2}S + \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \qquad S_{sh} = \sqrt{S_s^2 + (\tfrac{1}{2}S)^2}$$

The above formulas may be used whether shaft be solid or hollow by substituting the proper values for d. If shaft is round and hollow $(d_1^4 - d_2^4)/d_1$ should be substituted for d^3, where d_1 equals outer diameter and d_2 equals inner diameter.

Using the maximum strain theory the maximum "true" tensile—or compressive—stress is

$$T_t = \tfrac{1}{2}(1 - p)S + (1 + p)\sqrt{S_s^2 + (\tfrac{1}{2}S)^2}$$

in which p is Poisson's ratio. For steel p may be taken as $\frac{1}{3}$ and the above equation becomes

$$T_t = \tfrac{1}{3}S + 4/3\sqrt{S_s^2 + (\tfrac{1}{2}S)^2}$$

22. Miscellaneous Cases

Eccentric Forces are sometimes applied longitudinally to the ends of a simple beam, instead of along the axis, in order that they may not increase the deflection of the beam. Let P be an eccentric tensile force acting at the distance e above the centers of gravity of the end sections, let f be the deflection and M the bending moment before the application of P. Then $P(e+f) = M$, whence the distance e should be $e = M/P - f$. When P is a compressive force it is applied below the axis at the distance $e = M/P + f$. As an approximation for both cases $e = M/P$.

Curved Beams. Let the upper and lower sides of a beam, instead of being straight, be curved in circular arcs. Let the radii of the concave and convex sides be r_1 and r_2 the centers of the circular arcs being coincident. When

such a beam is of rectangular section, the neutral surface, after the application of a load, is between the central axis and the concave side. Let c_1 and c_2 be the distances from the concave and convex sides of the beam to the neutral surface, and $r_2 - r_1 = d$. Then,

$$c_1 = -r_1 + r \quad \text{and} \quad c_2 = r_2 - r \quad \text{where} \quad r = d/\,(\text{Nap log } r_2 - \text{Nap log } r_1)$$

Let b be the breadth and d the depth of the beam, or $d = r_2 - r_1$. Then,

$$S_1 = \frac{2c_1 M}{bdr_1\,(c_2 - c_1)} \quad \text{and} \quad S_2 = \frac{2c_2 M}{bdr_2\,(c_2 - c_1)}$$

are the unit stresses at the upper and lower surfaces due to the bending moment M. When $r_1 = r_2 = \infty$, then the beam is straight and above formulas reduce to $c_1 = c_2 = \frac{1}{2}d$, $S_1 = 6M/bd^2$. When r_1 is very small, as at the inside corner of a channel, then c_1 is very small and S_1 is very large. If $r_1 = 0$, then $c_1 = 0$, $c_2 = d$, $S_1 = \infty$, and $S_1 = 2M/bd^2$.

The high values of stress at the inside of beams with sharp curvature is especially important in beams subjected to repeated stress, in which case local over-stress may start a crack, which, spreading, may cause eventual failure.

The following table gives values of factors for obtaining the value of maximum stress in curved beams of circular and rectangular cross-section from the values of stress given for straight beams of the same cross-section by the common flexure formula $M = SI/c$:

Factors for Maximum Stress in Curved Beams

To obtain the maximum fiber stress compute the stress as for a straight beam, and multiply the result by the appropriate factor given in the table:

Ratio of depth or diameter of beam to radius of inner side	Factor for beams of rectangular section	Factor for beams of circular section
0.2	1.06	1.07
0.4	1.12	1.14
0.6	1.18	1.21
0.8	1.23	1.27
1.0	1.29	1.33
1.5	1.41	1.48
2.0	1.52	1.62
3.0	1.73	1.87
4.0	1.92	2.12

Helical Springs. A helical spring is formed by winding a wire closely around a cylinder which is then removed. Let d be the diameter of the wire, D the mean diameter of the helical coil, N the number of coils, P the longitudinal axial compressive load on the spring, and F the modulus of elasticity for shearing. The stress produced in the wire is one of shearing. The unit stress S_s and the shortening e of the spring are approximately

$$S_s = \frac{8DP}{\pi d^3} \qquad e = \frac{8ND^3P}{d^4F}$$

When P is a tensile load then e is the elongation of the spring. These formulas are only valid when S_s does not exceed the shearing elastic limit of the wire which is from 0.5 to 0.6 of the elastic limit in tension.

Circular Plates. A flat circular plate of radius r and thickness t supports a uniformly distributed load of p per square unit. The flexural unit stress S of tension or compression at the middle of the plate and the deflection f at that point are given by

$$S = m(r/t)^2 p \qquad f = n(r/t)^3 rp/E$$

where m and n are numbers which vary with the kind of material, and E is the modulus of elasticity.

	Wrought Iron and Steel	Cast Iron	Plain Concrete
Plate supported around the circumference	$m=1.00$ $n=0.67$	$m=1.12$ $n=0.74$	$m=1.35$ $n=0.86$
Plate fixt around the circumference	$m=0.67$ $n=0.17$	$m=0.75$ $n=0.18$	$m=0.90$ $n=0.19$

For a circular plate which carries at the middle a load of p lb per sq in uniformly distributed over a circle of radius r_0, the value of S is, for a supported plate,

$$S = m\left(1 + 2 \text{ Nap } \log\frac{r}{r_0}\right)(r_0/t)^2 p$$

in which m has same values as above. Example: when $r_0 = \frac{1}{4}r$, then S is $0.26(r/t)^2p$ for cast iron and $0.23(r/t)^2p$ for steel.

Elliptical Plates. Let $2a$ and $2b$ be the axes, t the thickness and p be the uniform load per square unit. The plate tends to crack along the longer axis, and the greatest unit stress is

$$S = m\frac{a^2b^2}{a^2+b^2}\frac{2p}{t^2}$$

in which the above values of m for circular plates may be used.

Rectangular Plates. Let $2a$ be the shorter and $2b$ the longer side; when the plate is fixed at the corners, then above formula for the elliptical plate may be used with above values of m for a circular supported plate. For a square plate of side $2a$ and supported on edges $S = \frac{9}{8}m(a/t)^2p$, while for one fixt on all edges $S = \frac{3}{4}m(a/t)^2p$ in which m has above values for supported and fixt circular plates.

Square Panels. Let an unlimited plate have many supports dividing the plate into many square panels, $2a$ being the side of each panel. The greatest unit stress S in the plate is $S = \frac{8}{9}m(a/t)^2p$ in which m has the values given above for circular fixt plates. This applies to the flat stayed plates of steam boilers.

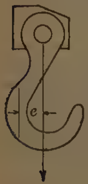


Fig. 49

Hooks. When a hook sustains a load P there exists in the bend of the hook combined tension and flexure. Let e be the distance from the line of action of P to the center of gravity of the cross-section at the bend, c the distance from that center of gravity to the inner edge of the bend, A the area and r the least radius of gyration of the cross-section then

$$S = k\frac{P}{A}\left(1 + \frac{ce}{r^2}\right)$$

gives the tensile unit stress at the inner edge of the bend in which k is a factor depending on the ratio of d the depth of the cross-section of the hook to R the inner radius of curvature of the hook. k varies for different sections but an approximate value may be taken from the following table:

d/R	k	d/R	k
0.2	1.06	1.0	1.31
0.4	1.13	1.25	1.39
0.6	1.19	1.50	1.44
0.8	1.25	1.75	1.50
		2.00	1.57

Circular Ring. When a circular ring of mean radius R is pulled in the direction of a diameter by two tensile forces each equal to P, the maximum bending moment is at the section, where P is applied, where it equals 0.318 PR and the unit stress is

$$S = 0.318k'PR/0.098d^3$$

in which k' is the factor for a curved beam of circular cross-section given in the table on p. 371 and d is the diameter of the cross-section of the ring.

TESTING AND INSPECTION

23. Testing Apparatus

Testing Machines for determining the strength of materials of construction are of various types. A testing machine consists of a mechanism for apply-

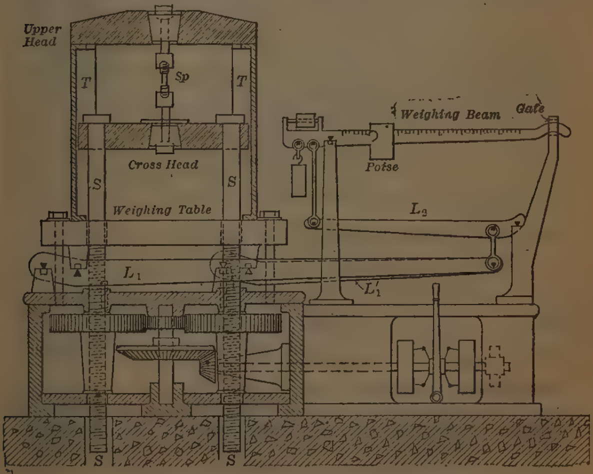


Fig. 50. Screw-power, Compound-lever Testing Machine

ing a large force to a test specimen and a mechanism for determining the magnitude of the force applied.

The Screw-power Compound-lever Testing Machine is the type in most common use in the United States, and it is shown in diagram in Fig. 50. Power is supplied by a belt drive or by a direct-connected motor, and the power is transmitted thru a series of gears to vertical screws which operate the cross-head of the machine. The force applied by the belt is multiplied many times by the gearing and the screws. This multiplied force is applied to the test specimen by the cross-head. If the specimen is a tension specimen it is placed between the crosshead and the upper head of the testing machine, if it is a compression specimen or a cross-bending specimen it is placed between the cross-head and the weighing table. In any event as the cross-head moves downward the specimen transmits downward pressure to the weighing table. The weighing table is supported on compound levers, L_1L_1', Fig. 50, which are fitted with knife-edge bearings, and these compound levers transmit the force, reduced, to a simple lever, L_2, Fig. 50, which, in turn, transmits the force, still further reduced, to the weighing beam. As load is applied the beam is kept in balance by moving the poise, the position of which indicates the load on the specimen, when the beam is in balance.

For Flexure Tests, I beams may be placed upon the weighing table, with suitable bearing blocks on their top flanges to support the beams to be tested, or the machines may be arranged with wings extending from the side and forming a part of the weighing table. The method of applying and registering the load is the same as in the compression test.

Other Types of Testing Machine. A type of testing machine frequently used for very large machines uses the principle of the hydraulic press for applying the load, and measures the load by some form of pressure gage. Machines of this type of moderate size are made with plungers so carefully ground to fit their cylinder that no packing is necessary, and such machines are very accurate. Large machines use plungers with packing, and the variation of friction of the packing renders their readings less accurate than machines with compound-lever weighing devices. The hydraulic press type of testing machine is simpler and cheaper than the screw-power, compound-lever type. Another type of testing machine weighs the load by means of the displacement given to a heavy pendulum. For small machines spring balances are sometimes used to measure the load. The accurate adjustment of a spring balance for zero load is a matter of some difficulty.

The Emery Testing Machine uses hydraulic pressure to apply the load to the specimen and transmits the load from the specimen to a hydraulic pressure cylinder, or pressure box, in which the plunger is replaced by a plate mounted on a thin metal diaphragm. This pressure box is filled with liquid and connected by means of a pipe to a smaller pressure box fitted with a plate and diaphragm, and the load on the smaller pressure box is weighed by means of a compound-lever balance. This hydraulic weighing device is extremely sensitive, and its accuracy is high. The Emery machine is very costly.

Calibration of Testing Machines. Testing machines are calibrated by means of calibrating levers, or by means of a standard tension test specimen. Calibrating levers consist of a pair of simple levers with known ratio of arms. These are placed so that their fulcra bear on the weighing table of the testing machine and the knife edges at the ends of the short arms press upward against the crosshead. Known standard weights are hung from the end of the long arms of the levers, and the poise reading on the balanced beam of the testing machine compared with the known pressure on the weighing table produced by the standard weights. This method of calibration is used for comparatively small loads.

For larger loads the reading of the beam of the testing machine is checked against the elastic stretch of a standard bar fitted with an extensometer. This method can be used to test machines to their full capacity, but the standard bar itself must be calibrated by loading in a standardized testing machine.

Accuracy and Sensitiveness of Testing Machines are two distinct characteristics. A machine is accurate if the readings of the machine agree closely with the actual loads applied by the machine. A testing machine is sensitive if a small change of load is indicated by a distinct movement of the beam or other weighing mechanism. A testing machine may be very sensitive, and yet very inaccurate.

Machines for Torsion Tests are of two types. In the Thurston machine one end of the specimen is held by a chuck in a horizontal spindle in the top of an A frame. The other end is held in a similar manner in a parallel frame. One spindle is rotated by means of a worm gear and crank. The motion is transmitted thru the specimen to the other spindle, to which is attached a weight on the end of a vertical bar. Any motion of this spindle will move the weight out of the vertical, and a torsional stress in the specimen results. The position of the weight will then be a measure of the stress, since its moment will be proportional to its deviation from the vertical.

Another form of machine consists of two parallel heads in which are chucks for holding the specimen, one head being operated by gears and the other attached to a weighing system, consisting of a compound lever. In order to keep the specimen central during the test, parallel levers are used between the weighing system and the head. The moment in inch-pounds is read from a scale beam.

Abrasion Machines are used for determining the toughness or resistance of road materials to wear. For testing macadam, the Deval machine is used. It consists of four cylinders 20 cm in diameter and 34 cm in depth, inside, mounted on a shaft at an angle of 30° with the axis of rotation. A charge of 5 kilograms of broken stone is placed in each cylinder, each charge consisting of 50 pieces. The cylinders are rotated 10 000 times at the rate of 30 to 33 per minute. The percentage of loss is computed from the amount of worn-off material that will pass thru a 1/16-mesh screen.

For Abrasion Tests of paving brick, the cylinder or rattler adopted by the National Brick Manufacturers' Association is used. This consists of a barrel 20 in in length, inside, and whose cross-section is a fourteen-sided polygon, 28 in in diameter, supported on either trunnions or rollers, but in no case does a shaft pass thru the barrel. The heads are made of cast iron, and the staves are of steel. The space between the staves must not exceed 5/16 in. The charge for bricks of "block-size" is ten bricks and with these bricks are placed in the rattler ten cast iron spheres 3¾ in in diameter and 300 lb of cast-iron spheres 1⅞ in in diameter. These cast-iron spheres are weighed after every ten tests and when a large sphere is found to weigh less than 7 lb, or a small sphere less than ¾ lb new spheres are substituted.

The charge is rotated 1800 times at the rate of 29.5 to 30.5 per minute, and the percentage of loss is calculated in terms of the weight of the dry brick composing the charge. The average of five tests on separate charges of brick is considered an official test. (See p. 386.)

Extensometers are used for measuring the small elastic deformations within the elastic limit, and for detecting the stress at which the elastic limit is exceeded. An extensometer consists essentially of two clamps which are fastened to the test specimen, and one or more micrometer devices by means of which a very small change of distance between the clamps can be measured. For all accurate measurement of elastic deformation of tension or compression specimens the extensometer should be attached so as to measure deformation along two or three symmetrically spaced axial lines on the specimen. Types of

micrometer devices used in extensometers are: the screw micrometer, in which a very small axial motion of a screw is accompanied by a large circumferential motion of a dial attached to the screw; clockwork dial gages; multiplying levers; microscopes; and the "optical lever" in which a very small angular motion of a mirror changes the direction of a reflected ray of light. Extensometers for general use usually measure deformations to the nearest ten-thousandth of an inch.

The Strain Gage is a special form of extensometer which can be attached to a specimen or moved from one gage line to another between readings. It consists of a pair of trammel points PP', Fig. 51, attached to a frame F. One

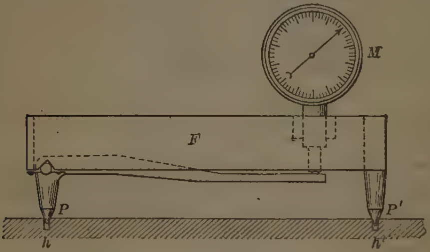


Fig. 51. Strain Gage

of the points, P, is movable and the motion of this point is measured by means of some micrometer device, M, usually a clockwork dial gage. The trammel points are conical in shape, and are made of hardened steel. On the specimen there are laid out gage lines along which deformation is to be measured, and at the ends of each gage line are drilled small holes hh' into which the trammel points fit. The strain gage may be clamped to a specimen, or, the reading of the strain gage may be taken under zero load, the instrument removed, and after an increment of load has been applied another reading taken, the difference in the readings being the deformation. Corrections for temperature deformations are made by taking readings on a "standard bar" which is not subjected to stress, but is subjected to the same temperature changes as the specimen or structure under test. In skillful hands deformations can be measured with a strain gage to the nearest ten-thousandth of an inch. The strain gage can be used to measure deformations in a structure or machine as well as in a test specimen. One instrument can be used for a large number of gage lines. In a test of stress determination in a reinforced concrete floor slab in the Soo Line Freight Terminal in Chicago measurements of deformation along over 1000 gage lines were made, and five strain gages were used. For detailed discussion of the use of the strain gage see Proceedings of the American Society for Testing Materials, Vol. XIII, p. 1019.

For measuring the dimensions of cross-section of a test specimen micrometer calipers are usually used. For measuring the relatively large deformations beyond the yield-point, and the elongation of tension test specimens after fracture measurements with dividers and steel scales are sufficiently accurate.

The Determination of the Elastic Limit according to its strict definition (see p. 323) is rarely carried out in commercial testing. In some U. S. Govern-

ment specifications it is required that a test specimen withstand a certain stress without a resulting permanent deformation greater than 0.002 in. In certain British specifications a similar requirement is made, except that the critical stretch is one which can be detected by the use of a pair of dividers. The proportional limit and the elastic limit of metals are practically identical, and the proportional limit is frequently determined and spoken of as the elastic limit. In certain tests of metals the Standards of the American Society for Testing Materials specify that the practical determination of the elastic limit shall be made as follows: "The elastic limit . . . shall be determined by an extensometer reading to 0.0002 in. The extensometer shall be attached to the specimen at the gage marks and not at the shoulders of the specimen nor to any part of the testing machine. When the specimen is in place and the extensometer attached, the testing machine shall be operated so as to increase the load on the specimen at a uniform rate. The observer shall watch the elongation of the specimen as shown by the extensometer, and shall note, for this determination, the load at which the rate of elongation shows a sudden increase."

The U. S. Bureau of Standards in connection with a series of column tests made for the American Society of Civil Engineers has proposed a "Useful Limit Point" for metal, which is to be determined from a stress-strain diagram by drawing tangent to the curve a line having one-half the slope of the initial straight portion of the diagram (Fig. 52). This is a modification of the method proposed by the late J. B. Johnson who used a tangent line having two-thirds the slope of the initial straight part of the stress-strain diagram. Both the "useful limit point" and Johnson's elastic limit involve the drawing of a stress strain diagram, either by an autographic attachment to the testing machine or by plotting observed values of load and deformation.

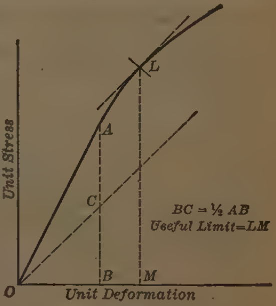


Fig. 52

It is probable that for actual materials no absolute elastic limit exists. The value obtained in actual testing depends on the precision of instruments and methods, and the method of determining the elastic limit should always be stated. The yield point is sometimes called the elastic limit. This is incorrect and should be avoided.

The Speed of Testing has no appreciable effect on the properties of iron and steel within the following limits, which conform to the standards of the American Society for Testing Materials.

In testing steel and wrought iron in gage lengths of 2 and 8 in in accordance with the specifications of the American Society for Testing Materials, the speed of the machine, by which is meant the speed of the cross-head when the machine is running idle, shall conform to the following requirements:

The cross-head speed of the testing machine shall be such that the beam of the machine can be kept balanced, but in no case shall the values given in the following table be exceeded:

Specified minimum tensile strength of material, lb per sq in	Gage length, in	Maximum cross-head speed for testing machine in determining, in per minute	
		Yield point	Tensile Strength
80 000 or under	2	0.50	2.0
	8	2.00	6.0
Over 80 000	2	0.25	1.0
	8	0.50	2.0

In determining the elastic limit by the method prescribed by the American Society for Testing Materials, described in the preceding paragraph, the cross-head speed for a specimen with a 2-in gage length shall not exceed 0.125 in per min. In determining the proportional limit the cross-head speed shall not exceed 0.025 in per in of gage length per min.

The Cost of Tests varies in different laboratories; average prices are:

Metals: Tensile test; breaking load only	$1.00
Tensile test with yield point	1.25
Tensile test, elongation and reduction of area	1.50
Transverse test (without deflections)	1.00
Compression test	1.50
Torsion test ultimate only	1.50
Cement: Determination of initial and final set, Vicat or Gilmore wires (cement required 1 lb)	0.75
Determination of Specific Gravity (1 lb required)	1.00
Determination of expansion, either cement or mortar, average of 3 specimens 6 months or less	5.00
Determination of tensile strength at 7 days, neat and sand, with fineness, time of set and soundness test (cement required 4 lb)	4.00
Ditto 7 and 28 days neat or sand	6.00
Ditto 7 and 28 days neat and sand	8.00
Paving brick: Two sets of rattler tests	8.00
Absorption test, 5 bricks	3.00
Transverse test, 10 bricks	5.00
Full standard test, National Brick Manufacturers' Association	20.00
Compression test, 1 sample	2.00
Compression test, 5 samples	6.00
Miscellaneous: Determination of voids in concrete aggregate	2.50
Determination of percentage of loam in sand	2.00
Determination of fineness of sand on 60, 50, 40, 30, and 20 mesh screen	2.00
Determination of fineness of gravel on 1/4, 1/2, 3/4, 1, 1 1/2, 2, and 2 1/2 in sieves	3.00
Determination of voids in sand	2.00
Stone: Absorption tests, 5 specimens	3.00
Compression test, 1 sample	1.50
Compression test, 5 samples	6.00
Specific gravity tests	2.00

Special discounts are given from the above prices for testing a series of samples or specimens. Tests of a special nature are charged at the rate of $3.00 per hour for use of testing machines, including services of attendant. Shop work for the preparation of specimens, $1.00 per hour.

24. Large Testing Machines in the United States

Owner	Capacity, pounds: Type	Weighing device	Power	Tension	Compression	Max. size of specimen, ft: Tension	Compression	Flexure
U. S. Bureau of Standards, Pittsburgh, Pa.	Vert.	Scale beam press. gage	Hydr.	....	10 000 000	......	6×6 ×30	
Am. Bridge Co., Ambridge, Pa.	Hor.	Mercury gage	Hydr.	4 000 000	....	42.5		
Pheonix Iron Co., Phoenixville, Pa.	Hor.	Mercury gage	Hydr.	2 600 000	2 600 000	50	55	
U. S. Bureau of Standards, Washington, D. C.	Hor.	Emery scale	Hydr.	1 200 000	2 400 000	33	33	
Boston Navy Yard, Boston, Mass.	Hor.	Scale beam	Hydr.	2 000 000	2 000 000	90		
Am. Chain Co., Norfolk, Va......	Hor.	Scale beam	Hydr.	2 000 000	2 000 000	90		
U. S. Steel Co., McKeesport, Pa.	Hor.	Mercury gage	Hydr.	1 200 000	800 000	40	32	
Renss. Poly. Inst., Troy, N. Y.	Vert.	Scale beam press. gage	Hydr.	....	1 200 000	......	2×2.5 ×3	
Pa. R. R., Altoona, Pa.	Vert.	Scale beam	Screw	1 000 000	1 000 000	8	8	10
Nat. Mall. Cast. Co., Sharon, O.	Vert.	Scale beam	Screw	1 000 000	1 000 000	8	8	10
Buckeye Steel Cast. Co., Columbus, O.	Vert.	Scale beam	Screw	1 000 000	1 000 000	10.5	10.5	10
W. H. Miner, Chicago, Ill.	Vert.	Scale beam	Screw	500 000	1 000 000	8	8	8
Am. St'l Foundries, Alliance, O.	Vert.	Scale beam	Screw	....	1 000 000	......	8	10
City of Phila., Phila., Pa.	Vert.	Gage	Hydr.	....	1 000 000	......	2×3 ×2.5	
U. S. Bureau of Std., Wash., D.C.	Vert.	Gage	Hydr.	....	1 000 000	......	8	
Lehigh Univ., S. Bethlehem, Pa.	Vert.	Scale beam	Screw	800 000	800 000	20	25	30
A. Leschen & Sons, St. Louis, Mo.	Vert.	Scale beam	Screw	800 000	800 000	19	19	
U. S. Ordnance Dp. Watertown, Mass.	Hor.	Emery scale	Hydr.	800 000	800 000	20	26	
Univ. of Illinois, Urbana, Ill.	Vert.	Scale beam	Screw	600 000	600 000	22	25	10
U. S. Bureau of Std. Pittsburgh, Pa.	Vert.	Scale beam	Screw	600 000	600,000	24	30	25
Renns. Poly. Inst., Troy, N. Y.	Vert.	Scale beam	Screw	600 000	600 000	22	24	20
Univ. of Penna., Phila., Pa.	Vert.	Scale beam	Screw	600 000	600 000	22	24	20
Univ. of Wis., Madison, Wis.	Vert.	Scale beam press. gage	Hydr.	450 000	600 000	10	12	20

The National Tube Co., Pittsburgh, Pa., owns a torsion machine with a capacity of 1 500 000 inch-pounds, which will take a specimen 8⅝ in diameter and 20 ft long.

25. Tests and Test Specimens for Metals

The paragraphs of this article that are enclosed in quotation marks are quoted from the 1916 "Standards" of the Am. Soc. for Test. Materials.

Tensile Specimens are either flat or round. A flat specimen is used for plates, and its standard size is 18 in in length, about 2 in wide along the ends, and 1½ in wide for a central length of about 9 in, while the thickness is the same as that of the plate from which it is cut. Fig. 54 shows a flat specimen with punch marks placed at every inch point. Wire and rods are frequently tested just as they come from the mill, the length between the jaws of the machine being usually more than ten times the diameter. Standard round specimens are cut from axles, shafts, beams, and other manufactured products. Prior

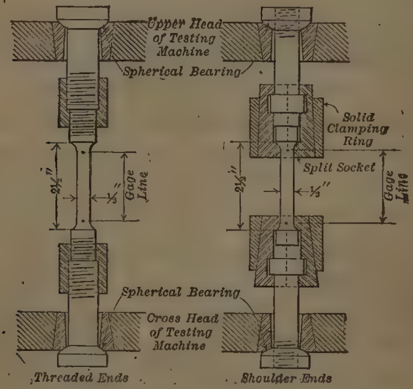


Fig. 53. Short Tension Specimens (2-in Gage Length)

to 1895 the standard size of the round specimen was 1 in in diameter along a central length of 9 in, while the ends were larger and provided with screw threads. Both flat and round specimens are frequently called 8-in specimens because two marks are placed upon them 8 in apart for the purpose of measuring the elongation. The standard round test specimen now in use in the United States is shown in Fig. 53; this is called the 2-in specimen, because the central part is a little more than 2 in long and the marks are placed upon it 2 in apart; the diameter of this specimen is 0.5 in or sometimes 0.505 in. This smaller specimen has the advantage that less material is required to be wasted in taking it from an axle or shaft, but percentages of elongation computed from it are greater than those determined from the 8-in specimen.

Fig. 54 shows the standard flat specimen and Fig. 53 the standard 2-in round specimen which is about 5¼ inches long, while the fillets connecting the body with the ends are of radius not less than ⅛ in. The ends may be of any form which will fit the holders of the testing machine.

Ultimate Elongation is determined by measuring the distance between the two end marks both before and after rupture and then dividing the increase in length by the gaged

length. Most records in technical literature give the elongation as determined from the 8-in specimen. Since the "necking down" of a specimen, or reduction of area of cross-section, extends over only a part of the gaged length, it follows that the computed unit elongations of different specimens are not comparable unless these are geometrically similar. For the 8-in specimen the ratio of gaged length to diameter is 10, for the 2-in specimen this ratio is 4, and accordingly the "necking down" occupies a proportionally larger part of the length, so that the computed ultimate elongation of the latter is the larger. Reports of ultimate elongation should always state the gaged length of the specimen. For specimens which have to be cut from plates or pieces so thin that the standard 2-in specimen cannot be used the gage length should be 4.5 times the square root of the area of cross-section, which ratio holds for the 2-in specimen.

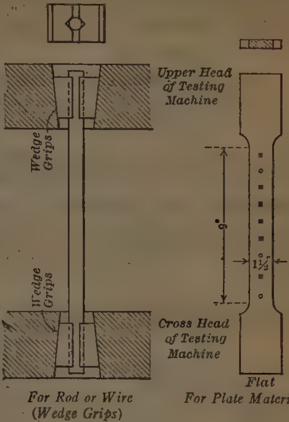


Fig. 54. Tension Specimens

Ultimate Tensile Strength or simply tensile strength is always computed by dividing the total breaking load by the original sectional area. This conventional method is adopted for the sake of uniformity, altho it does not, of course, give the actual unit stress which prevails at the small section at the time of rupture.

Reduction of Area is computed by finding the ratio of the diminution in sectional area to that of the original area. Or, instead of computing the areas the squares of the diameters may be used. Thus, for a certain 2-in tensile specimen the original sectional area was 0.1995 sq in, and the area of the ruptured section was 0.1064 sq in; hence the reduction of area was $0.0931/0.1995=0.467$, or 46.7 percent. Reduction of area is generally regarded as an index of the ductility of the material. It is probably a more reliable index than ultimate elongation, because the latter is liable to variation with the ratio of gaged length to diameter, whereas reduction of area is found to be more constant.

Gripping Test Specimens. "For specimens of rolled material, serrated grips, flat and V-shaped, should be adopted, the former for rectangular and the latter for round specimens. Serrated grips with curved faces appear to have no advantage, and to cause crushing of the material. Wedges with ball and socket do not seem to be necessary, and for commercial testing their use has been generally discontinued. Specimens of turned form, with threaded or shouldered ends, should be held in ball and socket bearings. Fig. 53 shows threaded-end and shouldered-end specimens. It is considered important for correct results that the specimen be located in the exact center of the heads, and to better secure this condition, the openings in the heads should be lined up with each other by means of a plumb bob and be tested for parallelism with a spirit level. Each pair of packing pieces and wedges that are to be used together in the same head should correspond exactly in thickness and other dimensions, and the wedges should be inserted an equal distance when the specimen is in place."

In Materials of Low Stretch, such as cast iron, it is important that special care should be exercised in the gripping of the specimens. In materials of a softer nature, the effect of improper gripping is largely local and probably does not extend to the portion of the specimen within the gage marks. For cast iron the form of test specimen shown in Fig. 53, held in ball and socket joints, is recommended.

Selections and Preparation of Specimens. "Specimens representative of steel castings may be cut out from the bottom of a sink head or riser, or from a coupon attached to the casting. In either case the part from which the specimen is taken should be relatively large in proportion to the size of the casting and should be annealed with it. Workmanship on specimens shall be of the most careful nature, and surfaces should be free from nicks and tool marks. All wire edges should be removed and corners generously

Fig. 55. Tension Specimen for Wire Rope

rounded. If specimens of rolled material are sheared in the rough from sections, at least ⅛ in of the material should be removed from the sheared edges in machining."

Compressive Tests of Metals. "The test specimen shall be a cylinder having plane ends truly normal to its axis. The diameter of the specimen shall be not less than 1 in nor greater than 1.13 in. A specimen 1 in in diameter is to be preferred. The length of the specimen should be between 2.5 and 4 diameters.

"No bedding should be used for the ends of the specimen. The bearing blocks which transmit the pressure from the testing machines should be truly normal to the plane ends of the specimen. To secure this one of the blocks should be provided with a hemispherical bearing which can turn freely." Fig. 56 shows a compression test specimen in place in a testing machine. The center of the spherical surface should lie in the plane of the end of the specimen. "The speed of compression should be slow, not exceeding 0.1 in per minute. Near the elastic limit and yield point the load should be increased very slowly."

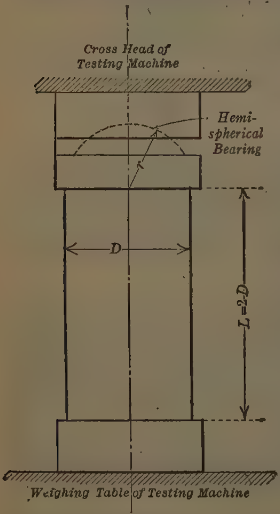


Fig. 56.

"For determining modulus of elasticity, the linear compression of the specimen should be observed by a precise compressometer which is attached to the specimen and does not touch the bearing blocks of the machine. Readings of the compressometer should be taken for three loads, the first at about one-fourth, the second at about one-half, and the third at about three-fourths of the elastic limit.

"To determine the proportional limit, several readings of the compressometer should be taken as that limit is approached for load increments of 1000 lb per sq in. The yield point shall be noted, for ductile materials, by the drop of the scale beam or by visible axial compression as shown by the use of a pair of dividers.

"The record of the test should mention any phenomena observed near the proportional limit and yield point. The manner of final failure should also be noted when the test is carried to this limit."

Flexure Tests are used in determining the strength qualities of cast iron and of malleable cast iron. For gray iron castings the standard test specimen is

an "arbitration bar," the specifications for which are given by the 1916 "Standards" of the Am. Soc. for Test. Materials as follows:

"The mold for the bars is shown in Fig. 57. The bottom of the bar is 1/16 in smaller in diameter than the top, to allow for draft and the strain of pouring. The pattern

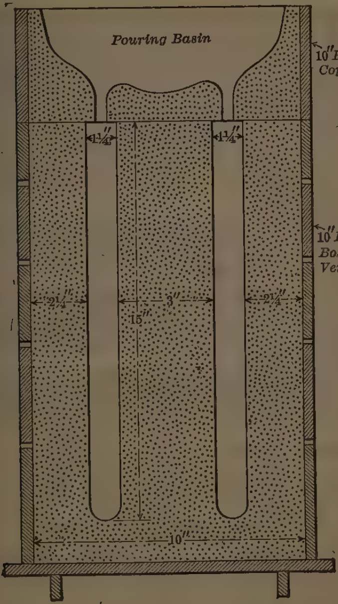


Fig. 57. Mold for Arbitration Bars.

should not be rapped before withdrawing. The flask shall be rammed with green sand, a little damper than usual, well mixt and put thru a No. 8 sieve, with a mixture of 1 to 12 bituminous facing. The mold shall be rammed evenly and fairly hard, thoroly dried and not cast until it is cold. The test bar shall not be removed from the mold until cold enough to be handled. It shall not be rumbled or otherwise treated, being simply brushed off before testing."

The arbitration bar is loaded at the center of a 12-in span, as shown in Fig. 58, and the load is applied at such a rate that 20 to 40 seconds are required to produce a deflection of 0.1 in.

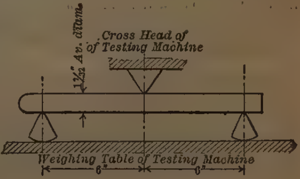


Fig. 58. Cross-bending Specimen for Cast Iron.

For cast-iron pipes the standard test specimen is 26 in long, 2 in wide, and 1 in thick, and is loaded at the middle of a 24-in span. For malleable cast iron the standard flexure specimen is 14 in long, 1 in wide, and either ½, ⅝, or ¾ in thick according to the thickness of the casting which it represents. It is loaded at the middle of a 12-in span.

Hardness Tests of Metals in common use are the Brinnell test and the Scleroscope test. In the Brinnell test a hardened steel ball of known diameter is pressed against the surface of the metal to be tested, the pressure exerted and the diameter (or depth) of the resulting permanent impression noted. The smaller the indentation the harder the metal. The hardness is indicated by a "hardness number," which is the quotient obtained by dividing the pressure by the spherical surface of the indentation. The standard steel ball used is 10 mm in diameter, and the standard pressure is 3000 kg for the harder metals, and 500 kg for the softer metals. A modification of the Brinnell test which is used to some extent is the Ludwik cone test, in which a hardened steel conical point is used in place of a steel ball.

The Scleroscope consists of a diamond-pointed plunger weighing 1 gram which drops vertically down the inside of a graduated glass tube to the surface of the metal to be tested, which is placed at the lower end of the tube. The height of rebound is measured, and is a measure of the hardness of the metal.

Brinnell tests and scleroscope tests possess the great advantage that they may be made without damage to the material which is to be used, instead of being vicarious tests on samples chosen to represent the material. They are especially useful in testing the uniformity of different parts of the same piece, for example, the uniformity of hardness of the different teeth of a gear wheel.

In a general way there seems to be a fairly well defined relation between the Brinnell hardness, the scleroscope hardness, and the tensile strength for the different grades of iron and steel. See paper by R. R. Abbott, Proc. Am. Soc. for Test. Materials, Vol. XV, Part II, p. 42.

Impact Tests of Metals are sometimes made for the purpose of determining toughness, that is, the ability to resist without actual rupture a combination of high stress and great deformation. Tough metals resist heavy accidental overload without shattering failure. In impact tests the amount of energy required to rupture or permanently deform the test specimen is the quantity measured rather than the stress. In general, the results of an impact test may be predicted by measuring the area under the load-deformation diagram for a static test of the specimen, though there may be some variation introduced by the very rapid application of load in an impact test.

The Master Car Builders' Drop Testing Machine is an impact machine consisting of a framework fitted with a hoist and a weight which can be varied from 1640 lb to 2000 lb, and which can be dropped from a height varying from 0 to 50 feet. The specimen is supported on an anvil weighing 20 000 lb, and the anvil is supported on heavy springs. With this machine tests are made of rails in flexure, of car couplers in tension, of car axles and locomotive axles in flexure. Impact tests of cast-iron car wheels are made on a machine somewhat similar to this. Drop tests are made by delivering on a test specimen one or more blows from a given height, and observing signs of failure in the test specimen.

The Charpy Impact Testing Machine is used for tests of small specimens of metal under impact. The specimen is held in a horizontal position and is in flexure over a short span. It rests against the vertical face of an anvil. The impact is furnished by a pendulum which is let fall from a predetermined height, striking the specimen at the bottom of its swing. The pendulum is of known weight, and is so hung that its center of percussion is at the point of impact against the specimen. After breaking the specimen the pendulum continues its swing, and an indicating apparatus shows the maximum height to which it rises. The energy supplied by the pendulum is the product of its weight and

its height of fall; the energy left after the fracture of the specimen is the product of its weight and the height of rise; the difference is the energy required to break the specimen. Specimens for the Charpy machine are notched at the middle of their length to insure a sharp plane of fracture, and the uniformity of size of specimen and of size and shape of notch is very important.

Repeated Stress Testing Machines are of various types. In the White-Souther machine a round flexure specimen is rotated while carrying a known weight; as the specimen rotates the fibers are alternately on the tension side and the compression side, and the stress is thus reversed. In the Upton-Lewis machine a small flexure specimen is bent back and forth against a pair of calibrated springs, and the bending force, as indicated by the amount of compression of the springs, is recorded on a strip of paper by a pencil. In the Smith machine, developed in England, a repeated stress is set up in a specimen by the variation in direction of the centrifugal force of a known weight rotating at a known speed with a known eccentricity.

Tests of Magnetic Permeability of steel and iron seem to give some indication of the hardness, strength, and uniformity. They possess the great advantage that the test does not destroy the sample tested, and hence may be made on the actual parts to be used. Magnetic tests as an indication of hardness or strength are not standardized yet, but hold promise of future usefulness.

Microscopic Examination of Metals is a most valuable means of studying their internal structure. The use of the microscope for the examination of metals involves great skill in handling, and reliable results can be obtained only by trained metallographists. The general process consists of polishing a small portion of the surface to be examined, using emery cloth followed by jewelers' rouge; etching the surface with acid to bring out the structure, and then examining and photographing the surface thru a microscope. Magnifications of about 100 times are in common use. The size of the "grains" or "crystals" of the metal can be determined, as well as the uniformity and plan of distribution of various ingredients, and the presence of small masses of impurities. Microscopic examination is the only sure way of distinguishing wrought iron from steel, by detecting the slag fibers always present in wrought iron.

26. Tests of Stone and Brick

The Specific Gravity Test for stone shall be made on a specimen that has been dried for 48 hours at a temperature of 230° to 250° Fahr. After removing all sharp corners and grains that are loose, the specimen shall be carefully weighed, then placed in water and put under the receiver of an air pump and all bubbles of air exhausted. The weight in water shall be noted and the specimen removed from the water and all surplus surface moisture removed either with a cloth or blotting paper. After again weighing, the specific gravity will be determined from the formula Sp. Gr. = Wt of dry stone in air ÷ (Wt of saturated stone in air − Wt of saturated stone in water). Where an air pump is not available, the specimen may be weighed in the water after an immersion of 24 hours, all surface air bubbles being removed with a small brush or feather.

Tests for Crushing Strength of stone shall be made on cubes, the size of which will depend upon the capacity of the testing machine. Specimens shall be sawed out and not chiseled. They shall be tested on their natural beds, and the opposite faces shall be ground parallel and tested between steel plates in a machine having a spherical socket in the crushing head. At least three specimens shall be tested, and the load at which the specimen first cracks and the ultimate resistance shall be noted in the report.

Transverse Strength of stone shall be determined by placing a beam whose length is 10 times its depth on slightly rounded knife-edges and applying a load at the center. The modulus of rupture shall be computed by the formula $R = 3\,Wl/2\,bd^2$, where W is the center load, l the span, b the width, and d the depth.

The Absorption Test shall be made by drying the stone for 48 hours at a temperature of 230° to 250° Fahr, carefully weighing, then placing in water for 48 hours. At the end of this period the stone shall be removed from the water, all surplus water wiped off and the stone reweighed. The percentage of water absorbed shall be noted in terms of the original dry weight of the stone.

In the Freezing Test, five clean stone cubes of the same size shall be dried, weighed, and immersed in water for 24 hours. They shall then be alternately frozen and thawed twenty-five times. After the final thawing, they shall again be dried in an oven, and all loose particles removed. The loss of weight shall then be noted.

The Quenching Test shall consist of heating the stone specimens to 500° to 600° Fahr, and plunging them while hot in water of a temperature of about 70°. The loss due to spalling or disintegration shall be noted in terms of the weight of the original dry stone.

The Acid Test shall consist of soaking small pieces of clean stone for four days in water containing 1 percent each of hydrochloric and sulphuric acid and agitating several times each day. At the end of the period the stone shall be washt, dried, and the percentage of loss computed in terms of the original dry weight.

Toughness Test for macadam rock. The following was adopted in 1908 by Amer. Soc. Testing Materials (Proceedings, vol. 8).

"Test pieces may be either cylinders or cubes, 25 mm in diameter and 25 mm in height, cut perpendicular to the cleavage of the rock. Cylinders are recommended, as they are cheaper and more easily made.

"The testing machine shall consist of an anvil of 50 kg weight, and placed on a concrete foundation. The hammer shall be of 2 kg weight, and dropt upon an intervening plunger of 1 kg weight, which rests on the test piece. The lower or bearing surface of this plunger shall be of spherical shape, having a radius of 1 cm. This plunger shall be made of hardened steel, and prest firmly upon the test piece by suitable springs. The test piece shall be adjusted so that the center of its upper surface is tangent to the spherical end of the plunger.

"The test shall consist of a 1 cm fall of the hammer for the first blow, and an increased fall of 1 cm for each succeeding blow until failure of the test piece occurs. The number of blows necessary to destroy the test piece is used to represent the toughness, or the centimeter-grams of energy applied may be used."

Paving Brick are tested for abrasion in the method described in Art. 23. Other tests specified are the following.

In the Transverse Test, the brick shall be placed on edge on two curved knife-edges which form the arc of a circle of 12 inches radius and rounded transversely to a radius of ¼ inch. They shall be placed 6 inches apart, and the load shall be applied at the center thru a knife-edge with a straight edge but rounded transversely to a radius of 1/16 in. Ten bricks shall be tested and the modulus of rupture computed from the formula $R = 3\,Wl/2\,bd^2$, where W is the average center broaking load, l the span, b the width and d the depth.

The Absorption Test shall be made by taking five of the brick that have past thru the abrasion test, drying them for 48 hours at a temperature of 230° to 250° Fahr, and immersing them in water for 48 hours. The percentage of water absorbed is computed in terms of the weight of the dry brick.

27. Miscellaneous Tests

Tests of Strength of Wood have not been standardized to an extent which allows their inclusion in specifications for timber, but compression tests, flexure tests, shearing tests, and impact tests are frequently made. Compression tests are made both along the grain and across the grain. Compression specimens

Special Tests of Materials, Structures, and Machines

Structural part, machine part, ructure, machine, r material tested	Test	Measurements
loor panel of building.	Proof test with dead load, not to destruction.	Deflections, tensile and compressive deformations in beams and columns.
ridge...........	Proof test with dead load or with moving load.	Deflections, tensile and compressive deformations in various members.
iveted joints.....	Tests of samples to destruction.	Ultimate load, slip of rivets.
ivets, metal plates	Shearing test of samples to destruction using special shearing tools.	Ultimate load, results depend on hardness of shearing tools used.
olts...........	Tests of samples to destruction in tension or torsion.	Ultimate load.
oilers..........	Tests with hydrostatic pressure. Proof tests with pressures somewhat above working pressures.	Observation of leaks, cracks, or permanent distortion of parts.
ar couplers and coupler yokes.	Proof tests, tests of samples to destruction under tension and impact tension.	Ultimate load, distortion under proof load.
Vire rope.........	Tests of samples to destruction in tension.	Ultimate load, observation of manner of fracture.
rake beams for railway cars.	Tests of samples to destruction in flexure.	Ultimate load, deflections.
Chain............	Proof tests of entire chain in tension, tests to destruction of sample sections.	Set after removal of proof load, ultimate strength.
arge pipe........	Hydrostatic pressure proof test.	Observation of leaks, cracks, or other evidence of failure.
Gear teeth........	Scleroscope test for uniform hardness.	Scleroscope hardness for various teeth.
ngraver's plates..	Brinnell or scleroscope tests for uniform hardness of samples.	Hardness at various points.
train insulators for electric transmission lines.	Proof load in tension, tension test of samples to destruction.	Evidences of failure under proof load, ultimate strength.
yebars...........	Tension test of sample eyebars to destruction.	Yield point, ultimate load, stretch.
Columns...........	Compression tests of models or samples to destruction.	Yield point, ultimate load, deflection, axial compression.

Tests of concrete and of cement are discussed in Section 5 of this Pocket Book. rther discussion or tests of road materials are given in Section 15.

e usually rectangular blocks, and should be tested with spherical-seated bearing ocks. Wood has a rather poorly defined proportional limit in compression. compression along the grain there is a well-defined ultimate. Shearing tests ong the grain are made by the use of shearing blocks placed in the testing achine, or for larger specimens by testing short, deep beams. The shearing rength of wood along the grain is of great importance. Flexure tests both of all selected specimens and of large beams are common. In flexure tests the ilure may be by longitudinal shear along the neutral axis, by compression along

Cold Bend Tests

Summarized from the 1916 "Standards" of the Am. Soc. for Test. Materials. t = thickness of specimen; D = diam. of pin round which specimen is to be bent; A = angle, in degrees, thru which specimen must bend without cracking on convex surface.

Material	D/t	A	Specimen
Splice Bars (R. R.):			
Low-carbon steel	0*	180	Unpunched bar.
Medium-carbon steel	2	180†	½ in × ½ in or rectangular with surface as rolled.
High-carbon steel	3	90†	
Extra-high carbon steel	3	60†	
Quenched high-carbon steel	3	90†	Rectangular with rounded edges.
Track bolts:			
Quenched carbon steel	1	45	Finished bolt.
Quenched alloy steel	1	90	Finished bolt.
Structural steel for bridges, buildings, locomotives, cars and ships.	0* for $t \leqq$ ¾ in. 1 for t = ¾–1¼ in 2 for $t >$ 1¼ in	180	Flat as rolled, machined to ¾ in thick if $t >$ 1½ in. For rounds, specimen for cold bend test is 1 in × ½ in.
Structural nickel steel	1 for $t \leqq$ ¾ in 2 for $t >$ ¾ in	180	Same as structural steel.
Rivet steel for ships, buildings, and boilers	0*	180	Rod as rolled.
Rivets	0*	180	Full-size rivet.
Billet-steel plain reinforcing bars:			
Structural steel grade	1	180	Rod as rolled.
Intermediate grade	2	180 for $t \leqq$ ¾ in 90 for $t >$ ¾ in	
Hard grade	3		
Billet-steel deformed:			
Reinforcing bars Structural steel grade	1 for $t \leqq$ ¾ in 2 for $t >$ ¾ in	180	Rod as rolled.
Intermediate grade	3	180 for $t \leqq$ ¾ in 90 for $t >$ ¾ in	
Hard grade	4		
Cold-twisted bars	2 for $t \leqq$ ¾ in 3 for $t >$ ¾ in	180	
Rail-steel reinforcing bars			
Plain	3	180 for $t \leqq$ ¾ in 90 for $t >$ ¾ in	Rod as rolled.
Deformed and hot-twisted	4		
Quenched and tempered steel axles.	For diam. $\leqq$ 7 in use flat mandrel 1 in thick with rounded edges; for diam. > 7 in use 1½ in mandrel with rounded edges.	180	½ in × ½ in with corners rounded to 1/16 in radius.

* Specimen bent flat on itself.
† Special alternative test may be made.

Cold Bend Tests—*Continued*

Material	D/t	A	Specimen
Cold-rolled steel axles..	Use mandrel 1 in diam.	[illegible]	Same as for quenched and tempered steel axles.
Soft steel castings.....	Use mandrel 1 in diam.	120	$1 \times \frac{1}{2}$ in with rounded corners.
Steel pipe	Diam of pipe $\times 18$	180	Full-size pipe.
Boiler and firebox steel	1 for $t \overline{\overline{<}} 1$ in. 2 for $t > 1$ in	180	Strip of plate as rolled.
Staybolts..............	0*.	180	Rod as rolled.
Engine-belt iron........	1	180	Rod as rolled for $t \overline{\overline{<}} 1\frac{1}{2}$ in; 1 in$\times$1 in for $t > 1\frac{1}{2}$ in
Refined wrought iron....	2	180	Rod as rolled if section $\overline{\overline{<}}$ 4 sq in.
Wrought-iron plates....	$1\frac{1}{2}$ for 1st qual. 3 for 2d quality	90	Strip as rolled; length parallel to direction of rolling.

* Specimen bent flat on itself.

the upper side of the beam, or by tension along the lower side. Impact flexure tests show the shock-resisting qualities of wood.

The Turner-Hatt impact testing machine is in common use in the U. S. for impact testing of wood. This machine consists of an anvil on which the specimen is placed, and of a weight which can be dropt from various heights. Attached to the weight is a pencil which draws a record on a rotating drum. From this record the deflections of the specimen can be measured, as the weight is dropt from successively increasing heights until rupture occurs, or until the deflections increase abnormally, showing that the proportional limit has been passed. Another method of making an impact test consists in dropping the weight from such a height that the specimen is fractured by one blow. The pencil attached to the weight traces a curve on the rotating drum whose steepness is a measure of the velocity of the falling weight. Measuring the velocity of the weight before and after fracture of the specimen the energy absorbed in fracturing the specimen can be determined.

Special Tests of materials, structures, and machines are often made. The table on page 387 gives some such tests.

28. Inspection

A Shop Inspector has duties suggested largely by the specifications governing the work. Primarily his duties are to see that workmanship, method of fabricating material, and finished product are all in accordance with the plans and specifications, a complete set of which should be furnished him. The tools needed are inside and outside calipers, micrometer calipers, rule, steel tape, testing hammer, and report blanks. In structural work he should measure up finished material, particularly with reference to field connections, examine abutting joints to see that they are full and square, test all rivets with a light hammer and have all loose or cocked-headed ones cut out and replaced, should see that all members are free from twists, kinks, or bends, that pin holes are at right angles to the web of the member, countersunk rivets are

chipped, that surfaces for rollers, splice plates, and bearing plates have been planed, that eyebars intended for the same pins should when piled all take the right pins at both ends at the same time, that threaded ends are wrapt with burlap, or otherwise protected against damage in transportation, etc. All accepted material should be marked with the inspector's private stamp.

The Cold Bend Test is of great importance as a shop test of ductility. It is made by bending cold a specimen of iron or steel flat on itself or round a mandrel of specified diameter. The bending may be accomplished either by pressure or by blows of a hammer. During and after bending the specimen must develop no cracks. A tabular statement of the requirements for cold bend tests of various grades of iron and steel as given by the 1916 " Standards " of the Am. Soc. for Test. Materials is given in the table on p. 388.

Flattening or Upsetting Tests are made on rivets, on boiler tubes, and on pipes. Boiler tubes are also subjected to flanging tests in which a flange is formed on a cold specimen of tube. In flanging, upsetting, or flattening tests the distorsion of the specimen must be accomplished without the development of cracks.

Hot Bend Tests and Quench Bend Tests are made on wrought iron, and quench bend tests are also made on material for boiler flues. Hot bend tests are made at a cherry red heat, and quench bend tests are made after heating the specimen to redness and cooling in water. For both tests the specimen must bend round a mandrel of specified size without cracking.

Nick Bend Tests are made on wrought iron to show the fibrous structure and the presence of scrap steel in the iron. A sharp nick is made across a test bar, which is then bent, opening out a cross-section. Wrought iron shows a gray, fibrous section, while the presence of scrap steel is indicated by bright, crystalline spots. This is a valuable shop test, but is not entirely conclusive.

The Punching Test is made to ascertain at what distance from its edge a bright red test specimen can be punched without breaking out. The width of the specimen should be more than five times its thickness and the diameter of the punch should be equal to its thickness.

The Drifting-out Test is made on a bright red specimen with width equal to five times its thickness. A hole is first made with a punch whose diameter is about twice that of the thickness of the specimen, and then drifted out until cracks begin to show.

Hammering Tests are made with a sledge hammer or a quick acting light steam hammer. The hot test specimen should be three times as wide as its thickness and be hammered until it is lengthened or widened 150 or 200 percent.

See Arts. 30-33 for other information in regard to tests.

IRON, STEEL, AND OTHER METALS

29. Cast Iron

Cast Iron is a saturated solution of carbon in iron, the amount of carbon varying ordinarily from a minimum of 1.5 percent to about 4 percent, depending upon the amount of silicon, sulphur, phosphorus, and manganese present in the solution. Other elements may also be present, but they are considered impurities.

Metallurgy. In the production of cast iron two steps may be necessary: First, the preparation of the ore for the blast furnace, involving the washing or dressing of the ore, and the calcination; and, secondly, the reduction in the blast furnace. In the case of the richer ores the first step is not always required, and this is the condition in the United States. The purpose of the roasting or calcination is the expulsion of volatile ingredients.

The reduction or smelting of the ore is accomplished in a blast furnace, which, as the molten metal is drawn off at the bottom thru the tapping hole, is kept filled to the top or throat by adding metal in the form of iron ore, fuel in the form of coke, and flux, usually in the form of limestone, allowing the charge to work down. By means of tuyeres near the base of the furnace, a supply of air, or "blast," under a pressure of from three to nine pounds, is provided to maintain combustion at sufficiently high temperatures to reduce the ore. The flux combines, in the process of reduction, with the earthy matter of the ore and of the fuel, forming the slag. The slag, being very much lighter than the iron, floats on the surface of the molten metal and is allowed to run away thru the cinder notch, or cinder fall, into trucks called cinder tubs, by which it is transported to the cinder heap.

Pig Iron is the term applied to the form in which cast iron is obtained from the blast furnace. Pigs are generally semicylindrical in form, about 5 in wide and about 36 in long, and weigh about one hundred pounds. In its more restricted sense, cast iron is the form assumed after it has been again melted and cast into the finished form.

Composition. Commercial pig iron varies widely in chemical composition, depending on the uses to which it is to be put. The tendency of the present day is to purchase pig iron according to analysis instead of by grades. The constituents of good commercial pig iron vary as follows: Carbon from 2 to 4 percent; silicon from 0.5 to 5.0 percent; sulphur from 0.005 to 0.3 percent; phosphorus from 0.62 to 1.50 percent; and manganese from 0.10 to 1.75 percent. But these limits are by no means fixt.

Carbon. The amount in cast iron is largely dependent on the presence of other elements. While 4 percent is the ordinary maximum, the carbon may run as high as 7 percent if much manganese is present. The presence of silicon in large proportions, on the other hand, may reduce the solubility of the carbon to as low as 1 percent. The percent of carbon present in cast iron in the combined form influences very largely the physical properties of the cast iron; thus, to get the maximum tensile strength, the combined carbon should be about 0.47 percent; for the maximum transverse strength it should be about 0.70 percent, and for the maximum crushing strength it should be over 1 percent. The hardness of cast iron increases regularly with the increase in the percentage of combined carbon. In Gray Iron the carbon exists almost wholly as graphite, having been precipitated as such in the process of solidifying. The graphite, known as "kish," gives the iron a somewhat spongy nature and a dark color. The condition is brought about in part by slow cooling, which tends to produce large crystals as well as graphite carbon. In White Iron the carbon is almost wholly combined and the iron has a more homogeneous texture, lighter appearance, and is composed of smaller crystals. Rapid cooling in solidifying tends to produce white iron. In Mottled Iron the proportions of combined and graphite iron are nearly equal, the fracture having, as the name indicates, a mottled appearance, due to the dark gray portions in the white matrix.

Silicon. The amount in cast iron determines, to some extent, the suitability of the material for various purposes. A certain amount is always desirable. Up to 0.8 percent it increases the hardness of the iron, but above that point it makes the iron soft and brittle. Silicon gives the iron a gray appearance, and if the proportion of silicon is large the iron is gray and highly crystalline. The shrinkage of cast iron is largely influenced by the presence of silicon, decreasing as the amount of silicon increases. A small proportion of silicon makes sound castings, free from blowholes.

Sulfur. The presence of sulfur is generally considered objectionable. A quantity greater than about 0.08 percent produces what is known as red shortness, that is, brittleness when in a heated condition. This condition makes the material unfit for use.

Phosphorus. The effect of phosphorus is to increase the fusibility and fluidity of the metal. Its presence, therefore, is desirable for light and ornamental castings, where well defined impressions in the mold are wanted. At the same time, it tends to increase the brittleness of the iron. The maximum amount should not exceed 0.7 percent in good foundry pig for ordinary castings.

Manganese. This element is present in most pig iron and the amount varies greatly. For foundry use its presence is of no benefit when it exceeds about 1.0 per cent. The value of the pig iron to the steel maker, however, increases in proportion to the amount

of manganese present. Its presence prevents the absorption of sulfur in remelting. Hardness and closeness of grain are produced by the presence of manganese. SPIEGELEISEN, so called on account of its white glistening fracture, is a pig iron containing a large proportion of manganese, that is, from 5 to 20 percent. It is very hard, resisting cutting by cast-steel tools. When the proportion of manganese rises above 20 percent reaching sometimes as high as 80 percent, it is known as ferro-manganese.

Other Elements that are often found in pig iron as the result of reduction, but in very small quarters, and which affect its properties to some extent, are tin, causing increased hardness, greater fusibility, unfitness for conversion into malleable iron, and rendering wrought iron cold short; copper, causing unfitness for conversion to malleable iron; vanadium, increasing the softness and ductility; titanium, increasing the strength; arsenic, chromium, aluminium, and zinc.

The Grades of Pig Iron. Pig iron was formerly classified by grades, such as No. 1 soft, No. 2 soft, No. 1 foundry, No. 2 foundry, forge pig, etc., but to-day the practise is to buy pig iron by chemical analysis rather than by trade names of grades. Specifications for buying pig iron by analysis are given in the "Standards" of the Am. Soc. for Test. Materials. The following table gives typical analyses of pig iron for various uses. All values are in percent:

Use	Graphite carbon	Combined carbon	Silicon	Sulfur	Phosphorus	Manganese
Bessemer pig, for making steel by the acid Bessemer process or the acid open-hearth process.	3.50	0.05–0.10	1.00–2.00	Not more than 0.05	Not more than 0.10	0.25
Basic pig, for making steel by the basic open-hearth process.	3.50	0.05–0.10	Not more than 1.00	Not more than 0.05	0.30–1.00	0.25
Malleable pig, for making malleable cast iron.	0.10	3.00	0.75–1.00	Not more than 0.05	Not more than 0.200	0.10
Foundry pig, for general casting.	3.15–3.90	0.05–0.10	0.75–4.00	0.02–0.08	0.10–1.00	0.20–0.25

Molding is the preparation of hollow molds to receive the molten metal. A wooden pattern of the shape of the finished piece is prepared and imbedded in molding sand placed in wooden boxes so arranged in two parts, called the lower and upper flasks, that after the molding sand is thoroly rammed and packed around the pattern, the upper flask may be taken off, the pattern withdrawn and the flask replaced and secured, leaving on the interior a hollow space of the size and shape of the finished piece. An opening is left thru which the molten metal is poured into the mold. Smaller holes are also provided connecting with the hollow interior, to allow for the escape of the air and other gases that may be generated during pouring. When the finished casting is to be hollow a core is employed.

Patterns are generally made of thoroly seasoned white pine or mahogany, the wood being carefully shellacked to keep the pattern from warping or being otherwise affected by moisture. If much used, patterns are often made of metal, in which case after being cast they are filed and scoured smooth, warmed and coated with wax. Patterns are sometimes also made of plaster of Paris,

especially for highly ornamental castings in architectural iron work. The SHRINKAGE of cast iron in cooling must be allowed for in the making of the patterns. The usual allowance is ⅛ inch per foot, but this cannot be laid down as a hard and fast rule, as the shrinkage varies with the relative dimensions of castings and with the character of the metal.

Molding Sand consists chiefly of silica, 90 to 95 percent, which makes the sand sufficiently refractory; alumina and magnesia, 3 to 8 percent, which furnishes the necessary cohesion and plasticity to the sand; oxide of iron, about 1.5 percent; lime about 0.5 percent; and sometimes a small quantity of coal dust. Fine sand compacts too much, preventing the gases from escaping readily and causing blowholes in the castings, while coarser sand lacks cohesion and makes inferior castings.

Green Sand Molds are made of molding sand and are the molds most generally used, being most readily made and cheap. DRY SAND MOLDS are made of a loamy sand, being first roughly molded into shape, then dried by heat and finished off with a tool. They can be made without a pattern, and are therefore not used when a pattern has been prepared, as in that case green sand molds are cheaper. LOAM MOLDS are generally used for large castings. They are built up of brickwork to the rough outline of the casting and are then finished off on the surface with loam laid on by a trowel.

Chills are metal molds used for certain castings, such as car wheels, where a hard surface is wanted, this being produced by the sudden cooling of the hot metal as it comes in contact with the comparatively cold surface of the mold.

Cores are used for the production of the hollow spaces in castings, and are made of baked sand and clay formed into shape and fixt in the molds. For large cylinders and other large castings, the cores are built up of brickwork to the approximate size and the surface finished off by facing with loam. For water pipes and similar castings the brickwork is often replaced by iron tubes.

The Cupola is the usual means for remelting pig iron to produce cast iron in its more restricted sense, namely, the finished castings. It consists of a cylindrical shaft provided with one or two rows of tuyeres near the base, thru which air is forced at a pressure of about one-half pound per square inch. The charge consists of pig iron and coke in the proportion of about 200 pounds of coke for each ton of metal. A little limestone is usually also introduced as a flux. The molten metal drawn off at the bottom is poured into the molds by means of cup-shaped ladles fitted with long handles, one of which has a cross bar for tipping the ladle in pouring, or, for larger work, in vessels carried on wheels and operated by mechanical means.

Malleable Castings are made in the same manner as ordinary castings, but are subjected to a further annealing process. Only small castings can be treated in this way. The castings are placed in cast-iron boxes called annealing pots, about 18 by 24 in and 4 ft high, with the decarbonizing material packed around them. For the decarbonizing agent an iron oxide in the form of hematite ore or forge iron scale is used. Only white iron low in sulfur can be used, and generally the best charcoal cast iron is selected. The castings, packed as stated, are placed in an oven in which the temperature is quickly raised and the castings kept at a cherry-red heat for three to five days, depending on their size, after which the furnace is allowed to cool slowly. The effect of this process is to make the castings " malleable " and to nearly double their strength. Castings of this kind may be bent cold, forged, or welded to a greater or less extent. They are used for pipe fittings, iron handles for tools, wheels, pinions, small parts of machinery, etc.

Semi-steel is the somewhat misleading trade name given to cast iron in the production of which 30 to 60 percent of steel scrap is used. With great care in foundry practise such " semi-steel " may be produced having somewhat higher strength than ordinary cast iron.

The Air Furnace which is sometimes used instead of a cupola to melt cast iron comprises a hearth on which fuel is burned and a separate chamber in which the pig iron and scrap are melted without direct contact with the fuel. Air furnace iron is somewhat

freer from impurities than is cupola iron, but is much more expensive. Except for producing malleable cast iron the air furnace is not very extensively used.

The Fusibility of Cast Iron is dependent on the percentage of carbon and some of the other elements. The average fusion point is about 2200° Fahr. Its heat conductivity is 35.9, silver being 100.

The Specific Gravity of cast iron varies with its composition from about 6.9 to 7.5. It is usually taken at 7.22 (water at 62° F. being unity) corresponding to a weight of 450 lb per cu ft. In a general way the specific gravity increases with the strength of the metal and the number of remeltings.

The Coefficient of Expansion of cast iron may be taken at 0.000 0062 for 1° Fahr. as an average. If exposed to continued heat, cast iron becomes permanently expanded 1½ to 3 percent, a fact that must be remembered in installing grate bars or other castings exposed to heat.

The Modulus of Elasticity varies from 12 000 000 to 14 000 000 lb per sq in for ordinary commercial cast iron, and from 16 000 000 to 18 000 000 for special grades of stronger cast iron, such as are used for ordnance.

The Elastic Limit. Cast iron has no clearly defined elastic limit either in tension or compression. For malleable iron the elastic limit in tension varies from 15 000 to 20 000 lb per sq in.

The Ultimate Strength in Tension for ordinary castings can be taken at 15 000 to 18 000 lb per sq in, and for better grades of castings at 20 000 to 30 000. Tests have shown as low as 9200 and as high as 46 450 lb per sq in. Malleable iron has a strength of about 40 000 lb per sq in.

Compressive Strength. 80 000 lb per sq in may be taken as the average ultimate strength in compression. Tests show as low as 44 500 and as high as 215 000 lb per sq in.

Modulus of Rupture. The average value may be taken at 35 000 lb per sq in, the tests varying from a minimum of 9700 to a maximum of 63 500.

Shear and Torsion. As the ultimate resistance to shearing 20 000 lb per sq in may be taken. Under torsion cast iron fails by tensile strain on an inclined section.

Defects. The most common defects in cast iron are (1) blowholes, caused by the formation of steam when the hot metal comes in contact with the damp molding sand: (2) sand holes, and (3) roughness of surface, due to breaking down of the mold in spots; (4) cold-shuts or cold-shorts, which are seams caused by the too rapid congealing of the metal so that it does not completely fill the mold, and (5) cracks resulting from uneven shrinkage in parts of the castings of unequal thickness. The last is sometimes not discoverable until the casting is put under load. Castings to be acceptable should present smooth, clean surfaces with all angles true and sharp, and should be soft enough to be dented on the edges by a hammer blow instead of breaking off.

For Cast-iron Pipe the metal shall be "of good quality and of such character as shall make the metal of the castings strong, tough and of even grain, and soft enough to satisfactorily admit of drilling and cutting." (Standard Specifications, American Society for Testing Materials.) For strength tests, see p. 384.

For Cast-iron Car Wheels the metal shall be soft, clean gray iron, closely approximating the following composition: Graphitic carbon 2.90 percent, combined carbon 0.60, silicon 0.70, manganese 0.40, phosphorus 0.50, sulfur 0.08 percent.

For Gray Iron Castings that is, ordinary castings, the following standard specifications have been adopted by the American Society for Testing Materials (1916 "Standards," p. 362):

Unless furnace iron is specified, all gray castings are understood to be made by the cupola process.

The sulfur contents to be not over following percentages: Light castings, 0.08; medium castings, 0.10, heavy castings, 0.12.

In dividing castings into light, medium, and heavy classes, the following standards have been adopted: Castings having any section less than ½ in thick shall be known as light castings. Castings in which no section is less than 2 in thick shall be known as heavy castings. Medium castings are those not included in the above classification.

Transverse Test. The minimum breaking strength of the "Arbitration Bar" under transverse load shall be not under 2500 lb for light castings, 2900 for medium castings, and 3300 for heavy castings. In no case shall the deflection be under 0.10 of an inch, (See p. 383.)

Tensile Test. Where specified, this shall not run less than 18 000 lb per sq in for light castings, 21 000 for medium castings, and 24 000 for heavy castings. The specimen is 1 in in minimum diameter, 3½ in long with threaded ends.

Borings from the broken pieces of the "Arbitration Bar" shall be used for the sulfur determinations. One determination for each mold made shall be required. In case of dispute, the standards of the American Foundrymen's Association shall be used for comparison.

Castings shall be true to pattern, free from cracks, flaws, and excessive shrinkage. In other respects they shall conform to whatever points may be specially agreed upon.

The inspector shall have reasonable facilities afforded him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall, as far as possible, be made at the place of manufacture prior to shipment.

For Malleable-Iron Castings the following standard specifications are given in the 1916 "Standards" of the Am. Soc. for Test. Materials, p. 359:

1. Manufacture. The castings shall be made from iron melted either in an air furnace, open-hearth furnace, or electric furnace.

2. Physical Properties and Tests. Tension test specimens specified in paragraph 5 shall conform to the following minimum requirements as to tensile properties: Tensile strength, 38 000 lb per sq in; elongation, 5 per cent in a 2-in gage length.

3. Tranverse test specimens specified in paragraph 5, tested with the cope side up on supports 12 in apart, pressure being applied at the center shall conform to the following minimum requirements as to transverse properties: Specimen ½ in thick, load at center 900 lb deflection at center, 1.25 in; specimen ⅝ in thick, load at center, 1400 lb, deflection at center 1 in; specimen ¾ in thick, load at center 2000 lb, deflection at centerr 0.75 in.

4. In addition to the tension and transverse tests the inspector representing the purchaser shall satisfy himself of the suitability of the iron used for castings by breaking a reasonable number of castings before annealing to examine for excessive mottling or graphite spots. In case of castings of special design or importance he may also require test lugs of a size proportional to the thickness of the casting, but not exceeding ⅝×¾ in in section. At least one of these lugs shall be left on the casting for final inspection.

5. (*a*) Tension test specimens shall be of the form and dimensions shown in Fig. 59. Transverse test specimens shall be 14 in in length by 1 in in width and either ½, ⅝, or ¾ in in thickness. The thickness of the specimen selected shall be in proportion to the thickness of the casting which it represents.

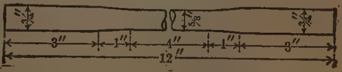


Fig. 59. Test Specimen for Malleable Castings.

(*b*) Two tension and two transverse test specimens shall be cast in each mold with risers of sufficient height at each end to secure sound bars. All specimens shall be cast without chills, and with ends perfectly free in the mold.

(*c*) Four molds shall be poured to represent each melt. When the entire melt is used for castings which are subject to these specifications, two molds shall be poured within

five minutes after tapping into the first ladle, and two molds from the last iron of the melt. When only part of the melt is required for such castings, two molds shall be poured from the first ladle of iron used and two molds after the required iron has been tapped.

(*d*) The molds shall be suitably stamped to identify the specimens.

The test specimens from one mold from the first and one mold from the last of the melt shall be annealed in the hottest part of the annealing oven, and the remaining specimens shall be annealed in the coldest part.

6. One tension and one transverse test specimen from each of the four molds representing a melt shall be selected for test. The remaining specimens shall be reserved, and shall be tested in case of failure to conform to the requirements specified.

7. If more than one tension or transverse test specimen from each of the two molds annealed in the two points in the oven specified in paragraph 5 (*d*) fails to meet the requirements as to tensile or transverse properties specified in paragraphs 2 and 3, the castings from that melt will be rejected.

8. Workmanship and Finish. The castings shall substantially conform to the sizes and shapes of the patterns, and shall be made in a workmanlike manner. A variation of $\overline{3/32}$ in per ft will be permitted.

9. The castings shall be free from blemishes, scale and shrinkage cracks.

10. Inspection. The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the castings ordered. The manufacturer shall afford the inspector, free of cost, all reasonable facilities to satisfy him that the castings are being furnished in accordance with these specifications. All tests and inspection shall be made at the place of manufacture prior to shipment, unless otherwise specified, and shall be so conducted as not to interfere unnecessarily with the operation of the works.

Working Unit Stresses for cast iron depend upon the character of the applied loads and upon the grade of metal. Common average values for tension fibers of flexure members are 4000 lb per sq in for steady loads and 3000 lb per sq in for variable loads, while for compression values about four times as great may be used. In shearing the working unit stresses may be taken as about the same as those for tension. The properties of castings depend upon the quality of the ores and upon the method of manufacture. Cold-blast pig produces stronger iron than hot-blast pig, but it is more expensive. Drying the air before admitting it to the blast tends to secure uniformity of product. The darkest grades of foundry pig make the smoothest castings, but they are apt to be brittle; the lightest grades make tough castings, but they are apt to contain blowholes or imperfections.

Very low unit stresses should be used when cast iron is subject to repetitive stresses alternating from tension to compression; probably 1000 lb per sq in is the highest value allowable. Cast iron is a brittle material which is unsuited to resist shocks, and since 1900 it has not been used in bridge construction. It is not generally used in direct tension.

30. Wrought Iron

Composition. Wrought iron is a product of the reverberatory furnace, and is composed principally of ferrite (pure iron) and slag (iron silicate); in these exist small amounts of impurities, an idea of the percentage of which can be obtained from the following analysis (Macfarlane):

	Common Wrought Iron	Best Wrought Iron
Carbon	0.05	0.06
Phosphorus	0.35	0.18
Sulfur	0.06	0.04
Silicon	0.23	0.20
Manganese	[illegible]	0.06
Slag	about 3.3	2.80

Metallurgy. Wrought iron is made in a reverberatory furnace from pig iron and less frequently from molten metal taken directly from the blast furnace. This method of manufacture is known as the puddling process. The furnace consists essentially of a firebox, puddling or working chamber, suitable draft openings and flue. The working chamber is provided with openings for the purpose of charging and working the metal, and for the removal of slag and the puddled metal. There are two distinct puddling processes, Wet Puddling and Dry Puddling.

In the Wet Process the hearth of the furnace is felted with high-grade iron ore or mill scale which acts as an oxidizing agent for reducing impurities. The reduction of impurities occurs in different stages, namely in the melting-down stage, during which most of the silicon and manganese and some of the phosphorus are removed; in the clearing stage, during which phosphorus and sulfur are removed, and in the boiling stage, during which carbon is removed and most of the remaining phosphorus and sulfur. Since pig iron melts at a much lower temperature than metallic iron, as the mass becomes purified it assumes a pasty condition. It is then collected in balls weighing about 80 lb, carried to the squeezers or a forge and most of the slag expelled. The resulting bars are then run thru roughing rolls, the rolled bars trimmed, and thereafter known as "muck bar." The muck bar is again heated to a welding heat and rerolled. This treatment cleans out most of the remaining slag, and the resulting product is commercial merchant bar. The operation of heating to a welding heat and rerolling is sometimes repeated several times, the effect being to improve the bars thus treated.

In the Dry Process, which is infrequently used, white pig iron is charged and subjected to the action of an oxidizing flame. The oxygen in this case is supplied by the furnace instead of from the fettling.

Appearance. A section of a wrought-iron rod or plate when polished shows more or less regular laminations of slag and iron. By notching one side of a bar and bending one end away from the notched side, the iron will break along the slag laminations and give a fracture known as "barking." If a bar is notched all around and then struck a blow heavy enough to cause it to break, the fracture will be coarsely crystalline; when broken in tension, the fractured section is generally irregular and fibrous.

To distinguish between Wrought Iron and Soft Steel (Iron Age, Dec. 23, 1909): The sample is cleaned from grease and scale and immersed in a solution of the following: water, 9 parts; sulphuric acid, 3 parts; muriatic acid, 1 part. The acids are poured into the water and the mixture allowed to cool. The specimen is allowed to remain in the solution for 15 or 20 minutes, when it is removed and rinsed in water. The fibers will now show plainly, or if not, the process is continued. Soft steel dissolves uniformly and without the fibrous structure found in wrought iron.

Grades. Wrought iron may be graded as follows: (1) Charcoal iron, the purest grade of wrought iron; (2) Puddled iron, classified according to quality into stay-bolt iron and merchant iron, grades A and B; (3) Busheled scrap, a heterogeneous product made from iron scrap; steel is frequently mixt with the iron scrap, causing considerable irregularity in the resulting product.

Properties. Wrought iron possesses the important qualities of toughness, ductility, malleability, and weldability, but it cannot be tempered.

Coefficient of expansion....	0.00000648 per degree F. (Clarke)
Electrical conductivity.....	0.16 (Cu = 100) (Lazare Weiler)
Melting temperature.......	2732° to 2912° F. (Pouillet, Claudel, Wilson)
Specific heat..............	0.1138 (Röntgen)
Specific gravity...........	7.4 to 7.9 (Kent)

Tension. The average results of a great many tensile tests made at the

Testing Laboratory of Columbia University on good wrought iron for general purposes give the following:

Yield point, lb per sq in....................	31 000
Ultimate strength, lb per sq in.............	51 000
Elongation in 8 in, percent................	21
Reduction of area, percent.................	30
Modulus of elasticity, lb per sq in..........	28 200 000

Shear and Torsion. J. Platt and R. F. Hayward (Proceedings Inst. C. E., Vol. 90) give the following values for " crown " best wrought iron which had an ultimate tensile strength of 48 400 lb per sq in.

Ultimate strength in single shear, lb per sq in..	42 050
Elastic limit in torsion, lb per sq in.........	20 530
Modulus of elasticity in torsion, lb per sq in.	12 800 000

Compression. The ultimate compressive strength of good wrought iron is not well defined. Practically, its yield point in compression should be considered as the ultimate for compression. This yield point is about the same as the yield point in tension.

The Strength of wrought iron is affected by its chemical composition, the mechanical work and heat treatment it has undergone, and also varies for different temperatures. Wrought iron has a well-defined yield point in both tension and compression which is from 2000 to 4000 lb per sq in higher than the proportional and elastic limit. Beyond the yield point wrought iron is a plastic material which flows rapidly as the maximum strength is approached. In compression it is difficult to determine the ultimate strength, and some engineers consider it best to regard this as coinciding with the yield point. The ultimate strength in tension increases with the amount of carbon, which, however, is rarely greater than $^1/_{10}$ of 1 percent. The strength of iron entirely free from carbon and phosphorus is probably between 39 000 and 40 000 lb per sq in.

Effect of Work of Rolling on Wrought-iron Plates (Holley)

Thickness in inches	Elastic limit, lb per sq in	Ultimate strength, lb per sq in	Elongation in 8 inches, percent	Reduction of area, percent
¼	32 400	51 800	11.2	18.4
½	31 180	49 760	14.2	22.0
⅝	30 775	50 200	15.5	22.5
¾	30 400	49 050	16.0	22.4

The effect of the work of rolling on circular sections exhibits the same variation, that is, a material increase in the strength of the bar results from a reduction of the cross-section.

Wrought-iron Wire is manufactured by cold drawing thru dies. Tensile tests on wire (U. S. Report on Tests of Metals, 1897) have given 110 000 lb per sq in for wire 0.01 inch diameter and 65 280 lb per sq in for wire 0.2 inch diameter. A rod 0.8 inch diameter of the same kind of material as the wire gave 50 000 lb per sq in.

The Effect of Temperature upon the ultimate strength of wrought iron is shown by the following table (M. Rudeloff, Trans. Internat. Soc. Testing Materials, 1909).

Temperature, degrees C.	Ultimate strength, lb per sq in	Elongation, percent	Temperature, degrees C.	Ultimate strength, lb per sq in	Elongation, percent
20	49 300	30.5	250	70 700	23.0
50	51 400	25.5	300	68 600	30.0
100	54 300	16.0	350	57 100	35.0
150	60 700	14.0	400	46 100	40.0
200	67 100	17.5	....	....	....

Effect of Reheating and Rerolling. Within limits, puddled iron is much improved in quality by being cut up, piled, reheated, and rerolled or hammered. However, it is found that only in special cases is it advantageous to reheat puddled iron more than twice. The following figures (Johnson) show the effect of reheating and rerolling on the tensile strength: The original bar had a tensile strength of 43 900 lb per sq in, after the second working this rose to 52 860 lb per sq in, and after the sixth working it became 61 820 lb per sq in; the tensile strength then diminished with the number of workings until after the twelfth working it became the same as that of the original bar.

Uses. Wrought iron is used for spikes, nails, bolts and nuts, wire, chain rod, horseshoe bars, sheets and plates, stay bolts, pipes and tubing, third rails, armatures, electro-magnets, and in the manufacture of crucible steel. Wrought iron is sold as "merchant bar" for subsequent working into various wrought shapes.

Wrought iron was extensively used in bridge and building construction prior to 1890, but since 1900 structural steel has entirely taken its place on account of being about 20 percent stronger, as also lower in price. CHAINS are made of wrought iron when the highest degree of reliability is required. The permissible tension P on a chain of the usual form is $P=0.4\,d^2S$ for open links and $P=0.5\,d^2S$ for stud links, where d is the diameter of the metal and S is the safe working unit stress (Goodenough and Moore, Bull. Univ. Illinois, No. 18, 1907).

Tests. Wrought iron used for rivets should be subjected to tensile and bending tests and also to an upsetting test. Boiler plate should be subjected to the tensile test, to bending tests, both hot and cold, as also to hot drifting-out tests (Art. 28). Iron wire is tested by tension, bending, and torsion. The bending should be performed in a hand vise, the jaws of which have a radius of curvature equal to double the diameter of the wire, and the quality of the metal is judged from the number of times it can be bent before fracture. In flexural tests wrought iron has no proper modulus of rupture, since the deflection increases indefinitely without rupture,

Behavior under Stress. Good wrought iron shows a fibrous structure when broken by tension or flexure. A stress exceeding the elastic limit causes a permanent set and raises the elastic limit higher than before. It is a fundamental rule that working unit stresses should not exceed the elastic limit. Under variable loads the allowable unit stress which is specified should seldom be greater than one-half of the elastic limit.

Detail Fracture is the name given to the small microscopic slippings which occur when a metallic specimen is subjected to repetitive stresses. These gradually increase in size and unite together, so that finally, after a large number of stresses, they may become visible to the eye. Chatelier has said (Proceedings Inter. Soc. Testing Materials, Feb., 1910): "If the existence of an elastic limit were rigidly exact a metal would be able to withstand repetitive stresses for any length of time, provided that limit were not exceeded. But every deformation of a body, however small, in fact gives rise to three distinct phenomena. When the stress which had called forth the strain ceases, we observe first a rapid

return of the metallic piece to its primitive dimensions; that is the manifestation of elasticity. But the specimen does not come back exactly to its initial dimensions; we can recognize this by means of methods of high precision. The specimen afterwards left to itself continues to undergo a slow deformation, approaching its initial dimensions more completely; these are viscosity phenomena. The specimen finally keeps a permanent deformation, extraordinary small if you like, but not rigorously nil. These residual deformations are of an absolutely negligible magnitude compared to the elastic deformations; they do not amount to the thousandth part of the latter, and they are therefore without importance to the static use of metals. But the repetition of the deformation can, by totalizing these parasitic phenomena, negligible so far as a single deformation is concerned, finally produce a profound alteration of the metal and even lead to its fracture. That is the elementary factor upon which the rupture of metals under alternating stress seems to depend." (See p. 330)

Defects. The principal defects in wrought iron as classified by Turner are rough edges, spilly places and blisters. To these might be added the presence of excess of slag. Rough edges are due to careless workmanship, imperfections in the rolls, and also to red shortness. Spilly places are spongy or irregular spotted parts, noticed particularly in sheets and occasionally in all kinds of wrought iron. They are generally attributed to imperfecting puddling.

Specifications. The following is a summary of the specifications for wrought iron as given in the 1916 "Standards" of the Am. Soc. for Test. Materials:

1. **Grades of Iron** recognized are: charcoal iron for boiler tubes and plates, staybolt iron, engine-bolt iron, refined wrought-iron bars, wrought-iron plates classes A and B.

2. **The Process of Manufacture** specified for each grade is as follows: For charcoal iron, the knobbling process, using charcoal as fuel, for staybolt iron the charcoal-fired knobbling process or the puddling process, for engine-bolt iron, wrought-iron bars and plates, the puddling process. For bars and plates the iron may be made from a mixture of muck bar and scrap, which, however, must be free from any admixture of steel.

3. **Physical Properties** are specified as given in the following table:

Specified Minimum Physical Properties of Wrought Iron

	Staybolt iron	Engine-bolt iron	Refined wrought-iron bars	Wrought-iron plates	
				Class A	Class B
Ultimate tensile strength, lb per sq in.	49 000–53 000	50 000–54 000	48 000	48 000–49 000	47 000–48 000
Yield point	0.6 Ult.	0.6 Ult.	25 000	26 000	26 000
Elongation in 8 in, percent	30	25	22	12–16	10–14
Reduction of area, percent	48	40			

The standard test specimen for bars is a piece of the bar as rolled. For plates the standard flat test specimen is used, as shown in Fig. 54; this is taken the full thickness of the plate as rolled.

4. **Cold Bend Tests** are made on specimens from bars and plates. These tests are summarized in the table of Cold Bend Tests, on p. 388.

5. **Quench Bend Tests** are made for charcoal-iron boiler tubes and for staybolt iron. A specimen the full size of bar or full thickness of plate after quenching in water from a red heat must bend flat on itself without cracking.

6. **Nick-bend Test** are made by nicking a bar with a tool having a 60° cutting edge, and breaking the bar crosswise, opening out a cross-section. This section must show a fibrous surface, wholly free from crystalline spots for charcoal iron, staybolt iron, and

engine-bolt iron; and with not over 10 percent of the fracture crystalline for other grades of iron.

7. **Threading Test.** Staybolt Iron shall be of such texture as to permit the cutting of a clear, sharp screw thread on a test bar.

8. **Finish.** All wrought iron shall be smoothly rolled and free from slivers, depressions, seams, crop ends, and evidences of being burnt.

Working Unit Stresses for wrought iron will depend upon the kind of loading, the highest being for steady loads and the lowest for alternating stresses or shocks. Following values are for wrought iron of an average quality and these may be increased 30 percent for the very best grades. All values are in pounds per square inch.

	Steady Stresses	Variable Stresses	Shocks
Tension	14 000	10 000	4000
Compression	13 000	9 000	3000
Shear	10 000	7 000	3500
Flexure	12 500	8 500	3500
Torsion	5 000	3 500	1500

In a rough general way the quality of wrought-iron can be estimated by the product of its tensile strength and ultimate unit elongation, this being an approximate measure of the work required to produce rupture. Or, the formula $K = \frac{1}{3}(S_1 + 2S)e$ is an approximate expression for this work per cubic unit of the material, S_1 being the elastic limit, S the ultimate strength, and e the ultimate unit elongation. For example, let $S_1 = 31\ 000$ $S = 58\ 000$ lb per sq in, and $e = 21$ percent, then $K = 10\ 290$ in lb per cu in. For another grade of iron let $S_1 = 25\ 000$, $S = 55\ 000$ lb per sq in, and $e = 28$ per cent; then $K = 12\ 600$ in-lb per cu in, and this is 22 percent higher than the other. Thus high ultimate strength is not advantageous when accompanied by low unit elongation.

Wrought iron has a decided "fiber" due to the slag entrained in the process of rolling. The above values for tensile strength and working unit stresses apply to those along the fiber, and about 12 percent is to be deducted from them for cases where the tension acts across the fiber.

31. Classification of Steel

Steel and Iron. Steel was originally produced directly from pure iron ore by the action of a hot fire, which did not remove the carbon to a sufficient extent to form wrought iron. The modern processes, however, involve the fusion of the ore, and the definition of the United States law is that "steel is iron produced by fusion by any process, and which is malleable." Chemically, steel is a compound of iron and carbon generally intermediate in composition between cast and wrought iron, but having a higher specific gravity than either.

	Percent of Carbon	Specific Gravity	Properties
Cast iron,	5 to 2	7.2	Not malleable, not temperable
Steel,	1.50 to 0.10	7.8	Malleable and temperable
Wrought iron,	0.30 to 0.05	7.7	Malleable, not temperable

It should be observed that the percentage of carbon alone is not sufficient to distinguish steel from wrought iron; also, that the mean values of specific gravity stated are in each case subject to considerable variation; further, only the hard steels are temperable, the softer grades resembling wrought iron.

Manufacture. The four principal methods, are the crucible process, the the electric-furnace process, the open-hearth process, and the Bessemer process. In the crucible process impure wrought iron or blister steel, with carbon and a flux, is fused in a sealed vessel to which air cannot obtain access; the best tool steels are thus made. In the electric-furnace process steel is refined out of contact with air by heat produced by means of an electric arc. In the open-hearth

process pig iron is melted, scrap and iron ore being added until the proper degree of refining is secured In the Bessemer process pig iron is completely decarbonized in a converter by an air blast and then recarbonized to the proper degree. The metal from the open-hearth furnace or from the Bessemer converter is cast into ingots which are rolled in mills to the required forms. The open-hearth process produces steel for machines, shafts, axles, springs, armor plates, rails, and for structural purposes the Bessemer process mainly produces steel for railroad rails and for the cheaper grades of steel. A combination method known as the duplex process is in wide use. In this process the refining action is started in a Bessemer converter, and after partial refining the molten steel is transferred to an open hearth furnace, where the refining process is finished. The following description of these methods is taken mainly from Kent's Mechanical Engineers' Pocket Book, 8th Edition, 1910:

Blister steel is a highly carbonized wrought iron, made by the "cementation" process which consists in keeping wrought-iron bars at a red heat for some days in contact with charcoal. Not over 2 percent of C is usually absorbed. The surface of the iron is covered with small blisters supposedly due to the action of carbon on slag. Other wrought steels were formerly made by direct processes from iron ore, and by the puddling process from wrought iron, but these steels are now replaced by cast steels. Blister steel is, however, still used as a raw material in the manufacture of crucible steel. Case-hardening is a process of surface cementation.

Crucible Steel is commonly made in pots or crucibles holding about 80 lb of metal The raw material may be steel scrap; blister steel bars; wrought iron with charcoal; cast iron with wrought iron or with iron ore; or any mixture that will produce a metal having the desired chemical constitution. Manganese in some form is usually added to prevent oxidation of the iron. Some silicon is usually absorbed from the crucible, and carbon also if the crucible is made of graphite and clay. The crucible being covered, the steel is not affected by the oxygen or sulfur in the flame. The quality of crucible steel depends on the freedom from objectionable elements, such as phosphorus, in the mixture, on the complete removal of oxide, slag, and blowholes by "dead-melting" or "killing" before pouring, and on the kind and quantity of different elements which are added in the mixture, or after melting, to give particular qualities to the steel, such as carbon, manganese, chromium, tungsten, and vanadium.

Bessemer Steel is made by blowing air thru a bath of melted pig iron. The oxygen of the air first burns away the silicon, then the carbon, and before the carbon is entirely burned away, begins to burn the iron. Spiegeleisen or ferro-manganese is then added to deoxidize the metal and to give it the amount of carbon desired in the finished steel. In the ordinary or "acid" Bessemer process the lining of the converter is a siliceous material which has no effect on phosphorus, and all the phosphorus in the pig iron remains in the steel. In the "basic" or Thomas and Gilchrist process the lining is of magnesium limestone, and limestone additions are made to the bath, so as to keep the slag basic, and the phosphorus enters the slag. By this process ores that were formerly unsuited to the manufacture of steel have been made available.

Open-hearth Steel. Any mixture that may be used for making steel in a crucible may also be melted on the open hearth of a Siemens regenerative furnace, and may be desiliconized and decarbonized by the action of the flame and by additions of iron ore, deoxidized by the addition of spiegeleisen or ferro-manganese, and recarbonized by the same additions or by pig iron. In the most common form of the process pig iron and scrap steel are melted together on the hearth, and after the manganese has been added to the bath it is tapped into the ladle. In the Talbot process a large bath of melted material is kept in the furnace, melted pig iron, taken from a blast furnace, is added to it, and iron ore is added which contributes its iron to the melted metal while its oxygen decarbonizes the pig iron. When the decarbonization has proceeded far enough, ferro-manganese is added to destroy iron oxide, and a portion of the metal is tapped out, leaving the remainder to receive another charge of pig iron, and thus the process is continued indefinitely. In the Duplex Process melted cast iron is desiliconized in a Bessemer converter, and then run into an open hearth, where the steel-making operation is finished. The open-hearth process, like the Bessemer, may be either acid or basic, according to the character of the

lining. The basic process is a dephosphorizing one, and is the one most generally available, as it can use pig irons that are either low or high in phosphorus.

Electric-furnace Steel. Instead of using the gas flame of an open-hearth furnace the heat of an electric arc may be used to heat a charge of steel, which can be kept out of contact with air under a protecting blanket of molten slag. The purifying ingredients, iron oxide, manganese dioxide, etc., are in the slag, and the impurities are absorbed by it. Electric heat is very uneconomical for producing low temperatures, but much more economical for producing high temperatures, such as are used in the final refining stages of steel making. The electric furnace is sometimes used for the final stages of the steel making process, the earlier stages being carried out in an open hearth furnace or a Bessemer converter.

The Physical Properties of steel depend upon both method of manufacture and chemical composition, the carbon having the controlling influence upon strength. Phosphorus increases strength, but it promotes brittleness; manganese increases strength in a less degree, and it promotes malleability; sulfur causes red-shortness or a tendency of the steel to crumble while being rolled; and silicon increases hardness. Acid steel is slightly stronger than basic steel having the same percentage of carbon, owing partly to the higher percentage of phosphorus and partly to the effect of the lime in forming slag in the basic steel. Since 1900 more than three-fourths of the open-hearth steel produced in the United States has been basic; the Bessemer product on the other hand being entirely acid.

The products described above are sometimes called carbon steel, because carbon is the controlling element in regard to strength. When the strength is largely governed by other elements the steels are said to be "special." Of these nickel steel (Art. 34) is the most important as a structural material, while chrome, vanadium, and tungsten steels (Art. 35) are used mainly for machinery and tools.

The Methods of Manufacture greatly influence the strength of steel. Forged steel is stronger and more reliable than cast steel. Heat treatment by annealing and tempering has also a marked influence. In fact, each manufacturer has special processes which are claimed to produce superior material.

Forging and Drawing greatly increase the strength of steel. Forging under a hammer or press renders the material more compact and increases both specific gravity and strength. The process of drawing steel bars into wire has a similar result, and wire has been made having a tensile strength of 250 000 lb per sq in, while the wire used for the cables of suspension bridges usually has a tensile strength of from 150 000 to 200 000 lb per sq in. By compressing steel while it is fluid, the strength may also be much increased, and this process is used for the steel from which large guns and hollow shafts are made.

Annealing consists in raising cold steel to a light red heat and then allowing it to cool for several days. This process reduces the ultimate strength, but it increases the ductility and also the capacity to resist shock. Tempering consists in plunging heated steel into a bath of water or oil, or by applying these fluids to its surface. The hardness of the steel and its ultimate strength are thereby much increased. Armor plate undergoes special processes of tempering or carbonization which render it excessively hard and tough.

Steel castings are extensively used for axle boxes, crossheads, and machine frames. They range in tensile strength from 60 000 to 90 000 lb per sq in and have an elastic limit of somewhat less than half the ultimate strength. Altho less reliable than steel forgings, they give excellent service after having been annealed so as to increase their ductility and their capacity to withstand shock and work.

Carbon is the controlling element in regard to strength, and the same is the case with respect to ultimate elongation. The higher the percentage of carbon, within a reasonable limit, the greater is the strength and the less the ultimate elongation. The product of strength and elongation is approximately

constant, and hence the ultimate elongation is approximately inversely proportional to the tensile strength. A rule frequently given is that the percentage of elongation equals 1 500 000/S_t; thus, for a tensile strength of 80 000 lb per sq in the ultimate elongation is about 19 percent. This rule, however, gives too high elongations for very strong steel.

General Classes. According to the percentage of carbon and its capacity for taking temper or being welded, steel may be classified as follows:

Soft, 0.05–0.20 C,	not temperable, easily welded
Medium, 0.15–0.40 C,	poor temper, weldable
Hard, 0.30–0.70 C,	temperable, welded with difficulty
Very hard, 0.60–1.00 C,	high temper, not weldable

Average Properties for carbon steel in lb per sq in are as follows:

	Soft	Medium	Hard	Very hard
Elastic limit in tension and compression	30 000	35 000	50 000	70 000
Ultimate tensile strength	50 000	60 000	90 000	120 000
Elastic limit in shear	17 000	20 000	28 000	40 000
Ultimate shearing strength	40 000	50 000	70 000	90 000
Modulus of elasticity, tension-compression	30 000 000 for all grades			
Modulus of elasticity, shear	12 000 000 for all grades			

The specific gravity is about 7.8, and the weight per cubic foot 490 or 491 lb. The coefficient of expansion is about 0.0000065 per Fahrenheit degree. Ultimate elongation ranges from 5 to 30 percent, the higher the amount of carbon the less the elongation. Reduction of area follows the same rule, ranging from 10 to 60 percent; see page 381.

Uses. Steel is frequently classified with reference to its uses, and the following is such a classification giving average elastic limits and ultimate tensile strengths in pounds per square inch and ultimate elongations in percentages. For the elastic limit a variation of about 2000 and for the ultimate strength a variation of 4000 or 5000 lb per sq in from these mean values may be expected. The ultimate elongations are subject to marked variation according to the ratio of the length of the test specimen to its diameter; those here given are for the standard 8-in specimen.

	Elastic limit	Tensile strength	Percent elongation in 8 in
Structural steel for rivets	30 000	55 000	30
Structural steel for beams and shapes	35 000	60 000	27
Boiler steel for rivets	25 000	50 000	30
Boiler steel for plates	30 000	60 000	26
Machinery steel	40 000	75 000	20
Gun steel	50 000	90 000	18
Axle steel	55 000	100 000	15
Spring steel	80 000	125 000	12
Cable wire steel	150 000	200 000	8

Formulas for Strength. Many formulas have been deduced to exhibit the relation between the tensile strength and the chemical composition, but none of these applies to all classes of steel. The rough rules,

$$S_t = 45\,000 + 108\,000\,C \qquad S_t = 45\,000 + 90\,000\,C$$

give an approximate idea of the influence of carbon in acid and basic unannealed open-hearth steel respectively, C being the percentage of carbon and S_t the tensile strength in pounds per square inch. Thus, acid steel with 0.40 percent of carbon has a tensile strength of about 88 000 lb per sq in, while basic steel has about 81 000 lb per sq in. When the percentages of phosphorus and manganese are also known, the following formulas, deduced from the exhaustive discussion of H. H. Campbell in 1905, may be used to give more reliable results,

$$S_t = 40\,000 + 68\,000\,C + 100\,000\,P + 80\,000\,CM$$
$$S_t = 38\,800 + 65\,000\,C + 100\,000\,P + 9\,000\,M + 40\,000\,CM$$

the first being for acid and the second for basic open-hearth steel. Here C is the percentage of carbon, P that of phosphorus, M that of manganese, and S_t the tensile strength in pounds per square inch. For example, acid steel having 0.344% of carbon, 0.045% of phosphorus, and 0.70% of manganese has a tensile strength of 87 200 lb per sq in; basic steel having 0.344% of carbon, 0.020% of phosphorus, and 0.35% of manganese has a tensile strength of 71 100 lb per sq in. These formulas do not apply to steel with a percentage of carbon higher than 0.75.

Strength of Various Kinds of Carbon Steel

W. L. Turner, Iron Age, July 2, 1908

Heat treatment	Kind	Carbon, %	Manganese, %	Elastic limit, lb per sq in	Tensile strength, lb per sq in	Elongation in 2 inches, %	Red. of area, %
R	Wrought iron	0.05	0.05	32 020	49 450	42.5	53.8
R	Old plate	0.24	0.42	58 160	70 840	25.5	53.4
A	Mild steel	0.18	0.40	39 460	60 650	35.0	62.6
OT	Mild steel	0.18	0.40	49 390	70 036	33.0	64.0
A	Forging steel	0.26	0.28	43 010	61 850	34.0	63.4
OT	Forging steel	0.26	0.28	52 230	77 310	28.0	65.3
A	Steel casting	0.18	0.65	34 690	58 800	28.0	44.9
A	Spring steel	1.00	0.30	63 800	125 000	8.5	15.2
OT	Spring steel	1.00	0.30	101 000	186 300	9.5	16.1

R = raw, A = annealed, OT = oil tempered.

32. Structural Steel

Specifications. The following clauses give essential features of the specifications for structural steel which have been adopted by Amer. Soc. for Testing Materials and Amer. Ry Eng. and Maint. Way Association:

1. Structural steel shall be made by the open-hearth process. (Structural steel for buildings may be made by the Bessemer process.)

2. Each of the three classes of structural steel shall conform to the following limits in chemical composition: Sulfur shall not exceed 0.05 percent; phosphorus shall not exceed 0.06 percent when the steel is made by the acid process and not exceed 0.04 percent when it is made by the basic process.

3. There shall be two classes of structural steel for bridges, buildings, locomotives, cars, and ships, namely, rivet steel and structural steel which shall conform to the following physical requirements: Rivet steel shall range in tensile strength from 46 000 to 60 000 lb per sq in, have a yield point not less than 0.5 of the tensile strength, and the percentage of elongation in 8 in shall be $\frac{1\,500\,000}{\text{tensile strength}}$. Soft steel shall range in tensile strength from 55 000 to 68 000 lb per sq in, have a yield point not less than 0.5 of the tensile strength, and the elongation in 8 in shall not be less than 25 percent.

4. For each increase of 1/8 in in a flat specimen above a thickness of 3/4 in, a deduction of 1 shall be made from the above specified percentage of elongation. For each decrease of 1/16 in below a thickness of 5/16 in, a deduction of 2½ shall be made from the above-specified percentage of elongation. For bridge pins the required percentage of elongation shall be 5 less than that above specified, as determined on a test specimen the center of which shall be 1 in from the surface of the pin.

5. Full-sized tests of eyebars shall show 15 percent elongation in 10 ft of the body of the eyebar, and the tensile strength shall not be less than 55 000 lb per sq in. Eyebars shall generally break in the body, but should an eyebar break in the head, and show 15 percent elongation in 10 ft and the tensile strength specified, it shall not be cause for rejection, provided that not more than one-third of the total number of eyebars tested break in the head.

6. Cold bend tests are summarized on p. 388.

7. The standard specimens are shown in Figs. 53 and 54. The flat specimen is used for plates and shapes, and the 2-in round specimen for pins, rollers, and bars. Rivet rounds and small bars are tested full size.

8. One tensile test specimen shall be taken from the finished material of each melt or blow, but in case this develops flaws, or breaks outside of the middle-third of its gage length, it may be discarded and another test specimen substituted therefor.

9. Material which is to be used without annealing or further treatment shall be tested for tensile strength in the condition in which it comes from the rolls. Where it is impracticable to secure a test specimen from material which has been annealed or otherwise treated, a full-size section of tensile-test specimen length shall be similarly treated before cutting the tensile-test specimen therefrom.

10. For the purpose of this specification, the yield point shall be determined by careful observation of the drop of the beam or halt in the gage of the testing machine.

11. In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of drillings taken from a small test ingot.

12. The variation in cross-section or weight of more than 2½ percent from that specified will be sufficient cause for rejection, except in the case of sheared plates, which will be covered by special allowances tabulated in the 1916 "Standards" of the Am. Soc. for Test. Materials.

When plates are ordered to gage, a variation of more than 1/100 in below that specified for any dimension will be sufficient cause for rejection.

14. Finished material must be free from injurious seams, flaws, defective edges, or cracks, and have a workmanlike finish.

15. Every finished piece of steel shall be stamped with the melt number and name of the manufacturer and steel for pins shall have a melt number stamped on the ends. Rivets and lacing steel, and small pieces for pin plates and stiffeners, may be shipped in bundles, securely wired together, with the melt number on a metal tag attached.

16. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

Structural Nickel Steel. The following is a summary of the principal items in the specifications for structural nickel steel as given in the 1916 "Standards of the Am. Soc. for Test. Materials:

1. The steel shall be made by the open-hearth process.

2. The phosphorus content shall not exceed: for acid structural steel, 0.05 percent; for basic structural steel, 0.04 percent; for acid rivet steel, 0.04 percent; for basic rivet steel, 0.03 percent. The sulfur content shall not exceed: for structural steel, 0.05 per

cent; for rivet steel, 0.45 percent. The nickel content shall not be less than 3.25 percent.

3. The following table summarizes the requirements as to tensile strength and ductility.

Properties considered	Rivet Steel	Plates, shapes and bars	Eyebars and rollers unannealed	Eyebars and rollers annealed
Tensile strength lb per sq in........	70 000–80 000	85 000–100 000	95 000–110 000	90 000–105 000
Yield point, min lb per sq in........	45 000	50 000	55 000	52 000
Elongation, in 8 in, percent.........	1 500 000 / tensile str.	1 500 000 / tensile str.	1 500 000 / tensile str.	20*
Elongation, in 2 in, percent.........	....	....	16	20†
Reduction of area, percent..........	40	25	25	35

* Flat test specimens from plates and shapes.
† Round test specimens from pins and rollers.

4. For plates, shapes, and unannealed bars over 1 in in thickness a deduction of 1 from the percentage of elongation specified in the above table shall be made for each increase of ⅛ in thickness above 1 in, to a minimum of 14 percent.

5. The test specimens are similar in size and form to those used for structural carbon steel.

6. The cold bend tests required are summarized on p. 388

Soft and Medium Steels resemble wrought iron in having a yield point which is from 2000 to 4000 lb per sq in above the elastic limit, while very hard steels have no yield point. The elastic limit in tension is a little higher than one-half of the ultimate strength. The modulus of elasticity is subject to little variation with the percentage of carbon, and the mean value of 30 000 000 lb per sq in may be used in computations for both tensile and compressive stresses that do not exceed the elastic limit. The modulus of elasticity for shearing is about two-fifths of that for tension and compression. Soft and medium steel will withstand a cold bend test similar to that mentioned in the last article, but some of the hard steels will fail to do so on account of their lack of ductility. The soft steels resemble wrought iron in being plastic under compressive stress exceeding the elastic limit, and some authorities regard the yield point as the compressive strength.

Steel Castings should not contain over 0.06 percent of phosphorus, or over 0.05 percent of sulfur. The minimum physical qualities prescribed by Amer. Soc. for Testing Materials are

	Soft castings	Medium castings	Hard castings
Tensile strength, lb per sq in.......	60 000	70 000	85 000
Yield point, lb per sq in...........	0.45 of the tensile strength		
Elongation, percent in 2 in.........	22	18	15
Contraction of area, percent........	30	25	20

Rivets. The ultimate strength of steel rivets as determined by various British authorities from experiments on riveted joints, assuming that the distribution of load was uniform on all rivets and that the friction of the plates is negligible, varied for iron from 42 582 to 62 362 lb per sq in, and for steel from 49 683 to 80 035 lb per sq in. Tests made at the University of Illinois in 1898 gave average bearing strengths of 36 950 for boiler rivet steel, 40 950 for structural rivet steel, and 37 950 lb per sq in for wrought iron. These values were, respectively, 76.4, 74.5, and 77.6 percent of the tensile strengths of

the same material. The tests were made on ½, ¾, and 1 in rivets in single and double shear. More recent tests (1905) made by the Am. Ry. Eng. and Maint. of Way Assn. on riveted steel joints gave for the rivets an average shearing strength of 48 580 lb per sq in.

Working Unit Stresses for members of roof and bridge trusses are given in Sec. 8. As a rough general rule, when specifications are not made, the working unit stresses for structural steel may be taken as 20 percent higher than those for wrought iron (Art. 30).

Alternating stresses require smaller working stresses than those used for steady loads. Following formulas were much used from 1880 to 1900 to give the unit stress S which would rupture a bar after an enormous number of alterations (40 000 000 or more):

Launhardt's formula, $$S = S_e\left(1 + \frac{S_u - S_e}{S_e}\frac{\min}{\max}\right)$$

Weyrauch's formula, $$S = S_e\left(1 - \frac{1}{2}\frac{\min}{\max}\right)$$

in which S_e is the unit stress at the elastic limit, S_u is the ultimate strength. In Launhardt's formula the bar under one kind of load only, either always in tension or always in compression, and min/max is the ratio of least of these loads to the greatest. In Weyrauch's formula the bar is under stress which ranges from tension into compression, or from compression into tension, and min/max is the same ratio without regard to sign. A further discussion of failure of metals under repeated stress is given on p. 330. For most members of roof and bridge trusses static strength is more important than strength to withstand repeated stress. For elevated railway structures and very busy bridges strength to withstand repeated stress may be the critical property of the material. Endurance under repeated stress is a prime requisite of material for machine parts, which frequently have to withstand hundreds of millions of repetitions of stress.

For working unit stresses the formulas of Launhardt and Weyrauch are usually divided by 3. Example: Let $S_e = 33\,000$ and $S_u = 66\,000$ lb per sq in; then for a bar whose load ranges from 30 000 to 120 000 lb in stress of one kind, the working unit stress is $S = 11\,000(1 + 1.0 \times 0.25) = 13\,700$ lb per sq in, while when the load ranges from tension to compression the working unit stresses is $S = 11\,000(1 - 0.5 \times 0.25) = 10\,600$ lb per sq in.

33. Nomenclature for Steel.

Steel Nomenclature as given by a committee at the International Congress held in 1909 (Proc. Inter. Assoc. Testing Materials, July, 1909), is shown in the table at top of page 409.

Definitions for Iron and Steel were also recommended by the same committee (H. M. Howe, chairman) as follows:

Alloy Cast Irons, those which owe their properties chiefly to the presence of an element (or elements) other than carbon.

Alloy Steels, those which owe their properties chiefly to the presence of an element (or elements) other than carbon.

Basic Pig Iron. In America, pig iron containing so little silicon and sulfur that it is suited for easy conversion into steel by the basic open-hearth process. It is restricted to pig iron containing not more than 1.90 percent of silicon. In England and on the Continent, pig iron containing so little silicon and sulfur that it can be converted into steel easily by the basic Bessemer or basic open-hearth process. It generally contains 1.00 percent or more of manganese and 1.50 to 3.00 percent of phosphorus, silicon averaging

English	French	Spanish	German
1. Soft or low-carbon steel, or ingot iron*	Acier doux, acier extra doux, fer fondu	Hierro fundido	Flusseisen ‡
2. Half-hard and hard or medium and high carbon steel, or ingot steel*	Acier fondu, acier mi-dur, acier dur	Acero fundido	Flussstahl ‡
Bessemer steel	Acier Bessemer	Acero Bessemer	Bessemer-Flusseisen Bessemer-Flussstahl
Open-hearth steel	Acier Martin Siemens, acier sur sole	Acero de Solera	Flammofen-Flusseisen Flammofen-Flussstahl
Crucible steel	Acier au creuset	Acero de crisoles	Tiegelflusseisen Tiegelflussstahl
Cast steel	Acier au creuset	Acero fundido	Gussstahl
Steel castings	Moulages d'acier	Piezas de acero colado	Flusswaren
Weld steel, or wrought steel †	Fer fort ou fer dur	Acero Soldado	Schweissstahl or Schweisseisen ‡
Blister steel, also called cemented and converted steel	Acier poule, acier cémenté, acier de cémentation	Acero cementado	Zementstahl
Shear steel	Acier rafiné une fois corroyé		Schweissstahl
Puddled steel	Acier puddlé	Acero Pudelado	Puddelstahl
Alloy steels	Alliages à base de fer, aciers spéciaux	Aleaciones de Acero	Sonderstahl

* These are cast initially into a malleable mass.

† This is capable of being greatly hardened by sudden cooling.

‡ According to Wedding cast metal having a tensile strength greater than 71 000 lb per sq in should be called Flussstahl, while one with a smaller tenacity should be called Flusseisen. Weld metal with a tensile strength exceeding 60 000 lb per sq in should be called Schweissstahl and one with a less tenacity Schweisseisen.

not more than 1.00 percent and sulfur 0.10 percent. Other varieties treated by the basic Bessemer or basic open-hearth process are not regarded as basic pig, but simply as phosphoric pig.

Bessemer Pig Iron, which contains so little phosphorus and sulfur that it can be used by itself for conversion into steel by the original or acid Bessemer process. In America this term is restricted to pig iron containing not more than 0.10 percent of phosphorus. In England this term is restricted to pig iron containing not more than 0.06 percent of phosphorus or sulfur.

Blown Metal, the red short metal made by purifying pig iron in the Bessemer converter without subsequently removing the oxygen which it absorbs during that purification.

Bessemer Steel, steel made by the Bessemer process, whether its carbon content is high, low, or intermediate.

Blister Steel, steel made by carburizing wrought iron by heating it in contact with carbonaceous matter. It might also be made by so carburizing a low-carbon steel. Much of the blister steel of commerce is made by cementing Swedish wrought iron in charcoal.

Cast Iron. Generally, iron containing so much carbon or its equivalent that it is not usefully malleable at any temperature. Specifically, cast iron in the form of casting other than pigs, or remelted cast iron suitable for casting into such castings, as distinguished from pig iron, that is, cast iron in pigs, etc. (See Pig Iron.) For instance, cast iron pigs, or pig iron, like lead in pigs, or pig lead, is remelted and cast into castings, such as columns, locks, gears, etc., of special shape suited to their special purpose; these are specifically

called "cast iron," and this is the usual restricted meaning of "cast iron" in trade language.

Cast Steel in the iron trade means "crucible steel." Obsolescent and undesirable because it might easily be understood to include other steels which have been cast.

Cemented Steel, the same as blister steel.

Charcoal Hearth Cast Iron, cast iron which has had its silicon and usually its phosphorus removed in the charcoal hearth, but still contains so much carbon as to be distinctly cast iron.

Converted Steel, the same as blister steel.

Crucible Steel, steel made by the crucible process, whether its carbon content is high, low, or intermediate.

Gray Pig Iron and Gray Cast Iron, pig iron and cast iron in the fracture of which the iron itself is nearly or quite concealed by graphite, so that the fracture has the gray color of graphite.

Hematite Pig Iron, originally pig iron made from the hematite ores of England, which happen to be so free from phosphorus and sulfur that the pig iron made from them can be used by itself for the acid Bessemer process. By association it has come to mean any pig iron thus relatively free from phosphorus and sulfur. The term is not used in America, and is undesirable.

Hot Metal or Direct Metal, the molten cast iron from the blast furnace before it has been allowed to solidify. The term is generally applied to molten metal taken direct from the blast furnaces to the steel-making plant.

Ingot Iron, steel cast into an initially malleable mass and containing so little carbon or its equivalent that it does not harden greatly when cooled suddenly and completely from a red heat. The word is rarely used in English, but "mild steel" or "low-carbon steel" or "soft steel" is generally used in its place. In America the line between soft steel and half-hard steel is usually drawn at a carbon content of about 0.20 percent.

Ingot Steel, steel cast into an initially malleable mass and containing so much carbon or its equivalent that it hardens greatly on sudden cooling. The word is rarely used in English, but "hard steel," "high-carbon steel," or "half-hard steel" are used in its place.

Malleable Castings, castings of malleable cast iron, which see.

Malleable Cast Iron, iron which when first made is cast in the condition of cast iron, and is made malleable by subsequent treatment without fusion. Altho the English name of this variety suggests that it is cast iron, it is not truly a variety of cast iron, but rather forms an independent species of iron, because it lacks the essential property of cast iron, namely, its extreme brittleness. Tho the term "malleable castings" is very common, the term "malleable cast iron" is very rarely used. The common but inexcusable term we regret to say is "malleable," pronounced "mallable," used as a substantive. Those with some respect for their mother tongue, if asked of what material a malleable casting was composed, would generally use a circumlocution.

Malleable Iron, the same as wrought iron. Used in Great Britain, but not in the United States, except carelessly as meaning "malleable cast iron" (vulgar "malleable").

Malleable Pig Iron, an American trade name for the pig iron suitable for converting into malleable castings thru the process of melting, treating when molten, casting in a brittle state, and then making malleable without remelting. The term should be used with care to avoid confusion. This material is also called in trade in America "malleable iron" but this use should be avoided, because "malleable iron" has the older and (in Great Britain) firmly established meaning of "wrought iron."

Mottled Pig Iron and Mottled Cast Iron, pig iron and cast iron the structure of which is mottled, with white parts in which no graphite is seen, and gray parts in which graphite is seen.

Open-Hearth Steel, steel made by the open-hearth process, whether its carbon content is high, low, or intermediate.

Pig Iron, cast iron which has been cast into pigs direct from the blast furnace. This name is also applied loosely to molten cast iron which is about to be so cast into pigs or is in a condition in which it could readily be cast into pigs.

Plate Iron, a name sometimes applied in Great Britain to refined cast iron.

Puddled Iron, wrought iron made by the puddling process.

Puddled Steel, steel made by the puddling process, and necessarily slag-bearing (see Weld Steel). It differs from wrought iron only in being richer in carbon. It differs from most other steels in containing much cinder.

Refined Cast Iron, cast iron which has had most of its silicon removed in the refinery furnace, but still contains so much carbon as to be distinctly cast iron.

Shear Steel, steel usually in the form of bars, made from blister steel by shearing it into short lengths, piling and welding these by rolling or hammering them at a welding heat. If this process of shearing, piling, etc., is repeated, the product is called "double shear steel."

Steel, iron which is usefully malleable at least in some one range of temperature, and in addition is either (a) cast into an initially malleable mass; or (b) is capable of hardening greatly by sudden cooling; or (c) is both so cast and so capable of hardening. Variety (a) includes also molten iron which if cast would be malleable, as do its two sub-varieties, "ingot iron" and "ingot steel." (Tungsten steel is malleable only when red-hot.)

Steel Cast (adjective) consisting of solid Bessemer, open-hearth, crucible or other slagless steel, and neither forged nor rolled; applied to steel castings. For instance, a "steel cast" gun is a gun which is a steel casting, that is, which has been neither forged nor rolled. To call it a "cast steel" gun would imply that it was made of crucible steel, to which the term "cast steel" is restricted.

Steel Castings, unforged and unrolled castings made of Bessemer, open-hearth, crucible, or any other steel. Ingots and pigs are in a sense castings; the term "steel castings" is used in a more restricted sense, excluding ingots and pigs and including only specially shaped castings, such as are generally used without forging or rolling. They may, however, later be forged, for example under the drop press, when they cease to be "castings" and become "drop forgings," or if only part is forged then they are partly forgings and partly castings.

Washt Metal, cast iron from which most of the silicon and phosphorus have been removed by the rich ferruginous slags of the Bell-Krupp process or its equivalent without removing much of the carbon, so that it still contains enough carbon to be classed as cast iron. The name "washt metal" is extended to cover this product even if its carbon is somewhat below the proper limit for cast iron.

Weld Iron, the same as wrought iron. Obsolescent and needless.

Weld Steel, iron containing sufficient carbon to be capable of hardening greatly by sudden cooling, and in addition slag-bearing because made by welding together pasty particles of metal in a bath of slag, as in puddling, and not later freed from that slag by melting. The term is rarely used.

White Pig Iron and White Cast Iron, pig iron and cast iron in the fracture of which little or no graphite is visible, so that their fracture is silvery and white.

Wrought Iron, slag-bearing, malleable iron, which does not harden materially when suddenly cooled.

Wrought Steel, the same as weld steel. Rarely used.

34. Nickel Steel

Nickel is used as an alloy for the steel of guns and armor plates and to a lesser extent for structural purposes and for railroad rails. It contains about 3¼ percent of nickel, and has an elastic limit of about 48 000 and a tensile strength of about 90 000 pounds per square inch. Nickel steel has been made with an elastic limit of 120 000 and a tensile strength of 277 000 pounds per square inch, the ultimate elongation being about 3 percent. Structural nickel steel has an elastic limit about 15 per cent higher and an ultimate strength about 25 per cent higher than common structural steel. Specifications for structural nickel steel are summarized on p. 406.

Nickel Steel Rivets were tested in 1910 by the Board of Engineers of the new Quebec bridge, giving an average shearing strength of 36 560 lb per sq in, an excess over the strength of carbon steel of 16.4 percent. Both rivets and plates in the tests were of nickel steel. The chemical analysis of the rivet steel was: carbon 0.126, phosphorus 0.010, manganese 0.410, sulfur 0.022, and nickel 3.290 percent. Tensile tests of the material showed an ultimate strength of 68 450 lb per sq in.

Nickel steel has been used for parts of the Queensboro bridge in New York and of the new bridge over the Mississippi river at St. Louis. Nickel steel costs about 1.5 cents

per pound more than plain carbon steel, when the price of nickel is 30 cents per pound. Owing to its greater strength it is possible to build very long span bridges cheaper when nickel steel is used.

For working unit stresses for nickel steel for bridges J. A. L. Waddell recommends the following: See Trans. A.S.C.E., Vol. 63 (1909): tension in eyebars, 30 000 lb per sq in; stress in plates and shapes, 28 000 lb per sq in; bending stress in pins, 50 000 lb per sq in; bearing stress on rivets, 30 000 lb per sq in; shear on pins, 25 000 lb per sq in; shear on rivets, 14 000 lb per sq in.

Coefficients of expansion of nickel steel decrease after 25 percent nickel is reached. The following mean values are for temperatures from 15° to 100° C.:

Percent nickel	Coefficient	Percent nickel	Coefficient
27.0	0.0000110	34.0	0.0000025
30.0	0.0000055	36.0	0.0000015
32.0	0.0000035	38.0	0.0000004

The alloy with 36.0 percent nickel content is known as "invar" steel, and is used for measuring tapes and scales, see p. 417.

35. Special Steels

Chrome Steel was used for structural purposes in the bridge erected by J. B. Eads at St. Louis in 1871, it being carbon steel with about 0.5 percent of chromium. Since 1890 nickel (Ni) and vanadium (Va) have been combined with chromium (Cr), carbon (C) and manganese (Mn) being also present.

Chromium-Nickel Steel having 0.36 per cent C, 0.43 percent Mn, 0.95 percent Cr, and 1.70 percent Ni, had an elastic limit of 56 500 and an ultimate strength of 81 400 lb per sq in when annealed, according to W. L. Turner; when tempered, however, its elastic limit rose to 134 500 and its ultimate tensile strength to 150 300, while the ultimate elongation dropt from 28.0 to 15.5 percent. (Iron Age, July 2, 1908). Steels of this class are used in automobile construction, for gun barrels, machine parts, gears, axles and shafts. Mayari steel is made from a chrome-nickel ore found in Cuba; its tensile strength is from 85 000 to 100 000 lb per sq in; it is used in the new bridge over the Mississippi river at Memphis.

Chromium-Vanadium Steel for springs may have a very high strength especially when tempered. An annealed specimen of spring steel tested by W. L. Turner, having 0.40 percent C, 0.77 percent Mn, 1.22 percent Cr, and 0.19 percent Va, had elastic limit 67 500 and ultimate strength 100 000 lb per sq in; after being oil tempered these rose to 195 000 and 208 500 lb per sq in. An annealed specimen of forging steel, having 0.26 percent C, 0.50 percent Mn, 1.00 percent Cr, and 0.16 percent Va had elastic limit 61 900 and ultimate strength 92 890 lb per sq in, but with oil tempering these rose to 141 600 and 151-700 lb per sq in. This steel is used for locomotive forgings, for shafts, piston rods, plates, tubes, tires, and even for wire.

Chromium-Nickel-Vanadium Steel is made by the crucible process. Two specimens tested by Turner had 0.30 per cent C, 0.27 percent Mn, 1.51 percent Cr, 3.45 percent Ni, and 0.08 percent Va. The annealed specimen gave elastic limit 69 100, ultimate strength 96 900 lb per sq in, and elongation of 28.5 percent, while the oil-tempered specimen gave elastic limit 152 300, ultimate strength 159 900 lb per sq in, and elongation of 17.0 percent.

Copper Steels are those having from 1 to 4 percent of copper, the carbon being less than 1 percent. They are said to equal nickel steel in tensile strength and to be no more brittle than nickel steels with the same proportions of nickel. A small copper content, about 1 percent, is claimed to render steel resistant to corrosion.

Manganese Steel has more than 3 percent of manganese, usually from 6 to 12 percent. It is very tough and hard, and has a high degree of resistance to wear, so that it is used for railway frogs and switches, rolls of crushers, and the teeth on steam-shovel dippers.

Tungsten Steel is used for tools for cutting metal and some kinds will cut when red hot. F. W. Taylor (Trans. Amer. Soc. Mech. Eng., Vol. 28) gives the following as the proportions which lead to greatest efficiency of tools: 18.19 percent tungsten, 5.47 percent chromium, 0.29 percent vanadium, 0.67 percent carbon, 0.11 percent manganese, 0.04 percent silicon.

Silicon Steel is a structural steel with high percentages of silicon, manganese, and carbon. Silicon steel tested by Griffith and Bragg contained: C, 0.35%; Mn, 0.83%; Si, 0.38%. In the design of the Metropolis bridge over the Ohio river the following stresses were allowed in silicon steel members: tension, 25 000 lb per sq in; tension in eyebars, 35 000 lb per sq in; compression, 30 000 lb per sq in. (See Engineering News-record for Dec. 20, 1917, p. 1141. U. S. Bureau of Standards, Tech. Paper 101 p. 22.)

36. Minor Metals

Aluminum is a white, malleable, and very light metal, its specific gravity being 2.75 when rolled and 2.55 when cast. It is almost non-corrodible, since even sulfuric acid has little effect upon it. Pure cast aluminum has a tensile strength of about 18 000 lb per sq in. When rolled into plates or drawn into wire this is raised to 30 000 or 40 000 lb per sq in with an elastic limit one-half as great. On account of the softness of the pure metal it is usually alloyed with copper, iron, tin, or zinc. (Art. 37.)

Common impurities in aluminum are silicon and iron. It can, however, be obtained 99 or 99.5 percent pure. In making castings the great shrinkage must be taken into account. The melting point of pure aluminum is 1150° F.

The Thermit Process used for welding iron and steel depends upon the affinity which powdered aluminum and iron oxide have for each other. These substances being finely mixt, and then ignited, the temperature suddenly rises to about 5400° and white-hot fused iron results which will melt any ordinary casting or forging. The process is mainly used for welding breaks in large pieces such as those in a locomotive frame. The chemical reaction which takes place is exprest by the equation $2Al+Fe_2O_3=Al_2O_3+2Fe$.

Copper is a reddish, malleable and ductile metal which can be drawn or rolled and also be cast; its specific gravity varies from 8.9 in the first form to 8.6 in the second. It unites with oxygen at a red heat and melts at about 1900° F. It does not corrode in dry air. Its most important use is for electric conductors.

Copper wire has a tensile strength of about 50 000 lb per sq in and the high elastic limit of over 40 000 lb per sq in owing to the stiffening due to drawing; by annealing it may be softened and both ultimate strength and elastic limit be thus lowered. Copper plates ¼ to ¾ in thick have tensile strength of 32 000 lb per sq in. Cast copper has a tensile strength of about 25 000 lb per sq in and an elastic limit of about one-third of this value. The strength of rods decreases as the temperature rises above 100°, the loss in strength being copper 16 percent at 500° F.

Small cylinders of copper are used to measure the high pressures developed by the explosion of powder in guns, the same being inferred from the shortenings which the cylinders undergo. In this way pressures as high as 30 000 lb per sq in have been noted.

Copper is alloyed with zinc, tin, and other metals to produce many useful alloys which are used in the arts and in engineering. (See Art. 37.)

Tin is a white, malleable metal of specific gravity 7.3. Commercial tin contains as impurities copper, iron, bismuth, and other metals. It melts at 450° F. and hence is often used for safety plugs in steam boilers. Its main commercial use is as a coating in the manufacture of the tin plates which are used for roofing, for household utensils, and for cans.

Roofing tin is thin sheet steel coated with an alloy of about 25 percent tin and 75 percent lead. A box of 112 sheets 14 by 20 in in size will cover approximately 192 sq ft of roof with the flat-seam method of laying. For the standing-seam method a box of 112 sheets 20 by 28 in in size will cover closely 370 sq ft.

Lead has a low strength and is almost devoid of elasticity, but is very plastic, so that it flows readily under stress. The weights per square foot of sheet lead ordinarily rolled are 2½, 3, 3½, 4, 4½, 5, 6, 8, 9, 10 lb and upward. Small lead pipes are often lined with tin in order to prevent the lead from dissolving in the water; the common commercial sizes of these are ¼ and ½ in. The use of lead pipes in bath rooms should usually be confined to the waste pipes. Lead melts at 625° F.

Zinc, called spelter when cast, has a tensile strength of about 9000 lb per sq in and about 24 000 lb per sq in rolled into thin plates. It is mainly used in coating iron and steel surfaces (galvanizing) and for making brass and other alloys. Its melting point is 780° F.

Nickel is a ductile, hard, and tough metal. It is mainly used for plating and in alloys. Its melting point is about 3000°, so that it is very difficult to fuse. Not corrodible from the atmosphere. (See Art. 34.)

Mercury is a silver-white metal which is liquid at common temperatures. It boils at 680° F. and freezes at − 38° F. When liquid its specific gravity ranges from 13.58 to 13.59. Its coefficient of cubical expansion for temperatures from 32° F. and 212° F. is 0.000101 per degree.

For other metals and for other properties of those above briefly described see Sec. 13, Art. 7. For atomic weights see Sec. 13, Art. 1.

37. Alloys

An alloy is a mixture of two or more metals which is made by combining them when in a molten condition. In a strict sense common steel is an alloy of iron and carbon, nickel steel is an alloy of iron and nickel, and all the special steels used for tools are also alloys, but usually the word is applied only to mixtures of copper, tin, zinc, and lead. The mixture is usually only mechanical, altho some slight chemical union may take place in special cases. It is impossible to predict the properties of an alloy from the properties and proportions of its constituent metals.

Brasses are alloys of copper and zinc, the most valuable of which have from 65 to 80 per cent of copper and from 35 to 20 percent of zinc. Sometimes a small percentage of tin is added, especially when it is to be turned or planed. Brass can be cast or rolled. It is harder and more ductile than copper. Tensile strength of castings is about 20 000 lb per sq in. Strength and hardness increase with the proportion of zinc. The mean specific gravity of cast brass is 8.95.

Delta metal is brass with a small amount of iron. Its strength and ductility when rolled are equal to those of medium steel; when cast, its tensile strength is about 45 000 lb per sq in. Its resistance to corrosion is high.

Tobin bronze is an alloy of copper, tin, and zinc. It is of high tensile strength, is very non-corrodible, and can be obtained either in castings or in rolled sheets.

Strength of Copper-Zinc Alloys (Brasses)

Based on the report of the U. S. Board to Test Iron, Steel, and Other Metals, the report of Roberts Austen to the British Alloys Research Committee, and tests by J. M. Lohr.

Composition, percent		Ultimate strength, lb per sq in		
		Tension		Compression cast
Copper	Zinc	Cast	Rolled	
0	100	9 000	24 000	5 000
20	80	9 000	10 000	55 000
40	60	7 000	7 000	......
50	50	30 000	65 000	115 000
55	45	47 000	77 000	90 000
60	40	48 000	61 000	75 000
65	35	38 000	53 000	65 000
70	30	37 000	50 000	55 000
75	25	35 000	51 000	45 000
80	20	33 000	50 000	39 000
85	15	32 000	45 000	37 000
90	10	31 000	41 000	30 000
100	0	25 000	40 000	40 000

Copper-Tin Alloys are called bronzes. The tin is added to harden the copper, and the alloy is denser, harder, and more fusible than copper. Bronze for making statues has about 87 percent copper, 7 percent tin, 3 percent lead, and 3 percent zinc. Bronze for medals has only 2 percent tin. Bronze is used to a slight extent for telegraph and telephone wires. Bronzes containing more than 24 percent of tin have insufficient strength for practical uses.

Abstract of Tests of Copper-Tin Alloys made by Thurston and Kent

From U. S. Tests of Metals, 1879 and 1882

Composition, percent		Ultimate percent of elongation in 5 in	Elastic limit, lb per sq in	Ultimate strength, lb per sq in		
Copper	Tin			Tension	Compression	Modulus of rupture
100	0	6.5	8 000	25 000	40 000	29 800
96	4	14.3	16 000	32 000	42 000	33 200
92	8	5.5	19 000	28 500	42 000	43 700
87	13	3.3	20 000	29 400	53 000	34 500
80	20	0.04	......	33 000	78 000	56 700
76	24	0.	22 000	22 000	114 000	32 000
70	30	0.	5 600	5 600	147 000	12 100
65	35	0.	2 200	2 200	84 700	4 800
57	43	0.	1 450	1 450	......	2 100
45	55	0.	3 000	3 000	35 800	4 800
9	91	6.9	3 500	6 400	9 800	5 300
4	96	12.3	2 751	4 800	9 800	6 900
0	100	35.5	......	3 500	6 400	3 700

The specific gravity of the copper was 8.874 and that of the tin was 7.293. The alloy having the highest specific gravity had 62.42 percent copper and 37.48 percent tin.

Strength of Various Non-ferrous Metals and Alloys

Average values for strength based on test data from various testing laboratories Data are lacking for values of strength in compression. In the absence of such data a safe practise would be to consider the ultimate in compression as having a value equal to the proportional limit in tension.

Metal	Approximate composition percent	Weight lb per cu in	Strength in tension lb per sq in	
			Elastic Limit	Ultimate
Copper, cast	Copper, 100	0.310	8 000	25 000
Hard drawn	Copper, 100	0.321	30 000	40 000
Zinc, cast	Zinc, 100	0.253	....	9 000
Rolled	Zinc, 100	0.253	4 000	24 000
Lead, rolled	....	0.411	800	2 700
Lead alloy, cast	Lead 95.5; antimony, 4.5	0.380	4 000	6 400
Aluminum, cast	Aluminum, 100	0.095	12 000	22 000
Hard-drawn	Aluminum, 100	0.095	18 000	30 000
Alloyed aluminum, cast	Aluminum, 92; copper, 8	0.100	15 000	20 000
Rolled	Aluminum, 92; copper, 8	0.100	17 000	29 000
Aluminum bronze, cast	Aluminum, 10; copper, 90	0.270	25 000	60 000
Rolled	Aluminum, 10; copper, 90	0.270	30 000	70 000
Cold-drawn	Aluminum, 10; copper, 90	0.270	80 000	90 000
Gun metal, cast	Copper, 88; tin, 10; zinc, 2	0.320	20 000	35 000
Phosphor bronze, cast	Copper, 80; tin, 10; lead, 10; phosphorus, trace.	0.330	16 000	30 000
Rolled	Copper, 95; tin, 4.9; phos., 0.1	0.310	60 000	65 000
Soft-gear bronze, cast	Copper, 88; tin, 10; lead, 2	0.320	18 000	32 000
Red brass, cast	Copper, 83; tin, 4; lead, 6; zinc, 7.	0.310	16 000	30 000
Manganese bronze, cast	Copper, 60; manganese, trace iron, 1.5; zinc, 38.5.	0.300	35 000	70 000
Rolled	Copper, 60; manganese, trace iron, 1.5; zinc, 38.5.	0.300	45 000	90 000
Tobin bronze, cast	Copper, 58; tin, 2; zinc, 40	0.290	....	60 000
Rolled	Copper, 58; tin, 2; zinc, 40	0.290	....	79 000
Delta metal, cast	Copper, 65; zinc, 30; iron, 5	0.300	25 000	45 000
Rolled	Copper, 65; zinc, 30; iron, 5	0.300	....	65 000

Phosphor Bronze is an alloy of copper and tin containing less than 1 percent of phosphorus. For hard castings of great strength from 4 to 10 percent of tin and from 0.5 to 1.0 percent of phosphorus is used. It is remarkable for its complete fluidity, so that most perfect castings can be made. It has been used for journal bearings, valve seats, and even for cannon. In the form of wire it has been used for telephone service. It is hard and tough, and its ultimate tensile strength may range from 40 000 to 100 000 lb per sq in.

Copper-Zinc-Tin Alloys have been made in large numbers and are also called bronzes, or sometimes composition metals. The strongest of these was found by Thurston to be that which had 55 percent of copper, 44.5 percent of zinc, and 0.5 percent of tin, the tensile strength of a cast bar being 68 900 lb per sq in. With 55 percent copper, 40 percent zinc, and 5 percent tin the tensile strength was 28 000 lb per sq in.

Aluminum Bronze has from 5 to 12 percent aluminum with from 95 to 88 percent copper. These alloys have high ductility and great strength. With

Composition of a Few Miscellaneous Alloys

Name	Copper	Zinc	Tin	Lead	Bismuth	Other metals
Naval brass........	62	37	1	....	....	
Bush metal........	80	10	5	5	....	
Gold bronze........	89.5	5.6	2.1	....	....	
Engine brass........	76.5	11.7	11.8	2.8	....	
Spring brass........	66	33	1	....	....	
German silver........	55	25	....	....	....	nickel, 20
Britannia metal....	1.9	81.9	....	....	....	antimony, 16.2
Fusible alloy *........	....	....	25	25	50	
Fusible alloy †......	....	....	13	27	50	cadmium, 10
Hard-type metal....	....	75	....	....	....	antimony, 25

* Fuses at 93° C. † Fuses at 60° C.

percent aluminum it has a modulus of elasticity of about 18 000 000 lb per q in. It has a high shrinkage in casting.

Manganese Bronze is an alloy with a copper content of about 60 percent, zinc content of about 38.5 percent, an iron content of about 1.5 percent, and a race of manganese. The manganese "cleanses" the metal of oxide. Manganese bronze has a high strength and also a high ductility. Its elongation in 2 in s about 25 percent. It resists corrosion remarkably well either in salt water or n fresh water, and is used for propeller blades and other submerged parts of hips.

Bearing Metals. A group of soft alloys of low strength is used mainly or forming the surfaces of bearings in machines. Steel rubbing on steel soon ives a rough torn surface. Steel rubbing on cast iron gives a smooth surface, ut cast iron is so brittle that it is in danger of cracking in service. For heavy oads brasses or bronzes are used for bearing metals. They serve excellently, ut are expensive. For bearings carrying light loads alloys of lead, antimony, nd tin are used. These alloys have a low melting point and can be cast in lace. One alloy in common use has the composition, lead 80 percent, antimony 20 percent. "Babbitt Metal," much used for high grade bearings, has he composition, tin 89 percent, copper 4 percent, antimony 7 percent.

Invar is an alloy of nickel and iron which has a very low coefficient of xpansion, which is used for steel tapes and other measuring instruments, nd for common standard of length. Its composition is iron 63 percent, nickel 6.0 percent, carbon 0.3 percent, manganese 0.7 percent. The mean coefficient f expansion for temperatures from 15° to 100° F. is about $^1/_{25}$ of that of steel. For all surveys except geodetic base lines invar tapes may be used without temperature corrections. (Eng. News, Aug. 13, 1908.)

Platinite is an alloy of iron with 42 percent of nickel which has the same coefficient of xpansion as glass and hence may be used in glass to prevent cracking under heat.

38. Corrosion of Metals

Rust forms rapidly upon exposed surfaces of iron or steel, especially in he presence of moisture or carbon dioxide, so that the life of an unprotected teel structure is very much shorter than that of one of stone or wood. The rocess of corrosion or rusting is the combination of oxygen (O) with iron Fe), but this does not occur with perfectly dry air. Analysis of rust from he bridge at Conway, Wales, gave 93.1 percent of Fe_2O_3 and 5.8 percent of FeO vith 0.9 percent of carbon dioxide (CO_2).

The carbon dioxide theory of rusting is that CO_2 acts upon iron to form iron carbonate ($FeCO_3$), then the action of free oxygen changes this into FeO and CO_2 so that there is a fresh supply of carbon dioxide by which the process is continued. Since it has been shown that rusting sometimes occurs when carbon dioxide is absent, it is plain that the theory is not a full and sufficient explanation.

The moisture theory of rusting is that water (H_2O) acting upon iron in the presence of free oxygen produces iron oxide (FeO) and peroxide of hydrogen (H_2O_2), but the facts in support of this chemical action are few. It is, however, certain, that water is one of the agencies which promote the quick and continued rusting of iron and steel, particularly when it contains acids or sulfurous matters.

The electrolytic theory assumes that rusting is caused by slight momentary currents of electricity which originate in the metal at points where there is non-homogeneity. Electrolysis produces rust on a large scale and with great rapidity, but ordinary rusting has nothing to do with such permanent action. It is rather due to small currents caused by a difference in electric potential of neighboring parts of the metal. The complete explanation, as worked out by A. S. Cushman in 1908, involves the concepts and the language of ions and cannot be presented here. The theory assumes that before iron can oxidize in the presence of moisture it must first pass into solution. While water is a solvent of high capacity it seems almost incredible that iron should dissolve in it, and yet many facts indicate that such is possible under the action of these infinitesimal currents. (Proc. Amer. Soc. Testing Materials, 1908.)

Structural Steel usually rusts more rapidly than cast iron. Overhead bracing of bridges exposed to the smoke of passing locomotives rusts rapidly, but old wrought-iron bridges often show less corrosion than do the modern steel structures. Metal subject to shocks and to overstrain is more liable to corrode than that subject to steady stresses. Smooth surfaces rust less than rough ones, due to greater homogeneity. Poor steel having blowholes and imperfections rusts rapidly from the same reason.

H. M. Howe states that direct experiments by a great many observers, in different countries and under different conditions, tend to show that there is no very great difference between the corrosion of wrought iron and steel. In unconfined sea water wrought iron rusts a little less than steel and its advantage seems to be still greater in the case of boiling sea water. In alkaline water wrought iron seems to have the advantage over steel, whereas in acidulated water steel seems to rust more slowly than wrought iron. (Proc. Amer. Soc. Testing Materials, 1906.)

Heyn says that the question whether wrought iron, mild steel or cast iron exhibits the greatest tendency to the formation of rust, is still an open one. In an experiment in undisturbed distilled water, for instance, it was found that: after a lapse of two months, mild steel was attacked most, wrought iron to a less, and cast iron to the least, degree; after seven months cast iron was attacked most and mild steel least, while after seventeen months mild steel and cast iron were attacked in almost equal degree. On a resumption of the tests, the conditions may happen to be different, so that it may be partly a matter of accident which kind of iron will, after a given duration of the test, show the greatest or least degree of attack. The differences in all these cases, however, are very small, and in presence of the other influences play quite a minor part. The statement that the solubility of the materials in sulfuric acid supplies a measure for the degree of attack by rust is not correct. (Proc. Inter. Soc. Testing Materials, Feb., 1910.)

Galvanizing. Zinc takes high rank as a preventive of corrosion in iron and steel. Its action depends upon excluding the steel from the atmosphere, and also upon the fact that, when two different kinds of metals are in contact, the one which is electro-positiv to the other will corrode while the electro-negativ one will be prevented from corrosion. Zinc is highly electro-positiv with respect to iron and hence the latter is uncorroded when surfaces of the two metals are in contact. Galvanizing is done by drawing a plate or wire through a bath of molten zinc or by the electroplating of steel with zinc. Sherardized iron is iron or steel coated with zinc by the condensation of volatilized zinc dust. The bond between the zinc coating and the iron or steel seems to be stronger for sherardized iron than for galvanized iron.

Life under Corrosion. Thwaite published data deduced from experiments made in England about 1870 from which the following comparative figures have been deduced for the life in years of unpainted iron or steel plates.

Years of Life of a Plate of a Certain Size

Material	Sea Water		Fresh Water		Air	
	Foul	Clear	Foul	Clear	Impure	Pure
Cast iron	15	16	26	88	21	88
Wrought iron	5	8	7	81	8	81
Steel	5	10	9	80	8	80
Cast iron, skin removed	4	11	14	90	12	90
Cast iron, galvanized	11	28	30	208	50	208

For surfaces painted at proper intervals probably the above figures may be multiplied by 2 or 3 if the metal is not subject to shock and wear.

Corrosion is promoted by moisture, by carbon dioxid, by sulfurous vapors, by smoke, by acid vapors or waters, by salt water, by sewage, by decaying animal and vegetable matter, and in general by all kinds of impurities; electrolysis and electrolytic action also originate and hasten it. Corrosion is prevented by excluding the atmosphere from the surfaces by preservative coverings of oils, paints, tar or bituminous coatings and deposits of tin or zinc.

Tin Plates are thin sheets of wrought iron or steel coated with tin which protect from corrosion. Pin holes are apt to occur in the tin, however, through which the atmosphere attacks the iron or steel and causes corrosion.

Painting is the usual method of preserving iron and steel surfaces from corrosion (see Art. 46 for Paints and Oils). Clean surfaces and the thorough application of the paint are two fundamental conditions which are necessary to be secured in order to produce best results.

Concrete protects from corrosion iron or steel which is surrounded or encased therein. Oil or oil paints should not be placed on steel to be thus encased (T. K. Thomson, Trans. Am. Soc. C. E., Vol. 71, 1911).

Chromic acid and its salts are said to be powerful protectives against rust. Even small additions of these chemicals to water have a protective effect.

The Minor Metals (Art. 36) are non-corrodible compared with iron or steel. Aluminum is one of the most non-corrodible. Lead dissolves in water to a slight extent, but carbonates or sulfates of lime deposited upon it from the water form a film on the surface which prevents further action. Nickel resists corrosion to a high extent. On zinc coatings a thin skin of zinc carbonate forms in the atmosphere which prevents corrosion.

The Alloys (Art. 37) are but slightly subject to corrosion. J. W. Richards gives following figures (Jour. Frank. Inst., 1895) as showing the losses in milligrams per square centimeter per day caused by the corrosion of surfaces of aluminum and its alloys when immersed in various liquids.

Liquid	89% Aluminum	2% Titanium	3% Nickel	3% German Silver	3% Copper
3% cold caustic potash	34.6	73.4	580.3	1534.4	265.0
3% cold hydrochloric acid	5.8	4.3	180.0	130.6	53.3
Cold strong nitric acid	9.6	18.6	83.0	97.7	36.1
Strong acetic acid 140° F.	0.15	0.20	0.75	0.6	0.4
Carbon dioxid water 77° F.	0.01	0.0	0.04	0.01	0.0

NON-METALLIC MATERIALS

39. Stone

Data for Building Stones of Good Quality

Values based mainly on test data from the Watertown (Mass.) Arsenal

Kind of stone	Weight lb per cu ft	Compressive strength lb per sq in	Shearing strength lb per sq in	Modulus of rupture lb per sq in	Modulus of elasticity, lb per sq in	Coeff. of expansion per deg. F.	Absorption of water percent of wght. of stone
Granite, range	160 to 170	15 000 to 26 000	1800 to 2800	1 200 to 2 200	5 900 000 to 9 800 000		
average	165	20 200	2300	1 600	7 500 000	0 0000040	0.5
Sandstone, range	135 to 150	6 700 to 19 000	1200 to 2500	500 to 2 200	1 000 000 to 7 700 000		
average	140	12 500	1700	1 500	3 300 000	0.0000055	5.0
Limestone, range	140 to 180	3 200 to 20 000	1000 to 2200	250 to 2 700	4 000 000 to 14 700 000		
average	160	9 000	1400	1 200	8 400 000	0.0000045	7.7
Marble, range	160 to 180	10 300 to 16 100	1000 to 1600	850 to 2 300	4 000 000 to 12 600 000		
average	170	12 600	1300	1 500	8 200 000	0.0000045	0.4
Slate, range	170 to 180	14 000 to 30 000	....	7 000 to 11 000	13 900 000 to 16 200 000		
average	175	15 000	....	8 500	14 000 000	0.0000058	0.5
Trap, average	185	20 000	....	....	....	....	....

Trap. This name is applied to a class of eruptive rocks, notably those of the Hudson River Palisades. Its most important properties are not among those in the table, for its most notable characteristics are resistance to wear and a high degree of binding power when powdered and moistened. Few tests have been made on its strength.

Artificial Building Stones. These as a rule are lower in compressive strength than the natural building stones, and without special test they may be rated with common building brick with an ultimate strength of 3000 to 4000 lb per sq in. The Arsenal Tests for 1906 give for four varieties of "manufactured stone" values ranging from 2570 to 3390 lb per sq in.

40. Brick and Terra Cotta

Bricks are commonly made of clay, whose chief characteristics are a plasticity when wet and a rocklike hardness after being heated to a high temperature. Pure clay, or kaolin, is white, and is employed in the manufacture of china and porcelain ware, while the lower-grade clays are used in making building brick. Bricks are also made from pulverized shale, which is simply a clay that has been consolidated thru geological processes. The more siliceous shales and clays are adapted to the manufacture of vitrified paving blocks. At the present time building bricks are also made from sand and lime.

Clay Bricks may be broadly classified as building and paving. The former include the common and the prest bricks Of more limited and yet important use are enamel brick, glazed brick and fire brick. The enamel brick are those that have a coating of enamel on one or two sides, while the body of the brick is usually of fire clay. Glazed bricks differ from enamel bricks in being coated with a transparent glass instead of an opaque enamel. Fire brick are ordinarily made of a mixture of flint clay and plastic clay. They are usually white, and are used in lining fire boxes and passages.

Defects. The finished brick may be defective because of errors in manufacture or because the clay contains harmful materials. Notable among these are limonite and pyrite. When the brick is fired, limonite concretions will cause fused blotches and weak spots, while pyrite burns away, leaving flaws in the brick.

Terra Cotta is a burnt clay product made in the same general way as brick. Hard terra-cotta blocks and tile are made by burning clay at a very high temperature. Porous or soft terra cotta, sometimes called terra-cotta lumber, is made by burning a mixture of clay and straw or sawdust. The straw or sawdust burns out, leaving a light, porous material. Nails and screws can be driven into porous terra cotta, and it can be cut with a wood saw. Terra-cotta lumber is weaker than hard terra cotta. Terra-cotta building blocks are made hollow with walls ¾ in to 1 in thick.

Sand-Lime Brick are made from an intimate mixture of sand and lime in proportion of about 16 to 1, molded in a press, and hardened in a large cylinder filled with steam at 125 lb pressure. The bonding is a result of the union between the sand and lime which forms calcium silicate.

Properties of Brick of an Average Good Quality

Kind	Weight per cu ft lb	Crushing strength, lb per sq in	Shearing strength, lb per sq in	Modulus of rupture, lb per sq in	Modulus of elasticity, lb per sq in	Absorption, percent of weight
Common	125	4 000	1000	600	2 000 000	15
Face	130	6 000	1000	800	3 000 000	10
Paving	150	10 000	1400	2000	7 000 000	2
Sand-lime	115	3 000	800	450	1 000 000	12
Terra cotta	....	4 000	....	800	....	13

Average Compressive Strength of Brick Masonry

Brick or block used	Mortar	Ultimate stress in compression on test pier, lb per sq in
Vitrified brick	1 : 3 Portland cement	2800
Face brick	1 : 3 Portland cement	2000
Face brick	1 : 3 lime mortar	1400
Common brick	1 : 3 Portland cement	1000
Common brick	1 : 3 lime mortar	700
Hard terra-cotta block	1 : 3 Portland cement	3000
Sand-lime brick *	1 : 3 Portland cement	750
Sand-lime brick *	1 : 3 lime mortar	500

* Estimated from the relative strength of individual sand-lime bricks and common bricks.

Strength of Brick Masonry. As in the case of stone masonry brick masonry is always used in compression. The strength in compression of brick masonry is much less than the strength of individual bricks. The strength depends not only on the strength of the bricks, but to a large degree on the strength of the mortar joints, and on the skill used in laying the brick. The preceding table, based on test data from the Watertown Arsenal, Cornell University, the U. S. Bureau of Standards, and the University of Illinois gives average values for ultimate strength in compression.

41. Timber

Classes of Timber Trees. Wood, as a building material, is produced by the spermatophyta, or seed-bearing trees, which may be divided into three groups, namely, the conifers, the broad-leaved, and the tropical trees. The conifers and the broad-leaved trees produce the structurally valuable timbers and give rise to the general classification of timber in the lumber trade into SOFT-WOODS (pines, spruce, cedar, cypress, larch, and fir) and HARD-WOODS (oak, walnut, maple, chestnut, hickory, ash, boxwood, whitewood, etc.). Of the tropical trees the products are the bamboos, palms, and rattans. Between the hard-woods and the soft-woods there is no sharply defined distinction in hardness, some of the hard-woods, such as the basswood, poplar, and sycamore, being softer than the pines.

The conifers and the broad-leaved trees are known as the outward-growing or exogenous trees. The structure consists of three parts, the bark, the sapwood, and the heartwood. On the outside of the tree trunk is found from ¼ to 2 in or more of bark or protective tissue. As a structural material this is valueless and is always removed soon after the tree is felled, as it hastens the decay of the wood. Inside of the bark there is a soft portion made up of thin-walled cells which constitute the living portion of the tree, called the sapwood. Arranged in a circle inside of this soft tissue are many fibrous bundles making up the middle of the stem, giving it strength and stiffness and known as the heartwood. As the stem grows, new and branching bundles of these hollow fibers appear under the bark and form each season an annular ring. At the end of the season growth stops, to be resumed the following spring; and the rapid open growth ofthe spring against the slow and condensed growth of the summer gives rise to the peculiar marking in the bundles which indicates each year's increase. The last few rings formed constitute the sapwood, usually from ½ to 4 in in thickness and light in color. The rings inside of the sapwood form the heartwood. The rings are interrupted by plates of tissue or radial cells communicating between the pith at the center of the tree and the soft tissue on the outside. These form the medullary rays. In the pine, the sapwood constitutes 40 to 60 percent of the cross-section of the tree, and the time required for sapwood to transform into heartwood varies from a few years in the case of the fir to many years in the oak tree. The approximate composition of all woods when dry is nearly uniform and consists by weight of the following elements: 49 percent of carbon, 6 percent of hydrogen, 44 percent of oxygen, and 1 percent of ash.

White Ash. Heavy, hard, very elastic, coarse-grained, and compact. Tendency to become decayed and brittle after a few years. Color, reddish-brown, with sapwood nearly white. Used for interior and cabinet work, but unfit for structural work.

Red Ash. Heavy, compact, and coarse-grained but brittle. Color, rich brown, with sapwood a light brown sometimes streaked with yellow. Used as a substitute for the more valuable white ash.

Green Ash. Heavy, hard and coarse-grained; brittle. Color, brown, with lighter sapwood. Used as a substitute for white ash.

Balsa. Extremely light, about half the strength of white pine. Appearance like poplar. Used for heat insulation and, when waterproofed, for life preservers.

White Cedar. Soft, light, fine-grained, and very durable in contact with the soil; lacks strength and toughness. Color, light brown, darkening with exposure. Sapwood very thin and nearly white. Used for water tanks, shingles, posts, fencing, cooperage, and boat building.

Red Cedar. Strong pungent odor repellent to insects. Very durable and compact, but easily worked and brittle. Color, dull brown tinged with red. Used as posts, sills, ties, fencing, shingles, and lining for chests, trunks, and closets.

Chestnut. Light, moderately soft, stiff, and of coarse texture. Shrinks and checks considerably in drying; works easily. Durable when exposed to weather. Color, heartwood dark and sapwood light brown. Used for cabinet work, cooperage, railway ties, telegraph poles, and exposed heavy construction.

Cypress. One of the most durable of woods, light, hard, close-grained but brittle. Easily worked, polishes highly and gives a satiny gloss. Color, bright clear yellow with nearly white sapwood. Used for interior finish and cabinet work, but used as extensively in the South as pine is in the North.

White Elm. Heavy, hard, strong and tough and very close-grained. Difficult to split and shape, but warps badly in drying. Capable of high polish. Color, light clear brown often tinged with red and gray, with broad whitish sapwood. Used for car, wagon, boat, and ship building, bridge timbers, sills and ties, and furniture, also barrel staves.

Greenheart. Very heavy, strong, durable heartwood, dark green to dark chestnut color, free from knots. Used for ship building, docks, implements, rollers.

Gum. Heavy, hard, tough, compact and close-grained. Tendency to shrink and warp badly in seasoning. Not durable if exposed. Takes high polish. Color, bright brown tinged with red. Used in the manufacture of furniture, wagon hubs, hat blocks.

Hickory. Medullary rays very numerous and distinct. Heaviest, hardest, toughest, and strongest of American woods. Very flexible. Color, brown, with very thin but valuable sapwood nearly white. Used for carriages sleighs, handles, and bent-wood implements. Unfit for building material because of extreme hardness and liability to attack of boring insects.

Hemlock. Brittle, splits easily and likely to be shaky. Soft, light, not durable, with coarse and uneven grain. Color, light brown tinged with red and often nearly white. Used for cheap rough framing timber.

Locust. Heavy, hard, strong, and close-grained. Very durable in contact with ground. Hardness increases with age. Color, brown and rarely light green, with yellow sapwood. Used for posts and turned ornaments.

Lignum Vitæ. Exceedingly heavy, hard, resinous, difficult to split and work, and has a soapy feeling. Color, rich yellow brown varying to almost black. Used for small turned articles, tool handles, and sheaves of block pulleys.

Hard Maple. Heavy, hard, strong, tough and close-grained. Medullary rays small but distinct. Curly and circular inflexion of fibers gives rise to "curly maple" and "bird's-eye maple." Susceptible of good polish. Color, very light brown to yellow. Used for flooring, interior finish, and furniture.

White Maple. Fine-grained, hard, strong, and heavy. Characteristics of grain the same as hard maple and more marked. Light colored. Used for flooring and furniture.

Mahogany. Strong, durable, and flexible when green, but brittle when dry. Free from shakes and less liable to attacks of dry rot or worms. Rapid seasoning causes deep shakes. Color, red-brown of various shades and degrees of brightness, often varied and mottled. Inferior qualities contain large numbers of gray specks. Used for interior finish, handrails, patterns, etc.

White Oak. Heavy, strong, hard, tough, and close-grained. Checks if not carefully seasoned. Well-known silver grain and capable of receiving high polish. Color, brown, with lighter sapwood. Used for framed structures, shipbuilding, interior finish, carriage and furniture making.

Chestnut Oak. Very durable in contact with soil. Color, dark brown. Used for railroad ties.

Live Oak. Very heavy, hard, tough, and strong. Difficult to work. Color, light brown or yellow, with sapwood nearly white.

Red and Black Oak. More porous than white oak and softer. Color, darker and redder than white oak. Used for interior finish and furniture.

Palmetto. Light but difficult to work when dry. Very durable under water and less subject to attacks of teredo. Color, light brown, with dark-colored fibers. Used for wharf piles, canes and handles.

White Pine. Light, soft and straight-grained and easily worked, but not very strong.

Color, light yellowish brown often slightly tinged with red. Used for interior finish and pattern making.

Red Pine (Norway Pine). Light, hard, coarse-grained, compact, with few resin pockets. Color, light red, with a yellow or white sapwood. Used for all purposes of construction.

Yellow Pine (Long-Leaf). Heavy, hard, strong, coarse-grained, and very durable when dry and well ventilated. Cells are dark colored and very resinous. Color, light yellowish red or orange. Cannot be used in contact with ground. Used for heavy framing timbers and floors. As house sills, sleepers, or posts it rapidly decays.

Yellow Pine (Short-leaf Pine). Varies greatly in amount of sap and quality. Cells broad and resinous, with numerous large resin ducts. Medullary rays well marked. Color, orange, with white sapwood. Used as a substitute for long-leaf pine.

Oregon Pine (Douglas Fir.) Hard, strong, varying greatly with age, conditions of growth, and amount of sap. Durable but difficult to work. Of two varieties, red and yellow, of which yellow is the more valuable. Color, light red to yellow, with white sapwood. Used in all kinds of construction.

Poplar (Whitewood). Soft, very close and straight-grained, but brittle and shrinks excessively in drying. Warps and twists exceedingly, but when dry will not split. Easily worked. Color, light yellow to white. Used in carpentry and joinery.

Redwood (California). Light, soft, coarse-grained, and easily worked. Durable in contact with soil, but brittle. "Shrinks lengthwise as well as crosswise." Color, dull red, resembling pine. Used for railroad ties, fence posts, telegraph poles, and general building material.

Alaska Spruce (Sitka Spruce). Light, soft, medium strength. Heartwood, light, reddish brown; sapwood, white; trees very large. Used for general structural purposes; also for airplane frame-work.

Red Spruce. Light, soft, close and straight-grained and satiny. Color, light red and often nearly white. Used for piles, lumber, and framing timber, submerged cribs and cofferdams, as it well resists decay and the destructive action of crustacea.

White Spruce. Similar to black variety, but not so common. Color, light yellow, sapwood indistinct. Used as lumber for construction.

Tamarack (Larch). Wood like pine in appearance, quality and uses. Used for telegraph poles, railway ties, and in ship building.

White Walnut (Butternut). Light, soft, coarse-grained, compact, and easily worked. Polishes well. Color, light brown, turning dark on exposure. Used for interior finish.

Black Walnut. Heavy, hard, strong, and checks if not carefully seasoned. Coarse-grained but easily worked. Color, rich dark brown, with light sapwood. Used for interior finish and cabinet work.

Standard Names for Structural Timber. The following classification of standard commercial names for structural timber is from the 1916 "Standards" of the Am. Soc. for Test. Materials:

1. Southern Yellow Pine. This term includes the species of yellow pine growing in the southern states from Virginia to Texas, that is, the pines hitherto known as long-leaf pine (*Pinus palustris*), short-leaf pine (*Pinus echinata*), loblolly pine (*Pinus taeda*), Cuban pine (*Pinus heterophylla*) and pond pine (*Pinus serotina*).

Under this heading, two classes of timber are designated: (*a*) dense southern yellow pine and (*b*) sound southern yellow pine. It is understood that these two terms are descriptive of quality rather than of botanical species.

(*a*) Dense southern yellow pine shall show on either end an average of at least six annual rings per inch and at least one-third summer wood, or else the greater number of the rings shall show at least one-third summer wood, all as measured over the third, fourth, and fifth inches on a radial line from the pith. Wide-ringed material excluded by this rule will be acceptable, provided that the amount of summer wood as above measured shall be at least one-half.

The contrast in color between summer wood and spring wood shall be sharp and the summer wood shall be dark in color, except in pieces having considerably above the minimum requirement for summer wood.

(*b*) Sound southern yellow pine shall include pieces of southern pine without any ring or summer-wood requirement.

2. Douglas Fir. The term "Douglas Fir" is to cover the timber known likewise as yellow fir, red fir, western fir, Washington fir, Oregon or Puget Sound fir or pine, northwest and west coast fir.

3. Norway Pine, to cover what is known also as "Red Pine."

4. Hemlock, to cover Southern or Eastern hemlock; that is, hemlock from all States east of and including Minnesota.

5. Western Hemlock, to cover hemlock from the Pacific coast.

6. Spruce, to cover Eastern spruce; that is, the spruce timber coming from points east of and including Minnesota.

7. Western Spruce, to cover the spruce timber from the Pacific coast.

8. White Pine, to cover the timber which has hitherto been known as white pine, from Maine, Michigan, Wisconsin and Minnesota.

9. Idaho White Pine, the variety of white pine from western Montana, northern Idaho, and eastern Washington.

10. Western Pine, to cover the timber sold as white pine coming from Arizona, California, New Mexico, Colorado, Oregon and Washington. This is the timber sometimes known as "Western Yellow Pine," or "Ponderosa Pine," or "California White Pine," or "Western White Pine."

11. Western Larch, to cover the species of larch or tamarack from the Rocky Mountain and Pacific coast regions.

12. Tamarack, to cover the timber known as "Tamarack," or "Eastern Tamarack," from States east of and including Minnesota.

13. Redwood, to include the California wood usually known by that name.

Specifications for Size. Sawed timbers shall be sound, of standard size, square-edged, and straight; shall be close-grained and free from defects, such as injurious ring shakes and cross grain, unsound or loose knots, knots in groups, decay, or other defects that will materially impair the strength.

Rough sawing to standard size shall mean that the timbers shall not be over ¼ in scant from the actual size specified; for instance, a 12 by 12 in timber shall measure not less than 11¾ by 11¾ in.

Standard dressing shall mean that not more than ¼ in shall be allowed for dressing each surface; for instance a 12 by 12 in timber after being drest on four sides shall measure not less than 11½ by 11½ in.

The standard lengths are multiples of 2 ft, running from 10 to 24 ft for boards, fencing, dimension, joists, and timbers. Longer or shorter lengths than those herein specified are special. Special and fractional lengths shall be counted as of the next higher standard length.

The standard of widths for lumber shall be multiplies of 1 in. All sizes 1 in or less in thickness shall be counted as 1 in thick.

Sawing. The manner in which the stick of lumber is sawed from the tree has a remarkable influence upon its qualities and behavior, and the selection of cutting is determined by the character of the wood and the purpose for which it is destined. FLAT SAWING consists in cutting the timber tangential to the annular rings. RIFT SAWING is cutting the boards out of the log in such a manner that the annular rings are cut thru as nearly as possible in a radial direction. Rift sawing and flat sawing give rise in the lumber trade to the terms edge grain and flat grain respectively. Rift sawing is done for the sake of the beauty of the grain thus obtained, as well as to expose the edge of the hard bands of summer wood. Edge-grain lumber shrinks and checks less, does not sliver, and wears more evenly and smoother than flat-grain lumber.

Natural Seasoning. In the preparation of lumber for construction purposes, it is necessary to expel the moisture from the pores of the wood by the process of seasoning. The drier the timber, the less likely is it to shrink and decay. Natural seasoning consists in exposing the planks and boards, after sawing, to a free circulation of the air. The lumber is placed on skids in large square piles under shelter in a dry place, the layers being separated by three or four narrow strips or boards laid in the opposite direction. The lowest layer should be at least two feet from the ground. At frequent intervals

the decayed pieces are removed and the timber repiled. The time required for thoro seasoning varies from one to three years, depending upon the character of the wood, the purpose to which it is to be adapted, and the dimensions.

Water Seasoning is another type of natural seasoning which consists in immersing the lumber in water. The soluble substances in the sapwood are removed, and the product is a timber less liable to warp and crack, but the heartwood becomes brittle and loses its elasticity. In this method the timber is immersed for about two weeks, then removed and thoroly dried with free access of air. If immersed too long, the wood when exposed to the air becomes brashy.

Artificial Seasoning or Kiln-Drying hastens the evaporation of the moisture but at the same time produces an inferior product, in that it causes a rapid drying of the surface and ends of the material and a slow or imperfect drying of the interior. This impairs both the strength and elasticity of the wood. The timber is stocked in a drying kiln and exposed to a current of hot air, the temperature depending on the kind and the dimension of the stock. Sometimes the heat is supplemented by the employment of vacuum pumps. The best temperature to be employed depends on the kind and dimensions of the lumber, and varies from 100° F. for oak to 200° F. for pine. The time required depends on the thickness of the stock. About four days is necessary for one-inch pine, spruce, or cedar boards. Hard woods are usually dried in air from three tc six months and then placed in the drying tank from six to ten days.

Shrinkage. The concentric annular cones of wood are made up of pores or cells enclosed by walls of cellulose and are pierced at right angles by plates of similar fibers. As the average width of cell is 1/100 of the length, the greatest shrinkage will take place in the cross-section of the fibers or tangentially to the annular rings. This is known as the circumferential shrinkage. By rift sawing the medullary rays are cut across the length, in which direction shrinkage is least. Flat sawing produces lumber which checks and cracks to a greater extent in shrinking. The average values for shrinkage in width are:

Light conifers (soft pine, spruce, cedar, cypress)	3 percent
Heavy conifers (hard pine, tamarack, yew, honey locust, box elder, wood of old oaks)	4 percent
Ash, elm, walnut, poplar, maple, beech, cherry, sycamore, and black locust	5 percent
Basswood, birch, chestnut, blue beech, young locust	6 percent
Hickory, young oak, especially red oak	up to 10 percent

The longitudinal shrinkage is usually less than 0.1 percent. The change in volume is therefore due to radial and tangential shrinkage, and exprest in percentage is approximately twice the figures given, as shrinkage takes place in two directions by approximately equal amounts.

The harder timbers are more compact in structure, with thicker cell walls, and therefore produce the greater shrinkage. The opposite effect to shrinkage is produced by the absorption of moisture, and protective checks must be resorted to in applying timber to construction; the expansion joints in wooden block pavement serve as checks. A roadway forty feet wide has been observed to expand eight inches.

Decay of Timber. The life of timber depends on its manner of felling, seasoning and working, and it is subject both in its growing and converted states to decomposition and attack of animal and vegetable life. The proper time of year for cutting the tree is important. In the spring and late summer the sapwood contains an abundance of moisture with starches, sugars, and oils in solution, which tend to hasten its decay. In the drier summer months and in winter the growing and conducting cells are less active or altogether dormant, and the best wood is secured if the tree is cut during those seasons. Oak is claimed to be more durable if cut just after the leaves have fallen.

Trees are felled with the ax or saw, and hewed lumber is commonly accepted as more durable than sawed lumber, as the former process closes the cells at the cut and prevents

the absorption of moisture. The agencies which produce the decay of wood may be classed as follows: alternate moisture and dryness, heat and confined air, bacteria and fungi, insects and worms. Well-seasoned wood in a uniform state of moisture or dryness, when well ventilated, should never decay. Timber kept constantly immersed may soften and weaken but does not decay; the elm, elder, oak, and birch in this condition possess great durability.

Dry Rot. This is caused by the fermentation and breaking down of the chemical compounds of wood when a certain fungus is introduced in the presence of moisture. These lower organisms excrete ferments which dissolve out parts of the cell wall and crumbling of the wood follows. Dry rot is prevalent in ill-ventilated places, such as the wall pockets at the ends of floor timbers, and is often prevalent in the core of timber columns in mill construction. The growth of the fungus is stimulated by moderate warmth, presence of damp, and lack of ventilation. In practise the decomposition is often accelerated by the use of unseasoned wood, the surfaces of such unprepared timber often being painted and tarred. Dry rot is indicated by the swelling of the timber, a change in color, the material gradually becoming covered with moldiness and emitting a musty odor. Sometimes reddish and yellowish spots appear on the timber and the fibers are gradually reduced to a powder. It is difficult to eradicate when once established, the only remedy being to remove all traces of the fungus and to disinfect the wood.

Wet Rot. Moisture, especially in the presence of warmth, will dissolve out the substance of the cell walls of sapwood and cause the decay of lumber. Wood felled between the months of April and October is especially subject to wet rot. Common Rot is manifested by the presence of external yellow spots on the ends of timber sticks and often by a yellowish dust in checks and cracks, especially where the pieces are in contact. The cause of common rot is improper seasoning in badly ventilated sheds.

Teredo. Of the worms most destructive to wood, the Teredo navalis, or shipworm, a species of mollusk, is the most active. It deposits its eggs on timber immersed in water, from which soon emerge small but rapidly developing worms. The head of the worm is equipped with a shell-like substance shaped like an auger, by means of which it bores its way into the timber in a direction generally parallel to the grain. As it progresses it lines the hole with a calcareous deposit and maintains communication with the outside thru two small lids at the external opening. The teredo continually increases in size as its boring progresses, and specimens varying from ¼ to ½ inch in diameter and 15 inches to 6 feet in length have been found. Its habitat is the salt water of the warmer climates, tho occasionally found in cold water. It avoids fresh water entirely, always prefers clear to muddy water, and is most active in the vicinity of calcareous shores. It may be found at work from half tide level down to the ground.

The Limnoria, or gribble (Limnoria terebrans), is a crustacean about the size of a grain of rice and resembles the wood louse. It is able to swim, crawl, and jump, and requires both air and water for existence. In contradistinction to the teredo, it works only to a depth of ½ an inch, devouring the wood by means of claws or mandibles. The surface of the wood becomes undermined and brittle and is finally washt away, exposing a new surface to the action of the worm. The limnoria devours wood at the rate of one inch to three inches per annum, and is always found in large colonies, as many as 20 000 having been observed on a surface 12 inches square. It is found both in cold and in warm water, being capable of inhabiting colder climates than the teredo. It has a preference for siliceous shores, and confines its work to a space between high and low water marks. It is usually most active in brackish waters at low-water level.

The Lycoris fucata is a little worm with numerous legs, somewhat similar to the centipede. It lives in the mud and crawls up piles inhabited by the teredo, against which

it is an active enemy. It enters the tunnel, devours the teredo, enlarges the entrance to the burrow and lives in it.

42. Data for Timber

Lumber is the general term used to include all material sawed from logs for structural or other commercial purposes. The term timber, in the trade, applies to the large sizes, such as beams and joists. To obtain the smaller sizes, lumber is resawed. This resawed lumber, when planed at the mill, is called drest lumber, and such may be had in great variety of dimensions adapted to all the uses of wood. Drest planks and boards free of all defects are called clear, and are produced in regular sizes ⅛ inch less in thickness than sawed lumber and ranging from ⅝ inch to 1⅞ inches in thickness. The term resawed lumber is applied to lumber sawed on four sides. Rough-edge, or flitch, is lumber sawed on two sides.

The Classification and Dimensions adopted by the New York Lumber Trades Association and other affiliated organizations are as follows: Flooring shall embrace four, five, and six quarter inches in thickness, by three to six inches in width, excluding 1½ by 6. Boards shall embrace all thicknesses under one and a half inches by over six inches wide. Plank shall embrace all sizes from one and one-half to under six inches in thickness by six inches and over in width. Scantling shall embrace all sizes exceeding one and one-half inches and under six inches in thickness, and from two to under six inches in width. Dimension sizes shall embrace all sizes six inches and up in thickness by six inches and up in width. Stepping shall embrace one to two and a half inches in thickness by seven inches and up in width. Rough edge, or flitch, shall embrace all sizes one inch and up in thickness by eight inches and up in width, sawed on two sides only.

Inspection. The inspection of timber must be undertaken with two viewpoints, namely, the quality of the stock and the dimensions of the pieces; and all condemned pieces must be plainly marked with paint or branding iron. Strong and durable timber possesses the following characteristics: It is obtainable from the slowest-growing trees, as indicated by the narrowness of the annular rings. The best timber comes from the heart of the tree with the sapwood entirely removed. Wood containing the least amount of resin or sap in the pores is the strongest and most durable. The wood should be uniform in substance, straight in fiber, free from large and dead knots and all flaws, shakes, and blemishes. Freshly cut, sound timber smells sweet and shows a firm and bright surface with silky luster when planed. The surface should never be woolly, and when cut should not clog the teeth of the saw. In highly colored woods, darkness of color is generally indicative of strength and durability. Sound timber when struck lightly or scratched at one end transmits the sound to the ear placed at the other extremity thru sticks of lumber 50 feet in length. Sound material is sonorous when struck, while decaying timber gives forth a dull sound. Dull, chalky appearance and disagreeable odor are signs of bad timber. In the absence of the usual external signs, dry rot may be detected, by boring test holes in the wood for the appearance and odor of the wood dust.

Tests. The U. S. Tenth Census enumerated and tested 412 species of timber in North America. They ranged in weight from 16 to 81 lb per cu ft and in compressive strength from 200 to 12 000 lb per sq in. A general rule is that the heaviest wood is the strongest. Tests on laboratory specimens give greater strength than those on the actual pieces used in construction. Flexural tests made by A. N. Talbot in 1909 on 112 large beams of long-leaf pine, short-leaf pine, loblolly pine, and Douglas fir gave average values for different series which ranged from 3823 to 4336 lb per sq in for elastic limit, from 4420 to 5430 lb per sq in for modulus of rupture, and from 300 to 354 lb per

sq in for ultimate horizontal shear along the neutral surface. The following conclusions were drawn from these tests (Bull. Univ. Illinois, Eng. Expt. Sta., No. 41):

1. The preponderance of failures in horizontal shear is very marked in every series of tests. The results emphasize the importance of the shearing resistance of the wood in beams of the dimensions of those tested. The cross-breaking failures were in sticks having bad knots and cross grain. A few of these defective sticks would not be called merchantable timber, but many of them might pass inspection as it is usually made. The influence of defects is very marked. Knots, cross grain, and seasoning checks act to give low strength. Any tendency to relax requirements and inspection should be avoided, unless accompanied with the use of lower working stresses.

2. The horizontal shearing resistance developed in the stringers is low in every series of tests. The averages range from 300 to 390 lb per sq in, and in individual cases the stress is as low as 200 lb per sq in. Comparatively little difference in shearing resistance is found for the different kinds and conditions of wood, for in the clear sticks of the stronger woods large seasoning checks act to prevent the utilization of the full cross-breaking strength of the timber.

3. For the cross-breaking failures the fiber stress developed was also generally low, averaging for the untreated timber from 3690 lb per sq in for the old Douglas fir to 5300 lb per sq in for the long-leaf pine.

Defects. The following are adopted as standard by the American Society for Testing Materials (1916 "Standards," p. 598). The diameters of knots and holes are mean or average diameters:

Knots. A SOUND KNOT is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixt by growth or position that it will retain its place in the piece. A LOOSE KNOT is one not firmly held in place by growth or position. A PITH KNOT is a sound knot with a pith hole not more than 1/4 in in diameter in the center. An ENCASED KNOT is one whose growth rings are not intergrown and homogeneous with the growth rings of the piece it is in. The encasement may be partial or complete; if intergrown partially or so fixed by growth or position that it will retain its place in the piece, it shall be considered a sound knot; if completely intergrown on one face, it is a watertight knot. A ROTTEN KNOT is one not as hard as the wood it is in. A PIN KNOT is a sound knot not over 1/2 in in diameter. A STANDARD KNOT is a sound knot not over 1 1/2 in in diameter. A LARGE KNOT is a sound knot more than 1 1/2 in in diameter. A ROUND KNOT is one which is oval or circular in form. A SPIKE KNOT is one sawn in a lengthwise direction.

Pitch Pockets are openings between the grain of the wood containing more or less pitch or bark. These shall be classified as small, standard, and large pitch pockets. A small pitch pocket is one not over 1/8 of an in wide. A standard pitch pocket is one not over 3/8 of an in wide, or 3 in in length. A large pitch pocket is one over 3/8 of an in wide, or over 3 in in length. A PITCH STREAK is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains, that is, the coarse-grained fiber, usually termed "Spring wood," is not saturated with pitch, it shall not be considered a defect. WANE is bark, or the lack of wood from any cause, on edges of timbers.

Shakes are splits or checks in timbers which usually cause a separation of the wood between annual rings. A RING SHAKE is an opening between the annual rings. A THRU SHAKE is one which extends between two faces of a timber.

ROT, DOTE, and RED HEART are forms of decay which may be evident either as a dark red discoloration not found in the sound wood, or as the presence of white or red rotten spots, and shall be considered as defects.

43. Preservation of Timber.

Preservative Processes. The average life of timber used in the United States which is subject to decay, is about 8 years; its life can be extended to 12 years either by a chemical impregnation of the wood cells or by an exterior application of a preservative coating which will penetrate the fibers.

Properties of Timber

Based mainly on Bulletin 556 of the U. S. Department of Agriculture (Forest Service) by Newlin and Wilson. Values are for wood tested green. Values for green timber are more reliable as a guide for structural use than values for dry timber, since in service the outer fibers of dry timber may readily reabsorb water. For working stresses see Sect. 7, p. 756.

Kind of timber	Weight, lb per cu ft	Strength in flexure, lb per sq in			Strength in compress'n, lb per sq in			Ultimate shearing strength, parallel to grain, lb per sq in	Ultimate tensile strength, perpendicular to grain, lb per sq in
		Fiber stress at elastic limit	Modulus of rupture	Modulus of elasticity in flexure	Fiber stress at elastic limit, parallel to grain	Fiber stress at ultimate, parallel to grain	Fiber stress at elastic limit, perpendicular to grain		
Ash	49	5500	9 900	1 490 000	3500	4200	800	1430	700
Cedar (white)	28	2600	4 200	640 000	1400	2000	290	620	240
Cedar (red)	27	3300	5 200	950 000	2500	2800	310	720	210
Chestnut	55	3100	5 600	930 000	2000	2500	380	800	430
Cypress	41	3800	6 500	1 070 000	2700	3200	440	820	270
Elm	52	3600	6 900	1 030 000	2300	2900	390	920	560
Gum (black)	45	4000	7 000	1 030 000	2400	3000	600	1100	570
Gum (blue)	70	7600	11 200	2 010 000	4900	5200	1020	1550	640
Hickory	64	5700	10 500	1 470 000	3500	4400	1270	930	680
Hemlock	45	3700	6 300	1 080 000	2500	3000	420	860	290
Locust	59	7200	12 000	1 570 000	4800	5600	1420	1710	850
Maple	67	4200	7 500	1 230 000	2500	3200	570	1150	630
Oak (white)	62	4700	8 300	1 250 000	3000	3600	830	1250	770
Oak (chestnut)	62	4600	8 000	1 370 000	2900	3500	660	1210	690
Oak (red and black)	64	3700	7 700	1 290 000	2300	3200	730	1120	740
Pine (white)	39	3400	5 300	1 070 000	2400	2700	310	640	260
Pine (western white)	39	3500	5 700	1 330 000	2800	3100	300	710	250
Pine (red or Norway)	42	3700	6 400	1 380 000	2500	3100	360	780	190
Pine (yellow long leaf)	50	5400	8 700	1 630 000	3800	4400	600	1070	290
Pine (yellow short lf.)	50	4500	8 000	1 450 000	3600	3800	480	890	330
Pine(western yellow)	46	3100	5 200	1 010 000	2100	2500	340	680	280
Pine (Oregon or Douglas fir	38	5000	7 800	1 580 000	3400	3900	530	910	200
Spruce (red)	34	3400	5 700	1 180 000	2400	2700	350	770	220
Spruce (white)	33	3300	5 400	980 000	2300	2400	270	670	200
Tamarack (or larch)	47	4400	7 300	1 300 000	3100	3600	520	890	240
Walnut	58	5400	9 500	1 420 000	3600	4300	600	1220	570

Bethel Process.—The green or partially seasoned timber on buggies is run into metal cylinders 8 to 9 ft in diameter and 150 ft long and the doors or heads bolted. The charge is then subjected to live steam raised to 20 lb per sq in in 30 to 50 minutes and maintained at this pressure from 1 to 5 hours. A vacuum of 18 to 26 in is then created and maintained for at least ½ hour, when creosote oil is introduced at a temperature of about 160° F. A pressure of 150 to 200 lb per sq in is then applied until the timber has absorbed about 5 lb of oil per cu ft.

Burnett Process.—This method of treatment is essentially the same as that previously described, except that zinc chloride is used as the preservative.

Wellhouse Process.—Because of the great solubility of zinc chloride, this method was devised to insure the permanency of the antiseptic salt in the wood cell. A ½% solution of glue and zinc chloride of 1½ to 3% strength is forced into the timber, after which a ½% tannin solution under pressure is injected as a separate treatment for about 2 hours and then withdrawn. A "leatheroid" coating is formed which prevents the subsequent absorption of water and consequent leeching out of the zinc chloride.

Card Process.—In this treatment, patented by J. P. Card in March, 1906, the timber is thoroughly air seasoned and run into the retorts, where it is subjected to a vacuum of 22 to 26 in for one hour. The liquid solution, consisting of an emulsion of chloride of zinc with 20% of creosote oil, is then introduced into the tanks at a temperature of 180° and under a pressure of 125 lb per sq in, maintained from 3 to 5 hours.

Allardyce Process.—This treatment consists in impregnating the timber with a 2% zinc chloride solution followed by an injection of about 3 lb of creosote oil per cubic foot of wood. The Card treatment injects both elements in the one treatment.

Curtis Boiling Process.—This method is extensively used on the Pacific Coast for the treatment of Douglas fir. The timber, usually green or partially seasoned, is run on trucks into the cylinder, which is then filled with creosote oil introduced at about 160° F. and raised to about 212° F. This hot bath is maintained for periods ranging from 4 to 8 hours, depending upon the size and moisture condition of the lumber. The water and volatilized light oils are driven off from the wood and the vapors are caught in a surface condenser. The water is decanted off and the oil is run back to the receiving tank. Creosote oil is then applied at a pressure of 100 to 140 lb per sq in, and at the same time the temperature is allowed to fall to 200° F. The pressure is maintained until the required amount of oil has been injected into the timber.

Ruping Process.—The timber should be either thoroughly air seasoned or steam-dried before treatment. The charge is run into the cylinder and subjected to an air pressure of 75 lb per sq in until all cells and cavities are filled. Creosote oil is then forced into the cylinder at a pressure of 80 to 85 lb per sq in, the air being allowed to escape in proportionate quantities from a valve at the top of the cylinder. After the timber is all covered with the preservative, the pressure is increased to 225 lb per sq in and maintained until the timber will not absorb any appreciable quantity of the liquid. The final absorption depends upon the relative air and solution pressures which may be varied or raised and lowered at the same time. The valves are then opened and the surplus oil is drained off into the receiving tanks.

Lowry Process.—Selected air-seasoned timber of same species and density is run into the retort on cars, but no compress air is used. The cylinder is filled from the charging tank with creosote oil at a temperature of 200° F., and pressure is applied until the timber takes up oil to a predetermined amount. The pressure is then released and the oil drained off and a strong quick vacuum is substituted to recover the free oil.

Buehler Process.—Green timber, stacked up in the cylinder with ½ in spaces between the pieces, is submerged in creosote oil at a temperature not less than 140° F., and kept covered during the entire seasoning period. Steam is then passed thru heating coils, until the temperature rises to 250° F., accompanied by volatilization and condensation in a hot well. The temperature is maintained until the condensation in the hot well does not exceed ⅙ lb of water per cu ft of charge. The pressure in the cylinder during seasoning should not exceed 5 lb per sq in. After seasoning, the steam pressure is released and the cylinder filled up with creosote oil from the measuring tank, and the temperature allowed to drop to 200° F. The pump pressure is applied until the timber absorbs the proper amount of oil. The cylinder is drained and air at a pressure of 20 lb per sq in applied so that the air penetrates the cells and drives out surplus oil. Seasoned timber is stacked in the cylinder and a vacuum is drawn until a minimum of 20 in is obtained and maintained for 20 minutes. Creosote oil is then introduced and pressure applied until the proper amount of oil is injected.

Non-Pressure Processes.—Preservatives of a penetrating nature, as tar oils, carbolineum, spirittine, etc., may be applied to the outside of wood either by spraying, by brush application, or by immersion in an open tank. The wood must be absolutely dry and seasoned so as to absorb a sufficient quantity of the preservative. All tar oil products should preferably be applied hot. Alternate hot and cold baths are occasionally employed. The timber, first peeled and thoroughly seasoned, is immersed in the preservative at about 200° F. for 5 to 6 hours, to drive off all the moisture.

Kyan's Process.—The timber is steeped in a solution of corrosive sublimate, 1 part of bichloride of mercury to 99 parts of water for a period (at least 5 to 10 days) sufficient to insure thorough penetration of the preservative. Sublimate is comparatively insoluble in water and remains in timber for a longer time than salts like zinc chloride.

Physical Properties.—Strength tests of the United States Forest Service in 1907 (H. F. Weiss, Amer. Wood Preservers' Assn., 1913) showed that both zinc chloride and creosote reduced the transverse strength of timber, tho only to a slight extent in the case of creosote oil. Transverse and compression tests made by H. B. MacFarland (Vol. XIV, Amer. Railway Eng. Assn., 1913) on long leaf pine showed greatly increased strength after one year for creosote treated wood. When tested immediately after treatment the strength was inferior. Thoroughly seasoned timber is stronger than green timber. Some processes tend to increase the moisture content and others to diminish it. If creosote is applied to green wood, the strengthening action of water evaporation is retarded. On the other hand, zinc chloride may cause a chemical dissolution of the wood fiber, thus weakening the structure. Zinc chloride is non-flammable, and wood so preserved is more fire-resisting than non-treated wood. (H. F. Weiss, Amer. Wood Preservers' Assn., 1913.) Zinc treated wood ignited at a temperature of 500° C., 19% of the wood by weight being consumed. Freshly creosoted wood ignited at 225° F., 27% of the wood by weight being consumed. Creosoted wood is adapted for exterior work subject to moisture and is especially effective against marine borers.

44. Cements and Plasters

For natural and portland cement see Sect. 5. For lime see Sect. 6, Art. 13, and Sect, 13, Art. 3. HYDRAULIC LIME is obtained by burning limestone containing from 10 to 20 percent of clay. MAGNESIAN LIME is the term applied to limes containing more than 5% (usually 30% and over) of magnesia. It has the characteristics of being slow slaking or cool, but sets more rapidly and makes a stronger mortar than the high-calcium limes. Lime is never used alone as a binding material, as it shrinks very much in drying. One part of lime paste is combined with 3 or 4 parts of sand, the resulting mortar being about equal in volume to that of the sand.

Grappier Cements are made by grinding lumps of unburned limestone and overburned material which remains when a hydraulic lime is slaked. If lime silicate predominates in the mixture, grappier cement approximates to Portland.

Lafarge Cement is a hydraulic grappier cement manufactured at Teil, France. It contains a small percentage of iron and soluble salts and does not stain porous stone masonry.

Plasters are classified by E. C. Eckel as (1) those produced by incomplete dehydration of gypsum ($CaSO_4$), the calcination being carried on at a temperature not exceeding 400° F, and (2) those produced by complete dehydration of calcium sulphate at a temperature exceeding 400° F. PLASTER OF PARIS is a product of the first class, no foreign material being added to the gypsum during or after calcination. CEMENT PLASTER is also of the first class, but is manufactured from an impure gypsum, or by the addition of certain impurities during manufacture to act as a "retarder" to the plaster. In the second class are included flooring plaster, commercially known as calcined plaster, being pure calcined gypsum, and also hard-finish plaster, obtained by adding alum or borax during manufacture. Structural gypsum weighs 80 lb per cu ft and cement plaster about 115 lb per cu ft.

Common or Lime Plaster is made of quick lime and sand in which hair or fiber is beaten up and thoroly incorporated with the lime paste. WALL PLASTERS are made by the addition of lime and retarder to the calcined plaster.

The Strength of Plaster as determined from tests by Woolson is shown by the following tables, see Proc. Am. Soc. for Test. Materials, Vol. X, p. 328:

Tensile Strength of Plaster

Values given in lb per sq in

Kind of lime	1 : 3 lime mortar				1 : 5 lime mortar			
Age	1 mo	3 mo	6 mo	12 mo	1 mo	3 mo	6 mo	12 mo
High-calcium (quicklime)........	27	34	47	60	25	36	38	39
High-calcium (hydrated lime)....	80	103	113	122	46	62	73	79
Dolomitic (quicklime)...........	56	60	74	125	51	72	93	124
Dolomitic (hydrated lime)	88	90	96	130	27	56	84	122

Compressive Strength of Plaster

Values given in lb per sq in

Kind of lime	1 : 3 lime mortar				1 : 5 lime mortar			
Age	1 mo	3 mo	6 mo	12 mo	1 mo	3 mo	6 mo	12 mo
High-calcium (quicklime)........	160	120	130	230	90	95	125	250
High-calcium (hydrated lime)....	180	750	740	750	175	320	405	485
Domomitic (quicklime)..........	370	350	340	355	170	260	350	530
Dolomitic (hydrated lime).......	270	530	710	1035	190	305	450	730

The Compressive Strength of Gypsum varies from 70 lb per sq in to 3000 lb per sq in depending on the amount of water used in mixing the gypsum paste, the completeness of drying out, the foreign ingredients in the gypsum, and the process of calcination used. For highest strength the least possible amount of water should be used in mixing. From 33 to 38 percent of water is necessary to make the gypsum paste sufficiently plastic to fill molds. From tests by W. A. Slater for the U. S. Gypsum Co. it was found possible to produce regularly gypsum having a compressive strength of 1400 lb per sq in when a few days old. The modulus of elasticity was found to be about 1 000 000 lb per sq in.

45. Miscellaneous Materials

Leather as an engineering material is mostly applied in the manufacturing of belting for transmission trains. The best quality of well-tanned ox-hide is cut into strips of from 4 to 6 ft long and usually about 3/16 in in thickness, which are scarfed, spliced, or cemented end to end to make the desired length of belt. According to the strength required, these are in turn cemented or riveted together in thickness to form " single " or " double " belts. Under light loads, the " single belt " gives the greater adhesion, but under heavy loads the " double belt " proves the more satisfactory. The " flesh side " or inside of the belt is customarily placed next to the pulley, as it gives the best wear, altho when placed grain side to the pulley the belt is less liable to slip.

The Weight of a hard well-tanned belt leather is about 62½ lb. per cu ft, while the tensile strength of a good quality is about 650 lb per in of width of single belt. When spliced or riveted, the tenacity is about one-half of the above figure, and when laced about one-third of the strength is developed. A safe working tension may be taken at about 50 lb per in of width.

Rawhide, or untanned leather, finds many applications in textile machinery connections, looms, ships' tiller ropes, etc. When sound, it is much stronger than tanned leather and gives greater resistance to violent impact. Its tensile strength may be taken as one-half greater than that of tanned leather.

Tensile Tests of Rubber Belting

From Watertown Arsenal Reports of 1893

Width and kind of belt	Actual dimensions, inches			Weight per ft	Sectional area	Tensile strength, pounds		Remarks on fracture
	Length	Width	Thickness	Lb oz	Sq in	Per sq inch	Per in of width	
2″ 4 ply	60.17	2.02	0.26	0 4.4	0.525	3276	851	At face of jaws.
6″ "	60.17	6.08	0.26	0 13.2	1.58	3227	839	3″ from jaws.
6″ "	60.12	6.13	0.26	0 13.6	1.59	3773	979	12″ from jaws.
6″ "	60.17	6.05	0.26	0 13.0	1.57	2739	711	At face of jaws.
12″ "	60.02	12.08	0.27	1 11.4	3.26	3037	819	10″ inside gaged length of 30″
12″ "	60.14	12.24	0.26	1 11.5	3.18	2987	776	Near middle.
2″ 6 ply	60.17	2.14	0.36	0 6.0	0.77	3104	1116	4″ from jaws.
6″ "	59.98	6.26	0.37	1 3.1	2.32	2737	1014	At jaws.
6″ "	60.08	6.27	0.36	1 2.9	2.26	3770	1358	Near middle.
12″ "	60.15	12.04	0.36	2 3.7	4.33	3436	1236	At face of jaws.
12″ "	60.17	12.16	0.34	2 3.7	4.13	3862	1311	At middle.
24″ "	60.13	24.11	0.41	4 14.2	9.89	2381	977	At face of jaws.
30″ "	60.04	30.18	0.40	6 2.3	12.07	2808	1123	16¾″ from face of jaws

Tensile Tests of Leather Belting

From Watertown Arsenal Reports of 1893

Width and kind of belt	Actual dimensions, inches			Weight per ft	Sectional area	Tensile strength, pounds		Remarks on fracture
	Length	Width	Thickness	Lb oz	Sq in	Per sq inch	Per in of width	
2″ single..	60.00	1.98	0.20	0 2.75	0.396	5045	1091	[illegible]″ from jaws.
6″ "	60.20	6.07	0.22	0 8.70	1.34	2537	560	At scarf joint.
6″ " *	60.11	6.08	0.24	0 9.6	1.46	2219	533	" " "
12″ "	60.11	12.05	0.18	0 14.9	2.17	3917	705	" " "
12″ "	60.30	12.05	0.24	1 3.2	2.89	1557	373	" " "
12″ " *	....	....	....	....	......	3598	863	5″ from jaws.
2″ double .	60.00	2.07	0.42	0 5.35	0.869	4025	1690	In jaws of machine at end of scarf joint.
4″ "	59.55	3.98	0.33	0 8.3	1.31	4931	1623	At scarf joint.
6″ "	60.18	5.91	0.47	1 1.0	2.78	4309	2027	" " "
6″ " *	59.93	6.00	0.40	1 0.6	2.40	5166	2066	" " "
12″ "	59.90	11.90	0.39	1 12.8	4.64	4090	1595	" " "
12″ "	60.06	11.93	0.36	1 14.0	4.29	4424	1591	4″ from jaws.
24″ "	60.00	23.90	0.47	4 8.0	11.23	2760	1297	At scarf joint.
30″ "	59.90	29.95	0.43	4 12.7	12.88	2717	1169	" " "

* Waterproofed.

India Rubber is employed in bands for belting, sheets for packing, tires, electrical insulation, etc. The rubber used by the engineer is generally vulcanized by heating and incorporating it with 20 to 30 percent of sulfur to render

it less readily softened by heat and hardened by cold. When vulcanized with 30 to 40 percent of sulfur, it forms the various qualities of ebonite used in the manufacture of rules, scales, curves, etc. India-rubber belts are made by weaving cotton canvas of the required length and width, which is then coated with the vulcanized rubber. These are made 2, 3, or 4 ply, according to the power to be transmitted requires the strength of 2, 3, or 4 thicknesses of belt. Rubber belts are superior to leather belts in strength, usually run truer and more smoothly, are perfectly impervious to water, and give a higher coefficient of friction. However, when overloaded they are apt to be injured by the tendency to slipping.

Glass. The tensile strength of common glass varies from 2000 to 3000 lb per sq in, and the compressive strength from 6000 to 10 000 lb per sq in. A series of transverse tests were undertaken on common glass at the Watertown Arsenal in 1902. Sheets of common window glass 2′ 6″ long by about 4.95 in wide and 0.121 in thick were subjected to a central load with an unsupported length of 2′ 0″. The modulus of rupture varied from 3000 to 4000 lb per sq in and the modulus of elasticity from 10 000 000 to 11 000 000 lb per sq in.

Glue. Thurston gives the following values for the absolute strength of well-glued joints in lb per sq in. For calculating the working strength of wooden surfaces joined with glue, the safe resistance may be figured at one-sixth of these values.

Kind of wood	Across grain, end to end	With grain
Beech	2133	1095
Elm	1436	1126
Oak	1735	568
Whitewood	1493	341
Maple	1422	896

Ice at 32° F. weighs 57.5 lb per cu ft, its specific gravity being 0.922 (water at 62° F. = 1). Its volume relative to water is 1.0855. Its melting point decreases from 32° F. at the rate of 0.0133° F. for each additional atmospheric pressure. Its specific heat is 0.504 (water = 1). Some German experiments made in 1885 gave a tensile strength of 142 to 223 lb per sq in. Tests made by U. S. Engineer Corps in 1880, on 6- and 12-inch cubes, gave crushing strengths varying from 100 to 1000 lb per sq in depending on the structure of the ice and the purity of water from which it was formed. Before crushing, ice in cubes will compress from 6 to 30 percent. The sustaining capacity of ice is not definitely determined; 2-in ice is considered safe for infantry, 4-in ice for cavalry or light guns, 6-in ice for heavy field guns, and 8-in ice for loads not over 1000 lb per sq ft on sledges. Railway trains have been run across ice which was 15 inches thick.

The expansive force of ice is given by Trautwine as probably not less than 30 000 lb per sq ft. The coefficient of expansion, as given by Ganot, is 0.000 052. By its expansion a sheet of ice 150 ft in width has been known to tip a masonry bridge pier weighing 1000 tons two inches out of plumb, and in another instance to move masonry piers on pile foundations from 2 to 12 inches out of line. The expansive effect in river or lake ice, however, does not make itself felt until the ice is at least 5 inches thick.

Freshly fallen snow weighs from 5 to 12 lb per cu ft; compacted or wet snow weighs from 15 to 50 lb per cu ft.

46. Paints and Oils

A Paint consists of a base, a vehicle, and a solvent; and many paints have also a stain and a drier added. The whites, as white lead and zinc white, form the base upon which nearly all tints are made by the addition of colored pigments called stains. Red lead and oxide of iron may also be used as bases for paint without using a white base or they may be used as stains. The usual VEHICLE for the base is linseed oil. Its function is to enable the paint to be spread and also to form a binder for the paint materials after drying. The SOLVENT most used is spirits of turpentine. Its function is to make the paint work more smoothly. The drying of the paint is often hastened by the addition of a drier, whose function is to hasten the solidification of the linseed oil by making its oxidation more rapid.

White Lead is a combination of lead carbonate and lead hydrate. The former gives the opacity or body, while the latter gives the saponifying and binding properties. White lead is supplied to the trade ground in linseed oil. It is a good drier of this oil, and very little artificial drier need be added to a white-lead paint.

Zinc White is the oxide of zinc. It is a permanent white pigment, and so is well adapted to interior decoration. Sulphureted hydrogen gases have no discoloring effect upon it, while such gases will darken a white-lead paint.

Red Lead is a double oxide of lead. It is used as a constituent of a priming paint for new woodwork, and its excellent anti-corrosive properties make it the best primer for coating ironwork. It excels all other pigments in withstanding abrasive wear. It is a strong drier of linseed oil, solidifying it in a short time, and it is for this reason that the pigment is sold in the dry powdered state.

Raw Linseed Oil is produced by pressing flaxseed. Boiled linseed oil is prepared by heating the raw oil either alone or with a drier such as red lead. Boiled oil dries in about one-half the time that the raw oil does. For this reason the boiled oil is much used for exterior work. For interior work and for grinding up colors the raw oil is used. Linseed oil is subject to adulteration by the addition of cotton-seed, resin, hemp, mineral, and fish oils. As substitutes, fish oil, cotton-seed oil, and Chinese tung oil are used.

Spirits of turpentine is a solvent or diluent used with paints and varnishes. It is also employed for mixing pigments in making flatting colors for interior decoration. It is often adulterated with mineral oil. As substitutes, benzine and naphtha are used.

Staining Colors or Pigments are used for tinting paints which have a white base such as white lead or white zinc. The most common black pigments are the soot and charcoal blacks. The best known red pigments are Venetian red, which is an earth with ferric oxide as the coloring agent, and red lead. The brown pigments are burnt umber, and the raw and burnt siennas, which resemble umber in composition. The green mostly used is chrome green, a pigment compounded of Prussian blue and chrome yellow. The standard commercial mixture of chrome green contains three parts of base to one of chrome green. Prussian blue is the common blue pigment which has great strength of coloring matter. Another important pigment is the ultramarine blue, an artificial chemical product of complex composition. As this latter pigment has some sulphur, it should not be mixt with white lead but should be used with zinc white as a base.

Graphite Paint is prepared by mixing graphite with boiled linseed oil to which a small percentage of drier has been added. It is an excellent paint for iron. One objection to this paint is that it mars easily under slight abrasion.

Asphalt paints are prepared by dissolving bitumen in paraffin, petroleum, naphtha, and benzine. This paint is used as a protection for ironwork against moisture.

Varnish is made by dissolving gums or resins in oil, turpentine, or alcohol. With the drying of the varnish, a smooth, solid, and transparent resin is left, forming the varnished surface.

STANDARD MARKET SIZES

47. Rolled Structural Beams

Structural Beams and Shapes should be ordered by weight per foot or by the thickness wanted. A variation of 2½ percent either way is allowed in the nominal weight of the shape. Unless otherwise arranged, structural shapes are cut to lengths as ordered with an extreme variation of ¾ in. For cutting with a less variation, an extra charge is made.

The increase of weight in I beams and channels is secured by increasing the web uniformly in thickness for the depth of the beam. This results also in an increased width of flange equal to the increase in web thickness. In angles the increase is made by increasing uniformly the thickness of the two legs. The effect of the spreading of the rolls to secure this additional thickness is to increase the length of the legs, amounting to about 1/16 in for each 1/16 in increase in thickness. As most sizes, however, are rolled in finishing grooves, the exact dimensions are generally maintained for different thicknesses. In Z bars the increase is made by increasing uniformly and by equal amounts the thickness of web and both legs. T shapes do not admit of any variation in weight, and are rolled only to fixt dimensions and weights.

Flanges of standard I beams and channels have a uniform slope of 2 in per ft, or 16⅔ percent. Flanges of Bethlehem I and girder beams have a uniform slope of 12½ percent. On standard I beams and channels small fillets have a radius of 0.6 of the minimum web thickness, and large fillets a radius of the minimum web thickness plus 0.1.

The Section Modulus of the tables is the moment of inertia of the shape divided by the distance of the center of gravity from the top or bottom of the section. It is used to determine the fiber stress per sq in in a beam or other shape, by dividing it into the bending moment exprest in inch-pounds. It may also be used to guide in the selection of a beam, or other shape, required to sustain a girder load, by dividing the bending moment, in inch-pounds, by the allowable fiber stress per sq in, and finding the required shape in the tables. In those shapes which are not symmetrical about the neutral axis there are, for each case, two section moduli; and in the tables the smaller is always given. The fiber stress calculated from it will be the greater stress and would generally be the one sought.

Compared with the lightest standard beams of like depth the lightest Bethlehem special I beams are designed so as to possess the same section moduli at weights uniformly 10 percent less. The Bethlehem girder beams are so designed that the lightest section of a given depth below 24 in possesses just twice the section modulus of the lightest standard beam of corresponding depth, whereas the weight of the former averages about 12.5 percent less than double the weight of the latter.

The Coefficient of Strength given in the last column of the tables is the calculated load in pounds that will produce a stress of 16 000 lb per sq in on the extreme fiber when the span is 1 ft. By dividing this coefficient by the span in feet, the safe load uniformly distributed in pounds is quickly found. This may be conveniently exprest in the following formula:

$$C = Wl = 8\,M = 8\,SI/12\,c$$

in which C = coefficient given in tables, W = safe load in pounds uniformly distributed, M = bending moment in ft-lbs, S = the extreme fiber stress in lb per sq in = 16 000 for values given in tables, I = moment of inertia of section, and c = distance of extreme fiber from center of gravity of section. Following are examples of the use of C.

(*a*) If the safe carrying capacity of a 20-in 80-lb I beam over a clear span of 20 ft is wanted, it is obtained by dividing 1 564 300, the corresponding coefficient, by 20, the span in ft, giving 78 215 lbs or 39.1 tons as the safe uniformly distributed load.

(*b*) Given a load of 25 tons to be carried on a girder of 16 ft span, the size of beam required may be obtained by multiplying the load in lb ($25 \times 2000 = 50\,000$) by the span in ft, 16, obtaining a coefficient of 800 000. A glance thru the tables shows that any of the following will safely sustain the load: 1 15-in 60-lb or 2 12-in 35-lb standard beams, or 1 15-in 54-lb Bethlehem I beam, or 1 12-in 70-lb Bethlehem girder beam, or 2 15-in 33-lb channels.

(*c*) Given a fireproof floor construction having a weight of 90 lb per sq ft, and intended for a live load of 60 lb per sq ft, the span between walls being 20 ft, what beams and spacing would be required to secure the most economical layout? The gross load to be carried is $90+60=150$ lb per sq ft. The uniformly distributed load that would come on each floor beam would be $150 \times s$, the spacing of the beams, multiplied by 20, their span; and the coefficient would be this product times the span, 20 ft. Exprest in form of an equation: $C = wl^2s = 150 \times 20 \times 20 \times s = 60\,000\,s$. The spacing of the floor beams (assumed to be uniform) would depend on the length of the floor. Assuming that at 48 ft, we could have 12 spaces at 4 ft, 11 spaces at 4.36 ft, 10 spaces at 4.8 ft, 9 spaces at 5.33 ft, 8 spaces at 6 ft, 7 spaces at 6.86 ft, or 6 spaces at 8 ft. These would give respectively, coefficients of 240 000, 267 000, 288 000, 319 800, 360 000, 411 600, and 480 000, by means of which the required beams could be selected from the tables. When the corresponding standard or Bethlehem beams have been selected and their weights divided by the spacing, it will appear that the most economical arrangement would be either 12-in 28.5-lb Bethlehem I beams spaced 6 ft, or 15-in 38-lb Bethlehem I beams spaced 8 ft.

(*d*) Conversely, the safe carrying capacity of a certain floor construction might be quickly determined by dividing the coefficient of strength of the beams used by the square of the span in ft and again by the spacing in ft, thus: If the floor consists of 12-in 31.5-lb standard beams, spaced 6 ft on centers, having a span of 16 ft, the gross floor capacity will be $383\,700 \div (16 \times 16 \times 6) = 249.8$ lb per sq. ft. Taking from this the dead weight of the floor construction, the live load capacity could be obtained.

Where the load on the girder is not uniformly distributed, the coefficient may still be used to advantage by determining the maximum bending moment in ft-lbs and multiplying by 8. The result is the coefficient to be sought in the table.

For any other fiber stress than 16 000 the values of C can be obtained by direct proportion; thus for a fiber stress of 12 500 lbs per sq in, used for moving loads as on bridges, the value of the coefficient C for a 15-in 60-lb standard I beam would be 125/160 of 866 100, or 676 600 (to nearest 100).

In the values for C it is assumed that the shapes are secured against deflecting sideways by lateral supports at intervals not exceeding 20 times the width of the compression flange. If not so supported the values of C should be reduced as follows for the unsupported lengths indicated: For 30 times flange width use 0.9 tabular load, for 40 times use 0.8, for 50 times use 0.7, for 60 times use 0.6, and for 70 times use 0.5.

Unsymmetrical Loading of Beams. It should be noted that the stresses in rolled structural shapes given by the following tables are correct only if the loads are applied in a direction parallel to or perpendicular to an axis of symmetry of the cross-section. If the loads are in any other direction the stresses set up may be much higher than the stresses given by the tables. The computation of stresses for unsymmetrical loading of beams is very complicated, see article by L. J. Johnson in the Trans. A. S. C. E. Vol. LVI, p. 169. Unsymmetrical loading of beams is the exception rather than the rule, but is of importance in such cases as roof purlins, and for angles and Z-bars in flexure.

Properties of I Beams—**Continued on p. 440**

Dimensions are given in inches

Depth of beam	Lb per foot	Area of section	Thickness of web	Width of flange	Neutral axis perpendicular to web at center			Neutral axis coincident with center line of web		Coefficient of strength for fiber stress of 16 000 lb per sq in, C
					Moment of inertia, I	Radius of gyration, r	Section modulus, I/c	Moment of inertia, I'	Radius of gyration, r'	
27	83.00	24.41	0.424	7.500	2888.6	10.88	214.0	53.1	1.47	2 282 400
	115.00	33.98	0.750	8.000	2955.5	9.33	246.3	83.2	1.57	2 626 900
*24	110.00	32.48	0.688	7.938	2883.5	9.42	240.3	81.0	1.58	2 563 300
	105.00	30.98	0.625	7.875	2811.5	9.53	234.3	78.9	1.60	2 499 000
	100.00	29.41	0.754	7.254	2380.3	9.00	198.4	48.56	1.28	2 115 800
	95.00	27.94	0.692	7.192	2309.6	9.09	192.5	47.10	1.30	2 052 900
24	90.00	26.47	0.631	7.131	2239.1	9.20	186.6	45.70	1.31	1 990 300
	85.00	25.00	0.570	7.070	2168.6	9.31	180.7	44.35	1.33	1 927 600
	80.00	23.32	0.500	7.000	2087.9	9.46	174.0	42.86	1.36	1 855 900
24	69.50	20.44	0.390	7.000	1928.0	9.71	160.7	39.3	1.39	1 714 000
*21	57.50	16.85	0.357	6.500	1227.5	8.54	116.9	28.4	1.30	1 246 700
	100.00	29.41	0.884	7.284	1655.8	7.50	165.6	52.65	1.34	1 766 100
	95.00	27.94	0.810	7.210	1606.8	7.58	160.7	50.78	1.35	1 713 900
*20	90.00	26.47	0.737	7.137	1557.8	7.67	155.8	48.98	1.36	1 661 600
	85.00	25.00	0.663	7.063	1508.7	7.77	150.9	47.25	1.37	1 609 300
	80.00	23.73	0.600	7.000	1466.5	7.86	146.7	45.81	1.39	1 564 300
	75.00	22.06	0.649	6.399	1268.9	7.58	126.9	30.25	1.17	1 353 500
20	70.00	20.59	0.575	6.325	1219.9	7.70	122.0	29.04	1.19	1 301 200
	65.00	19.08	0.500	6.250	1169.6	7.83	117.0	27.86	1.21	1 247 600
	90.00	26.47	0.807	7.245	1260.4	6.90	140.0	52.0	1.40	1 493 300
*18	85.00	25.00	0.725	7.163	1220.7	6.99	135.6	50.0	1.42	1 446 200
	80.00	23.53	0.644	7.082	1181.0	7.09	131.2	48.1	1.43	1 399 400
	75.00	22.05	0.562	7.000	1141.3	7.19	126.8	46.2	1.45	1 352 500
	70.00	20.59	0.719	6.259	921.3	6.69	102.4	24.62	1.09	1 091 900
	65.00	19.12	0.637	6.177	881.5	6.79	97.9	23.47	1.11	1 044 800
18	60.00	17.65	0.555	6.095	841.8	6.91	93.5	22.38	1.13	997 700
	55.00	15.93	0.460	6.000	795.6	7.07	88.4	21.19	1.15	943 000
*18	46.00	13.53	0.322	6.000	733.2	7.36	81.5	19.90	1.21	869 300
	100.00	29.41	1.184	6.774	900.5	5.53	120.1	50.98	1.31	1 280 700
	95.00	27.94	1.085	6.675	872.9	5.59	116.4	48.37	1.32	1 241 500
*15	90.00	26.47	0.987	6.577	845.4	5.65	112.7	45.91	1.32	1 202 300
	85.00	25.00	0.889	6.479	817.8	5.72	109.0	43.57	1.32	1 163 000
	80.00	23.81	0.810	6.400	795.5	5.78	106.1	41.76	1.32	1 131 300
	75.00	22.06	0.882	6.292	691.2	5.60	92.2	30.68	1.18	983 000
	70.00	20.59	0.784	6.194	663.6	5.68	88.5	29.00	1.19	943 800
*15	65.00	19.12	0.686	6.096	636.0	5.77	84.8	27.42	1.20	904 600
	60.00	17.67	0.590	6.000	609.0	5.87	81.2	25.96	1.21	866 100
	55.00	16.18	0.656	5.746	511.0	5.62	68.1	17.06	1.02	726 800
15	50.00	14.71	0.558	5.648	483.4	5.73	64.5	16.04	1.04	687 500
	45.00	13.24	0.460	5.550	455.8	5.87	60.8	15.00	1.07	648 200
	42.00	12.48	0.410	5.500	441.7	5.95	58.9	14.62	1.08	628 300
*15	36.00	10.63	0.289	5.500	405.1	6.17	54.0	13.5	1.13	575 900
	55.00	16.18	0.822	5.612	321.0	4.45	53.5	17.46	1.04	570 600
*12	50.00	14.71	0.699	5.489	303.3	4.54	50.6	16.12	1.05	539 200
	45.00	13.24	0.576	5.366	285.7	4.65	47.6	14.89	1.06	507 900
	40.00	11.84	0.460	5.250	268.9	4.77	44.8	13.81	1.08	478 100
12	35.00	10.29	0.436	5.086	228.3	4.71	38.0	10.07	0.99	405 800
	31.50	9.26	0.350	5.000	215.8	4.83	36.0	9.50	1.01	383 700
*12	27.50	8.04	0.255	5.000	199.6	4.98	33.3	8.7	1.04	355 200

* Beams marked with a * are listed by some steel makers as special beams, command a slightly higher price per pound than standard beams, and, in small lots, are not always available to the customer.

Properties of I Beams—Continued
Dimensions are given in inches

Depth of beam	Lb per foot	Area of section	Thickness of web	Width of flange	Neutral axis perpendicular to web at center: Moment of inertia, I	Radius of gyration, r	Section modulus, I/c	Neutral axis coincident with center line of web: Moment of inertia, I'	Radius of gyration, r'	Coefficient of strength for fiber stress of 16 000 lb per sq in, C
10	40.00	11.76	0.749	5.099	158.7	3.67	31.7	9.50	0.90	338 500
	35.00	10.29	0.602	4.952	146.4	3.77	29.3	8.52	0.91	312 400
	30.00	8.82	0.455	4.805	134.2	3.90	26.8	7.65	0.93	286 300
	25.00	7.37	0.310	4.660	122.1	4.07	24.4	6.89	0.97	260 500
*10	22.00	6.52	0.232	4.670	113.9	4.18	22.8	6.4	0.99	243 200
9	35.00	10.29	0.732	4.772	111.8	3.29	24.8	7.31	0.84	265 000
	30.00	8.82	0.569	4.609	101.9	3.40	22.6	6.42	0.85	241 500
	25.00	7.35	0.406	4.446	91.9	3.54	20.4	5.65	0.88	217 900
	21.00	6.31	0.290	4.330	84.9	3.67	18.9	5.16	0.90	201 300
8	25.50	7.50	0.541	4.271	68.4	3.02	17.1	4.75	0.80	182 500
	23.00	6.76	0.449	4.179	64.5	3.09	16.1	4.39	0.81	172 000
	20.50	6.03	0.357	4.087	60.6	3.17	15.1	4.07	0.82	161 600
	18.00	5.33	0.270	4.000	56.9	3.27	14.2	3.78	0.84	151 700
*8	17.5	5.15	0.210	4.330	58.3	3.37	14.6	4.5	0.93	155 700
7	20.00	5.88	0.458	3.868	42.2	2.68	12.1	3.24	0.74	128 600
	17.50	5.15	0.353	3.763	39.2	2.76	11.2	2.94	0.76	119 400
	15.00	4.42	0.250	3.660	36.2	2.86	10.4	2.67	0.78	110 400
6	17.25	5.07	0.475	3.575	26.2	2.27	8.7	2.36	0.68	93 100
	14.75	4.34	0.352	3.452	24.0	2.35	8.0	2.09	0.69	85 300
	12.25	3.61	0.230	3.330	21.8	2.46	7.3	1.85	0.72	77 500
5	14.75	4.34	0.504	3.294	15.2	1.87	6.1	1.70	0.63	64 600
	12.25	3.60	0.357	3.147	13.6	1.94	5.4	1.45	0.63	58 100
	9.75	2.87	0.210	3.000	12.1	2.05	4.8	1.23	0.65	51 600
4	10.50	3.09	0.410	2.880	7.1	1.52	3.6	1.01	0.57	38 100
	9.50	2.79	0.337	2.807	6.7	1.55	3.4	0.93	0.58	36 000
	8.50	2.50	0.263	2.733	6.4	1.59	3.2	0.85	0.58	33 900
	7.50	2.21	0.190	2.660	6.0	1.64	3.0	0.77	0.59	31 800
3	7.50	2.21	0.361	2.521	2.9	1.15	1.9	0.60	0.52	20 700
	6.50	1.91	0.263	2.423	2.7	1.19	1.8	0.53	0.52	19 100
	5.50	1.63	0.170	2.330	2.5	1.23	1.7	0.46	0.53	17 600

*Beams marked with a * are listed by some steel makers as special beams, command a slightly higher price per pound than standard beams, and, in small lots, are not always available to the customer.

For shearing stress factors for I-beams, see p. 458.

Comparative Tests on standard I beams and Bethlehem special I beams and girder beams, made at the University of Pennsylvania in 1909, gave following values of the modulus of rupture in lb per sq in as an average of three specimens in each case. The 15-in beams were 15 ft in span, the others were 20 ft in span.

Standard I,	15-in, 42-lb,	central loading,	$S = 42\ 200$
Standard I,	15-in, 42-lb,	quarter point loading,	$S = 34\ 700$
Standard I,	24-in, 80-lb,	quarter point loading,	$S = 33\ 000$
Bethlehem I,	15-in, 38-lb,	central loading,	$S = 46\ 100$
Bethlehem I,	15-in, 38-lb,	quarter point loading,	$S = 37\ 900$
Bethlehem I,	24-in, 72-lb,	quarter point loading,	$S = 34\ 600$
Girder beam,	15-in, 73-lb,	central loading,	$S = 53\ 900$
Girder beam,	15-in, 73-lb.	quarter point loading,	$S = 41\ 100$
Girder beam,	24-in, 120-lb,	quarter point loading,	$S = 34\ 300$
Girder beam,	30-in, 175-lb,	quarter point loading,	$S = 31\ 000$

The modulus of elasticity for the 31 beams tested was nearly constant, its average value being 26 300 000 lbs per sq in. This value. deduced from the full-size tests, was somewhat less than that obtained from the tensile tests on specimens cut from the flange, web and root of the several beams; this average was 28 700 000.

The Bethlehem Structural Beams are wide-flange I sections rolled by the Grey universal beam mill. Instead of the horizontal grooved rolls of the ordinary beam mill, the Grey mill has both horizontal and vertical rolls, by which the flanges and web of an I beam shape are each produced by simultaneous combined rolling operations acting at right angles. This method of rolling makes it possible to obtain wider flanges than can be produced by the ordinary beam mill.

Properties of Bethlehem I Beams

Dimensions are given in Inches.

Depth of beam	Lb per foot	Area of section	Thickness of web	Width of flange	Neutral axis perpendicular to web at center			Neutral axis coincident with center line of web		Coefficient of strength for fiber stress of 16 000 lbs per sq in, C
					Moment of inertia, I	Radius of gyration, r	Section modulus, I/c	Moment of inertia, I'	Radius of gyration, r'	
30	120.0	35.30	540	10.500	5239.6	12.18	349.3	165.0	2.16	3 726 000
28	105.0	30.88	.500	10.000	4014.1	11.40	286.7	131.5	2.06	3 058 400
26	90.0	26.49	.460	9.500	2977.2	10.60	229.0	101.2	1.95	2 442 800
24	84.0	24.80	.460	9.250	2381.9	9.80	198.5	91.1	1.92	2 117 300
24	83.0	24.59	.520	9.130	2240.9	9.55	186.7	78.0	1.78	1 991 900
24	73.0	21.47	.390	9.000	2091.0	9.87	174.3	74.4	1.86	1 858 700
20	82.0	24.17	.570	8.890	1559.8	8.03	156.0	79.9	1.82	1 663 800
20	72.0	21.37	.430	8.750	1466.5	8.28	146.7	75.9	1.88	1 564 300
20	69.0	20.26	.520	8.145	1268.9	7.91	126.9	51.2	1.59	1 353 500
20	64.0	18.86	.450	8.075	1222.1	8.05	122.2	49.8	1.62	1 303 600
20	59.0	17.36	.375	8.000	1172.2	8.22	117.2	48.3	1.66	1 250 300
18	59.0	17.40	.495	7.675	883.3	7.12	98.1	39.1	1.50	1 046 900
18	54.0	15.87	.410	7.590	842.0	7.28	93.6	37.7	1.54	997 900
18	52.0	15.24	.375	7.555	825.0	7.36	91.7	37.1	1.56	977 700
18	48.5	14.25	320	7.500	798.3	7.48	88.7	36.2	1.59	946 100
15	71.0	20.95	.520	7.500	796.2	6.16	106.2	61.3	1.71	1132 400
15	64.0	18.81	.605	7.195	664.9	5.95	88.6	41.9	1.49	945 600
15	54.0	15.88	.410	7.000	610.0	6.20	81.3	38.3	1.55	867 600
15	46.0	13.52	.440	6.810	484.8	5.99	64.6	25.2	1.36	689 500
15	41.0	12.02	.340	6.710	456.7	6.16	60.9	24.0	1.41	649 400
15	38.0	11.27	.290	6.660	442.6	6.27	59.0	23.4	1.44	629 500
12	36.0	10.61	.310	6.300	269.2	5.04	44.9	21.3	1.42	478 600
12	32.0	9.44	.335	6.205	228.5	4.92	38.1	16.0	1.30	406 200
12	28.5	8.42	.250	6.120	216.2	5.07	36.0	15.3	1.35	384 400
10	28.5	8.34	.390	5.990	134.6	4.02	26.9	12.1	1.21	287 100
10	23.5	6.94	.250	5.850	122.9	4.21	24.6	11.2	1.27	262 200
9	24.0	7.04	.365	5.555	92.1	3.62	20.5	8.8	1.12	218 300
9	20.0	6.01	.250	5.440	85.1	3.76	18.9	8.2	1.17	201 800
8	19.5	5.78	.325	5.325	60.6	3.24	15.1	6.7	1.08	161 600
8	17.5	5.18	.250	5.250	57.4	3.33	14.3	6.4	1.11	153 000

Shearing Stress Factors for Bethlehem I Beams and Bethlehem Girder Beams. The maximum shearing stress in lb per sq in at the neutral axis of a beam is given by multiplying the total shear in pounds by the shearing factor in the table on p. 442 for the particular beam used.

Bethlehem I Beams

Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor
30	120.0	0.071	20	64.0	0.127	15	41.0	0.221
				59.0	0.151		38.0	0.258
28	105.0	0.082				12	36.0	0.304
			18	59.0	0.131		32.0	0.282
26	90.0	0.095		54.0	0.155		28.5	0.374
				52.0	0.180			
24	84.0	0.103		48.5	0.196	10	28.5	0.314
	83.0	0.092					23.5	0.480
	73.0	0.121	15	71.0	0.147	9	24.0	0.352
20	82.0	0.101		64.0	0.129		20.0	0.500
	72.0	0.132		54.0	0.185	8	19.5	0.441
	69.0	0.114		46.0	0.176		17.5	0.565
Bethlehem Girder Beams								
30	200.0	0.050	24	140.0	0.077	15	104.0	0.126
	180.0	0.055		120.0	0.087		73.0	0.173
28	180.0	0.058	20	140.0	0.086	12	70.0	0.203
	165.0	0.062		112.0	0.100		55.0	0.251
			18	92.0	0.129	10	44.0	0.360
26	160.0	0.068				9	38.0	0.412
	150.0	0.069	15	140.0	0.095	8	32.5	0.481

Properties of Bethlehem Girder Beams

Dimensions are given in Inches.

Depth of beam	Lb per foot	Area of section	Thickness of web	Width of flange	Neutral axis perpendicular to web at center			Neutral axis coincident with center line of web		Coefficient of strength for fiber stress of 16 000 lbs per sq in
					Moment of inertia, I	Radius of gyration, r	Section modulus, I/c	Moment of inertia, I'	Radius of gyration, r'	C
30	200.0	58.71	.750	15.00	9150.6	12.48	610.0	630.2	3.28	6 507 100
30	180.0	53.00	.690	13.00	8194.5	12.43	546.3	433.3	2.86	5 827 200
28	180.0	52.86	.690	14.35	7264.7	11.72	518.9	533.3	3.18	5 535 000
28	165.0	48.47	.660	12.50	6562.7	11.64	468.8	371.9	2.77	5 000 100
26	160.0	46.91	.630	13.60	5620.8	10.95	432.4	435.7	3.05	4 611 900
26	150.0	43.94	.630	12.00	5153.9	10.83	396.5	314.6	2.68	4 228 800
24	140.0	41.16	.600	13.00	4201.4	10.10	350.1	346.9	2.90	3 734 600
24	120.0	35.38	.530	12.00	3607.3	10.10	300.6	249.4	2.66	3 206 500
20	140.0	41.19	.640	12.50	2934.7	8.44	293.5	348.9	2.91	3 130 300
20	112.0	32.81	.550	12.00	2342.1	8.45	234.2	239.3	2.70	2 498 300
18	92.0	27.12	.480	11.50	1591.4	7.66	176.8	182.6	2.59	1 886 100
15	140.0	41.27	.800	11.75	1592.7	6.21	212.4	331.0	2.83	2 265 200
15	104.0	30.50	.600	11.25	1220.1	6.32	162.7	213.0	2.64	1 735 300
15	73.0	21.49	.430	10.50	883.4	6.41	117.8	123.2	2.39	1 256 600
12	70.0	20.58	.460	10.00	538.8	5.12	89.8	114.7	2.36	957 800
12	55.0	16.18	.370	9.75	432.0	5.17	72.0	81.1	2.24	768 000
10	44.0	12.95	.310	9.00	244.2	4.34	48.8	57.3	2.10	521 000
9	38.0	11.22	.300	8.50	170.9	3.90	38.0	44.1	1.98	405 000
8	32.5	9.54	.290	8.00	114.4	3.46	28.6	32.9	1.86	305 100

48. Rolled Structural Shapes

Properties of Bethlehem Rolled Steel H Column Sections—Continued on p. 444

Dimensions are given in inches

Section Number	Weight lb per foot	Depth of Section d	Width of Flange b	Thickness of Web	Area of Section A	For XX as neutral axis (see Fig. 60) Moment of Inertia I	Section Modulus I/½d	Radius of Gyration r	For YY as neutral axis (see Fig. 60) Moment of Inertia I′	Section Modulus I′/½b	Radius of Gyration r′
H 8	32.0	7⅞	8.00	.31	9.17	105.7	26.9	3.40	35.8	8.9	1.98
	34.5	8	8.00	.31	10.17	121.5	30.4	3.46	41.1	10.3	2.01
	39.0	8½	8.04	.35	11.50	139.5	34.3	3.48	47.2	11.7	2.03
	43.5	8¼	8.08	.39	12.83	158.3	38.4	3.51	53.4	13.2	2.04
	48.0	8⅜	8.12	.43	14.18	177.7	42.4	3.54	59.8	14.7	2.05
	53.0	8½	8.16	.47	15.53	197.8	46.5	3.57	66.3	16.3	2.07
	57.5	8⅝	8.20	.51	16.90	218.6	50.7	3.60	73.1	17.8	2.08
	62.0	8¾	8.24	.55	18.27	240.2	54.9	3.63	80.2	19.4	2.09
	67.0	8⅞	8.28	.59	19.66	262.5	59.2	3.65	87.1	21.0	2.11
	71.5	9	8.32	.63	21.05	285.6	63.5	3.68	94.4	22.7	2.12
	76.5	9⅛	8.36	.67	22.46	309.5	67.8	3.71	101.9	24.4	2.13
	81.0	9¼	8.39	.70	23.78	333.5	72.1	3.75	109.2	26.0	2.14
	85.5	9⅜	8.43	.74	25.20	359.0	76.6	3.77	117.2	27.8	2.16
	90.5	9½	8.47	.78	26.64	385.3	81.1	3.80	125.1	29.6	2.17
H 10	49.0	9⅞	9.97	.36	14.37	263.5	53.4	4.28	89.1	17.9	2.49
	54.0	10	10.00	.39	15.91	296.8	59.4	4.32	100.4	20.1	2.51
	59.5	10⅛	10.04	.43	17.57	331.9	65.6	4.35	112.2	22.3	2.53
	65.5	10¼	10.08	.47	19.23	368.0	71.8	4.37	124.2	24.6	2.54
	71.0	10⅜	10.12	.51	20.91	405.2	78.1	4.40	136.5	27.0	2.56
	77.0	10½	10.16	.55	22.59	443.6	84.5	4.43	149.1	29.4	2.57
	82.5	10⅝	10.20	.59	24.29	483.0	90.9	4.46	162.0	31.8	2.58
	88.5	10¾	10.24	.63	25.99	523.5	97.4	4.49	175.1	34.2	2.60
	94.0	10⅞	10.28	.67	27.71	565.2	103.9	4.52	188.6	36.7	2.61
	99.5	11	10.31	.70	29.32	607.0	110.4	4.55	201.7	39.1	2.62
	105.5	11⅛	10.35	.74	31.06	651.0	117.0	4.58	215.6	41.7	2.64
	111.5	11¼	10.39	.78	32.80	696.2	123.8	4.61	229.9	44.3	2.65
	117.5	11⅜	10.43	.82	34.55	742.7	130.6	4.64	244.4	46.9	2.66
	123.5	11½	10.47	.86	36.32	790.4	137.5	4.67	259.3	49.5	2.67

The clear distance between the flange fillets or the depth of the flat surface of the web available for connections, is 6.14 in for the H 8 sections, 7.67 in for the H 10 sections, 9.21 in for the H 12 sections, and 11.06 for the H 14 sections.

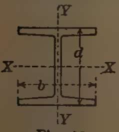


Fig. 60.

All columns having the same section number are from the same rolls. Whenever possible it is advisable to confine the selection of column to the same section number.

The section modulus is needed when the sections are to be used as beams, and also when columns are subject to bending. When used as beams the coefficient of strength (p. 437) can be found by multiplying the section modulus by ⅔S, where S is the allowable fiber unit stress.

These sections are widely used for columns in buildings. The allowable unit load recommended in the handbook of the Bethlehem Steel Co. is 13 000 pounds per square inch when the unsupported length of the column is less than 55 times the radius of gyration. For greater lengths the unit load is recommended to be $16\,000-55\ (l/r')$, where l is the unsupported length and r' is the least radius of gyration.

Properties of Bethlehem Rolled Steel H Column Sections (Continued)

Dimensions are given in inches

Section Number	Weight lb per foot	Depth of Section d	Width of Flange b	Thickness of Web	Area of Section A	For XX as neutral axis (see Fig. 60) Moment of Inertia I	Section Modulus I/½d	Radius of Gyration r	For YY as neutral axis (see Fig. 60) Moment of Inertia I′	Section Modulus I/½b	Radius of Gyration r
	64.5	11¾	11.92	.39	19.00	499.0	84.9	5.13	168.6	28.3	2.98
	71.5	11⅞	11.96	.43	20.96	556.6	93.7	5.15	188.2	31.5	3.00
	78.0	12	12.00	.47	22.94	615.6	102.6	5.18	208.1	34.7	3.01
	84.5	12⅛	12.04	.51	24.92	676.1	111.5	5.21	228.5	37.9	3.03
	91.5	12¼	12.08	.55	26.92	738.1	120.5	5.24	289.2	41.3	3.04
	98.5	12⅜	12.12	.59	28.92	801.7	129.6	5.27	270.1	44.6	3.06
H 12	105.0	12½	12.16	.63	30.94	866.8	138.6	5.30	291.7	48.0	3.07
	112.0	12⅝	12.30	.67	32.96	933.4	147.9	5 33	313.6	51.4	3.08
	118.5	12¾	12.23	.70	34.87	1000.0	156.9	5.36	335.0	54.8	3.10
	125.5	12⅞	12.27	.74	36.91	1069.8	166.2	5.38	357.7	58.3	3.11
	132.5	13	12.31	.78	38.97	1141.3	175.6	5.41	380.7	61.9	3.13
	139.5	13⅛	12.35	.82	41.03	1214.5	185.0	5.44	404.1	65.4	3.16
	146.5	13¼	12.39	.86	43.10	1289.4	194.6	5 47	428.0	69.1	3.15
	153.5	13⅜	12.43	.90	45.19	1366.0	204.3	5.50	452.2	72.8	3.16
	161.0	13½	12.47	.94	47.28	1444.3	214.0	5.53	477.0	76.5	3.18
	83.5	13¾	13.92	.43	24.46	884.9	128 7	6.01	294.5	42.3	3.47
	91.0	13⅞	13.96	.47	26.76	976.8	140.8	6.04	325.4	46.6	3.49
	99.0	14	14.00	.51	29.06	1070.6	153.0	6.07	356.9	51.0	3.50
	106.5	14⅛	14.04	.55	31.38	1166.6	165.2	6.10	387.8	55.2	3.52
	114.5	14¼	14.08	.59	33.70	1264.5	177.5	6 13	420.3	59.7	3.53
	122.5	14⅜	14.12	.63	36.04	1364.6	189.9	6 16	453.4	64.2	3.55
	130 5	14½	14.16	.67	38.38	1466.7	202.3	6.18	486.9	68.8	3.56
	138.0	14⅝	14.19	.70	40.59	1568.4	214.5	6.21	519.7	73.3	3.58
	146.0	14¾	14.23	.74	42.95	1674.7	227.1	6.24	554.4	77.9	3.59
	154.0	14⅞	14.27	.78	45.33	1783.3	239.8	6.27	589.5	82.6	3.61
	162.0	15	14.31	.82	47.71	1894.0	252.5	6.30	626.1	87.5	3.62
H 14	170.5	15⅛	14.35	.86	50.11	2007.0	265.4	6.33	662.3	92.3	3.64
	178.5	15¼	14.39	.90	52.51	2122.3	278.3	6.36	699.0	97.2	3.65
	186.5	15⅜	14.43	.94	54.92	2239.8	291.4	6.39	736.3	102.1	3.66
	195.0	15½	14.47	.98	57.35	2359.7	304.5	6.41	774.2	107.6	3.68
	203.5	15⅝	14.51	1.02	59 78	2481.9	317.7	6.44	812.6	112.0	3.69
	211.0	15¾	14.54	1.05	62.07	2603.3	330.6	6.48	849.8	116.9	3.70
	219.5	15⅞	14.58	1.09	64.52	2730.2	344.0	6.51	889.3	122.0	3.71
	227.5	16	14.62	1.13	66.98	2859.6	357.5	6.53	929.4	127.1	3.72
	236.0	16⅛	14.66	1.17	69.45	2991.5	371.0	6.56	970.0	132.3	3.74
	244.5	16¼	14.70	1.21	71.94	3125.8	384.7	6.59	1011.3	137.6	3.75
	253.0	16⅜	14.74	1.25	74.93	3262 7	398.5	6.62	1053.2	142.9	3.76
	261.5	16½	14.78	1.29	76.93	3402.1	412.4	6.65	1095.6	148.3	3.77
	270.0	16⅝	14.82	1.33	79.44	3544.1	426.4	6.68	1138.7	153.7	3.79
	278.5	16¾	14.86	1.37	81.97	3688.8	440.5	6.71	1182.4	159.1	3.80
	287.5	16⅞	14.90	1.41	84.50	3836.1	454.7	6.74	1226.7	164.7	3.81

Properties of Angles with Equal Legs—Continued on p. 446

Dimensions are given in inches

See paragraph on Unsymmetrical Loading of Beams, p. 438

Size	Thickness	Pounds per foot	Area of section	Distance of center of gravity from back of flange	Moment of inertia, neutral axis thru center of gravity parallel to flange I	Section modulus, neutral axis as before I/c	Radius of gyration, neutral axis as before r	Least radius of gyration, neutral axis thru center of gravity at angle of 45° to flanges r'
8×8	1 1/8	56.9	16.73	2.41	97.97	17.53	2.42	1.55
8×8	1 1/16	54.0	15.87	2.39	93.53	16.67	2.43	1.56
8×8	1	51.0	15.00	2.37	88.98	15.80	2.44	1.56
8×8	15/16	48.1	14.12	2.34	84.33	14.91	2.44	1.56
8×8	7/8	45.0	13.23	2.32	79.58	14.01	2.45	1.57
8×8	13/16	42.0	12.34	2.30	74.71	13.11	2.46	1.57
8×8	3/4	38.9	11.44	2.28	69.74	12.18	2.47	1.57
8×8	11/16	35.8	10.53	2.25	64.64	11.25	2.48	1.58
8×8	5/8	32.7	9.61	2.23	59.42	10.30	2.49	1.58
8×8	9/16	29.6	8.68	2.21	54.09	9.34	2.50	1.58
8×8	1/2	26.4	7.75	2.19	48.63	8.37	2.50	1.58
6×6	1	37.4	11.00	1.86	35.46	8.57	1.80	1.16
6×6	15/16	35.3	10.37	1.84	33.72	8.11	1.80	1.16
6×6	7/8	33.1	9.74	1.82	31.92	7.64	1.81	1.17
6×6	13/16	31.0	9.09	1.80	30.06	7.15	1.82	1.17
6×6	3/4	28.7	8.44	1.78	28.15	6.66	1.83	1.17
6×6	11/16	26.5	7.78	1.75	26.19	6.17	1.83	1.17
6×6	5/8	24.2	7.11	1.73	24.16	5.66	1.84	1.18
6×6	9/16	21.9	6.43	1.71	22.07	5.14	1.85	1.18
6×6	1/2	19.6	5.75	1.68	19.91	4.61	1.86	1.18
6×6	7/16	17.2	5.06	1.66	17.68	4.07	1.87	1.19
6×6	3/8	14.9	4.36	1.64	15.39	3.53	1.88	1.19
*5×5	1	30.6	9.00	1.61	19.64	5.80	1.48	0.96
*5×5	7/8	27.2	7.99	1.57	17.75	5.17	1.49	0.96
*5×5	3/4	23.6	6.94	1.52	15.74	4.53	1.51	0.97
*5×5	5/8	20.0	5.86	1.48	13.58	3.86	1.52	0.97
*5×5	1/2	16.2	4.75	1.43	11.25	3.15	1.54	0.98
*5×5	3/8	12.3	3.61	1.39	8.74	2.42	1.56	0.99
4×4	7/8	21.2	6.23	1.31	8.59	3.20	1.17	0.77
4×4	13/16	19.9	5.84	1.29	8.14	3.01	1.18	0.77
4×4	3/4	18.5	5.44	1.27	7.67	2.81	1.19	0.77
4×4	11/16	17.1	5.03	1.25	7.17	2.61	1.19	0.77
4×4	5/8	15.7	4.61	1.23	6.66	2.40	1.20	0.77
4×4	9/16	14.3	4.18	1.21	6.12	2.19	1.21	0.78
4×4	1/2	12.8	3.75	1.18	5.56	1.97	1.22	0.78
4×4	7/16	11.3	3.31	1.16	4.97	1.75	1.23	0.78
4×4	3/8	9.8	2.86	1.14	4.36	1.52	1.23	0.79
4×4	5/16	8.2	2.40	1.12	3.71	1.29	1.24	0.79
3½×3½	7/8	18.3	5.36	1.19	5.53	2.39	1.02	0.67
3½×3½	13/16	17.1	5.03	1.17	5.25	2.25	1.02	0.67
3½×3½	3/4	16.0	4.69	1.15	4.96	2.11	1.03	0.67
3½×3½	11/16	14.8	4.34	1.12	4.65	1.96	1.04	0.67
3½×3½	5/8	13.6	3.98	1.10	4.33	1.81	1.04	0.67
3½×3½	9/16	12.4	3.62	1.08	3.99	1.65	1.05	0.68
3½×3½	1/2	11.1	3.25	1.06	3.64	1.49	1.06	0.68
3½×3½	7/16	9.8	2.87	1.04	3.26	1.32	1.07	0.68
3½×3½	3/8	8.5	2.48	1.01	2.87	1.15	1.07	0.69
3½×3½	5/16	7.2	2.09	0.99	2.45	0.98	1.08	0.69

* Angles marked with a * are listed by some steel makers as special angles, command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the customer.

Properties of Angles with Equal Legs—Continued

Dimensions are given in inches

See paragraph on Unsymmetrical Loading of Beams, p. 438

Size	Thickness	Pounds per foot	Area of section	Distance of center of gravity from back of flange	Moment of inertia, neutral axis thru center of gravity parallel to flange I	Section modulus, neutral axis as before I/c	Radius of gyration, neutral axis as before r	Least radius of gyration, neutral axis thru center of gravity at angle of 45° to flanges r'
3×3	5/8	11.5	3.36	0.98	2.62	1.30	0.88	0.57
3×3	9/16	10.4	3.06	0.95	2.43	1.19	0.89	0.58
3×3	1/2	9.4	2.75	0.93	2.22	1.07	0.90	0.58
3×3	7/16	8.3	2.43	0.91	1.99	0.95	0.91	0.58
3×3	3/8	7.2	2.11	0.89	1.76	0.83	0.91	0.58
3×3	5/16	6.1	1.78	0.87	1.51	0.71	0.92	0.59
3×3	1/4	4.9	1.44	0.84	1.24	0.58	0.93	0.59
*2¾×2¾	1/2	8.5	2.50	0.87	1.67	0.89	0.82	0.25
*2¾×2¾	7/16	7.6	2.22	0.85	1.51	0.79	0.82	0.53
*2¾×2¾	3/8	6.6	1.92	0.82	1.33	0.69	0.83	0.53
*2¾×2¾	5/16	5.6	1.62	0.80	1.15	0.59	0.84	0.54
*2¾×2¾	1/4	4.5	1.31	0.78	0.93	0.48	0.85	0.55
2½×2½	1/2	7.7	2.25	0.81	1.23	0.73	0.74	0.47
2½×2½	7/16	6.8	2.00	0.78	1.11	0.65	0.74	0.48
2½×2½	3/8	5.9	1.73	0.76	0.98	0.57	0.75	0.48
2½×2½	5/16	5.0	1.47	0.74	0.85	0.48	0.76	0.49
2½×2½	1/4	4.1	1.19	0.72	0.70	0.40	0.77	0.49
2½×2½	3/16	3.1	0.90	0.69	0.55	0.30	0.78	0.49
*2¼×2¼	1/2	6.8	2.00	0.74	0.87	0.58	0.66	0.43
*2¼×2¼	3/8	5.3	1.55	0.70	0.70	0.45	0.67	0.43
*2¼×2¼	1/4	3.7	1.06	0.66	0.51	0.32	0.69	0.44
2×2	1/2	6.0	1.75	0.68	0.59	0.45	0.58	0.38
2×2	7/16	5.3	1.56	0.66	0.54	0.40	0.59	0.39
2×2	3/8	4.7	1.36	0.64	0.48	0.35	0.59	0.39
2×2	5/16	4.0	1.15	0.61	0.42	0.30	0.60	0.39
2×2	1/4	3.2	0.94	0.59	0.35	0.25	0.61	0.39
2×2	3/16	2.5	0.72	0.57	0.28	0.19	0.62	0.40
*1¾×1¾	7/16	4.6	1.30	0.59	0.35	0.30	0.51	0.33
*1¾×1¾	3/8	4.0	1.17	0.57	0.31	0.26	0.51	0.34
*1¾×1¾	5/16	3.4	1.00	0.55	0.27	0.23	0.52	0.34
*1¾×1¾	1/4	2.8	0.81	0.53	0.23	0.19	0.53	0.34
*1¾×1¾	3/16	2.2	0.62	0.51	0.18	0.14	0.54	0.35
1½×1½	3/8	3.4	0.99	0.51	0.19	0.19	0.44	0.29
1½×1½	5/16	2.9	0.84	0.49	0.16	0.162	0.44	0.29
1½×1½	1/4	2.4	0.69	0.47	0.14	0.134	0.45	0.29
1½×1½	3/16	1.8	0.53	0.44	0.11	0.104	0.46	0.29
1½×1½	1/8	1.3	0.36	0.42	0.08	0.070	0.46	0.30
*1¼×1¼	5/16	2.4	0.69	0.42	0.09	0.109	0.36	0.23
*1¼×1¼	1/4	2.0	0.56	0.40	0.077	0.091	0.37	0.24
*1¼×1¼	3/16	1.5	0.43	0.38	0.061	0.071	0.38	0.24
*1¼×1¼	1/8	1.1	0.30	0.35	0.044	0.049	0.38	0.25
*1×1	1/4	1.5	0.44	0.34	0.037	0.056	0.29	0.19
*1×1	3/16	1.2	0.34	0.32	0.030	0.044	0.30	0.19
*1×1	1/8	0.8	0.24	0.30	0.022	0.031	0.31	0.20
*⅞×⅞	3/16	1.0	0.29	0.29	0.019	0.033	0.26	0.18
*⅞×⅞	1/8	0.7	0.21	0.26	0.014	0.023	0.26	0.19
*¾×¾	3/16	0.9	0.25	0.26	0.012	0.024	0.22	0.16
*¾×¾	1/8	0.6	0.17	0.23	0.009	0.017	0.23	0.17

* Angles marked with a * are listed by some steel makers as special angles, and command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the customer.

Properties of Angles with Unequal Legs—Continued on p. 448

Dimensions are given in inches

See paragraph on Unsymmetrical Loading of Beams, p. 438

Size	Thickness	Lb per foot	Area of section	Perpendicular distances from center of gravity to back of flanges: To back of l'nger fl'nge	Perpendicular distances from center of gravity to back of flanges: To back of sh'ter fl'nge	Moments of Inertia, I: Neutral axis parallel to longer flange	Moments of Inertia, I: Neutral axis parallel to shorter flange	Section Moduli, I/c: Neutral axis parallel to l'nger flange	Section Moduli, I/c: Neutral axis parallel to sh'ter flange	Radii of gyration, r: Neutral axis parallel to l'nger flange	Radii of gyration, r: Neutral axis parallel to sh'ter flange	Radii of gyration, r: Least radius
*8×6	1	44.2	13.00	1.65	2.65	38.78	80.78	8.92	15.11	1.73	2.49	1.28
*8×6	15/16	41.7	12.25	1.63	2.63	36.85	76.59	8.43	14.27	1.73	2.50	1.28
*8×6	7/8	39.1	11.48	1.61	2.61	34.86	72.32	7.94	13.41	1.74	2.51	1.28
*8×6	13/16	36.5	10.72	1.59	2.59	32.82	67.92	7.44	12.55	1.75	2.52	1.28
*8×6	3/4	33.8	9.94	1.56	2.56	30.72	63.42	6.92	11.67	1.76	2.53	1.28
*8×6	11/16	31.2	9.15	1.54	2.54	28.56	58.82	6.40	10.77	1.77	2.54	1.29
*8×6	5/8	28.5	8.36	1.52	2.52	26.33	54.10	5.88	9.87	1.77	2.54	1.29
*8×6	9/16	25.7	7.56	1.50	2.50	24.04	49.26	5.34	8.95	1.78	2.55	1.30
*8×6	1/2	23.0	6.75	1.47	2.47	21.68	44.31	4.79	8.02	1.79	2.56	1.30
*7×3½	1	32.3	9.50	0.96	2.71	7.53	45.37	2.96	10.58	0.89	2.19	0.88
*7×3½	3/4	24.9	7.31	0.87	2.62	6.08	35.99	2.31	8.22	0.91	2.22	0.88
*7×3½	1/2	17.0	5.00	0.78	2.53	4.41	25.41	1.62	5.68	0.94	2.25	0.89
6×4	1	30.6	9.00	1.17	2.17	10.75	30.75	3.79	8.02	1.09	1.85	0.85
6×4	15/16	28.9	8.50	1.14	2.14	10.26	29.26	3.59	7.59	1.10	1.86	0.85
6×4	7/8	27.2	7.99	1.12	2.12	9.75	27.73	3.39	7.15	1.11	1.86	0.86
6×4	13/16	25.4	7.47	1.10	2.10	9.23	26.15	3.18	6.70	1.11	1.87	0.86
6×4	3/4	23.6	6.94	1.08	2.08	8.68	24.51	2.97	6.25	1.12	1.88	0.86
6×4	11/16	21.8	6.41	1.06	2.06	8.11	22.82	2.76	5.78	1.13	1.89	0.86
6×4	5/8	20.0	5.86	1.03	2.03	7.52	21.07	2.54	5.31	1.13	1.90	0.86
6×4	9/16	18.1	5.31	1.01	2.01	6.91	19.26	2.31	4.83	1.14	1.90	0.87
6×4	1/2	16.2	4.75	0.99	1.99	6.27	17.40	2.08	4.33	1.15	1.91	0.87
6×4	7/16	14.3	4.18	0.96	1.96	5.60	15.46	1.85	3.83	1.16	1.92	0.87
6×4	3/8	12.3	3.61	0.94	1.94	4.90	13.47	1.60	3.32	1.17	1.93	0.88
6×3½	1	28.9	8.50	1.01	2.26	7.21	29.24	2.90	7.83	0.92	1.85	0.74
6×3½	15/16	27.3	8.03	0.99	2.24	6.88	27.84	2.74	7.41	0.93	1.86	0.74
6×3½	7/8	25.7	7.55	0.97	2.22	6.55	26.38	2.59	6.98	0.93	1.87	0.75
6×3½	13/16	24.0	7.06	0.95	2.20	6.20	24.89	2.43	6.55	0.94	1.88	0.75
6×3½	3/4	22.4	6.56	0.93	2.18	5.84	23.34	2.27	6.10	0.94	1.89	0.75
6×3½	11/16	20.6	6.06	0.90	2.15	5.47	21.74	2.11	5.65	0.95	1.89	0.75
6×3½	5/8	18.9	5.55	0.88	2.13	5.08	20.08	1.94	5.19	0.96	1.90	0.75
6×3½	9/16	17.1	5.03	0.86	2.11	4.67	18.37	1.77	4.72	0.96	1.91	0.75
6×3½	1/2	15.3	4.50	0.83	2.08	4.25	16.59	1.59	4.24	0.97	1.92	0.76
6×3½	7/16	13.5	3.97	0.81	2.06	3.81	14.76	1.41	3.75	0.98	1.93	0.76
6×3½	3/8	11.7	3.42	0.79	2.04	3.34	12.86	1.23	3.25	0.99	1.94	0.77
5×3½	15/16	24.2	7.09	1.06	1.81	6.52	16.49	2.67	5.17	0.96	1.53	0.75
5×3½	7/8	22.7	6.67	1.04	1.79	6.21	15.67	2.52	4.88	0.96	1.53	0.75
5×3½	13/16	21.3	6.25	1.02	1.77	5.89	14.81	2.37	4.58	0.97	1.54	0.75
5×3½	3/4	19.8	5.81	1.00	1.75	5.55	13.92	2.22	4.28	0.98	1.55	0.75
5×3½	11/16	18.3	5.37	0.97	1.72	5.20	12.99	2.06	3.97	0.98	1.56	0.75
5×3½	5/8	16.8	4.92	0.95	1.70	4.83	12.03	1.90	3.65	0.99	1.56	0.75
5×3½	9/16	15.2	4.47	0.93	1.68	4.45	11.03	1.73	3.32	1.00	1.57	0.75
5×3½	1/2	13.6	4.00	0.91	1.66	4.05	9.99	1.56	2.99	1.01	1.58	0.75
5×3½	7/16	12.0	3.53	0.88	1.63	3.63	8.90	1.39	2.64	1.01	1.59	0.76
5×3½	3/8	10.4	3.05	0.86	1.61	3.18	7.78	1.21	2.29	1.02	1.60	0.76
5×3½	5/16	8.7	2.56	0.84	1.59	2.72	6.60	1.02	1.94	1.03	1.61	0.76
5×3	13/16	19.9	5.84	0.86	1.86	3.71	13.98	1.74	4.45	0.80	1.55	0.64
5×3	3/4	18.5	5.44	0.84	1.84	3.51	13.15	1.63	4.16	0.80	1.55	0.64
5×3	11/16	17.1	5.03	0.82	1.82	3.29	12.28	1.51	3.86	0.81	1.56	0.64
5×3	5/8	15.7	4.61	0.80	1.80	3.06	11.37	1.39	3.55	0.82	1.57	0.64
5×3	9/16	14.3	4.18	0.77	1.77	2.83	10.43	1.27	3.23	0.82	1.58	0.65

*Angles marked with a * are listed by some steel makers as special angles, and command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the purchaser.

Properties of Angles with Unequal Legs—Continued

Dimensions are given in inches

See paragraph on Unsymmetrical Loading of Beams, p. 438

Size	Thickness	Lb per foot	Area of section	Perpendicular distances from center of gravity to back of flanges		Moments of Inertia, I		Section Moduli, I/c		Radii of gyration, r		
				To back of l'nger fl'nge	To back of sh'ter fl'nge	Neutral axis parallel to longer flange	Neutral axis parallel to shorter flange	Neutral axis parallel to l'nger flange	Neutral axis parallel to sh'ter flange	Neutral axis parallel to l'nger flange	Neutral axis parallel to sh'ter flange	Least radius
5×3	½	12.8	3.75	0.75	1.75	2.58	9.45	1.15	2.91	0.83	1.59	0.65
5×3	7/16	11.3	3.31	0.73	1.73	2.32	8.43	1.02	2.58	0.84	1.60	0.65
5×3	⅜	9.8	2.86	0.70	1.70	2.04	7.37	0.89	2.24	0.84	1.61	0.65
5×3	5/16	8.2	2.40	0.68	1.68	1.75	6.26	0.75	1.89	0.85	1.61	0.66
*4×3½	13/16	18.5	5.43	1.11	1.36	5.49	7.77	2.30	2.92	1.01	1.19	0.72
*4×3½	¾	17.3	5.06	1.09	1.34	5.18	7.32	2.15	2.75	1.01	1.20	0.72
*4×3½	11/16	16.0	4.68	1.07	1.32	4.86	6.86	2.00	2.56	1.02	1.21	0.72
*4+3½	⅝	14.7	4.30	1.04	1.29	4.52	6.37	1.84	2.35	1.03	1.22	0.72
*4×3½	9/16	13.3	3.90	1.02	1.27	4.17	5.86	1.68	2.15	1.03	1.23	0.72
*4×3½	½	11.9	3.50	1.00	1.25	3.79	5.32	1.52	1.93	1.04	1.23	0.72
*4×3½	7/16	10.6	3.09	0.98	1.23	3.40	4.76	1.35	1.72	1.05	1.24	0.72
*4×3½	⅜	9.1	2.67	0.96	1.21	2.99	4.18	1.18	1.50	1.06	1.25	0.73
*4×3½	5/16	7.7	2.25	0.93	1.18	2.59	3.56	1.01	1.26	1.07	1.26	0.73
4×3	¾	16.0	4.69	0.92	1.42	3.28	6.93	1.57	2.68	0.84	1.22	0.64
4×3	11/16	14.8	4.34	0.89	1.39	3.08	6.49	1.46	2.49	0.84	1.22	0.64
4×3	⅝	13.6	3.98	0.87	1.37	2.87	6.03	1.35	2.30	0.85	1.23	0.64
4×3	9/16	12.4	3.62	0.85	1.35	2.66	5.55	1.23	2.09	0.86	1.24	0.64
4×3	½	11.1	3.25	0.83	1.33	2.42	5.05	1.12	1.89	0.86	1.25	0.64
4×3	7/16	9.8	2.87	0.80	1.30	2.18	4.52	0.99	1.68	0.87	1.25	0.64
4×3	⅜	8.5	2.48	0.78	1.28	1.92	3.96	0.87	1.46	0.88	1.26	0.64
4×3	5/16	7.2	2.09	0.76	1.26	1.65	3.38	0.74	1.23	0.89	1.27	0.65
3½×3	¾	14.7	4.31	0.96	1.21	3.15	4.70	1.54	2.05	0.85	1.04	0.62
3½×3	11/16	13.6	4.00	0.94	1.19	2.96	4.41	1.44	1.91	0.86	1.05	0.62
3½×3	⅝	12.5	3.67	0.92	1.17	2.76	4.11	1.33	1.76	0.87	1.06	0.62
3½×3	9/16	11.4	3.34	0.90	1.15	2.55	3.79	1.21	1.61	0.87	1.07	0.62
3½×3	½	10.2	3.00	0.88	1.13	2.33	3.45	1.10	1.45	0.88	1.07	0.62
3½×3	7/16	9.1	2.65	0.85	1.10	2.09	3.10	0.98	1.29	0.89	1.08	0.62
3½×3	⅜	7.9	2.30	0.83	1.08	1.85	2.72	0.85	1.13	0.90	1.09	0.62
3½×3	5/16	6.6	1.93	0.81	1.06	1.58	2.33	0.72	0.96	0.90	1.10	0.63
3½×2½	11/16	12.5	3.65	0.77	1.27	1.72	4.13	0.99	1.85	0.67	1.06	0.53
3½×2½	⅝	11.5	3.36	0.75	1.25	1.61	3.85	0.92	1.71	0.69	1.07	0.53
3½×2½	9/16	10.4	3.06	0.73	1.23	1.49	3.55	0.84	1.56	0.70	1.08	0.53
3½×2½	½	9.4	2.75	0.70	1.20	1.36	3.24	0.76	1.41	0.70	1.09	0.53
3½×2½	7/16	8.3	2.43	0.68	1.18	1.23	2.91	0.68	1.26	0.71	1.09	0.54
3½×2½	⅜	7.2	2.11	0.66	1.16	1.09	2.56	0.59	1.09	0.72	1.10	0.54
3½×2½	5/16	6.1	1.78	0.64	1.14	0.94	2.19	0.50	0.93	0.73	1.11	0.54
3½×2½	¼	4.9	1.44	0.61	1.11	0.78	1.80	0.41	0.75	0.74	1.12	0.54
3×2½	9/16	9.5	2.78	0.77	1.02	1.42	2.28	0.82	1.15	0.72	0.91	0.52
3×2½	½	8.5	2.50	0.75	1.00	1.30	2.08	0.74	1.04	0.72	0.91	0.52
3×2½	7/16	7.6	2.22	0.73	0.98	1.18	1.88	0.66	0.93	0.73	0.92	0.52
3×2½	⅜	6.6	1.92	0.71	0.96	1.04	1.66	0.58	0.81	0.74	0.93	0.52
3×2½	5/16	5.6	1.62	0.68	0.93	0.90	1.42	0.49	0.69	0.74	0.94	0.53
3×2½	¼	4.5	1.31	0.66	0.91	0.74	1.17	0.40	0.56	0.75	0.95	0.53
2½×2	½	6.8	2.00	0.63	0.88	0.64	1.14	0.46	0.70	0.56	0.75	0.42
2½×2	7/16	6.1	1.78	0.60	0.85	0.58	1.03	0.41	0.62	0.57	0.76	0.42
2½×2	⅜	5.3	1.55	0.58	0.83	0.51	0.91	0.36	0.55	0.58	0.77	0.42
2½×2	5/16	4.5	1.31	0.56	0.81	0.45	0.79	0.31	0.47	0.58	0.78	0.42
2½×2	¼	3.7	1.06	0.54	0.79	0.37	0.65	0.25	0.38	0.59	0.78	0.42
2½×2	3/16	2.8	0.81	0.51	0.76	0.29	0.51	0.20	0.29	0.60	0.79	0.43

* Angles marked with a * are listed by some steel makers as special angles, and command a slightly higher price per pound than standard angles, and, in small lots, are not always available to the purchaser.

Properties of Standard Channels

Dimensions are given in inches. See p. 458 for shearing stress factors.

Depth of channel	Lb per foot	Area of section	Thickness of web	Width of flange	Neutral axis perpendicular to web at center: Moment of inertia, I	Radius of gyration, r	Section modulus, I/c	Neutral axis parallel with center line of web: Moment of inertia, I'	Radius of gyration, r'	Coefficient of strength for fiber stress of 16 000 lb per sq in, C
	55.00	16.18	0.818	3.818	430.2	5.16	57.4	12.19	0.868	611 900
	50.00	14.71	.720	3.720	402.7	5.23	53.7	11.22	.873	572 700
	45.00	13.24	.622	3.622	375.1	5.32	50.6	10.29	.882	533 500
15	40.00	11.76	.524	3.524	347.5	5.43	46.3	9.39	.893	494 200
	35.00	10.29	.426	3.426	320.0	5.58	42.7	8.48	.908	455 000
	33.00	9.90	.400	3.400	312.6	5.62	41.7	8.23	.912	444 500
	40.00	11.76	.758	3.418	197.0	4.09	32.8	6.63	.751	350 200
	35.00	10.29	.636	3.296	179.3	4.17	29.9	5.90	.757	318 800
12	30.00	8.82	.513	3.173	161.7	4.28	26.9	5.21	.768	287 400
	25.00	7.35	.390	3.050	144.0	4.43	24.0	4.53	.785	256 100
	20.50	6.03	.280	2.940	128.1	4.61	21.4	3.91	.805	227 800
	35.00	10.29	.823	3.183	115.5	3.35	23.1	4.66	.672	246 400
	30.00	8.82	.676	3.036	103.2	3.42	20.6	3.90	.672	220 300
10	25.00	7.35	.529	2.889	91.0	3.52	18.2	3.40	.680	194 100
	20.00	5.88	.382	2.742	78.7	3.66	15.7	2.85	.696	168 000
	15.00	4.46	.240	2.600	66.9	3.87	13.4	2.30	.718	142 700
	25.00	7.35	.615	2.815	70.7	3.10	15.7	2.98	.637	167 600
9	20.00	5.88	.452	2.652	60.8	3.21	13.5	2.45	.646	144 100
	15.00	4.41	.288	2.488	50.9	3.40	11.3	1.95	.665	120 500
	13.25	3.89	.230	2.430	47.3	3.49	10.5	1.77	.674	112 200
	21.25	6.25	.582	2.622	47.8	2.77	11.9	2.25	.600	127 400
	18.75	5.51	.490	2.530	43.8	2.82	11.0	2.01	.603	116 900
8	16.25	4.78	.399	2.439	39.9	2.89	10.0	1.78	.610	106 400
	13.75	4.04	.307	2.347	36.0	2.98	9.0	1.55	.619	96 000
	11.25	3.35	.220	2.260	32.3	3.11	8.1	1.33	.630	86 100
	19.75	5.81	.633	2.513	33.2	2.39	9.5	1.85	.565	101 100
	17.25	5.07	.528	2.408	30.2	2.44	8.6	1.62	.564	92 000
7	14.75	4.34	.423	2.303	27.2	2.50	7.8	1.40	.568	82 800
	12.25	3.60	.318	2.198	24.2	2.59	6.9	1.19	.575	73 700
	9.75	2.85	.210	2.090	21.1	2.72	6.0	0.98	.586	66 800
	15.50	4.56	.563	2.283	19.5	2.07	6.5	1.28	.529	69 500
6	13.00	3.82	.440	2.160	17.3	2.13	5.8	1.07	.529	61 600
	10.50	3.09	.318	2.038	15.1	2.21	5.0	0.88	.534	53 800
	8.00	2.38	.200	1.920	13.0	2.34	4.3	0.70	.542	46 200
	11.50	3.38	.477	2.037	10.4	1.75	4.2	0.82	.493	44 400
5	9.00	2.65	.330	1.890	8.9	1.83	3.5	0.64	.493	37 900
	6.50	1.95	.190	1.750	7.4	1.95	3.0	0.48	.498	31 600
	7.25	2.13	.325	1.725	4.6	1.46	2.3	0.44	.455	24 400
4	6.25	1.84	.252	1.652	4.2	1.51	2.1	0.38	.454	22 300
	5.25	1.55	.180	1.580	3.8	1.56	1.9	0.32	.453	20 200
	6.00	1.76	.362	1.602	2.1	1.08	1.4	0.31	.421	14 700
3	5.00	1.47	.264	1.504	1.8	1.12	1.2	0.25	.415	13 100
	4.00	1.19	.170	1.410	1.6	1.17	1.1	0.20	.409	11 600

For explanation of this table see Art. 47. In computing the weights and areas in the ables the fillets are disregarded.

Properties of Standard T Shapes

Dimensions are given in Inches.

Size, flange by stem	Lb per foot	Area of section	Distance of center of gravity from outside of flange	Neutral axis thru center of gravity parallel to flange			Neutral axis thru center of gravity coincident with center line of stem		
				Moment of inertia, I	Least section modulus, I/c	Radius of gyration, r	Moment of inertia, I'	Section modulus, I/c	Radius of gyration, r'
5×3	13.6	3.99	0.75	2.6	1.18	0.82	5.6	2.22	1.19
5×2½	11.0	3.24	0.65	1.6	0.86	0.71	4.3	1.70	1.16
4½×3½	15.9	4.65	1.11	5.1	2.13	1.04	3.7	1.65	0.90
4½×3	10.0	3.00	0.75	2.1	0.94	0.86	3.1	1.38	1.04
4½×2½	9.3	2.79	0.60	1.2	0.65	0.68	3.1	1.38	1.08
4×5	12.3	3.54	1.51	8.5	2.43	1.56	2.1	1.06	0.78
4×4½	11.6	3.36	1.31	6.3	1.98	1.38	2.1	1.06	0.80
4×4	10.9	3.21	1.15	4.7	1.64	1.23	2.2	1.09	0.84
4×3	9.3	2.73	0.78	2.0	0.88	0.86	2.1	1.05	0.88
4×2½	7.4	2.16	0.60	1.0	0.55	0.70	1.8	0.88	0.91
4×2	7.9	2.31	0.48	0.6	0.40	0.52	2.1	1.05	0.96
4×2	6.7	1.95	0.51	0.54	0.34	0.51	1.8	0.88	0.95
3½×4	12.8	3.75	1.25	5.5	1.98	1.21	1.89	1.08	0.72
3½×4	10.0	2.91	1.19	4.3	1.55	1.22	1.42	0.81	0.70
3½×3½	11.9	3.45	1.06	3.7	1.52	1.04	1.89	1.08	0.74
3½×3½	9.3	2.70	1.01	3.0	1.19	1.05	1.42	0.81	0.73
3½×3	11.0	3.21	0.88	2.4	1.13	0.87	1.88	1.08	0.77
3½×3	8.7	2.49	0.83	1.9	0.88	0.88	1.41	0.81	0.75
3½×3	7.7	2.28	0.78	1.6	0.72	0.89	1.18	0.68	0.76
3×4	11.9	3.48	1.32	5.2	1.94	1.23	1.21	0.81	0.59
3×4	10.6	3.12	1.32	4.8	1.78	1.25	1.09	0.72	0.60
3×4	9.3	2.73	1.29	4.3	1.57	1.26	0.93	0.62	0.59
3×3½	11.0	3.21	1.12	3.5	1.49	1.06	1.20	0.80	0.62
3×3½	9.8	2.88	1.11	3.3	1.37	1.08	1.31	0.88	0.68
3×3½	8.6	2.49	1.09	2.9	1.21	1.09	0.93	0.62	0.61
3×3	10.1	2.94	0.93	2.3	1.10	0.88	1.20	0.80	0.64
3×3	9.0	2.67	0.92	2.1	1.01	0.90	1.08	0.72	0.64
3×3	7.9	2.28	0.88	1.8	0.86	0.90	0.90	0.60	0.63
3×3	6.8	1.95	0.86	1.6	0.74	0.90	0.75	0.50	0.62
3×2½	6.2	1.80	0.68	0.94	0.52	0.73	0.75	0.50	0.65
2¾×2	7.4	2.16	0.53	1.1	0.75	0.71	0.62	0.45	0.54
2½×3	6.2	1.80	0.92	1.6	0.76	0.94	0.44	0.35	0.51
2½×2¾	5.9	1.71	0.83	1.2	0.60	0.83	0.44	0.35	0.51
2½×2½	5.6	1.62	0.74	0.87	0.50	0.74	0.44	0.35	0.52
2½×1¼	3.0	0.84	0.29	0.094	0.09	0.31	0.29	0.23	0.58
2¼×2¼	4.2	1.20	0.66	0.51	0.32	0.67	0.25	0.22	0.47
2×2	3.7	1.08	0.59	0.36	0.25	0.60	0.18	0.18	0.42
2×1½	3.2	0.90	0.42	0.16	0.15	0.42	0.18	0.18	0.45
1¾×1¾	3.2	0.90	0.54	0.23	0.19	0.51	0.12	0.14	0.37
1½×1½	2.0	0.54	0.44	0.11	0.11	0.45	0.06	0.07	0.31
1¼×1¼	1.7	0.45	0.38	0.06	0.07	0.37	0.03	0.05	0.26

Properties of Standard Z Bars

Dimensions are given in Inches

See Paragraph on Unsymmetrical Loading of Beams, p. 438

Depth of web	Width of flange	Thickness of metal	Lb per foot	Area of section	Moments of inertia, I: Neutral axis thru center of gravity perpendicular to web	Moments of inertia, I: Neutral axis thru center of gravity coincident with web	Section moduli, I/c: Neutral axis thru center of gravity perpendicular to web	Section moduli, I/c: Neutral axis thru center of gravity coincident with web	Radii of gyration, r: Neutral axis thru center of gravity perpendicular to web	Radii of gyration, r: Neutral axis thru center of gravity coincident with web	Least radius, neutral axis diagonal
6	3½	3/8	15.6	4.59	25.32	9.11	8.44	2.75	2.35	1.41	0.83
6 1/16	3 9/16	7/16	18.3	5.39	29.80	10.95	9.83	3.27	2.35	1.43	0.84
6 1/8	3 5/8	½	21.0	6.19	34.36	12.87	11.22	3.81	2.36	1.44	0.84
6	3½	9/16	22.7	6.68	34.64	12.59	11.52	3.91	2.28	1.37	0.81
6 1/16	3 9/16	5/8	25.4	7.46	38.86	14.42	12.82	4.43	2.28	1.39	0.82
6 1/8	3 5/8	11/16	28.0	8.25	43.18	16.34	14.10	4.98	2.29	1.41	0.84
6	3½	¾	29.3	8.63	42.12	15.44	14.04	4.94	2.21	1.34	0.81
6 1/16	3 9/16	13/16	31.9	9.40	46.13	17.27	15.22	5.47	2.22	1.36	0.82
6 1/8	3 5/8	7/8	34.6	10.17	50.22	19.18	16.40	6.02	2.22	1.37	0.83
5	3¼	5/16	11.6	3.40	13.36	6.18	5.34	2.00	1.98	1.35	0.75
5 1/16	3 5/16	3/8	13.9	4.10	16.18	7.65	6.39	2.45	1.99	1.37	0.76
5 1/8	3 3/8	7/16	16.4	4.81	19.07	9.20	7.44	2.92	1.99	1.38	0.77
5	3¼	½	17.9	5.25	19.19	9.05	7.68	3.02	1.91	1.31	0.74
5 1/16	3 5/16	9/16	20.2	5.94	21.83	10.51	8.62	3.47	1.91	1.33	0.75
5 1/8	3 3/8	5/8	22.6	6.64	24.53	12.06	9.57	3.94	1.92	1.35	0.76
5	3¼	11/16	23.7	6.96	23.68	11.37	9.47	3.91	1.84	1.28	0.73
5 1/16	3 5/16	¾	26.0	7.64	26.16	12.83	10.34	4.37	1.85	1.30	0.75
5 1/8	3 3/8	13/16	28.3	8.33	28.70	14.36	11.20	4.84	1.86	1.31	0.76
4	3 1/16	¼	8.2	2.41	6.28	4.23	3.14	1.44	1.62	1.33	0.67
4 1/16	3 1/8	5/16	10.3	3.03	7.94	5.46	3.91	1.84	1.62	1.34	0.68
4 1/8	3 3/16	3/8	12.4	3.66	9.63	6.77	4.67	2.26	1.62	1.36	0.69
4	3 1/16	7/16	13.8	4.05	9.66	6.73	4.83	2.37	1.55	1.29	0.66
4 1/16	3 1/8	½	15.8	4.66	11.18	7.96	5.50	2.77	1.55	1.31	0.67
4 1/8	3 3/16	9/16	17.9	5.27	12.74	9.26	6.18	3.19	1.55	1.33	0.69
4	3 1/16	5/8	18.9	5.55	12.11	8.73	6.05	3.18	1.48	1.25	0.66
4 1/16	3 1/8	11/16	20.9	6.14	13.52	9.95	6.65	3.58	1.48	1.27	0.67
4 1/8	3 3/16	¾	23.0	6.75	14.97	11.24	7.26	4.00	1.49	1.29	0.69
3	2 11/16	¼	6.7	1.97	2.87	2.81	1.92	1.10	1.21	1.19	0.55
3 1/16	2¾	5/16	8.4	2.48	3.64	3.64	2.38	1.40	1.21	1.21	0.56
3	2 11/16	3/8	9.7	2.86	3.85	3.92	2.57	1.57	1.16	1.17	0.55
3 1/16	2¾	7/16	11.4	3.36	4.57	4.75	2.98	1.88	1.17	1.19	0.56
3	2 11/16	½	12.5	3.69	4.59	4.85	3.06	1.99	1.12	1.15	0.55
3 1/16	2¾	9/16	14.2	4.18	5.26	5.70	3.43	2.31	1.12	1.17	0.56

49. Bars and Plates

Commercial Sizes of Steel Bars in Inches (Carnegie Steel Co.). ROUNDS: 7/32 1¾ advancing by 64ths; 1 25/32 to 3½ advancing by 32ds; 3 9/16 to 7 advancing 16ths. SQUARES: 3/16 to 2 advancing by 64ths; 2 1/32 to 3½ advancing by

Square and Round Steel Bars

Side or diameter, inches	Pounds per linear foot		Area in square inches		Circumference of round bar, square inches	Side or diameter, inches
	Square	Round	Square	Round		
1/16	.013	.010	.0039	.0031	.1963	1/16
1/8	.053	.042	.0156	.0123	.3927	1/8
3/16	.119	.094	.0352	.0276	.5890	3/16
1/4	.212	.167	.0625	.0491	.7854	1/4
5/16	.333	.261	.0977	.0767	.9817	5/16
3/8	.478	.375	.1406	.1104	1.1781	3/8
7/16	.651	.511	.1914	.1503	1.3744	7/16
1/2	.850	.667	.2500	.1963	1.5708	1/2
9/16	1.076	.845	.3164	.2485	1.7671	9/16
5/8	1.328	1.043	.3906	.3068	1.9635	5/8
11/16	1.608	1.262	.4727	.3712	2.1593	11/16
3/4	1.913	1.502	.5625	.4418	2.3562	3/4
13/16	2.245	1.763	.6602	.5185	2.5525	13/16
7/8	2.603	2.044	.7656	.6013	2.7489	7/8
15/16	2.989	2.347	.8789	.6903	2.9452	15/16
1	3.400	2.670	1.0000	.7854	3.1416	1
1 1/16	3.833	3.014	1.1289	.8866	3.3379	1 1/16
1 1/8	4.303	3.379	1.2656	.9940	3.5343	1 1/8
1 3/16	4.795	3.766	1.4102	1.1075	3.7306	1 3/16
1 1/4	5.312	4.173	1.5625	1.2272	3.9270	1 1/4
1 5/16	5.857	4.600	1.7227	1.3530	4.1233	1 5/16
1 3/8	6.428	5.049	1.8906	1.4849	4.3197	1 3/8
1 7/16	7.026	5.518	2.0664	1.6230	4.5160	1 7/16
1 1/2	7.650	6.008	2.2500	1.7671	4.7124	1 1/2
1 9/16	8.301	6.520	2.4414	1.9175	4.9087	1 9/16
1 5/8	8.978	7.051	2.6406	2.0739	5.1051	1 5/8
1 11/16	9.682	7.604	2.8477	2.2365	5.3014	1 11/16
1 3/4	10.41	8.178	3.0625	2.4053	5.4978	1 3/4
1 13/16	11.17	8.773	3.2852	2.5802	5.6941	1 13/16
1 7/8	11.95	9.388	3.5156	2.7612	5.8905	1 7/8
1 15/16	12.76	10.02	3.7539	2.9483	6.0868	1 15/16
2	13.60	10.68	4.0000	3.1416	6.2832	2
2 1/8	15.35	12.06	4.5156	3.5466	6.6759	2 1/8
2 1/4	17.22	13.52	5.0625	3.9761	7.0686	2 1/4
2 3/8	19.18	15.07	5.6406	4.4301	7.4613	2 3/8
2 1/2	21.25	16.69	6.2500	4.9087	7.8540	2 1/2
2 5/8	23.43	18.40	6.8906	5.4119	8.2467	2 5/8
2 3/4	25.71	20.20	7.5625	5.9396	8.6394	2 3/4
2 7/8	28.10	22.07	8.2656	6.7771	9.2284	2 7/8
3	30.60	24.03	9.0000	7.0686	9.4248	3
3 1/4	35.92	28.20	10.563	8.2958	10.210	3 1/4
3 1/2	41.65	32.71	12.250	9.6211	10.996	3 1/2
3 3/4	47.82	37.56	14.063	11.045	11.781	3 3/4
4	54.40	42.73	16.000	12.566	12.566	4

32ds; 3 9/16 to 5 1/2 advancing by 16ths. HEXAGONS: 1/4 to 1 3/16 advancing by 32d 1 1/4 to 3 1/16 advancing by 16ths. SQUARE EDGE FLATS: 3/8 to 3 in wide, any thickness up to width; 3 to 5 in wide, any thickness 1/4 to 3 in; 5 to 7 in wide, an thickness 1/4 to 2 in.

Weight of Rolled Steel Plates in Pounds per Linear Foot—Continued on p. 454

Width in inches	Thickness of plate in inches									
	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
1	0.638	0.850	1.06	1.28	1.49	1.70	1.92	2.12	2.34	2.55
1¼	0.797	1.06	1.33	1.59	1.86	2.12	2.39	2.65	2.92	3.19
1½	0.957	1.28	1.59	1.92	2.23	2.55	2.87	3.19	3.51	3.83
1¾	1.11	1.49	1.86	2.23	2.60	2.98	3.35	3.72	4.09	4.47
2	1.28	1.70	2.12	2.55	2.98	3.40	3.83	4.25	4.67	5.10
2¼	1.44	1.91	2.39	2.87	3.35	3.83	4.30	4.78	5.26	5.75
2½	1.59	2.12	2.65	3.19	3.72	4.25	4.78	5.31	5.84	6.38
2¾	1.75	2.34	2.92	3.51	4.09	4.67	5.26	5.84	6.43	7.02
3	1.91	2.55	3.19	3.83	4.46	5.10	5.74	6.38	7.02	7.65
3¼	2.07	2.76	3.45	4.15	4.83	5.53	6.22	6.91	7.60	8.29
3½	2.23	2.98	3.72	4.47	5.20	5.95	6.70	7.44	8.18	8.93
3¾	2.39	3.19	3.99	4.78	5.58	6.38	7.17	7.97	8.76	9.57
4	2.55	3.40	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20
4¼	2.71	3.61	4.52	5.42	6.32	7.22	8.13	9.03	9.93	10.84
4½	2.87	3.83	4.78	5.74	6.70	7.65	8.61	9.57	10.52	11.48
4¾	3.03	4.04	5.05	6.06	7.07	8.08	9.09	10.10	11.11	12.12
5	3.19	4.25	5.31	6.38	7.44	8.50	9.57	10.63	11.69	12.75
5¼	3.35	4.46	5.58	6.69	7.81	8.93	10.04	11.16	12.27	13.39
5½	3.51	4.67	5.84	7.02	8.18	9.35	10.52	11.69	12.85	14.03
5¾	3.67	4.89	6.11	7.34	8.36	9.77	11.00	12.22	13.44	14.67
6	3.83	5.10	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30
6¼	3.99	5.31	6.64	7.97	9.29	10.63	11.95	13.28	14.61	15.94
6½	4.14	5.53	6.90	8.29	9.67	11.05	12.43	13.81	15.20	16.58
6¾	4.30	5.74	7.17	8.61	10.04	11.48	12.91	14.34	15.78	17.22
7	4.46	5.95	7.44	8.93	10.41	11.90	13.39	14.87	16.36	17.85
7¼	4.62	6.16	7.70	9.25	10.78	12.32	13.86	15.40	16.94	18.49
7½	4.78	6.36	7.97	9.57	11.16	12.75	14.34	15.94	17.53	19.13
7¾	4.94	6.58	8.23	9.88	11.53	13.18	14.82	16.47	18.12	19.77
8	5.10	6.80	8.50	10.20	11.90	13.60	15.30	17.00	18.70	20.40
8¼	5.26	7.01	8.76	10.52	12.27	14.03	15.78	17.53	19.28	21.04
8½	5.42	7.22	9.03	10.84	12.64	14.44	16.26	18.06	19.86	21.68
8¾	5.58	7.43	9.29	11.16	13.02	14.87	16.74	18.59	20.45	22.32
9	5.74	7.65	9.56	11.48	13.40	15.30	17.22	19.13	21.04	22.94
9¼	5.90	7.86	9.83	11.80	13.76	15.73	17.69	19.65	21.62	23.59
9½	6.06	8.08	10.10	12.12	14.14	16.16	18.18	20.19	22.21	24.23
9¾	6.22	8.29	10.36	12.44	14.51	16.58	18.65	20.72	22.79	24.86
10	6.38	8.50	10.62	12.75	14.88	17.00	19.14	21.25	23.38	25.50
10¼	6.54	8.71	10.89	13.07	15.25	17.42	19.61	21.78	23.96	26.14
10½	6.70	8.92	11.16	13.39	15.62	17.85	20.08	22.32	24.54	26.78
10¾	6.86	9.14	11.42	13.71	15.99	18.28	20.56	22.85	25.13	27.42
11	7.02	9.34	11.68	14.03	16.36	18.70	21.02	23.38	25.70	28.05
11¼	7.17	9.57	11.95	14.35	16.74	19.13	21.51	23.91	26.30	28.68
11½	7.32	9.78	12.22	14.68	17.12	19.55	22.00	24.44	26.88	29.33
11¾	7.49	10.00	12.49	14.99	17.49	19.97	22.48	24.97	27.47	29.97
12	7.65	10.20	12.75	15.30	17.85	20.40	22.95	25.50	28.05	30.60
12¼	7.82	10.42	13.02	15.62	18.23	20.82	23.43	26.03	28.64	31.25
12½	7.98	10.63	13.28	15.94	18.60	21.25	23.96	26.56	29.22	31.88
12¾	8.13	10.84	13.55	16.26	18.97	21.67	24.39	27.09	29.80	32.52

Approximate rules for computing weights of bars, plates, and prisms: A wrought-iron ar one yard long and one square inch in section weighs 10 pounds. Steel is about 2 pernt heavier than wrought iron. Cast iron is about six percent lighter than wrought

Weight of Rolled Steel Plates in Pounds per Linear Foot — Continued

Width in inches	Thickness of plate in inches									
	13/16	7/8	15/16	1	1 1/16	1 1/8	1 3/16	1 1/4	1 5/16	1 3/8
1	2.76	2.98	3.19	3.40	3.61	3.83	4.04	4.25	4.46	4.6
1 1/4	3.45	3.72	3.99	4.25	4.52	4.78	5.05	5.31	5.58	5.8
1 1/2	4.14	4.47	4.78	5.10	5.42	5.74	6.06	6.38	6.69	7.0
1 3/4	4.84	5.20	5.58	5.95	6.32	6.70	7.07	7.44	7.81	8.1
2	5.53	5.95	6.38	6.80	7.22	7.65	8.08	8.50	8.93	9.3
2 1/4	6.21	6.69	7.18	7.65	8.13	8.61	9.09	9.57	10.04	10.5
2 1/2	6.90	7.44	7.97	8.50	9.03	9.57	10.10	10.63	11.16	11.6
2 3/4	7.60	8.18	8.77	9.35	9.93	10.52	11.11	11.69	12.27	12.8
3	8.29	8.93	9.57	10.20	10.84	11.48	12.12	12.75	13.39	14.0
3 1/4	8.98	9.67	10.36	11.05	11.74	12.43	13.12	13.81	14.50	15.2
3 1/2	9.67	10.41	11.16	11.90	12.65	13.39	14.13	14.87	15.62	16.3
3 3/4	10.36	11.16	11.95	12.75	13.55	14.34	15.14	15.94	16.74	17.5
4	11.05	11.90	12.75	13.60	14.45	15.30	16.15	17.00	17.85	18.7
4 1/4	11.74	12.65	13.55	14.45	15.35	16.26	17.16	18.06	18.96	19.8
4 1/2	12.43	13.39	14.34	15.30	16.26	17.22	18.17	19.13	20.08	21.0
4 3/4	13.12	14.13	15.14	16.15	17.16	18.17	19.18	20.19	21.20	22.2
5	13.81	14.87	15.94	17.00	18.06	19.13	20.19	21.25	22.32	23.3
5 1/4	14.50	15.62	16.74	17.85	18.96	20.08	21.20	22.32	23.43	24.5
5 1/2	15.19	16.36	17.53	18.70	19.87	21.04	22.21	23.38	24.54	25.7
5 3/4	15.88	17.10	18.33	19.55	20.77	21.99	23.22	24.44	25.66	26.8
6	16.58	17.85	19.13	21.40	21.68	22.95	24.23	25.50	26.78	28.0
6 1/4	17.27	18.60	19.92	21.25	22.58	23.91	25.23	26.56	27.90	29.2
6 1/2	17.95	19.34	20.72	22.10	23.48	24.87	26.24	27.62	29.01	30.3
6 3/4	18.65	20.08	21.51	22.95	24.39	25.82	27.25	28.69	30.12	31.5
7	19.34	20.83	22.32	23.80	25.29	26.78	28.26	29.75	31.23	32.7
7 1/4	20.03	21.57	23.11	24.65	26.19	27.73	29.27	30.81	32.35	33.8
7 1/2	20.72	22.32	23.91	25.50	27.10	28.68	30.28	31.88	33.48	35.0
7 3/4	21.41	23.05	24.70	26.35	28.00	29.64	31.29	32.94	34.59	36.2
8	22.10	23.80	25.50	27.20	28.90	30.60	32.30	34.00	35.70	37.4
8 1/4	22.79	24.55	26.30	28.05	29.80	31.56	33.31	35.06	36.81	38.5
8 1/2	23.48	25.30	27.10	28.90	30.70	32.52	34.32	36.12	37.93	39.7
8 3/4	24.17	26.04	27.89	29.75	31.61	33.47	35.33	37.20	39.05	40.9
9	24.86	26.78	28.69	30.60	32.52	34.43	36.34	38.26	40.16	42.0
9 1/4	25.55	27.52	29.49	31.45	33.41	35.38	37.35	39.31	41.28	43.2
9 1/2	26.24	28.26	30.28	32.30	34.32	36.34	38.36	40.37	42.40	44.4
9 3/4	26.94	29.01	31.08	33.15	35.22	37.29	39.37	41.44	43.52	45.5
10	27.62	29.75	31.88	34.00	36.12	38.25	40.38	42.50	44.64	46.7
10 1/4	28.32	30.50	32.67	34.85	37.03	39.21	41.39	43.56	45.75	47.9
10 1/2	29.00	31.24	33.48	35.70	37.92	40.17	42.40	44.63	46.86	49.0
10 3/4	29.69	31.98	34.28	36.55	38.83	41.12	43.40	45.69	47.97	50.2
11	30.40	32.72	35.06	37.40	39.74	42.08	44.42	46.76	49.08	51.4
11 1/4	31.08	33.47	35.86	38.25	40.64	43.04	45.42	47.82	50.20	52.5
11 1/2	31.76	34.21	36.66	39.10	41.54	44.00	46.44	48.88	51.32	53.
11 3/4	32.46	34.95	37.46	39.95	42.45	44.94	47.45	49.94	52.44	54.
12	33.15	35.70	38.25	40.80	43.35	45.90	48.45	51.00	53.55	56.
12 1/4	33.83	36.44	39.05	41.65	44.25	46.86	49.46	52.06	54.67	57.
12 1/2	34.53	37.19	39.84	42.50	45.16	47.82	50.46	53.12	55.78	58.
12 3/4	35.22	37.93	40.64	43.35	46.06	48.77	51.48	54.19	56.90	59.

iron Stone is about one-third the weight of wrought iron. Brick is about one-fourth weight of wrought iron. Timber is about one-twelfth the weight of wrought iron. C crete is slightly lighter than stone.

50. Bolts and Nuts

U. S. Standard Screw Threads

Dimensions in Inches.

Threads per inch	Dia. of root of thread	Width of flat	Area of bolt body	Area of root of thread	Short dia. rough	Short dia. finish	Long dia. rough hexagonal	Long dia. rough square	Thickness rough	Thickness finish
20	0.185	0.0062	0.049	0.027	1/2	7/16	37/64	7/10	1/4	3/16
18	.240	.0074	.077	.045	19/32	17/32	11/16	10/12	5/16	1/4
16	.294	.0078	.110	.068	11/16	5/8	51/64	63/64	3/8	5/16
14	.344	.0089	.150	.093	25/32	23/32	9/10	1 7/64	7/16	3/8
13	.400	.0096	.196	.126	7/8	13/16	1	1 15/64	1/2	7/16
12	.454	.0104	.249	.162	31/32	29/32	1 1/8	1 23/64	9/16	1/2
11	.507	.0113	.307	.202	1 1/16	1	1 7/32	1 1/2	5/8	9/16
10	.620	.0125	.442	.302	1 1/4	1 3/16	1 7/16	1 49/64	3/4	11/16
9	.731	.0138	.601	.420	1 7/16	1 3/8	1 21/32	2 1/32	7/8	13/16
[illegible]	.837	.0156	.785	.550	1 5/8	1 9/16	1 7/8	2 19/64	1	15/16
7	.940	.0178	.994	.694	1 13/16	1 3/4	2 3/32	2 9/16	1 1/8	1 1/16
7	1.065	.0178	1.227	.893	2	1 15/16	2 5/16	2 53/64	1 1/4	1 3/16
[illegible]	1.160	.0208	1.485	1.057	2 3/16	2 1/8	2 17/32	2 3/32	1 3/8	1 5/16
[illegible]	1.284	.0208	1.767	1.295	2 3/8	2 5/16	2 3/4	3 23/64	1 1/2	1 7/16
5 1/2	1.389	.0227	2.074	1.515	2 9/16	2 1/2	2 31/32	3 5/8	1 5/8	1 9/16
5	1.491	.0250	2.405	1.746	2 3/4	2 11/16	3 3/16	3 57/64	1 3/4	1 11/16
5	1.616	.0250	2.761	2.051	2 15/16	2 7/8	3 13/32	4 5/32	1 7/8	1 13/16
4 1/2	1.712	.0277	3.142	2.302	3 1/8	3 1/16	3 5/8	4 27/64	2	1 15/16
4 1/2	1.962	.0277	3.976	3.023	3 1/2	3 7/16	4 1/16	4 61/64	2 1/4	2 3/16
4	2.176	.0312	4.909	3.719	3 7/8	3 13/16	4 1/2	5 31/64	2 1/2	2 7/16
4	2.426	.0312	5.940	4.620	4 1/4	4 3/16	4 29/32	6	2 3/4	2 11/16
3 1/2	2.629	.0357	7.069	5.428	4 5/8	4 9/16	5 3/8	6 17/32	3	2 15/16
3 1/2	2.879	.0357	8.296	6.510	5	4 15/16	5 13/16	7 1/16	3 1/4	3 3/16
3 1/4	3.100	.0384	9.621	7.548	5 3/8	5 5/16	6 7/64	7 39/64	3 1/2	3 7/16
3	3.317	.0413	11.045	8.641	5 3/4	5 11/16	6 21/32	8 1/8	3 3/4	3 11/16
3	3.567	.0413	12.566	9.963	6 1/8	6 1/16	7 3/32	8 41/64	4	3 15/16
2 7/8	3.798	.0435	14.186	11.329	6 1/2	6 7/16	7 9/16	9 3/16	4 1/4	4 3/16
2 3/4	4.028	.0454	15.904	12.753	6 7/8	6 13/16	7 31/32	9 3/4	4 1/2	4 7/16
2 5/8	4.256	.0476	17.721	14.226	7 1/4	7 3/16	8 13/32	10 1/4	4 3/4	4 11/16
2 1/2	4.480	.0500	19.635	15.763	7 5/8	7 9/16	8 27/32	10 49/64	5	4 15/16
2 1/2	4.730	.0500	21.648	17.572	8	7 15/16	9 9/32	11 23/64	5 1/4	5 3/16
2 3/8	4.953	.0526	23.758	19.267	8 3/8	8 5/16	9 23/32	11 7/8	5 1/2	5 7/16
2 3/8	5.203	.0526	25.967	21.262	8 3/4	8 11/16	10 5/32	12 3/8	5 3/4	5 11/16
2 1/4	5.423	.0555	28.274	23.098	9 1/8	9 1/16	10 19/32	12 15/16	6	5 15/16

The U. S. Standard for Screw Threads, recommended by the Franklin titute in December, 1864, and often called the Seller's System, has been y generally adopted in the United States, but the proportions recommended nuts and bolt heads have not found general acceptance because of the odd es of bar, not usually rolled by the mills, required to make the nut.

n the above table the first six columns refer to the bolt, the latter six columns to the d and nut. The threads are not cut to a sharp angle at either point or root but are tened, and the amount is termed the "width of flat." Where not otherwise specified data on heads and nuts apply to both square and hexagonal. The angle formed by surfaces of the thread is 60°. In the Whitworth or English system the angle is 55° l the point and root of the thread are rounded.

Nuts and Bolt Heads in the U. S. Standard are determined by the follow- rules, which apply to both square and hexagon nuts:

Upsets on Square and Round Steel Bars

Dimensions in inches.

Round bars				Size of upset						Square bars			
Dia.	Area, sq in	Addition for upset	Excess of area at root of thread %	Dia.	Length	Dia. at root of thread	Area at root of thread	No. of thr'ds per inch	Weight of one turnbuckle	Side of square	Area, sq in	Addition for upset	Excess of area at root of thread
5/8	0.307	4½	36.8	7/8	4	0.731	0.420	9	2½	...	.....	....	......
3/4	0.442	3⅞	24.4	1	4	0.837	0.550	8	3½	...	.....	....	......
....	......	.....	......	1⅛	4	0.940	0.694	7	4	3/4	0.563	3½	20.6
7/8	0.601	5	48.3	1¼	4	1.065	0.891	7	5¼	7/8	0.766	4	16.3
1	0.785	4⅜	34.7	1⅜	4	1.160	1.057	6	6	...	.....	....	......
1⅛	0.994	3⅞	30.3	1½	4	1.284	1.295	6	7½	1	1.000	4	29.5
1¼	1.227	3⅞	23.5	1⅝	4½	1.389	1.515	5½	8½	1⅛	1.266	4½	19.7
1⅜	1.485	3½	17.4	1¾	4½	1.490	1.744	5	10	...	.....	....	......
....	......	.....	......	1⅞	4½	1.615	2.049	5	11½	1¼	1.563	4½	31.1
1½	1.767	4⅝	30.3	2	5	1.712	2.302	4½	13	1⅜	1.891	4⅛	21.7
1⅝	2.074	4¼	27.8	2⅛	5	1.837	2.651	4½	15	...	.....	....	......
1¾	2.405	4	25.7	2¼	5	1.962	3.023	4½	18	1½	2.250	4¾	34.
1⅞	2.761	4⅛	23.9	2⅜	5½	2.087	3.410	4	20	1⅝	2.641	4⅝	29.6
2	3.142	3⅞	18.3	2½	5½	2.175	3.716	4	24	1¾	3.063	4¼	21.3
2⅛	3.547	3⅝	17.1	2⅝	5½	2.300	4.155	4	28	...	.....	....	......
....	......	.....	......	2¾	6	2.425	4.619	4	30	1⅞	3.516	5⅛	31.4
2¼	3.976	4⅝	28.5	2⅞	6	2.550	5.107	3½	34	2	4.000	4¾	27.7
2⅜	4.430	4⅜	22.6	3	6	2.629	5.430	3½	38	2⅛	4.516	4⅜	20.2
2½	4.909	4⅜	21.3	3⅛	6½	2.754	5.957	3½	50	...	.....	....	......
2⅝	5.412	4¼	20.3	3¼	6½	2.879	6.510	3½	50	2¼	5.063	5⅛	28.6
2¾	5.940	4¼	19.3	3⅜	7	3.004	7.088	3¼	65	...	.....	....	......
....	......	.....	......	3½	7	3.100	7.548	3¼	65	2⅜	5.641	6⅛	33.8
2⅞	6.492	5½	25.9	3⅝	8	3.225	8.170	¼	....	2½	6.250	6¼	30.7
3	7.069	5¼	22.2	3¾	8	3.317	8.641	3¼	....	...	.....	....	......
3⅛	7.670	5⅛	21.3	3⅞	8	3.442	9.305	3	....	2⅝	6.891	6¾	35.0
3¼	8.296	4⅞	20.7	4	8	3.567	9.9935	3	....	2¾	7.563	6	25.1
3⅜	8.946	6	26.6	4¼	9	3.798	11.33	2⅞	....	2⅞	8.266	8	37.0
3⅝	10.32	4½	23.6	4½	9	4.028	12.75	2¾	....	3	9.000	7½	41.7

Short diameter of rough nut = 1½ × diameter of bolt + ⅛ in.
Short diameter of finished nut = 1½ × diameter of bolt + 1/16 in.
Thickness of rough nut = diameter of bolt.
Thickness of finished nut = diameter of bolt − 1/16 in.
Short diameter of rough head = 1½ × diameter of bolt + ⅛ in.
Short diameter of finished head = 1½ × diameter of bolt + 1/16 in.
Thickness of rough head = ½ short diameter of head.
Thickness of finished head = diameter of bolt − 1/16 in.

The long diameter of a hexagon nut may be obtained by multiplying the short diameter by 1.155 and the long diameter of a square nut by multiplying the short diameter by 1.414.

Upsetting reduces the strength of iron, so that bars having the same diameter at root of thread as that of the bar, invariably break in the screw end, when tested to destruction, without developing the full strength of the bar. It is therefore necessary to make up for this loss in strength by an excess of metal in the upset screw ends over that in the bar. Lengths of upset ends for use with turnbuckles of standard length (6 in between heads) and with clevises should be 1 in longer than specified in table, and additions for upset should be correspondingly increased.

Standard Cast Washers are proportioned to the bolt as follows: The diameter of the larger or bearing surface is 4 times the diameter of the bolt plus ¼ in; the diameter of the

Weight in Pounds of 100 Bolts with Square Heads and Nuts

Length under head to point, inches	Diameter of bolts, inches												
	¼	5/16	⅜	7/16	½	⅝	¾	⅞	1	1¼	1½	1¾	2
1½	4.0	7.0	10.5	15.2	22.5	39.5	63.0	.....	.....	....	....	....	....
1¾	4.4	7.5	11.3	16.3	23.8	41.6	66.0	.....	.....	....	....	....	....
2	4.8	8.0	12.0	17.4	25.2	43.8	69.0	109.0	163	....	....	....	....
2¼	5.2	8.5	12.8	18.5	26.5	45.8	72.0	113.3	169	....	....	....	....
2½	5.5	9.0	13.5	19.6	27.8	48.0	75.0	117.5	174	....	....	....	....
2¾	5.8	9.5	14.3	20.7	29.1	50.1	78.0	121.8	180	....	....	....	....
3	6.3	10.0	15.0	21.8	30.5	52.3	81.0	126.0	185	358	589	900	1312
3½	7.0	11.0	16.5	24.0	33.1	56.5	87.0	134.3	196	375	613	934	1355
4	7.8	12.0	18.0	26.2	35.8	60.8	93.1	142.5	207	392	638	967	1399
4½	8.5	13.0	19.5	28.4	38.4	65.0	99.1	151.0	218	409	662	1001	1442
5	9.3	14.0	21.0	30.6	41.1	69.3	105.2	159.6	229	426	687	1034	1486
5½	10.0	15.0	22.5	32.8	43.7	73.5	111.3	168.0	240	443	711	1068	1529
6	10.8	16.0	24.0	35.0	46.4	77.8	117.3	176.6	251	460	736	1101	1573
6½	....	....	25.5	37.2	49.0	82.0	123.4	185.0	262	477	760	1135	1616
7	....	....	27.0	39.4	51.7	86.3	129.4	193.7	273	494	785	1168	1660
7½	....	....	28.5	41.6	54.3	90.5	135.0	202.0	284	511	809	1202	1703
8	....	....	30.0	43.8	59.6	94.8	141.5	210.7	295	528	834	1235	1747
9	....	....	....	46.0	64.9	103.3	153.6	227.8	317	562	883	1301	1835
10	....	....	....	48.2	70.2	111.8	165.7	244.8	339	596	932	1368	1922
11	....	....	....	50.4	75.5	120.3	177.8	261.9	360	630	982	1435	2009
12	....	....	....	52.6	80.8	128.8	189.9	278.9	382	665	1031	1502	2096
Per inch additional	1.4	2.1	3.1	4.2	5.5	8.5	12.3	16.7	21.8	34.1	49.1	66.8	87.2

smaller surface (against which the bolt head or nut bears) is twice the diameter of the bolt plus ¼ in; the thickness is equal to the diameter of the bolt; and the bolt hole is ⅛ inch larger than the diameter of the bolt.

Limit Gages. Rods generally run oversize and it has been found necessary to fix an allowable variation from required sizes. Limit gages have been proposed, and it can be then specified that the iron should enter the larger and not enter the smaller opening. The limits, as adopted by the Master Car Builders Association in 1883, are as follows (all figures in inches):

Size of iron	Maximum limit	Minimum limit	Difference	Size of iron	Maximum limit	Minimum limit	Difference
¼	0.255	0.245	0.010	⅝	0.6330	0.6170	0.016
5/16	0.318	0.307	0.011	¾	0.7585	0.7415	0.017
⅜	0.381	0.369	0.012	⅞	0.8840	0.8660	0.018
7/16	0.444	0.431	0.013	1	1.0095	0.9905	0.019
½	0.507	0.493	0.014	1⅛	1.1350	1.1150	0.020
9/16	0.570	0.555	0.015	1¼	1.2605	1.2395	0.021

Bauer's Standard. It has been found advisable in practise to make the taps somewhat larger in outside diameter than the nominal size. An examination of rough iron shows that much of it is rolled out of round more than the limit of variation allowed. C. A. Bauer therefore devised a standard which meets more readily the commercial conditions and is also interchangeable with the U. S. Standard. In this system the actual diameter of the bolt is equal to the nominal diameter plus the fraction 0.2165 divided by the number of threads per inch; the diameter of bolt at bottom of thread is the nominal diameter of bolt minus the fraction 1.29904 divided by the number of threads per inch; and the depth of thread is equal to 0.7577 divided by number of threads per inch.

Shearing Stress Factors for Beams

The maximum shearing stress in lb per sq in at the neutral axis of a beam is given by multiplying the total shear in pounds by the shearing factor in the table for the particular shape used.

Standard I Beams

Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor
27	83.0	0.099	18	65.0	0.105	9	35.0	0.187
24	115.0	0.066		60.0	0.118		30.0	0.235
	110.0	0.072		55.0	0.141		25.0	0.322
	105.0	0.077		40.0	0.193		21.0	0.440
	100.0	0.067	15	75.0	0.092	8	25.5	0.280
	95.0	0.072		70.0	0.102		23.0	0.333
	90.0	0.078		65.0	0.118		20.5	0.412
	85.0	0.088		60.0	0.133		18.0	0.534
	80.0	0.097		55.0	0.123		17.5	0.661
	69.5	0.120		50.0	0.143	7	20.0	0.375
21	57.5	0.148		45.0	0.169		17.5	0.477
20	100.0	0.068		42.0	0.189		15.0	0.659
	95.0	0.074		36.0	0.257	6	17.25	0.422
	90.0	0.079	12	55.0	0.124		14.75	0.556
	85.0	0.089		50.0	0.144		12.25	0.832
	80.0	0.098		45.0	0.172	5	14.75	0.483
	75.0	0.094		40.0	0.211		12.25	0.668
	70.0	0.104		35.0	0.224		9.75	1.090
	65.0	0.115		31.5	0.275	4	10.5	0.735
18	90.0	0.083		27.5	0.364		9.5	0.872
	85.0	0.091	10	40.0	0.165		8.5	1.120
	80.0	0.099		35.0	0.202		7.5	1.510
	75.0	0.115		30.0	0.264	3	7.5	1.128
	70.0	0.094		25.0	0.370		6.5	1.595
				22.0	0.478		5.5	2.260

Standard Channels

Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor	Depth in	Weight lb per foot	Shearing stress factor
15	55.0	0.105	10	20.0	0.323	7	12.25	0.541
	50.0	0.118		15.0	0.494		9.75	0.806
	45.0	0.135	9	25.0	0.234	6	15.50	0.382
	40.0	0.158		20.0	0.309		13.00	0.480
	35.0	0.191		15.0	0.465		10.50	0.645
	33.0	0.202		13.25	0.572		8.00	0.987
12	40.0	0.143	8	21.25	0.275	5	11.50	0.537
	35.0	0.168		18.75	0.322		9.00	0.748
	30.0	0.203		16.25	0.391		6.50	1.253
	25.0	0.262		13.75	0.496	4	7.25	0.950
	20.5	0.353		11.25	0.673		6.25	1.200
10	35.0	0.161	7	19.75	0.292		5.25	1.653
	30.0	0.192		17.25	0.346	3	6.00	1.143
	25.0	0.240		14.75	0.423		5.00	1.600
							4.00	2.390

51. Sheet Metal, Board Measure

Dimensions of Sheets of Corrugated Iron

Width of corrugation, w	Depth of corrugation, D	Number of corrugations to sheet	Covering width with a lap of one corrugation	Width of sheet after corrugation	Length of longest sheets
2½	⅝ inch	10	24 inches	26 inches	10 feet
1¼	½ inch	19½	24 inches	26 inches	8 feet
¾	¼ inch	34½	25 inches	26 inches	8 feet

Weight of Corrugated Iron

Weight Calculated for Sheets 30½ inches wide before Corrugating.

Number by Birmingham gage	Thickness, inches	Weight per sq foot, flat pounds	Weight per sq foot, corrugated pounds	Weight per square of 100 sq feet when laid, allowing 6″ lap in length and 2½″ or one corrugation lap in width for sheet lengths of:						Galvanized, weight per sq foot, flat pounds
				5 feet	6 feet	7 feet	8 feet	9 feet	10 feet	
16	.065	2.61	3.28	365	358	353	350	348	346	2.95
18	.049	1.97	2.48	275	270	267	264	262	261	2.31
20	.035	1.40	1.76	196	192	190	188	186	185	1.74
22	.028	1.12	1.41	156	154	152	150	149	148	1.46
24	.022	.88	1.11	123	121	119	118	117	117	1.22
26	.018	.72	.91	101	99	97	97	96	95	1.06

Number of Square Feet of 2½-Inch Corrugated Iron, Required to Lay One Square of 100 Square Feet with Side Lap of One Corrugation

Length of sheet, feet	Length of end lap					
	1 inch	2 inches	3 inches	4 inches	5 inches	6 inches
5	110	112	114	116	118	120
6	110	111	113	115	117	118
7	110	110	112	114	115	117
8	109	110	112	113	114	115
9	109	110	112	113	114	115
10	108	109	110	111	112	113

The United States standard gage, adopted by act of Congress in 1893, is in general use by manufacturers of sheet steel, and the above table gives the thickness and weight of corrugated iron in accordance with that standard. For weight per square, 5 percent should be added to figures given in the table of weights when sheets are laid with one and one-half laps, and 10 percent when laid with two laps. If the corrugated iron is to be painted, .2 lb per sq ft should be added to weights given in table. The 2½-inch corrugation is the one generally employed for roofing and siding, and the regular lengths of sheets are 6, 7, 8, 9, and 10 feet.

The transverse strength of a sheet of corrugated iron is found by the formula $w = 90000\, tbd/l$, in which W = the uniformly distributed load in lbs that will produce failure, t is the thickness of the sheet in inches, b is the width of the sheet in inches, d is the depth of corrugations in inches, and l is the unsupported length of the sheet in inches.

For Roofing and Siding of buildings, corrugated iron is applied directly upon steel purlins or studding by means of clips of hoop iron, placed not more than 12 inches apart,

United States Standard Gags for Sheet Iron and Steel

Adopted as standard by Amer. Rail. Mas. Mechan. Asso. and the Asso. of Amer. Steel Manufacturers.

Number of gage	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Approximate thickness in millimeters	Weight per sq ft in pounds avoirdupois of iron	Weight per sq ft in pounds avoirdupois of steel	Weight per sq meter in kilograms of steel	Number of gage
0000000	1/2	.5	12.70	20.	20.4	99.601	0000000
000000	15/32	.46875	11.91	18.75	19.125	93.376	000000
00000	7/16	.4375	11.11	17.50	17.85	87.151	00000
0000	13/32	.40625	10.32	16.25	16.575	80.926	0000
000	3/8	.375	9.53	15.	15.3	74.701	000
00	11/32	.34375	8.73	13.75	14.025	68.476	00
0	5/16	.3125	7.94	12.50	12.75	62.251	0
1	9/32	.28125	7.14	11.25	11.475	56.026	1
2	17/64	.265625	6.75	10.625	10.8375	52.913	2
3	1/4	.25	6.35	10.	10.2	49.800	3
4	15/64	.234375	5.95	9.375	9.5625	46.688	4
5	7/32	.21875	5.56	8.75	8.925	43.575	5
6	13/64	.203125	5.16	8.125	8.2875	40.463	6
7	3/16	.1875	4.76	7.5	7.65	37.350	7
8	11/64	.171875	4.37	6.875	7.0125	34.238	8
9	5/32	.15625	3.97	6.25	6.375	31.125	9
10	9/64	.140625	3.57	5.625	5.7375	28.013	10
11	1/8	.125	3.18	5.	5.1	24.900	11
12	7/64	.109375	2.78	4.375	4.4625	21.788	12
13	3/32	.09375	2.38	3.75	3.825	18.675	13
14	5/64	.078125	1.98	3.125	3.1875	15.563	14
15	9/128	.0703125	1.79	2.8125	2.86875	14.006	15
16	1/16	.0625	1.59	2.5	2.55	12.450	16
17	9/160	.05625	1.43	2.25	2.295	11.205	17
18	1/20	.05	1.27	2.	2.04	9.960	18
19	7/160	.04375	1.11	1.75	1.785	8.715	19
20	3/80	.0375	0.953	1.50	1.53	7.470	20
21	11/320	.034375	0.873	1.375	1.4025	6.848	21
22	1/32	.03125	0.794	1.25	1.275	6.225	22
23	9/320	.028125	0.714	1.125	1.1475	5.603	23
24	1/40	.025	0.635	1.	1.02	4.980	24
25	7/320	.021875	0.556	.875	.8925	4.358	25
26	3/160	.01875	0.476	.75	.765	3.735	26
27	11/640	.0171875	0.437	.6875	.70125	3.424	27
28	1/64	.015625	0.397	.625	.6375	3.113	28
29	9/640	.0140625	0.357	.5625	.57375	2.801	29
30	1/80	.0125	0.318	.5	.51	2.490	30
31	7/640	.0109375	0.278	.4375	.44625	2.179	31
32	13/1280	.01015625	0.258	.40625	.414375	2.023	32
33	3/320	.009375	0.238	.375	.3825	1.868	33
34	11/1280	.00859375	0.218	.34375	.350625	1.712	34
35	5/640	.0078125	0.198	.3125	.31875	1.556	35
36	9/1280	.00703125	0.179	.28125	.286875	1.401	36
37	17/2560	.006640625	0.169	.265625	.2709375	1.323	37
38	1/160	.00625	0.159	.25	.255	1.245	38

which encircle the purlin or stud. Numbers 20 and 22 are the gages most frequently used for roofs, and numbers 22 and 24 for siding. The sheets are either painted or galvanized, preferably the latter.

Section Areas of Rectangular Plates, in Square Inches

Width of Plate, in Ins.	Thickness of Plate, in Inches									
	⅛	¼	⅜	½	⅝	¾	⅞	1	1¼	1½
1	.1250	.2500	.3750	.500	.6250	.9375	.8750	1.00	1.250	1.500
1½	.1875	.3750	.5625	.750	.9375	1.125	1.312	1.50	1.875	2.500
2	.2500	.5000	.7500	1.00	1.250	1.500	1.750	2.00	2.500	3.000
2½	.3125	.6250	9375	1.25	1.562	1.875	2.188	2.50	3.125	3.750
3	.3750	.7500	1.125	1.50	1.875	2.250	2.625	3.00	3.750	4.500
3½	.4375	.8750	1.312	1.75	2.188	2.625	3.062	3.50	4.375	5.250
4	.5000	1.000	1.500	2.00	2.500	3.000	3.500	4.00	5.000	6.000
4½	.5625	1.125	1.687	2.25	2.812	3.375		4.50	5.125	6.750
5	.6250	1.250	1.875	2.50	3.125	3.750	3.838	5.00	6.250	7.500
5½	.6875	1.375	2.062	2.75	3.438	4.125	4.812	5.50	6.875	8.250
6	.7500	1.500	2.250	3.00	3.750	4.500	5.250	6.00	7.500	9.000
6½	.8125	1.625	2.438	3.25	4.062	4.875	5.688	6.50	8.125	9.750
7	.8750	1.750	2.625	3.50	4.375	5.250	6.125	7.00	8.750	10.50
7½	.9375	1.875	2.812	3.75	4.688	5.625	6.562	7.50	9.375	11.25
8	1.000	2.000	3.000	4.00	5.000	6.000	7.000	8.00	10.00	12.00
8½	1.062	2.225	3.188	4.25	5.312	6.375	7.438	8.50	10.62	12.75
9	1.125	2.250	3.375	4.50	5.625	6.750	7.875	9.00	11.25	13.50
9½	1.188	2.375	3.563	4.75	5.938	7.125	8.312	9.50	11.88	14.25
10	1.250	2.500	3.750	5.00	6.250	7.500	8.750	10.0	12.50	15.00
10½	1.313	2.625	3.938	5.25	6.563	7.875	9.188	10.5	13.12	15.75
11	1.375	2.750	4.125	5.50	6.875	8.250	9.625	11.0	13.75	16.50
11½	1.438	2.875	4.312	5.75	7.188	8.625	10.06	11.5	14.38	17.25
1[illegible]	1.500	3.00[illegible]	4.500	6.0[illegible]	7.50[illegible]	9.000	10.50	12.0	15.00	18.00
12½	1.563	3.225	4.688	6.25	7.812	9.375	10.94	12.5	15.62	18.75
13	1.625	3.250	4.875	6.50	8.125	9.750	11.38	13.0	16.25	19.50
14	1.750	3.500	5.250	7.00	8.750	10.50	12.25	14.0	17.50	21.00
15	1.875	3.750	5.625	7.50	9.375	11.25	13.13	15.0	18.75	22.50
16	2.000	4.000	6.000	8.00	10.00	12.00	14.00	16.0	20.00	24.00
17	2.125	4.250	6.375	8.50	10.62	12.75	14.88	17.0	21.25	25.50
18	[illegible]50	4.500	6.75[illegible]	9.00	[illegible]	13.50	15.75	18.[illegible]	22.50	27.00
19	2.375	4.750	7.125	9.50	11.88	14.25	16.63	19.0	23.75	28.50
20	2.500	5.000	7.500	10.0	12.50	15.00	17.50	20.0	25.00	30.00
21	2.625	5.250	7.875	10.5	13.12	15.75	18.38	21.0	26.25	31.50
22	2.750	5.500	8.250	11.0	13.75	16.50	19.25	22.0	27.50	33.00
23	2.875	5.750	8.625	11.5	14.38	17.25	20.13	23.0	28.75	34.50
24	3.000	6.000	9.000	12.0	15.00	18.00	21.00	24.0	30.00	36.00
26	3.250	6.500	9.750	13.0	16.25	19.50	22.75	26.0	32.50	39.00
28	3.500	7.000	10.50	14.0	17.50	21.00	24.50	28.0	35.00	42.00
30	3.750	7.500	11.25	15.0	18.75	22.50	26.25	30.0	37.50	45.00
32	4.000	8.000	12.00	16.0	20.00	24.00	28.00	32.0	40.00	48.00
34	4.250	8.500	12.75	17.0	21.25	25.50	29.75	34.0	42.50	51.00
36	4.500	9.000	13.50	18.0	22.50	27.00	31.50	36.0	45.00	54.00
38	4.750	9.500	14.25	19.0	23.75	28.50	33.25	38.0	47.50	57.00
40	5.000	10.00	15.00	20.0	25.00	30.00	35.00	40.0	50.00	60.00

This multiplication table is applicable to any kind of material, but it is especially useful in computations on the plates which are used in the beams and members of steel structures for roofs and bridges.

Feet Board Measure, for Stuff 1 inch thick

Width, in Ins.	Length, in Feet									
	1	2	3	4	5	6	7	8	9	12
1	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	1.00
1¼	.1042	.2083	.3125	.4167	.5208	.6250	.7292	.8333	.9375	1.25
1½	.1250	.2500	.3750	.5000	.6250	.7500	.8750	1.000	1.125	1.50
1¾	.1458	.2917	.4375	.5833	.7292	.8750	1.021	1.167	1.313	1.75
2	.1667	.3333	.5000	.6667	.8333	1.000	1.167	1.333	1.500	2.00
2¼	.1875	.3750	.5625	.7500	.9375	1.125	1.313	1.500	1.688	2.25
2½	.2083	.4167	.6250	.8333	1.042	1.250	1.458	1.667	1.875	2.50
2¾	.2292	.4583	.6875	.9167	1.146	1.375	1.604	1.833	2.063	2.75
3	.2500	.5000	.7500	1.000	1.250	1.500	1.750	2.000	2.250	3.00
3¼	.2708	.5417	.8125	1.083	1.354	1.625	1.896	2.167	2.437	3.25
3½	.2917	.5833	.8750	1.167	1.458	1.750	2.042	2.333	2.625	3.50
3¾	.3125	.6250	.9375	1.250	1.563	1.875	2.188	2.500	2.812	3.75
4	.3333	.6667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	4.00
4¼	.3541	.7083	1.062	1.416	1.771	2.125	2.479	2.833	3.187	4.25
4½	.3750	.7500	1.125	1.500	1.875	2.250	2.625	3.000	3.375	4.50
4¾	.3953	.7917	1.188	1.584	1.979	2.375	2.776	3.167	3.562	4.75
5	.4167	.8333	1.250	1.667	2.083	2.500	2.917	3.333	3.750	5.00
5½	.4583	.9167	1.375	1.833	2.292	2.750	3.208	3.667	4.125	5.50
6	.5000	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	6.00
6½	.5417	1.083	1.625	2.167	2.708	3.250	3.792	4.333	4.875	6.50
7	.5833	1.167	1.750	2.333	2.917	3.500	4.083	4.667	5.250	7.00
7½	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5 000	5.625	7.50
8	.6667	1.333	2.000	2.667	3.333	4.000	4.667	5.333	6.000	8.00
8½	.7083	1.417	2.125	2.833	3.542	4.250	4.958	5.667	6.375	8.50
9	.7500	1.500	2.250	3.000	3.750	4.500	5.250	6.000	6.750	9.00
9½	.7917	1.583	2.375	3.167	3.908	4.750	5.542	6.333	7.125	9.50
10	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	10.0
10½	.8750	1.750	2.625	3.500	4.375	5.250	6.125	7.000	7.875	10.5
11	.9167	1.833	2.750	3.667	4.583	5.500	6.417	7.333	8.250	11.0
11½	.9583	1.917	2.875	3.833	4.792	5.750	6.708	7.667	8.625	11.5
12	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	12.0
12½	1.042	2.083	3.125	4.167	5.280	6.250	7.292	8.333	9.375	12.5
13	1.083	2.167	3.250	4.333	5.417	6.500	7.583	8.666	9.750	13.0
14	1.167	2.333	2.500	4.667	5.833	7.000	8.167	9.333	10.50	14.0
15	1.250	2.250	3.750	5.000	6.250	7.500	8.750	10.00	11.25	15.0
16	1.333	2 667	4.000	5.333	6.667	8.000	9.333	10.67	12.00	16.0
17	1.417	2.833	4.250	5.667	7.083	8.500	9.917	11.33	12.75	17.0
18	1.500	3.000	4.500	6.000	7.500	9.000	10.50	12.00	13.50	18.0
19	1.583	3.167	4.750	6.333	7.917	9.500	11.08	12.67	14.25	19.0
20	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	20.0

A board 1 in thick, 1 ft wide, and 1 ft long contains 1 foot board measure (1.00 ft BM). This table gives ft BM for lumber of different widths and lengths. For stuff more than 1 in thick, multiply tabular values by the thickness in inches; thus, for a plank 3 in thick, 10½ in wide, and 1 ft long, $3 \times .875 = 2.625$ ft BM. Also, for 418 lineal ft of such plank $418 \times 2.625 = 1097$ ft BM.

The number of feet BM in a lot of lumber may also be found by taking 12 times its contents in cubic feet. Or, conversely, the number of cu ft is one-twelfth of the number of ft BM.

52. Pipes and Fittings

Standard Welded Pipe

Dimensions as manufactured by National Tube Co. and by Crane Co.

Diameter			Thickness, in	Transverse areas			Length of pipe containing one cubic foot, feet	Nominal weight per foot, pounds	Number of threads per inch of screw
Nominal internal, in	Actual external, in	Actual internal, in		External, sq in	Internal, sq in	Metal, sq in			
Butt welded									
1/8	0.405	0.27	.068	.129	.0573	.0717	2513.	.241	27
1/4	0.54	0.364	.088	.229	.1041	.1249	1383.3	.42	18
3/8	0.675	0.494	.091	.358	.1917	.1663	751.2	.559	18
1/2	0.84	0.623	.109	.554	.3048	.2492	472.4	.837	14
3/4	1.05	0.824	.113	.866	.5333	.3327	270.	1.115	14
1	1.315	1.048	.134	1.358	.8626	.4954	166.9	1.668	11½
1¼	1.66	1.38	.14	2.164	1.496	.668	96.25	2.244	11½
Lap welded									
1½	1.9	1.611	.145	2.835	2.038	.797	70.66	2.678	11½
2	2.375	2.067	.154	4.43	3.356	1.074	42.91	3.609	11½
2½	2.875	2.468	.204	6.492	4.784	1.708	30.1	5.739	8
3	3.5	3.067	.217	9.621	7.388	2.243	19.5	7.536	8
3½	4.0	3.548	.226	12.566	9.887	2.679	14.57	9.001	8
4	4.5	4.026	.237	15.904	12.73	3.174	11.31	10.665	8
4½	5.0	4.508	.246	19.635	15.961	3.674	9.02	12.34	8
5	5.563	5.045	.259	24.306	19.99	4.316	7.2	14.502	8
6	6.625	6.065	.28	34.472	28.888	5.584	4.98	18.762	8
7	7.625	7.023	.301	45.664	38.738	6.926	3.72	23.271	8
8	8.625	7.982	.322	58.426	50.04	8.386	2.88	28.177	8
9	9.625	8.937	.344	72.76	62.73	10.03	2.29	33.701	8
10	10.75	10.019	.366	90.763	78.839	11.924	1.82	40.065	8
11	11.75	11.000	.375	108.4	95.03	13.37	1.51	45.0	8
12	12.75	12.000	.375	127.6	113.0	14.6	1.27	49.0	8

Tests on lap-welded iron pipe, made in 1895, showed as follows: The external diameter of pipe was 7⅝ inches and the thickness ⅝ in. Specially designed cast-iron flanges, weighing about 61 lb per pair, were used for joints. Eight tests, in which the hydraulic pressure varied from 800 to 3000 lb per sq in, were made on six lengths of pipe bolted together at the flanges. In all but one test failure occurred in the flanges. In the single case a section of pipe burst on the side opposite the weld at a pressure of 2400 lb per sq in, leaving a longitudinal rent about 18 inches long. At the point of rupture the metal had the same thickness as elsewhere but contained a blister about 12 inches long.

Butt-welded pipe is tested to 300 lb per sq in, and lap-welded is tested to 500 lb per sq in.

Soft steel is being largely used in place of wrought iron for pipes, altho the latter is considered by some authorities as preferable on the ground that it is less liable to corrosion. Tests made by Howe in 1897, developed the following average bursting pressures in lb per sq in under hydrostatic pressure:

	2-in line pipe	2-in tubing	5⅝-in casing
Wrought iron	2 918	4 106	931
Steel	4 733	5 800	2 038
Excess strength of steel over wrought iron	62 percent	41 percent	119 percent

Extra Strong Welded Pipe

Dimensions as manufactured by National Tube Co. and by Crane Co.

Diameter			Thickness, in	Transverse areas			Nominal weight per foot, pounds	Number of threads per inch of screw
Nominal internal, in	Actual external, in	Actual internal, in		External, sq in	Internal, sq in	Metal, sq in		
Butt welded								
⅛	.405	.205	.100	.129	.033	.096	.29	27
¼	.540	.294	.123	.229	.068	.161	.54	18
⅜	.675	.421	.127	.358	.139	.219	.74	18
½	.840	.542	.149	.554	.231	.323	1.09	14
¾	1.050	.736	.157	.866	.425	.441	1.39	14
1	1.315	.951	.182	1.358	.710	.648	2.17	11½
1¼	1.660	1.272	.194	2.164	1.271	1.893	3.00	11½
Lap welded								
1½	1.900	1.494	.203	2.835	1.753	1.082	3.63	11½
2	2.375	1.933	.221	4.430	2.935	1.495	5.02	11½
2½	2.875	2.315	.280	6.492	4.209	2.283	7.67	8
3	3.500	2.892	.304	9.621	6.569	3.052	10.25	8
3½	4.000	3.358	.321	12.566	8.856	3.710	12.47	8
4	4.500	3.818	.341	15.904	11.449	4.455	14.97	8
4½	5.000	4.280	.360	19.635	14.387	5.248	18.22	8
5	5.563	4.813	.375	24.306	18.193	6.113	20.54	8
6	6.625	5.751	.437	34.472	25.976	8.496	28.58	8
7	7.625	6.625	.500	45.664	34.472	11.192	37.67	8
8	8.625	7.625	.500	58.426	45.664	12.762	43.00	8
9	9.625	8.625	.500	72.760	58.426	14.334	48.25	8
10	10.750	9.750	.500	90.763	74.662	16.101	54.25	8
12	12.750	11.750	.500	127.68	108.43	19.25	65.00	8

Boiler Tubes (National Tube Works), of charcoal-iron, lap-welded, come in standard sizes of 1 in to 4 in external diameter, advancing by quarter inches. There is also a 4½-inch size. Above that, beginning at 5 in, they advance by inches to 21 in.

For estimating the effective steam-heating or boiler surface of tubes, the surface in contact with air or gases of combustion (whether internal or external) is to be taken. For heating liquids by steam, superheating steam, or transferring heat from one liquid or gas to another, the mean surface is to be taken. The square feet of surface, S, in a tube of a given L feet long and d inches in diameter may be obtained by the formula $S=0.2618\ dL$.

The Standard Pipe Fittings given in the tables were adopted by the A.S.M.E. in 1914 after conferences between a committee of that society, a committee representing the manufacturers of pipe fittings, and a committee of the National Assn. of Master Steam and Hot Water Fitters.

Standard Pipe Flanges—Continued on p. 466

(Trans. A.S.M.E., Vol. XXXVI, p. 29)

For 125 lb per sq in pressure—Dimensions are in inches

Dia. of pipe	Minimum Thickness (Fractions of an in)	Stress on pipe lb per sq in	Dia. of flange	Thickness of flange	Width of flange face	Dia. of bolt circle	No. of bolts	Dia. of bolts	Stress, lb per sq in on bolt metal	Dia. of bolt holes
1	7/16	143	4	7/16	1⅛	3	4	7/16	264	9/16
1¼	7/16	178	4½	½	1⅝	3⅜	4	7/16	412	9/16
1½	7/16	214	5	9/16	1¾	3⅞	4	½	438	⅝
2	7/16	286	6	⅝	2	4¾	4	⅝	486	¾
2½	7/16	357	7	11/16	2¼	5½	4	⅝	750	¾
3	7/16	428	7½	¾	2¼	6	4	⅝	1093	¾
3½	7/16	500	8½	13/16	2½	7	4	⅝	1488	¾
4	½	500	9	15/16	2½	7½	8	⅝	972	¾
4½	½	562	9¼	15/16	2⅜	7¾	8	¾	823	⅞
5	½	625	10	15/16	2½	8½	8	¾	1016	⅞
6	9/16	667	11	1	2½	9½	8	¾	1463	⅞
7	⅝	700	12½	1 1/16	2¾	10¾	8	¾	1991	⅞
8	⅝	800	13½	1⅛	2¾	11¾	8	¾	2600	⅞
9	11/16	818	15	1⅛	3	13¼	12	¾	2194	⅞
10	¾	833	16	1 3/16	3	14¼	12	⅞	1948	1
12	13/16	923	19	1¼	3½	17	12	⅞	2805	1
14	⅞	1000	21	1⅜	3½	18¾	12	1	2915	1⅛
15	⅞	1072	22¼	1⅜	3⅝	20	16	1	2510	1⅛
16	1	1000	23½	1 7/16	3¾	21¼	16	1	2856	1⅛
18	1 1/16	1059	25	1 9/16	3½	22¾	16	1⅛	2865	1¼
20	1⅛	1111	27½	1 11/16	3¾	25	20	1⅛	2829	1¼
22	1 3/16	1158	29½	1 13/16	3¾	27¼	20	1¼	2660	1⅜
24	1¼	1200	32	1⅞	4	29½	20	1¼	3166	1⅜
26	1 5/16	1238	34¼	2	4⅛	31¾	24	1¼	3096	1⅜
28	1⅜	1273	36½	2 1/16	4¼	34	28	1¼	3078	1⅜
30	1 7/16	1304	38¾	2⅛	4⅜	36	28	1⅜	2985	1½
32	1½	1333	41¾	2¼	4⅞	38½	28	1½	2775	1⅝
34	1 9/16	1360	43¾	2 5/16	4⅞	40½	32	1½	2741	1⅝
36	1⅝	1385	46	2⅜	5	42¾	32	1½	3073	1⅝
38	1 11/16	1407	48¾	2⅜	5⅜	45¼	32	1⅝	2924	1¾
40	1¾	1428	50¾	2½	5⅜	47¼	36	1⅝	2880	1¾
42	1 13/16	1448	53	2⅝	5½	49½	36	1⅝	3175	1¾
44	1⅞	1467	55¼	2⅝	5⅝	51¾	40	1⅝	3136	1¾
46	1 15/16	1484	57¼	2 11/16	5⅝	53¾	40	1⅝	3428	1¾
48	2	1500	59½	2¾	5¾	56	44	1⅝	3393	1¾
50	2 1/16	1515	61¾	2¾	5⅞	58¼	44	1¾	3195	1⅞
52	2⅛	1530	64	2⅞	6	60½	44	1¾	3456	1⅞

Standard Pipe Flanges—Continued

(Trans. A.S.M.E., Vol. XXXVI, p. 29)

For 125 lb per sq in pressure—Dimensions are in inches

Dia. of pipe	Minimum thickness (Fractions of an in)	Stress on pipe lb per sq in	Dia. of flange	Thickness of flange	Width of flange face	Dia. of bolt circle	No. of bolts	Dia. of bolts	Stress, lb per sq in on bolt metal	Dia. of bolt holes
54	2 3/16	1543	66¼	3	6⅛	62¾	44	1¾	3726	1⅞
56	2¼	1555	68¾	3	6⅜	65	48	1¾	3674	1⅞
58	2 5/16	1567	71	3⅛	6½	67¼	48	1¾	3941	1⅞
60	2 7/16	1538	73	3⅛	6½	69¼	52	1¾	3892	1⅞
62	2½	1550	75¾	3¼	6⅞	71¾	52	1⅞	3538	2
64	2 9/16	1561	78	3¼	7	74	52	1⅞	3770	2
66	2⅝	1572	80	3⅜	7	76	52	1⅞	4010	2
68	2 11/16	1582	82¼	3⅜	7⅛	78¼	56	1⅞	3952	2
70	2¾	1591	84½	3½	7¼	80½	56	1⅞	4188	2
72	2 13/16	1600	86½	3½	7¼	82½	60	1⅞	4136	2
74	2⅞	1609	88½	3⅝	7¼	84½	60	1⅞	4368	2
76	2 15/16	1617	90¾	3⅝	7⅜	86½	60	1⅞	4608	2
78	3	1625	93	3¾	7½	88¾	60	2	4325	2⅛
80	3 1/16	1633	95¼	3¾	7⅝	91	60	2	4549	2⅛
82	3⅛	1640	97½	3⅞	7¾	93¼	60	2	4779	2⅛
84	3 3/16	1647	99¾	3⅞	7⅞	95½	64	2	4702	2⅛
86	3¼	1653	102	4	8	97¾	64	2	4928	2⅛
88	3 5/16	1660	104¼	4	8⅛	100	68	2	4857	2⅛
90	3⅜	1667	106½	4⅛	8¼	102¼	68	2⅛	4416	2¼
92	3½	1643	108¾	4⅛	8⅜	104½	68	2⅛	4615	2¼
94	3 9/16	1649	111	4¼	8½	106¼	68	2⅛	4817	2¼
96	3⅝	1655	113¼	4¼	8⅝	108½	68	2¼	4401	2⅜
98	3 11/16	1661	115½	4⅜	8¾	110¾	68	2¼	4587	2⅜
100	3¾	1667	117¾	4⅜	8⅞	113	68	2¼	4776	2⅜

NOTES:—Bolt holes should straddle center lines. Flanges should be plain faced. Square head bolts with hexagonal nuts are recommended. For bolts 1⅝ in diameter and larger stud, with a nut, at each end is satisfactory.

Hexagonal nuts for pipe sizes 1 to 46 in can be conveniently pulled up with open wrenches of minimum design of heads. Hexagonal nuts for pipe sizes 48 to 100 in can be conveniently pulled up with box or socket wrenches.

RULES approximately followed in compiling above data:

Bolt circle $= 1.10\,D + 3$
Flange thickness $= 0.0315\,D + 1.25$ (for sizes 26 to 100 in)
D = inside diameter of pipe

Flanges to be spot bored for nuts for sizes 32 to 100 inclusive.

Standard Pipe Flanges

(Trans. A.S.M.E., Vol. XXXVI, p. 29)

For 250 lb per sq in pressure—Dimensions are in inches

Dia. of pipe	Minimum thickness (Fractions of an in)	Stress on pipe lb per sq in	Dia. of flange	Thickness of flange	Width of flange face	Dia. of bolt circle	No. of bolts	Dia. of bolts	Stress, lb per sq in on bolt metal	Dia. of bolt holes
1	½	250	4½	11/16	1¾	3¼	4	½	389	⅝
1¼	½	312	5	¾	1⅞	3¾	4	½	609	⅝
1½	½	375	6	13/16	2¼	4½	4	⅝	547	¾
2	½	500	6½	⅞	2¼	5	4	⅝	972	¾
2½	9/16	555	7½	1	2½	5⅞	4	¾	1016	⅞
3	9/16	667	8¼	1⅛	2⅝	6⅝	8	¾	731	⅞
3½	9/16	778	9	1 3/16	2¾	7¼	8	¾	995	⅞
4	⅝	800	10	1¼	3	7⅞	8	¾	1300	⅞
4½	⅝	900	10½	1 5/16	3	8½	8	¾	1646	⅞
5	11/16	909	11	1⅜	3	9¼	8	¾	2032	⅞
6	¾	1000	12½	1 7/16	3¼	10⅝	12	¾	1950	⅞
7	13/16	1077	14	1½	3½	11⅞	12	⅞	1909	1
8	13/16	1230	15	1⅝	3½	13	12	⅞	2493	1
9	⅞	1285	16¼	1¾	3⅝	14	12	1	2410	1⅛
10	15/16	1333	17½	1⅞	3¾	15¼	16	1	2231	1⅛
12	1	1500	20½	2	4¼	17¾	16	1⅛	2546	1¼
14	1⅛	1555	23	2⅛	4½	20¼	20	1⅛	2773	1¼
15	1 3/16	1579	24½	2 3/16	4¾	21½	20	1¼	2473	1⅜
16	1¼	1600	25½	2¼	4¾	22½	20	1¼	2814	1⅜
18	1⅜	1636	28	2⅜	5	24¾	24	1¼	2968	1⅜
20	1½	1666	30½	2½	5¼	27	24	1⅜	3096	1½
22	1 9/16	1760	33	2⅝	5¼	29¼	24	1½	3058	1⅝
24	1⅝	1846	36	2¾	5¾	32	24	1⅝	3110	1¾
26	1 13/16	1793	38¼	2 13/16	6⅛	34½	28	1⅝	3126	1¾
28	1⅞	1866	40¾	2 15/16	6⅜	37	28	1⅝	3629	1¾
30	2	1875	43	3	6½	39¼	28	1¾	3615	1⅞
32	2⅛	1882	45¼	3⅛	6⅝	41½	28	1⅞	3501	2
34	2¼	1889	47½	3¼	6¾	43½	28	1⅞	3952	2
36	2⅜	1894	50	3⅜	7	46	32	1⅞	3877	2
38	2 7/16	1948	52¼	3 7/16	7⅛	48	32	1⅞	4320	2
40	2 9/16	1953	54½	3 9/16	7¼	50¼	36	1⅞	4255	2
42	2 11/16	1953	57	3 11/16	7½	52¾	36	1⅞	4691	2
44	2 13/16	1955	59¼	3¾	7⅝	55	36	2	4587	2⅛
46	2⅞	2000	61½	3⅞	7¾	57¼	40	2	4512	2⅛
48	3	2000	65	4	8½	60¾	40	2	4913	2⅛

Notes:—Bolt holes should straddle center lines. Flanges should have 1/16-in. raised face for gaskets.

Square head bolts with hexagonal nuts are recommended. For bolts 1⅝ in diameter and larger stud with a nut at each end is satisfactory.

Hexagonal nuts for pipe sizes 1 to 16 in can be conveniently pulled up with open wrenches of minimum design of head 18 to 48 in can be conveniently pulled up with box or socket wrenches.

Rules approximately followed in compiling above data:

Bolt circles $= 1.171\,D + 3.75$

Flange thickness $= 0.0546\,D + 1.375$ (for sizes 10 to 48 in)

D = inside diameter of pipe.

Distance between inside edge of bolt holes and raised face to be 1/32 in.

Thickness of flange given in table includes raised face.

Flanges to be spot bored for nuts.

53. Ropes and Chains

Weight and Strength of Manila Rope

Eng. News, Dec. 6, 1890.

Diameter in inches	Circumference in inches	Weight of 100 feet of rope in pounds	Ultimate tensile strength of rope in pounds, calculated by the formulas of Hunt	Miller
3/16	9/16	2	230	280
5/16	1	4	630	790
3/8	1 1/8	5	900	1 140
1/2	1 1/2	7 2/3	1 620	2 020
5/8	2	13 1/3	2 880	3 380
13/16	2 1/2	20	4 500	5 030
1	3	28 1/3	6 480	7 020
1 1/8	3 1/2	38	8 820	9 370
1 5/16	4	52	11 500	12 000
1 1/2	4 1/2	65	14 600	14 900
1 5/8	5	80	18 000	18 100
2	6	113	25 900	25 200
2 1/4	7	153	35 300	34 300
2 5/8	8	211	46 100	44 800
3	9	262	58 300	56 700
3 1/4	10	325	72 000	70 000

"Stevedore" Rope is made by lubricating the fibers with plumbago mixt with sufficient tallow to hold it in position, so as to prevent the internal chafing and wear due to the friction between the several strands and the yarns of these strands in passing over sheaves. After running awhile the exterior of the rope becomes comprest and coated with the lubricant. The breaking strength (according to Hunt), in pounds, may be taken at 720 times the square of the circumference in inches; and the weight in pounds per foot at 0.032 times square of circumference in inches.

Tests of Cordage

Tests of Metals, Watertown Arsenal, 1906.

Circumference, Nominal	Circumference, Actual inches	Actual diameter, inches	Number of strands	Yarns per strand	Lay, one turn in inches	Tensile strength, pounds
		Manila rope				
6 threads	0.99	.29	[illegible]	2	.90	620
9 "	1.22	.37	[illegible]	3	1.02	1 250
9 "	1.30	.42	3	3	1.25	1 260
15 "	1.55	.49	[illegible]	5	1.42	1 860
15 "	1.64	.52	[illegible]	5	1.60	1 620
21 "	1.80	.57	3	7	1.90	2 640
1 inch	1.10	.34	[illegible]	3	1.02	820
1 1/2	1.64	.56	3	6	2.04	2 550
2	2.53	.84	4	8	2.72	4 80
2 1/2	2.85	.92	4	11	3.16	5 790
3	3.52	1.10	4	16	3.54	8 05
3 1/2	4.30	1.38	4	23	4.18	11 80
4	4.60	1.52	4	30	4.24	14 700
4 1/2	5.02	1.66	4	35	4.40	17 05
5	5.32	1.80	4	40	4.70	18 100
5 1/2	6.02	2.00	4	50	5.40	18 750
6	7.10	2.33	4	62	5.80	25 300
7	8.02	2.63	4	90	6.60	29 400
		Hemp rope				
2 1/2	3.00	.96	3	15	2.80	3 800
3	3.20	1.02	4	19	2.70	5 140
3	3.33	....	4	17	....	5 960
3 1/2	3.87	1.26	4	26	3.16	6 100
3 1/2	3.93	....	4	23	....	7 850

Crane Chains

Dimensions				"D.B.G." special crane			Crane		
Size of chain, in	Pitch approximately, in	Weight per foot approximately, lb	Outside width, in	Proof test, lb	Average breaking load, lb	Ordinary safe load for general use, lb	Proof test, lb	Average breaking load, lb	Ordinary safe load for general use, lb
1/4	25/32	0.875	7/8	1 932	3 864	1 288	1 680	3 360	1 120
5/16	27/32	1.000	1 1/16	2 898	5 796	1 932	2 520	5 040	1 680
3/8	31/32	1.700	1 1/4	4 186	8 372	2 790	3 640	7 280	2 427
7/16	1 5/32	2.000	1 3/8	5 796	11 592	3 864	5 040	10 080	3 360
1/2	1 11/32	2.500	1 11/16	7 728	15 456	5 182	6 720	13 440	4 480
9/16	1 15/32	3.200	1 7/8	9 660	19 320	6 440	8 400	16 800	5 600
5/8	1 23/32	4.125	2 1/16	11 914	23 828	7 942	10 360	20 720	6 907
11/16	1 27/32	5.000	2 1/4	14 490	28 980	9 660	12 600	25 200	8 400
3/4	1 31/32	5.875	2 1/2	17 388	34 776	11 592	15 120	30 240	10 080
13/16	2 3/32	6.700	2 11/16	20 286	40 572	13 524	17 640	35 280	11 760
7/8	2 7/32	8.000	2 7/8	22 484	44 968	14 989	20 440	40 880	13 627
15/16	2 15/32	9.000	3 1/16	25 872	51 744	17 248	23 520	47 040	15 680
1	2 19/32	10.700	3 1/4	29 568	59 136	19 712	26 880	53 760	17 920
1 1/16	2 23/32	11.200	3 5/16	33 264	66 538	22 176	30 240	60 480	20 160
1 1/8	2 27/32	12.500	3 3/4	37 576	75 152	25 050	34 160	68 320	22 773
1 3/16	3 5/32	13.700	3 7/8	41 888	83 776	27 925	38 080	76 160	25 387
1 1/4	3 7/32	16.000	4 1/8	46 200	92 400	30 800	42 000	84 000	28 000
1 5/16	3 15/32	16.500	4 3/8	50 512	101 024	33 674	45 920	91 840	30 613
1 3/8	3 5/8	18.400	4 9/16	55 748	111 496	37 165	50 680	101 360	33 787
1 7/16	3 25/32	19.700	4 3/4	60 368	120 736	40 245	54 880	109 760	36 587
1 1/2	3 31/32	21.700	5	66 528	133 056	44 352	60 480	120 960	40 320

Crane Chains. The distance from center of one link to center of next is equal to the inside length of the link, but in practise 1/32 inch is allowed for weld. This is approximate, and where exactness is required chains should be made so. For chain sheaves the diameter, if possible, should be not less than 20 times the diameter of chain used.

Wrought-iron Chain Cables. The United States Committee on Tests of Chain Cables found that the ultimate strength of a chain is equal to 163 percent of the bar from which the links are made. The short open links were found to be stronger than the long stud links, although Weisbach states that stud chains are 4/3 times as strong as open link chains. The proof test of a chain cable, which should be carried without deformation, should be about one-half of the ultimate strength of the weakest link. The Pennsylvania R.R. specifications (1899) prescribe testing to destruction a piece 2 feet long out of each 200 feet, and provide for an elongation of 10 percent.

The ultimate resistances, determined by the U. S. committee from a great number of tests, are given for wrought-iron chain cables corresponding to diameters of bars from which cables are made, as follows:

Inches	Pounds	Inches	Pounds	Inches	Pounds
1	71 172	1 3/8	128 129	1 3/4	200 074
1 1/16	79 544	1 7/16	139 103	1 13/16	213 475
1 1/8	88 445	1 1/2	150 485	1 7/8	227 271
1 3/16	97 731	1 9/16	162 283	1 15/16	241 463
1 1/4	107 440	1 5/8	174 475	2	256 040
1 5/16	117 577	1 11/16	187 075	....	....

54. Wire and Wire Gages

Sizes and Weights of Steel Wire

American Steel and Wire Co.'s Gage

Number of Gage	Diameters			Sectional area sq in	Weight		Number of feet per pound
	Fractions of inch	Decimals of inch	Milli-meters		Pounds per foot	Pounds per mile	
....	½	.5000	12.70	.19635	.6668	3521.	1.500
0000000	....	.4900	12.45	.18857	.6404	3381.	1.562
....	15/32	.46875	11.91	.17257	.5861	3094.	1.706
000000	....	.4615	11.72	.16728	.5681	2999.	1.76
....	7/16	.4375	11.11	.15033	.5105	2696.	1.959
00000	....	.4305	10.93	.14556	.4943	2610.	2.023
....	13/32	.40625	10.32	.12962	.4402	2324.	2.272
0000	....	.3938	10.00	.12180	.4136	2184.	2.418
....	⅜	.3750	9.525	.11045	.3751	1980.	2.666
000	....	.3625	9.2075	.10321	.3505	1851.	2.853
....	11/32	.34375	8.731	.092806	.3152	1664.	3.173
00	....	.3310	8.407	.086049	.2922	1548.	3.422
....	5/16	.3125	7.938	.076699	.2605	1375.	3.839
0	....	.3065	7.785	.073782	.2506	1323.	3.991
1	....	.2830	7.188	.062902	.2136	1128.	4.681
....	9/32	.28125	7.144	.062126	.2110	1114.	4.74
2	....	.2625	6.668	.054119	.1838	970.4	5.441
....	¼	.2500	6.350	.049087	.1667	880.2	5.999
3	....	.2437	6.190	.046645	.1584	836.4	6.313
4	....	.2253	5.723	.039867	.1354	714.8	7.386
....	7/32	.21875	5.556	.037583	.1276	673.9	7.835
5	....	.2070	5.258	.033654	.1143	603.4	8.750
6	....	.1920	4.877	.028953	.09832	519.2	10.17
....	3/16	.1875	4.763	.027612	.09377	495.1	10.66
7	....	.1770	4.496	.024606	.08356	441.2	11.97
8	....	.1620	4.115	.020612	.07000	369.6	14.29
....	5/32	.15625	3.969	.019175	.06512	343.8	15.36
9	....	.1483	3.767	.017273	.05866	309.7	17.05
10	....	.1350	3.429	.014314	.04861	256.7	20.57
....	⅛	.125	3.175	.012272	.04168	220.0	24.00
11	....	.1205	3.061	.011404	.03873	204.5	25.82
12	....	.1055	2.68	.0087417	.02969	156.7	33.69
....	3/32	.09375	2.381	.0069029	.02344	123.8	42.66
13	....	.0915	2.324	.0065755	.02233	117.9	44.78
14	....	.0800	2.032	.0050266	.01707	90.13	58.58
15	....	.0720	1.829	0040715	.01383	73.01	72.32
16	1/16	.0625	1.588	.0030680	.01042	55.01	95.98
17	....	.0540	1.372	.0022902	.007778	41.07	128.60
18	....	.0475	1.207	.0017721	.006018	31.77	166.20
19	....	.0410	1.041	.0013203	.004484	23.67	223.00
20	....	.0348	.8839	.00095115	.003230	17.05	309.60
21	....	.0317	.8052	.00078924	.002680	14.15	373.10
....	1/32	.03125	.7938	.00076699	.002605	13.75	383.90
22	....	.0286	.7264	.00064242	.002182	11.52	458.40
23	....	.0258	.6553	.00052279	.001775	9.374	563.30
24	....	.0230	.5842	.00041548	.001411	7.45	708.70

For iron wire multiply columns 6 and 7 by 0.98.

For copper wire multiply columns 6 and 7 by 1.12.

For other wire gages see next page.

Comparison of Standard Gages

Number of Gage	Thickness in decimals of an inch							Number of gage
	Birmingham	Browne & Sharpe	United States Standard Plate Iron and Steel	British Imperial	American Steel & Wire Co.	Trenton Iron Co.	Stubs Steel Wire	
0000000	....	....	.500	.500	.4900	....	......	0000000
000000	....	.58	.46875	.464	.4615	....	......	000000
00000	.500	.5165	.4375	.432	.4305	.45	......	00000
0000	.454	.46	.40625	.400	.3938	.40	......	0000
000	.425	.40964	.375	.372	.3625	.36	......	000
00	.380	.3648	.34375	.348	.3310	.33	......	00
0	.340	.32486	.3125	.324	.3065	.305	......	0
1	.300	.2893	.28125	.300	.2830	.285	.227	1
2	.284	.25763	.265625	.276	.2625	.265	.219	2
3	.259	.22942	.25	.252	.2437	.245	.212	3
4	.238	.20431	.234375	.232	.2253	.225	.207	4
5	.220	.18194	.21875	.212	.2070	.205	.204	5
6	.203	.16202	.203125	.192	.1920	.190	.201	6
7	.180	.14428	.1875	.176	.1770	.175	.199	7
8	.165	.12849	.171875	.160	.1620	.160	.197	8
9	.148	.11443	.15625	.144	.1483	.145	.194	9
10	.134	.10189	.140625	.128	.1350	.130	.191	10
11	.120	.090742	.125	.116	.1205	.1175	.188	11
12	.109	.080808	.109375	.104	.1055	.1050	.185	12
13	.095	.071961	.09375	.092	.0915	.0925	.182	13
14	.083	.064084	.078125	.080	.0800	.0800	.180	14
15	.072	.057068	.0703125	.072	.0720	.0700	.178	15
16	.065	.05082	.0625	.064	.0625	.0610	.175	16
17	.058	.045257	.05625	.056	.0540	.0525	.172	17
18	.049	.040303	.05	.048	.0475	.0450	.168	18
19	.042	.03589	.04375	.040	.0410	.0400	.164	19
20	.035	.031961	.0375	.036	.0348	.0350	.161	20
21	.032	.028462	.034375	.032	.03175	.0310	.157	21
22	.028	.025347	.03125	.028	.0286	.0280	.155	22
23	.025	.022571	.028125	.024	.0258	.0250	.153	23
24	.022	.0201	.025	.022	.0230	.0225	.151	24
25	.020	.0179	.021875	.020	.0204	.0200	.148	25
26	.018	.01594	.01875	.018	.0181	.0180	.146	26
27	.016	.014195	.0171875	.0164	.0173	.0170	.143	27
28	.014	.012641	.015625	.0148	.0162	.0160	.139	28
29	.013	.011257	.0140625	.0136	.0150	.0150	.134	29
30	.012	.010025	.0125	.0124	.0140	.0140	.127	30
31	.010	.008928	.0109375	.0116	.0132	.0130	.120	31
32	.009	.00795	.01015625	.0108	.0128	.0120	.115	32
33	.008	.00708	.009375	.0100	.0118	.0110	.112	33
34	.007	.006304	.00859375	.0092	.0104	.0100	.110	34
35	.005	.005614	.0078125	.0084	.0095	.0095	.108	35
36	.004	.005	.00703125	.0076	.0090	.0090	.106	36
37	....	.004453	.006640625	.0068	.0085	.0085	.103	37
38	....	.003965	.00625	.0060	.0080	.0080	.101	38
39	....	.003531	....	.0052	.0075	.0075	.099	39
40	....	.003144	....	.0048	.0070	.0070	.097	40

In the U. S. Standard, the numbers correspond to the weight in ounces per sq ft and an equal number of 640ths of an inch in thickness.

See p. 460 for fuller data of U. S. standard gage.

Strength of Steel-Wire Rope

Circumference in inches	Diameter in inches	Weight per foot in pounds	Ultimate strength in tons of 2000 lb	Working load in tons of 2000 lb	Circumference of hemp rope of equal strength, inches	Advisable minimum size of drum or sheave, feet
Cast-Steel Hoisting Rope, 6 strands of 19 wires each						
8⅝	2¾	11.95	243	48.6	....	11
7⅞	2½	9.85	200	40.0	....	10
7	2¼	8.00	133	27	....	9
6¼	2	6.30	106	21	....	8
5½	1¾	5.25	85	19	15¾	7½
5	1⅝	4.10	72	17	14½	6
4¾	1½	3.65	64	15	13½	5½
4¼	1⅜	3.00	56	12	12¼	5¼
4	1¼	2.50	47	10	11½	5
3½	1⅛	2.00	38	8	10	4½
3⅛	1	1.58	30	6	9½	4
2¾	⅞	1.20	23	5	8	3¾
2⅜	¾	0.88	18	3½	6½	3½
2	⅝	0.60	14	2½	5¼	3
1¾	9/16	0.48	9	1¾	4¾	2¾
1½	½	0.39	7½	1½	4½	2
1⅜	7/16	0.29	6	1¼	4	1¾
1¼	⅜	0.23	4½	⅞	3½	1½
1	5/16	0.16	3	¾	3	1¼
¾	¼	0.10	2½	44/100	2½	1
Standing Rope for Derricks, 6 strands of 7 wires each						
4¾	1½	3.37	62	13	15	....
4¼	1⅜	2.77	52	11	13	....
4	1¼	2.28	44	9	12	....
3½	1⅛	1.82	36	7	10¾	....
3⅛	1	1.50	30	6	10	....
2¾	⅞	1.12	22	4½	8½	....
2⅜	¾	0.92	17	3½	7¼	....
2⅛	11/16	0.70	14	2¾	6½	....
2	⅝	0.57	11	2	5½	....
1¾	9/16	0.41	8	1¾	5	....
1½	½	0.31	6	1¼	4¾	....
1⅜	7/16	0.23	5	1	4¼	....
1 3/16	⅜	0.21	4	⅞	3¾	....
1	5/16	0.16	3	¾	3¼	....
⅞	9/32	0.12	2¾	⅝	2¾	....

The above tables give strength values for ropes of crucible steel wire. The strength of ropes of other material may be found by multiplying the values given by the following factors:

Ropes of iron wire	0.50
Ropes of open-hearth steel wire	0.75
Ropes of plow steel wire	1.33
Ropes of special plow steel wire	1.50

Wire Rope is made of iron, open-hearth steel, crucible steel, or plow steel wires, and accordingly has an ultimate strength per sq in of 45 000 to 100 000 lb for iron, 50 000 to 130 000 lb for open-hearth steel, 130 000 to 190 000 lb for crucible steel, and 190 000 lb for plow steel.

The wires are either laid parallel to each other as in suspension bridge cables, or twisted together into strands as in smokestack guys or cases where only moderate flexibility is

eded. Wire strands consist of 4, 7, 12, 19, or 37 wires depending on the work intended, ne most pliable rope, for hoisting and transmission, contains 19 wires to the strand. hile ropes of 12 or 7 wires to the strand are better adapted for use as standing ropes, ys, or rigging.

Wire rope must not be coiled or uncoiled like hemp rope. When mounted on a reel, e latter should be so mounted that the rope may be paid off. When furnished in a coil should be rolled over the ground like a wheel, and the rope run off in that way. All twisting and kinking must be avoided.

The Flexibility of Wire Rope depends largely on the size of the individual wires ed in making up the strands of the rope, the smaller these wires (and consequently the ore numerous) the more flexible the rope. For guy wires and other wire ropes not bent und sheaves wire rope is usually made up of 6 strands of 7 wires each. For hoisting pe which is to pass around sheaves the rope in commonest use has 6 strands of 19 wires ch. The make up of two extra flexible hoisting ropes and the approximate ratio of eir strength to that of 6 strand 19 wire rope of the same material and the same nominal e is:

8 strand 19 wires	0.87
6 strand 37 wires	0.95

Bending Stresses in Wire Ropes which pass around sheave wheels are sometimes considerable magnitude. Sometimes it is not feasible to use sheaves as large as those ven in the table on p. 472. If a straight rod or wire of diameter d is bent into a circle diameter D there is set up a bending stress S

$$S = E\,d/D$$

is the modulus of elasticity of the material.

In a wire rope bent round a sheave each individual wire is bent into the arc of a circle d in the outer fibers of each wire there are set up bending stresses. The extreme ers on the outer side of each wire must withstand the combined tensile stress due bending and the direct tensile stress due to the pull on the wire. If a wire breaks, its are of the direct tensile stress is transferred to the remaining wires, but the bending ress in the outer fibers of the other wires is not increased. Owing to the twisting of the rands the length of wire subjected to bending in a wire rope is greater than that of a raight rod bent to the same size circle. This increased length decreases the stiffness the wire rope and has the same effect on the bending stress as reducing the value of E the above formula. For 6-strand ropes tests by the American Steel and Wire Co. ve for bending stresses in lb per sq in:

$$S = 12\,000\,000\,d/D$$

is the diameter in inches of the sheave wheel to the center of the rope, d is the diameter the largest individual wires in the rope. In determining allowable tensile load on a wire pe passing round a sheave the stress due to bending should be deducted from the lowable working stress for a straight rope, so that no fiber of any wire shall be over-essed.

Strength of Wire Rope Fastenings. The best fastening for a wire rope is a steel cket with the wire rope "leaded" into it. When such sockets are fitted by skilled work-en under shop conditions their strength may be made equal to the strength of the wire pe itself. Made under field conditions the strength of such sockets will hardly be greater an 75 percent of the values given for wire rope if the work is carefully done, and the rength may fall far below this if the fitting of the rope to sockets is not carefully done. emporary fastenings for wire rope in the form of clips or clamps are often used. To velop the maximum strength of such a fastening four clips should be used for small-zed ropes and six clips for the larger sizes. The holding power of clips or clamps depends a large extent on the care with which all the clips are tightened and kept tight. In any ent clips or clamps weaken the rope by "crimping" it at the point of application. Care-lly made clip or clamp fastenings may develop 60 to 75 percent of the full strength of a re rope.

55. Terra Cotta

Hollow Terra-Cotta Flat Arches

(National Fireproofing Co.)

The following table is applicable to all shapes of tile. Generally speaking, hollow tile of various shapes, but of the same depth and cross-sectional area, have equal strength and, therefore, the strength of arches of equal depth, is directly proportional to their weights. The weight of the arch has not been deducted from safe loads in table below; therefore this and other dead load must be deducted to obtain the net safe live load for any arch and span.

Table of Safe Loads—(Dead and Live) [1]

Factor of Safety of 7

Arches	6 in	7 in	8 in	9 in	10 in	12 in	15 in
Average weight per sq ft	26	29	32	35	38	42	50
Spans, ft and in	Lb	Lb	Lb	Lb	Lb	Lb	Lb
3–0	482	617	767	933	1114	1524	2255
3–3	410	525	654	795	950	1299	1922
3–6	354	453	563	685	819	1120	1657
3–9	308	394	491	597	713	975	1443
4–0	271	347	431	525	627	857	1268
4–3	240	307	382	465	555	759	1124
4–6	214	274	341	414	495	677	1002
4–9	192	246	306	372	444	608	900
5–0	173	222	276	336	401	548	812
5–3	....	201	250	304	364	497	736
5–6	....	183	228	277	331	453	671
5–9	....	168	208	254	303	415	614
6–0	....	....	191	233	278	381	563
6–3	....	....	176	215	256	351	519
6–6	....	....	163	198	237	324	480
6–9	....	....	....	184	220	301	445
7–0	....	....	....	171	204	280	414
7–6	....	....	....	....	178	243	360
8–0	....	....	....	....	....	214	317
8–6	....	....	....	....	....	190	281
9–0	....	....	....	....	....	169	250
9–6	....	....	....	....	....	....	225
10–0	....	....	....	....	....	....	203

Example: What load will an 8-in arch carry with a factor of safety of 5 in a span of 5 ft 6 in, the arch having a weight of 36 lb per sq ft. In the table, 8-in arch has a strength of 228 lb for a weight of 32 lb. Therefore, 32 : 36 : : 228 : 256 and $256 \times 7 \div 5 = 358$ lb total load. $358 - 36$ lb dead load $= 322$ lb live load, which must be further reduced by the weight of fill over the arch, finish flooring and plastering to get the net safe live load.

Terra Cotta is either dense, semi-porous, or porous. The dense tile is very hard and is used for floor arches because of its high crushing strength. Porous and semi-porous terra cotta is made by mixing sawdust with the clay, the sawdust being destroyed during the burning operation. In the porous tile the proportion of sawdust is about 25 to 35 percent; in the semi-porous tile it is about 20 percent. Porous terra cotta can be cut with a saw or edge tools, and nails may be easily driven into it.

In a series of tests at Columbia University for the New York Building Department, terra-cotta blocks secured in the open market developed a net crushing strength for dense tile of 5820 lb per sq in, and for semi-porous terra cotta of 3292 lb per sq in, the figures in each case being the average results of ten tests. See also p. 421.

Safe Loads, Hollow Terra-Cotta Segmental Arch

(National Fireproofing Co.)

Given for tile with the following sectional areas (per foot of arch parallel with beams): 6-in, 336 sq in; 8-in, 43 sq in; 10-in, 47 sq in. Factor of safety, 7.

Rise in inches per foot of span. Example: Rise 1½ for 12 ft span = 18 in.

Note.—The weight of the arch tile has been deducted in table so that only the dead load of concrete fill, plastering, etc., must be deducted to obtain net live load.

Spans, ft and in	Rise, in	6-in arch, lb	8-in arch, lb	10-in arch, lb	Spans, ft and in	Rise, in	6-in arch, lb	8-in arch, lb	10-in arch, lb	Spans, ft and in	Rise, in	6-in arch, lb	8-in arch, lb	10-in arch, lb
4	¾	902	1078	1178	8-6	¾	411	491	536	13	¾	261	312	341
	1	1184	1414	1545		1	551	658	719		1	351	419	458
	1¼	1485	1774	1939		1¼	678	810	885		1¼	437	522	570
	1½	1740	2079	2272		1½	806	963	1052		1½	519	620	677
	1¾	1986	2373	2593		1¾	926	1106	1208		1¾	596	712	778
	2	2233	2667	2915		2	1037	1239	1354		2	670	801	875
4-6	¾	792	946	1034	9	¾	386	461	504	14	¾	240	287	313
	1	1044	1247	1363		1	518	619	677		1	326	390	426
	1¼	1313	1568	1713		1¼	645	770	842		1¼	406	485	530
	1½	1539	1838	2009		1½	758	906	990		1½	482	575	629
	1¾	1775	2121	2318		1¾	871	1041	1137		1¾	553	661	722
	2	1975	2359	2578		2	977	1167	1275		2	619	740	808
5	¾	709	847	926	9-6	¾	364	435	475	15	¾	225	268	293
	1	957	1143	1249		1	489	584	638		1	302	361	394
	1¼	1172	1400	1530		1¼	608	726	793		1¼	377	450	491
	1½	1379	1647	1800		1½	721	862	942		1½	447	534	583
	1¾	1592	1902	2078		1¾	823	983	1074		1¾	515	616	673
	2	1773	2118	2315		2	923	1102	1204		2	577	690	754
5-6	¾	641	766	837	10	¾	344	411	449	16	¾	209	249	272
	1	864	1032	1128		1	462	552	603		1	281	336	367
	1¼	1062	1269	1387		1¼	576	688	751		1¼	353	421	460
	1½	1266	1512	1652		1½	683	816	892		1½	419	500	546
	1¾	1439	1719	1879		1¾	784	937	1024		1¾	481	575	628
	2	1619	1933	2113		2	879	1050	1147		2	540	645	705
6	¾	585	699	764	10-6	¾	331	396	432	17	¾	194	232	254
	1	788	941	1028		1	438	523	572		1	265	316	345
	1¼	969	1157	1265		1¼	546	652	713		1¼	330	394	430
	1½	1154	1379	1507		1½	648	774	846		1½	392	468	512
	1¾	1315	1570	1716		1¾	744	889	972		1¾	452	540	590
	2	1476	1763	1927		2	832	994	1086		2	506	605	661
6-6	¾	551	658	719	11	¾	315	376	411	18	¾	182	218	238
	1	724	864	944		1	421	503	550		1	248	296	324
	1¼	902	1077	1177		1¼	519	621	678		1¼	310	370	404
	1½	1058	1264	1382		1½	617	737	805		1½	370	442	482
	1¾	1218	1455	1590		1¾	709	847	925		1¾	425	507	554
	2	1358	1622	1772		2	794	948	1036		2	477	570	623
7	¾	508	606	662	11-6	¾	299	358	391	19	¾	173	206	225
	1	669	799	873		1	401	480	524		1	233	279	304
	1¼	834	996	1089		1¼	499	596	652		1¼	293	350	382
	1½	981	1171	1280		1½	592	707	773		1½	348	416	455
	1¾	1127	1346	1471		1¾	680	812	887		1¾	402	480	524
	2	1264	1510	1650		2	761	909	993		2	451	539	589
7-6	¾	471	563	615	12	¾	285	341	372	[illegible]	¾	163	194	212
	1	621	741	810		1	383	458	500		1	221	265	289
	1¼	774	925	1011		1¼	477	569	622		1¼	277	331	361
	1½	920	1099	1201		1½	566	676	738		1½	330	395	431
	1¾	1049	1253	1369		1¾	649	776	848		1¾	381	455	497
	2	1176	1405	1536		2	727	869	949		2	427	510	558
8	¾	439	525	573	12-6	¾	273	326	356	21	¾	[illegible]	183	200
	1	588	703	768		1	366	437	478		1	209	250	273
	1¼	724	864	944		1¼	456	545	595		1¼	263	315	344
	1½	859	1026	1122		1½	541	646	706		1½	314	375	409
	1¾	987	1179	1288		1¾	621	742	811		1¾	361	432	472
	2	1099	1312	1434		2	696	832	909		2	406	485	530

Hollow Terra-Cotta Blocks

(National Fireproofing Co.)

Partition blocks

Thickness, in	Weight per sq ft, lb	Size of block, in
2	14	6×12
2	14	8×12
2	14	12×12
3	17	6×12
3	17	8×12
3	17	12×12
4	18	6×12
4	18	8×12
4	18	12×12
5	20	8×12
5	20	12×12
6	26	8×12
6	26	12×12
7	29	12×12
7	29	12×12
8	32	12×12
9	36	12×12
10	38	12×12
12	42	12×12

Wall-furring

Thickness, in	Weight per sq ft, lb	Size of block, in
1½	9	12×12
2	10	12×12

Hollow Brick Haverstraw Size

Brick	Size, in	Weight lb
Stretcher	2⅛×3½×8	3
Header....	2⅛×3½×7¼	2½
Porous.... stretcher	2¼×3¾×8	2½
Solid porous stretcher	2¼×3¾×8	3⅛

Roof blocks

Thickness, in	Weight per sq ft, lb	Size of block, in
3	20	12×18
3	20	12×20
3	20	12×24
4	22	12×24

Ceiling blocks

Thickness, in	Weight per sq ft, lb	Size of block, in
2	12	12×16
2	12	12×18
2	12	12×20
3	20	12×16
3	20	12×18
3	20	12×20
3	20	12×24

56. Gypsum Tiles and Slabs

Sizes and Weights of Gypsum Tile

(U. S. Gypsum Co.)

Size of gypsum tile, in	For ceiling heights up to	Weight tile, lb per sq ft	Weight mortar, lb per sq ft	Weight plaster, one side, lb per sq ft	Total weight plastered one side, lb per sq ft	Weight plaster, two sides, lb per sq ft	Total weight plastered two sides, lb per sq ft
1½-in split (1½×12×30)	Furring	4.9	1.00	3	7.9	6	10.9
2-in split (2×12×30)	Furring	6.4	1.00	3	9.4	6	12.4
2-in solid (2×12×30)	10 ft	9.4	1.00	3	12.4	6	15.4
3-in hollow (3×12×30)	13 ft	9.9	1.2	3	12.09	6	15.9
3-in solid (3×12×30)	15 ft	12.4	1.2	3	15.4	6	18.4
4-in hollow (4×12×30)	17 ft	13.0	1.63	3	16.00	6	19.0
5-in hollow (5×12×30)	25 ft	15.6	2.04	3	18.60	6	21.6
6-in hollow (6×12×30)	28 ft	16.6	2.45	3	19.60	6	22.6
8-in hollow (8×12×30)	40 ft	22.4	3.26	3	25.40	6	28.4

Gypsum Floor Tile

(U. S. Gypsum Co.)

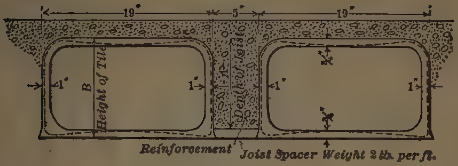


Fig. 61

	Size of floor tile			
A—Depth of joist	6 in	8 in	10 in	12 in
B—Height of tile	7 in	9 in	11 in	13 in
Weight per ft	24 lb	27 lb	30 lb	33 lb

Each tile 24 in long. All weights shown are per lineal foot.

Gypsum Roof Tile (Reinforced)

(U. S. Gypsum Co.)

Dimensions of tile thickness, 3 in solid or 4 in hollow; width, 12 in; length, 30 in. Weight of tile, 13 lb per sq ft. 40 tile will cover 100 sq ft of roof. Spacing of T-iron purlins, 2 ft 6½ in centers. Safe uniformly distributed load on tile 100 lb per sq ft.

Gypsum Roof Slabs (Reinforced)

(U. S. Gypsum Co.)

Safe loads 50 lb per sq ft (factor of safety, 4)

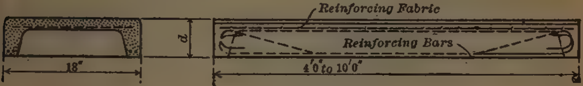


Fig. 62. Gypsum Roof Slab.

Span in ft	4	5	6	7	8	9	10
Depth, *d*, in	5	5	5	6	6	7	7
Weight, lb per sq ft	16.5	16.5	16.5	18.5	18.5	20.0	20.0

57. Slates and Shingles

Number of Slates and Pounds of Nails per Square of 100 Square Feet

Size of slate, inches	Number of inches exposed when laid	Number per square	Weight of galvanized nails per square	lb	oz	Size of slate, inches	Number of inches exposed when laid	Number per square	Weight of galvanized nails per square	lb	oz
14 × 24	10½	98	4d.	1	6	12 × 16	6½	185	3d.	2	2
12 × 24	10½	115	4d.	1	10	10 × 16	6½	222	3d.	2	8
12 × 22	9½	126	4d.	1	12	9 × 16	6½	247	3d.	3	0
11 × 22	9½	138	4d.	1	15	8 × 16	6½	277	3d.	3	2
12 × 20	8½	142	4d.	2		10 × 14	5½	262	3d.	3	0
10 × 20	8½	170	4d.	2	6	8 × 14	5½	328	3d.	3	12
12 × 18	7½	160	3d.	1	13	7 × 14	5½	374	3d.	4	4
10 × 18	7½	192	3d.	2	3	8 × 12	4½	400	3d.	4	9
9 × 18	7½	214	3d.	2	7	7 × 12	4½	458	3d.	5	3
						6 × 12	4½	533	3d.	6	1

Number and Weight of Pine Shingles per Square of 100 Square Feet

Length, inches	Width, inches	Number of inches exposed to weather	Number of shingles per square	Weight of shingles per square, pounds	Pounds of nails per 1000 shingles
16	4	4	900	188	5 (approx.)
16	4	4½	800	172	5
16	4	5	720	156	5
16	4	5½	655	140	5
16	4	6	600	124	5

Roofing Slate is supplied commercially in thicknesses of ⅛, $\frac{3}{16}$, ¼, increasing by 8ths to 1 inch. Slate roofing as laid weighs about 6.5 lb per sq ft for $\frac{3}{16}$ inch thickness, and 8.75 lb for ¼ inch thickness, the smaller sizes weighing more on account of lap.

Wood Shingles are made from cypress, redwood, cedar, pine, and spruce, this being the order of their durability. Redwood is much less inflammable than any of the others. For hip roofs, 5% should be added to quantities in table to allow for cutting, and for irregular roofs with dormer windows 10% should be added.

Common shingles are of random widths, varying from 2 to 14 inches. They come in bundles, four of which contain a "thousand" or the equivalent of 1000 shingles 4 inches in width. Above table makes no allowance for waste.

SECTION 5

PLAIN AND REINFORCED CONCRETE

BY

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CEMENT AND CEMENT MORTAR

PLAIN CONCRETE

REINFORCED CONCRETE BUILDINGS

BRIDGES AND OTHER STRUCTURES

CEMENT AND CEMENT MORTAR

1. Kinds and Uses of Cement

Kinds of Cement. In engineering literature the term cement is understood to mean the finely pulverized product obtained by the burning of a suitable mixture of argillaceous and calcareous materials, or by an artificial mixture of such materials after burning, which will possess the property of hardening into a solid mass when mixt with water. The essential ingredients in the manufacture of cement are calcium carbonate ($CaCO_3$), silica (SiO_2) and alumina (Al_2O_3); the last two, combined in various proportions, constitute the argillaceous material. The characteristic property of cement is that of hardening by the addition of water, and therefore its ability to harden when excluded from the air. By reason of this property it is also commonly called Hydraulic Cement, of which there are three varieties, Portland Cement, Natural Cement and Puzzolan Cement. Hydraulic lime is a fourth material which possesses characteristics similar to the hydraulic cements.

Portland Cement is the finely pulverized product resulting from the calcination to incipient fusion of an intimate, artificial mixture of properly proportioned argillaceous and calcareous materials. It has a definite chemical composition varying within comparatively narrow limits. There are three distinct stages in the process of its manufacture, (1) the preparation of the correct mixture by the selection, mixing and grinding of the ingredients, (2) the burning of the mixture to a clinker, and (3) the pulverizing of the burned clinker to a fine powder. Portland cement is distinguished by uniformity of quality and high strength which it rapidly acquires.

Portland cement should be used in structures where strength is of special importance, or where the structure is exposed to the action of the elements. It should be used invariably in reinforced concrete construction.

Natural Cement is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature below fusion. The proportions of lime and clay in the raw material may vary between much wider limits than in the case of portland cement. Natural cement does not develop its strength as quickly nor is it as strong or uniform in composition as portland cement.

Natural cement is suitable for use in structures where mass and weight rather than strength are the essential features. It is thus adapted for use in many underground structures, such as sewers, conduits, massive foundations and foundations of street pavements, and in the unexposed portions of large and massive structures above ground, such as dams, retaining walls, piers and abutments. Used alone, or mixt with lime mortar, it also makes a suitable mortar for ordinary brick and stone masonry.

Puzzolan or Slag Cement is the finely pulverized product resulting from grinding a mechanical mixture of fused argillaceous material and hydrated lime. The argillaceous substance may consist of natural puzzolanic material, such as volcanic ash, trass or allied igneous material, or of artificial materials, such as blast-furnace slag or burnt clay. Puzzolan cement is not as strong or reliable as portland or natural cement and is not extensively used in the United States. Its use should be confined to unexposed work where great strength is not required.

Hydraulic Lime is a name given to a natural cement in which the proportion of free lime is sufficient to cause the mass to slake in the same manner as common lime. While much used in Europe, it is not manufactured or

ed in the United States. GRAPPIER CEMENT is made by grinding the lumps material remaining unslaked in the manufacture of hydraulic lime. An ported Grappier cement called LA FARGE is used in the United States as a n-staining cement. Its strength approaches that of portland cement.

The Hardening of Cement is due chiefly to the decomposition of the compounds lime upon the addition of water, resulting in the formation of calcium hydrate, which crystallization binds together the other solid ingredients. The initial setting is due ncipally to the decomposition of the aluminates, while the final hardening depends re upon the action of the silicates.

2. Composition and Manufacture of Cement

Composition of Portland Cement. Portland Cement is of complex mposition, but consists for the most part of the tricalcium silicate and rious calcium aluminates, these compounds resulting from the burning gether of the three ingredients, calcium carbonate, silica and alumina. cording to Newberry the correct theoretical proportions are represented by

$$\text{lime} = 2.8 \times \text{silica} + 1.1 \times \text{alumina}.$$

here is also always present a small amount of ferric oxide (Fe_2O_3), which ts in a manner similar to alumina, but requires only a factor of 0.7 for lime.

Typical Analyses of Portland Cement

Locality	Material	Silica (SiO_2)	Alumina (Al_2O_3)	Iron (Fe_2O_3)	Lime (CaO)	Magnesia (MgO)	Sulphuric Acid (SO_3)
Lehigh District	Cement Rock and Limestone	19.06	7.47	2.29	61.23	2.83	1.34
Sandusky, O.	Marl and Clay	23.08	6.16	2.90	62.38	1.21	1.66
Chicago, Ill.	Slag and Limestone	23.62	8.21	2.71	61.92	1.78	1.32
Germany	Chalk and Clay	24.90	8.00	3.22	59.38	0.38	1.46

The Proportion of Lime. An excess of lime causes unsoundness, while too low a oportion yields a quick-setting cement of inferior strength. The higher the lime the eater the strength so long as it all exists in the combined form. A high proportion lime, however, requires high temperatures in burning and very thoro mixing and inding, thus rendering the process more difficult. To avoid an excess it is necessary keep the lime content considerably below that represented by the theoretic formula.

The Proportion of Alumina controls largely the clinkering temperature, the higher e alumina the lower the temperature necessary to cause the proper combination of lime th the silica and alumina. Increased proportions of alumina, however, tend to make e cement quicker setting and of lower ultimate strength, so that good results require is element to be as low as consistent with economy of manufacture. From 5 to 10 % usual. High proportions of alumina are especially undesirable where the cement ll come in contact with sea water, as it appears to render the cement more liable to sintegration under such conditions.

Other substances generally present in portland cement are magnesia and various lfur compounds. The effect of magnesia in large amounts is not well determined, t up to 4 or 5 % no deleterious effect results. Sulfate of calcium, in the form of psum or plaster of Paris, is always added to a cement after burning to delay the set. he amount of such material is usually limited to 2 or 3%. The standard specifications ace the limit for the sulfuric acid (SO_3) content at 1.75%.

Manufacture of Portland Cement. A great variety of materials is ailable for the manufacture of portland cement. The principal combina- ons in use are as follows:

	Argillaceous.	Calcareous.
(1)	Argillaceous limestone (cement rock).	Pure limestone.
(2)	Clay or shale.	Pure limestone.
(3)	Clay or shale.	Chalky limestone or chalk.
(4)	Clay.	Marl.
(5)	Clay.	Alkali waste.
(6)	Blast-furnace slag.	Pure limestone.

Two general processes are used in the grinding and mixing of the ra material, the dry and the wet. In the dry process the material is thorol dried and then finely ground, the latter stages of grinding being done after th mixing. Where slag is used it is first granulated by being run into water, whic causes it to form into small, easily crusht particles. In the wet process th material is mixt and ground together into a "slurry" containing 60 to 70% of water. This slurry is then pumped directly to the burning kiln. Th wet process is generally used with clay and marl.

After this preparation the material is burned to a clinker in a kiln of whic there are two general types, the fixt vertical kiln and the continuous rotar kiln. The latter is used in most American plants, and to its development due, largely, the growth of the cement industry in this country. From 10 to 200 pounds of coal are required per barrel of product, and the productio per kiln is generally from 100 to 300 barrels.

The final grinding of the burned clinker is of the greatest importance. A it is only the very finest particles (the "flour") that possess active propertie the advantage of fine grinding is obvious; this is done in two or three stage The necessary calcium sulfate is generally added before the final grinding.

Composition and Manufacture of Natural Cement. Natural cemen is made from a cement rock in which the proportion of clay is from 20% t 75% greater than is required in the more exact mixture used for portlan cement. The cement rock also generally contains a relatively large propor tion of magnesia, which acts in much the same manner as the lime. In th process of manufacture the rock is selected so as to give as uniform a produc as practicable, this often requiring a mixing of materials from different strat It is then burned at a temperature of about 1200° F., which, altho considerabl below that used in the portland cement process, is beyond that required fo calcination and is sufficient to cause the formation of silica compounds wit the lime and magnesia. The higher the clay content the lower the temper ature required for burning, but the weaker is the resulting product. Th burning is done in vertical kilns which consume from 10 to 15 pounds of co per 100 pounds of cement. The grinding of a natural cement is not carrie as far as that of portland, and the entire cost of manufacture is much less.

Typical Analyses of Natural Cements

Chemical Composition	Utica, Ill.	Louisville, Ky.	Rosendale, N. Y.
Silica (SiO_2)	27.60	22.54	27.30
Alumina (Al_2O_3)	10.60	8.24	7.14
Iron Oxide (Fe_2O_3)	0.80	2.14	1.80
Lime (CaO)	33.04	44.31	35.98
Magnesia (MgO)	17.26	5.39	18.00
Alkalies (K_2O, Na_2O)	7.42	2.82	6.80
Carbon Dioxide (CO_2) } Water	2.00	....	2.98

Slag Cement is manufactured at several blast-furnace plants in the United ates. In its composition the proportion of the argillaceous material is r.ied still higher than in the natural cement. In its manufacture only sic slag is used, containing approximately equal amounts of lime and clay. e slag is granulated, mixt with about 25% of hydrated lime and the mix- re then ground. Slag cements are normally slow in setting. One method regulating this is by the addition of caustic soda.

Production of Cements. The production of cements in the United States 1912 was as follows, portland cement, 92 097 131 bbls; natural cement, 4 658 bbls; puzzolan cement, 107 313 bbls.

3. Testing of Cement

Standard Cement Tests. In all important work the cement should be ly specified and regularly tested. For unimportant work, however, there little danger to be feared in using well-known brands of cement without ting. Results of test depend largely upon the method of making them; it is nce very important that definite and uniform methods be used. The kinds tests usually made are as follows: (1) Specific gravity. (2) Fineness. (3) me of setting. (4) Tensile strength of neat cement and of sand mortar. (5) undness, or constancy of volume. A chemical analysis is also desirable on ge work. Of much importance in securing uniform and reliable results are methods of sampling, the consistency of the mortar, temperature of the t rial, character of sand used and of apparatus employed. The standard thods of testing given below have come into general use in the United States.

Standard Methods of Cement Testing devised and recommended by a nt committee of the Am. Soc. C.E. and the A.S.T.M. (1916) are as follows emical analysis omitted):

Sampling. 1. Tests may be made on individual or composite samples as may be ered. Each test sample should weigh at least 8 lb.

. (*a*) Individual Sample. If sampled in cars one test sample shall be taken from h 50 bbl or fraction thereof. If sampled in bins one sample shall be taken from each bbl.

b) Composite Sample. If sampled in cars one sample shall be taken from one sack each 40 sacks (or 1 bbl in each 10 bbl) and combined to form one test sample. If npled in bins or warehouses one test sample shall represent not more than 200 bbl.

. Cement may be sampled at the mill by any of the following methods that may be cticable, as ordered:

a) From the Conveyor Delivering to the Bin. At least 8 lb of cement shall taken from approximately each 100 bbl passing over the conveyor.

b) From Filled Bins by Means of Proper Sampling Tubes. Tubes inserted tically may be used for sampling cement to a maximum depth of 10 ft. Tubes inserted izontally may be used where the construction of the bin permits. Samples shall be en from points well distributed over the face of the bin.

c) From Filled Bins at Points of Discharge. Sufficient cement shall be drawn m the discharge openings to obtain samples representative of the cement contained in bin, as determined by the appearance at the discharge openings of indicators placed the surface of the cement directly above these openings before drawing of the cement tarted.

. Samples preferably shall be shipped and stored in air-tight containers. Samples ll be passed through a sieve having 20 meshes per linear inch in order to thoroughly the sample, break up lumps and remove foreign materials.

Specific Gravity. 5. The determination of specific gravity shall be made with a ndardized Le Chatelier apparatus. This apparatus is standardized by the United

States Bureau of Standards. Kerosene free from water, or benzine not lighter than 62° Baumé, shall be used in making this determination.

6. The flask shall be filled with either of these liquids to a point on the stem between zero and 1 cc, and 64 g of cement, of the same temperature as the liquid, shall be slowly introduced, taking care that the cement does not adhere to the inside of the flask above the liquid and to free the cement from air by rolling the flask in an inclined position. After all the cement is introduced, the level of the liquid will rise to some division of the graduated neck; the difference between readings is the volume displaced by 64 g of the cement.

The specific gravity shall then be obtained from the formula

$$\text{Specific gravity} = \frac{\text{Weight of cement (g)}}{\text{Displaced volume (cc)}}$$

7. The flask, during the operation, shall be kept immersed in water, in order to avoid variations in the temperature of the liquid in the flask, which shall not exceed 0°.5 C. The results of repeated tests should agree within 0.01.

8. The determination of specific gravity shall be made on the cement as received; if it falls below 3.10, a second determination shall be made after igniting the sample.

Fineness. 9. Wire cloth for standard sieves for cement shall be woven (not twilled) from brass, bronze, or other suitable wire, and mounted without distortion on frames not less than 1½ in below the top of the frame. The sieve frames shall be circular, approximately 8 in in diameter, and may be provided with a pan and cover.

10. A standard No. 200 sieve is one having nominally an 0.0029-in opening and 200 wires per inch standardized by the U. S. Bureau of Standards, and conforming to the following requirements:

The No. 200 sieve should have 200 wires per inch, and the number of wires in any whole inch shall not be outside the limits of 192 to 208. No opening between adjacent parallel wires shall be more than 0.0050 in in width. The diameter of the wire should be 0.0021 in and the average diameter shall not be outside the limits 0.0019 to 0.0023 in. The value of the sieve as determined by sieving tests made in conformity with the standard specification for these tests on a standardized cement which gives a residue of 25 to 20% on the No. 200 sieve, or on other similarly graded material, shall not show a variation of more than 1.5% above or below the standards maintained at the Bureau of Standards.

11. The test shall be made with 50 g of cement. The sieve shall be thoroughly clean and dry. The cement shall be placed on the No. 200 sieve, with pan and cover attached, if desired, and shall be held in one hand in a slightly inclined position so that the sample will be well distributed over the sieve, at the same time gently striking the side about 150 times per minute against the palm of the other hand on the up stroke. The sieve shall be turned every 25 strokes about one-sixth of a revolution in the same direction. The operation shall continue until not more than 0.05 g passes through in 1 minute of continuous sieving. The fineness shall be determined from the weight of the residue on the sieve expressed as a percentage of the weight of the original sample.

12. Mechanical sieving devices may be used, but the cement shall not be rejected if it meets the fineness requirement when tested by the hand method described in Par. 11.

13. A permissible variation of 1 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 22%.

Mixing Cement Pastes and Mortars. 14. The quantity of dry material to be mixed at one time shall not exceed 1000 g nor be less than 500 g. The proportions of cement or cement and sand shall be stated by weight in grams of the dry materials; the quantity of water shall be expressed in cubic centimeters (1 cc of water = 1 g). The dry materials shall be weighed, placed upon a non-absorbent surface, thoroughly mixed dry if sand is used, and a crater formed in the center, into which the proper percentage of clean water shall be poured; the material on the outer edge shall be turned into the crater by the aid of a trowel. After an interval of ½ minute for the absorption of the water the operation shall be completed by continuous, vigorous mixing, squeezing and kneading with the hands for at least 1 minute. During the operation of mixing, the hands should be protected by rubber gloves.

15. The temperature of the room and the mixing water shall be maintained as nearly as practicable at 21° C. (70° F.).

Normal Consistency. 16. The Vicat apparatus consists of a frame bearing a movable rod weighing 300 g one end being 1 cm in diameter for a distance of 6 cm, the other having a removable needle 1 mm in diameter, 6 cm long. The rod is reversible, and can be held in any desired position by a screw and has midway between the ends a mark which moves under a scale (graduated to millimeters) attached to the frame. The paste is held in a conical, hard-rubber ring 7 cm in diameter at the base, 4 cm high, resting on a glass plate about 10 cm square.

17. In making the determination, 500 g of cement, with a measured quantity of water, shall be kneaded into a paste, as described in Par. 14 and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in apart; the ball resting in the palm of one hand shall be pressed into the larger end of the rubber ring held in the other hand, completely filling the ring with paste; the excess at the larger end shall then be removed by a single movement of the palm of the hand; the ring shall then be placed on its larger end on a glass plate and the excess paste at the smaller end sliced off at the top of the ring by a single oblique stroke of a trowel held at a slight angle with the top of the ring. During these operations care shall be taken not to compress the paste. The paste confined in the ring, resting on the plate, shall be placed under the rod, the larger end of which shall be brought in contact with the surface of the paste; the scale shall then be read, and the rod quickly released. The paste shall be of normal consistency when the rod settles to a point 10 mm below the original surface in ½ minute after being released. The apparatus shall be free from all vibrations during the test. Trial pastes shall be made with varying percentages of water until the normal consistency is obtained. The amount of water required shall be expressed in percentage by weight of the dry cement.

18. The consistency of standard mortar shall depend on the amount of water required to produce a paste of normal consistency from the same sample of cement. Having determined the normal consistency of the sample, the consistency of standard mortar made from the same sample shall be as indicated in the table, the values being in percentage of the combined dry weights of the cement and standard sand:

Percentage of Water for Standard Mortars

Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement, three standard Ottawa sand	Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement, three standard Ottawa sand
15	9.0	23	10.3
16	9.2	24	10.5
17	9.3	25	10.7
18	9.5	26	10.8
19	9.7	27	11.0
20	9.8	28	11.2
21	10.0	29	11.3
22	10.2	30	11.5

Soundness. 19. A steam apparatus, which can be maintained at a temperature between 98 and 100° C. is recommended.

20. A pat from cement paste of normal consistency about 3 in in diameter, ½ in thick at the center, and tapering to a thin edge, shall be made on clean glass plates about 4 in square, and stored in moist air for 24 hours. In molding the pat, the cement paste shall be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center.

21. The pat shall then be placed in an atmosphere of steam at a temperature between 98 and 100° C. upon a suitable support 1 in above boiling water for 5 hours.

22. Should the pat leave the plate, distortion may be detected best with a straight edge applied to the surface which was in contact with the plate.

Time of Setting. 23. The following are alternate methods, either of which may be used as ordered:

24. The time of setting shall be determined with the Vicat apparatus.

25. A paste of normal consistency shall be molded in the hard-rubber ring as described in Par. 17 and placed under the movable rod, the smaller end of which shall then be carefully brought in contact with the surface of the paste, and the rod quickly released. The initial set shall be said to have occurred when the needle ceases to pass a point 5 mm. above the glass plate in ½ minute after being released; and the final set, when the needle does not sink visibly into the paste. The test pieces shall be kept in moist air during the test. This may be accomplished by placing them on a rack over water contained in a pan and covered by a damp cloth, kept from contact with them by means of a wire screen or they may be stored in a moist closet. Care shall be taken to keep the needle clean as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste receives, but by the temperature and humidity of the air, and its determination is therefore only approximate.

26. The time of setting shall be determined by the Gillmore needles.

27. The time of setting shall be determined as follows: A pat of neat cement paste about 3 in in diameter and ½ in in thickness with a flat top mixt to a normal consistency shall be kept in moist air at a temperature maintained as nearly as practicable at 21° C. (70° F.). The cement shall be considered to have acquired its initial set when the pat will bear, without appreciable indentation, the Gillmore needle 1/12 in in diameter, loaded to weigh ¼ lb. The final set has been acquired when the pat will bear without appreciable indentation, the Gillmore needle 1/24 in in diameter, loaded to weigh 1 lb. In making the test, the needles shall be held in a vertical position, and applied lightly to the surface of the pat.

Tension Tests. 28. The form of test piece shown in Fig. 1 shall be used. The molds shall be made of non-corroding metal and have sufficient material in the sides to prevent spreading during molding. Molds shall be wiped with an oily cloth before using.

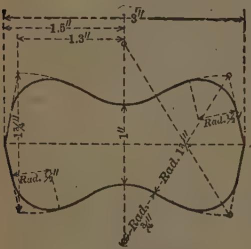

Fig. 1.—Cement Briquette.

29. The sand to be used shall be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve. This sand may be obtained from the Ottawa Silica Co., at a cost of two cents per pound, f. o. b. cars, Ottawa, Ill.

30 This sand, having passed the No. 20 sieve, shall be considered standard when not more than 5 g pass the No. 30 sieve after one minute continuous sieving of a 500-g sample.

31. The sieves shall conform to the following specifications:

The No. 20 sieve shall have between 19.5 and 20.5 wires per whole inch of the warp wires and between 19 and 21 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0165 in and the average diameter shall not be outside the limits of 0.0160 and 0.0170 in.

The No. 30 sieve shall have between 29.5 and 30.5 wires per whole inch of the warp wires and between 28.5 and 31.5 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0110 in and the average diameter shall not be outside the limits 0.0105 to 0.0115 in.

32. Immediately after mixing, the standard mortar shall be placed in the molds, pressed in firmly with the thumbs and smoothed off with a trowel without ramming. Additional mortar shall be heaped above the mold and smoothed off with a trowel; the trowel shall

drawn over the mold in such a manner as to exert a moderate pressure on the material. e mold shall then be turned over and the operation of heaping thumbing and smooth- off repeated.

33. Tests shall be made with any standard machine. The briquettes shall be tested soon as they are removed from the water. The bearing surfaces of the clips and bri- ettes shall be free from grains of sand or dirt. The briquettes shall be carefully cen- ed and the load applied continuously at the rate of 600 lb. per minute.

34. Testing machines should be frequently calibrated in order to determine their curacy.

35. Briquettes that are manifestly faulty, or which give strengths differing more than % from the average value of all test pieces made from the same sample and broken at same period, shall not be considered in determining the tensile strength.

Storage of Test Pieces. 36. The moist closet may consist of a soapstone, slate or ncrete box, or a wooden box lined with metal. If a wooden box is used, the interior ould be covered with felt or broad wicking kept wet. The bottom of the moist closet ould be covered with water. The interior of the closet should be provided with non- sorbent shelves on which to place the test pieces, the shelves being so arranged that ey may be withdrawn readily.

37. Unless otherwise specified all test pieces, immediately after molding, shall be placed the moist closet for from 20 to 24 hours.

38. The briquettes shall be kept in molds on glass plates in the moist closet for at least hours. After 24 hours in moist air the briquettes shall be immersed in clean water in rage tanks of non-corroding material.

39. The air and water shall be maintained as nearly as practicable at a temperature 21° C. (70° F.).

4. Specifications for Cement

Standard Specifications for Portland Cement. Adopted by the American ciety for Testing Materials, Sept. 1, 1916:

Definition. 1. Portland cement is the product obtained by finely pulverizing clinker oduced by calcining to incipient fusion, an intimate and properly proportioned mixture argillaceous and calcareous materials, with no additions subsequent to calcination cepting water and calcined or uncalcined gypsum.

Chemical Properties. 2. The following limits shall not be exceeded:

Loss on ignition, percent	4.00
Insoluble residue, percent	0.85
Sulfuric anhydride (SO_3), percent	2.00
Magnesia (MgO), percent	5.00

Physical Properties. 3. The specific gravity of cement shall not be less than 3.10. .07 for white Portland cement). Should the test of cement as received fall below this quirement a second test may be made upon an ignited sample. The specific gravity st will not be made unless specifically ordered.

4. The residue on a standard No. 200 sieve shall not exceed 22% by weight.

5. A pat of neat cement shall remain firm and hard, and show no signs of distortion, acking, checking, or disintegration in the steam test for soundness.

6. The cement shall not develop initial set in less than 45 minutes when the Vicat edle is used or 60 minutes when the Gillmore needle is used. Final set shall be attained ithin 10 hours.

7. The average tensile strength in pounds per square inch of not less than three standard ortar briquettes composed of one part cement and three parts standard sand by weight, all be equal to or higher than the following:

Age at test, days	Storage of briquettes	Tensile strength lb per sq in
7	1 day in moist air, 6 days in water	200
28	1 day in moist air, 27 days in water	300

8. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

Packages, Marking and Storage. 9. The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. A bag shall contain 94 lb net. A barrel shall contain 376 lb net.

10. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

Inspection. 11. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinbefore prescribed. The 28-day test shall be waived only when specifically so ordered.

Rejection. 12. The cement may be rejected if it fails to meet any of the requirements of these specifications.

13. Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100° C. for 1 hour it meets this requirement.

14. Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter.

15. Packages varying more than 5% from the specified weight may be rejected; and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

Standard Specifications for Natural Cement. Adopted by the American Society for Testing Materials, 1904; revised 1908, 1909:

Definition. 1. Natural cement is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

Physical Properties. 2. The residue on a standard No. 100 sieve shall not exceed 10%, and on a standard No. 200 sieve shall not exceed 30%, by weight.

3. Pats of neat cement about 3 in in diameter, ½ in thick at center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat shall then be kept in air at normal temperature.

(*b*) Another pat shall be kept in water maintained as near 70° F. as practicable.

These pats shall be observed at intervals for at least 28 days, and, to satisfactorily pass the tests, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

4. The cement shall not develop initial set in less than 10 minutes, using the Vicat needle. Final set shall be attained in not less than 30 minutes nor more than 3 hours, using the Vicat needle.

5. The minimum requirements for tensile strength for briquettes 1 sq in in cross-section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Age	Neat cement	Strength
24 hours in moist air		75 lb
7 days (1 day in moist air, 6 days in water)		150 lb
28 days (1 day in moist air, 27 days in water)		250 lb
	One Part Cement, Three Parts Standard Ottawa Sand	
7 days (1 day in moist air, 6 days in water)		50 lb
28 days (1 day in moist air, 27 days in water)		125 lb

Packages, Marking and Storage. 6. The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon. A bag shall contain 94 lb net. A barrel shall contain 282 lb net.

7. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

Inspection. 8. (*a*) Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test.

(*b*) The cement shall be tested in accordance with the methods contained in the Standard Specifications and Tests for Portland Cement of the American Society for Testing Materials

Rejection. 9. The cement may be rejected if it fails to meet any of the requirements of these specifications.

10. Cement failing to meet the 7-day requirements may be held awaiting the results of the 28-day tests before rejection.

5. Sand

General Requirements. Sand should be composed of hard siliceous material, free from vegetable loam, clay, sticks, and organic matter. It should preferably be of coarse grain, or of graded size with the coarse grains predominating Fine sand can be used, but it requires more cement for a given strength and more thoro mixing. The best sand as to size is one which is so graded from coarse to fine as to reduce the percentage of voids to a minimum, but such a sand has the appearance of a very coarse sand, as the amount of fine material required is small. The common requirement of "sharpness of grain" is unnecessary. Sands with rounded grains are likely to contain a smaller percentage of voids, as they are more readily compacted; they give equally as good results in mortar tests as sands with angular or sharp grains.

Importance of Good Sand. The quality of the sand for mortar or concrete is generally not given the attention it deserves. The manufacture of portland cement is well controlled and the product is very uniform and reliable, so that in practise the results are likely to be much less affected by variations in the cement than by variations in the quality of the sand.

Mineral Composition. Sand should preferably consist of grains of silica, but a considerable proportion of other minerals may be present without detriment. The physical condition is of more importance than the chemical composition. Occasionally a sand is found which, altho of satisfactory size, produces a very poor mortar. This may be due to its soft friable character, or to the presence of organic matter or too much clay, or of some partially decomposed mineral which acts as a lubricant to the mass. Mica is a very objectionable constituent, a small portion causing a serious reduction in the strength of the mortar. While a clean sand is usually specified, the presence of 5 or even 10% of clay or loam, if finely divided, appears to cause no deleterious effects.

Analysis of Sand. For the purpose of determining the relative value of a sand as regards the size of its particles, two methods are in use: (1) By determining the proportion of voids in the sand, and (2) by separating the sand into several sizes by means of sieves and determining the proportion of the various sizes.

The Voids in Sand may be determined by dropping a known volume of sand into a measuring vessel containing water. The volume displaced by the sand, subtracted from the original volume of the sand, will give the volume of voids. Pouring water into sand gives unreliable results, as the air cannot easily be driven out of the sand to admit the water.

The percentage of voids can also be found very closely for most sands by weighing a measured quantity of the dry material and determining the volume of solid material present by applying a specific gravity of 2.65 to the weight. Nearly all materials of which sand is composed have a specific gravity between 2.6 and 2.7. Assuming a specific gravity of 2.65, let W = weight of a cubic foot of the dry sand, V = absolute volume of the solid material, and P = percentage of voids; then $V = W/(62.5 \times 2.65) = W/166$ and $P = 100\ (1 - W/166)$. The space occupied by the moisture in the sand can be found, if desired, by first weighing the moist sand and then the dry sand. The difference is the weight of the water, from which its volume is found.

The volume of sand is much affected by moisture and by its degree of compactness so that for accurate comparative purposes the sand should be thoroly dry before measuring and definite methods of compacting employed. Generally the voids are determined with reference to the dry material well shaken.

The Percentage of Voids in dry sand will range from 40 to 45% for a very uniform natural or screened sand to about 28% for a coarse, well-graded natural sand. Generally the voids in ordinary good coarse sand will range from 30 to 35%.

Effect of Moisture upon the Volume and Weight of Sand. The specific gravity of sand grains being nearly constant, the weight of dry sand varies only as the percentage of voids varies. Moist sand not packed weighs less than dry sand, the difference depending upon the size of the sand grains and amount of moisture. Ordinary moist sand contains 2 to 4% of moisture and will weigh from 90 to 95 lbs per cu ft in normally loose condition Thoro tamping may easily reduce the volume and increase the weight 15 to 20%. Proportioning of sand and cement by loose volume of sand and packed cement will generally give a somewhat smaller proportion of sand than when proportioned by weight.

The Mechanical Analysis of Sand. The best method of determining the value of a sand with reference to its size is by means of a mechanical analysis made by sifting the sand thro several different sieves. These are made of standard size, 8 in in diameter by 2¼ in high. For openings exceeding 1/10 in in diameter sheet brass is used with drilled circular openings of the desired size. For smaller openings woven brass wire sieves are employed. The woven sieves are known by numbers corresponding to the number of meshes per lineal inch, altho the actual number of meshes will vary somewhat. They are made so as to fit together in nests. For analyzing cement and sand the following sizes are desirable:

Commercial No.	10	20	30	40	50	80	100	200
Approximate size of hole, inches	0.073	.034	.022	.015	.011	.007	.0055	.0026

For screening out large material from sand a screen with ¼-inch openings is generally employed.

Graphic Representation of Analysis. The sand being separated the total percentage smaller than each size is plotted upon a diagram, abscissas representing size, and ordinates percentage. Fig. 2 represents the analysis of four natural sands. (Bull. 331, U. S. G. S., 1908.) No. 21 is very fine sand, over 80% passing No. 30 sieve; No. 10 is also a fine sand but is not so uniform

in size as No. 21. It is better graded from fine to coarse. The percentages of voids in these two sands are 40.9 and 31.6 respectively, and the tensile strength of 1 : 3 mortar is 380 and 488 lbs per sq in at 180 days. No. 20 is a medium sand of good gradation in size, and No. 4 is a very coarse sand, also well graded. The voids are 28.0% in each case, and strengths of 1 : 3 mortar, 670 and 731 lbs per sq in.

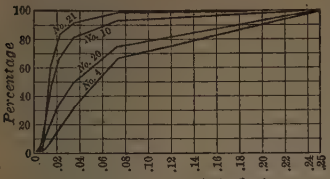

Fig. 2. Mechanical Analysis of Sands

Coarseness of sand is indicated by large abscissas, and good grading by approach to a straight line. The effective size of a sand cannot be determined by mere inspection. A sand which appears to be coarse may contain so much fine material as to be in effect a fine sand. It rarely happens that a coarse sand contains too small an amount of fine material.

Size of Sand. Sand containing coarse gravel must be screened when it is to be used in mortar for laying masonry. For use in concrete, screening is not necessary, but where the sand contains a considerable amount of such material, better and more uniform results will be secured by screening and remixing. For this purpose a ¼-in screen is generally employed, all material passing such sieve being considered as sand, or fine aggregate, and all material larger than this size being classed as coarse aggregate.

A sand containing more than 30% of particles that will pass a No. 40 sieve is a fine sand and is generally undesirable. Such a sand should be used only after careful study of its behavior in mortar.

Standard Sand. For comparative purposes in making tests of mortars a standard sand is necessary. That recommended by the Committee of the Am. Soc. C. E. is a natural bank sand obtained at Ottawa, Ill., and screened to size by means of a No. 20 and a No. 30 sieve. The percentage of voids of this sand is about 37%. The grains are rounded and it is readily compacted. Other sands of smaller grain or larger percentage of voids are likely to give somewhat lower results in mortar tests. Results from this sand are given in Figs. 8 and 11 of Art. 7.

Gravel Screenings usually make excellent sand. The material is apt to grade quite coarse and to contain very little fine material, but the grains pack readily and the percentage of voids is not likely to be high.

Stone Screenings, free from clay, constitute a good material to use in place of sand. They are likely to grade coarser than sand and to have about the same void space. Fig. 3 shows the mechanical analysis of two samples each of gravel and stone screenings, Nos. 4 and 6 relating to gravel and Nos. 2 and 17 to stone screenings. (Bull. 331, U. S. G. S., 1908.)

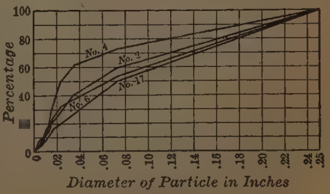

Fig. 3. Mechanical Analysis of Screenings

6. Proportioning of Cement Mortars

Cement Mortar is a mixture of cement with sand, or its equivalent, and water. CEMENT PASTE is a mixture of cement and water.

Proportioning of Mortar. The most exact method of regulating the proportions of sand and cement is by weight, and this method is generally followed in laboratory tests. In practise the proportions are determined by volume. For this purpose the cement should not be measured loosely but the barrel taken as the unit at some definite specified volume. Packed cement will increase as much as 30% in volume when piled loosely, hence the necessity of definite rules. Sand is measured by volume as shoveled into barrows or measuring boxes.

Weight and Volume of a Barrel of Cement. A barrel of portland cement contains 376 lbs of cement. A bag is one-fourth of a barrel. When packed in the barrel the cement occupies about 3.2 cu ft and when measured loose about 4.0 to 4.2 cu ft. A considerable difference in practise exists regarding the volume which is to be assumed for a barrel, but the value most commonly used is 3.8 cu ft. A cubic foot of cement will then weigh 100 lbs.

A barrel of natural cement varies in weight according to the locality. Generally a barrel of eastern cement contains 300 lbs of cement and a barrel of western cement 265 lbs. A standard weight of 282 lbs is also used. A bag of natural cement is usually one-third of a barrel. The volume of a barrel of eastern cement may be taken as 3.8 cu ft, the same as portland cement.

Proportions Used in Practice. For use in the construction of stone or brick masonry the usual proportions for portland-cement mortar are 1 : 2 or 1 : 3, and for natural-cement mortar 1 : 1 or 1 : 2. For use in concrete a wider range is employed, depending upon the requirement. For imperviousness the voids in the sand must be entirely filled, which will ordinarily require proportions of 1 : 2 or 1 : 2½, but for strength a leaner proportion is often sufficient. Large proportions of sand render a mortar difficult to handle with a trowel. The addition of 10 or 15% of lime paste makes an easier worked mortar, tends to increase its imperviousness and does not materially affect its strength. The quantities of cement and sand required for one cubic yard of mortar, based on a volume of 3.8 cu ft for one barrel of cement and the use of a fairly well graded sand, are given closely by the following table:

Quantities for One Cubic Yard of Cement Mortar

Proportions	Cement, bbls	Sand, cu yds	Proportions	Cement, bbls	Sand cu yds
neat paste	7.50	0	1 : 3	2.35	1.00
1 : 1	4.88	0.70	1 : 4	1.84	1.05
1 : 1½	3.85	0.82	1 : 5	1.52	1.08
1 : 2	3.20	0.90	1 : 6	1.30	1.12
1 : 2½	2.70	0.96	1 : 8	1.00	1.15

For natural cement, weighing 275 lbs per bbl the quantity of cement should be increased 10%. Variations in consistency, fineness of sand, and character of cement will modify the amount of cement required as much as 10% in either direction from the values given in the table.

Mixing of Mortar, if done by hand, should be done on a water-tight platform to prevent the escape of cement. Large quantities of mortar are preferably mixt in mechanical concrete mixers. The following is the specification for mortar of the Am. Ry. Eng. and M. W. Assoc.:

The sand and cement should be mixt dry and in small batches in proportions as directed, on a suitable platform, which must be kept clean and free from all foreign matter; then water is to be added, and the whole remixt until the mass of mortar is thoroly homogeneous and leaves the hoe clean when drawn from it. It must not be retempered after it has begun to set.

7. Strength of Cement Mortars

General Laws Governing Strength. The chief influences affecting the strength of a mortar are: (1) the proportions of cement used, (2) the size and grading of the sand, (3) the amount of water used, and (4) the degree of compactness of the product.

The effect of all these factors can be exprest by two general laws: (1) The strength of a mortar increases with increased proportions of cement. (2) The strength of a mortar of given proportions of cement increases with its density. The second law expresses in a general way the effect of sand, amount of water and compactness of material. Feret gives the following as expressing approximately the comparative strength of mortars of different densities:

$$P = K\left(\frac{C}{1 - S}\right)^2$$

in which K = a constant; C = absolute volume of cement in a unit of mortar; S = absolute volume of sand in a unit of mortar; P = compressive strength of the mortar. For the cement employed by Feret, K = 28 000 and P = strength at 5 months in lbs per sq in.

Uniform Conditions in Testing must be carefully observed in order to secure uniform or comparable results. The strength of mortar is greatly affected by the temperature of the air or water, the thoroness of gaging and manner of testing. These conditions necessitate the adoption of uniform methods of testing fully setting forth in much detail all parts of the process. Such specifications are given in Art. 3. The conditions of testing being fixt and uniform it is then possible to determine the influence of the character of the cement and sand employed in the mortar.

The Density of a Mortar or Concrete is the ratio of the volume of actual solid material (cement, sand and coarse material) to the total volume of the hardened mortar. In determining density the absolute solid material in the cement, sand and stone is to be taken. This may generally be obtained very closely by weighing the separate ingredients and calculating their volume, using a specific gravity of cement of 3.1 and of sand and stone of 2.65.

Effect of Size of Sand. The effect of size and grading of the sand grains follows the general law of density. In general, that sand which will give the densest mortar will give also the strongest. This requires (1) that the percentage of voids be small, and (2) that the sand be generally of coarse grain. The first requires a mixture of large and small particles, the large particles predominating. The percentage of pore space depends upon the grading of the sand and not upon the actual size of grain. But with the same percentage of pore space a denser mortar can be made with a coarse than with a fine sand because the former, having less superficial area to coat with water and cement, requires less water to give it the required consistency.

The effect of size of sand is so great that it will often be profitable to ship a coarse sand a considerable distance in preference to using a local fine sand.

Feret has made an exhaustive study of effect of size of sand upon density and strength. Artificial mixtures of three sizes of sand were made in which all ranges of proportions of the three sizes were represented. The sizes used were known as fine (0 to 0.5 mm), medium (0.5 to 2 mm) and coarse (2 to 5 mm). The results as to density and compress-

ive strength are concisely shown in Figs. 4 and 5. In these figures the result for any particular mortar is recorded in the triangle at such distances from the three base lines as will represent the proportions of each size used. Lines of equal strength or density are then drawn on the diagram. The maximum strength of 3500 lbs per sq in was obtained from a mixture containing about 85% of coarse sand and 15% of fine sand with very little of the medium size. The region of maximum strength is seen to be very near the region of maximum density, and thruout, the strength curves follow very closely those for density.

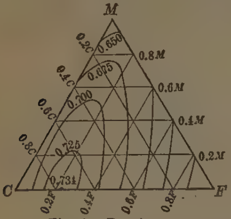

Fig. 4. Density

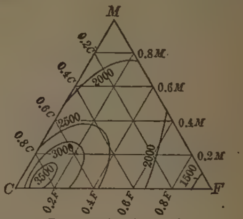

Fig. 5. Compressive Strength, 1 year

Properties of 1 : 3 Mortars made of Different Mixtures of Sand

These triangles show clearly that the proportion of fine grains in a sand should not be large. They show also that if a sand is uniform in size a coarse sand is better than a medium and a medium is better than a fine. The effect of size of sand is also shown by the table on p. 496.

Effect of Consistency or Amount of Water. From 20 to 22% of water is required to produce the normal consistency in neat cement paste of portland cement as specified in Art. 3. Much greater strength can be secured by using a drier consistency and by hard tamping, especially in short-

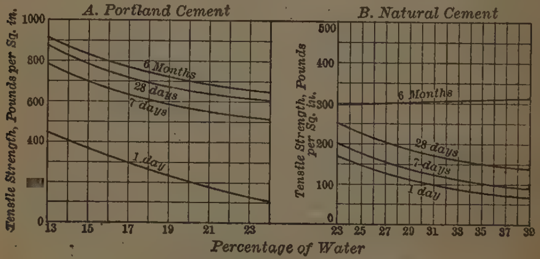

Fig. 6. Effect of Consistency on Tensile Strength

time tests, but such results are not likely to be as uniform, and in practice the consistency employed corresponds more nearly to that described in the specifications. The effect of amount of water is relatively less on long-time tests and within certain limits may be favorable to the wetter mixture. Very wet mixtures will remain the weaker.

Fig. 6 gives results of tests by Edward S. Larned on neat cement mixt with different proportions of water (Eng. News, Aug. 6, 1903).

The percentage of water also affects greatly the time of setting, increasing it in the case of Fig. A from 207 to 912 min and Fig. B from 59 to 1057 min.

Tensile Strength of Portland-Cement Mortars. Under normal conditions the strength of portland-cement mortars increases very rapidly during the first few days, the rate of change gradually falling off. In seven days its strength is generally half to two-thirds its ultimate, which is practically attained in two or three months. The compressive strength varies according to a somewhat different law.

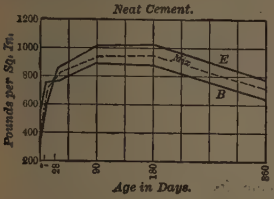

Fig. 7

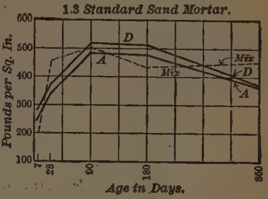

Fig. 8

Variation of Tensile Strength with Age

Fig. 7 gives some of the results of tests on 7 brands of portland cement made by the Technologic Branch of the U. S. Geological Survey. The three curves show the maximum and minimum results and the results from a mixture of the entire 7 brands. The results are averages of 10 tests each. (Bull. No. 331, U. S. G. S., 1908.) Fig. 8 gives similar results of tests of 1 : 3 mortars made with standard Ottawa sand. Other values at 180 days with natural sands are given on p. 496. For the better grades of material they are somewhat greater than those obtained with the standard sand.

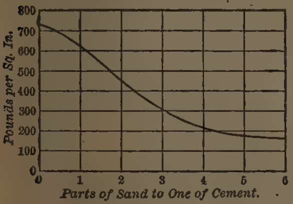

Fig. 9. Tensile Strength of Mortars One Month Old

Fig. 9 shows approximately the tensile strength at one month of portland-cement mortars containing various proportions of sand.

The Retrogression in Strength shown in Fig. 7 is a common experience but is not permanent. It does not occur in the corresponding compression tests. Generally cements of low strength at 7 days show greater gain thereafter than those of high strength, so that the long-time test shows less relative variation. Excessively high 7-day tests are not to be desired.

Transverse Strength of Portland-Cement Mortars. The transverse strength is primarily a function of the tensile strength, but the modulus of rupture (fiber stress) calculated from the formula $S = Mc/I$ is considerably greater than the strength in direct tension. The value of this ratio will ordinarily range from 1.8 to 2. Transverse tests on mortar prisms 1 in square by 12 in long are given in the table on p. 496.

Compressive Strength of Portland-Cement Mortars. The compressive strength of mortars of artificially graded sands are given in Fig. 5, as determined on prisms of the same height as width. Average results on 2-inch cubes from 7 brands of cement, and on a mixture of all, are given in Figs. 10 and 11. Other values for mortars of sand, gravel, and stone screenings are given on p. 496. For average crushing strength at 30 days see Fig. 15.

The ratio between compressive and tensile strength is very variable and

cannot be used as a basis for calculations. The compressive strength steadily advances with age.

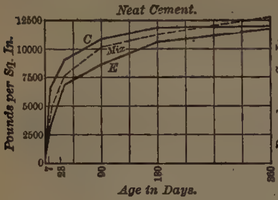

Fig. 10

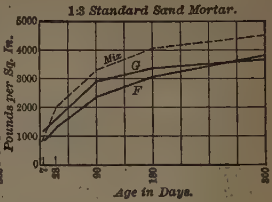

Fig. 11

Variation of Compressive Strength with Age

Relative Strength of Mortars made from Sand, Gravel, and Stone Screenings. In the following table are given results of tests on mortars made from various sands and from material screened from gravel and broken stone. (Bull. No. 331., U. S. G. S., 1908.) The poorest results were obtained from the very fine sands and from sands having a large percentage of voids. With this exception the results are about the same on the three varieties of materials.

Strength of Portland-Cement Mortar

No.	Kind of Material	Percentage Passing No. 20 Sieve	Percent of Voids	Weight per cu ft, pounds	Density of Mortar	Strength of 1 : 3 Mortar at 180 days, pounds per square inch		
						Tensile	Compressive	Transverse
	Sand							
4	Bank Gravel......	32	28.0	116.4	0.808	731	6105	1224
13	Washed Bank Sand	38	28.9	119.5	.754	668	7183	1314
3	River Sand........	50	36.1	103.3	.752	495	5280	1182
20	Bank Sand.........	50	28.0	116.5	.794	670	6200	......
5	River Sand........	60	34.6	104.8	.752	420	4752	984
6	Bank Sand.........	68	31.6	113.5	.766	614	4225	1482
7	Bank Sand.........	77	31.6	116.0	.730	603	5733	1038
10	Bank Sand.........	82	31.6	110.0	.743	488	4639	......
1	River Sand........	86	32.5	109.3	.742	415	3677	1056
21	River Sand........	90	40.9	89.0	.700	380	2892	......
	Gravel Screenings							
9	Bank Gravel......	0	30.7	117.8	0.791	654	8567	......
8	River Gravel......	3	34.8	111.5	.741	476	6825	......
3	River Gravel......	18	38.6	102.6	....	426	4654	......
1	River Gravel......	27	29.0	115.0	.771	601	7460	......
5	Bank Gravel......	36	29.7	115.3	.756	647	6873	......
12	Bank Gravel......	50	26.5	120.3	.782	733	6892	......
	Stone Screenings							
15	Chats.............	8	37.0	109.5	0.726	660	4681	......
6	Chats.............	22	33.1	109.5	.760	750	8048	1326
23	Limestone.........	25	38.2	99.7	.737	545	4362	......
17	Granite...........	30	34.7	108.8	....	575	5313	1014
1	Limestone.........	40	39.4	103.5	.740	707	5236	......
12	Limestone.........	45	35.1	105.3	.733	717	6193	1446
3	Limestone.........	55	37.0	103.5	.733	809	6500	1410
21	Limestone.........	80	41.0	97.4	.655	683	3948	......

Effect of Clay. Clay is generally considered deleterious, and specifications require sand to be clean. Under some circumstances, however, considerable quantities of clay (up to 10 or 15%) added to the sand increase the strength and imperviousness of the mortar. It appears that a lean mortar will be thus improved but that a rich mortar is likely to be injured. In lean mortar it helps to fill the voids, but in rich mortars whose voids are already full it reduces the cohesion without increasing the imperviousness. If clay is added or permitted it should be finely divided and mixing be thoroly done.

Effect of Lime. Lime acts in a manner similar to clay. It tends to increase the imperviousness, and, in the case of poor mortars, it appears also to increase the strength. The amount which may be added without reducing the strength may be taken approximately as that which acts to increase the density of the mortar or which is sufficient to fill the pore spaces. Somewhat more lime may be used without decreasing the strength below that of the unlimed mortars. The addition of lime also makes a cement mortar easier to use with a trowel. For use of lime to produce impermeable mortar see Art. 15.

Lazell found that 15 % of hydrated lime could be added to 1 : 3 mortar without decreasing the strength, and as high as 30 % to 1 : 5 mortars.

Strength of 1 : 3 Portland-Cement Mortars

From Proc. Am. Soc. Test. Materials, 1908.

Proportions Used			Tensile Strength, lbs per sq in			
Portland Cement	Hydrated Lime	Sand	7 days	28 days	6 mo	12 mo
1.0	0	3	209	266	382	630
0.95	0.05	3	203	258	312	456
0.9	0.1	3	205	255	304	513
0.85	0.15	3	209	245	281	642
0.8	0.2	3	133	203	229	553
0.75	0.25	3	112	170	211	444
0.7	0.3	3	141	225	219	327

Effect of Regaging. Regaging or remixing of cement mortar after setting has begun should not generally be permitted. Experiments on the effect of regaging give various results, but little or no effect on strength is shown on portland-cement mortar when regaged after a lapse of not more than two hours. Natural cements are more likely to be affected injuriously. A regaged mortar is slower setting than a fresh mortar and appears to contract less in hardening. It works better under the trowel and is preferred by masons for plastering. Special circumstances may make it desirable to permit the use of regaged mortar under suitable regulations.

Effect of Low Temperature. The effect of temperature on the rate of setting and hardening of cement is very great and often requires special consideration in the process of construction. Removal of forms and the loading and exposure of newly completed work will thus ofteh depend upon the prevailing temperature. At or near 32° F. the rate of setting and hardening is very slow. Experiments on natural-cement mortars indicate a rate of hardening at 40° about one-half that at 80° on sixty-day tests. Tests on portland cement show that at a temperature of 40° the strength at 30 and 60 days is about two-thirds that attained at a temperature of 70°. At or near 32° the time of setting is greatly prolonged. Cement setting in 8½ hrs at a temperature of 65° required 38 hrs at 32°.

Effect of Freezing. The freezing of natural mortars injures them seriously and should be wholly avoided. Portland-cement mortar or concrete does not appear to be injured by freezing, altho the hardening is greatly retarded. Troweled surfaces or surfaces where free moisture is produced are injured by freezing and will scale. Frozen concrete will gain little strength even after the lapse of several weeks and hence must be given special attention when warm weather arrives. While successful work may be done in freezing weather, the difficulty of properly mixing and placing the material makes it undesirable to work in very cold weather. Thin or superficial coats of mortar such as plaster should not be applied in freezing weather.

Effect of Salt. Salt added to the water lowers its freezing point and thus, within limits, prevents freezing. Experiments on the effect of salt show that up to 10% at least the ultimate strength of mortar is not reduced, altho the result of short-time tests may show some reduction. The amount of salt required to lower the freezing temperature is given by Tetmajer as equal to 1% of the weight of the water per degree F. below 32°.

Rise of Temperature in Setting. In setting, the chemical changes cause a rise of temperature which in the interior of large masses may reach 100° F. In experiments on 12-in cubes of portland-cement mortar Howard observed a rise of temperature in the interior of about 115° in the case of neat cement, and 45° in the case of 1 : 1 mortar (Tests of Metals, 1901). A rise of temperature of 90° has been observed in the interior of a wall of concrete 11.5 ft thick. This increase in temperature effectually prevents freezing in the interior of large masses.

Contraction and Expansion Due to Hardening. Cement mortar when hardened in air will contract, and when hardened in water will maintain a nearly constant volume or show a slight swelling. The richer the mortar the greater the effect. Experiments by Considère and others indicate that 1 : 3 mortar will shrink 0.05 to 0.15% when hardened in air two to four months, and neat cement from two to three times as much.

Adhesive Strength. Adhesive strength of portland-cement and natural-cement mortar to sawn limestone is shown by the experiments of Wheeler to be as follows (Rept. Chief of Engrs., 1895):

Adhesive Strength of Mortars

Kind of Cement	Mixture	Adhesive Strength, lbs per sq in	Cohesive Strength, lbs per sq in
Portland-cement Mortar	neat	270	686
	1 : 1/3	233	710
	1 : 1	221	747
	1 : 2	169	487
Natural-cement Mortar	neat	94	183
	1 : 1/2	104	198
	1 : 1	116	218
	1 : 2	66	186

The adhesive strength of mortar to brick surfaces was found to be about 50 lbs per sq in for neat, and 30 lbs per sq in for 1 : 2 mortar, for both portland and natural cement mortar. For adhesion to steel rods see Art. 21.

Strength of Natural-Cement Mortars. Natural-cement mortars gain strength less quickly but continue to increase in strength for a longer period

than portland-cement mortars. Fig. 12 shows average results of time experiments on good brands of natural cements. Different brands show great

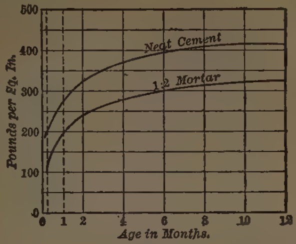

Fig. 12. Tensile Strength of Natural-Cement Mortars

differences in strength, and variations as much as 30% above or below the values given may be expected. For compressive strength of various mortars at one month see Fig. 15.

PLAIN CONCRETE

8. The Aggregate

Concrete is an artificial stone made by mixing together cement, water, and an aggregate consisting of large and small particles, such as sand or screenings and broken stone or gravel. The aggregate is generally made up of two distinct or separate materials, the fine aggregate and the coarse aggregate. The use of a variety of materials and sizes for both fine and coarse aggregates necessitates careful definition and specification. FINE AGGREGATE is generally taken as that part of the material which will pass a screen having ¼-inch diameter holes. It consists generally of sand, but may also consist of gravel or stone screenings. (See Art. 5.) COARSE AGGREGATE consists of material larger than ¼ inch, and may be broken stone, gravel, slag or any hard, durable material. The particles should be clean, hard and free from deleterious material.

Materials for Coarse Aggregates. Any stone is suitable for concrete which is durable and has sufficient strength so that the strength of the concrete will not be limited by the strength of the stone. Beyond this minimum, great strength is of little advantage. Granites, traps and limestones are generally employed. Many sandstones and shales are of deficient strength and should be used only after determination of their strength. Soft, flat or elongated particles do not make a satisfactory material for coarse aggregate.

Screened gravel constitutes generally a satisfactory coarse aggregate but may contain particles of soft, friable material which will seriously reduce the strength of the concrete.

Cinders may be used in concrete for fire-proofing purposes and for very low stresses. As generally furnished, cinders are composed largely of ashes and are very unsuitable for concrete. They should consist of hard, clean, vitreous clinker, free from sulphides, unburned coal and ashes.

Size of Aggregate. The maximum strength of concrete will be secured for a given quantity of cement when the aggregates are so proportioned as to size as to reduce the percentage of voids in the mixture to a minimum, due

attention being given to the effect of size of sand (Art. 5). To reduce the proportion of voids to a minimum the aggregate should consist of particles of different sizes so that the smaller will fill the interstices between the larger. A uniform size of broken stone, filled with sand, does not make as dense or strong concrete as one in which the coarse aggregate consists of large and small stones so that the small stones may partly fill the larger interstices.

Screened stone of a single size is not desirable. Crusher-run material contains less voids and is more satisfactory, provided the fine material is free from clay. Frequently the crusher-run material is used with the dust screened out, but repeated experiments show that this fine material, if clean, is entirely satisfactory. Its use reduces the proportion of sand required, but the amount of this reduction should be ascertained by a determination of voids.

If gravel is used it should be screened to at least two sizes, separated by a ¼-inch screen, and remixt, in order that the proportions may be reasonably definite. Natural gravel is sometimes so uniformly mixt as to be suitable for use without screening, either with or without additional sand, but generally it is advisable to screen and remix. This is especially important in the case of reinforced work.

The Maximum Size of the Coarse Aggregate depends upon the use to be made of the concrete. It should be such as will enable the concrete to be readily placed in all parts of the forms and to leave a reasonably smooth finish. Where used in massive walls the maximum size may well be 2½ to 3 inches, but for thin sections and reinforced work a size of 1 inch, or even less, is necessary. To a limited extent the strength of the concrete increases with increase in size of stone, hence the size should not be less than necessary to secure good work.

Results of an extended inquiry regarding practise in respect to size led the Masonry Committee of the Am. Ry. Eng. and M. of W. Assn. to the following conclusion: Considering plain concrete only, and assuming that the aggregate will range in size from ¼ inch to the maximum named, a preference is shown for the following maximum sizes: for foundations, 2½ in; for abutments, 2 in; for arch rings, 1½ in; for coping, bridge seats and thin walls, 1 in.

Specific Gravity of Stone. The specific gravity of hard stone suitable for concrete varies generally within narrow limits, and can often be used to advantage in calculating weight and proportions of voids. Specific gravities of various materials are about as follows:

Trap	2.8 to 3.0	Sandstone	2.3 to 2.6
Granite	2.65 to 2.75	Sand and gravel	2.6 to 2.7
Limestone	2.6 to 2.7		

Determination of Voids. Voids in coarse aggregates may be determined closely by measuring the amount of water required to fill the voids in a given volume of aggregate. Or it may be determined somewhat more accurately by weighing a known volume and calculating the volume of solid material by means of its specific gravity. If an appreciable amount of moisture is present this must first be eliminated by drying. If V = volume of the material, W = weight of volume V, G = specific gravity of solid stone, p = percentage of voids, then

$$p = 1 - W/V(G \times 62.5)$$

For aggregate containing much fine material the first method is not reliable on account of the difficulty of excluding all the air.

Much care is required in filling the vessel with the aggregate so that the degree of compactness will be uniform. Generally the voids are determined

with the materials simply shoveled into the receptacle or slightly compacted by dropping a short distance. Moderate ramming or shaking will readily reduce the volume 10%, heavy ramming as much as 15%.

Voids in Coarse Aggregates will range from 30 to 55% depending upon the uniformity of the stone, its shape and degree of compactness. Voids in loose broken stone, crusher-run, with the fine material screened out, will generally measure from 43 to 48%. If well shaken or rammed, as in the placing of concrete, the volume can readily be reduced 10%, thus reducing the voids to from 37 to 42% respectively. Stone screened to a uniform size will contain from 50 to 55% of voids. Gravel of the same size as broken stone will contain less voids, as it packs more readily. With the sand screened out it will generally contain from 35 to 40% voids.

I. O. Baker obtained the following results from experiments on three varieties of limestones when dropt several feet into the measuring box (Bul. No. 23, Univ. of Ill. Eng. Exp. Sta., 1908):

Voids in Loose Limestone

Kind of Stone	Size of Stone	Percent of Voids	
		By Pouring in Water	By Specific Gravity Method
Chester Limestone	⅜-in screenings	40.9	46.8
	¾-in screenings	43.0	45.6
	2-in to ¾-in	46.6	46.6
	3-in to 2-in	46.1	45.1
Joliet Limestone	½-in screenings	42.2	47.1
	2-in to ½-in	47.9	46.2
	3-in to 2-in	47.5	46.1
Kankakee Limestone	⅜-in screenings	39.6	46.1
	1¼-in to ⅜-in	45.7	44.7
	2¼-in to ⅜-in	44.3	42.9
	2¼-in to 1¼-in	46.2	43.4

Voids in Loose Materials

Authority	Kind of Stone	Percent of Voids
Wm. M. Hall	Green River limestone, 2½-in and smaller, dust screened out	48
"	Hudson River trap, 2½-in and smaller, dust screened out	50
"	River gravel, 1½-in and smaller, sand screened out	35
H. Von Schon	Sandstone, 1-in to 1½-in	45.3
"	Crusht granite boulders, 1-in to 1½-in	48.7
"	Gravel, 1½-in and smaller	34.1
G. W. Rafter	Portage (N. Y.) sandstone	43.3
Taylor & Thompson	Hard trap, 2½-in to 1-in	54.5
"	Hard trap, 2½-in to dust	45.0
"	Gravel, 2½-in to ½-in	36.5
W. E. McClintock	Trap, 1½-in to ½-in	50.2
"	Trap, 3-in to 1½-in	48.1
"	Trap, ½-in screenings	46.5

Weight of Broken Stone and Gravel

Broken stone is generally sold by the cubic yard, but the stone is measured by weight, assuming a certain standard weight per cu yd. A cu yd of stone of 2.7 specific gravity and 45% voids will weigh almost exactly 2500 lbs, a value frequently assumed.

Percentage of Voids	Specific Gravity				
	2.6	2.7	2.8	2.9	3.0
	Weight, lbs per cu ft				
35	108	110	114	118	122
40	97	101	105	109	112
45	89	93	96	100	103
50	81	84	87	91	94
55	73	76	79	82	84

Specifications for Aggregate. Stone shall be round, hard and durable, crusht to sizes not exceeding two inches in any direction. For reinforced concrete, sizes usually are not to exceed three-quarters (3/4) in in any direction, but may be varied to suit the character of the reinforcing material.

Gravel should be composed of clean pebbles of hard and durable stone of sizes not exceeding two inches in diameter, and shall be free from clay and other impurities except sand. Where containing sand in any considerable quantity, the amount of sand per unit of volume of gravel shall be determined accurately, to admit of the proper proportion of sand being maintained in the concrete mixture. (Am. Ry. Eng. and M. W. Assn.)

9. Proportioning of Concrete

Principles of Proper Proportioning. In the proportioning of concrete the object to be aimed at is to so proportion the fine and coarse materials that a given amount of cement will be as effective as possible in filling the remaining voids and binding together the particles of the aggregate. This requires that the proportion of voids be reduced to a minimum, avoiding, however, the use of much very fine material. Having a minimum proportion of voids an amount of cement is then used which will give the requisite strength or degree of imperviousness. Such a concrete will be of maximum density for the given amount of cement. Often an amount of cement less than sufficient to fill the voids will produce a concrete of ample strength and satisfactory for the purpose, altho somewhat porous.

Proportioning is commonly done by rule of thumb, using certain standard proportions. Better results with greater economy can often be secured by the use of more accurate methods of proportioning by the determination of voids or by a mechanical analysis.

Proportioning with Reference to the Coarse Aggregate. By this method the voids in the coarse aggregate are determined and an amount of mortar then used, of the desired strength, sufficient to fill these voids. To insure this requires some excess of mortar, but this excess should be as small as practicable, since it increases the cost without increasing the strength. A fine aggregate will require more mortar than a coarse aggregate. The yield of mortar from the given sand should be separately determined.

Generally the run of the crusher with the fine material screened out is used for the coarse aggregate, and the best local coarse sand for the fine aggregate. In extensive work, however, it may well pay to make an artificial mixture of two or more materials of different sizes, or to screen and recombine a material, such as a natural gravel. In this way it is possible to produce the most dense and economical concrete. By a reasonable degree of grading a dense and impervious concrete has been made of proportions 1 : 3 : 7 where

a 1 : 2 : 4 is the usual practise. By proper grading the voids in the coarse aggregate can be reduced to 35%, or less, with corresponding saving in cement.

The proportion of sand may also be directly determined, but less accurately, by basing it upon the percentage of voids in the coarse aggregate, each being measured loosely. A slight excess of sand will be required to insure filling in the case of a fine broken stone or a lean mortar.

Experience in the handling of concrete enables one to judge quite readily whether the mortar is deficient or not. An excess of mortar is preferable to a deficiency, as a concrete deficient in mortar is difficult to place, and is especially to be avoided in reinforced work, or work designed to be impervious.

Use of Arbitrary Standard Proportions. As the voids in ordinary crusher-run broken stone, from which the fine material has been screened out, will range generally between 45 and 50 %, a proportion of one volume of sand to two volumes of stone will generally give but little surplus of sand unless the mortar is quite rich. Thus for ordinary sand, the proportions of mortar to stone, when various mortars are used and the sand is equal to one-half of the stone are about as follows:

Mortar....................	1:1	1:1½	1:2	1:2½	1:3	1:3½	1:4
Stone....................	2	3	4	5	6	7	8
Percent of mortar to stone ..	72	61	55	52	50	49	48

For all proportions of mortar of 1 : 2, or poorer, the relative amount of mortar is closely equal to 50% of the stone. For richer mortars the ratio of sand to stone may be reduced somewhat below one-half. For a 1 : 1½ mortar the stone may be at least 3½ parts.

Proportioning with Reference to the Mixt Aggregate. A more exact method of proportioning is to first grade the aggregates, both coarse and fine, so as to reduce the voids in the mixture to a minimum. The proportion of aggregates having been determined, the amount of cement required will then depend upon the strength needed or the degree of imperviousness. An amount necessary to fill the voids can be estimated by determining the percentage of voids in the mixture of aggregates, well compacted or settled by shaking. If a concrete of maximum density be desired, then an amount of cement should be used slightly in excess of that necessary to fill the voids. For this calculation the cement paste may be assumed equal to the volume of cement used, on the basis of 3.8 cu ft per barrel.

Proportioning by Mechanical Analysis is the most exact and effective method of studying the character of the aggregates and of calculating the effect of various mixtures. For this purpose the material is separated into sizes by screening as for sand (Art. 5) and the results plotted on a diagram. The following sizes of screens are desirable, altho a very useful analysis may be made with fewer screens: 3-in, 2¼-in, 1½-in, 1-in, ¾-in, ½-in, and ¼-in. The percentage by weight passing each size is then plotted as abscissæ and the size as ordinates. Fig. 13 illustrates the analysis of a bank gravel and a broken stone. (Trans. Am. Soc. C. E., vol. 59, 1907.) A straight line indicates a uniform grading of size.

In Fig. 14 are given the analysis curves of the same gravel shown in Fig. 13, screened to two sizes, and also when recombined in proportions of 66% coarse to 34% fine. The dotted line shows the ideal mixture, according to experiments by Fuller. This line consists of a straight line tangent to an elliptical curve, the point of tangency having an abscissa equal to one-tenth the maximum size of stone and an ordinate equal to about 30 to 33%. This signifies that the aggregate should be uniformly graded from the maximum size down to a size one-tenth of this, and that the amount of material finer than this size should equal about 30% of the total. The exact proportion of

the finer aggregate depends upon the closeness of packing of the coarse material and somewhat upon the proportion of cement used, a large proportion of cement replacing a part of the fine sand. Including the cement the

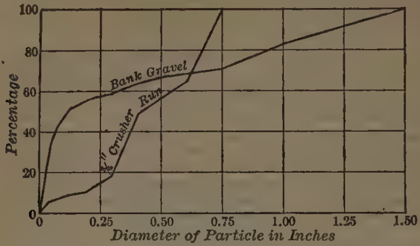

Fig. 13. Mechanical Analysis of Two Sizes of Aggregate

total amount of fine material must be sufficient to fill the voids, which, with the graded mixture assumed, is about 38%.

Aggregates of unsuitable proportions may be studied in this way, and be screened to two or more sizes and remixt so as to give a resulting aggregate requiring a minimum amount of cement.

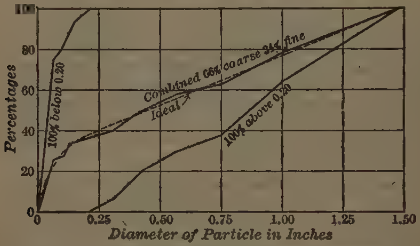

Fig. 14. Effect of Combining Two Sizes of Aggregate

Proportions Used for Various Classes of Work are about as follows:

Foundations, and structures requiring mass rather than strength	1 : 3 : 6	to 1 : 4 : 8
Structures requiring greater strength, such as piers, abutments and massive reinforced work	1 : 2½ : 5	to 1 : 3 : 6
Tanks, buildings, thin walls and reinforced concrete; also impervious construction	1 : 2 : 4	to 1 : 2¼ : 4½
Reinforced columns and parts requiring extra strength	1 : 1 : 2	to 1 : 1½ : 3

Cinder Concrete may properly be used for filling vacant spaces, for fire-proofing and for certain parts of structures in which the stresses are very small. Owing to the variable and unreliable character of the material it cannot be recommended for structures in which strength is the controlling factor in the design. The proportions commonly used are 1 : 1 : 3 and 1 : 2 : 5.

The Weight of Concrete of the usual proportions will vary from 140 to 150 lbs per cu ft, depending upon the degree of compactness and the specific gravity of the materials. Variation of proportions will effect the weight but little if the proper ratio of sand and stone be maintained, but a wet concrete when dried out will weigh less than a well-compacted concrete containing originally less water. The average values are about as follows in pounds per cubic foot:

Limestone or gravel concrete	142 to 148
Trap concrete	148 to 155
Cinder concrete	110 to 115

10. Strength of Concrete

The Compression Specimen. The compression test specimen is preferably made in the form of a cylinder 6 to 8 in in diameter and about two diameters high. Formerly the cube of 4 in to 12 in in size was the standard form but, since the introduction of reinforced work a study of the elastic properties of concrete has become of increased importance and for this purpose a prismatic specimen is preferable to the cube and is largely used. The results of tests are not appreciably affected by size of specimen if not too small, but it should be large enough to give a fairly homogeneous material. Small specimens are apt to show greater variation and somewhat lower values than large specimens. The larger the stone the larger the specimens should be. The compressive strength of the cylindrical form is about 80% of the cube form. The standard age for testing is usually 30 to 60 days, the latter being preferable and giving 75 to 80 per cent of the strength at six months.

In the preparation of compression specimens great care must be taken to secure smooth and parallel surfaces at bottom and top, preferably by means of a thin coat of cement. The specimens should be molded on smooth bases, and it is advisable to place on the top a block of cast iron or steel carefully leveled up and allowed to remain until the concrete has set. This is of especial importance in column tests. Molds may be of wood, but for a large amount of work cast-iron molds, made to separate on a diagonal or a diameter, are better and quite inexpensive.

The Compressive Strength of Concrete depends primarily upon the relative proportion of cement in a unit of volume. A given quantity of cement is, however, more effective the less the voids to be filled, so that it may be said that the strength varies directly with the amount of cement and inversely with the percentage of voids in the mixture of aggregates. It depends also to a considerable extent upon the amount of water used, upon the character of the aggregate and upon the quality of the cement, altho the last named element is probably the most constant and reliable of all the factors involved.

Fig. 15 gives the results of a study of a large number of compression tests of concrete and mortar. The average curve represents what may reasonably be expected under favorable conditions, for stone or gravel concretes in which the aggregates are fairly balanced so that the remaining voids are not excessively high. The consistency is assumed to be rather soft, such as is the practise in most construction and especially in reinforced work. The minimum and maximum curves are not infrequently reached under conditions favoring extreme results.

For the common proportions used in practise it may be concluded that under reasonably good conditions as to character of material and workmanship an average strength of about 2000 lbs per sq in may be expected of a 1 : 2 : 4

concrete in 30 to 60 days, on cylindrical specimens, the rate of hardening depending upon the consistency and the temperature. For 1 : 3 : 6 concrete a strength of about 1600 lbs per sq in may be expected. Poorer results will be obtained from poor material or poor workmanship, but better results may also be had where the conditions are especially favorable.

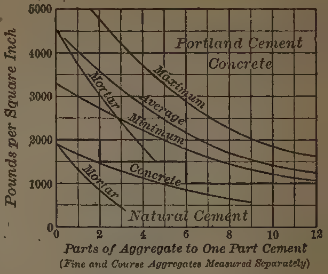

Fig. 15. Compressive Strength of Mortar and Concrete at One Month

Results of Individual Tests. An important series of tests is that made at the Watertown Arsenal for Mr. George A. Kimball, Chief Engineer of the Boston Elevated Railway Company (Tests of Metals, Watertown Arsenal, 1899.) The concrete was made of five brands of portland cement, coarse, sharp sand and broken stone up to 2½-inch size, having 49.5% voids. The concrete was well rammed into the molds, water barely flushing to the surface. The specimens were buried in wet ground after being taken from the molds. They were in the form of 12-in cubes. The average results were as follows:

Compressive Strength of Concrete

Mixture	Brand of Cement	Strength, pounds per square inch			
		7 Days	1 Month	3 Months	6 Months
1 : 2 : 4	Saylor	1724	2238	2702	3510
	Atlas	1387	2428	2966	3953
	Alpha	904	2420	3123	4411
	Germania	2219	2642	3082	3643
	Alsen	1592	2269	2608	3612
	Average	1565	2399	2896	3826
1 : 3 : 6	Saylor	1625	2568	2882	3567
	Atlas	1050	1816	2538	3170
	Alpha	892	2150	2355	2750
	Germania	1550	2174	2486	2930
	Alsen	1438	2114	2349	3026
	Average	1311	2164	2522	3088

The following are average results obtained on 6-inch cubes in the laboratory of the U. S. Geolog. Survey. (Bul. No. 344, U. S. G. S., 1908.) The concrete was machine-mixt and of medium consistency.

Aggregate	Compressive Strength, pounds per sq in		
	One Month	Three Months	Six Months
Granite	3156	4992	4949
Gravel	3547	4989*	4808
Limestone	2975	3939	3896

* Mixt damp consistency.

The strength of cylindrical specimens of 1 : 2 : 4 concrete as determined from a large number of tests made at the University of Wisconsin in 1907 was as follows: Twenty-five cylinders 10 in × 24 in, of machine-mixt concrete, 30 days old, gave an average strength of 1940 lbs per sq in. The sand was fairly fine, and specimens were cured in air

but kept moist by sprinkling. Further tests of 44 similar specimens in 1908 gave an average value in 60 days of 2150 lbs per sq in. The consistency was soft, such as is suitable for reinforced work, but not excessively wet.

The Increase of Compressive Strength with Age is shown by the results above given and also by Figs. 10, 11, Art. 7. At 60 days the strength of portland-cement concrete is a large fraction of its ultimate strength, being generally from 80 to 90% of its strength at 1 year. At 30 days the strength is considerably less, and is more variable, depending to a greater degree upon temperature and consistency.

Effect of Consistency. In general a somewhat stronger concrete will be secured when mixt fairly dry, and thoroly tamped until moisture appears on the surface, than if more water is used. A wetter consistency is found to give better results in practise and is necessary for reinforced work. Wet concrete will show a lower strength than dry concrete, especially for the earlier periods, but the difference becomes less with lapse of time, and a fairly soft, plastic concrete will acquire about the same strength as dry concrete within three or four months. A very wet concrete will, however, continue to be somewhat weaker than one containing less water, and while such a concrete may, on the whole, be desirable, its deficiency in strength as compared to maximum values should not be overlookt.

The following average results of a large number of tests made for G. W. Rafter (Tests of Metals, 1898) show the relative strengths of dry, plastic, and wet concrete at the age of about twenty months. The dry mixtures were only a little more moist than damp earth and required much ramming; the plastic mixtures required a moderate amount of ramming to bring water to the surface; the wet mixtures quaked like liver under moderate ramming. Five brands of cement were used.

Consistency	Mean Compressive Strength
Dry	2348 lbs per sq in
Plastic	2203 lbs per sq in
Wet	2129 lbs per sq in

The following results were obtained by tests made at the University of Wisconsin on cylinders of 1 : 2 : 4 limestone concrete.

Consistency	Percent water by weight	Compressive Strength, lb per sq in		
		14 days	70 days	350 days
Dry	6	1774	2635	4000
Quaking	7	1945	3126	4320
"Mushy"	8	1709	2927	4500
"Soupy"	10	1283	2578	3070

Effect of Size of Stone. With equally good grading of size, the actual size of stone has little if any effect upon the strength of the concrete. Generally, however, a small size is less well graded, and grades less favorably with the sand than a larger size, and therefore gives somewhat less strength. To insure maximum strength the maximum size of stone should be as large as convenient to use. The tests quoted in the next paragraph show a marked effect of size of stone for the sizes less than 1 inch.

Tests by Wm. B. Fuller relative to effect of size of stone gave the following results on 1 : 9 concrete (Trans. Am. Soc. C. E., vol. 59, 1907):

Maximum Size of Stone, inches	Average Modulus of Rupture at 90 days.		Average Compressive Strength at 140 days	
	Lbs per sq in	Ratio to Max	Lbs per sq in	Ratio to Max
2¼	252	1.00	1391	1.00
1	229	0.91	1153	0.83
¼	189	0.75	1008	0.72

Gravel vs. Broken Stone. Tests generally show that broken stone makes a somewhat stronger concrete than gravel of the same proportions. Tests by E. Candlot using gravel with 40% voids, and broken stone with 47.4% voids and of 1½-in maximum size, showed an average excess of crushing strength of 20% for the broken stone over the gravel concrete for 1-month and 6-months tests. At the end of the year the average difference was 9%.

Tests at the Watertown Arsenal gave the following results, using broken stone and pebbles of a uniform size. The broken stone was trap rock and the proportions used were 1 : 1 : 3, the tests being made on 12-inch cubes. (Tests of Metals, 1898.)

Broken Stone				Gravel			
Size	Age, Days	Crushing Strength, lbs per sq in: Individual Tests	Crushing Strength, lbs per sq in: Average of Groups	Size	Age, Days	Crushing Strength, lbs per sq in: Individual Tests	Crushing Strength, lbs per sq in: Average of Groups
½-inch ...	7 19 32	1391 2220 2800	2137	⅝-inch ...	7 21 34	1298 2600 2992	2297
¾-inch ...	8 20 32	1900 2769 3200	2623				
1-inch ...	17 29 34	3390 4254 4917	4187				
1½-inch ..	11 26 41	3189 4006 4562	3919	1½-inch ..	7 22 29	2276 3186 3817	3093
2½-inch ..	7 22 32	2400 4143 4140	3561	3-inch	11 26 41	2800 3400 4200	3467

Effect of Strength of Aggregate. If the strength of the coarse aggregate is too low it will reduce the strength of the concrete, but whether this result follows depends also upon the richness of the mixture. The fracture of rich concretes is likely to be thru the stone, in which case a stronger stone will give greater strength, altho not in proportion to the strength of the stone. If the fracture occurs around the stone then no further increase in strength of stone will produce any effect. Since the strength of sound limestone is generally above 10 000 lbs per sq in, and granite and trap still higher, the strength of stone is usually ample and need not be considered. Sandstones average much lower, some being very soft and friable. Cinders are still weaker and give a weak concrete. Other material, such as broken brick, old concrete, etc., should be investigated relative to strength before being utilized.

Effect of Mica upon the Strength of Mortar. The presence of mica in sand is very objectionable, as it increases the voids and greatly reduces the strength of the mortar. Results of tests by W. A. Willis on 1 : 3 mortar, in which sand was used containing varying percentages of mica, are given in Fig. 16. (Eng. News, Feb. 6, 1908.)

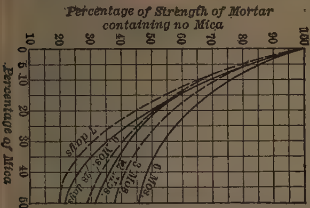

Fig. 16. Effect of Mica on Tensile Strength of Mortar

Strength of Plain Concrete Columns. The strength of concrete in the form of short columns, of lengths up to 10 or 15 diameters, is from 10 to 20% less than that of short prismatic specimens. For such ratios, however, the effect of length is so small that no law relative thereto is observable in the results of tests.

In the following table are given results of tests made at the Watertown Arsenal (Tests of Metals, 1904, 1905). All columns were 8 ft high and ranged from 10 in diameter to 12 in square. The age of the concrete ranged from 5 to 8 months.

Kind of Concrete	Crushing Strength, lbs per sq in	
	Results of Individual Tests*	Average Crushing Strength
1 : 1 mortar	5011+ 4320	4665
1 : 2 mortar	3652 2488	3070
1 : 3 mortar	2062 2692	2377
1 : 4 mortar	1564 1471 1050	1362
1 : 5 mortar	1038 1082	1060
1 : 1 : 2 (pebbles)	1525 1720	1622
1 : 1 : 2 (trap rock)	3900	3900
1 : 2 : 4 (pebbles)	1506 1710	1608
1 : 2 : 4 (trap rock)	1750 1990 1413	1718
1 : 3 : 6 (pebbles)	462 700 1260	807
1 : 3 : 6 (trap rock)	1350 750 1446	1182
1 : 2 : 4 (cinders)	871	871
1 : 3 : 6 (cinders)	1060 698	879

* Where two lines of values are given, those in the first line are results obtained in the 1904 series, those in the second line are from the 1905 series.

A. N. Talbot obtained the results given in the adjacent table on columns 12 in in diameter and 10 ft long (Bul. Univ. of Ill., No. 20, 1907).

Kind of Concrete	No. of Tests	Average Age, days	Average Strength, pounds per sq in
1 : 1½ : 3	2	64	2300
1 : 2 : 4	7	65	1740
1 : 2 : 4	6	192	2025
1 : 2 : 3¾	2	420	2710
1 : 3 : 6	2	61	1030
1 : 4 : 8	2	63	575

Tests by M. O. Withey gave the results in the table below (Bul. Univ. of Wis., 1909):

Size of Column		Kind of Concrete	No. of Tests	Average Strength, pounds per sq in	Compressive Strength of Small Compression Specimen, pounds per sq in
Length	Cross-sec.				
10 ft	144 sq in	1 : 2 : 4	4	2070	2360
8½ "	118 "	1 : 2 : 4	1	1880	2230
8½ "	86.6 "	1 : 2 : 4	3	2600	2410

Tensile Strength. Tensile tests of concrete are difficult to make. The specimen should be as large as practicable and made in the form of a prism with gradually reduced section at the center. The specimen may be held by means of rods embedded therein, which should be centrally placed and provided with link joints so as to avoid bending stresses.

The tensile strength is generally from one-tenth to one-twelfth of the compressive strength, but this ratio varies considerably. The character of the material and workmanship has probably a greater influence upon the tensile strength than upon the compressive. The tensile strength of well-made concrete is about as follows:

1 : 2 : 4 concrete .... 175 to 250 lbs per sq in
1 : 3 : 6 concrete .... 125 to 200 lbs per sq in

Tests by W. H. Henby gave the results shown in the adjacent table (Jour. Assn. Eng. Soc., Sept., 1900):

Mixture	Compressive Strength, lbs per sq in	Tensile Strength, lbs per sq in
1 : 2 : 4	3000	180
1 : 3 : 6	1800	115

Tests by W. K. Hatt gave the results shown in the table below (Jour. Assn. Eng. Soc., Sept., 1900):

Kind of Concrete	Age, days	Compressive Strength, lbs per sq in	Tensile Strength, lbs per sq in
1 : 2 : 4 (broken stone)........	30	....	311
1 : 2 : 5 " "	90	2413	359
1 : 2 : 5 " "	28	2290	237
1 : 5 (gravel)...............	90	2804	290
1 : 5 "	28	2400	253

Tests by Ira H. Woolsen on 1 : 2 : 4 mixtures 5 to 7 weeks old gave an average tensile strength of 161 lbs per sq in, compared to 1753 lbs per sq in compressive strength.

A. N. Talbot obtained values for 1 : 3 : 6 concrete from 50 to 84 days old of 178, 160, and 170 lbs per sq in. (Bul. Univ. of Ill. No. 1, 1904.)

Transverse Strength. For transverse tests the specimens should be as large as 6 in × 6 in in section in order to secure homogeneous and representative material. The modulus of rupture determined in a transverse test will be nearly twice the strength of the concrete in tension.

Average results of a large series of tests by Wm. B. Fuller on 6-in × 6-in beams 33 to 35 days old are shown in the left-hand table on the following page. Average results of a series of tests made by H. Von Schon, on beams 6 in × 6 in in cross-section, 60

days old, made from 1-in to 1½-in stone are shown in the right-hand table below. (Trans. Am. Soc. C. E., Vol. 42, 1899.)

Mixture (by weight)	Modulus of Rupture, lbs per sq in
1 : 2 : 4	439
1 : 2 : 5	380
1 : 3 : 5	285
1 : 3 : 6	226
1 : 3 : 7	239

Mixture	Average Modulus of Rupture, lbs per sq in	
	Sandstone Aggregate	Granite Aggregate
1 : 2.4 : 5.3	176	340
1 : 2.4 : 4.8	200	353
1 : 2.4 : 4.4	275	385
1 : 2.4 : 4.0	320	402

Results of tests by the U. S. Geolog. Survey on beams 6 in × 6 in in section are given below. The concrete was of medium consistency, and each result is the average of three tests. (Bul. No. 344, U. S. G. S., 1908.)

Aggregate	Modulus of Rupture, pounds per sq in		
	One Month	Three Months	Six Months
Granite.............	475	536	566
Gravel.............	451	477	520
Limestone..........	458	541	566

Shearing Strength. The shearing strength of concrete as stated by various authors and experimenters varies widely, due mainly to variation in methods employed and in the use of the term itself. It will here be used to denote the strength of the material against a sliding failure when tested as a rivet or bolt would be tested for shear; that is, when the maximum shearing stresses are confined to a single plane.

Tests made under the direction of C. M. Spofford on cylinders 5 inches in diameter with ends securely clamped in cylindrical bearings gave results as follows:

Mixture	Shearing Strength, lbs per sq in	Compressive Strength, lbs per sq in	Ratio of Shearing to Comp. Strength
1 : 2 : 4	1480	2350	.63
1 : 3 : 5	1180	1330	.89
1 : 3 : 6	1150	1110	1.04

Tests made at the University of Illinois on rectangular specimens tested in a similar manner gave the following average results:

Mixture	Shearing Strength, lbs per sq in	Compressive Strength, lbs per sq in	Ratio of Shearing to Comp. Strength
1 : 2 : 4	1418	3210	.44
1 : 3 : 6	1250	2290	.57

Tests made by punching thru plates gave shearing strengths varying from 37 to 90% of the compressive, the value depending upon the form of the test-piece. (Bul. No. 8, Univ. of Ill., 1906.)

Tests by M. Feret on mortar prisms gave results for shearing strength equal to about one-half the crushing strength. Considering that the ordinary crushing failure is really a failure by shearing, a theoretical consideration of the question indicates that the shearing strength per unit area is approximately equal to one-half the crushing strength as usually stated. It may then be concluded, both from theory and from tests, that the shearing strength of concrete, in the sense here used, is nearly one-half the crushing strength. It is in fact so large that it will need to be considered only in exceptional cases.

"Shearing stress" and "shearing strength" are used by some authors with reference to the combined shearing and tensile action which occurs in the web of a beam. In the case of a material like concrete, which is relatively weak in tension, failure of web occurs by tension, at values about equal to the tensile strength of the material. If this is considered as a shearing failure, then the shearing strength is about the same as the tensile strength determined by a transverse test. It is on this basis that shearing strength is frequently given as equal to but 1½ to 2 times the tensile strength.

Compressive Strength of Natural-Cement Concrete. The compressive strength of natural-cement concrete of various proportions is given approximately by Fig. 15. The values given represent average results.

Strength of Cinder Concrete. The following table of results indicates fairly well the strength of cinder concrete. The age of the specimens varied from 30 to 100 days. (Tests of Metals, 1898.)

Mixture			Average Crushing Strength, lbs per sq in	
Cement	Sand	Cinders	One Month	Three Months
1	1	3	1540	2050
1	2	3	1098	1634
1	2	4	904	1325
1	2	5	724	1094
1	3	6	529	788

11. Elastic Properties of Concrete

The Elastic Limit. Concrete like other non-homogeneous materials does not possess such perfect elasticity under low stresses as do steel and many other metals. In a compressive test small permanent deformations or set will occur at comparatively low loads, and the stress-deformation diagram will be slightly curved almost from the beginning as represented in Fig. 17. Unless the load be excessive a repetition of the load will cause the material to become more perfectly elastic and the subsequent diagram will approach a straight line up to the amount of stress which is repeated.

The elastic limit of concrete is, for the reasons above mentioned, not well defined. In the usual sense the material has no well-defined elastic limit. There appears to be, however, a limit to the stress which can be repeated indefinitely without continuing to add to the deformation, and this limit may be taken as the elastic limit for practical purposes. From experiments by Bach and others, this limit appears to be from 50 to 60% of the ultimate strength.

Modulus of Elasticity in Compression. Since the material is not per-ectly elastic the modulus of elasticity is somewhat indefinite. However, or most purposes, the modulus is used in connection with total deformations from the initial unloaded condition, and it is therefore desirable to calculate its value on the same basis. On this basis it is equal to the stress per unit area at any given load, divided by the total deformation per unit length for such load. Graphically the modulus for stress *DB* (Fig. 17) is the ratio of the stress *DB* to the deformation *OD*. If the permanent set *OC* is first deducted from the elongation *OD* the resulting modulus will be considerably higher.

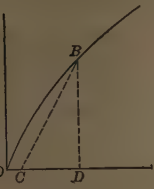

Fig. 17

The value of the modulus of concrete varies greatly, as determined by different experimenters and for different kinds of concrete. As a rule the denser and older the concrete the higher the modulus. For working loads the modulus of ordi-ary concrete ranges from 2 500 000 to 3 500 000 lbs per sq in, depending upon the aixture and age of the concrete.

Among the most careful experiments are those by Bach. (Zeit. V. dt. Ing., 1895.) The ollowing are some average results: The specimens were 25 cm in diameter and 100 cm igh, and were from three to four months old.

Kind of Concrete	Modulus of Elasticity, lbs per sq in		
	Based on Elastic Deformation		Based on Total Deformation
	At 114 lbs per sq in	At 570 lbs per sq in	At 570 lbs per sq in
1 : 2½ : 5 (broken stone)	4 660 000	3 590 000	3 440 000
1 : 2½ : 5 (gravel)	3 170 000	2 520 000	2 200 000
1 : 3 : 6 (broken stone)	3 870 000	2 990 000	2 570 000
1 : 3 : 6 (gravel)	3 000 000	2 240 000	2 110 000

Average values obtained in tests made at the Watertown Arsenal (1899) were as ollows: These results were calculated by using the total deformation minus the set. If ie total deformation be used the values would be reduced in most cases 10 to 20 %.

Kind of Concrete	Modulus of Elasticity between Loads of 100 and 600 lbs per sq in, based on Elastic Deformation		
	7 to 10 Days	1 Month	3 Months
1 : 2 : 4	2 090 000	2 620 000	3 650 000
1 : 3 : 6	1 970 000	2 590 000	3 210 000

The Modulus of Elasticity of Cinder Concrete has been determined at 2 540 000 bs per sq in for 1 : 1 : 3 concrete and 1 400 000 lbs per sq in for 1 : 2 : 4 concrete.

Modulus of Elasticity in Tension. Bach found for 1 : 4 concrete an verage value of the modulus of 3 800 000 lbs per sq in at a stress of 80 lbs er sq in, and 3 100 000 at a stress of 135 lbs per sq in. The ultimate tensile trength was 185 lbs per sq in. The modulus in compression for the same oncrete was 3 850 000 at 80 lbs per sq in. (Mitt. über Forsch. a. d. Gebeit. es Ing., 1907, Heft 54–47.)

W. K. Hatt determined values from 2 000 000 to 5 000 000 lbs per sq in, which were generally about equal to the values in compression. (Proc. Am. Soc. Test. Mat., 1902.) These and other tests indicate that the initial moduli in tension and in compression are about the same, and as the working limit in tension is very low they may be assumed as equal.

Coefficient of Expansion. Experiments by W. D. Pence (Jour. West. Soc. Eng., Vol. VI, 1901, p. 549) on 1 : 2 : 4 concrete gave an average value of the coefficient of expansion of 0.0000055 per degree Fahrenheit, there being little variation among the several tests. Tests made at Columbia University on 1 : 3 : 6 concrete gave values of about 0.0000065. Other experiments give somewhat higher results. A value of 0.000006 may be assumed.

Contraction and Expansion in Hardening. Results of experiments show that generally when mortar or concrete is hardened in air there will be more or less shrinkage, but when hardened in water there is likely to be some swelling, altho the results in this respect are not entirely consistent. The richer the concrete, the greater the change in dimensions. The shrinking appears to be due primarily to the drying out of the concrete, and nearly the same degree of shrinkage occurs when a concrete hardened in water is dried out. The amount of shrinkage has been found to be from 0.02 to 0.04% for 1 : 2 : 4 concrete.

12. Preparation and Storage of Materials

Storage of Cement In order to allow ample time for inspection and testing it is necessary on all important work to store the cement for a period of ten or twelve days. Storehouses must be weather-tight and have tight floors placed well above the ground. Each carload should be stored separately so as to permit convenient access for sampling, counting of packages and removal. If kept dry, cement will not be injured by long storage, on the contrary, a period of storage diminishes the danger of unsoundness.

Screening of Sand and Gravel may be done by hand or by machinery. Hand screening is adapted to small jobs and light work as, for example, where a small amount of gravel is to be screened out of sand. Screening of gravel, or of sand containing large amounts of coarse material, can be done much more cheaply by mechanical means, using either fixt screens placed on an incline or revolving screens. The fixt screen is suited to locations where the natural topography supplies the necessary elevation and where power is not conveniently had. Revolving screens are the most efficient and economical for large quantities. The material is conveyed to the screens by bucket elevators, and from the screens it passes to bins provided with convenient gates for loading. Slopes for bins should be made about 45°.

Washing of Sand or Gravel may be necessary. Gravel is sometimes coated with a film of clay which if not removed will greatly reduce the strength of the concrete. Sand having over 5% of clay or loam should generally be washt.

A method of washing, suitable for small quantities, is to shovel the sand into a sloping V-shaped box and wash by means of a hose. The clean sand may be drawn off at the bottom by means of a gate. For large quantities of materials a combination of ejector and sloping trough has been used successfully for washing filter sand at Yonkers, N. Y. As high as 10% of silt and clay, together with a large amount of coarse gravel, was removed in one operation using a trough 54 feet long. Water was forced in at the lower end thru three 3-inch pipes supplied by a centrifugal pump. The washer handled

200 cu yds per 10 hrs (Eng. Record, vol. 59, 1909, p. 805). The cost of washing, including handling, will range from 10c to 25c per cu yd.

Stone Crushing. Crushers are of two kinds, the jaw crusher and the gyratory crusher. The former is better adapted to small or portable plants, while the latter is generally used in large stationary plants. A convenient size of jaw crusher for a semi-portable plant is about 10 by 16 in. This will crush from 50 to 100 cu yd per day, depending upon the character of the stone and size desired. Revolving screens for broken stone are made in sections from 3 to 5 ft long, and of a diameter of 2 to 4 ft. For concrete, only two sizes are needed, the maximum size and a ¼-in screen to remove the dust.

The Cost of Quarrying and Crushing varies greatly with the nature of the stone and the size desired. A. J. Noyes gives the cost of quarrying and crushing hard trap rock as 90c per cu yd, not including interest on plant. At the same place the cost of crushing trap cobble stones was 45 cents per cu yd. The cost of quarrying and crushing limestone will generally range from 50 to 75c per cu yd, including plant charges.

The Cost of Hauling Broken Stone may be estimated on the basis of 2 to 2½ cu yd per load for paved streets, and 1 to 1½ cu yd for country roads with no steep hills. A team will travel 2½ miles per hour. About 5 min per load is required for loading and the same for dumping. An estimated rate of 2 miles net per hour will approximately cover delays at both ends.

Water used in making concrete should be free from oil, acid, strong alkalies or large amounts of vegetable matter.

13. Mixing of Concrete

Measurement of Materials. Cement is measured by counting the bags, assuming a standard volume per bag or barrel. The standard recommended is 3.8 cu ft per bbl or 0.95 cu ft per bag. Sand and stone are very commonly measured in barrows, but this method needs careful watching to maintain uniform conditions. The barrows should be relatively deep and of uniform size. Their capacities should be determined in cu ft by actual measurement of the heaped-up contents in a barrel or shallow bottomless box. The cement being then measured by the bag, the results are likely to be satisfactory, as the most important element (the measurement of cement) is guarded. Where accurate results are desired, and especially with three or more sizes of aggregate, the use of a bottomless box 8 to 10 in deep is recommended.

Hand Mixing. As the strength of the concrete is very largely dependent upon the thoroness of mixing, much care is needed in this part of the work. Concrete may be mixt as thoroly by hand as by machine, but the process is laborious and expensive and such results are not likely to be secured unless special and constant attention is given to the work.

In hand mixing the sand and cement should be thoroly mixt dry, fine sand requiring much more work than coarse. After this is completed the mixing in of water and stone may be accomplished in various ways, generally the water being partially mixt with the sand and cement before the stone is added. After the stone is added the entire mixture should be turned not less than three times, and an improvement in quality will result by further turning. Care is required to prevent separation of cement by too rapid flushing. The mixing platform should be water-tight. Thoro mixing will produce a more plastic concrete with a given amount of water than inadequate mixing.

Quantities of Material for One Cubic Yard of Compacted Concrete

Based on 3.8 cu ft per bbl of cement. Sand and stone measured loose

Proportions by Volume			Ratio: Mortar/Stone	Quantities of Material		
Cement	Sand	Stone		Cement, Bbls	Sand, Cu Yds	Stone, Cu Yds
1	1	1½	0.98	3.10	0.44	0.65
		2	0.72	2.75	0.39	0.78
		2½	0.58	2.48	0.35	0.88
		3	0.48	2.25	0.32	0.95
		3½	0.42	2.05	0.29	1.01
1	1½	2	0.92	2.40	0.51	0.68
		2½	0.73	2.20	0.47	0.78
		3	0.61	2.00	0.42	0.85
		3½	0.53	1.85	0.39	0.91
		4	0.46	1.72	0.36	0.97
1	2	3	0.74	1.85	0.52	0.78
		3½	0.63	1.72	0.49	0.85
		4	0.55	1.60	0.45	0.90
		4½	0.49	1.48	0.42	0.94
		5	0.44	1.39	0.39	0.98
1	2½	3½	0.75	1.60	0.56	0.79
		4	0.66	1.48	0.52	0.83
		4½	0.58	1.38	0.49	0.87
		5	0.52	1.30	0.46	0.91
		5½	0.47	1.22	0.43	0.95
		6	0.44	1.17	0.41	0.99
1	3	4	0.75	1.40	0.59	0.79
		4½	0.67	1.30	0.55	0.82
		5	0.60	1.22	0.52	0.86
		5½	0.55	1.16	0.49	0.90
		6	0.50	1.10	0.46	0.93
		6½	0.46	1.04	0.44	0.95
		7	0.43	1.00	0.42	0.99
1	4	5	0.78	1.10	0.62	0.77
		6	0.65	1.00	0.56	0.84
		7	0.55	0.92	0.52	0.91
		8	0.48	0.85	0.48	0.96
		9	0.43	0.80	0.45	1.01
1	5	9	0.52	0.73	0.51	0.93
		10	0.47	0.68	0.48	0.96
		11	0.42	0.64	0.45	1.00
1	6	10	0.55	0.63	0.53	0.89
		12	0.45	0.58	0.49	0.98

Specification for Hand Mixing. Tight platforms shall be provided of sufficient size to accommodate men and materials for the progressive and rapid mixing of at least two batches of concrete at the same time. Batches shall not exceed one cubic yard each, and smaller batches are preferable, based upon a multiple of the number of sacks of cement to the barrel.

Spread the sand evenly upon the platform, then the cement upon the sand and mix thoroly until of an even color. Add all the water necessary to make a thin mortar and spread again; add the gravel if used, and finally the broken stone, both of which if dry, should first be thoroly wet down. Turn the mass with shovels or hoes until thoroly incorporated and all the gravel and stone covered with mortar; this will probably require the mass to be turned four times.

Another approved method, which may be permitted at the option of the engineer in charge, is to spread the sand, then the cement, and mix dry, then the gravel or broken stone; add water and mix thoroly as above. (Am. Ry. Eng. & M. of W. Assn.)

Machine Mixing. Machine mixing is greatly to be preferred to hand mixing and should generally be required. Efficient and portable mixers of various sizes are so readily obtainable and operated that hand mixing need seldom be resorted to. In machine mixing all the materials, including the water, are generally introduced at once without intermediate mixing, but better results are likely to be secured, even with machine mixers, by first mixing the materials dry. It is desirable to regulate closely the amount of water per batch, as better results can thus be secured, and, with machine mixing, this can very readily be done.

Tests of the relative strength of concrete mixt by hand and by machine, made at the University of Wisconsin by M. O. Withey, showed a marked superiority of the machine-mixt over the most thoro hand mixing, this difference amounting in many cases to 25% and reaching as high as 50%.

Machine Mixers are of two general types, (1) the batch mixer and (2) the continuous mixer. In the batch mixer the proper amounts of material for a single " batch " of concrete are placed in the mixer and the contents are then mixt either by means of moving paddles or blades, or by the rotation of the receptacle itself, in which are generally placed deflectors to aid in the mixing. In the continuous mixer the operation is continuous, more or less adequate provision being made for maintaining the proper proportions of materials.

The gravity mixer is a type of continuous mixer into which the material is introduced at the top and is mixt by striking various obstructions or deflectors in its descent. Continuous power mixers utilize some form of screw or paddle blades.

While good results may be secured by either type, the correct proportions are more readily secured in the batch mixer and it is generally to be preferred especially for small plants where the supervision is likely to be inadequate.

The former practise of using concrete of dry consistency and then tamping thoroly has generally given place to the use of concrete of wet or " mushy " consistency which requires little or no tamping. Use of excessive water should be carefully avoided, as the strength and density of concrete rapidly falls off when the consistency is reduced below that which will enable the concrete to flow sluggishly in the forms. Segregation of the material is also likely to occur if the concrete is too wet. Dry concrete is advantageous for first layers in wet excavations.

14. Transporting and Placing Concrete

Transporting Concrete to Place should proceed promptly, otherwise there is likely to be some separation of stone from the mortar, especially if made very wet. Danger from setting is not likely. For long distances a tramway or cable-way can be economically used. In buildings where the concrete is elevated and wheeled on level ways a cart holding 5 or 6 cu ft is convenient and economical. For large masses of concrete such as piers and abutments a method of placing successfully used is to lift the concrete by vertical elevators to a height considerably above the working surface and thence to distribute it thru an inclined chute.

Tamping and Puddling. Dry concrete should be tamped in layers of 6 to 8 in in thickness. Iron tampers weighing about 30 to 40 lbs and having a surface area of 40 to 50 sq in are commonly used. Wet or mushy concrete

requires no ramming, but a small amount of cutting or spading with shovel or other suitable tool to remove air bubbles and to bring the concrete into all corners of the forms and produce a homogeneous mass. To give a smooth surface finish the tool should be worked up and down along the forms. A common spade is a good tool, but in narrow walls a straight strip of steel mounted on a long handle, or a scantling sharpened to an edge, is satisfactory.

Cost of Mixing and Placing Concrete. The labor cost of hand mixing and placing soft concrete, requiring spreading but not ramming, when the material is convenient and is handled in barrows, will range from $1.00 to $1.25 per cu yd. On large jobs machine mixing will save from 20 to 30 cents per cu yd. Wheeling long distances adds about 5 cents per cu yd for each additional 100 ft. Ramming of very dry concrete may cost as high as 30 cents per cu yd; ordinary ramming costs about 15 cents per cu yd.

For large quantities of concrete the materials may be handled more economically by mechanical means, using bucket elevators or belt conveyors for sand and stone and measuring the material in hoppers filled from bins. The measured material may be carried in cars to the mixer and the mixt concrete conveyed in cars or by derrick or elevator to the work.

Placing Concrete in Freezing Weather should be avoided if practicable, but good work can be done if proper precautions are observed. Various means are used to secure this end. The essential requirement is to remove the frost from the material and to prevent freezing until the concrete is deposited in place. Subsequent freezing does not apparently injure portland-cement concrete except as to surface finish. Structures in which the surface finish is important must not be permitted to freeze until the concrete is well set.

Freezing of concrete can be prevented or sufficiently delayed by warming the materials or by adding salt to the water, or by both means combined. A steam plant enables the material to be conveniently warmed either directly by the steam or by means of steam coils. The latter is preferable for sand, as it leaves the material in a dry condition. Direct heating of sand and stone is conveniently done by means of a heater consisting of heavy sheet iron bent to a half cylinder and braced. This is placed upon the ground and the fire built near one end. A piece of large sheet-iron pipe forms also a convenient heater.

To reduce the freezing temperature by adding salt requires an amount equal to about 1% of the weight of water for each degree F.

In massive walls the rise of temperature due to chemical action in setting will be sufficient to enable the concrete in the interior to set before freezing, so that such walls will acquire considerable strength during cold weather. Thin walls if allowed to freeze will gain very little strength during continued cold and must be loaded with caution.

Concrete may sometimes be prevented from freezing over night by a covering of straw, manure or similar material. A temporary housing is sometimes employed on important work, but such means increase considerably the cost of the work.

Protection of Concrete from too Rapid Drying is important, especially in the case of narrow sections and reinforced work. Keeping the concrete wet prevents to a large degree contraction in hardening. Specifications relative to this point of the New York Board of Water Supply are as follows:

Every precaution shall be taken to prevent concrete from drying until it shall have become so thoroly set and hardened that there can be no danger of cracking from lack of moisture. . . . Concrete shall be kept moist for at least two weeks after the removal of the forms, or until covered with earth, unless otherwise directed.

Placing Concrete under Water can be successfully accomplished where there is little or no current. The contact of the concrete with the water does not affect the interior of the mass deposited but does wash out some of the cement at and near the surface. This detrimental effect should be reduced

to a minimum by depositing the concrete in large quantities at a time, and by a minimum amount of disturbance of the concrete after depositing and of the water adjacent thereto. The concrete should be of such consistency as to settle into place by its own weight. This method of placing concrete should be used only for relatively massive work and where great strength is not required. Porous and imperfect concrete may be expected around the margin of the work.

In placing the concrete it may be deposited either from a box or bucket opened at the bottom or side, or thru a large tube (12 in to 24 in in diameter), called a *tremie*, resting on the deposited concrete below and extending above the water level. The tube is kept full of concrete, which is allowed to escape at the bottom by raising and shifting the tube along the working surface. Much care is required to secure a steady movement of the concrete. Concrete may also be deposited in bags of cloth or paper. Sometimes very large bags are employed. If loosely filled they will flatten out after placing and will cement together more or less.

Laitance. Wherever the surface of concrete is exposed to the action of water a portion of the cement will be washt out, thus weakening the concrete at that point. Accompanying this action there appears also to be a disintegration of some of the cement, forming a whitish gelatinous substance called "*laitance*" which, while of about the same composition as the cement, has little or no hardening properties. Its presence on the surface of or adjacent to deposited concrete weakens the bond between old and new material. When practicable, laitance should be removed before placing fresh concrete.

Joining of Old and New Concrete. In joining fresh concrete to concrete which has become hard, or to old masonry, the old surface should be thoroly cleaned of all loose material, dirt, or laitance. A rough surface is desirable. If special strength of bond is required the surface should be picked rough or cleaned with dilute acid and water and then should be coated with a thick coat of rich cement mortar. For impervious walls, a tongue and groove joint should be made by setting temporary pieces of timber into the wall at the end of the day's work. Permanent tongues of sheet steel have been successfully used for such joints, extending 6 or 8 in into the wall on each side. The older and drier the concrete the weaker the joint with the new.

Contraction Joints should be provided in long walls in order that shrinkage from hardening and temperature changes shall not produce unsightly cracks. These are generally made by merely forming a smooth vertical face on the wall, against which new concrete is not placed until after the old has set. This forms a plane of weakness which opens up more readily than cracks will form elsewhere. In massive work, such as retaining walls, abutments, etc., built without reinforcement, joints should be provided approximately every 50 feet thruout the length of the structure. To provide against the structure being thrown out of line by unequal settlement, each section of the wall may be tongued and grooved into the adjoining section.

Contraction joints are difficult to make in work designed to be impervious. A common method used in the construction of reservoir floors is to use asphalt filling in narrow joints placed at intervals in the concrete. This is not very successful. An effective method suitable especially for vertical walls is the use of sheet lead folded at the joint as shown in Fig. 18 so as to permit longitudinal movement. To provide against unsightly cracks, due to unequal settlement, a joint should be made at all sharp angles.

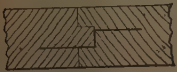

Fig. 18

Surface Finish of Concrete should be made upon the solid material itself. Plaster coats are not generally durable and are not to be recommended. Marking concrete to resemble stone is unartistic and unsatisfactory.

The untreated surface of concrete will show the marks of forms even if they

are planed and grooved, but with reasonably close work, and care in spading the concrete next to the forms, the resulting appearance is quite satisfactory for massive structures. For finer work, or for structures to be viewed at close range, some special treatment of the surface is desirable.

Various methods of finishing are used. One method is to hammer or pick the surface before it has become hard so as to give it a uniformly rough appearance. With a pneumatic tool a laborer can cover 50 to 60 sq ft per day. For fine work the use of a fine aggregate, dressed with a stone hammer, gives good results but is more expensive than the use of the pick.

Another method successfully used on surfaces accessible to a trowel is to fill against the forms with a thin layer of very dry mortar, made of coarse granite or gravel screenings. The forms are removed as soon as possible and the surface then brushed with a stiff brush. This removes some of the cement and leaves a rough surface free from marks of forms. The same general effect is also obtained by dissolving out some of the cement by applying to the surface a wash of dilute sulfuric acid and rubbing with a stiff brush. This is then followed up with an alkaline wash and finally washt with water.

The following is a specification for a rubbed finish, satisfactorily used in factory construction (Eng. Record, Dec. 28, 1907):

After the forms are removed, the concrete shall be thoroly wet with a brush and then rubbed with a coarse carborundum stone, No. 16, bringing the surface to a lather. After this stone has been used sufficiently to take off the rough projections, the lather shall be washt off with a brush and the concrete again wet, and then dusted with a mixture of dry sand and cement, the proportions being one part of cement to two parts of sand. This shall be rubbed into the surface with the coarse No. 16 stone. Care shall be taken not to allow any of the mortar to remain on the surface. To give the final finish, a No. 30 carborundum stone shall be used and the whole surface well rubbed.

Moldings and ornamental pieces may often be poured of thin mortar, using accurately finished molds, or casting in sand.

Rubble Concrete is concrete in which are embedded stones of large size, handled and placed separately. Where such stone is readily obtainable, as frequently is the case in dam construction, a considerable saving can be effected over the cost of ordinary concrete; under ordinary conditions the saving will be small if any. This construction is suitable only for massive work where the walls are not less than three or four feet thick. The joints should be at least four inches thick and thoroly filled with wet concrete, or the stone sunk well into the soft concrete. The size of stone is limited only by ability to transport and the width of wall.

15. Imperviousness

Impervious Concrete. Concrete may be rendered practically impervious but success requires much care. Four general methods are employed.

Use of Rich Concrete. The concrete itself can be made of such density as to be practically impervious under moderate pressures. For this purpose a rich mixture should be used of such proportion of sand and stone as to produce the maximum density. The concrete should be of wet consistency and well spaded or puddled. Thoro mixing and curing are essential to good results. Proportions of 1 : 2 : 4 will generally give an impervious mixture, and, with well graded aggregates, a somewhat leaner concrete can be used. Cracks must be prevented and an effective bond made between successive day's work. Good results may be secured by the use of a thin steel plate 4 to 6 ins

wide embedded in the concrete at the junction of successive day's work. This method of securing impervious concrete is generally satisfactory where conditions can be easily controlled, and where cracks are not likely to form.

Use of Waterproofing Materials. Waterproofing by the use of bituminous material is frequently employed for floors and walls of underground structures, and for bridges and retaining walls where imperviousness is desired. Bituminous mastic, or asphalt, may be used alone for horizontal or slightly inclined surfaces; but generally it is applied with alternate layers of roofing felt or burlap to secure it in place and to make a continuous unbroken layer over the entire surface. If subject to disturbance or wear the asphaltic coat should be placed between layers of concrete. From three to six layers of felt were used in waterproofing the New York Subway.

Where a hard wearing surface must be made impervious, a form of bitumen emulsion mixt with cement mortar has been successfully used. Brick laid in asphaltic mortar is also frequently used.

Mixing Foreign Matter with the Concrete. Various materials may be mixt with the concrete to reduce its porosity. There are many patented materials on the market for this purpose which possess more or less merit. Hydrated lime is a good material to employ for this purpose, and clay will improve a lean mortar. Tests by S. E. Thompson showed that a water-tight concrete can be made by the use of the following proportions of dry hydrated lime, based on the weight of the dry cement (Am. Soc. Testing Materials, 1908):

For 1 : 2 : 4 concrete.............. 8% hydrated lime,
For 1 : 2½ : 5 concrete.............12% hydrated lime,
For 1 : 3 : 6 concrete.............16% hydrated lime.

The permeability of an 8-inch wall under a pressure of 60 lbs per sq in is shown in Fig. 19. Tests quoted in Art. 10 show that the strength of the concrete is little affected by the use of such proportions of lime.

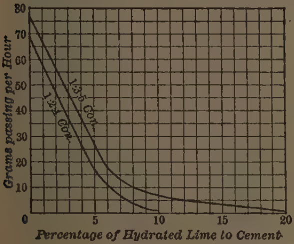

Fig. 19. Effect of Lime on Permeability of Cement

Alum and soap, used either as a wash or mixt with the concrete, will aid in rendering concrete impervious. The proper proportions are 2.2 parts soap to 1 part alum. This mixture may be dissolved in the water used in mixing up to 2 or 3%. While the concrete is rendered more impervious by this process, it is also somewhat weakened. In the Sylvester process alternate washes of alum and soap solutions are applied to the dry surface of the concrete, the soap solution being hot. Each is allowed to dry before the next coat is applied. By applying several pairs of coats very good results may be obtained.

Plastering with Rich Mortar is effective, provided cracking can be prevented. This is very difficult to accomplish unless the plaster is applied to the concrete before it is set, as the variation in temperature and moisture between concrete and plaster is almost certain to cause a separation. Structures which will be permanently submerged or moist may be plastered.

16. Durability of Concrete

Effect of Fire. Severe fire tests show that when concrete is subjected to the temperature of red-hot iron (about 1700° F.) for three or four hours and then is quenched by hose streams, it is likely to show pitting, but that it will still offer a sufficient protection to steel encased by it. In the Baltimore fire of 1904, the value of concrete as a fire-proofing material was fully demonstrated. C. L. Norton of the Insurance Engineering Experiment Station, after a careful study of the damage done by the fire, states as follows: "Where concrete floor arches and concrete steel construction received the full force of the fire it appears to have stood well, distinctly better than the terra-cotta." (Eng. Record, June 2, 1904.) The reason for this he considers to be the fact that terra-cotta expands about twice as much as steel, while concrete expands about the same amount. Little difference was observed between stone and cinder concrete.

In a report of a committee of members of the American Society of Civil Engineers on the effects of fire in the San Francisco conflagration of 1906, similar conclusions were reached as to the value of concrete as a fire-proofing material. It was also found preferable to tile for floors. With respect to the injury to the concrete itself the committee was of the opinion that it was sufficient in many cases to require reconstruction. (Trans. Am. Soc. C. E., vol. 59, 1907.)

Effect of Acids and Oils. Concrete of first-class quality thoroly hardened is affected appreciably only by strong acids which seriously injure other materials. When concrete is properly made and the surface carefully finished and hardened it resists the action of petroleum and ordinary engine oils.

Effect of Sea Water. Sea water frequently has a disintegrating effect upon concrete, but the exact conditions which control and modify this action are not clearly understood. The sulphates of lime and magnesia appear to be the main cause of trouble, but the presence of magnesium chloride seems to accelerate the action. For best results, the concrete should be dense and impervious and allowed to harden as much as practicable before being immersed. Concrete continuously submerged is less affected than where alternately submerged and exposed. Long-time tests and observations indicate that the best concrete is apt to be somewhat affected in ten to fifteen years, but that under favorable conditions no serious effect may result in forty to fifty years. The use of fine sand is particularly objectionable for concrete of this character. If seepage occurs thru the concrete, disintegration is likely to be rapid.

Effect of Alkali. Many instances have been observed where concrete located so as to be partly submerged in alkali water has disintegrated near the water line. Sodium salts do not appear to have any deleterious effect, the disintegration being generally due to the presence of sulphates, as in sea water. Sometimes in sewage tanks the formation of sulphuric acid from the hydrogen sulphide is sufficient to affect the concrete. The use of a sand containing alkali should be avoided.

17. Forms for Concrete

General Requirements. Forms for concrete should be strong and rigid and should be tight enough to prevent leaking of the mortar. The material should be planed on one side and be of even thickness if a smooth face is desired. This is generally advisable for convenience in handling. Material which warps readily, such as hemlock or oak, is not desirable. Pine or spruce is commonly the best available material. If forms are to be used repeatedly, oiling will do much to prevent the absorption of water and shrinkage and

will enable them to be removed more readily. For repeated use in built-up sections, tongued and grooved material is to be preferred, or the forms may be covered with galvanized sheet steel. The latter is preferable where a smooth finish is desired.

Pressure of Concrete. The pressure of wet concrete, such as is frequently used, will be the full hydraulic pressure of a liquid weighing about 150 lb per cu ft, until initial setting begins. If the forms are to be filled before this occurs they must be designed for the full pressure. If setting begins while the work is in progress, then the maximum pressure will be less than the full hydrostatic pressure, this maximum limit depending upon the rapidity of filling and rate of setting of the concrete. It will therefore depend largely upon the existing temperature. Results of experiments made by F. R. Shunk on the maximum pressure of concrete are shown in Fig. 20. This diagram gives the maximum pressure which occurred for various rates of filling and at various temperatures. The tests were made by means of a pressure board 9.23 inches in diameter, set into the side of a form supporting a large mass of concrete. The concrete was made of 1 : 3 : 5½ proportions and was very wet, the workmen sinking into it about 18 inches. (Eng. News, Sept. 9, 1909.)

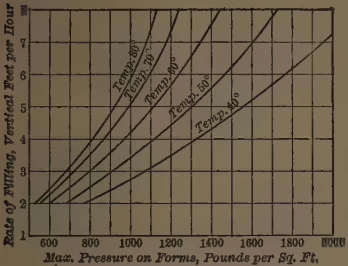

Fig. 20. Pressure of Concrete

Design of Forms. Either 1-inch or 2-inch stuff is suitable for lagging, but, excepting for small parts, 2-inch is preferable. One-inch stuff requires supports spaced 18 to 24 in apart and 2-inch stuff a spacing of 4 to 5 ft. Studding or joists should be 2 in by 4 in to 2 in by 5 in for 1-inch lagging, and 4 in by 6 in up to 4 in by 10 in for 2-inch lagging. Forms for opposite faces of walls of ordinary thickness should be connected together by ties designed to take the full pressure. Thus connected, the outside bracing need be only sufficient to steady the forms as a whole while the concrete is being placed. Forms are conveniently connected by wire twisted into a sort of turn-buckle for adjustment. Iron rods fitted with nuts are more readily adjusted. These may be past thru small pipes cut slightly shorter than the thickness of the wall, which are afterwards filled with cement.

Fig. 21. Form for Coping of Wall

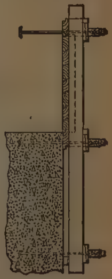

Fig. 22. Sectional Form for Massive Walls

Where no cross ties are used the entire pressure must be borne by outside props placed, generally, as diagonal struts. Great pains must be taken to secure the ends of such struts from settlement. For light work, forms may be

made up into sections of convenient size for handling. Where the vertical rate of construction is small such forms may be supported entirely by the concrete already in place. An arrangement of this sort adapted to the construction of a coping is shown in Fig. 21. For high massive walls, the forms are conveniently fastened to the concrete itself, projecting upwards in the form of a cantilever as illustrated in Fig. 22.

Removal of Forms. In the case of vertical walls constructed in warm weather, forms can generally be removed the second day after the placing of the concrete. Cool weather will require an additional day or more of hardening. Concrete to support loads requires a much longer time to harden, and the removal of forms should be entrusted only to skilled supervision. For reinforced work this is of special importance.

18. Concrete Blocks

Use of Concrete Blocks. Blocks of molded concrete are well adapted to the construction of walls that are relatively thin or that sustain only light loads, such as building walls, walls between reinforced framework, partitions and the like. For such purposes solid concrete is not so well adapted on account of the expense of forms, the difficulty of securing a satisfactory finish and of preventing the formation of unsightly shrinkage cracks. The concrete blocks are generally made of such shape as to form a wall containing hollow spaces, thus increasing the stability and giving a warmer and drier wall than one of solid concrete. In general concrete blocks may be used as a substitute for ordinary brick or stone masonry in building construction.

Forms of Concrete Blocks. Figs. 23, 24, 25 illustrate three common forms of hollow blocks. Fig. 24 is known as a two-piece block. In laying

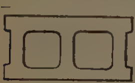

Fig. 23

Fig. 24

Fig. 25

Three Forms of Concrete Blocks

it, successive courses break joints, thus bringing the heavy central web of an outside block directly over the web of the inside block below, giving a good bond to the wall. In Fig. 25 a double air space is provided for the purpose of more thoroly preventing the penetration of moisture. To produce a more pleasing appearance to the wall the blocks are frequently made of several sizes but generally arranged in some systematic manner.

Cement Brick are sometimes used as a substitute for clay brick. They are made of about the same proportions of material as concrete hollow blocks.

Processes of Manufacture. A great variety of molding machines are in use for the manufacture of concrete blocks. They vary with respect to the method employed for compacting the concrete and the consistency of the material used. In one process the material is mixt to a damp consistency and is tamped into the molds either by hand or by pneumatic or mechanical tamper. Other machines use a somewhat wetter mixture and apply pressure mechanically. A third process consists in pouring wet concrete into molds. In the first two processes the material is sufficiently dry to enable the mold to be removed immediately from the block, but in the last process the mold cannot be removed until the concrete is set. If the tamping method is em-

ployed, care must be taken to secure a sufficient amount of compacting and to avoid the use of too dry a mixture. The ease with which a good-appearing block can be made with dry material and little tamping has led to much poor work by this method. Thoroly satisfactory results may however be obtained.

Casting blocks in sand molds is also used to some extent. It is expensive, but an excellent finish can be produced by this process, and it is well suited to the molding of ornamental parts.

Materials Employed. The materials for block construction should be selected and mixt according to the same principles as are applied in making ordinary concrete. A strong and fairly impervious material is desired, to secure which requires a proportion of sand not to exceed 2½ or 3 parts. The coarse aggregate should be of clean broken stone or gravel screened thru a ¾-inch screen and retained on a ¼-inch screen. From 3 to 4 parts of coarse aggregate may be used without decreasing the strength of the product. Frequently the coarse aggregate is omitted and as many as 5 parts of sand used. This is bad practise and incorrect in principle; the omission of the coarse aggregate should not permit the use of increased quantities of fine material unless the latter contains a considerable proportion of large particles.

Surface Finish. The surface is finished in a variety of ways. Frequently a facing layer of fine material is used next to the face mold, this becoming intimately bonded with the body of the block in the process of filling. Special effort is made to render the face coat impervious. Variety in color may be secured by the use of stone screenings of the desired color. For massive work the body concrete makes a satisfactory finish without a special face coat. The form of the surface should be determined as a part of the work of the architect. Walls of concrete blocks are susceptible of very satisfactory architectural treatment, but the common pitch face finish with blocks of uniform size is very unsatisfactory.

Specifications for Concrete Blocks. The following abstracts from the specifications adopted by the National Association of Cement Users cover the most important features of the manufacture and testing of concrete blocks, and represent good practise:

Proportions. For exposed exterior or bearing walls: (a) Concrete hollow blocks, machine made, using a semi-wet concrete mortar, shall contain one part cement to not exceeding three parts sand and to not exceeding four parts stone, of character and size before stipulated. When the stone is omitted, the proportion of sand shall not be increased unless it can be demonstrated in each case that the percentage of voids and tests of absorption and strength allow greater proportions with equally good results. (b) When said blocks are made of slush concrete in individual molds and allowed to harden undisturbed in same before removal, the proportions may be one part cement to not exceeding three parts sand and five parts stone, but in this case also, if the stone be omitted, the proportion of sand shall not be increased, except as specified in (a).

Molding. Due care shall be used to secure density and uniformity in the blocks by tamping or other suitable means of compression. Tamped blocks shall not be finished by simply striking off with a straight-edge, but, after striking off, the top surfaces shall be troweled or otherwise finished to secure density and a sharp and true arris.

Curing. Every precaution shall be taken to prevent the drying out of the blocks during their initial set and first hardening. A sufficiency of water shall first be used in the mixing to perfect the crystallization of the cement, and, after molding, the blocks shall be carefully protected from wind-currents, sunlight, dry heat or freezing, for at least five days, during which time additional moisture shall be supplied by approved methods, and occasionally thereafter until ready for use.

Ageing. Concrete hollow blocks in which the ratio of cement to sand is one-third (one part cement to three parts sand) shall not be used in the construction of any building until they have attained the age of not less than three weeks. Concrete hollow blocks in which the ratio of cement to sand is one-half (one part cement and two parts sand)

may be used in construction at the age of two weeks, with the special consent of the Bureau of Building Inspection and the architect or engineer in charge. Special blocks of rich composition, required for closures, may be used at the age of seven days with the special consent of the same authorities. The time herein named is conditional, however upon maintaining proper conditions of exposure during the curing period.

Thickness of Walls. The thickness of bearing walls for any building where concrete hollow blocks are used may be 10% less than is required by law for brick walls. For curtain walls or partition walls the requirements shall be the same as in the use of hollow tile, terra-cotta or plaster blocks.

Party Walls. Hollow concrete blocks shall not be permitted in the construction of party walls, except when filled solid.

Limit of Loading. No wall, nor any part thereof, composed of concrete hollow blocks, shall be loaded to an excess of eight tons per superficial foot of the area of such blocks, including the weight of the wall, and no blocks shall be used in bearing walls that have an average crushing strength less than 1000 lb per sq in of area, at the age of 28 days; no deduction to be made in figuring the area for the hollow spaces.

Girders or Joists. Wherever girders or joists rest upon walls so that there is a concentrated load on the block of over two tons, the blocks supporting the girder or joists must be made solid for at least 8 in from the inside face. Where such concentrated load shall exceed 5 tons, the blocks for at least three courses below, and for a distance extending at least 18 in each side of said girder, shall be made solid for at least 8 in from the inside face. Wherever walls are decreased in thickness, the top course of the thicker wall shall afford a full solid bearing for the webs or walls of the course of blocks above.

Sills and Lintels. Concrete sills and lintels shall be reinforced by iron or steel rods in a manner satisfactory to the Bureau of Building Inspection, or the architect or engineer in charge, and any lintels spanning over 4 ft 6 in shall rest on block solid for at least 8 in from the face next the opening and for at least three courses below the bottom of the lintel.

Hollow Space. The hollow space in building blocks, used in bearing walls, shall not exceed the percentage given in the following table for different height walls, and in no case shall the walls or webs of the block be less in thickness than one-fourth their height, except that the Department of Buildings, architect or engineer may specially approve thinner construction after having past the prescribed tests. The figures given in the table represent the percentage of such hollow space for different height walls.

Stories	1st	2nd	3rd	4th	5th	6th
1 and 2	33	33	..	..	..	..
3 and 4	25	33	33	33	..	..
5 and 6	20	25	25	33	33	33

Test Requirements. Concrete hollow blocks must be subjected to the following tests: transverse, compression and absorption; and may be subjected to freezing and fire tests; but the expense of conducting the freezing and fire tests will not be imposed upon the manufacturer of said blocks. The test samples must represent the ordinary commercial product, of the regular size and shape used in construction. The samples may be tested as soon as desired by the applicant, but in no case later than 60 days after manufacture. In calculating results no deduction is to be made for the hollow spaces in the concrete blocks.

Transverse Test: The modulus of rupture for concrete blocks at 28 days must average 150 lbs, and must not fall below 100 lbs in any case.

Compression Test: The ultimate compressive test at 28 days must average 1000 lbs per sq in, and must not fall below 700 lbs in any case.

Absorption Test: The percentage of absorption (being weight of water absorbed divided by the weight of the dry sample) must not average higher than 15%, and must not exceed 22% in any case.

Cement Brick. Cement brick may be used as a substitute for clay brick. They shall be made of one part cement to not exceeding four parts clean sharp sand, or one part cement to not exceeding three parts clean sharp sand and three parts broken stone or gravel passing the ½-in and refused by the ¾-in mesh sieve. In all other respects cement brick must conform to the requirements of the foregoing specifications.

REINFORCED CONCRETE BUILDINGS

19. Qualities of Concrete

Reinforced Concrete is concrete in which iron, steel or other metal is mbedded in such a manner as greatly to increase its strength, especially with espect to tensile stresses.

The Quality of Concrete for reinforced work should generally be of elatively high grade. In this form of construction the strength of the material s much more important than in many forms of plain concrete construction. t is especially important that the concrete be uniform in quality and free from oids, as the stability of the structure is dependent upon the integrity of every art. Thoroly sound concrete is also required in order to insure good adesion to the reinforcement and thoro protection of the metal from corrosion nd from fire. This requires great care in the preparation and placing of the naterial. Portland cement only should be used and should be carefully pecified. In this form of construction the rapidity of hardening of the ement requires special attention.

The sand should be clean and preferably of a coarse grade. A fine sand equires more cement than a coarse sand for equal strength and more water or a like consistency. It is very important that regular and systematic ests of the material as actually used be made during the progress of the work.

The maximum desirable size of stone or gravel depends upon the thickness f the sections and the size and spacing of the reinforcement. It is desirable o use as large a size of aggregate as will admit of convenient working. Maxmum sizes of ¾ inch to 1¼ inches are common, but on heavy work with rods videly spaced there is no objection to the use of somewhat larger sizes.

Proportions of Ingredients. The proportions commonly used vary from bout 1 : 1½ : 3 to 1 : 3 : 6. The use of the latter proportions requires areful grading of the material to produce satisfactory results. Occasionally vhere great strength is desired a mortar or concrete of richer proportions than he one first mentioned is desirable. Customary proportions, such as 1 : 2 : 4, hould not be blindly adopted. In any important work a careful study of he materials and of the best proportions to use for economy and strength vill be well repaid. There must be no unfilled voids in the stone and few r none in the sand, but the former is of more importance than the latter.

Consistency. In reinforced concrete work reliability is more important han maximum strength, and is promoted by using concrete of such consistncy that it can readily be worked into place in the forms and around the einforcement. Dry concrete is not satisfactory. In practise the consistency aries from that which will involve considerable tamping and working to that vhich will enable the concrete to flow into place.

Strength. Various data on the strength of concrete are given in Art. 16 or the usual proportions employed. With reference to reinforced work, it generally desirable to use a concrete having a strength of not less than 800 to 2000 lb per sq in at 60 days. This will be obtained ordinarily by the se of a 1 : 2 : 4 or 1 : 2½ : 5 mixture, or their equivalent. Where the usual roportions give lower results than those named, it will generally be advisable o use a richer concrete rather than reduce the working stresses.

Elastic Properties. In a design of reinforced concrete it is necessary o know the value of the modulus of elasticity in compression and the relation f stress to deformation at various loads. Considering the various results

given in Art. 11, the value of E for ordinary concrete and for working loads will range from 2 000 000 to 3 000 000 lb per sq in, depending upon the mixture and the age of the concrete. For use in calculations relating to strength, a value of 2 000 000 is generally assumed and on some accounts is more satisfactory than a higher value.

The Form of the Stress Deformation Diagram which expresses the relation between stress and deformation is shown in Fig. 17 as a curved line. In theoretical analyses involving high stresses, it may be assumed to be a parabola with vertex at the origin.

Weight of Reinforced Concrete. The weight of concrete of the usual proportions will generally vary from 145 to 150 lb per cu ft, depending upon the degree of compactness and the specific gravity of the materials. The addition of reinforcing steel in the usual proportions will add from 3 to 5 pounds, so that the weight of reinforced concrete may be taken at 150 to 155 lb per cu ft.

Elongation of Concrete when Reinforced. Results of early experiments by Considère indicated that concrete on the tension side of a beam elongates much more before final rupture occurs than when not reinforced, and that the resistance of the concrete is nearly constant and at its maximum value for some time previous to rupture. Later experiments made at the University of Wisconsin and confirmed by Bach have shown, however, that concrete on the tension side of a beam will begin to crack at about the same deformation as plain concrete. Very fine cracks will occur at elongations of 0.00006 to 0.0001 part, corresponding to a stress in the adjacent steel of 2000 to 3000 lb per sq in.

Tests under repeated loads made at the University of Pennsylvania, have shown that these cracks will become plainly visible and will gradually extend towards the neutral axis under loads producing stresses of 10 000 to 18 000 lb per sq in in the steel.

These results show that the tensile strength of the concrete should not be considered in any calculations where the stress in the adjacent steel is to exceed above 2000 to 3000 lb per sq in.

Relative Contraction and Expansion of Concrete and Steel. The contraction of mortar and concrete due to hardening is given in Art. 11. Some experiments indicate that the contraction of reinforced concrete is considerably less. Considère observed a shrinkage in 1 : 6 mortar, reinforced with 5½% of steel, of only 0.01%. In practise it is found that unless thoroly reinforced, concrete will shrink sufficiently to develop unsightly cracks which cannot be permitted in work designed to be impervious. Temperature changes affect both the steel and the concrete. The coefficient of expansion of steel is about 0.0000065 and of concrete about 0.000006. The relative change is therefore small and the two materials will be but slightly strest because of any difference in rate of expansion.

20. Reinforcing Material

Reinforcing Steel must be of such form and size as can be readily incorporated into the concrete so as to make a monolithic structure. Generally the reinforcement is used in the form of rods or bars, varying in size from about ¼ to ⅜ in for thin floors, up to 1½ to 2 in as a maximum size for heavy beams or columns. A riveted skeleton of bars and shapes is also employed and is particularly advantageous where it is desired to have the steel work self-supporting. Whatever form may be used, the general requirement of adequate strength of adhesion and good distribution of stress must be met.

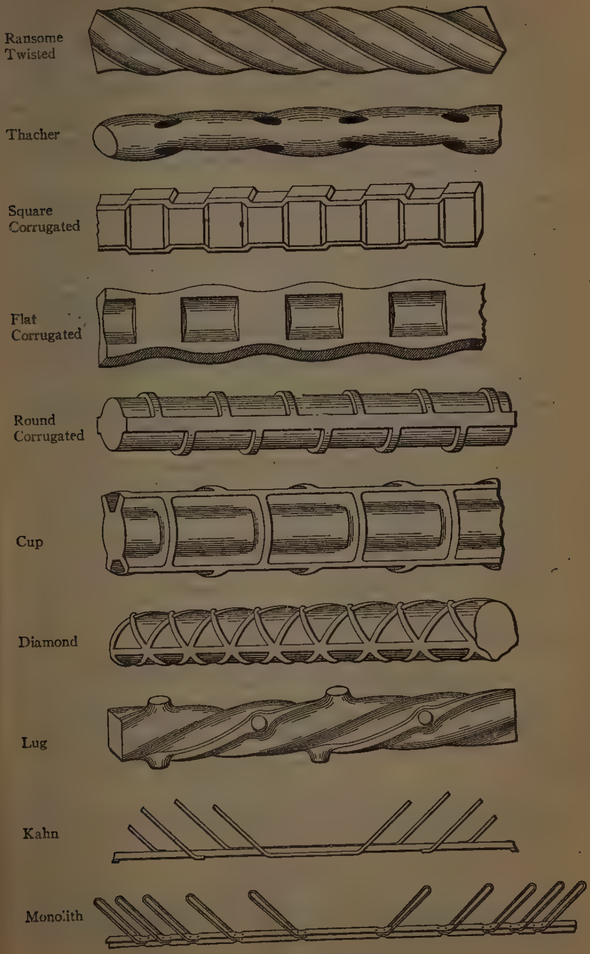

Fig. 26. Ten Forms of Reinforcing Bars

Forms of Bars. Plain round and square rods are largely used, the adhesion of the steel and concrete being depended upon to furnish the necessary bond strength. Plain flat bars are undesirable unless used in connection with riveted reinforcement, as their adhesion to the concrete is much less than that of round or square bars. Many special forms of bars have been devised, the principal object of which is to furnish a bond with the concrete independent of adhesion, a mechanical bond as it is usually called. Some of the most common types of such bars are illustrated in Fig. 26. Many other devices are employed to a greater or less extent to provide a mechanical bond, and numerous combinations of forms are used as patented " systems " in the construction of beams, floors, and columns.

Quality of Steel. Steel bars used in reinforced concrete are not usually subjected to as severe treatment as ordinary structural steel. They must, however, be capable of being bent cold to the desired form. These conditions lead to the use by many engineers of high elastic limit material, while other engineers, probably the majority, prefer the use of standard structural material. The Joint Committee on Reinforced Concrete recommends material complying with the specifications of the American Society for Testing Materials for billet steel of structural grade. The American Society for Testing Materials specifications also provide for two other grades, intermediate and hard; the latter having a yield point strength of 50 000 lbs per sq in. These specifications are as follows:

Standard Specifications. Steel may be made by the Bessemer or open-hearth process. The bars should be rolled from new billets. No re-rolled material will be accepted.

Chemical and Physical Requirements

Phosphorus	Bessemer	Maximum 0.10%
	Open-hearth	Maximum 0.05%

Tensile Properties

Properties Considered	Plain Bars			Deformed Bars			Cold Twisted Bars
	Structural Steel Grade	Intermediate Grade	Hard Grade	Structural Steel Grade	Intermediate Grade	Hard Grade	
Tensile strength, lb per sq in	55 000 to 70 000	70 000 to 85 000	80 000 min.	55 000 to 70 000	70 000 to 85 000	80 000 min.	Recorded only
Yield point, min., lb per sq in	33 000	40 000	50 000	33 000	40 000	50 000	55 000
Elongation in 8 in, min., percent [1]	1 400 000 / Ten. str.	1 300 000 / Ten. str.	1 200 000 / Ten. str.	1 250 000 / Ten. str	1 125 000 / Ten str.	1 000 000 / Ten. str.	5

[1] See modification for thickness below.

The yield point shall be determined by the drop of the beam of the testing machine.

For plain and deformed bars over ¾ in in thickness or diameter, a deduction of 1 from the percentages of elongation specified shall be made for each increase of ⅛ in in thickness or diameter above ¾ in. For plain and deformed bars under 7/16 in in thick-

ness or diameter, a deduction of 1 from the percentages of elongation specified shall be made for each decrease of 1/16 in in thickness ordiameter below 7/16 in.

Bend-test Requirements. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion, as follows:

Thickness or Diameter of Bar	Plain Bars			Deformed Bars			Cold Twisted Bars
	Structural Steel Grade	Intermediate Grade	Hard Grade	Structural Steel Grade	Intermediate Grade	Hard Grade	
Under ¾ in.....	180° $d=t$	180° $d=2t$	180° $d=3t$	180° $d=t$	180° $d=3t$	180° $d=4t$	180° $d=2t$
¾ in or over....	180° $d=t$	90° $d=2t$	90° $d=3t$	180° $d=2t$	90° $d=3t$	90° $d=4t$	180° $d=3t$

Explanatory Note.—d = the diameter of pin about which the specimen is bent; t = the thickness or diameter of the specimen.

21. Adhesion of Concrete and Steel Reinforcement

The Adhesion or Bond Strength between the concrete and steel rods embedded therein is, in the sense here used, the resistance which such rods offer to longitudinal motion. It may be called the Tangential Adhesion but is generally known as the Bond Strength. The adhesive strength is largely frictional resistance and varies greatly with the roughness of the bars. It also varies with the quality of the concrete and the method of conducting the test. Usually the test is made by embedding the rod in a block of concrete and pulling it therefrom, the rod being stressed in tension and the concrete in compression. It has been found, however, in a series of tests by M. O. Withey, that the local compression to which the concrete in the ordinary block specimen is subjected tends materially to increase the bond strength, so that the results from the usual tests are considerably higher than those made directly on beams.

Results of Tests. Results of numerous tests show that for ordinary round or square bars, not too smooth, the bond strength varies from 200 by 300 lb per sq in, depending upon richness of mixture, age of cement, and roughness of bar, with a frictional resistance of about two-thirds this amount; a much smaller value is shown for very smooth bars and also for flat bars. The maximum bond resistance does not develop until a small amount of slip (about .01 in) has taken place.

The table on p. 532 contains in condensed form the results of some of the most important tests made by direct tension.

Individual results show little or no effect due to differences in size of rod, but the adhesion of flat bars is much less than that of round or square bars. In general the stronger the concrete the greater the bond strength. The effect of consistency within ordinary limits is not great, but a small amount of corrosion of the steel tends to increase the value. Tests by Withey in which the rods were arranged as in a beam gave results considerably below those obtained in usual tension tests. Average results for 1 : 2 : 4 concrete 60 days old are as follows (Eng. Record, vol. 57, 1908):

Diam. of rod =	⅜	½	⅝	¾	1 in
No. of tests =	3	3	9	3	3
Bond strength =	278	286	256	276	163 lb per sq in

Tests of Bond Strength

Plain bars; concrete 1 : 2 : 4; 60 to 90 days old

Authority	Reinforcement		Depth Embedded	Bond Resistance, lb per sq in		
	Kind	Size in		At slip of .0005 in	At slip of .001 in	Maximum
Withey: Bul. Univ. of Wis., No. 175, 1907.	Round	3/16 to 3/4	6 in	....	....	400
			8 in	....	....	310
Van Ornum: Eng. News, Vol. LIX, 1908, p. 142.	Round	1/2 to 1 1/4	25 dia.	....	....	410
			40 dia.	....	....	390
Abrams: Bul. No. 71, Univ. of Ill., 1913.	Round	1/2	8 in	323	339	381
		5/8	8 "	266	295	405
		3/4	8 "	275	303	387
		1	8 "	247	281	385
		1 1/4	8 "	269	296	397
	Flat	1/2×1	6 "	359	395	459
		1/4×2	4 "	239	263	293
	Round polished	1	5 "	149		152
		3/4	5 "	137	146	160
		3/4	6 "	170	192	255

Bach found average values of about 290 lbs per sq in on beams 6 months old. Similar results have been obtained by other experiments.

Effect of Water. Tests by H. C. Berry on specimens immersed in water for 20 to 24 months showed no weakening of bond resulting therefrom. The bond improves somewhat with age.

Frictional Resistance. In bond tests it is found that after the adhesion has failed, the rod still offers much resistance to movement due to friction alone. This frictional resistance varies from 50% to about 80% of the initial bond strength. Hatt found a frictional resistance, after starting, of 50 to 70% of the initial strength, and Moersch reports such resistance as about two-thirds the initial. Abrams found the average value for round rods embedded 8 in to be two-thirds the bond strength at a slip of 0.1 in.

Mechanical Bond. The ultimate bond strength of bars with indented surfaces is very high, but to develop high resistance requires a considerable slip. Up to a slip of about 0.01 n the action of deformed bars is about the same as plain bars, but for increased slip the resistance continues to increase so that for a slip of 0.1 in the resistance is about double that at 0.01 in and double the maximum for plain bars. For twisted bars the resistance increases only slightly beyond a slip of 0.01 in.

Efficiency of Hooked Ends. Bach has found that the initial slip of smooth bars is but slightly retarded by bending into a short right-angle bend at the end of the rod, but the ultimate bond strength is increased about 50%. These experiments were made on rods 3/4 to 1 inch in diameter and with an embedded length of 20 inches. When hooked ends are used they should consist of bends of 180° with a short length of straight rod beyond the bend. Such hooks increase the bond strength about 100%.

22. General Principles of Design

Use and Advantages of Reinforced Concrete. Steel is a material especially well suited to resist tensile stresses, and for such purposes the most economical form, the solid compact bar, is well adapted. A serious disadvantage in the use of steel in many locations is its lack of durability, thus rendering it necessary to add a protective covering to prevent corrosion and injury from fire. Concrete is characterized by low tensile strength, relatively high compressive strength, and great durability. It is a good fire-proof material, and therefore serves as a good fire-proof covering for steel. A combination of steel and concrete constitutes a form of construction possessing in a large degree the advantages of both materials without their disadvantages.

For those structural members carrying purely tensile stresses steel must be employed, but it may be surrounded by concrete as a protection against corrosion and fire, or for the sake of appearance. For large and compact compression members plain concrete may be used. For more slender members, however, such as long columns, plain concrete is too brittle a material, and therefore too much affected by secondary and unknown stresses to be satisfactory; and for such members steel alone, or the two materials in combination, will preferably be used. For those structural forms in which both tension and compression exist, that is to say, in all forms of beams, the combination of the two materials is particularly advantageous. In this form the tensile stresses are carried by the steel and compressive stresses by the concrete.

Some of the most important types of construction in which reinforced concrete can be advantageously employed are the following: — in buildings, for floors or for the complete structure; in foundations, especially where broad footings are required; in culverts and small beam bridges; in retaining walls, dams and abutments; in arch bridges; in bins and tanks for coal, grain and other material; in conduits and pipe lines subjected to low pressures; in chimneys and towers; and in separate structural forms such as piles, railroad ties, poles, etc.

General Assumptions in Calculations. In calculations of stresses and sections of reinforced concrete structures the following general principles are usually followed:

(1) Calculations are made with reference to working stresses and safe loads rather than with reference to ultimate strength and ultimate loads, altho some prefer to calculate ultimate loads and then apply a safety factor.

(2) Perfect adhesion is assumed between concrete and reinforcement. Under compressive stresses the two materials are therefore strest in proportion to their moduli of elasticity. Under tensile stresses this relation also holds up to the ultimate strength of the concrete.

(3) Inasmuch as the extensibility of concrete is small, its tensile resistance is usually neglected and the entire tensile stress is assumed to be taken by the reinforcement. Where the stresses are very small, as frequently occurs in arches, the resistance of the concrete is sometimes considered. Certain beam formulas also take account of the tensile resistance of the concrete.

(4) Under working stresses the modulus of elasticity of concrete in compression is constant and the variation of compressive stress on a section of a beam is therefore rectilineal. The variation of tensile stress is also rectilineal and at the same rate. For ultimate loads a curvilinear law is usually assumed, the parabola being a convenient approximate curve to use.

(5) In beams, a plane section before bending remains plane after bending.

(6) Initial stress in the reinforcement due to contraction or expansion in the concrete is neglected.

(7) The ratio of the modulus of elasticity of the steel to that of the concrete is variously assumed at from 10 to 20. A value of 15 for working loads is commonly used. For deflection calculations 8 or 10 is to be preferred.

23. Theory of Reinforced Beams

General Arrangement of the Reinforcement. The purpose of steel reinforcement is to carry the principal tensile stresses, the concrete being depended upon for the compressive and direct shearing stresses. If no steel were present the concrete would tend to rupture on lines perpendicular to the direction of maximum tension, as shown in Fig. 27, and hence we may conclude that the ideal

Fig. 27

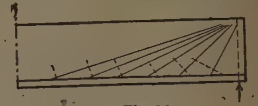

Fig. 28

tension reinforcement would require the steel to be distributed in the beam along the lines of maximum tension.

Fig. 28 shows by the dotted lines the lines along which failure of the concrete in a reinforced beam tends to occur. The inclined full lines show how the beam may be reinforced against such failures.

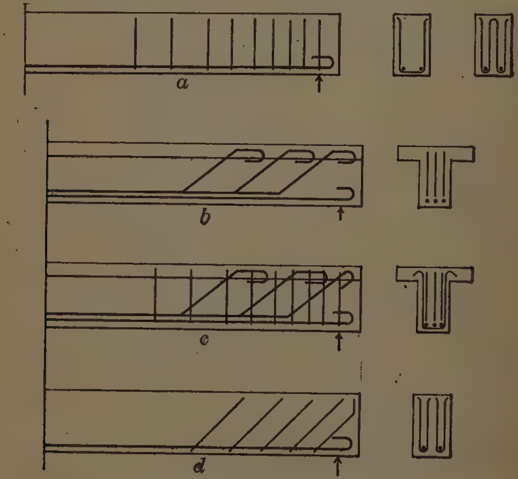

Fig. 29.—Methods of Shear Reinforcement.

Where the shearing stresses exceed about 50 lbs per sq in some form of shear reinforcement is necessary. Fig. 29 shows various methods of arranging such reinforcements. Figs. (*a*) and (*c*) show the use of vertical stirrups and Fig. (*d*) inclined stirrups Inclined stirrups must be firmly fastened to the horizontal rods. For maximum strength form (*c*) or (*d*) should be used.

Generally speaking, it is more economical to carry compressive stresses by concrete than by steel, but limitations as to size sometimes make it desirable to strengthen the compressive side of a beam. In cases, also, where both positiv and negativ moments exist in the same beam, either as alternating stresses or as simultaneous stresses at different points, steel reinforcement will be used on both sides of the beam, and its value on the compressive side needs to be known. The effectiveness of steel in compression has sometimes been questioned, but results of tests on beams and columns indicate that, in ordinary proportions at least, the steel does its share of work.

Varieties of Flexure Formulas. Many different flexure formulas have been proposed for beam calculations. These differ according to the different assumptions made with reference to the value of the tensile stress in the concrete and the law of variation of the compressive and tensile stresses on the cross-section. The principal varieties of these assumptions are represented in Fig. 30, in which the shaded areas represent the assumed stress variation in the concrete.

In Fig. (a) the rectilineal law is assumed and the tension in the concrete is neglected; this is the assumption generally used for working conditions. In (b) the parabolic curve is generally used; this theory is adapted to ultimate loads. In (c) the tensile stresses are considered for small deformations near

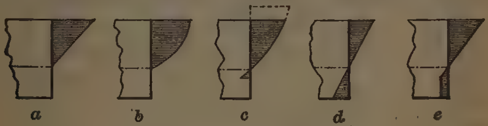

Fig. 30. Varieties of Flexure Theories

the neutral axis. In (d) the stresses vary as in an ordinary beam; this assumption is proper only for very small tensile stresses. In (e) the variation in tensile stresses accords with the theory of Considère, which assumes a constant resistance of the concrete for distortions beyond the ultimate distortion of plain concrete.

24. Flexure Formulas for Working Loads

Assumptions. The formulas below are based upon the assumptions of Fig. 30 (a). The rectilineal law of stress variation is assumed and the tension in the concrete is neglected. The notation used is as follows:

For Rectangular Beams:

S_s = tensile unit stress in steel. S_c = compressive unit stress in concrete.

E_s = modulus of elasticity of steel. E_c = modulus of elasticity of concrete.

M = moment of resistance, or bending moment in general.

M_c and M_s = moments of resistance with respect to the concrete and the steel.

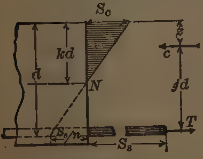

Fig. 31

A = steel area. b = breadth of beam.

d = depth of beam to center of steel.

k = ratio of depth of neutral axis to effective depth d.

z = depth of resultant compression below top.

j = ratio of lever arm of resisting couple to depth d.

$jd = d - z$ = arm of resisting couple.

$n = E_s/E_c$. $m = S_s/S_c$. $p = A/bd$.

For T Beams (in addition to preceding):

b = width of flange. b' = width of stem.

t = thickness of flange.

p = steel ratio based on circumscribing rectangle bd.

For Beams Reinforced for Compression (in addition to preceding):

A' = area of compressive steel. p' = steel ratio for compressive steel.
S_s' = compressive unit stress in steel.
C = total compressive stress in concrete. C' = total compressive stress in steel.
d' = depth to center of compressive steel. z = depth to resultant of C and C'.

Formulas for Rectangular Beams. (a) For investigating a given beam:

Position of neutral axis, $k = \sqrt{2\,pn + (pn)^2} - pn$.
Arm of resisting couple, $jd = d\,(1 - \tfrac{1}{3}k)$.
Resisting moments, $M_s = S_s A \cdot jd = S_s pj \cdot bd^2$; $M_c = \tfrac{1}{2}\,S_c kj \cdot bd^2$.

Fiber stresses, $S_s = \dfrac{M}{pj\,bd^2}$ $\quad S_c = \dfrac{2\,M}{jk\,bd^2}$ $\quad \dfrac{S_s}{S_c} = \dfrac{k}{2p} = m$

(b) For designing a beam under specified working stresses:

$$p = \frac{n}{2m(m+n)} \qquad bd^2 = \frac{(m+n)^2}{n\,(3m+2n)}\left(\frac{6\,M}{S_c}\right)$$

Diagram for Rectangular Beams. The values of k, j and M/bd^2 for any given value of p, or of S_c and S_s, are given by the diagram of Fig. 32.

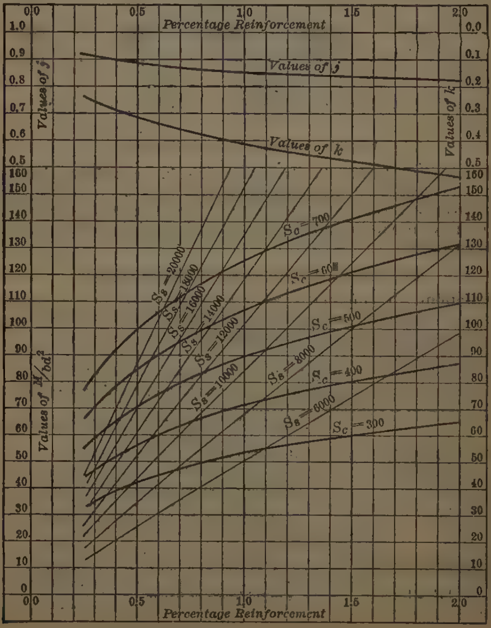

Fig. 32. Diagram for Reinforced Rectangular Beams. ($n=15$)

ormulas for T Beams. (Fig. 33.) CASE I, NEUTRAL AXIS IN THE FLANGE: e formulas for rectangular beams should be used.

ASE II, NEUTRAL AXIS IN THE STEM: The following formulas neglect compression in the stem: Position of neutral axis, $kd = \dfrac{2\,ndA + bt^2}{2\,nA + 2\,bt}$.

osition of resultant compression, $z = \dfrac{3\,kd - 2\,t}{2\,kd - t} \cdot \dfrac{t}{3}$.

rm of resisting couple, $jd = d - z$.

esisting moment, $M_s = S_s A jd \qquad M_c = S_c \dfrac{bt(kd - \frac{1}{2}t)\,jd}{kd}$

iber stresses, $S_s = \dfrac{M}{Ajd} \qquad S_c = \dfrac{Mkd}{bt\,(kd - \frac{1}{2}t)\,jd} \qquad \dfrac{S_c}{S_s} = \dfrac{k}{n\,(1-k)}$.

he following formulas take into account the compression in the stem; y are recommended where the flange is small compared with the stem.

osition of neutral axis,

$$kd = \sqrt{\frac{2\,ndA + (b-b')t^2}{b'} + \frac{nA + (b-b')t^2}{b'}} - \frac{nA + (b-b')t}{b'}.$$

osition of resultant compression, $z = kd - \dfrac{2}{3} \cdot \dfrac{b(kd)^3 - (b-b')(kd-t)^3}{b(kd)^2 - (b-b')(kd-t)^2}$;

rm of resisting couple, $jd = d - z$.

iber stresses, $S_s = \dfrac{M}{Ajd} \qquad S_c = \dfrac{2Mkd}{b(kd)^2 - (b-b')(kd-t)^2}$.

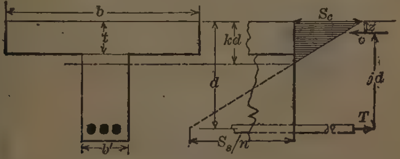

Fig. 33

Diagram for T Beams. The diagram, Fig. 34, gives the position of the itral axis for various ratios t/d and various steel ratios p. Corresponding ios of S_s/S_c are given along the right-hand margin.

Examples. (1) A T beam has the following dimensions: $b = 48$ in, $t = 4$ in, $d = 22$ $b' = 10$ in. The steel consists of six ¾-in rods. Assume $S_s = 15\,000$ and $S_c =$ lb per sq in, respectively. Find the resisting moment of the beam.

The steel area = 2.65 sq in and $p = 2.65/(48 \times 22) = 0.0025$. The ratio $t/d =$ 2 = 0.182. From Fig. 34, for $p = 0.25\%$ and $t/d = 0.182$ we find $k = 0.25$ and 0.93. Then $kd = 5.5$ in and $jd = 20.5$ in. The resisting moments with respect to steel and the concrete are

$$M_s = 15\,000 \times 2.65 \times 20.5 = 815\,000 \text{ in-lb, and}$$

$$M_c = 600 \times \frac{48 \times 4\,(5.5 - 2)\,20.5}{5.5} = 1\,510\,000 \text{ in-lb}$$

(2) Suppose that the diameter of the rods in example (1) is 1 in, and that the beam is jected to a bending moment of 1 250 000 in-lb. Compute the working stresses in steel and concrete.

In this case $A = 4.71$ sq. in and $p = 0.00445$. From the diagram, $k = 0.33$ and $j = 0.92$. Then $kd = 7.26$ in, $jd = 20.2$ in, and

$$S_s = 1\ 250\ 000 \div (20.2 \times 4.71) = 13\ 200 \text{ lb per sq in},$$

$$S_c = 13\ 200 \times \frac{0.33}{15\,(1 - 0.33)} = 434 \text{ lb per sq in}$$

Beams Reinforced for Compression. Position of neutral axis,

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p')$$

Position of resultant compression, $z = \dfrac{\frac{1}{3}k^3d + 2p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)}$.

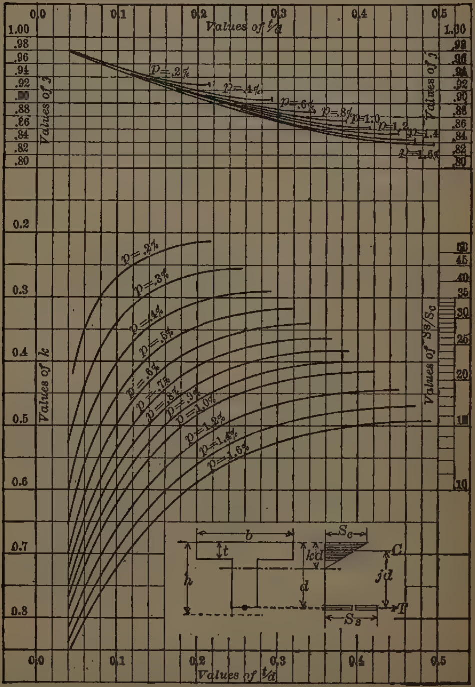

Fig. 34. Diagram for Reinforced T Beams. $(n = 15)$

Arm of resisting couple, $jd = d - z$.

Fiber stresses,

$$S_c = \frac{6M}{bd^2} \Big/ \left[3k - k^2 + \frac{6p'n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right]$$

nd

$$S_s = \frac{M}{pjbd^2} = nS_c\frac{1-k}{k} \qquad S_s' = nS_c\frac{k - d'/d}{k}$$

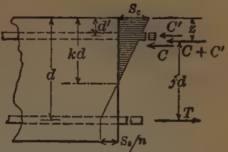

Fig. 35

Shear and Bond Stress. Let V = total shear, S_v = maximum shearing iit-stress, S_u = bond stress per unit area of bar, o = circumference or perieter of bar, Σo = sum of the perimeters of all bars. In the following rmulas Σo refers only to the bars constituting the tension reinforcement at ie section in question and jd is the lever arm of the resisting couple at the ction. For approximate results j may be taken at $\frac{7}{8}$.

For rectangular beams $S_v = V/bjd$.
For T beams $S_v = V/b'jd$.
For both rectangular and T beams $S_u = V/jd.\Sigma o$.

25. Test of Beams

Methods of Failure. A reinforced concrete beam tested to destruction ill usually fail in one of three ways: (a) By the yielding of the steel at or near ie section of maximum bending moment. (b) By the crushing of the con-

Tests of Reinforced T Beams

Number	Width of Flange, inches	Percentage Reinforcement	Number and Kind of ¾-inch Rods	Total Breaking Load, pounds	Maximum Shearing Stress on Section 8″×10″, lbs per sq in	Stress in Steel, lbs per sq in
1		1.05	3 Corrugated	46 700	345	64 300
4	16	1.10	4 Plain round	32 410	240	41 500
7		1.10	4 Plain round	30 100	222	38 100
3		0.93	4 Corrugated	55 700	410	57 500
6	24	0.92	5 Plain round (2 bars bent up)	39 300	290	40 700
8		0.92	5 Plain round (2 bars bent up)	40 100	295	41 200
2		1.05	6 Corrugated	80 500	592	55 700
5	32	1.05	6 Corrugated (2 bars bent up)	83 300	612	57 400
9		0.97	7 Plain round (3 bars bent up)	50 900	375	37 600

Concrete, 1 : 2 : 4, age about 60 days, compressive strength of cubes = 1820 lbs per in. Steel: yield point of plain round = 38 300 lb per sq in, of corrugated bars = 53 800 s per sq in. Size of beams: thickness of flange = 3¼ in, thickness of web = 8 in, depth center of steel = 10 in, total length = 11 ft, span length = 10 ft width of flange varied shown in the table. Stirrups: made of ½-in corrugated bars, five stirrups at each d, spaced 6 in apart. Loads applied at third points. All failures were steel tension ilures.

crete at the same place. (c) By a diagonal tension failure of the concrete at a place where the shear is large.

General Results of Tests. From the results of tests made by various experimenters the following general conclusions may be drawn:

(1) The elastic limit or, more strictly speaking, the yield point of the steel may safely be taken as its ultimate strength in reinforced beams.

(2) The crushing strength of concrete as determined by tests on cubes hardened under similar conditions will be fully realized in the beam.

(3) The usual shearing strength for beams reinforced with straight rods only is from 100 to 150 lbs per sq in, but this can readily be increased by the use of proper web reinforcement to 300 or 400 lbs per sq in.

(4) Steel used on the compressive side of beams is strest in accordance with the usual assumptions.

Tests of T Beams Showing Stresses in the Steel are given in the table. on p. 539. The table also contains information relative to the efficiency of shear reinforcement. All failures were due to over stressing of steel. (Univ. of Ill., Bul. No. 12, 1907.)

Tests of Rectangular Beams Showing Strength in Shear. Results of tests on beams in which only straight rods were used are given below. All failures were shearing failures. (Univ. of Ill., Bul. No. 29, 1909. Both smooth and corrugated bars were used and the amount of reinforcement varied from 0.98 to 2.21%.

Tests of Rectangular Beams

Reinforced with Straight Rods Only

Number of Tests	Kind of Concrete	Shearing Stress at Failure, $S_v = V/bjd$, lb per sq in			Ratio of Shearing Strength to Crushing Strength
		Min	Max	Average	
4	1 : 1 : 2	134	229	184	.044
2	1 : 1½ : 3	142	181	162	.049
24	1 : 2 : 4	128	214	164	.066
2	1 : 3 : 6	104	170	137	.098
6	1 : 4 : 8	65	113	90	.069
7	1 : 5 : 10	37	82	63	.067

Tests of T Beams Showing Strength in Shear are given in the table on p. 541. The shearing stresses at the first diagonal crack are approximately the same as the ultimate tensile strength of the concrete. The maximum shearing stresses indicate limits which may readily be attained in practise. None of the beams failed from weakness in shear except G_1 and G_2, in which the stirrups were too small to support the load. (Univ. of Wis., Bul. No. 2, 1908.)

26. Design of Beams and Floor Slabs

Loads. The loads or forces to be resisted consist of (1) The dead load, which includes the weight of the structure and fixt loads and forces. (2) The live load or the loads and forces which are variable. The dynamic effect of the live load will often require consideration. Any allowance for the dynamic effect is preferably taken into account by adding the desired amount to the live load or to the live load stresses. The working stresses usually employed are intended to apply to the equivalent static stresses so deterrmined.

In the case of buildings an allowance for impact will be necessary only in special cases, as, for example, in the case of floors supporting heavy machinery,

Tests of Reinforced T Beams

No. of Beam	Kind of Reinforcement	At First Diagonal Crack		Tensile Strength of Concrete, lbs per sq in	At Maximum Load	
		Load, lbs	Shearing Stress, V/bjd, lbs per sq in		Load, lbs	Shearing Stress, V/bjd, lbs per sq in
A_1	Four cor. bars (2 bent), ¾ in	24 000	128	256	66 600	361
A_2	Fourteen cor. stirrups, ¼ in	32 000	171	167	66 200	357
B_1	Four cor. bars (2 bent), ¾ in	42 000	226	260	65 600	357
B_2	Sixteen round stirrups, ¼ in	26 000	150	157	62 400	354
C_1	Five round rods (3 bent), ¾ in	34 000	181	197	60 000	322
C_2	Sixteen round tirrups ¼ in	34 000	185	216	57 400	316
D_1	Six cor. bars (3 bent), ¾ in	46 000	247	182	96 200	526
D_2	Fourteen cor. stirrups, ⅜ in	58 000	306	...	101 400	544
E_1	Six cor. bars (3 bent), ¾ in	46 000	244	148	92 800	505
E_2	Twenty-four cor. stirrups, ⅜ in	40 000	215	184	88 000	485
F_1	Four cor bars (2 bent), ¾ in	30 000	158	142	67 600	386
F_2	No. 11 wire mesh, 1 in	28 000	150	174	65 400	352
G_1	Four cor. bars (straight), ¾ in	24 000	126	184	48 000	259
G_2	Sixteen round stirrups, ¼ in	28 000	149	164	48 200	260

Concrete 1 : 2 : 4, age 28 days, compressive strength = 1940 lbs per sq in. *D* and *E* ad 24-in flanges, all other 16-in. Depth of flange 3½ in, depth of web 10 in. Span ngth = 10 ft; loaded at third points. Stirrups uniformly spaced between loads and ipports. All beams failed in tension except G_1 and G_2, which failed by breaking of irrups.

r where large loads are moved in a body. Such allowance may properly ary all the way from 25% to 100%, depending upon the proportion of the pecified live load which may be subject to motion.

Working Stresses. In selecting working unit-stresses for beams it is ɔ be noted that the ultimate strength of a beam, if properly designed to resist ie shearing stresses, will be determined by the compressive strength of the oncrete or by the elastic limit of the reinforcement; the elastic limit of the eam will be determined by the elastic limit strength of the concrete or that f the steel reinforcement. Working stresses in the concrete are generally aken at about one-half its elastic limit, or from 25% to 30% of its ultimate trength. Working stresses in the steel should pr ferably be less than one-alf its elastic limit, otherwise the ultimate strength as well as the elastic mit of the beam will be reached at a load only double the working load. A tress of about 40% of the elastic limit (taken as the yield point) is a suitable alue for medium steel. For high elastic limit material this gives a value of bout 20 000 lbs per sq in, a value which is sometimes used, but a lower value is eneral'y to be preferred. Many engineers consider that a value of 16 000 os per sq in should not be exceeded.

The Joint Committee on Reinforced Concrete recommends (1916) for working stresses the following percentages of the crushing strength of the concrete t 28 days when tested in the form of cylinders 8 in by 16 in:

Compression on extreme fibre of beams 32.5%. Adjacent to the support of continuous beams this may be increased 15%.

Shearing stresses, determined by the formula $S = V/bjd$,

(a) with straight reinforcement only, 2% of compressive strength;

(b) with shear reinforcement 4½% to 6% of compressive strength, depending on the design.

Bond stress:

(a) Smooth bars, 4% of compressive strength;

(b) Deformed bars, 5% of compressive strength.

Steel reinforcement, 16 000 lbs per sq in.

Value of n: 15 when crushing strength is between 800 and 2200 lb per sq in.
12 when crushing strength is between 2200 and 2900 lb per sq in.
10 when crushing strength is greater than 2900 lb per sq in.

Stresses in Continuous Beams and Slabs. When the beam or slab is continuous over its supports, reinforcement should be fully provided at points of negativ moment. In computing the positiv and negativ moments in beams and slabs continuous over several supports, due to uniformly distributed loads, the following rules will give results sufficiently close in all ordinary building construction:

(a) For floor slabs the bending moments in lb-ft at center and at support may be taken at $wl^2/12$ for both dead and live loads, where w represents the load per lineal foot and l the span length in feet.

(b) For beams, the bending moment at center and at support for interior spans may be taken at $wl^2/12$ and for end spans at $wl^2/10$, for center and adjoining support, for both dead and live loads.

(c) In the case of beams and slabs continuous for two spans only, or spans of unusual length, more exact calculations should be made. Special consideration is also required in the case of heavy concentrated loads.

A common rule used in building regulations is $wl^2/10$, but this is unnecessarily large for floor slabs of interior panels. Where beams are reinforced on the compression side, the steel may be assumed to carry its proportion of stress. In the case of continuous beams, tensile and compressive reinforcement over supports must extend sufficiently beyond the support to develop the requisite bond strength.

Floor Slabs. Reinforced concrete floors are well adapted to use where the framework is of steel columns and beams, as well as where the entire structure is of concrete.

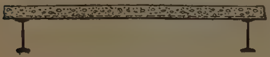

Fig. 36

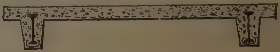

Fig. 37

Figs. 36 to 39 illustrate various designs of floors supported on steel I beams. In Figs. 36 and 37 the reinforcement may be of small rods or a metal fabric of some sort, the latter being convenient for short spans. In

Fig. 38

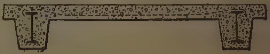

Fig. 39

Figs. 38 and 39 the reinforcing bars are hooked around the flange of the beam, thus providing anchorage.

Fig. 40 illustrates floor arch construction in the case of a very heavy floor

to support a load of 1 500 lbs per sq ft. In this case the shearing stresses were

Fig. 40

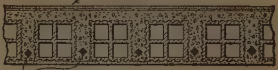

Fig. 41

so great that a sufficient bond strength could not readily be obtained without anchorage. When such anchorage is used it changes the beam to an arch, and enables the thickness of the concrete to be greatly reduced near the center of the span. Fig. 41 shows a common form of floor construction in which terra-cotta tile is used with concrete. The ribs of concrete form essentially small T beams.

Where concrete beams are used the slab and beam are built simultaneously. For short spans a metal fabric is convenient, as in Fig. 37. For longer spans, Fig. 42 shows common arrangement of reinforcement, the negativ moments being provided for by bending up a part of the rods as in (a) or (b). The result may also be arrived at by using separate short straight rods over the support.

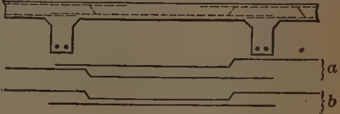

Fig. 42

Reinforcement of Slabs in Two Directions. If the length of a slab exceeds 1.5 times its width the entire load should be carried by transverse reinforcement. Square slabs may well be reinforced in both directions. The exact distribution of load on square and rectangular slabs, supported on four sides and reinforced in both directions, cannot readily be determined. The following method of calculation will give results on the safe side. The distribution of load is to be determined by the formula

$$r = \frac{l^4}{l^4 + b^4}$$

in which r = proportion of load carried by the transverse reinforcement, l = length, and b = breadth of slab. For various ratios of l/b the values of r are as follows:

l/b =	1.0	1.1	1.2	1.3	1.4	1.5
r =	0.50	0.59	0.67	0.75	0.80	0.83

Using the values above specified each set of reinforcement is to be calculated in the same manner as slabs having supports on two sides only. The spacing of rods so determined may safely be increased somewhat for the portions of the slabs between the edges and the quarter points.

Beams and Girders. The arrangement of columns, girders and beams is determined according to the same principles as in steel construction. Where the spacing of girders is not large (12 to 15 ft) and where cross beams are not needed to secure lateral stiffness, the latter may be entirely omitted or used only at columns so as to form a panel which is square or nearly so. Reinforcement in two directions is not economical for oblong panels. Generally where cross beams are used they should be spaced from 5 to 8 feet apart.

Fig. 43

If the panels are square or nearly so, the distribution of load on the beam may be

assumed in accordance with Fig. 43. Under this assumption the bending moment at the center is equal to $wl^2/12$, where w = load per sq ft and l = length of beam. Strictly speaking, the distribution is somewhat more uniform than here indicated, so that the bending moment given by this formula is somewhat on the safe side.

T Beams. Beams and girders are usually designed as T beams, utilizing a portion of the floor slab as a part of the beam. The proportions of the beam

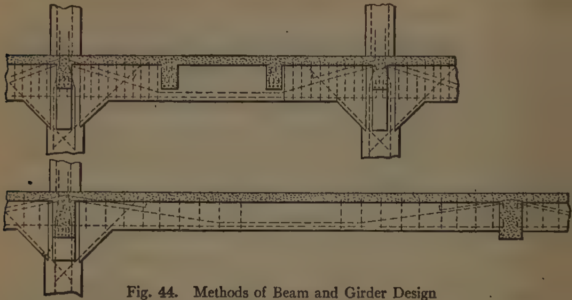

Fig. 44. Methods of Beam and Girder Design

below the slab will be determined by considerations of strength, economy, requirement of head-room, and space for reinforcing material. Generally the ratio of depth to width will vary from 1 to 3 as extreme values, the larger ratio being suitable for very large beams. Deep beams are economical of concrete, but cost more for forms than shallow beams. An effective bond should be provided at the junction of the beam and slab. When the principal slab reinforcement is parallel to the beam, transverse reinforcement should be used extending over the beam and well into the slab. The width of slab which may be considered as a part of the beam should not exceed about one-fourth the span of the beam, and each overhanging flange should not exceed 4 or 5 times the thickness of the slab.

If designed as continuous, the beam will be a rectangular beam at the support and will require strengthening at that point by compressive reinforcement, or by increase of depth. An increase of compressive stress at the support, amounting to 10 or 15%, is often permitted. Fig. 44 illustrates the main features of beam and girder design.

Bond Strength. Adequate bond strength should be provided in accordance with the formulas of Art. 24. Where high bond resistance is required, the deformed bar is a suitable means of supplying the necessary strength. Adequate bond strength thruout the length of a bar is preferable to end anchorage, but such anchorage may properly be used in special cases. Anchorage furnished by short bends at a right angle is much less effective than hooks consisting of turns thru 180 degrees. The lateral spacing of parallel bars should not be less than two and one-half diameters, center to center, nor should the distance from the side of the beam to the center of the nearest bar be less than two diameters. The clear spacing between two layers of bars should not be less than ½ inch.

Shear or Web Reinforcement. Fig. 29 illustrates various methods of reinforcing a beam against failure by diagonal tension or shear. The most

nmon design is the use of bent rods in case of small beams, and bent rods th some form of inclined or vertical stirrup in large beams. Where inclined mbers are used, the connection to the horizontal reinforcement should be :h as to insure against slip.

In the calculation of web reinforcement the concrete may be counted upon carrying a portion (one-fourth to one-third) of the shear. The remainder ıst be provided for by means of metal reinforcement consisting of bent bars stirrups, or both. The requisite amount of such reinforcement may be imated on the assumption that the entire shear on a section, less the amount umed to be carried by the concrete, is carried by the reinforcement in a gth of beam equal to its depth. The longitudinal spacing of stirrups or nt rods should not exceed three-fourths the depth of the beam. It is portant that adequate bond strength be provided to develop fully the umed strength of all shear reinforcement.

Length of Rods to Resist Moment. In determining the length of the rious horizontal rods necessary to resist the bending moment, the same thod may be used as in the design of plate girder flanges. If the bending ment is due to a uniform load the parabolic formula may be used. It is

$$x_n = \frac{l}{\sqrt{A}}\sqrt{a_1 + a_2 + \ldots + a_n}$$

which x_n = length of the nth rod in the order of length, counting the shortest number one; l = length of span; A = total steel area at center; and a_1, etc., = area of each rod up to the nth rod. For unsymmetrical loading maximum moments at various sections will need to be determined and the gths obtained therefrom.

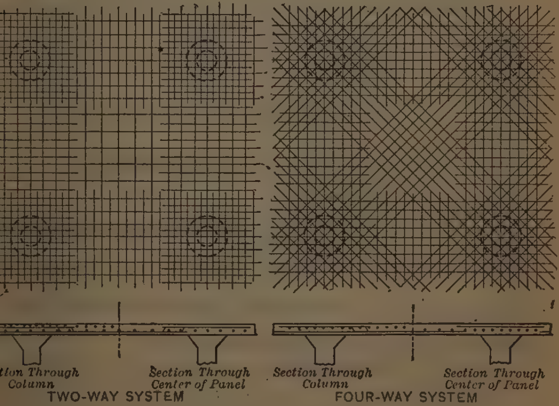

Fig. 44a. Flat Slab Reinforcement.

Flat Slab Floors. The "flat slab floor" is a type of floor in which a slab supported directly upon the columns. No beams nor girders are used, the slab

acting as a continuous plate built into its supports. Two general arrangements of reinforcement of the slab are in common use: the *two-way*, in which the rods are placed in two directions only, and the *four-way*, in which diagonal bars are also used. Fig. 44*a* illustrates the general arrangement of rods in the two systems.

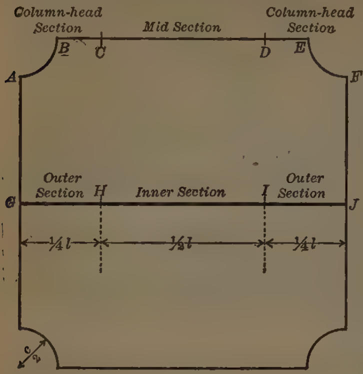

Fig. 44*b*.

In order to extend the area of support the columns are usually enlarged at the top, forming "capitals." The slab is also often made thicker over and near the column, forming a "dropped panel."

Bending Moments. The bending moments in the slab can be only approximately determined. For a square slab, Fig. 44*b*, surrounded by a number of similar square slabs in each direction, the Joint Committee recommends the following set of moment values:

Total negativ moment on section $ABCDEF = 1/_{15}\, wl(l - \tfrac{2}{3}c)^2$;
Total positiv moment on section $GJ = 1/_{25} wl(l - \tfrac{2}{3}c)^2$, in which
c = diameter of column capital;
w = sum of live and dead load in lbs per unit area.
l = panel length c to c.

The distribution of these moments is estimated as follows:

Negativ moment:
For the mid section, at least 20%.
For the two column head sections, at least 65% (for dropped panels, 80%).
Positiv moment:
For the inner section, at least 25%.
For the two outer sections, at least 55% (for dropped panels 60%).
For oblong panels the total moments become:
Negativ moment $= 1/_{15}\, wl_1(l_2 - \tfrac{2}{3}\, c)^2$.
Positiv moment $= 1/_{25}\, wl_1(l_2 - \tfrac{2}{3}\, c)^2$
in which l_1 and l_2 are respectively the dimensions of the slab parallel and perpendicular to the section along which the moment is calculated.

In calculating reinforcement all bars crossing the section are considered effective. For diagonal bars the effective area is taken as the sectional area multiplied by the sine of the angle between the bar and the straight portion of the section considered.

Minimum Thickness of slab is recommended to be:

For slab without dropped panels, $t = .024l\sqrt{w} + 1\tfrac{1}{2}$;

For dropped panels, $t = 0.2l\sqrt{w} + 1$;

where t = total thickness in inches, l = panel length in feet and w = total load in lbs per sq ft.

In arranging the bars adequate provision must be made for bond resistance, and due attention must be paid to the bending moments at intermediate sec-

ns. Points of bends should be staggered so as to provide resistance to both gativ and positiv moments for a width of at least $^1/_{15}$ of the panel length.

Diagonal Tension should be taken care of in the same manner as in beams, t stirrups will seldom be needed. For calculating the shearing stresses as a easure of diagonal tension the formula recommended by the Joint Committee $v=0.25W/bjd$ for slabs of uniform thickness, and $v=0.30W/bjd$ for slabs th dropped panels, where W is the total load on a panel, b is half the lateral mension of the panel c to c and jd is the lever arm of the stress couple. Punch- g shearing stress may be calculated on the assumption that the total load on a lumn is uniformly distributed over the section of the slab around the peri- ery of the column capital or around the periphery of the dropped panel.

Wall Panels. For wall panels the moment coefficients should be increased %. For irregular designs and for structures having only two or three rows of nels the coefficients should be modified in about the same manner as the corre- onding coefficients for continuous girders of the same span lengths.

Unit Frames. In executing work a practical difficulty of considerable im- rtance is that of placing and keeping all bars in their proper position until e concrete is in place. Very considerable labor is required in wiring bars position, or in providing other means of support, and careful supervision necessary during construction to see that they remain in place. To avoid ese difficulties various arrangements have been devised for fastening together

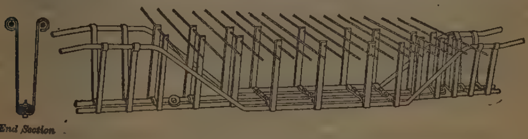

Fig. 45. Unit Frame

, or a part, of the rods of a single span into a group which can be handled a unit, giving rise to the so-called "unit frame." Fig. 45 illustrates a form unit frame.

Separately Molded Members of reinforced concrete have been used to a limited ent in the construction of buildings. Each member is cast separately and handled the same manner as members of wood or steel. Floor and roof slabs are also cast in tions. This method saves a large part of the cost for forms, but it is more difficult to ure sufficient strength and rigidity of connections. It is best adapted to low buildings uiring little lateral stiffness.

27. Columns

Methods of Reinforcement. Columns may be reinforced by means of gitudinal bars, by bands or hoops, by bands or hoops together with longi- dinal bars, or by structural forms which in themselves are sufficiently rigid to t as columns. The general effect of bands or hoops is greatly to increase e "toughness" of the column and its ultimate strength, but hooping has tle effect upon its behavior within the limit of elasticity. It renders the ncrete a safer and more reliable material and permits the use of a some- at higher working stress.

Tests of Columns. The following is a summary of a series of tests on columns, nforced with longitudinal rods only, made at the Mass. Inst. of Tech. The concrete

was 1 : 3 : 6 proportions, the rods from ¾ in to 1¼ in diameter, partly plain rods and partly twisted. The columns ranged from 6 ft to 17 ft and were about 30 days old.

For 8 by 8 inch columns, average length 12.4 ft:

Reinforcement, percent	= 1.56	2.44	3.51	6.25
Average strength, lb. per sq in	= 1904	2267	2535	3140

For 10 by 10 inch columns, average length 11.0 ft:

Reinforcement, percent	= 1.00	1.56	2.25
Average strength, lb per sq in	= 2145	2452	2870

The results of an extensive series of tests of reinforced columns made by M. O. Withey at the University of Wisconsin are given in the table on p. 549. (Proc. Am. Soc. Test. Materials, 1909.) Columns *A* were unreinforced, *E* were reinforced by longitudinal bars only, with wire ties widely spaced, *B* were reinforced by latticed angles forming a square 8 by 8 inches in section, the remaining columns by spiral wire and varying proportions of longitudinal reinforcement wired to the spirals on the inner side. The yield point of the steel reinforcement was 38 000 to 45 000 lb per sq in for the longitudinal steel and 80 000 to 105 000 lb per sq in for the spiral steel. The length of columns *A* to *C* was 120 in, of *H* to *U* 102 in. Age was 2 months.

The yield point is taken to be the point on the stress-strain diagram where the rate of deformation increases rapidly. It is very nearly the deformation at which plain concrete would fail. It was found that this "yield point" is closely given by the formula $P_1 = A[(1 - p) S_c + pS_s]$, in which S_c is the ultimate strength of the concrete, S_s is the yield point of the longitudinal steel, and p is the steel ratio for the longitudinal steel. The spiral steel is not included as its effect is very small. The ultimate strength is given approximately by the formula $P = A\left[(1 - p) S_c + pS_s + 0.12\, S_s' \sqrt{p'}\right]$, in which p' is the steel ratio for the spiral steel and S_s' is the yield point of this steel.

Experiments by A. N. Talbot on columns reinforced only by spiral or hoop reinforcement gave results shown in the following table. The bands were 1 in wide and the spiral reinforcement was 1 in pitch. The concrete was 1 : 2 : 4 and from 56 to 69 days old. The columns were 10 feet long and 12 inches diameter.

Tests of Hooped Columns

Reinforcement				No. of Tests	Average Strength, lb per sq in
Kind	Size and Spacing between centers	Yield Point, lb per sq in	Per-cent		
Electrically welded bands	No. 16, 2 in	48 000	1.07	3	2239
	No. 12, 2 in		2.09	3	2877
	No. 8, 2 in		3.21	3	3202
	No. 12, 3 in		1.39	1	2735
	No. 12, 4 in		1.02	2	2226
High carbon wire spiral	No. 7,	60 000	0.85	2	2505
	¼ in	115 000	1.69	3	3437
Mild steel wire spiral	No. 7,	38 500	0.84	3	2168
	¼ in	54 000	1.67	2	2736

The average increase per 1% of steel for the banded columns was about 570 lb per sq in; for the high carbon wire 960; and for the mild steel wire 555 lb per sq in.

General Results of Tests. The results above given and other tests indicate (1) that in columns reinforced longitudinally the distribution of stress between concrete and steel is in proportion to the moduli of elasticity; (2) that hooping of columns, either with or without longitudinal steel, greatly increases the ultimate strength of the column but does not greatly modify its yield point; (3) that great gain in strength is secured by increasing the proportion of cement in the concrete.

Tests on Reinforced Columns

No.	Reinforcement: Kind	Reinforcement: Per cent Vertical	Per cent Lateral	Concrete: Mix	Concrete: Compressive St'n'th of Cyl. lb per sq in	Concrete: Cross-section, sq in	Average Strength of Columns, lb per sq in: At Yield Point	Average Strength of Columns, lb per sq in: At Max. Load	No. of Tests
A	None............	0	0	1 : 2 : 4	2420	86.6	......	2600	3
E	Nine 5/8" rods with 1/4" ties, 1 ft c to c	2.35	0.11	1 : 2 : 4	2300	118	......	2470	3
B	Four 2"×2"×5/16" latticed angles..	4.5	......	1 : 2 : 4	2200	64	2860	3740	2
D	Nine 5/8" rods and 1/4" spiral, 1" pitch............	3.50	2.00	1 : 2 : 4	2250	78.5	3580	4750	4
C	1/4" spiral, 1" pitch	0	2.0	1 : 2 : 4	2180	78.5	1950	3660	1
H	No. 7 wire spiral, 2" pitch........	0	0.5	1 : 2 : 3½	1750	78.5	1850	2230	2
G	Eight 1/2" rods and No. 7 wire spiral, 2" pitch........	2.0	0.5	1 : 2 : 3½	1760	78.5	2710	3300	2
I	Eight 11/16" rods and No. 7 wire spiral, 2" pitch..	3.78	0.5	1 : 2 : 3½	2180	78.5	3470	4160	2
J	Eight 7/8" rods and No. 7 wire spiral 2" pitch........	6.11	0.5	1 : 2 : 3½	2050	78.5	4240	5120	2
L	No. 7 wire spiral, 1" pitch........	0	1.0	1 : 2 : 3½	1770	78.5	1370	2640	2
K	Eight 1/2" rods and No. 7 wire spiral, 1" pitch........	2.0	1.0	1 : 2 : 3½	2000	78.5	2610	3900	2
N	Eight 11/16" rods and No. 7 wire spiral, 1" pitch..	3.78	1.0	1 : 2 : 3½	1800	78.5	3370	4190	2
M	Eight 7/8" rods and No. 7 wire spiral, 1" pitch........	6.11	1.0	1 : 2 : 3½	1680	78.5	3760	4680	2
P	Eight 1" rods and No. 7 wire spiral, 1" pitch........	8.0	1.0	1 : 2 : 4	2360	78.5	5660	6920	2
O	Eight 7/8" rods and 1/4" spiral, 1" pitch	6.11	1.96	1 : 2 : 4	2480	78.5	4480	6580	2
R	Eight 1" rods and 1/4" spiral, 1" pitch	8.0	1.96	1 : 2 : 4	2380	78.5	5190	6960	2
Q	Eight 1 1/8" rods and 1/4" spiral, 1" pitch........	10.12	1.96	1 : 2 : 4	2300	78.5	5760	7090	2
S	No. 7 wire spiral, 1" pitch........	0	1.0	1 : 1 : 2	4070	78.5	4050	5850	2
T	Eight 7/8" rods and No. 7 wire spiral, 1" pitch........	6.11	1.0	1 : 1 : 2	4400	78.5	5760	7290	2
V	No. 7 wire spiral, 1" pitch........	0	1.0	1 : 3	4870	78.5	3570	5340	2
U	Eight 7/8" rods and No. 7 wire spiral, 1" pitch........	6.11	1.0	1 : 3	4550	78.5	5950	8150	2

Formulas and Working Stresses. (a) For columns reinforced by longitudinal steel only, the safe strength may be calculated by the formula

$$P = S_c A\ [1 + (n - 1)\ p]$$

in which P = total safe strength of column, S_c = working stress in concrete, n = ratio of moduli E_s/E_c, and p = ratio of steel cross-section to total cross-section A. For most purposes n may be taken at 15.

(b) For columns reinforced by hoops only the steel cannot be counted on as taking much stress at loads below the elastic limit, but inasmuch as the bands or hoops increase the capacity of the column to deform at loads beyond the elastic limit, a somewhat higher working stress may be employed than for plain concrete columns.

(c) For columns reinforced by longitudinal steel and by hoops also, the formula given under (a) may be used, the value of p being determined with reference to the longitudinal steel only.

Working Stresses for concrete recommended in the Report of the Joint Committee on Concrete and Reinforced Concrete, Nov., 1916, are as follows. They are given in terms of crushing strength of cylinders at 28 days. (a) For plain concrete piers, whose length does not exceed four diameters or for columns with longitudinal reinforcement only, 22.5% of crushing strength. (b) For columns reinforced with not less than 1% and not more than 4% of longitudinal bars, and with bands or hoops amounting to not less than 1% of the volume, a unit stress 55% higher than given for (a), provided the ratio of length to diameter of hooped core is not more than 10. The total strength of the columns under (b) is to be calculated by the formula for column strength given above, considering the longitudinal steel only.

Formula for Hooped Columns of the French Commission. This authority recommends that the hooping be counted upon directly to a much larger extent than the longitudinal reinforcement. The formula recommended is

$$S = S_c(1 + 15p + 32p')$$

in which S_c is the safe strength of plain concrete, taken at 28% of the ultimate strength in the form of cubes, p = ratio of longitudinal reinforcement, and p' = ratio of spiral reinforcement. It is also recommended that the maximum stress shall not exceed 0.6 of the ultimate strength of the concrete. These values are based chiefly on a consideration of ultimate strength.

Details of Column Design. (1) Columns of slender proportions should be avoided; it is preferable to limit the ratio of unsupported length to least width to 15. (2) In building construction, if special fire-proofing is not provided a certain thickness of the concrete must be considered as protective covering and omitted in the strength calculations. This allowance should be at least 1½ in on all sides. (3) Bars composing longitudinal reinforcement should be straight and have sufficient lateral support to be securely held in place until the concrete has set. (4) Where bands or hoops are used, the total amount of such reinforcement should be not less than 1% of the volume of the columns enclosed. The clear spacing of such bands or hoops should be not greater than one-fourth the diameter of the enclosed column. Adequate means must be provided to hold bands or hoops in place so as to form a column, the core of which will be straight and well centered. (5) Bending stresses due to eccentric loads must be provided for by increasing the section until the maximum stress does not exceed the values above specified.

The size of bars will depend somewhat on the proportion of reinforcement desired, but will usually range from ½ to 1½ in. Great care should be taken in the splicing of

rods in columns extending thru several stories. All rods should be spliced at the floor level. Rods of ¾ in, or larger, should be milled square and butted together in a secure manner. At foundations, planed bearing plates should be provided.

High percentages of reinforcement are undesirable, as the strength of such columns is not well determined. If great strength is required an increase in the proportion of cement is preferable to a high percentage of steel. If large amounts of steel are required, the riveted column unit has advantages over the separate rod reinforcement, inasmuch as the position and arrangement of the steel are secure and certain.

28. Roofs

General Design. Reinforced concrete roofs are designed and constructed in the same manner as floors. For steep slopes the concrete in the slabs must be laid quite dry, or else top forms must be used to retain the concrete. The latter method is slow and expensive. Cinder concrete may frequently be used to advantage as the loads are relatively light.

Imperviousness. Concrete roofs may be designed merely as the supporting area for a roof covering of other material, or may be designed to be impervious and complete without such covering. To secure absolute imperviousness is difficult. It requires special precautions as noted in Art. 15, and a maximum amount of reinforcement against shrinkage cracks (0.5 to 1.0%). If designed to support a separate roof covering the construction is much simplified and the design can be made very economical. To support the roof covering nailing strips of wood are embedded at suitable intervals.

Adaptability. Reinforced concrete is well adapted to roof construction where a fire-proof construction is desired. To support the roof, beams may be employed for spans up to 40 or 50 feet and trusses for still longer spans. The arch type of construction can also be readily carried out in reinforced concrete. Domes have been built of reinforced concrete in several instances. The stresses are resisted chiefly by circumferential tension rods and by the compressive stress in the concrete in a radial direction.

In the construction of domes at the Cincinnati Zoological Gardens forms were dispensed with by using a stiff framework of small angles and expanded metal and plastering first with a coat of hard plaster. This coat on hardening supported the concrete.

If a special roof covering is employed the concrete merely replaces the usual support of steel or wood and is adapted to any structure. Used without such covering it is well adapted and economical for all structures where absolute imperviousness is not essential, such as train sheds, storage sheds, reservoir roofs, coal bins, and "out of door" structures in general. It has also been successfully used in other types of structures and a water-tight roof secured without separate covering.

Separately Molded Slabs of reinforced concrete have been used as tile or shingles and laid with lap joints. These slabs may readily be made water-tight.

29. Walls and Partitions

Concrete or Reinforced Concrete is well adapted to the construction of all walls where considerable strength is desired. For walls supporting light loads, such as curtain walls between the framework of a reinforced concrete building, or partitions, the requirements of strength are met by the use of very thin walls, but such walls become relatively expensive on account of the cost of forms and do not make a dry or warm construction.

Walls of Double Thickness. Concrete walls are often made of double thickness with an air space enclosed, each part being 3 to 4 inches thick. The air spaces are formed by core boxes which are drawn up with the out-

side forms as the work proceeds. Cheap pipe of thin metal may also be used and left in place.

Brick Facing. Walls of single thickness are often finished with a single layer of face brick with satisfactory results, both in appearance and dryness.

Reinforcement of walls should be sufficient to prevent cracks, unless the wall is to be faced. A small amount is desirable in any case to insure adequate strength.

Partitions may be made as thin as 2 or 3 inches, if the concrete is slightly reinforced. Hollow concrete tile or metal lath and plaster will, however, generally be more economical for partitions than poured concrete.

Separately Molded Walls have been successfully used in house construction. A section of the wall for one story is molded on its side in a horizontal position and after hardening is raised into place. The process simplifies the construction of forms and has other advantages.

Walls of Storage Bins and other structures where great strength is required are designed in the same manner as floor slabs. The vertical posts and floor girders may be used also as beams to support the lateral pressure against the wall slab.

30. Durability of Reinforced Concrete

Protection of Steel from Corrosion. A continuous coating of portland cement has been found by experience to be a practically perfect protection of steel against corrosion. The rusting of iron requires the presence of moisture and carbon dioxide (CO_2). Portland cement not only forms a coating which excludes the moisture and CO_2, but in hardening it absorbs CO_2, tending to remove any of this gas which may be present.

Many cases have been cited of steel removed from concrete after the lapse of 20 years or more and found to be in perfect condition. A covering of dense concrete of as little as 1 in appears to give satisfactory protection against corrosion for concrete exposed to ordinary atmospheric conditions or where constantly submerged. Where exposed alternately to the effect of air and water, especially where structures are exposed to the action of sea water, more difficulty is experienced and special pains must be taken to make the protective covering dense and thick.

Electrolysis of Steel Reinforcement has occurred in some cases to a serious extent, and there is good evidence that concrete is not an adequate protection of steel from this action. When the work is well executed, and the steel well covered by the cement, the electrolytic action is greatly retarded but not entirely prevented, as indicated by tests in which direct connection has been made with the steel reinforcement. In foundation work it is important that where electrolytic action is possible, the steel reinforcement be very carefully placed and that it be covered at all points by the concrete.

Fire-proofing of Steel Reinforcement. The effect of fire upon concrete is discussed in Art. 16. The necessary thickness of concrete to furnish adequate protection to enclosed steel in reinforced work depends somewhat upon the character and importance of the member. Such members as main girders, where a failure would involve a considerable portion of the building and where the steel is concentrated in a few rods, should be more thoroly protected than floor slabs of small span, where a few local failures would be of no importance and where additional covering would add largely to expense.

The following are the conclusions of the American Joint Committee on Concrete and Reinforced Concrete: (1) It is recommended that the metal in girders and columns be protected by a minimum of two inches of concrete;

that the metal in beams be protected by a minimum of one and one-half inches of concrete, and that the metal in floor slabs shall be protected by a minimum of one inch of concrete. (2) It is recommended that the corners of columns, girders, and beams be beveled or rounded, as a sharp corner is more seriously affected by fire than a round one.

While satisfactory protection of the steel can thus be secured, the effect of fire upon the concrete itself and its usefulness after more or less calcination, is a question of much importance. Where a sufficient allowance has been made for such damaged material it would appear that the removal of the soft or loosened portions and replastering by cement mortar would generally secure effective repair.

Reinforcing against Shrinkage and Temperature Stresses. Where a reinforced structure is unrestrained by outside forces the only stresses arising from shrinkage and temperature changes are those due to the mutual action of steel and concrete. As the two materials have nearly equal rates of expansion, temperature changes will cause very little stress. Shrinkage in hardening will cause more important stresses, but still not unduly large unless the steel ratio is very high. When the structure is restrained by outside forces so that it is not free to contract or expand, as in the case of a long wall, then the resulting stresses are likely to be high. When not reinforced, concrete will, under such circumstances, crack at intervals, its maximum deformation under stress not being equal to its maximum temperature deformations.

The size and distribution of the cracks will depend upon the amount of steel used and upon the strength of bond furnished by the rods. The size and the spacing of the cracks will vary inversely with the bond strength and amount of steel per unit of concrete section. For reinforcement against shrinkage and temperature stresses, a high elastic limit steel is desirable, and in order to distribute the deformation as much as possible a mechanical bond is advantageous. The amount of steel necessary for such reinforcement depends upon the thickness and exposure of the structure of the walls. For thin walls and exposed locations 0.4 to 0.5% is required, while under very favorable conditions as little as 0.1% has been found to be sufficient. The reinforcement for this purpose should be placed close to the exposed faces of the concrete. In floor slabs longitudinal bars of ¼ in to ½ in diameter, spaced about 2 ft apart, is customary.

31. Forms for Reinforced Concrete

General Requirements. Forms should be as simple in design as possible, and this requirement should be well considered in the design of the structure itself. They should be durable and designed to be easily removed and re-erected. They should be so arranged as to permit the removal first of the column forms, then the sides of the beams and the floor forms and, lastly, the bottoms of the beam forms. Forms should be designed to sustain a live load of 50 to 75 lb per sq ft in addition to the weight of the concrete. Rigidity is as important as strength, and deflections of forms should be limited to very small amounts.

Column Forms should be made of 2-in plank and well clamped together to resist the heavy pressures involved. They are conveniently held together by wooden blocks connected by bolts and adjusted by wedges as shown in Fig. 46. Care must be taken to see that the bottoms of column forms are thoroly clean before pouring concrete. To render this easy and certain it is advantageous to arrange a small removable panel at the base of the form.

Beam Forms are preferably made of 2-in stuff. The bottoms should be well supported and the sides held together by wedges or clamps. Wedges are also conveniently used at top or bottom of the supporting struts.

Slab Forms are made of 1-in or 2-in stuff. If the former is used it should be supported on joists spaced not more than 2 ft apart. Joists are conveniently supported on 2-in strips fastened to the sides of the beam forms or may be independently supported. Fig. 47 shows a typical arrangement of beam and slab forms.

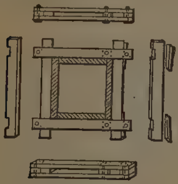

Fig. 46. Form for Column

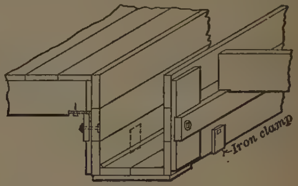

Fig. 47. Form for Beam

Adhesion of Concrete to the forms can be prevented by oiling the forms with crude oil, soap or other greasy material. This aids also in preventing warping. If not oiled the forms should be thoroly wet before placing the concrete. This is especially necessary in hot dry weather.

Wall Forms. Forms for thin walls are generally made to be supported by the concrete already in place. Fig. 48 shows a common arrangement. The bolts and slotted holes enable the sections to be raised conveniently as the work proceeds.

Fig. 48. Form for Wall

Removal of Forms should be done only under expert supervision. In warm weather column forms can be removed in 2 to 4 days, slab forms in 10 to 15 days, and beam forms in 2 to 3 weeks. Low temperatures require much more time for hardening. Forms should be removed very carefully and gradually and the concrete examined before general removal is ordered.

32. Cost

The Cost of Reinforced Concrete per unit of volume depends greatly upon the cost of forms. The cost of forms depends chiefly upon the area supported and not upon the volume, altho total cost is commonly estimated per unit of volume of concrete. FORMS will generally cost from 8 to 15 cents per square foot of surface, depending upon the simplicity of the design. CONCRETE will vary from \$6.00 to \$10.00 per yard in place, depending upon the massiveness of the walls. PLAIN STEEL RODS cost 3 to 4 cents per lb, deformed bars about 0.2 to 0.3 cent additional. The cost of placing will generally range from ¼ to ¾ cent per lb.

Total Cost per Cubic Foot. The amount of reinforcement is generally

from 1 to 1½%. With this amount of reinforcement and an average thickness of concrete of 6 in the total cost per cu ft will be from 50 cents to 80 cents.

Cost of Various Structures. The following table gives minimum, maximum and average costs (not including profit) for a large number of structures as given by L. C. Wason. (Eng. Record, Jan. 16, 1909.)

Cost of Reinforced Concrete

Form of Construction	No. of Structures	Cost of Forms, cts per sq ft			Cost of Concrete, cts per cu ft		
		Min.	Max.	Average	Min.	Max.	Average
Concrete columns	9	7.5	18.1	13.0	27.1	34.0	30.1
Beam floors	18	6.7	27.5	11.6	20.2	47.0	35.4
Flat slab floors	3	10.6	11.8	11.1	25.2	37.4	31.5
Slabs between steel beams	13	4.9	18.4	9.5	27.2	42.8	35.9
Building walls	17	7.9	17.6	12.8	17.4	44.6	30.1
Foundation walls	14	5.6	19.3	10.3	14.8	59.9	26.9
Footings and mass foundations	10	1.8	19.8	9.3	18.1	27.5	22.9

The cost of handling steel reinforcement for 21 structures ranged from 0.13 cent to 0.82 cent per lb, averaging 0.426 cent.

BRIDGES AND OTHER STRUCTURES

33. Foundations

Reinforced Concrete is especially well suited for the construction of footings for walls and columns and the foundations of any structure where the bearing area must be considerably larger than the superstructure resting thereon, such as towers, chimneys, lighthouses, arches, and, frequently, bridge piers and abutments. To secure the necessary bearing area by the use of plain concrete or other masonry requires great depth of foundation, increasing the expense and, often, in the case of buildings, occupying much valuable space. Grillage foundations, consisting of steel I beams protected by concrete, are more expensive than slabs of reinforced concrete, as they do not utilize the compressive resistance of the concrete; generally, therefore, the reinforced slab is to be preferred. Reinforced concrete is also used extensively for bearing piles and also for sheet piling, caissons, and other forms of foundation work.

Footings. The problem of the design of reinforced concrete footings is in general the same as that of floors. For single footings of ordinary size a single square slab is most convenient. For larger footings and for footings carrying more than one column, a combination of beam and slab, similar to floor construction, is often most economical. For very soft soils this may need to spread as a floor over the entire foundation area, forming a monolithic structure.

For a SQUARE SLAB (Fig. 49) the pressures should be carried as directly as possible from the extremities to the center. Two sets of main reinforcing rods, aa' and bb' should be used, as shown in the figure. The reinforcing of the remaining corners can best be done by sets of diagonal rods dd'. If these do not cover the area, then a few short

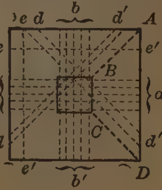

Fig. 49

cross-rods may be used. Reinforced in this way the total pressure on the area $ABCD$ may be assumed to be carried to the line BC, where the bending moment and shear will be a maximum. Figured as a free cantilever the resulting stresses will be higher than actually exist. If the entire square be reinforced by rods in two directions only, as ee', then a considerable part of such rods in the corners of the square are ineffective.

In CANTILEVER BEAMS, used as footings, the maximum shearing stress is near the base of the wall where the moment is large. Shear cracks tend to form on the dotted lines a, a, Fig. 50. Bent rods, if used, should be bent just outside the column base, and not at the end of the beam, and stirrups must be spaced closely at this point. The beam being short the bond stress may require special attention. The shearing stresses in the concrete are likely to be high and exceed the limit generally allowed for beams. In this case, since the depth of the beam is relatively great, the dangerous section will be some distance from the face of wall. This distance may be taken equal to the effective depth of the beam and the rules for maximum shear then applied.

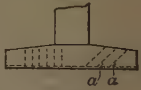

Fig. 50

For large INDIVIDUAL FOOTINGS a beam and slab may be economical. To secure the benefit of a T-section and to give a flat upper surface the beam may be placed under the slab as shown in Fig. 51. This arrangement requires some attention to the connection of slab to beam, as the upward pressure against the slab tends to pull it away from the beam. The use of a horizontal rod in the top of the main beam, bonded by stirrups, will give a thoroly good anchorage for the transverse rods of the slab. For still larger areas a system of girders and beams may be adopted, constituting a floor reversed as to loads.

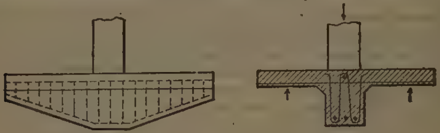

Fig. 51. Large Individual Footings

For WALL FOOTINGS a simple slab construction with transverse reinforcement is suitable.

Stresses in Footings Eccentrically Loaded. If for any reason (eccentric load, earth pressure, wind pressure, etc.) the resultant of the forces acting above the foundation level intersects that level eccentrically, the pressures upon the earth below are not uniform but vary in intensity from a minimum at one edge to a maximum at the other. It is customary, and sufficiently accu-

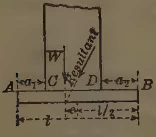

Fig. 52

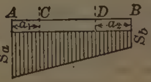

Fig. 53

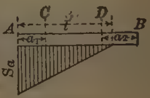

Fig. 54

rate for the purpose, to assume that in such a case the pressure varies uniformly from one side to the other. In Fig. 52 let e = eccentricity of resultant pressure, l = length of base, W = total vertical load per lineal foot on foundation, and S = pressure per unit area at any point. Then for rectangular areas, if e is less than $\frac{1}{6}l$, the variation in pressure is as represented in Fig. 53. The values of the intensity of pressure at A and B are as follows:

For edge A, $$S_a = \frac{W}{l}\left(1 + \frac{6e}{l}\right)$$

For edge B, $$S_b = \frac{W}{l}\left(1 - \frac{6e}{l}\right)$$

If e is greater than $\frac{1}{6}l$ the pressure is distributed as in Fig. 54 over a length l', equal to $3(\frac{1}{2}l - e)$, and the pressure at A is $S_a = 2W/l' = 4W/3(l-2e)$. The bending moments at C and D are then given by the formulas:

For Fig. 53, $$M_C = \frac{W}{2l^3}(l^2 + 6el - 4ea_1)a_1^2$$

and $$M_D = \frac{W}{2l^3}(l^2 - 6el + 4ea_1)a_1^2$$

For Fig. 54, $$M_C = \frac{2Wa_1^2}{3(l-2e)^2}(9l - 18e - 2a_1)$$

and $$M_D = \frac{W}{54(l-2e)^2}(2a_2 + l - 6e)^3$$

Reinforced Concrete Piles are generally advantageous where piles of timber would be subject to decay or to destruction by the action of marine worms. They are especially advantageous for foundations on land where the permanent ground water is at a considerable depth. In such cases if wooden piles are used the excavation must be carried down sufficiently to enable the piles to be cut off below water level, while with concrete piles this is unnecessary. Another advantage of the concrete pile is the fact that it can be made of any desired shape or size and thus made to meet special conditions more satisfactorily than wooden piles.

(a) Piles Molded in Place. Two varieties are widely used, the Simplex pile and the Raymond pile. In the former a hollow steel shell of the desired diameter and made of ¾-in material is driven into the ground to the necessary depth. This shell is then gradually filled with concrete and at the same time is withdrawn, thus leaving the molded concrete in place. The driving point may be a cast-iron point left in place or a hinged cutting edge which opens as the casing is withdrawn. In very wet soil a permanent casing of thin sheet steel is used to prevent washing or displacement of the concrete. In the Raymond pile a thin shell of steel is driven into the ground by means of a collapsible driving core. After driving, the latter is removed and the shell filled with concrete. This pile is generally made with a large taper, an 8-in point and 20-in butt being common.

(b) Piles Molded on the Surface may be made of any desired shape or size. Lengths up to 60 ft have been successfully used. The reinforcement should be sufficient to prevent cracking in handling, and will depend upon the length of pile and other local conditions. Well-distributed reinforcement in the form of wire netting or mesh is more effective than large longitudinal rods. A form of molded pile much used, is made by rolling up the concrete as a layer spread on a metal fabric; this avoids the use of forms.

The Strength of Concrete Piles, when driven to solid bottom, is dependent upon the compressive strength of the material and the end conditions. A value of 15 short tons per sq ft, or about 200 lb per sq in, should not be exceeded unless the conditions are exceptionally favorable. The safe strength of a con-

crete pile is somewhat less than that of a wooden pile of the same size and under the same conditions, but experience indicates that a concrete pile is less likely to be injured in driving than a wooden pile. Where the bearing capacity depends largely upon frictional resistance the strength of the concrete pile may be taken the same as that of the wooden pile. Tapered piles appear to have a bearing capacity in soft material considerably greater than a straight pile of the same average diameter.

In the construction of piers in sandy beaches molded piles having an enlarged base have been very successfully used. These have been jetted into place. The same form is well adapted generally to foundations in sand where the jet process can be used.

In driving molded piles a driving head is used in which a cushion of sand, rope, or some other convenient material is interposed between a driving block of wood and the concrete. Experience indicates that any injury caused in driving rarely extends more than a few inches from the base of the pile. The jet process may often be used as in the case of wooden piles. In this case the end of the pile is preferably made square in order to provide a maximum of bearing surface. Where piles of considerable length are required the lower portion may often be advantageously made of wood and the upper portion of concrete doweled to the wooden pile. A considerable saving in cost is thus possible. In open water a hollow concrete pile driven over a wooden pile and to a firm bearing into the ground has given very satisfactory results.

The cost of concrete piles is generally from two to four times as much as that of wooden piles, so that the former are not likely to be economical where the conditions are favorable to the life of the wooden pile.

Sheet piling may also be made of concrete to advantage wherever it is desirable to leave it in place as a part of the permanent structure.

Repairs to Wooden Pile Structures have been successfully made by enclosing the upper portion of the wooden pile within a concrete box constructed of slabs of reinforced concrete.

34. Arch Bridges

Advantages of Reinforcement. In ordinary masonry or concrete arches tensile stresses are not permissible. The ring must therefore be designed so that the line of pressure will not pass outside the middle third. In reinforced arches this limitation does not exist. The arch rib is a beam, and if properly reinforced it may carry heavy bending moments involving tensile stresses in the steel. Besides this advantage, reinforcement renders an arch a much more secure and reliable structure, it greatly aids in preventing cracks due to any slight settlement, and by furnishing a form of construction of greater reliability makes possible the use of working stresses in the concrete considerably higher than is used in plain masonry. Furthermore, in long-span arches where the dead load constitutes by far the larger part of the load, any increase in average working stress counts greatly towards economy. It affects not only the arch itself but also the abutments and foundations.

Methods of Reinforcement. Since the arch is a beam subject to either positiv or negativ bending moment it is essential that it should be reinforced on both sides, but the shearing stresses due to beam action are relatively small, so that little is needed in the way of web reinforcement. It is desirable, however, that the inner and outer reinforcements be tied together, somewhat as in a column, altho in this case the necessity therefor is much less. A large proportion of the arches which have been constructed have been built according to some one of the various "systems" that have been devised. The most important of these systems are the Monier and the Melan.

In the Monier system, invented about 1865, the reinforcement consists of wire netting, one net being placed near the intrados and one near the extrados. The longitudinal wires

re made smaller than those following the arch ring, as they serve only to aid in equalizing ie load and in preventing cracks. A large number of bridges have been built in Europe.

In the Melan type, invented about 1890, the steel is in the form of ribs of rolled I-sec- ons, or of built-up lattice girders, which are spaced two or three feet apart. The flanges onstitute the principal reinforcement, but the web enables the steel frame to be self- ipporting and to carry shearing stresses, and in the open lattice type it furnishes a good ond with the concrete. The Melan arch has been built extensively in this country. rgely under the direction of Edwin Thatcher.

Many arches have also been constructed in which reinforcing bars of any satisfactory rm are employed without reference to any particular system, being used in accordance ith the requirements of the case.

Calculation of Reinforcement. In the design of a reinforced arch the oments, stresses, shears and thrusts are found in the same manner as for ny form of solid arch as described in Section 7. The elastic theory is referable in this case. In the application of this theory the value of the oment of inertia to employ is that for the combination of concrete and steel. I_c = moment of inertia of the concrete at a given section and I_s = moment inertia of the steel reinforcement, then the total moment of inertia is

$$I = I_c + nI_s$$

here n is the ratio of the moduli of elasticity, commonly taken at 15. In ie calculation of an arch it is necessary to assume a design by the use of some npirical formula and then to calculate the stresses in this arch. The design then adjusted to bring the stresses close to the specified values. The cal- ilations are then repeated if necessary.

The amount of steel used is very variable and may, indeed, be varied at will: or short spans, the steel is serviceable mainly for the purpose of making the ch more secure against cracking. Where foundations are excellent it is ardly needed at all, and no economy is secured by its use. For long spans,

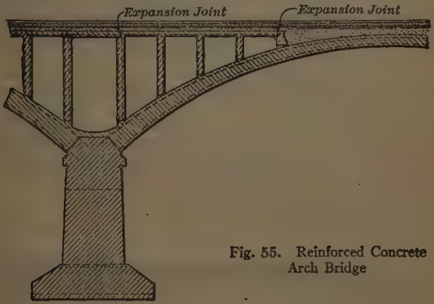

Fig. 55. Reinforced Concrete Arch Bridge

here the dead load becomes relatively large, the use of steel becomes econom- al, as it enables a large saving to be made in weight by concentrating the arch aterial into separate deep ribs of high resistance. In practise the amount reinforcement along extrados and intrados thus varies from ½% to 4 or 5%.

Details of Design. Advantage is generally taken of the adaptability of reinforced oncrete in the various details of the arch. The roadway is generally supported by pandrel arches, or by beam and slab construction resting on piers or cross walls, which

in turn rest upon the main arch or arch ribs. The design of these parts is carried out as for other beam and column designs. The various parts should be especially well bonded together to resist temperature changes. The abutments can generally be made hollow with economy, the roadway being carried on a reinforced concrete floor. This relieves the spandrel walls of lateral pressure and enables them to be made of minimum thickness of 12 to 18 in. All walls should be reinforced both vertically and horizontally with 0.1 to 0.2% of steel in order to prevent contraction cracks. Spandrel walls should be provided with expansion joints above or on each side of the pier, otherwise cracking is very likely to occur, due to rise and fall of the crown. Fig. 55 illustrates the principal features of design of large arches.

35. Girder Bridges, Trestles and Culverts

Beam or Girder Bridges are designed in the same manner as other heavy concrete floors. Spans up to 20 or 30 ft may well be made in the form of a simple slab of uniform thickness spanning the opening. For railroad structures the loads are relatively so large that shearing stresses will usually require careful attention. For longer spans a gain in economy will result by the use of main horizontal girders of relatively great depth, with a floor supported by the girders and reinforced transversely. The bridge may be made either a "thru" or "deck" girder according to the requirements of the case, the latter being the more economical. The details are arranged in a variety of ways, but the calculation and design of the reinforcement to meet the given conditions require no special consideration. The proper allowance for impact is an important point in this connection. Durability is an important factor favorable to the use of reinforced concrete for bridge floors.

As compared to steel construction the cost of reinforced concrete girders compares favorably up to spans of 30 to 50 feet, depending upon the loading, larger spans being feasible for light loads. A saving in the abutments is effected, as the cover acts as a lateral support, permitting them to be designed as beams.

Trestles. Where several short spans are required and concrete is used for both the girders and the piers, the latter may usually be made of comparatively small cross-section, — much smaller than possible if ordinary masonry be used. The structure then approaches the ordinary floor and column construction in the relation of its parts. The piers may generally consist of two or more columns connected by a suitable portal. Where the piers are small, as here assumed, they must be built rigidly in connection with the girders of one or more spans, as are the columns in a building. The girders must be designed with proper reference to their continuity, and the piers must be reinforced somewhat near the top to enable them to resist a part of the bending moment due to the continuity of beams and piers.

Expansion joints should be provided at intervals not exceeding 75 to 100 ft, otherwise the movement due to contraction and expansion will be concentrated at the ends of the structure, resulting in severe stresses in the end columns. Fig. 56 illustrates the general

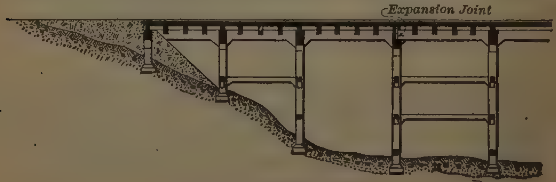

Fig. 56. Concrete Trestle

arrangement of a large concrete trestle. Low trestles may be made without longitudinal bracing, the bending resistance of the columns giving the necessary stability. The end piers or abutments must be designed to act also as retaining walls.

Pile Trestles, in which reinforced concrete piles are used, are advantageous in many locations. They are somewhat more expensive than wooden trestles, but are durable and are well adapted to ballast floor construction. In their construction the concrete piles are driven into place and then caps of reinforced concrete molded about the heads of the piles. On the caps are then placed slabs, forming the supporting floor for ballast.

Pipe and Box Culverts. For small openings the monolithic pipe or box form is very advantageous. Considerable settlement, as a whole, may be permissible, and hence solid foundations may not be needed. The cross-section may be circular, elliptical or rectangular. Theoretically, the elliptical form is the best, as it corresponds more nearly to the requirements for resisting the earth pressure. The circular is practically as good for small openings, while for large openings the rectangular form will often be the best on account of its simplicity and the smaller head-room required. Where the culvert is manufactured at a shop and transported to the site, the circular or the elliptical form will usually be the most advantageous.

The loads coming upon such structures cannot be very closely estimated, but except for small spans and high embankments the entire weight of filling and live load is usually assumed to act directly upon the structure. The stresses may be calculated on the assumption that this load acts vertically and is uniformly distributed over the width or diameter of the culvert. The lateral pressure should not be considered as an aid in supporting the vertical load. On this basis the maximum bending moment in a circular culvert is given by $M=\frac{1}{16}pd^2$, where M = bending moment at crown and at center of base for a unit length of culvert, p = load per unit area and d = diameter of culvert. For rectangular culverts the bending moments may be taken as for a simply supported beam, and in the vertical walls the moments may be calculated for a lateral pressure equal to ⅓ the vertical load.

Reinforcement of Culverts. In the circular form a wire mesh is convenient, especially for small diameters. A single mesh is sufficient, placed near the extrados at the sides and crossing the central axis at about the quarter point. In the rectangular form, if reinforcement for negativ moments at the corners is omitted, then the four sides will act as simple beams.

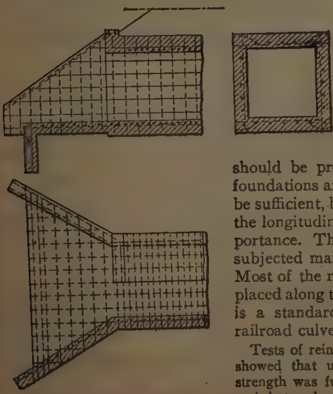

Fig. 57. Standard Box Culvert

Longitudinal reinforcement should be provided to some extent. Where foundations are good a very small amount will be sufficient, but if settlement is likely to occur the longitudinal reinforcement becomes of importance. The entire culvert will act as a beam subjected mainly to positiv bending moments. Most of the reinforcement should therefore be placed along the bottom of the culvert. Fig. 57 is a standard design for a rectangular box railroad culvert.

Tests of reinforced culvert pipe by A. N. Talbot showed that under concentrated loads the actual strength was fully equal to the theoretical strength; and that under a load distributed by means of a sand

box the actual effective strength was relatively somewhat greater than for concentrated loads. Complete failure in the latter tests was generally prevented by lateral support from the sand box.

36. Retaining Walls

Forms of Walls built of reinforced concrete are made such that advantage is taken of the weight of the material supported to increase the stability of the wall against overturning. Fig. 58 represents in outline the usual type of reinforced wall. It consists of a vertical wall AE attached to a floor DC. For low walls the upright part AE may act simply as a cantilever; and likewise the parts EC and ED. For higher walls the part AE is tied to EC at intervals by back walls, AEC, in the form of narrow transverse walls with tension reinforcement. The projecting portion ED may still act as a cantilever, or it, also, may be connected to the vertical wall AE by means of buttresses. In either case the earth pressures act in essentially the same manner, and the necessary width of base is found in the same way. The first type is adapted to heights of 12 to 18 ft. For higher walls the second type will be more economical.

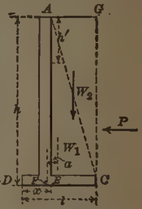

Fig. 58

Stability of Wall. The stability of a retaining wall is usually determined with reference to its overturning. For safety in this regard the common requirement is that the resultant line of pressure shall intersect the base within the middle third. This gives a distribution of pressure such that there is some compression over the entire section. Besides stability against overturning, the pressure on the foundation must be investigated and if found excessive the footings must be extended until the pressures are reduced to safe values.

In Fig. 58 let h = height of wall, l = width of base, x = distance from toe to back of wall AE, $k = x/l$, P = total horizontal pressure of earth against the vertical section GC, w = weight of earth filling per cubic foot, W_1 = weight of masonry per lineal foot, W_2 = weight per lineal foot of earth above the floor EC, a = lever arm of W_1 about point F, the edge of the middle third. Then in order that the resultant pressure shall cut the outer edge of the middle third the value of l is given by

$$l = \sqrt{\frac{2Ph - 6W_1a}{wh(1 + 2k - 3k^2)}}$$

If the moment of the masonry, W_1a, be neglected the value of k which will give a minimum value of l is ⅓. Using this value of k and taking w = 100 lbs per cu ft the value of l becomes $l = \frac{1}{8}\sqrt{P}$ in which l is to be exprest in feet and P in pounds.

For other values of x/l, or k, the width of base must be increased over that required for $k = \frac{1}{3}$ by the following percentages:

For $x/l = k =$	0.5	0.25	0.15	0.10	0.0
Percent increase =	2	2	4	6	15

Compared to masonry walls the reinforced wall with l/h = ¼ to ⅛ has about the same stability as a solid masonry wall of the same width of base, and of the common type in which the outer face is battered 1 to 2 in per ft. Under ordinary conditions a width of 0.4 to 0.5 the height will be required.

Foundation Pressures. If the pressures on the foundations need to be considered the value of these pressures is obtained as described for footings, after first determining where the resultant of W_1, W_2 and P cuts the base.

This feature of the design is too often neglected and most of the failures of retaining walls have been due to excessive pressures at the outer toe. The pressure can be reduced to safe limits by widening the foundation or the foundation may be strengthened by means of piles.

Stresses in the Wall. Form (*a*), Fig. 59. The bending moment at the base of the upright portion AE is $Ph/3$. At any other section, F, it is $P'h'/3$, where P' is the pressure for depth h'.

Only a portion of the reinforcing rods need be carried up the full height. At the base the vertical rods have an insufficient length below the point of maximum moment to develop their full strength, and therefore they should be anchored in a substantial manner. This may be done by screw-ends and nuts, or by looping the rods around anchor bars near the bottom of the floor DC.

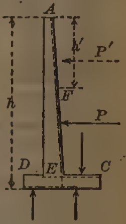

Fig. 59

The cantilever DE must be treated in the same manner as the upright cantilever. The pressures will be much heavier and the shear and bond stress may need attention. The cantilever EC is acted upon by an upward and a downward force as shown in the figure. The maximum moment will be at E and will be negativ. It is provided for by reinforcement as shown. The bending moments in DE and EC, due to upward earth pressure, can be readily found by means of the moment formulas given under the subject of footings for foundations.

Form (*b*), Fig. 60. The external pressures are practically the same as in the case previously considered. The pressure against the longitudinal wall AE is carried laterally for the most part and given over to the inclined back walls. The wall AE must therefore be designed as a slab supported along the lines AE, $A'E'$, and EE', Fig. (*b*).

The floor EC is supported by the back wall AEC and is therefore reinforced longitudinally as a floor slab in accordance with the resultant pressure at any point. The back wall ACE acts as a cantilever beam anchored to the floor. It is also a T beam, the flange being the longitudinal wall AE. The tension along the edge AC is carried by rods near this edge, whose stress at any point

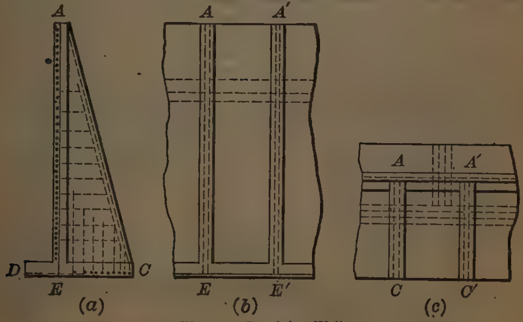

Fig. 60. Retaining Walls

is found with sufficient accuracy by an equation of moments taken about the center of the front wall. The maximum stress will be at the bottom. The main tension rods in AC should be well anchored to the reinforcing rods of the floor EC. A few additional vertical rods should also be put in to insure thoro bonding of floor to wall.

Fig. 61

A horizontal beam may be made of the coping at A, thus giving some support to the wall AB along its upper edge. A downward projection may be necessary at the toe D, or at some other point in the base, in order to increase the resistance against forward sliding.

To prevent unsightly cracks, long walls must be provided with expansion joints at intervals of 50 to 75 ft, or must be reinforced longitudinally by 0.1 to 0.2% of steel. Long walls have thus been successfully built without expansion joints.

The form of retaining wall shown in Fig. 60 was extensively used on the Great Northern R.R. at Seattle, Wash. (Eng. News, Vol. 53, 1905). An estimate by C. F. Graff of the amounts of material per lineal foot required in reinforced and plain concrete walls, made in connection with this design, gave the following results, the steel being included by adding its concrete equivalent.

Comparison of Plain and Reinforced Walls

Height of Wall, Feet	Amount of Concrete, Cubic Feet per Lineal Foot of Wall		Percent of saving by Reinforced Wall
	Plain Wall	Reinforced Wall	
40	396.4	218	45
30	226	127.8	43.3
20	110	69.9	36.4
10	44	34.9	20.4

Bridge Abutments. The second type of retaining wall above described is readily modified to serve as an abutment for a steel superstructure as shown in Fig. 61.

37. Dams

Two Types of Reinforced Concrete Dams are shown in Figs. 62 and 63. The first is suitable for locations where little or no water passes over the crest, the second is designed to act as a spillway. In both cases the dam consists of a water-tight reinforced floor on the upstream side, supported by solid buttresses placed 10 to 15 feet apart and 12 to 24 inches thick. The upper wall may be supported directly on the cross walls and reinforced with longitudinal rods, or longitudinal beams may be used as shown and the slab supported on these. The pressure on the foundation is determined by considering the

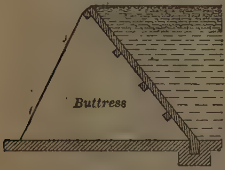

Fig. 62

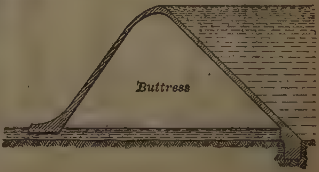

Fig. 63

resultant of water pressure and weight of dam. The buttresses or cross walls are subjected to compressive stresses only. Ample longitudinal reinforcement should be provided to thoroly bind the structure together.

Where the foundation is solid rock the buttresses may rest directly upon it, but where a pile foundation is required a continuous floor should be provided under the entire structure and covering the heads of the piles. If well bonded together a reinforced concrete dam is exceedingly strong and stable and must fail as a unit if at all. Great care must be exercised in securing a thoro connection between the foundation and dam by means of a deep concrete connection at the upper toe. When founded on piles all seepage of water underneath the dam must be cut off by a line of sheet piling at the upper toe. This piling should be well covered by the concrete. To prevent upward pressure on the floor, or upon the lower face, drain openings should be provided connecting the interior with the tail-water level.

38. Dock and Harbor Walls. Breakwaters

Forces to be Considered. Dock and harbor walls not subjected to wave action are designed as retaining walls subjected to the earth pressure on the land side with surcharge due to loads on docks or quays. The action of heavy ice may require consideration, both with respect to its pressure and its wearing effect. In the case of outer protecting works, such as breakwaters, piers or jetties, the action of the waves is the principal force to be considered.

Force of Waves. The pressure or force exerted by waves depends upon the velocity of the impinging water and its depth. The former depends upon the exposure, wind velocity, depth of water and shape of bottom. The velocity in shallow water is approximately proportional to the square root of the height of wave and the depth of water. The pressure due to impact is proportional to the square of the velocity and therefore is also approximately proportional to the height of wave and depth of water.

Pressures due to impact of high waves have been measured in several cases. For waves 10 ft high, pressures of 1800 to 3000 lbs per sq ft have been observed, while for extremely high waves of 25 to 30 ft, pressures of 6000 to 7000 lbs per sq ft have been noted. In exposed locations large blocks of concrete weighing 3000 tons have been moved, requiring an estimated average pressure of over 2 tons per sq ft. The maximum intensity occurs at about the mean water level, is zero at the crest and gradually diminishes towards the bottom. The impact of waves on either vertical or sloping walls causes large masses of water to be thrown upward which, on falling, strikes severe blows upon the upper surface of the wall or breakwater, tending to rupture it.

Fig. 64

Fig. 65

Breakwaters are constructed in various ways. Timber cribs filled with stone are suitable for moderate wave action. The same construction, capped with concrete, as shown in Fig. 64, forms a strong combination. The struc-

ture is built to above low-water level by the use of large concrete blocks. The whole is then capped with mass concrete. Fig. 65 illustrates the same general

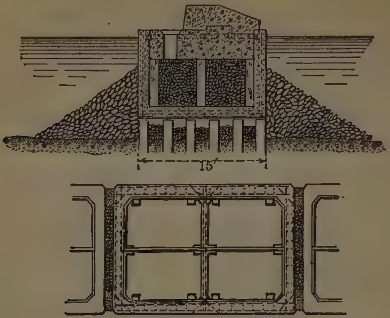

Fig. 66. Reinforced Concrete Caisson

type in which the base is built of mass concrete deposited under water within temporary frames. Separate concrete blocks are often dovetailed or clamped together to give additional stability.

Foundations for Breakwaters, not located on solid rock, are commonly prepared by first depositing a broad layer of rubble stone at a flat slope and leveling up the top by the aid of careful soundings or by means of divers. Piles are also used, which are cut off below water level and protected by heavy riprap. Bags of cement have also been frequently employed for the purpose.

The Reinforced Concrete Caisson furnishes perhaps the most satisfactory method of construction yet devised. Fig. 66 illustrates such a form used by W. V. Judson on Lake Michigan. The caisson is built at shore and towed to place. It is then sunk by filling one or more compartments with stone, and the whole is capped with concrete, sufficiently reinforced to preserve the monolithic character of the structure. Similar caissons have been employed for quay walls and breakwaters in European works. At Rotterdam, caissons were employed of dimensions 131 ft × 32 ft × 43 ft high. These were built in dry docks, towed to place and brought into alinement by means of a tongue-and-groove joint. Some cells were filled with concrete, others with sand or rock. Reinforced concrete possesses great advantages for construction of this sort over ordinary masonry or concrete construction by reason of its monolithic character.

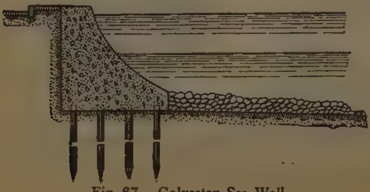

Fig. 67. Galveston Sea Wall

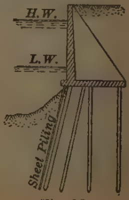

Fig. 68

Concrete sheet piling, fitted with tongue and groove, has been used successfully for breakwaters. Two rows of such piling are driven, after which the intervening space is filled with sand and the whole capped with a superstructure of reinforced concrete.

Sea Wall. Fig. 67 illustrates the Galveston sea wall. It is founded on piles and constructed of mass concrete, but this is reinforced near the front face, as shown.

Dock and Harbor Walls are built, in the main, like ordinary retaining walls. Frequently a pile foundation is necessary. If the piles are driven in

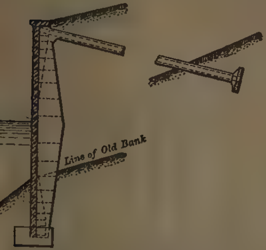

Fig. 69. An Anchored Wall

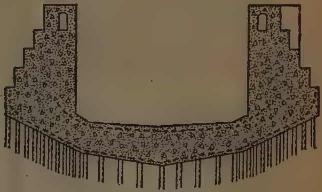

Fig. 70. Canal Lock

soft material, one or more rows should be driven at a considerable inclination in order to resist the horizontal component of the pressure. The walls are commonly constructed of concrete, either in loose blocks or of mass concrete, plain or reinforced. Where the wear from ice is heavy a concrete wall may well be faced with granite or with other hard stone. Reinforced concrete possesses the same advantages in this type of wall as for ordinary retaining walls and is built of the same general form. Fig. 68 shows the essential features.

Anchored walls of reinforced concrete are often advantageous, especially in the construction of new walls along a river front where anchorage is readily secured. Fig. 69 illustrates such a construction as carried out at Berne, Switzerland. The wall consists of a series of anchored beams connected by slab construction. The stability of the wall is measured by the stability of the entire mass between the wall and the anchorage.

Dry Docks and Canal Locks. In this construction reinforced concrete is of advantage, not only in the construction of the walls, but also in the floors, enabling them to be readily designed against upward pressure and in one piece with the walls. Fig. 70 is a section thru the lock on the Charles River, Boston, showing this feature.

39. Reservoirs, Tanks, Bins

For Covered Reservoirs reinforced concrete is very well adapted. The rectangular form with flat cover is usually the most convenient; its design

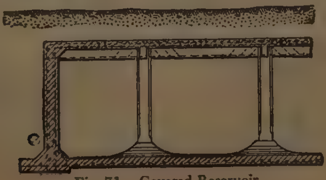

Fig. 71. Covered Reservoir

involves the same features as building design with the additional one of imperviousness. Fig. 71 illustrates a typical design of this kind for Indianapolis. (Eng. News, Oct. 15, 1908.) Especial care is required in such structures to prevent contraction cracks by the use of a liberal amount of reinforcement Imperviousness may be secured by methods described in Art. 15.

Fig. 72 illustrates the general form of construction used in a large elevator at Baltimore. The walls of the bins are 8 in thick and are reinforced with vertical rods of ½-in round steel, spaced 3 ft apart, and with horizontal bars ⅜ in by 1 to 1¾ in, spaced 12 in apart. The hopper bottoms, the tunnels for the conveying machinery and the foundation floor resting upon the piles are all of reinforced concrete. (Eng. Record, Feb. 20, 1909.)

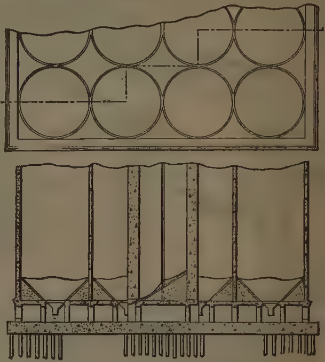

Fig. 72. Grain Elevator

Elevated towers and tanks may also be made of concrete, but high pressures are difficult to deal with. Bins and coal pockets are structures for which concrete is well adapted. For the storage of coal bins of unprotected steel are not durable, but reinforced concrete furnishes an almost ideal material, lending itself readily to the necessary form for strength and furnishing the desired durability.

Reinforced concrete is advantageously used in other minor forms of structures and structural elements. Noteworthy among such uses is its employment for fence posts and poles for various purposes. In building construction it is found that sills, lintels, steps and staircases are usually cheaper in reinforced concrete than in stone or metal.

40. Conduits, Sewers, Pipes

For Conduits not under pressure, large sewers and the like, reinforced concrete lends itself to convenient and economical construction. As to the

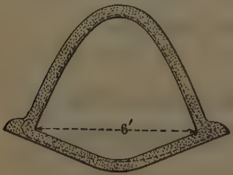

Fig. 73

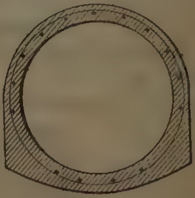

Fig. 74

analysis and design, these structures are only special cases of the monolithic pipe or box which was discust in Art. 35. The character of the foundation

and convenience in construction will lead to various modifications of design Fig. 73 is a typical cross-section of a large sewer for Harrisburg, Pa. A mesh of expanded metal is used for reinforcement, arranged to resist positiv moments excepting at bottom and corners. Fig. 74 illustrates a design of a circular pipe or conduit molded in place.

Reinforced concrete has also been used to some extent for pipes under pressure, but it is very difficult to secure imperviousness under heads of considerable magnitude. In pressure pipes the tensile stress is entirely taken by the steel, the concrete furnishing merely the impervious layer and resisting bending due to earth loading.

Pipes of reinforced concrete separately molded in sections have been successfully used for moderate pressures with a considerable economy over metal. The joining of such pipes is accomplished by overlapping of the reinforcement. One method of doing this is illustrated in Fig. 75.

41. Chimneys

General Design. A reinforced concrete chimney consists of the outer main shell, the inner shell or lining, and the base. The outer shell is made 4 to 6 in thick at the top and 8 to 12 in at the bottom, depending upon the height and diameter. The inner shell is generally made 4 in in thickness, and of concrete or fire brick, the latter being preferable for high temperatures. The amount of reinforcement required is small, as the load is purely vertical and of small amount. Half-inch rods spaced 1 to 2 ft apart are ample. The lining is generally carried up about one-third the height, but the large amount of cracking which has occurred in some chimneys just above the lining indicates the desirability of more extensive linings. The outer shell is designed to resist wind pressure and also with some reference to the temperature stresses. The base must be of such an area that the maximum earth pressure due to vertical load and wind pressure will not exceed safe limits. The thickness is then determined with reference to the bending and shearing stresses which are caused by these pressures.

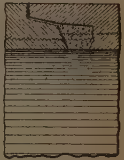

Fig. 75

Wind Stresses. The stresses at any given section, distance h below the top, will be determined. The notation is as follows:

A = area of chimney section under consideration, A_s = total area of steel sections,
W = weight of superincumbent portion of chimney,
P = wind pressure on that portion, M = bending moment at the section,
e = distance from the center of the section to where the resultant of the weight and wind pressure cuts the section ("eccentric distance"),
S_c = unit stress in concrete adjacent to the steel at lee side,
S_c' = unit stress in concrete adjacent to steel at windward side,
h = distance from top to section considered, r = mean radius of section,
S_s = unit stress in steel at the windward side, p = steel ratio, A_s/A,
m = a coefficient such that $S_c = mW/A$, m' = a coefficient such that $S_c' = m'W/A$.
n = ratio of modulus of elasticity of steel to that of concrete, taken at 15.

The value of e is first to be determined by the formulas:

$$M = \tfrac{1}{2} Ph \qquad e = M/W = Ph/2W$$

If e is greater than about $\tfrac{1}{2} r$ there will be tension on the windward side. In this case it is assumed that the steel will carry all the tension. If the tensile stress is small, however, the concrete will not be overstrest and it may be desirable to calculate its stress on this assumption. Two cases will therefore

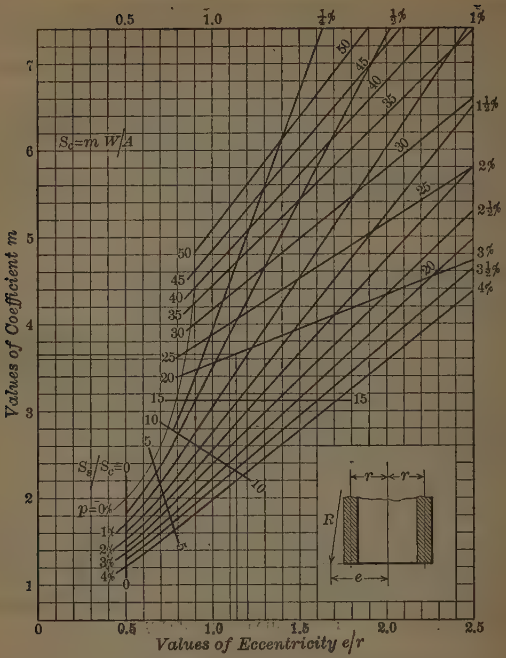

Fig. 76. Diagram for Reinforced Concrete Chimneys

be considered: (1) Where the stresses are all compressive, or the tension on the windward side is small (less than 50 lbs per sq in, say), and (2) where there are large stresses on the windward side and the concrete is neglected.

Case 1. Having found e as above described the unit stresses are found by

$$S_c = \frac{1 + 2\,e/r}{1 + np}\,\frac{W}{A}, \quad \text{and} \quad S_c' = \frac{1 - 2\,e/r}{1 + np}\,\frac{W}{A}$$

Case 2. Practical formulas for this case cannot be stated, but the curved lines in Fig. 76 give values of m for different values of e/r and of p. Then $S_c = m\,W/A$. The straight lines give the ratio of S_s/S_c for the given value of e/r and p; from this ratio the value of S_s is determined.

The maximum concrete stress is slightly greater than the values calculated by the above formulas, as these give the stresses adjacent to the steel. The results are, however, sufficiently accurate.

The Wind Pressure commonly assumed is 25 to 50 lbs per sq ft on the

projected area, but this is probably higher than necessary. Experiments made at the Eiffel Tower and at the National Physical Laboratory of England show that the pressure per sq ft on square, flat surfaces, from 10 to 100 square feet in extent, is 0.0032 times the square of the wind velocity in miles per hour. Taking the velocity at 100 miles per hour and the pressure on a cylindrical surface at ⅔ that on the projected plane, a value of 20 lbs per sq ft is deduced, which should be ample.

Temperature Stresses. Both theoretical considerations and evidence from practice indicate that temperature stresses are likely to be high, especially near the top of the lining. They occur in directions both vertically and circumferentially and are due to the high temperature of the interior as compared to the exterior. Both vertical and horizontal cracks of the exterior are of frequent occurrence and are doubtless due to this cause. Assuming the coefficient of expansion to be the same at all temperatures, a theoretical analysis indicates that for 1% of circumferential reinforcement the compressive stress in the concrete on the interior is as much as 350 lbs per sq in for each 100° difference in temperature between exterior and interior surfaces; the corresponding stress in the steel is about 5000 lbs per sq in. Approximately the same stresses are caused in the vertical reinforcement. The stresses in the concrete increase somewhat with increase in reinforcement, but decrease in the steel; they are proportional to the differences in temperature of outer and inner surfaces. A difference in temperature of 200° will therefore bring the compressive stress well up to safe limit. So far as temperature stresses are concerned it is advantageous to use a small amount of steel.

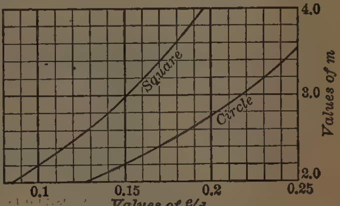

Fig. 77

Bases. The bases of chimneys are designed in the same manner as column footings. In this case, however, the earth pressures must be determined with reference to the effect of wind pressures and vertical load combined. The resultant of the wind pressure, the weight of chimney, the earth filling above the base and the base itself is to be first determined. Suppose the resultant cuts the base at a distance e from the center. Let $W=$ total vertical load, $M=$ wind moment at bottom of base, $A=$ area of base, $d=$ maximum diameter of rectangular base, or diameter of circular base, and $p=$ maximum pressure on foundation per unit area. Then $e=M/W$. The values of p are as follows:

For rectangular bases, wind acting in the direction of the diagonal, and for values of e less than $d/12$, $p=W/A+12\,M/Ad$. For circular bases, and values of e less than $d/8$, $p=W/A+8\,M/Ad$. For either rectangular or circular bases, and values of e greater than the limits above given, $p=mW/A$, in which m is to be taken from Fig. 77.

42. Lighthouses

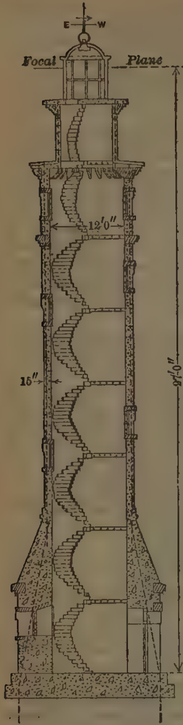

Fig. 78

The Forces to be Considered in lighthouse construction which are of special importance are wind and wave pressures. The wind pressures assumed should be a maximum, as lighthouses are commonly exposed to severe gales. Wind velocities reaching 130 to 150 miles per hour have been measured in the hurricanes which are not uncommon along certain coasts. The corresponding pressures will probably reach 60 to 80 lbs per sq ft. Where located off shore, lighthouses must be constructed to resist wave action in the same manner as breakwaters. The relative importance of such structures requires, however, a somewhat more ample margin of safety.

The Foundations are built similar to those of breakwaters. A heavy rubble foundation surmounted by a crib or caisson is a common arrangement. The reinforced concrete caisson is well adapted to this work. In any case the upper mass of the foundation should be of such a character as to enable the superstructure to be readily anchored thereto, and of sufficient weight and size to give ample stability to the entire structure.

The Superstructure in exposed locations is generally built of stone or concrete masonry. The necessity of securing a monolithic structure, especially in the lower portion, makes reinforced concrete a particularly satisfactory material for this work. When stone masonry is used the desired result must be secured by the use of dovetail joints and iron clamps. When designed in reinforced concrete the necessary anchorage to the foundation and the reinforcement of the superstructure are readily calculated in a manner similar to that employed in chimney design.

Fig. 78 illustrates the reinforced concrete lighthouse constructed at Maniguin Island, P. I. (Trans. Am. Soc. C. E., Vol. 58, 1907.) This is built on shore and is anchored to solid rock by the vertical reinforcing bars. The vertical reinforcement consists of ¾-in bars spaced 6 inches apart up to the floor of the watch room and 12 inches apart above. Circular reinforcement is also provided of ½-in bars spaced 12 inches apart. All floors and stairways are also of reinforced concrete, as well as door and window trimmings. The factor of safety against wind pressure is estimated to be 7.

SECTION 6

MASONRY, FOUNDATIONS, EARTHWORK

BY

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STONE QUARRYING AND CUTTING

STONE AND BRICK MASONRY

FOUNDATIONS ON LAND

FOUNDATIONS UNDER WATER

EARTHWORK

STONE QUARRYING AND CUTTING

1. Building Stone

Classification. Building stones may be classified according to their geological position, physical structure, or chemical composition. Geologically rocks are divided into igneous, metamorphic, and sedimentary. Basalt, diorite, and granite are examples of igneous rocks; gneiss, marble, and slate of metamorphic; and sandstone, limestone, and shale, of sedimentary. The geological position of a rock has but little connection with its properties as a building material, altho as a rule the more ancient rocks are the stronger and more durable. Physically rocks are divided into stratified and unstratified. Most of the limestones and sandstones are examples of the first class and granite, trap, basalt, and lava are examples of the second. Chemically stones are divided into siliceous, argillaceous, and calcareous. Siliceous stones are those in which silica is the predominating constituent. Usually the structure is crystalline granular; and the strength and durability of the stone is dependent upon that of the material which cements the grains together. Argillaceous or clayey stones are those in which alumina gives the stone its characteristic properties, slate being the best example; as a rule they are not durable. Calcareous stones are those in which carbonate of lime predominates. The durability of stones of this class depends upon their compactness, the more porous being acted upon by the acids in the atmosphere and by rain-water, and are disintegrated by freezing.

Trap is the strongest of building stones, and is exceedingly durable; but it is little used, owing to the great difficulty with which it is quarried and wrought. It is exceedingly tough rock; and, being generally without cleavage or bedding, is especially intractable under the hammer or chisel.

Granite is the strongest and most durable of all the stones in common use. It generally breaks with regularity, and may be quarried in simple shapes with facility; but it is extremely hard and tough, and therefore can be wrought into elaborate forms only with a great expenditure of labor.

Limestone is widely distributed and is the most common building stone. There are many varieties which differ in color, composition, and value for engineering and building purposes, owing to the differences in the character of the deposits and chemical combinations entering into them. Large quantities of limestones and dolomites are quarried in nearly all of the Western States. These are mostly of a dull grayish color, and their uses are chiefly local. The light-colored oolitic limestone of Bedford, Indiana, is, however, an exception to this rule. Not only are the lasting qualities fair and the color pleasing, but its fine even grain and softness render it admirably adapted for carved work.

Sandstones vary greatly in color and fitness for building purposes, but they include some of the most beautiful, durable, and highly valued materials used in construction. Whatever their differences, they have this in common, that they are chiefly composed of grains of quartz which are to a greater or less degree cemented and consolidated. The durability of sandstones varies with both their physical and chemical composition. Sandstones composed of nearly pure silica which is well cemented are as resistant to weather as granite, and are very much less affected by the action of fire. Taken as a whole, they may be regarded as among the most durable of building materials. When first taken from the quarry and saturated with quarry water, they are frequently very soft, but on exposure become much harder by the precipitation of the soluble silica contained in them.

As a general rule, the densest, hardest, and most uniform stone will most nearly meet the requisites for a good building stone. The fitness of stone for structural purposes can be determined approximately by examining a fresh fracture. It should be bright, clean, and sharp, without loose grains, and free from any dull, earthy appearance. The stone should contain no "drys," that is, seams containing material not thoroly cemented together, nor "crow-foots," that is, veins containing dark-colored, uncemented material.

2. Tests for Building Stone

Weight or Density is an important property, since upon it depends to a large extent the strength and durability of the stone. To find the exact weight per cubic foot of a given stone, it is generally easier to find its specific gravity first, and then multiply by 62.4, the weight, in pounds, of a cubic foot of water. This method obviates, on the one hand, the expense of dressing a sample to regular dimensions, or, on the other hand, the inaccuracy of determining the volume of a rough, irregular piece. Notice, however, that this method determines the weight of a cubic foot of the solid material, which will be a little more than the weight of a cubic foot of the stone as used for structural purposes. In finding the specific gravity there is some difficulty in getting the correct displacement of porous stones, and all stones are more or less porous. The following method is most frequently used. All loose grains and sharp corners having been removed from the sample and its weight taken, it is immersed in water and weighed there after all bubbling has ceased. It is then taken out of the water, and, after being compressed lightly in bibulous paper to absorb the water on its surface, is weighed again. The specific gravity is found by dividing the weight of the dry stone by the difference between the weight of the saturated stone in air and in water.

Average values for weights of building stones are as follows, in pounds per cubic foot: 167 for granites, 158 for limestones, 170 for marbles, 139 for sandstones, 174 for slates.

Crushing Strength is determined by applying measured force to prisms until they fail. The test specimen is usually a cube, altho theoretically it should be a prism having a height one and a half to two times the least lateral dimension, so as to allow free shearing action in the crushing. The strength of a cube is about 9 or 10 percent stronger than a prism one and a half times as high as broad. The strength of a cube per square inch of bed area is independent of the size, considerable literature to the contrary notwithstanding.

The position of the test specimen with reference to the bedding planes of the rock in its native position has a very important relation to the strength of the stone. The direction in which a stone splits most easily is called the rift, and the next easiest the grain, while the direction in which the resistance to splitting is the greatest is called the head. The test specimen will be the strongest, if the pressure is applied perpendicular to the rift. In most cases the rift is horizontal, i.e., is parallel to the natural bed; but in some cases the rift makes an angle with the natural bed. Horizontally bedded stones are quarried by blasting or wedging along the natural lines of cleavage, or by channeling and wedging along the laminations; but if the rift is not horizontal, it is common practise to cut out blocks without reference to the natural seams, since the quarrying machines run most easily upon horizontal tracks, and since it is desirable to maintain a level quarry floor. In the last case, then, there is a difference between the rift and the "natural bed"; and in preparing test specimens care should be taken to have two sides of the cubes parallel to the rift, which should be marked so that the pressure may be applied on these faces. It is not sufficient to have the faces of the cubes parallel and perpendicular to the broadest face of the original block, for the latter may not have been cut out with reference to the rift. The rift can be determined by a careful examination of the block. The failure to apply the pressure perpendicular to the rift doubtless accounts for part of the large difference in strength of different specimens found by most experimenters.

If the specimen is drest by hand, the concussion of the tool greatly affects its internal conditions, particularly with test specimens of small dimensions. With 2-inch cubes. the tool-drest specimen usually shows only about 60% of the strength of the sawed sample. The sawed sample most nearly represents the conditions of actual practise. The disintegrating effect of the tool in dressing is greater with small than with large specimens.

Tests of the Crushing Strength of blocks of stone are useful only in comparing different stones, and give no idea of the strength of structures built of such stone or of the crushing strength of stone in large masses in its native bed. Then, since it is not possible to have the stone under the same conditions while being tested that it is in the actual structure, it is best to test the stone under conditions that can be accurately described and readily duplicated. Therefore it is rapidly coming to be the custom to test the stone between metal pressing surfaces. To obtain definite and precise results, these surfaces should be rubbed or ground perfectly smooth; but as this is tedious and expensive, it is quite common to reduce the bed-surfaces to planes by plastering them with a thin coat of plaster of paris, and inverting the cube on a sheet of plate glass or allowing the plaster to set under a small pressure between the metal pressing surfaces of the testing machine. With the stronger stones, specimens with plastered beds will show less strength than those having rubbed beds, and this difference will vary also with the length of time the plaster is allowed to harden. With a stone having a strength of 5000 to 6000 lb per sq in, allowing the plaster to attain its maximum strength, this difference varied from 5 to 20 percent, the mean for ten trials being almost 10 percent of the strength of the specimen with rubbed beds.

Compressive Strength of Stones

Cubes set in plaster of paris

Quarries	Tests	Stone	Crushing strength, lb per sq in		
			Min.	Mean	Max.
16	37	Granite	2045	19 379	27 738
15	35	Limestone	3634	9 438	24 121
10	21	Marble	6872	12 709	17 780
19	24	Slate	4353	9 333	15 163

The above table shows the average compressive strength in pounds per square inch for all the stones tested at the Watertown Arsenal from 1883 to 1905, except two stones noted below. The quantities in the columns headed Min. represent the strength of the weakest stone of each particular kind, and are the mean of three or more tests; and similarly for the quantities in the columns headed Max. The results in the table are for test specimens prepared in the usual way; but the results for two test specimens which were prepared with special care are not included in the table. The test specimens of one of these stones, granite from Salisbury, N.C., were drest out to 2½ inches on a side and then reduced on a rubbing bed to 2 inches; and the mean crushing strength of six cubes was 49 457 lb per sq in, the maximum for a single cube being 51 900. The results for this stone are the highest known to have been obtained for the crushing strength of any stone or brick, and are probably largely due to the manner of dressing the specimens and also to unusual care in selecting the sample, in bedding the cubes, and in placing them in the testing machine; and hence are not to be taken as representative. For similar reasons the argillaceous limestone from which Rosendale natural cement is made gave a crushing strength of 40 875 lb per sq in.

3. Durability of Building Stone

By Two Methods a judgment can be formed as to the durability of a building stone, which may be distinguished as the natural and the artificial. The former is the better, since it involves the exact agencies concerned in

the attack on the stone and also long periods of time. The NATURAL METHOD of studying durability is to examine the surfaces of old buildings, bridge piers, monuments, tombstones, etc., which have been exposed to atmospheric influences for years. A durable stone will retain for a long time the tool marks made in working it, and preserve its edges and corners sharp and true. The ARTIFICIAL METHOD of testing durability is based upon the assumption that the relative durability of stones is proportional to their crushing strength, their absorptive power, their resistance to freezing, and their solubility in acids; but in making the tests each element acts by itself, while in the structure the stone is exposed to the combined action of all these methods of attack, and their action may be, and frequently is, different from that when acting separately. The principal tests made to determine the durability of a stone are the absorptive power, the solubility in sulfuric and hydrochloric acids separately and combined, the resistance to fire, the loss due to freezing and thawing, and the artificial freezing test, which consists in soaking the sample in a solution of sulfate of soda and then hanging it up to dry, the crystallizing of the salt in the pores of the stone having an effect somewhat similar to that of freezing of water. None of these tests give wholly satisfactory results.

INCREASE IN DURABILITY may be secured by proper seasoning and by finishing the surface in a suitable manner. All stones, and especially limestones and sandstones, when first quarried contain considerable quarry sap; and when full of sap the stone works considerably easier under the tool than when well seasoned. If a stone freezes while full of quarry sap, it is nearly certain to crack; but if it is first allowed to season, it is not likely to be appreciably damaged by a single freezing. The hardening by seasoning also adds materially to the weathering qualities of the stone. The increased strength and durability caused by seasoning are due to the fact that the sap holds in solution a small amount of calcareous or siliceous matter, and by the process of seasoning this material is drawn to the surface and is deposited in the pores of the stone by the evaporation of the sap, the matter in solution thus becoming an additional cementing material and binding the grains more firmly together. It is surprising that an otherwise inappreciable amount of liquid can produce such a marked effect. The method of dressing a stone has an important bearing upon its durability. If the surface is finished with a tool similar to the bush hammer or the patent hammer, the heavy blows break the grains and produce minute fissures and render it much more susceptible to the action of frost. Granite and other compact crystalline rocks are most durable with a rock-face finish, that is, a surface untouched by chisel or hammer; while the softer and more absorbent stones are usually most durable when finished with a sawed or rubbed surface.

Many methods have been devised for preventing or checking the action of the weather upon building stones; but none of them are satisfactory or very efficient. These preservatives consist of some liquid into which the stone may be dip. or which may be applied with a brush to its outer surface, to fill the pores and prevent the access of moisture. Paint, coal tar, linseed oil, paraffin, and numerous chemical preparations have been used. To resist the effects of both pressure and weathering a stone should be placed on its natural bed, which precaution adds considerably to the durability of a laminated stone.

4. Methods of Quarrying

Hand Tools. When the stone is thin-bedded, it may be quarried by hand tools alone. The principal tools are pick, crowbar, drill, hammer, wedge, and plug and feathers. The plug is a narrow wedge with plane faces, and the feathers are wedges flat on one side and rounded on the other.

When a plug is placed between two feathers, the three will slip into a cylindrical hole; and if the plug is then driven, it exerts a great force. If these plugs and feathers are placed a few inches apart in a row, and all driven at the same time, the stone will be cracked along the line of the holes, even tho it be comparatively thick. The drill ordinarily used to cut the holes for the plug and feathers is a bar of steel furnished with a wide edge sharpened to a blunt angle and hardened. It is operated by one man, who holds the drill with one hand and drives it with a hammer in the other, rotating the drill between blows. The holes are usually from ⅜ to ¾ of an inch in diameter. Sandstones and limestones occurring in layers thin enough to be quarried as above are usually of inferior quality, suitable only for slope walls, paving, riprap, and concrete.

When the stone is hard a pneumatic drill is sometimes used to make the holes for the plug and feathers. It is handled by one man, and the power may be furnished by a fixt compress-air plant or a portable gasoline-driven compressor.

Channeling and Wedging. Channeling consists in cutting long narrow

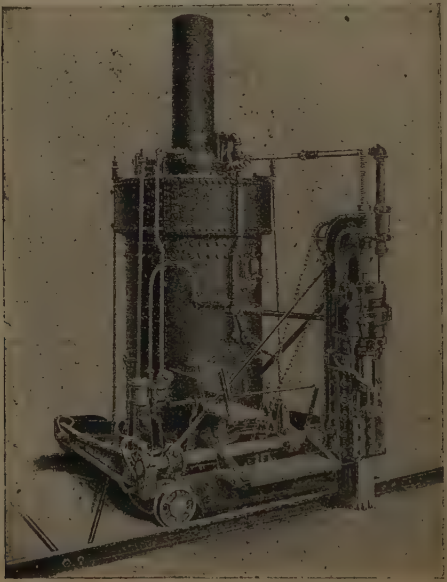


Fig. 1. Single-swivel Channeler with Boiler.

channels in the rock to free the sides of large blocks of stone. A channeling machine consists of an engine running on a track on the floor of the quarry and operating a drill bit which cuts the channel as the machine is run back

and forth along the track (see Figs. 1 and 2). The drill is sometimes a rotary steel or diamond bit, but is usually a percussion bit. The percussion channeler is ordinarily employed for marble, massive limestone, and thick-bedded sandstones; and the diamond drill for the harder stones. With the percussion channeler, the channels are 1 to 3 inches wide according to depth, and may be 10 to 14 feet deep, different lengths of drill rods being used as the depth of the channel increases. The diamond channeler first bores a series of holes close together, and then bores out the partitions between the holes; and can

Fig. 2. Percussion Rock Drill.

operate at any angle with the vertical, even horizontal. If the rock is not in layers, after the necessary channels have been cut around the block, it is necessary to undercut the block in order to release it. This is accomplished by drilling a series of holes along the bottom, called "gadding" by quarry-men. The block is then split from its bed by means of wedges or plug and feathers.

Quarrying with Explosives. In this method drill-holes are put down to the depth to which the rock is to be split, and the requisite amount of powder or other explosive put in, covered with sand, and fired by a fuse. Sometimes numerous charges in a line of drill-holes are fired simultaneously by means of electricity. Coarse gunpowder is the explosive ordinarily used, since the quick-acting explosives, like nitroglycerine and dynamite, have a

tendency to shatter the stone and break it in many directions. For quarrying each class of rock there is a characteristic method employed, which is, however, varied in detail in different quarries. The minor details of quarry methods are as various as the differences existing in the textures, structures, and modes of occurrences of the rocks quarried. Even such an apparently unimportant matter as the form of the bottom of the drill-hole into which the explosive is put has a very marked effect. If bored with a hand-drill, the hole is generally triangular at the bottom, and a blast in such a hole will break the rock in three directions. In some quarries the lines of fracture are made to follow predetermined directions by putting the charge of powder into canisters of special forms.

Drills are of three forms. The JUMPER is a drill similar to that used for drilling holes for plugs and feathers, except that it is larger and longer. It is usually held by one man, who rotates it between the alternating blows from hammers in the hands of two other men. CHURN-DRILLS are long, heavy drills, usually 6 to 8 feet in length. They are raised by the workmen, let fall, caught on the rebound, raised and rotated a little, and then dropt again, thus cutting a hole without being driven by the hammer. They are more economical than jumpers, especially for deep holes, as they cut faster and make larger holes than hand-drills. MACHINE ROCK-DRILLS bore much more rapidly than hand-drills, and also more economically, provided the work is of sufficient magnitude to justify the preliminary outlay. They drill in any direction, and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam directly, or usually by air comprest by steam, electric or water power.

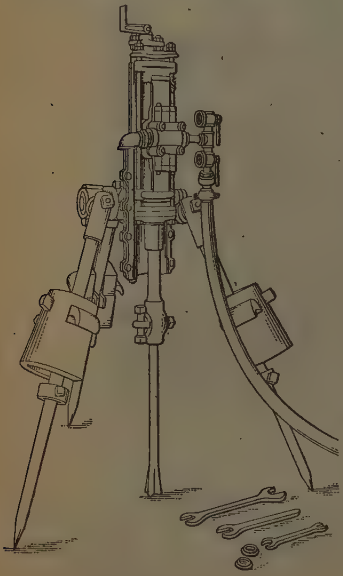


Fig. 3. Percussion Rock Drill.

Rock-drilling machines use either percussion or rotary drills. The method of action of the PERCUSSION-DRILL is the same as that of the churn-drill already described. The usual form is that of a cylinder, in which a piston is moved by steam or compress air, and the drill is attached to this piston so as to make a stroke with every complete movement of the piston. An automatic device causes it to rotate slightly at each stroke. Fig. 3 shows one of the best-known air-driven percussion rock-drills. This machine is often called a tripod drill. Fig. 4 shows a section of the drill and also an electric motor for compressing the air. This form is usually called an electric air-drill.

Of the ROTARY DRILLS there are two forms, one in which the cutting is done by teeth on the end of a hardened steel tube and the other by diamonds. In either case the drill-

rod is a long tube, revolving about its axis, kept in contact with the rock, and by its rotation cuts in it a cylindrical hole, generally with a solid core in the center. The drill-rod is fed forward, or into the hole, as the drilling proceeds. The débris is removed from the hole by a constant stream of water which is forced to the bottom of the hole thru the hollow drill-rod, and which carries the débris up thru the narrow space between the outside of the drill-rod and the sides of the hole. The DIAMOND DRILL is the only form of rotary rock-drill extensively used in America. The tube has a head at its lower end, in which are set a number of carbons or black diamonds. The diamonds usually project slightly beyond the circumference of the head, which is perforated to permit the ingress and egress of the water used in removing the débris from the hole and at the same time prevent the head from binding in the hole. When it is desirable to know the precise nature and stratification of the rock penetrated, the cutting points are so arranged as to cut an annular groove in the rock, leaving a solid core, which is broken

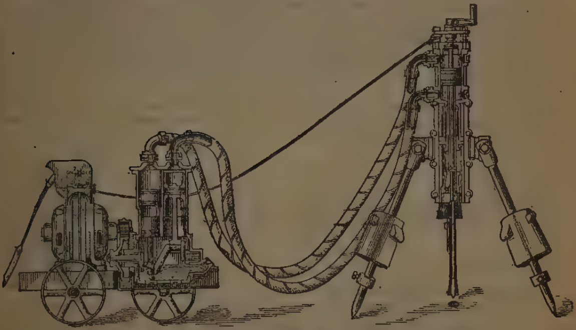


FIG. 4. Electric-air Percussion Rock-Drill.

off and lifted out whenever the head is brought up. Where it is not desired to preserve the core intact, a solid boring-bit is used instead of the core bit. They are made of any size up to 15 inches in diameter.

Dynamite is the name given to any explosive which contains nitroglycerine mixt with a granular absorbent. If the absorbent is inert, the mixture is called true dynamite; if the absorbent itself contains explosive substances, the mixture is called false dynamite. Dynamite is exploded by means of sharp percussion, which is applied by means of a cap and fuse. The cap is a hollow copper cylinder, about ¼ inch in diameter and an inch or two in length, containing a cement composed of fulminate of mercury and some inert substance. The cap is called single-force, double-force, etc., according to the amount of explosive it contains. TRUE DYNAMITES must contain at least 50% of nitroglycerine, otherwise the latter will be too completely cushioned by the absorbent, and the powder will be too difficult to explode. FALSE DYNAMITES, on the contrary, may contain as small a percentage of nitroglycerine as may be desired, some containing as little as 15%. The added explosive substances in the false dynamites generally contain large quantities of oxygen, which are liberated upon explosion, and aid in effecting the complete combustion of any noxious gases arising from the nitroglycerine. The false are generally inferior to the true dynamites, since the bulk of the former is increased in a higher ratio than its power; and as the cost of the work is largely dependent upon the size of the drill-holes, there is no economic gain. High-power

dynamites are required for hard and refractory rock, but for the softer rock a low-power is better. In quarrying, 40% dynamite is ordinarily used.

Gunpowder is sold in kegs of 25 lbs, costing about $2.00 to $2.25 per keg, exclusive of freight, which is very high, owing to the risk. Dynamite is sold in cylindrical, paper-covered cartridges, from ⅞ of an inch to 2 inches in diameter and 6 to 8 inches long, or longer, which are packed in boxes containing 25 or 50 pounds each. They are furnished, to order, of any required size. The price per pound ranges from 15 cents for 15% nitroglycerine to 20 cents for 75% nitroglycerine.

5. Tools for Stonecutting

A knowledge of the tools used in stonecutting is necessary for an intelligent understanding of the methods of preparing stones and is also necessary for a recognition of the names of the various kinds of drest surfaces. The following names and descriptions of tools were first proposed in 1877 by a committee of the American Society of Civil Engineers and have been widely adopted and used:

The Double-Face Hammer (Fig. 5) is a heavy tool weighing from 20 to 30 pounds, used for roughly shaping stones as they come from the quarry and for knocking off projections. This is used only for the roughest work.

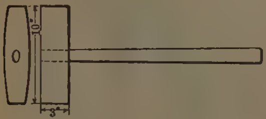


Fig. 5. Double-Face Hammer

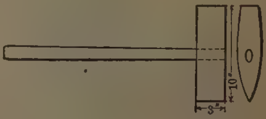


Fig. 6. Face Hammer

The Face Hammer (Fig. 6) has one blunt and one cutting end, and is used for the same purpose as the double-face hammer where less weight is required. The cutting end is used for roughly squaring stones, preparatory to the use of finer tools.

The Cavil (Fig. 7) has one blunt and one pyramidal, or pointed, end, and weighs from 15 to 20 pounds. It is used in quarries for roughly shaping stone for transportation.

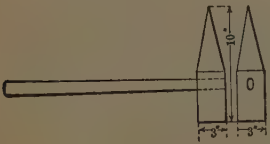


Fig. 7. Cavil

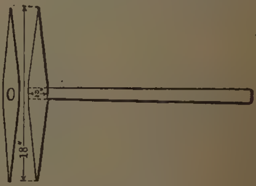


Fig. 8. Pick

The Pick (Fig. 8) somewhat resembles the pick used in digging, and is used for rough dressing, mostly on limestone and sandstone. Its length varies from 15 to 24 inches, the thickness at the eye being about 2 inches.

The Ax or Peen Hammer (Fig. 9) has two opposite cutting edges. It is used for making drafts around the arris, or edge, of stones, and in reducing faces, and sometimes joints, to a level. Its length is about 10 inches, and the cutting edge about 4 inches It is used after the point and before the patent hammer.

The TOOTH AX (Fig. 10) is like the ax, except that its cutting edges are divided into teeth, the number of which varies with the kind of work required. This tool is not used on granite and gneiss.

Fig. 9. Ax or Peen Hammer.

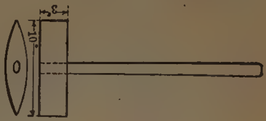


Fig. 10. Tooth Ax

The BUSH HAMMER is a square prism of steel whose ends are cut into a number of pyramidal points. The length of the hammer is from 4 to 8 inches, and the cutting face from 2 to 4 inches square. The points vary in number and in size with the work to be done. One end is sometimes made with a cutting edge like that of the ax.

The CRANDALL, Fig. 11, is a malleable-iron bar about two feet long, slightly flattened at one end. In this end is a slot 3 inches long and ⅜ inch wide. Thru this slot are passed ten double-headed points of ¼ inch square steel, 9 inches long, which are held in place by a key.

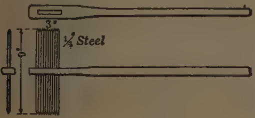


Fig. 11. Crandall

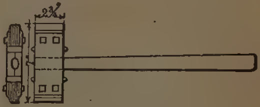


Fig. 12. Patent Hammer

The PATENT HAMMER (Fig. 12) is a double-headed tool so formed as to hold at each end a set of wide thin chisels. The tool is in two parts, which are held together by the bolts which hold the chisels. Lateral motion is prevented by four guards on one of the pieces. The tool without the teeth is 5½ by 2¾ by 1½ inches. The teeth are 2¾ inches wide. Their thickness varies from 1 1/12 to ⅙ of an inch. This tool is used for giving a finish to the surface of stones.

The PITCHING CHISEL (Fig. 13) is usually of 1⅛-inch octagonal steel, spread on the cutting edge to a rectangle of ⅛ by 2½ inches. It is used to make a well-defined edge to the face of a stone, a line being marked on the joint surface to which the chisel is applied and the portion of the stone outside of the line broken off by a blow with the hand hammer on the head of the chisel.

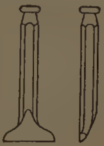


Fig. 13. Pitching Chisel

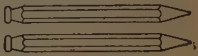


Fig. 14. Point

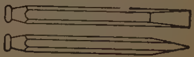


Fig. 15. Chisel

The POINT (Fig. 14) is made of round or octagonal rods of steel, from ¼ inch to 1 inch in diameter. It is made about 12 inches long, with one end brought to a point. It is used until its length is reduced to about 5 inches. It is employed for dressing off the irregular surface of stones, either for a permanent finish or preparatory to the use of the ax. According to the hardness of the stone, either the hand hammer or the mallet is used with it.

The CHISEL (Fig. 15) of round steel of ¼ to ¾ inch in diameter and about 10 inches long, with one end brought to a cutting edge from ¼ inch to 2 inches wide is used for cutting drafts or margins on the face of stones.

The TOOTH CHISEL (Fig. 16) is the same as the chisel, except that the cutting edge is divided into teeth. It is used only on marbles and sandstones.

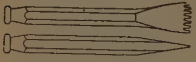


Fig. 16. Tooth Chisel

Fig. 17. Splitting Chisel

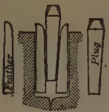


Fig. 18. Plug and Feather.

The SPLITTING CHISEL (Fig. 17) is used chiefly on the softer stratified stones, and sometimes on fine architectural carvings in granite.

The PLUG, a truncated wedge of steel, and the FEATHERS of half-round malleable iron (Fig. 18), are used for splitting unstratified stone. A row of holes is made with the DRILL (Fig. 19) on the line on which the fracture is to be made, in each of these holes two feathers are inserted, and the plugs are lightly driven in between them. The plugs are then gradually driven home by light blows of the hand hammer on each in succession until the stone splits.

Fig. 19. Drill

6. Dressing the Surface

Building stones are divided into three classes according to the finish of the surface, viz.: unsquared stone, squared stone, and cut stone.

Unsquared Stones include all stones which are used as they come from the quarry without other preparation than the removal of very acute angles and excessive projections from the general figure. The term " backing," which is frequently applied to this class of stone, is inappropriate, as it properly designates material used in a certain relative position in a wall, whereas stones of this kind may be used in any position.

Squared Stones include all stones that are roughly drest on beds and joints. The dressing is usually done with the face hammer or ax, or in soft stones with the tooth hammer. In gneiss it may sometimes be necessary to use the point. The distinction between this class and the third lies in the degree of closeness of the joints. Where the dressing on the joints is such that the distance between the general planes of the surfaces of adjoining stones is one-half inch or more, the stones properly belong to this class. Three subdivisions of this class may be made, depending on the character of the face of the stones: QUARRY-FACED STONES are those whose faces are left untouched as they come from the quarry. PITCH-FACED STONES are those on which the arris is clearly defined by a line beyond which the rock is cut away by the pitching chisel, so as to give edges that are approximately true. DRAFTED STONES are those on which the face is surrounded by a chisel draft, the space inside the draft being left rough. Ordinarily, however, this is done only on stones in which the cutting of the joints is such as to exclude them from this class. In ordering stones of this class the specifications should state the width of the bed and end joints which are expected, and also how far the surface of the face may project beyond the plane of the edge. In practise, the projection varies between 1 inch and 6 inches. It should also be specified whether or not the faces are to be drafted.

Cut Stones include those with smoothly drest beds and joints. As a rule, all the edges of cut stones are drafted, and between the drafts the stone

is smoothly drest. The face, however, is often left rough where the construction is massive. In architecture there are a great many ways in which the faces of cut stone may be drest, but the following are those that will usually be met in engineering work.

When it is necessary to remove an inch or more from the face of a stone, it is done by the pick or heavy point until the projections vary from $\frac{1}{2}$ inch to 1 inch. The stone is then said to be rough-pointed (Fig. 20). In dressing limestone and granite, this operation precedes all others. If a smoother finish is desired, rough pointing is followed by fine pointing, which is done with a fine point. Fine pointing (Fig. 21) is used only where

Fig. 20. Rough-Pointed

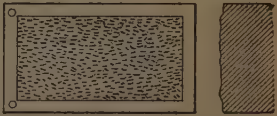


Fig. 21. Fine-Pointed

the finish made by it is to be final, and never as a preparation for a final finish by another tool. Crandaling is a speedy method of pointing, the effect being the same as fine pointing, except that the dots on the stone are more regular. The variations of level are about $\frac{1}{8}$ inch, and the rows are made parallel. When other rows at right angles to the first are introduced, the stone is said to be cross crandaled. Fig. 22 shows a crandaled and also a cross-crandaled surface. These two vary only in the degree of smoothness of the surface which is produced (Fig. 22). The number of blades in a patent ham-

Fig. 22. Crandaled.

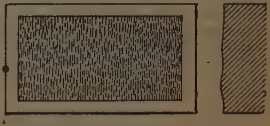


Fig. 23. Axed.

mer varies from 6 to 12 to the inch; and in precise specifications the number of cuts to the inch must be stated, as 6-cut, 8-cut, 10-cut, or 12-cut. The effect of axing is to cover the surface with chisel marks, which are made parallel as far as practicable. Axing is a final finish (Fig. 23).

The tooth-ax is practically a number of points, and it leaves the surface of a stone in the same condition as fine pointing. It is usually, however, only a preparation for bush-hammering, and the work is then done without regard to appearance so long as the surface of the stone is sufficiently leveled. The roughnesses of a stone are pounded off by the bush hammer, and the stone is then said to be bushed (Fig. 24). This kind of finish is dangerous on sandstone or other soft stone, as experience has shown that stone thus treated is likely to scale off. In dressing limestone which is to have a bush-hammered finish, the usual sequence of operations is (1) rough-pointing, (2) tooth-axing, and (3) bush-hammering.

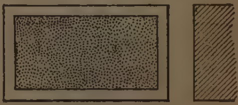


Fig. 24. Bush-Hammered.

In dressing sandstone and marble, it is very common to give the stone a plane surface at once by the use of the stone-saw. Any roughnesses left by the saw are removed by

rubbing with grit or sandstone. Such stones, therefore, have no margins. They are frequently used in architecture for string courses, lintels, door jambs, etc.; and they are also well adapted for use in facing the walls of lock chambers and in other localities where a stone surface is liable to be rubbed by vessels or other moving bodies. Sometimes the space between the margins is sunk immediately adjoining them and then rises gradually until the four planes form an apex at the middle of the panel. In general, such panels are called diamond panels, and the one just described is called a sunk diamond panel. When the surface of the stone rises gradually from the inner lines of the margins to the middle of the panel, it is called a raised diamond panel. Both kinds of finish are common on bridge quoins and similar work. The details of this method should be given in the specifications.

Forming the Surface. The surfaces most frequently required in stonecutting are plane and cylindrical; but sometimes warped, conical, spherical, and irregular surfaces are required.

To secure a PLANE SURFACE, the stonecutter draws a line with iron ore or black lead, on the edges of the stone, to indicate as nearly as possible the required plane surface. Then with the hammer and the pitching-tool he pitches off all waste material above the lines, thereby reducing the surface approximately to a plane. With a chisel he then cuts a draft around the edges of this surface, that is, he forms narrow plane surfaces along the edges of the stone. To tell when the drafts are in the same plane, he uses two straight-edges having parallel sides and equal widths (Fig. 25). The projections on the surface are then removed by the pitching chisel or the point, until the straight-edge will just touch the drafts and the intermediate surface when applied across the stone in any direction. The surface is usually left a little slack or concave, to allow room for the mortar.

Fig. 25. Plane Surfaces

To form a second plane surface at right angles to the first one, the workman draws a line on the cut face to form the intersection of the two planes; he also draws a line on the ends of the stone approximately in the required plane. With the ax or the chisel he then cuts a draft at each end of the stone until a steel square fits the angle. He next joins these drafts by two others at right angles to them, and brings the whole surface to the same plane. The other faces may be formed in the same way. If the surfaces are not at right angles to each other, a bevel is used instead of a square, the same general method being pursued.

To form a CYLINDRICAL SURFACE, the stone is first reduced to a parallelopipedon, after which the curved surface is produced in either of two ways: (1) by cutting a circular draft on the two ends and applying a straight-edge along the rectilinear elements (Fig. 26); or (2) by cutting a draft along the line of intersection of the plane and cylindrical surface, and applying a curved templet perpendicular to the axis (Fig. 27).

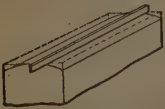


Fig. 26. Cylindrical Surfaces.

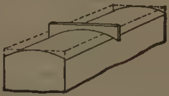


Fig. 27. Cylindrical Surfaces.

Fig. 28. Warped Surfaces.

Warped Surfaces include what the stonecutter calls twisted surfaces, as well as the general warped surface. The former is seldom used in masonry, and the latter almost never. The method of forming a surface equally twisted right and left is shown in Fig. 28. Two twist rules are required, the angle between the upper and lower edges being half of the required twist. Drafts are then cut in the ends of the stone until the tops of the twist rules, when applied as in Fig. 28, are in plane. The remainder of the projecting face is removed, until a straight-edge, when applied parallel to the edge

of the stone, will just touch the end drafts and the intermediate surface. If the surface is to be twisted at only one end, a parellel rule and a twist rule are used.

7. Dry Stonework

Riprap is stone laid, without mortar, about the base of bridge piers and abutments to prevent scour, and sometimes on banks to prevent wash, altho the latter are usually protected by stone paving. Often riprap is dumped in promiscuously, the size of the stone depending upon the material at hand and the velocity of the current, in extreme cases stones of 15 to 20 cu ft being used. In the most careful work, the stones are placed by hand so as to fill up the greater cavities and secure a tighter and more stable mass.

Slope Wall is a stone-block pavement laid upon the sloping earth bank of a river, canal or reservoir to protect it from the erosive action of the current and waves. It is usually made of thin-bedded stones 6 to 12 inches wide set on edge in a bed of fine gravel or coarse sand. The gravel or sand is necessary to prevent the finer native soil from being displaced by the action of the water. The thickness of the bed of gravel will depend upon the regularity of the width of the stones, but usually a thickness of 4 to 8 inches is sufficient. The stones should break joints horizontally so that if a stone is displaced those above will bridge over the opening and prevent the raveling of the whole wall. The joints should be made reasonably close to prevent the washing out of the building material, but ordinarily a width of 1 or 1½ inches is sufficient. The length and width of the face of the stone are unimportant, and will depend upon the way the stone quarries out. Ordinarily a slope wall can be built cheaper and better of stones that one man can lift readily, than of two-men stones or of those that must be placed with a derrick. The paving should be laid approximately to a plane and in fairly regular courses. Sometimes a slope wall is made of cobble stones or rounded boulders, but such stones are more troublesome to place, and do not make as desirable or stable a wall as stratified stones.

STONE AND BRICK MASONRY

8. Kinds of Stone Masonry

Classifications of stone masonry are: (1) according to the finish of the face of the stones, (2) according to whether the horizontal joints are more or less continuous, (3) according to the care which is employed in dressing the beds and joints.

(1) **Quarry-faced Masonry** is that in which the face of the stone is left as it comes from the quarry (Fig. 29). PITCH-FACED MASONRY is that in which the face edges of the beds are pitched to a right line (Fig 30). The outer edge of a horizontal joint of pitch-faced masonry is straight, while in quarry-faced it is not. CUT-STONE MASONRY is that in which the face of the stone is finished by any one of the methods described in Art. 6, as rough-pointed fine-pointed, crandaled, axed, bush-hammered, rubbed.

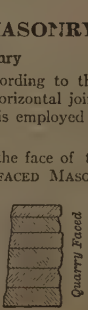


Fig. 29

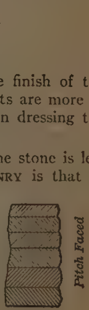


Fig. 30

(2) **Range Masonry** is that in which a course is the same thickness thruout its length (Fig. 31). BROKEN RANGE is that in which the course is of uniform thickness for only parts of its length (Fig. 32). RANDOM MASONRY is that which is not laid in courses at all (Fig. 33);

it is sometimes designated as one against-two or two-against three, the first term indicating that there is one stone on one side of a vertical joint and two on the other, and similarly for the second term.


Fig. 31. Range


Fig. 32. Broken Range


Fig. 33. Random

(3) **Ashlar Masonry** is composed of any of the kinds of cut-stones (Art. 6). It is usually held that when the dressing of the joints is such that the distance between the general planes of the surfaces of adjoining stones is one-half inch or less, the masonry belongs to this class. From its derivation ashlar apparently means large, square blocks; but practise seems to have made it synonymous with cut stone, and this secondary meaning has been retained for convenience. The coursing of ashlar is described by prefixing range, broken range, or random; and the finish of the face is described by prefixing a name to designate the finish of the face of the stone of which the masonry is composed, as for example fine-pointed ashlar, rubbed ashlar. Dimension Stone Masonry is that composed of cut-stones all of whose dimensions have been fixt in advance. Ordinarily the specifications of ashlar are so written as to prescribe the dimensions of the stones to be used, and hence it is seldom or never necessary to make a new class of masonry composed of dimension stones.

Squared-Stone Masonry is that in which the stones are roughly squared and roughly drest on beds and joints. The distinction between squared-stone masonry and ashlar lies in the degree of closeness of the joints. "When the dressing on the joints is such that the distance between the general planes of the surface of adjoining stones is one-half inch or more, the stones properly belong to this class."

Rubble Masonry is composed of unsquared stones, and it may be coursed or uncoursed. Coursed Rubble is masonry composed of unsquared stone which is leveled off at specified heights to an approximately horizontal surface (Fig. 35). Uncoursed Rubble is masonry composed of unsquared stones laid without any attempt at regular courses Fig. 34). The specifications for rubble may require that the stones shall be roughly shaped with the hammer. Rubble is sometimes designated as one-man or two-man rubble, according to the number of men required to handle a stone.


Fig. 34. Uncoursed Rubble


Fig. 35. Coursed Rubble

General Rules. The following general principles apply to all classes of stone masonry: (1) The largest stones should be used in the foundation to give the greatest strength and lessen the danger of unequal settlement. (2) A stone should be laid upon its broadest face, since then there is better opportunity to fill the spaces between the stones. (3) For the sake of appearance, the larger stones should be placed in the lower courses, the thickness of the courses decreasing gradually toward the top of the wall. (4) Stratified stones should be laid upon their natural bed, i.e., with the strata perpendicular to

the pressure, since they are then stronger and more durable. (5) The masonry should be built in courses perpendicular to the pressure it is to bear. (6) To bind the wall together laterally, a stone in any course should break joints with or overlap the stone in the course below; that is, the joints parallel to the pressure in two adjoining courses should not be too nearly in the same line. This is briefly comprehended by saying that the wall should have sufficient lateral bond. (7) To bind the wall together transversely, there should be a considerable number of headers extending from the front to the back of thin walls or from the outside to the interior of thick walls; that is, the wall should have sufficient transverse bond. (8) The surface of all porous stones should be moistened before being bedded, to prevent the stone from absorbing the moisture from the mortar and thereby causing it to become a friable mass. (9) The spaces between the back ends of adjoining stones should be as small as possible, and these spaces and the joints between the stones should be filled with mortar. (10) If it is necessary to move a stone after it has been placed upon the mortar bed, it should be lifted clear and be reset, as attempting to slide it is likely to loosen stones already laid and destroy the adhesion, and thereby injure the strength of the wall.

9. Ashlar Masonry

Ashlar is the best quality of stone masonry, and is employed in all important structures. It is used for piers, abutments, arches, and parapets of bridges; for hydraulic works; for facing quoins, and string courses; for the coping of inferior kinds of masonry and of brickwork; and, in general, for work in which great strength and stability are required. The dimensions of the blocks should vary with the character of the stone employed. With the weaker sandstone and granular limestones the length of any stone should not be greater than three times its depth, as otherwise it is likely to be broken across; but with the stronger stones the length may be four or five times the depth. With the weaker stones the breadth may range from one and a half to two times the depth; and for the stronger stones it may range from three to four times the depth.

Dressing. The dressing consists in cutting the side and bed joints to plane surfaces, usually at right angles to each other. The accurate dressing of the bed joints to a plane surface is exceedingly important. If any part of the surface projects beyond the plane of the chisel draft, that projecting part will have to bear an undue share of the pressure, the joint will open at the edges, and the whole will be wanting in stability. On the other hand, if the surface of the bed is concave, having been drest down below the plane of the chisel draft, the pressure is concentrated on the edges of the stone, to the risk of splitting them off. Such joints are said to be flushed. They are more difficult of detection, after the masonry has been built, than open joints; and are often executed by design, in order to give a neat appearance to the face of the building. Their occurrence must therefore be guarded against by careful inspection during the progress of the stone-cutting.

Great smoothness is not desirable in the joints of ashlar masonry intended for strength and stability, for a moderate degree of roughness adds at once to the resistance to displacement by sliding and to the adhesion of the mortar. When the stone has been drest so that all the small ridges and projecting points on its surface are reduced nearly to a plane, the pressure is distributed nearly uniformly for the mortar serves to transmit the pressure to the small depressions. Each stone should first be fitted into its place dry, in order that any inaccuracy of figure may be discovered and corrected by the stonecutter before it is finally laid in mortar and settled in its bed.

The entire bed area of a stone should be drest to a plane; but, unless the wall is so

thin that the stones extend clear through, it is not necessary to dress the entire area of the ends of the stones; and it is not necessary to dress any portion of the back side of the stones. The specifications should state the distance back from the face of the stone that the end is to be drest to a plane surface. This distance is sometimes stated in inches and sometimes as a fractional part of the thickness of the course. Sometimes specifications permit the vertical joints to be wider than the bed joints. This decreases the cost of cutting, and may not materially reduce the strength of the masonry; but may slightly affect the durability and the architectural appearance.

The thickness of mortar in the joints of the very best ashlar masonry is about 1/8 inch; in first-class railroad masonry the joints are from 1/4 to 1/2 inch. A chisel draft 1 1/2 or 2 inches wide is usually cut at each exterior corner. In the best work, as fine cut-stone buildings, all projecting courses, as window sills, water tables, cornices, etc., have grooves or "drips," cut in the under surface a little way back from the face, so as to cause rain-water to drop from the outer edge instead of running down over the face of the wall and disfiguring it.

Bond. The bond is the arrangement or overlapping of the stones to tie the wall together longitudinally and transversely, and is of great importance to the strength of the wall. No joint of any course should be directly above a joint in the course below; but the stones should overlap, or break joint, from one to one and one-half times the depth of the course, both along the face of the wall and also from the front to the back. The effect is that each stone is supported by at least two stones of the course below, and assists in supporting at least two stones of the course above. The object is twofold: first, to distribute the pressure, so that inequalities of load on the upper part of the structure (or of resistance at the foundation) may be transmitted to and spread over an increasing area of bed in proceeding downwards (or upwards); and second, to tie the building together, both lengthwise and from face to back.

The strongest bond is that in which each course at the face of the structure contains a header and a stretcher alternately, the outer end of each header resting on the middle of a stretcher of the course below, so that rather more than one-third of the area of the face consists of ends of headers. This proportion may be deviated from when circumstances require it, but in every case it is advisable that the ends of headers should not form less than one-fourth of the whole area of the face of the structure. A header should be over the middle of the stretcher in the course below. In a thin wall a header should extend entirely thru the wall.

Where very great resistance to displacement of the masonry is required (as in the upper courses of bridge piers, or over openings, or where new masonry is joined to old, or where there is danger of unequal settlement), the bond is strengthened by dowels or by cramp irons of, say 1 1/4-inch round iron set with cement mortar.

Backing. Ashlar is usually backed with rubble masonry which in such cases is specified as coursed rubble. Special care should be taken to secure a good bond between the rubble backing and the ashlar facing. Two stretchers of the ashlar facing having the same width should not be placed one immediately above the other. The proportion and the length of the headers in the rubble backing should be the same as in the ashlar facing. The "tails" of the headers, or the parts which extend into the rubble backing, may be left rough at the back and sides; but their upper and lower beds should be drest to the general plane of the bed of the course. These tails may taper slightly in breadth, but should not taper in depth. The backing should be carried up at the same time with the face-work, and in courses of the same depth; and the bed of each course should be carefully built to the same plane with that of the ashlar facing. The rear face of the backing should be lined to a fair surface.

Pointing. In laying masonry of any character, whether with lime or cement mortar, the exposed edges of the joints will naturally be deficient in

density and hardness. The mortar in the joints near the surface is especially subject to dislodgment, since the contraction and expansion of the masonry are liable either to separate the stone from the masonry or to crack the mortar in the joint, thus permitting the entrance of rain-water, which upon freezing forces the mortar from the joint. Therefore, it is usual, after the masonry is laid, to refill the joints as compactly as possible, to the depth of at least an inch, with mortar prepared especially for this purpose. This operation is called pointing.

The very best cement mortar should be used for pointing, as the best becomes dislodged all too soon. Clear portland-cement mortar is the best, altho 1 volume of cement to 1 of sand is frequently used in first-class work. The mortar, when ready for use, should be rather incoherent and quite deficient in plasticity.

Before applying the pointing, all mortar in the joint should be dug out to a depth of at least 1 inch; or, better, in setting the stones, the mortar should be kept back an inch or more from the face, and thus save the labor of digging out the joints preparatory to pointing. For the bed joints this may be accomplished by keeping the mortar back from the face of the wall about 3 inches, and then when the stone is put into place the mortar will probably be forced out to about 1 or 1½ inches from the face of the joint, and consequently little or no labor will be required to dig out the mortar. Frequently in laying a stone the mortar is spread to the very edge of the joint; and then when the pointing is done, it is so difficult to dig out the mortar that the joint is cleared only about half an inch deep, which depth does not give the pointing sufficient hold, and consequently it soon drops out. The difficulty of digging out the mortar from the vertical joints may be obviated by bending a strip of tin or thin steel to the form of a U having one leg considerably longer than the other, and nailing the long leg to the side of a light strip of wood so that the closed end of the U will project beyond the edge of the wood a distance equal to the depth of the pointing, and then inserting the closed end of the U in the vertical joint before it is filled with mortar.

When the surplus mortar has been removed, the joint should be cleansed by scraping and brushing out all loose material and then it should be well moistened. The mortar is applied with a mason's trowel, and should be well "set in" with a calking iron and hammer. The joint should be rubbed smooth and finished even with the pitch line or with the face of the stone. In the very best work, the joint is also rubbed smooth with a steel polishing tool. Walls should not be allowed to dry too rapidly after pointing; and therefore pointing in hot weather should be avoided.

There are four general forms of finishing the edges of the horizontal joints of cut-stone masonry, whether or not they are formally pointed as above described. Fig. 36 shows these four forms. When the horizontal joints are finished as in either of the first two examples in Fig. 36 it is customary to finish the vertical joints by the first method; but when either of the last two methods is employed, it is used for both the vertical and the horizontal joints. Occasionally in cut-stone masonry, and frequently in brick masonry, the weather joint is improperly made to slope in the opposite direction, due to the fact that the mason stands at the back of the wall and "strikes" the joint by reaching down and resting the edge of the trowel on the stone below the joint. If the mason stands behind the wall, it is not comfortable to make the weather joint as shown in Fig. 36 at the time the masonry is laid. The grooved joint is frequently called a tuck-pointed joint, and is sometimes made with a V-like face. The beaded joint is not very durable, since the projecting portion soon becomes detached. In making the beaded joint, the beading tool is sometimes guided by a straight-edge, called a "rod," and the joint is then said to be "rodded."

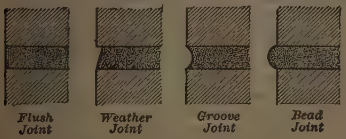


Fig. 36. Finish of Horizontal Joints

The Amount of Mortar required for ashlar masonry varies with the size of the blocks, and also with the closeness of the dressing. With ⅜- to ½-inch

joints and 12- to 20-inch courses, there will be about 2 cubic feet of mortar per cubic yard; with larger blocks and closer joints, i.e., in the best masonry, there will be about 1 cubic foot of mortar per yard of masonry. Laid in 1 to 2 mortar, the former will require ¼ to ⅓ of a barrel of cement per cubic yard of masonry exclusive of the rubble backing, and the latter about half as much.

10. Squared-Stone and Rubble Masonry

Squared-Stone Masonry. Squared-stone masonry is distinguished from ashlar in having less accurately drest beds and joints; and from rubble in being more carefully constructed. In ordinary practise, the field covered by this class is not very definite. The specifications for "second-class masonry" as used on some railroads usually conform to the above description of quarry-faced range squared-stone masonry; but sometimes this grade of masonry is designated "superior rubble." Squared-stone masonry is employed for the piers and abutments of highway bridges, for small arches, for box culverts, for basement walls, etc. The quoins and the sides of openings are usually reduced to a rough-smooth surface with the face-hammer, the ordinary ax, or the tooth-ax. This work is a necessity where door or window frames are inserted; and it greatly improves the general effect of the wall, if used wherever a corner is turned.

The remarks concerning size of stones and backing under ashlar above apply substantially also to squared-stone masonry. As the joints of squared-stone masonry are thicker than those of ashlar the pointing should be done proportionally more carefully while as a rule it is done much more carelessly. The mortar is often thrown into the joint with a trowel, and then trimmed top and bottom to give the appearance of a thinner joint. Such work is called ribbon pointing. Trimming the pointing adds to the appearance but not to the durability. When the pointing is not trimmed, it is called dash pointing.

Rubble Masonry. This is the lowest grade laid with mortar; and is built of stones as they come from the quarry without other preparation than the removal of very acute angles and excessive projections from the general figure. The stone used for rubble masonry is prepared by simply knocking off all the weak angles of the block. It should be cleansed from dust, etc., and moistened, before being placed on its bed. This bed is prepared by spreading over the top of the lower course an ample quantity of good, ordinary-tempered mortar in which the stone is firmly embedded. The vertical joints should be carefully filled with mortar. The interstices between the larger masses of stone are filled by thrusting small fragments or chippings of stone into the mortar. Careful attention should be given to bonding the wall laterally and transversely. It is frequently specified that one-fourth or one-fifth of the mass shall be headers. The corners and jambs should be laid with hammer-dressed or cut stones.

A very stable wall can be built of rubble masonry without any dressing, except a draft on the quoins by which to plumb the corners and carry them up neatly, and a few strokes of the hammer to spall off any projections or surplus stone. This style of work is not generally advisable, as very few mechanics can be relied upon to take the proper amount of care in leveling up the beds and filling the joints; and as a consequence, one small stone may jar loose and fall out, resulting probably in the downfall of a considerable part of the wall. Some of the naturally bedded stones are so smooth and uniform as to need no dressing or spalling up; and a wall of such stones is very economical, since there is no expense of cutting and no time is lost in hunting for the right stone and yet strong massive work is assured. However, many of the naturally bedded stones have inequalities on their surfaces, and in order to keep them level in the course it becomes necessary to raise one corner by placing spalls or chips of stone under the bed, and to fill the vacant spaces well and full with mortar, and here the disadvantage of this style of work becomes apparent.

When carefully executed with good mortar, rubble possesses all the strength and dura-ility required in structures of an ordinary character, and is much less expensive than ther ashlar or squared-stone masonry. But it is difficult to get rubble well executed. 'he most common defects are (1) not bringing the stones to an even bearing; (2) leaving rge unfilled vertical openings between the several stones; (3) laying up a considerable eight of the wall dry, with only a little mortar on the face and back, and then pouring ortar on the top of the wall; (4) using insufficient cement, or that of a poor quality. The nly way to prevent the first defect is to have an inspector on the job all the time. The econd and third defects can be detected by probing the wall with a small pointed steel od. To prevent the fourth defect it is customary for the owner to furnish the cement the contractor. Apparently it is commonly believed that the rougher the stones and e poorer the grade of masonry, the poorer the cement or the leaner the mortar should e. The principal object of the mortar is to equalize the pressure; and the more nearly e stones are reduced to closely fitting surfaces, the less important is the mortar. Con-equently, when a substantial rubble is required, it would not be amiss to use a first-clas ement mortar, particularly if the stones are comparatively small.

The Amount of Mortar required for rubble masonry varies greatly with he character of the surfaces with which the stone quarries out. If the stone stratified sandstone or limestone yielding flat-bedded stones with good end urfaces, the rubble may not require much if any more mortar than ashlar uilt of the more refractory stones; but if the rubble is built of stone that uarries out in irregular chunks and is difficult to dress, a very large percent f mortar may be required. The amount of mortar required can be consid-rably reduced by packing spalls into the vertical spaces between the stones, proceeding that is always economical since spalls are always much heaper than cement mortar. However, when the cement is furnished by he owner, the mason is apt to fill the joints entirely with mortar since it equires less time.

If rubble masonry is composed of small and irregularly shaped stones, bout one-third of the mass will consist of mortar; and if laid in 1 : 2 mortar vill require about 1 barrel of portland cement per cubic yard, and if laid in : 3 about 0.8 barrel. If the stones are large and regular in form, one-fifth o one-quarter of the mass will be mortar; and the rubble will require about .7 barrel per cubic yard for 1 : 2 mortar and 0.6 barrel for 1 : 3.

Rubble Concrete is ordinary concrete in which large irregular stones, sometimes alled plums, are embedded. This form of masonry is adapted to moderately massive onstruction. The plums decrease the cost of crushing the stone and also decrease the mount of cement required, and increase the density of the mass. The plums are usually imited to about 40% of the entire volume, to insure that they shall be surrounded by oncrete. If the concrete is wet, there is little or no trouble in getting the large stones horoly bedded, and consequently this form of masonry is as good as or better than rdinary concrete.

Cyclopean Masonry. Sometimes in building large structures, as dams, the rubble s made of very large irregular rocks and wet concrete is used instead of mortar. The arge stones are placed in the wall by means of a derrick, and concrete is deposited from bottom-dump bucket. This form of construction was first used about 1900, and is spe-ially applicable in building a dam, in which the faces are laid in ashlar, squared-stone, r rubble, and serve as forms in which to place the rock and concrete. The term con-rete-rubble is sometimes used for this kind of filling.

11. Safe Pressures for Stone Masonry

Strength of Stone Masonry is a subject about which there is almost no efinite information. The strength of a mass of stone masonry depends pon the strength of the stone, the accuracy of the dressing, the proportion f headers to stretchers, the amount and the strength of the mortar. A ariation in any one of these items may greatly change the strength of the nasonry. For example, if the mortar is of insufficient strength, it will be

squeezed out laterally and cause the stone to fail by tension, or improper bedding of the stone may cause it to fail by flexure, for neither of which methods of failure is the stone as strong as in compression.

No experiments have ever been made upon the strength of stone masonry under the conditions actually occurring in masonry structures, owing to the lack of a testing machine of sufficient strength. Experiments made upon brick piers 12 inches square and from 2 to 10 feet high, laid in mortar composed of 1 volume portland cement and 2 sand, show that the strength per square inch of the masonry is only about one-sixth of the strength of the brick. An increase of 50 % in the strength of the brick produced no appreciable effect on the strength of the masonry; but the substitution of cement mortar (1 portland and 2 sand) for lime mortar (1 lime and 3 sand) increased the strength of the masonry 70 %. The method of failure of these piers indicates that the mortar squeezed out of the joints and caused the brick to fail by tension. Since the mortar is the weakest element, the less mortar used the stronger the wall; therefore the thinner the joints and the larger the blocks, the stronger the masonry, provided the surfaces of the stones do not come in contact.

The only practicable way of determining the actual strength of masonry is to note the loads carried by existing structures. However, this method of investigation will give only the load which does not crush the masonry, since probably no structure ever failed owing to the crushing of the masonry.

Pressure Allowed. Early builders used much more massive masonry, proportional to the load to be carried, than is customary at present. Experience and experiments have shown that such great strength is unnecessary. The load on the monolithic piers supporting the large churches in Europe does not usually exceed 30 tons per sq ft (420 lb per sq in), or about one-thirtieth of the ultimate strength of the stone alone, altho the columns of the Church of All Saints at Angers, France, is said to sustain 43 tons per sq ft (600 lb per sq in). The stone-arch bridge of 140 ft span at Pont-y-Prydd, over the Taff, in Wales, erected in 1750, is supposed to have a pressure of 72 tons per sq ft (1000 lb per sq in) on hard limestone rubble masonry laid in lime mortar. The granite piers of the Saltash bridge sustain a pressure of 9 tons per sq ft (125 lb per sq in). The maximum pressure on the granite masonry of the towers of the Brooklyn bridge is about 28½ tons per sq ft (about 400 lb per sq in). The maximum pressure on the limestone masonry of this bridge is about 10 tons per sq ft (125 lb per sq in). The face stones ranged in cubical contents from 1½ to 5 cu yd; the stones of the granite backing averaged about 1½ cu yd, and of the limestone about 1¼ cu yd per piece. The mortar was 1 volume of Rosendale natural cement and 2 of sand. The stones were rough-axed or pointed to ½-inch bed-joints and ½-inch vertical face-joints. These towers are very fine examples of the mason's art.

In the Rookery Building, Chicago, granite columns about 3 feet square sustain 30 tons per sq ft (415 lb per sq in) without any signs of weakness. In the Washington Monument, Washington, D. C., the normal pressure on the lower joint of the walls of the shaft is 20.2 tons per sq ft (280 lb per sq in), and the maximum pressure brought upon any joint under the action of the wind is 25.4 tons per sq ft (350 lb per sq in). The pressure on the limestone piers of the Eads bridge, St. Louis, Mo., was, before completion, 38 tons per sq ft (527 lb per sq in); and after completion the pressure was 19 tons per sq ft (273 lb per sq in) on the piers and 15 tons per sq ft (198 lb per sq in) on the abutments. The limestone masonry in the towers of the Niagara suspension bridge failed under 36 tons per sq ft, and were taken down; however, the masonry was not well executed. At the South Street bridge, Philadelphia, the pressure on the limestone rubble masonry in the pneumatic piles is 15.7 tons per sq ft (220 lb per sq in) at the bottom and 12 tons per sq ft at the top. The maximum pressure on the rubble masonry (laid in cement mortar) of some of the large masonry dams is from 11 to 14 tons per sq ft (154 to 195 lb per sq in). The new Croton dam was designed for a maximum pressure of 16⅔ tons per sq ft (230 lb per sq in) on massive rubble masonry in best hydraulic cement mortar.

Safe Pressures. In the light of the preceding examples it may be assumed that the

safe load for the different classes of masonry is about as follows, provided each is the best of its class:

	Net tons per sq ft	Lb per sq in
Rubble	10 to 15	140 to 200
Squared-stone	15 to 20	200 to 280
Limestone ashlar	20 to 25	280 to 350
Granite ashlar	25 to 30	350 to 400
Concrete	30 to 40	400 to 550

A large committee composed of the leading architects and engineers of Chicago recommended, in 1907, the following values for the building laws of that city.

Kind of Masonry	Lb per sq in
Rubble, uncoursed, in lime mortar	60
in portland-cement mortar	100
Rubble, coursed, in lime mortar	120
in portland-cement mortar	200
Ashlar, limestone in portland-cement mortar	400
granite in portland-cement mortar	600
Concrete, portland cement, 1 : 2 : 4, hand mixt	350
1 : 2 : 4, machine mixt	400
1 : 3 : 6, hand mixt	250
1 : 3 : 6, machine mixt	300
Concrete, natural cement, 1 : 2 : 5	150

12. Qualities of Brick

Good Bricks have the following qualities to recommend them as a building material: (1) Bricks are practically indestructible, since they are not acted upon by fire, the weather, or the acids in the atmosphere. (2) Bricks may be had in most localities of almost any shape, size, or color. (3) Bricks are comparatively easy to put into place in the wall. (4) In most localities brick masonry is cheaper than stone masonry, even rubble, and under some conditions is a competitor with concrete. The disadvantages of brick as a building material are: (1) Owing to the smallness of the unit, bricks are comparatively expensive to lay, and require considerable skill to secure a strong and good-appearing wall. (2) Ordinarily brick masonry is not durable, since a considerable part of the face of the wall is mortar, which is not as durable as the brick; but by making thin joints or using superior mortar in the exterior edges of the joints, a reasonably durable wall may be constructed.

Clay Brick. Until about 1900 the word brick always meant a prism of burned clay; but at about that date bricks composed of sand and lime were put upon the market, and at present many such brick are used annually, altho their number is very small in comparison with that of ordinary clay brick. Ordinarily the word brick means a burned-clay brick, and a brick composed of sand and lime is called a sand-lime brick. Clay brick is made by submitting clay which has been prepared properly and molded into shape to a temperature which converts it into a semi-vitrified mass. Building brick are usually made from surface clay, and paving brick from shale (a fine-grained and indurated clay), since the latter gives a tougher, denser, and stronger brick.

The method of molding gives rise to the following terms.

Soft-mud Brick: One molded from clay which has been reduced to a soft mud by adding water. It may be either hand-molded or machine-molded. Stiff-mud Brick: One molded from clay in the condition of stiff mud. It is always machine-molded. Prest Brick: One molded from dry or semi-dry clay. Re-prest Brick: Usually a stiff-mud brick which has been subjected to an enormous pressure to render the form more regular and to increase its strength and density. It is doubtful whether the re-pressing increases either the strength or the density. Occasionally in the

East, and more formerly than at present, a soft-mud brick, after being partially dried, is re-prest, which process greatly improves the form and also the strength and the density. A re-prest brick is sometimes, but inappropriately called a prest brick. Slop Brick: In molding brick by hand, the molds are sometimes dipt into water just before being filled with clay, to prevent the mud from sticking to them. Brick molded by this process is known as slop brick. It is deficient in color, and has a comparatively smooth surface, with rounded edges and corners. This kind of brick is now seldom made. Sanded Brick: Ordinarily, in making soft-mud brick, sand is sprinkled into the molds to prevent the clay from sticking; the brick is then called sanded brick. The sand on the surface is of no serious advantage or disadvantage. In hand-molding, when sand is used for this purpose, it is certain to become mixt with the clay and occurs in streaks in the finished brick, which is very undesirable; and owing to details of the process, which it is here unnecessary to explain, every third brick is especially bad. Machine-made Brick: Brick is frequently described as "machine-made"; but this is very indefinite, since all grades and kinds are made by machinery.

When bricks were usually burned in the old-style up-draft kiln, the classification according to position was important; but with the new styles of kilns and improved methods of burning, the quality is so nearly uniform thruout the kiln that the classification is less important. Three grades of brick are taken from the old-style kiln: arch brick, body brick, and salmon brick. Arch or Clinker Bricks: Those which form the tops and sides of the arches in which the fire is built. Being overburned and partially vitrified, they are hard, brittle, and weak. Body, Cherry, or Hard Bricks: Those taken from the interior of the pile. The best bricks in the kiln. Salmon, Pale, or Soft Bricks: Those which form the exterior of the mass. Being underburned, they are too soft for ordinary work, unless it be for filling. The terms "salmon" and "pale" refer to the color of the brick, and hence are not applicable to a brick made of a clay that does not burn red. Altho nearly all brick clays burn red, yet the localities where the contrary is true are sufficiently numerous to make it desirable to use a different term in designating the quality.

The form of the brick gives rise to the following terms. Compass Brick: One having one edge shorter than the other. Feather-edge Brick: One having one edge thinner than the other. Used in arches; and more properly, but less frequently, called voussoir brick. Face Brick: Those which, owing to uniformity of size and color, are suitable for the face of the wall of buildings. Sometimes face bricks are simply the best ordinary brick; but generally the term is applied only to re-prest or prest brick made specially for this purpose. Sewer Brick: Ordinary hard brick, smooth, and regular in form. Paving Brick: Very hard, ordinary brick. A vitrified clay block, very much larger than ordinary brick, is sometimes used for paving, and is called a paving brick, but more often a brick paving-block. Vitrified Brick: The introduction of brick for street pavements about 1890 led to a new grade, one burned to the point of vitrifaction and then annealed or toughened by slowly cooling. Vitrified brick and paving blocks, though originally made for paving purposes, are now much used in building and engineering structures.

The Size of common brick varies widely with the locality and also with the maker, and with the same maker the brick are likely to be larger as the working season advances, owing to the wear of the molds or the die. Hard-burned bricks are smaller than soft-burned ones, owing to the greater shrinkage in burning; and this difference varies with the different kinds of clays. The standard sizes adopted by the National Brick Makers' Assoc. in 1889 are 8¼ by 4 by 2¼ inches for common, 8½ by 4 by 2½ for paving, 8⅜ by 4 by 2⅜ for prest, 12 by 4 by 1½ for Roman, and 12 by 4 by 2⅜ for Norman. In Cuba the standard size is 11 by 5½ by 2⅝ and in England it is 8¾ by 4⅜ by 2¾ in.

Cost. In 1905 the average selling price for 1000 bricks at the kiln was $6.25 for common, $10.07 for paving, and $13.12 for prest. (Mineral Resources of the U. S., 1905, p. 957.) The highest prices were nearly double and the lowest about two-thirds of the average.

Form. A good brick should have plane faces, parallel sides, and sharp edges and angles. In regularity of form re-prest brick ranks first, dry-clay brick next, then stiff-mud brick, and soft-mud brick last. Regularity of

form depends largely upon the quality of the clay and the method of burning. A good brick should not have depressions or kiln marks on its edges caused by the pressure of the brick above it in the kiln.

Texture. A good brick should have a fine, compact, uniform texture; and should contain no fissures, air bubbles, pebbles, or lumps of lime. It should give a clear ringing sound when struck a sharp blow with a hammer or another brick. A brick which gives a clear ringing sound is strong and durable enough for any ordinary work.

The compactness and uniformity of texture, which greatly influence the durability of brick, depend mainly upon the method of molding. As a general rule, hand-molded bricks are best in this respect, since the clay in them is more uniformly tempered before being molded; but this advantage is partially neutralized by the presence of sand seams. Machine-molded soft-mud bricks rank next in compactness and uniformity of texture. Then come machine-molded stiff-mud bricks, which vary greatly in durability with the kind of machine used in their manufacture. By some of the machines, the brick is molded in layers (parallel to any face, according to the kind of machine), which are not thoroly cemented, and which separate under the action of frost. In compactness the dry-clay brick comes last. However, the relative value of the products made by the different processes varies with the nature of the clay used.

Formerly it was believed that the capacity of a building brick to absorb water had an important effect upon its ability to resist destruction by frost; but experiments and a more careful study of experience has shown that this has little or nothing to do with its durability. The absorptive capacity varies with the chemical composition of the clay, and there seems to be no close relation between the absorptive power and the strength of a brick or the loss of strength by freezing.

The Crushing Strength is valuable only in comparing different brands, and gives no idea of the strength of walls built of such bricks. The crushing strength of brick is of relatively less importance than that of stone, since owing to the relatively smaller size of the brick and consequently the relatively larger proportion of mortar, the strength of brick masonry is more dependent upon the strength of the mortar than is stone masonry. The strength of the brick is of relatively small importance unless the mortar is nearly as strong as the brick.

Soaking a brick in water decreases its compressive strength, apparently because the water acts as a lubricant on the plane of rupture. In a series of experiments with the United States testing machine at Watertown, of thirty tests upon ordinary building brick from ten localities, all but two showed a loss of strength due to immersion in water for one week; and the wet half of a brick gave an average crushing strength of only 85 % of the strength of the dry half.

Some experiments with the testing machine at the United States Arsenal at Watertown, to determine the relative strength of hard-burned face brick tested flatwise, edgewise, and endwise, gave averages for four tests each of 11 174, 8978 and 6972 lb per sq in respectively. The prest surfaces were set in plaster of paris.

Brick sent from ten states to the Columbian Exposition at Chicago in 1892 and afterwards crusht at the Watertown Arsenal flatwise with the prest surfaces set in plaster of paris, gave results varying from 1311 to 22 561 lb per sq in. Illinois brick ranged from 5828 to 12 269 lb per sq in.

Five samples each of fourteen lots of Hudson River brick gave an average crushing strength of 3943 lb per sq in for half bricks tested flatwise, the range for the averages of the several lots being from 2701 to 5416, and the range for the individual brick being from 1607 to 8944. (Eng. News, 1905, Vol. 53, p. 384.) The highest known crushing strength of any brick is 38 446 lb per sq in.

Sand-lime Brick consist of a mass of sand cemented together with lime. There are two classes of sand-lime brick: one in which the binding material is carbonate of lime, and the other in which it is silicate of lime.

The first is virtually a brick made of ordinary lime mortar, molded as are soft-mud clay brick, and hardened in the open air or in an atmosphere rich in carbon dioxide (CO_2),

either with or without pressure. This form may properly be called a lime-mortar brick. It is the older form of sand-lime brick, and was formerly made in a small way where sand and lime were cheap and clay and fuel were expensive; but the brick is so weak and friable that it has not given satisfaction, and needs no further consideration here.

The second kind is made from a mixture of sand and lime which is molded in a press and hardened by being subjected to steam under pressure. In this case the binding material consists chiefly of hydrosilicate of lime. Probably part of the lime is converted into carbonate by absorbing carbon dioxide; but the most of the lime combines with the silica of the sand and forms hydrosilicate of lime, a stable and comparatively strong cementing material. This form is the only one to which the term sand-lime brick is now applied; but in consulting the past literature on the subject, a careful distinction should be made between the two forms of so-called sand-lime brick. This form of sand-lime brick was first manufactured in Germany about 1880, and was introduced into America about 1900. There are localities where this form of brick is an important factor in building operations.

Sand-lime brick are made which in appearance and quality are the equal of dry-clay (prest) brick; but the ordinary run of sand-lime brick are not as strong as the usual clay building brick, and many of them are deficient in resistance to frost and weather.

13. Lime and Lime Mortar

Lime Mortar, a mixture of lime paste and sand, is generally used for brick masonry because of its cheapness and the comparative ease with which it is used. There are two classes of lime on the market, high-calcium lime and magnesian or dolomitic lime. The former is made of nearly pure limestone, and the latter of a limestone containing a considerable quantity of magnesia; the latter slakes more slowly, evolves less heat, expands less, sets more slowly, and makes the stronger mortar. The former is known as hot or quick lime, and the latter as cool or slow-setting lime.

The lime must be slaked before being mixed with the sand. Lime is usually slaked by placing the lumps in a layer 6 or 8 inches deep in either a water-tight box or a basin formed in the sand to be used in mixing the mortar, and pouring upon the lumps a quantity of water 2½ to 3 times the volume of the lime. If the quantity of water added is just right, the lime will be reduced to a thick paste; but if too much water is used, the lime will be reduced to a semi-liquid condition and a considerable part of its binding quality will be destroyed.

With a high-calcium or quick-slaking lime the best results are obtained when all the water is added at once; but with a magnesian or slow-slaking lime only a little water should be added at first, and then after the lime and water are hot, more water may be added gradually so as not to chill the mixture and retard the slaking. The slaking proceeds more rapidly and is more complete if the mass is hot. The lime absorbs the water, and the chemical action generates heat enough to change part of the absorbed water into steam which bursts the lumps of lime apart and thus exposes new surfaces to the action of the water; but if cold water is added after the slaking has begun, it chills the mass, prevents the formation of steam and the consequent bursting of the lumps, and hence the slaking is not complete, and the amount of paste formed is less than it should be. Further, when the slaking has been thus retarded, a thin paste forms on the outside of the fragments of the unslaked lime, which excludes the water from the interior or unslaked portion of the lump; and hence it is difficult, if not impossible, to thoroly slake lime that has ever been chilled in the slaking. Partial air slaking is harmful in much the same way, since the slaked lime on the outside of a lump prevents the free access of the water to its interior.

Stirring the lime while slaking chills the mass and thereby retards the slaking; but, on the other hand, stirring breaks up the friable lumps and thereby aids the slaking. Therefore if the mass is stirred at all, the stirring should be done in such a manner as to cool the mass as little as possible. The swelling of the lime in the lower portion of the mass frequently lifts some of the lumps out of the water, the heat in the lump causes a column of steam to rise from it, and the lump is said to "burn." This burning is detrimental, since a film of slaked lime is formed on the surface of the unslaked portion which tends to prevent complete slaking. Therefore it is important that lumps which are burning

should be pushed back into contact with the water. Burning can be pr ering the box with boards or a tarpaulin to retain the heat and the moi

Lime slakes spontaneously when exposed to the air by absorbing atmosphere; and lime that is thoroly air slaked is as good. or even slaked in the usual way, a popular prejudice to the contrary notwithstanding lime that is only partially air slaked is undesirable, since it is more difficult to sla the ordinary process than lime that is not partially air slaked.

Slaked lime is sold under the name of hydrated lime. It is a dry powder, and is usually packed in paper sacks. In certain classes of work, hydrated lime is of decided advantage, since it is ready for immediate use without waiting to slake it. Lime is sold in bulk and in barrels of about 200 lb net, the price usually being from 50 to 65 cents per barrel in bulk, and 15 to 20 cents more in barrels. A barrel of high-calcium lime will make about 2½ bbl of stiff lime paste; and 1 bbl of paste and 3 bbl of good sand will make about 3 bbl (0.4 cu yd) of mortar. One barrel of unslaked lime will make about 0.95 cu yd of 1 : 3 mortar.

Mortar. Sand is added to the lime paste for four reasons: (1) to divide the paste into thin films and make the mortar more porous, thus allowing the penetration of the air and facilitating the absorption of the carbonic acid which causes the setting of the mortar; (2) to prevent excessive cracking of the mortar owing to shrinkage due to the evaporation of the water in the lime paste; (3) to give greater strength to the mortar against crushing (practically the only stress that comes upon mortar), since sand has a greater resistance to compression than lime paste either before or after it has set; and (4) to reduce the amount of lime necessary to make a given bulk of mortar, thus decreasing the cost. See Sect. 5 for the requisites of good sand, and also for cement mortar which is used in brickwork of the highest class.

After the lime is slaked, the sand is spread evenly over the paste, and the ingredients are mix with a shovel or hoe, a little water being added occasionally if the mortar is too stiff. The mixing should be thoro, i.e., it should be continued until the mortar is of a uniform color.

To determine whether the proportion of sand is right, hold the hoe handle nearly horizontal and lift up a hoeful of mortar. If the mortar will not of itself slide from the hoe, it does not contain enough sand; and if a hoeful of mortar cannot be thus lifted up, it contains too much sand. The brick-mason on the wall by a somewhat similar process checks the proportions of the mortar by the way in which it slips from the trowel. If there is an excess of sand, the mortar will be "brash" or "short," and will drop from the trowel so abruptly as to make it impossible to "string out the mortar," namely, to spread the mortar over several bricks by simply allowing it to flow from the trowel as the latter is drawn along. On the other hand, if there is an excess of paste, the mortar will not flow from the trowel, at least in sufficient quantity to make the joint. This method of proportioning gives a mortar that works well under the trowel, and with reasonably clean sand also a mortar of practically maximum strength.

If the sand is very fine and contains a good deal of finely pulverized clay, the above test may be satisfied when the mortar contains too little lime; but lime paste is so cheap, and lime mortar is so weak, that a sand with any considerable amount of clay should not be used in lime mortar, since the clay is a source of weakness.

14. Laying the Brick

Wetting the Brick. Since most bricks have a great avidity for water, it is best to dampen them before laying. If the mortar is stiff and the bricks are dry, the latter absorb the water so rapidly that the mortar does not set properly, and will crumble in the fingers when dry. Neglect in this particular is the cause of most of the failures of brickwork. Since an excess of water in the brick can do no harm, it is best to thoroly drench them with water before laying. Lime mortar is sometimes made very thin, so that the brick will not absorb all the water. This process interferes with the adhesion of the mortar to the brick. Watery mortar also contracts excessively in drying, which causes undue settlement and, possibly, cracks or distortion. Wetting the

brick before laying will also remove the dust from the surface, which otherwise would prevent perfect adhesion. When the very strongest work is desired, as in brick sewers, it is customary to require that the brick shall be immersed in wate- for 3 to 5 min before being laid. Wetting in the pile is not as effective as immersion, since in the pile the water is not likely to reach all the surfaces of all of the bricks. Masons very much dislike to lay wet brick, since the water softens the skin on their fingers and causes it to wear away rapidly. The softer the bricks the more necessary that they should be thoroly wet when laid. In freezing weather, care should be taker that the water does not form a film of ice on the brick.

Laying. The bricks should not be merely laid, but every one should be prest down in such a manner as to force the mortar into the pores of the brick and produce the maximum adhesion. This is more important and also more difficult to accomplish with cement than with lime mortar. The increased value of the cement mortar can be attained only by bringing the brick and the mortar into close contact; and this is more difficult to do, since cement mortar is not as plastic as that made with lime. The mason is apt either (1) to butter the edges of the brick, and thus secure a joint that looks well after the brick is laid; or (2) to place insufficient mortar to make a full bed joint of the required thickness, run the point of his trowel thru the middle of the mass, making an open channel with a sharp ridge of mortar on each side, and then lay the brick upon the top of these two ridges, thus leaving the center of the brick unsupported. The first method is the one employed with thin joints, which is a reason why they should not be required; the second method is popular because it requires less exertion and is more rapid than fully bedding the brick.

If strength or imperviousness is a matter of any moment, care should be taken to see that the vertical joints are filled solidly full of mortar; this is called slushing the joints. Unless slushing is insisted upon, masons are apt to butter the end joints, lightly bed the brick, throw a little mortar into the top of the vertical joints, and scrape off the excess above the top of the brick, thus leaving the major portion of the vertical joints open; and sometimes little or no attempt is made to fill the vertical joint between adjacent tiers of stretchers, thus leaving also long and high unfilled vertical spaces.

For the best work it is specified that the brick shall be laid with a "shove joint"; that is, that the brick shall first be laid so as to project over the one below, both at the end and the side, and be prest into the mortar, and then be shoved into its final position. Masons are very reluctant to lay brick with a shove joint, partly because it is hard work and partly because many of them have not acquired the art. If brick are not laid with a shove joint, it is highly improbable that the lower part of the vertical joints will be filled with mortar, and consequently the wall will not be as strong or as impervious to water, air and heat as it would otherwise be.

Pointing. In laying inside walls that are to be plastered, the mortar that is forced out when the brick is prest into position is merely cut off with the trowel; but for outside walls and also for inside walls that are to be left exposed, the joints should be more carefully finished. In laying common brick the mortar in the vertical joints is simply prest back with the flat face of the trowel; but there are three methods of pointing or finishing the bed joints, viz.: (1) flush joints, (2) struck joints, and (3) weather joints.

Flush pointing consists in pressing the mortar flat with the trowel, thus making the edge of the joint flush with the face of the wall (Fig. 37). The struck joint is formed by resting the lower edge of the blade of the trowel upon the edge of the brick below the joint and drawing the trowel along the joint, which smoothes the face of the joint and slightly consolidates the mortar, and leaves the joint as shown in the center of Fig. 37. The weather joint is formed, as shown in right-hand side of Fig. 37 by pressing the mortar back with the upper edge of the trowel. This form of finish is much more durable than the

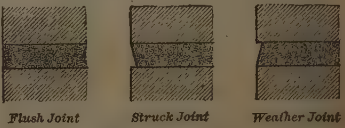


Fig. 37. Pointing Bed Joints of Common Brickwork

struck joint, since water will not lodge in the joint and soak into the mortar and on freezing dislodge the mortar; but this form of joint is much more difficult to make, since the mason stands above and back of the brick he is laying. If the weather joint is desired, it must be distinctly specified and the inspector must be watchful to see that it is secured. Brick masonry is usually laid with lime mortar or with lime-cement mortar, the lime giving cohesive strength to the mortar so that enough mortar stays in the joint to permit of its being successfully struck; but when cement mortar or mortar containing but little lime is used, the mortar is so lacking in cohesion that enough does not remain in the joint to permit of striking it, and hence with cement mortar it is necessary to formally point the masonry.

Prest brick are usually laid with a mortar made of one volume of stiff lime paste (called lime putty) and one volume of fine sand; and when this mortar is used, the brick is buttered, i.e., a little mortar is spread upon only the edges of the brick before it is laid. If the above mortar were spread over the entire surface of the brick, the joint could not be made as thin as is usually specified; but some of the better architects specify thicker joints for prest work so that the bricks can be laid otherwise than by being buttered. If the mortar is to be spread in the usual way, it should consist of 1 volume of lime paste to about 2 volumes of rather fine sand. Some architects specify 1 volume lime paste, 1 volume natural cement, and 2 volumes of fine sand. Some contractors prefer to substitute at their own expense a rich natural-cement mortar and lay thicker joints rather than lay thin buttered joints, since the brick-mason can lay more brick with the former than with the latter.

The joints of prest-brick work are finished by grooving or beading (Fig. 36, p. 591), the former being more common. The grooved joint is preferred to the flush joint, because of the variation in light and shade that the former gives to the face of a wall.

Bond is the arrangement of the bricks in successive courses to tie the wall together both longitudinally and transversely. The primary purpose of bond is to give strength to the masonry, but architects employ various longitudinal bonds to improve the appearance of the wall. Altho numerous bonds are employed for artistic effect, in the construction of ordinary brick masonry only three bonds are used, the common, the English, and the Flemish. The common bond consists of four to seven, usually five or six, courses of stretchers to one of headers. The proportionate numbers of the courses of headers and stretchers should depend on the relative importance of transverse and longitudinal strength. The proportion of one course of headers to two of stretchers is that which gives equal tenacity to the wall lengthwise and crosswise. English bond consists of alternate courses of stretchers and headers, and Flemish bond consists of a header and stretcher alternately in each course so placed that the outer end of each header lies on the middle of a stretcher in the course below.

If the wall is more than one brick thick, it should be bonded transversely as well as longitudinally. The exact arrangement of the transverse bond varies with the thickness of the wall, but is easily worked out if a little attention is given to it. The face bond is likely to receive more attention than the transverse bond, and it can be readily inspected after the completion of the wall; but the transverse bond cannot be examined after a course is laid on top of it, and therefore it should be carefully lookt after as the work progresses.

15. Brick Masonry

Crushing Strength. The table below shows the crushing strength of two kinds of brick masonry with three kinds of mortar, as determined with the testing machine at the United States Arsenal at Watertown, Mass.

All tests show that brick masonry gives evidence of distress when the load is about half the ultimate strength, hence the factor of safety should be based upon this value rather than upon the load producing complete collapse. The nominal pressure that may be safely allowed upon brick masonry depends upon (1) the quality of the materials employed; (2) the degree of care with which the work is executed; whether it is for a temporary or permanent, an

important or unimportant structure; and (3) the care with which the nominal maximum load is estimated.

The pressure at the base of a brick shot-tower in Baltimore, 246 feet high, is estimated at 6½ short tons per sq ft (about 90 lb per sq in). The pressure at the base of a brick chimney at Glasgow, Scotland, 468 ft high, is estimated at 9 tons per sq ft (about 125 lb per sq in); and in heavy gales this is increased to 15 short tons per sq ft (210 lb per sq in) on the leeward side. In 1890 the leading architects of Chicago were counted as good authorities in such matters, and did not consider it safe to allow more than 10 tons per square foot (139 lb per sq in) on the best brick laid in 1 : 2 portland-cement mortar; but since 1905 this value is frequently greatly exceeded.

Crushing Strength of Brick Piers

Age when tested 6 months, except as noted

Ref. No.	Pounds per Square Inch			Percent of the Mean Crushing Strength of the Brick		
	Neat Portland	1 Portland 3 Sand	1 Lime Paste 3 Sand	Neat Portland	1 Portland 3 Sand	1 Lime Paste 3 Sand
		Face Brick				
1	4021	2410*	1420	31	19	11
2	2880*	2400	1517	26	21	13
3	1925	1670	1260	28	25	19
		Common Brick				
4	4700*	1800*	994	42	16	9
5	1969	1800	733	44	40	16
6	1400	1411	718	24	24	12
7	1510*	1519	732	23	23	11
8	1061	1224	465*	20	23	9
Mean	2063	1671	1065			

* Strength at one month.

A representative committee of leading architects and civil engineers of Chicago in 1908 recommended the following safe working pressures in pounds per square inch for incorporation in the building laws of that city.

Paving brick in 1 : 3 portland-cement mortar	350
Prest and sewer brick having a crushing strength of 5000 lb per sq in, in 1 : 3 portland mortar	250
Select hard common brick having a strength of 2500 lb per sq in:	
in 1 : 3 portland-cement mortar	200
in 1 portland cement, 1 lime paste and 3 sand	175
Common brick having a strength of 1800 lb per sq in:	
in portland-cement mortar	175
in natural-cement mortar	150
in lime and cement mortar	125
in lime mortar	[illegible]

Estimates. If the brick be of standard size (8¼ by 4 by 2¼ inches) and laid with ½- to ⅝-inch joints, a cubic yard of masonry will require about 410 brick; or a thousand brick will lay about 2½ cubic yards. If the joints are ¼ to ⅜ inch, a cubic yard of masonry will require about 495 brick; or a thousand brick will lay about 2 cubic yards. With face brick (8¾ by 4⅛ by 2¼ inches) and ⅛-inch joints, a cubic yard of masonry will require about 496 brick; or a thousand face brick will lay about 2 cubic yards. An allowance

must be made for breakage, and for waste in cutting brick to fit angles, etc. With good brick, in massive work this allowance need not exceed 1 or 2 percent; but in buildings 3 to 5 percent is none too much.

Mortar. With the standard size of brick (8¼ by 4 by 2¼ inches), a cubic yard of masonry, laid with ½- to ⅝-inch joints, will require from 0.35 to 0.40 cu yd of mortar; or a thousand brick will require 0.80 to 0.90 cu yd. If the joints are ¼ to ⅜ inch, a cubic yard of masonry will require from 0.25 to 0.30 cu yd of mortar; or a thousand brick will require from 0.45 to 0.55 cu yd. If the joints are ⅛ of an inch, a cubic yard of masonry will require from 0.10 to 0.15 cu yd of mortar; or a thousand brick will require from 0.15 to 0.20 cu yd. Ordinarily 0.75 barrel of unslaked lime or 1 barrel of lime paste and 0.75 cu yd of sand will lay a thousand bricks.

Cost. The number of days' work required to lay one cubic yard of brick masonry was as follows for four large massive jobs:

High Bridge enlargement, N. Y. City:	
Lining wall and flat arches laid with very close joints..............	0.714
Washington (D. C.) Aqueduct:	
Circular conduit, 9 feet in diameter with walls 12 inches thick......	0.439
St. Louis Waterworks:	
Semicircular conduit, 6 feet in diameter..........................	0.364
New York City Storage Reservoir:	
Lining of gate-house walls and arches, rough work.................	0.304

In the United States Government buildings the cost of labor per thousand, including tools, etc., is estimated at the wages of mason and helper for 8.75 hours.

The following table shows the cost of the labor for five brick buildings forming part of a large manufacturing plant. Buildings No. 1 and 2 were long and low, with about equal amounts of 9-inch and 13-inch walls; buildings No. 3 and 4 had large proportion of 13 inch walls; building No. 5 contained more brick than any of the others, and had 13-inch walls, with some 17-inch and 22-inch walls.

The total average cost per cubic yard of the brick masonry for No. 1 was $7.68, the items being as follows: masons $2.08, helpers $0.93, carpenters $0.39, common labor $0.58, brick $2.89, cement $0.44, lime $0.20, sand $0.17.

Cost of Labor per 1000 Brick

Kind of Labor and Price	Building No.					Average
	1	2	3	4	5	
Bricklayers, 60 cts per hour* ..	$5.56	$4.49	$4.57	$4.68	$3.68	$4.16
Helpers, 17½ cts per hour ...	1.95	1.67	2.14	1.95	2.00	1.87
Carpenters, 21¼ cts per hour .	.70	.71	.88	1.15	.67	.77
Handling materials...........	1.16	1.16	1.16	1.16	1.16	1.16
Total for labor..........	$9.37	$8.03	$8.75	$8.94	$7.51	$7.96

* On Building No. 1 bricklayers received 50 cts per hr.

Efflorescence is a white deposit which frequently disfigures the surface of brick masonry, especially in a moist climate or in damp places. This deposit generally originates with the mortar, but frequently spreads over the entire face of the wall. The water which is absorbed by the mortar dissolves the salts of soda, potash, magnesia, etc., contained in the lime or cement, and on evaporating deposits these salts as a white efflorescence on the surface.

With lime mortar the deposit is frequently very heavy, particularly on plastering; and, usually, it is heavier with natural than with portland cement. The efflorescence sometimes originates in the brick, particularly if the brick was burned with sulfurous coal, or was made from clay containing iron pyrites; and when the brick gets wet the water dissolves the sulfates of lime and magnesia, and on evaporating leaves the crystals of these salts on the surface. Frequently the efflorescence on the brick is due to the absorption by the brick of the impregnated water from the mortar. This efflorescence is objectionable chiefly because of the unsightly appearance which it often produces, but also because the crystallization of these salts within the pores of the mortar and of the brick or stone causes disintegration which is in many respects like frost.

As a palliative, Gillmore recommended the addition of 100 lb of quicklime and 8 to 12 lb of any cheap animal fat to each barrel of cement. The lime is simply a vehicle for the fat, and should be thoroly incorporated with the cement before slaking. The object of the fat is to saponify the alkaline salts. The method is not entirely satisfactory, since the deposit is only made less prominent and less effective, and not entirely removed or prevented.

As a preventive, make the wall as impervious as possible by using a rich mortar (preferably of portland cement), mixing it well, and filling all the joints solidly full. If the wall stands in damp ground, one or more of the horizontal joints should contain a layer of tarred paper or bituminous felt to prevent the wall's absorbing moisture from below. Particular care should be taken during the erection of the building to see that the roof, cornice, and gutters are made water-tight; and all ducts that carry water or steam pipes should be water-proofed on their inner surfaces. After the building is finished, if the efflorescence appears, all leakage of water into the wall must be stopt; and if the efflorescence is due to the penetration of rain-water through the exterior face of the wall, then the face may be rendered impervious by the application of one or more pairs of the Sylvester washes which will not materially darken or discolor the bricks.

The Sylvester washes consist of an alum solution made by dissolving 1 pound of alum per gallon of water, and a soap solution made by dissolving 2.2 pounds of reasonably pure hard soap per gallon of water. The brick masonry should be clean and dry, and not colder than about 50° F. The soap wash should be applied boiling hot, but the alum solution may be 60° to 70° F. when applied. One wash should be put on and allowed to dry for 24 hours, when the other is applied. The above washes have long been used for rendering masonry impervious; but instead of using alum as above, it is better to use aluminium sulfate (sometimes, but improperly, called alum). The aluminium sulfate is cheaper than alum, and only two-thirds as much is required.

Efflorescence will gradually be blown away by the winds and be washt off by the rains, but it can be entirely removed with scrubbing-brushes and hydrochloric acid mixt with at least four or five times its volume of water. Before applying the acid, the wall should be well dampened; and after being scrubbed, the wall should be thoroly washt with clear water.

FOUNDATIONS ON LAND

16. Examining the Site

If the nature of the soil has not already been revealed to a considerable depth by excavations, it will be necessary to make an examination of the subsoil preparatory to deciding upon the details of the method of constructing the foundation. Except for the heaviest structures, it will usually be sufficient, after having dug the foundation pits or trenches, to examine the soil by driving a steel rod or boring a hole with a post-auger from 3 to 5 feet further, the depth depending upon the nature of the soil and the weight and importance of the intended structure, but for the largest structures it is necessary to

examine the soil to greater depths, in which case the following devices may be employed.

In Soft Soil, soundings 20 or 30 feet deep can be made by driving a rod or sections of gas-pipe with a hammer or maul from a temporary scaffold, the height of which will of course depend upon the length of the rod or of the sections of the pipe. Good judgment is required in interpreting the results of such tests, particularly if the structure is to be a heavy one or a bridge abutment or pier in a stream liable to scour. A layer of compact sand or cemented gravel, which may be scoured away, may be mistaken for a ledge of rock; but the difference can usually be detected by striking the rod or pipe with a hammer, since rock will give a decided rebound, while gravel or sand will not. A bowlder may be mistaken for bed rock; but the difference can usually be detected by making one or more additional tests, and accurately noting the depths at which rock is struck. If samples of the soil are desired, use a 2-inch pipe open at the lower end. If much of this kind of work is to be done, it is advisable to fit up a hand pile-driving machine, using a block of wood for the dropping weight.

Borings 50 to 100 feet deep can be made very expeditiously in common soil or clay with a common wood-auger turned by men with levers 3 or 4 feet long. Or the boring may be made with any one of several earth augers having a spoon-like form for bringing up samples of the soil. An auger will bring up samples sufficient to determine the nature of the soil, but not its compactness, since it will probably be comprest somewhat in being cut off. When the testing must be made thru sand or loose soil, it may be necessary to drive down a steel tube to prevent the soil from falling into the hole. The sand may be removed from the inside of this tube with an auger, or with the "sand-pump" used in digging artesian wells.

Water Jet. In soft soil or clay that can be washt with a stream of water, a hole can be sunk rapidly by driving a pipe, inserting a smaller pipe inside of it, and forcing water down the inside pipe, the débris and water flowing up between the two pipes.

Drilling. When the subsoil is composed of various strata, particularly if there are strata of hard soil or rock, it is necessary to use a percussion drill in connection with some form of core drill; and in extreme cases the diamond drill is employed. Great care is needed in interpreting the results of such borings. In using the percussion drill, care must be taken that a stratum sufficiently hard to serve as a foundation is not past by unnoticed. This can be prevented by taking dry cores at frequent intervals. In using a core drill care must be taken to discriminate between erratic bowlders and native ledge rock.

17. Bearing Capacity of Soils

Test Load. A stick of timber, say a foot square, may be set in a vertical position and a platform built around it; then the platform may be loaded and the settlement observed. Or a stout frame or table may be constructed, the legs of which are heavy timbers that rest upon foot-plates 1 foot square. In interpreting the results the fact should not be overlookt that a small area will bear a larger load per unit of area for a short time than a larger area perpetually; and hence the area tested should be as large as practicable and the test should continue as long as possible.

The best method of determining the load a specific soil will bear is by direct experiment; but good judgment and experience, aided by a careful study of the nature of the soil, will enable one to determine with reasonable accuracy its probable supporting power. The terms "bearing power" and "supporting power" are often used as synonymous

with bearing capacity, the numerical measure of each being usually exprest in short tons per square foot.

Rock. The ultimate compressive strength of stone, as determined by crushing cubes, ranges from 150 tons per square foot for the softest stone to 2000 tons per square foot for the hardest. The crushing strength of slabs is much greater than that of cubes; and the crushing strength of slabs when the pressure is concentrated on only a portion of the upper surface is much greater than for a load uniformly distributed over the entire upper surface. Therefore it is safe to say that any ordinary rock in its native bed will bear any load that can be brought upon it by an artificial structure.

Clay Soils vary from slate or shale, which will support any load that can come upon it, to a soft, wet clay which will squeeze out in every direction when a moderately heavy pressure is brought upon it. Foundations on clay should be laid at such depths as to be unaffected by the weather, since clay, at even considerable depths, will gain and lose considerable water as the seasons change. The bearing capacity of clayey soils can be very much improved by drainage, or by preventing the penetration of water. If the foundation is laid upon undrained clay, care must be taken that excavations made in the immediate vicinity do not allow the clay under pressure to escape by oozing away from under the building. When the clay occurs in strata not horizontal, great care is necessary to prevent this flow of the soil. When coarse sand or gravel is mixt with the clay, its supporting capacity is greatly increased, being greater in proportion as the quantity of these materials is greater. When they are present to such an extent that the clay is just sufficient to bind them together, the combination will bear nearly as heavy loads as the softer rocks.

Experiments made on the clay under the piers of the bridge across the Missouri River at Bismarck, with surfaces 1½ feet square, gave an average ultimate bearing power of 15 tons per sq ft. Clay in thick compact beds, without any admixture of loam or vegetable matter, has carried 10 short tons per sq ft without appreciable settlement. In the case of the Congressional Library, Washington, D. C., the ultimate supporting power of "yellow clay mix with sand" was 13½ short tons per sq ft; and the safe load was assumed to be 2½ short tons per sq ft. From the experiments made in connection with the construction of the capitol at Albany, N. Y., the conclusion was drawn that the extreme supporting power of that soil was less than 6 tons per sq ft, and that the load which might be safely imposed upon it was 2 short tons per sq ft. "The soil was blue clay containing from 60 to 90 per cent of alumina, the remainder being fine siliceous sand. The soil contains from 27 to 43, usually about 40, percent of water; and various samples of it weighed from 81 to 101 lb per cu ft." At Chicago it was formerly the custom to found upon the clay, and the load ordinarily put on a thin layer of clay (hard above and soft below, resting on a thick stratum of quicksand) was 1½ to 2 short tons per sq ft; and the settlement, which usually reached a maximum in a year, was about 2 to 2½ inches per short ton of load.

Sandy Soils vary from coarse gravel to fine sand. The former when of sufficient thickness forms one of the firmest and best foundations; and the latter when saturated with water is practically a liquid. Sand when dry, or wet sand when prevented from spreading laterally, forms one of the best beds for a foundation. Porous, sandy soils are, as a rule, unaffected by stagnant water, but are easily removed by running water; in the former case they present no difficulty, but in the latter they require extreme care at the hands of the constructor, as will be considered later.

Compact gravel or clean sand, in beds of considerable thickness, protected from being carried away by water, may be loaded with 8 to 10 short tons per sq ft with safety. In an experiment in France, clean river-sand compacted in a trench supported 100 short tons per sq ft. Fine sand well cemented with clay and compacted, if protected from water,

will safely carry 4 to 6 short tons per sq ft. The piers of the Cincinnati Suspension Bridge are founded on a bed of coarse gravel 12 feet below low water, altho solid limestone was only 12 feet deeper; if the friction on the sides of the pier be disregarded, the maximum pressure on the gravel is 4 short tons per sq ft. The New York pier of the Brooklyn Suspension Bridge is founded 44 feet below the bed of the river, upon a layer of sand 2 feet thick resting upon bed-rock, the maximum pressure being about 6¾ short tons per sq ft. At Chicago sand and gravel about 15 feet below the surface are successfully loaded with 2 to 2½ short tons per sq ft. At Berlin the safe load for sandy soil is generally taken at 2 to 2½ short tons per sq ft. The Washington Monument, Washington, D. C., rests upon a bed of very fine sand two feet thick underlying a bed of gravel and bowlders, the ordinary pressure on certain parts of the foundation being not far from 11 short tons per sq ft, which the wind may increase to nearly 14 short tons per sq ft.

Semi-Liquid Soils as mud, silt, alluvium, or quicksand, have little or no supporting power; and with any one of these soils it is customary (1) to remove it entirely, or (2) to sink piles, tubes, or caissons thru it to a solid substratum, or (3) to consolidate the soil by adding earth, sand, stone, etc. The method of performing these operations will be described later. Soils of a soft or semi-liquid character should never be relied upon for a foundation when anything better can be obtained; but a heavy superstructure may be supported by the upward pressure of a semi-liquid soil, in the same way that water bears up a floating body.

It is difficult to give results of the safe bearing power of soils of this class. A considerable part of the supporting power is derived from the friction on the vertical sides of the foundation, and hence the bearing power depends to a considerable degree upon the area of the side surface in contact with the soil; and with this class of soils it is particularly important that the area tested should be as large as possible. Furthermore, it is difficult to determine the exact supporting power of a plastic soil, since a considerable settlement is certain to take place with the lapse of time.

According to Rankine a building will be supported when the pressure per unit of area at its base is $wh \tan^4 (45° + ½ \alpha)$, in which w is the weight of a unit volume of the soil, h is the depth of immersion, and α is the angle of repose of the soil. If $\alpha = 5°$, then according to the preceding relation the supporting power of the soil is 1.4 wh per unit of area; if $\alpha = 10°$, it is 2.0 wh; and if $\alpha = 15°$, it is 2.9 wh. The weight of soils of this class varies from 100 to 130 lb per cu ft. Rankine gives this formula as being applicable to any soil; but since it takes no account of cohesion, for most soils it is only roughly approximate, and gives results too small. The following experiment seems to show that the error is considerable. "A 10-foot square base of concrete resting on mud whose angle of repose was 5 to 1 or $\alpha = 11⅓°$, bore 700 lb per sq ft." This is 2½ times the result by the above formula, using the maximum value of w. Experience at New Orleans with alluvial soil and a few experiments that have been made on quicksand seem to indicate that with a load of ½ to 1 short ton per sq ft the settlement will not be excessive.

A Summary of the preceding discussion is presented in the following table for convenient reference. It is well to notice that there are some practical

Safe Bearing Capacity of Soils in Short Tons per Square Foot

Kind of Material.	Min.	Max.
Rock, the hardest, in thick layers in native bed	200	..
Rock equal to best ashlar masonry	25	30
Rock equal to best brick masonry	15	20
Rock equal to poor brick masonry	5	10
Clay in thick beds, always dry	6	8
Clay in thick beds, moderately dry	4	6
Clay, soft	1	2
Gravel and coarse sand, well cemented	8	10
Sand, dry, compact and well cemented	4	6
Sand, clean, dry	2	4
Quicksand, alluvial soils, etc.	0.5	1

considerations that modify the pressure which may safely be put upon a soil. For example, the pressure on the foundation of a tall chimney should be considerably less than that of the low massive foundation of a fire-proof vault. In the former case a slight inequality of bearing power, and consequent unequal settling, might endanger the stability of the structure; while in the latter no serious harm would result. The pressure per unit of area should be less for a light structure subject to the passage of heavy loads than for a heavy structure subject only to a quiescent load, since the shock and jar of the moving load are far more serious than the heavier quiescent load.

In foundations for buildings, it may be necessary to provide a safeguard against the soil's escaping by being prest out laterally into excavations in the vicinity. For example, in Chicago some of the largest and finest buildings have settled owing to the flow of the plastic clay into foundations opened across the street. In New York City one of the largest buildings settled because of the pumping of fine sand from an artesian well on the site in getting water for the boilers of the building.

18. Improvement of Bearing Capacity

Increasing the Depth. The simplest method of increasing the bearing capacity is to dig deeper. Ordinary soils will bear more weight the greater the depth reached, owing to their becoming more condensed from the superincumbent weight. Depth is especially important with clay, since it is then less liable to be displaced laterally owing to other excavations in the immediate vicinity, and also because at greater depths the amount of moisture in it will not vary so much. However, occasionally the soil grows more moist as the depth increases beyond a moderate distance, in which case increasing the depth is undesirable. For example, in Chicago the clay grows softer after a depth of about 12 to 14 feet below the sidewalk is reached. In any soil, the bed of the foundation should be below the reach of frost. Even a foundation on bed-rock should be below the frost line, else water may get under the foundation thru fissures and, freezing, do damage.

Drainage. Another simple method is to drain the soil. The water may find its way to the bed of the foundation down the side of the wall, or by percolation thru the soil, or thru a seam of sand. In most cases the bed can be sufficiently drained by surrounding the building with a tile drain laid a little below the foundation. In more difficult cases, the expedient is employed of covering the site with a layer of gravel, the thickness depending upon the plasticity of the soil, the gravel serving the double purpose of distributing the concentrated loads of the footings to a larger area of the native soil and of improving the drainage of the bed of the foundation. In extreme cases, it is necessary to enclose the entire site with a puddle-wall to cut off drainage water from a higher area.

Adding Sand. The simplest method of improving the bearing capacity of a compressible soil is to spread sand or gravel or broken stone over the bed of the foundation, and pound it into the soil, thus forming a comparatively compact stratum upon which to found the structure. This method is not very effective, since at best the effect of the blow cannot extend very deep, while the heavy masses of the masonry make themselves felt at great depths. A more efficient way is to make an excavation a little larger than the proposed structure and cover the bed of the foundation with a layer of sand or gravel. The sand should be deposited in successive layers, each of which should be thoroly tamped before laying the next. The sand should be moist, so it will pack well. Sand, when used in this way, possesses the valuable property of assuming a new position of equilibrium and stability should the soil on which

it is laid yield at any of its points; and not only does this take place along the base of the sand bed, but also along its edges or sides. The bed of sand or gravel must be thick enough to distribute the pressure on its upper surface over the entire base of the trench.

Wood Piles. If the soil is very soft, it can be consolidated to a considerable depth by driving wood piles, for which purpose many small ones are preferable to fewer but larger ones. It is customary to employ piles about 6 feet long and about 6 inches in diameter, since this size can be driven with a hand maul or by dropping a heavy block of wood with a tackle attached to any simple frame, or by a hand pile-driver. They may be driven as close together as necessary, altho 2 to 4 feet in the clear is usually sufficient. Clay is compressible, while sand is not; and hence this method of consolidating soils is not applicable to sand, and is not very efficient in soils largely composed of it.

When the piles are driven primarily to compact the soil, it is customary to load them and also the soil between them, either by cutting off the piles near the surface and laying a tight platform of timber on top of them, or by depositing a bed of concrete between and over the heads of the piles. If the soil is very soft or composed largely of sand, this method is ineffective; in which case long piles are driven as close together as is necessary, the supporting power being derived either from the resting of the piles upon a harder substratum or from the buoyancy due to immersion in the semi-liquid soil.

Sand Piles. Experiments show that in compacting the soil by driving wood piles it is better to withdraw them and immediately fill the holes with sand, than to allow the wooden piles to remain. This advantage is independent of the question of the durability of the wood. When the wooden pile is driven, it compresses the soil an amount nearly or quite equal to the volume of the pile, and when the latter is withdrawn this consolidation remains, at least temporarily. If the hole is immediately filled with sand this compression is retained permanently, and the consolidation may be still farther increased by ramming in the sand in thin layers, owing to the ability of the latter to transmit pressure laterally. And further, the sand pile will support a greater load than the wooden pile; for, since the sand acts like innumerable small arches reaching from one side of the hole to the other, more of the load is transmitted to the soil on the sides of the hole. To secure the best results, the sand should be fine, sharp, clean, and of uniform size.

The Compressol System has recently been employed to a considerable extent in Europe It consists in forming a hole in compressible soil by dropping a heavy conical iron weight, or "perforator," from a considerable height, and then filling the hole with concrete upon which is to rest a column or beam which carries the superstructure. The perforator usually has a base of 2½ to 3 feet, and weighs about 2 tons; and frequently has a fall of 20 to 30 feet. Holes have been sunk 50 feet deep by this process. The perforator compresses the soil laterally, and thereby greatly increases its water-tightness; and by dropping a little lime paste or wet clay into the hole before each fall of the perforator, it is usually possible to make the hole absolutely water-tight, since the lime or clay is plastered and compacted on the sides of the hole. Sometimes bowlders are rammed into the soil at the bottom of the hole, by dropping a pear-shaped weight upon them, thus still further consolidating the soil. Finally the hole is filled with concrete, the thoro tamping of which enlarges the hole and still further consolidates the soil, the amount of concrete put into the hole frequently being three or four times the original volume of the hole.

This method of founding has a number of marked advantages. (1) No excavations are required, and therefore there is no danger of disturbing the equilibrium of the soil. (2) It eliminates all danger to men working below the surface of the ground. (3) It is

comparatively cheap, since all the operations are performed by machinery. (4) It is quite rapid, since a hole from 25 to 30 feet deep can be sunk and filled in 3 or 4 hours. (5) It is possible to sink a hole as deep and as large as required by the desired bearing capacity.

19. Loads for Designing Foundations

Spread Foundations are those in which the width is increased until the load can be safely carried, and for soft or compressible soils the loads must be accurately determined. The structures requiring the most careful consideration are buildings, since usually they give a greater unit pressure upon the soil and are more likely to settle unevenly and crack. The load consists of three parts, that of the building itself, the movable loads on the floors, and the part of the load that may be transferred from one part of the foundation to the other by the wind.

Dead Load. The weight of the building is ascertained by calculating the cubical contents of all the various materials in the structure. If the weight is not equally distributed, care must be taken to ascertain the proportion to be carried by each part of the foundation. For example, if one vertical section of the wall is to contain a number of large windows while another will consist entirely of solid masonry, it is evident that the pressure on the foundation under the first section will be less than that under the second. In this connection it must be borne in mind that concentrated pressures are not transmitted, undiminished, thru a solid mass of masonry in the line of application, but spread out in successively radiating lines; and hence, if any considerable distance intervenes between the foundation and the point of application of this concentrated load, the pressure will be nearly or quite uniformly distributed over the entire area of the base.

Ordinary lathing and plastering weighs about 10 lb per sq ft. The weight of floors is approximately 10 lb per sq ft for dwellings, 25 for public buildings, and 40 or 50 for warehouses. The weight of the roof varies with the kind of covering, the span, etc.; but a shingle roof may be taken at 10 and a roof covered with slate or corrugated iron at 25 lb per sq ft. Weights of different kinds of masonry in pounds per cubic foot are as follows:

Brickwork, prest brick, thin joints	145
Brickwork, ordinary quality	125
Brickwork, soft brick, thick joints	100
Concrete, 1 cement, 3 sand, and 6 broken stone	140
Granite, 6 percent more than the corresponding limestone	...
Limestone, ashlar, largest blocks and thinnest joints	160
Limestone, ashlar, 12- to 20-inch courses and ⅜- to ½-inch joints	155
Limestone, squared-stone	150
Limestone, rubble, best	140
Limestone, rubble, rough	135
Sandstone, 14 percent less than the corresponding limestone	...

Live Load. The movable load on the floors depends upon the nature of the building. For dwellings, it does not exceed 10 lb per sq ft; for large office buildings, it is usually taken at 30 lb per sq ft, but is seldom if ever that high; for churches and theaters, the maximum load, a crowd of people, seldom reaches 100 lb per sq ft; for stores, warehouses and factories, the load will be from 100 to 400 lb per sq ft, according to the purposes for which they are used. These live loads are to be used in determining the strength of the floor, and not in designing the footings; for there is no probability that each and every square foot of floor will have its maximum load at the same time. The amount of moving load reaching the footings in any particular case is a matter of judgment. At Chicago in designing tall steel-skeleton office build-

ings, hotels, and retail stores, it is the practise to assume that nearly all of the maximum live load reaches the girders, that a smaller percent reaches the columns of the upper story and a decreasing amount the columns of the succeeding stories downward, and that no live load reaches the footings. In wholesale stores and warehouses a portion of the total live load is assumed to reach the footings, the exact amount being a matter of judgment and varying with the circumstances. In many cities the building law specifies the proportion of live load to be assumed as reaching the footing.

On a compressible soil it is very important that the live load assumed as reaching the footings shall be neither over- nor under-estimated. The dead load can be estimated with sufficient accuracy, and as the load on the footings under the walls is chiefly dead load, this part of the foundation is likely to receive the assumed load. But the possible maximum on the footings of interior columns is made up largely of live load, and if the live load reaching these footings is taken too large, the footings are likely to be made too great and consequently the columns will not settle as much as the walls; and on the other hand, if the live load reaching the column footings is taken too small, the columns will settle more than the walls. Experience in Chicago in founding upon a compressib e soil shows that the settlements of the columns and the walls of eight- and ten-story office buildings, hotels, and retail stores are almost exactly the same when designed on the assumption that no live load reaches the footings.

Attention must be given to the manner in which the weight of the roof and floors is transferred to the walls. For example, if the floor joists of a warehouse run from back to front, it is evident that the back and front walls alone will carry the weight of the floors and of the goods placed upon them, and this will make the pressure upon the foundation under them considerably greater than under the other walls. Again, if a stone-front is to be carried on an arch or on a girder having its bearings on piers at each side of the building, it is manifest that the weight of the whole superincumbent structure, instead of being distributed equally on the foundation under the front, will be concentrated on that part of the foundation immediately under the piers.

20. Unit Pressure on Foundations

Area of Foundation. This is determined by dividing the weight of the structure by the pressure which may safely be brought upon the soil. Then having found the area of foundation, the base of the structure must be extended by footings of masonry, concrete or timber, so as to cover that area and distribute the pressure uniformly over it. The object is not so much to secure an absolutely unyielding base as to secure one that will settle as little as possible, and uniformly. All soils will yield somewhat under the pressure of any building, and even masonry itself is comprest by the weight of the load above it. The pressure per square foot should, therefore, be the same for all parts of the building, and particularly of the foundation, so that the settlement may be uniform. This can be secured only when the axis of the load (a vertical line thru the center of gravity of the weight) passes thru the center of the area of the foundation. If the axis of pressure does not coincide exactly with the axis of the base, the ground will yield most on the side which is prest most; and, as the ground yields, the base assumes an inclined position, and carries the lower part of the structure with it, thus producing unsightly cracks, if nothing more.

The coincidence of the axis of pressure with the axis of resistance is of the greatest importance. The principle is almost self-evident, and yet the neglect to observe it is the most frequent cause of failure in the foundations of buildings. Fig. 38 is an example of one way in which this principle is violated. The shaded portion represents a heavily loaded exterior wall, and the unshaded portion a lightly loaded interior wall. The foundations of the two walls are rigidly connected at their intersection. The center of the load is under the shaded section, and the center of the resisting area is at some point farther to the left; consequently the exterior wall is caused to incline outward producing cracks at or near the corners of the building. The two foundations are con-

nected in the belief that an increase of the bearing surface is of advantage; but the true principle is that the coincidence of the axis of pressure with the axis of resistance is of more importance. Fig. 39 is another illustration of the same principle. The foundation is continuous under the opening, and hence the center of the foundation is to the left of the center of pressure; consequently the wall inclines to the right, producing cracks, usually over the opening.

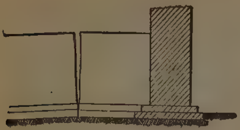


Fig. 38.

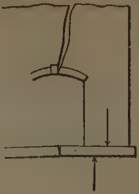


Fig. 39.

One conclusion to be drawn from the above examples is that the foundation of a wall should never be connected with that of another wall either much heavier or much lighter than itself, as both are equally objectionable. A second conclusion is that the axis of the load should strike a little inside of the center of the area of the base, to make sure that it will not be out ide. Any inward inclination of the wall is rendered impossible by the interior walls of the building, the floor beams, etc.; while an outward inclination can be counteracted only by the bond of the masonry and by anchors. A slight deviation of the axis of the load outward from the center of the base has a marked effect, and is not easily counteracted by anchors.

The center of the load can be made to fall inside of the center of foundation by extending the footings outwards, or by curtailing the foundations on the inside. The latter finds exemplification in the properly constructed foundation of a wall containing a number of openings. For example, in Fig. 40, if the foundation is uniform under the entire front, the center of pressure must be outside of the center of the base; and consequently the two side walls will incline outward, and show cracks over the openings. If the width of the foundation under the openings be decreased, or if this part of the foundation be omitted entirely, the center of pressure will fall inside of the center of base and the walls will tend to incline inwards, and hence be stable.

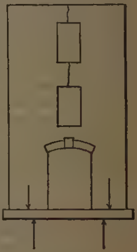


Fig. 40

An Important Principle derived from the above conclusions is the following: Foundations should be so constructed as to compress the ground slightly concave upwards, rather than convex upwards. On even slightly compressible soils, a small difference in the pressure on the foundation will be sufficient to cause the bed to become convex upwards. At Chicago, in buildings founded upon soft clay an omission of 1 to 2 percent of the weight (by leaving openings) usually causes sufficient convexity to produce unsightly cracks. With very slight differences of pressure on the foundation, it is sufficient to tie the building together by careful bonding, by hoop-iron built in over openings, and by heavy bars built in where one wall joins another.

The art of constructing foundations on a compressible soil was brought to a high degree of development by the architects of Chicago between 1870 and 1890, when the principal buildings were founded upon a bed of soft clay. The special feature of the practice in that city is what is called "the method of independent piers"; that is, each tier of columns, each pier, and each wall has its own independent foundation, the area of which is proportioned to the load on that part. The interior walls are fastened to the exterior ones by anchors which slide in slots.

The opposite extreme is to rest the structure upon a platform of concrete, timber, or steel beams so strong as to resist local settlement. This method is not usually successful; and when it is successful, it is exceedingly expensive and usually needlessly extrava-

gant. The post-office building erected in Chicago in 1875 rested upon a bed of concrete 3 feet thick over the entire site; but the concrete was insufficient to resist the unequal loading, and the building settled so badly and so unevenly that it was necessary to demolish it after it had stood only seventeen years. It is said that some noted buildings in Europe rest upon beds of concrete 8 or 10 feet thick.

Effect of Wind. On chimneys and tall structures the action of wind increases the pressure on one side and decreases it on the other. The maximum horizontal pressure of the wind is usually taken as 50 lb per sq ft on a flat surface perpendicular to the wind, and on a cylinder at about 30 lb per sq ft of the projection of the surface. In Fig. 41 let $ABED$ represent a vertical section of the chimney; a is a point horizontally opposite the center of the surface exposed to the pressure of the wind and vertically above the center of gravity of the chimney; C is the position of the center of pressure when there is no wind; N is the center when the wind is acting. Let H = the total pressure of the wind against the exposed surface, W = the weight of the structure above the section considered, l = the distance AB, h = the distance aC, d = the distance NC found from $d = hH/W$. When there is no horizontal force acting, the load on AB is uniform; but when the wind blows from the right, the pressure is greatest near A and decreases towards B. The maximum pressure at A will be that due to the weight of the chimney plus the compression due to flexure; and the pressure at B will be the compression due to the weight minus the tension due to flexure. For a rectangular base with length l normal to the width b the average pressure is W/lb, and

$$\text{Maximum unit-pressure at } A = \frac{W}{lb}\left(1 + 6\frac{d}{l}\right)$$

$$\text{Minimum unit-pressure at } B = \frac{W}{lb}\left(1 - 6\frac{d}{l}\right)$$

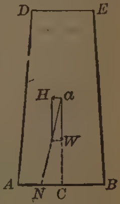


Fig. 41

These equations show that when $d = NC = \frac{1}{6}l$, the maximum pressure at A is twice the average, and that the pressure at B is zero. This is equivalent to what is known, in the theory of arches, as the principle of the middle-third. It shows that as long as the center of pressure lies in the middle-third, the maximum pressure is not more than twice the average pressure, and that there is no tendency to produce tension at B. The average pressure per unit on AB is supposed to have been adjusted to the safe bearing power of the soil, and if the maximum pressure at A does not exceed the ultimate bearing power, the occasional maximum pressure due to the wind will do no harm; but if this maximum exceeds or is dangerously near the ultimate strength of the soil, the base must be widened.

21. Footings

The footing is the bottom course or courses of masonry, timber, or steel beams employed to increase the area of the foundation. Whatever the character of the soil, footings should extend beyond the face of the wall (1) to add to the stability of the structure and lessen the danger of the work being thrown out of plumb, and (2) to distribute the weight of the structure over a larger area and thus decrease the settlement due to the compression of the ground. To serve the first purpose, footings must be securely bonded to the body of the wall; and to produce the second effect, they must have sufficient strength to resist the transverse strain to which they are exposed.

In Masonry Footings the portion of the footing that projects beyond the one above it acts as a cantilever beam uniformly loaded. To determine the safe offset let P = the pressure in short tons per sq ft at the bottom of the

footing course under consideration, R = the modulus of rupture of the material in lb per sq in, o = the greatest possible offset or projection of the footing course in inches, t = the thickness of the footing course in inches, f = the factor of safety. Then with sufficient accuracy, $o = \frac{1}{6} t\sqrt{R/Pf}$.

The factor of safety will depend upon the care used in computing the loads, in selecting the materials for the footing courses, and in bedding and placing them. If all the loads have been allowed for at their probable maximum value, and if the material is to be reasonably uniform in quality and laid with care, then a comparatively small factor is sufficient; but if all the loads have not been carefully computed, and if the job is to be done by an unknown contractor, and neither the material nor the work is to be carefully inspected, then a large factor is necessary. It is wiser to assume, for each particular case, a factor of safety in accordance with the attendant conditions of the problem than to blindly use the result deduced by the application of some arbitrarily assumed factor. The following table shows the results obtained by the above formula, using 10 as a factor of safety. It gives values of o/t for three values of P. For example, if P is 2 tons per sq ft, then for granite $o/t = 1.6$, and hence if t is 12 inches the offset o should be 19 inches.

Safe Offset for Masonry Footing Courses

Kind of Stone	R in lb per sq in	P in short tons per sq ft		
		0.5	1.0	2.0
Bluestone, North River	5026	5.2	3.7	2.7
Granite	1849	3.2	2.2	1.6
Limestone	1377	2.8	2.0	1.4
Sandstone	1378	2.3	2.0	1.4
Good building brick in poor 1 : 2 natural-cement mortar, age 60 days	120	0.8	0.6	0.4
Under-burned building brick in 1 : 3 portland-cement mortar, age 60 days	706	2.0	1.4	1.0
Vitrified building brick in 1 : 3 portland-cement mortar, age 60 days	3560	5.0	3.2	2.2
Concrete, 1 : 2 : 4 portland, at 1 month	300	1.3	0.9	0.6
Concrete, 1 : 2 : 4 portland, 6 months	400	1.6	1.1	0.7

The above computation when applied to STONE-MASONRY footings is strictly correct only for the lower offset, and then only when the footing is composed of stones whose thickness is equal to the thickness of the course and which project less than half their length, and which are also well bedded. The resistance of two or more courses to bending, if bedded in good cement mortar, probably varies about as the square of their combined depth, and the bending due to the uniform pressure on the base increases as the square of the sum of the projections; and therefore the successive offsets should be proportional to the thickness of the course; or, in other words, the values as above are applicable to any of the several projecting courses, provided no stone projects more than half its length beyond the end of the top course. The preceding results will be applicable to built footing courses only when the pressure above the course is less than the safe crushing strength of the mortar. The proper projection for rubble masonry lies somewhere between the values for stone and for concrete. If the rubble consists of large stones well bedded in good strong portland-cement mortar, then the values for this class of masonry will be but little less than those given for stone; but if the rubble consists of small irregular stones laid with portland-cement mortar, the projection should not much exceed that given for concrete. Footing courses should not be laid of small stones in either natural-cement or lime mortar. The offsets for BRICKWORK are the combined offsets of one or more projections in terms of the total thickness of the one or more projections. A CONCRETE footing should be built as a monolith for its full depth, since the deeper the beam the greater its strength; but the outer upper corner may be stept to save concrete, provided the combined projection in any case does not exceed that given in the table.

After the safe length of the offset of the footing has been determined, it should be

examined to see if it is safe against failure by shearing. Since footings are subject to heavy loads and consequently to great shearing stress, and since the allowable shearing stress for stone and brick is only about 10 to 12 percent of the crushing strength as determined in the usual way, the shearing strength of the footing should be considered.

Timber Footings may be used where the ground is soft provided it is always wet. The advantage of timber over masonry is that the former may have a greater projection, and therefore will occupy less space in the cellar, and will also give less weight on the foundation, which is an important consideration with a small bearing power. The safe projection can be computed by the equation on the preceding page. For oak or yellow pine, $P = 1000$, and the above equation gives a safe projection of 7.5 times the thickness of the course when the pressure on the soil is 0.5 short ton per sq ft, 5.3 times for a pressure of 1 short ton, and 3.7 times for a pressure of 2 short tons. The above formula is not applicable to two or more courses of timber if one is transverse to the other, since the deflection of the timber materially affects the distribution of the pressure on the different courses of the footing. Timber footings were once much used, but now reinforced concrete (see Sect. 5, Art. 33) would be employed instead.

Steel-Beam Footings are specially useful in the foundations (particularly of the columns) of a heavy building supported upon a compressive subsoil. The footing consists of a row of steel beams placed side by side and embedded in rich concrete; on top of this and at right angles to it is a shorter row; and above this, one and sometimes two other rows. The advantages of such footings are that they decrease the area of the foundation and at the same time decrease the weight of the footing and increase the usable space in the cellar. Steel-beam footings were first used in Chicago in 1878; and at first, on account of the artificially high price of steel I-beams, railroad rails were used, but later I-beams have been employed, as they have a more economical cross-section.

Reinforced-Concrete Footings are sometimes more economical than steel-beam footings. See Section 5, Art. 33.

Inverted Arches are sometimes built under and between the bases of piers, as shown in Fig. 42. Employed in this way, the arch simply distributes the pressure over a greater area; but it is not well adapted to this use, for it is nearly impossible to prevent the end piers of a series from being pushed outward by the thrust of the arch, and it is generally impossible, with inverted arches, to make the areas of the different parts of the foundation proportional to the load to be supported. The only advantage the inverted arch has over masonry footings is in the shallower foundations obtained.

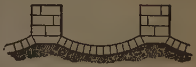


Fig. 42

22. Piles and Pile Driving

Timber Piles are widely used for foundations, the softer varieties for easy driving and the harder varieties for difficult driving. Piles should never be less than 6 and preferably not less than 8 inches in diameter at the small end; never more than 18 and preferably not more than 14 inches at the large end. To prevent bruising and splitting in driving, 2 or 3 inches of the head is usually chamfered off. As an additional means of preventing splitting, the head is often hooped with a strong iron band, 2 to 3 inches wide and ½ to 1 inch thick. The expense of removing these bands and of replacing the broken ones, and the consequent delays, led to the introduction of a hood or cap for the protection of the head of the pile. The hood consists of a cast-iron block with a

tapered recess above and below, the chamfered head of the pile fitting into the lower recess and a cushion piece of hard wood, upon which the hammer falls, fitting into the upper one. The hood preserves the head of the pile, adds to the effectiveness of the blows, and keeps the pile head in place to receive the blows of the hammer.

A further advantage of the pile hood is that it saves the piles by preventing the brooming and consequently the necessity of successively cutting off the pile when the driving is hard. Sometimes, particularly in stony ground, the point is protected by an iron shoe, but some engineers claim that a shoe is no advantage. The shoe may be only two V-shaped loops of bar iron placed over the point, in planes at right angles to each other, and spiked to the piles; or it may be a wrought- or cast-iron socket, of which there are a number of forms on the market.

Cost. In 1909, in the North Central States, prices were about as follows: White or burr oak, 6-inch top and 12-inch butt, 20 to 30 ft long, 16 to 18 cents per foot; same, 40 to 60 ft long, 21 to 25 cents according to length. Long-leaf yellow pine, 40 to 60 ft long, 18 to 23 cents; and short-leaf pine, 14 to 15 cents; other soft woods 1 to 2 cents less.

Concrete Piles are of two types, those molded before driving and those molded in place. The *first type* must be reinforced to permit of handling, and the reinforcement may be any of the forms used for reinforced concrete columns. Such piles may be driven with a drop hammer or a water jet. In the former case, the pile is provided with a cast-iron point, and the top is provided with a cushioned head to prevent the crushing of the pile. If the pile is to be sunk by a water jet, either an iron pipe is set in the center of the concrete or a hole is there molded. The most common form is the Chenoweth pile which is made by plastering a woven wire-net with cement mortar or concrete and then rolling the combination about a mandrel to form a cylinder. However, piles molded in place are more common. Of the *second type* there are three common forms. For the Simplex and the Raymond, see Sec. 5, Art. 33. The Pedestal concrete pile is formed by driving a steel casing, dropping concrete into it, and ramming the concrete out into the soil by driving a steel core into the casing. Concrete is added and rammed successively until the projecting part has any desired diameter, usually 2 or 3 feet. Finally the casing is filled and withdrawn in successive stages.

When exposed to the air or in sea-water infested with marine borers, concrete is much more durable than wood. In some cases concrete tops have been placed upon wooden piles to prevent the decay of the wood above the water line or to prevent the attack of sea worms above the ground line. Concrete piles on account of their size will support a greater load, it being claimed that usually one concrete pile will support as much load as two or three wooden ones; but it is not always wise or possible to decrease the number of piles and proportionally increase the load on each. Concrete piles are superior to wooden ones for foundation work in that they need not be cut off below the water's surface, and hence the more expensive masonry structure need not start as low with concrete piles as with wooden ones. The following shows the actual cost in 1904 of the concrete pile foundations for the physics building of the United States Naval Academy at Annapolis, Md., and also the estimated cost of an equivalent wood-pile foundation (Eng. Record, 1905, vol. 51, p. 277).

Items	Concrete Piles	Wood Piles
Piles	855 at $20.00 = $17 100.00	2193 at $9.50 = $20 835.50
Excavation, cu yd	1038 at .40 = 415.00	4542 at .40 = 1 816.80
Concrete, cu yd	986 at 8.00 = 7 888.00	3250 at 8.00 = 26 000.00
Steel I beams, lb		5222 at .04 = 208.88
Shoring and pumping		4 000.00
Total	$25 403.00	$52 861.18

The Drop-hammer Pile Driver consists of a heavy block of iron, called a ram, monkey, or hammer, which is carried by a rope or cable passing over a

pulley fixt at the top of an upright frame and allowed to fall freely on the head of the pile. The machine is generally placed upon a car or a scow. The frame consists of two uprights, called leaders, from 10 to 60 feet long, placed about 2 feet apart, which guide the falling weight in its descent. The leaders are either wooden beams or steel channel-beams, usually the former. The hammer is generally a mass of iron weighing from 500 to 4000 pounds (usually about 2000), with grooves in its sides to fit the guides and a staple in the top by which it is raised. The rope employed in raising the hammer is usually wound up by a steam engine placed on the end of the scow or car, opposite the leaders.

There are two methods of detaching the weight or of letting the hammer fall. The NIPPER consists of a block which slides freely between the leaders and which carries a pair of hooks, or tongs, projecting from its lower side. The tongs are so arranged that when lowered onto the top of the hammer they automatically catch in the staple in the top of the hammer, and hold it while it is being lifted until they are disengaged by the upper ends of the arms striking a pair of inclined surfaces in another block, the trip, which may be placed between the leaders at any elevation, according to the height of fall desired. The FRICTION CLUTCH method consists in attaching the rope permanently to the staple in the top of the hammer, and dropping the hammer by setting free the winding drum by the use of a friction clutch. The advantages of this method are that the hammer can be dropt from any height, thus securing a light or heavy blow at pleasure, and that no time is lost in waiting for the nipper to descend or in adjusting the trip.

The Steam-hammer Pile Driver consists essentially of a steam cylinder (stroke about 3 feet), the piston-rod of which carries a striking weight of 3000 to 5000 pounds. The steam cylinder is fastened to and between the tops of two I beams 8 to 10 feet long, the beams being united at the bottom by a piece of iron in the shape of a frustum of a cone, which has a hole thru it. The under side of this connecting piece is cut out so as to fit the top of the pile. The striking weight, which works up and down between the two I beams as guides, has a cylindrical projection on the bottom which passes thru the hole in the piece connecting the feet of the guides and strikes the pile. The steam to operate the hammer is conveyed from the boiler thru a flexible tube.

The whole mechanism can be raised and lowered by a rope passing over a pulley in the top of the leaders. After a pile has been placed in position for driving, the machine is lowered upon the top of it and entirely let go, the pile being its only support. When steam is admitted below the piston, it rises, carrying the striking weight with it, until it strikes a trip, which cuts off the steam, and the hammer falls. At the end of the down stroke the valves are again automatically reversed, and the stroke repeated. By altering the adjustment of this trip-piece, the length of stroke (and thus the force of the blows) can be increased or diminished. The admission and escape of steam to and from the cylinder can also be controlled directly by the attendant, and the number of blows per minute is increased or diminished by regulating the supply of steam. The machine can give 60 to 80 blows per minute.

There are three types of steam pile drivers, the single-acting, the double-acting, and the percussion pile hammer. The first has long been in use; but the second and third are comparatively recent inventions. In the first the steam simply raises the ram or hammer and it falls by its own weight. In the second the steam raises the ram and is also applied to drive the ram down. The third is somewhat similar to the rock-drill and the pneumatic percussion hammer. In a general way the first usually has a heavier ram and a longer stroke, and makes fewer blows per minute than the second; and the third differs from the second in having a lighter ram and a shorter stroke, and making more blows per minute. The greater rapidity of blows in a measure compensates for the lighter ram. Any of the steam pile drivers may be operated with comprest air.

In Water-jet Pile Driving a jet of water is discharged below the point of the pile, thus loosening the soil and allowing the pile to sink by its own weight or with very light blows of a drop hammer. The water may be conveyed to the point thru a hose or pipe loosely tied or stapled to the pile, or, if the pile is a concrete one, thru a hole molded in the center for that purpose. It makes very little difference, either in the rapidity of the sinking or in the accuracy with which the pile preserves its position, whether the nozzle is exactly under the middle of the pile or not. The efficiency of the jet is greatest in clear sand, mud or soft clay; and it is almost useless in gravel, or in sand containing a large percentage of gravel, or in hard clay.

Screw Piles are employed chiefly in anchoring buoys and signal stations in marine surveying, in founding small lighthouses on a sandy sea-shore, for piers, and for supports for light bridges. A screw pile usually consists of a rolled-iron shaft of 3 to 10 inches diameter, having at its foot one or two turns of a cast-iron screw, the blades of which may vary from 1½ to 5 ft in diameter. The piles ordinarily employed for lighthouses exposed to moderate seas or to heavy fields of ice have a shaft of 3 to 5 inches and blades 3 to 4 ft diameter, the screw weighing from 600 to 700 lb. For bridge piers, the shafts are from 6 to 10 in and the blades from 4 to 5 ft diameter, the screw weighing from 1500 to 4000 lb. See Eng. News, Vol. 13, p 210, and Vol. 28, p 116, for illustrated accounts of the founding of a railroad bridge on screw piles.

For founding beacons and buoys, the screw pile has the special advantage of not being drawn out by the upward force of the waves against the superstructure. Even when all cohesion of the ground is destroyed in screwing down a pile, a conical mass, with its apex at the bottom of the pile and its base at the surface, would have to be lifted to draw the pile out. The supporting capacity also is considerable, owing to the increased bearing surface of the screw blade. Screw piles have, therefore, an advantage in soft soil. They could also be used advantageously in situations where the jar of driving ordinary piles might disturb the equilibrium of adjacent structures.

These piles are usually screwed into the soil by men working with capstan bars. Sometimes a rope is wound around the shaft and the two ends pulled in opposite directions by two capstans; and sometimes the screw is turned by attaching a large cog-wheel to the shaft by a friction-clutch, which is rotated by a worm-screw operated by a hand crank. Horse-power, steam, and hydraulic power have been used for this purpose. The screw will penetrate most soils. It will pass through loose pebbles and stones without much difficulty, and push aside bowlders of moderate size. Ordinary clay does not present much obstruction; but clean, dry sand gives the most difficulty. The danger of twisting off the shaft limits the depth to which they may be sunk. Screw piles with blades 4 ft in diameter have been screwed 40 feet into a mixture of clay and sand. The resistance to sinking increases very rapidly with the diameter of the screw; but under favorable circumstances an ordinary screw pile can be sunk very quickly. Screws 4 ft diameter have, in less than two hours, been sunk by hand labor 20 ft in sand and clay, the surface of which was 20 ft below the water. However, for depths of 15 to 20 ft, an average of 4 to 8 ft per day is good work for wholly hand labor.

Disk Piles differ but little from screw piles, a flat disk, instead of a screw, being keyed or bolted on at the foot of the iron stem. Disk piles are sunk by the water-jet, the water being usually forced down the hollow shaft and out thru a hole in center of the disk. One of the few cases in which disk piles have been used in this country was in founding an ocean pier on Coney Island, near New York City. The shafts were wrought iron, lap-welded tubes, 8⅝ inches outside diameter, in sections 12 to 20 ft long; disks were 2 ft diameter and 9 inches thick, and were fastened to the shaft by set-screws. Many of the piles were 57 ft long, of which 17 ft were in the sand. For a detailed description of this work, see Trans. Am. Soc. C. E., vol. 8 (1879), p. 227–37.

23. Safe Bearing Loads for Piles

Two cases must be distinguished: that of columnar piles or those whose lower end rests upon a hard stratum, and that of ordinary bearing piles. In the first case the load is limited by the strength of the pile considered as a column; and in the second place it is carried by the friction of the earth on the sides of the pile.

For a Friction Pile there are two types of formulas in use: (1) theoretical formulas, those based wholly upon a theoretical consideration of the principles involved; and (2) empirical formulas, those based chiefly upon experience or experiments. A number of theoretical or rational formulas have been proposed, but none are of any considerable value because of the impossibility of including therein all of the essential conditions; and most of the empirical formulas are unreliable because of the failure to consider important conditions. The only formulas in anything like general use are known as the Engineering News formulas. They are:

For a pile driven with a drop hammer, $$P = \frac{2\,Wh}{s + 1}$$

For a pile driven with a single-acting steam hammer, $$P = \frac{2\,Wh}{s + 0.1}$$

For a pile driven with a double-acting steam-hammer, $$P = \frac{2\,h(W + a\,p)}{s + 0.1}$$

in which P is the safe load in pounds, W the weight of the hammer in pounds, h the fall of the hammer in feet, a the effective area of the piston in square inches, p the mean effective steam pressure in pounds per square inch, and s the penetration or sinking in inches under the last blow, assumed to be sensible and at an approximately uniform rate. The sinking s must be measured only when there is no visible rebound of the hammer and only when the last blow is struck upon practically sound wood. The first two of the above formulas have been generally accepted for many years; but the last has only recently been proposed (Eng. News, Vol. 75 (1916), p. 33–34, 372–73). The first two formulas are supposed to give a factor of safety of six; and are also claimed to be safe, for ordinary weights of hammer and the usual height of fall. These formulas were deduced for wood piles; but they are the best there are for concrete piles. The following table compares the first of these formulas with nine observed cases.

Comparison of Actual and Computed Safe Loads of Wood Piles

L'th of Pile, ft	Wt of Hammer, W, lbs	Height of Fall, h, ft	Last Penetration, s, inches	Load, lbs		Character of Soil	Locality
				Observed	Computed		
53	2000	4.0	8.5	13 333	1 690	Almost fluid mud.....	Aquia Creek, Va.
25	1600	25.0	3.0	22 400	19 750	Soft muddy bottom...	East St. Louis, Ill.
35	1700	25.0	2.0	44 800	28 300	Mud 30 ft deep.......	Perth Amboy, N. J.
35	910	5.0	0.35	62 500	6 740	Mud, sand, and clay ..	Proctorsville, La.
....	1900	29.0	1.5	75 000	44 100	Stiff clay............	Buffalo, N. Y.
91	2300	22.0	1.75	75 000	36 800	Mud 60 ft, fine sand 6 ft	Annapolis, Md.
73	2300	33.5	3.75	34 000	32 500	Water 12 ft, mud 61 ft.	Annapolis, Md.
30	2300	22.0	2.0	38 000	33 700	Sand.................	Annapolis, Md.
33	2300	22.0	1.0	110 000	50 600	Sand.................	Annapolis, Md.

In making the test blow there should be little or no bouncing of the hammer. If the hammer bounces to any considerable extent, the fall is too great, or the pile has struck a solid obstacle, or the hammer is too light. Under such circumstances, careful trials and discriminating judgment are required to determine the cause of the bouncing. Frequently, decreasing the fall will decrease the bouncing and also increase the effectiveness of the blow. If the pile has struck an impenetrable stratum, and the driving is continued, it is probable that there will be a small and continuous apparent penetration due to the mashing of the foot of the pile. Not infrequently when piles are dug out or pulled up, the foot is found badly bruised, and sometimes the body of the pile is crusht. Of course, after the point is bruised or the body crusht, further driving is useless. In hard driving there is likely to be a little rebound of the hammer, owing to the elastic compression of the pile; but in making the test blow there should be only a very little bouncing.

The penetration to be used in the formula should not be taken unless it has been at a reasonably uniform or uniformly decreasing rate. If the penetration is at an uneven rate it is probable that the pile is passing bowlders or logs. If the penetration is practically zero, it is probable that the pile is against an impenetrable stratum or is already crusht. When the penetration has reached a small amount, say 1/4 or 1/2 inch per blow, and the hammer rebounds considerably, it is safe to conclude that the limit of safe driving of that pile has been reached. Of course, the apparent penetration due to the brooming of the head, or to the crushing of the body of the pile, or to the bruising of the point should not be used in the formula for computing the bearing power. Care should be taken that the test blow is struck on sound wood, as otherwise the observed penetration may be much too small and consequently the computed supporting power be much too great.

24. Deep Foundations

This is a method of supporting the load by extending the foundation to an underlying hard substratum by driving steel pipes or sinking wells which are afterward filled with concrete. A pile foundation may be classed as a deep foundation if the piles are driven to a hard substratum; and the compressol system (Art. 18) may be employed for a so-called deep foundation by sinking a pit under each column footing to a hard underlying stratum. Sometimes a building is supported by forcing steel pipes or tubes down to a hard substratum either by hydraulic pressure or by the use of the water jet, with or without filling the tube with concrete.

Concrete Piers. If the soil is a water-tight clay, open wells may be sunk to the hard substratum and afterwards be filled with concrete, a column of the steel frame of the building resting upon each concrete pier. At Chicago this method of putting in foundations has been brought to a high state of development. The shafts are sunk as open wells 3 to 8 feet in diameter, and are usually lined with 2- by 6-inch tongue-and-groove planks from 4 to 6 feet long, which are supported by two and sometimes three interior iron sectional rings. A section about 6 feet deep is excavated and then lined. The intention is to make the excavation only large enough to get the lagging into place, to prevent settlement of adjoining buildings; and if the excavation is accidentally made too large, clay is packed behind the lagging as the latter is put into position. If beds of quicksand or other soft material are encountered, steel sheet piles or steel cylinders are used instead of wood lining. The bearing power of the concrete column may be increased by belling out the lower end of the well. After the excavation is completed, the hole is filled with concrete. The rings are taken out as the concreting progresses, except in soft swelling clay; but the lagging is usually left in place. Since about 1890 this method has been employed in Chicago almost exclusively for all large buildings. The usual practise there is to mix the concrete rather dry and put it into wells 60 to 100

feet deep by shoveling it in at the top and allowing it to drop freely, an attempt being made to drop it from the shovel in such a manner that the shovelful will go down without being broken up. Such columns safely carry 20 to 25 tons per square foot of top area, usually the former.

Hydraulic Caissons. A few deep foundations of buildings in New York City have been sunk to bed-rock by the hydraulic caisson method. This consists of sinking steel cylinders without interior excavation by means of hydraulic jets. A riveted steel cylinder is attached at its lower end to an annular cast-iron cutting edge of hollow triangular cross-section having numerous small perforations along its lower edge; and the hollow cast-iron cutting section is connected with a force pump by pipes and flexible hose. The cylinder is heavily loaded with cast iron, the pump is started, and the numerous jets of water issuing from the cutting edge scour away the soil and form an annular trench into which the cylinder descends. As the sinking progresses, another section of the cylinder is added at the top. When a hard substratum is reached, the pump is stopt, the soil in the interior of the cylinder is dug or dredged or "washt" out. If the cutting edge of the cylinder stops in clay, probably little or no water will leak into the cylinder; but if it stops upon bed-rock which is irregular or not level, it may be necessary to seal the cylinder by depositing concrete under water.

The objection to this method is that in some cases it does not permit an inspection and proper preparation of the bed of the foundation. The pneumatic method (Art. 31), which is primarily a method of founding under water, is sometimes employed in sinking the foundation of buildings, particularly in New York City since about 1895, because it permits inspection and proper preparation of the underlying hard stratum. The only advantage of the hydraulic caisson over the pneumatic method is that the former is sometimes the cheaper.

FOUNDATIONS UNDER WATER

25. Divers and Diving

In laying foundations under water it is sometimes necessary to employ a diver to examine the site, to prepare the bed, or to lay the masonry. A diver is also sometimes required in other engineering operations, as in tunneling to erect or remove a bulkhead; in water-supply engineering to inspect and repair a conduit, clear an inlet, or take out pumps; in wrecking operations, etc. With the best modern apparatus it is possible for any able-bodied man to prosecute any ordinary kind of work under a moderate head of water without serious risk of personal injury. No man suffering from chronic disease, alcoholic excess, ear or heart troubles, having a sluggish circulation or a great excess of fat, should attempt the work of a diver; and if the work is to be carried on at a greater depth than 35 to 40 feet, the man should undergo a thoro examination by a physician before attempting diving.

A Diver's Outfit consists of a metal helmet or head covering, a breast plate which rests upon the chest and back, and an air-tight flexible diving suit which envelops the body from the breast down. The helmet is provided with at least one window in front, and usually also one on each side, and sometimes one above the face; and is also provided with a valve in either the head piece or the breast plate for receiving air, and also a safety valve and a regulating valve. The diving suit is closed at the feet, but is open at the hands, which are provided with rubber bands that fit closely around the wrists. The helmet is connected by a hose to an air pump above the water. A rope, called the life line, passing around the diver's waist and reaching to the surface, is used in raising and lowering him. To overcome buoyancy weights are hung to the

diver's waist, and his shoes have lead or iron soles. The weights upon the feet are more particularly intended to prevent the diver from losing his upright position by undue buoyancy of the lower portion of his diving suit, caused by an excessive air pressure on the inside. The air pressure in the diving dress may be controlled by the diver by means of the regulating valve. To prevent the possibility of the diver being blown to the surface by the inflation of the lower part of his diving dress, the British navy has introduced a new type of dress having the legs tightly laced up, which prevents the air from getting into the legs when the diver suddenly stoops over or if he falls down; and consequently if he loses his upright position, he is able to right himself. The air pump is always operated by hand power, and may be any one of several forms, according to the amount of work or the depth: one, two, or three cylinders; single or double acting; lever or fly-wheel type; with or without a water-jacketed cylinder. The diver communicates with his attendant by jerks upon the life line thru a code of signals, or by a telephone the wires of which are embedded in the life line or in the air hose. Except in very clear water, the surface light does not penetrate very deep, and hence the diver usually works in the dark, altho sometimes he is provided with an electric light.

To Prepare for Diving the diver takes off all his own clothing, puts on a heavy flannel shirt, a pair of drawers which must be carefully adjusted outside of the shirt and be well secured to prevent slipping down, and then a pair of heavy stockings. If the water is cold, he may put on two or more of each of these articles. He also puts on a woolen cap, drawing it well down over his ears. Usually a shoulder pad is put on and tied under the arms. The diver then gets into the diving suit and draws it well up to the waist; he next puts his arms into the sleeves, and an assistant opens the cuffs, by means of the cuff expander provided with the apparatus, so that the diver can push his hand thru the cuff. Sometimes the diver wears rubber mittens; and if so, they are next put on by first inserting a ring into the cuffs, and then each mitten is fastened in place by a clamp. Canvas overalls are usually next put on over the diving suit to preserve it from injury. The diver now sits down, and the shoes are put on. Next the inner cape is drawn up around his neck and tied loosely with a yarn string. The breast plate is then put on, and the rubber collar of the diving suit is put over the projecting screws of the breast plate; and the sectional clamping piece is put over the projecting screws and fastened with the thumb nuts. The helmet, with the front plate or window removed, is next put on and screwed fast to the breast plate; but before putting the helmet over the diver's head, the attendant should place it over his own head and place his mouth over the place where the air escapes and blow, to see whether the safety valve is in working order. The life line is looped around the diver's waist, brought up in front of the man's body, and secured with a small rope passing around his neck or is fastened to the stud on the helmet. The waist belt is then buckled on with the knife pocket at the left. The end of the air hose is past thru the ring on the belt on the diver's left and up to the inlet valve of the helmet, and secured to it and then the upper part of the hose is made fast by lashing it to the loop on the helmet. A ladder or a rope must be provided which reaches from the surface to the bottom, for the use of the diver in descending and ascending; and must be heavily weighted at the bottom. The ladder, usually a rope one, is the better for the inexperienced man altho an expert diver prefers a single rope. The diver steps on the ladder, and weights are hung to his waist and fastened in place. Two men take their places at the pump. When the attendant is sure that everything is right and the diver understands the code of signals, he gives orders for the pump to start, screws the front plate into place, takes hold of the life line, and gives the signal for the diver to descend. While the diver is under water, an attendant should give constant attention to air hose and life line, keeping the former free and clear of kinks and the latter just taut enough that any signals by the diver can be easily felt; no talking, laughing or distracting noise should be allowed on the surface.

The diver should descend slowly, and as soon as his head is under water should stop to see that everything is satisfactory, particularly the escape valve. The valve is kept closed by a spring, and the farther the valve is screwed up the less air escapes. If too much air escapes, breathing is difficult; and if too little escapes, the diving dress may become inflated to such an extent that the diver will rise. The proper adjustment is that which just takes the pressure of the weights from the diver's shoulders. As he descends, if he feels

any pain in his ears, he should shut his mouth and inflate his cheeks, or blow thru his nose, or swallow several times, to equalize the pressure on the inside of the drum of his ear. He should continue his descent slowly, and not demand too much air. If the pain is not relieved or if he feels opprest, he should slowly rise a yard or two, and remain at that point a minute or two, and inflate his cheeks or swallow or do both; and if the oppression, or the singing in the ears, or the headache should continue, he should slowly return to the surface. However, unless there is some serious physical defect a man with a cool head and good judgment is not likely to have any trouble in diving. Arriving at the bottom, the diver signals his attendant, usually by one jerk on the life line to indicate "all right." The diver takes down in his hand a coil of small rope, one end of which is fast to his waist, and the other end of which he attaches to the bottom of the ladder or rope down which he descended, so that he can always find his way back to the ladder when he wishes to return to the surface.

If the diver has been down some time, particularly in deep water, the liquids and tissues of his body have gradually become saturated with comprest air and other gases, and consequently when he ascends it should be slowly to allow time for desaturation. Remaining down too long or ascending too rapidly is likely to cause a form of paralysis, which in pneumatic caisson work (Art. 32) is called by the physicians caisson disease and by the workmen "bends." It occurs only after returning to the normal atmosphere; and in ordinary cases in diving can be prevented by care in ascending. The desaturation is more rapid and less harmful if the diver ascends say halfway and then stops for a time (the exact time depending upon the depth and the time under pressure), and then continues the ascent, again stopping when halfway up. This is what is known in the British navy as stage decompression, and an elaborate series of tables have been established for that service which give the time to be consumed in the ascent for different depths and for various times down. For a depth of 36 ft and unlimited time down, the time of ascent is 1 minute; for a depth of 60 ft, the time of ascent after being down 20 minutes is 2 minutes, and after being down 2 or 3 hours it is 32 minutes; and for a depth of 120 ft, after being down 15 minutes the time of ascent is 15 minutes, and after being down 30 minutes it is 33 minutes.

The Nominal Maximum Depth for a diver is 100 ft, and the ordinary maximum for a strong and expert diver is 150 ft, and the maximum ever reached is 202 ft. Strong and experienced divers can stay under 2 hours at 100 ft, and 5 or 6 hours at 90 ft, but the maximum depth at which a diver can do any considerable work is 60 to 80 ft. It must not be forgotten that the work of a diver is much less efficient than that of a man above water, owing to restraint of his diving apparatus, the resistance of the water, the seeming tendency of his tools to rise, etc. The usual wages for a diver, his apparatus and one attendant is $40 or $50 per day with expenses for himself and attendant. There are not many professional divers: for example, there seem to be only five in Chicago, a city of 2 000 000 inhabitants, having numerous deep intake cribs and water tunnels, shipbuilding, lake commerce, harbor improvements, etc. The catalog price of a diving outfit varies from $300 to $800, the former being for an outfit for a brief examination in shallow water and the latter for the best apparatus suitable for the deepest work.

26. The Coffer-Dam Process

A coffer-dam is a water-tight wall built to enclose a proposed foundation. This method consists in constructing a coffer-dam around the site of the proposed foundation, pumping out the water, preparing the bed of the foundation by driving piles or otherwise, and laying the masonry on the inside of the coffer-dam; and is applicable only where the soil at the bottom of the dam is nearly impervious, for if there is much of an inflow of water it will be impossible, or at least expensive, to pump it out. The difficulties in the use of the coffer-dam method increase rapidly with the depth of the water.

There are various methods of constructing the coffer-dam. In still, shallow water, a well-built bank of clay and gravel is sufficient. If there is a slow current, a wall of bags

partly filled with clay and gravel does fairly well, and a row of cement barrels filled with gravel and banked up on the outside has also been used. If the water is too deep for any of the ab .ve methods, a single or double row of plank may be driven and banked up on the outside with a deposit of impervious soil sufficient to prevent leaking. If ther is much current, the puddle on the outside will be washt away; or, if the water is deep, a large quantity of material will be required to form the puddle-wall; and hence the preceding simple methods are inapplicable where there is much current or where the water is more than 3 or 4 ft deep. The more elaborate coffer-dams may consist of a wall of either wood or steel sheet piles, or of two rows of sheet piles with a puddle-wall between them, or of a timber crib.

Puddle-Wall Coffer-Dam. Before the introduction of interlocking steel sheet piles, the usual method of constructing a coffer-dam in deep water was to drive two lines of wood sheet piles and fill in between them with impervious soil, called puddle. The general form of such a dam is shown in Fig. 43. The area to be enclosed is first surrounded by two rows of ordinary piles, *m*, *m*. On the outside of the main piles, a little below the top, are bolted two longitudinal pieces, *w*, *w*, called wales; and on the inside are fastened two similar pieces, *g*, *g*, which serve as guides for the sheet piles, *s*, *s*, while being driven. A rod, *r*, connects the top of the opposite main piles to prevent spreading when the puddle is put in. The timber, *t*, is put on primarily to carry the footway, *f*, and is sometimes notched over, or otherwise fastened to, the pieces *w*, *w*, to prevent the puddle space from spreading. *b* and *b* are braces extending from one side of the coffer-dam to the other. These braces are put in position successively from the top as the water is pumped out; and as the masonry is built up, they are removed and the sides of the dam braced by short struts resting against the pier.

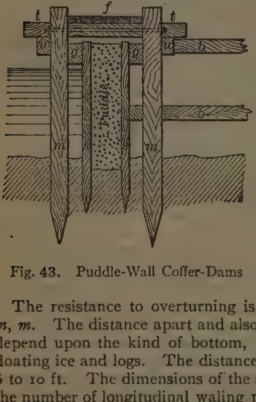


Fig. 43. Puddle-Wall Coffer-Dams

The resistance to overturning is derived principally from the main piles, *m*, *m*. The distance apart and also the depth to which they should be driven depend upon the kind of bottom, the depth of water, and the danger from floating ice and logs. The distance between the piles in a row is usually from 6 to 10 ft. The dimensions of the sheet piles will depend upon the depth and the number of longitudinal waling pieces used. Two thicknesses of ordinary 2-inch plank are generally employed; but sometimes, for the deeper dams, the sheet piles are timbers 10 or 12 inches square.

The thickness of the dam will depend upon (1) the width of gangway required for the workmen and machinery, (2) the thickness required to prevent overturning, and (3) the thickness of puddle necessary to prevent leakage thru the wall. The thickness of shallow dams will usually be determined by the first consideration; but for deep dams the thickness will be governed by the second or third requirement. An old rule for the thickness of the puddle-wall, which is frequently quoted, is: "For depths of less than 10 ft make the width 10 ft, and for depths over 10 ft give an additional thickness of 1 ft for each additional 3 ft of wall." Another rule, also frequently quoted, is: "Make the thickness of the puddle-wall three-fourths of its height, but in no case is the wall to be less than 4 ft thick." Judged by ordinary experience both of the above rules are extravagant, for numerous coffer-dams from 20 to 30 ft deep have been built in which the thickness of the puddle varied from 3 to 5 ft, or say one-sixth of the depth.

The puddle should consist of impervious soil, of which gravelly clay is best. It is a common idea that clay alone, or clay and fine sand, is best. With pure clay, if a thread

of water ever so small finds a passage under or thru the puddle, it will steadily wear a larger opening. On the other hand, with gravelly clay, if the water should wash out the clay or fine sand, the larger particles will fall into the space and intercept first the coarser sand, and next the particles of loam which are drifting in the current of water; and thus the whole mass puddles itself better than the engineer could do it with his own hands. Before putting in the puddling, all soft mud and loose soil should be removed from between the rows of sheet piles, for the most common cause of trouble with puddle-wall coffer-dams is a leak between the natural surface and the puddle. The puddling should be deposited in layers, and compacted as much as is possible without causing the sheet piles to bulge so much as to open the joints.

The introduction of interlocking steel sheet-piles has nearly done away with the use of the puddle-wall coffer-dam.

Crib Coffer-Dam. Coffer-dams are sometimes made by building a crib and sinking it. For shallow water, the crib is sometimes made of uprights framed into caps and sills, and covered on the outside with tongued-and-grooved planks. Or the crib may be made of squared timbers laid one on top of the other and drift-bolted together. The joints between the timbers may be made water-tight by placing cement grout between the timbers during the construction of the crib, or by driving oakum into the joints after the crib is built. The crib is built on land, launched, towed to its final place, and sunk by piling stones on top or by throwing them into cells constructed for that purpose. The dam is made water-tight at the bottom either by driving sheet piles outside it or by using canvas. The upper edge of the canvas may be nailed to the crib near the bottom or above the water; and the outer edge may be spread out upon the river bed and be loaded with stone or sand. The chief advantage of the above form of construction is that the area of possible leakage is reduced to the space below the crib, where leakage may be prevented by the use of sheet piles and clay or of canvas.

Coffer-Dam on Rock. Sometimes when the bed of the river is rock, or rock covered with but a few feet of mud or loose soil, a coffer-dam only sufficiently tight to keep out the mud is constructed. The mud at the bottom of the inclosed area is then dredged out, and a bed of concrete deposited under the water. Before the concrete has set, another coffer-dam is constructed inside of the first one, the latter being made water-tight at the bottom by settling it into the concrete or by driving sheet piles into the concrete. However, under such circumstances it is usually not wise to employ the coffer-dam method in laying the foundation.

Sheet-Pile Coffer-Dams. *Wood Sheet-Piles* for a coffer-dam in shallow water may be simply planks, and for greater depths either thick tongued-and-grooved planks or Wakefield piles. Ordinarily wood sheet-piles are simply thick planks, sharpened and driven edge to edge. Sometimes a thinner plank is driven outside of the thick one to cover the joint and prevent leakage; and sometimes two rows of thick planks are driven. Formerly, when greater strength was required than one or two thicknesses of plank, heavy sawed timbers were employed as sheet piles, wooden blocks or iron lugs being fastened on the edges to assist in guiding them into position, or a tongue and groove was formed by nailing two strips on the edges of one side of the pile and one strip in the middle of the other edge; but now the Wakefield pile is generally used when greater strength is required than that afforded by a single plank. Fig. 44 shows the Wakefield pile, the planks being usually 10 or 12 in wide and 10 to 16 ft long.

Wood sheet-piles should be sharpened wholly from one side, and the long edge should be placed next to the last pile driven to cause the piles to crowd together and make closer joints In hard soil at small depths or in soft soil at

moderate depths, the sheeting may be driven by hand with a wooden maul or an iron sledge; but for any considerable depth a power pile-driver must be employed, altho sometimes where only a few piles are to be driven a hand pile-driver is rigged up with a block of wood for a hammer. The sheeting should be driven at least a foot or two below the lowest excavation inside of the dam; and in soft soil the sheet piles should be driven at least 3 or 4 feet below the proposed excavation to prevent leakage under the bottom.

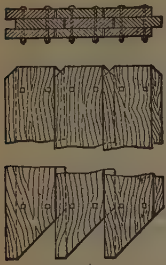


Fig. 44.
Wakefield Sheet-Pile

If the sheet piles are to resist a head of more than 4 or 5 ft of water or a semi-fluid soil, their tops should be supported by wales (horizontal timbers on the inside of the dam against the top of the sheeting), which in turn are braced at their ends by the wales on the adjacent sides of the dam or at intermediate points by horizontal timbers across the dam; and in deep dams similar wales and cross braces are inserted at vertical intervals as the excavation progresses. Sometimes, in comparatively shallow water and with a suitable bottom, the waling pieces are supported by ordinary bearing piles driven inside of the dam, thus eliminating the braces across the dam, and therefore facilitating the excavation and the laying of the masonry. Sometimes the top and the bottom waling pieces are framed together, and the upper and lower waling frames are separated from each other by small vertical posts placed between them and joined to them. This frame is sunk in the desired position, and the sheet piles are driven around it.

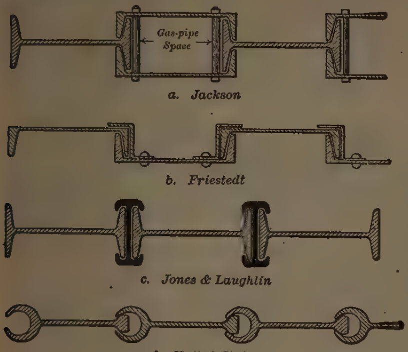
The thickness of the sheet piling required in any particular case is usually a matter of judgment based upon past experience; but the strength required can be approximated by regarding the sheet pile as a beam either fixt at one end and free at the other, or as supported at both ends. The amount of the lateral pressure against the pile is one-half of the continued product of the weight of a cubic foot of water, the width of the pile in feet, and the depth of the water in feet; and the point of application of this pressure is two-thirds of the depth of the water from the top. Of course, the weight of liquid mud is more than that of water; but extreme accuracy is impossible, and hence the above method is probably sufficient for the purpose.

Steel Sheet-Piles may be divided into two general classes: those built up from standard structural shapes, and those consisting of special shapes. Fig. 45 shows several typical forms. Form *a* is the first steel sheet-pile ever made, being used in Chicago in 1901. It may be made of any size of channels and I beams, altho the 12-inch or the 15-inch are ordinarily used. With this form, the space between the channels can be tamped full of clay and thus make the wall watertight. Form *b* is made with two weights of 12-inch and also with two weights of 15-inch channels. There is another variety of this form in which there is an angle on each edge of the intermediate channel to form a calking joint, but such a joint is seldom necessary. Form *c* is made of a 12-inch I beam and a 5-in locking piece or a 15-in beam and a 7-in locking piece. Form *d* is made in three sizes, 12-in 40-lb, 12-in 35-lb, and 6-in 11-lb. With this form of pile a half-round wood strip may be driven in the joint, which upon absorbing water expands and prevents leakage, but it is seldom required. Form *e* in its essential features is substantially the same as form *c*. It is claimed that forms *d* and *e* are deficient in lateral stiffness, and to meet this objection form *f* was devised.

By the use of special corner pieces, a right-angled corner can be turned with any of the steel sheet-piling; and some of the regular forms can be used to inclose a comparatively small circular area. Steel sheet-piles are ordinarily driven by allowing the pile hammer to strike directly against the end of the pile; but in hard driving a cast-iron or steel hood,

into the under side of which fits the top end of the pile, is sometimes used. These hoods are furnished by the makers of the piles.

Steel sheet-piles are superior to wooden ones in that they make tighter work, are easier to drive, may be used repeatedly, and some forms have nearly their original value as standard sections when no longer required as piles, and all forms have value as scrap when they can no longer be used as piles. Steel sheet-piles are more easily pulled out than wooden ones. Steel sheet piles require less bracing across the coffer-dam than wooden ones. Steel sheet piles may be spliced bp driving one section on top of another; and, by varying the length of the members, a stout wall of almost any depth may be built. Most forms of steel sheet pile speedily become water-tight in muddy water; and usually all are easily made tight in clear water by throwing sawdust, paper bulp, manure, etc., near a leak.

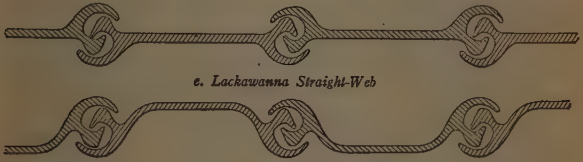


a. Jackson

b. Friestedt

c. Jones & Laughlin

d. United States



e. Lackawanna Straight-Web

f. Lackawanna Arched-Web

Fig. 45. Typical Forms of Steel Sheet-Piles

Coffer-Dam on Permeable Bottom. The ordinary coffer-dam process i.e., the process in which the floor of the coffer-dam is kept reasonably free from water, is practically limited to impervious soil; but sometimes a modification of this process is applied in pervious soil. The site is dredged, and enclosed with a sheet-pile coffer-dam. Before or after driving the sheet piles a group of bearing piles are driven upon which the proposed pier or other structure is to rest; and then the bottom of the coffer-dam is sealed by depositing concrete under water with a tremie around and over the heads of the piles, after which the coffer-dam is pumped out and the structure is completed in the dry.

Sometimes in this modification of the usual coffer-dam process, the coffer-dam is a reinforced-concrete shell which is molded on shore, floated to place, and sunk. The concrete shell may be floated by suspending it between barges or by inserting a temporary wooden bottom. For an example of the latter method, see Engineering and Contracting, vol. 49 (1918), p. 390–91.

For a somewhat similar method of procedure, see Art. 29.

Cost. The introduction of interlocking steel sheet piles has materially cheapened the cost of coffer-dams, and has also increased the depth to which

a coffer-dam may be economically sunk. For wood sheet piles it is usually claimed that the limiting depth is 30 or 35 feet. Steel sheet piles were introduced so recently that practice has not established a corresponding limit, but they have been successfully used in coffer-dams at more than twice the above depth. However, at such great depths, some other method of constructing the foundation is usually preferable. Estimates for the cost of foundations under water are very unreliable, and none are more so than those contemplating the use of a coffer-dam. The difficulties are due to striking logs or bowlders in driving the sheet piles and to leakage.

The following (Eng.-Con., May 27, 1909) gives the details of the actual cost, exclusive of contractor's profits, of a coffer-dam and concrete pier on a pile foundation in water averaging 5 ft deep. The coffer-dam consisted of triple-lap sheet piling of the Wakefield pattern, the planks being 2 in thick and giving a coffer-dam wall 6 in thick. The coffer-dam inclosed an area 14 by 20 ft, giving a clearance of 1 ft all around the base of the concrete pier, and a clearance of 2 ft between the coffer-dam and the outer edge of the nearest pile. The sheet piles were 18 ft long, were driven 11 ft deep into sand, and projected 2 ft above the surface of the water. There were twenty-four wood piles, 40 feet long, driven 33 feet. Upon the heads of the piles rested a concrete base containing 100 cu. yd. The cost of the work was as follows:

Coffer-dam: Lumber, 7900 ft B.M., at $20.00	$158.00	
Labor, $16.00 per M of lumber	126.00	
Total, $36.00 per M, exclusive of salvage		$284.00
Excavation: 58 cu. yd at 57 ct. per cu yd		33.00
Foundation piles: Material, 960 lin ft at 10 ct.	$96.00	
Driving, 8⅓ ct. per lin ft	80.00	
Total, 960 ft at 18⅓ ct. per lin ft		176.00
Forms: Material, 2400 ft B.M. plank at $25.00	$60.00	
1000 ft B.M. studding at $20.00	20.00	
Nails, wire, etc.	2.00	
Labor 8 days at $3.00	24.00	
Total, 100 cu yd concrete at $1.06 exclusive of salvage		106.00
Concrete: Materials at $3.20	$320.00	
Labor at $1.46	146.00	
Total, 100 cu yd at $4.66		466.00
Plant: Transportation	$20.00	
Setting up and taking down	70.00	
Rental, 20 days at $5.00	100.00	
Total for plant		190.00
Total cost, exclusive of salvage		$1255.00

27. Leakage and Pumping

Leakage. A serious objection to the use of a coffer-dam is the difficulty of preventing leakage thru or under it. It is nearly impossible to prevent considerable leakage, unless the bottom of the crib rests upon an impervious stratum or the sheet piles are driven into such stratum. Water will find its way thru nearly any depth or distance of gravelly or sandy bottom, and seams of sand are very troublesome. Logs or stones under the edge of the dam are also a cause of considerable annoyance. The object of a coffer-dam is not to prevent all infiltration, but only so to reduce it that a moderate amount of pumping will keep the water out of the way.

The method to be employed in removing the water will vary greatly with the amount present, the depth of the excavation, the appliances at hand, etc. If the excavation is shallow and the amount of water small, it can be bailed out; but usually some form of pump must be employed. The pumps generally used for this kind of work are the direct hand-lift foundation-pump, the diaphragm pump, the steam siphon, the pulsometer, and the centrifugal pump.

The Direct Hand-lift Foundation-Pump consists of a straight tube at the bottom of which is fixt a common flap valve, and in which works a piston carrying another flap valve. The tube is either a square wooden box or a sheet-iron cylinder, usually the latter, since it is lighter and more durable. The pump is operated by applying the power directly to the upper end of the piston-rod, the pump being held in position by wooden stays or ropes. The only advantage of the wood-box hand-lift pump is that it may be improvised on the job; and the disadvantage for foundation work of all pumps having flap valves is the danger that straw, sticks, mud, etc., will interfere with the action of the valves.

The Diaphragm Pump is the usual form of hand pump for foundation work. This pump consists of a short cast-iron cylinder having a rubber hose connected to its lower end, and being divided about midway of its height by a flexible horizontal rubber diaphragm. The central portion of the diaphragm is connected to a bent-lever handle, and there is a valve in the center of the rubber disk. The rise and fall of the center of the disk acts as a piston. A pump of this form throws a large amount of water, allows sand and gravel to pass without choking, is not easily clogged by straw, leaves, etc., and is easily unclogged. It is made in various sizes, the smallest having a capacity of 25 gallons per minute and usually costing about $20.

Recently various forms of diaphragm pumps driven by a gasoline-engine have been put upon the market. They are much superior to the hand pump.

The Steam Siphon is the simplest of all pumps, since it has no movable parts whatever. It consists essentially of a discharge pipe, open at both ends, thru the side of which enters a smaller pipe having its end bent up. The lower end of the discharge pipe dips into the water; and the small pipe connects with a steam boiler. The steam, in rushing out of the small pipe, carries with it the air in the upper end of the discharge pipe, thus tending to form a vacuum in the lower end of that pipe; the water then rises in the discharge pipe and is carried out with the steam. The steam siphon is limited practically to lifting water only a few feet; its cheapness and simplicity are recommendations in its favor, and its efficiency is not much less than that of other forms of pumps. One of the advantages of the steam siphon is that frequently it can be improvised on the work from ordinary pipe and fittings. Several forms and sizes are upon the market, ranging in capacity from 5 to 200 gallons per minute, and are much better than one made from pipe. A steam siphon, or jet pump as it is usually called by the manufacturer, having a capacity of 100 to 125 gallons per minute can usually be had for $35.

The Pulsometer is an improved form of the steam siphon. It may be properly called a steam pump which dispenses with all movable parts except the valves. The height to which it can lift water is practically unlimited, and it is in very common use for pumping out coffer-dams.

A Centrifugal Pump must be used when very large quantities of water must be handled. The distance from the water to the pump is limited by the height to which the ordinary pressure of the air will raise the water; but the height to which a centrifugal pump can lift the water is limited only by the velocity of the outer ends of the revolving blades. Since there are no valves in action while the pump is at work, the centrifugal pump will allow sand and large gravel to pass. Pumps having a 6-inch to 10-inch discharge pipe are the sizes most frequently used in foundation work.

28. Crib and Open-Caisson Process

This method of constructing a foundation consists in building the pier inside of a strong, water-tight box, called a CAISSON, whose vertical sides and bottom are composed of heavy timbers and which sinks as the masonry is added until it rests upon the bed prepared for it. Sometimes the bottom of the box consists of a timber crib-work of such thickness that the masonry will extend only a foot or two below low water, in which case the timber work below the caisson proper is called a CRIB. The crib may be sunk before the caisson, or the two may be sunk as a single mass. A noteworthy feature of this method is that the permanent structure contains a considerable amount of timber. The timber is employed chiefly because it is more easily put into place than masonry; and in the past timber has usually been cheaper than masonry. Timber when always wet is as durable as masonry. Because of the recent decrease in the cost of concrete and the increase in the cost of timber, the latter is not likely to be used for this purpose as much as formerly, altho the essential feature of the open caisson method will probably continue in use.

The Crib is the timber structure below the caisson, which transmits the pressure to the bed of the foundation. If the pressure is great, the crib is built of successive courses of squared timbers in contact; but if the pressure is small, it is built more or less open. In either case, if the crib is to rest upon a soft bottom, a few of the lower courses are built open so that the higher portions of the bed may be squeezed into these cells, and thus allow the crib to come to an even bearing. If the crib is to rest upon an uneven rock bottom, the site is first leveled up by throwing in broken stone, altho this is a poor method. If the bottom is rough or sloping, the lower courses of the crib are sometimes made to conform to the bottom as nearly as possible, as determined from soundings; but this method requires care and judgment to prevent the crib from sliding off from the inclined bed, and should be used with great caution, if at all.

The crib is usually built afloat. Owing to the buoyancy of the water, about one-third of a crib made wholly of timber would project above the water, and would require an inconveniently large weight to sink it; therefore, it is best to incorporate considerable stone in the crib-work. If the crib is more or less open, this is done by putting a floor into some of the open spaces or pockets, which are then filled with stone. If the crib is to be solid, about every third timber is omitted and the space filled with broken stone. The timbers of each course should be securely drift-bolted to those of the course below to prevent the buoyancy of the upper portion from pulling the crib apart, and also to prevent any possibility of the upper part's sliding on the lower.

The Caisson differs in construction according to the depth of the water. The simplest form is made by erecting studding by toe-nailing or tenoning them into the top course of the crib and spiking planks on the outside. For a caisson 6 or 8 feet deep, which is about as deep as it is wise to try with this simple construction, it is sufficient to use studding 6 inches wide, 3 inches thick, and 6 to 8 feet long, spaced 3 feet apart, mortised and tenoned into the deck course of the crib. The sides and floor (the upper course of the crib) should be thoroly calked with oakum. The sides may be braced from the masonry as the sinking proceeds. When the crib is grounded and the masonry is above the water, the sides of the box or caisson are knocked off.

When the depth of water is more than 6 to 8 feet, the caisson is constructed somewhat after the general method shown in Fig. 46. The sides are formed of timbers framed together and a covering of thick planks on the outside. The joints are carefully calked to make the caisson watertight. In deep

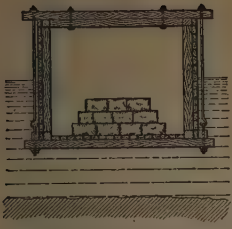


Fig. 46. Open Caisson and Pier

caissons, the sides can be built up as the masonry progresses, and thus not be in the way of the masons. The sides and bottom are held together only by the heavy vertical rods; and after the caisson has come to a bearing upon the soil, and after the masonry is above the water, the rods are detached and the sides removed, the bottom only remaining as a part of the permanent structure.

The caisson should be so contrived that it can be grounded, and afterwards raised in case the bed is found not to be accurately leveled. To effect this, a small sliding gate is sometimes placed in the side of the caisson for the purpose of filling it with water at pleasure. By means of this gate, the caisson can be filled and grounded; and by closing the gate and pumping out the water, it can be set afloat. Since the caisson is a heavy, unwieldy mass, it is not possible to control the exact position in which it is sunk; and hence it should be larger than the base of the proposed pier, to allow for a little adjustment to bring the pier to the desired location.

The crib may rest upon a group of piles cut off below the water, altho piles and a crib are used together; or the crib may be placed directly upon the bottom, or in a hole dredged in the bed of the stream. The excavation may be made with any form of dredge; and if the bottom is sand, mud, or silt, the soil may be removed by pumping it with the water thru an ordinary centrifugal pump, the suction hose of which is kept in contact with the bottom.

If the stream is shallow, the current swift, and the bottom soft, the site may be excavated or scoured out by the river itself. To make the current scour, construct two temporary wing-dams, which diverge up-stream from the site of the proposed pier. The wings can be made by driving stout stakes or small piles into the bed of the stream, and by nailing ordinary boards to light uprights, against the piles with their lower edge on the bottom. The wings concentrate the current at the location of the pier, increase its velocity, and cause it to scour out the bed of the stream. This process requires a little time, usually one to three days, but the cost of construction and operation is comparatively slight.

When the water is too deep for the last method, it is sometimes possible to suspend the caisson a little above the bed of the stream, in which case the current will remove the sand, and silt from under it. At the bridge over the Mississippi at Quincy, Ill., a hole 10 ft deep was thus scoured out. However, if the water is already heavily charged with sediment it may drop the sediment on striking the crib and thus fill up instead of scour out. Notwithstanding the hole is liable to be filled up by the gradual action of the current or by a sudden flood before the crib has been placed in its final position this method is frequently more expeditious and less expensive than using a coffer-dam.

The crib and open caisson was formerly the only process that could be employed on a permeable bottom; but the introduction of the interlocking steel sheet-pile and of the method of sealing the bottom of a coffer-dam by depositing concrete under water, has practically eliminated this method of sinking foundations under water.

29. Dredging thru Wells

This method consists in sinking a timber crib, a cast-iron cylinder, a steel shell, a mass of concrete, or a masonry well by excavating the soil thru compartments left for that purpose, thus undermining the crib or shell and causing it to sink. As soil below the caisson is excavated, the weight of the crib or caisson causes it to sink. When the crib or caisson has reached a hard stratum, the working chamber and also the shaft leading from it to the surface are filled with concrete.

Although this method is primarily for sinking a foundation under water, it

may be employed also for dry excavation. The soil, if dry, may be removed by loading it by hand into wheelbarrows or buckets and hoisting it out; or, if under water, the soil may be removed by a hydraulic mud-pump (see page 638) or by an orange-peel, clam-shell or grab-bucket dredge.

The chief advantage of this method for foundations under water is that it is applicable to greater depths than any other method except the freezing process; but the disadvantage is that the descent of the crib or cylinder is liable to be stopt by logs, bowlders, etc. For shallow depths the method is both economical and expeditious, and is applicable to nearly all soil conditions. It is the only method employed for depths which much exceed 100 ft, the limit of the pneumatic process.

The **timber crib** was formerly much used. The most noted example is the bridge across the Hudson river at Poughkeepsie, N. Y. It was erected in 1886-87, and is remarkable both for the size of the cribs and for the depth of the foundations. There are four river piers. The crib for the largest is 100 ft long, 60 ft wide at the bottom and 40 ft at the top, and 104 ft high. It is divided by one longitudinal and six transverse walls, into fourteen compartments thru which the dredge worked. The side and division walls terminate at the bottom with a 12 by 12 in oak stick, which served as a cutting edge. The exterior walls and the longitudinal division wall were built solid, of triangular cross-section for 20 ft above the cutting edge, and above that they were hollow. The gravel used to sink the crib was deposited in these hollow walls. The longitudinal walls were securely tied to each other by the end and cross division walls, and each course of timber was fastened to the one below by 450 1-in drift-bolts 30 in long. The timber was hemlock, 12 in square. The fourteen compartments in which the clam-shell dredges worked were 10 by 12 ft in the clear. The cribs were kept level while sinking by excavating from first one and then the other of the compartments. Gravel was added to the pockets as the crib sunk. When hard bottom was reached, the dredging pockets were filled with concrete deposited under water from boxes holding one cubic yard each and opened at the bottom by a latch and trip-line.

After the crib was in position the masonry was started in a floating caisson which finally rested upon the top of the crib. Sinking the crib and caisson separately was a departure from the ordinary method. Instead of using a floating caisson, it is generally considered better to construct a coffer-dam on top of the crib in which to start the masonry. If the crib is sunk first the stones which are thrown into the pockets to sink it are likely to be left projecting above the top of the crib and thus prevent the caisson from coming to a full and fair bearing. The largest crib was sunk thru about 53 ft of water, 20 ft of mud, 45 ft of clay and sand, and 17 ft of sand and gravel. It rests, at 134 ft below high water, upon a bed of gravel 16 ft thick, overlying bed-rock. The timber work is 110 ft high, including the floor of the caisson, and extends to 14 ft below high water (7 ft below low water), at which point the masonry commences and rises 39 ft. On top of the masonry a steel tower 100 ft high is erected. The masonry in plan is 25 by 87 ft, and has nearly vertical faces. The lower chord of the channel span is 130 ft and the rail is 212 ft above high water.

Cast-iron cylinders were sunk by this process for the Morgan City bridge over the Atchafalaya bayou or river, in Louisiana. Two cast-iron cylinders 8 ft in diameter braced together form the pier between two 250-ft spans of a railroad bridge. The cylinders were sunk 120 ft below high water, from 70 to 115 ft below the mud line, by dredging the material from the inside with an orange-peel excavator.

Steel shells were used for the piers of the Hawkesbury bridge, in southeastern Australia, which is remarkable for the depth of its foundations. This bridge is founded upon elliptical iron caissons 48 by 20 ft at the cutting edge, which rest

upon a bed of hard gravel 126 ft below the river bed, 185 ft below high water, and 227 ft below the track on the bridge. The soil penetrated was mud and sand. The caissons were sunk by dredging thru three tubes, 8 ft in diameter, terminating in bell-mouthed extensions, which met the cutting edge. The spaces between the dredging tubes and the outer shell were filled with gravel as the sinking progressed. The caissons were filled to low water with concrete, and above with cut-stone masonry. As these caissons were to be sunk to an unprecedented depth, it was considered wise to construct them with a flare at the bottom, that is, to make the bottom larger than the upper portion, so as to decrease the resistance due to friction. Experience showed that making the bottom larger was a mistake, since it seriously increased the difficulty of guiding the caisson in its descent.

A Brick Cylinder was sunk 256 ft by this process in Germany for a coal shaft. A cylinder 25½ ft in diameter was sunk 76 ft thru sand and gravel, when the frictional resistance became so great that it could be sunk no farther. An interior cylinder, 15 ft in diameter, was then started in the bottom of the larger one, and sunk 180 ft farther thru running quicksand. The soil was removed without exhausting the water. Brick and stone masonry cylinders have often been sunk by this process in both dry and water-bearing soils. To lessen the friction a ring of masonry has been built inside of a thin iron shell.

Reinforced-concrete caissons have recently been substituted for timber cribs or metal shells. The mass of concrete used in this method is called a caisson. Concrete was first used for this purpose in the construction of the bridge across the Mississippi river at Thebes, Illinois, in 1901–07. Fig. 47 and 48 show two comparatively shallow reinforced-concrete caissons employed by the Chicago, Burlington and Quincy Railroad (L. W. Skov. Jour. Western Soc. of Engrs., vol. 23, pp. 383–403).

In this form of construction the bottom of the side walls and interior struts are tapered to a width of from 3 to 12 in to form a cutting edge. Where caissons are sunk through clay or sand, it is customary to leave the concrete cutting edge unprotected; where gravel, rip rap or other hard materials are encountered, the cutting edge is protected by steel angles or hardwood timbers. Steel-protected cutting edges are not used except where the material penetrated is very hard and offers a great resistance, as the hardwood cutting edge is found to give better protection to the concrete and on account of its greater width gives a much stronger caisson wall near the bottom. Where the excavation is done by hand, openings may be left in the interior walls to allow the passage of men and tools from one chamber to the other. Where the material is removed by grab buckets or pumps, the interior walls are usually stopped off about 2 ft above the cutting edge to allow the water to pass from one compartment to another. In Fig. 47 the outside of the walls have been tapered off toward the top to reduce the skin friction in sinking; but it has been found that the difference in frictional resistance between an inclined and vertical surface is not very great, and hence the battered wall has been discontinu d on account of the greater cost of forms for it. Notice that the side walls in Fig. 47 are only 15 ft high, and are tapered; while those of Fig. 48 are 54 ft high, and are not tapered. In the latter case the caisson was constructed to a height of 10 ft, when the forms were removed and the sinking was begun, additional sections each 8 to 10 ft high being constructed as the sinking progressed. The forms were made of wood, and were used five or six times. After the caisson is sunk the working chamber and the shaft are filled with concrete. Reinforced concrete caissons have been sunk through dry soil and also through water and mud.

The advantages of the concrete caisson over the timber crib are: (1) the concrete caisson is smaller, and hence requires less excavation and offers less frictional resistance to sinking; (2) the concrete caisson weighs more, and hence

does not require less extraneous load to sink it; (3) the cutting edge can be shaped to fit the bed-rock bottom; and (4) the entire substructure is an unyielding mass of solid concrete.

When concrete is employed in the construction of the crib or caisson, this method has some advantages over the coffer-dam process, viz.: (1) the crib or

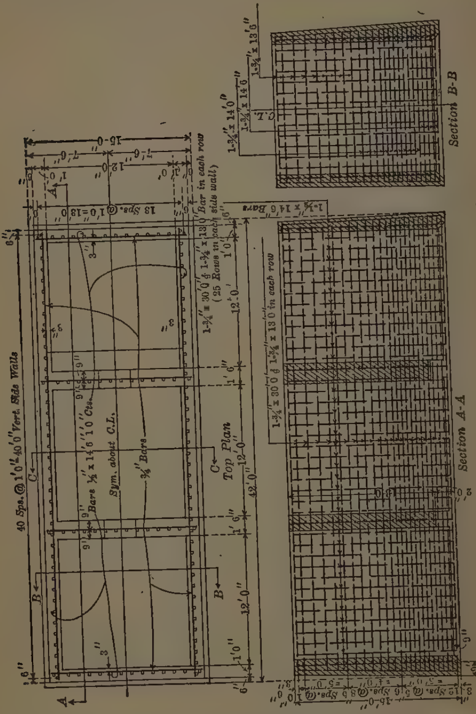


Fig. 47. Shallow Reinforced-Concrete Open Caisson

caisson is smaller than the corresponding coffer dam, and hence less excavatio required; (2) the concrete is absolutely watertight, and hence less pumpin required; (3) the equipment for driving and pulling sheet-piles is elimina required; (4) the work is more expeditious and certain; and (5) less timber is required making the forms of the concrete caisson than for building coffer-dam.

The Fractional Resistance varies greatly with the material of the tube and the c acter of the soil. During the construction of the bridge over the Seine at Oriv cast-iron cylinder, standing in an extensive and rather uniform bed of gravel, and ha ceased to move for thirty-two hours, gave a frictional resistance of nearly 200 lb per

t a bridge over the Danube near Stadlau, a cast-iron cylinder sunk 18.75 ft into the soil he lower 3.75 ft being "solid clay") gave a frictional resistance of 100 lb per sq ft. ccording to some European experiments, the friction of cast-iron cylinders in sand and ver mud was from 400 to 600 lb per sq ft for small depths, and 800 to 1000 for depths om 20 to 30 ft. At the first Harlem River bridge, New York City, the frictional resist-

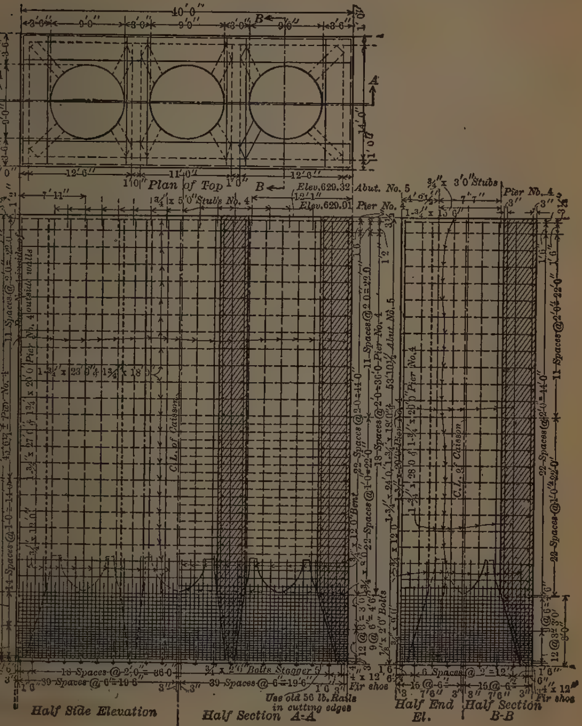


Fig. 48. Deep Reinforced-Concrete Open Caisson

ce of a cast-iron pile, while the soil around it was still loose, was 528 lb per sq ft of sur-e; and later 716 lb per sq ft did not move it. From these two experiments, McAlpine, engineer in charge, concluded that "1000 lb per sq ft is a safe value for moderately e material." At the Omaha bridge, a cast-iron pile sunk 27 feet in sand with 15 feet sand on the inside, could not be withdrawn with a pressure equivalent to 254 lb per sq of surface in contact with the soil; and after removal of the sand from the inside, it oved with 200 lb per sq ft. A wrought-iron pile, penetrating 19 ft into coarse sand at

the bottom of a river, gave 280 lb per sq ft; another, in gravel, gave 300 to 335 lb per sq ft. In the silt on the Clyde, the friction on brick and concrete cylinders was about 3½ tons per sq ft. The friction on the brick piers of the Dufferin (India) bridge, thru clay, was 900 lb per sq ft.

30. The Pneumatic Process

This is a method of constructing a foundation under water by utilizing the difference between the pressure of air inside and outside of an air-tight chamber. The chamber may be either a tube or a box, open below and air-tight elsewhere, the first being a pneumatic pile and the latter a pneumatic caisson. Pneumatic piles were once considerably used for bridge piers, but have now been superseded for that purpose by pneumatic caissons. Since 1894 the compress-air process has been frequently employed in constructing foundations for tall buildings on Manhattan Island, New York City, and in some cases the pneumatic pile has been used, altho it is there usually called a caisson.

A Pneumatic Caisson is an immense box, open below but air-tight and water-tight elsewhere, upon the top of which the masonry pier is built. The essential difference between the pneumatic pile and the pneumatic caisson is one of degree rather than one of quality. Sometimes the caisson envelops the entire masonry of the pier; but in the usual form the masonry envelops the iron cylinder and rests upon an enlargement of the lower end of it. The pneumatic pile is sunk to the final depth before being filled with concrete or masonry; but with the caisson the masonry is built upward while the whole pier is being sunk downward, the masonry thus forming the load which forces the caisson into the soil. A pneumatic caisson is, practically, a gigantic diving-bell upon the top of which the masonry pier rests. The essential principle of the pneumatic caisson is that compress air is pumped into the working chamber and drives out the water and permits men to work therein.

The New Memphis Bridge (1915) is a recent example of the pneumatic process in constructing the foundation of a bridge pier under water. The bridge carries a steam railroad over the Mississippi river. Ralph Modjeski was the designing engineer. Fig. 49 shows the construction of the caisson and crib for pier IV, one of the smallest piers, the sheathing being removed in the end elevations; ○ indicates a screw bolt, and × a drift bolt. The construction is typical of that of all the other piers. The bottom portion, the AIR-CHAMBER and its roof and sides, constitute the CAISSON. Above the roof of the caisson is an open timber CRIB which is filled with concrete as the sinking proceeds. The crib extends to a point a little below low water. The construction of the caisson for the new (or Harahan) Memphis bridge differs from that usually employed previously in that the timber roof of the air chamber is thinner. In the large caisson of the Metropolis, Ill., bridge across the Ohio river (designed in 1916 by Mr. Modjeski) the roof is stiffened by steel trusses embedded in the concrete of the crib. Sometimes the roof of the air chamber is reinforced concrete, and sometimes the entire caisson is made of that material, as for example in the McKinley bridge over the Mississippi river at St. Louis.

Usually a COFFER-DAM is constructed on top of the crib to facilitate sinking. If the roof of the caisson is not to go very far below low water, the crib is omitted; and sometimes it is omitted to decrease the obstruction to the flow of the water. If the soil is stiff clay and the range of low and high water is not great, the coffer-dam is sometimes omitted; but when the coffer-dam is not used, it is necessary to regulate the speed of sinking by the speed with which the masonry can be had, which is liable to cause inconvenience and delay, and further it is necessary to go on constructing the masonry whether or not the additional

weight is needed in sinking the caisson. The workmen descend to and ascend from the air-chamber thru an AIR-SHAFT, about mid-height of which is an AIR-LOCK. The air-lock consists of a short steel cylinder, elliptical in horizontal cross-section, connecting on one side of the bottom with a steel shaft to the air-chamber and on the other side at the top with a similar shaft to the outer air, each shaft being closed by a door opening downward. The SUPPLY-SHAFT is a steel tube down which concrete is passed in filling the air-chamber.

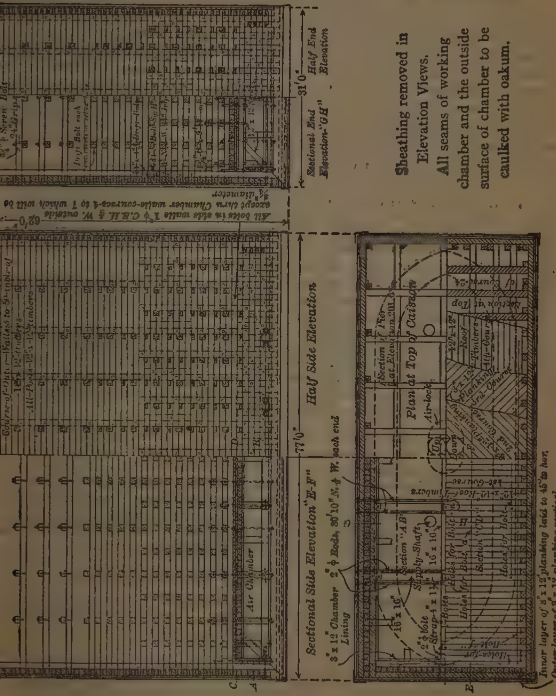


FIG. 49. Pneumatic Caisson and Crib for Second Memphis Bridge

The Air-Lock is often placed at the top of the air-shaft, altho this position is objectionable because the men must then climb all the way out in compressed air, which is exceedingly fatiguing. On the other hand, if the lock is placed at the bottom and there is sudden in-rush of water owing to an unusual blow-out of air, not an infrequent occurrence, there is danger that the men in the working chamber may be drowned before they get into the air-lock; while if the lock is at the top, they can quickly escape by climbing up in the air-shaft. If the air-lock is placed part way up, the construction of the lock is more expensive.

Removal of the Earth is accomplished in several ways. One device is the SAND LIFT, which consists of a pipe, reaching from the working chamber to the surface, controlled by a valve in the working chamber. The sand is heapt up around the lower end of the pipe, the valve opened, and the comprest air in the working chamber forces a continuous stream of air and sand up and out. Mud or semi-liquid soil may be removed by this means by immersing the lower end of the tube and opening the valve; but this method is most effective with sand. When used for dry sand, the sand-lift is usually called dry blow-out; and when used for mud, the wet blow-out. Altho the sand-lift is efficient there are some objections to it: (1) forcing the sand out by the pressure in the caisson decreases the pressure, which causes, particularly in pneumatic piles or small caissons, the formation of vapors so thick as to prevent the workmen from seeing; (2) the diminished pressure allows the water to flow in under the cutting edge; and (3) if there is much leakage, the air-compressors are unable to supply the air fast enough. These objections are now satisfactorily overcome by furnishing a liberal supply of comprest air.

The MUD PUMP was invented by Eads in sinking the foundations of the steel-arch bridge across the Mississippi river at St. Louis, to overcome the above objections to the sand lift. It is based upon the principle of the induced current, and this principle is utilized by discharging a stream of water with a high velocity on the outside of a small pipe, which produces a partial vacuum in the latter, when the pressure of the air on the outside forces the mud thru the small pipe and into the current of water by which the mud is carried away. The mud pump is not now much used.

The CLAY HOIST is a device for hoisting material in a bucket by means of comprest air, and is particularly useful when excavating stiff clay, which can not easily be removed with either the sand-lift or the mud pump. It was invented by George S. Morison in sinking the foundations for the bridge across the Mississippi River at Memphis, and consisted of a cylinder and piston placed at one side of the top of the material shaft. The piston was actuated by air pressure, and was connected to a cable to which was attached a bucket working up and down thru the material shaft. At the top of the shaft were two doors operated by levers from the outside, to facilitate the passage of the buckets. The buckets held 6½ cu ft; and the device was very effective.

MORAN'S AIR-LOCK consists of a lock at the top of the material shaft, closed at both top and bottom by a pair of sliding doors so arranged as to permit of hoisting buckets of material out of the air-chamber by means of a derrick and cable. The doors are moved by comprest air, and are interlocked so that one can not be opened until the other is closed. On the cable is a stuffing box which fits into a semi-circular groove in the edges of the two halves of the upper door, and permits the bucket to be raised or lowered while the upper door is closed. This lock is very effective, since in ordinary operations the bucket usually passes the lock with only about 5 seconds delay, and can do it in 2 seconds.

The WATER COLUMN, a device used at the Brooklyn bridge in the early history of the pneumatic process, consists of a material shaft, open top and bottom, which projects a little below the cutting edge, and is kept full of water, the greater height of water in the column balancing the pressure of the air in the chamber. The workmen simply push the material under the edge of the shaft, from whence it is excavated by an orange-peel or clam-shell dredge.

Sinking the Caisson. To guide the caisson in its descent, after it is floated to the site, it is usually suspended by screws to a framework resting upon piles or pontoons. In a strong current or in deep water, it may be necessary to support the caisson partially in order to govern its descent after the cutting edge reaches the soil; but ordinarily, the suspension is needed only until the caisson is well imbedded in the soil. The caisson may be protected from the current by constructing a breakwater above and producing dead water at the pier site. After the soil has been reached, the caisson can be kept in its course by removing the soil from the cutting edge on one side or the other of the caisson. In case the caisson does not settle down after the soil has been removed from under the

cutting edge, a reduction of a few pounds in the air pressure in the working chamber is usually sufficient to produce the desired result. At the Havre de Grace bridge, it was found that by allowing the discharged material to pile up against the outside of the caisson, the latter could be moved laterally almost at will. The top of the caisson was made 3 feet larger, all round, than the lower course of masonry, to allow for deviation in sinking. The deviation of the caisson, which was founded 90 feet below the water, was less than 18 inches, even tho neither suspension screws nor guide piles were employed.

In sinking the foundations for the bridge over the Missouri River near Sibley, Mo., it was necessary to move the caisson considerably in a horizontal direction without sinking it much farther. This was accomplished by placing a number of posts 12 inches square in an inclined position between the roof of the working chamber and a temporary timber platform resting on the ground below. When these posts had been wedged up to a firm bearing, the air pressure was released. The water flowing into the caisson loosened the soil on the outside, and the weight of the caisson coming on the inclined posts caused them to rotate about their lower ends, which forced the caisson in the desired direction. In this way, a lateral movement of 3 or 4 feet was secured while sinking about the same distance.

The Frictional Resistance on the timber sides of the pneumatic caisson at Havre de Grace was 280 to 350 lb per sq ft for depths of 40 to 80 feet, the soil being silt, sand, and mud; when bowlders were encountered, the resistance was greater, and when the air escaped in large quantities the resistance was less. At the bridge over the Missouri River near Blair, Neb., the frictional resistance usually ranged between 350 and 450 lb per sq ft, the soil being mostly fine sand with some coarse sand and gravel and a little clay. At the Brooklyn bridge, the frictional resistance at times was 600 lb per sq ft. At Cairo, in sand and gravel, the normal friction was about 600 lb per sq ft. At Memphis, in sand, the friction was 400 lb per sq ft.

Rate of Progress. The work in the caisson usually continues day and night, winter and summer. At the Havre de Grace bridge the average rate of progress was 1.37 ft per day; at the Plattsmouth bridge, 2.22 ft; and at the Blair bridge 1.75 ft per day. Speeds of 6 and 8 ft per 24 hours have been maintained for a few consecutive days with large caissons. The above rates are for bridge piers, and were greatly exceeded in the case of the small caissons for column foundations of tall buildings. In constructing the Broad Exchange Building, New York City, 88 caissons were sunk an average of 30 ft in 47 days, one being sunk 27 ft in 20 hours and 2 ft in 1 hour.

The Maximum Depth to which the pneumatic process has been applied is 113.0 ft, in 1911 in caisson No. 4 of the Municipal bridge over the Mississippi at St. Louis; and the next deepest was at the east abutment of the Eads bridge in St. Louis. At the first Memphis bridge the depth was 106.4 ft. At the Williamsburg bridge over East River in New York City, the maximum depth reached was 107.5 ft, but this depth extended over only a very small area and the maximum pressure was for only a few minutes. Except in these instances, the compres:-air process has never been applied at a greater depth than about 90 feet. The pneumatic process is limited to depths not much greater than 100 feet owing to the deleterious effect of the compress air upon the workmen. When the caisson has reached the required depth, the bottom is leveled off, by blasting if necessary, and the working chamber and shafts are filled with concrete. At the Eads bridge at St. Louis, only enough concrete was placed in the bottom to seal the chamber water-tight, and then the remaining space was filled with sand, the sand being pumped in from the river with the pump previously used for excavating the material from under the caisson.

31. Pneumatic Foundations for Buildings

The pneumatic process was devised for laying foundations of bridges under considerable depths of water or water-bearing soil, and for a number of years

was used exclusively for that purpose and in tunneling; but since 1894 the pneumatic process has been used extensively in laying foundations for buildings in New York City, and has there been carried to a great degree of refinement. In addition to the advantages of the pneumatic process for foundations in general, the two conditions which primarily led to the introduction of the pneumatic process for building-foundations were: (1) the tall buildings required so great a supporting power that it was necessary to carry the foundation to bed-rock, which is from 60 to 80 ft below the street surface, and (2) the necessity of using a process of excavating through the water-bearing soil that would not disturb the soil under adjacent buildings.

In some cases, the so-called caisson was virtually a pneumatic pile made of wood, stone masonry, or steel plates; but in most cases the foundation consists of a pneumatic caisson proper made of wood or steel plates surmounted by brick masonry or concrete. The steel caisson is usually preferred because the thinner sides give greater space in the working chamber, and also because the caisson can be brought to the building ready for sinking. The method of operation is the same as in bridge foundations except in three particulars: (1) Since the caissons are comparatively small, have vertical sides, and are sunk all the way thru earth, the weight of the masonry upon them is insufficient to overcome the friction and the upward pressure of the compress air, and hence extra weight is usually required to sink the caissons. (2) To prevent the soil from escaping from under the shallower foundations of adjacent buildings, it is necessary to make the excavation without reducing the air pressure. (3) The sinking must be continuous, as otherwise the soil will settle around the caisson and make it impossible to start again without releasing the air pressure.

For an account of the pneumatic-foundation work for the Singer Building in New York City, see Trans. Am. Soc. C. E., vol. 63 (1909), pp. 1-52. For an account of recent improvements in the details of pneumatic foundations for buildings, see Trans. Am. Soc. C. E., Vol. 61 (1908), pp. 211-37. The more important of these improvements are: (1) the elimination of the roof of the caisson, (2) the doing away with the coffer-dam, (3) the elimination of the shaft-lining, (4) the substitution of cylindrical for rectangular caissons, and (5) surrounding the foundation area with a row of pneumatic caissons which together act as a coffer-dam.

32. Physiological Effect of Compress Air

After entering the air-lock, as the pressure increases, the first sensation experienced is one of great heat. As the pressure is still further increased a pain is felt in the ear, arising from the abnormal pressure upon the ear-drum. The tubes extending from the back of the mouth to the bony cavities over which this membrane is stretched are so very minute that compress air cannot pass thru them with a rapidity sufficient to keep up the equilibrium of pressure on both sides of the drum (for which purpose the tubes were designed by nature), and the excess of pressure on the outside causes the pain. These tubes can be distended, thus relieving the pain, (1) by the act of swallowing, or (2) by closing the nostrils with the thumb and finger, shutting the lips tightly, and inflating the cheeks, or (3) by taking enough snuff to cause sneezing. Either action facilitates the passage of the air thru these tubes, and establishes the equilibrium desired. The relief is only momentary, and the act must be repeated from time to time as the pressure in the air-lock increases. This pain is felt only while the air in the lock is being "equalized," and is most severe the first time compress air is encountered, a little experience generally removing all unpleasant sensations. A drop of oil in each ear is a material help in obstinate cases. The passage thru the lock, both going in and coming out, should be slow; that is to say, the compress air should be let in and out gradually, to give the pressure time to equalize itself thruout the various parts of the body.

When the lungs and whole system are filled thoroly with the denser air, the general effect is rather bracing and exhilarating. The increased amount of oxygen breathed in compress air very much accelerates the organic functions of the body, and hence labor in the caisson is more exhaustive than in the open air; and on getting outside again a reaction with a general feeling of prostration sets in. At moderate depths, however, the laborers in the caisson, after a little experience, feel no bad effects from the compress air, either while at work or afterwards. In passing thru the air-lock on leaving the air-chamber, the workman experiences a great loss of heat owing (1) to the expansion of the atmosphere in the lock, (2) to the expansion of the free gases in the cavities of the body, and (3) to the liberation of the gases held in solution by the liquids of the body. Hence, on coming out the men should be protected from currents of air, should drink a cup of hot strong coffee, dress warmly, and lie down for a short time.

Remaining too long under heavy pressure causes a form of paralysis, called by the physicians caisson disease and by the workmen bends, which is sometimes fatal. The attack occurs only after returning to atmospheric pressure, and particularly after coming thru the air-lock quickly; and ordinary cases are cured or greatly relieved by returning to the compress air and coming out very slowly. With reasonable care, the pneumatic process can be applied at depths less than 80 or 90 feet without serious consequences. At great depths the danger can be greatly decreased by observing the following precautions, in addition to those referred to above: (1) in hot weather cool the air before it enters the caisson; (2) in cold weather warm the air in the lock when the men come out; (3) raise and lower them by machinery; and (4) pass the men thru the lock slowly, especially coming out. For the maximum depth coming thru the lock should consume forty-five minutes. To prevent the men from passing thru the lock too rapidly, an automatic constant-rate decompression valve is sometimes used (Trans. Amer. Soc. of C. E., vol. 65 (1909), p. 9). If after coming out of the compress air, a man is attacked by the caisson disease, he is placed in a hospital lock (a chamber containing compress air) for several hours, after which the pressure is reduced exceedingly slowly. A hospital lock is now usually established in connection with all pneumatic work, and is quite effective.

On account of the effect of compress air upon the workmen it is generally held that the pneumatic process is limited to depths not much exceeding 100 feet. For a valuable article on Caisson Disease and its Prevention, see Trans. Amer. Soc. of C. E., vol. 65 (1909), p. 1-37.

The Working Time in compress air for depths less than 40 or 50 ft is usually eight hours per day, with a visit to the open air for lunch at the middle of the shift; but when the pressure becomes greater the working time is materially shortened. At the Eads bridge, at a pressure from 45 to 50 lb per sq in, corresponding to a theoretical depth of 104 to 115 ft, the men were able to remain in the compress air only four hours per day in shifts of two hours each, and even then they worked only part of the time they were in the air-chamber. At Memphis the time was: between 80 and 90 ft depth, two shifts of two hours each; and below 90 ft, three shifts of one hour each. At the Williamsburg bridge across the East River, New York City, the working time was eight hours for less than 55 ft below mean high water, six hours from 55 to 70 ft, fours hours from 70 to 80 ft, two hours from 80 to 90 ft, and one hour below 90 ft.

33. Cost of Pneumatic Bridge Foundations

The following table gives the cost of the pneumatic foundation of moderate depth for the pivot pier of a swing bridge. The dimensions at the cutting edge of the caisson were 30 by 30 ft, the height of the caisson 15 ft, the depth sunk below water 55 ft, the depth sunk below ground 45 ft, and the displacement below ground 1500 cu yd. Two rest piers of the same bridge, which were smaller but were sunk the same distance, cost $10.27 and $12.50 per cu yd.

Cost of Pneumatic Bridge Foundation at Moderate Depth *

Items	Total	Per Cu Yd.
Plant, proportionate cost	$2 525	$1.68
Platform and derrick, setting up	100	.07
Pipe left in caisson	130	.09
Iron left in caisson @ 5 cents per lb	300	.20
Lumber in caisson @ $20 per M	1 576	1.05
Lumber in coffer-dam @ $20 per M	180	.12
Iron in cutting edge @ 4½ cents	675	.45
Rods, drift-bolts, etc., @ 2½ cents	230	.16
Boat spikes, etc	172	.11
Oakum @ 4 cents per lb	80	.05
Rubber packing @ 70 cents per lb	70	.05
Building coffer-dam @ $2.97 per day	235	.16
Building caisson @ $2.96 per day	1 439	.96
Sinking caisson @ $2.99 per day	5 094	3.39
Coal @ $3.00 per ton	660	.44
Piles @ 10 cents per lin ft	60	.04
Driving piles @ 12 cents per ft	72	.05
Concrete @ $4.25 per cu yd	2 805	1.93
Supplies	185	.12
Supt. and office expenses	700	.47
Total	$17 288	$11.59

* Compiled from Engineering-Contracting, 1907, vol. 27, pp. 204–5, 220–21.

The following table gives details for a deep foundation and it will be noted that the cost per cu yd is practically twice as great as in the one of moderate depth. This difference is partly due to higher wages paid in the air chamber on account of the greater depth, the wages in the first case being $2.99 for 8 hours and in the second ranging from $2.50 for 8 hours above 55 ft to $3.75 for one hour below 100 ft; and part of the difference is due to difference in the material excavated.

The table on p. 644 gives the cost of the pneumatic foundations and piers of the second (Harahan) bridge across the Mississippi River at Memphis, Tenn., constructed in 1913–14, under the direction of Ralph Modjeski. Fig. 49, p. 637 shows the crib and caisson of pier IV. The piers above the crib or caisson, except number V, are of granite backed with concrete except that the beltfng courses and courses next below are backed with granite. The copings are entirely of granite. The face stones have a rock or quarry face. Pier V is built entirely of reinforced concrete.

Except in very shallow or very deep water, the comprest-air process has almost entirely superseded all others. The following are advantages of this method. (1) It is reliable, since there is no danger of the caisson's being stopped before reaching the desired depth, by sunken logs, bowlders, etc., or by excessive friction, as in dredging thru tubes or shafts in cribs. (2) It can be used regardless of the kind of soil overlying the rock or ultimate foundation. (3) It is comparatively rapid, since the sinking of the caisson and the building up of the pier go on at the same time. (4) It is comparatively economical, since the weight added in sinking is a part of the foundation and is permanent, and the removal of the material by blowing out or by pumping is as uniform and rapid at one depth as at another, the cost only being increased somewhat by the greater depth. (5) This method allows ample opportunity to examine the ultimate foundation, to level the bottom, and to remove any disintegrated rock. (6) Since the rock can be laid bare and be thoroly

Cost of North Pier of Williamsburg Bridge, New York City *

Items	Quantity		Rate	Cost
Caissons:				
Timber, yellow pine, ft B.M.	980	M	$18.50	$18 120
Iron: bolts, rods, etc.	90.5	tons	35.00	3 168
Oakum, pitch, paint, tar	980	M	.31	302
Labor, building, calking, and launching	980	M	13.61	13 362
Plant, rental and labor	980	M	2.68	2 617
General expenses, superintendence, etc.	980	M	3.83	3 757
Total cost of caisson	980	M	42.12	$41 326
Per gross cu yd	8766	cu yd	4.83	
Coffer-dam:				
Timber, yellow pine	228	M	$19.00	$4 335
Iron: rods, bolts, etc	23.5	tons	35.00	823
Oakum	48	bales	2.50	120
Labor, building and removing	228	M	25.40	5 779
Plant	228	M	1.54	351
General expenses, 10 per cent			5.00	1 141
Total for coffer-dam	228	M	55.00	$12 549
Concrete (1 P. C. : 2½ S. : 6 B. S.):				
Above roof	5692	cu yd	$4.73	$26 935
In shaft holes	576	cu yd	6.47	3 737
In working chamber	1566	cu yd	11.73	18 362
Total cost of concrete				$49 034
Sinking:				
Mud, sand, gravel, depth 51–57 feet	1714	cu yd	$2.35	$4 035
Fine sand, depth 57–69 feet	2175	cu yd	2.31	5 021
Clay and stratified clay, depth 69–73 feet	866	cu yd	4.50	3 896
Stratified clay, depth 73–81 feet	1493	cu yd	7.20	10 752
Stratified clay, depth 81–90 feet	1621	cu yd	9.25	15 002
Gneiss, sound rock, depth 83–91 feet	84	cu yd	78.40	6 581
Stratified clay, depth 91–95 feet	702	cu yd	17.55	12 324
Gneiss, benching, depth 91–95 feet	253	cu yd	65.80	16 709
Stratified clay, depth 95–107.5 feet †	435	cu yd	25.00	11 337
Incidentals				715
Total	9343	cu yd	$9.03	$84 372
Total cost of foundation	8766	cu yd	$21.36	$187 281

* Compiled from Engineering-Contracting, 1906, vol. 26, pp. 33–36, 40–41, 46–48, which gives the cost in minute detail.

† Maximum depth over only a small area. Only 2 cu yd below 106 ft., and only 21 cu yd below 104 ft.

washt, the concrete can be commenced upon a perfectly clean surface; and hence there need be no question as to the stability of the foundation.

34. The Freezing Process

The presence of water has always been the great obstacle in foundation work and in shaft-sinking, and it is only comparatively recently that any one thought of transforming the liquid soil into a solid wall of ice about the space to be excavated. The method of doing this consists in inclosing the site to be excavated, by driving into the ground a number of tubes thru which a freezing mixture is made to circulate. These consist of a large tube, closed at the lower end, inclosing a smaller one, open at the lower end. The freez-

Cost of Pneumatic Foundations of Harahan Bridge Across Mississippi River at Memphis, Tenn.*

For Drawing of Crib and Caisson, see Fig. 49, p. 637

Items	Pier I	Pier II	Pier III	Pier IV	Pier V
VOLUMES, IN CUBIC FEET:					
Crib and caisson-cutting edge to masonry	189 440	192 780	193 725	147 278	75 150
Sinking—cutting edge to river bed	243 616	195 642	281 666	182 772	98 088
Masonry—above top of crib	101 986	173 632	180 074	86 282	19 062
COST, PER CUBIC YARD:					
Crib and caisson	$ 3.81	$ 8.37	$ 5.56	$ 3.75	$2.94
Sinking	12.66	20.49	17.04	13.74	13.34
Foundation, in place	17.52	29.00	25.14	18.41	13.72
Masonry, above crib	21.20	19.47	20.44	24.03	10.02
Total	$18.82	$24.46	$22.80	$20.47	$15.34

* Compiled from Report of Chief Engineer, p. 9.

ing mixture is forced down the inner tube, and rises thru the outer one. At the top, these tubes connect with a reservoir, a refrigerating machine, and a pump. The freezing liquid is cooled by an ice-making machine, and then forced thru the tubes until a wall of earth of sufficient thickness to stand the external pressure is frozen around them, when the excavation can proceed as in dry ground.

This method was invented by F. H. Poetsch, of Aschersleben, Prussia, in 1883. The process has been used many times in sinking shafts in mining operations. "Shaft Sinking in Difficult Cases," by J. Reimer, translated from the German by J. W. Brough (1907), gives 64 examples, most of which are in Germany and France, only one being in the United States. One shaft has been sunk by this process 816 ft, and several have been sunk over 300 ft. The Transactions of the American Society of Civil Engineers, Vol. 52, pp. 365-450, give a résumé of the literature of the freezing process to February, 1904, and also a discussion of the same. This process seems to have been valuable for sinking shafts thru quicksand and other water-bearing soil under difficult circumstances, but has not been applied in foundation work.

Two methods of applying this process for foundations under water have been proposed. One of these consists in combining the pneumatic and freezing processes. A pneumatic caisson is to be sunk a short distance into the river bed; and then the congealing tubes are applied, and the entire mass between the caisson and the rock is frozen solid. When the freezing is completed, the caisson will be practically sealed against the entrance of water, and the air-lock can be removed and the masonry built up as in the open air. The other method consists in sinking an open caisson to the river bed, and putting the freezing tubes down thru the water. When the congelation is completed, the water can be pumped out and the work conducted in the open air.

It is claimed for this process that it is expeditious and economical, and also that it is particularly valuable in that it makes possible an accurate estimate of the total cost before the work is commenced, a condition of affairs unattainable by any other known method in equally difficult ground. It has an advantage over the pneumatic process in that it is

not limited by depth. Two difficulties are anticipated in applying it to sink foundations for bridge piers in river beds; viz. (1) the difficulty in sinking the pipes, owing to striking sunken logs, bowlders, etc.; and (2) the possibility of encountering running water, which will thaw the ice-wall. These difficulties are not insurmountable, but experience only can demontrate how serious they are.

EARTHWORK

35. Loosening and Shoveling

Loosening. If the earth is not to be handled by steam power, it must first be loosened, unless it is sand or sandy loam. The loosening is done with a pick, a mattock, or a plow. The PICK is used in trenches and other confined positions, and the plow elsewhere. The pick is much more economical than a mattock; and the latter should be used only in trimming a surface. The amount that a man can loosen in a given time with a pick varies greatly with the man, the supervision, the character of the soil, the depth of breast, etc.; but is usually about as follows: hard pan or cemented gravel, 0.5 to 1.0 cu yd per hour, and ordinary loam 3.0 to 5.0. The PLOW is usually drawn by two horses or mules in ordinary loam, four in stiff clay and gravel, and six in hard pan. In each case there is a driver and a plowman, and in the last also one or two men to ride the plow-beam to keep the plow in the ground. The amount loosened is in ordinary loam 40 to 50 cu yd per hour, in stiff clay 25 to 30, and in hard pan 15 to 20. Assuming the daily wages of a plow, pair of horses and a driver to be $3.50, and the plow holder $1.50, or a total of 50 cents per hour, the cost of loosening light loam is 1¼ cents per cu yd. If the plow worked continuously and straight along, it could loosen a great deal more than this; but the plowing must usually be done in short sections, and for various reasons much time is lost.

Shoveling. The shovel employed may be either square-end or round-end, and may have either a long or a short handle. The square-end shovel should be used, except possibly in very stiff clay; and the long-handled shovel is usually more economical, unless the shovelers are crowded closely together. A man will shovel well-loosened earth into a wagon at the rate of 1.2 to 1.5 cu yd per hour; if the wagon is comparatively low the larger amount can be realized, and if the wagon is high the smaller amount is a good day's work. If the man shovels earth from a platform he can handle 2 or 2.5 cu yd per hour. In soil that can be spaded easily, a man can dig and load more solid earth with a tile spade than of loose earth with a shovel. Ordinary men can spade and load as task work 20 cu yd of brick clay per day; and experienced skilful men have spaded and loaded month after month 40 cu yd in 8 to 9 hours, and occasionally 56 cu yd. A man can pick and load about 1 cu yd of loam per hour, about ¾ cu yd of stiff clay, and ⅓ to ½ cu yd of hard pan.

36. Scrapers

There are four kinds of drag scrapers: the drag scoop, or "slip" or "slusher," the Fresno, the buck, and the flat-bottom tongue scraper.

The Drag-scoop Scraper, the form of drag scraper most frequently used, consists of a solid steel bowl with two handles by which to load and dump it, and of a bail to which to hitch the team. It is made in three sizes, which vary a little with the maker. The smallest, for one horse, has a rated capacity of about 3 cu ft, and the larger sizes, for two horses, have nominal capacities of about 5 and 7 cu ft, respectively. The drag-scoop scraper is admirable

for borrowing at the sides of embankments and for wasting from cuts or ditches, and also for opening the mouths of large cuts; but is not economical except for short distances. There is no danger of the scraper getting out of order until it is worn out and unfit for use, and the manner of using it is quickly learned by any one.

Drag or slip scrapers, in the usual short hauls, are operated in gangs of three, each with a man to load the scrapers; and the teams, even on the shortest hauls, travel in a circuit of about 150 ft. Usually the driver dumps the scraper; but when an embankment is being built, there is sometimes a man to keep the dump level, who also aids in dumping the scrapers. If the earth is sand or sandy loam and unobstructed with grass or tree roots, it may be scraped without plowing; but usually it is economical to plow before scraping, since then the scrapers are filled more nearly full. Sometimes a man is required to grub roots, etc. The "drag" is not economical to move earth more than about 200 ft, and some contractors claim that at more than 100 ft the wheel scraper is more economical than the drag.

Gillette's rule (Earthwork and its Cost, p. 54) for the cost of moving earth with a drag-scoop scraper is as follows:

"Add together the following items: 1/20 of an hour's wages of team with driver and that of the plowman, for plowing; 1/20 of an hour's wages of team with driver, for time lost in loading and dumping; 1/20 of an hour's wages of laborer, for loading scrapers; 1/20 of an hour's wages of team with driver, for extra travel in turning; 1/10 of an hour's wages of team with driver, for each 100 ft of lead (the distance from center of cut to center of fill), for transporting.

"With wages at 15 cts per hour for laborer, and 35 cts for team with driver, the above rule becomes: To a fixt cost of 6 cts per cu yd, add 4 cts for each 100 ft of lead. For example, with a lead of 25 ft, the cost will be $6 + \frac{1}{4} \times 4 = 7$ cts per cu yd. Fairly tough clay will cost one-third more."

The Fresno Scraper differs from the ordinary drag-scoop scraper in the form of the bowl and in having adjustable runners upon which the bowl is carried in dumping and returning, and which permit the scraper to distribute its load in dumping. The Fresno scraper is made in three sizes, the cutting edge being 3½, 4 and 5 ft, and having nominal capacities of 8, 10 and 12 cu ft, respectively. The distance from the cutting edge is comparatively small, which enables it to be easily loaded to its full capacity; and the runners enable it to deposit its load in layers. It is usually drawn by four horses abreast. On account of its form, it will follow up a steep bank without dumping. It is much used on the Pacific slope; but not as much elsewhere as its merits warrant. The cost is usually $17, $18, and $19, respectively.

The Fresno scraper is always more economical than the drag-scoop scraper, since it is more easy to fill and since considerable earth may be pushed along in front of it. The large size scraper under ordinary conditions may be counted upon to move ½ cu yd of compacted earth per load; and with light damp soil on short hauls a load may be 25 to 50% more than the contents of the bowl, and in soil difficult to load 25% less. The economical limit of haul is 200 to 300 ft; but with soil that drifts well in front of the scraper, it is more. The amount moved by a Fresno scraper varies from 60 to 120 cu yd per day, according to the character of the earth and length of haul.

The following is the range of cost of building a number of large irrigation canals in Nevada (Eng.-Contr., Nov. 3, 1909, p. 370). In making a narrow irrigation ditch in which the excavation made the banks, the distance from bottom of ditch ranging from 6 to 7½ ft, and the soil being sand and light loam, the cost was 5.06 cts per cu yd; and the amount moved per scraper was 125 to 130 cu yd per day. In "difficult earth thoroly mixt with stones up to 5 cu ft," the cost of making a side-hill ditch was 17½ cts per cu yd, the amount moved per scraper per day being 58.5 cu yd.

For the larger sizes of the Fresno scraper the fixt cost is 6 cts per cu yd; and the cost of transportation is 2 to 3 cts per cu yd per 100 ft of lead (distance from center of cut to center of fill), the smaller sum being for short hauls and the latter for long hauls, since with long hauls part of the load shakes out.

The Buck Scraper in its simplest form consists of an upright board, 4 to 6 ft long and 12 to 18 inches wide, with handles attached; and is employed in replacing earth in a trench, a team being hitched to it and pulling perpendicular to the trench on one side and two men holding the scraper on the opposite side. In its most elaborate form it consists of an upright board about 8 ft long and 2 ft wide, shod on its lower edge with iron, provided with a tail piece or platform upon which the driver stands while the scraper is loading or hauling. The earth is simply pushed along in front of the scraper; and the scraper automatically dumps when the driver steps off. This form of the scraper may or may not have a tongue, but is always provided with curved pieces upon which the scraper runs in dumping and returning. The efficiency of the scraper decreases with the steepness of the slope up which the earth is hauled; but is quite economical on slopes of 1 to 4 or less.

The buck scraper is much used in the West in building levees and irrigation ditches and canals, and is usually drawn by four horses. In California in placing 364 000 cu yd in a levee 12 ft high and 6 ft wide on top and 90 ft wide at the base, from borrow pits 100 ft wide on each side, a buck scraper moved on the average 89.5 cu yd per day, about 90% of the material being drifted up a 1 to 4 slope, the cost being 9.6 cts per cu yd when the levee was being started and 11.4 as the bank became higher; and in excavating an irrigation canal in a side hill, a buck scraper making a total round trip of 400 ft horizontally and 40 ft vertically, averaged 1.3 cu yd per load and 128 cu yd per day, and the cost was 6.2 cts per cu yd, the material being favorable and most of it being simply pushed down a steep slope (Eng. News, 1885, Vol. 14, p. 115).

The Flat-Bottomed Tongue scraper is an iron-shod wood or solid-metal scoop employed in leveling off the bottom of an excavation, particularly in pavement and macadam road construction; and is made in two sizes, 36 and 48 inches wide, which cost about $6 and $7 at the factory.

The Wheel Scraper is a common and exceedingly valuable machine for moving earth, and consists of a steel box mounted on wheels and furnished with levers for raising, lowering, and dumping. All of the movements may be made without stopping the team. There are two forms, the two-wheel and the four-wheel. The former has long been in use; the latter was first made in 1909, and is rapidly coming into use.

The *two-wheel scraper* is made in three sizes, Nos. 1, 2, and 3, having a rated capacity of 9, 12, and 16 cu ft, respectively. Some manufacturers make an automatic front end-gate which adds materially to the load the scraper will carry, particularly on a rough or down-hill road. The two-wheel scraper is drawn by two horses; but it is usually necessary with the larger sizes to hitch another team, called a SNATCH TEAM, ahead to aid in loading. One man, and in tough soil two, in addition to the driver is required in loading the largest scraper.

Except under the most favorable conditions the wheel scraper is not entirely filled owing to the difficulty of forcing the earth to the back of the bowl, and before the load has gone far considerable earth is lost from the front of the bowl; and hence it is not usually safe to count upon the scraper placing in the dump more than half its nominal capacity. Sometimes when the haul is long, the bowl is filled heaping full by men with shovels as the scraper leaves the pit. The smallest wheel scraper is used when the haul is short and the rise is steep; and is usually more economical than the drag-scoop scraper, but where there are many stones the latter is the better.

According to Gillette (Handbook of Cost Data, p. 87) when the wages of common

labor are 15 cts per hour and that of team and driver 35 cts, the cost of moving average earth with a two-wheel scraper is as follows:

The fixt cost for the three sizes of scrapers is:

Items.	No. 1	No. 2	No. 3
Plowing, cents per cu yd	1.7	1.7	1.7
Man for loading scrapers	0.8	0.8	1.5
Time lost by team in loading and dumping	1.5	1.2	0.8
Extra travel of team in turning	1.5	1.2	0.7
Snatch team loading	...	1.5	1.5
Man dumping	...	...	0.8
Total fixt cost, cents per cu yd	5.5	6.4	7.0
Average load hauled, cu yd	1/5	1/4	4/10

"To the above fixt cost add the following for each 100 ft of lead (the distance from the center of the cut to the center of the fill): No. 1, 2.75 cts per cu yd; No. 2, 2½ cts; and No. 3, 1⅜ cts. The cost of foreman and repair of scrapers will usually add 1 ct per cu yd more. For example, the cost with a No. 2 scraper and a 25-ft lead is: $6.4 + \frac{1}{4} \times 2.5 = 7$ cts per cu yd."

In constructing the Chicago Sanitary Canal, the average output for 300 000 cu yd having a cut ranging from 4 to 8 ft deep, a vertical lift of 10 to 20 ft and a horizontal haul of 400 ft, ranged from 39 to 50 cu yd per scraper per 10 hours, and from 27 to 35 cu yd per team; and on another section the average output per 10 hours was 46 cu yd per scraper, and 30 cu yd per team.

The *four-wheel scraper* is a steel box or scoop suspended from a frame supported upon four wheels, Fig. 50. It is made in two sizes, a half yard and a

Fig. 50. Four-wheel Scraper

yard capacity. It is usually loaded by either a traction or a hoisting engine. The larger size usually requires a 20 H.P. steam engine or a 30–60 H.P. gasoline tractor to load it; but it is hauled by a two-horse team. The four-wheel scraper is economical because it is self-loading and self-dumping, and on long hauls also because of the larger load carried. Of course enough scrapers must be employed to keep the tractor busy.

37. Grading Machines

The Scraping Grader is a machine primarily for smoothing a roadway, and incidently for moving small quantities of earth toward the crown. It consists of a cutting blade sliding upon the ground or of a cutting blade adjustable in height and direction supported upon two wheels or suspended from a frame carried by four wheels. In addition to its use in smoothing the trackway and restoring the crown of an earth road, the scraping grader is

used in pavement and macadam-road construction in smoothing up the surface of an excavation, and also in backfilling trenches.

The Elevating Grader consists of a frame resting upon four wheels, from which are suspended a plow and a frame carrying a wide traveling belt. The plow loosens the soil and throws it upon the traveling inclined belt, which delivers it upon the embankment direct or into wagons. The carrier is built in sections and its height is adjustable. The larger carrier will deliver earth 14, 17, 19, or 20 feet horizontally and 8 ft vertically from the plow; while the smaller size delivers 14 and 17 ft horizontally and 7 ft vertically. The smaller machine is designed for highway work. The elevating grader is an effective machine for building open ditches, earth embankments, or filling wagons. The large machine is usually propelled by twelve horses, eight in front and four behind, and the smaller by eight in front; but often a traction engine is cheaper than horses. There is upon the market, but not in common use, an elevating grader in which the elevating mechanism is attached directly to a traction engine, the plow being under the middle of the boiler. The elevating grader cannot be used in sand or gravel, since the plow will not throw the material upon the elevator; and it is not suitable for use where there are roots or stones enough to interfere with the work of the plow. The cost of the machine is about $1200; and in ordinary loam it will usually load about 50 cu yd per hour into wagons, and will place 60 to 75 in a road embankment.

The daily cost of operating the elevating grader is as follows:

10 horses at $1.00	$10.00
3 drivers at $1.50	4.50
2 men to operate the grader at $2.00	4.00
Rent of machine (say)	5.00
Total cost per day	$23.50

If the output is 500 cu yd, the cost is 4.7 cts per cu yd. It is more economical to pull the grader with a traction engine than with horses or mules, provided an engine can be rented for $7.50 per day or less, since ordinarily an engine runner will cost $3.00, coal $2.00, and attendance $2.00 per day; and usually the engine can do more work than 10 horses, particularly in warm weather.

The Shuart Grader consists of a scraping steel blade attached to a frame borne upon four low wheels, having at each side a guard that enables the blade to push considerable earth along in front of it. The height of the blade may be adjusted by levers. It is used in the West in preparing the ground for irrigation. If the guards are removed, this machine can be used as a grader, that is, to smooth off a surface and to push small quantities of earth horizontally sidewise.

38. Loading and Hauling

Wheelbarrows are never economical where teams can be used. A man will pick and load into a wheelbarrow 1 cu yd of ordinary loam per hour. A wheelbarrow load will make about 1/15 cu yd in the settled embankment. The time consumed in going and returning is about 1⅓ minutes per 100 ft, since the load is usually taken up a rather steep grade; or say 20 minutes per cu yd per 100 ft. For each load the time lost in dumping, fixing runway, and changing the position of the barrow is ½ minute per load, or 7½ minutes per cu yd. If wages are 15 cts per hour, the fixt cost of loading and lost time is practically 17 cts per cu yd; and the cost of transporting is 5 cts per cu yd per 100 ft.

Carts drawn by one horse are used in some parts of the country for transporting earth. They are not economical except where the haul is so short

that a driver can tend two carts by taking one to the dump while the other is being loaded. The loading is usually done by four men casting in at the back end of the cart. The chief advantage of the cart is the ease with which it is dumped, especially into hoppers or scows; but usually either the wheeled scraper or the wagon is preferred. A load is ½ cu yd for level hauls, and ¼ for steep ascents, or say ⅓ cu yd per load on the average; and the speed is 200 ft per minute, or 3 minutes to transport 1 cu yd 100 ft. It requires about 3 minutes for 4 men to load a cart, and about 1 minute to dump it; or the lost time is 4 minutes per load, equivalent to 12 minutes per cu yd. If wages are 15 cts per hour for a driver and $1.00 per day for a horse and cart, and if it is assumed that there is a driver for each cart, the fixt cost according to the kind of soil is 10 to 15 cts per cu yd for loading plus 5 cts for lost time of cart and driver, a total of 15 to 20 cts per cu yd; and the cost of transportation is 1¼ cts per cu yd per 100 ft. If it is assumed that a driver takes care of two carts, the fixt cost is 10 to 15 cts for loading plus 3½ cts lost time, or a total of 13½ to 18½ cts per cu yd; and the cost of transportation is practically 0.8 ct per cu yd per 100 ft.

Wagons. There are three general forms of wagons made specially to transport earth: the slat-bottom, the drop-bottom, and the end-dump wagon. The ordinary farm wagon is unsuitable for the purpose, because the box or bed is so light as soon to be knocked to pieces by being struck with the shovels, and also because the load must be shoveled out of it. A SLAT-BOTTOM WAGON-BOX on an ordinary farm wagon is much used; but it is objectionable since at best considerable time is required to dump it, and if it is provided with end gates still more time is required. The slat-bottom wagon-box with end gates holds 1 to 1½ cu yd according to the length and the depth of the sides, and has been found advantageous on small jobs because of its availability.

Of the DROP-BOTTOM WAGONS there are two forms: a box which is placed upon the running gears of an ordinary wagon, and a wagon made especially for hauling earth. In each the bottom of the box consists of two doors, usually hinged at the sides, which drop down to discharge the load. The doors are kept in place by chains which are wound around a spool by means of a lever operated by the driver; and when the chains are released the load drops. The load can be dumped almost instantly by the driver without stopping the team, and the bottom can be closed while the wagon is returning for another load. The drop-bottom box holds 1½ to 2 cu yd of loose earth, and the drop-bottom wagon 1 to 5 cu yd, although 1½ to 2½ are the most common. The larger sizes of drop-bottom wagons are made of steel, and are drawn by three horses and sometimes by four. The drop-bottom wagon is better than the drop-bottom box on ordinary running gears, since (1) it has larger capacity, (2) there is no coupling pole to interfere with the dumping device, (3) the front wheels go under the bed, thus greatly facilitating the turning of the wagon, and (4) it has wider tires and hence draws easier.

The END-DUMP wagon is used only where it is necessary to dump into hoppers, or onto barges, or into railroad cars. This form of wagon is too heavy and too high for use in ordinary earthwork construction.

Loading Wagons by Hand. If the haul is comparatively short, a wagon is left to be loaded while the team takes another to dump, whereby the team and driver are kept busy, and besides this gives an opportunity to fix the responsibility for the amount of output. The number of shovelers and extra wagons will depend upon the length of haul. If no extra wagons are used, and the haul is short, it is important that the team be kept on the road as much as possible, which requires that as many men as possible should be employed in loading. If the haul is extremely long, it is not important that the wagon be loaded quickly, since the team then needs a little rest. Ordinarily ten men, four on each side and two at the rear, are as many as can profitably be used. The height of the wagon materially affects the cost of loading by hand.

The top of the box of a dump wagon is about 4½ or 5 ft high, and at this height a man can load 15 cu yd per day; but for greater heights the amount will be decreased about 10% for each 6 inches of additional height. The loading of wagons can be greatly facilitated by first excavating narrow cuts 5 or 6 ft deep, and then placing the wagons in these trenches while being loaded. The earth at the sides of the trenches can be shoveled into the wagons much more rapidly than if the men stood on the same level as the wagon. As the sides are shoveled off, the trench can be deepened. The slopes of the material at the sides of the wagon should not be steeper than about 2 to 1, or it will be difficult to operate the plow on them or for the men to stand on them. If the wagon must be loaded from only one side, and particularly if the haul is long, an extra side board should be placed upon the opposite side of the box to increase the size of the load. The cost with a 1-yard slat-bottom wagon and two-horse team, assuming labor at 15 cts per hour and team and driver at 35 cts, is: Loosening and loading 12 cts per cu yd; time of team and driver in pit 5 minutes, or 3 cts per cu yd; time dumping 2 minutes, or for the time of team, driver, and helper say 2 cts per cu yd; or a total fixt cost of 17 cts per cu yd. If the speed of the team is 2½ miles per hour, this is equivalent to a cost of ½ cent per cu yd per 100 ft for transportation. If the capacity of the wagon is more or less than 1 cu yd, the fixt cost per cu yd need not vary, while cost of transportation will vary inversely proportionally.

Loading Wagons with Scrapers. An inclined wood runway is built high enough to allow the wagon to stand under it, the scrapers are driven loaded up one side of the incline, dumped at the top thru a hole in the floor, and then driven empty down the other side. This device, called a TRAP, is frequently employed. The incline need not be wider than 8 ft, and should not be steeper than 1 to 4 or 5; and its height can be materially diminished by excavating a trench into which to drive the wagons. For wheel scrapers the hole in the top may be 3 ft wide and 4 ft long, and for a slip a little narrower, and for a Fresno a little wider. With drag-scoop scrapers a cleat should be nailed at the front edge of the hole to catch the edge of the scraper in dumping. With the data in the preceding portion of this article, it is easy to compute the cost of loading wagons with scrapers. On account of the cost of the incline the method is not economical unless there is a considerable quantity of earth to be moved; and on account of the time and cost to change the incline, it is not economical unless there is also a considerable depth of earth to be removed.

Loading Wagons with an Elevating Grader. Drop-bottom dump wagons are driven under the discharge end of the elevator, and travel along with the grader until filled, and then the grader is stopt a few seconds while the loaded wagon is driven out and an empty wagon is driven in. Assuming that 2-yd wagons are used and that the average load is 1½ cu yd loose or 1¼ cu yd compacted, and assuming that the wagon is drawn by three horses, and assuming a 3-horse team driver and wagon to cost 50 cts per hour, and also assuming that wagons would not be used for a lead (distance from center of cut to center of fill) of less than 200 ft and that a wagon travels 500 ft for a lead of 200 ft, the fixt cost per cu yd is as follows:

Services of grader, as previously stated	4.7 cts
Time of wagon and driver while loading	1.5 "
Extra travel in loading and turning	2.5 "
Time lost in dumping and waiting	1.0 "
Man on dump	1.0 "
Total fixt cost per cu yd	10.7 cts

The cost of transportation is 0.6 ct per cu yd for each 100 ft of lead more than 200 ft.

Wagons are also loaded with a steam shovel, in which case the earth should preferably be first dumped into a hopper under which the wagon is driven to load, as otherwise there is time lost in centering the wagon under the shovel and there is much dribbling around the wagon, and besides the shovel is likely to strike and damage the wagon. The hopper should rest upon a trestle which is mounted upon wheels so that it may move forward as the shovel advances. Wagons are also loaded with an orange-peel excavator or grab bucket swung from a derrick.

39. Steam-Shovel Work

This is the most economical method for loosening and loading earth, provided the quantity of earth to be moved will justify the expense of installation, and provided a considerable depth is to be removed. A steam shovel, called a NAVVY in England, consists essentially of a bucket or dipper attached to a dipper handle which is supported by a boom and which can revolve thru a full semicircle and in some types a complete circle. The boom is operated thru a system of chains and sheaves by an engine and winding drum located on a car or platform which also carries the steam boiler and all other operating machinery. The car or platform upon which the machinery is mounted usually runs upon a railroad track, ordinarily standard gage, and is sometimes self-propelling; but the smaller sizes are sometimes mounted upon wheels similar to those of a traction engine, and are called TRACTION SHOVELS, and are self-propelling. With most forms at least two men are required to operate a steam shovel; one, called a cranesman, to manipulate the dipper; and one, called the engineer, to run the engine and operate the winding drum. The size of a steam shovel is stated by the capacity of the bucket or by the total weight of the entire machine. In a very rough way, the total weight is about 30 tons for each cubic yard of capacity of the bucket. The cost of a steam shovel ranges from $100 to $125 per ton. The smaller the shovel the greater the cost per ton; and the more the total weight the greater the weight per cubic yard of bucket. The capacity and reach of steam shovels differ greatly. The rated capacity of the bucket varies from ½ to 5 cu yd, but those having a nominal capacity of 1½ to 2½ cu yd are most common. The width of cut varies from 15 ft for the smallest to 40 ft for the largest. The maximum height of the cutting edge of the bucket is usually limited to 12 or 15 ft above the track upon which the car runs; but a steam shovel by undermining can dig down banks much higher, and not infrequently works on faces 20 to 25 ft high, and in loose soil even higher. The smallest sizes of steam shovels can operate with reasonable economy, as compared with other methods, on a face only 1½ or 2 ft high, in which case a traction shovel would be used, altho of course a proportionally large amount of time is lost in moving the shovel forward.

A steam shovel will excavate almost any material except solid rock; and will load rock that has been broken into pieces small enough to go into the bucket. The output of a steam shovel is usually limited by the arrangements for carrying away the excavated material. The earth is usually taken away (1) in large drop-bottom wagons, or (2) by dump cars holding from 1½ to 5 cu yd running upon a light track and being drawn by horses or small locomotives, or "dinky engines," or (3) by full-sized locomotives drawing ordinary flat or gondola cars or large dump cars.

For general specifications for a steam shovel for railroad work, see the Manual of Recommended Practise of the American Railway Engineering and Maintenance of Way Association, 1907, pp. 30–32; and for the method of handling steam-shovel work and a description of the equipment to be employed, see same, pp. 32–37.

On the Chicago Sanitary Canal between 60 and 70 steam shovels from six different manufacturers were employed, and the average output of a 2-yd bucket was approximately as follows: In stiff clay from 50 to 70 cu yd per

hour; in very hard clay, thickly filled with bowlders and requiring blasting, from 25 to 35 cu yd per hour for the hardest, and from 30 to 40 cu yd per hour for the medium material; in cemented gravel, requiring blasting, from 25 to 50 cu yd, depending on the hardness; and in handling blasted rock, from 25 to 30 cu yd per hour. The minimum daily cost of operating a 2-yd shovel is about as follows:

One foreman	$4.00	
One engineman	4.00	
One cranesman	3.50	
One fireman	2.00	
Four pitmen at $1.50	6.00	
Total for wages of crew		$19.50
One ton coal	$3.00	
Oil and waste	.75	
Water (1 gallon per lb of coal)	.50	
Total for fuel and supplies		$ 4.25
Interest on capital, $6000 at 6%	$1.00	
Depreciation at 10%	2.00	
Repairs	1.00	
Total for plant		$ 4.00
Total daily expense		$27.75

In average earth the average output of the above shovel would be about 500 cu yd per 10 hr, equivalent to a cost of $27.75/500 = 5.6 cts per cu yd, which should be regarded as the minimum cost. If the water must be pumped by hand and be hauled by wagon, the cost of this item may be four to six times that given above. In hard or stony earth the cost of repairs may be three or four times that stated. Again, the interest charge above assumes that the shovel works every day of the year; and if it works only part time, as is very probable, the interest charge should be proportionally increased. Finally, it may be noted that the above statement does not include the cost of installing and of removing the shovel.

The following, from Bulletin No. 81 (1906) of the American Railway Engineering and Maintenance of Way Association, p. 31–50, are the particulars of moving 251 711 cu yd of wet clay with a 2½-yd steam shovel and dump cars by a railway company's forces, the material excavated being finally dumped from a temporary trestle.

The shovel was on the job from April 27 to November 2, a total of 190 days; and worked night shifts from June 20 to October 2, a total of 129 nights. The shovel actually worked 140 10-hr shifts by day and 88 at night, a total of 228 shifts. The time lost by shovel was: 57 shifts due to Sundays and rains, 23 shifts due to moving shovel and shovel failures, 11 waiting for grading of temporary track, a total of 25% lost time. The output for day shifts was 160 121 cu yd, night shifts 91 590, total 251 711 cu yd, "based upon cross-section measurement." The average output per day shift was 1144 cu yd, per night shift 1040 cu yd, the latter being 92% of the former. The average load per car was 3.35 cu yd. The average length of haul was 1½ miles. Average depth of cut 15 ft. The temporary trestle was 2961 ft long, and average height 40 ft. The cost of the earthwork was:

Equipment:	Total	Cu Yd
1 second-hand 65-ton shovel, depreciation 10% of $5000	$500.00	$0.002
2 second-hand 30-ton locomotives, 5% of $4400	220.00	.001
43 second-hand 5-yd cars, 10% of $5052.50	505.25	.002
1 Jordan spreader, 5% of $1800	90.00	.000
Bunk houses, material and labor	1144.86	.004
Water supply, material and labor	272.60	.001
Total for equipment	$2732.71	$0.011

Steam-Shovel Service:	Total	Cu Yd
Steam-shovel service	$6 228.54	.024
Engine service	6 417.33	.026
Repairs	771.47	.003
Lighting	184.62	.001
Dumping	4 265.51	.017
Total for steam-shovel service	$7 867.47	$0.071
Supplies:		
1040 tons coal for shovel at $1.48	$1 539.20	$ .006
1339 tons coal for locomotive at $1.48	1 987.70	.008
Other supplies	962.62	.004
Total for supplies	$4 483.52	$0.018
Trestle (average height 40 ft):		
Material (per lin ft = $1.74) and labor (per lin. ft = $1.30)	$9 007.80	$0.036
Track Work:		
Labor	$11 582.31	$ .046
Supplies, depreciation and labor upon	856.11	.003
Total for track work	$12 438.42	$0.049
Supervision and engineering	$ 610.38	$ .002
Total cost	$47 140.30	$0.187

The above work was unusually expensive because the clay was very wet which made the dumping expensive, and also because the cars were too small. The above cost was almost exactly the same as that for another cut on the same division of the same road in which 39% of the material was hardpan and 61% clay.

40. Shrinkage and Settlement

Shrinkage and Settlement. In all operations involving the moving of earth, it should not be forgotten that the act of excavation so breaks up the earth that it occupies more space after excavation than before; but when the material has been placed in an embankment, it will usually occupy less space than in its original position. The expansion due to excavation is usually 8 to 12% of the volume, and in extreme cases may be 40%; but in placing the material in the embankment, it is compacted by the weight of the embankment itself, by the pounding of the hoofs, and by the action of the wheels, until usually the final volume is less than the original. Ordinary earth in its original position is more or less porous owing to its soluble portions having been carried away by the percolating water, to the penetration of vegetable roots which subsequently decay, and to the continued action of frost. There is usually also more or less earth lost in transporting it from cut to fill. The amount of shrinkage depends chiefly upon the character of the material and the means by which it is put into the embankment, and somewhat upon the moisture of the soil, the rainfall conditions while the work is in progress and soon afterwards, and the depth to which frost usually penetrates. If the soil is moist when placed in the bank, it will become more compact than if it were dry. Rain greatly affects the shrinkage; and embankments put up during a rainy season will be more compact than those built during a dry time. Soil from above the usual frost line is more porous than that not subject to the heaving effect of alternating freezing and thawing, and consequently shrinks more when put into an embankment. The shrinkage of the ordinary soils is in the following order: (1) sand and sandy gravel least, (2) clay and clayey soil intermediate; and (3) loams most. The shrinkage

according to the method of handling is in the following order beginning with the greatest: (1) wheelbarrows, (2) cars, (3) wagons, (4) wheel scrapers, and (5) drag scrapers. The usual allowance for shrinkage for drag scraper work is as follows: gravel 8 %, gravel and sand 9 %, clay and clayey earth 10 %, loam and light sandy earth 12 %, loose vegetable surface soil 15 %. The above results are for ordinary earth, and do not apply to such unusual materials as "buck-shot," gumbo, very fibrous soil. etc., which have a much greater shrinkage. Solid rock will expand 40 to 50 %.

The shrinkage of earthwork referred to above takes place chiefly during construction; but the continued action of the weight of the embankment and the effect of rain and traffic will usually cause a comparatively small settlement after completion. Sand or gravel embankments built with wheel scrapers will usually settle 1 to 2% after completion, and clay or loam embankments about 2 to 3%. With drag scrapers the settlement will usually be a little less than the above; and with dump carts or wagons a little more. With wheelbarrows the settlement is usually about 10 %, but may be as much as 25 %. The settlement of steam-shovel work depends upon the method of dumping, the length of time the work is in progress, the season, and the soil. The Manual of Recommended Practice of the American Railway and Maintenance of Way Association recommends the following allowance for the shrinkage of a green embankment built by steam shovel and cars: for black earth, trestle filling 15%, raising under traffic 5%; clay, trestle filling 10%, raising under traffic 5%; sand, trestle filling 6%, raising under traffic 5%.

41. Levee Construction

A Levee is a bank of earth thrown up along the side of a stream to prevent the overflow of the adjoining land. The cross section of a levee depends upon the material of which it is constructed and the length of time it will be subjected to the flood. To prevent the levee from being overtopped by waves, the crown should be 3 ft above the highest water, if along a stream of any considerable width. A levee usually has a top width of 3 to 8 ft, a slope on the water side of 1 to 3 or 4, and a slope on the land side of 1 to 2 or 3. If the material of the levee is very light, the slopes should be flatter than the above. The above slopes are easily constructed, are readily kept clear of weeds and brush by the use of a mowing machine, do not slide when tramped over by stock when wet, are not difficult to get set in grass, and resist wave action reasonably well. It is important to avoid sharp corners, and flat curves should be used even if arable land is thereby lost; and on the curves the levee should be thicker and have flatter slopes than on the straight portions.

The standard levee of the U. S. Mississippi River Commission is shown in Fig. 51. The banquette (reinforcing bank) is constructed where the founda-

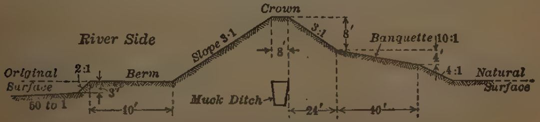


Fig. 51. Standard Levee of U. S. Mississippi River Commission

tion is weak and where the height is more than 10 or 12 ft. The chief object of the banquette is to keep the levee from sloughing off when saturated by water, which it does by keeping the surface of saturation well within the embankment.

After the alinement is fixt, the surface should be freed from trees, logs, brush, and débris; and all buried logs, stumps, and roots of any considerable size should be removed. For the Mississippi river levee all logs, stumps, and roots are removed for a depth of 8 ft

and for a width extending 5 ft each side of the base of the levee. After clearing the foundation the ground is plowed deeply, and any unsuitable soil is removed. If the soil is not reasonably uniform, the foundation is tested by sinking pits at various points, and any defects are remedied as far as practicable. A muck ditch is usually cut on the river side of the center line, to explore the foundation and to aid in preventing the seepage of water along the plane of the base of the levee. It should extend thru the porous surface soil; and should be filled with the best soil available, preferably clay or clayed gravel, which is thoroly tamped into place. The standard dimensions of the muck ditch of the Mississippi river levees are 12 ft wide on top, 8 ft wide at the bottom, and 8 ft deep, these dimensions being selected so that the muck ditch can be dug and filled with wheel scrapers. The width of the muck ditch may vary with the quality of the soil used for filling; if pure clay is employed, the width need not be more than 1 or 2 ft except for the highest levees.

The levee is constructed in layers usually with slips or wheel scrapers, not more than 2 ft thick extending over the entire base. Only unfrozen earth free from sticks, leaves, and straw should be used. The earth is taken when practicable from borrow pits on the stream side of the levee. For the Mississippi river levees the borrow pits on the river side must be at least 40 ft from the toe of the levee, the side slope of the pit must not be steeper than 1 on 2, the depth next to the levee must not exceed 3 ft, the bottom of the pit must have a uniform slope toward the river, the depth on the farther side must not exceed 6 ft, and traverses (tongues of undisturbed earth) must be left at intervals of 300 to 400 ft. The traverses are to prevent destructive currents along the base of the levee; and should be pierced at the river end by drainage ditches, since standing water affords refuge for crawfish, muskrats, etc., which endanger the levee. The berm between the borrow pit and the levee should be kept intact, and any depressions in it should be filled to the level of the natural surface. Ditches and borrow pits on the inside are very objectionable; and if the earth is borrowed from the land side, the pit should be well back from the levee. On the Mississippi the distance from the toe of the levee to the edge of a borrow pit on the land side must be at least 100 ft.

When the levee has been carried to the proper height and the crown and side slopes have been drest to planes, tufts of Bermuda grass 2 to 4 inches square are planted at intervals of about 2 ft, which soon spread over the entire embankment and effectually protect it from the wash of the rain and to a considerable extent from the wash of waves and the current. Weeds and bushes should not be permitted to grow on levees, since weeds die and blow over, thereby loosening the soil, and bushes by shading the ground kill the grass and their roots loosen the earth. The injurious effect of wave action is greatly increased by the presence of weeds and bushes, as their roots break the surface and cause erosion to begin. Large trees should not be permitted to grow near the levee, as their roots will penetrate the base of the levee and give an opportunity for seepage; and when the tree is cut or dies its roots will decay and leave large openings which may cause a crevasse.

Levees are usually built in the East with drag-scoop or wheel scrapers and on the Pacific slope with buck or Fresno scrapers, altho in both localities elevating graders and dump wagons are used for the larger levees. Elevating graders discharging directly upon the embankment have been employed for small levees and for portions of large ones, but are not usually satisfactory since with them the borrow pits are too close to the base of the levee. A steam shovel on a barge has been tried, but is satisfactory only for small levees, since either the boom must be unmanageably long or the borrow pit must be too near the levee. Besides, the floating steam shovel takes up such large quantities of water with the earth that the material will not stand at the desired slope, and consequently it is usually necessary to go over the work a second time; and the same difficulty frequently occurs with a steam shovel on a track, and it is practically impossible to go over the work a second time with this machine. A steam shovel on a track along the berm gives fairly good results when the ground will support the track. Hydraulic and also

elevator dredges have been tried, but usually without success. The drag-line bucket has recently been used with great success in building levees. It may be operated from a boom or a cableway (see p. 658).

Drains are sometimes constructed thru levees to carry away storm water during a low stage of the stream, but such openings are always a menace to the stability of the levee. If used at all, they should have substantial and adequate walls of concrete at both ends and the barrel should be or iron or vitrified pipe so laid that the joints will not leak. The lower end should be provided with a substantial automatic gate, usually a flap valve, for closing at the time of high water in the stream; and the inner end should be provided with a sliding gate to be moved by hand, for use in case the outer valve fails to work at the time of high water.

Roads along or across the top of levees are objectionable since ruts and chuck holes cause low places, and the edges of the crown are cut off by the wheels. If the crown is to be used as a roadway the levee should be given an additional width, and should be carefully and frequently inspected. Further, water collects in ruts and flowing down the slope does considerable damage.

For a Bibliography of Levees see Report of the Flood Commission of Pittsburg, Pennsylvania, 1911, p. 403-05; and for references to later articles see indexes to current engineering literature.

42. Dredges and Dredging

Classification. Dredges may be classified according to the method of handling the material, as (1) dipper dredges, (2) elevator or ladder dredges, (3) hydraulic or suction dredges, (4) the grapple or grab-bucket dredges, and (5) scraper or drag-line bucket dredges. Dredges may be classified according to the way in which they are moved forward while working, as (1) floating dredges, (2) traction dredges, (3) roller dredges, (4) drag dredges, and (5) walking dredges, of which the last four are used chiefly in ditch work. In harbor work the dredge usually discharges into a scow moored alongside; but sometimes the barge carrying the dredging machinery is provided with pockets large enough to hold several hours' output, and when these are filled the dredge steams out to sea and dumps its load. The latter type of dredge is called a SEA-GOING or HOPPER DREDGE; and is used only where the water is too rough to fasten scows alongside of the dredge.

A Dipper Dredge is simply a steam shovel (see Art. 39, page 652) mounted upon a scow; and differs from an ordinary steam shovel chiefly in having a longer dipper-handle and a longer boom. In extreme cases, for ditch and levee work, the dipper-handle is 50 ft and the boom 75 ft. The dipper ranges in size from ½ to 5 cu yd. It is better adapted to all kinds of work than any of the other types, since it can handle any material any other type can, and it can excavate material too tough for any of the other forms; but when large quantities of earth are to be moved and when other conditions are favorable, some of the other types are more economical. Dipper dredges are much used for harbor and ditch work.

Usually in dredging the conditions are more favorable for a large output than in dry excavation. On the Chicago Sanitary Canal a 2-yd dipper dredge in soft clay averaged 700 cu yd, place measurement, per 10-hr shift for 6 months, in comparison with an output of about 500 cu yd for dry excavation under nearly similar conditions, and may be considered fairly representative of good work. On the same work in hard clay the average for five 2-yd dipper dredges for five months was 636 cu yd, place measurement, per 10-hr shift.

The Elevator Dredge, also called a ladder dredge and a chain-and-bucket dredge, consists of a series of scraper buckets attached to a chain, which scoop up the material and deliver it at the top of the ladder, where it is dis-

charged into a chute or onto a belt conveyor. The buckets have capacities of 3 to 15 cu ft, are spaced from 3 to 6 ft apart, and travel at a speed of 40 to 60 ft per minute. The first machine of this type was made in France in 1859 for use on the Suez Canal. This type is more common in Europe than in the United States; but is rapidly gaining favor here. For certain kinds of work where the material is free from roots and stones this machine is rapid and economical of power. It can work in deeper water than the dipper dredge, and is more economical than the grapple or drag-line excavator; but is more complicated and more likely to get out of repair. The elevator principle is used chiefly in marine dredges, but has been employed in dry work, particularly for trenches, and also in gold dredging (Eng. News, 1913, vol. 95, p. 1079).

Suction Dredge. The essential feature of the suction or hydraulic dredge is a centrifugal pump which draws water and suspended earth thru a suction pipe and discharges them thru a line of pipe floated on buoys or pontoons. With light alluvium or fine sand, a reasonable amount of solid material is drawn up with the water, if the lower end of the suction pipe is nearly in contact with the soil; but in any other soil some form of agitator must be employed to loosen and stir up the material to be excavated. With medium soils the material can be stirred up sufficiently with one or more water jets; but in tough material a mechanical agitator, usually in the form of a hollow rotary cutter, is required. The agitator and the suction pipe are swung from side to side, and the whole dredge is moved forward for each sweep of the suction pipe. Sometimes the whole boat is swung from side to side by means of lines anchored at the sides. Under favorable conditions the proportion of earth to water may be 40 %, but 10 to 15 % is the usual and more economical proportion. The discharge pipe is often one thousand feet long, and sometimes five or six thousand.

The suction dredge seems to have been developed in San Francisco, Cal., about 1880 to 1885; but has since been widely used elsewhere. It is the most economical machine for excavating material that is easily displaced and conveying it considerable distances. It is well adapted to filling tidal flats and low lands that are surrounded by an embankment. Apparently the largest dredges of this class are those used by the U. S. Mississippi Commission in removing bars from the river. In some of the larger of these boats the runner of the centrifugal pump is 6 to 7 ft in diameter, the discharge pipe is 30 to 34 inches in diameter, the velocity of discharge is 10 to 14 ft per second, and the capacity is 1000 to 2000 cu yd per hour thru 1000 ft of pipe at a cost of 1 cent per cu yd. For details of these dredges see Trans. Am. Soc. C. E., 1898, Vol. 40, p. 215.

In the Chicago Sanitary Canal two suction dredges having a 6-ft centrifugal pump driven by a 250-hp engine excavated 1 500 000 cu yd of soft black loam and discharged it thru an 18-in pipe, the average per 10-hr shift being 1732 cu yd and the maximum 446 cu yd per hr (Engineering News, vol. 32 (1894), p. 190). In Oakland, California, Harbor a hydraulic dredge averaged 30 000 cu yd per month for eight months, the delivery pipe being 1100 ft long with a lift of 20 ft, at a cost of 10 cts per cu yd. (Trans. Amer. Soc. C. E., vol. 13 (1884).)

Specially designed suction dredges were used on the new Barge Canal of New York State; for description and costs of operation, see Eng. News, 1913, vol. 69, p. 710.

A Grapple or Grab-Bucket Dredge consists of a self-filling bucket suspended from a swinging or rotary boom; and is often called a CLAM-SHELL DREDGE from the former form of the two halves of the bucket. At present there are only two forms of bucket in use, the orange-peel and the grab-bucket. The former consists of three or more curved triangular spades which when closed form a hemisphere; and the latter consists of two quadrants of a cylinder which when closed form half of a short cylinder. These buckets are suspended by two chains or wire cables, one to close the bucket in loading it and one

to open it in discharging. Both forms of buckets are made of rated capacities varying from ½ to 10 cu yd loose measurement. The great advantage of the grapple dredge over the dipper form is the greater length of boom permissible. A grapple dredge has done excellent work with a boom 120 ft long. A long boom is sometimes of great advantage in ditch or levee work. The orange-peel bucket fills itself only in comparatively soft ground; and consequently the grab-bucket must be employed in hard or tough ground, for which work it is sometimes provided with teeth on the cutting edge and with extra power for closing. This form of dredge is peculiarly adapted to very deep dredging or to work in confined places like the inside of a cofferdam or even a pier cylinder. The grapple excavator is sometimes economical in excavating trenches; and can be used to load wagons or cars in dry excavation, and is particularly advantageous in cellar excavation, since the wagons can remain on the natural surface. In good material it can deliver a bucket load every minute. A 10-yd 15-ton 2-part bucket working in water 65 ft deep with 10 men and 5 tons of coal averaged 4000 cu yd in 10 hours, and in another case 2300 cu yd in 10 hours (Engineering News, 1899, vol. xli, p. 66).

A Drag-Line Bucket Dredge is a form in which material is handled with a scoop roughly resembling the ordinary drag-scoop scraper suspended from a swinging boom, the scoop being drawn toward the machine by a line attached to the front and a second line at the rear holding it at the proper angle to slice the earth away as it is moved forward. When the scoop is filled, it is lifted to the point of the boom, both lines being kept taut, and is then swung around; when on slacking the drag or hauling line, the scoop dumps automatically. The bucket or scoop is lighter than an orange-peel or grab bucket of equal size, and hence the whole machine may be lighter. It will excavate either soft or hard material. There are several slightly different forms of bucket on the market. An important advantage of the drag-line excavator is the wide reach possible by the use of a long boom. The drag-line excavator is at present the favorite machine for widening and deepening river channels, and for building levees and earth dams.

The engineers of the Miami Conservancy District, Dayton, Ohio, under the direction of Arthur E. Morgan, Chief Engineer, made an elaborate investigation of different earth-excavating machines, and adopted the drag-line excavator for that mammoth work, and have already (1919) had extensive experience in the use of this machine in excavating channels, and in building levees and earth dams. The District has nineteen drag-line excavators at work, ranging from a 40-ft boom and a 1½-yd bucket to a 160-ft boom and a 5-yd bucket. The following are the advantages, as stated by C. A. Bock, Division Engineer, which lead to the adoption of the drag-line excavator on this work almost to the exclusion of other devices.

Advantages. 1. The shape of the bucket and the method of loading make it possible to excavate boulders and also to pull stumps and large roots. The capacity of the buckets in use range from ¾ to 5 cu yd. Clam-shell and orange-peel buckets may be used. 2. The drag-line has greater reach than steam shovels or any type of dredge. The horizontal radius for excavating or dumping may be 20 to 30 ft beyond the end of the boom; and the vertical reach may be as low as 50 ft below the machine bed and as high as 30 ft above. 3. While the drag-line ordinarily digs toward itself, with comparatively simple rigging it may be arranged to dig away from itself. 4. Although primarily a land machine, it is capable of digging under water from a bank or an island or from a barge or scow; and in these respects it surpasses the dipper dredge, and under certain conditions also the suction dredge. 5. It is efficient in loading cars, in some cases even surpassing the steam shovel. 6. The drag-line excavator may often be

used to advantage as a movable revolving derrick for lifting track and equipment in conjunction with its own operation, which frees it from many interruptions to which other types are subject. 7. The lighter drag-line machines may be mounted on caterpillar traction, or may be provided with walking devices, which gives greater speed and more freedom of movement than any other type of excavating machine. 8. The heavier types are usually mounted on skids and rollers or on wheels and track, in which case they have greater range of action and require less track shifting than steam shovels or other types of dredges. 9. The drag-line picks up, moves ahead, and relays, its own track.

Disadvantages. The disadvantages of the drag-line excavator are: 1. In narrow cuts there is not sufficient dragging distance to properly fill the bucket; and hence under this condition the drag-line is not as efficient as the steam-shovel or the dipper dredge. 2. The drag-line ordinarily leaves the bottom of cuts and borrow pits quite rough, although a skillful operator can secure fairly satisfactory results.

Performance. The following data are furnished by C. A. Bock, Division Engineer of the Miami Conversancy District; and are from reports of actual operations, and are representative of the work being accomplished by the District.

On one job of **river improvement** (Engineering News-Record, Vol. 81 (1918), p. 814) in 503 ten-hour shifts under unfavorable conditions and in shallow digging, two electric-driven machines mounted on trucks and track and having 125-ft and 135-ft booms and 3½-yd buckets excavated 360 500 cu yd of gravel and clay from the river channel in 4300 working hours, an average of 84 cu yd per hour. Of the total time, 730 hours or 14.2% were consumed in major repairs and outside delays. Most of this work was done with an average power consumption of 1.00 kwh per cu yd, and the lowest average for any one month was 0.75 kwh. Machines of this type on straight digging will average 40 swings per hour, and will run as high as 60 or 65 under favorable conditions and a swing of not more than 110°. It was found that these machines would climb and work on a 5% grade, and that the bucket could be thrown to excavate 25 ft and to dump 30 ft beyond the end of the boom. They will travel on a curve with a minimum radius of 135 ft, measured from the center of the machine.

On a more favorable job of **river improvement** a similar machine, mounted on skids and rollers and having a 100 ft boom and a 3½-yd bucket, averaged 1400 cu yd per ten-hour shift for twenty-three days of two shifts each, at a power consumption of 0.52 kwh per cubic yard.

The following is the record of a contract job consisting of 58 609 cu yd of **levee embankment.** The work consisted for the most part of raising an old levee 4 to 5 ft, and required the placing of an average of about 530 cu yd per 100 ft of levee. The earth was dug from a shallow borrow pit separated from the toe of the levee slope by a 30-ft berm, and about 15 ft below the top of the levee. The machine was a walking-type steam drag-line excavator with a 70-ft boom and a 2½ yd bucket. This machine had been in service for seven years on other work. The material handled was loam and gravel, the greater part of it being dry. Four men worked 34 days on the erection of the machine. Two hundred and four days elapsed between the start and finish of the actual digging. In this time there were 26 Sundays and holidays on which there was no work, 12 days were used in crossing a railroad embankment and passing under a bridge, and for 41 days the machine was idle due to lack of coal. On 125 days there was actual digging, and on 33 of these days two shifts worked. This made a total of 1580 working hours, 1159 hours on 73% of which were consumed in actual running time. The remaining 421 hours were consumed in delays due to repairs, waiting

for coal delivery, and bad weather. The average digging was 372 cu yd per shift. The greatest amount in any one shift was 1150 cu yd. The coal consumed was 400 tons, or 12.5 lb per cu yd excavated. The usual crew consisted of an operator, fireman, pump-man, two laborers, and a team for hauling coal and supplies. The machine could travel up a maximum grade of 7%, and could excavate, revolve, and place material on this grade.

On **dams** the drag-line excavators have been used in excavating the conduit trenches, building coffer-dams, digging gravel for the screening plant, and placing permanent fill in the dams. The material was wet and dry loam, gravel, clay, and blasted stratified limestone. Due to the irregular nature of this work and the numerous interruptions, only a few performance records have been obtained. In two winter months one steam machine with a 75-ft boom and a 1½-yd bucket handled 33 400 cu yd of gravel and loam in a total of 891 working hours, 12% of the time being used in major repairs and delays. One hundred and seventeen tons of coal were consumed.

During the same time an electric machine with a 100-ft boom and a 4½-yd bucket, loading 12-yd dump cars, handled 27 330 cu yd of blasted rock and gravel in 1005 working hours, with 25% of the time out for delays and major repairs. Another electric machine with a 100-ft boom and a 2½-yd bucket handled 28 500 yd during July, 1918, with a power consumption of 0.85 kwh per cubic yard.

The usual crew on the machines consists of one operator, one oiler or fireman, and three laborers placing the track and spreading the material dumped. Since the output of the excavator depends largely upon the skill and care with which it is handled, the operator is paid a high rate, varying from $175 to $220 per month

43. Ditching Machinery

It has been estimated that there are still in this country something like 100 000 000 acres, or over 150 000 square miles, of swamps and wet lands. This area is about two and a half times as large as all the New England States. A considerable part of the redemption of this vast tract will consist of building levees and dredging main drainage channels. The building of levees and the construction and operation of dredges have been described in Arts. 42 and 43, and the principles of drainage are discussed in Section 17, pages to . It is proposed to briefly consider here some of the machines employed in constructing medium-sized drainage and irrigation ditches.

The **dipper dredge** (see Art. 42) is an important machine in the drainage of swamp or overflowed land. There are four forms of this machine specially adapted to ditch work, viz.: the floating type, the land type, the walking dredge, and the drag boat.

The **floating type** is a steam shovel mounted upon a wooden or steel hull. It may work either up or down stream; but the latter is much the more common, since then dams are not necessary behind the dredge to retain water in which to float it. The capacity of the dipper varies from 0.5 to 5 cu yd. The maximum size of ditch that can be dug economically with this type is about 40 ft wide on the bottom and 10 ft deep; and the minimum about 10 ft wide and 5 ft deep. The cost of dipper-dredge work in 1913 varied from 5 to 10 cents per cu yd, depending chiefly upon the cross-section of the ditch and the size of the job.

The **land type** of dipper dredge consists of a boom, a dipper, and the necessary machinery mounted upon a frame which straddles the ditch and runs upon wheels resting directly on the banks or upon a track. Some machines have caterpillar traction. The capacity of the dipper varies from ½ to 1½ cu yd, and the span from 15 to 50 ft. The land type can be dismantled and moved more easily than the floating form; but in stumpy ground

it is not as good as the latter, since it lacks power to up-root large stumps. The land type is objectionable unless the banks of the ditch are firm. With soft banks the caterpillar traction is better than either broad wheels or a track. The chief advantage of the land dredge is that all of the excavation is in sight, and hence a better ditch can be dug than when the excavation is done under water as with the floating type. The cost of excavation with the land type of dipper dredge in 1912–14 was 8 to 20 cents per cu yd, the greater cost in comparison with the floating dredge being due to the smaller cross-section of the ditches and also to the smaller jobs.

The **walking dredge** straddles the ditch. Its distinguishing characteristic is that each corner of the frame rests upon a built-up foot whose horizontal cross-section is about 6×8 ft, and in the middle of each side of the frame is a similar foot about 6×12 ft. When the machine is to be moved forward, it raises itself and also the four corner feet until it is entirely supported on the two center feet; and then the machine slides itself about 6 ft forward on the middle supports. The corner feet are then lowered, and the weight of the form thrown upon them; and next the center supports are raised, and brought forward ready for another move. The chief advantage of the walking device is that it enables the whole machine to move itself across country without dismantling, thus saving much time and also the cost of dismantling and rebuilding.

The **drag-boat ditching machine** is mounted on a hull whose transverse cross-section is somewhat like that of the ditch to be dug. The whole machine is moved forward by winding up a cable anchored ahead. This type has booms from 25 to 30 ft long, and dippers of ½ to ¾ cu yd capacity; and will excavate a ditch having a bottom width of 4 to 6 ft. It cannot be used if the banks of the ditch will not stand without caving, since that may wedge the hull fast.

The **drag-line excavator** (see Art. 42) is in many ways the most developed type of machine for ditch work. It may follow the center line of the ditch, backing away from the completed work, or it may travel on one side of the ditch. The first method is usual for small ditches, and the second for large ditches; and sometimes for large ditches the machine travels down one side and back on the other, excavating half of the channel each trip. Drag-line excavators have been made with booms 40 to 150 ft long and buckets holding from 1 to 5 cu yd; but the longer booms and larger buckets are not very common. The comparatively wide reach of the drag-line excavator fits it for excavating channels wider than can be dug economically with the floating dipper-dredge, but the cost of ditch work with a drag-line excavator is usually more than with a dipper dredge. The drag line can cut the side slopes of the ditch more accurately and more economically than the dipper dredge and also can leave a wider berm, both of which are important advantages.

When arranged specially for ditch work, the drag-line excavator may be mounted upon traction wheels, or on skids and rollers, or on car wheels on a track; or it may be mounted as a walking machine like the walking dipper-dredge previously described. It may also be mounted on a tractor, and is then specially adapted to ditches of small cross-section.

A form of drag-line scraper is called the **templet excavator.** It has two buckets, back to back, working on a frame transverse to the ditch in such a manner that one bucket shaves off a thin slice down one slope and across the bottom and up the other slope, and then dumps. One scraper does the excavating one way and the other in the opposite direction. The frame carrying the buckets is lowered automatically as the ditch increases in depth, so that the slopes and the bottom are cut accurately to specifications. When the desired depth is reached, the frame is raised and the machine moved forward or backward, as the case may be, for a new cut.

SECTION 7

MASONRY AND TIMBER STRUCTURES

BY

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DATA FOR MASONRY DESIGN *

MASONRY WALLS AND DAMS *

MASONRY ARCHES AND PIERS *

SIMPLE TIMBER CONSTRUCTION

TIMBER FALSEWORK AND TRUSSES

DOMES OF MASONRY †

* Thomas C. J. Baily, Jr., and E. E. Halmos assisted in writing these chapters.

† E. E. Halmos, C.E., assisted in writing these articles.

DATA FOR MASONRY DESIGN

1. Working Unit Stresses

Maximum Safe Unit Stresses for masonry design are given in the following tables. The values are based upon good material and workmanship in accordance with the best practice. It is assumed that the masonry is to be properly bonded and the mortar ingredients suitably tested, also that the foundation is properly designed to carry all the forces which the masonry transmits to it. The masonry is assumed to be laid in mortar of 1 part cement to 2 parts sand, or 1 part lime paste to 2½ or 3 parts of sand. The unit stresses given are for masonry which is at least 3 months old if laid in warm weather and 6 months old if laid when the temperature is below or near freezing.

For large structures, such as dams over 70 ft high and arches over 150 ft span, or in a series of arches where the amount of masonry is great and where each unit of masonry is dependent for its stability upon each and every other unit, it is advisable to use lower working unit stresses than those in the tables. For such cases it is recommended that 75 per cent of the tabulated values be employed.

If, for special reasons, it is desirable to consider masonry laid in warm weather one month old, or that laid when the temperature is below or near freezing, two months old, it is best to use 60 per cent of the values given in the tables.

Ashlar Masonry. The following are safe working compressive stresses recommended to be used in the design of structures of ashlar masonry laid in 1:2 portland cement mortar with joints not exceeding ½ in. in width.

Kind of ashlar	Pounds per square inch	Short tons per square foot
Granite, Syenite, Gneiss......	700	50.4
Limestone, hard............	650	46.8
Limestone, medium; Marble..	600	43.2
Limestone, soft; Sandstone...	500	36.0

For ashlar laid with 1 : 2 portland cement mortar, and having joints greater than ½ in and less than 1 in, it is best to use 450 lb per sq in (32.4 tons per sq ft) for all kinds of sound building stone.

For ashlar laid in lime mortar with joints not exceeding ½ inch, it is best to use 225 lb per sq in (16.2 tons per sq ft) for all kinds of sound building stone.

For ashlar laid in lime mortar with joints greater than ½ in and less than 1 in, it is best to use 150 lb per sq in (10.8 tons per sq ft) for all kinds of sound building stone.

For Brick Masonry in portland cement mortar, with joints not exceeding ⅜ in, the following working unit stresses are recommended: for hard best quality brick 350 lb. per sq. in (25.2 tons per sq ft), for hard to medium brick 300 lb per sq in (21.6 tons per sq ft).

For brick masonry in lime mortar use 40 percent of these values. If the bricks have been tested and show a uniform ultimate compressive strength of at least 7000 lb per sq in, and they are laid with great care in portland cement mortar, the safe working compressive strength may be taken at 400 lb per sq in (28.8 tons per sq ft). Masonry of soft brick should not be used except for abnormally light loads, its safe working compressive strength being taken at 40 lb per sq in (2.9 tons per sq. ft).

For Rubble Masonry laid in portland cement mortar, the following working compressive unit stresses are recommended:

Kind of rubble	Pounds per square inch	Short tons per square foot
Flat or scabbled stones..........	250	18.0
Irregular stones, not scabbled....	200	14.4
Very irregular stones (field stones)	100	7.2

For rubble masonry in lime mortar about 40 percent of the preceding values should be used as working unit stresses.

For Plain Concrete Masonry made with portland cement the following safe working compressive stresses in lb per sq in are recommended: 450 for 1:2:4 concrete, 350 for 1:3:6, and 250 for 1:4:8. For plain concrete made with natural cement it is best to use only 25 per cent of these values.

The above unit stresses apply to compression from dead loads, and do not apply to masonry where only a small portion of the area is under direct compression as under the footings of steel columns. (See Art. 7, Fig. 17*a*.) For shocks lower values should be used. Under bed plates of steel bridges use 50 per cent of above values.

The Working Shearing Unit Stress for all stone masonry should be taken as one-quarter of the working compressive unit stresses above given. For concrete masonry experiments have shown that it may be taken as six-tenths of the compressive unit stress provided diagonal tension does not accompany the shear or is properly provided for by steel reinforcement.

For Stone Slabs or single blocks of stone, the following working unit stresses are recommended, all being in pounds per square inch:

Kinds of stone	Compression	Tension*	Shear	Weight, lbs. per cu ft
Granite, Syenite, Gneiss.......	....	150	200	170
Hard.....................	1500	....	....	....
Medium.................	1200	....	....	....
Soft.....................	1000	....	....	....
Limestone.................	....	125	150	165
Hard.....................	1000	....	....	....
Medium.................	800	....	....	....
Soft.....................	700	....	....	....
Marble.....................	....	125	150	165
Hard.	900	....	....	....
Soft.....................	700	....	....	....
Sandstone.................	700	75	150	150
Blue stone-flagging..........	1500	200	....	....

* Values in this column apply to both direct and flexural tension.

If the shearing stress occurs without diagonal tension, the working shearing unit stress may be taken at 4/10 of the working compressive unit stress, but with detached blocks and slabs of stone shear is almost always accompanied by diagonal tension.

Tension, whether direct or flexural, should not occur in stone masonry, and working tensile unit stress should be taken as zero in the computations of designing. In the analysis of existing structures, where the mortar is found to be strong and adhesive, a tensile unit stress of 15 lb per sq in may be allowed for masonry laid in portland cement mortar, 10 lb per sq in for that in natural cement mortar and 5 lb per sq in for that in lime mortar.

The resistance of masonry joints against tension is often wholly or partially destroyed by erection stresses, by shrinkage of the mortar in setting, and by expansion and contraction of the mass under changes of temperature. The mortar in the vertical joints, as a rule, wholly loses its adhesion from these causes. The mortar of the bed joints, however, sets under pressure and hence is more or less available to transmit tension. Since the shrinkage of mortar is less below ground, due to ground moisture and a uniform temperature, and since expansion and contraction are also less, masonry below ground is stronger in tension than that in air.

When it is necessary to build masonry to take tension, either direct or flexural, special care should be given to the bond. In the design of concrete footings a tensile unit stress not to exceed 8 per cent of the safe compressive unit stresses may be allowed.

Masonry Pressures at the Top of the Foundation (From Corthell)

Name and location	Short tons per sq ft	Lb. per sq in
American Surety Building, New York	7.25	101
Atlantic Mutual Insurance Bldg., New York	15.00	210
Cincinnati & Covington Ry. Bridge	6.50	91
Croton Dam, New York	15.00	210
Manhattan Life Insurance Bldg., New York	10.80	150
Missouri River Bridge, Omaha	14.40	200
Mutual Life Insurance Bldg., New York	36.00	500
St. Charles Bridge, St. Charles, Mo.	19.00	266
South Street Bridge, Philadelphia	15.70	218
Washington Monument, Washington, D. C.	9.00	126
Williamsburg Bridge, New York	5.00	70

Allowed Working Compressive Stresses in Short Tons per Square Foot for Masonry According to Building Laws

Kind of masonry	New York, 1917	Philadelphia 1916	Washington, 1919	Boston, 1919	Cincinnati, 1917	Cleveland 1914	Louisville, 1913	Baltimore, 1908
Granite ashlar	43	......	*72–173	60		9–12–15–20	......	72–173
Gneiss ashlar	43	......	94	....		9–12–15–20	......	......
Limestone ashlar	43	......	*50–166	40	†	9–12–15–20	......	72
Marble ashlar	43	......	*43–86	40		9–12–15–20	......	......
Sandstone ashlar	21.5	......	*29–115	30		9–12–15–20	......	......
Rubble masonry in portland cement	10	10	10	....	12	10–11	......	9
Rubble masonry in natural cement	8	......	8	....	9	7–9	12	7.2
Rubble masonry in lime and cement	7	8	7	....	......	5–7	......	5
Rubble masonry in lime	......	5	5	....	6	4–6	......	4
Brickwork in portland cement	18	15	18	....	18	14–25	18	18
Brickwork in natural cement	15	......	15	15	12	13–18	12	15
Brickwork in lime and cement	11.5	12	11.5	12	......	11	......	11.5
Brickwork in lime	8	8	8	8	8	7	8	8
Concrete, portland cement 1 : 2 : 4	36	15	29	....	15	29	......	28
Concrete, natural cement 1 : 2 : 4	15	......	9	....	......	8	......	9

* According to test. † According to mortar.

2. Unit Weights and Other Constants

Unit Weights of Masonry are slightly less than those of the materials of hich it is composed. Average values are given in the following table; the ird column also gives the approximate moduli of elasticity and the fourth e coefficients of expansion for one degree Fahrenheit.

Physical Properties of Masonry

Kind of masonry	Weight. Lb per cu ft	Modulus E. Lb per sq in	Coefficient e of expansion
Ashlar: granite, syenite, gneiss	165	4 000 000	0.000 0035
limestone, marble	160	4 000 000	0.000 0035
Sandstone	140	4 000 000	0.000 0035
Mortar rubble: granite, syenite, gneiss	155	2 000 000	0.000 0035
Limestone	150	2 000 000	0.000 0035
Sandstone	130	2 000 000	0.000 0035
Dry rubble: granite, syenite, gneiss	130	....	....
Limestone	125	....	....
Sandstone	110	....	....
Brick: prest, thin joints	140	2 000 000	0.000 0030
common, ⅜-in joints	120	2 000 000	0.000 0030
soft, ⅜-in joints	100	....	....
Concrete: Broken Stone, 1 : 2 : 4	145	2 500 000	0.000 0060
broken stone, 1 : 3 : 6	145	2 000 000	0.000 0060
cinder	110	....	....
Cyclopean: masonry with maximum volume of stone	155	....	....

Slabs or detached block stone have values of E much higher than those in the table. pproximate values are 7 000 000 for granite, syenite and gneiss, and the harder limeones, 8 000 000 for hard marble, 5 500 000 for soft limestone, 2 800 000 for sandstone. oefficients of expansion are 0.000 0040 for the granitic rocks, 0.000 0037 for limestone d 0.000 0050 for sandstone.

Values in the last two columns are used in investigating the temperature stresses which ay come upon masonry arches (Art. 24) and for computing their deformations.

Weights of other Materials which may bring pressure upon masonry walls d arches are as follows, in pounds per cubic foot.

Weights of Miscellaneous Materials.

Sand dry clean	90	Cinders, bituminous, dry compact	45
Sand, wet	115	Ashes, anthracite dry compact	30
Gravel, clean	100	Paving in place:	
Broken Stone	95	Asphalt top and binder	107
Clay, dry, compact	100	Asphalt block	145
Clay, plastic	100	Granite block	155
Sand, Gravel, and Clay, mixed:		Wooden block	50
dry, compact	100	Brick	140
wet	115	Macadam	105
Mud	110	Water, fresh	62 5
Rock, rotten, soft compact	110	Water, salt	64
Rock, hard, loose	100	Snow, fresh	8

The weight of snow to be used in designing is generally assigned by specification, as is so the lateral wind pressure, the latter being usually 30 lb per sq ft of vertical surface.

The Slope of Repose of a bank of loose earth, in its natural state, is a factor hich governs the lateral pressure which the earth may bring to exert against

a retaining wall. In the following table the slope is the ratio of horizontal to vertical projection. It will be noted that the weight per cubic foot of filling material generally varies between narrow limits when the slope of repose varies between wide limits.

Slopes of Repose and Weights for Loose Earth

Kind of earth	Slope of repose	Angle of repose	Weight Lb per cu ft
Sand, clean	1.5 to 1	33° 41′	90
Sand and clay	1.33 to 1	36 53	100
Clay, dry	1.33 to 1	36 53	100
Clay, damp, plastic	2 to 1	26 34	100
Gravel, clean	1.33 to 1	36 53	100
Gravel and clay	1.33 to 1	36 53	100
Gravel, sand and clay	1.33 to 1	36 53	100
Soil	1.33 to 1	36 53	100
Soft rotten rock	1.33 to 1	36 53	110
Hard rotten rock	1 to 1	45 00	100
Bituminous cinders	1 to 1	45 00	45
Anthracite ashes	1 to 1	45 00	30

The Angle of Repose given in the third column of this table is the angle ϕ which the sloping face of a bank of loose earth makes with the horizontal (Fig. 1). The cotangent of this angle is the slope ratio given in the second column; thus, 1.5 is the cotangent of 33° 41′. In general, if s is the slope of repose, or the ratio of horizontal to vertical projection, and ϕ the angle of repose, then $s = \cot \phi$. The term "natural slope" is sometimes used as synonymous with "slope of repose." The tangent of this angle is the coefficient of friction for earth upon earth, or $f = \tan \phi$ (Art. 3).

In the theory of retaining walls (Art. 4), the coefficient of friction along planes not on the surface but within the mass of the backing material is of interest. The angle corresponding to this coefficient is termed the angle of internal friction.

It has been found by experiment, that, for non-cohesive materials, the angle of internal friction is generally larger than the angle of repose given for loose earth in the above table, but, in determining the theoretic pressure upon retaining walls, the values of ϕ given in this table are recommended and are on the safe side. Non-cohesive materials, such as sand, gravel, broken stone, cinders, etc., consist of granular particles and resist shearing forces through friction only.

In the case of cohesive backing materials, such as clays which possess a degree of shearing (cohesive) strength, the angle of internal friction is considerably smaller than the angle of repose given in the above table and varies from 0° to 16°. The following table (used in Art. 4) was taken from A. L. Bell's paper (Proc. Inst. Civ. Eng., 1915):

Cohesive Strength and Angle of Internal Friction in Clays

Description of material	Cohesive strength tons per sq ft c	Angle of internal friction ϕ_1
Puddle clay, wet	0.2–0.45	0°–5½°
Stiff clay puddle	0.62	2°
Sandy clay, wet	0.6	2½°
Stiff sandy clay	0.5	10°
Moderately firm boulder clay	0.7	6½°
Very stiff, boulder clay, fairly dry	1.6	16°

Material Excavated by a wet or a dry process, and dumped into water, as at the back of a sea wall, has weights and slopes approximated as follows:

Kind of material	Slope of repose	Angle of repose	Weight lb per cu ft
Sand, clean	2 to 1	26° 34′	60
Sand and clay	3 to 1	18 26	65
Clay	3½ to 1	15 57	80
Gravel, clean	2 to 1	26 34	60
Gravel and clay	3 to 1	18 26	65
Gravel, sand and clay	3 to 1	18 26	65
Soil	3½ to 1	15 57	70
Soft rotten rock	1 to 1	45 00	65
Hard rock, riprap	1 to 1	45 00	65
River mud	8 to 1	0 00	90

When the material is excavated by suction dredging and pumped back of a retaining wall which has efficient drains to carry off the water, the weight per cubic foot may be taken at 110 pounds and the slope of repose as 2 to 1 for sand and clay, clay and gravel, or clay, gravel, and sand combined. River mud may be taken at 100 lb per cu ft with a slope of 3 to 1.

3. Data Regarding Friction

The Law of Friction for one stone beginning to move on another along their plane of contact is $F = fN$, in which F is the force parallel to the plane, N is the normal pressure on the surface of contact, and f is the coefficient of friction. Fig. 2 shows a stone about to move along a surface inclined to

Coefficients and Angles of Friction

Kind of surface	Coefficient of friction f	Angle of friction ϕ
Granite, Limestone, Marble:		
Soft dressed upon soft dressed	0.70	35° 00′
Hard dressed upon hard dressed	0.55	28 50
Hard dressed upon soft dressed	0.65	33 00
Stone, brick or concrete:		
Masonry upon masonry	0.65	33 00
Masonry upon wood (with the grain)	0.60	31 00
Masonry upon wood (across the grain)	0.50	26 40
Masonry upon dry clay	0.50	26 40
Masonry upon wet or moist clay	0.33	18 20
Masonry upon sand	0.40	21 50
Masonry upon gravel	0.60	31 00
Soft stone upon steel or iron	0.40	21 50
Hard stone upon steel or iron	0.30	16 40

The angle of friction is materially lessened by jar, shock, or vibration caused by blasts, the passage of trains, or motion of attached machinery. For such cases it is recommended that in masonry design the angles of friction be taken as 5° less than those in the table and that the coefficients of friction be likewise modified by the relation $f = \tan \phi$.

For clay foundations it is best to use the values given for wet clay, unless it is reasonably certain that water cannot reach the bed.

the horizontal at the angle ϕ, the conditions being exactly the same as those seen in Fig. 1 for a particle of earth on the face of a sloping bank. In both cases $F = fN$ and $f = \tan \phi$, where ϕ is the angle of repose, which in the case of stone surfaces is often called the "angle of friction." When the plane of contact is horizontal, as in Fig. 3, motion is about to begin when the resultant force R against that plane makes with the normal an angle ϕ such that $\tan \phi = f$. The angle of repose ϕ and the coefficient of friction f depend upon the roughness of the surface of contact.

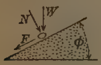


Fig. 1

Fig. 2

Fig. 3

Fig. 4

The preceding table assumes that the surfaces of the stones are rougher than rubbed work, that is, they are for fine pointed, bush hammered, crandalled or sawed surfaces, while the wood surfaces are those of undrest yellow pine. For rubbed surfaces of stone the angles of repose are about 5° less than those given, and for polished stone about 10° less.

Moisture usually increases the angle ϕ, so that wet wood has a larger value than dry wood. For stone surfaces with fresh wet mortar between them, the angle ϕ is about 5° less than that in the table, the value given corresponding to the consistency generally used.

Arch Masonry built upon timber centering has usually a greater angle of friction than given in the table, due to mortar setting between adjacent pieces of lagging; in the case of concrete there is also a mechanical bond with the lagging, since when the lagging is removed the imprint of the grain can always be seen upon the concrete surface.

When a plane makes with the horizontal an angle β less than ϕ, the horizontal force H (Fig. 4) which is required to slide a body up the plane is $H = fN/(\cos \beta - f \sin \beta)$, in which N is the pressure normal to the surface. This formula gives also the horizontal force required to slide the body down the plane, if the minus sign be changed to plus.

4. Lateral Pressure of Earth

A wall backed with earth is subject to a lateral pressure which tends to overturn it as well as to cause it to slide. This pressure is the greater the heavier

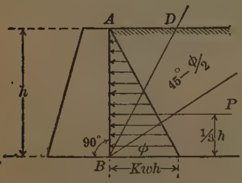


Fig. 5a

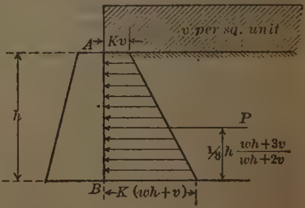


Fig. 5b

the earth and the less its angle of repose. Many attempts have been made to determine the direction, magnitude and point of application of the resultant lateral pressure by experiment, but the results have been of general value only.

Engineers, therefore, are limited in design to the use of certain theoretical nethods or to the recorded dimensions or previously built walls. All theories f earth pressure on retaining walls assume that the backing material is of uniform ranular consistency without cohesion and that the angle of internal friction ϕ_1 Art. 2) is the same at every point of the mass, the pressure intensities, therefore, varying as the depth as shown by the arrows in Fig. 5*a*.

Rankine's theory of earth pressure upon the back of retaining walls has been most comnonly used. It is based on principles of elasticity applied to granulous materials with imitations, the pressure being determined by means of the ellipse of stress. Coulomb's heory assumes that the maximum thrust on the wall is exerted by a wedge of the backing naterial, this wedge being bounded on one side by the back of the wall and on the other ide by a plane called the plane of rupture (Fig. 6*a*) upon which the wedge tends to slide vhen the retaining wall fails. Theories involving cohesion are not treated here. For uch theories see Résal's "Poussée des Terres."

Direction of Earth Pressure. Different authorities make different assumpions as to the direction of the earth pressure acting upon the back of retaining valls. Rankine makes the direction dependent on the slope of the retained bank and on the inclination of the back of the wall. This theory leads to the ollowing. If the retained bank is level or sloping downward (negative surcharge) and the back of the wall is vertical, the direction of the earth pressure is iorizontal. If the back is vertical and the bank is sloping upwards (positive urcharge) the direction of the pressure is parallel to the slope of the bank. If he back of the wall is inclined to the vertical, the direction of the pressure is lifferent for different slopes of surcharge and for different inclinations of the vall, with the exception of the case of walls leaning toward the bank, the retained arth having a downward slope (negative surcharge) which always results accordng to Rankine's theory in a horizontal earth pressure. Coulomb's theory gives he direction of the earth pressure upon the back of the wall as arbitrary between he horizontal and a downward pointing line making an angle with the normal o the back of the wall, this angle not exceeding the angle of friction of earth ipon masonry. (Art. 3.)

Experiments made to determine the direction of earth pressure are comparatively ew and show widely divergent results. Engels' experiments (1896) show that the lirection of the lateral pressure of earth is always horizontal for walls with backs vertical or leaning toward the bank, while it is inclined downward in the case of walls with backs eaning away from the bank. The direction of the resultant earth pressure as deternined by Engels' experiments will be used in what follows; it is within the limits given by Coulomb and agrees with the direction determined by Rankine's theory except in the case of upward sloping (positively surcharged) banks. Engels' direction of the resultant, vhere it differs from Rankine, gives the greater overturning effect and since the angles of epose given in Art. 2 are subject to much variation, it seems reasonable to use, in design, ssumptions which err, if they err at all, on the side of safety. When a vertical retaining vall begins to fail, it is probable that the pressure is not exactly horizontal, but design hould be made for conditions of stability and not for those of failure. It will also be ssumed that liveload placed upon the bank does not change the direction of the earth pressure.

Magnitude and Point of Application. Formulas have been deduced giving he magnitude of the resultant earth pressure for all inclinations of the back of he wall and slope of the bank. Rankine and Coulomb agree in the formula on page 672 for the magnitude of the lateral pressure of level banks upon walls vith vertical backs. Formulas for other conditions are complicated and for hese, Rebhann's graphical method, based on Coulomb's theory, is hereafter given and recommended. It gives results practically in accord with Rankine and is more simple. In the following a slice of wall of unit length will be considered.

Wall with Vertical Back. Level Bank. For conditions shown on Fig. 5*a*, let w be the weight of the earth per cubic unit; ϕ its angle of repose, and h the height of the wall. The magnitude of the resultant earth pressure is

$$P = \tfrac{1}{2}wh^2 \tan^2 (45^\circ - \phi/2)$$

For a similar wall, but the bank uniformly loaded with a weight v per unit of surface (Fig. 5*b*)

$$P = (\tfrac{1}{2}wh^2 + vh) \tan^2 (45^\circ - \phi/2)$$

For a backing of clay, denoting the value of the cohesional resistance per unit surface by c and the angle of internal friction by ϕ_1 (Art. 2.) Bell proposes the following formula for the resultant lateral pressure.

$$P = \tfrac{1}{2}wh^2 \tan^2 (45 - \tfrac{1}{2}\phi_1) - 2ch \tan (45^\circ - \phi_1/2)$$

In these formulas P is expressed in the same unit of weight as w and v.

Intensity and Point of Pressure Application. In the value of P, $\tfrac{1}{2}wh^2$ and $(\tfrac{1}{2}wh^2 + vh)$ represent the magnitude equivalent to a hydrostatic pressure resulting from a head h of a fluid weighing w per unit volume and loaded with v if any; $\tan^2(45^\circ - \phi/2)$ is the ratio of the true lateral pressure of earth to the hydrostatic pressure and is a constant for the same material. It is a fraction which increases when ϕ decreases; in other words, the flatter the angle of repose the greater the thrust. Denoting this ratio by K, the earth pressure P in the case of an unloaded bank is $\tfrac{1}{2}wh^2K$ which is equal to the area of a triangle of height h and base Kwh (Fig. 5*a*); in the case of a loaded bank $P = (\tfrac{1}{2}wh^2 + vh)K$ which is equal to the area of a trapezoid of height h, the upper side equal Kv and the base equal $K(wh + v)$ (Fig. 5*b*). Horizontal lines drawn in these diagrams will represent the intensity of the earth pressure at any point of the back of the wall. The resultant earth pressure will pass through the center of gravity of the intensity diagrams, its distance from the base, therefore, being $\tfrac{1}{3}h$ for the unloaded bank and $\tfrac{1}{3}h(wh + 3v)/(wh + 2v)$ for the loaded bank. This latter value is greater than $\tfrac{1}{3}h$, but the most excessive load cannot raise it as high as $\tfrac{1}{2}h$.

Graphic Determination of Earth Pressure. In an embankment made of granulous material possessing friction but no cohesion and supported by a retaining wall, the plane of maximum shear (plane of rupture) is located as follows: If, in Fig. 6*a*, AB represents the back of the wall, BC the slope of repose and ADC the surface of the bank, then BD is the plane of rupture if area ABD is equal the area of triangle BDF, DF being perpendicular to BC. Make $FG = DF$. The magnitude of the earth pressure is given by the product of the area, $DFG = \tfrac{1}{2}DF^2 = \tfrac{1}{2}a^2$, and the unit weight of the earth w In the case of vertical back and level bank (Fig. 5*a*) the plane of rupture bisects the angle $90^\circ - \phi$ and this graphic method leads to the same formula as given above or $P = \tfrac{1}{2}wh^2 \tan^2 (45^\circ - \phi/2)$.

In the case of a wall with inclined back or supporting a sloping bank or both, the plane of rupture and the magnitude of the earth pressure can be obtained by a cut and try method making area $ABD = BDF$ on either side of the plane of rupture BD, or by the following construction. In Fig. 6*b* a wall with the back slightly inclined toward the fill supports a bank with a positive surcharge. Draw BC to make angle ϕ with the horizontal, C being the intersection of the slope of repose with the surface of the ground and draw a semicircle with diameter BC. Draw AH perpendicular to BC and make $BF = BH$. Then draw $DF = a$ at right angles to BC and make $FG = DF$. The areas BAD and BDF are then equal and therefore BD is the plane of rupture. The area of triangle $DFG = \tfrac{1}{2}a^2$ multiplied by w will give the magnitude of the resultant earth pressure acting upon AB in a horizontal direction and at $\tfrac{1}{3}h$ above the base. $P = \tfrac{1}{2}a^2w$. The intensity diagram will be a triangle the base of which is $BM = 2P/h$. This solution is perfectly general

and applies for all slopes of the bank and inclinations of the back of the walls. In the case of a wall leaning away from the bank, however, the weight W resting upon the back

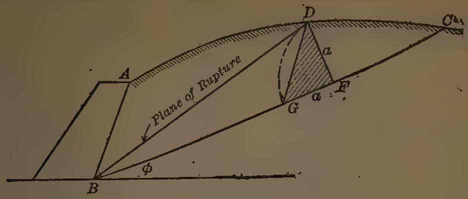


Fig. 6a

of the wall (ABL in Fig. 6c) should be regarded as part of the wall and the earth pressure upon the vertical BL determined. Combine P graphically with W to get the pressure P' upon the back of the wall. P' will be inclined downward.

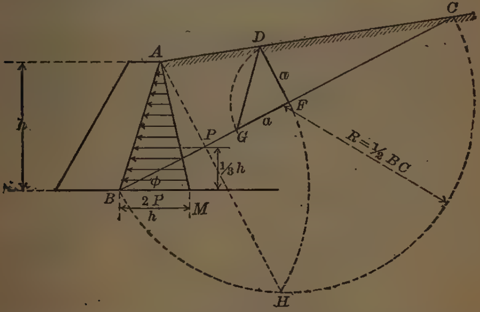


Fig. 6b

If the surcharge (Fig. 6d) is parallel to the slope of repose of the retained bank, the above (Fig. 6b) construction is not necessary, the distance between the lines AD and BC being equal to a in $P = \frac{1}{2}a^2w$.

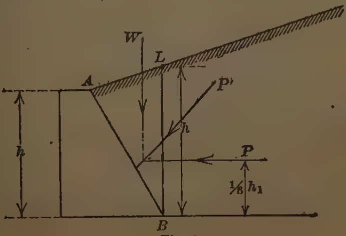


Fig. 6c

If the bank is loaded with a load of v per unit surface, draw an imaginary slope, parallel to the true slope and at distance $h_1 = v/w$ above same (Fig. 7); find P_1 for the extended wall A_1B by the construction shown in Fig. 6b and construct the intensity triangle for wall A_1B, the base of the triangle being $2P_1/(h+h_1)$. Then the area of trapezoid $ANMB$ will represent the pressure P on wall AB, being applied at the center of gravity of the trapezoid.

If the surface of the retained bank is broken, as in Fig. 8, locate the plane of rupture BD

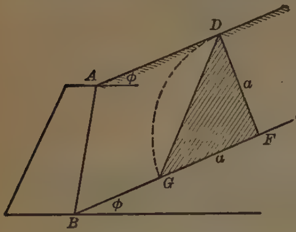


Fig. 6d

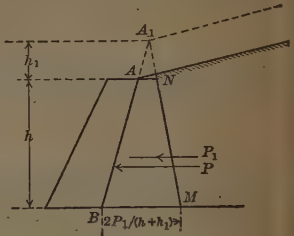


Fig. 7

by trial and successive approximation so that area $BAMND$ shall equal area BDF. Then $DF = a$ and $P = \frac{1}{2}a^2W$. In this case the point of application of the resultant earth pressure is not at $\frac{1}{3}h$ from the base, the intensities not being a linear function of the depth. The actual distribution of the pressure on the back of wall AB and consequently the distance of the resultant pressure from the base cannot be determined graphically in any simple way but for this distance the use of $0.4h$ is recommended.

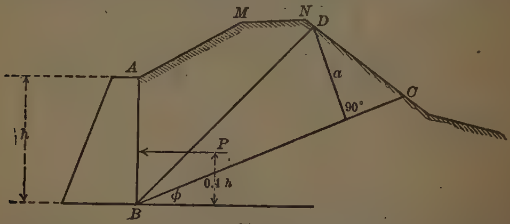


Fig. 8

If the bank has a plane top surface, but is only partially loaded, apply the same method as shown above (Fig. 8) the loading transformed into fill resulting in a broken surface.

In computations h is taken in feet, w in pounds per cubic foot and v in pounds per square foot, so that values of P are in pounds for one linear foot of wall. For example, let $h = 18$ ft. $w = 100$ lb per cu ft, $\phi = 34°$. For an unloaded level bank retained by a wall with vertical back (Fig. 5a) P is 4580 lb and its point of application is 6 ft above the base. If this bank is loaded (Fig. 7) so that $v = 120$ lb. per sq ft, then P is 5190 lb and its point of application is 6.35 ft above the base.

Experiments on Earth Pressure. Many retaining walls which have stood for years, if investigated by means of the foregoing theories would be found inadequate. This fact shows that the theories furnish excessive values for the lateral pressure of earth, due possibly to the neglect of cohesion, which to a greater or lesser amount, exists in the backfill, unless granulous. During the last 35 years many experiments were made to obtain data upon which more accurate theories than those of Rankine and Coulomb might be based. Some

of the more important experimenters, divided according to nationalities are: American; Steel, Goodrich. English; Baker, Darwin, Bell. French; Leygue, Siegler. German; Engels, Muller-Breslau. M. M. L. Leygue's experiments (Annales des Ponts et Chaussées, 1885), made on small models with dry sand showed that the amount of cohesion existing in the dry sand is negligible and that the surface of rupture is not plane but convex.

Sir Benjamin Baker's experiments (Min. Proc. Inst. C. E., Vol. 65) showed that the magnitude of the lateral pressure is in inverse ratio to the degree of coarseness of the material.

G. H. Darwin's experiments (Min. Proc. Inst. C. E., Vol. 12) are interesting chiefly because they tend to show that the internal friction is not constant in the retained granulous mass, but varies considerably with the depth.

H. Engels' experiments (1896) were primarily made to determine the influence of friction and thrust of earth on the sides of foundations upon the bearing value of same. The results of these experiments applicable to retaining walls are given at the beginning of this article.

E. P. Goodrich's experiments are published in Trans. A. S. C. E., Vol. 53. All of his experiments, except the one marked "against retaining wall," were carried on in a laboratory. A hollow cylinder 6 in diameter and 5 in deep was fitted with a piston upon which the loads were applied by a machine. Near the bottom of the cylinder a circular opening 1 in in diameter was fitted with a plug. The cylinder was filled with the material to be tested. Loads were applied to the piston and the side pressure exerted on the plug measured by a scale-beam apparatus. The retaining wall experiment was made on construction work. A wooden box 15 ft high, 6 ft long and 1 ft wide was placed against the back of a wall, so arranged that the deflection of any point of the back of the box could be obtained. Then the filling material was dumped back of the wall and the deflection of various points of the back of the box measured. From these data the lateral pressure exerted by the filling material was computed. In the table the ratios are for various loads between 2500 lb and to 10 000 lb per sq ft.

Average Ratio of Lateral to Vertical Earth Pressure as Obtained by Experiment

Kind of material	Experimenter	Vertical pressure in pounds			
		2500	5000	7500	10 000
Coal, shingle, ballast or macadam	Baker	0.10	......	......	......
Rip-rap, under water	Goodrich	0.15	......	......	......
* Gravel 1/2 inch	Goodrich	0.41	......	......	......
* Gravel 1/4 inch	Goodrich	......	......	......	0.50
Hard coal cinders	Goodrich	0.40	0.42	0.44	0.44
* Sand 1/20″ to 1/30″ mesh	Goodrich	0.72	0.71	0.72	0.71
* Sand 1/30″ to 1/50″ mesh	Goodrich	0.60	0.60	0.60	0.60
* Sand 1/50″ to 1/100″ mesh	Goodrich	0.36	0.40	0.40	0.40
Quicksand 1/60″ to 1/100″ mesh	Goodrich	0.24	0.26	0.30	0.32
Quicksand 1/100″ up	Goodrich	0.20	0.22	0.25	0.28
Bank sand	Goodrich	0.10	0.18	0.24	0.25
Earth	Steel	0.26	......	......	......
Earth	Baker	0.15	......	......	......
Bank sand	Wilson	......	......	......	0.26
Bank sand	Goodrich	0.33	......	......	......
† Bank sand	Goodrich	0.50	......	......	......
* Clay	Goodrich	0.40	0.40	0.40	0.40
Clay	Baker	0.22	......	......	......

* Observations incomplete. † Against retaining wall.

Experiments on the lateral and vertical pressure of earth are, as a rule, made with dry material. The pressures observed in practice are usually those of earth containing more or less moisture.

5. Pressures of Water

Hydrostatic Pressure against a surface immersed in water is wAd, where w = weight of a cubic unit of water, A = area of surface, and d = depth of the center of gravity of the surface below the water level. When dimensions are in feet, the total pressure in pounds is 62.5 Ad for fresh water and 64 Ad for salt water. For an immersed plane the direction of this pressure is always normal to the surface.

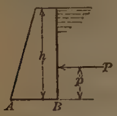


Fig. 9

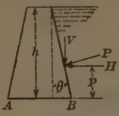


Fig. 10

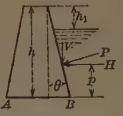


Fig. 11

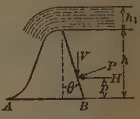


Fig. 12

Pressures on a Dam. In Figs. 9 and 10 the water level is at the top of the dam, in Fig. 11 it is below and in Fig. 12 it is above the top. The horizontal pressure H and the height of its point of application above the base are given by

For Figs. 9 and 10 $H = \frac{1}{2}wh^2$ $\qquad p = \frac{1}{3}h$

For Fig. 11 $H = \frac{1}{2}w(h - h_1)^2$ $\qquad p = \frac{1}{3}(h - h_1)$

For Fig. 12 $H = \frac{1}{2}wh(h + 2h_1)$ $\qquad p = \frac{1}{3}h\dfrac{h + 3h_1}{h + 2h_1}$

Also the normal pressure and its vertical component are

$$P = H \sec \theta \qquad V = H \tan \theta$$

The notation is shown on the figures and P is for unit length of dam. The distribution of unit pressures on the back of dams is exactly that for earth illustrated in Figs. 5a and 5b. Thus, at the base the unit pressure for Fig. 9 and 10 is wh, for Fig. 11 it is $w(h - h_1)$ and for Fig. 12 it is $w(h + h_1)$.

An Overflow Dam, or spillway dam, is one where the water flows over the top, as in Fig. 12. The effect of the water above the top is both to increase the pressure P and to raise its point of application. The following are the percentages of increase in P and p over those for the case of Fig. 10, due to different ratios of h_1 to h;

h_1/h	0.5	0.10	0.20	0.20	0.30	0.40	0.50
For P	10	20	30	40	60	80	100
For p	4.5	8.3	11.5	14.3	18.8	22.2	25.0

The flow of water over a spillway dam may cause a partial vacuum beneath the falling water sheet or nappe if it is not in contact with the masonry. If the atmospheric pressure between the masonry and the falling sheet of water is thus reduced from 14.7 lb per sq in to a value q over the whole front face, the practical effect is to add a unit pressure $14.7 - q$ lb per sq in to the back of the dam and thus to cause an additional normal pressure 144 $(14.7 - q)$ h lb, h being expressed in feet, whose point of application may be as high as $\frac{1}{2}h$. The possibility of decrease of atmospheric pressure between the dam and the falling water sheet should be kept in mind by a designer, but it is obvious that only a small percent of the theoretically possible total pressure need be used in design. (Art. 17.)

For a Dam Backed with Earth, the hydrostatic pressure upon the back of the dam may be increased or decreased by the back-filling material depending upon whether or not the material is porous or so non-porous that it will not permit the transmission of water pressure. A puddled bank of sand, clay and

gravel in which the clay fills the voids of the harder materials would materially reduce the pressure when compacted. Silt and detritus deposited behind a dam may increase the pressure. Sand, gravel and broken stone (riprap) backing increased the pressure due to water alone. For this case, in addition to the full hydro-static pressure, the pressure of the backfill must be calculated by the methods given in Art. 4 using the values of w and ϕ as given in Art 2 for material excavated by a wet process and dumped into water.

Water Below the Base exerts an upward pressure upon the base which has the effect of diminishing the weight of the masonry, thereby reducing its resisting value against both overturning and sliding. This is one reason why it is very important to prevent water from entering below the base of a dam as far as it is practicable. To entirely exclude such water and completely prevent uplift is generally difficult, expensive, requires experience and a thorough investigation of the foundation bed and underlying strata. Where the dam rests upon solid unfissured rock the problem is fairly simple but in the average case of a dam resting on ordinary rock, hardpan or more pervious material the problem is difficult. One must allow for uplift of an intensity dependent upon the character of the foundation bed and upon the cost of the special construction necessary to prevent or diminish it. The magnitude of this upward pressure depends upon the head, the character of the foundation, design and construction and no general rule can be given for its intensity and distribution (Art. 15). Specifications often call for full upward pressure acting at the heel B (Fig. 10) and diminishing to zero at the toe A. This is the extreme case if there is no apron as in Fig. 10. For this case the unit upward pressure at the heel is wh, the total uplift $\frac{1}{2}whb$, where b is the base width, and the force is applied at a distance $\frac{1}{3}b$ from the heel B. If there is an apron, the unit upward pressure at the toe is greater than zero as in Fig. 44. When uplift is considered as acting upon the base, no diminution of the weight of the masonry is to be made in the computations.

6. Internal Stresses in Walls and Dams

The external forces which act on a masonry structure are its weight and the loads that it carries, the lateral pressures of earth, water and wind, if such exist and the reactions of the foundation. The deformation caused by these external forces induces internal stresses of compression, shear and sometimes tension within the structure. For the purpose of determining the internal stresses, the wall will be considered as having a uniform cross-section, uniformly acted upon at every unit of its length by the same vertical or vertical and lateral (horizontal or inclined) forces so that to determine the actual stress in the wall, any unit of its length may be considered. It will be assumed in computing stresses, that the masonry of the wall is homogeneous and obeys (within the limits of working stresses) Hooke's law for elastic solids, that is, stress is proportional to strain. A horizontal joint of a wall, AB in Figs 13 to 16, will be investigated. It forms a parallelogram of width $AB=b$, length unity and area b.

Vertical Loads, Normal Stresses on Horizontal Joints. Let W be the resultant of the vertical loads acting on joint $AB=b$, c the middle point of AB, e the eccentricity of W or the distance of the point of application of W from c, and x the distance of any point of the joint from c, positiv if measured in the direction of W and negativ for the other half of the joint. Then the normal stress per unit area at any point x, expressed in the same units as W and b

$$S=\frac{W}{b}\left(1+12\frac{ex}{b^2}\right)$$

which formula is called the law of the trapezoid. At the middle point C, where $x = o$, $S_c = W/b$ for all values of e, in other words at the middle of the joint the unit stress is always equal to the average stress along the joint.

At the edges of the joint where $x = +b/2$ or $-b/2$, the unit normal stress is a maximum (S_1) or minimum (S_2) for any value of e and

$$S_1 = \frac{W}{b}\left(1 + \frac{6e}{b}\right); \quad S_2 = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

S_1 is always positiv (compression) for any value of e. S_2 is positiv (compression) as long as $e < 1/6b$ (Fig. 13). It becomes zero if $e = 1/6b$ (Fig. 14). If the resultant falls outside the middle third, $e > 1/6b$, the formula furnishes a negativ value (tension) for S_2 (Fig. 15) and this formula does not apply unless the masonry is capable to develop tension. This case is discussed below. Usually no dependence should be placed upon masonry taking tension.

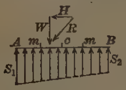


Fig. 13

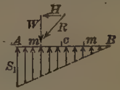


Fig. 14

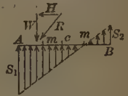


Fig. 15

Since, in design, the maximum and minimum stresses largely govern these formulas for said stresses are more generally used and will be further discussed. If $e = o$, the resultant load is applied at the middle point of the joint and $S_1 = S_2 = W/b$ (compression) i.e., the stress is uniform along the joint. If $e = 1/6b$, the resultant load is applied at the middle third point and $S_1 = 2W/b$ (compression) or twice the average value of the stress, and $S_2 = o$. If $e > 1/6b$ and the joint cannot take tension, part of it will not be stressed at all, while the other part will transmit compressive stresses only. This condition of stress distribution is shown in Fig. 16, where the distance e of the point of application g of

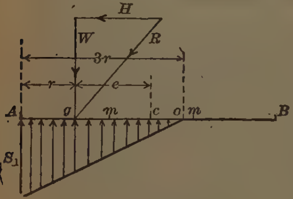


Fig. 16

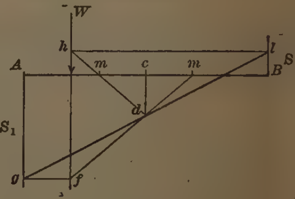


Fig. 16a

the resultant load is greater than $cm = 1/6b$. Let r be the distance of W from the nearest edge of the joint A in Fig. 16, then the point of zero stress O will lie at a distance $3r$ from this edge and $S_1 = 2W/3r$; for $r = o$, S_1 becomes infinity. Substituting the safe unit stress s for S_1 we get $r = 2W/3s$. This gives the minimum distance r from the edge of the joint to the resultant permissible with a given safe unit stress.

If tensile stresses are considered in a masonry wall the general formulas given above for S_1 and S_2 should be used even though the resultant falls outside the middle third. For such a case, if $e = \frac{1}{2}b$, the resultant is applied at the edge of the joint, $S_1 = 4W/b$ (compres-

sion) or four times the average stress, and $S_2 = 2W/b$ (tension) or twice the average stress.

Graphically the stresses along AB may be found by the following construction. Lay off (Fig. 16*a*) $cd = W/b$ the average stress, connect the middle third points m and m with d and produce lines md to intersect with W in f and h. Draw horizontal fg and hl to intersect verticals A and B in g and l. Then Ag and Bl will be the maximum and minimum normal stresses and the stress at intermediate points will be obtained by scaling the vertical intercept between AB and gl, ordinates above AB indicating tension.

Example. Let $W = 30\,000$ lb and $b = 6$ ft. Then for W applied at the middle of the joint, $e = 0$, $S_1 = S_2 = 5000$ lb per sq ft. For W at the middle third point, $e = 1$ ft, $S_1 = 2W/b = 10\,000$ lb per sq ft, $S_2 = 0$. For W at $\frac{1}{3}b$ from the center, $e = 2$ ft, $S = 3W/b = 15\,000$ lb per sq ft, $S_2 = -Wb = -5000$ per sq ft. If the joint cannot take tension, $r = 1$ ft, $S_1 = 2W/3r = 20\,000$ lb per sq ft. If the masonry is rubble, with a safe compressive stress of 18 tons per sq ft how near in safety can the resultant come to the edge of the joint? $r_{\min} = 2W/3S = 2 \times 30\,000/3 \times 36\,000 = 5/9$ ft.

Vertical and Lateral Loads. Normal Stresses on Horizontal Joints. If lateral forces act upon a wall, the resultant of these and the vertical loads will be inclined to the joint as R in Figs. 13 to 16. In order to find the normal stresses on the joint, resolve R into a vertical and a horizontal component, W and H at the point of application of R and treat W exactly as shown in the preceding discussion. H, the horizontal component of R, will produce compression on vertical planes and shearing stresses along the joint, if it is capable of taking shear. If not, the stability of the structure is dependent upon the friction developed along the joint. Designers should not add the safe shearing value to the safe frictional resistance, because the latter will not come into action unless the shearing resistance has been overcome.

Shearing Stresses in Walls. Careful consideration should be given to the shearing stresses in the design of dams over 50 ft in height but they need rarely be considered in the design of other walls. The magnitude and distribution of shear in walls cannot be determined by the customary formulas for beams since they generally are not prismatic and are always subject to axial loads. The only case in which shearing stresses do not exist in horizontal and vertical planes is when the wall has a vertical face and back and carries an axial or an evenly distributed vertical load, but in this case there are shearing stresses along every other plane.

Let a prismatic wall (Fig. 16*b*) be subjected to an axial compressive force P, then the intensity of the compressive stress on joint AB, normal to the axis will be $= P/a$, if a is the area of joint AB. On a plane CD whose normal is inclined to the axis at an angle ϕ we have a stress in the direction of the axis and therefore oblique to the plane CD, of intensity P/a_1 where a_1 is the area of joint CD, that is, $a_1 = a/\cos\phi$. The whole stress P on CD may be resolved into two components, one normal to CD and the other a shearing stress tangential to CD. The normal component is $P\cos\phi$, the tangential component is $P\sin\phi$. The unit intensity of normal compression on CD, therefore, is $P/a\cos^2\phi$ and the intensity of shearing stress is $P/a\sin\phi\cos\phi$. This expression makes the shearing stress a maximum when $\phi = 45°$; surfaces inclined at 45° to the axis are called surfaces of maximum shearing stress, the intensity of shearing stress on them is $\frac{1}{2}P/a$.

In the case of retaining walls and dams which are subjected to horizontal loads besides vertical ones and generally have a wedge-shaped cross-section, it is obvious that shearing stresses exist on every horizontal joint, the total of such stresses being equal to the resultant horizontal force above the plane of the joint. The intensity of the horizontal shear will vary from point to point along the joint. Its distribution can be determined by applying the principle, that, at any point of the structure, the horizontal and vertical shear intensities must be equal. Let Fig. 16*c* represent the cross-section of a dam, a slice of unit thickness being considered. According to the law of trapezium, the distribution of the normal pressure on joint AB may be represented by $ADEB$; and if forces are expressed in masonry units (the unit of weight being the weight of one cubic unit of masonry) and the same scale used for dimensions and forces, the area

ADEB will be equal to the area *AHCKB*. Similarly the distribution of thrust on joint *HK* will be represented by *HFGK*, which is equal to *HCK*. The weight of the layer between *AB* and *HK* will be represented to the same scale by *AHKB*.

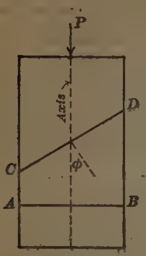


Fig. 16b.

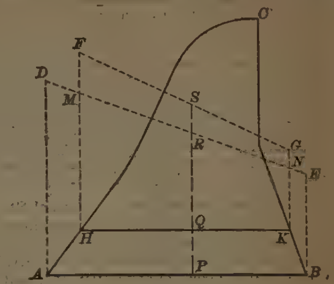


Fig. 16c

Consequently, $ADMH + KNEB = MFGN$. Take any vertical *PS*. The difference between the vertical forces to the right of *PS* and acting on *AB* and *HK* respectively $= RSGN - KNEB$, and that to the left of $PS = ADMH - MFSR$. These two resultant forces are equal but of opposite design, and either of these differences represents the resultant of the vertical forces on either side of the layer *AHKB*, to the right or left of *PQ*, and, therefore, the shear on *PQ*. Hence the intensity of shearing stress at *P* (horizontal and vertical) is approximately either $(RSGN - KNEB)/PQ$ or $(ADMH - MFSR)/PQ$. The approximation will be closer the smaller is *PQ* (Unwin, Engineering 1905, Vol. 79).

Applying the foregoing method to a triangular dam with vertical back, the distribution of the shearing stresses is found to be uniformly varying from zero at the back to a maximum at the face, the shear diagram being a triangle. In the case of a rectangular dam the shear diagram is a parabola with vertical axis; the shear intensity is zero at either face and maximum at the center where its value is $q_{max} = 3h^2/4\gamma b$, h being the depth of water to the joint in question, b the width of the dam and γ the density of the masonry relative to water.

For the important practical case of a dam with vertical back and inclined face, E. P. Hill (Proc. Inst. Civ. Eng., 1908, Vol. 172) gives the following formula for the intensity of shear on horizontal or vertical planes at any point of a horizontal joint:

$$q = \left[4Wm(b - 3d) + \frac{h^2(3b - 2mh)}{\gamma}\right]\frac{x}{b^3} + \left[6Wm(3d - b) - \frac{3h^2(b - mh)}{\gamma}\right]\frac{x^2}{b^4}$$

In this formula W is the total weight above the joint in masonry units, b the width of the joint and h the depth of the water above it, d the distance of the center of gravity of W from the water edge of the joint, x the distance of the point where q is to be determined, m the tangent of the angle between the face of the dam and the vertical and γ the specific gravity of the masonry. The formula shows that the shear intensity on horizontal (or vertical) planes is practically proportional to the distance from the back of the dam at which point it is zero. The experiments of Ottley, Brightmore, Wilson and Gore (Proc. Inst. Civ. Eng., 1908, Vol. 172) have verified this with the exception of the joints at and near the base, where, on account of the rigidity of the fixing of the dam, the shear intensities are practically uniformly distributed along the joint. The shearing stresses to be considered in the design are not those at the base but higher up and near the face. For

practical purposes, except in the case of a very high dam (for which the closer analysis is necessary), the shearing stress along horizontal joints more than $1/5h$ above the base, can be taken as uniformly varying from zero at the back to a maximum at the face at which point it equals $2H/b$, H being the magnitude of the resultant of the horizontal components of the lateral forces above the joint considered and equal to the total shear along the joint. For the joints near to and at the base, the shearing stress should be taken as uniformly distributed having an average value of H/b. (See example, Art. 16.)

Horizontal Pressures on Vertical Planes. The lateral external forces (water or earth pressure) also generate normal stresses on vertical planes. For a dam with vertical back and inclined face, Hill gives the following formula to determine the intensity p' of these stresses, the notation being the same as in the preceding paragraph:

$$p' = \frac{h}{\gamma} + \left[\frac{2Wm^2}{b^2}\left(2 - \frac{9d}{b}\right) - \frac{3h(b-mh)^2}{\gamma b^4} + \frac{m}{b}\right]x^2$$

$$+ \left[\frac{6Wm^2}{b^4}\left(\frac{4d}{b} - 1\right) + \frac{2h(b-mh)(b-2mh)}{\gamma b^5} - \frac{m}{b^2}\right]x^3$$

The horizontal stress intensity on vertical planes, therefore, increases from the back. For $x = a$, $p' = h/\gamma$, or in other words, at the back of the dam p' equals the unit water pressure. For all walls, except very high structures, where a closer investigation is necessary, the normal stress intensity on vertical planes, for every point of a horizontal joint, can be taken as constant and equal to the intensity of the lateral forces at the elevation of the joint under investigation. (See example, Art. 16.)

Maximum and Minimum Stresses. Both normal and shearing stresses exist at every plane passing through any point of an elastic solid which is in equilibrium under the influence of external forces. These stresses vary in magnitude for the same point according to the direction of the plane and there invariably exists one plane on which the normal stress intensity is a maximum, and another plane perpendicular to the first, on which the normal stress intensity is a minimum. These limiting values of the normal stresses are termed principal stresses and they can be determined both as to magnitude and direction if the normal and shearing stresses at a certain point are known on two planes at right angles to each other. At any point of the horizontal joint investigated, let S and p' be the intensities of the normal stresses on horizontal and vertical planes and let q be the intensity of the shear on these two planes, both kind of stresses being determined by the methods given above (compressions taken as positive and tensions as negative), then the magnitude of the principal stresses at the same point is given by the formula:

$$f = \frac{S + p' \pm \sqrt{(S + p')^2 - 4(Sp' - q^2)}}{2}$$

the positiv sign before the radical furnishing the maximum and the negativ sign the minimum principal stress.

Let θ be the inclination of the principal stress to the horizontal, then

$$\cot\theta = (f - p')/q$$

These formulas show that even in the case when the resultant cuts the horizontal joint within the middle third, in other words when S is compression along the whole joint, tensile stresses will occur at some points in a certain direction, if $Sp' < q^2$. According to Levy in order that tensile stresses should entirely be eliminated, the design should be such that not only shall the resultant lie within the middle third of the joint, but that S_2 (minimum normal stress) shall be not less than the intensity of the horizontal components

of the lateral forces at the elevation of the joint investigated. It can also be shown that Sp' is always greater than q^2 at the toe of retaining walls and dams designed for the requirement that the resultant shall fall within the middle third of the horizontal joints and, therefore, no tension can exist at and near the face of such walls. The maximum shearing stresses in walls with vertical face and back and vertical loading only are inclined 45° to the vertical and equal to $S/2$ for every point of the horizontal joint, S being the intensity of the constant normal stress upon the joint. For other methods of loading according to Wilson and Gore, the maximum shearing stress can be approximately expressed by the formula $q_{max} = S/2 \cos^2 \phi$ where S is the normal stress on the horizontal plane at the point considered and ϕ the angle between the resultant R and the vertical. According to Bouvier, the maximum stresses in dams of the usual cross-section are about 13/9 times the stresses on horizontal joints. This rule gives good approximate values also for other types of walls for preliminary design.

The above discussion of stresses and the formulas do not take in account the factor of lateral deformation of the masonry which is probably very small. (See example, Art. 16.)

Joints in Arches. The above discussion has been made for the horizontal joints of walls and dams, but it applies equally well to the radial joints between the voussoir of stone arches. Against such a joint there acts a resultant force R, and the component of this parallel to the joint gives the total shear H, while the component normal to the joint may be called N. Then the preceding formulas apply to such a radial joint if W is replaced by N, and the distributions of the normal stresses are exactly like those shown in Figs. 13–16.

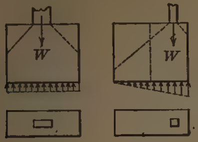
7. Local and Eccentric Loads

A Bridge Pier which sustains only vertical loads whose resultant coincides with the axis of the pier has the compressive unit stresses uniformly distributed on the base. The least area for the base is $A = W/S$, where W is the total vertical load on that base and S is the allowable unit stress from Art. 1. Practical considerations, however, usually necessitate a larger section. Small piers in buildings should have this least area, unless their length is such that columnar action exists, and then S must be taken smaller. (Art. 14.)

Under a Concentrated Load on the top of a pier, the compressive unit stress may be taken higher than for a section of the pier lower down. If the area under local compression is 0.3 of the total area of masonry over which the compression is distributed, the values in Art. 1 may be increased 60%; if that area is 0.1 of the masonry area, those values may be increased 200%. For other values of the ratio r the increase may be made in direct proportion, or, for r between 0.1 and 0.3, percent increase = $270 - 700\,r$, for r between 0.3 and 1.0, percent increase = $600\,(1 - r)/7$. These percentages apply when the concentrated load is at the middle of the top of pier as at (*a*) in Fig. 17; thus, for a steel column having a base 15 by 15 inches and a masonry pier 24 × 24 inches the value of r is 0.4, and the allowable S of Art. 1 may be increased 51 percent under the base of the column.

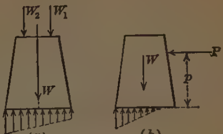
An Eccentric Load is one placed so that its line of action does not coincide with the gravity axis of the pier as at (*b*) in Fig. 17. Here the distributions of unit stresses on the base are like those shown in Art. 6, and when the eccentricity e is large enough the stresses on one side may be tensile. For this case the working unit stress S cannot be reduced to the full extent indicated in the last paragraph, and when the load is close to the edge of the pier top no reduction should be made. A specially unfavorable condition is that in the first diagram of Fig. 19*a*, where there is an eccentricity in both horizontal directions, so that the opposite corner of the pier should be anchored down in order to prevent it from rising or from cracking by tension.

The inclined lines in Fig. 17*a* show the manner in which the local compression spreads out until it finally becomes uniformly distributed over the horizontal cross-section; the inclination of these lines is about 45°. For the eccentric load in Fig. 17*b*, one of the inclined lines quickly reaches the edge of the pier, indicating that the unit stress under the column must be taken lower for an eccentric load than for a central one.



(*a*) (*b*)

Fig. 17



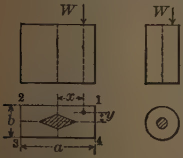
(*a*) (*b*)

Fig. 18

A Bridge Pier under Vertical Loads (Fig. 18*a*) is subject to non-uniform compression on the base when the two adjacent spans of the bridge differ in length or when only one of two equal spans is covered with live load. Thus in Fig. 18*a* let W be the weight of the pier itself above the base, and W_1, and W_2 the weights brought upon the top of the pier from the two spans of the bridge, these being applied at the distance c from the middle of the top. Here the resultant vertical pressure cuts the base at a distance e from the center which is found from $e = c\,(W_1 - W_2)/(W + W_1 + W_2)$, and the distribution of compressive unit stresses on the base is the same as in Art. 6. When $W_1 > W_2$ in Fig. 18*a*, this point is at the right of the center; when $W_1 < W_2$, it is at the left of the center. The proper formula of Art. 6 enables the maximum compressive unit stress to be found.

Wind blowing on a tower or chimney, as shown in Fig. 18*b*, causes a similar eccentricity of the resultant on the base; for this case, the value of $e = Pp/W$. Wind blowing at right angles to the line of a bridge causes a similar longitudinal eccentricity on the base of a pier, while the lateral eccentricity due to live loads is the same as given in last paragraph. Two horizontal wind forces may act on a bridge pier, P_1 for wind blowing against the bridge spans and P_2 for wind against the pier itself; here the above equation also holds if Pp is replaced by $P_1p_1 + P_2p_2$.

The Kern is an area on the base of the pier within which the resultant force that acts on the base must lie in order that no part of the base may be subject to tension. For a rectangular section with sides a and b, the kern is a rhombus which has the diagonals $\frac{1}{3}a$ and $\frac{1}{3}b$, the corners of this rhombus pointing toward the middle of the sides of the rectangle as shown in the plan of Fig. 19*a*. For a square with side d, the kern is a smaller square which has the diagonal $\frac{1}{3}d$ and whose sides are parallel to the diagonals of the given square. To prevent tension on any corner or side of the base of a masonry pier, wall, or dam, the line of action of the resultant must cut the base within the kern. For a circular section of diameter d the kern is a smaller concentric circle of diameter $\frac{1}{4}d$ as in Fig. 19*b*. The resultant is the weight W above the base in case no lateral force acts upon the structure, otherwise it is the R of Art. 6, which is found by combining W and H.



(*a*) (*b*)

Fig. 19

A Rectangle with sides a and b (Fig. 19d) is the most common case. Let x and y be the eccentricities with respect to axes of the rectangle parallel to the sides a and b. Then the unit stresses at the four corners are

$$S_1 = \frac{W}{ab}\left(1 + 6\frac{x}{a} + 6\frac{y}{b}\right) \qquad S_3 = \frac{W}{ab}\left(1 - 6\frac{x}{a} - 6\frac{y}{b}\right)$$

$$S_2 = \frac{W}{ab}\left(1 - 6\frac{x}{a} + 6\frac{y}{b}\right) \qquad S_4 = \frac{W}{ab}\left(1 + 6\frac{x}{a} - 6\frac{y}{b}\right)$$

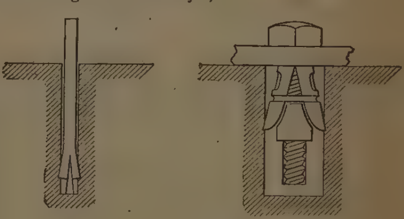
When W lies within the kern all of these are compression; the formulas hold, however, even if W is without the kern, provided the masonry can take tension near the corners and edges.

For a Circular Cross-Section of diameter d, the maximum and minimum unit stresses, due to a load having the eccentricity e, are

$$S_1 = \frac{W}{A}\left(1 + 8\frac{e}{d}\right)$$

$$S_2 = \frac{W}{A}\left(1 - 8\frac{e}{d}\right)$$

in which A is the area of the circle. The unit stress S_2 is tensile if the load is without the kern, but the formula gives it correctly when the masonry can resist tension. The radius of the kern is $\frac{1}{8}d$ (Fig. 19b).



Fox Bolt *Expansion Bolt*

Fig. 20

If the material is unable to develop tensile stresses and W acts outside the kern in one of the main axes of the figure comprising the horizontal joint of the pier, the following methods to determine the distribution of normal stresses are recommended.

For a pier rectangular in plan as in Fig. 19 and W being applied on the axis parallel to side a at a distance r from the nearest short side of the joint, $S_1 = 2W/3rb$.

For a horizontal joint having any symmetrical figure (as, for instance, a chimney section), according to Mohr the following construction will give the compressive stress distribution, Fig. 19c. A is the point of application of W lying on the symmetry axis BMB_1 of figure F. Draw an equilibrium polygon for area F with an arbitrary pole distance H. Draw a perpendicular to axis BB_1 at A to intersect the tangent $B'O$ in A'. Draw $B'N$ so that the area of triangle $B'A'N$ shall be equal to area $B'CN$, in other words, that the two shaded areas shall be equal. Then the line NN drawn at right angle to BB_1 is the zero line from which the compressive stresses increase in proportion to the distances. Make $B'D = W/H$ = weight : Area (H being measured in the same unit as F), draw DE parallel to $B'N$, then for the point B, furthest from NN, the unit stress $S_1 = EE_1$ which is drawn parallel to BB_1 from point E. EE_1 is measured in the same unit as W/H.

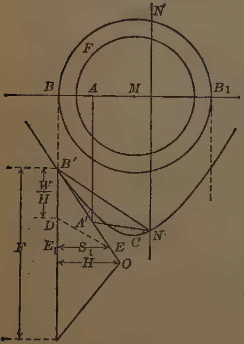


Fig. 19c

If the point of application of the resultant does not lie on an axis of symmetry of the joint, or if the joint has an unsymmetrical figure, the determination of the purely compressive stresses becomes very complicated. The following method is recommended. Connect the point of application with the center of gravity of the joint and find in this direction the conjugate axis of the ellipse of inertia of the figure. Then the line of zero stress will be parallel to the so-found direction and the location of same should be determined by trial, remembering that the stresses will be proportional to the distance from the zero line and that the center of gravity of the intensity wedge (the volume of which is equal to the vertical component of the resultant) must fall in the same vertical with the point at which the resultant cuts the joint.

Anchor Bolts. Masonry structures sometimes are required to be anchored to a rock foundation, especially where tension may occur under eccentricity of loading. A fox bolt is a straight iron bar, split for about an inch at the lower end, the split allowing an iron wedge to enter as the bolt is driven down (Fig. 20); the end of the bolt then expands and is held in the hole by friction aided by cementing material. The expansion bolt (Fig. 20) is the best form of anchor bolt for fastening timber or steel members to masonry or rock. Ordinary straight round rods are also used, they being cemented in place with lead, babbitt metal or hydraulic cement, the last being most common.

M. Ferret found. for plain round rods in 1 : 2 : 4 concrete, that the holding capacity was 237 lb per sq in of embedded surface. A. N. Talbot found 424 and W. O. Withey 452 lb per sq in. Plain square rods give slightly higher values. A working unit stress of 75 lb per sq in is recommended for plain round rods in 1 : 2 : 4 concrete, and for flats one-third of this value.

For fox and expansion bolts E. S. Wheeler found 264 lb per sq in when embedded in 1 : 2 portland mortar, 843 lb per sq in when in sulphur and 485 lb per sq in when in lead. A working unit stress of about one-fifth of these values should be used. Sulphur should not be used where exposed to rain or other water, as acid will be formed which rusts the steel and often disintegrates the stone near the top of the hole.

MASONRY WALLS AND DAMS

8. Principles of Stability

Definitions. A Bearing Wall is a vertical wall which supports a building and is subject mainly to vertical pressures. A Retaining Wall is subject to its own weight and to the lateral pressure of earth which is deposited behind it after it is built. A Brest Wall, or face wall, is one built to prevent the fall of an undisturbed bank of earth. A Buttress is a projection of masonry built into the front of the wall to strengthen it, while a Counterfort is a projection of masonry built into the back of the wall. Relieving Arches (Fig. 21) are those built into the back of the wall, with their axes perpendicular to the wall, in order to strengthen it against lateral pressure, while they also resist overturning by their weight and the weight of the earth resting upon them.

A dam is a structure to obstruct the flow of a stream or to impound water, thus creating a storage of water, or head, or both. Masonry dams are, as a rule of the gravity type, resisting the lateral water pressure by virtue of their weight. Arched masonry dams are dams curved in plan and for their resistance against water pressure rely partly or entirely upon arch action.

The front of a wall or dam is the face seen after it is finished, while the back is the face which is acted upon by earth or water. The lowest point of the front, at the top of the foundation, is called the toe, while the lowest point of the back is called the heel.

The Batter of the face of a pier, wall or dam is its inclination to the vertical, measured by the ratio of horizontal to vertical projection, which is usually exprest in inches per foot of height. The following table gives the angle θ

(Fig. 10) between the inclined face and the vertical for various batters and also trigonometric functions of those angles:

Angles and Functions for Batters

Batter. Inches per ft	Angle θ	Sin θ	Cos θ	Sec θ	Tan θ
0	0′ 00′	0.0000	1.0000	1.0000	0.0000
½	2 23	0.0416	0.9991	1.0009	0.0417
1	4 46	0.0831	0.9965	1.0035	0.0833
1½	7 08	0.1242	0.9923	1.0078	0.1250
2	9 28	0.1645	0.9864	1.0138	0.1667
2½	11 46	0.2039	0.9790	1.0210	0.2083
3	14 02	0.2425	0.9702	1.0308	0.2500
3½	16 15	0.2798	0.9601	1.0416	0.2915
4	18 26	0.3162	0.9487	1.0541	0.3333
4½	20 33	0.3510	0.9364	1.0680	0.3750
5	22 37	0.3846	0.9231	1.0825	0.4166
5½	24 37	0.4166	0.9091	1.1000	0.4582
6	26 34	0.4472	0.8944	1.1181	0.5000

The last column in this table gives the batter exprest as a decimal, that is, the horizontal projection for a vertical projection of unity. The column next to the last gives the length of the inclined face for a vertical projection of unity

Failure of a masonry wall or dam usually occurs in three ways: (1) by sliding along the joint at the base, (2) by overturning or rotating, (3) by crushing of the masonry. In order that failure may not occur, the design should be so made that the structure may have such size and weight that sliding, rotation or crushing cannot occur.

Section *Elevation*

Fig. 21

Failure of retaining walls by tension seldom if ever, occurs, but such was the cause of some of the most disastrous failures of dams. The failure of the Habra dam in Algiers and that of the Bouzey dam in France was due primarily to tension fracture and subsequent shearing action. (Unwin, Min. Proc. Inst. Civ. Eng., Vol. 172, page 160.)

The most common cause of failure of walls and dams is a defective foundation. A washout around the foundation of a wall due to excessive rainfall may cause its destruction. Water entering beneath the base of a masonry dam or around its ends may impair the foundation as well as exert an upward pressure. In this section it is assumed that the foundations are securely built and that precautions have been taken to prevent the action of water beneath it.

Sliding Stability is secured by giving the structure a sufficient weight so that there is no danger of motion along a joint at the base or any other horizontal joint. For the horizontal joint in Fig. 22*a* let H be the horizontal pressure and W the sum of all the vertical weights above the joint. Motion occurs when $H = fW$ where f is the coefficient of friction. In order that there may be no motion, let n be a number greater than unity, called the SAFETY FACTOR, then if W is sufficiently large so that $nH = fW$, stability is secured. A common value of n for use in designing is 2, so that the formula $2H = fW$ secures a proper degree of security, and then $W = 2H/f$. For an existing wall or dam the safety factor is $n = fW/H$.

For an inclined joint (Fig. 22*b*) let α be its angle with the horizontal, N the

normal pressure on the joint and F the force acting parallel to it; then for motion about to begin $F = fN$, and for stability $nF = fN$. For this case

$$W = H\ (n - f \tan \alpha)/(n \tan \alpha + f)$$

gives sufficient security if the safety factor n be taken as 2.

Another method of consideration is by means of the angle which the resultant R makes with the normal to the joint. In Fig. 22*b*, motion will occur when the angle β equals the angle of repose ϕ, for masonry upon masonry. (Art. 3.) For stability the angle β should be less than ϕ, and proper security requires that $n \tan \beta = \tan \phi$, or $\tan \beta = f/n$.

Stability against Rotation is secured by giving the wall or dam such size and weight that R, the resultant of all the external forces above the joint, cuts the base AB well within the joint. (Fig. 22*a*.) When R passes thru A failure occurs by rotation, and if R passes near A the material may begin to crush so that rotation is imminent. Some writers require that R shall cut AB at the edge of the middle-third or even within same; others require that R shall cut the base at A when the acting horizontal force H is doubled. The first method is used for dams and the second for walls. The SAFETY FACTOR against rotation is defined as the number by which the horizontal pressure must be multiplied in order that R shall pass through the toe. Let l_1 and l_2 be lever arms of H and W with respect to A in Fig. 22*b*; then the safety factor is $n = Wl_2/Hl_1$ for an existing wall or dam. For designing the value of W must be

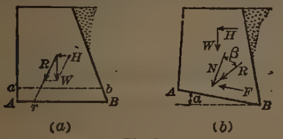


Fig. 22

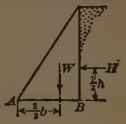


Fig. 23

such that $W = nHl_1/l_2$ and n may be taken as 2 for walls and between 2 and 3 for dams.

For a triangular wall or dam with vertical back (Fig. 23) a safety factor of 2 causes R to cut the base at the edge of the middle-third. For a trapezoidal section of top width a and base width b and a vertical back, the safety factor required in order that R may cut the edge of the middle-third is $n = 2 + a^2/(b^2 + ab - a^2)$, which shows that n should be 3 for a rectangular dam and between 2 and 3 for a trapezoidal one.

Stability against Crushing is secured by making the greatest compressive unit stress S_1 much less than the crushing strength of the masonry. For S_1 the safe working value given in Art. 1 should be used in design, the wall or dam being given such size that this shall not be exceeded.

The Resistance Line is a line drawn within the cross-section of a wall or dam which shows for every horizontal joint the point where the resultant of the external forces above it cuts the joint. Let ab in Fig. 22*a* be any horizontal joint in a masonry wall, H the resultant of the horizontal forces above ab and W the weight of the masonry above ab; then the resultant R of H and W cuts ab at a point r. If ab is conceived to move up and down, the point r describes the line called the resistance line. Figs. 24–27 show four forms of walls in each of which two broken lines are drawn dividing each horizontal joint into three equal parts, the middle part being the "middle-third." (Art. 6.) Fig. 24 is a rectangular pier or wall subject only to its own weight, and the vertical line rr is the

resistance line. Fig. 25 is a vertical rectangular wall subject to the horizontal pressure of earth or water, and the curved line *rr* is the resistance line. Fig. 26 is a triangular wall with vertical back under horizontal pressure, and the straight line *rr* which cuts the base at the middle is the resistance line. Fig. 27 is a

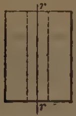


Fig. 24

Fig. 25

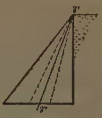


Fig. 26

Fig. 27

trapezoidal wall with vertical back under horizontal pressure, and the resistance line *rr* is curved.

The conditions of stability discussed in this article apply not only to the base joint but also to every horizontal joint of the cross-section of a wall or dam. In the case of stone and brick structures, the shearing strength of the joints is disregarded, and friction alone is assumed to resist sliding. In the design of concrete walls, including dams, the tendency to slide is assumed to be resisted by the shearing strength of the masonry. Both friction and shear should not be assumed to be simultaneously available to resist sliding because frictional resistance will not come into action until the section has failed in shear.

The conditions of stability (against sliding, rotating and crushing) here discussed apply also to radial joints of masonry arches, where a normal pressure N acts upon the joint and a shearing force H along the same.

9. Moments for Trapezoidal Sections

The Overturning Moment M for a wall with vertical back is $P \times \frac{1}{3}h$, where P = horizontal pressure and $\frac{1}{3}h$ is lever arm with respect to the toe For a level bank of earth Art. 4 gives for this overturning moment

$$M = \tfrac{1}{6}wh^3 \tan^2 (45° - \phi/2)$$

and values of M are given in table (A) on p. 689 for earth having w = 100 lb per cu ft and for various values of ϕ. These values of M are in thousands of pound-feet. This table is useful in making approximate computations and estimates for walls with backs nearly vertical. It does not apply to banks which have upper surfaces inclined.

The table may be used to find M for earth less than 100 lb per cu ft by multiplying the tabular values by the ratio of the unit-weight to 100; for example, for cinders weighing 45 lb per cu ft, multiply by 0.45. Interpolation may be made for earths having slopes different from those given. For a height of wall intermediate between the heights given in the table, take the overturning moment for a wall 100 ft high and multiply it by the ratio of the cube of the height of the desired wall to the cube of 100. For example, for a wall 55 ft high and a slope of repose 1.33 : 1, the overturning moment = 4 167 000 $\times$ (55/100)^3 = 692 000 lb-ft.

For a Loaded Level Bank of Earth (Art. 4, Fig. 5*b*), the overturning moment M may be found by multiplying the tabular values by $1 + 3v/100\ h$, where v is the weight of the load per square foot of level surface and h equals height of wall in feet. When the load consists of merchandise or building materials back of a wall, v may be taken as from 200 to 400, the higher the wall the greater to be the value of v. For walls retaining railroad embankments v should be taken as 500. The tables are not directly applicable to a dock wall or to any wall in which the retained earth is saturated with water.

(A) Overturning Moments for Retaining Walls in Units of 1000 Pound-Feet

Angle of repose of earth	Slope of repose of earth	Height of wall in feet									
		10	20	30	40	50	60	70	80	90	100
45° 00′	1 : 1	2.9	22.9	77.1	182.8	357.1	617.0	980	1463	2083	2 857
39 50	1.2 : 1	3.7	29.2	98.6	233.6	456.3	788.4	1252	1869	2661	3 650
37 20	1.3 : 1	4.1	32.4	109.4	259.2	506.3	874.8	1389	2074	2953	4 050
36 50	1.33 : 1	4.2	33.3	112.5	266.7	520.8	900.0	1429	2133	3038	4 167
35 30	1.4 : 1	4.4	35.4	119.3	282.9	552.5	954.7	1516	2263	3222	4 420
33 40	1.5 : 1	4.8	38.2	128.8	305.3	596.3	1030	1636	2442	3477	4 770
26 40	2 : 1	6.4	50.8	171.3	406.1	793.1	1371	2176	3249	4626	6 345
18 30	3 : 1	8.6	69.1	233.3	553.0	1080	1866	2964	4424	6299	8 640
For water		10.4	83.3	281.3	666.7	1302	2250	3573	5334	7594	10 417

(B) Resisting Moments for Walls of Type B in Units of 1000 Pound-Feet.

Ratio of base to height	Height of wall in feet										
	10	15	20	25	30	40	50	60	70	80	90	100
0.33	8.2	26.5	65.3	127.6	220.5	522.7	1021	1762	2802	4182	5954	8 168
0.34	8.7	28.1	69.1	135.5	234.1	554.9	1084	1873	2974	4439	6320	8 670
0.35	9.2	29.8	73.5	143.5	248.1	588.0	1149	1985	3151	4704	6697	9 188
0.36	9.7	31.5	77.8	151.9	262.4	622.1	1215	2100	3334	4977	7086	9 720
0.37	10.3	33.3	82.1	160.1	277.2	657.1	1283	2218	3522	5257	7485	10 268
0.38	10.8	35.1	86.6	169.2	292.4	693.1	1354	2339	3715	5545	7895	10 830
0.39	11.4	37.0	91.3	178.2	308.0	730.1	1426	2464	3913	5841	8316	11 408
0.40	12.0	38.9	96.0	187.5	324.0	768.0	1500	2592	4116	6144	8748	12 000
0.41	12.6	40.0	100.4	197.0	240.4	806.9	1576	2723	4324	6455	9191	12 608

(C) Resisting Moments for Retaining Walls of Type C in Units of 1000 Pound-Feet

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.1	60.2	199.0	469	909	1567	2482	3 692	5 374	7 208
0.40	10.3	78.0	259.0	609	1185	2042	3236	4 824	6 861	9 403
0.50	15.8	120.7	402.2	933	1846	3113	5045	7 527	10 759	14 670
0.60	22.3	172.7	577.0	1361	2652	4574	7254	10 819	15 393	21 103
0.70	30.1	205.1	781.9	1849	3618	....	....	.....	.....	.....

(D) Resisting Moments for Walls of Type D in Units of 1000 Pound-Feet.

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.6	61.0	193.9	444.0	848	1443	2265	3352	4 740	6 465
0.40	11.0	78.0	249.0	572.0	1095	1866	2933	4344	6 147	8 390
0.50	16.5	118.0	379.5	876.0	1683	2874	4526	6712	9 509	12 990
0.60	23.0	166.0	537.0	1244	2395	4098	6461	9592	13 599	18 590
0.70	30.5	222.0	721.5	1676	3233	....	....	....	.....	.....

(E) Resisting Moments for Walls of Type E in Units of 1000 Pound-Feet

Ratio of base to height	Height of wall in feet									
	10	20	30	40	50	60	70	80	90	100
0.35	8.4	62.7	205.6	480.0	929	1595	2521	3 751	5 326	7 290
0.40	10.8	81.0	266.5	623.3	1208	2075	3282	4 883	6 915	9 498
0.50	16.5	124.7	412.0	966.0	1874	3224	5103	7 599	10 799	14 790
0.60	23.3	177.7	589.0	1383	2687	4625	7324	10 911	15 510	21 248
0.70	33.3	240.0	797.5	1875	3645	....	....	.....	.....	.....

Interpolation is not satisfactorily made except approximately by an arithmetic method in tables (B), (C), (D), (E), for heights intermediate between those given. For a case of this kind, it is recommended that interpolation be made by help of a curve drawn thru plotted points whose ordinates represent values taken from the table.

The Resisting Moment of a wall or dam is defined as its weight multiplied by the lever arm of that weight with respect to the toe. Tables (B), (C), (D), (E) give resisting moments in thousands of pound-feet for masonry walls 1 foot long of four types, and for various ratios of height to base and for top width between 2 and 3 feet. These types, also called B, C, D, E, are shown in Figs. 28-31. Walls B and C may have a vertical face or one with a batter not exceeding 1 in 3. The weight of the masonry was taken at 150 lb per cu ft in computing these tables. If the unit weight w of the masonry is materially less, multiply the tabulated resisting moments by $w/150$. The resisting moment will be called M_1. The weight of earth upon the back of walls, type C and E, was taken as effectiv. Assumed weight of earth 100 lbs.

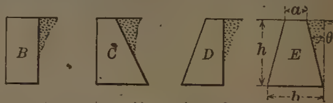


Fig. 28 Fig. 29 Fig. 30 Fig. 31

The Unit Stress S_1 at the toe of a trapezoidal wall or dam is given approximately in the following table. It applies to each of the four types (Figs. 28-31) when the resisting moment M_1 is twice as great as the overturning moment M. The values are in pounds per square foot when h is taken in feet.

(F) Unit Stress S_1 for Toe of Wall

Type of wall	Height of wall in feet		
	5 to 10	10 to 20	20 to 100
B	400 h	400 h.	400 h.
C	392 h	392 h to 378 h	378 h to 362 h
D	263 h	263 h to 198 h	195 h to 158 h
E	319 h	319 h to 272 h	270 h to 237 h

Example. To find the compressive unit stress at the toe of a wall of type B, the height being 20 ft. The table gives $S_1 = 400 \times 20 = 8000$ lb per sq ft = 56 lb per sq in. When the value of S_1 approaches the working value (Art. 1), the foundation must be widened by an offset on the front side.

Formulas for Use with the Tables. Let a = width of top, b = width of base, h = height, e = distance from middle of base to point where resistance line cuts it; all in feet; f = coefficient of friction.

Horizontal pressure = $P = M/\frac{1}{3}h$.
Weight of wall, types B and D = $W = 75\,h\,(a + b)$.
Weight of wall and earth on back, type C = $W = 25\,h\,(a + 5\,b)$.
Weight of wall and earth on back, type E = $W = 50\,h\,(a + 2\,b)$ approx.

Eccentricity e of resultant for type B = M/W.
Eccentricity e of resultant for type C = M/W approximately.
Eccentricity e of resultant for type D = $M/W - b/6$ approximately.
Eccentricity e of resultant for type E = M/W approximately.
Unit compression at toe = $S_1 = 2\,W/3(\frac{1}{2}b - e)$.
Safety factor against rotation = $n = M_1/M$.
Safety factor against sliding = $n = fW/P$.

Centers of Gravity. Let a be the top width, b the base width and h the height of any trapezoid whose back face makes the angle θ with the vertical, this face being inclined forward as in Fig. 31. Then the distance t from the toe to the vertical passing thru the center of gravity of the trapezoid is given by

$$t = \tfrac{2}{3}b - \tfrac{1}{3}h\tan\theta - \frac{a(a + h\tan\theta)}{3(a+b)}$$

When the back is vertical tan θ is zero. For a rectangular section $b = a$ and $t = \frac{1}{2}a$. For a triangular section $a = 0$ and $t = \frac{2}{3}b$. When the back inclines backward tan θ is to be taken negativ.

10. Investigation of Retaining Walls

To Investigate a wall is to determine its degree of stability against sliding, rotation, and crushing. Often this may be done by inspection in the case of an existing wall, for if it has stood for many years and no signs of failure are seen, it is to be regarded as safe. The problem, however, generally occurs in the case of a proposed design, where all dimensions and data are given and it is required to ascertain whether these satisfy the conditions of stability. The process of investigation should determine at least these three things: (1) the factor of safety against sliding on the base, (2) the point where the resultant cuts the base, (3) the compressive unit stress S_1 which exists at the toe.

The height of retaining walls seldom exceeds 30 ft and a height of 60 ft is about the limit. The condition of stability against rotation, with a proper safety factor, will usually insure against excessive stresses; further, in retaining walls it is not imperative that the whole joint should be under compression. It is not necessary, therefore, to apply for retaining walls, the investigation discussed in the latter part of Art. 6. The vitally important theoretic determination is the maximum unit pressure upon the foundation bed at the toe.

A Graphic Analysis is often convenient and satisfactory. First, the pressure P acting on the back of the wall is to be determined by Art. 4, using for the earth which is to be put behind the wall values of w and ϕ which are most unfavorable to stability. Second, the weight W of the wall and the weight of earth resting upon the back, if any, is computed and its line of action determined, this passing thru the center of gravity of the combined cross-section of the wall and earth upon it. Then a drawing of this cross-section is made to scale and the resultant R of P and W determined, Fig. 22a; this gives the point r where the resultant cuts the base or the lowest point of the resistance line. The ratio W/P may be computed, or be scaled from the drawing by taking W as unity, and then $n = fW/P$ is the factor of safety against sliding. The base b being divided into three equal parts, it is at once seen if the resultant falls within the middle-third, in which case there is ample security against rotation. Lastly, the distance between the center of the base and the point r is measured, and this gives e (Art. 6), from which S_1 is computed, and if S_1 does not exceed the allowable unit stress there is full security against crushing.

For a wall with vertical back, top width a and base width b, the distance from the toe to the vertical line thru the center of gravity is $(2b^2 + 2ab - a^2)/3(a + b)$.

A Better Graphic Method is that shown in Fig. 32, where the force and equilibrium polygons are used. W_1 is the weight of the rectangular part of the wall and W_2 that of the triangular part. The force polygon is seen on the right, R_1 being the resultant of P and W_2, and R the resultant of P and $W_1 + W_2$. The equilibrium polygon is seen within the wall, P being produced to m_2, thru which m_2m_1 is drawn parallel to R_1; then thru m_1 the line m_1r is drawn parallel to R, thus determining r, the point in the line of resistance where the resultant cuts the base. Finally e is found by measuring the distance from r to the middle of the base. This method, altho here illustrated for only two weights, is a general one readily applied to Fig. 33 or to any number of weights (Section 12). It obviates the necessity of computing the line of action of the resultant of the given weights or of finding it by a geometric construction.

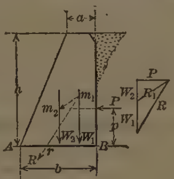


Fig. 32

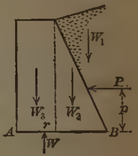


Fig. 33

The Analytic Method computes e by the principles of mechanics, the cross-section being divided into rectangles and triangles whose centers of gravity are known. For example, take the wall with vertical back in Fig. 32 which is held in equilibrium by the forces shown, W_1 being the weight of the rectangle, W_2 that of the triangle, and $W_1 + W_2 = W$ the vertical reaction due to the total weight, its point of application being at a distance e from the center of the base or at $\frac{1}{2} b - e$ from the toe A. Here the equation of moments with respect to the toe is

$$Pp - W_1(b - \tfrac{1}{2} a) - W_2 \tfrac{2}{3}(b - a) + W(\tfrac{1}{2} b - e) = 0.$$

Inserting for W_1 and W_2 their values vah and $\frac{1}{2} v(b - a) h$, where v is the weight of a cubic unit of the masonry, there results

$$e = \tfrac{1}{2} b - [vh(2ab + 2b^2 - a^2) - 6Pp] / 3vh(a + b)$$

which is a general formula for computing e for a trapezoidal wall or dam with vertical back. The weight W and the safety factor against rotation are

$$W = \tfrac{1}{2} vh(a + b) \qquad n = vh(2ab + 2b^2 - a^2)/6Pp.$$

Example. A wall of good rubble with a vertical back retains a level bank of earth, the data being $w = 100$ lb per cu ft, $\phi = 34°$, $h = 18$ ft, $a = 2$ ft, $b = 5$ ft, $v = 140$ lb per cu ft, $f = 0.65$. From Art. 4, $P = 4580$ lb per lin ft of wall and $p = 6$ ft. Then the formula gives $e = 2.47$ ft, so that the resultant comes outside the middle-third. The safety factor against rotation is $n = 1.02$, which is too small. For sliding, $W = 8820$ lb, and $n = 0.65 \times 8820/4580 = 1.25$, which is too low a degree of security, since n should be at least 2 (Art. 8). The thickness of this wall is too small to give proper security against either sliding or rotation.

For the Trapezoidal Wall in Fig. 33, let W_1 be the weight of the triangle of earth above the inclined back, W_2 and W_3 the weights of the masonry, and let p_1, p_2, p_3 be their lever arms with respect to the toe A, while p is the lever arm of the horizontal pressure P. Then the equation of moments is

$$Pp - W_1p_1 - W_2p_2 - W_3p_3 + W(\tfrac{1}{2}b - e) = 0$$

the solution of which gives the value of e. Here W is the total vertical force $W_1 + W_2 + W_3$. The factor of safety against sliding is $n = fW/P$, and that against rotation is $(W_1p_1 + W_2p_2 + W_3p_3)/Pp$.

After e is found, the compressive unit stress S_1 at the toe is ascertained by

the proper formula of Art. 6. For the example on page 692, where the resultant comes without the middle-third, $S_1 = 2W/3(\frac{1}{2}b - e)$ = 196 000 lb per sq ft = 1360 lb per sq in, which is much greater than the allowable.

Approximate investigations may be quickly made by the help of the tables in Art. 9 if interpolation is not required. Example: Let h = 60 ft, b = 24 ft, a = 3 ft for a wall of type C which retains earth having a slope of 1.3 to 1. Table (A) gives M = 874 800 lb-ft, and table (C) gives M_1 = 2 042 000 lb-ft, so that the safety factor against rotation is $n = M_1/M_2 = 2.3$. Then P = 874 800/20 = 43 740 lb, $W = 25 \times 60\ (3 + 5 \times 24)$ = 184 500 lb. The factor of safety against sliding if f = 0.65 is n = 184 500 × 0.65/43 740 = 2.8. Lastly, the compressive unit stress at the toe, from table (F), is approximately 370 × 60 = 22 230 lb per sq ft. Table (F) was made for a factor of safety against rotation of 2, and should not be applied, except for design or roughly approximate pressures, when factor of safety exceeds or is less than 2.

To find the resistance against sliding upon the foundation bed, by the help of the tables of Art. 9, use $F = f(W_1 + M_1/Kb)$, in which F = resistance in pounds, M_1 = resisting moment taken from the proper table, b = width in feet of base of wall as previously determined, W_1 = weight of foundation in pounds, f = coefficient of friction of masonry upon the material of which the foundation bed is composed; also K = 0.50 for wall B, K = 0.47 for wall C, K = 0.66 for wall D, and K = 0.57 for wall E.

Bulletin No. 108 of the American Railway Engineering and Maintenance of Way Association, February, 1909, gives the following information relative to existing retaining walls. R is the ratio of base to height.

Data concerning Retaining Walls

Character of load	Height, ft	Base, ft	Batter	R	Remarks
*Railway, level top......	17.6	10.0	vertical	0.57	has not moved out at top
*Railway, level top......	18.0	10.0	vertical	0.55	has not moved out at top
*Railway, level top......	21.7	10.0	vertical	0.46	moved out 2½ ins
*Railway, level top......	14.2	6.0	vertical	0.43	moved out 4 ins
*Railway, level top......	14.7	6.0	vertical	0.40	moved out 11 ins
Railway, level top......	30.0	8.5	1 : 24	0.43	moved out several ins
Railway, level top......	23.5	8.6	?	0.37	moved out 15 ins
†Railway, small surcharge	18.2	8.2	vertical	0.45	fell over.
‡No track, level top......	21.0	9.3	1 : 24	0.44	no movement
No track, level top.....	21.8	9.5	1 : 48	0.46	moved out 5 ins
Railway, level top......	17.0	7.2	vertical	0.43	moved out 7½ ins
Railway, level top......	24.0	10.3	vertical	0.43	no movement
Railway, level top......	24.0	11.3	1 : 24	0.47	moved out 4 ins

* Toe pressure excessive. † Rock foundation. ‡ Pile foundation.

11. Design of Retaining Walls

General Data. The top width of a retaining wall should not be less than 2 ft where frost penetrates into the ground more than 3 ft, and not less than 1.5 ft for smaller penetrations. For railway walls these widths should be increased to 3 ft and 2.5 ft. The front face of the wall should have a batter of at least 1 in 24, preferably 1 in 12. The foundation masonry should be offset at least 6 inches both at heel and toe. Offsets to reduce the unit compression on the foundation bed should always be at the toe (Fig. 34), never at both heel and toe, since the compression at the heel will be small under the horizontal pressure of the earth. A factor of safety of 2 against rotation is recommended, altho this always brings some tension in the back joints over the heel.

Fig. 34

Convenient Approximate Data on retaining walls of type C for preparation of estimates and use in the field. Let h = the height of a wall in feet, P = the horizontal pressure in lbs applied at $h/3$ above the heel, for a wall one foot long, W = the weight of the wall including earth upon its back in lbs for a wall one foot long, W_1 = weight of foundation masonry in lbs for a wall one foot long, b = width in feet of base of the wall at top of foundation, b_1 = width in feet of bottom of foundation masonry, S_1 = the pressure upon the toe of the wall in tons per sq ft, S_0 = the safe load upon the foundation base in tons per sq ft, f_1 = the coefficient of friction of masonry upon rock = 0.66, f_2 = the coefficient of friction of masonry upon sand and gravel = 0.50, f_3 = the coefficient of friction of masonry upon clay = 0.33.

For approximate design, the width of the base may be taken as $b = 0.45\,h$ for ordinary walls retaining level banks, $b = 0.66\,h$ for railway walls, $b = 0.70\,h$ for surcharged walls, that is, walls retaining an inclined bank. The batter of the foundation masonry with a vertical line may be taken as 1:0.9. Also $b_1 = \frac{1}{2}b\,(1 + 0.18\,h/S_0)$, and $P = 13\,h^2$ for ordinary unsurcharged walls, $P = 20\,h^2$ for railway walls, $P = 22\,h^2$ for surcharged walls, $W = 50\,h^2$ for ordinary unsurcharged walls, $W = 80\,h^2$ for surcharged walls, $W = 85\,h^2$ for railway walls, and $S_1 = 0.2\,h$. The resistance to sliding $= f(W + W_1)$, which must be greater than P.

To determine, in the field, the necessary width of base and of foundation, proceed as in the following example of a design made for estimating purposes (Fig. 35). Let h = 40 ft for a surcharged wall which is to be built upon a foundation having a safe bearing capacity of 5 tons per sq ft. The above approximate formulas then give $b = 0.70\,h = 28.0$ ft, and $b_1 = 14.0\,(1 + 1.44) = 34.2$ ft.

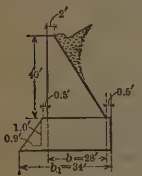


Fig. 35

A retaining wall is probably slightly stronger if the back is built in steps. This is the most convenient form of construction for ashlar and rubble walls, but when the wall is to be built of concrete, either the stept or unstept back may be properly used. In calculating the stability of a wall with a step back, it may be regarded as having a uniform batter. The placing of earth back of a retaining wall in layers, and ramming it, may or may not reduce the horizontal pressure against the wall, but the calculated width of wall should not be reduced if the earth is to be so compacted. The ramming of soft rotten rock or earth which is a combination of clay, sand and gravel, may reduce the earth pressure.

Designing by Tables. As an example take a retaining wall 30 ft high for a highway bridge approach which for architectural reasons must have a vertical face. The masonry will be taken at 150 lb per cu ft and the earth at 100 lb per cu ft. The slope of repose of the earth will be taken at 1.33 : 1. A factor of safety 2 will be used for rotation and hence the overturning moment in Table (A) must be doubled. Entering table (A) for a slope of repose of 1.33: 1 and a height of 30 ft, M is found to be 112 500 lb ft, and hence the overturning moment for the proper factor of safety is 225 000 lb ft. As a vertical faced wall is desired, either type B or C must be used. Type B is a very uneconomical type and should be used only for dry walls. Therefore type C will be adopted in the design.

Entering table (C) with the multiplied overturning moment, the nearest resisting moment is 259 000 lb-ft for a wall 30 ft high, and the required ratio of the base of the wall to its height is found to be $^4/_{10}$ and therefore a base 12 ft wide is needed. Entering table (F), the compression at the toe of the wall is found to be intermediate between that for a 20-ft and a 100-ft wall.

By proportion the expression for the compression upon the toe of a 30-ft wall is found to be $376\,h$ or 11 280 lb per sq ft or 5.6 + tons per sq ft. If the foundation bed is rock or hard rotten rock and the bed is rough so that the wall will not slide, the design is completed. Offsets of 6 inches should be made at the top of foundation for reasons heretofore given. The stability of this design may now be investigated in detail by the methods of Art. 8.

The capacity of a granite ashlar wall to resist overturning, as compared with a concrete wall, is generally overestimated. Since both kinds of walls resist the overturning thrust of the earth by gravity alone, the granite ashlar wall is little stronger than the concrete one. Calling the resistance of a concrete wall 1, the resistance of a granite ashlar wall for types B, C and D would be 1.07, 1.05 and 1.07 respectively. A brick wall is only 0.85 as strong as a concrete one. A sandstone ashlar wall has about the same strength as a concrete one. On the basis of the recommended factor of safety 2 against overturning, the maximum unit compression stresses are so low that, except for masonry laid in lime mortar, they need not be considered. The maximum compression in a retaining wall 100 ft high is given in table (F) as $400\,h = 20$ tons per square foot.

Walls with Vertical Backs. The top width a is assumed and the horizontal earth pressure P computed by Art. 4. Then for the case where the resultant cuts the base at the edge of the middle-third, so that the compression at the heel is zero, the width of base is

$$b = -\tfrac{1}{2}a + \tfrac{1}{2}\sqrt{5\,a^2 + 8\,P/v}$$

where v = unit weight of the masonry. When the condition is inferred that the resultant cuts the toe when P is doubled, the formula is

$$b = -\tfrac{1}{2}a + \tfrac{1}{2}\sqrt{3\,a^2 + 8\,P/v.}$$

The first formula gives the greater value of b, and it should be used when it is important that no tension should exist at the heel. See Art. 12 for a comparison of designs made by these two methods. For rectangular sections, where $b = a$, these formulas become $b = \sqrt{2\,P/v}$ and $b = 2\sqrt{P/3\,v}$.

In Construction a trench is excavated in which to lay the foundation of the wall. Fig. 36 shows a wall whose foundation $aebd$ is at a considerable depth below the ground surface sg, while m_1m_2 and n_1n_2 are the sides of the trench which will be excavated in order to lay the foundation masonry $aebd$. It is important that upon the completion of the retaining wall the remaining trench spaces m_1aem_2 and bn_1n_2d be filled with well selected earth thoroly rammed into place. This gives an additional factor of safety against overturning. If the wall leans forward it will meet resistance at the front, which is shown by P_1 applied at $\tfrac{1}{3}ae$ measured down from a and also at P_2 applied at $\tfrac{1}{3}af$ measured up from f. The values of P_1 and P_2 cannot usually be approximated, but the most extreme case is that where $P_1 = 0$, and then the height of the wall would be that of the top above the foundation bed. In construction, however, it is important that back-filling the space between the masonry and the sides of the trench should be well done. In very deep foundations the trench sheeting should be left in place so as not to disturb the earth back and front of the foundation masonry.

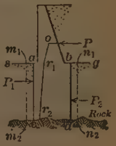


Fig. 36

Deep foundations for retaining walls, however, are very seldom used, it being more economical, as a rule, to use piles to transmit the loads to soil of sufficient bearing capacity.

In pile foundations it is important in the design to take care of the horizontal thrust by land ties or battered piles.

In the case of clay foundations the proper back-filling of the trench is also important, as it tends to keep water away from the foundation.

Sliding on the Base is to be prevented by making the same as rough as possible. For a wall on a timber grillage, some of the top timbers well bolted to the grillage may project above the level of the top of the grillage, or bolts well fastened to the timbers may project into the vertical masonry joints. When this is not possible the base should be placed on an inclined grillage. Fig. 22*b* of Art. 8 applies to this case and there will be no force F parallel to the incline $A\ B$ if $\tan\ \alpha = H/W$, and it is usually easy to secure such an inclination in construction. Bonding the stones together will usually give ample security against sliding of a portion of a wall, on a lower portion. The designer should provide such methods of preventing sliding rather than to widen the base and thereby increase the weight in order to meet the theoretic condition of stability against failure by this cause.

Waterproofing and Drainage. Fig. 37 shows a design of a retaining wall to be built of concrete or of concrete faced with ashlar. If it is important that the face of the wall shall be dry, the treads of the steps of the back must have a batter with the horizontal, as shown in Fig. 37, of at least one inch.

Fig. 37

The back of the wall should be given two coats of coal tar pitch, and immediately adjacent to the back a stone drain, marked, *d–d* in the figure, should be built. Near the bottom of the wall, but at such height above the surface of the ground as will permit free discharge of the water back of the wall, drain pipes not less than 3 in in diameter should be laid in the masonry. These drains should be at every 25 ft of the length of the wall, and at their inside ends piles of broken stone, from ¼ cu yd to 1 cu yd in volume, should be placed, depending upon the height of the wall. This broken stone prevents the pipes from becoming stopped up with earth.

In order to guard against seepage of water through cracks which may result from temperature changes, a concrete wall may be built with vertical expansion joints at intervals of 50 ft which extend from the foundation bed thru the coping. This method of construction causes the temperature cracks to occur at known, vertical lines. The wall may also be built without such expansion joints, provided it be reinforced with sufficient steel to take care of the tension in the concrete resulting from the lengthening of the wall under rise of temperature. Water may be prevented from seeping thru these joints by forming a rectangular vertical recess, as the wall is built up, which is filled with plastic clay well rammed in place. Fig. 38 is a plan of a portion of the length of a wall showing a horizontal section of the vertical expansion joint and the recess *a a a* into which the clay is packed. Even tho there should be a slightly uneven settlement of the adjacent sections, the clay will be effective in stopping the seepage of water.

Fig. 38

A Frost Batter is a forward inclination of the back face near the top of the wall, as shown in Fig. 32. The object of this is to allow the earth to lift upward under the action of frost, and thus prevent an additional horizontal pressure at the top of the wall. A drainage ditch, at a few feet parallel to the wall away from the top, is often provided to carry off rain water before it percolates into the earth.

If the pressure upon the toe of the wall, as obtained from table (F), exceeds the safe

unit pressure which may be placed upon the foundation bed, the foundation masonry must be widened by an offset in front of the toe of the wall. The necessary width of foundation may be obtained by the empirical formula $b_1 = \frac{1}{2} b (1 + S_1/S_0)$, in which b_1 = the necessary width in ft of the bottom of the foundation, b = the width in ft of the bottom of the wall, previously determined, S_1 = pressure in tons per sq ft upon the toe of the wall, S_0 = the safe pressure in tons per sq ft upon the foundation bed.

12. Comparison of Retaining Walls.

The Profile of a wall or dam means the shape of its cross-section as shown by the bounding lines. An economic profile is one which has the least material consistent with proper stability and with the local conditions where it is built. Retaining walls are usually of trapezoidal form, but the batter of the faces has much influence upon economy of material. A sea wall may have its front face more or less curved, the curve being introduced to deflect upward the waves that strike it and thus to lessen the lateral pressure which may be caused by their impact.

The Relative Economy of retaining walls of different types may be approximately ascertained by comparing their resisting moments given in the tables of Art. 9. In type C the weight of the earth upon the back of the wall adds to the resistance. The efficiency of earth above the back in resisting overturning of the wall has been demonstrated by the successful construction of reinforced concrete walls with counterforts where stability against overturning depends almost entirely upon the weight of the earth resting upon the wall. It should be noted in table (F) that the pressures upon the toe of walls of type D are much less than for C or in fact any other type, and therefore type D is a desirable one when the carrying capacity of the foundation bed is low. This type, however, cannot often be used in cities, since its front batter encroaches upon valuable land. Type B should rarely be used except for dry rubble walls, since it is a very uneconomical type. It has one advantage, however, that due to the greater weight, it offers the most resistance to sliding, of all the types. Even in this respect, however, type B has little advantage over type C. Type D offers the least resistance to sliding. Type E is really a modification of C, and when the front batter of E is small compared with its back batter, type E becomes type C. Type E need be given little consideration in practice.

Relieving Arches at the Back of Walls (Fig. 21) are not economical in American practice. Before the reinforced concrete counterfort type of wall came into use, they may have been economical in Europe, where the cost of materials of construction is relatively high compared to the cost of labor. Counterforts on the back are advisable only when built of reinforced concrete, because of the practical impossibility of taking care of the tension at the junctions of the back face and base with the counterforts, especially in walls of rubble or ashlar. Buttresses should rarely be used except when needed to prevent failure of a wall which is seen to be weak. A buttressed wall has no place in engineering except for architectural effect.

A Wall with Back Inclined Backwards (Fig. 39) is not usually an economical one, owing to the added cost of construction, altho theoretically such a form tends toward economy of material. Fig. 40 shows an excellent form of wall with a short and steep batter *aa* at the front a little above the surface of the ground, while the greater part of the front is vertical. In walls of types B and C the batter of the front face should be at least

Fig. 39

Fig. 40

Theoretic Comparison of Walls. The following is a comparison of the width of base computed for several retaining walls with vertical backs all being 18 ft high masonry 150 lb per cu ft, and retaining a level bank of earth weighing 100 lb per cu ft and having a slope of repose of 34°. The top width a in feet in the first line being assumed, the base width b in feet in the second line is computed under the condition that the distance e (Art. 6) shall be 1/6 b, thus giving a high degree of stability against rotation.

a =	0	1	2	3	4	5	6	7	7.8 ft
b =	7.8	7.4	7.1	7.0	7.0	7.1	7.3	7.5	7.8 ft
% =	50	53	58	65	71	78	85	93	100

The last line shows the relative amounts of materials in these walls as percentages of the rectangular wall in the last column. The triangular wall (a = 0) has 50% of the material in the rectangular one.

The following shows a comparison for the condition that there shall be a factor of safety of 2 against rotation, that is, that the resultant cuts the toe of the wall when the pressure P is doubled:

a =	0	1	2	3	4	5	6	6.4 ft
b =	7.8	7.4	7.0	6.7	6.6	6.5	6.4	6.4 ft
% =	60	66	70	76	83	90	97	100

The width of base is here less than for the more rigid requirement, that for the rectangular wall being 1.4 ft less. The triangular wall has here only 60% the material of the rectangular one. In practice 1.5 ft is the least allowable width of the top of a retaining wall (Art. 11).

13. Brest Walls and Dock Walls

A Brest Wall, often called a face wall, is one built against a bank of earth or rock to prevent it from falling. Since nothing is known regarding the horizontal pressure that may be exerted, experience and practice are the only guides. The following rules may serve as a basis for thickness needed for different materials.

Hard rock and hard firm rotten rock will usually stand with a vertical face or with a slight batter 1:10 to 1:4 for an indefinite length of time. If the dip of the rock is away from the face, a wall is seldom necessary. If the dip of the rock is toward the face, a base of wall greater than 4/10 its height is seldom necessary, provided however that drains are built into the wall so as to prevent the wall being forced out by the formation of ice between the face of the rock and the back of the wall. Brest walls for all classes of material should have drains not only at the bottom, but also at frequent intervals between the bottom and the top. Soft and seamy rotten rock, the dip of which is away from the face, seldom requires a wall whose base is greater than 1/4 its height. Soft and seamy rotten rock, the dip of which is towards the face. seldom requires a wall whose base is greater than 1/3 its height. Firm sand and combinations of clay, sand and gravel seldom require a wall whose base is greater than 1/3 its height. If the dip of the stratum, however, is steep and toward the face, a base at least 4/10 the height of the wall should be used. If the walls are surcharged, these ratios should be increased to 4/10 and 5/10 respectively.

Clay and other materials resting upon a moist stratum of clay back of the wall, whose dip is toward the face, may exert enormous pressures, particularly when the wall is surcharged. If, for this case, the earth back of the wall is level with its top, a base larger than 6/10 its height is seldom required, and for a level top, if the dip is away from the face, a base larger than 5/10 its

height is seldom required. If the wall is surcharged and the dip of the moist clay stratum is away from the face, a base 6/10 its height is usually sufficient. If the wall is surcharged and the dip of the moist clay stratum is towards the face and making an angle with the horizontal not exceeding 15°, a base 6/10 its height is usually sufficient. If the wall is surcharged and the dip of the moist clay is toward the face and over 15°, but under 30°, a base equal its height is usually sufficient. If the dip exceeds 30°, the ratio should be increased to 3/2.

A Dock Wall differs from an ordinary retaining wall in that water pressure is exerted against its front face in the lower portion of the wall, while water, and earth submerged in water, exerts pressure upon its back in the lower portion of the wall; also the weight of the masonry below the water line is reduced by the weight of the volume of water which the wall displaces.

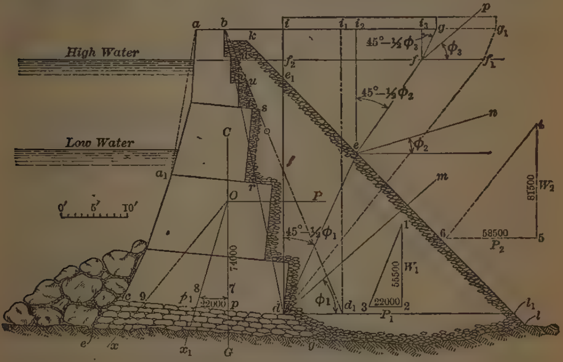


Fig. 41. Concrete Dock Wall on Rock

Assumptions for Design. The water pressure upon the face of the wall should be regarded as balancing the water pressure upon its back. When the wall is built in tidal water, particularly when the range of tide is large, there is probably a lag in the tide back of the wall, and therefore it is advisable above low water to increase the width of the wall arbitrarily above the average back batter line, as shown in Fig. 41 by the dotted lines marked *rs*, *tu* and *vw*. The weight of the earth on the inside of the wall and below high water is decreased by the weight of the volume of water displaced. The angles of repose of submerged earth will generally be much less than for earth above high water. The angle of repose of rip-rap or large cobble stone is not increased by submergence and this fact makes both materials desirable as filling, back of a sea wall. Art. 2 gives the weights and angles of repose of submerged earth. It should be noted that the weight of river mud is given at 90 lb per cu ft and its angle of repose as 0°. River mud should therefore be regarded as a liquid whose weight is 90 lb per cu ft exerting a horizontal pressure against the back of the wall of 45 h_1^2, in which h_1 equals the vertical distance from the heel of the wall to high water. This horizontal pressure is applied at 1/3 h_1 above the heel. To get the net or effective pressure against the back of the wall, due to river mud, subtract from the

liquid mud pressure the pressure of the water against the front, which gives $13.8\,h_1^2$ for fresh water and $13\,h_1^2$ for salt water. The pressure of liquid mud against the back of a wall need seldom be considered, but in the case of high walls, and where the earth filling is dumped from the shore toward the wall (Fig. 42), the river mud becomes banked against the back of the wall and exerts great pressure. The wedge $b_1\ b_2\ a$ will be a mixture of river mud and earth with angle of repose of practically 0°. Due to the filling from the shore, a wave of mud, caused by the sudden settlement of the fill ab_3b_4 may add impact to the river mud and exert enormous pressure. Therefore when the depth of river mud is great, the filling should be made from the top of the wall, but even if this be done, it is advisable to regard the slope of repose of the submerged earth filling as having its base increased 30%; because the mud mixes with earth filling and the result is a flattening of the angle of repose. For example, rip-rap should be regarded as having a slope of repose of 1.3:1, and clay, sand and gravel as having a slope of 2:1.

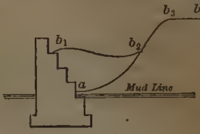


Fig. 42

The pressure at the back of a dock wall is considered, in what follows, to be applied at 4/10 the height of the wall above the heel. The point of application of the horizontal pressure is raised above that recommended in the design of ordinary retaining walls because the submerged portion of the earth back of the wall is generally semi-liquid in character, that is, it has a flat angle of repose. If the back fill were a liquid, under an abnormally large live load, the point of application of the horizontal pressure would never be higher than ½ the height of the wall above the heel. The effective weight of a masonry dock wall, below high water, equals its weight in air less the weight of the volume of water displaced by the wall, or the weight per cu ft of masonry below high water should be taken as its weight in air less 62½ lb for fresh water and 64 lb for salt water.

Masonry Docks have walls of three general types: (1) A gravity masonry wall resting upon rock or other hard foundation or upon piles in firm material extending to the pile tops. Fig. 41 shows this type of wall founded upon rock. (2) A masonry wall resting upon a timber platform, called a relieving platform, the top of which is generally from 12 to 18 inches above low tide, resting upon a pile foundation, Fig. 43. (3) A rubble wall resting upon rip-rap; this is seldom used for dock walls, but may be so used by the construction of a timber wharf in front. It is more generally used in the reclamation of land from the sea.

Dock Wall of Type 1. Fig. 41 shows a type of dock wall founded upon rock. The rock should be cleaned off by dredge and divers, and the foundation of concrete in bags be laid by derrick boat and divers. Assume the safe unit compressive stress of well-bonded portland cement bag concrete as 10 tons per sq ft. If the wall is built in salt water where the temperature of the air falls below 32° F. the face of the dock wall between the top and 2 ft below low tide should be faced with ashlar masonry, which will withstand the action of salt water and frost. Type 1 is generally built of large portland cement concrete blocks from the top of the foundation concrete to several feet above low tide. These blocks weigh from 15 to 80 tons, depending upon the size of the wall, and are generally cast on the shore, hauled by boat to the site of the wall and lowered by a derrick boat. The blocks are made with their beds beveled approximately 10:1, so that the bed joints are approximately normal to the resultant pressure. Grooves or recesses are molded in the

vertical joints of each block and these grooves or recesses are connected by a hole passing thru the block. By means of chains passing thru these grooves and holes, the blocks are lowered into place, and when in place the chains are removed. The vertical grooves are made to match, and the square recesses thus formed by adjacent blocks are packed with concrete. Above low water the concrete is molded in place. The top of the wall should be at least three feet wide to withstand shock from vessels. The front of the wall may be built with an increasing batter forming a curve as in Fig. 41. The back of the wall should be built in steps to agree with the heights of the concrete blocks, or the back of each block may be divided into several steps. If the wall rests upon a foundation bed of material that may be scoured out if unprotected, a bank of rip-rap should be placed in front of the wall to protect its foundation.

The resultant pressure P for a section one foot long in Fig. 41 is found as follows. The wall is to be built of concrete weighing 150 lbs per cu ft, and in sea water weighing 64 lb per cu ft. It will be backed with rip-rap, and will have a live load back of it equal to 750 lb per sq ft. The filling back and above the rip-rap will be a combination of gravel, sand and clay. All material above high water will be regarded as dry and below high water will be regarded as submerged. The following data are here applicable:

Kind of material	Slope	Angle	Lb per cu ft
Hard rock or rip-rap submerged	1 : 1	45° 00′	65
Hard rock or rip-rap dry	1 : 1	45 00	100
Gravel, sand and clay, submerged	3 : 1	18 30	65
Gravel, sand and clay, dry	1.33 : 1	36 50	100

Draw an inclined line bd, which is an average line for the stept back. This line should pass thru the heel d always; thru d draw a vertical line di and a horizontal line dl_1 to intersect with kl, the line which represents the limiting surface of the rip-rap which is placed back of the wall to reduce the intensity of the earth pressure; thru d draw de making an angle $45° - \phi_1/2$ with the vertical, in which ϕ_1 is for rip-rap and e is a point in kl; at e draw ef making an angle $45° - \phi_2/2$ with the vertical, ϕ_2 being for submerged gravel, sand and clay, while f is a point in the line of high water; at f draw fg making an angle $45° - \phi_3/2$ with the vertical, ϕ_3 being for dry gravel, sand and clay. Then d-i-g-f-e is the wedge which tends to slide down the lower plane de and cause a horizontal pressure P against the back of the wall, this being applied at a distance $4/10\ di$ measured from d. To the weight of the wedge must be added the live load of 750 pounds per sq ft, or, since a wall one foot long is being considered, a load of 750 lb per lin ft. The weight of the sliding wedge in pounds equals area $de_1 \times 65 +$ area $e_1f_2fe \times 65 +$ area $f_2igf \times 100 + 750\ ig = 55\ 500$ lb.

To find the horizontal pressure P lay off to any scale of intensity 55 500 on the line 1-2; at 1 draw line 1-3 parallel to de and at 2 draw horizontal line 2-3 to intersect 3. Then length of line 3-2 gives 22 000 lb = horizontal pressure P. The weight of the material between the back of the wall and di will be regarded as helping the wall to resist the overturning moment of the horizontal pressure. The center of gravity of this material can be found analytically or graphically. It should be remembered in getting the weight of this material and its center of gravity that submerged and unsubmerged materials have different weights. In finding the center of gravity of the masonry and its weight also, it should be remembered that the masonry above high water weighs 150 lb per cu ft and below high water $150 - 64 = 86$ lb per cu ft. CG is a vertical passing thru the combined center of gravity of the wall and the earth bid resting upon it. The total weight acting thru CG is 74 000 lb.

To find the resultant pressure P, lay off on any scale the line 0-7 equal to 74 000 lb, and from 7 lay off line $7\text{-}8 = P = 22\ 000$ lb. Then line 0-8 gives the resultant in direction and magnitude, and this intersects the base at p_1. Dock wall designs need not be investigated for pressures at the toe, because their weight is materially reduced by the buoyancy of the water. Their foundation pressures and their tendency to slide upon their foundations should be investigated as in retaining walls and all designs of dock walls should be investigated for overturning. The factor of safety against overturning

for this example equals the ratio of the distances pc to pp_1 or 3.4. For dock walls the factor of safety against overturning should be at least 2.5.

If no rip-rap is placed back of the wall, df_1 is the plane of rupture down which the wedge dig_1f_1 tends to slide and the live load is applied to ig_1. The line 4-5, to the same scale as line 1-2, represents the weight of wedge dig_1f_1 and its live load, while line 5-6 equals the horizontal pressure of this wedge. The line Ox gives the position and direction of the resultant pressure for the wall without rip-rap. The factor of safety against rotation of the wall without rip-rap equals the ratio of the distances pc to pg, or 1 : 1.3. The wall is therefore not safe, and either rip-rap should be placed back of it or its base should be increased to cd_1, for a factor of safety of 2.5 against overturning. The foregoing analysis shows the value of rip-rap in decreasing the pressure upon the back.

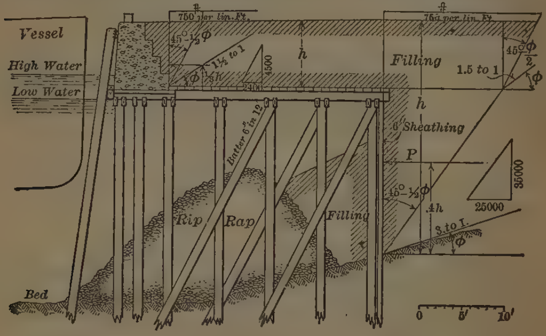


Fig. 43. Relieving Platform Dock Wall

Dock Wall of Type 2. (Fig. 43.) This wall is built where rock or other hard foundation bed lies at considerable depth, and for permanent work, but not where the piles are subject to the action of the teredo. It is also used where a hard foundation cannot be reached by piles, that is, where the wall is floated in mud; in this case, no analysis is of much value in making or checking a design, but the wall should be designed so as to permit of settlement and forward displacement without demolition, and land ties be used to keep the wall from excessive forward movement. Fig. 43 shows the method of finding the horizontal pressure against the back of the masonry wall and also that against the timber sheeting; these methods are similar to those for type 1. For discussion of the timber substructure see Art. 37.

Dock Wall of Type 3. This wall may be used for any class of foundation bed, and is well adapted to mud flotation. The rip-rap walls settle and go out of line, but cannot be overturned, and, if possible, the construction of the masonry wall should be delayed for a year or two after the rip-rap is placed. The rip-rap extends up to the low water line, its slope on the land side being 1 to 1 and on the water side 1½ to 1. The rubble wall on top of the rip-rap is about 6 ft in width. This wall is generally the cheapest type of permanent sea wall, but is unsightly, and vessels cannot approach so close to it as to the other types. When floated on mud, dock walls cannot be subjected to computation, but they require extraordinary precautions in design and construction. See paper by S. W. Hoag in Proc Munic Engrs of City of New York, 1905.

14. Walls and Piers in Buildings

A Building Bearing Wall may fail (1) by the crushing of the masonry, (2) by tension of the masonry over openings, (3) by overturning about any horizontal joint due to the thrust of arches adjacent to corners, (4) by overturning of an exterior wall about any horizontal joint, due to wind, (5) by failure of foundation. For (1) the same methods of preventing failure are to be used as for retaining walls (Art. 8). For (2) arches of masonry or lintels of stone, reinforced concrete, or metal must be provided to support the load of the masonry over openings. For (3) tie rods must be provided to take care of the horizontal pressure when the width of wall outside of the arch is insufficient to act as an abutment. For (4) the walls must have a thickness not less than hereinafter recommended.

Failures are liable to occur during construction due to the application of concentrated loads, as a rule eccentrically applied and when the mortar is not well set. Many cracked stone lintels are to be found over openings, but these seldom cause failure except of a local nature. Eccentric loads resulting from placing the bearing plates of beams and lintels very close to the inside edges of the masonry walls may cause failure by shearing the masonry or by overturning the walls. The flexure of beams and lintels causes an inclined reaction which tends to overturn the walls.

Thickness in Inches of Exterior Walls of Buildings as required by the Building Laws of Washington, D.C.

No. of the story	Height of wall in feet								
	110 to 130	100 to 110	90 to 100	80 to 90	70 to 80	60 to 70	45 to 60	35 to 45	35* or less
11	13	......	......	......	......	......	......	......	......
10	13	13	......	......	......	......	......	......	......
9	18	13	13	......	......	......	......	......	......
8	18	13	13	13	......	......	......	......	......
7	18	18	13	13	13	......	......	......	......
6	22	18	18	13	13	13	......	......	......
5	22	22	18	18	13	13	13	......	......
4	22	22	18	18	18	13	13	13	......
3	26	26	22	18	18	18	13	13	13
2	30	26	22	22	18	18	18	13	13
1	30	30	26	26	22	18	18	18	13
Basement	34	30	26	26	22	22	18	18	13

* For walls, 25 ft or less, 9-inch walls may be used above the basement.

This table applies to walls of brick, dimension stone or concrete. For warehouses or buildings used for storage or mercantile purposes the walls shall be at least 4½ in thicker than given in the table. For rubble masonry add one-third to the thickness given. Reinforced concrete walls may be made ⅔ of the thickness given. For cross walls extending from exterior wall to exterior wall, a thickness ⅔ of that shown in table.

Thin ashlar facing shall not be counted in determining the thickness of walls. If stone facing is used with bond courses alternately, not less than eight inches thick, on the beds, then such facing may be counted as forming part of the wall, and the total thickness of the wall and facing shall not be required to be more than that herein specified for walls.

The height of stories for all given thicknesses of walls must not exceed eleven feet in the clear for basements, eighteen feet in the clear for the first story, fifteen feet in the clear for the second story, fourteen feet in the clear for the third and fourth stories, and fourteen feet in the clear average height for any upper stories, unless the walls of such story and all the walls below the same shall be increased four inches in thickness additional to the thickness already mentioned.

When girders are supported at the ends by masonry walls the center of the bearing plates shall be practically concentric with the center of the wall. Small joists of steel or wood need not have more bearing in the wall than is necessary to reduce the unit stresses of the bearing within safe limits.

Stone Beams may be designed by $w = bd\,(0.2\,Sd/l^2 - 1)$, where w = safe total uniform load in lb per lin ft, b = width and d = depth of beam in inches, l = clear span in feet, S = specified modulus of rupture in lb per sq in; for a concentrated load at middle use $\frac{1}{2}\,w$. For depth of a stone lintel in inches use $d = 4 + a\sqrt{l^3}$, where l = span in ft, and a = 0.65 for granite, 0.75 for limestone and marble and 0.85 for sandstone.

Piers in Buildings should not have a greater ratio of unbraced height to least width than 15 for monoliths of stone and portland cement concrete and ashlar piers or blocks having the full horizontal dimensions of the piers, 12 for brick and well-bonded ashlar piers laid in portland cement mortar, 10 for brick and well-bonded ashlar piers laid in natural cement mortar, 8 for brick and well-bonded ashlar piers laid in lime mortar, 8 for rubble piers of flat or scabbled stones laid in portland cement mortar, 6 for rubble piers of unscabbled stones laid in portland cement mortar, and 4 for rubble piers of unscabbled stones laid in natural cement or lime mortar.

The Unit Stresses of Art. 1 should be reduced for piers when the ratio of the unbraced height to least width exceeds 10 for monoliths of stone and portland cement concrete and ashlar blocks having the full horizontal dimensions of the piers, 8 for ashlar masonry, 6 for brick masonry, 4 for rubble masonry. This may be done by multiplying the S of Art. 1 by $(1 + 0.1c/d - 0.1h/d^2)$, in which h is height of pier in ft, d is the least width of pier in ft, h/d is the ratio of unbraced height to least width, and c = 10 when S is from 700 to 500, c = 8 when S is from 500 to 300, c = 6 when S is from 300 to 200, c = 5 when S is from 200 to 100, c = 4 when S is below 100 lb per sq in. When brick piers are faced with prest brick, thin ashlar or terra-cotta, the facing should be neglected in figuring the area of the piers.

Example. Find the safe compressive unit stress for a brick pier of hard brick laid in portland cement mortar, the unbraced height being 16 ft and least width 16 inches. From Art. 1, S_1 = 350 lb per sq in, c = 8, h/d = 12, h = 16 and d = 1.33. Therefore the pier may be built of the stated dimensions, but the unit stress must be reduced by the above method to 245 lb per sq in.

15. General Data for Masonry Dams

Dams built up to the middle of the 19th century apparently were designed without theoretical analysis. The principles upon which safe and economical dams should be designed were laid down first by Sazilly in 1853 and after him by such authorities as Graeff, Delocre, and Rankine. According to these principles the requirements of a safe dam are:

a. At all horizontal sections, the resultant of the acting forces shall strike at or within the middle-third for all conditions of the water elevation behind the dam. This principle implies the condition that no tensional stresses are permissible in a masonry structure of this character, the assumption being made that the pressure intensities along the section or joint vary uniformly, according to the law of trapezium. (Art. 6.)

b. At no point along any horizontal joint or the base shall the unit normal stress upon the plane be more than a specified safe limit. (Art. 1.)

Investigation of the old Spanish dams, standing for over three hundred years show a maximum compression stress on horizontal joints of from 11 to 14 tons per sq ft.

c. The friction developed by the weight of the structure at any horizontal joint or at

the base shall be greater than the resultant horizontal force acting upon the joint tending to slide the part of the dam above it. (Art. 3.)

No exact mathematical expression has been found as yet which would determine the outline of the dam section in accordance with the three requirements enumerated and with given data as for permissible stress and specified factor of safety against sliding and overturning. If only condition *a* is considered, the economical section will be a right triangle with back vertical and base width $= h/\sqrt{\gamma}$, where h is the height of water above the base and γ the specific weight of the masonry. The authorities mentioned and a number of others have endeavored to find the best modification of this triangle form to comply with requirements *b* and *c*, the result being a series of proposed forms of more or less complex outline, but none having an unimpeached general value and most being entirely impracticable. A very complete record of these forms and of the dams built according to the different theories can be found in E. Wegmann's "The Design and Construction of Dams." The "practical profiles" recommended by him (Art. 17) represent a valuable addition to the profiles mentioned.

In 1895 M. Levy and following him in 1904 Karl Pearson and Atcherley made thoro analytical studies of stresses in gravity dams of masonry and came to the conclusion that in such dams the planes subjected to maximum compressive and tensional stresses are not horizontal as a rule; that the direction of the maximum pressure intensity is parallel to the face of the dam and the pressure is greatest near the outer profile; and that, in every case, in a dam designed according to the heretofore accepted principles tensional stresses occur on inclined planes near the heel. These analytical deductions have been verified by the famous experiments of Ottley and Brightmore on plasticine models and by the experiments of Wilson and Gore on India rubber models. To eliminate tension at or near the heel, Levy proposes to design dams so that "the normal pressure on every joint at the upstream face should be greater than the hydrostatic pressure at that point." Recently built French dams are all designed on this principle. German engineers use a clay or loam fill at the back of high dams, to assist in the plastering of the masonry on the waterside and to fill every crack possibly developed by tensional stresses. Wilson and Gore pointed out the advantage of a rounded or projecting inner heel to decrease tensions. In American practice this latter method is extensively used.

It is recommended that for the lower one-fifth of the height of dams, the safe working stresses of Art. 1 shall be reduced 30 per cent if the dam is designated without considering the maximum stresses resulting from the combination of normal and shearing stresses. (Art. 6.)

Horizontal Pressure on Dams. While the physical properties of water are well known (Art. 5) it takes a good deal of judgment to determine the horizontal forces exerted by water upon a dam. As a rule the strata upon which the dam is built is covered by more or less pervious or impervious layers of sand, gravel, loam or clay. The dam generally extends thru this overlying material and the question arises how far down and to what extent shall the horizontal water pressure be considered. In the case of spillway dams the back pressure from the water downstream (tailwater) may decrease the net pressure upon the back and this pressure may also be exerted through the material overlying the foundation.

How much numerical value should be given to this underground pressure, is entirely dependent on the kind of overlying material. Clean gravel will practically not reduce the hydrostatic pressure at all, fine sand will reduce it to a great extent, while puddled alluvial soil may not transmit any water pressure at all, but such soil quickly disintegrates with a running leak. The intensity of this horizontal underground water pressure cannot be accurately determined but its possibility should not be overlooked. The overlying material also exerts a horizontal thrust upon the back of the dam which should always be considered in the design (Art. 5), although usually this pressure is practically negligible.

The active thrust and passive resistance of the overlying material at the down-stream side of the dam on the other hand should generally be disregarded.

Uplifting Pressure on Dams. Masonry dams of the gravity type are preferably built on rock foundation, but they have been built successfully on gravel and sand, which materials, if compact (hardpan) or confined so that no erosion can take place under the base, have a sufficient bearing capacity to support moderate size dams (say up to 60 ft).

In the design of all dams, except those whose foundations will rest on solid, impervious and unfissured rock, the most careful consideration should be given to the possibility of uplift upon the base of the dam due to the water being forced under the base of the dam through veins, fissures or interstices. Neglect to consider this very great force has resulted in disasters. The designer must allow for uplift or upward pressure on the base of the dam on all but the best rock foundations or he must provide an adequate impervious cutoff that will either prevent uplift entirely or make it negligible. In "Professional Memoirs," Jan Feb. 1915, Capt. W. A. Mitchell describes many experiments and observations made on the uplift on dams, especially those made very recently on the Ohio River dams built by the Federal Government. The following is largely taken from the statements of the paper referred to.

With foundations on rock, the amount of upward pressure is not easily determined. It is evident, however, that there is such pressure. Experiments by Francis, Col. Shunk and Maj. Jervey show that water percolates through joints between concrete and rock, and travels in small veins in these joints. If allowed free exit, the pressure varies irregularly between upper and lower pools; if the exit is closed the pressure quickly becomes that of the upper pool as soon as enough water has passed through the veins to fill the test holes. The amount of space of these small veins, that is, the area of upward water pressure, varies from nearly zero in excellent granite foundations to 50 per cent or more in rotten shale. The final height of the rise of water in the test holes when free exit is provided below the dam varies roughly as the distance from the lower pool relative to the distance between pools. If efficient drains are introduced, the upward water pressure beyond such drains is practically negligible in good rock. In this connection it is well to note that the drains must be effective, must not be too small and must not be stopped. With rock foundations, the Ohio River Board in the general case uses the following rules:

a. Upward water pressure varying at a constant ratio from upper pool at upstream edge to lower pool at drains.

b. Drains to be located in the rock on the downstream side of the key or cutoff.

c. Pressure under 50 per cent of base for firm, solid rock and under 100 per cent of base for weak rock.

d. Neglecting upward pressure, the center of pressure to fall within the middle-third of the base; including uplift, the center of pressure to fall far enough inside the toe so that the maximum compression shall not exceed 400 lb per sq in for good rock and 70 lb per sq in for poor or doubtful rock.*

Cutoff Walls and Drains. It is customary and proper to build a cutoff of concrete as shown in Fig. 43*a*, the concrete being made of the best quality, most carefully placed so as to prevent the water getting under the dam or at least to increase the travel of the water by the length *abcd*. The higher the dam and the more pervious or fissured the strata the deeper this cutoff should be. Sometimes the cutoff wall is made quite deep, the rock being channeled out by machine. It is good practice to provide at *d* (Fig. 43*a*) a continuous drain with proper openings *p* extending to the face of the dam, so that any water that

* The writer believes that the center of pressure should fall within the middle-third for reasonable assumptions of uplift.

passes the cutoff may not exert uplift except of little intensity, but will flow out at e. On account of the possibility of these small drains and discharge pipes getting stopped up it is recommended in large dams to build instead of a small drain as in Fig. 43a, a gallery (Fig. 51) of such height that a man may walk in same from one end to the other to inspect and clean the drains so that the real uplifting pressure will be known. This gallery should be reached thru manholes located at or near the top of the dam.

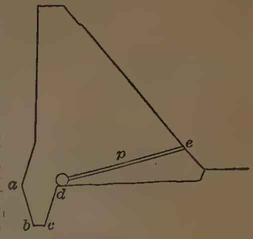


Fig. 43a

Experiments made on the Gatun dam and the Ohio River at Dam No. 18 show that water pressure is transmitted thru concrete and is transmitted thru joints between old and new concrete more freely than thru solid concrete. If allowed free exit, the pressure is practically negligible. Mitchell suggests that in order to utilize the full value of the weight of the masonry, drainage should be provided from near the back of the dam at other levels than the base only or if such drainage is not provided for, in the design allowance should be made for the upward pressure at every horizontal joint investigated, but a smaller relative intensity can be assumed than that considered at the base. Uplift on joints above the base or drains above it have rarely been considered. Certainly if entirely disregarded, the line of pressure above the base should lie well within the middle-third.

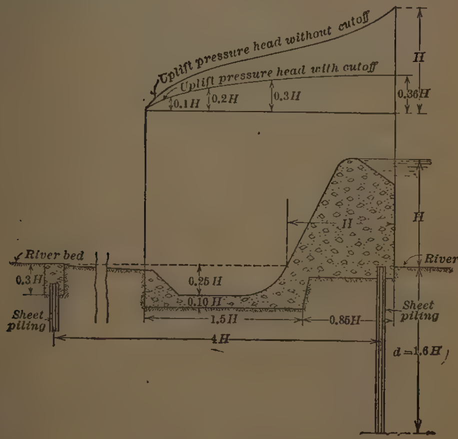


Fig. 44.—Spillway Dam on Sand Foundation

Dams on Sand Foundation. The uplifting pressure of water below the base is most serious in the case of dams built on sand or gravel foundation. Water pressure is transmitted more or less freely thru washed sand, gravel aggregate and washed river gravel. Sand offers the greatest, clean gravel the

least resistance to uplifting action. Besides being dependent on the physical properties of the foundation strata, the magnitude and the method of distribution of the upward pressure are functions of the depth of the cutoff or sheet piling, of the location of same and of the width of the base and apron. Watertight sheet piling or cutoff wall is most effective if located at or near the heel of the dam. Another line of sheet piling placed near the toe will be effective in confining the material below the base but will increase the uplifting pressure if made watertight. The results of a thoro analytical investigation of the matter are published in Trans. Am. Soc. Civ. Eng., 1911, vol. 73, by G. E. Smith, in discussing A. C. Koenig's paper on dams on sand foundation. Figs. 44 and 45 are taken from this article, illustrating the influence of the depth of the cutoff wall and the width of the base of the dam and the length of the apron upon the distribution of upward water pressure. The diagram above the dam section, Fig. 44, shows, that for the relative dimensions of this particular instance the total of uplift is reduced by the sheet piling to about one-half of that which would prevail without cutoff. For other dimensions the uplift can be estimated with the help of the diagram, Fig. 45. Koenig, in the paper above referred to, proposes the following formula for the depth d of the necessary penetration of the sheet piles.

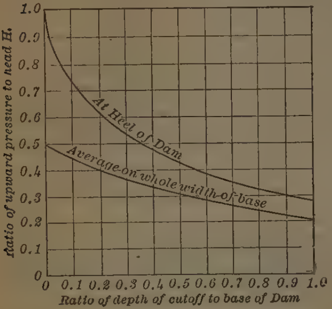


Fig. 45.—Variation of Upward Pressure with Change in Depth of Cut off

$d = h/sx$ where h = the maximum head of water on the upstream side above the bed of the stream, s = the specific gravity of the material penetrated and x = the proportion of solids in the strata. In sand with $s = 2.65$ and $x = 0.6$, $d = 0.629\,h$, which is the absolute minimum depth needed. For low heads, Koenig suggests $d = 2\,h$ and for medium heads up to 30 ft $d = h$.

A number of failures of dams occurred due to the uplifting action of water upon the aprons of dams. The floor of the Lost River dam, Cal., and that of the Narrora weir on the Ganges was destroyed by being lifted up by upward water pressure.

Percolation. The flow of water under dams on porous foundations can not be entirely stopped and this is the origin of a frequent cause of failure of such dams, the "blow out" at the toe, which is due directly to a higher rate of percolation than permissible for the material in question. If the percolating water has a greater velocity than a certain limit, it will carry with it parts of the material lining the vein of its passage, thus increasing the size of the vein. This action being progressive, the size of the vein continues to increase until it becomes a large hole and the dam collapses from lack of support. The blowout always starts with the appearance of a boil at the toe of the dam, that is a strong spring carrying sediment. The time between the first appearance of the boil and the blowout varies from a few hours to several days.

The only way to prevent a blowout in pervious foundations is to cut down the velocity of the percolating water below that necessary to disturb the particles of the foundation bed, by artificially making the underground travel of the water from headwater to tail-

water as long as is necessary for this purpose. The ratio of this length of underground travel to the head of water is called the percolation factor. The most efficient way to create a large percolation factor is by a cutoff or sheet-piling.

W. G. Bligh (Eng. News, 1910) proposes the following percolation factors: Fine sand and silt 18, fine micaceous sand, 15; ordinary coarse sand, 12; gravel and sand, 9; boulders, gravel and sand 4–6. The percolation factor may also be increased by a substantial impervious apron downstream or an impervious mat upstream of the dam, which latter may be an upstream concrete apron or a protected clay fill or the dam may be given a large inclination on the upstream face. These types have been mostly developed for dams across streams in India, where many failures occurred before the importance of the use of a proper percolation factor was recognized. The knowledge there acquired was used to good advantage on some dams in the United States. The Laguna weir across the Colorado River, with a head of 15 ft has a total base width of 244 ft, while the Madaya weir in Burmah, India, for a head of 11 ft is 288 ft wide. In the example shown on Fig. 44 the sheet piling, the base of the dam and the apron take care of a percolation factor of about 6, corresponding to a foundation strata of gravel and sand.

Ice Pressure acting at the top of the dam is sometimes required to be taken into account. John D. Van Buren in 1895 advocated the use of 40 000 lb per lin ft. The board of experts for the Quaker Bridge dam (report of Aqueduct Com., 1889) recommended that the ice pressure should be taken at 43 000 lb. per lin ft of dam. For the design of the Kensico Dam 47 000 lb, for the Cross River dam 24 000 lb, and for the Croton Falls dam 30 000 lb, per lin ft of dam have been used. These widely differing specified values of the ice pressure are mostly based on the crushing strength of the ice, but it is obvious that before pressures of such magnitude could be developed, the ice sheet would fail as a long column; moreover the creeping of the ice up the generally sloping shores of the reservoir will prevent any considerable thrust upon the dam. It can be shown (Trans. Am. Soc. Civ. Eng., Vol. 75) that the consideration of ice pressure of such intensities as quoted above would result in absurdly large dimensions for low dams while the dimensions of high dams would be only slightly affected. It is believed unnecessary to consider ice pressure in the design of dams unless the reservoir is very short in the direction perpendicular to the axis of the dam and the sides slope abruptly.

Pressure due to the shock of waves can be entirely ignored in the design of dams where the water reaches to the top of the dam or spills over and may be generally ignored except in the case of a low dam impounding a large and exposed sheet of water where relatively high waves are possible. Wave pressure may be taken into account by using Stevenson's formula. (Section 16.)

Temperature changes often seriously affect dams, and expansion joints should be provided perpendicular to the axis of the dams at intervals of about 30 to 60 ft.

If expansion joints are not provided at about these intervals, cracks may form which will be unsightly, will leak and may be injurious to the full strength of the dam. Further, if expansion joints are not provided, vertical cracks parallel to the axis of the dam may occur especially near the center line or near the downstream face and such cracks would endanger the stability of the dam. For further information see Section 11.

Comparative cross-sections of several high masonry dams are shown in Fig. 46, and data regarding them and a large number of other dams are given in Section 11.

Cost of Masonry for Retaining Walls and Dams. The cost of masonry for retaining walls and dams varies with the locality, size of wall, the difficulties of foundation, construction and, in the case of dams, with the difficulties

of handling the water during construction. The average cost per cu yd is about as follows: Ashlar facing, first class \$20.00, second class and backing \$12.00 to \$14.00, rubble \$8.50 to \$10.00, concrete \$6.50, concrete walls over 15 ft high with ashlar facing \$8.00 to \$10.00.

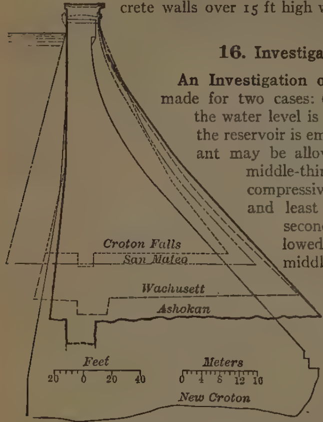


Fig. 46. Four American Dams

16. Investigation of Masonry Dams.

An Investigation of a Proposed Section must be made for two cases: (1) when the reservoir is full and the water level is at its maximum height; (2) when the reservoir is empty. In the first case, the resultant may be allowed to come at the edge of the middle-third nearest the toe, and then the compressive unit stress is greatest at the toe and least at the heel. (Art. 6.) In the second case, the resultant may be allowed to come at the edge of the middle-third nearest the heel, and then the compressive unit stress is greatest at the heel and least at the toe. Maximum economy of material will result when the resistance line for reservoir full coincides with the limit of the middle-third nearest the front and when the resistance line for reservoir empty coincides with the limit of the middle-third nearest the back. For other heights of water the resistance line will lie well within the middle-third.

A safety factor against rotation is not generally used in discussing high masonry dams, but an expression for it is $n = Wt/Hp$, where W = weight above any horizontal plane considered, t = distance from toe to point where resistance line cuts that plane, H = horizontal water pressure above the plane, p = height of H above the plane.

A Trapezoidal Section, such as is generally used for low dams, can be investigated by the methods given for retaining walls in Art. 10. The resultant must cut the base within the middle-third for all cases. This will always occur when the reservoir is empty, so that the entire base is then under compression. For reservoir full to the top of the dam, the resultant cuts the base on the other side of the middle, and it should also lie within the middle-third in order that the entire base may be under compression.

Example. Fig. 32 may represent a section of a trapezoidal ashlar masonry dam of 30 ft height, 5 ft top, and 18 ft base, the back face plumb and the foundation sound, impervious rock. The weight of the masonry will be taken at 160 lb per cu ft. Let a be the top width, b the base width and h the height, all in feet; then the weight of a section one foot long is $80\,h\,(a+b) = 55\,200$ pounds. The line of action of this weight passes thru the center of gravity of the trapezoid whose horizontal distance from the toe A is

$$t = (2b^2 + 2ab - a^2)/3\,(a+b) = 11.64 \text{ ft}$$

therefore the resisting moment about the toe is $M_1 = 55\,200 \times 11.64 = 642\,530$ lb-ft. For Reservoir Full the water surface is taken as level with the top of the dam, and its horizontal pressure P is $31.25 \times 30^2 = 28\,125$ lb. The overturning moment about the

toe is $M = P \times \frac{1}{3}h = 281\ 250$ lb-ft. Hence the factor of safety against rotation is $M_1/M = 2.3$, which is ample provided the resultant cuts the base within the middle third. That this is the case is found by using the formula

$$e = \tfrac{1}{2}b - t + \tfrac{1}{3}Ph/W. = 2.45 \text{ ft}$$

where t is the distance 11.64; the value of e is less than $\frac{1}{6}b$, so that the resultant lies within the middle-third. The ratio $6\,e/b$ is $6 \times 2.45/18 = 0.817$, and $W/b = 3070$; hence (Art. 6) the maximum and minimum pressures on the base are $S_1 = 3070\,(1 + 0.817) = 5580$ lb per sq ft, and $S_2 = 3070\,(1 - 0.817) = 560$ lb per sq ft. The coefficient of friction required to prevent sliding on the base with a safety factor of 2 is $f = 2\,P/W = 1.02$, so that the rock should be made very rough. For Reservoir Empty, $e = 9.0 - 11.64 = 2.64$ ft on the other side of the middle, so that here $6\,e/b = 0.88$, and hence the pressure at the heel is slightly greater than at the toe for reservoir full, and the pressure at the toe is slightly less than at the heel for reservoir full, but the line of resistance here also falls within the middle-third. For ice pressure see Article 17. The formulas for walls with a vertical back (Art. 10) could have been used in this example.

Fig. 32 shows a simple graphic analysis for this case by the use of the force and equilibrium polygons. the trapezoid being divided into a rectangle and a triangle whose centers of gravity are known without computation and the lines of action of the weights W_1 and W_2 being drawn thru these centers.

A Dam with Curved Front (Fig. 47) is 60 ft high, 40 ft wide on the base, top width 10 ft, built of granite ashlar weighing 160 lb per cu ft, and is to be founded on impervious rock. The first step in its investigation is to divide the cross-section into subdivisions A, B, . . . G by horizontal lines, these being sufficiently near together so that the subdivisions may be regarded as trapezoids. The areas and centers of gravity of each subdivision are next found, the latter marked on the drawing by small circles. The horizontal lines at a, b, . . . g are called joints for brevity, altho no joints may really exist. P_a, P_b . . . P_g denote the horizontal water pressure acting above the joints a, b, . . . g, each of these being applied at a distance above the joint equal to one-third of the depth of the joint below the water level at O.

Resistance Line for Reservoir Full. To determine the point where the resistance line cuts any joint as b in Fig. 47 a the weight of $A + B$ is to be laid off vertically thru its center of gravity and P_b produced until it intersects this vertical at the point marked by the small square, then the resultant of the weight $A + B$ and the pressure P_b is determined and this, being produced until it intersects the joint, locates a point in the line of resistance. The steps of the graphic analysis are as follows:

(1) In Fig. 47 b the resultant forces acting against the joints are found. The load line O_1g_1 is first laid off to scale to give the different weights, O_1a_1 being that of A, $a_1\,b_1$ that of B, and so on. On the horizontal line at the top the distances O_1P_a, O_1P_b, etc., are laid off to represent the horizontal forces P_a, P_b, etc. Joining corresponding points P_a, R_b, etc., are the resultants in direction and magnitude which act on the joints a, b, etc.

(2) In Fig. 47 b a force polygon is constructed with pole at any point O, as shown by the broken lines, this to be used for locating lines thru the centers of gravity of the areas $A + B$, $A + B + C$, etc., the points where these lines intersect the horizontal forces P_b, P_c, etc., being indicated by small squares.

(3) To find these vertical lines, Fig. 47 (c) is used where verticals thru a', b'. etc., are first drawn at the same horizontal distances apart as the small

circles on Fig. 47*a*. Usually a larger scale is used for Fig. 47*c* and the horizontal distances laid off to this scale from a reference line, these corresponding to the horizontal distances from a vertical thru the heel *g* to the small circles. Then an equilibrium polygon a_2, b_2, . . . g_2 is constructed and its sides produced to cut the first side in the points h_a, h_b, h_c, . . . h_g. The distances from these points to the reference line are the distances to be laid off on Fig. 47*a* from the vertical thru the heel, proper consideration

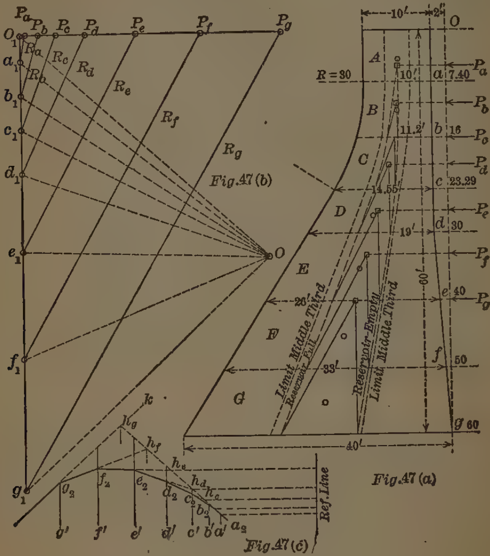


Fig. 47. Graphic Analysis of a Masonry Dam

being given to the different scales, in order to find the verticals thru the centers of gravity of $A + B$, $A + B + C$, etc.

(4) Producing the horizontal directions of P_a, P_b, . . . P_g to these last vertical lines the points of intersection are marked by small squares. Thru these points draw lines parallel to the resultants R_a, R_b, . . . R_g in the force polygon, and the intersection of these with the joints *a*, *b*, . . . *g* will give points on the resistance line for reservoir full. A curve is then drawn thru these points, and it is seen that the resistance line for reservoir full lies well within the middle-third, so that there is full security against rotation and against any occurrence of vertical tension at the heel of the dam.

Resistance Line for Reservoir Empty. Thru the small squares draw verticals to cut the corresponding joints and pass a curve thru the points thus determined. The resistance line thus determined is also within the middle-

third but nearer to its limit than for the preceding case. Strictly speaking, then, the tendency of the dam to rotate forward under full water pressure is less than that to rotate backward when the reservoir is empty.

Stresses at the Base Joint. The weight of the dam, equal to the full load line O_1g_1 in Fig. 47*b* is 201 050 lb and hence the average normal pressure on the base is 5026 lb per sq ft. The resistance lines for reservoir full and empty cut the base at distances of 5.5 and 6.0 from the middle, so that the ratios $6e/b$ are 0.825 and 0.9. Then for reservoir full, using formula $S = W/b\,(1 \pm 6e/b)$, (Art. 6), the maximum vertical pressure intensity is at the toe and is equal to $5026\,(1 + 0.825) = 9170$ lb per sq ft while for reservoir empty the maximum vertical pressure intensity is at the heel and is equal to $5026\,(1 + 0.9) = 9550$ lb per sq ft. Both these are very low for both the masonry and the rock foundation. The value of the principal stresses will now be found for the case of reservoir full. The following data are necessary:

Vertical pressure intensity at the toe	$S_1 = 9170$ lb per sq ft
Vertical pressure intensity at the heel	$S_2 = 880$ lb per sq ft
Intensity of water pressure $p' = 60 \times 62.5$	$= 3950$ lb per sq ft
Average intensity of shear $q = \frac{1}{2} \times 60^2 \times 62.5/40$	$= 2810$ lb per sq ft

Then the principal stresses at the toe will be (Art. 6)

$$S_{max} = \tfrac{1}{2}(9170 + 3950 + \sqrt{(9170 + 3950)^2 - 4(9170 \times 3950 - 2810^2)}) = +10\,370 \text{ lb per sq ft}$$

$$S_{min} = \tfrac{1}{2}(9170 + 3950 - \sqrt{(9170 + 3950)^2 - 4(9170 \times 3950 - 2810^2)}) = +2750 \text{ lb per sq ft}$$

Both principal stresses being positiv at the toe, no tension exists there on any plane.

The principal stresses at the heel will be

$$S_{max} = \tfrac{1}{2}(880 + 3950 + \sqrt{(880 + 3950)^2 - 4(880 \times 3950 - 2810^2)}) = +5620 \text{ lb per sq ft}$$

$$S_{min} = \tfrac{1}{2}(880 + 3950 - \sqrt{(880 + 3950)^2 - 4(880 \times 3950 - 2810^2)}) = -790 \text{ lb per sq ft}$$

This latter value being negativ a tensile stress of about 5.5 lb per sq in, which is so small as to be negligible, exists on a plane at the heel of the dam, which plane is inclined to the horizontal at an angle θ determined by the relation

$$\cot\theta = \frac{-790 - 3950}{2810} = -1.69, \text{ from which } \theta = 120^\circ\ 30'$$

Security against Sliding. The base of masonry dams on rock foundation is usually placed several feet below the surface of the rock (Fig. 51). If the base line of the dam here investigated (Fig. 47*a*) represents the joint at the level of the original rock surface, sliding of the dam is prevented by the shearing resistance of the masonry. If the dam were placed with its base upon the surface of the rock it would slide on the base if the tangent of the angle between the resultant pressure on the base and a vertical line is greater than the coefficient of friction between the materials of the dam and the foundation, or if the coefficient of friction were less than $P/W = 0.65$. For full security the base should be roughened so that the frictional resistance will be greatly augmented. In

high dams "stepped shoulders" should be cut in the rock in sufficient depth and number so that independent of the frictional resistance at the base the entire horizontal thrust might be taken by these shoulders without too high stresses on the rock shoulders or the abutting concrete, thus assuring at least a factor of safety of two against sliding. If the rockbed slopes downward from the heel of the dam, these steps are especially necessary. In high dams or poor laminated rock the possibility of the rock bulging up in front of the dam should be considered. Make doubly sure that the dam will not slide. Serious dam failures have been quite usually due in whole or part to failure to provide adequately for the transfer of the horizontal thrust to the foundation bed.

17. Details of Design of Masonry Dams

The Back of the Dam is often vertical or has a uniform batter, this being always the case for a low dam. For a high section, however, the profile of the back as well as the front of the section is often formed by straight lines of different inclinations (Fig. 50), but more often the front is curved. The water pressure is normal against a battered back, but its horizontal and vertical components may be used instead. (Art. 5.) It is customary to neglect the vertical component V (Art. 5), since such neglect is on the side of safety, and since the percolation of water beneath the heel would entirely annul it. For a uniform inclination above an assumed joint the horizontal water pressure $H = \frac{1}{2} wh_1^2 \cos\theta$, where h_1 = depth of joint below water level and w = weight of water per cubic unit. For further security it is best to consider θ as 0, and then $H = \frac{1}{2} wh_1^2$, which is strictly true for a vertical back.

Above the assumed joint there are hence only two forces acting, the horizontal water pressure and the weight of the masonry. The resultant of these should cut the base within the middle-third. The point where the resultant cuts the planes of several joints gives points thru which the resistance line can be drawn. This may be done either graphically (Art. 16) or analytically, the latter leading to complicated formulas.

Trapezoidal Dams with vertical backs are designed by the same methods as those given in Art 10 for retaining walls, the water pressure being taken horizontal and equal to $\frac{1}{2} wd^2$ when no overflow occurs, or in pounds $P = 31.25d^2$, if d be in feet; d is height from water line to base, this being usually less than the height h of the dam. The resultant should cut the base at or inside the edge of the middle-third for reservoir full, and for the resultant at the edge the required width of base is $b = -0.5a + \sqrt{1.25a^2 + d^2/s}$, where s is the specific gravity of the masonry. This formula applies for water pressure only and takes no account of the action of ice or waves.

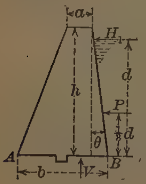


Fig. 48

For any Trapezoid (Fig. 48) let a = top width, h = height, d = height of water line above base, θ = inclination of back to vertical, w = unit weight of water, v = unit weight of masonry, H_1 = ice or wave pressure acting at water line on one unit of length of dam. Then proper base width so that resistance line cuts base at edge of the middle-third is $b = (A^2 + B)^{\frac{1}{2}} - A$, in which

$$A = \tfrac{1}{2}(a - h\tan\theta) \qquad B = a^2 + 2ah\tan\theta + (6H_1 + wd^2)\,d/vh$$

Example. For Fig. 32, let $a = 5$, $h = d = 30$ ft, $\theta = 0°$, $w = 62.5$, $v = 160$, $H_1 = 0$; then $A = 2.5$, $B = 376.5$, and the formula gives $b = 17.1$ ft. If,

however, $H_1 = 43\ 000$ lb for ice pressure, then $B = 537.8$ and formula gives $b = 20.8$ ft. The amount of material in this dam is increased over 30% by taking ice pressure into account.

A trapezoidal dam with vertical back requires less material than one with inclined back if vertical water pressure on the inclined back is ignored. For example: $h = 60$ ft, $d = 57$ ft, $a = 9$ ft, $v = 150$ lb per cu ft, $H_1 = 0$. Then

for $\tan\theta = 0$	$b = 32.7$ ft,	area = 1251 sq ft,	percent = 100
for $\tan\theta = 1/12$	$b = 36.2$ ft,	area = 1356 sq ft,	percent = 108
for $\tan\theta = 1/6$	$b = 39.8$ ft,	area = 1664 sq ft,	percent = 117

When a trapezoid is used for a high dam, the compression at the toe may become greater than the allowable value even tho the resultant cuts the base at the edge of the middle-third. When this is found to be the case by computation or graphic analysis, the section is too small and the base must be widened. Let S be the safe working compressive stress, and the other quantities as above; then the required width of the base is $b = C + \sqrt{C^2 + D}$, in which $C = \frac{1}{2} vh^2 \tan\theta / S$ and $D = (wd^3 + vha^2 + 2\,vah^2 \tan\theta)/S$.

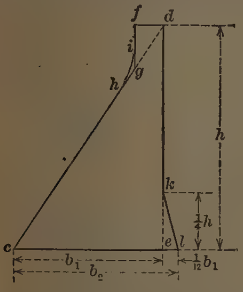


Fig. 49

A Pentagonal Section (Fig. 49) is a modification of a triangular one by giving to the top a width of 5 feet or more and then making the front face fg vertical until it reaches the inclined line dc. For a triangular section the necessary width of the base is $b = 7.91\, h\sqrt{1/v}$. For the pentagonal section the width of base should be $b_1 = b/\sqrt{1 + 2r^2 - 2r^3}$, where r is the ratio of the width of the top to the computed b. A short back batter of height $\frac{1}{4}h$ above the heel, and base width of $\frac{1}{12} b_1$ may here, as in other cases, be advantageously used to diminish any tendency to tension at the heel. This formula does not include the effect of ice or wave pressure at the water line.

Water under the Base reduces the unit weight of masonry by 62.5 lb per cu ft if this produces a full uniform pressure. It is best to keep the unit weight of masonry at its full value, however, and to introduce beneath the base such upward pressures as may be specified. For discussion of the uplift see Art. 15.

Dams over 60 Feet High may be designed by means of Fig. 50, known as Wegmann's "Practical Type No. 2," and with the help of the following table, these being taken from Wegmann's "Design and Construction of Dams" (1909) and modified in columns 4 and 5 in order to diminish the tendency to tension at the heel (Art. 6). The figure gives this profile for a 200-foot dam, the masonry of which was assumed to weigh 145.8 lb per cubic foot. This profile is drawn for a top width of 20 feet or one-tenth the height of the dam, and the table is made for the same ratio of top width to base. For a dam of the same top

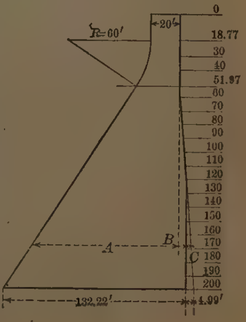


Fig. 50

width cut off the section at any desired height. For a dam of less top width, for example 12 ft and 100 ft high, draw a dam 12 ft × 10 or 120 ft in height, by reducing the given profile in the proportion of 120/200 and cut off a height of 100 ft. If the width of the top of the dam is less or greater than 20 ft, the pressures in columns 6 and 7 must be reduced or increased proportionately.

Elements of Wegmann's Practical Profile No. 2
Slightly modified in fourth and fifth columns.

Depth of water below top of dam, feet	Length of joint in feet				Maximum pressure. Tons per sq ft		Tangent of resultant with vertical
	A	B	C	Total	Reservoir full	Reservoir empty	
18.77	20.00	0	0	20.00	1.89	1.36	0.20
30	21.07	0	0	21.07	3.68	2.37	0.31
40	23.89	0	0	23.89	5.03	3.53	0.41
51.97	30.04	0	0	30.04	5.53	4.91	0.50
60	35.38	0	0	35.38	5.59	5.63	0.54
70	42.03	0.62	0	42.65	5.94	6.11	0.58
80	48.68	1.25	0	49.93	6.45	6.59	0.61
90	55.33	1.87	0	57.20	7.02	7.09	0.62
100	61.98	2.50	0	64.48	7.62	7.61	0.63
110	68.63	3.12	0	71.75	8.26	8.15	0.63
120	75.28	3.74	0	79.02	8.90	8.69	0.63
130	81.93	3.74	0.62	86.29	9.55	9.46	0.64
140	88.58	3.74	1.25	93.57	10.22	10.22	0.64
150	95.23	3.74	1.87	100.84	10.89	10.96	0.64
160	101.88	3.74	2.49	108.11	11.56	11.71	0.64
170	108.53	3.74	3.12	115.39	12.25	12.44	0.64
180	115.18	3.74	3.74	122.66	12.95	13.18	0.64
190	121.83	3.74	4.37	129.94	13.63	13.91	0.64
200	128.48	3.74	4.99	137.21	14.32	14.65	0.64

For a dam having a top width of 12 ft and a height of 120 ft, the maximum pressure on the base for reservoir full is 14.32 × 0.6 = 8.59 tons per sq ft, and for reservoir empty 14.65 × 0.6 = 8.79 tons per sq ft. The minimum width of the top of a dam over 60 feet should be at least 8 ft, and this width must often be arbitrarily increased by the designer on account of the pressure of ice. The height of the dam above high-water level varies from 2 ft in low dams to 10 ft in high dams. The tables and profile recommended are for dams upon impervious rock no upward water pressure being considered. For dams upon pervious rock proceed as in trapezoidal and pentagonal dams. The profile recommended is made without regard to the effect of ice pressure in tending to overturn the dam, except that the upper section has been arbitrarily increased so as to provide for this pressure. For heavy ice pressures the specified pressure per linear foot at the water line can be taken into account in a graphic analysis.

The Ashokan Dam, completed in 1912, in the lower Catskill region, for impounding the additional water supply of the city of Greater New York, has some novel features in design which are shown on Fig. 51. The length of the central and masonry section of the dam is 1000 ft, hight 220 ft, width of base 190.2 ft, width of top 23, which is corbeled to 26.3 ft. The dam is built of cyclopean concrete faced with solid concrete blocks laid up as ashlar. The interior of the masonry is drained into galleries approximately level, one at the water level and the other about 40 ft above the base. The galleries are connected at intervals by vertical porous concrete drains. This drainage

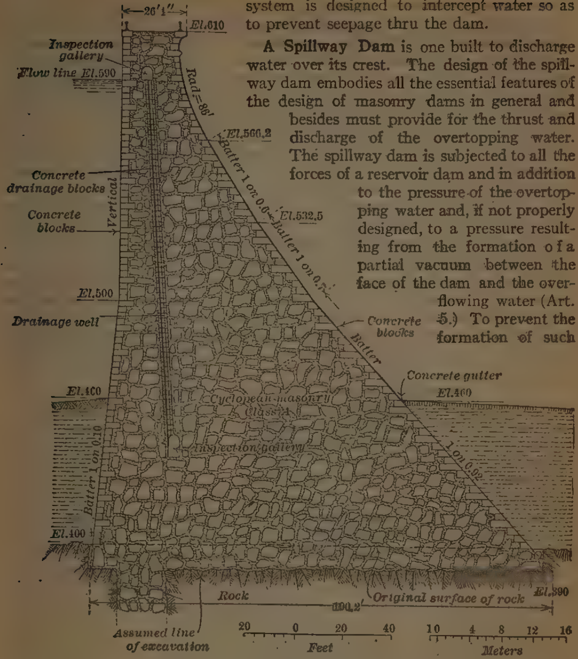


Fig. 51. Maximum Section of Ashokan Dam

system is designed to intercept water so as to prevent seepage thru the dam.

A Spillway Dam is one built to discharge water over its crest. The design of the spillway dam embodies all the essential features of the design of masonry dams in general and besides must provide for the thrust and discharge of the overtopping water. The spillway dam is subjected to all the forces of a reservoir dam and in addition to the pressure of the overtopping water and, if not properly designed, to a pressure resulting from the formation of a partial vacuum between the face of the dam and the overflowing water (Art. 5.) To prevent the formation of such a vacuum the face of the dam must have a sufficiently flat slope so that the falling sheet of water will not separate from it in transit from the top to bottom. Special care must be taken to so design the bottom of the dam that the water will leave the dam causing a minimum amount of erosoin. This latter is accomplished by so designing the base of the dam that the delivery of the water is tangential to the stream bed below the dam. If it is not of the hardest rock, the lower stream bed adjacent to the dam should be paved with concrete or masonry and in some cases a pool is created at the toe of the dam to absorb the shock of the falling water.

The Form of the Crest, as shown by the Cornell experiments on weirs, has an important bearing on the discharging capacity and, therefore, on the economy of spillway dams. It is often difficult, in narrow valleys of large drainage area, to get sufficient length of spillway to discharge freshets without a great depth of water passing over the dam. Every effort should be made to keep this depth a minimum because it increases the pressure on the dam and the scour at the bottom and increases back water and thereby floods more land.

Flat-topped rectangular dams up to 1 ft top-width discharge practically the same volume of water as the standard sharp-crested weir. With wider crests, the discharge diminishes rapidly, a 6-ft horizontal crest passing only 80 percent of the standard discharge. So-called compound crests (Fig. 52), with a sloping top (dotted line in figure) discharge more than standard weirs, the volume of discharge being about 20 percent more for an inclination of 45° of the sloping top, which is connected by a circularly or parabolically rounded portion to the downstream face. This face should be inclined about 30° to the vertical (Fig. 52) so that the nappe of the overflowing water will adhere. For any dam the precise profile to insure an "adherent nappe" may be computed by the ordinary principles of hydraulics, assuming the depth of the overflow.

The sloping top is also useful in preventing injury of the upper part of the dam, due to ice and logs, and also reduces ice pressure of the reservoir if full, since the ice will slide up the slope without exerting pressure.

If to the compound crest and sloping face a smooth transition to the river bed or to an apron protecting same is added (*ik* in Fig. 52) the familiar ogee section results. This section which, in masonry spillway dams, is of general use, not only approximates the theoretical economical section, but possesses several practical advantages. The lower curved part helps to keep the eddies and back waves at a distance from the toe of the dam and thereby protects the dam against the impact of logs and ice revolving with great velocity in these eddies and waves. The inclined face and the rounding of the down stream top make the falling sheet of water adhere to the dam, eliminating the possibility of a vacuum below the nappe, and helps somewhat to destroy by friction a part of the energy of the falling water. This energy may be very large and is the cause of the greatest danger to spillway dams by the scouring action upon the bed of the stream at and near the toe. The destruction of this energy is sometimes attempted by building the downstream face in steps as for instance at the Pedlar River dam in Virginia (Fig. 53). For the same reason the face of the Bassano dam (height of spillway 38 ft, 13 ft of water over the crest) is equipped with two staggered rows of baffle piers built like snow plows and pointed upstream. The baffle piers are not designed to take the shock of the falling water, but to split and divert the stream into interfering jets and so create eddies and disturbances within the body of the downstream pool which is 50 ft long. The Gatun dam (height of spillway 59 ft, 18 ft of water over the crest) has two staggered rows of baffle piers on the apron close to the toe. These piers have vertical upstream faces, the upper row consisting of piers 18 ft wide and 8 ft high. They are designed to take the shock and are faced with heavy cast-iron protection. The destruction of the kinetic energy of the falling water is almost complete.

The necessity of adequate protection of a dam with free overfall is shown at natural waterfalls where at the foot a deep pool is formed in the stream bed. The erosion of the bed stops as soon as the pool becomes deep enough for the contained water cushion to take up and dissipate the shock of the falling water. A similar artificial pool, the bottom and sides well protected by heavy concrete lining should, therefore, be built at the toe of free overfall dams, except in case of low dams on hard rock, the dimensions of the pool and its lining being the functions of the material of the river bed, the height of the fall and the volume of the discharged water.

In the case of a spillway dam with an ogee face or any other form with adherent nappe, it is customary to protect the bed of the stream with a concrete or masonry apron for a considerable distance below the toe of the dam. The design of this apron on any but the hardest rock foundation is a matter of the greatest importance. Besides its function of protecting the bed from scouring, it also increases the percolation factor of dams built on sand, gravel or other pervious foundations. (Art. 15.) The apron should extend far enough downstream so that the river bed would be protected at least to the point where the velocity of the flow becomes normal, in other words where the velocity of the flow

becomes the same as it was before the dam was built. The transition of the high velocity at the toe of the dam to the much lower normal velocity can happen gradually if the apron is inclined at a certain angle, but for this the apron must be long and consequently expensive. If the apron is built horizontal or nearly so, the transition takes place suddenly by a jump, and the protection need not be as long as in the former case. The distance from the toe of the dam to the point where the jump takes place, for any given dam, is a function of the volume of the water discharged and is, therefore, variable. It is shorter at small discharges and longer at great ones. It is good practice to extend a concrete apron for the requirements of a moderate flood discharge and to use good riprap for the protection of that part of the stream bed where the jump would occur only rarely at exceptional floods. The thickness of the apron must be so proportioned that it will more than equalize, by its weight, the uplifting force from below, or it will "blow up." Weep-holes can be used if the foundation is seamy rock, but they should be avoided in material which easily erodes. It is best to give the apron a form similar to Fig. 44, where an artificial pool of moderate depth is formed. Such an apron is very efficient in dissipating the kinetic energy of the falling water and brings the location of the jump to the point desired, i.e., to the lower end of the pool. Baffle walls on the ogee or on the apron will be found necessary only in the case of very high dams or if a very large volume of water is discharged.

Rarified air space forms between the nappe and the face of the dam if the nappe is not adherent and the space between the falling sheet of water and the dam is not connected

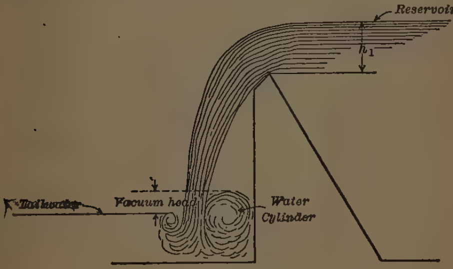


Fig. 51*a*

with the outside atmosphere. Air ducts are occasionally provided for to equalize the pressure of the air, as shown in Fig. 53 illustrating the Pedlar River dam. There is, however, a tendency among engineers to attribute too much importance to the vacuum which may possibly occur behind the nappe (Art. 5) some authors stating that the hydrostatic pressure may thereby increase by as much as the full atmospherical pressure equivalent to 34 ft of additional head. Observations do not admit of the possibility of such a condition. When the air is gradually exhausted from behind the nappe by action of the falling water, a water cylinder is pushed up into the space between the nappe and the dam from the foot of the overfall. The height to which this cylinder, Fig. 51*a*, rises is a measure of the rarification of the air and of the additional pressure upon the dam. The cylinder, however, reaches only a certain limiting height which has been found to depend upon the quantity of the discharge over the spillway. When this height is reached, the nappe is pierced very soon and air admitted through it so that the pressure beneath the nappe again becomes atmospheric. In one case, where the water stood about 2½ ft over the crest, the nappe was pierced every 10 seconds with a thunderlike report, at which time the pressure of the air in the enclosed space under the nappe was diminished by about $1/20$ atmosphere. The experiments were repeated with different amounts of the overflow and showed that the limit of the rarification is proportional to the thickness of the nappe. It is recommended, that for a dam with non-adherent nappe and the space between same

and the face not ventilated, that an increase of pressure on the back, equivalent to 60 percent of the height of the water above the crest be considered.

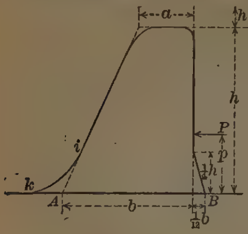


Fig. 52

The spillway dam is usually treated as a masonry dam with trapezoidal cross-section, the curved position *Aik* (Fig. 52) being disregarded in design and investigation. For the magnitude of the acting forces and their point of application, see Art. 5 and Art. 15.

The spillway dam at Austin, Tex., which failed in 1900, was 60 ft high and had an effective width at the base of 44 ft, the height of the water above the crest being 11 ft at the time of the failure. Sliding occurred on the defective rock foundation and also shearing on four vertical cross-sections so that two pieces of the dam, each about 250 ft long, were pushed out of the main portion and moved downstream without overturning. For full details, see Eng. News, Apr. 14, 1910.

Dams Curved in Plan. When a masonry dam is to be built in a gorge with firm rock at its sides, it may be curved in plan and its ends abut against the rock. Many dams have been thus built, but usually the cross-section is computed as if no arch action existed. An arch dam designed as a gravity dam is stronger than the computations show. It is also more pleasing to the eye and gives to the mind a feeling that the structure is one of great security. Since the base is really fastened to the foundation rock, no arch action can exist there, but for the top of the dam, computations may be made by assuming that the arch is to carry a part of the water load which acts normal to its back. For a small radius, say 200 feet, the dam for a few feet below the top might be considered as resisting entirely by arch action. (For Arched Dams, see Art. 23½.)

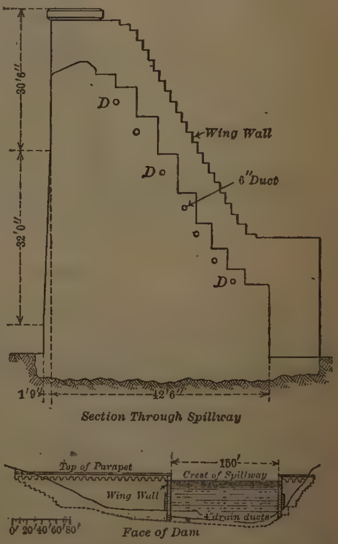


Fig. 53. Pedlar River Dam

The San Mateo dam (California, 1888) is 170 ft high, 180 ft wide on the base, and the radius in plan is 637 ft. The Cheesman dam (Denver, Colorado, 1900) is 227 ft high, 175.9 ft wide on the base, and its radius in plan is 399.9 ft. The method of analysis used for the Cheesman arched dam is given by Harrison and Woodard in Trans. Am. Soc. C. E., 1904, vol. 53.

MASONRY ARCHES AND PIERS

18. General Data for Arches

Definitions. Fig. 54 shows most of the technical terms relating to the construction of stone arches. The SPAN is the horizontal distance between abutments. The SOFFIT is the under or concave surface of an arch. The BACK is the upper or convex surface of an arch, marked *BBB* in figure. The RISE is the vertical distance between the lowest and highest points of the soffit.

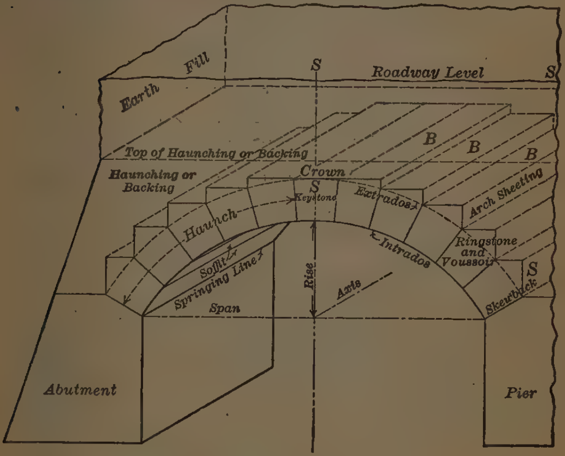


Fig. 54. Technical Terms for Masonry Arches

The CROWN is the highest part of the arch ring. The SKEWBACK is the inclined surface or joint upon which the end of an arch rests. The SPRINGING LINE is the inner edge of the skewback. The AXIS is an imaginary horizontal line, parallel to the abutments, passing thru the middle point of a line joining the springing lines; if the arch is built to withstand horizontal pressure only, as for a shaft, the axis will be vertical. The INTRADOS is the line of intersection of the soffit with a vertical plane perpendicular to the axis. The EXTRADOS is the line of intersection of the back with a vertical plane perpendicular to the axis. The ARCH RING is the entire arch included between the skewbacks, the soffit and the back. The HAUNCH is the portion of the arch ring between the springing line and the crown; haunching or backing is masonry, with bed joints nearly horizontal, which is placed above the haunch. The SPANDREL is the space between the extrados and the roadway, marked *SSS* in figure. SPANDREL FILLING is earth deposited between the back and roadway. A VOUSSOIR is one of the wedge-shaped blocks of stone or concrete of which the arch is composed; in design the voussoir is regarded as having a depth equal to the depth of the arch ring, but in construction the

voussoir may be of less depth; the KEYSTONE is the highest voussoir. RING STONES are voussoirs which show at the ends or faces of the arch ring. ARCH SHEETING comprises all voussoirs except the ring stones; in concrete arches, which are not faced with stone voussoirs, the term arch sheeting includes the entire arch ring.

Kinds of Arches. HINGED ARCHES are those with stone, steel or lead hinges at crown and springing line; hinged masonry arches are always built with three hinges. SOLID ARCHES are those constructed without hinges. A FULL-CENTERED ARCH is one whose intrados is a semicircle. A SEGMENTAL ARCH is one whose intrados is less than a semicircle. A POINTED ARCH is one in which the intrados consists of two arcs of radii greater than the

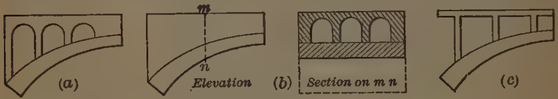


Fig. 55. Spandrel Arches and Columns

half-span, these arcs intersecting at the crown. A RIGHT ARCH is one terminated by vertical planes perpendicular to its axis. A SKEW ARCH is one terminated by vertical planes oblique to its axis. SPANDREL ARCHES are those which rest upon the back of the larger or main arch; they are called transverse when their axes are parallel to that of the main arch (Fig. 55*a*) and longitudinal when their axes are at right angles to that of the main arch (Fig. 55*b*). Spandrel columns are shown in Fig. 55*c*.

Loads. Dead load includes the weight of the masonry itself, the weight of the spandrel filling and that of the roadway. The live load to be used in investigating a masonry arch is the greatest that comes or is liable to come upon its roadway, while for design those given in the specifications are to be employed. Since the spandrel filling distributes the load to a large extent, uniform live loads are often used, about 150 lb per sq ft for heavy highway traffic and about 700 lb per sq ft for railroad traffic. Culverts and small arches are designed for a uniform live load over the entire span. Larger arches are to be discussed not only for full uniform load, but also for a live load extending over certain portions.

Earth-filled arches should be designed for uniform live load over the span and half-span. Arches with the roadway supported on spandrel arches or spandrel columns should be designed for uniform live load over the span, half-span, middle-third of the span, the outer thirds of the span, and narrow highway arches with spans under 100 ft. and all railway arches for concentrated loads. Impact of live loads need not be generally considered except for short span arches with arched or column spandrels. In northern latitudes a snow load of 25 lb per sq ft over the entire span should be considered for a highway bridge if a uniform live load of less than 100 lb per sq ft is provided for. Arches for aqueducts must be figured for the live load of the water. Wind loads are rarely taken into account for the arch, but may be needed for a pier.

In computations an arch one unit in width parallel to the axis is considered. The total live load over the portion considered being divided by the width of arch gives the weight per unit of width, and this may be reduced, for convenience of representation on the drawings, to a rectangle whose height is that of an equivalent weight of masonry.

A Horizontal Thrust H (Fig. 59, Art. 19) is produced at the crown when the arch is loaded symmetrically. For non-symmetrical loading an inclined pressure P acts at the crown, and its horizontal component H is also called the horizontal thrust for that loading. The semi-arch is held in equilibrium by the systems of forces consisting of the horizontal thrust, the vertical loads

and the reaction of the skewback. For any joint 2-2 the part of the arch on the right is held in equilibrium by the horizontal thrust H, the vertical loads W_1 and W_2 and the reaction of the arch below the joint. This reaction is equal and opposite to the resultant of H and the loads W_1 and W_2, and the point r_2 where this resultant cuts the joint is a point in the resistance line, the definition of this term being the same as in Art. 8, except that in the arch the joints are not horizontal. The resistance line itself is a curve joining the points $o\ r_1 r_2 r_3 r_4 r_5$.

A Linear Arch is a curve which is in equilibrium under the action of a given system of loads; it is an equilibrium polygon with an infinite number of sides. In an actual arch the resistance line is a linear arch for the actual loading. Each kind of loading has a linear arch which corresponds to it and which holds that load in equilibrium. The three simplest cases are the following: (1) The linear arch for water pressure is a circle if the axis of the

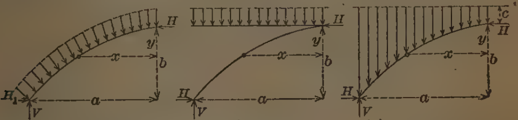


Fig. 56. Loadings for Linear Arches

arch is vertical or if the depth of water over the crown is very large compared to the rise of arch (Fig. 56*a*). (2) The linear arch for uniform horizontal load is the common parabola (Fig. 56*b*). (3) The linear arch for a load of uniform density between a horizontal roadway and the curve itself is the transformed catenary (Fig. 56*c*). Let a be the half-span and b the rise of a linear arch. Let x and y be abscissa and ordinate of any point in the curve with respect to the crown as an origin. Let c be the depth of the load at the crown, and $n = (b + c)/c$. Then the equations of these linear arches are

for circle, $y = r - \sqrt{r^2 - x^2}$, where $r = \frac{1}{2} b + a^2/2b$
for parabola, $y = bx^2/2a^2$
for transformed catenary, $y = c\,(\cosh(\beta x/a) - 1)$

in which β is the Naperian logarithm of $n + \sqrt{n^2 - 1}$. Let w = load per linear unit at the crown. Then the horizontal thrust at the crown and the vertical reaction at the skewback are

for circle, $H_1 = wr - wb$ $H = wr$ and $V = wa$
for parabola, $H = wa^2/2b$ and $V = wa$
for transformed catenary, $H = wa^2/c\beta^2$ and $V = (wa/\beta)\sinh\beta$

The resultant thrust at the skewback is the square root of the sum of the squares of H and V. (For tables of naperian logarithms and hyperbolic functions see Sect. 1.)

Curves for Arches. The central curve of an arch, that is, the curve lying half-way between intrados and extrados, may be the circle, the parabola, the transformed catenary, the ellipse, or various combinations of these. In Fig. 57 are shown the first three curves for five different ratios of rise to span, these having been plotted from the above equations. The transformed catenary curves are for a height of roadway above the crown equal to 1/20 of the span, or $c = 1/10\,a$. When the ratio of rise to

span is 1/10 or less, the three curves are practically the same. ECONOMIC CURVES, or those giving a minimum amount of masonry for various loadings, have been determined to be as follows: (1) A circle should be used for water or other fluid material under a head which is large compared with the span. (2) A parabola should be used for a uniform load on the horizontal projection, which is closely the case for an arch having spandrel columns or spandrel arches. (3) A transformed catenary should be used for vertical loads like those of Fig. 56c or for an earth-filled arch having a ratio of rise to span less than 1/4. (4) An ellipse should be used for an earth-filled arch having a ratio of rise to span greater than 1/4.

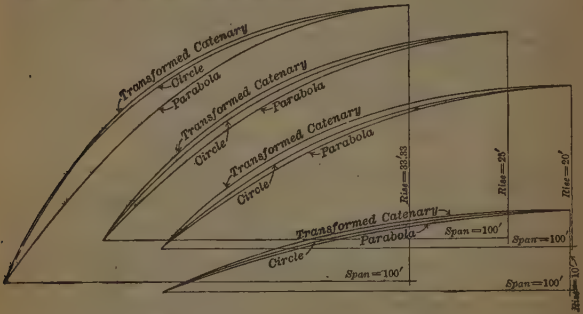


Fig. 57. Comparison of Curves for Arches

The transformed catenary is difficult to lay out and is unsightly, but an approximate catenarian curve consisting of a compound curve made up of circles of different radii may be used. The parabola may be modified for the sake of appearance by short circular curves at its ends, made tangent to the parabola and to the vertical side of the pier.

When arches are built of cut-stone voussoirs, the cost of cutting each stone to a different curve more than offsets the saving in material resulting from using the theoretically correct curves. Many designers for this reason prefer the segmental arch, and for flat arches use the same depth of voussoirs from the crown to the springing line. The economic curves may be used advantageously for concrete arches, because false-work may be laid out as easily for a parabolic, elliptical or multi-centered arch as for a segmental arch.

When the maximum waterway for a given span and height of crown is desired th ellipse must be used. The flatter the ellipse the greater the area of waterway. The flatter the arch, however, the greater the thrust and the larger and more expensive the requisite abutments. For economic abutment design the ratio of the rise of the arch to its span should be as large as possible.

The Span and Rise are, as a rule, determined by physical conditions. A creek or river must be spanned and ample waterway provided. Roadways must pass under the bridge in fixt positions. Railway rights of way must be left unobstructed. Piers and abutments must be located, if possible, where the cost of foundation is a minimum. Where the cost of foundation is uniform from end to end of bridge, a series of arches should for economy be of equal spans. When unequal spans are mandatory, the spans should be made as nearly equal as possible. The rise of the soffit of the arch is always governed by the necessary clearance under the arches or the necessary level of the roadway or by both.

The length of span and the curve of an arch are often determined by the necessities of river travel. The position of the piers should be made parallel to the thread of the current; this condition may necessitate a skew arch, and the clearance over certain portions of the river must be a maximum for clearance of vessels, or the waterway must be a maximum on account of freshets or ice gorges.

Appearance is sometimes the governing factor in selecting the curve of an arch. The semicircular arch best satisfies this demand, and next in sequence are the segmental, modified parabolic and basket-handle arches. A series of arches should, for appearance, always be of equal spans, unless the profile of the ground demands unequal spans; if of unequal spans, a symmetrical arrangement of spans is desirable. The number of spans should, for appearance, always be odd unless the number exceeds seven, in which case it may be odd or even. To increase the length of spans from the ends of the bridge to its middle, for aesthetic effect merely, does not give effective results and is very uneconomical on account of the necessary increased width of piers.

Economic Types. If the arches are of low rise or of high rise and under 100 ft span, the earth-fill bridge is generally the cheapest type. However, if the foundations are poor, regardless of span, the three-hinged ribbed arch (Fig. 60) is the safer one and therefore the better type to use. If the foundations are neither poor nor first-class, the arches should be heavily reinforced with steel, to provide better for possible deformation of the arches due to settlement, however small.

If the arches are of high rise and spans over 100 ft, the arch with arched or column spandrels is generally cheaper. This is due to the great cost of the retaining walls of earth-filled bridges. However, for very wide bridges where the cost of the retaining walls per foot width of bridge is small, comparative designs should be made before selecting the type of design. The three-hinged ribbed type of bridge is generally expensive, due to the cost of the hinges and the added cost of two-faced masonry, or, if concrete, due to the added cost of form work. The column spandrel is used only for reinforced concrete, but it is cheaper than the arched spandrel.

Dimensions for Designs. The approximate thickness at the crown of an arch built of masonry, with portland cement mortar, is given in following table in feet, l being the span of the arch in ft. The thickness at the springing line may be approximated by the following empirical rules in which the percentages are to be added to the crown thickness found from the table: (1) Add 50% for circular, parabolic and catenarian arches having a ratio of rise to span less than 1/4. (2) Add 100% for circular, parabolic, catenarian and three-centered arches having a ratio of rise to span greater than 1/4. (3) Add 150% for elliptical, five-centered, and seven-centered arches. These thicknesses should be measured along radial joint as in Fig. 58, namely at aa for cases (1) and (2) and at bb for case (3).

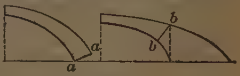


Fig. 58

Thickness in Feet at Crown for Highway Arches (Original)

Kind of masonry	Span in feet			
	Under 20*	20 to 50*	50 to 150†	Over 150‡
First-class ashlar...	0.04 $(6+l)$	0.020 $(30+l)$	0.00012 $(11\,000+l^2)$	0.018 $(75+l)$
Second-class ashlar or brick.........	0.06 $(6+l)$	0.025 $(30+l)$	0.00016 $(11\,000+l^2)$	0.025 $(75+l)$
Plain concrete.....	0.04 $(6+l)$	0.020 $(30+l)$	0.00014 $(11\,000+l^2)$	0.020 $(75+l)$
Reinforced concrete	0.03 $(6+l)$	0.015 $(30+l)$	0.00010 $(11\,000+l^2)$	0.016 $(75+l)$

* For culverts under a high fill, add 60%; for railroad arches, add 25%. † For railroad arches add 20%. ‡ For railroad arches add 15%.

The Top Thickness of a Pier capable of supporting adjacent arches of equal span may be taken as 3½ times the crown thickness. In a series of arches of more than five spans, abutment piers capable of resisting the entire thrust of one adjacent arch should be introduced at every third or fifth span, so that in case of failure of the foundations of one arch the entire series would not fail. The top width of abutment piers may be assumed at five times the crown thickness.

Cost of an Arch Bridge in 1910 was approximately given in dollars as follows:

For a bridge of concrete, plain or reinforced, Cost $= 0.8\,bl\sqrt{d}$.

For a bridge of ashlar, Cost $= 1.4\,bl\sqrt{d}$,

in which b = the width in feet of the bridge, l = its length in feet, and d = the average depth in feet of the bed of the stream below the roadway level. These formulas cannot be used when foundation work is expensive.

Long Span Ashlar Masonry Arches

Name	Location	Span in feet	Rise in feet	Thickness at crown in feet
Bellfield	Pittsburg, Pa	150.0	36.6	4.0
Elyria	Elyria, Ohio	150.0	27.0	3.8
Main Street	Wheeling, W. Va	159.0	28.4	4.5
Tyne	Near Newcastle, Eng	159.9	79.9	4.6
Gignac	Gignac, France	160.0	44.0	6.5
Navaur	Near Navaur, France	160.5	65.0	6.3
Vieile Brioude	Brioude, France	183.7	60.0*	5.3
Wiesen	Wiesen, Switzerland	180.0	68.0	5.9
Coppel	Near Coppel, Germany	187.0	55.8	5.9
Gour Noir	Near Uzerche, France	196.8	52.8	5.6
Grosvenor	Chester, England	200.0	42.0	4.5
Thur	Thur, Switzerland	207.6	45.4	5.9
Bogenhausen †	Bogenhausen, Bavaria	209.9	21.4	3.4
Jaremcze	Jaremcze, Austria	213.0	59.0	6.9
Cabin John	Washington, D. C.	220.0	57.3	4.2
Sidi Rached	Algiers	227.0	82.0	4.9
Trezzo ‡	Near Trezzo, Italy	251.0	87.8	4.0
Montangas	France	262.8	65.9	4.9
Luxemburg	Luxemburg	277.7	101.7	4.7
Salcano	Near Trieste, Austria	278.9	78.0	6.6
Plauen	Plauen, Saxony	295.3	56.4	4.9

* Approximate. † Lead in joints at ⅓ span from abutment. ‡ Destroyed in 1427.

Before laying out the spans and the curves of the arches, a contour map and profile of the site of the bridge should be made. On the profile the depth and character of the foundation and its overlaying material should be indicated. The position of the piers should be located, with reference to economic foundation, clearance of waterway, roadway and right of way; the spans to be made equal unless impossible or, on account of the cost of foundation, manifestly uneconomical to do so. The most economical type of bridge should then be selected and also the curves of the arches requiring a minimum amount of material, due consideration being given, however, to cost of labor of cutting stones, clearances and appearance.

19. Equilibrium and Stability

Methods of Failure. A masonry arch may fail in the following ways. (1) By crushing of the masonry. (2) By sliding of one voussoir upon another.

Long Span Concrete Arches

Name	Location	Span in feet	Rise in feet	Thickness at crown in feet
Plain Concrete:				
Connecticut Avenue	Washington, D. C.	150.0	75.0	5.0
Neckarhausen *	Neckarhausen, Germany	165.0	13.5	2.8
Almandares †	Havana, Cuba	190.0	32.5	5.5
Walnut Lane	Philadelphia, Pa.	233.0	70.3	5.5
Rocky River	Cleveland, O.	280.0	79.1	6.0
Monroe St.	Spokane, Wash.	281.0	115.0	6.75
Reinforced Concrete:				
Colorado St.	Pasadena, Cal.	204.0	77.4	3.7
Meadow St.	Pittsburg, Pa.	209.0	46.1	5.0
Sitter	Gmunden, Switzerland	259.3	87.0	4.1
Halen	Halen, Switzerland	286.0	112.0	3.8
Larimer Ave.	Pittsburg, Pa.	300.4	67.0	6.5
Langwitz	Langwitz, Switzerland	315.0	137.8	4.0
Grafton *	Grafton, New Zealand	320.0	90.0	5.5
Tiber	Rome, Italy	328.1	32.8	0.94

* Three-hinged arch. † Built on pile foundations.

(3) By one voussoir or section of masonry overturning about an adjacent voussoir or section. (4) By shearing in a horizontal or vertical plane, this applying to solid concrete arches and not to voussoirs. (5) As a column when the ratio of the unsupported length of an arch to its least width is greater than 12. (6) From striking the centering before the mortar is hard or when the arch, tho stable under the full load, is not stable under its weight alone. (7) By striking the centering or loading the arch during construction unsymmetrically. (8) By settlement of the foundations. (9) By sliding upon the foundations. (10) By overturning about any point in the pier or abutment. Methods (8) and (9) are the most common ways of failure. All methods of failure, however, must be guarded against in design.

Just before an arch fails the forces acting upon it are in equilibrium, but there is no stability. For any degree of stability, however, the forces acting on the arch are in equilibrium. The conditions of stability are, in general, the same as those explained in Art. 8 for walls and dams.

Conditions of Stability corresponding to the above methods of failure are as follows: (1) The unit compression must not be greater than the safe unit compressive stress given in Art. 1. (2) The angle between the resistance line at any joint and a normal to the joint should be less than the angle of repose of masonry upon masonry. (3) The resistance line should lie within the middle-third of the arch ring; in reinforced concrete arches it may depart a small distance outside the middle-third, but there should be sufficient steel to take care of the tension which will develop upon the portion of such joints farthest from the resistance line. (4) The greatest tendency to shear is in a horizontal plane for a high arch, and in a vertical plane for a low arch; it is sufficient to calculate the shear for points near the springing line, and the frictional resistance in the vertical or horizontal plane should

be regarded as an additional factor of safety against failure by shear. (5) When the span is more than twelve times the width of the arch at the crown, either the crown width must be widened or the faces of the arch ring must be battered so as to reduce the ratio; for an arch formed of independent ribs, the ribs must be well bonded together by transverse walls. (6) The exposed mortar in the joints should always be inspected before striking the center, and in case of large concrete arches it is best to drill holes thru the last concrete placed, to make sure the concrete is set up within the interior of the arch ring; an analysis should be made to find whether the arch ring is stable under its own weight, and if not, the centering should not be struck until the total dead load rests upon the arch. (7) If the masonry above the arch is to be built after the center is struck, it should be built symmetrically; if for any reason this appears impractical, the arch must be analyzed for the desired unsymmetrical load. (8) Foundations for solid arches must be practically unyielding, and the allowed unit load should not, as a rule, exceed 50% of that for ordinary foundations. (9) The horizontal thrust on the foundation should be less than the vertical load upon the foundation multiplied by the coefficient of friction of masonry upon the material of which the foundation bed is composed, or in case of hard or rotten rock, the foundation bed should be roughened; when piers and abutments rest upon piles, the piles should be driven parallel to the line of the resultant thrust or they should be braced by means of horizontal struts to unyielding foundation. (10) The line of thrust must lie within the middle-third of the piers and abutments, including the foundation masonry.

The External Forces (Fig. 59) holding a semi-arch in equilibrium are the vertical loads W_1, W_2, etc., the horizontal thrust H, a vertical reaction V_1 at the skewback, and a vertical shear V_0 at the crown which is due to the action of the other semi-arch. When both semi-arches are loaded equally and symmetrically then $V_0 = 0$. For the case where the position of the resistance line within the arch ring is known, so that the half-span a and the rise b of the linear arch (Art. 18) are also known, the forces V_1, V_0, H are found as follows: first, the vertical reaction V_1 is computed in exactly the same way as for vertical loads on a simple beam, by taking the center of moments at the right end of the arch; second, the shear V_0 is equal to V_1 minus all the loads on the semi-arch under consideration; third, the horizontal thrust H is found by taking moments about r_5 at the left end, or $Hb - V_0a = \Sigma Wl$, in which the last term is the sum of the moments of the loads.

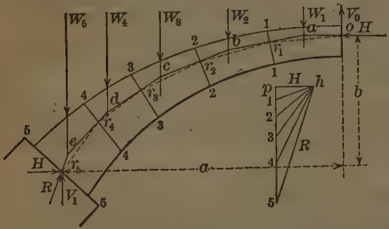


Fig. 59

Example. For arch loaded symmetrically, let $a = 30$ and $b = 21$ ft: $W_1 = 3$, $W_2 = 4$, $W_3 = 7$, $W_4 = 10$, $W_5 = 12$ tons, their lever arms being $l_1 = 28$, $l_2 = 16$, $l_3 = 10$, $l_4 = 4$, $l_5 = 1.5$ ft; here $V_1 = 36$ tons and $V_0 = 0$; $H \times 21 = 3 \times 28 + 4 \times 16 +$ etc. $=$ 276 ton-ft, whence $H = 13.1$ tons. For arch with no load on right-hand span and above loads on left-hand span: $V_1 \times 60 = 3 \times 32 + 4 \times 44 +$ etc. $= 1882$ ton-ft, whence $V_1 = 31.4$ tons; $V_0 = 31.4 - 36.0 = -4.6$ tons, which acts downward upon the right half-span or upward upon the left half-span; $H \times 21 - (-4.6 \times 30) = 276$ ton-ft, whence $H = 6.6$ tons.

For a Three-Hinged Arch (Fig. 60) the above method gives the correct values for V_1, V_0 and H, since a is the half-span between end and middle hinge and b the rise of the middle hinge. For a common unhinged arch it is usual to take a as the horizontal and b as the vertical distance between the

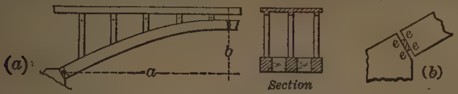


Fig. 60. Three-Hinged Arch

middle of skewback and middle of crown, but the values found are approximate only, since the assumption that the resistance line passes thru the middle points of crown and skewback joints is rarely correct, and since the method of computing V_1 is not perfectly exact.

The three-hinged arch in Fig. 60*a* consists of three ribs, which support the roadway on spandrel columns. Each rib has hinges at both ends and one at the crown, the hinge being a steel pin set in a pedestal and upon which abuts the steel shoe which holds the end of the concrete arch. Another form is that in Fig. 60*b* where a lead plate or a steel plate covered with lead is placed between two abutting concrete surfaces; this form limits the resistance line to a smaller area, but can scarcely be called a real hinge. The lead covered steel plate hinge should be used only for arches erected as hinged arches but afterwards converted into solid arches by encasing the hinge in mortar or concrete.

Stability against Sliding along any joint will be secured when the resultant of all the forces on each side makes an angle β with the normal to the joint such that $2 \tan \beta$ is less than $\tan \phi$, where ϕ is the angle of friction of masonry upon masonry (Arts. 3 and 8). Or, let F and N be the components of the resultant parallel and normal to a joint (Fig. 61) and f the coefficient of friction; then fN/F should not be less than 2. After V_0 and H have been found, F and N for any joint are computed by $F = H \sin \theta - (W - V_0) \cos \theta$, $N = H \cos \theta + (W - V_0) \sin \theta$, in which W is the sum of all the loads between the crown and the joint, and θ is the angle which the joint makes with the vertical. When F is positiv it acts upward; when negativ it acts downward.

Stability against Rotation at any joint is secured when the resultant cuts the joint ab within the middle-third mm (Fig. 61), for the entire joint being then under compression, no tension or opening of the edges of the voussoirs will occur (Arts. 6 and 8).

Stability against Crushing of the material is secured if the compressive unit stress S_a at the edge nearest the resultant is less than the allowable value given in Art. 1. The average unit stress on the joint is N/A, where A is the area of the joint, and the unit stress at a is found by multiplying N/A by $1 + 6\,e/b$, where b is the length of the joint and e is the distance from its middle to the point where the resultant R cuts it (Fig. 61). When R cuts the joint without the middle-third, this rule must be modified unless the joint can take tension (Art. 6).

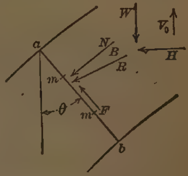


Fig. 61

At each and every joint thruout the arch the above conditions must be satisfied in order that security may prevail thruout. The points cut by the resultants at all joints being connected by a curved line, the resistance line is constructed. The most important of the above conditions is that the resistance line must lie within the middle-third. Altho the word joint has been used in this discussion, all the conclusions apply to the imaginary radial joints of a plain concrete arch.

20. Static Analysis of an Arch

The Static Method of investigating an arch is that outlined in a general way in Art. 19, and is so called because only the principles of statics are used. To determine the horizontal pressure H by that method it is necessary to assume its point of application at the crown joint and also the point of application of the resultant at the skewback joint. Then H can be computed and the resistance line be drawn within the arch ring, but there is no assurance that this is the true resistance line. It is, however, generally accepted that an arch has proper stability against rotation if a resistance line can be constructed which will lie within the middle-third of the arch ring at all radial joints. Hence, different assumptions as to points of application are to be made and different resistance lines to be constructed, and the design should be pronounced deficient in stability if a resistance line cannot be found which lies within the middle-third.

The static method was used in America almost exclusively prior to 1900. While the elastic method (Art. 22) has the advantage that it determines both H and its point of application, yet the static method is still of great value in preliminary investigations.

The Resistance Line can be constructed, after H has been computed, by help of the force polygon. In Fig. 59 the thrust H is laid off horizontally and the loads vertically in succession; then from h as a pole the rays are drawn to the points of division between the loads, the last ray thus giving the magnitude and direction of the resultant R on the joint 5-5. An equilibrium polygon is then constructed, the first side being in the line of H produced, the second parallel to the first ray, and so on until the last side thru e gives the position of R. The points r_1, r_2, r_3, r_4, r_5, where these sides cut the joints, are points in the resistance line, and the resistance line itself is the curve joining them.

This force polygon is for the case of symmetrical loading on the arch so that the crown shear V_0 is zero. The method, however, is perfectly general and may be used to give closely approximate results for unsymmetrical loading by drawing V_0 as the first vertical force in the force polygon, laying off its value upward if the same is positiv and downward if it is negativ.

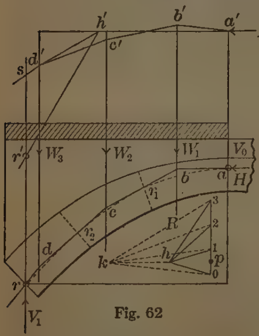


Fig. 62

A Graphic Method of finding the horizontal thrust H is shown in Fig. 62 for a simple case where only three loads are used for the sake of clearness. The arch has the live load on the half-span shown, while the other half-span has only dead load. The horizontal thrust is assumed to act at the crown at the upper middle-third limit, and the resultant at the skewback is assumed to act at the lower middle-third limit. The correct horizontal thrust H on the foregoing assumptions is that which will, with the three loads W_1, W_2, W_3, give an equilibrium polygon passing thru the assumed point a at the crown and the assumed point r at the skewback. The procedure is as follows:

(1) The reaction V_1 is computed in exactly the same way as for a simple beam. The crown shear V_0 is found by subtracting the sum of the three loads from V_1. The load line for the force polygon is laid off from p, the distances 3-2, 2-1, 1-0, op being made equal on a proper scale to the values of W_3, W_2, W_1, V_0 respectively.

(2) Assume pk in the force polygon to be the value of H; with k as a pole draw the rays ko, $k1$, $k2$, $k3$, the last of which should be the resultant at the skewback if the assumed H is correct. Above the arch, in order not to confuse the drawing, extend upward the lines of action of the loads and place the points a' and r' above a and r, and at the same vertical distance apart. Then construct the equilibrium polygon $a'b'c'd's$, each side being parallel to the corresponding ray in the force polygon. The last side $d's$ of this trial equilibrium polygon does not pass thru r'. Extending its line of direction upward until it meets that of H', there is found at h' the point thru which the resultant of the vertical forces acts. Then a line joining r' and h' gives the direction of the true resultant.

(3) In the force polygon, draw R parallel to $r'h'$ then ph is the value of the horizontal thrust H on the scale employed. From h draw a new set of rays, and in the arch ring itself construct the equilibrium polygon $abcdr$, which must now pass thru the point r on the skewback joint.

(4) This equilibrium polygon cuts the two radial joints at the points r_1 and r_2, these being points in the resistance line, which is a curve joining a_1, r_1, r_2, and r. This curve lies everywhere within the middle-third of the arch ring, and hence there is full stability against rotation. The direction and magnitude of the resultants acting on these joints are given by the rays $h1$ and $h2$ in the force polygon.

An Actual Investigation of a proposed design by the static method involves no principles not explained above. The practical procedure is as follows: (1) Vertical lines are drawn dividing the arch structure into an even number of parts, about 16 parts being used for a span of 80 ft. (2) The weights of the masonry and earth filling for each of these parts is found for an arch one unit in width, as also the line of action of that weight; earth is usually reduced to an equivalent height of masonry, and then the center of gravity of an area is the point thru which its weight acts. (3) For loads W_1, W_2, etc., which act thru these centers of gravity the horizontal thrust may be found and the resistance line be located as above explained. (4) The points assumed at crown and skewback may be the middle points for the first investigation, but the upper limit of the middle-third at one joint and the lower limit at the other joint may be used for other resistance lines. (5) One set of lines should be found for full dead and live load over the whole span, another for the case when the live load is on the left half-span only. Lines for other loadings may be sometimes necessary (Art. 18).

The True Resistance Line for a given loading is one that has the least average deviation from the neutral axis of the arch, this being the central curve half-way between intrados and extrados. After having drawn a resistance line which passes outside of the middle-third at one or more places, an attempt should be made to find another one which lies within it. For this purpose find on the drawing the two joints where the resistance line departs most widely from the neutral axis and select two points A_1 and A_2 on those joints which are nearer that axis, A_1 being on the joint which is the nearer to the crown. Let P_1 and P_2 be the sum of all loads between the crown and A_1 and A_2 respectively, a_1 and a_2 be the horizontal distances from A_1 and A_2 to the lines of action of P_1 and P_2, h = vertical distance from crown to A_2, and h' = vertical distance between A_1 and A_2; then the horizontal thrust H' for the new resistance line and the distance t from the crown to its point of application are (Cain's Voussoir Arches, 1904)

$$H' = (P_2a_2 - P_1a_1)/h' \qquad\qquad t = h - P_2a_2/H'$$

With this new horizontal thrust a second resistance line may be drawn and this should pass thru the points A_1 and A_2.

After the true resistance line is found, the degree of stability of the arch with respect to that joint is determined for overturning, crushing, and sliding tendencies in the same manner as explained in Art. 19. The resultant R for each joint is given by the corresponding ray in the force polygon, and its component N normal to the joint may be constructed graphically for use in the equations which refer to compression and sliding.

To Design an Arch by this static method, a drawing is made which seems to give good proportions and dimensions for the given local conditions and loads, the thickness of the arch ring being assumed from the data in Art. 18. Then several resistance lines are constructed, and, if none can be found which lies within the middle-third, the proportions or thicknesses are unsafe ones and must be changed. Another drawing with different dimensions is then made and resistance lines constructed for it. When several drawings or designs all furnish proper stability for the given loads, then that one should be chosen which can be built for the minimum cost.

Cain's Method of finding the crown thrust H and crown shear V_0 for a case of unsymmetric loading is as follows (Fig. 63). P_1 = weight of left half of arch and the load upon it, P_2 = weight of right half of arch and the load upon it, P_3 = weight of the portion of the arch and load between a joint rr and the crown, other notations in the figure. It is necessary to assume the points of application of the resultants on the lowest joints mm_1 and nn_1 and the point of application of the resultant upon some other intermediate joint as at L, while I, L and K are points thru which it is desired to pass the resistance line. All of the points should be taken within the middle-third of their respective joints. If I and L are on the more heavily loaded half of the span and K therefore on the more lightly loaded half, I should be assumed a little below the middle of its joint and L and K a little above the middle of their joints. If I and L are on the lighter half and K on the heavier half, I should be taken a little above the middle of its joint and L and K a little below the middle of their joints. The following formulas (Cain's Voussoir Arches, 1904) give the horizontal and vertical components of the inclined crown thrust and its point of application:

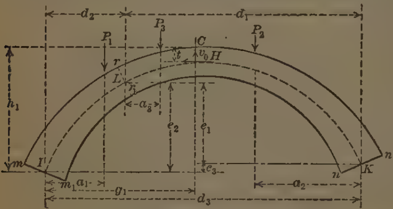


Fig. 63. Unsymmetric Loading

$$V_0 = \frac{a_1e_1P_1 - a_2e_2P_2 + a_3e_3P_3}{e_2d_3 - e_3d_2} \qquad H_0 = \frac{a_1d_1P_1 + a_2d_2P_2 - a_3d_3P_3}{e_2d_3 - e_3d_2}$$

$$t = h_1 - (a_1P_1 - g_1V_0)/H$$

Limitations of Static Method. The old method of design gives curves of resistance which agree closely with those determined by the elastic method. The maximum unit stresses determined by the old method agree with those found by the elastic method within less than 10%, and usually the difference is less than 5%. It is therefore a safe method of design, and may, in general, be used for the design of an arch. As a check upon the final design the elastic method should also be applied in all important arches. The static method, however, ignores temperature stresses (Art. 24). Inclined loads, resulting from pressure of earth, may usually be treated more simply by the elastic method.

Longitudinal Walls and masonry haunching resting upon the back of the arch do not exert their full weight upon the back of the arch, but, for safety, the full weight of walls and haunching should be used in determining the line of resistance.

A Retaining Wall resting upon the back of the arch exerts greater pressure upon the arch at the toe of the wall than at the heel, and the pressure varies between the heel and toe, but it is exact enough to regard the pressure upon the base of the wall as uniformly distributed upon the back of the arch. The ring stones should be tied by iron cramps to the sheeting back of the ring stones, otherwise the horizontal thrust upon the back of the retaining wall may separate them from the arch sheeting.

Change in Spandrel Loading. The line of resistance may be shifted in position so as to lie closer to the neutral axis, by changing the spandrel load-

ing. For instance in Fig. 64, which is an arch of an earth-filled bridge, the resistance line will be changed if transverse hollow arches are formed thru the haunches on each side of the arch; such hollow tunnels are used in the longest masonry arch in the world, that at Plauen in Saxony. Changes in the spacing of spandrel columns will also modify the resistance line; such changes, however, should be mainly confined to cases where errors in design are detected after the arch is erected and before the spandrel columns are built. In an earth-filled arch it is manifestly impractical to change the line of resistance by changing the unit weight of the earth fill at selected points.

Fig. 64

Transverse Spandrel Arches (Fig. 65). The spandrel columns or walls of transverse arches should not generally be spaced parallel to the span at intervals exceeding about 1/7 of the span. The spandrel columns should not generally be spaced parallel to the axis of the arch, transverse to the span,

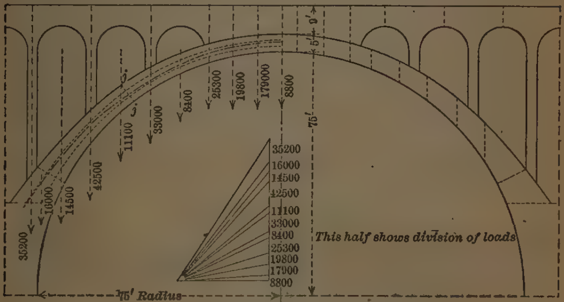


Fig. 65. Connecticut Ave. Arch, Washington, D. C.

at intervals exceeding about two and one-half times the thickness of the arch ring upon which the columns rest. When the columns are so placed, the load brought upon the back of the arch may be regarded as distributed uniformly across the transverse section of the arch. In the case of a reinforced concrete arch this transverse spacing of the columns may exceed two and one-half times the thickness of the arch.

Fig. 65 shows the method of determining the resistance line for transverse spandrel arches, this being shown by a heavy dot and dash line, while the limits of the middle-third are shown by light dotted lines. This arch is one of the main spans of the Connecticut Avenue bridge in Washington, D. C. The 5-foot arch ring is of concrete blocks and the arch sheeting of plain monolithic concrete. The arch is a full-centered one of 150 ft span, and the span of transverse arches is 14 ft. The analysis shown is for a full uniform load over the entire span. The resistance line comes nearest the edge of the middle-third at the joint *jj*, and the maximum compression there is 350 lb per sq in.

21. Three-Hinged Arches

Advantages. (1) If the foundations are compressible, such as pile or clay foundations, the three-hinged arch offers the best type, as it permits of considerable settlement without injury to the arch. (2) If the arch has a rise

of less than ¼ the span, on account of the high temperature stresses in a solid arch, the three-hinged arch is the better. (3) Since the temperature stresses are practically nothing, and as the arch is statically determinate, higher unit stresses may be used than in the solid arch.

Approximate Design. The crown thickness t_0 may be taken a little less than those given by empirical rules for the solid arch. The thickness at the springing line may be approximately 1.25 t_0, and that at the middle of the haunch approximately 1.5 t_0. Having selected these dimensions, the arch should be drawn to a large scale and be analyzed according to the static method. The cheapest type of the three-hinged arch is one of detached ribs supporting spandrel columns, altho there appears to be no reason why an arch with continuous sheeting should not be used. In what follows the lower hinges will be assumed at the same level and the crown hinge at the middle of the arch.

Dead and Live Load over the Entire Span. The weights and lines of application of the loads may be found as in the static method (Art. 20). As the loads are symmetrical, only the left-hand half of the arch need be considered. The crown shear is here zero. The crown thrust H, which is horizontal for symmetrical loading, may be computed from $H = \Sigma Wz/b$, in which W = any load applied upon the left half of the arch, z = horizontal distance from it to the left hinge, b = rise of arch measured from the horizontal line passing thru the lower hinges to the crown hinge. Knowing H, lay off the load line, construct the force and equilibrium polygons as in the static method, applying H at the center of the crown hinge. If the graphical work is correct, the last line of thrust will pass thru the center of the lower hinge.

For dead load on entire span and live load on the left half, let W = any dead load of the left half-span, W' any live load, and z and z' be the horizontal distances from them to the left end hinge. Then

Thrust $H = (\Sigma Wz + \frac{1}{2}\Sigma W'z')/b$ Crown shear $V_0 = + \Sigma W'z'/2a$

in which a = half-span. For dead load over entire span and live load on the right half the same formulas apply, except that V_0 is negativ. The force polygon for either case is then constructed as in Fig. 62, and the equilibrium polygon drawn, taking H and V_0 as applied at the middle hinge; the last line giving the direction of the resultant thrust at the skewback should pass thru the end hinge.

Special Method for Thickness of Arch Ring. The hinges may be drawn at the given points and the approximate loads of the spandrels and arch be applied as shown in Fig. 66. These loads and their point of application may be determined by a practically complete design of the spandrels

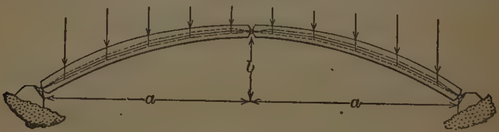


Fig. 66. Three-Hinged Arches

made prior to the arch design. The weight of the arch ring may be approximated, using the dimensions recommended, but the arch ring itself need not be drawn. After computing H and V_0 for each case construct the equilibrium polygons or thrust lines for (1) dead and uniform live load over the entire span, (2) for dead and uniform live load over the right half-span,

(3) for dead and uniform live load over the left half-span. The first line is shown in Fig. 66 as a full line, the second by a line of dashes and the third by a line of dots. After the lines are drawn, knowing, from the force polygon, the intensity of the resultant thrust at any point, the correct thickness of the arch ring at any point may be determined by keeping all thrust lines within the middle-third and the unit stresses within safe limits. If the assumed thicknesses are found to be wrong, a second analysis may be necessary.

This method, considering only dead load, or dead and uniform load over the entire span, may be used to determine the curve of the neutral axis of a solid arch, which will closely approximate the linear arch. This approximate linear arch would be the one requiring a minimum amount of material for any given loading.

The Hinges may be structural or cast steel pins (Fig. 60*a*), or flat lead plates or steel plates covered with lead (Fig. 60*b*). The steel hinge requires to be kept painted, but is advantageous because it locates the line of thrust more precisely than the other types. The lead hinge requires no maintenance. Lead-covered steel hinges should be used only when, for foundation reasons, it is desirable to build the arch as three-hinged and convert it into a solid arch, after the full load is upon the foundations, by filling the spaces around them with solid grout. Lead and lead-covered steel hinges should not have a greater pressure upon them than 1600 lb per sq in. Adjacent to the hinges, blocks of hard stone should be placed to withstand the high unit pressure, or if concrete is used, it should be rich and heavily hooped.

There has been some doubt as to the efficiency of hinges for spans of over 150 ft, but nothing has been proven to their detriment, and the tables in Art. 18 mention two three-hinged arches which have long spans. A two-span ashlar bridge near Munich, Germany, which had arches of 144 ft span and 16 ft rise, failed in 1904, due to the masonry of both arches slipping off the skewback hinges. Stone hinges have been used, but not to an extent which would warrant their general adoption.

22. Elastic Method of Analysis

The True Resistance Line for an arch ring is found in the static method by a series of approximations which start with assumed points on the crown and skewback joints. In the elastic method this true line is found without such approximations in its exact location. This can be done entirely by computation, but graphic work may also be used to draw the equilibrium polygon within the arch ring after H and its point of application have been found. An arch one unit in length is considered, as in the static method, and the NEUTRAL AXIS of the flexural forces is represented by a central line drawn half-way between intrados and extrados. The thickness of the arch at any joint being called t, the moment of inertia of the surface of that joint for an arch of a unit length about the neutral axis is $I = 1/12\, t^3$. The BENDING MOMENT for any point on the neutral axis is the algebraic sum of the moments of all the forces on one side of that point. A moment is positiv when it tends to increase the compression on the back of the arch, this being the same convention as for beams (Sect. 4). The RESISTING MOMENT at any joint is the product of the normal pressure N acting upon the joint and its distance e from the middle of that joint. The word "moment" when used without qualification applies to the bending moment.

Notation (Fig. 67). At the crown: V_0 = vertical shear, H = horizontal thrust. M_0 = bending moment, e_0 = distance of H from crown of the neutral axis, He_0 = resisting moment. For any point: x = horizontal distance from it to crown, y = vertical distance of it below crown, V = vertical shear, M = bending moment, R = resultant thrust on joint, N = component of R normal to joint, e = distance of N from middle point of joint or from the neutral axis, Ne = resisting moment.

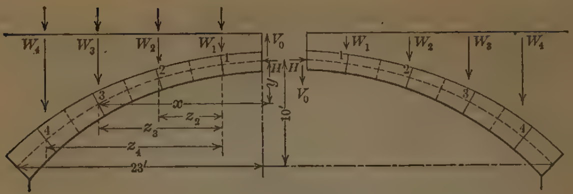


Fig. 67. Data for the Elastic Method

Preliminary Steps. (1) The arch ring is to be drawn to scale and be divided into an even number of parts by the broken radial lines shown in Fig. 67, while half-way between these are drawn solid radial lines to represent joints. When the arch ring is of constant thickness, the arch divisions are equal in length; when it increases in thickness from crown to skewback, the lengths of the divisions are to be determined so that the ratio I/s shall be the same for all, s being the length of any division and I the average of the moments of inertia of the two limiting joints. Let the middle points of the joints be marked 1, 2, 3, etc., and the coordinates x and y be found for each point by computation or measurement. (2) Let Σx = sum of the values of x for these points, Σx^2 = sum of the squares of the abscissas, Σy = sum of their ordinates, Σy^2 = sum of the squares of these ordinates. (3) For a load W, placed at one of these points, let z denote the distance from it, toward the nearest skewback, to another middle point; also let Σz = sum of all these distances, that is, the sum of the distance of W from each of the points nearer the nearest skewback, Σzx = sum of the products of all values of z by the corresponding x, and Σzy = sum of all products of z by the corresponding y; that is, each z in the last two summations is multiplied by the x or y of the point back of W which corresponds to z.

For a Single Load W on the left semi-arch of Fig. 67, the elastic theory of the arch furnishes the following formulas, n being the number of parts into which the semi-arch is divided.

For horizontal thrust, $$H = \tfrac{1}{2} W \frac{n\Sigma zy - \Sigma y.\Sigma z}{n\Sigma y^2 - (\Sigma y)^2} \qquad \text{(A)}$$

For moment at crown, $$M_0 = (\tfrac{1}{2} W\Sigma z - H\Sigma y)/n \qquad \text{(B)}$$

For shear at crown, $$V_0 = \tfrac{1}{2} W\Sigma zx/\Sigma x^2 \qquad \text{(C)}$$

Summations are for the half arch only. The value of H is always positiv; that of M_0 may be either positiv or negativ, depending upon the position of W. The formula for V_0 gives positiv values only, this force acting upward with respect to the left semi-arch, the load being on the left of the crown, but downward with respect to the right one.

When W is on the right semi-arch the same formulas apply if z is measured from W toward the right-hand skewback. For this case, however, the value of V_0 is to be taken as negativ with respect to the left semi-arch, and positiv with respect to the right one.

For two Symmetric and Equal Loads, such as W on the left and W on the right in Fig. 67, let W be the weight of each; then the horizontal thrust and crown moment due to both loads are double those found by the above formulas, while the crown shear V_0 is zero.

For Several Loads, the formulas are to be applied to each in succession and the results added. Thus if one load produces a crown moment of 640 lb-ft and another −870 lb-ft, then for both loads M = −230 lb-ft.

The denominator in formula (A) depends only on the coordinates of the curve and is independent of the span. Hence, when H is to be computed for several loads, the different numerators should be added and then their sum be divided by the constant denominator. Art. 23 gives an example.

For any Joint whose middle point is at horizontal distance x from the crown, values of the moment and shear may be computed, after H, M_0, V_0 are known, from the formulas

$$M = M_0 + Hy + V_0 x - \Sigma Wz \qquad V = V_0 - \Sigma W \qquad \text{(D)}$$

where ΣW is sum of all loads between that joint and the crown, and ΣWz is the sum of the moments of those loads with respect to the middle of the joint. The components of the resultant thrust normal and parallel to the joints are

$$N = H \cos\theta - V \sin\theta \qquad F = H \sin\theta + V \cos\theta \qquad \text{(E)}$$

where θ is the angle which the plane of the joint makes with the vertical (Fig. 61). The resultant thrust itself is $R = \sqrt{H^2 + V^2}$.

Resistance Lines should be determined, in the design of an important arch, for at least three cases: (1) For dead load only. (2) For dead load plus live load over the entire span. (3) For dead load plus live load on left of crown only. In each of the cases let M_0 and H be found for the crown, and M and N for any other joint. Then the distances from the neutral axis to the resistance line are

$$\text{At the crown, } e_0 = M_0/H \qquad \text{At any joint, } e = M/N \qquad \text{(F)}$$

and thus that line may be located at every joint. Or, after having found e_0, an equilibrium polygon can be drawn as in the static method, but here no approximation is needed, since H and e_0 being correctly found for the crown, the force and equilibrium polygons are immediately drawn in correct magnitude and position.

The coefficient of friction which is necessary in order that there may be full security against sliding along the joint is $f = nF/N$, in which F and N are to be computed from E, and n should be 2 or greater.

A common masonry arch is a statically indeterminate structure like a continuous beam or like a beam fixt at its ends. The elastic theory, by which the above formulas are deduced, makes the following assumptions: (1) that the material is elastic and obeys Hooke's law; (2) that the material is homogeneous so that modulus E is constant; (3) that the arch is fixt at its ends so that a tangent there remains unchanged under the loading, (4) that the loads cause no change in the length of the span; (5) that the ends remain in the same horizontal plane under all loadings. Altho it is often difficult in practice to secure the complete observance of these assumptions, the theory gives correct results if they are fulfilled. When an arch is built upon piles or compressible soil the conditions (3), (4), (5) may be fulfilled only partially, and for such cases the three-hinged arch (Art. 21) may be preferable. For rock foundations the elastic theory is entirely satisfactory, and in an important case the old static method of trial and approximation ought not to be used, except as a check.

23. Example of the Elastic Method

Data. It is required to design a plain concrete arch for a double-track railroad which shall have a width of 24 ft, a span of 46 ft between centers of skewback joints, and a rise of 10 ft from the springing line to the center of the crown joint. The railroad track is to be 3.6 ft above the center of the crown joint, and the filling above the back of the arch will be earth. The live load per linear foot of track is specified as 4000 lb. Assume the thickness of the arch as 1.42 ft at the crown and 2.14 ft at the skewback. Thru the given end and middle points draw a curve approximating a parabola,

and lay off these two radial joints. Then draw the extrados and intrados curves so that the radial distance between them shall increase uniformly from middle to end, that is, so that the thickness of the arch at horizontal distance x from crown is $1.42 + 0.0313\,x$. Following is an investigation of this proposed design (Fig. 67).

(1) Divide the central curve of one semi-arch into four parts, of unequal length, those nearest the crown being the shortest. Let the lengths of these divisions, measured on the curve, be represented by s_1, s_2, s_3, s_4. Let radial lines be drawn thru the middle points of these divisions and their lengths t_1, t_2, t_3, t_4 be found. Compute the ratios t_1^3/s_1, t_2^3/s_2, t_3^3/s_3, t_4^3/s_4; if these are equal, the division is correctly made; if not, the process must be repeated until four points 1, 2, 3, 4 are found for which these ratios t^3/s have approximately the same value.

(2) Let the final results of this work be that the thickness of the four joints are $t_1 = 1.52$, $t_2 = 1.72$, $t_3 = 1.90$, $t_4 = 2.06$ ft; that the abscissas of their middle points are 3.22, 9.47, 15.20, 20.50 ft, and that the corresponding ordinates are 0.21, 1.84, 4.62, 8.31 ft. These coordinates are entered in the second and third columns of the table below.

(3) Drawing vertical lines at the limits of the four divisions, the loads for the points 1, 2, 3 are, for an arch one foot in length;

For dead load, $W_1 = 4500$ $W_2 = 5400$ $W_3 = 7000$ lb
For live load, $W_1 = 2100$ $W_2 = 2000$ $W_3 = 1800$ lb

The last load, W_4 in this case, is not used, because by this method of division and computation it does not produce stresses in the arch; in reality, it does, of course, cause stresses, but they are small on account of its nearness to the skewback. In Fig. 67 the dead loads are shown for both halves of the span, while the live loads are shown only for the left-hand half.

(4) The summations and quantities involving only x and y, which are required for use in the formulas of Art. 22, are computed and entered in the following table; the denominator in the formula for H has the constant value 150.9 for all loads in all positions.

Point	x	y	y^2	x^2
1	3.22	0.21	0.04	10.4
2	9.47	1.84	3.39	89.7
3	15.20	4.62	21.34	231.0
4	20.50	8.31	69.06	420.2
$(\Sigma y)^2 = 224.4$ $n = 4$ $n\Sigma y^2 - (\Sigma y)^2 = 150.9$		$\Sigma y = 14.98$	$\Sigma y^2 = 93.83$	$\Sigma x^2 = 751.3$

(5) The products and summations involving z, which depend upon both position of load and the other points of division, are computed and entered in a second table. Since the load W_1 is taken as applied at point 1, the values of z for it are found by subtracting the first value of x from each of the following ones; also for W_2 the second value of x is subtracted from the following ones. The distances z_2, z_3, z_4, shown in Fig. 67 are the values of z for the load W_1.

Point	Values of z			Values of zy			Values of zx		
	W_1	W_2	W_3	W_1	W_2	W_3	W_1	W_2	W_3
1	0	......	......	0	......	......	0	......	......
2	6.25	0	......	11.5	0	......	59.2	0	......
3	11.98	5.73	0	55.3	26.5	0	182.1	87.1	0
4	17.28	11.03	5.30	143.6	91.7	44.0	354.2	226.1	108.6
$\Sigma z =$	35.5	16.8	5.3	......	......	......	......	......	......
$\Sigma y \cdot \Sigma z =$	531.8	251.7	79.4	......	......	......	......	......	......
$\Sigma zy =$	......	......	......	210.4	118.2	44.0	......	......	......
$\Sigma zx =$	......	......	......	......	......	......	595.5	313.2	108.6

(6) Formula (A) of Art. 22 is now used to obtain the horizontal thrust caused by each load, and then Formulas (B) and (C) to find the crown moment and crown shear. For example, the work for the first load is

$$H = \tfrac{1}{2} W_1 (4 \times 210.4 - 531.8)/150.9 = 1.026\, W_1$$
$$M_0 = (\tfrac{1}{2} W_1 \times 35.5 - 1.026\, W_1 \times 14.98)/4 = 0.595\, W_1$$
$$V_0 = \tfrac{1}{2} W_1 \times 595.5/751.3 = 0.396\, W_1$$

This value of V_0 is positiv if W_1 is on the left and negativ if it is on the right of the crown, it being understood that the left semi-arch is the one for which the analysis is to be made. For each load in Fig. 67 the computed results for the crown now are

for W_1, $H = 1.026\, W_1$ $\quad M_0 = 0.595\, W_1$ $\quad V_0 = 0.396\, W_1$
for W_2, $H = 0.733\, W_2$ $\quad M_0 = -0.645\, W_2$ $\quad V_0 = 0.208\, W_2$
for W_3, $H = 0.320\, W_3$ $\quad M_0 = -0.535\, W_3$ $\quad V_0 = 0.072\, W_3$

(7) Only one case of loading will be here investigated, namely, that when the live load covers the left semi-arch only, as in Fig. 67. For dead load over the whole span, the values $W_1 = 4500$, $W_2 = 5400$, $W_3 = 7000$ lb are to be inserted above, the products added and the sums doubled for H and M, while those for V cancel each other on account of the double sign. For live load over left semi-arch only, the values $W_1 = 2100$, $W_2 = 2000$, $W_3 = 1800$ lb are to be inserted and the products added. Thus, for

dead load, $\quad H = 21\,600$ $\quad M_0 = -9100$ $\quad V_0 = 0$
for half live load, $\quad H = 4200$ $\quad M_0 = -1000$ $\quad V_0 = +1400$

Lastly, the addition of these gives the final values for the case of loading shown in Fig. 67, namely, $H = 25\,800$ lb, $M_0 = -10\,100$ lb-ft, $V_0 = 1400$ lb. The negativ sign of M_0 shows that it tends to produce tension on the back of the arch.

(8) These final crown moments and shears reduce formula (D) of Art. 22 to

$$M = -10\,100 + 25\,800\,y + 1400\,x - \Sigma Wz, \qquad V = 1400 - \Sigma W$$

and the values of the moment and shear for each joint are placed in the table below. Also the first formula (E) takes the form $N = 25\,800 \cos\theta - V \sin\theta$, and the computed normal pressures for each joint are given in the last column.

Joint	x feet	y feet	$\cos\theta$	$\sin\theta$	M pound-ft	V pounds	N pounds
Crown	0.0	0.0	1.00	0.00	− 10 100	+ 1 400	25 800
1	3.2	0.2	0.99	0.12	− 460	+ 1 400	25 370
2	9.5	1.8	0.94	0.34	+8390	− 5 200	26 020
3	15.2	4.6	0.86	0.50	+ 8390	− 12 600	28 490
4	20.5	8.3	0.79	0.61	− 9570	− 21 400	33 440
Skewback	23.0	10.0	0.77	0.64	− 19 210	− 21 400	33 560

(9) Finally, the eccentricity e, or the departure of the resistance line at the neutral axis, is obtained for each joint by formula (F) of Art. 22, namely $e = M/N$. Following table shows that the resistance line is within the middle-third of the arch ring except at the crown and at joint 2 and skewback; the values of $\tfrac{1}{6}t$ are distances from neutral axis to limits of middle-third. The maximum compression S_1 and the maximum tension S_2, using the formulas of Art. 6 and using only the normal component N of the thrust, are

Joint	Thickness, t, feet	Middle-third, $\tfrac{1}{6}t$, feet	Eccentricity, e, feet	Maximum unit stress, lb per sq in		
				Compress. S_1	Tension, S_2	S_1
Crown	1.42	0.237	− 0.391	330	80	370
1	1.52	0.253	− 0.018	120	....	.....
2	1.72	0.287	+ 0.322	220	13	230
3	1.90	0.317	+ 0.294	200	....	.....
4	2.06	0.343	− 0.286	210	....	.....
Skewback	2.14	0.357	− 0.572	280	65	310

computed for the joints under the assumption that no rupture of the material occ under the tensile stress; if such occurs, then the compression on the other side is increa to the value shown in the last column.

Other cases of loading cannot here be investigated, but a live load over the entire sp or perhaps live loads W_1 and W_2 acting on both sides of the crown, may produce a grea moment at the crown and raise the resistance line higher above the neutral surface. T result of the investigations thus far made shows that the resistance line passes outs the middle-third at three points, and hence the design must be modified in order to satisfactory. This may be done by increasing the rise, by increasing the thickness the arch ring at the crown, or by introducing steel reinforcement.

Above method, in its essential features, was first given in America by Howe (Arc 1890), the formulas being materially simplified by Turneaure and Maurer (Reinfor Concrete, 1907); both formulas and tabulations are here given in somewhat differe form. The way of obtaining the loads and applying them above the centers of the joi is regarded as not perfectly satisfactory. There is, however, nothing in the theory wh requires the loads to be applied at the points 1, 2, 3 in Fig. 67; they may be applied any other positions, but the x's for any W must be measured from the position of W all the points between it and the skewback.

23½. Arches under Water Pressure

For an Arch with Vertical Axis the water pressure upon the extrados uniform for each arch ring or horizontal section. In circular arches the press is radial and produces uniform compression thruout each horizontal arch ri In a circular arch with a vertical axis let p denote the intensity of the wa pressure at a certain elevation, r = radius of the arch center line, T = the thr in the arch ring, then $T = pr$. In the design of arched dams the radius of upstream face is usually taken as r. For an arch ring of unit height, S being allowable compression of the material per unit area, the necessary thickness the arch ring at the height corresponding to p will then be $t = pr/S$. I example, an arched concrete dam 20 ft high having a radius of 60 ft should hav thickness at base $t = 20 \times 62.5 \times 60/250 \times 144 = 2.09$ ft if the allowable u compression is 250 lb per sq in.

Practical considerations and the influence of indeterminate stresses will increase thickness. The fixing of the arch at the abutments and possibly at the bottom is considered in the formula. Such fixing, together with unfavorable temperature stre may increase the stresses developed in an arched concrete dam by as much as 100 perc of those given by the cylindrical formula.

For an Arch with Horizontal Axis the intensity of the water pressure any point of the extrados depends only on the depth of that point below water surface and its direction is normal to a tangent to the extrados at tl point. If the arch is circular, bending moments as well as thrusts are develop

For Symmetrical Arches, the loading (both dead and live loads) will symmetrical and the formulas to be used for finding the thrust and moment the crown are

$$H = \frac{n\,\Sigma my - \Sigma m \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} \qquad M_0 = \frac{\Sigma m - H\Sigma y}{n} \qquad V_0 = 0$$

the summations being taken for the half-arch only; here m is the cantile moment at any point of the half arch, while the other symbols are the sa as given in the notation on page 735.

Example: To design a plain concrete circular arch, which shall have a sp of 30 ft between centers of skewback joints and a rise of 8 ft from center springing line to center of crownjoint, to support a waterload the water surf

of which is 6 ft above the middle of the crownjoint. The arch ring to have uniform thickness.

(1) An arch ring 1 ft wide will be considered. Assume a thickness of 18 in. Divide the half arch (Fig. 67*a*) into 4 equal parts by dotted radial lines, while half way between these are drawn solid radial lines to represent joints. The weight of the arch and the water pressure are assumed to act at the centers of the equal arch divisions (that is, at the middle of the joints). The water pressure is calculated by taking its intensity at the intersection of the extrados and the joints, and by multiplying this by the length of the division measured at the extrados. The direction of the water pressure so calculated is radial, since the pressure acts normal to the extrados. By taking n sufficiently great, we can come very near to the exact magnitude of the resulting water pressure.

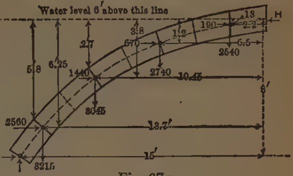


Fig. 67*a*

(2) With the given data the radius of the center line is 18.05 ft, and for the extrados 18.80 ft. The length of one division at the center line equals 4.42 ft and at the extrados 4.60 ft. The central angle for the half span equals 56° 10′. The weight of one arch division equals 4.42 × 1.5 × 150 or 995 lb. The radial water pressures are 1555, 1870, 2505. 3395 lb for joints 1, 2, 3, 4 respectively, the depths below the water surface of these joints at the extrados being 5.4, 6.5, 8.7 and 11.8 ft as scaled from the drawing. Taking horizontal and vertical components for each of these, the following table results:

Joint	Weight of Arch lb	Vertical Component of Water Pressure, lb	Total Vertical Force, lb	Horizontal Component of Water Pressure, lb
1	995	1545	2540	190
2	995	1745	2740	670
3	995	2050	3045	1440
4	995	2220	3215	2565

(3) The so-called cantilever moments m are now computed for each joint, using only the forces on the right of that joint. Fig. 67*a* shows that all these moments are positiv. Thus for joint 2

$$m_2 = 2540\ (6.5-2.2) + 190\ (1.2-0.13) = 11\ 130 \text{ lb-ft}$$

The following table gives values of m and other quantities needed in the computations:

Point	x	y	y^2	m	my
1	2.2	0.13	0.02	0	0
2	6.5	1.2	1.44	11 130	13 360
3	10.45	3.3	10.90	33 550	110 720
4	13.7	6.25	39.06	67 630	422 690
Σ		10.88	51.42	112 310	546 770

Inserting the values of the last line in the formulas on p.740 for H and M_0, the horizontal thrust of the crown is $H = 11\ 050$ lb, and the bending moment at the crown is $M_0 = -1980$ lb-ft. Under the given water pressure, then, the stress at the crown is tension at the extrados and compression at the intrados.

(4) The bending moment M at any joint is now computed from the formula $M = M_0 + Hy - m$. Then the normal pressure N at each joint is computed as in Art. 22 or found graphically, and finally the eccentricity $e = M/N$.

(5) The line of pressure can now be laid out by measuring the distances e along the joints from the middle in the direction as indicated by the sign + or −, and by connecting the points so found. The unit stresses are highest at the springing, 147 lb per sq in compression and 27 lb per sq in tension. Since concrete unreinforced cannot transmit tension, either the depth of the arch ring must be increased or it must be reinforced.

Arched Dams. Where the slopes of the valley are steep and are of rock suitable for abutments, dams consisting of a single arch span with vertical axis can be advantageously built under certain conditions. Such dams are designed on the assumption that the water pressure is resisted by arch action only, the resistance by the weight of the dam being neglected in the computations and regarded as an additional factor of safety. The formula giving the necessary thickness of the arch (see the beginning of this article) shows that this thickness increases with the radius, the length of which can be expressed in terms of the thickness of the dam, the given water pressure and the safe compressive stress of the masonry: $r = ts/p$. It is obvious, that for economy the thickness of the arch must be less than that of a gravity dam of the same height. Therefore, if t_1 is the thickness of the gravity section at the elevation corresponding to p, the arched dam will be economical only if the abutments can be conveniently joined with a radius considerably smaller that t_1S/p; if the distance between abutments would necessitate a longer radius, the gravity type of dam would be the cheaper.

The thickness of the arch, on the other hand, according to the formula, decreases with increasing values of the permissible unit stress. The records of the existing arched dams show a wide divergence in the compressive stress they are subjected to. These stresses vary from 155 lb per sq in in the Parkes dam, N. S. W., to 825 lb per sq in in the Bear Valley dam, Cal.; the recently built arched dams are strest from 300 to 350 lb per sq in. With the latter stress it can be shown that the limiting value of the radius is about 500 ft; the economical subtended angle being about 120°, the limiting length of the arched dam, measured at the crest is about 800 ft. In V-shaped valleys very considerable saving over arched dams with constant upstream radius can be effected by keeping the subtended angle constant and accordingly reducing the radius at the lower parts of the dam.

The Bear Valley dam (Calif., 1884), is a remarkable instance of the efficiency of arch action, being only 3.2 ft. wide on the top and 8.4 ft wide at a depth 48 ft below the crest. The resistance line for the case of reservoir full, obtained by considering it as a gravity dam, runs out of the section 12.5 ft below the top; this is a rubble dam, every stone of which was thoroughly bedded in cement mortar. The Salmon River dam, Alaska, is a constant angle concrete dam, 163 ft high, 44 ft wide at the bottom. It is described in Trans. Am. Soc. Civ. Eng., Vol. 78, by L. R. Jorgensen, the originator of this type of dams. Its maximum radius is 331 ft on the top, its minimum radius 147.5 ft at the base, the length of the crest is about 620 ft. The maximum compressive stress developed is 330 lb per sq in at a point about 100 ft below the top.

Multiple Arch Dams. Instead of building an arched dam with a single span it will more often be economical to use several arches having spans from 20 to 60 ft. Fig. 67*b* shows a dam designed by Wm. Barclay Parsons, for the Garoga river, its maximum height being 56 ft. On the left is a gravity spillway dam, and on its right are seen three vertical arches of 60 ft span, 3 ft thick at top and 5½ ft thick at bottom. These arches form the water front of the dam and the thrust from their water load is carried back by four buttresses.

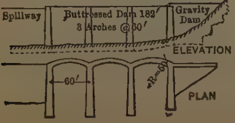


Fig. 67*b*.

Great care should be taken to prevent such dams from sliding. Sometimes the water

front is inclined instead of vertical so that the normal water pressure may increase the load on the base of the buttresses and thus also increase the frictional resistance to sliding.

24. Temperature and Deformation

General Effect of Temperature. If a ring of any elastic material, as in Fig. 68, is rigidly fastened to fixt abutments, the expansion of the ring under a rise of temperature exerts a horizontal thrust against the abutments. The abutments being immovable, the ring distorts, and the points *dd* retreat from the horizontal diameter *ab*. If the temperature falls, the ring is also distorted, but the points *dd* approach *ab*. Since the ring is still in equilibrium, the stresses at all sections will have undergone a change. The removal of the lower half of the ring would evidently not affect the distortion of the upper

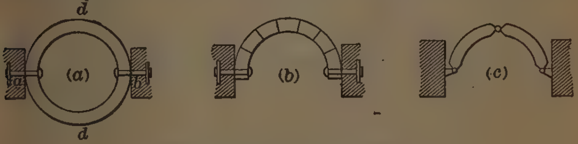


Fig. 68. Influence of Temperature

half under the change of temperature, provided the upper half is rigidly fastened to the immovable abutments. The condition of the upper portion of the ring is approximately that of a solid masonry arch built upon a rock foundation. If the upper portion of the ring is of loose voussoirs (Fig. 68*b*), that is, without mortar in the joints, it is evident that the arch would adjust itself with but little strain. In practice, however, the arch ring is held down by the loads upon it so that the stresses due to the loads are increased or diminished by a change in temperature. Fig. 68*c* shows a three-hinged arch which, disregarding friction of the hinge, is free to move under change of temperature without developing temperature stresses.

Changes in Crown Thrust and Moment occur when the temperature varies from the standard under which the design was made. When the temperature rises above the standard, the horizontal thrust is increased; when it falls below the standard, the horizontal thrust is decreased. Let H' = the change in H due to a change of T degrees; let e = the coefficient of expansion and E = modulus of elasticity of the material (Art. 2); let n = the number of divisions into which the half arch ring is divided and m = the constant ratio I/s for each division (Art. 22); let Σy and Σy^2 be the sum of the ordinates and the sum of their squares for the middle points of the several joints; let a = half-span of the arch. Then, for a rise of T degrees,

$$\text{Horizontal thrust} = H' = \frac{ETenma}{n\Sigma y^2 - (\Sigma y)^2}$$

$$\text{Crown Moment} = M_0' = -H'\Sigma y/n \qquad \text{Crown shear } V_0 = 0$$

The bending moment at any point due to this rise in temperature is $M' = M_0' + H'y$. The computed values of H', M_0' and M are to be combined with those found for H, M_0 and M in Art. 23 for the case of loading under consideration. For a fall of temperature T is to be taken negativ.

The standard temperature is usually assumed in the design at 50° Fahrenheit and a range of from 20° to 45° above and below it is allowed, or $T = \pm$ 20° for a spandrel filled arch and $T = \pm$ 45° for an isolated arch ring. The values of M due to temperature are

most conveniently taken from a diagram, being the ordinates between the neutral axis and a straight horizontal line drawn at a distance $(\Sigma y)/n$ below the crown if the ordinate below the crown represents on a certain scale the value of M_0'.

Example. For the arch discussed in Art. 23, the value of m is 0.45. Then for $T = 45°$ and $E = 2\,500\,000 \times 144$ lb per sq ft, the horizontal thrust due to temperature is $H' = \pm$ 27 000 lb. Hence under dead load the horizontal thrust ranges from 21 600 + 27 000 = 48 600 lb at 95° Fahr. to 21 600 − 27 000 = − 5400 lb at 5° Fahr.

Temperature changes will often, on the basis of the formulas, cause the line of thrust to pass outside of the middle-third of the arch ring. It does not, however, seem necessary, when temperature is considered, that the line of thrust at all joints, based upon the maximum moments, shall lie inside the middle-third. The following tabulation shows the stresses, in lb per sq in, as given by the formulas, for an arch of 150 ft span with a uniform thickness of 4 ft, the range of temperature being 40° Fahr. from the normal.

Ratio of Rise to Span	Crown			Skewback		
	Concrete*	Ashlar†	Ashlar‡	Concrete*	Ashlar†	Ashlar‡
½	40	35	65	70	65	110
¼	110	105	180	210	190	340
⅙	150	140	240	270	250	445
1/10	340	320	550	625	580	1010
1/20	770	720	1260	1375	1280	2245

* $E = 2\,500\,000$ lb per sq in and $e = 0.0000060$. † For average joints, $E = 4\,000\,000$ and $e = 0.0000035$. ‡ For very thin joints, $E = 7\,000\,000$ and $e = 0.0000035$.

Shortening of the Arch Ring under the compressive stresses would produce the same effect as a decrease in temperature if the compression were uniform thruout. If S = average compressive unit stress in the arch ring, this being found by taking the average of several values, or known in advance by specification, then the shortening per unit of length is S/E instead of eT, and accordingly

$$\text{Horizontal thrust } H' = -\frac{Snma}{n\Sigma y^2 - (\Sigma y)^2}$$

while the expression for crown moment is same as before and crown shear is zero. For the numerical example of Art. 23, this value of H' is − 5300 lb, or about 25 % of the thrust due to dead load.

The Deflection of the Crown under load may be closely found by the following formula. For dead load, or for full live load, it is

$$f = (M_0 \Sigma x + H \Sigma xy - \Sigma W zx)/mE$$

in which the summations are for the half span only, as in Art. 23. For any loading on the right of the crown the deflection of the crown is

$$f = (M_0 \Sigma x + H \Sigma xy + V_0 \Sigma x^2 - \Sigma W zx)/mE$$

and this expression applies also to any loading on the left of the crown if the negativ sign be used before V_0.

For Temperature Changes the deflection of the crown is much greater than that due to the loads, and it may be computed from

$$f_1 = \frac{eTa\,(n\Sigma xy - \Sigma x \Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$$

For very flat arches this deflection should be computed and allowance be made for the same in the cambering of the centering. For arches laid up in alternate blocks, the shrinkage of the concrete may cause more deflection than that due to the loads.

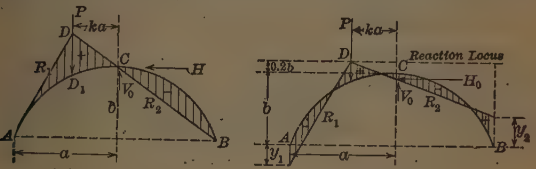
Tests made by the Austrian Society of Engineers on masonry arches ranging in span from 6 to 75 feet, have confirmed the correctness of the above formulas and hence also the validity of the modern elastic theory. See Engr. News, Nov. 21, 1895, and April 9, 1896.

25. Diagrams and Influence Lines

Formulas for a Three-hinged Arch (Fig. 69*a*). For a single load on left of the crown,

$$H = Pa\,(1 - k)/2\,b \qquad V_0 = P\,(1 - k)/2 \qquad M_0 = 0$$

The moments are shown by the vertical lines above and below the arch curve. The moments at all hinges are 0. If the ordinate DD_1 to some scale equals



(*a*) *Three-Hinged Arch* (*b*) *Solid Arch*

Fig. 69. Moments due to Load P

the moments at D_1, then the moment at any other point equals the length of the vertical line at that point included between the neutral axis ACB and the resultant R_1 or R_2, measured to the same scale. Or the moment at any point equals H multiplied by the vertical line or ordinate included between ACB and R_1 or R_2, measured to the same scale as the arch. For uniform load over the entire arch, $H = wa^2/2\,b$, while $V_0 = 0$ and $M_0 = 0$. The resistance line is a curve passing thru the hinges; it may coincide with the neutral axis or may lie above or below this axis.

Formulas for a Solid Parabolic Arch (Fig. 69*b*). For a single load P at a distance ka from the crown,

$$y_1 = 2\,b(1-5\,k)/15(1-k) \qquad y_2 = 2\,b(1+5\,k)/15(1+k) \qquad H = 15\,Pa(1-k^2)^2/32\,b$$
$$V_0 = P(2-3\,k+k^3)/4 \qquad M_0 = -Pa(3-16\,k\times 18\,k^2-5\,k^4)/32$$

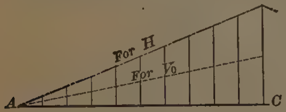
The reaction locus, or the locus of the intersection of all loads with their resultants, is a horizontal line 0.2 b above the crown of the neutral axis. The moments are shown graphically at all points of the arch. This may be drawn for any load by laying off the reaction locus and y_1 and y_2 computed by formulas. This moment diagram should be interpreted in the same manner as explained for the three-hinged arch. For uniform load over the entire arch, $H = wa^2/2\,b$, while $V_0 = 0$ and $M_0 = 0$.

The following Table for Solid Parabolic Arches is useful in obtaining H, V_0, and M_0 for a single load P at a distance ka to the left of the crown. The numbers of each column should be multiplied by the factors shown in the first line. This table may be used for the design of earth-filled bridges or for arches with spandrel arches or columns, if the loads are applied at tenth points. By use of formulas this table may be extended to cover all classes of parabolic arch design. The table may be used as a close approximate check for all positions of loading and for non-parabolic arches in which b/a is less than 0.5.

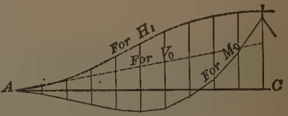
Thrust, Crown Shear, and Moment for Solid Parabolic Arches

k	H	V_0	M_0
0	0.4687 Pa/b	+ 0.5000 P	+ 0.0938 Pa
0.1	0.4593	+ 0.4252	+ 0.0494
0.2	0.4320	+ 0.3520	+ 0.0161
0.3	0.3881	+ 0.2818	− 0.0069
0.4	0.3308	+ 0.2160	− 0.0203
0.5	0.2636	+ 0.1562	− 0.0254
0.6	0.1920	+ 0.1040	− 0.0240
0.7	0.1219	+ 0.0608	− 0.0181
0.8	0.0607	+ 0.0280	− 0.0103
0.9	0.0169	+ 0.0072	− 0.0031
1.0	0.0000	+ 0.0000	− 0.0000

An Influence Line is a line whose ordinates represent the values of a function as a single load travels over the span, the ordinates being drawn at the positions of the load. Thus in Fig. 70*a* let a load P travel from the left end A of the span to the middle C. Then the influence line for the horizontal thrust H is constructed by laying off from AC an ordinate at each



(*a*) *Three-Hinged Arch*



(*b*) *Solid Arch*

Fig. 70. Influence Lines

position of the load to represent the value of H for that position. Since H varies as the first power of k, the influence line is straight and it is only necessary to determine the ordinates for $k = 0$ and $k = 1$. For the three-hinged arch both H and V_0 are 0 when the load is at the springing line, and reach their maximum values at the crown, while M_0 is always 0. For the solid arch with fixt ends, Fig. 70*b* shows influence lines for H_1, V_0 and M_0 which have been constructed by the help of the preceding table. For both cases the ordinates for the other half span have the same values as for CA, except that those for V_0 are negativ, or the load travels from C to B. The influence lines clearly show that loads near the crown produce the greatest thrusts, shears, and moments.

26. Construction and Erection

Materials of Construction. In selecting the material of which the arch is to be built, the designer must be governed by the available materials in the local market, due consideration being given to labor conditions. If cement, sand, and broken stone are cheaper than cut stone, the arch should be built of concrete, provided, however, architectural conditions do not demand stone. For arches over 150 ft span, in which the cost of the false-work or centering may exceed $5.00 per cu yd of the arch masonry, the stone arch may be cheaper than the concrete arch, but usually this will not be so. Occasionally when labor rates of cutting and setting stone are low and stone can be quarried at abnormally low rates, the stone arch may be cheaper for all spans.

If the foundations are compressible the reinforced concrete arch with 1% of steel is preferable, as it can better withstand deformation without failure. It is not correct to

figure the reinforcement as carrying the total bending moment at all joints as is occasionally done, while the concrete is assumed to take the thrusts only. Such assumptions result in an absurdly high percentage of steel. However, when the cost of false-work is abnormally high, as for an arch of large span over a deep gorge, the percentage of reinforcement may be made high so as to reduce the amount of concrete and therefore the load upon the false-work. In such cases the cost of the false-work governs the design. As steel can never be strest higher than 15 times the maximum compression in the concrete, or about 7500 lb per sq in, a large percentage of steel is not generally economical nor is a steel of high strength advantageous.

In a large work where the amount of arch construction is small, it is often better to build arches of stone, as they may not get the inspection care necessary for concrete. When stonecutters and setters are first-class and labor poor, the stone arch is the safer arch to build, altho if first-class inspection can be had, concrete work can be well done under nearly all conditions.

In order to decrease the cost of work at the quarries, it is common practise to make the voussoirs of a stone bridge of the same depth from the crown to the springing line, as in Fig. 77, and to haunch the arch with concrete or rubble. When this is done the arch should generally be designed regarding the haunching as load and indirectly as a factor of safety. By bonding this haunching with stone voussoirs, however, they may be regarded as acting together and the combined masonry may be analyzed as an arch, in which case it is suggested that the safe working unit compressive stresses be taken as 70% of those recommended in the tables of Art. 1, for the weaker material.

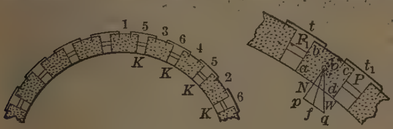


Fig. 71. Concrete Voussoirs

For arches over 100 ft span it is not good practise to use cut ring stones with the arch sheeting of rubble, concrete or brick, due to the great difference in the modulus of elasticity. Where large concrete arches, which should be built in alternate sections, are faced with stone voussoirs, the difficulties of erection are much increased.

The Architectural Details for all masonry bridges should be simple and logical. The accentuation of the roadway by an ornamental but simple coping and parapet, the accentuation of the ring stones and springing blocks by projection and simple ornamentation, the apparent strengthening of the abutment adjacent to the arch by projection, all tend to enhance the appearance of the bridge. The paneling and ornamentation of the spandrels, more commonly seen in concrete bridges, injure the appearance of the structure and lower the magnitude of its scale. No amount of ornamentation will relieve the awkward appearance of a bridge where spans and arch curves have been illogically selected. The ornamentation of a bridge depends upon its location. Where appearance is of paramount importance, a bridge should be designed so as to be in harmony with the surrounding landscape, present and future, whether it be rural or formal

Drainage. The roadway of a masonry bridge should have drains at either side of the roadway at intervals of 30 to 40 feet, for a level roadway, and 100 feet when the roadway is on a grade. These drains should have a minimum diameter of 2 inches and preferably 3 inches. The minimum area of a drain in square inches should be $a = A/200$, where A = area of the surface drained in sq ft. If a drain is placed at every 40 ft of each gutter for a bridge 50 ft wide, $A = 50/2 \times 40 = 1000$ sq ft and $a = 1000/200 = 5$ sq in. If the drains are at 100-ft intervals, $A = 50/2 \times 100 = 2500$ and $a = 2500/200 = 12.5$ sq in. The drains should be designed with an intake trap, and the entire drainage

system carrying the water to the ground should be so designed as to permit of easy cleaning.

False-work. The lower portion of an arch of large rise up to about 30° from the horizontal, known as the umbrella, needs comparatively little vertical support during its construction and therefore may be usually built without centering. Arches are generally built upon a centering of timber or steel. If of timber, the load of the arch is generally transmitted to the foundations by posts which may be vertical or approximately normal to the soffit, altho occasionally the centering is built as a Howe truss or timber arch. The later types are used when a large clearance under the arch is necessary during construction. The trussed center is generally expensive, due to expensive carpentry work and the fact that the timbers are more severely injured and have less market value after their use. The trussed center is undesirable because of its excessive deflection under the added loads of the arch during construction. The crown of a trussed center of the arch type rises as the lower arch loads are built upon it and its lower portion falls. When the upper arch loads are placed, the reverse is true, and therefore there is a tendency for the upper loads to crack the lower portion of the arch at about the middle of the haunch. To prevent this it is better to load such centering at the crown with a temporary load or to build the arch in alternate sections as described hereafter.

Steel centering is more expensive than timber unless in a large series of arches it may be used several times. It is generally built of the arch trussed type and is therefore subject to the foregoing criticism. Further, it is materially affected by temperature, and would better not be used for large arches unless the arch is built in alternate sections.

The centering should be designed for the lowest practical amount of settlement and deformation. Concrete arches have been built without centering and with a portion of the arch load, during construction, carried by the steel reinforcement, in which the reinforcement was built as an arch. These methods have no general application and result in economy only in abnormal cases.

Methods of Construction. All arches must be built symmetrically. That is, the centering must be loaded equally on either side of its middle. Stone arches under 75 ft span may be built continuously from springing line to crown, but unless the centering is abnormally unyielding for spans over 40 ft hair-line cracks will occur at one or more joints, showing that the tensional value of the mortar is destroyed. Where the span is over 75 ft, these cracks may amount to ⅛ inch. As a result, when the centering is struck there will be a settlement of the crown of at least one inch. Therefore for spans of over 75 ft the masonry work should be laid up in alternate sections as shown in Fig. 71. The sections are numbered in the general order in which they should be built. The sections last built, called the keying sections and marked K, should be as small as will permit the men to work. The length of the larger sections should not exceed about 15 ft. In arches built of voussoirs each of which is composed of more than one stone, the masonry blocks should be "racked back." This is not shown in the figure.

Plain and reinforced concrete arches under 40 ft span should be built as continuous work between umbrella points if possible. Arches of plain concrete over 40 ft and under 75 ft may be built continuously from the springing line to the crown; if over 75 ft the alternate block system should be used.

Reinforced concrete arches over 40 ft and under 100 ft span may be built in longitudinal ribs 3 or 4 ft wide, so arranged as to incorporate 2 or more ribs or lines of reinforcing steel. If so built the centering should be made very stiff, so that having one rib complete, the adjacent rib will not so deform the center as practically to strike the center under the first rib before the concrete of that rib is hardened. If built in ribs, transverse reinforcement should be used to tie the ribs together. Reinforced concrete arches over 100 ft span would better be built in alternate blocks because of the danger of an adjacent longi-

tudinal rib striking the centering under the completed ribs. The longitudinal rib method, however, has been successfully used in spans over 100 ft. As all centers settle and deform, the steel reinforcement becomes more or less buckled in construction and therefore will not be found in its theoretical position. In the construction of large arches it appears better to have the reinforcing steel in lengths not exceeding 30 ft, the lengths to be spliced after the large blocks have been placed. The splice may be an overlap, a bolted or a riveted joint. In concrete arches the concrete should be laid in horizontal courses or, where the inclination of the soffit is not too great, in courses parallel to the soffit. If alternate concrete blocks are used, the size of each block should be such that it can be completed as continuous work. Several keying blocks can usually be made in a day.

When arches are built in alternate sections the large blocks must be supported by stone, concrete or steel piers or columns until the key block masonry is in place (Fig. 71), or tie rods may be used as shown. The stresses in these piers or tie rods may be computed by resolving the weight of the blocks normal and tangent to the soffit (Fig. 71) and deducting the friction. $W = oq =$ weight of block. $N = op =$ normal thrust against the centering. $T = pq =$ tangential component. of is a line making an angle with $N =$ the angle of friction of masonry upon wood. Then fq is the total stress in the pier P, due to the weight of block $abcd$. When the angle poq = or $<$ pof, the block exerts no pressure upon P. The upper strut P_1 may bring additional load upon P, due to the tendency of the upper blocks to slide down the arch centering. If tie rods are used, the stresses are found in a similar manner, but in that case the accumulated stress in the tie rod t is due to the lower blocks. As the arch centering settles or deforms, the block above the pier P rotates about the foot of the pier P and in consequence the pressure at the top and bottom of the pier is applied close to the edge. It is therefore suggested that when stone or concrete piers are used the unit compressive stress be taken at 40% of those given in Art. 1.

Striking or Lowering of the Centering should be done gradually. The wedges or sand boxes should be lowered symmetrically, beginning at the crown and working to the springing lines. In a series of arches the centering between abutments or abutment piers should be struck simultaneously.

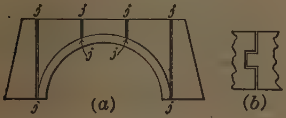


Fig. 72

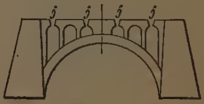


Fig. 73

The centering, which should be designed to carry the load of the arch, only, may be struck before the portion of the bridge above it is built, if the arch is stable under its own weight. If this is not done, expansion joints should be provided in the upper masonry, otherwise this masonry will crack, should there be any settlement of the crown of the arch, upon striking the center. Fig. 72*a* shows such joints for an earth-filled bridge, the expansion joints being marked *jj*, and 72*b* is a cross-section of one of these joints. Fig. 73 shows expansion joints in arched spandrels; they need only be left open, the spandrel arch centering remaining in place, until the large arch is struck. As the rise and fall of an arch, due to temperature, has a similar tendency to crack the masonry above the arch, such expansion joints are desirable to prevent temperature cracks. The joints of Fig. 73, when made permanent, should be about 1 inch wide, and the spandrel arches should be reinforced with steel so as to act as beams and cantilever beams. The spandrel expansion joints of the Walnut Lane Bridge, described in Trans. Am. Soc. of C. E., vol. 65, p. 423, are of excellent design. Expansion joints should extend from the back of the arch to the top of the parapet.

The permanent paving of earth filled bridges should not be laid for two years after the earth fill is placed, as the settlement will crack and warp the pavement. The filling should be spread in layers and thoroly rammed to decrease the amount of settlement.

27. Large American Masonry Arches

Cabin John Bridge at Washington, D.C., completed in 1864, is the longest stone arch and the one having the largest ratio of span to width of bridge in America. It is further notable as the first American example of the use of hollow or cellular abutments. The arch was built up continuously from springing line to crown. The clear span is 220 ft and the rise is 57.3 ft.

Walnut Lane Bridge at Philadelphia, Pa. (Fig. 74), completed in 1908, is the largest masonry arch in America. The main span consists of two detached parallel plain concrete arch ribs each supporting spandrel arches upon which rest low spandrel walls. These walls support steel I beams encased in concrete and the concrete jack arches of the floor. The main arch was built in alternate blocks.

The Edmondson Avenue Bridge at Baltimore, Md. (Fig. 75), completed in 1909, has piers and arches of plain concrete. The spandrels and floor system are of reinforced concrete. The order of building the large arch in alternate transverse blocks is shown in the sectional elevation.

The Bellfield Bridge at Pittsburg, Pa. (Fig. 76), is a stone arch built in 1900. It is the only large masonry bridge in America built with longitudinal spandrel arches. The necessary great width of the outside walls which must resist the spandrel arch thrust limits this type of construction to special cases. The face wall in section *C-C* is marked abutment wall. The Bellfield arch was built continuously from springing line to crown.

The Watertown Railway Bridge at Watertown, Wis. (Fig. 77), was completed in 1904. It consists of a series of four stone arches one of which is shown in the figure. It is built on pile foundations. The arch ring is of uniform depth from crown to springing line.

The Pennsylvania R.R. Bridge at New Brunswick, N. J. (Fig. 78), consists of a series of stone arches having spans between 51′ 00″ and 72′ 00″. Two of these arches are shown in the figure.

Connecticut Avenue Bridge, Washington, D. C. (Fig. 79), was completed 1907. It consists of five 150-ft spans and two 82-ft spans. It is built of plain concrete. The coigns, belt courses, copings and other ornamental details were cast upon the ground, drest and set in place as ashlar blocks. These details are indicated, in part, in the figure. Each spandrel arch nearest a pier and nearest the crown of the main arch has an expansion joint at its crown. These spandrel arches are therefore not true arches, and were reinforced with steel so as to act as simple and cantilever beams.

28. Piers and Abutments

River Piers are subject to the action of ice and floating matter which impinges against them in times of flood, this causing injury to the masonry, while the current may scour around the foundations and cause settlement or even overturning. To give full security against these causes the foundations must be properly made (Sect. 6) and the shape of the pier itself should be such as to deflect floating matter. The upstream end of the pier is often made triangular in plan below the high-water line in order to split and deflect laterally floating ice; this form of nose is largely used in latitudes where the stream freezes or becomes choked with floating ice. For rivers where there

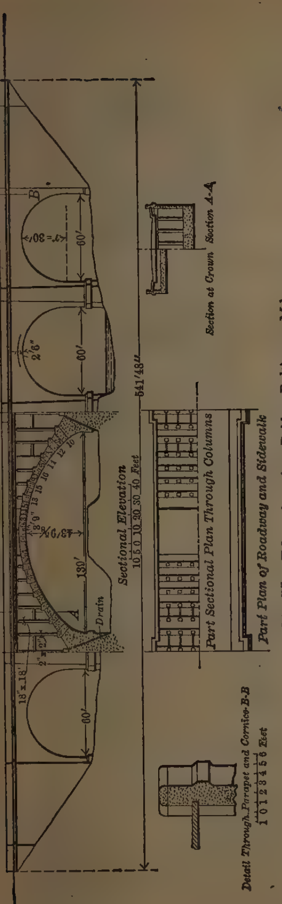


Fig. 75. Edmondson Ave. Bridge, Baltimore, Md.

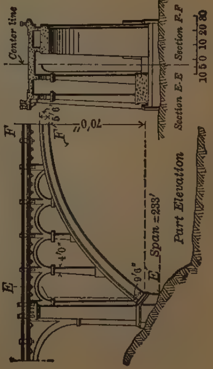


Fig. 74. Walnut Lane Bridge. Philadelphia, Pa.

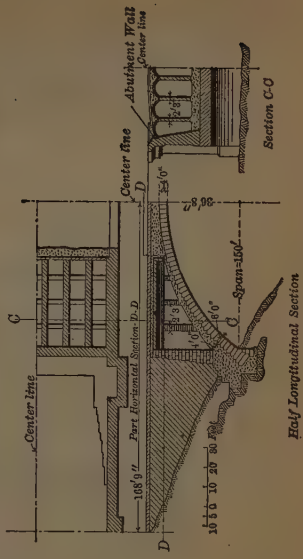


Fig. 76. Bellfield Bridge, Pittsburg, Pa.

is but little ice a rounded end is considered preferable, since then the current splits at a farther distance upstream, and thus floating matter is deflected before it strikes the end of the pier.

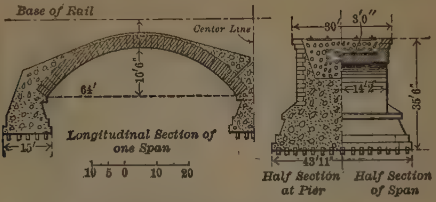


Fig. 77. Railroad Bridge, Watertown, Wis.

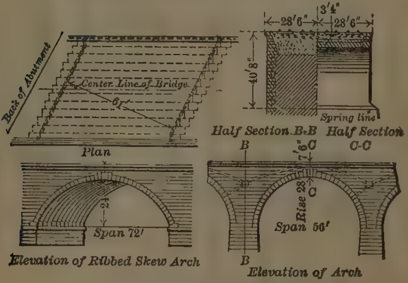


Fig. 78. Railroad Bridge, New Brunswick, N. J.

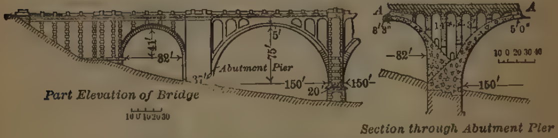


Fig. 79. Connecticut Ave. Bridge, Washington, D. C.

The Cross-section of a Pier is usually much larger than is required to carry the vertical loads only, since the above causes and the action of wind upon a high pier are considered in designing it. Unless the pier is founded upon impervious rock and the masonry is so built as to prevent water entering below the base, the unit weight of the masonry below the water line is to be diminished by 62½ lb per cubic foot. In what follows the base of the pier is considered as the section at the top of the foundation.

Example. A river pier of concrete for a bascule bridge is shown at G in Fig. 80. One-half the length of the bascule leaf is seen in its elevated position, the middle being at C, which falls toward the right as the leaf closes. The bascule is operated from pier F, 50 ft

from G and the operating strut makes the leaf stable against wind pressure when the span is open. The length of the pier is 30 ft, its average thickness 12 ft, its height above top of foundation 40 ft, and high and low water is 30 and 10 ft, respectively, above the base. The pier is assumed to be built upon pervious rock, so that its effective weight must be reduced from 150 lb per cu ft to $150 - 62 = 88$ lb per cu ft. Then the weight of the pier W_1 for high water is 745 tons and for low water 968 tons. The wind pressure being 30 lb per sq ft, the horizontal pressure P_1 against the pier is 4.5 tons for high water and 13.5 tons for low water, the point of application of above the base being (p_1) 35 ft and 25 ft, respectively. The wind pressure P_2 against the leaf is 33.8 tons and the point of application of same is $d = 37.5$ ft above the top of the pier The weight of the leaf W_2 is 24 tons. The condition of stress at the base AB of pier G will be as follows: On account of pin connection at the top of the pier, only direct (compression or tension) and shearing forces can be transmitted from the leaf to the pier. The moment P_2d will have the effect of diminishing the weight of pier G by $33 \times 37.5/50 = 25.4$ tons and of adding the same amount to the weight of pier F. At high water, therefore, $W_1 = 719.6$ tons and at low water 942.6 tons. The total horizontal shear transmitted at the top of the piers P_2, can be regarded as being divided equally between piers G and F, through the steel strut shown on the figure. The shear acting on top of pier G, therefore, is $P_2/2 = 16.9$ tons. The equation of moments for computing e, the distance from the middle of the base of pier G to the point where this resultant cuts the base is $We = P_1p_1 + P_2/2h$, where $W = W_1 + W_2$, the weight of the pier plus that of the leaf and h is the height of the pier. For the case of high water, this gives $e = (4.5 \times 35 + 16.9 \times 40)/743.6 = 1.12$ ft and for low water $e = (13.5 \times 25 + 16.9 \times 40)/966.6 = 1.05$ ft both coming within the middle-third of the base. The average compression on the base is $743.6/360 = 2.06$ tons per sq ft at high water and $966.6/360 = 2.69$ tons at low water. From Art. 6, the maximum compression on the base at A is $2.06\ (1 + 6 \times 1.12/12) \doteq 3.22$ tons per sq ft for high water and $2.69\ (1 + 6 \times 1.05/12) = 4.10$ tons per sq ft for low water.

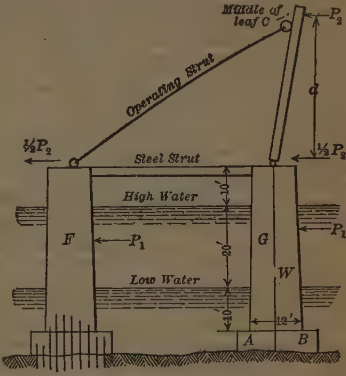


Fig. 80.—Piers of Bascule Bridge

If the leaf when open were fixt with respect to pier G, that is without an operating strut, as shown to pier F, the stress conditions would be different from those just determined. In that case the wind moment would be transmitted from the leaf to pier G and there would be no reduction in weight due to wind.

Wind and Eccentric Loads. Art. 7 explains the general principles and an application to a river pier of a double-track bridge will now be made. Fig. 81 represents the loads W_1, W_2, W_3, W_4, applied at a distance c from the middle of the length of the pier and at a distance c' from the middle of the width. General expressions for the longitudinal eccentricity e_x and the transverse eccentricity e_y on the base of the pier are

$$e_x = [P_1p_1 + (W_1 + W_1 - W_2 - W_3)c]/W \qquad e_y = [P_2p_2 + (W_1 + W_2 - W_3 - W_4)c']/W$$

in which W denotes the total weight on the base, P_1 is wind pressure on the end of the pier applied halfway between water line and pier top, while P_2 is wind pressure on the side of the bridge transferred to the pier top thru the bed plates. The full wind pressure of 30 lb per sq ft cannot act in both of these directions at the same time, but a wind blowing diagonally might pro-

duce about 21 lb per sq ft in both directions at the same time. However in high piers supporting long spans many cases of wind direction should be considered, as also many cases of span and track loading, in order to obtain the condition which produces the maximum and minimum unit stresses on the rectangle base which are computed by the formulas of Art. 7 after x and y have been found.

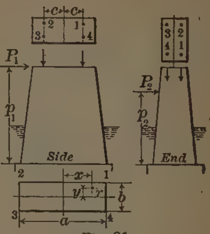


Fig. 81

The most dangerous case will generally be that when the loads W_1 and W_2 of the longer span include live load, W_3 and W_4 being dead load only, while P_1 is due to full transverse wind pressure on both spans. For example, let there be two double-track spans, one 400 and the other 200 ft long, dead load being 2000 and live load 4000 lb per lin ft per track, so that $W_1 = 600$, $W_2 = 600$, $W_3 = 200$, and $W_4 = 200$ short tons. The wind pressure is 450 lb per lin ft for the loaded span and 200 for the unloaded one, so that for transverse wind $P_1 = 90$ tons and $P_2 = 0$. Let the pier be 16 by 24 ft on base, height = 21 ft and weight = 320 tons after subtracting the weight of displaced water. Then $W = 1920$ tons and, for $c = 6$ ft and $c' = 3$ ft, the formulas give $e_x = 0.98$ ft and $e_y = 1.25$ ft. Finally, from the formulas of Art. 7, the stress at corner 1 of the pier is $S_1 = 9.6$ tons per sq ft compression and at the opposite corner $S_4 = 1.4$ tons per sq ft compression. The resultant lies within the kern of the base, and hence compression everywhere prevails.

Tendency to slide or rotate may be diminished when the pier is on a rock foundation, by the use of anchor bolts (Art. 7). If the current is very strong, or if it is probable that the river may become choked with ice, round dowel rods, extending for at least 40 diameters into both rock and pier, may be used as seen in pier F of Fig. 80.

Piers of Arch Bridges are subject to horizontal thrust from the skew-backs of the adjacent spans, and the difference of these acts at the top and causes an overturning tendency. The ordinary arch pier should be analyzed for one adjacent arch without live load and the other adjacent arch with live load over the whole span. It is not necessary to consider thrust due to temperature or rib shortening except when two reinforced concrete arches of widely different spans rest upon the same pier and the larger arch has a rise of less than 1/6 the span; even then the consideration of these thrusts may be an unwarranted refinement. If the pier is an abutment pier it should be analyzed for either arch standing and the other arch removed and for both arches standing. Every third or fourth pier in a series of arches should be built wider than the others so as to act as an abutment in case of failure of one or more of the arches.

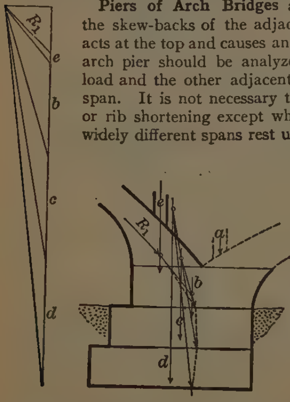


Fig. 82

Fig. 82 shows graphic analysis for an abutment pier for the case of the left arch standing and the right arch removed. The pier is arbitrarily divided into horizontal courses for the purpose of analysis. The resultant pressures are shown for each joint, and the resistance line passes thru the points where the resultants cut the joints. For each joint the distance e from the middle to the resistance line is measured and then the maximum stresses may be found from the

formulas of Art. 6. The resistance line should lie within the middle-third of all joints. The inclination of the resultant thrust at each joint and at the foundation should be noted and the safety factor against sliding be determined.

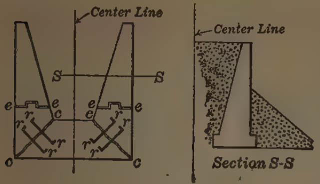


Fig. 83

Abutments for Arches. The method of analyzing the portion of an arch abutment which takes the arch thrust is similar to that used for piers, except that for an earth-filled bridge the earth pressure against the abutment should be combined with the other forces. For a single span, the cost of the abutments often exceeds the cost of the arch. The U abutment, Fig. 83, is uneconomical except for a very wide or very low bridge. If built upon a compressible foundation a crack *cc* will usually form at the junction of the transverse and longitudinal walls. This may be prevented by steel rods *rr* or by providing an expansion joint at *ee*.

In the design of U abutments the earth pressure against the back of the longitudinal walls may be less than against an ordinary retaining wall, since the wedge of earth which exerts pressure against the back of the wall is smaller. At the approach end of the abutment, the earth at the outside of the wall is effective in resisting the earth pressure at the back of the wall. It is not believed advisable, however, to materially decrease the masonry at the approach end on this account, as the outside fill may be disturbed by future excavation. As an additional factor of safety against rotation, the longitudinal walls may be tied together by steel rods.

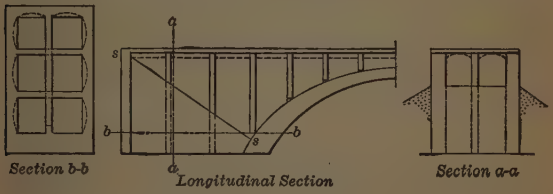


Fig. 84. Cellular Type of Abutment

A good type of abutment is shown in Fig. 84 for the case where the earth pressure is against the outside of the walls. The interior is cellular as shown in the plan or horizontal section *bb*. The earth thrust is taken by the interior walls acting as buttresses, *ss* being a line at the top of the outside earth-filled slope. The cells below *ss* may be partially filled with earth to help resist the pressure of the outside earth. Above this line the masonry walls may be changed to columns, as they carry only vertical loads of the roadway. The abutment may be built of stone, plain concrete, or reinforced concrete. If built of stone or plain concrete, the masonry should be arched as shown by dotted lines in the sections *aa* and *bb*; if built of reinforced concrete the arches may be changed to flat slabs as shown in full lines.

Owing to lack of knowledge regarding the material that may be deposited behind abutments by contractors, no very definite computations regarding earth thrust can generally be made. Probably a horizontal pressure equal to one-third of that of water will be sufficient for most cases.

SIMPLE TIMBER CONSTRUCTION

29. Working Unit Stresses

The Recommended Unit Stresses for railway bridges and trestles, which were adopted by the Association of Railway Superintendents of Bridges and Buildings, 1895, and modified in 1904, are given in the following table.

Safe Allowable Working Unit Stresses in Pounds per Square Inch

Kind of timber	Tension		Compression			Transverse		Shearing	
	With grain	Across grain	With Grain: End bearing	With Grain: Columns under 15 diams	Across grain	Extreme fiber stress	Modulus of elasticity $E/2$	With grain	Across grain
Factor of safety	Ten	Ten	Five	Five	Four	Six	Two	Four	Four
White oak.....	1200	200	1400	1000	500	1200	750 000	200	1000
White pine....	700	50	1100	800	200	700	500 000	100	500
Southern long-leaf pine....	1200	60	1400	1000	350	1200	750 000	150	1250
Douglas fir....	800	....	1200	900	200	800	750 000	130	......
Short-leaf yellow pine....	900	50	1100	800	250	1000	600 000	100	1000
Red pine (Norway pine)	800	50	1000	750	200	800	565 000	.....	......
Spruce and eastern fir...	800	50	1200	900	200	700	600 000	100	750
Hemlock......	600	....	1100	800	150	600	450 000	100	600
Cypress.......	600	....	1000	750	200	800	450 000	.....	......
Cedar........	700	....	1100	750	200	700	350 000	100	400
Chestnut......	850	....	.....	800	250	800	500 000	150	500
California redwood....	700	....	.....	800	150	750	350 000	100	......
California spruce......	.....	....	.....	800	....	800	600 000	.....	......

For timber containing large or loose knots, the unit stresses recommended are high.

The unit stresses are for unseasoned timber. They may be used without considering impact and may be increased 20% for highway bridges and trestles and 30% for protected timber not subjected to impact, as in a building. The average ultimate breaking unit stresses may be obtained by multiplying the working unit stresses by the factor of safety at the top of columns.

The unit stresses for columns whose ratio of unbraced length to least width is 15 or over must be reduced by column formulas. The modulus of elasticity for unseasoned timber should be taken as given in the table based on a factor of safety of 2. E, for seasoned timber, may be used with a factor of safety of one or double that given in the table, unless in designing it is important that the flexure of a beam or the shortening of a column shall not exceed a requisite limit. For instance, in a building, beams supporting an ornate plastered ceiling would better be figured, using the value $E/2$ given in the table. E cannot be used to determine the shortening of timber when the pressure is applied to side grain, such as the pressure of a post upon a cap or sill.

For continuous heavy loading, or loading causing reversal of stress in a member, 80% of the table stresses are suggested. The strength of timbers in old exposed structures should not be assumed higher than 80% of the table stresses, and only the section exclusive of rotted or partially rotted portions should be considered as effective. Old timbers should be bored at frequent intervals, in order to determine the effective section.

Permissible Unit Stresses for Timber in Pounds per Square Inch

From Building Laws of Cities.

Kind of stress	Kind of timber	Baltimore 1908	Boston 1907	Chicago 1900	Cincinnati 1917	District of Columbia 1909	New York 1917
Tension	Yellow pine	1800	......	......	1200	1200	1200
	White pine	1000	......	......	800	800	700
	Spruce & Va. Pine	1200	......	......	800	800	800
	Oak	1500	......	......	1000	1000	1200
	Hemlock	800	......	......	600	......	600
	Chestnut	......	......	......	600	......	......
Compression with the grain	Oak	1000	......	......	900	900	1400
	Yellow pine	1000	......	......	1000	1000	1600
	White pine	800	......	......	800	800	1000
	Spruce & Va. Pine	800	......	......	800	800	1200
	Locust	1200	......	......	1200	1200	1200
	Chestnut	......	......	......	500	500	......
	Hemlock	600	......	......	500	......	800
Compression across the grain	Oak	600	600	250	800	800	1000
	Yellow pine	600	500	250	600	600	1000
	White pine	400	250	150	400	400	800
	Spruce & Va. Pine	400	250	150	400	400	800
	Locust	1000	......	......	1000	1000	1000
	Chestnut	......	......	......	1000	1000	......
	Hemlock	500	......	......	500	......	800
Transverse bending	Yellow pine	......	1500	1250	1200	1200	1600
	White & Va. pine & spruce	1350	1000	650	800	800	1200
	Oak	1500	1000	1000	1000	1000	1200
	Locust	......	......	......	1200	1200	......
	Chestnut	......	......	......	800	800	......
	Hemlock	1000	......	......	600	......	800
Shear with the grain	Yellow pine	100	100	100	70	70	150
	White pine	85	80	80	40	40	100
	Spruce & Va. Pine	90	80	80	40	50	100
	Oak	100	150	150	100	100	200
	Locust	......	......	......	100	100	......
	Hemlock	75	......	......	40	......	100
	Chestnut	......	......	......	......	......	......
Shear across the grain	Yellow pine	500	......	......	500	500	1000
	White pine	350	......	......	250	250	500
	Spruce & Va. Pine	350	......	......	250	320	500
	Oak	720	......	......	600	600	1000
	Locust	......	......	......	720	720	......
	Hemlock	350	......	......	270	......	600
	Chestnut	150	......	......	150	......	......

In Designing only the net section of drest timber should be considered. Undrest seasoned timber is generally a little scant of the listed or nominal cross sectional dimensions. Dimension sticks are generally accepted when the dimensions do not underrun more than ¼ inch. Joists may underrun ¼ inch in width and ⅛ inch in depth.

30. Properties of Timber

Unit Weights of various kinds of timber are given in Section 4. As an average value 40 pounds per cubic foot or 3.3 lbs per foot board measure may be used; this is closely the actual weight of commercially seasoned southern long-leaf pine and also of chestnut.

Sizes and Specifications. In designing small structures, the size and lengths generally available in the local market should be used when possible. Special sizes and long lengths are expensive. Standard lengths are in multiples of two feet and usually do not exceed 20 to 24 ft. Standard cross-sectional sizes of undrest timber are in multiples of 1 or 2 ins.

The standard sizes and lengths, classification and specifications vary thruout the United States. Bulletin 71, published in 1906, by the U. S. Forest Service, gives the best available information on this subject. There are about fifteen lumber-manufacturing associations in the United States, each of which has its own rules and specifications for sizing and classifying or grading timber. The meaning of timber terms and the names of timbers vary in different sections, and it is usually only by inquiring of the local merchants that the buyer can be sure that he is ordering what is wanted.

Indentation under Side Grain Compression. The information contained in the following table has been taken from the Census Report of 1880, Volume 9, Forests of North America. The table is based on seasoned dry wood, having the pressure uniformly applied. In practise, due to reasonable carpentry inexactness, the pressure is not uniformly applied, nor are the timbers dry or seasoned with laboratory care. The writer has noted that the indentation of commercially seasoned white oak, long-leaf yellow and short-leaf pine and white pine in exposed structures is about 0.05 inch for the allowable side grain unit pressures given in Art. 29 and approximately 0.10 inch for double the allowed pressures. It is suggested that in computing the amount of side grain indentation or compression 0.05 inch be used for all exposed timbers under the safe load unit pressures and 0.10 inch for double these unit pressures. For thoroly seasoned timber in a housed structure and for first-class carpentry work, it is suggested that the indentation be computed as 0.01 inch for one-half of the loads given in the first column of the following table.

Pressures in Pounds per Square Inch Required to Produce given Side Grain Indentations in Timber

Pressures are applied across the grain.

Kind of wood	Indentation in inches			Relative hardness
	0.01	0.05	0.10	
White oak	1450	2700	3310	1.00
White pine	690	1080	1180	0.40
Yellow pine, long-leaf	1330	1980	2350	.75
Douglas fir	980	1460	1730	.65
Yellow pine, short-leaf	970	1500	1730	.65
Red pine (Norway)	530	1010	1170	.40
Spruce and Eastern fir	73	1010	1200	.50
Hemlock	710	1140	1250	.50
Cypress	600	1120	1330	.40
Cedar	560	980	1030	.40
Chestnut	920	1480	1770	.65
California redwood	600	830	970	.40

Coefficients and Angles of Friction

Kind of surface. Position of fibers with reference to direction of sliding	Coefficient of friction	Angle of friction	
Wood on wood, undrest, side grain, fibers parallel	0.45	24°	10′
Wood on wood, undrest, side grain, fibers at right angles	0.35	19	20
Wood on wood, undrest, end grain on side grain	0.30	16	40
Wood, undrest, on rough metal	0.45	24	10
Wood, undrest, on smooth metal	0.30	16	40
Wood on wood, drest smooth	0.35	19	20
Wood drest smooth on smooth metal	0.25	14	00
Wood on masonry. See Art. 3			

These coefficients are average values for dry seasoned wood as determined by comparison of published data. Fundamental Ideas of Mechanics by Morin contains good data on this subject. Soft, wet or unseasoned timber usually gives higher coefficients, altho very green timber under small loads gives lower values.

When very heavy loads are applied, the fibers interlock and give higher values. End grain timber on side grain and metal on wood under heavy loads bite into the wood, tearing the fibers (Fig. 85) and causing mechanical restraint against sliding.

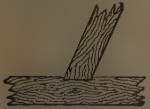


Fig. 85

If unguents are used and the pressures are small, the coefficients of friction may be reduced 50 or 75% of the tabulated ones. If the pressures are as great as those given in Art. 29, the unguent is squeezed out, the fibers bite into each other and the coefficient of friction will not be reduced.

Friction should not generally be regarded in the design of framed structures except as an added factor of safety. For temporary structures or simple members of permanent ones, such as battered posts of a highway trestle, it may properly be considered, but it is suggested that not more than 50% of the tabulated coefficients be used.

31. Beams and Columns

Wooden Beams are generally of rectangular section and of uniform depth, and only such beams are considered in what follows. A beam should be designed so that the extreme fiber stress due to bending, the maximum horizontal shear, and the compression across the grain, at the end bearings, do not exceed the allowed unit stresses, and so that the maximum deflection does not exceed a limit which is fixt by the use to which the structure is to be put. For highway bridges and trestles this limit is generally 1/200 of the span, for railway bridges and trestles 1/300, for plastered ceilings 1/360 and for ceilings supporting shafting 1/700. When a beam is laterally unbraced and the ratio of length to width exceeds 20, the safe extreme fiber stress in Art. 29 should be reduced as follows:

Ratio of length to width	20 to 30	30 to 40	40 to 50	50 to 60
Percentage of reduction	25	34	42	50

Wooden beams are often continuous over several supports and may be so figured, but most designers compute all timber beams as simple beams.

To design a beam for any loading, the depth *d* should be assumed, consistent with the general governing conditions and the fact that beams over 12 inches deep cost more per board foot than shallower ones. Having selected *d*, the width of beam can be determined by the proper formulas, selecting values for the unit stresses from Art. 29. The greater width should be used. For long and shallow beams the flexure formula will generally govern, for short and deep beams the shear formula.

If the amount of deflection is important, formulas for depth in terms of deflection and unit stress should be used. The end bearing area A should not be less than V/S_a, in which S_a is one-half of the allowed unit stress across the grain; this is recommended because flexure of the beam makes the pressure upon the front of the bearing greater than that at the back.

Safe Loads in Pounds Uniformly Distributed for Rectangular Beams One Inch Wide

For an allowable fiber stress of 1000 pounds per square inch.

Span in feet	Depth of beam in inches.										
	4	6	8	10	12	14	16	18	20	22	24
4	440	1000	1780	2780	4000	5440	......	......	......	......	......
5	360	800	1420	2220	3200	4360	......	......	......	......	......
6	300	670	1190	1850	2670	3630	......	......	......	......	......
7	250	570	1020	1590	2290	3110	......	......	......	......	......
8	220	500	890	1390	2000	2720	......	......	......	......	......
9	200	440	790	1240	1780	2420	3160	4000	4940	5980	7110
10	180	400	710	1110	1600	2180	2840	3600	4440	5380	6400
11	160	360	650	1010	1460	1980	2590	3270	4040	4890	5820
12	150	330	590	930	1330	1820	2370	3000	3700	4480	5330
13	140	310	550	860	1230	1680	2190	2770	3420	4140	4920
14	130	290	510	790	1140	1560	2030	2570	3180	3840	4570
15	120	270	470	740	1070	1450	1900	2400	2960	3590	4270
16	110	250	440	690	1000	1360	1780	2250	2780	3360	4000
17	110	240	420	650	940	1290	1670	2120	2610	3160	3770
18	100	220	400	620	890	1210	1580	2000	2470	2990	3560
19	90	210	370	590	840	1150	1500	1900	2340	2830	3370
20	90	200	360	560	800	1090	1420	1800	2220	2690	3200
21	85	190	340	530	760	1040	1350	1710	2120	2560	3050
22	80	180	320	510	730	990	1290	1640	2020	2440	2910
23	80	170	310	480	700	950	1240	1570	1930	2340	2780
24	......	160	300	460	670	910	1190	1500	1850	2240	2670
25	......	160	280	440	640	870	1140	1440	1780	2130	2560
26	......	150	270	430	610	840	1090	1380	1710	2070	2460
27	......	150	260	410	590	810	1050	1330	1650	1990	2370
28	......	140	250	400	570	780	1020	1290	1590	1920	2290
29	......	140	240	380	550	750	980	1240	1530	1850	2210
30	......	130	240	370	530	730	950	1200	1480	1790	2130
31	......	130	230	360	520	700	920	1160	1430	1730	2060
32	......	125	220	350	500	680	890	1130	1390	1680	2000
33	......	120	210	340	480	660	860	1090	1350	1630	1940
34	......	120	210	330	470	640	840	1060	1310	1580	1880
35	......	110	200	320	460	600	810	1030	1270	1540	1830

The weight of the beam need be considered only when the ratio of span to depth of beam is large. For concentrated loads at the middle of a beam, divide table safe loads by 2. For fiber stresses other than 1000 lbs, correct safe loads of table.

Safe Loads Uniformly Distributed for Rectangular Beams One Inch Wide

For an allowable horizontal shearing stress along the grain of 100 lbs per square inch.

Depth in inches....	4	6	8	10	12	14	16	18	20	22	24
Safe load, pounds	530	800	1070	1330	1600	1870	2130	2400	2670	2930	3200

The weight of the beam need not be considered, because when the shearing strength of a beam governs the design the weight of the beam is negligible. For short and deep beams these values will generally govern rather than those in the preceding table.

The Deflection formula for a beam 1 in wide under uniform load may be written $f = 270\ (l/d)^3 W/E$. Following table gives values of $270\ (l/d)^3$ for various spans and widths. In this formula, l is exprest in ft, f and d in in, W in lb of total load in beam divided by width in inches.

Values of C in the Formula, Deflection = CW/E for Beams One Inch Wide and Uniformly Loaded

Span in feet	Depth in inches										
	4	6	8	10	12	14	16	18	20	22	24
5	530	160	66	34	20	12	.....	.....	.....	.....	.....
6	910	270	110	58	34	20	14	.....	.....	.....	.....
7	1 450	430	180	93	54	33	23	.....	.....	.....	.....
8	2 160	640	270	140	80	50	34	.....	.....	.....	.....
9	3 080	910	380	200	110	71	48	.....	.....	.....	.....
10	4 220	1 250	530	270	160	98	66	46	34	25	20
11	5 620	1 660	700	360	210	130	88	61	45	33	26
12	7 310	2 160	910	470	270	170	110	79	59	43	34
13	9 270	2 750	1160	590	340	220	150	100	75	55	43
14	11 580	3 430	1450	740	430	270	180	130	93	69	53
15	14 240	4 320	1780	910	530	330	220	160	120	84	66
16	17 280	5 120	2160	1110	640	400	270	190	140	100	80
17	......	6 140	2590	1330	770	480	330	230	170	120	96
18	......	7 290	3070	1580	910	570	390	270	200	150	110
19	......	8 570	3610	1850	1070	670	450	320	230	170	130
20	......	10 000	4220	2160	1250	780	530	370	270	200	160
21	......	......	4880	2500	1450	910	610	430	320	230	180
22	......	......	5610	2880	1660	1050	700	490	360	270	210
23	......	......	6410	3280	1900	1200	800	560	410	300	230
24	......	......	7290	3730	2160	1350	910	640	470	350	270
25	......	......	......	4220	2440	1540	1030	720	530	390	310
26	......	......	......	4740	2740	1730	1160	810	590	440	340
27	......	......	......	5310	3070	1930	1300	910	660	500	390
28	......	......	......	5930	3420	2160	1450	1020	740	550	430
29	......	......	......	6590	3810	2390	1610	1130	820	620	480
30	......	......	......	7290	4210	2650	1780	1250	910	680	530
31	......	......	......	......	......	......	1970	1380	1010	750	580
32	......	......	......	......	......	......	2160	1520	1110	830	640
33	......	......	......	......	......	......	2370	1660	1220	900	710
34	......	......	......	......	......	......	2590	1820	1330	990	770
35	......	......	......	......	......	......	2830	1980	1450	1080	840
36	......	......	......	......	......	......	3080	2160	1580	1170	910

Since E varies considerably for woods of the same species, and for different degrees of seasoning and for variations in moisture content, the constants for intermediate depth may be selected by inspection. For a concentrated load at the middle of a beam multiply the value of C by 1.6.

To Design a Beam for a span of 15 ft under a uniform load of 4000 lbs, deflection not to exceed $1/200 \times$ span, the beam to be of white oak. Art. 29 gives an extreme fiber stress of 1200 lbs per sq in for white oak. Using the first table of this article, assuming a depth of 12 inches the safe load is 1070 lb, but as this table is for a fiber stress of 1000 lb per sq in, the safe load for an oak beam 1 inch wide $= 1070 \times 1200/1000 =$ 1280 lb. As beams come in widths which are generally in multiples of 1 inch, select a beam 4 inches wide.

The allowable horizontal shearing stress, with the grain, for white oak, is 200 lbs per sq in. The preceding tabulation, based on shear, gives for an oak beam 1 inch wide and 12 inches deep $1600 \times 200/100 = 3200$ lb. For a beam 4 inches wide, the safe load = 12 800 lb. Therefore the dimensions of the beam are governed by flexure rather than by shear.

From Art. 29, $E = 1\ 500\ 000$ lb if a factor of safety of 1 is used. For the given span and depth $C = 530$ and $f = 530 \times 1000 / 1\ 500\ 000 = 0.35$ in, which is less than $1/200 \times$ span, and hence satisfactory. The safe compression across the grain is 500 lb per sq in, and using one-half of 500 lb, the area of the end bearing must be at least $4000/2 \times 250 = 8$ sq in. Since the beam is 4 inches wide, the length of bearing required is 2 inches.

If the ratio of length of unbraced span to width exceeds 20, the safe loads of the table must be reduced by the percentages given in the first part of this article. For example, the ratio $= 180/4 = 45$. Therefore the safe load $= 1280 \times 0.58 \times 4 = 2970$. If this beam is detached so that it has no lateral support in its length, the width should be increased to 6 ins. If the beam supports joists or stringers or a floor fastened to it, it may be regarded as braced laterally and the 4-inch width may be used. A timber beam which is not braced at one or more points very seldom occurs in practise.

Columns. The following formula adopted by the American Railway Engineering and Maintenance of Way Association, 1907, is recommended;

$$S_1 = S\ (1 - l/60\,d)$$

in which S_1 = safe working stress for the column, S = safe end-bearing stress, compression with the grain (Art. 29), l = length of column, d = least side of column, and l and d are exprest in the same unit. Columns may be designed by the formula or by the following table which is based on it. Columns abutting against side-grain lumber, at top or bottom, must usually be provided with caps or shoes of hard wood or metal, otherwise the pressure upon the side grain will be excessive.

Safe Loads for Square Columns in Units of 1000 Pounds

Based on safe end-bearing compression of 1000 pounds per square inch.

Unbraced length in feet	Size of column in inches						
	4×4	6×6	8×8	10×10	12×12	14×14	16×16
4	16.0	......	......	......	......	......	......
6	11.2	36.0	......	......	......	......	......
8	9.6	26.3	......	......	......	......	......
10	8.0	24.1	64.0	......	......	......	......
12	6.4	21.6	44.8	100.0	......	......	......
14	4.8	19.1	41.6	72.0	130.4	......	......
16	......	16.9	38.4	68.0	105.0	196.0	......
18	......	14.4	35.2	63.0	100.8	145.0	256.0
20	......	11.9	32.0	60.0	96.5	141.1	192.0
22	......	9.7	28.8	57.0	90.6	133.2	185.6
24	......	......	25.6	52.0	86.4	127.3	179.1
26	......	......	22.4	48.0	82.1	123.5	172.8
28	......	......	19.2	43.0	76.3	117.6	166.4
30	......	......	......	40.0	72.0	111.7	160.0
32	......	......	......	36.0	67.6	107.8	153.5
34	......	......	......	33.0	61.9	101.9	147.2
36	......	......	......	......	57.6	94.2	140.8
38	......	......	......	......	53.3	90.2	134.4
40	......	......	......	......	47.5	84.3	128.0
42	......	......	......	......	......	78.4	121.7
44	......	......	......	......	......	72.5	115.2
46	......	......	......	......	......	66.6	108.9
48	......	......	......	......	......	......	102.4
50	......	......	......	......	......	......	96.0

For any other unit end-bearing stress the safe load can be obtained by proportion.

Example. Design a white oak column 20 ft long to support a load of 100 000 lbs. From Art. 29 the safe end-bearing compression for white oak = 1400 lb per sq inch.

Therefore the safe load for an oak column = 1.4 × table safe load. From the sixth column select the nearest larger size, which gives a safe load of 100 000/1.4 = 71 400 lb, for a 12 by 12 inch section. Or the safe load of a 10 by 10 for one inch width = 60 000/10 = 6000 lb. Dividing, 71 400/6000 = 11.9 in. A column 10 by 12 inches can therefore be used.

Notched Beams. Beams are frequently notched at their ends in order to cut down clearance and to bring the top surfaces of adjacent beams to the same level. They are also occasionally notched at intermediate points to clear other parts of a structure, such as the top lateral rods of a highway deck bridge. When notched at the ends, the strength of the beam is materially decreased if the depth of the notch is a large percentage of the depth. For beams whose depth is great compared to their length, the effect of notching is greatest because such beams fail in horizontal shear. It is suggested that in designing beams with end notches the actual end depth be used to determine the horizontal shearing stresses, or the safe load based upon horizontal shear, and that only 80% of the allowed horizontal shearing stresses given in Art. 29 be used.

When the notches are at or near the middle of the beam, the depth of the beam, in determining the extreme fiber stresses or the safe loads based on same, should be regarded as the net depth. It is further recommended that only 80% of the allowed fiber stresses and the safe loads based thereon be used. Report of the Chief of Engineers, U. S. A., 1883, p. 1496, contains valuable tests upon the subject of notches.

Knots. Beams containing large or loose knots should not be used, unless an allowance be made for same. Beams containing knots should be placed with the knots in the compression side of the beam, that is for simple beams, the knots should be at the top.

32. Fastenings

Nails for ordinary structural work are of two general types, cut nails of rectangular cross-section tapering from head to point and wire nails of circular cross-section without taper. Both types are usually of steel. Cut and wire nails of larger cross-section than common nails are called spikes. Boat spikes, used for very heavy timber work, have larger cross-sections than ordinary spikes. A clinch nail is similar to a cut nail, but is so made that it may be clinched or bent down, so as to better withstand shock or vibration. They are often made of wrought iron.

Screws are preferable to nails for permanent work and work subject to vibration or shock, when a joint or connection of great strength is desired, and for lumber which splits readily under the driving of nails.

The common sizes of cut and wire nails and spikes and the approximate number per pound are given in the following tables. The numbers per pound are approximate, as different manufacturers use slightly different standards of cross-section, taper, and head. Nails are usually designated in size by the "penny" system. For instance, a 3-inch nail called a ten-penny nail (written 10d.), probably because such nails once were sold at ten pence per 100. A keg of nails or spikes weighs 100 lb.

Square Steel Boat Spikes

Approximate number in a keg of 200 pounds

Size, inch	Length of spikes in inches											
	3	4	5	6	7	8	9	10	11	12	14	16
3/8	1320	1140	940	800	650	600	530	480	.....	.....	.....	.....
7/16	......	......	.....	600	590	510	400	360	320	280	.....	.....
1/2	......	......	.....	450	380	340	300	280	260	240	.....	.....
5/8	......	......	.....	.....	.....	260	240	220	210	190	180	160

Nails and Spikes

	Steel cut nails and spikes				Steel wire nails and spikes				
Size	Length in inches	Number common nails per pound	Number spikes per pound	Number clinch nails per pound	Size	Length in inches	Number common nails per pound	Number spikes per pound	Number clinch nails per pound
2d.	1	740	....	400	2d.	1	900	....	622
3d.	1¼	460	....	260	3d.	1¼	615	....	412
4d.	1½	280	....	180	4d.	1½	322	....	267
5d.	1¾	210	....	125	5d.	1¾	250	....	230
6d.	2	160	....	100	6d.	2	200	....	156
7d.	2¼	120	....	80	7d.	2¼	154	....	110
8d.	2½	88	....	68	8d.	2½	106	....	98
9d.	2¾	73	....	52	9d.	2¾	85	....	86
10d.	3	60	....	48	10d.	3	74	37	66
12d.	3¼	46	....	40	12d.	3¼	57	32	57
16d.	3½	33	17	34	16d.	3½	46	29	46
20d.	4	23	14	24	20d.	4	29	23	35
25d.	4¼	20	12	....	30d.	4½	23	18	....
30d.	4½	16	10	....	40d.	5	17	13	....
40d.	5	12	9	....	50d.	5½	14	10	....
50d.	5½	10	8	....	60d.	6	10	9	....
60d.	6	8	7	....	....	[illegible]	....	8	....
....	6½	....	6	....	....	7	....	7	....
....	7	....	5	....	....	8	....	6	....
....	....	....	....	....	....	9	....	5	....
....	....	....	....	....	....	10	....	4	....
....	....	....	....	....	....	12	....	3	....

Screws for structural work are divided into two general types, wood screws and lag screws. Lag screws (Fig. 93*q*) are used for heavy timber work. Lag screws are often used instead of bolts where it is difficult or impossible to place thru bolts.

Wood Screws

Length in inches	Gage numbers
3	6 to 26
3½	8 to 26
4	8 to 30
4½	12 to 30
5	12 to 30
6	12 to 30

Flat heads are commonly used.

Approximate Screw Gages

Number of gage	Diameter in inches
6	0.14
10	0.19
15	0.26
20	0.32
25	0.39
30	0.45

Other gages may be approximated by proportion.

A hole should be bored, for a wood screw, less than the diameter of the shank and about one-half its depth before inserting the screw. The screw should not be hammer driven. For a lag screw a hole should be bored slightly larger than the diameter of the unthreaded shank and for its length. A second hole should then be bored at the bottom of the first hole, less than the diameter of the threaded shank and about one-half its length.

Lag Screws

Lengths in inches	Diameters in inches
3	5/16 to 7/8
3½	5/16 to 1
4	5/16 to 1
4½	5/16 to 1
5	5/16 to 1
5½	5/16 to 1
6	5/16 to 1
6½	7/16 to 1
7	7/16 to 1
7½	7/16 to 1
8	7/16 to 1
9	7/16 to 1
10	½ to 1
11	½ to 1
12	½ to 1

Lag screws may be obtained with hexagonal or square heads.

Strength of Nails. Nails are seldom used in permanent structural work where the strain is parallel to the length of the nail, but for scaffolding such a fastening is not uncommon. The following formulas for safe loads on a nail were determined from a comparison of tests on the resistance to pulling out. They are for nails having a depth of penetration of at least $\frac{9}{10}$ the length of the nail and when driven across the grain.

Cut nails in oak, $P = 7\,d$ Cut nails in yellow pine, $P = 4\,d$
Wire nails in oak, $P = 4\,d$ Wire nails in yellow pine, $P = 2\frac{1}{2}\,d$

in which d is the number of penny weight of nail. For example a 60*d*. cut nail in oak, if driven perpendicular to the fiber, has a safe resistance to pulling out of $7 \times 60 = 420$ pounds.

If driven parallel to the grain use 50% of the above values. If subject to shock, the resistance of the nail to pulling out is very low as was shown by W. C. Riddick in 1890 and as may be observed when a nailed plank is struck by a sledge in the demolition of timber structures. Nails should never be depended upon for important joints, in either temporary or permanent structures.

The Safe Lateral Resistance of Nails driven across the grain may be approximated by the following formulas, in which P is in pounds when d is the penny weight.

Cut nails in oak, $P = 12\,d$ Cut nails in yellow pine, $P = 7\frac{1}{2}\,d$
Wire nails in oak, $P = 10\frac{1}{2}\,d$ Wire nails in yellow pine, $P = 7\,d$

Nails should be driven at right angles to the contact surfaces. When nails are driven parallel to the grain, use 60% of the tabulated values. For shock see preceding paragraph. The nails should not be driven closer to each other than about ½ their length nor closer to the edge of the timber than about ¼ their length. It is generally impractical to develop the full strength of timber by the use of nails.

Safe Resistance to Pulling Out, in Pounds per Linear Inch, of Wood Screws when Inserted across the Grain

Kind of wood	Gage number							
	4	8	12	16	20	24	28	30
White oak......	80	100	130	150	170	180	190	200
Yellow pine.....	70	90	120	140	150	160	180	190
White pine.....	50	70	90	100	120	140	150	160

These values are exprest in pounds per lineal inch of the threaded portion. For wood screws inserted with the grain use 60% of these values. The lateral resistance of screws is unquestionably much higher than nails, and it is thought that safe design will result if the safe resistance of a No. 20 gage screw be assumed at double that given for nails having the same length, provided the full length of screw thread penetrates the supporting piece of the two pieces connected.

Safe Resistance to Pulling Out, in Pounds per Linear Inch of Thread, of Lag Screws when Inserted across the Grain

Kind of wood	Diameter in inches				
	½	⅝	¾	⅞	1
White oak.......	590	620	730	790	900
Douglas fir......	310	330	390	450	570

As Douglas fir and Yellow pine are quite similar woods, it is thought that the values recommended for the former may be safely used for the latter.

A Dowel or dowel pin is a pin of wood or steel extending into adjacent parts of a structure (Figs. 86 and 87) to keep them from being displaced.

Wood dowels are also called tree nails. Steel dowels are troublesome in renewals. Dowel pins are usually not computed to transmit stress. They should be made a tight fit and extend at least 4 diameters into side-grain and

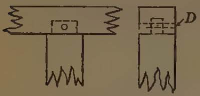


Fig. 86

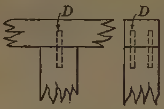


Fig. 87

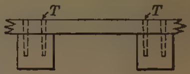


Fig. 88

6 diameters into end-grain timbers It is better to use two rather than one to a joint. Tree nails are occasionally computed to transmit stress, the strength of the connection being regarded as ½ the shearing strength of the cross-section of the nail. They are often substituted for bolts in lumber work below high water. In Figs. 86 and 87, the dowels or tree nails are marked *D*. In Fig. 88, the tree nails fastening grillage planks are marked *T*.

Drift Pins or drift bolts are long pins of steel or wood, usually the former, driven thru the timber and into an adjacent timber to hold them together and to transmit stress. Drift pins are made with or without heads. Round pins are preferable and whether round or square, a hole should be driven having a diameter from 70 to 80% of the diameter of the bolt and for the full length of the bolt. Boat spikes may be used for drift pins.

Safe Resistance to Pulling Out, in Pounds per Linear Inch, of Drift Bolts when Driven across the Grain

Kind of wood	Diameter in inches				
	½	⅝	¾	⅞	1
White oak.......	300	400	490	640	810
White pine......	80	110	130	180	220
Douglas pine....	270	350	390	470	560

For yellow pine it is suggested that the values of Douglas fir be used. For drift bolts driven with the grain use 50% of the tabular values.

Common Bolts (see Sect. 4) are generally preferable to nails, screws or drift pins. The holding power of bolts will be given in the design of joints. Strap bolts are bolts with a strap at one end. They are used for the

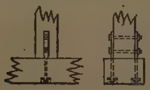


Fig. 89

Fig. 90

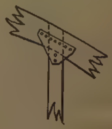


Fig. 91

purpose of fastening the end of one timber to the side of another timber, Fig. 89. Common bolts are often used with steel straps as in Fig. 90, and steel gussets or plates as in Fig. 91.

Washers are metal disks placed under the heads and nuts of bolts to distribute the pressure upon the wood over such an area that the unit compression will not exceed a safe limit. Washers are of three types, cast iron (Fig. 92, *a*, *d*, *e*, *f*), steel or wrought iron (Fig. 92 *b*, *c*) or malleable iron (Fig. 92*g*). Washers of standard sizes and shapes may be bought from the manufacturers. For bolts over two inches in diameter, special cast-iron washers (Fig. 92*d*)

are often used. This type of washer is very expensive unless a large number of the same size are required. Beveled, cast-iron washers (Fig. 92, *e*, *f*) of almost any angle of bevel may be purchased from the manufacturers.

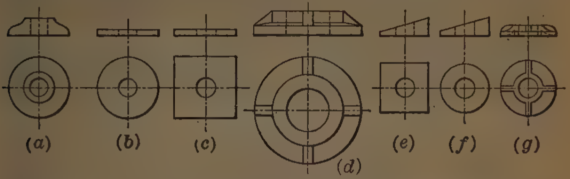


Fig. 92. Types of Metal Washers

When the tension in a bolt is at right angles to the base of the washer, the washer should be designed so that the compression upon the wood, due to tension in the bolt, will not exceed the allowable compressive unit stresses of Art. 29 by more than 50%. Standard washers, preferably cast-iron ones, can be used when the bolts are not strest high in tension. In many joints the bolt is not strest in tension, and standard steel-plate or cast-iron washers can be used. The manufacturers' standards for wrought and steel-plate washers are usually too thin and too small in area to develop the full strength of the bolts. The circular steel-plate washer is particularly lacking in this respect.

Wrought-iron or Steel-plate Round Washers

Dia. in inches	Size of hole in inches	Approximate number in a pound	Area of washer in ins
1⅜	9/16	27.0	1.3
1½	⅝	22.0	1.5
1¾	11/16	13.0	2.0
2	13/16	10.0	2.6
2¼	15/16	8.6	3.3
2½	1 1/16	6.2	4.0
2¾	1¼	5.2	4.5
3	1⅜	4.0	5.6
3¼	1½	2.8	6.5
3½	1⅝	2.5	7.6
3¾	1¾	2.4	[illegible].6
4	1⅞	2.2	9.8
4¼	2	1.9	11.0
4½	2⅛	1.7	12.4
4¾	2⅜	1.0	13.3
5	2⅝	1.0	14.2

The size of hole in washer is 1/16 in larger than the diameter of the bolt up to and including bolts 1 in in diameter. For larger bolts the hole is ⅛ in larger.

Wrought-iron or Steel-plate Square Washers

Width in inches	Size of hole in inches	Approximate number in a pound	Area of washer in ins
2	9/16	5.0	3.8
2¼	¾	3.2	4.6
2½	⅞	2.5	5.7
3	1	1.7	8.2
3½	1⅛	.9	11.3
4	1⅛	.7	15.0
4½	1¼	.5	19.0
5	1½	.4	23.2
6	1⅝	.3	34.0

For size of bolts see round plate washers.

Cast-iron Washers

Diameter in inches	Size of hole in inches	Approximate number in a pound	Area of washers in ins
2½	⅝	2.00	4.6
2¾	¾	1.60	5.5
3	⅞	1.30	6.5
3½	1	.80	8.8
4	1⅛	.62	11.6
4½	1¼	.44	14.7
5	1⅜	.33	18.2
6	1¾	.20	25.9

The size of hole in washer is ⅛ in larger than the diameter of the bolt up to and including bolts 1¼ in in diameter. For larger bolts the hole is ¼ in larger.

To design a washer, divide the tension in the bolt by the safe unit compression of the wood and add the area of the hole of the washer, which gives the gross area required. Its thickness, if of wrought iron or steel, should be at least 1/8 its width or diameter. If of cast iron (Fig. 92*a*), the thickness should be not less than about 1/4 its diameter. If of the type shown in Fig. 92*d*, it may be designed similar to cast-iron shoes, the unit pressure upon the bottom of the washer being the allowable compression upon the wood. If the tension in the bolt is small use standard washers.

If the washer is bevelled (Fig. 93*n*), the pull on the washer should be resolved into the components shown by the arrows and the pressure upon the wood in either direction P or N should not exceed the allowed safe limits of Art. 29. The washer should be so designed that the line of pull and the line of resistance P and N meet in a point.

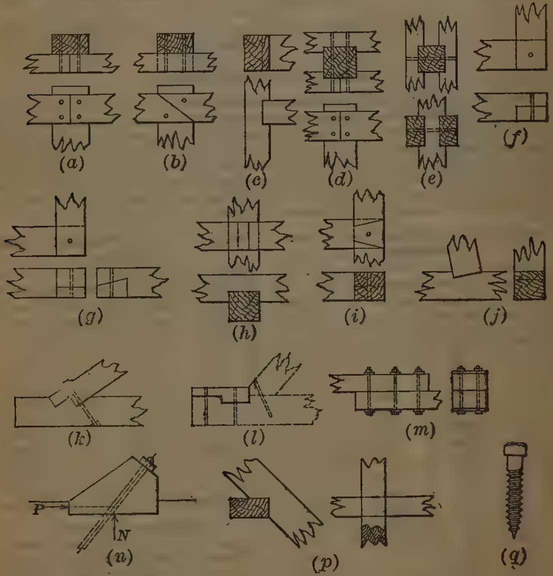


Fig. 93. Common Timber Joints

Common Joints are shown in Fig. 93. *a* is a butt joint. *b* is a bevel joint. *c* is housing. *d* is single notching. *e* is double notching. *f* is halving. *g* is beveled halving. *h* is cogging. *i* is dovetail halving. *j* is a dapped joint. *k* is a double-step dapped joint. *l* is a head block. *m* is a lap joint. *p* is a bird's-mouth joint. *q* is a lag screw

33. Compression Joints

The Butt Joint, shown in Fig. 94, is used to splice columns and struts. The ends of adjacent sections should be sawed at right angles, and in important work the ends should be drest to an even surface. If the contact surface or ends are not flush and true, the column is loaded eccentrically. This

joint need not be computed, but braces should hold the lower or upper section of the joint to correct alignment, so as to prevent buckling of the column at the joint.

Fig. 94

A Column with beams framed into its sides (Fig. 95*a*) must not be reduced at section *A*, so that the unit compression exceeds the safe end-bearing compression. The beveled dap may not give sufficient end bearing for the beams, in which case the beam bearings may be increased as in Fig. 95*a*, by use of angles (Fig. 95*b*) by bolted side blocks or by use of side and bent plates (Fig. 95*c*).

Example (Fig. 95*a*). Girder 10 × 10 inches, columns 10 × 10 inches, both southern long-leaf pine. Load on upper column = 70 000 lb. End beam reaction each = 10 800 lb. The allowed compression across the grain = 350 lb per sq in. It is suggested that only 50% of the allowed unit compression be used, as the flexure of the beam causes greater compression at the front of the support than at the back. The minimum bearing area of the beams = 10 800/175 = 61.7 sq in. As the beam is 10 inches wide, the length of bearing should be 6 in. As the load on the upper section of column is 70 000 lb and the safe bearing on end grain is 1400 lb per sq in, the minimum reduced area of the column = 70 000/1400 = 50 sq in. Therefore the length of bearing on the column for each beam may be 2½ in. The additional length of end bearing needed for each beam = 6 − 2½ = 3½ in. This may be provided by using 3½ × 3½ × ⅜ inch angles as in Fig. 95*a*. The load on each angle will be 10 800 × 3.5/6 = 6300 lb. The bolts are in single shear.

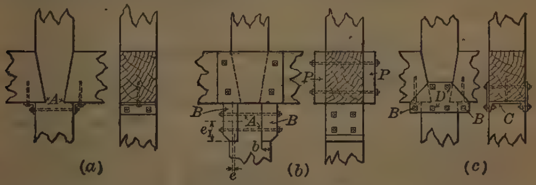


Fig. 95. Struts Framed into Beams

Bolts should be selected of such diameter that the shear will not exceed safe limits and so that the compression upon the wood brought by the bolts will not exceed the allowed stresses of Art. 29. At least two bolts should be used. The shear on the ends of each bolt = 6300/2 = 3150 lb. ¾-inch bolts will be safe against shearing and bearing of the angles on the bolts. As the bolts bear on the end grain of the wood, the safe bearing for the two bolts is = 2 × ¾ × 10 × 1400 = 21 000 pounds, which is much greater than the load. The proper size of bolts may be more easily selected from the tables.

The joint of Fig. 95*b* should be designed similar to Fig. 95*a*. The load on block *B* is transmitted to its end *b*. The lines of action of the pressure at the top and bottom of the block *B* are not coincident. The eccentricity *e* causes a couple whose moment equals the load multiplied by *e*. This couple is resisted by a couple consisting of the bolts acting in tension and the adjacent wood acting in compression parallel to the length of the bolts. The arm of this resisting couple may be considered as equal to ⅞ e_1 in which e_1 equals the distance from a point halfway between the bolts to the shoulder *b*. The washers should be designed to develop the tension in the bolts without stressing the timber higher than allowed in Art. 29.

In the joint of Fig. 95*c*, the end-bearing area of the beams is increased by bent plates or channels marked *C*. The load on these bent plates is transmitted to the side plates thru bolts *B*, and the load on the side plates is transmitted to the post by bolts *D*.

Joints at Top and Bottom of Columns. When beams rest upon the top of a column, a hardwood bolster marked *B*, Fig. 96, may be used to

decrease the side-grain compression upon the ends of the beams. In this case the unit side-grain compression may be excessive upon the bottom of the bolster. To decrease this compression, the tops of the columns may be widened as in Fig. 95*b*. Cast-iron caps of the type shown in Fig. 97, *a*, *b*, are often used at the tops of columns of buildings. The compression upon the side grain of caps and sills, at the top and bottom of columns, will generally be excessive if the columns be designed to take the maximum safe loads and the top and bottom of the columns are not widened similar to Figs. 95, 96. In trestles it is often cheaper to design the columns for loads which, without widening the top and bottom, do not

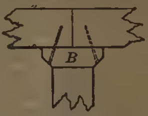


Fig. 96

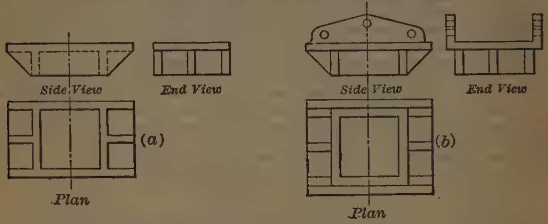


Fig. 97. Metal Caps for Timber Columns

produce excessive side-grain compression upon the caps and sills. Joints like Fig. 95*b* exposed to the elements hold water and rot quickly.

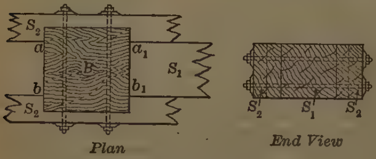


Fig. 98. Dapped Compression Joint

Dapped Compression Joints (Fig. 98) may be designed as follows. Assume that a load of 80 000 lbs is to be transferred from the middle 12 in × 12 in white pine timber marked S_1 to two outside white pine timbers marked S_2. The length of the block B should be such that it will not shear along the lines aa_1 and bb_1. Using a white oak block which has a horizontal shearing value of 200 lbs per sq in, and assuming that the depth of the side pieces S_2 will be 12 inches to correspond with the middle pieces S_1, the length of the block must be at least $80\,000/2 \times 12 \times 200 = 17$ in. The length of bearing in each side timber, since the safe end-bearing compression on white pine = 1100 lb per sq in, must be at least $80\,000/2 \times 12 \times 1100 = 3$ in. A sufficient number of bolts with standard washers should be used to clamp the wood closely together and prevent displacement of the block. The outside timbers S_2 may have the

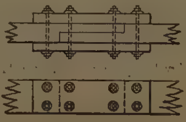


Fig. 99

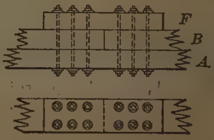


Fig. 100

same cross-section as the middle timber S_1, unless l/d for $S_2 > l/d$ for S_1, in which l/d is the ratio of the length of the timbers to their least width.

Fig. 99 shows a half-lap compression joint with side or fishplates. Fig. 100 shows a compression joint in which the timber A is continuous and timber B is spliced. F is a fishplate and A acts as a fishplate in splicing B.

34. Tension Joints

Plain Fishplate Joints (Figs. 102 and 103) can be economically used for small timbers or large timbers under small unit stresses. The fishplates marked F may be of wood or steel. The design of a joint with metal plates (Fig. 103) is similar to that for wood plates, excepting that the strength of the bolts in shear and bearing on the metal must be carefully considered. Round bolts are generally used and are the only ones considered in what follows. Square bolts make the cost of the joint greater, but as they offer greater bearing area for the same diameter or width, they are slightly advantageous in reducing the number of the necessary bolts. When planks less than three inches thick are joined by bolts they may be designed without considering the bending of the bolt.

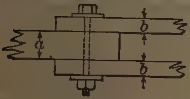


Fig. 101

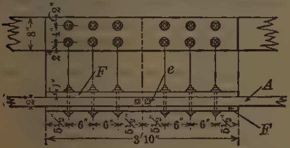


Fig. 102

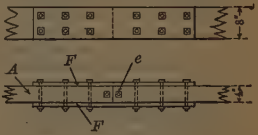


Fig. 103

The following table may be used in determining the safe loads upon bolts of plain fishplated joints. In this class of joints, the bolts are not in tension, and therefore standard washers, preferably of cast iron, may be used. The bolts should be taut so as to bring the pieces tightly together, to prevent buckling of the side plates and to exclude moisture. Both values given by the table should be considered and the lesser safe load be used.

Safe Bearings and Uniform Loads for Bolts

Diameter of bolt, inches	Safe bearing load, lb per lin in		Safe uniform load in pounds for dimension in inches as shown in Fig. 101.					
	End grain	Side grain	$a=2$ $b=1$	$a=3$ $b=1\frac{1}{2}$	$a=4$ $b=2$	$a=6$ $b=3$	$a=8$ $b=4$	$a=10$ $b=5$
½	500	50	1 100	740	550	370	280	220
⅝	625	63	2 160	1 440	1080	720	540	430
¾	750	75	3 720	2 490	1860	1250	930	750
⅞	875	88	5 920	3 940	2960	1970	1470	1180
1	1000	100	8 820	5 880	4410	2940	2210	1760
1⅛	1125	113	12 580	8 390	6290	4190	3140	2510
1¼	1250	125	17 250	11 500	8640	5750	4300	3450

The safe loads per lineal inch of bolt in the second and third columns are based on a safe end-grain bearing of 1000 lbs per sq in and a safe side-grain bearing of 100 lbs per sq in. It is evident from these values that the direction of stress in a bolted connection

should be parallel to the grain. For other allowed unit stresses the safe loads may be obtained by proportion.

The safe uniform loads in the table are for the entire bolts, based upon an extreme safe fiber stress of 22 500 lb per sq in. For higher or lower assumed safe fiber stresses the safe load can be obtained by proportion.

Design of a Plain Fishplate Joint with wooden fishplates (Fig. 102). The timber A to be spliced is a 2 in × 8 in southern long-leaf pine tension member, strest to 1200 lb per sq in of net section, see Art. 29. It will be assumed, in order to get the approximate net section, that two bolts will be necessary in the same plane at right angles to the length of the member and that the area of each hole equals 1 sq in. Therefore the approximate net section equals 16 − 4 = 12 sq in. The total tension in the member = 1200 × 12 = 14 400 lbs. Using the same kind of wood for the fishplates, as for timber, A, their gross area must be 16 in. Therefore use two 1 in × 8 in fishplates.

The Number of Bolts required may be obtained by assuming any convenient diameter of bolt and finding the safe load on the bolt as a restrained beam, taking the length of the beam as the distance between the center lines of fishplates and finding also the safe load upon the end grain of the timbers against which the bolts bear. The smaller safe load should be used. Dividing the smaller safe load upon the bolt into the total stress 14 400 lb, the number of bolts necessary may be obtained. This work is much simplified by using the table on p. 771.

Assuming that a ¾ in bolt will be used, the table shows that 2 inches of the length of the bolt will bear upon the end grain of the tension member and also of the fishplates. The safe end-bearing compression for long-leaf pine = 1400 lbs per sq in. Therefore the safe load upon a ¾-in bolt = 750 × 2 × 1400/1000 = 2100 lb.

From the table the safe uniform load for ¾-in bolt = 3720 lb. Using the lesser value, or 2100 lb, the number of bolts required = 14 400/2100 = 7. As the fishplates are wide there should be two lines of bolts. If one line were used the splice plates might buckle and the splice would be very long. Use six ⅞-inch bolts, which are safe in bearing for a safe load of 6 × 875 × 2 × 1400/1000 = 14 700 lb.

The distance between adjacent bolts of the same line from each other and from the end of the tension member and the fishplates, must be such that the unit horizontal shearing stress will not exceed 80% that allowed in Art. 29. The reduction of 20% is made on account of a splitting action transverse to the line of tension when round bolts are used. For square bolts this reduction need not be made. Bolts tend to shear the wood in two planes tangent to the top and bottom of the bolt. There are six ⅞-inch bolts on either side of the joint and the length of the bolt in the tension member and in the fishplates is 2 in. The safe horizontal shear with the grain is 150 lb per sq in. Therefore the distance between the bolts must not be less than 14 400/6 × 2 × 2 × 150 × 0.8 = 5 in. Add 1 inch for the bolt hole, which is assumed as ⅛-inch larger than the bolts. The distance from the ends may be ½ inch less than between bolt holes, as only half of the bolt hole need be added. Use 5½ inches for end distances.

Tabled fishplate joints are commonly used for heavy timber work. They offer the best type of tension joints and should be used in permanent work where practicable.

Design of a tabled plate joint with wooden fishplates (Fig. 104). Southern long-leaf pine will be used in both the tension member, A, which is to be spliced and the fishplates. From Art. 29, the safe stresses per sq in for long-leaf pine are: for tension with the grain 1200 lb; for end-bearing compression 1400 lb; for side-bearing compression 350 lb, and for shear with the grain 150 lb. The fishplates F are to transmit the full tension of the member across the joint, which in this example is 56 000 lb. The size of the tension member is 8 × 10 in.

The tension to be transferred to and carried by one fishplate = 56 000/2 = 28 000 lb. The net area therefore must be 28 000/1200 = 23.3 sq in. A bolt should be placed

at h, the end of each timber A, to hold it firmly and to keep the fishplates from buckling. Assume all of the bolts in the joint to be ¾ in and their holes ⅞ in. The net width of the fishplate is then 10 − ⅞ = 9⅛ in. Dividing the net area by this net width gives the net depth, thus 23.3/9.125 = 2.56 in. This should be made the same at f and g The fishplate receives its tension thru the end grain bearing of the shoulders of the tables $T\,T\,T\,T$, which bear against the end grain of the tables of the main tension member A. Assume 4 tables in each fishplate, two either side of the middle of the joint. Two tables should be used only for small timbers. The net area of each shoulder should therefore be 28 000/2 × 1400 = 10 sq in. Since the tables are the same width as the depth of timber A, or 10 in, the depth of the tables should be 10/10 = 1 in. The total depth of the fishplates must therefore be 2.56 + 1 = 3.56 in. The nearest standard size timber is 4 in, which should be used.

The length of the tables on the fishplates is determined by the necessary shearing strength, with the grain along lines bb. Since there are two tables on either side of the joint, the required area for each table is 28 000/(2 × 150) = 93.3 sq in. If x is the length of the table, then 10 x is the gross area, from which must be deducted 2 × 0.60, the area of two bolt holes, since two lines of bolts are necessary because of the great width of the fishplates, which might buckle if only one line were used. Then $10x - (2 \times 0.60) = 93.3$ from which $x = 9.45$ in. The tables of the main timbers A are generally made the same length as those of the fishplates. When no bolts go thru the tables of the timber, they may be made shorter than those of the fishplates.

The bolts do not transmit the stresses by bending as in the plain fishplate joint, but by tension. The tension in the fishplate and the compression upon the shoulders, shown by arrows in Fig. 104, do not act in the same plane, thus causing a couple, = (28 000/2) e

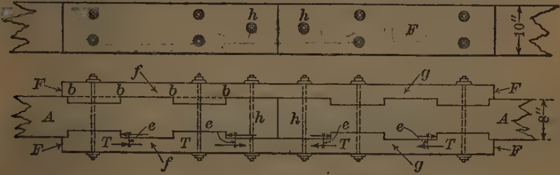


Fig. 104. Tabled Fishplate Joint

for each table, where e = one-half the net thickness of the fishplate + ½ the depth of the shoulder, or 1½ + ½ = 2.0 in. The moment of the couple for each table = (28 000/2) × 2.0 = 28 000 lb-in. This moment must be resisted by a couple consisting of tension in the adjacent bolts and compression in the adjacent portion of the fishplate and timber A acting in a direction parallel to the bolt. The arm of this couple may be taken at ½ the length of the table or 4.73 in. The stress in each bolt hence = 28 000/(2 × 4.73) = 2960 lb. A ¾-inch bolt has a net area of 0.302 sq in, and therefore, based on the allowed tension, the strength of the bolt = 4830 lb, hence ¾-inch bolts can be used. The bolts should be tightened to their full working strength when constructed, and as this is difficult to determine in practise, an excess of strength is desirable. For the same reason it is suggested that for this type of joint the side grain unit compression under the washers should not exceed that allowed in Art. 29.

The washers must have a net area of 2960/350 = 8.5 sq in. A standard cast-iron washer may therefore be used. If a standard washer cannot be used, the gross area should be determined by adding the area of the hole in the washer, and from this gross area the diameter or width of the washer computed.

Fig. 105 shows a modified type of Fig. 104. This joint is costly, but as the bolts are not located at the minimum section of the main timber or fishplates, the area of the bolts need not be deducted in figuring its strength. Fig. 106 shows a common eccentric tabled splice. Timber A is continuous in this joint, while B is spliced by C.

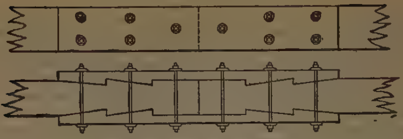


Fig. 105. Scarf-Tabled Joint

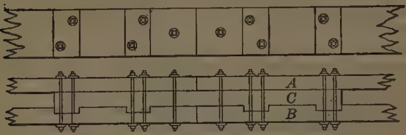
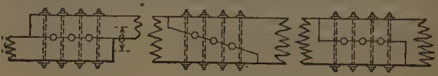


Fig. 106. Eccentric Tabled Joint

Lap Joints (Fig. 107*a*) are often used in temporary work; they are inefficient but quickly and inexpensively made. To compute this joint either in tension or compression assume that the bolts take a bending moment = the



(*a*) (*b*) (*c*)

Fig. 107. Scarf Joints

tension or compression × the distance e. The shear in the bolts and the compression on the timber brought by the bolts must also be computed. The following table is convenient in finding the safe resisting moment and safe shear in the bolt, while the safe bearing on the bolts may be obtained from the table on p. 771. The shearing resistance may be increased by the use of hardwood or iron keys, shown in the figure.

Scarf Joints (Figs. 107, *b, c*) are used to transmit tension or compression and sometimes flexure. They are less efficient for tension than fishplated joints, but may be used when the tension is small. They are more generally used to transmit compression stresses with possible small reversal of stress

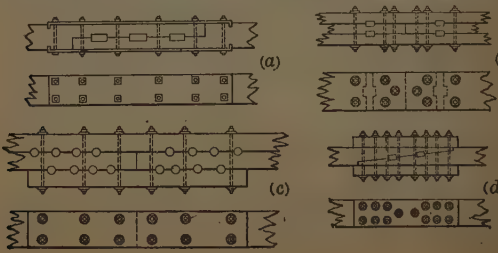


Fig. 108. Four Types of Tension Joints

Safe Stresses for Bolts when Used in Timber Connections

Bolt	Area in gross in	Net area in inches	For tension	For single shear	For resisting moment
½	0.196	0.126	2 020	1 960	275
⅝	0.307	0.202	3 230	3 073	540
¾	0.442	0.302	4 830	4 420	930
⅞	0.601	0.420	6 720	6 010	1 480
1	0.785	0.550	8 800	7 850	2 205
1⅛	0.994	0.694	11 100	9 940	3 145
1¼	1.227	0.893	14 290	12 300	4 310
1⅜	1.485	1.057	16 910	14 800	5 745
1½	1.767	1.295	20 720	17 700	7 460
1¾	2.405	1.746	27 940	24 000	11 835
2	3.142	2.302	36 830	31 400	17 670
2¼	3.976	3.023	48 370	39 800	25 155
2½	4.909	3.719	59 510	49 100	34 475
2¾	5.940	4.620	73 920	59 400	45 935
3	7.069	5.428	86 850	70 700	59 635

Unit stresses used in this table are: for tension 16 000, for single shear 10 000, and for resisting moment 22 500 lb per sq in.

Values for tension are for net area, values for shear for gross area.

35. Joints in Flexure

Built-up Truss Members are occasionally designed for both compression and flexure. Cases of this kind can usually be avoided and should be whenever possible. Compound beams built of two pieces of timber, one above the other (Figs. 109, 110), have been used, but in present practice trussed beams are generally employed if beams of single timbers cannot be obtained of large enough dimensions. The general method of designing a compound beam is given below. When such a beam is in compression, as the top chord of a Howe truss, directly supporting floor beams intermediate between panel points, the direct compression stresses must be considered in determining the cross-sections.

Fig. 109

A Compound Beam should be designed so that the horizontal shear between the adjacent timbers is properly taken care of, so that when the beams bend under the applied loads they will not slide upon each other. If no such provision were made, the strength of the beam would be equal only to the sum of the strengths of each timber acting independently. Compound beams cannot in practice be made as strong as those of single timbers of equal dimensions, and the deflection of the former exceeds the deflection of the latter. For properly designed compound beams of well-seasoned timber carefully framed together and keyed with metal wedges the difference will not be material.

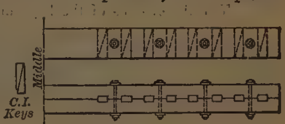


Fig. 110

For temporary structures the two timbers may be fastened together by a continuous sheeting of planks, well spiked to the main timbers as in Fig. 109. The thickness of each side piece should be at least ⅛ of the width of the main timbers and the nails should extend at least ½ their length into the timbers. On the basis of Kidwell's tests (Trans Am Inst M E, vol. 27) it is recommended that the strength of this type of compound beam be taken at 70% of a solid beam of equal dimensions.

For permanent work, keys of hard wood or metal should be used and the pieces fastened together by bolts as in Fig. 110. The efficiency of keyed beams on the basis of Kidwell's tests will be taken at 75% for hardwood keys and 80% for metal keys. The keys are generally made of wedges or tapered pieces so that they may be made a driving fit; they are also made of flat plates or cast iron blocks without taper. It is thought that a stronger and stiffer beam will result by the use of wedged or tapered keys.

To Design a Compound Beam with iron keys, since the efficiency is taken at 80% that of a solid beam of the same dimensions, multiply the load upon the beam by 1.25, adding the weight of the beam, determined by approximate design, and use this increased load to determine the stresses and necessary dimensions. From the bending moment determine the necessary dimensions of the beam, making the width of beam approximately $\frac{6}{10}$ its depth. Compute the vertical external shears at governing points and draw the vertical shear diagram. Determine the total horizontal shear between the point of zero shear and the ends of the beam. The number of keys in each half of the beam may be assumed, and five is a desirable number. Making all keys of the same size, the compression to be resisted by each key equals the total horizontal shear in the left or right portion of the beam divided by the number of keys. As the horizontal shear will be different to the right and left of the point of zero shear, in an eccentrically loaded beam, the compression to be resisted, per key, or the number of keys, may be different in the right and left portions of the beam.

To find the positions of the keys, divide the right and left areas of the shear diagram by vertical lines into as many equal areas as there are keys to the right and left of the point of o shear. This may be done graphically or by trial. Fig. 111 shows a shear diagram for an eccentrically loaded beam divided by vertical lines into equal areas. The position of each key is fixt by the vertical lines of division. The keys should be placed back towards the ends, and not in front of the vertical lines. They are shown on the horizontal axis of the diagram. The dimensions of the keys must be such that the safe end compression on the wood is not exceeded. The distance between keys must be such that the safe horizontal shearing stresses are not exceeded. The end keys may be moved back of the point of support to decrease the unit horizontal shear between the first and second keys from the end.

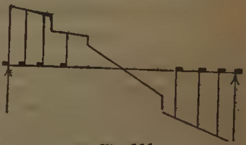


Fig. 111

Stresses in the bolts and their sizes may be determined as in the design of a tabled fishplate joint. Additional bolts should be used to bind the timbers together. The friction resulting from the tightening of the bolts increases the strength of the beam, but to an indeterminate amount, and should not be depended upon in design.

If the keys are of wood, the bearing of the end grain of the timbers against the side grain of the keys will govern their cross-sectional dimensions. If the keys are inclined, the stress in the bolts and also the available shearing area between keys will be increased. For inclined keys, the shearing area of the timber for any key may be regarded as extending from its outer end to the middle of the next key. The breadth of a key should not be less than twice and preferably not less than 2½ times its thickness.

36. Trussed Beams

For Spans exceeding 20 ft or when the loads are unusually large and where head room permits, trussed beams may economically be used. For economy, the trusses should be generally as deep as head room permits. Trussed beams are of two types (Fig. 112). Type 1 is used when available head room is

below the beam seats and type 2 when the head room is above. The member aca of both types may be of a single timber or of two or more timbers placed side by side. Usually each timber aca is a single unspliced piece extending from end support to end support. When it is of a single timber, the tie rod or rods of type 1 pass thru oblique holes bored thru the ends of

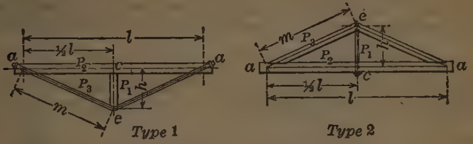


Fig. 112. King Types of Trussed Beams

the piece. If it is of two or more pieces, they are separated so as to allow the tie rod or rods to pass between them. Type 1 should be designed with ends of tie rods made adjustable, or with an intermediate portion of rods, made adjustable by turn-buckles, or preferably with both adjustments. The adjustable tie rods should be made taut when erected, which causes initial stresses in the beam, and if the tightening is not carefully done, the strut ce may be pulled out of the vertical. The base of ce in type 1 should be made wide to better withstand unequal tightening of adjacent tie rods.

For Spans over 25 ft or when the head room is small or when loads are concentrated at two intermediate points, types 3 and 4 (Fig. 113) are preferable

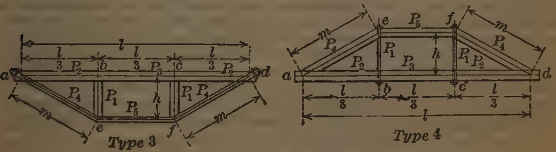


Fig. 113. Queen Types of Trussed Beams

to types 1 and 2. The struts be and cf are generally placed vertical in these types, but they would be more efficient if placed so as to bisect the angle between the horizontal and inclined portions of the rods. In the design of all beams, the line of action of all pieces intersecting in a joint should meet in a point, or secondary stresses must be considered.

Type 1 (Fig. 112) may be designed by the following formulas, in which W in lbs = total load on beam. (1) WHEN W IS UNIFORMLY DISTRIBUTED over the length of beam, P_1 = compression in $ce = \frac{5}{8}W$; P_2 = compression in $ac = 5Wl/32\,h$; P_3 = tension in $ae = 5Wm/16\,h$; and $S_2 = P_2/bd$ = unit compression in ac, due to direct compression. To find the maximum unit compression in ac, add to S_2 the flexural or bending unit extreme fiber stress, due to the load acting upon ac as a beam. The extreme fiber stress due to bending $S_3 = 2.25\,Wl/bd^2$. To find the maximum unit fiber stress add S_2 to S_3, or use the formula $S = Wl\,(5/h + 72/d)\,/32\,bd$. All lengths are exprest in feet, except that b and d, the width and depth of the chord aca, are exprest in inches. P_1, P_2 and P_3 are exprest in pounds, and S_2, S_3 and S in lb per sq in. (2) WHEN W IS CONCENTRATED at the middle of the beam, P_1 = compression in $ce = W$; P_2 = compression in $ac = Wl/4\,h$; P_3 = tension

in $ae = Wm/2\,h$. For this loading there are no flexural stresses except those due to dead load of the beam, which, excepting in long spans, under small loads, may usually be ignored.

Type 2. The stresses will be of the same intensity as in type 1, but the character of the stresses will be reversed. *ce* and *ac* will be in tension, and *ac* in compression. If the load is uniformly distributed, the foregoing flexural formulas may be used, but the maximum unit stress in *ac* will be tension.

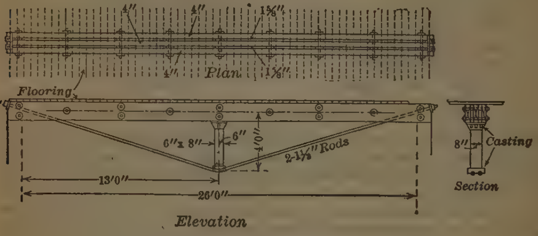


Fig. 114. Beam Trussed with Steel Rods

Example, Fig. 114. To design a trussed beam, type 1, span 26 ft, depth of truss 4 ft load, exclusive of weight of truss, 1500 lb per lin ft, using long-leaf yellow pine for top chord and vertical post. The compression in $ce = 5/8 \times 1500 \times 26 = 5/8 \times 39\,000 = 24\,375$ lb. Assuming that the ratio of length of *ce* to its least width will be less than 15, the cross-sectional area of $ce = 24\,375/1400 = 17.4$ sq in. The direct compression in $ac = 5\,Wl/32\,h = 39\,600$ lb. The necessary cross-sectional area of *ac* therefore $= 39\,600/1400 = 28.3$ sq in. The extreme fiber stress in *ac* acting as a beam, $S_3 = 2.25\,Wl/bd^2$. Use a safe extreme fiber stress 80% of that given in Art. 29 in order to allow approximately for direct compression. From the above formula $d = \sqrt{2.25\,Wl/bS_3}$. Assume $b = 12$ in, and for S_3 substitute eight-tenths of the allowed extreme fiber stress or $1200 \times 0.8 = 960$ lbs per sq in. Then $d = \sqrt{2.25 \times 39\,000 \times 26/12 \times 960} = 14.1$ in. The maximum fiber stress per sq in, due to direct compression and flexure, is $S = Wl\,(5/h + 72/d)/32\,bd = 1160$ lb per sq in. The weight of the truss per lineal ft = approximately 1.3 of the weight per lineal ft of the top chord *aca* or $1.3 \times 12 \times 14/12 \times 3.3 = 60$ lb, where $12 \times 14/12$ equals the number of board ft, and 3.3 equals weight of a board ft of timber. As this weight is only 4% of the load, it may be ignored. Instead of using 80% of the allowed extreme fiber stress, the necessary size of *ac* due to bending may be found by formula, using the allowed extreme fiber stress and adding the necessary area for direct compression.

For the top chord 3 planks 4×14 inches will be used, separated so as to allow two steel rods to pass between them. This chord will be assumed to be stiffened laterally by a plank floor. Allowing 16 000 lb per sq in in the tie rods, the net section *A* in sq in for $S = 16\,000$ and $P_3 = 5Wm/16\,h = 41\,740$ is $A = 2.6$ sq in, or the two rods having a diameter of 1 ½ inches may be used if the ends are not upset. If the ends are upset, two rods of 1 ⅜ diameter may be used, as they have a combined area slightly in excess of 2.6 sq in. The pieces of the top chord should be separated, so as to permit the insertion of the rods and their upset ends.

As the necessary cross-sectional area of *ce* based on end compression is 17.4 sq in, a timber 3 by 6 inches might be used if the columnar reduction is not too great. The safe load for a 6×6 long-leaf pine column 8 ft long $= 26\,300 \times 1400/1000 = 36\,800$ lb. Therefore the safe load for a 3×6 column 4 ft long $= 36\,800/2 = 18\,400$ lb. A larger-sized member must therefore be used, which might be readily selected from a table. For stiffness and simplicity of end connections, 6 by 8 inch timber will be used.

A cast-iron cap may be advantageously used at the top of the vertical strut to transmit the pressure to the side grain of the top chord. The width of the top must be sufficient

to engage the top timbers, and its length must be sufficient to reduce the unit side-grain compression to that allowed in Art. 29. A metal foot of cast iron or steel should be used to reduce the unit compression upon the end grain of *ce* within a safe limit. The width of *ce* was, in this case, increased to 8 inches in order to receive the rods, which, on account of the top chord construction, are 5½ inches apart on centers. The washer at the ends of the tie rods must be of such size that the end-grain compression will not exceed the assigned safe limits.

The Design of Type 3 (Fig. 113), when the three panel lengths are all equal, may be made as follows. W = total load on beam in pounds. (1) W UNIFORMLY DISTRIBUTED. The stresses P_1 in the vertical struts $= 11/30\ W$. The end reactions brought by *ab* and *cd* acting as beams $= 4/30\ W$. The stresses in the various members acting as a truss may be determined graphically or analytically. To the stresses so determined the flexural beam stresses in the top chord of type 3 and in the bottom chord of type 4 should be added. The following approximate formulas are easily applied when the panels are of equal lengths. They are sufficiently accurate for practical purposes. P_1 = compression in *be* or $cf = 1/3\ W$; $P_2 = P_3$ = compression in $ad = P_8$ = tension in $ef = Wl/9\ h$; P_4 = tension in *ae* and $fd = Wm/3\ h$. The extreme fiber stress per sq in, in the top chord panels acting as beams $= S_3 = Wl/bd^2$. The maximum extreme fiber stress $S = Wl/bd\,(1/9\ h + 1/d)$. (2) FOR ½ W ON EACH VERTICAL STRUT. Let P_1 = compression in *be* or $cf = 1/2\ W$; $P_2 = P_3$ = compression in $ad = P$ = tension in $ef = Wl/6\ h$; P_4 = tension in *ae* and $fd = Wm/2\ h$. For this loading there are no flexural stresses except those due to dead load of beam, which may be usually ignored.

Design of Type 4. The stresses will be of the same intensity as in type 3, but the character of the stresses will be reversed. *be*, *cf* and *ad* will be in tension and *ae*, *ef* and *fd* in compression. If the load is uniformly distributed, the flexural formulas of type 3 may be used, but in this case the maximum stress in *ad* will be tension.

37. Grillages and Wharves

Grillages of Timber are used on the tops of piles or upon foundation beds to support masonry. Fig. 115 shows a design of a grillage supporting a masonry retaining wall. The top of the grillage blocks B should not be higher than about 18 inches above low tide and the bottom timbers should not be so low that they must be laid by divers. The sizes of timbers may be computed by means of the beam formulas and tables. As the timber is under water the unit side-grain compression should be taken at 75% of that given in Art. 29. The caps C and blocks B, which help prevent the masonry sliding on top of the grillage, should be drift bolted or lag screwed, using long special screws, to the top of the piles. The grillage plank A should be fastened to the caps with ship spikes.

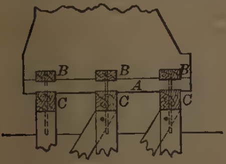


Fig. 115

Grillages supporting Building Bearing Walls are of the type shown in Fig. 116. These grillages should not be used except where they will be permanently wet. The bottom course of plank should be at least 2 inches thick and the top course at least 3 inches thick, provided that the clear spacing of the middle timber does not exceed 18 in. This middle timber must be designed so that the extreme fiber stresses do not exceed those specified in Art. 29. The bearing of the top plank upon the middle timber will not

need to be investigated, except for soft wood. As in the preceding design, only 75% of the specified unit side-grain compression should be used.

The middle timber should be figured as a cantilever beam whose length is the portion extending beyond the outer edge of the wall. The load upon this cantilever beam equals the allowed safe load on the foundation bed $\times l_1 \times c$ (Fig. 116). If the depth of the middle timber be assumed, its width may be determined by

$$b = 36\,wl_1^2c/Sd^2$$

and

$$b_1 = 3\,wl_1c/2\,S_hd$$

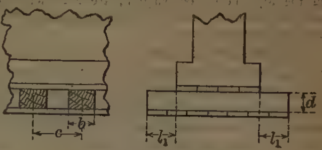


Fig. 116. Grillage for Wall

where b = width of timber in inches, based on extreme fiber stress, b_1 = width of timber in inches, based on horizontal shearing stress, l_1 = portion of middle timber extending beyond the edge of wall in ft, d = depth of beam in inches, c = distance between consecutive middle timbers in ft, w = safe load on foundation in lb per sq ft, S = safe extreme fiber stress in lb per sq in, S_h = safe horizontal shearing stress in lb per sq in. For shallow beams the width b will generally govern, and for deep beams the width b_1; the greater width should be used.

Wharves. The width of a wharf or pier is usually governed by local and commercial conditions, and its length by such conditions and regulations of the United States government. The direction of the length of a wharf, as a rule, should be perpendicular to the bulkhead wall or shore line. In narrow rivers this direction may be made oblique to the shore line, the outer end being farther down stream than the shore end. This arrangement facilitates the entrance of vessels into slips between adjacent wharves. Timber wharves are of two general types: (1) having the floor system or deck supported on piles; (2) having the floor system or deck supported upon timber cribs. The first type is generally the better and cheaper and should be built wherever conditions permit. It cannot be built where the bottom of the river is too hard for pile penetration. When a small penetration, from 4 to 10 feet, may be gotten, the requisite stiffness and holding power of the piles may be obtained by using riprap. The riprap, if of small stones, may be placed, in whole or part, in advance of the pile driving. This is unusual practice, and if adopted, the pile points should be shod with iron.

The top of the deck of a wharf must be above high water, but low enough to permit the economical loading of vessels at low tide. The wharf should be designed with the greatest practical free waterway, so as not to obstruct the flow of water, sewage or ice. Pile piers are usually built with piles in transverse rows 8 to 12 ft on centers, the piles in the rows being 3 to 10 ft on centers. The close spacing is for heavy loads or where the pile resistance is small. When timber cribs are used they extend for the transverse width of the wharf and are usually spaced 15 to 35 ft on centers. Where cost of cribs and their foundations is high, trussed spans may be found economical for distances much over 25 ft. A wharf need not, as a rule, be braced longitudinally, as the impact of vessels in this direction may be taken care of by the bulkhead wall or the earth at the shore end, and as the length of a wharf is generally much greater than its width, the force of impact is distributed over a greater number of piles than when the force acts in a transverse direction.

The lines of transverse piles should be placed as nearly parallel to the thread of the current as possible. They should be strongly sway braced, the braces being carried down at least as low as low tide. If the piles are to be driven into soft bottom or deep water or in a rapid current, riprap or batter piles should be used to stiffen the wharf against the impact of vessels. A horizontal stick of 12 by 12 inch timber, 20 ft long, neglecting its weight, acting as a cantilever, will deflect ¼ inch under a load of 100 lbs

applied at its end; if 40 ft long, the deflection will be about 2 in. Wharf piles if unbraced and under the impact of vessels, are cantilever beams whose length equals the distance from their top to firm river bottom. If the upper portion of the piles is sway braced sufficiently, the length of pile, acting as a cantilever under vessel impact, is the portion below the bottom of the sway bracing. As the deflection under a given load varies as the cube of the length, it is evident that the sway bracing should be carried down as low as possible.

Guard or fender piles should be used to protect the structure from abrasion of vessels lying alongside, and clusters of piles should be placed near corners to protect them. In deep water, or where the river mud is deep and for large vessels, the clusters should consist of from 10 to 50 piles. The corners of wharves for large vessels should be rounded and strongly reinforced. The rounded corner makes it easier for boats to enter slips between the adjacent wharves.

A timber wharf in fresh water or in salt water where marine wood borers do not exist, lasts about 30 years, with minor repairs and floor renewals. To increase the life of a wharf all joints and bearings should be treated with a preservative; drift bolts should be driven below the tops of the timber, and the hole thus formed filled with tar or asphaltic cement. Large washers should be used under all bolt heads and nuts and the bolts should not be tightened so as to break the outer wood fibers. When wood pieces are in contact and it is possible to force them into closer contact with bolts in order to exclude water, this should generally be done. Beams built up of two or more pieces and all joints should be packed at least 1 inch apart, using metal packing pieces or separators. The timber should, in general, be so designed as to permit as much circulation of air around each piece as is possible. Wooden floors should be built so as to drain the rain water to outlets. Leaving open joints between adjacent floor planks in order to get rid of rain water decreases the life of a floor subject to vehicular travel, due to the pounding of the wheels across the joints and the abrasion of the edges by horses' feet. Exposed timbers will last longer if beveled so as to shed water freely. Creosoting lumber, if well done, increases the life of the structure.

In fresh water the portion of the pile between about 1 ft above low water and high water usually rots before the rest of the structure. By making the caps continuous over several piles, the rotted pile tops may be removed and replaced without removing the top timbers. In fresh water, piles generally last longer with bark removed above low tide. In salt water, piles in warm climates are attacked by marine borers which generally enter the piles between high tide and a point about 4 ft below low tide. They may continue their destructive work above high tide and many feet below low tide. They never enter the wood below the mud line. Wood borers are classified by the Forest Service as Teredo, Xylotrya and Limnoria, Chelura and Sphaeroma. The first two are commonly called Teredo and the latter three Limnoria. These borers do not live in fresh water nor in salt water containing much sewage, but may exist in brackish water. The average length of time in which the Teredo and Xylotrya have destroyed barked and unprotected piles in the Gulf of Mexico ranges from 1 to 5 years, the minimum time being sometimes only a few months. Circular 128, U. S. Forest Service, gives methods of protection.

Live Loads for Wharves range between 75 and 750 lb per sq ft for concentrated loads of wagons, unloading apparatus, trains, and posts of wharf buildings. The lighter loads are used for excursion boat landings, the heavier for commercial wharves. A commercial wharf at Washington, D.C., was designed for a uniform load of 350 lbs per sq ft. The necessary sizes of timbers were determined from the preceding beam formulas and tables. All the parts of the structure should be well bolted, drift bolted, spiked or lag screwed together, so that the wharf will act as a unit under the impact of vessels. As the piles between high and low water decay before the other portions, except the floor plank, the unit stresses in the piles should be taken at about 50% of those recommended in Art. 29. This will usually be taken care of by the proper application of pile formula.

The cost of pile wharves varies between \$0.75 and \$2.00 per sq ft of floor, depending upon the depth of water, nature of foundation necessity of batter piles or riprap and the loads to be carried. Wharves of the crib bridge type cost between \$1.50 and \$3.00 depending upon similar conditions.

FALSEWORK AND TRUSSES

38. Trestles

Timber Trestles are generally used for semi-permanent work when their first cost is less than an earth fill or an earth fill and culvert, if a stream of water is to be taken care of. When permanent work is desirable, there is usually no sound reason for timber trestle construction, other than the lack of funds, because, due to the high cost of lumber and timber maintenance, an earth fill or steel structure will be cheaper in the end. Trestles usually consist of a series of bents spanned by beams or stringers. Each bent generally consists of two or more vertical posts, depending upon the width of roadway and the load to be carried, two outside battered posts, a cap and sill, and sway bracing. The outside posts are given a batter of from 1½ to 3 inches in 12 inches, and the bent is sway braced to resist lateral force such as wind or rack of a locomotive. The roadway or track is carried upon stringers, which should be continuous over two spans. Fig. 117 shows a common form of trestle less than 30 ft in height as used in 1906 on the Baltimore and Ohio R.R. Trestles over 30 ft high are generally built in stories of from 15 to 30 ft in height, as in Fig. 118, which shows the 1908 standard of the New York, New Haven and Hartford R.R. In this design the bent is framed into two separate stories, the second story being erected upon the cap of the completed story below. This is the more common design and is easy to erect, but occasionally the vertical and batter posts are made continuous from top to bottom of trestle. This latter method reduces to a minimum the number of joints where end grain bears upon side grain. This is desirable, as the side grain is often injured by such compression, and in the case of sills dampness is retained by the broken fibers and rot takes place rapidly. Trestles over 50 ft high are sometimes built with pairs of bents framed into towers at intervals of 20 to 30 ft, the stringers being compound or trussed beams.

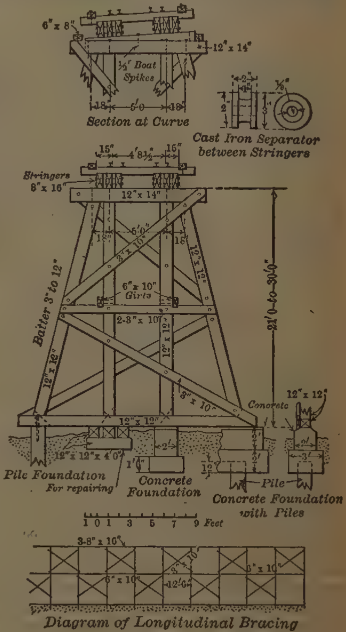


Fig. 117. Trestle, Baltimore and Ohio R.R.

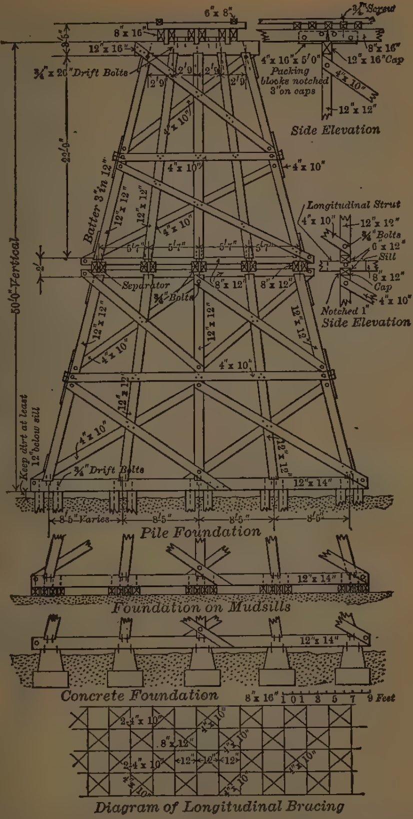


Fig. 118. High Trestle, New York, New Haven and Hartford R. R.

Longitudinal bracing for railway bridges must be designed to take the thrust of a braking train, which may amount to 75 000 to 100 000 lbs. The longitudinal bracing should not only consist of horizontal struts between bents to take compression, but the bents should also be X braced as shown in Fig. 117. For highway trestles the bracing may be of smaller cross-section and the X bracing at less frequent intervals. Where the trestle is of short length and there are solid abutments of earth, timber or masonry at either end, the X bracing may be safely reduced in cross-section.

Trestle Design. The sizes of the stringers and posts can be obtained from the timber tables and formulas, using the specified loads given in Sect. 8,

but it is advisable to increase the dimensions so determined, to allow for decay. Stringers rot quickest where the ties or flooring rest upon them and where they rest upon the caps, as dampness lodges in such joints. The stringers of railway bridges should be packed 1 to 2 inches apart, so as to permit circulation of air, using metal separators or packing spools, as in Fig. 117. Bolts should pass thru the spools, having large washers at their ends so as to permit the bolts to be tightened without injury to the fibers. The sizes of the posts depend upon the safe allowable compression upon the side grain of the caps and sills, see Art. 29. It is therefore desirable to use hardwood caps and sills when possible. The caps and sills should have about the same dimensions as the posts. As a rule the posts can be so placed with reference to the stringers that there will be practically no bending stress in the caps, and similarly there will be none in the sills, the function of caps and sills being to hold the posts in position and transmit shear. The posts may be drift bolted or mortised to the caps and sills.

The bottoms of sills should be supported by masonry so as to be at least 12 in above the ground. If the trestle consists of piles driven into dry ground, the piles at ground level will rot badly in 5 or 6 years, and as the life of a trestle is from 10 to 14 years, it is advisable to double, or nearly double, the number of piles per bent, in order to allow for decay, or cap the piles with masonry, as in the extreme right footing of Fig. 117.

Bracing. Good examples of sway bracing are given in Figs. 117 and 118. For highway bridges the sways may be lighter. Use ¾- or preferably 1-inch bolts for fastening sways. The design of this bracing for trestles under 30 ft high is generally made on the basis of precedent, the effect of wind being small and the other stresses indeterminate. For higher trestles the wind stresses should be computed as guide in the selection of proper size and in the design of joints. The horizontal struts or girts of the logitudinal bracing should not, for railway work, depend upon a single bolt for the transmittal of stress, but such stress should be transmitted by end bearing. For railway trestles the struts should be from 6 by 8 to 8 by 12 inches. For highway trestles, the span being longer, the size should not generally be less than 6 by 6 in. The longitudinal X bracing need not be used in every panel, except for railway trestles on curves over 7°. In high railway trestles on curves over 8°, the centrifugal force of the moving train should be further guarded against by additional braces on the convex side of the trestle. The sizes of the X braces cannot be determined by computation, but should be selected from existing designs. The details of all trestle work should be simple. When pieces are in contact they should be drawn tight, to exclude water, and where possible they should be separated at least 1 in and preferably 2 in to allow circulation of air. The cost of labor should not exceed about $15.00 per 1000 ft B.M. In the design of high trestles, in order to decrease the number of bents the span of the beams is reduced by means of corbels and A frames as shown in Fig. 119, thereby permitting a greater distance center to center of columns.

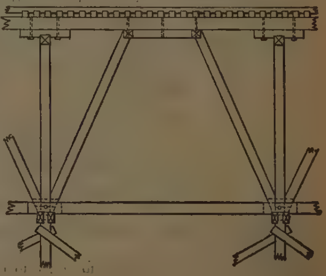


Fig. 119

Life of Trestles may be increased by the use of wood preservatives, such as creosote, and all surfaces in contact or which have been disturbed by ax or adze, all screw and bolt holes and the tops of all drift bolts should be thoroly coated with similar preservatives. The tops of caps and stringers are often covered with sheet zinc or iron to protect them from water and hot cinders. These coverings are unquestionably of value as fire protection, but as they decrease the circulation of air, it is doubtful if the life of the caps and stringers is prolonged thereby.

Construction Trestles are for temporary use, and therefore in selecting the sizes for posts the side-grain compression may be taken 50% higher than allowed in Art. 29, and no allowance need be made for rot. Trestles for light industrial tracks and small gage construction tracks require only light transverse and longitudinal bracing. In making earth fills from temporary trestles constructed therefor and from old trestles constructed for semi-permanent use, as railway trestles, the lateral and longitudinal bracing, and in high fills even the caps and sills, may be broken by the fall or weight of the material or its settlement. Such fills should be brought up evenly on either side of the axis of the trestle so as to diminish the possibility of large and sudden movements of the fill, which occur most frequently in fills made upon side hills. It is often advantageous to remove the bracing when it is reached by the fill so as to prevent the pull in the bracing, under the load of the fill, from distorting the posts and even breaking them. This is a matter which can be determined only after the character of the fill and contours of the ground are known. If the bracing is old and rotted it is often best to put on a new and substantial bracing before starting the fill.

When earth fills are made from trestles of piles driven into 15 to 20 ft of mud, sudden movements of the mud and newly made fill frequently not only break the bracing but snap off the piles and demolish the entire structure. Such trestles should be built of piles of large diameter, driven to hard pan and heavily sway braced, and the sway bracing removed when reached by the fill. But even with these precautions sudden failure may take place.

39. Floors for Trestles and Bridges

Floors for Steel Highway Bridges are illustrated in Fig. 27 of Sect. 8. In designing the roadway joists concentrated wheel loads may be regarded as distributed over two joists, when the distance center to center of joists does not exceed 2½ ft and the flooring is at least 3 inches thick. In maintenance work, when the center distances do not exceed 2 ft and the wheel load is not applied near the outer line of stringers, it may be regarded as distributed over three joists, provided the flooring is at least 3 inches thick. The front roller of a steam roller may be computed as equally distributed over all joist which it can cover or partially cover. The wheel loads upon cross ties of electric railways may be regarded as distributed over three ties.

Roadway Flooring may consist of a single thickness of 3- or 4-in plank, the distance between centers of stringers not exceeding 9 times this thickness; 3-in flooring should be used for light and infrequent travel and 4-in where it is heavy and continuous. Roadway plank should be laid with a transverse grade of from 1 to 1¾ inches in 10 ft, to drain the water to the sides of roadway. In bridges over 16 ft wide, the plank may be cut in two lengths and the level of the roadway raised at the middle. FLOOR PLANK should be laid snug, as the planks at open joints are torn by the horses' shoes and the floor is made very rough and wears quickly. They may be laid transverse to the bridge or oblique; the former method is slightly cheaper and appears equally satisfactory. Planks need only be drest on one side, but all joints should be

adzed smooth when the floor is laid. Roadway plank should have a uniform width of 12 in, altho if ordered in variable widths of 8, 10 and 12 inches, a saving of about $3.00 per 1000 B.M. results. When variable widths are used it is necessary in making repairs to select particular widths of plank for each defective one replaced, which is troublesome. For bridges having little travel the life of the flooring depends upon the rotting of the timber, and therefore timber which rots slowly, such as long-leaf pine, is desirable. For bridges carrying heavy and continuous travel the floor does not rot but is worn out, and therefore the harder and tougher woods such as white oak, chestnut oak or rock oak are best.

Three-inch flooring should be fastened to the joists with 6-in nails and four-inch with 7-in nails. Flat-head wire nails are more easily driven than round-head nails. Well-seasoned hardwood floors often require holes to be bored in advance of driving nails, but most commercially seasoned oak and all pine flooring may be nailed down without such holes. All flooring should have a one-heart face. The heart face should be laid down for light travel and up for heavy. The general rule is to put the heart face down, but experience with bridges in Washington, D. C., shows, that as a rule, it should be up.

The life of the flooring under light travel is from 4 to 5 years, and under heavy travel 3½ to 4 years. In the latter case probably 10% of the plank will have to be replaced before a general replacement is made, using a low grade of timber. The cost of tearing up and relaying plank floors for roadways should not exceed $6.00 per 1000 B.M. The cost of maintaining bridge floors, including cost of timber joist, assumed to last 12 years, and on the basis of timber at $33.00 per 1000 B.M. and labor of relaying at $6.00 per 1000 B.M., is about 5 cents per square foot per annum.

Double floors consisting of a 3- or 4-in creosoted subfloor, laid with ½- to 2-in open joints and having a wearing floor of from 1½ to 2 in, are sometimes used. They should never be used with untreated plank in the subfloor. A creosoted subfloor lasts 12 or 15 years. The lower planks are laid transverse or oblique to the bridge and the upper ones at an angle of from 45° to 60° with the ones below. The subfloors should be nailed to the joists with 60d nails, using two per joist, and the top planks nailed to the lower floor with two 40d nails at intervals of 2 ft. This type of floor has the advantage of cheap resurfacing at frequent intervals, and therefore usually gives a smoother roadway than single planking. The first cost and the difficulty of getting properly creosoted timber militate against the use of this type.

A Roadway Joist should be at least 3 inches thick and should not deflect more than 1/400 of the span under the full load. The general sizes of joists used are 3 by 12, 3 by 14, 4 by 14 and 4 by 16 in. When they rest upon the tops of floor beams they may lap or butt, if there is sufficient bearing. When lapped, an air space of ½ inch should be left between the lapping ends to permit circulation of air. Neither bridging nor fastening of the joist to the floor beams is necessary, unless planks thinner than 3-inch are used. The floor will be too heavy to be disturbed by wind.

Long-leaf pine and oak joist last from 12 to 15 years, provided they are selected of such depth that they may be turned over once. Joists rot where the planks rest upon them, and when this depth reaches an average of one inch they should be turned and used until the top of the joist, in the new position, rots about the same amount. Long-leaf pine joists are better than oak joist, as they do not warp, and pine rots from the outside and oak often from the inside. White pine joist have lasted from 25 to 30 years in Washington, D. C. The painting of the top of joists with a timber preservative is recommended.

Sidewalk Flooring should be 2 inches thick, drest one side, 6 or 8 inches wide and supported by joists not over 2 ft on centers. They should be laid snug, as open joints make rough floors and the public object to seeing thru the open cracks of floors of high bridges. Pine floor planks may be spiked with two 20d nails per joist and oak ones with two 30d nails. Floors should be given a transverse grade of ⅜ inch per ft. SIDEWALK JOISTS are usually 3 by 12 or 3 by 14 inch, placed 2 ft on centers. These joists should be fastened

to the floor beams, as sidewalks are usually narrow, of light weight per sq ft, and if the joists are not fastened the floor may be disturbed by wind, altho cases of this kind are extremely rare.

When steel stringers are used the planks in small country bridges are often fastened by clinching the nails under the flanges. The best arrangement is to bolt spiking strips 4 to 5 inch thick to the flanges of the beams, countersinking the bolt heads.

Wheel Guards should be provided on either side of the roadway to protect the adjacent trusses or to act as a curb protection for the sidewalks. They should consist of 4 by 6 or 6 by 6-in string pieces supported upon two- or three-inch shims, 6 to 12 inches long placed at intervals of 4 to 6 ft on centers. The roadway water should drain thru the openings left between the shims. Guard rails should be bolted to the joists or floor beams, the bolts passing thru the shims. Joints in the string pieces should be made at the shims and with halved laps. These rails should be protected with steel angle facings when laid upon a steep grade, as they are often used by teamsters as brakes.

Electric Railway Tracks may be supported upon cross ties or directly upon longitudinal stringers, the latter method being used when the bridge also carries vehicles. Cross ties should not be less than 6 by 6 in when the stringers are 6½ ft on centers. For a greater distance they should be designed to carry the maximum wheel load, assuming a distribution of this load over three ties, the fiber stress not to exceed 1200 lbs per sq in. The openings between ties should not exceed 6 inches. Every fifth tie should be bolted to the stringers and every fourth tie to the guard rail. Ties should be notched from ½ to 1 inch over the stringers and made a tight fit. Guard rails 6 by 8 in should be placed on either side of the track at a distance of 3 ft 7 in from the center of track and notched 1 in over ties. Splices should be made over ties with bolted half laps.

Solid Timber Floors covered with ballast have been used for trestles and have given satisfactory results. The timber is creosoted and not afterwards cut if it can be avoided. When cut the end surfaces are painted with creosote. This type of floor is expensive to build and maintain and is somewhat difficult to inspect, but is regarded as offering a good protection against fire, decreases noise and gives a better track than the common type of floor. Arguments for and against this type of floor are given with interesting detail in the Proc of Assoc of Railway Superintendents of Bridges and Buildings, Oct., 1906. Fig. 120 shows a ballast trestle floor on the El Paso and South Western R.R.

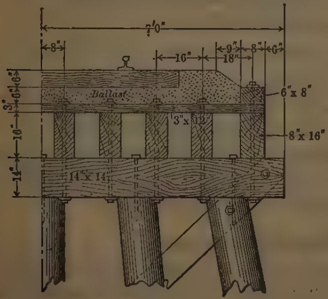


Fig. 120

Railway Trestle and Bridge Floors. Figs. 117 and 118 show typical trestle timber floors and Fig. 121 shows floors for Howe bridges. The specifications of the American Bridge Company for 1900 give the following clauses relating to such floors:

The floor stringers shall be placed generally 6½ ft between centers, the standard distance between centers of tracks being 13 ft.

The floor shall consist of cross ties 8 by 8 inches if the stringers are placed 6½ ft between centers. For greater distances they are to be proportioned to carry the maximum wheel load distributed over three ties, the fiber strain on the timber not to exceed

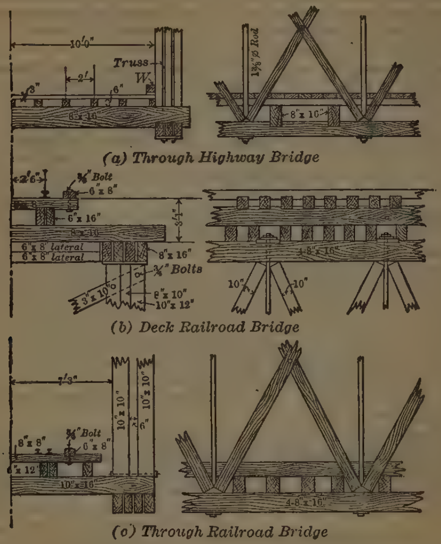


Fig. 121. Floors for Howe Bridges.

1000 pounds per square inch. The ties shall be spaced with openings not exceeding 6 inches, and shall be notched down ½ inch and have a full bearing on stringers.

Every fifth tie shall be fastened to the stringer by a ¾-inch bolt.

There shall be guard timbers 6 by 8 inches on each side of each track, with their inner faces not less than 3 ft 3 in from center of track. They shall be notched 1 inch over every tie, and shall be fastened to every third tie and at each splice by a ¾-inch bolt. Splices shall be over floor timbers with half and half joints of 6 inches lap.

40. Falsework for Bridges

Truss Bridges are generally erected upon timber trestles or scaffolds consisting of vertical timber posts and outside batter posts, supporting caps upon which rest longitudinal stringers (Fig. 122). The bottom chord of the bridge is laid upon the falsework and blocked several inches above its correct position to allow for settlement of the falsework under the load of the bridge. Bents are usually placed at panel points of the bridge, but where clearance makes it necessary the panel loads may be carried at any point of the stringers.

Small and light bridges may be erected by gin poles, which are timber masts guyed in a vertical position and carrying a block and tackle at their tops, by means of which the separate truss members are hoisted into their positions. Most bridges are erected by overhead travelers consisting of two or more bents braced together to form a tower. The traveler rests upon steel wheels which travel upon rails laid upon the timber stringers. The falsework must be designed to carry the weight of the bridge and traveler. Where there is danger from injurious freshets or where there must be large clearances maintained as for navigation, trussed falsework, generally of the Howe bridge type, is used. This falsework may be erected upon temporary

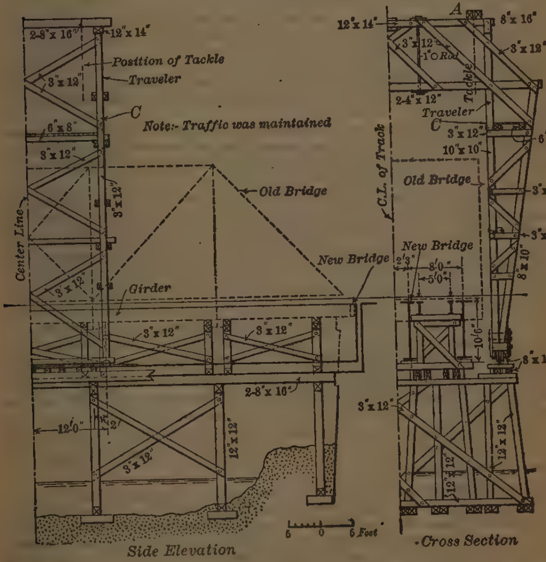


Fig. 122. Bridge Erection, Baltimore and Ohio R.R.

timber trestles or upon pontoons. Fig. 122 shows the falsework and traveler used on the Baltimore and Ohio R.R. in erecting a small bridge.

Design of Falsework. The loads to be carried by the falsework consist of its weight, which, except for trussed falsework, is negligible; the weight of the bridge, including the girders, but not necessarily the stringers and floor resting upon same, and the weight of the traveler and operating engine. In the design consideration must be given to wind pressure exerted upon the bridge and the traveler, and where a stream is to be bridged the effect of ice and débris carried by freshets must be regarded and preferably the erection

completed within a period of time when ice gorges or serious freshets are least probable. Where trussed spans are used, slightly higher unit stresses may generally be used than are allowed for permanent work, as the period of erection is not sufficient for a material deterioration of the timber. The trusses should be of first-class carpentry work so as to reduce the truss deflection to a minimum and should be strongly sway-braced.

When viaducts or bridges overland are erected, the falsework may consist of a series of ordinary trestle bents resting upon either mud sills or spread timber footings. The trestles for water work are generally formed, in whole or part, of piles. When the trestles are over 30 ft high they are generally built in stories of from 15 to 30 ft high, the timbers of adjacent stories being tied together by fishplates, so as to transmit tension resulting from wind pressure upon the bridge and traveler.

For light bridges the trestle sills, posts and caps may be of 6 by 6 or 8 by 8 in timbers, the sway and longitudinal bracing of 3 by 8 in timbers and the stringers of 6 by 12 in timbers. For most bridges the sills, posts and caps should be 12 by 12 in, the sway and longitudinal bracing 4 by 10 in, and the stringers 8 by 16 in. The approximate dimensions of the timbers should be determined by computation, but as stiffness and sometimes weight are governing factors, the sizes should generally be larger than theoretically required.

All falsework should be designed so as to permit of economical erection and demolition and so that the timbers will not be materially injured for future use. Bolts should be used instead of spikes, and beveled, mortised or other expensive joints avoided. The cost of erection should not generally exceed $12.00 per 1000 ft B.M.

The Settlement of the Falsework under its full load should be a minimum consistent with economy. A practically unyielding foundation should be constructed; the number of horizontal joints and particularly the number of joints where end grain bears upon side grain should be reduced to a minimum and the unit compression upon side grain should not exceed those allowed in Art. 29. The probable settlement of the falsework, exclusive of foundations, may be approximated as follows: (1) By figuring the columnar shortening, c, in inches, by means of the following formula, $c = 24\ Sl/E$, in which S is the unit compression per sq in, l = length of column in ft, and E the modulus of elasticity. When pile foundations are used the length of piles should be added to obtain the total columnar length to be used in the formula. (2) By allowing $\frac{1}{16}$ in settlement for each horizontal joint for "taking up" and an additional $\frac{1}{16}$ in at each horizontal joint where end grain bears upon side grain for "biting" of the grain into the side grain, provided the unit stresses of Art. 29 are not exceeded. The total settlement will be the total of that obtained from (1) and (2).

In railway bridge renewals or other bridge erection where the falsework carries heavy locomotive or vehicular loads the locomotives or heavy vehicles may cause settlements of the falsework foundation so much greater than the deformation of the timber, that the latter is negligible. Frequently, even in highway bridge construction, falsework foundation settlements of from 6 to 12 inches occur without serious mishap. The panel points are wedged up as the falsework settles, but in design such settlements should be guarded against.

Design of Travelers. The traveler should be designed to carry the heaviest bridge members and withstand the pull of the tackle and wind pressure. The bridge members are lifted by means of block and tackle fastened to the top timbers, as seen in Fig. 122. The loads are transmitted to the falsework thru the columns, marked C. The wheels and wheel bearings at the feet of the columns need not be designed to carry the gross load, as the traveler is blocked up when hoisting. The bracing should be heavy so as to withstand the racking action of the hoist. It is not subject to refined computation, and hence the designs of previously constructed efficient travelers should be consulted.

41. Falsework for Arches

Masonry Arches are built upon temporary falsework, called centers or centering, usually of timber consisting of parallel frames, bents or ribs with top members whose upper surface is cut to the curve of the arch soffit. The top members may be joists, or bows, also called back pieces, may be used. Upon the top members plank called lagging are laid which support the stone voussoirs or concrete sheeting of the arch. Wedges are placed at convenient points of the falsework to lower sections of it after the arch is built. The wedges may be located at any point between the soffit of the arch and the falsework foundation. All arches should be built up symmetrically and the centering designed for such loading.

The Centering for Short Span Arches, when the span is under 10 ft and the arch is flat, may be built of single planks on edge with tops cut to curve of soffit, supporting lagging drest on one side and supported at the ends by posts resting upon wedges, as at *a* in Fig. 123. If the centering is built for a stone or brick arch, the lagging may be laid with open joints not wider than ⅓ the width of voussoirs or bricks; if for concrete, tongued and grooved lagging should be used. For spans over 6 ft, the top member or bow may be in two pieces tied by small battens (*b* in Fig. 123), the curve of the arch being cut out of the top piece. To save labor of cutting bows when the barrel of the arch is long enough, the bows may be placed 4 or 5 ft on centers and 2- or 3-in lagging used. The lagging and bows are computed as beams and the posts as columns under the full load of the arch. In buildings, 1-in lagging is generally used, supported on two or three bows about 12 inches on centers. When the arch is flat and the span is over 10 ft, intermediate props or posts are generally used to shorten the beam length of the bow; 2- to 4-in lagging may be used, the bows being placed 4 to 6 ft on centers.

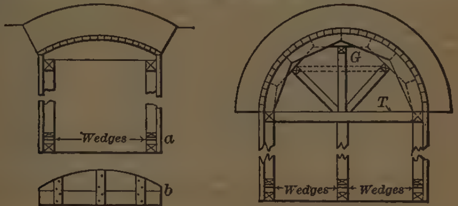


Fig. 123 Fig. 124

Centers for Short Span Arches

When the arch has a rise of over ⅙ the span, centering of the form of Fig. 124 may be used. The bow may be made up of two or three planks breaking joints and nailed together. For economy the bows should be as far apart as possible, preferably 4 to 6 ft, using from 2- to 4-in lagging. For the larger spans, the thicker lagging is the more economical. A center post should be used for spans over 10 ft, and when this cannot be done, the bottom piece, *T*, should be stiff and strong enough to carry the load brought by the radial braces. This load is indeterminate, but in design it may be assumed at ¼ of the total load of the arch, the bow also carrying ¼ of the load as an arch and the balance being carried by the masonry arch acting as a compression member.

This type of centering may be used up to spans of 30 ft, but for spans over 15 ft one or two lines of horizontal girts should be used for each frame to take care of the horizontal thrust of the arch load. A girt, marked *G*, is shown in dotted lines in Fig. 124.

Unbraced arch centering of type shown in Fig. 125 should never be used, except where other forms are impossible, and then only for spans under 30 ft, as it deforms badly, rising at the crown when loaded at the haunches and sinking at the crown under the crown loads and is at best risky. By placing a temporary load at the crown, the change in the form of the curve can be lessened. If used at all, the carpentry work should be first class, the abutting sections of the bows in intimate contact, wedges of shingles being used just prior to loading if the shrinkage of the timber has caused the joints to open.

Fig. 125. Bow Centering

Centering for Arches over 30 ft span may be of four types. (1) Arches, (2) Howe trusses, (3) Bowstring trusses, (4) Bents. The first is expensive to build, gives small salvage for lumber, deforms badly under loading and therefore requires temporary crown loading and causes cracks at the haunches during the arch construction unless loaded in alternate transverse sections. Where a deep gorge is to be spanned it may be economical. The second type is objectionable for general use for the foregoing reasons, except that the crown will not rise when the haunches are loaded. Maximum stresses should be determined for all web members under various stages of loadings. There is no reason for using this type except to utilize existing available trusses. The curve of the arch must be built up by additional timbers resting upon the top chord at panel points. For the design of this type see Art. 44.

Bowstring Centering, shown in Fig. 126, has been used extensively for spans up to 100 ft, and may be necessary for arches over streams, but should never be used where intermediate supports can be gotten, as in Fig. 127, or where vertical posts may be carried to intermediate foundations. It has all the objections given for type 1, but is cheaper to build and deforms less. This type is preferable to the Howe truss bridge type. Bowstring centering should be built with Warren bracing, as in Fig. 126. The stresses in the various members should be computed for the centering, one-third, one-half, two-thirds, and fully loaded, and each member designed for the maximum stress. Reversal of stress will take place in the bracing, and all joints of this bracing should be made with straps or gussets to transmit tension as in Fig. 126. The crown of this centering should always be loaded with a temporary load of about ⅙ the total arch load, but even with this precaution hair cracks will generally develop at the haunches, showing that the tensional value of the mortar has been destroyed.

Fig. 127 shows a modification of the bowstring centering which was used in the construction of the Massachusetts Ave. culvert, Washington, D. C. The bowstring truss was designed to be moved ahead in sections upon the bottom bents or A frames. Eight bows were struck at one time and skidded upon pipe rollers resting upon a stringer track. One additional A frame was erected in advance in order to permit of this movement.

The Bent Type of Centering is recommended for the construction of all arches where clearance does not prohibit its use. Its first cost is generally least, it deforms less than the other types, and gives the greatest salvage of lumber. Its design should not be begun until the method of building the arch has been determined. If the arch is to be built up continuously from the springing line to the crown, it should deform or settle less than if built in either longitudinal rings or alternate transverse sections. But however strongly built, if the arch is built up continuously cracks will develop at the

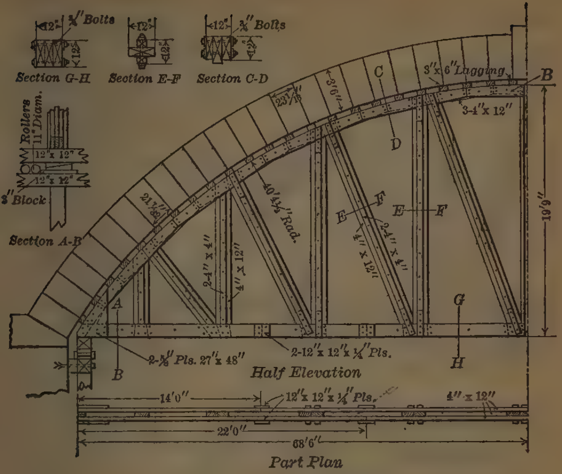


Fig. 126. Bowstring Centering

aunches when the crown is built. This of course will not be so if the arch small enough to be built complete in a day. The stiffer the center, the naller the cracks. These cracks are not serious, except in the case of a einforced concrete arch they may permit the infiltration of water and the isting of the steel, and they are the cause of the abnormal sinking of the rown of the arch when the centering is lowered. Where the cracks are rger than hair cracks, the masonry above the extrados of the arch should ot be built until the centering has been lowered sufficiently to close up the racks, otherwise the closing up of the cracks in the arch may crack the upper asonry.

If the arch is to be built in longitudinal rings three or four feet wide, each ring to be ompleted in a day, which is common practice in the construction of reinforced concrete rches, the centering should deform or settle less than if built in transverse blocks, but ay deform more than if built continuously for the full width of the bridge. When uilt in alternate transverse blocks, a settlement equal to 1¼ in per 100 ft and proporonally for larger or smaller span, is good practice, but a less settlement is desirable. or arches in longitudinal rings, the settlement should be less than half this amount, nd for arches built up continuously from the springing line to the crown a smaller settleent should be gotten if possible. Since the settlement of the falsework varies almost rectly as the distance from the foundation to the crown, arches of large span and rise nd those over deep gorges or valleys should be built in alternate sections, whereas arches small span and rise over flat ground may be built according to the other methods.

Two Types of Centerings of Bents. The first type (Fig. 130), which is sed for concrete arches, generally is designed so that the load upon the lagging transmitted to joists spaced from 10 to 24 inches on centers, supported y transverse caps resting upon vertical or inclined posts. In the second ype, Fig. 128, which is generally used for stone arches, the lagging rests

directly upon bows spaced from 3 to 6 ft on centers, the bows being supported by the posts. In Fig. 128 the voussoirs are shown supported by wedges, but usually lagging is used in lieu of wedges. The first type is preferable for concrete bridges, because, upon the completion of the arches and the striking of the centering, lagging, studs, and wales of proper sizes are available for the balance of the concrete construction. The second type is better for stone arches, because by laying the lagging with open joints the bottom or soffit joints of the voussoirs are accessible and can be "packed with rope or rags" for grouting, the packing removed and the work pointed prior to the removal of the lagging. Further, this second method permits the placing of wedges under each voussoir, allowing small errors in the top curve of the centering or the top curve may be omitted and the curve of the soffit made by shimming pieces and wedges of variable heights, as in Fig. 128.

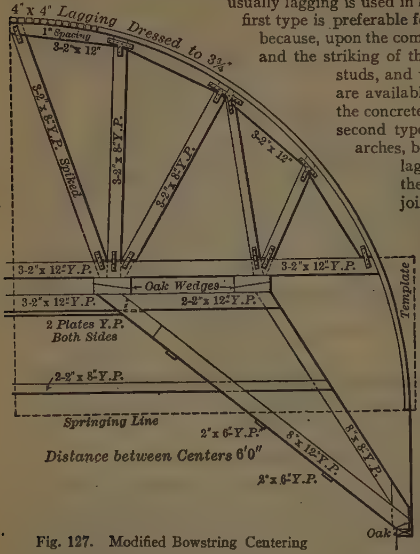


Fig. 127. Modified Bowstring Centering

Loads and Stresses. The centering should be designed to carry the weight of the arch ring alone, except in the case hereafter noted. In the case of concrete the nature of the load varies, the concrete being deposited as a semi-liquid. For unit weights see Art. 2. The weight of the falsework is negligible, except in very high falsework. It is suggested that, in determining the load upon the centering, friction be neglected, as this is on the safe side and other assumptions reduce to an absurdity. Fig. 129 shows a voussoir or section of stone or concrete, *abcd*, supported by the lagging and by the masonry or strut *SS* below it. The weight *W* is resolved parallel and perpendicular to a tangent to the lagging at its middle point, giving a normal component *N*, which must be carried by the centering. Assuming that there is friction along *ab*, which reduces the intensity of *N*, it is evident that the reduction cannot equal *T* times the coefficient of friction,

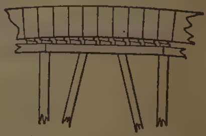


Fig. 128

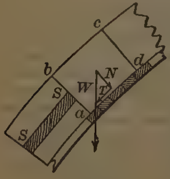


Fig. 129

ecause, if such were the case, the arch ring from the joint *ab* to the springing ne would have to be capable of supporting itself. Concrete when deposited eing in a semi-liquid form would cause little friction along the joint *ab*, nd the setting of the concrete could not modify the initial stresses.

In addition to the loads of the arch ring, the effect of wind pressure upon the falsework, rior to the keying of the arch, should be considered, and where the arch is over a stream, nasonry footings should extend to above the high-water line, or other precautions taken to rotect the falsework from injury, due to the pressure of ice or débris.

Information upon the Design of Centerings. The lowest 30° of a semi-ircular arch and of other arches of less rise exert little vertical pressure upon the falsework and may be built upon a template or light falsework. In applying this general rule to an arch which is not semicircular, the 30° point is one at which a perpendicular to the tangent to the intrados makes an angle with the horizontal equal to 30°. Fig. 127, which is a design used in the construction of a brick arch, shows that a template was used in the lower portion of the arch.

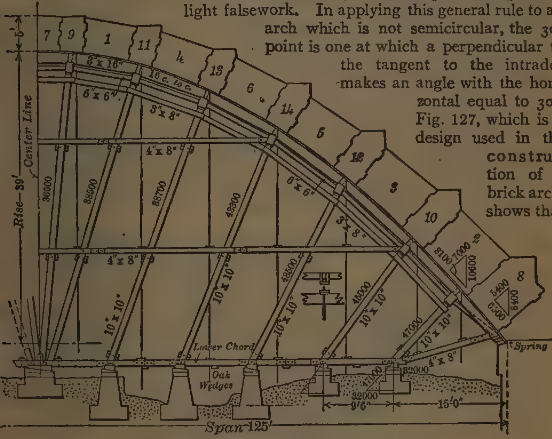


Fig. 130. Centering for Voussoir Arch

The settlement of the centering may be computed approximately by using he rules given in Art. 40, but as arch centering should be, and generally is, f a better grade of carpentry work, an allowance is suggested of 1/32 inch for 'taking up" and 1/32 inch additional for each joint, where end grain bears pon side grain, provided the unit stresses of Art. 29 are not exceeded. If articular care is not taken with the joints, such as adzing off rough places, he settlement may more nearly equal 1/16 inch for "taking up" and 1/32 inch or "biting."

At the Piney Branch Bridge, Washington, D. C., there were used six joints between he soffit and foundation, two of which were in end bearing (Fig. 130). Assuming first-lass carpentry work, which was obtained, the settlement due to "taking up" and "biting" vould be $6 \times 1/32 + 2 \times 1/32 = 1/4$ in. To this must be added the columnar shortening vhich, on the basis of the formula in Art. 40, gives a total of 9/16 in, which agrees closely vith the actual measurements. Measurements of other centerings show that the limits ie between 1/16 and 1/32 inch, depending upon the grade of carpentry work. However, vhen the unit compression upon side grain exceeds the allowable unit stresses, the settle-nent of the centering is greater. In some parts of the Connecticut Ave. and the Edmondson

Ave. bridge centerings where the stresses were 50% higher than allowed in Art. 29, due in part to defective framing, the "bite" was 1/8 inch per joint.

When the posts are placed vertically, and not radially or approximately radially, there is a lateral deformation of the centering as well as the vertical. When posts are vertical, struts must bear against the transverse caps, to prevent displacement under the radial pressure, or the joists resting upon the caps must be notched out and shimmed tight against the caps. Centerings are frequently designed with only one-half as much load on the outside posts of the same bent as upon the middle and intermediate ones and the cross-section of all timbers is made the same. As a result the center settles and the face forms, if the arch is of concrete, go out of plumb often as much as 1/2 inch in 5 ft. and if the arch is wide a crack may be formed. When the arch is to be built of concrete, the caps should extend about 3 ft beyond the face of the arch ring to support the forms for the face of the arch and also to provide a walk way.

Sway Bracing need not be computed for wind pressure, except for high arches. It should, as a rule, be heavier than such computations indicate to be necessary. A study of successful designs is suggested. Very little longitudinal bracing is required, except such girts as are necessary to take the horizontal component of the load of the arch when vertical posts are used. The use of steel, except for sway rods and in the form of plates or channels to increase end bearing, is not generally recommended. Steel is greatly affected by temperature and should not generally be used. However, in centerings of high arches the posts for 1/3 the height of the centering may be of steel, as their expansion or contraction will have comparatively little effect upon the total height of the falsework.

Since centerings are designed for temporary work, the less permanent woods, such as hemlock and short-leaf pine, may be used for posts, lagging, and sway-bracing. Only hard wood should, as a rule, be used for sill, caps, and wedges, altho long-leaf pine is sufficiently hard for caps. Long-leaf pine is best for joists.

Layout. Having determined upon a centering of bents, the types of foundation, span and curve of the arch govern the

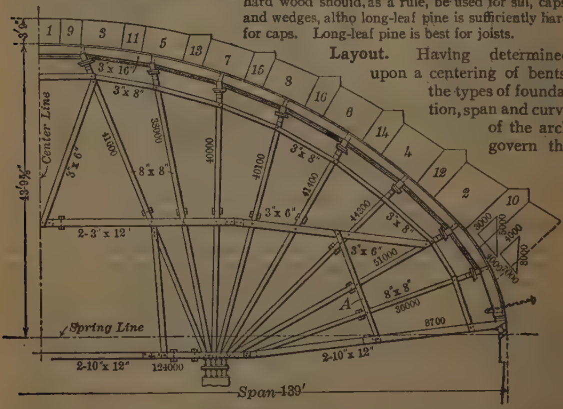


Fig. 131. Centering for Voussoir Concrete Arch

layout. The distance of the posts apart at the top of the centering should be, in general, as great as the available depths of joist or, in a case of a stone arch centering, the available depths of timber for the bows, permit. Joists cannot generally be gotten of larger dimensions than 6 × 16 or 4 × 16 and sometimes only 4 × 14 in. If rock or other suitable foundation can be gotten

at shallow depths, Fig. 130 gives a good layout. If suitable foundation is at considerable depth and foundations are expensive, Fig. 131, which shows the center for the main arch of the Edmondson Avenue bridge at Baltimore, gives a good layout. In this design the railway track near the center of the arch was cleared, which somewhat modified the layout.

Fig. 127 shows a good way of supporting a centering upon an offset in the foundation masonry. Small arches upon high piers may be supported in a similar manner upon a projecting coping or upon projecting blocks of concrete or masonry.

Details for a Concrete Arch Centering. The lagging should be of tongued and grooved flooring, such as second-class Virginia pine flooring, drest one side and having a thickness of from ⅞ to 1¾ inches, depending upon the distance center to center of joists. The deflection of the lagging under full load should not exceed ⅛ inch per ft and preferably should be less. Joist may have their tops sawed to the curve of the soffit or a spiking strip 2 inches wide may be used. This spiking strip may be of a low grade of lumber, and its use will result in a saving, particularly where the radius of the curve of the arch is less than 150 ft. Posts, when inclined, as in the case of the lower posts of Fig. 131, should be supported at one or more points by timber marked *A* in figure, otherwise they will sag materially and be inefficient as columns. Where end grain bears upon side grain, the end-bearing area may be increased by angles, as in Fig. 130, or by steel plates or channels. The number of joints in side-grain compression should be reduced to a minimum, and therefore for high centering continuous posts are preferred to independent stories.

It is expensive to cut bevels and miters at the ends of timbers and notches at intermediate points, and since the salvage of the timber is less when so cut, they should be omitted when possible. The difference in the cost of framing may be increased from $15 per 1000 B.M., which is an average price, to $40 by failure to consider this point in design. Caps should extend over three or more posts. They should be drawn tight upon the posts by means of bolts or other fastenings and be tied in place by fishplates and bolts, by notches in the joists above them, or by continuous struts, as in Fig. 130.

Wedges may be placed at the tops of posts, as in Fig. 131, or at the bottom, as in Fig. 130. When placed at the top, they permit of easier adjustment of the top before concreting, as there is less load to lift. When at the top they should be placed at right angles to the length of the cap, as in this position they are more easily driven than when placed parallel to the cap. Wedges at or near the top should not be loaded with more than 15 short tons per wedge. At the bottom they may be loaded to 20 short tons. With this loading they may be loosed with a 12-lb sledge hammer. Wedges should be of hard oak with a bevel of 1:5 to 1:10, preferably the former. They need not be spiked in order to hold them in correct position relative to each other. Lubricants do not appear to facilitate striking, as they are forced out under the pressure of the arch load, and the friction to be overcome is due largely to the mechanical interlocking of the fibers. At Piney Branch Bridge the coefficient of friction of white-oak wedges was found to be about 0.4.

Screw Jacks may be used to lower arches, but they are generally too expensive. It is proposed in the construction of the Hendrick Hudson Memorial Bridge, New York, to use hydraulic jacks, not only to strike the centering, but to jack it up if there is any settlement of the falsework foundations.

Sand Boxes have been used extensively in Europe, but only in a few cases in America. They permit the centering to be lowered without jar and quickly if desired. Their first cost is high, but in large arches the cost of striking wooden wedges is so great that it is thought sand boxes result in economy. They cannot be used to raise the centering before it is loaded, in order to

adjust the curve, and great care must be taken to keep the sand dry during the entire construction. The sand should be thoroly washed and dried before being placed in the boxes. Fig. 132 shows details of the sand boxes used in the main arch of the Edmondson Avenue Bridge, Baltimore.

This sand box consisted of a cylindrical shell built of 3/16-in sheet steel inclosing, at the bottom of the cylinder, an oak bottom, which fitted snugly against the steel. The box was then filled with dry clean sand, upon which rested a circular oak plunger, which supported the bottom of the posts of the centering. Near the bottom of the box was a 1-in circular hole, closed by a wooden plug, which was removed when it was necessary to lower the centering, permitting the sand to run out of the hole. The top of the cylinder was covered with several thicknesses of tar paper, in order to exclude rain. The maximum pressure upon the sand was 620 lbs per sq in. The stress in the steel shell was figured on the basis of this pressure applied to each sq in of the shell, or on the basis of water pressure.

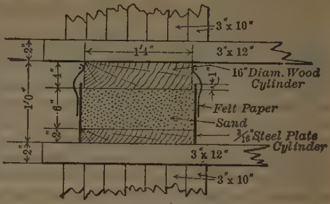


Fig. 132. Sand Box

Erection. A full-size drawing of the centering should be laid out upon a level floor. using transit and steel tape. Templates of thick paper or preferably of white pine should be made of all curved pieces and of the ends of all members having beveled or mitered ends, and the full length of each piece measured, intermediate notches being accurately located. The separate pieces of the centering should be laid out, using these templates and measurements. In making the layout drawing, the true curve should be first laid out and then a second curve drawn, adding sufficient camber to allow for the probable settlement of the falsework.

All carpentry work should be the best class of heavy timber work. Care should be taken to see that all saw cuts are true, and surfaces of abutting timber should be adzed off to make a snug fit. Just before the centering is loaded it should be inspected and all joints which have opened up should be shimmed tight, using shingles or thin wedges. During the construction of the arch similar wedging should be done, as the settlement of the falsework may open joints.

As large centerings are expensive and if destroyed by fire, not only the centering, but the arch also is lost, they are occasionally insured and given fire protection. At Connecticut Avenue Bridge, the centering for which cost about $50 000, a 4-in water main was laid along the top of the construction trestle and a hose connection with hose attached provided opposite each large arch. The centering was watered every evening as an additional precaution. The wetting of the centering, however, tends to cause softening of the wood fiber of the side grain, and when used, somewhat lower stresses upon side grain should be used than are allowed in Art. 29.

For concrete arches the tongued and grooved lagging should be laid with a board omitted every 10 ft, so as to allow for the swelling of the flooring when it rains, otherwise much of the flooring will buckle and have to be relaid. If continuous forms are built to form the face of the arch ring, they should be sawed thru with radial saw cuts at intervals of about every 25 ft, otherwise when the centering settles these forms will twist and warp and an unsightly face results.

It is customary to speak of lowering the centering, but when the wedges at the top of the falsework are loosened, the centering actually rises. It has been held down by the weight of the arch, and when the wedges are driven out, due to the elastic resilience of the wood, the centering rises. If the wedges are at the bottom of the falsework, the general movement of the falsework is downward, but the falsework is not really lowered until the timber is no longer under restraint.

It seems to be the common opinion that when wedges are moved and still remain tight the arch is lowering and following the centering. This usually is not so. Arches of concrete, built of alternate blocks, do settle very slightly, due to the fact that the concrete of each block shrinks in setting up, but in arches from 60 to 150 ft, of moderate rise, the settlement is seldom over ⅛ in. Arches built in longitudinal rings settle less.

If the temperature rises after keying the arch, the expansion of the concrete may be sufficient to develop arch action while the arch is still on the centering, and if so, probably no settlement will take place. Or if the falsework is weak, the arch will key itself tight, due to its own load. If the falsework is very wet when keyed, the arch may also develop arch action by the shrinkage of the timber due to drying out. If the temperature of the air falls 15° or 20° for any length of time after keying the arch and the centering is stiff, the contraction of the arch masonry may cause cracks to appear in the arch while still upon the centering, unless the arch is reinforced with steel.

If an arch is built in alternate transverse sections, the falsework may have to carry loads in excess of the weight of the arch, due to the fact that on account of the shrinkage of the concrete in the several sections the arch ring consists of a number of separate blocks which are not quite in contact. A load such as that at A, Fig. 131, if placed upon the arch before striking the wedges, may have to be carried by the centering. At both the Connecticut Ave. arch and the Edmondson Ave. arch a settlement of about ¼ inch took place in the crown when the load A was placed. After this there was no appreciable lowering of the crown, except upon striking the settlement may have been as much as 1⁄16 in.

Details of a Stone Arch Centering. All the foregoing data are generally applicable to centering for stone arches. The lagging for stone arches may consist of timber from 3 to 6 inch thick and laid with open joints, as in Figs. 126–128. The wedges are placed under the lagging or at lower points. These figures also show typical bows which may be used for the trussed or the bent type of centering. As a considerable portion of the arch stones may be placed temporarily upon the falsework to decrease deformation without inconvenience to the contractor or added cost, the bowstring centering is somewhat better adapted to stone than to concrete arches.

42. Roof Trusses

Definitions and Description. The top chords of roof trusses, sometimes called the main rafters, usually support timbers called purlins, laid at right angles to the planes of the trusses. These purlins support timbers called common rafters, laid at right angles to the purlins or parallel to the main rafters. Upon the common rafters are laid boards called sheathing, and upon the sheathing a roof covering, such as tin, shingles, or terra cotta. The purlins may be laid close together and the common rafters omitted, or heavy planking may be used to span between main rafters and both purlins and common rafters omitted. This latter method is uneconomical and seldom used.

Trusses are usually placed from 12 to 16 ft on centers, purlins 8 to 10 ft, and common rafters 1½ to 2⅓ ft. Purlins should be placed at or near panel points so as not to cause flexure in the top chord members. When the common rafters are omitted, the top chord members should always be computed for both compression and flexure. Purlins may be placed with their sides either vertical or at right angles to the main rafters. The latter method is slightly more expensive, except where the purli s are supported by steel hangers.

Roof trusses are designed to support the roof and snow load and withstand wind pressure and occasionally to support a ceiling or lower floors, which are hung from the trusses by means of hanger rods. For loads and stresses see Sect. 8.

Weights of Trusses. The following formula, recommended by H. S Jacoby (Structural Details, 1910), is based upon 121 designs of English and Belgium roof trusses of spans ranging from 48 to 196 ft, with a rise of one-sixth to one-third of the span, spacing of trusses 8 to 12 ft, a snow load of 25 lb per sq ft and a normal wind load corresponding to a pressure of 40 lbs per vertical sq ft.

$$W = \tfrac{1}{2}al\ (1 + 0.15\,l)$$

in which W is the weight of a truss in pounds, a the distance between centers of trusses, and l the span, both a and l being exprest in feet. The above approximate weight of truss is suggested for use in design, but the weight of the truss, when designed, should be estimated and compared with the approximate one used.

Common Types of Roof Trusses are shown in Fig. 133. In these diagrams the heavy lines indicate timbers, the lighter ones iron or steel ties, the light dotted lines counterbracing and the heavy dotted lines auxiliary timbers. When timbers are used as tension members they are marked with a plus sign. The ends of timber ties should be detailed to transmit tension, using steel straps or gusset plates and bolts for this purpose. Trusses with all tension members of iron or steel are called combination trusses.

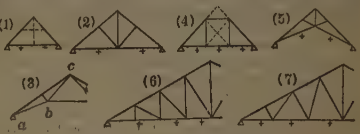


Fig. 133. Types of Roof Trusses

Type (1) is used for temporary buildings such as sheds, the spans not exceeding 24 ft. The horizontal tie is generally supported at its middle by a tie rod or plank, shown by the dotted vertical line. An intermediate horizontal plank, shown by the dotted horizontal line, spiked to the main rafters, may be used to stiffen the truss. Type (2) is a king-rod truss, which is a modification of type (1), the rafters being supported by timber, struts at their middle points. This type is used for 24 to 36 ft spans. Type (3) is a combination truss with main rafters supported by struts at middle points, which may be used for spans of 25 to 45 ft. Joint b should be pin-connected, and ties ab and bc should not be in one length. Type (4) is a queen-rod truss suitable for spans between 24 and 36 ft. The middle panel should be counterbraced, as shown by light dotted lines, to take care of the wind pressure upon the roof and eccentric loading. When this type is used with a pitch roof, rafters are placed above the top chord, as shown by heavy dotted lines. For spans between 36 and 45 ft struts should be placed to support the main rafters at their middle points. Type (5), called a scissor truss, is suitable for spans 24 to 30 ft and where a high ceiling is desired. Modifications of this are sometimes used for slightly larger spans. Type (6), called an English roof truss, is used for spans from 40 to 60 ft, or for larger spans with an increased number of panels. For the shorter lengths it is built in 6 or 8 panels. Type (7), called the Belgian roof truss, may be used for the same spans as Type (6). In this truss the struts are perpendicular to the rafters and there are two less panels in the lower chord than in the upper one.

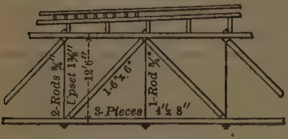


Fig. 134. Howe Roof Truss

Howe trusses are generally used for flat roofs, for spans from 40 to 130 feet. They are built with depths equal to from 1/7 to 1/10 of the span. Fig. 134 shows the middle portion of one of these trusses. The upper chord is horizontal, but the roof is made to have a low pitch by using short vertical struts of different lengths. There are no counterbraces, since there is no rolling load. This figure will apply to roofs of from 85 to 125 feet span.

43. Design of a Roof Truss

Data of Design. (Fig. 135.) The truss will have a span of 48 ft, pitch 30°, distance center to center of trusses 16 ft. The roof covering will be of 3/16-in slate laid upon wooden sheathing. The sheathing will be supported by common rafters resting upon purlins at or near panel points of the top chord. Slate roofing is selected in preference to tin, corrugated iron, or wooden shingles because of its permanency and low cost of maintenance, and in preference to clay tile because appearance is not desirable and slate is the cheaper. For slate roofs the pitch should not be less than 1:2 or 26° 20″. This rule for pitch may be applied to the other kinds of covering mentioned. The trusses were spaced 16 ft apart on centers, as this is generally the maximum limit of span for untrussed purlins. This spacing therefore is the economical one. The roof has been designed to carry the dead load, a snow load of 10 lbs per sq ft of horizontal area and a wind pressure of 30 lbs per sq ft of vertical area, which for a 30° pitch gives a pressure normal to the roof of 19.9 lbs per sq ft (see Sect. 8). The weight of the slate covering and sheathing was taken at 11 lbs per sq ft of roof.

Unit Stresses. Long-leaf pine was used thruout. The allowable unit stresses in Art. 29 may be increased 30% for protected timber not subjected to impact as in a building. On this basis the allowable stresses in lbs per sq inch are: tension with the grain 1560, compression with the grain 1820, compression in short columns 1300, compression across the grain 455, extreme fiber stress 1560, shearing with the grain 195 and shearing across the grain 1625.

Common Rafters. The rafters will be placed 2 ft centers, so that 7/8-in tongued and grooved sheathing drest one side may be used. Occasionally the sheathing may have to be figured as a beam to carry the load upon it, but usually it may be selected without computation. The vertical load upon the rafter consists of the weight of roof covering and sheathing, snow load and the weight of rafter assumed at 3.5 lb per lineal foot; or $2 \times 9.25 \times 11 + 2 \times 8 \times 10 + 3.5 \times 9.25 = 396$ lb. The wind load acting perpendicular to the top of the rafter is $2 \times 9.25 \times 19.9 = 368$ lb.

The total load normal to the top of the rafter equals $368 + 396 \times \cos 30° = 711$ lb. The load parallel to the length of the rafter $= 396 \times \sin 30° = 198$ lb (Fig. 135*c*). This latter component is transmitted direct to the purlin, altho a part may be carried up thru the common rafters to the apex of the roof. In this design it will be assumed that the entire component is taken by the purlins. To select the size of rafter, enter the beam table (Art. 31) with a load of $711 \times 1000/1560 = 460$ lb, since the table is based on a fiber stress of 1000 lb. For a span of 9 feet a 1×6 in rafter would be almost strong enough, but for nailing the sheathing a 2×6 in should be used. If the rafter be investigated for bearing at the ends, horizontal shear and deflection, it will be found large enough.

Purlins. Since the load upon the purlins (Fig. 135*f*) is inclined, it is necessary to find the inclination of the neutral axis in order to find the true stresses. The following formulas from Jacoby's "Structural Details" are recommended:

$$\operatorname{Tan} \beta = (d^2/b^2) \tan \alpha \quad I = 1/12\, bd\,[(d \cos \beta)^2 + (b \sin \beta)^2]$$

$$c = 1/2\,(d \cos \beta + b \sin \beta) \qquad S = Mc/I$$

in which α is the angle made by the load with the longer side of the beam and β is the angle between the neutral axis and the shorter side of the beam. d and b are the depth and width of the beam in inches, I is the moment of inertia, c is the distance from the neutral axis to the most remote fiber, M equals the bending moment in inch-lbs and S equals the maximum fiber stress per sq in.

The load normal to the top of a purlin exclusive of its weight, since there are 8 rafters in each length of purlin, $= 711 \times 8 = 5688$ lb, and the load normal to the side of the purlin is $198 \times 8 = 1584$ lb. The weight of the purlin is approximately 12 lb per ft, or 192 lb.

Resolving this weight normal to the top and side of the purlin gives respectively 167 lb and 96 lb to be added to the foregoing loads, or the total load normal to the top = 5890 lb. and that normal to the side = 1680 lb. The total load on the purlin equals 6130 lb, and then $M = 6130 \times 16 \times 12/8 = 147\,000$, the load being assumed as uniformly distributed. Tan $\alpha = 1680/5890$. Assuming the dimensions of the purlin to be 8 by 10 in, and the 10-in side normal to the main rafter, $\beta = 24° - 10$, $I = 625$, $c = 6.19$, $M = 147\,000$ inch-lb and $S = 1450$ lb, which is within allowable unit stress.

The equation for the moment of inertia shows that when β is small, I is small, and as I varies directly as b/d, a purlin in which this ratio is small, as in the case of one 3 by 10 in, is uneconomical. For this reason the placing of the purlins close together and omitting the common rafters is not generally economical.

Allowable Compression. Jacoby's equation for the allowable unit compression S upon a surface which makes an angle θ with the fibers is $S = S_1 \sin^2 \theta + S_2 \cos^2 \theta$, in which S_1 and S_2 are respectively the allowable unit stresses in end bearing and across the grain. For long-leaf pine used in buildings $S_1 = 1820$ lbs, $S_2 = 455$ lbs, and for these stresses the equation may be written $1365\,(1.33 - \cos^2 \theta)$. This is a convenient form for use in designing. As every washer and timber in bearing must be figured for compression, the table in Art. 44 may be used to reduce labor in designing wooden bridges, and it may be applied to buildings by increasing the tabular values 30 per cent.

Truss Members. The maximum stresses are given in Fig. 135*c*. For the TOP CHORD the maximum stress is 36 000 lb compression. The length of the chord per panel is 9 ft 3 in. The allowed stress per sq in for columns under 15 diameters is 1300 lb. The approximate net area hence is 36 000/1300 = 28 sq in. Use a 6 by 6 inch timber in order to allow for the cutting of daps and notches, holes for bolts and tie rods. While theoretically smaller-sized timbers may be used in the upper panels, it will be cheaper to make the main rafter or top chord of one length, and even where a single splice is necessary, due to the length of rafter, the use of the same size of timber thruout the length is economical. For the BOTTOM CHORD the maximum stress is 31 300 lb and the allowed stress per sq in 1560 lb. To allow for the cutting of daps and bolt holes, particularly at the end joint, *a*, use a 6 by 6 in timber.

The STRUT *Bc* has a maximum compressive stress of 7200 lb. Its length is 9 ft 3 in. Since the column length is small, the side-grain compression of the chord by this member will determine its size. For the purpose of selecting the requisite size it will be sufficiently accurate to assume that the end of the timber bears against the side grain of the top chord, the angle between the top chord and *Bc* being 60°. The requisite size is 7200/455 = 16 sq in. Since this member will sag slightly, due to its inclination, use a 4 by 6 in timber.

THE STRUT *Cd* has a maximum compressive stress of 9500 lb. Its length is 12.0 ft. This strut, on account of its length, will be first computed as a column. Enter the column table (Art. 31) with a load of $9500 \times 1000/1820 = 5300$ lb, since this table is based on a stress of 1000 lb per sq in and the allowable end bearing is 1820 lb per sq in. A 4 by 4 in timber is theoretically safe as a column, but on account of the bearing against the side grain of the top chord the timber must be at least 9500/455 = 21 sq in. The nearest commercial size timber is 4 by 6 in, but as the member is not vertical and will sag slightly, use a 6 by 6 in.

THE TIE ROD *Bb*, if there is no ceiling to be supported as in this design, carries very little stress, its function being to prevent any sag in *ac*; a ⅝-inch rod will be used. The TIE ROD *Cc* is strest 3600 lb. To select the proper steel rod, enter table in Art. 34 for recommended stresses in bolts. A ⅝-in rod is too small, therefore use a ¾-in rod which is good for 4830 lb. It should be noted, in this table, that the gross area of a ⅝-in rod is the same as the net area of a ¾-in rod therefore if the ends of a ⅝-in rod are upset it will be strong enough. A ¾-in rod without upset ends will be used because in small trusses where only a few rods are required they are often made in small local shops and the welds are not first class. The tie rod *Dd* may be selected in a similar manner, a 1¼-in rod being used; if the ends are upset a 1⅛-in rod will answer.

JOINT *C*. The washer for the tie rod should be a beveled one (Art. 32). The end of the strut should be cut as shown in the figure so that the short bevel bisects the angle

between the top chord and strut Cd. If so cut this bevel will make the same angle with the fiber of both the top chord and Cd, and the allowable unit stress will be the same for both the top chord and strut. If cut at any other angle the allowable stress will be less for one member and greater for the other and the lesser will govern. The angle made by the surface of the small bevel with the fiber of both chord and Cd equals 50°, and the safe allowable pressure, from table in Art. 44, is 1260 lb per sq in. The pressure on the bevels is found by resolving the stress in the strut, 9500 lb, into two components, one perpendicular to the short bevel and the other perpendicular to the long bevel. This gives a pressure on the short bevel of 4700 lb and on the long bevel of 6600 lb. Therefore the length of the short bevel must be $4700/1260 \times 6 = \frac{5}{8}$ in. Make the bevel $\frac{3}{4}$ in to allow for inaccurate framing. The surface of the long bevel makes an angle with the top chord of 7° 30′ and the safe compression from the table of Art. 44 is about 480 lb per sq in. The length of the bevel is $5\frac{1}{2}$ inches, and the pressure per sq in therefore equals $6600/6 \times 5\frac{1}{2} = 200$ lb, which is safe.

Joint D. The stress in the vertical rod is 14 400 lb, and the washer makes an angle of 30° with the fibers of the chord. The safe bearing from table 1 is 790 lb per sq in. The net area of washers required $= 14\,400/790 = 18$ sq in. A standard cast-iron washer can therefore be used, see Art. 32. The horizontal pressure at the joint is 18 700 lbs, and the angle of the joint surface with the fibers is 60°. From the table the allowable compression is 1470 lb per sq in. The net area of contact, deducting the area cut out by the bolt hole, must be at least $18\,700/1470 = 12.7$ sq in. The actual area is much greater than this.

Joint d. The stress in the vertical rod is 14 400 lb. The washer presses against the side grain of the bottom chord, and therefore its net area must be at least $14\,400/455 = 31.6$ sq in. Use a $6 \times 6 \times \frac{3}{4}$ washer (Art. 32). The bevel surface of the angle block makes an angle of 40° with the fiber of the block. The allowable compression from the table is therefore 1010 lb per sq in, and the stress in Cd being 9500 lb the necessary area is $9500/1010 = 9.4$ sq in. The actual area is 36 sq in. The bearing of the bottom of the block upon the side grain of the bottom chord is evidently sufficient. The dap in which the angle block is seated must be deep enough so that the difference between the horizontal components of the stress in Cd and in C_1d can be transmitted when the wind blows on either side of the truss. The horizontal component equals 2700 lb. Now since both the block and chord bear on end grains, the depth of the dap must be at least $2700/1820 \times 6 = \frac{1}{4}$ in. Use $\frac{1}{2}$-in dap to allow for the inaccurate framing. In this design it is not necessary to compute the actual horizontal shearing stress in the block, due to the pressure of 2700 lbs, as it is evidently very low.

Joint c. The washer was designed similar to those of joints D and d. The short bevel at the end of the timber is on the line bisecting the angle between the lower chord and Bc, see joint C. The angle made by the surface of the short bevel with both the chord and Bc is 75° and the allowable compression for both chord and strut is 1715 lb per sq in. Therefore since the stress Bc is 7200 lb, the length of the short bevel must be $7200/1715 \times 6 = \frac{7}{10}$ inch, but 1 inch will be used to allow for inaccurate framing.

Joint a. The stress in the tie equals 31 300 lb. The allowable unit tension equals 1560 lb per sq in. Therefore the net cross-section of the tie must be at least $31\,300/1560 = 20$ sq in, which requires a net depth of timber of at least $3\frac{1}{3}$ in. Using $3\frac{1}{2}$ in, the length of the heel bevel of the top chord will be $2\frac{1}{2}/\cos 30$, or $2\frac{7}{8}$ in. The toe bevel should be a little less, and in this design will be made $1\frac{1}{2}$ in. The total area of end bearing therefore equals $(2\frac{7}{8} + 1\frac{1}{2})\,6 = 26\frac{1}{4}$ in. The angle made by the bevels with the fiber of the bottom chord equals 60°. Therefore, from table in Art. 43, the allowable compression is 1470 lb per sq in. Therefore the total allowable pressure on the bearing is $1470 \times 26\frac{1}{4} = 38\,600$ lb, which is greater than the thrust of the top chord. In order to take care of the horizontal component of the thrust of the top chord, the distance from the heel to the end of the lower timber must be such as to develop the necessary horizontal shear, or the length must be at least $31\,300/6 \times 195 = 27$ in, as shown in the figure.

In this design the bolt tying the chords together is not figured to transmit stress, but is designed to hold the members together. The bolt at the end of the timber is introduced to increase the resistance against horizontal shear.

The bearing area on the post must be able to take the total reaction of the truss, or 21 600 lb. Since side grain bears on side grain, this area must be at least $21\,600/455$, or 48 sq in. The block should therefore be at least 8 in wide. However, since the truss will de-

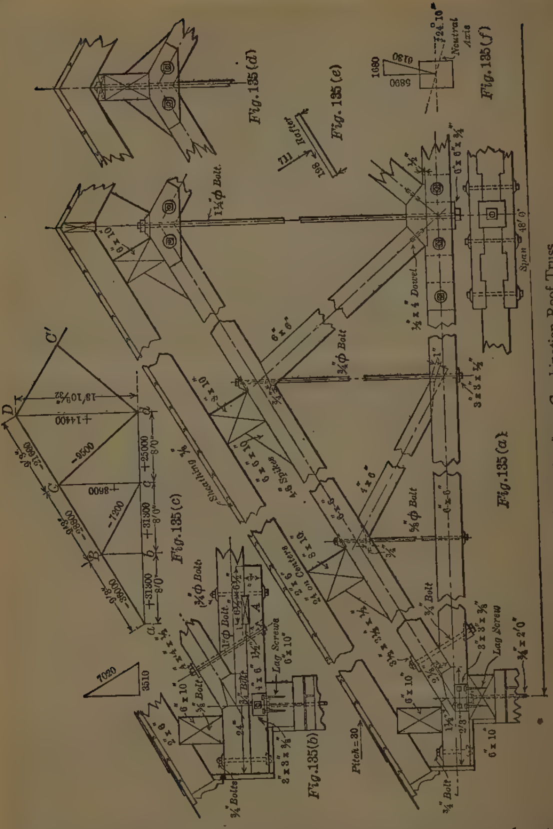


Fig. 135. Design for a Combination Roof Truss

flect under its load, the pressure on the inside of the block will be greater than on the outside. The pressure is not uniform, but the exact difference is unknown. An area of 25% in excess of that theoretically required is recommended, or $48 \times 1.25 = 60$ sq in. Since the chord is 6 inches wide, the block should be at least 10 inches long and a 6×10 in block will be used.

If it is impractical to get a length of 27 in to take care of the horizontal shear, a bolt or bolts may be used to take part of the shear as at Fig. 135*b*. Assume that the length is 24 in, then the shear to be taken by the bolt equals $3 \times 6 \times 195$ or 3510 lb. Resolving this horizontal stress parallel to the bolt and perpendicular to the tie member, as in Fig. 135*b*, gives a stress in the bolt of 7020 lb. Enter table of recommended stresses for bolts (Art. 34), selecting a 1-in bolt, which is a little larger than necessary. The top

washer, which bears on side grain, should have a net area of 7020/455 or 15.5 sq in. Use a $4 \times 4 \times \frac{1}{2}$ in square plate washer.

A bolster will be used under the end of the truss so that it will not be necessary to cut into the bottom chord timber in order to get a proper bearing for the lower washer. This bolster will stiffen the truss and is particularly valuable when the support cannot be placed directly under the intersection of the neutral axes of the top and bottom chord timbers. The bolster will be made of a 4×6 in timber. It must transmit to the lower chord not only the horizontal component of the pull of the bolts, but must also take the horizontal component of the wind pressure.

The design of the bottom washer is similar to those in joints C and D. A key of long-leaf pine will be provided at A to transmit the horizontal pressure, which amounts to $3510 + 4200 = 7710$ lb. The area of bearing of the key in both the bolster and lower chord members, since both the key and adjacent timbers bear on end grain, must be at least $7710/1820 = 4.2$ sq in, and the depth of the key must be $2 \times 4.2/6 = 1.4$ in. A depth of 1½ in will be used. The length of the key should be $7710/195 \times 6 = 6.6$ in, but 6½ in may be used. The distance of the end of the key from the end of the bolster must also be 6½ in.

The key tends to rotate, the moment being equal to $7710 \times \frac{3}{4}$, or 5785 in-lbs, the arm of the horizontal forces acting on the top and lower halves of the key being ¾ in. This moment compresses the top and bottom of the key and the adjacent chord and bolster timbers. The maximum compression $S = 6\,M/bl^2$ in which M is the rotating moment in in-lbs, b is the width of the key and l its length in inches, or $S = 6 \times 5785/(6 \times 6\frac{1}{2} \times 6\frac{1}{2}) = 136$ lb per sq in, which is safe. The rotating moment also causes tension in the adjacent bolt. This bolt acts with a leverage of at least ½ the length of the key, or 3¼ in. The stress in the bolt will not exceed $5785/3\frac{1}{4}$ or 1800 lb. A ¾-inch bolt, altho stronger than theoretically necessary, will be used.

44. Howe Bridge Trusses

General Data. Timber bridges are generally of the Howe type, shown in Fig. 136. All members of this type are of timber except the vertical tension members and the lateral and sway tension members. The depth of the trusses of thru bridges is generally governed by the necessary overhead clearance, which is 14 ft for highways and from 17 to 21 ft for railways. The weights of Howe trusses and of Howe bridges are about ⅛ larger than those of steel bridges of the same span, character of floor and class of loading. The stresses in the Howe truss are similar to those in the Pratt type, except that the diagonals carry compression instead of tension.

Compression on Inclined Joints. The following table, based on the first table of Art. 29 and on Jacoby's formula given in Art. 43, will be found useful in designing the joints of timber bridges.

Allowable Compression in Pounds per Square Inch on Surfaces Inclined to the Fibers

Kind of timber	Inclination θ between surface and fiber									
	0°	10°	20°	30°	40°	50	60°	70°	80°	90°
White oak	500	530	610	720	870	1030	1170	1290	1370	1400
White pine	200	230	310	410	570	730	870	990	1070	1100
Southern long-leaf pine	350	380	470	610	780	970	1130	1270	1370	1400
Douglas fir	200	230	320	450	610	790	950	1080	1170	1200
Short-leaf yellow pine	250	270	350	460	600	750	890	1000	1080	1100
Red pine (Norway pine)	200	220	300	400	530	670	800	900	980	1000
Spruce and eastern fir	200	230	320	450	610	790	950	1080	1170	1200
Hemlock	150	180	270	390	540	710	860	990	1070	1100
Cypress	200	220	300	400	530	670	800	900	980	1000
Cedar	200	230	310	410	570	730	870	990	1070	1100

For roof trusses and buildings these values may be increased 30 per cent, owing to the absence of shocks due to live loads.

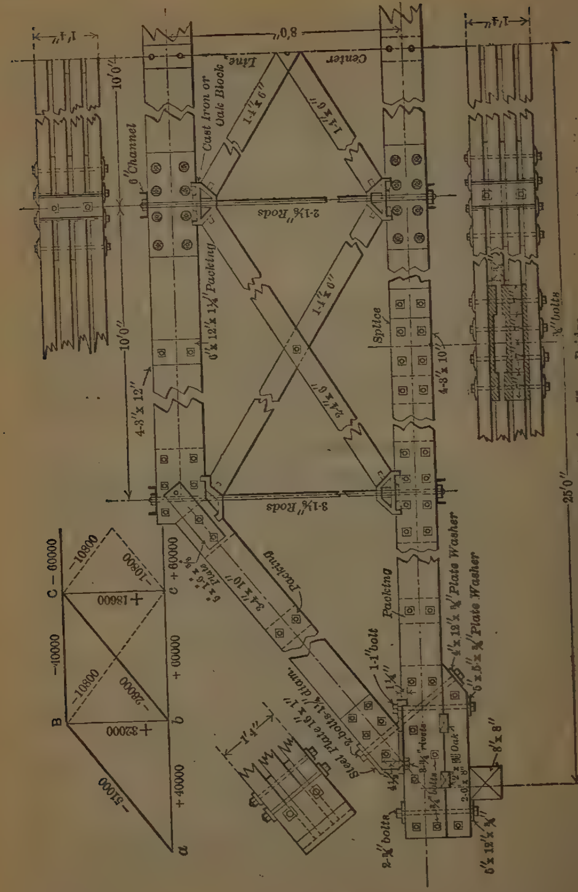


Fig. 136. Design for a Howe Bridge

Chords. The top and bottom chord sections of the trusses are built of from two to four pieces of timber varying from 3 by 8 inches to 8 by 16 inches in section. The greatest available length of these timbers should be used in order to decrease the number of splices. Timbers 65 feet long have been used for chord timbers. Fig. 136 shows a common method of packing the

p and bottom chord timbers. Not more than one timber should be spliced the same panel. Splices should be made at packing blocks. The simplest wer chord joint is the tabled fishplate type, and other styles are shown in t. 34.

Angle Blocks. Hollow or solid cast-iron or oak angle blocks are used to stribute the compression brought by the web members upon the side grain the chords, and channel washers as in Fig. 136 are often used in lieu of ate washers to transmit to the chords the pull of the vertical tie rods. ypical floors for timber bridges are shown in Fig. 121.

Design of a Howe Truss. The truss in Fig. 136 was designed for a ghway bridge having a span of 50 ft and a width of 16 ft. The bridge was signed for a live load of 100 lb per sq ft and a 15-ton road roller. Long-leaf ne was used thruout. The allowable unit stresses are those of the first ble in Art. 29 and those of the table in Art. 44. The sizes of the members ere determined in a similar way to those of the roof truss (Art. 43). The me sizes of timbers are used in all panels of the top chord and all panels of e bottom chord, altho theoretically uneconomical. In using the column bles or applying the column formula to the top chord or other built-up mpression members, the separate pieces should be regarded as detached lumns. When the separate pieces are well packed and bolted, they may t together as a single column, but on account of the shrinkage of the lumber is not safe to depend upon such action. That chord which supports e floor beams, unless these are placed at panel points, should be computed r both flexure and direct stress. In this design, the floor beams are placed panel points only.

Joint *C*. The stress on the vertical tie rod is 18 600 lb. The allowable bearing on the le grain of the top chord from the table of Art. 44 is 350 lb per sq in. The necessary t area of washer is 18 600/350 = 54 sq in. Allowing for the bolt holes, the necessary oss area is 57 sq in. A 6-in channel the full width of the top chord, or 16 inches long, used, as this size washer is needed at joint *B* and the same size will be used thruout.

A cast-iron angle block is shown in the figure, but in this design oak angle blocks might used. When cast-iron blocks are used, their webs should be designed to transmit e thrust of the main brace and counterbrace. Counterbraces are shown by dotted es in Fig. 136*b*. Each lug should be designed so as to be able to transmit the entire rizontal component of the main brace to the top chord.

The vertical component of the stress in the brace is 18 600 lb, and therefore the area of e base of the angle block should be at least 18 600/350 = 54 sq in. In order to provide aring for the brace and counterbrace the area of the base will be made much larger. ne horizontal component of the stress in the brace being 21 000 lb, the depth of the gs, assuming that the packing pieces take their part of the horizontal component of e brace, must be at least 21 000/16 × 1400 = 1 inch. Ignoring the packing pieces, the pth should be 1¼ in, which depth will be used.

DOMES OF MASONRY

45. General Data for Domes

Definitions. A dome (Fig. 136*a*) is a spherical or spheroidal vault, a solid of evolution with vertical axis. The section of the inner surface of the dome soffit) may be semi-circular (Fig. 137), pointed (Fig. 138) or segmental (Fig. 39); in the last case the dome is also termed a "cupola." Domes may be losed, as that shown in Fig. 142, or open on the summit (Fig. 136*a*), the opening eing termed the "eye" of the dome. The rim of the eye frequently supports structure called a lantern (Fig. 140). The section of the outer surface (ex-

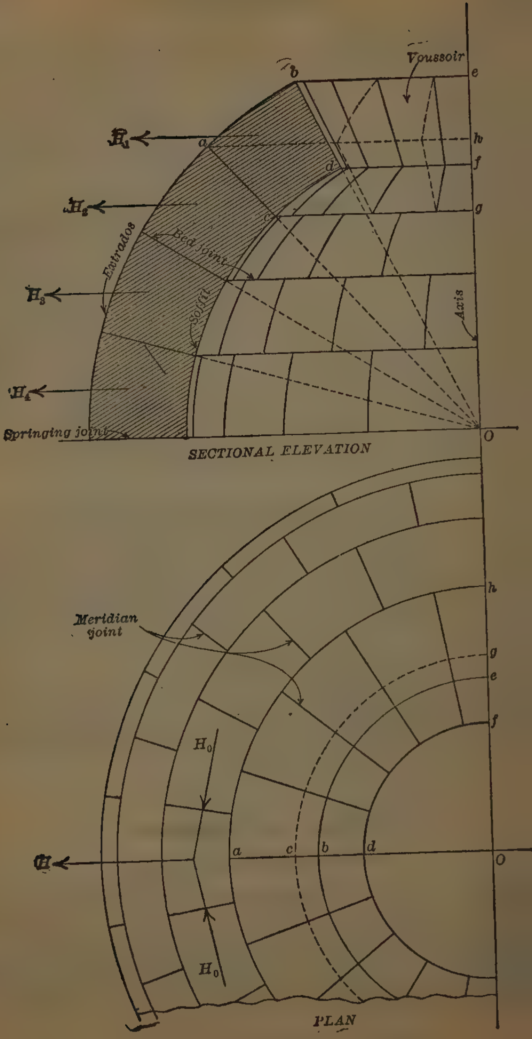


Fig. 136a.—Plan and Elevation of a Dome

trados) is usually similar to that of the soffit, but may have a different curve. The solid bounded by the soffit and extrados is the shell of the dome. The bed

Fig. 137.—Hemispherical Dome

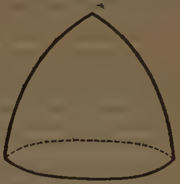


Fig. 138.—Pointed Dome

joints are conical. Annular parts of the shell between bed joints (true or imaginary) are called crowns. In Fig. 136a the points a, b, c, d, e, f, g, h, inclose one-

Fig. 139.—Segmental Dome (Cupola)

Fig. 140.—Open Dome with Lantern

fourth of a crown. Voussoirs are parts of the crowns included between meridian joints, that is, vertical planes passing through the axis of the dome.

The plan of the dome is always a circle; it may be supported on a circular wall (drum) or carried over a square of polygonal area, resting upon walls or arches whose plan circumscribes the plan of the dome. The vaulted areas between the arches or walls and the base of the dome are called "pendentives" (Fig. 141). The joint between the shell of the dome and the supporting walls is the springing joint.

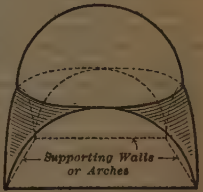


Fig. 141.—Hemispherical Dome and Pendentives

Data and Precedents. The dome is a remarkably stable form of structure; it can be shown analytically to be stable with a uniform thickness of 23/1000 of its diameter, which is less than 1/3 of the necessary thickness of a semi-circular masonry arch of uniform thickness carrying only its own weight. If the thickness of the shell is tapered from the springing joint toward the summit, it need only have a volume of 9/16 of the thinnest uniform dome of the same span. If enough steel bands are provided at the springing joint, the upper portion of the shell extending down 26° from the summit can be made abnormally thin and yet be stable. The dome of the Cathedral of St. John The Divine, in New York City, built by Gustavino, is an excellent example of this. It has a diameter of 110 ft, and is built of tile, having a thickness of 4 in for 29°

from the summit or $^3/_{1000}$ of the span. For the next 19° it is 6 in and from there to the springing line, 12 in thick.

According to Rondelet the following thicknesses are usually given to domes built of brick:

For spans up to 13 ft, a uniform thicknses of 4 in.
For spans from 13 to 20 ft, a uniform thickness of 8 in.
For spans from 20 to 25 ft, 12 in at the spring, 8 in at the summit.
For spans from 25 to 35 ft, 16 in at springing, 8 in at the summit.

Pointed domes are generally stronger than hemispheres; a dome generated by the revolution of an equilateral arc (60°) requires, to be self-supporting, a thickness of only ⅝ of that of a hemispherical dome of the same span.

The dome of the Pantheon in Rome, built about A.D. 112 is the largest existing hemispherical dome of masonry, its diameter being 142 ft. It rests on a circular wall nearly 20 ft thick which, however, is interrupted by 8 large and 8 small niches. The dome is open on the top, the "eye" being about 30 ft in diameter. The lower courses of the dome, built of Roman tile, were all laid horizontal up to about 40° from the springing line. Above this the construction is not known but is said to consist of tile arches and ribs of concrete. The outside of the dome shows a strong masonry backing beginning at the springing line and extending about half way up.

The dome of Santa Sofia in Constantinople, built by Justinian A.D. 532–537 from the designs of Anthemius of Tralles and Isidorus of Miletus, has a diameter of 104 ft and is carried on pendentives over a square area. The shell consists of 40 stone ribs and the space between same (about 6 ft at the springing line) is filled with brick masonry. The dome is closed at the summit and has 40 windows between the ribs.

The dome of St. Peter's Cathedral in Rome, built by Michelangelo, has an inside diameter of 140 ft, and is built of brick. The lower quarter of the shell is of solid construction 9 ft thick. The upper ¾ of the height consists of a double shell. The outer shell, which carries a lantern, is raised higher than the inner one and the two shells are connected by 16 stone ribs. St. Paul's Cathedral in London, designed by Sir Christopher Wren, has three shells; the innermost is hemispherical, open at the summit, the middle one is conical and carries the lantern, the outer dome is framed in timber and covered with lead.

46. Conditions of Stability

The dome is usually built without centering, by constructing the crowns in succession, beginning at the springing joint. Every complete crown is self-supporting; consequently open domes may be stable. This stability is the result of tangential pressures acting normally on the meridian joints (H_0 in Fig. 136*a*). Pressures of this character do not exist in simple arches. In Fig. 142, two meridian planes are shown in plan making a small angle ϕ with each other. They cut from the dome two lunes having contact only in a line at O, or, if the dome were open at the summit, the lunes would have no common point at all. The difference between the stability of the arch and the dome is manifest. The arch for its stability depends upon the balancing of the horizontal thrust of each half on either side of the crown or summit. In case of the dome the contact of symmetrically opposite sections (lunes) is in a line without area and incapable, therefore, of transmitting horizontal thrust which would counteract the tendency of the opposing sections to fall into the space below. Therefore the stability of the lunes must be the result of tangential forces or forces acting upon their sides (that is, upon the meridian joints), and normal to them.

These forces, called crown thrusts, are horizontal and act upon both sides of each voussoir (H_0 in Fig. 136*a*). They result in the forces H or H_1, H_2, etc., which act radially outward in a horizontal direction. The forces H are termed resultant crown thrusts. Consider first the uppermost crown of a dome. To insure stability, in other words to prevent the voussoir from rotating or sliding upon its bed joint, the magnitude of H_1 must be such that if it is combined with

the weight of the voussoir W_1, the resultant force P_1, called the meridian thrust shall cut the bed joint ac and its direction shall not be inclined to the normal to the bed joint by more than the angle of friction of masonry upon masonry, say 30°. The meridian thrust acting upon the bed joint ac of the first voussoir will be transmitted to the crowns below and at each voussoir new forces H_2, H_3, etc., must exist to maintain stability. In the case of domes having a high rise there will be found a bed joint called the joint of rupture below which no additional forces H occur, the above requirements of stability (as to the resultant) being fulfilled without the action of such additional horizontal forces. From this joint downward, the dome can be treated as a simple arch.

47. The Line of Pressure

Crown Thrust. In the analysis of stresses in domes of masonry it is assumed that the least horizontal force, which is sufficient to keep the voussoirs from rotating and sliding will be the resultant crown thrust H (Fig. 136a) if its combination with the weight of the voussoir, the meridian thrust, does not cut the bed joint so close to the edge as to crush the masonry, or due to the elasticity of the masonry, cause the joint to open on the opposite side.

The Meridian Thrusts, P_1, P_2, etc., in Fig. 142, result from combining weight W of the voussoir and the resultant crown thrust H. In order to make the latter a minimum, H must be applied as near to the extrados as possible and P must cut the bed joint as near the soffit as possible, consistent with the foregoing requirements. This may readily be proven graphically by assuming any other point of application of these forces. Theoretically in order that no joint may open, the resultant force upon any joint, shall not lie outside the kern or the middle-third (Art. 6 and 7). In the case of domes, however, investigation of domes which have been standing for centuries, shows that the limits of the middle half of the meridian and bed joints can be safely used as points of application for both crown thrust and meridian thrust. The use of these limits furnish smaller dimensions than the use of the middle-third points and is here recommended for practical design. Therefore the point of application of the minimum force H should be taken at the upper middle half limit of the shell and the meridian thrust should be applied at the lower middle half point of the bed joint providing that the condition of stability against sliding permits this location of the meridian thrust. If with the help of the so-found forces H a line of pressure can be drawn that will be within the middle half of the shell, the dome will be stable, providing that the unit stresses on both the meridian and the bed joints are not excessive.

48. Graphic Analysis

In Fig. 142 a plan of a lune and a meridian half section of a closed dome are shown. The soffit of the dome is segmental, the radius of the soffit being 8 ft and its rise 5 ft. The central angle ϕ of the lune is 15°. The shell of the dome is 1 ft 3 in thick throughout. The section is divided by full radial lines into voussoirs 1 ft 9 in long on the center line. These radial lines form the bed or conical joints. The middle half limits of the shell are shown by the dotted curves and their intersections with the bed joints give the middle half limits of these joints. The weight of each voussoir acts at its center of gravity, which may be taken, with sufficient accuracy, as coincident with the geometrical center of the voussoir, except at the topmost wedge-shaped one in which the center of gravity is taken at the lower third. The weights of the voussoirs, including the superimposed loads if any may be determined graphically or analytically. In the

example given (Fig. 142), the weights of the voussoirs, W_1, W_2, W_3, etc., are proportional to the lengths of the arcs W_1, W_2, etc., of the plan which pass

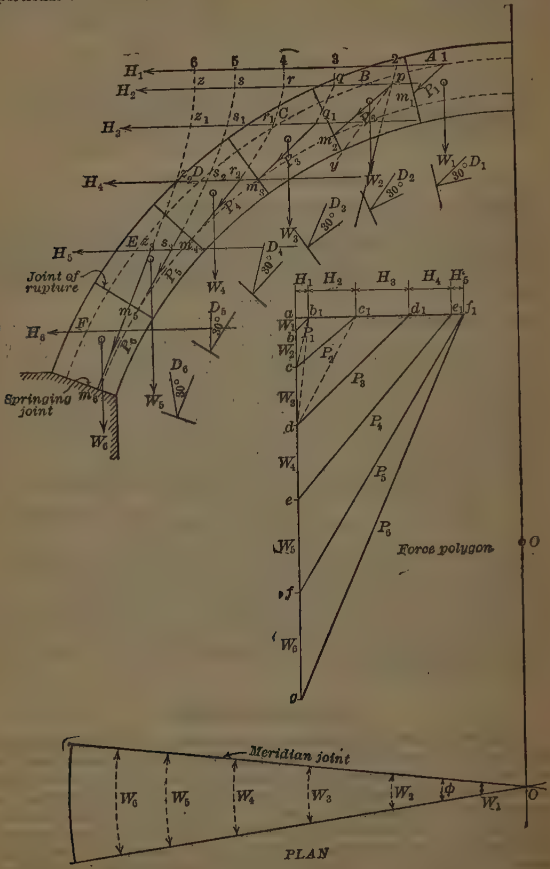


FIG. 142. Graphic Analysis of a Dome

through the geometrical centers of the voussoirs. The length of these arcs are used directly in the force polygon to represent the weights of the voussoirs. The true magnitude of all the forces shown on the force polygon will, therefore,

be found by measuring their lengths to the same scale as that of the plan of the dome, and multiplying these scaled lengths by the length of the voussoir × the thickness × the weight per unit volume of the masonry W_1, W_2, to W_6 shows on the section the lines of action of the vertical force.

The lines of action of the horizontal resultant crown thrusts H_1, H_2, etc., are assumed to be and are shown as applied at points A, B, C, D, E, and F, which points are located at the upper limit of the middle half of the shell and on the same radii as the centers of gravity of the corresponding voussoirs. The lines of action of the combined weights $W_1 + W_2$, $W_1 + W_2 + W_3$, etc., may be determined graphically or analytically and are shown in Fig. 142 as passing through points 2 to 6, inclusive. For example, the line of action of combined loads $W_1 + W_2 + W_3 + W_4$ passes through point 4. At D_1, D_2, etc., the limiting directions of the meridian thrusts (which must not be inclined to the normal of any bed joint by more than 30°) are shown for every conical joint.

Point 1 is the intersection of the weight W_1 of the first voussoir and of the resultant crown thrust H_1. The resultant of these two forces must pass through this point and its direction, if the dome be stable, must be such that it will intersect the first bed joint between the middle half points and it must not be steeper than the limiting direction D_1. Draw $1m_1$ (in section) and also bb_1 (force polygon) parallel to D_1; bb_1 is the probable magnitude of the meridian thrust P_1, because if its direction were steeper it would not fulfill the condition necessary to prevent sliding; if it were less inclined, it would require a greater resultant crown thrust than the minimum. Therefore, m_1 is a point of the probable line of pressure and ab_1 (force polygon) is the magnitude of the resultant crown thrust H_1 acting on the first voussoir.

The combined weight of voussoirs 1 and 2 passes through point 2. If no additional crown thrust were present the meridian thrust on the second bed joint would be obtained by combining H_1 and $W_1 + W_2$, in other words by drawing line $2x$ through 2 parallel to b_1c (force polygon). This line $2x$ falls outside the second bed joint and is too steep for stability against sliding. Therefore, at point p (the intersection of $2x$ and H_2) an additional horizontal force acts if the dome is stable. If through point p, a line py, parallel to the limiting direction D_2, is drawn, this line is also seen to fall outside the joint. The minimum resultant crown thrust H_2 acting at the second voussoir and consistent with the requirements of stability will therefore be such as to make the meridian thrust pass through the lower middle half limit m_2 of the second bed joint.

Draw pm_2 in which m_2 is a second point of the line of pressure and draw cc_1 (force polygon) parallel to pm_2. Then cc_1 is the magnitude of the meridian thrust P_2 and b_1c_1 the magnitude of the resultant crown thrust H_2.

A similar construction is carried out for the other voussoirs to determine meridian thrusts P_3, P_4, P_5, and the resultant crown thrusts H_3, H_4, H_5. At voussoir 6 it is found that the sum of the horizontal forces H_1 to H_5 is sufficient to insure the stability of the voussoir and therefore the value of H_6 reduces to zero. Bed joint 5 then is the joint of rupture of this dome.

The magnitude of the crown thrust H_0 (the normal force acting on the meridian joints Fig. 136a) can be calculated by the formula: $H_0 = \frac{1}{2}H/\sin\phi/2$ or if ϕ is small $H_0 = H/\phi$, ϕ being measured in radians and H being the value of H_1, H_2, etc., for the voussoir in question. After the magnitude of both the crown thrusts and meridian thrusts has been determined, the maximum unit stresses upon the bed joints and the meridian joints can be obtained by the methods given in Art. 6 and Art. 7.

It is sometimes necessary, in order to insure the stability of a dome or its supporting walls, to eliminate the transmission of the resultant crown thrust either at the joint of rupture or at the springing joint. This may be done by hoops of iron or steel. The magnitude of the tension for which these hoops must be figured can be determined by the above equation, where, in this case, H_0 denotes the tension in the band (hoop tension) and H denotes the sum of the values H_1, H_2, etc., at and above the joint in question.

Numerical Example. How large is the crown thrust H_0 acting on the meridian joint of voussoir 3 (Fig. 142) and what is the maximum compressive stress on this joint? By

scaling (force polygon) H_8 is found to be 1.0 ft, the length of the voussoir is 1.75 ft its thickness is 1.25 ft, and the weight of the masonry is 165 lb per cu ft. Then

$$H_8 = 1 \times 1.75 \times 1.25 \times 165 = 360 \text{ lb}$$

ϕ being 15°, $H_0 = 360/0.262 = 1385$ lb. By Art. 7,

$$S_{max} = 2H_0/3rb$$

where $r = \frac{1}{4} \times 1.25$ and $b = 1.75$ ft for the case here discussed.

$$S_{max} = 8 \times 1385/3 \times 1.25 \times 1.75 = 1740 \text{ lb per sq ft.}$$

What is the required section of iron bands necessary to take the hoop tension at the springing joint so as to make the thrust at the springing vertical? $\Sigma H = 3.15$ (force polygon) $\times 1.75 \times 1.25 \times 165 = 1135$ lb. $H_0 = 1135/0.262 = 4335$ lb. If the allowable unit stress for iron is 8000 lb per sq in, the iron section required $= 4335/8000 = 0.54$ sq in.

SECTION 8

STEEL STRUCTURES

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ROOFS AND BUILDINGS †

GENERAL DATA FOR BRIDGES

BEAM AND GIRDER BRIDGES

SIMPLE TRUSS BRIDGES

MOVABLE AND CANTILEVER BRIDGES

ARCH AND SUSPENSION BRIDGES

MISCELLANEOUS STRUCTURES

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ROOFS AND BUILDINGS

1. Data for Roof Trusses

The constituent parts of a roof are the COVERING (including covering proper and any sheathing or slab that is used), the PURLINS and the TRUSSES.

Roof Coverings are of many kinds, the most familiar being wood, slate, tin, sheet metal, tile, concrete, gypsum and the various composition roofings. It is highly important that the choice of covering be adapted to the slope of the roof or vice versa. The lack of this adaptation is a frequent cause of leaky roofs.

The **Slope** of a roof is the natural tangent of the inclination with the horizontal and is best expressed in inches of rise to one horizontal foot. The preferable slope for tar and gravel on sheathing is from ¾ to 1¼ in per ft; reinforced cement tile of the "Bonanza" and similar types, 5 in per ft; corrugated sheet metal, 6 in per ft; slate, 7 in per ft. The kind of waterproofing used on concrete and gypsum roofs will determine the slope.

Purlins are either wooden or steel beams, extending horizontally from truss to truss and carry the roof loads to the trusses. For long spans it is sometimes economical to space trusses far apart and to use deep purlins supporting intermediate beams called JACK RAFTERS.

Trusses are used to support roofs covering openings from 20 to 200 ft, while for spans exceeding 200 ft arches are generally used. A TRUSS is an assemblage of bars, or members, forming a structure to carry transverse loads, and which under vertical loads has vertical reactions; the bars being so joined together at their ends that they bear only direct tension or compression when the external loads are applied at the joints. A RIVETED TRUSS (Fig. 6) is one in which the members are connected together at the joints by being riveted to connecting steel plates, and a pin truss or PIN-CONNECTED TRUSS has the members joined together at their ends by means of one steel cylinder or pin at each joint. Purlins usually rest on the top surface of the upper chord, which is the upper bar of a truss, and should be placed directly over the joints whenever possible. In some roofs without jack rafters the purlins must be spaced so close together that some of them rest on the upper chord between the joints, thus causing the upper chord to act as a beam in addition to its direct stress.

A Panel of a roof truss is the space between two consecutive joints of the top chord, and a panel length is the distance from one of these joints to the next. A BAY is the space between two adjacent trusses, and the bay length is the distance from center to center of trusses.

Forms of Trusses vary to suit the span length, outline of roof and clear head-room required underneath. The Fink and Warren trusses shown in Figs. 1 and 2 are suitable for spans up to about 120 ft. When the short web member of a Fink truss is replaced by two members, the Fink truss is converted into a fan truss as shown at (*a*) and (*b*) Fig. 2.

Loads on Roof Trusses are, dead, snow, wind; also ceilings, a crowd of people on a gallery or floor, or machinery suspended from the truss. Dead loads consist of weight of roof covering, sheathing, rafters, purlins, trusses and suspended ceilings.

Weights of roofing per square foot of roof surface.

1. Coverings: wood shingles, 2 to 3 lb; tin sheets or shingles, 1 lb; corrugated steel, 2 to 3 lb; slates, 7 to 10 lb; tiles, 8 to 25 lb; felt and gravel, 8 to 10 lb; skylight glass, 3/16 to ½ in, including iron frames, 4 to 10 lb.

2. Sheathing: spruce, white pine or hemlock boards, 1 in thick, 3 lb; hard pine, 1 in thick, 4 lb.

3. Rafters: wood, 2 by 4 to 2 by 8 in spaced 16 to 24 in on centers, 1.5 to 5 lb.

4. Purlins: wood, 1 to 3 lb; steel, 1½ to 5 lb.

5. Ceilings: plastered, 10 lb per square foot of ceiling area.
6. Cinder concrete 7½ lb per in of thickness; stone concrete 12 lb per in of thickness.

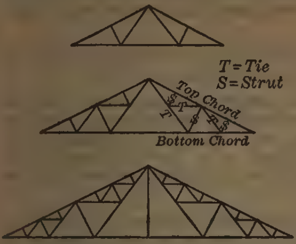


Fig. 1. Fink Roof Trusses

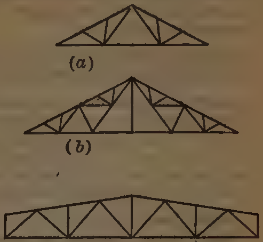


Fig. 2. Fan and Warren Trusses

Weights of Fink Trusses in Pounds

Slope of Top Chord, 6 In on 1 Ft

Span c to c of Bearings, ft	Load in lb per linear ft of Top Chord, Uniformly Distributed					
	500	600	700	800	900	1000
25	610	620	650	730	740	820
30	760	860	880	960	1030	1140
35	1050	1100	1250	1440	1500	1650
40	1250	1400	1620	1670	1830	1920
45	1500	1740	1920	1970	2220	2320
50	1840	2100	2170	2400	2650	3000
55	2100	2340	2420	2850	3050	3340
60	2360	2620	2920	3200	3600	3800
65	2700	3000	3500	3650	4140	4200
70	3100	3450	3930	4260	4540	4900
75	3460	4100	4560	4800	5200	5560
80	3920	4430	4800	5500	5960	6350

Weights of Warren Trusses in Pounds

Slope of Top Chord, ¾ in on 1 ft for 25 to 35 ft-spans; 1 in on 1 ft for 40 to 55-ft spans; 1¼ in on 1 ft for 60 to 80-ft spans

Span c to c of Bearings, ft	Load in lb per linear ft of Top Chord, Uniformly Distributed					
	500	600	700	800	900	1000
25	680	700	730	840	900	980
30	840	1000	1020	1150	1200	1350
35	1100	1270	1360	1530	1640	1870
40	1230	1420	1620	1760	1940	2100
45	1600	1800	2080	2180	2420	2640
50	1950	2230	2400	2660	2920	3280
55	2230	2680	2730	3180	3600	3700
60	2380	2720	3270	3360	3840	3900
65	2920	3300	3620	4170	4220	4770
70	3230	3920	4030	4620	4980	5350
75	3770	4360	5000	5280	5730	5800
80	4200	4940	5320	6060	6120	6850

Weights of Steel Trusses are often determined from empirical formulas, but such formulas are not trustworthy as they neglect many of the variable factors entering into the weights. The preceding tables are for the two most common forms of trusses:

The weights of ordinary Fink trusses with slopes 5 in and 4 in on 1 ft are from 5 to 15% more than those with 6 in on 1 ft given in the table.

Snow Loads on roofs vary with the latitude and slope of roof. The following values for weight of snow per horizontal square foot are commonly specified: New England and Michigan, 30 lb; New York City and Chicago, 20 lb; Baltimore, Cincinnati and St. Louis, 10 lb. Snow loads need not be used on roof surfaces having an inclination of 45 degrees or more from the horizontal, if there are no snow guards.

Wind Loads for roofs vary with the locality and slope of the roof and should be taken as acting horizontally at 30 lb per sq ft on vertical surfaces of the most exposed structures and 15 to 25 lb per sq ft on vertical surfaces of less exposed structures. On inclined surfaces only the normal component of the wind pressure need be considered, and this varies with the inclination of the roof.

Duchemin formula for wind pressure on inclined surfaces:

$$N = P\frac{2 \sin \theta}{1 + \sin^2 \theta}$$

in which N = normal pressure in pounds per square foot;
P = horizontal pressure on a vertical plane (taken at 20 lb per sq ft);
θ = degrees of inclination to the horizontal.

θ	N	Slope	θ	N
5°	3.46	1 in on 1 ft	4° 45′ 49″	3.30
10°	6.76	2 in on 1 ft	9° 27′ 45″	6.39
15°	9.63	3 in on 1 ft	14° 2′ 10″	9.14
20°	12.25	4 in on 1 ft	18° 26′ 6″	11.50
25°	14.35	5 in on 1 ft	22° 37′ 12″	13.42
30°	16.00	6 in on 1 ft	26° 33′ 54″	14.88
35°	17.28	7 in on 1 ft	30° 15′ 24″	16.06
40°	18.20	8 in on 1 ft	33° 41′ 25″	16.95
45°	18.88			
to 90°	20.00			

For a pressure other than 20 lb per sq ft the above values are to be changed proportionally.

The Total Load for an ordinary roof truss may be either

Dead + Wind + ½ Snow

or Dead + ½ Wind + Snow

It is improbable that both the maximum snow and wind loads will be upon the roof at the same time. Sufficient accuracy is obtained in designing roofs up to 100-ft span by assuming a total load per sq ft of exposed surface large enough to include the dead, wind and snow loads. This combined load may be considered applied vertically. Unless governed by a local building code, for spans

up to and including 90 ft and in climates corresponding to that of New York, the total minimum uniform load in lb per sq ft of roof surface for different kinds of covering may be taken as follows:

Corrugated metal on steel purlins	40
Gravel or composition on wood sheathing	50
Slate on boards	50
Tile on steel purlins	55
Gravel or composition on cinder concrete	65
on stone concrete	75
Slate or tile on cinder concrete	70
Fireproof buildings of the mill building class when slope is less than 2 in per ft	90

For roof spans over 90 ft the above cited loads should be increased 1% for every 2 feet increase of span. For roofs in climates where snow is excessive 5 to 10 lb should be added, and in climates where there is not liable to be snow 10 lb may be deducted. Additions should be made for any special loads such as shafting, ceilings or trolleys carried by the bottom chord, and tanks, elevator supports or similar loads above the roof.

Stresses are oftentimes found more quickly by analytic than by graphic methods. For roof trusses the simplest analytic solution is by the method of joints or method of moments. In applying the METHOD OF JOINTS a section is passed around a joint in such a manner as to cut all the bars meeting at that joint and then applying the relation from statics that the algebraic sum of all the vertical components meeting at the joint must = 0, or that the algebraic sum of the horizontal components meeting at the joint must = 0. Thus, by passing a section around joint A, Fig. 3, it is seen at once that the vertical component of $P_1 = 3\frac{1}{2}W$ and therefore the stress in $P_1 = 3\frac{1}{2}W \sec\theta$, where θ is the angle the bar P_1 makes with the vertical. Similarly, the horizontal component of P_8, that is, stress in $P_8 =$ horizontal component of $P_1 = 3\frac{1}{2}W \tan\theta$. The METHOD OF MOMENTS is that method of computing stresses in which a section is first passed thru the truss, cutting in general not more than three members, one of which shall contain the stress desired; and then the equation of moments about the intersection of the other two bars cut is written and solved for the desired stress. Thus, in Fig. 3, to compute stress P_{10} by method of moments, pass a vertical section thru P_4, P_{14}, P_{10} and consider the left side of this section. The equation of moments about B, the origin of moments for P_{10}, is $(3\frac{1}{2}W)\,l/2 - W6\,l/8 - P_{10}\,h = 0$. In this equation P_{10} is assumed to be in tension, and since in the solution of the equation both sides are positiv the bar P_{10} is in tension and its value $= Wl/h$. The section passed may be straight or curved, vertical or inclined. In Fig. 3 the dotted section for P_{12} is passed in such a way as to cut 6 bars, but by taking the origin of moments at B and writing the equation of moments of all forces above the section there results $-Wl/8 + P_{12}\,h/2 = 0$, whence $P_{12} = Wl/4\,h$.

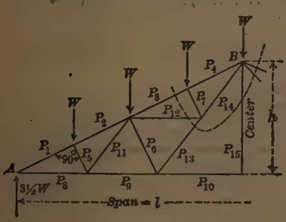


Fig. 3

By using the method of moments or method of joints the stresses in the Fink truss of Fig. 3 have been computed and the accompanying table prepared for a span of l, a height of h and a vertical load W at each upper joint. The load W is called a panel or apex load.

Coefficients for Stresses in Fink Truss. Fig. 3

Bar	Kind of Stress	$l/h = 3$	$l/h = 3.46$ 30°	$l/h = 4$	$l/h = 5$	$l/h=6$
P_1	Comp.	6.31	7.00	7.83	9.42	11.07
P_2	"	5.75	6.50	7.38	9.05	10.75
P_3	"	5.20	6.00	6.93	8.68	10.43
P_4	"	4.65	5.50	6.48	8.31	10.12
P_5	"	0.83	0.87	0.89	0.93	0.95
P_6	"	1.66	1.73	1.79	1.86	1.90
P_7	"	0.83	0.87	0.89	0.93	0.95
P_8	Tension	5.25	6.06	7.00	8.75	10.50
P_9	"	4.50	5.19	6.00	7.50	9.00
P_{10}	"	3.00	3.46	4.00	5.00	6.00
P_{11}	"	0.75	0.87	1.00	1.25	1.50
P_{12}	"	0.75	0.87	1.00	1.25	1.50
P_{13}	"	1.50	1.73	2.00	2.50	3.00
P_{14}	"	2.25	2.60	3.00	3.75	4.50
P_{15}	"	0.00	0.00	0.00	0.00	0.00

The upper chord panel loads are equal. To obtain stress in pounds in any bar of the truss, multiply the coefficient by the panel load in pounds.

2. Design of Roof Trusses

Working Stresses. For designing the structural steel of roof trusses and buildings (when not governed by a local building code) the following unit or working stresses in lb per sq in are recommended:

Tension, net section, rolled steel	16 000
Direct compression, rolled steel and steel castings	16 000
Bending; on extreme fibers of rolled shapes, built sections, girders, and steel castings	16 000
Bending, on extreme fibers of pins	24 000
Shear on shop rivets and pins	12 000
Shear on bolts and field rivets	10 000
Shear, average, on webs of plate girders and rolled beams, gross section	10 000
Bearing pressure on shop rivets and pins	24 000
Bearing on bolts and field rivets	20 000
Axial compression on gross section of columns and struts	16 000–70 l/r
With a maximum of	13 000

Where l = effective length of member in inches;
r = least radius of gyration of section in inches.

For combined stresses due to wind and other loads the above-mentioned unit stresses may be increased 50%, provided the section thus obtained is not less than that required if wind forces be neglected.

The effective length of main compression members should not exceed 125 times their least radius of gyration, and those for wind and lateral bracing 160 times their least radius of gyration.

Spacing of Trusses. Theoretically, the economic spacing of roof trusses is about one-quarter the span, but in practice the spacing may be governed by conditions peculiar to the structure under consideration. If the covering or sheathing rests on purlins a spacing of about 16 ft is commonly used for spans up to 65 ft; beyond that the spacing may be one-quarter of the span. Where plank sheathing rests directly on the trusses the spacing may be 8 to 10 ft for 2-in and 10 to 12 ft for 3-in tongued and grooved plank. If spans are of unusual

length or roof loads are very heavy it may be found economical to use jack rafters resting on latticed or beam girders framed into the roof trusses. Sub-purlins can rest on top of the jack rafters or wood sheathing can extend from rafter to rafter.

Proportioning Purlins. Steel purlins are usually made of L's, Z bars, [̲'s or I beams, altho for long spans they are made of truss form. Where the purlin is placed so as to deflect in the plane of the resultant load acting upon it, as when an I beam is set with its web vertical to carry a vertical load, the size of the purlin is computed by the formula, $M/S = I/c$, where M is the maximum bending moment on the purlin in inch-pounds; S, the allowable unit stress in pounds per square inch; I, the moment of inertia about axis perpendicular to web; c, half depth of beam in inches. Having computed M, S is assumed, the value of I/c (called the section modulus) is found by dividing M by S, and the size of a beam having a section modulus equal to or slightly greater than this is chosen from a table giving properties of structural shapes.

If the purlin is placed so that the deflection is not in the plane of the resultant load carried by it, as when an I beam supports a vertical load and rests on an inclined chord of a truss, hence having its web inclined, a trial size of the beam is taken and the maximum fiber stress in the outermost fiber is computed by the formula

$$S = \frac{M_1 c_1}{I_1} + \frac{M_2 c_2}{I_2}$$

where S = maximum fiber unit stress, M_1 = bending moment due to that component of load which is normal to plane of roof, I_1 = moment of inertia of section of purlin about a

Section Moduli of Z bars and Angles at Right Angles to Roofs; Loading Vertical

Purlin	Slope of Roof in Inches on 1 Foot								
	0	1	2	3	4	5	6	7	8
3 × ¼ Z	0.66	0.72	0.79	0.87	0.98	1.11	1.29	1.52	1.83
3 × 5/16 Z	0.83	0.90	0.98	1.08	1.21	1.37	1.59	1.87	2.23
4 × ¼ Z	1.15	1.26	1.39	1.57	1.80	2.11	2.53	3.12	3.52
4 × 5/16 Z	1.41	1.55	1.71	1.93	2.21	2.59	3.09	3.82	4.49
5 × 5/16 Z	2.04	2.28	2.58	2.98	3.56	4.36	5.61	5.05	4.19
5 × ⅜ Z	2.41	2.70	3.05	3.53	4.21	5.14	6.60	6.17	5.12
6 × ⅜ Z	3.30	3.74	4.31	5.11	6.26	7.99	7.98	6.35	5.38
6 × 7/16 Z	3.82	4.30	5.00	5.89	7.21	9.19	9.48	7.55	6.39
2 × 2 × ¼ L	0.18	0.19	0.20	0.21	0.22	0.24	0.26	0.29	0.31
2 × 2 × ⅜ L	0.27	0.28	0.29	0.31	0.33	0.35	0.37	0.40	0.42
2½ × 2 × ¼ L	0.30	0.31	0.33	0.35	0.38	0.41	0.44	0.46	0.49
2½ × 2 × ⅜ L	0.40	0.42	0.45	0.48	0.52	0.57	0.61	0.65	0.69
2½ × 2½ × ¼ L	0.31	0.32	0.33	0.35	0.37	0.39	0.42	0.45	0.48
2½ × 2½ × ⅜ L	0.42	0.44	0.46	0.49	0.52	0.55	0.60	0.64	0.68
3 × 2½ × ¼ L	0.44	0.46	0.49	0.52	0.56	0.60	0.65	0.69	0.74
3 × 2½ × ⅜ L	0.64	0.67	0.71	0.76	0.82	0.89	0.95	1.00	1.06
3½ × 2½ × ¼ L	0.56	0.59	0.64	0.70	0.76	0.83	0.89	0.96	0.84
3½ × 2½ × ⅜ L	0.84	0.89	0.96	1.04	1.14	1.22	1.29	1.37	1.19
4 × 3 × ¼ L	0.75	0.80	0.86	0.93	1.02	1.11	1.18	1.26	1.27
4 × 3 × ⅜ L	1.11	1.18	1.26	1.35	1.47	1.58	1.70	1.81	1.76
5 × 3½ × 5/16 L	1.48	1.55	1.66	1.80	1.96	2.12	2.30	2.39	2.06
5 × 3½ × ⅜ L	1.76	1.86	1.99	2.15	2.34	2.52	2.71	2.80	2.42
6 × 4 × ⅜ L	2.50	2.66	2.87	3.10	3.41	3.70	4.00	3.64	3.17
6 × 4 × ½ L	3.30	3.52	3.79	4.10	4.52	4.84	5.18	4.65	4.09

gravity axis parallel to roof, c_1 = the distance from this axis to fiber on which stress is maximum, M_2 = bending moment due to component of load which is parallel to roof, I_2 = moment of inertia of section about a gravity axis normal to roof, and c_2 = the distance from this axis to fiber on which stress is a maximum. This method does not apply to purlins of ∟ and Z sections, but the moments must be resolved into the planes of their principal axes. By connecting the purlins together with SAG RODS running up both sides of a sloping roof to the same connection at the peak of the roof the second term of the preceding equation is negligible in the design of all purlins except the one or two at the peak carrying the pull from the sag rods. Sag rods are usually made 5/8 or 3/4 in in diameter and are spaced 6 to 7 ft apart. To prevent undue deflection the depth of a rolled beam purlin should not be less than 1/32 of its span length.

To determine the strength of the ordinary sizes of ∟'s and Z bars for purlins without sag rods and free to bend in any direction, the section moduli can be taken from the preceding table and used in the fundamental formula for flexure.

It should be noted that ∟ and Z purlins set as in Fig. 4 are stronger than those of Fig. 5. This table is based on purlins being set as shown in Fig. 4.

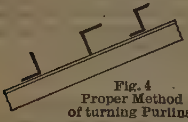


Fig. 4. Proper Way of Turning Purlins

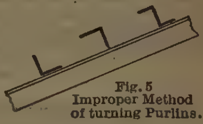


Fig. 5. Improper Way of Turning Purlins

Proportioning Tension Members. A tension member or bar of a truss, also called a tie, is one that carries a pull or tensile stress. For riveted trusses 2 ∟'s, and for pin trusses 2 eye-bars are commonly used for each tension member. For design of eye-bars see Art. 28. In riveted trusses carrying light loads the 2 ∟'s should be placed side by side with a space between them equal to the thickness of the gusset plates, Fig. 6, and should have their vertical legs up. These ∟'s should be connected together at points from 2 to 3 ft apart thruout their lengths by placing a circular steel washer between the ∟'s and driving a rivet thru the 2 ∟'s and the washer. These rivets are called tack or STAY RIVETS, and simply hold the ∟'s together during transportation and erection and make the ∟'s act together when under stress.

To proportion the cross-section of a tension member of 2 ∟'s divide the total stress in the member by the allowable unit tensile stress, 16 000 lb per sq in being commonly used, thus obtaining the required net area of the 2 ∟'s, and then choose from a table of properties of ∟'s, 2 ∟'s of the same size, each having a net area equal to or slightly larger than 1/2 the required net area. The net area of an ∟ is found by deducting from the gross area of the cross-section the greatest sectional area cut out by rivet holes in any one section. In COMPUTING NET AREAS the diameter of a rivet hole is assumed 1/8 in larger than the diameter of the cold rivet before driving. An angle is sometimes connected to a gusset plate by rivets in one leg only, in which case there would usually be only one row of rivets in the connecting leg, and hence only one rivet hole area, equal to diameter of hole times thickness of piece, would be deducted from the gross section. Where there are two rows of rivets in the ∟ at one end the rivets should be staggered so that there would be only one rivet hole in any one section. Best design requires that provision be made for taking stress out of both legs by having one leg riveted directly to the gusset plate and by connecting the outstanding leg to the gusset by means of a short piece of ∟ called a lug. Where the lug is not used it is sometimes specified that only one leg of the main ∟ shall be counted in computing the net area.

Proportioning Compression Members. A compression member or column in a truss is one which carries a compressive stress, and for roof trusses

is made of one or two ∟'s, Fig. 6, two ∟'s with a plate between, or of two [’s. In light work one or two ∟'s are nearly always used for each member, while for heavy trusses the top chord is composed of two [’s and the remaining members of 2 [’s or two or more ∟'s. To proportion the cross-section of a compression member subjected to compression only, the simplest way is to assume a trial cross-section and compare the actual average stress per square inch on this section with the allowable average stress per square inch as computed from a column formula. If the actual and allowable stresses are equal or if the actual is only slightly less than the allowable the section is used, but if the actual is greater than the allowable a new trial area is chosen and the process repeated.

The Actual Average Stress per square inch is found by dividing the total stress in the member by the gross area of the cross-section. The allowable average stress is found from a column formula, the following being commonly used for steel roof members. $S = 16\,000 - 70l/r$, where S is the allowable average stress in pounds per square inch, l = length of member from center to center of connections, r = least radius of gyration of section. For ratios of l/r less than 43 the corresponding values of S in the formula may be taken at 13 000 lb per sq in.

The following example will illustrate the method of proportioning the inclined top chord of a Fink truss having a panel length along this chord of 7 ft, a direct compression of 46 500 lb, and made of 2 ∟'s spaced 3/8 in apart for the gusset plates, the allowable unit stress being given by the formula $S = 16\,000 - 70l/r$. Assume two 4 by 3 by 5/16 in ∟'s and place the 4 in legs vertical and 3/8 in apart. From a steel company's hand-books (or p. 448), the area of the 2 ∟'s is found to be 4.18 sq in, and the least radius of gyration being that about the gravity axis parallel to the short legs. Dividing 46 500 lb by the area 4.18, the actual average unit stress is found to be 11 120 lb per sq in, and by substituting $l = 84$ in and $r = 1.27$ in in the column formula the value of the allowable unit-stress, S, is 11 370 lb per sq in. Hence the 2 ∟'s assumed are of correct size. These ∟'s must be riveted together at intervals not exceeding $0.65 \times 84/1.27 = 43$ in; 0.65 being the least radius of gyration of one ∟ and 1.27 the least for 2 ∟'s. It is customary to place tack rivets every 2 or 3 ft apart thruout the length of these members.

Purlins resting between joints on the top chord of a truss cause CROSS-BENDING therein, which, in addition to the compression acting, must be considered in the design of the chord. In such cases the shortest method for designing the chord is to assume a section, usually two ∟'s or two ∟'s and a vertical plate for light trusses, and compare the maximum unit stress with the allowable. If the section thus assumed is too large or too small a new trial section is chosen and the process repeated until the section is satisfactory. The maximum unit-stress in a continuous chord due to combined direct compression and bending is with sufficient accuracy given by

$$S = \frac{P}{A} + \frac{Mc}{I} \Big/ \left(1 - \frac{Pl^2}{10\,EI}\right)$$

where S is the maximum unit stress on the most strained fiber, in lb per sq in; P the direct compressive load in pounds; A the area of cross-section in square inches; M the maximum bending moment on the chord acting as a beam of panel length l and continuous over the joints; c the distance from gravity axis to most strained fiber under consideration; I the moment of inertia of cross-section; E the modulus of elasticity, for steel say 30 000 000 lbs per sq in.

Designing Joints. Since most roof trusses are of the riveted type, only riveted joints will be considered. For the design of pin joints see Art. 30. In laying out the members of the truss their center of gravity lines should meet at a point at each joint and should coincide with the truss diagram. Rivets should be placed in each member to transmit the stress from that member into the gusset plate, and they should be placed if possible so that the center of gravity of the rivets coincides with the gravity axis of the member and so that the stress will follow the shortest and most direct path. As an illustration of the method

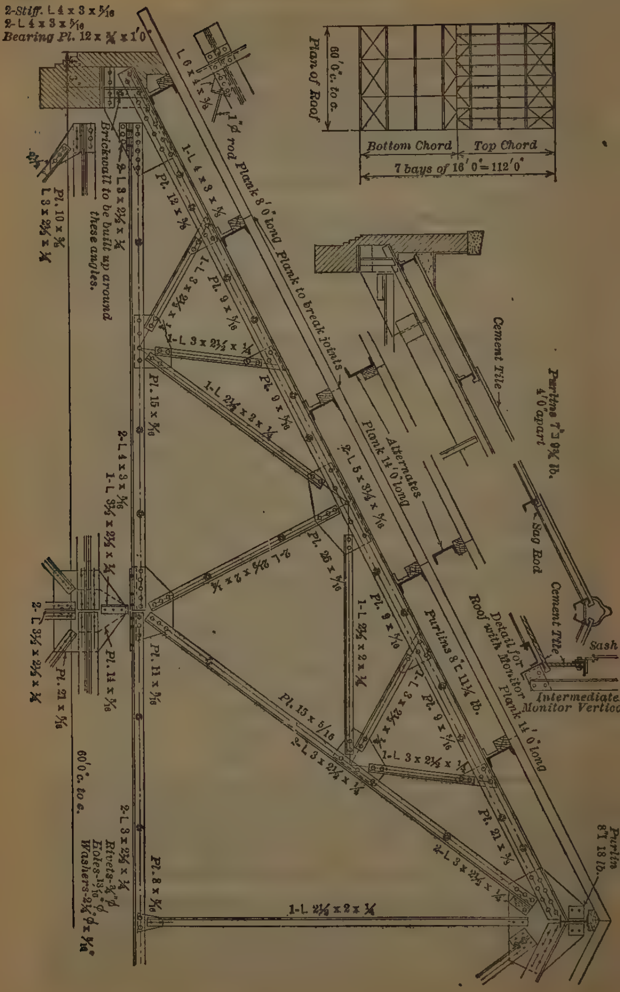


Fig. 6. Fan Roof Truss with Details

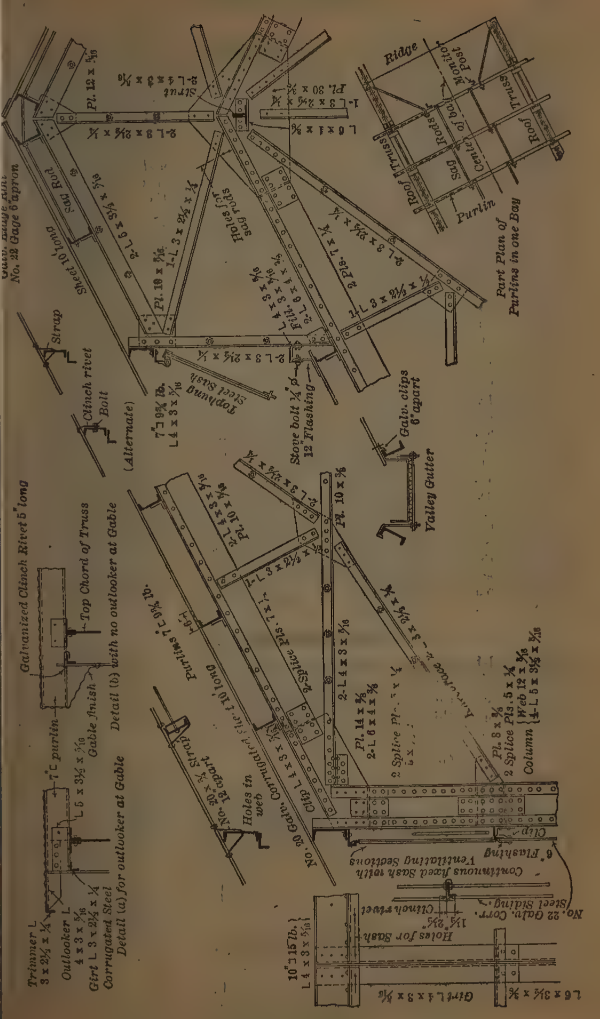


Fig. 7. Details for Corrugated Steel Covered Building

of computing the number of rivets consider a member carrying a direct stress of 46 500 lb and made of 2 L's 4 by 3 by $5/16$ in, one on either side of a $3/8$-in gusset. The rivets connecting the L's to the gusset are in double shear, that is, they must be sheared on two sections before failure can occur, they are in bearing on the $3/8$-in-gusset plate and in bearing in the opposite direction on 2 thicknesses of $5/16$-in L. The DOUBLE SHEARING VALUE of one rivet is equal to twice the area of its cross-section multiplied by the permissible shearing stress per square inch, or, assuming rivets $3/4$ in in diameter, the double shearing strength is $2 \times 0.44 \times 12\,000 = 10\,560$; the 12 000 being the permissible shear in pounds per square inch. The BEARING VALUE of a rivet on a piece is the product of the diameter of the rivet in inches, the thickness of the piece in inches and the permissible bearing pressure in pounds per square inch. In the case here assumed the bearing value of the $3/4$-inch rivet on the $3/8$ gusset plate is $3/4 \times 3/8 \times 24\,000 = 6750$ lb, and since this is smaller than either the double shearing value or the bearing on the two $5/16$-in L's, 6750 lb is the allowable stress on one rivet; 24 000 being the permissible bearing strength per square inch. The number of rivets in the member at this joint is 46 500/6750, or 7.

The Permissible Unit-Stresses used in the problem just solved, namely 12 000 lb per sq in for shearing and 24 000 lb per sq in for bearing, are safe values for shop, that is machine-driven rivets. For field rivets, that is, rivets driven by hand, or for bolts, the safe shearing and bearing values are 10 000 and 20 000 lb per sq in respectively. Allowable shearing and bearing values for rivets of different diameters are given in Art. 23.

Fig. 6 shows a fan roof truss with details.

Fig. 7 shows details of an ordinary steel building with covering of corrugated steel.

Details. Even tho to make what is usually considered the design of a structure may be easy, to detail it may be difficult. The ideal draftsman is a designer, a designer of details. Seldom has a structure given way for lack of strength in the main members but faulty details have invited and brought disaster. Theoretically, in laying out the members of a roof truss the center of gravity lines should meet at a point at each joint and should coincide with the truss diagram. However, in practice the rivet lines are generally used. Details are thus simplified. Eccentric rivet connections should be reduced to a minimum. Economy will be gained by having as many duplicate parts as possible. Connections that induce secondary stresses should be avoided. Details should be designed in accordance with shipping facilities, cost of transportation and ease of erection. While it may lessen the cost of field work to fabricate trusses in few sections, the transportation charges may be enough larger to consume all saving. Again, while it may lessen freight charges to ship a truss in many pieces, the cost of field riveting will be increased.

3. Steel Mill-Building Frames

Steel Mill-buildings in general are divided into three types. Those of the first type have steel frames carrying the roof loads as well as the weight of the walls which protect the frames; the walls being constructed of corrugated steel sheets or of thin concrete supported by the steel frames. The essential

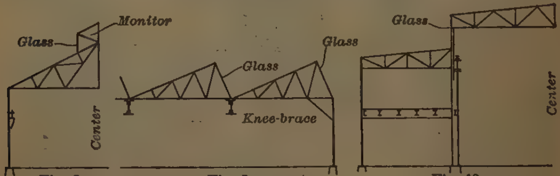


Fig. 8 Fig. 9 Fig. 10

Steel Mill-building Frames

parts of the steel work are the roof trusses, columns and bracing; a truss together with the columns which support it constituting a bent of which Figs.

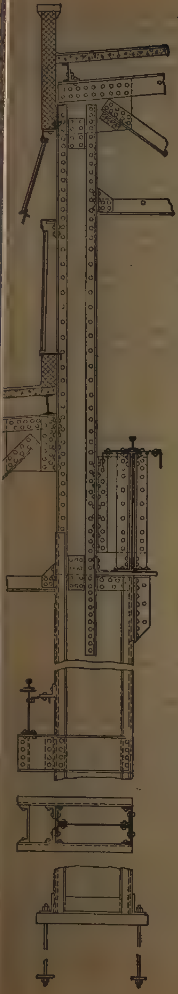


Fig. 11. Typical Column Details

8, 9 and 10 represent typical cases. In the second type of steel mill-building the columns are braced principally by thin masonry walls, thus eliminating most of the steel bracing between columns, while in the third type the steel roof trusses are supported on masonry walls instead of on steel. Light is admitted to the interior of mill-buildings thru windows in the side and end walls and in addition thru skylights in the roof proper or in the monitor roof, or thru windows in clerestory of the monitor. A MONITOR, Fig. 8 is that portion of building extending above the main roof for the purpose of ventilating or lighting the interior of a building. Fig. 9 shows SAW-TOOTH roof construction. Roofs of this form of construction are extensively used for wide areas. The glazed portions should of course face the north; a diffused light thus illuminating the floor below without casting shadows.

Columns should be proportioned to resist (1) direct stress due to roof, crane or other loads, (2) stress due to wind and (3) stress due to eccentric loads. A great number of column formulas have been suggested for the design of steel members in compression. The "straight-line" formula $S = 16\,000 - 70\,l/r$ is used thruout this section.

Rivets in tension should be avoided when practicable. Carrying the crane load on a column flange directly under the web of the runway girder is preferable to carrying it on a bracket. By connecting roof trusses to columns with gussets greater stiffness is obtained than by resting them on column caps. Various column details are shown in Figs. 11 and 12.

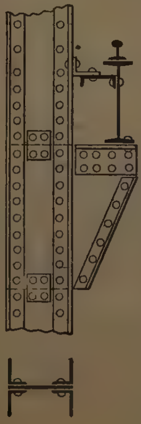


Fig. 12. Connection of Crane Bracket to Column

Girts, sometimes called side purlins, are beams, usually of angles or channels, fastened to the columns to support the side covering and to resist wind pressure. For corrugated steel siding the wind pressure is the greater load and the longer leg of the angle or the web of the channel should be laid horizontal. To prevent deflection sag rods 5 to 8 ft apart fastened to the girts and running to the eave struts are used.

The Bracing of a mill-building should be so designed that wind and vibratory stresses are carried to the foundations. Diagonal bracing should be intro-

duced into the plane of both top and bottom chords for stiffness as well as for calculated stresses. This applies also to roof trusses resting on brick walls. Adjustable rods may be used for top chord bracing, the purlins acting as struts, but the bracing of the bottom chord should be angles or other rigid shapes. Diagonals between the steel columns in each end of the building and in an occasional side bay are usually sufficient to carry the induced stresses to the foundations. When it can be done an excellent way is to carry the transverse thrust from the wind as well as that from the cranes to the ends of the building and thence by diagonals to the ground. The usual way is to design each bent as a unit for both the vertical and horizontal loads of one-half the adjoining bays. In such cases the kneebrace is an important member. A KNEEBRACE is a short diagonal, usually a pair of angles, used to connect a truss or beam to a column. Traveling cranes running thru a building often bar the use of transverse kneebraces. The gusset plates connecting the trusses to the columns should then be as large as possible and calculations made accordingly. Angles may be added to stiffen the edges of the plates. Kneebraces should be used between crane girders and their supporting columns.

Electric Traveling Cranes are an important part of the equipment of most mill-buildings and are often the dominant factor in the design. Ten ton cranes are the most common. Cranes of 50 short tons capacity and under are

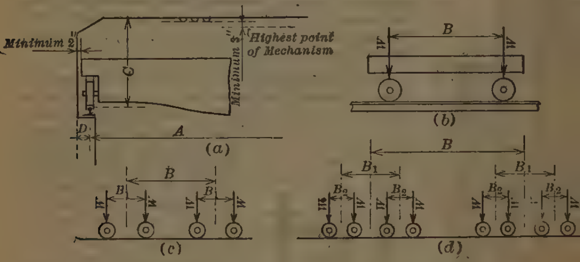


Fig. 13. Wheel Diagrams for Traveling Cranes

usually carried on a two-wheel carriage at each end. Those of 60 to 125 short tons capacity should have four-wheel carriages in order to avoid excessively concentrated loads on the runway girders. Cranes of 150 short tons capacity and more are built with end carriages of both four and eight wheels. The latter reduces the concentrated load on the runway girders. The wheel bases, maximum wheel loads and clearance of cranes vary with the design of the maker. The figures give in the table on next page represent average values.

Stresses due to the wheel loads given in the table should be increased 25% for vibration and impact except for hand power cranes where 10% is sufficient. For two cranes in action on the same girder, no impact need be added provided the stress obtained is larger than the stress due to a single crane with impact. In addition to the vertical loads the top flanges of crane girders should be designed to resist a transverse horizontal thrust on each carriage, applied to the wheels, of 10% of the lifting capacity of the crane. The traction stress due to starting or stopping the crane should be assumed at 10% of each wheel load and may be considered as distributing itself along the entire length of the runway.

Loads and Clearances for Electric Cranes See Fig. 13

Capacity in tons	Span in ft c to c Rails (A)	Wheel Base B ft	B in	B_1 ft	B_2 ft	B_2 in	Max. Load in lbs for each wheel (W)	Vertical Clearance (C) ft	in	Side Clearance (D) in	A.S.C.E. Rail in lbs per yd
5	40	9	0	......	......		13 000	6	0	10	50
	60	10	0	......	......		15 000	6	0	10	50
	80	11	0	......	......		18 000	6	0	10	50
10	40	9	0	......	......		20 000	6	6	12	60
	60	10	0	......	......		23 000	6	6	12	60
	80	11	0	......	......		26 000	6	6	12	60
15	40	10	0	......	......		30 000	7	0	12	60
	60	11	0	......	......		33 000	7	0	12	60
	80	12	0	......	......		36 000	7	0	12	60
20	40	10	6	......	......		35 000	7	6	12	80
	60	11	6	......	......		39 000	7	6	12	80
	80	12	6	......	......		43 000	7	6	12	80
30	40	11	0	......	......		50 000	8	0	13	80
	60	12	0	......	......		54 000	8	6	13	80
	80	13	0	......	......		59 000	8	6	13	80
40	40	12	0	......	......		66 000	8	6	14	100
	60	13	0	......	......		71 000	9	0	14	100
	80	14	0	......	......		77 000	9	0	14	100
50	40	13	0	......	......		80 000	9	6	15	100
	60	13	6	......	......		85 000	10	0	15	100
	80	14	6	......	......		90 000	10	0	15	100
75	60	12	0	5	......		65 000	12	0	16	100
	80	14	0	5	......		70 000	12	6	16	100
100	60	12	0	5	......		90 000	14	0	18	150
	80	14	0	5	......		95 000	15	0	18	150
125	60	13	0	5	......		110 000	15	0	20	175
	80	15	0	5	......		120 000	16	0	20	175
150	60	18	0	5	......		130 000	16	0	22	175
	80	20	0	5	......		140 000	17	0	22	175
175	60	14	0	7	3	6	65 000	17	0	24	150
	80	19	0	7	3	6	75 000	18	0	24	150
200	60	14	0	7	3	6	75 000	17	0	24	150
	80	19	0	7	3	6	85 000	18	0	24	150
Loads and Clearances for Hand Cranes											
2	30	4	0	......	......		3 500	4	0	7	30
	50	5	0	......	......		4 000	4	0	7	30
4	30	4	0	......	......		5 500	4	6	8	30
	50	5	0	......	......		6 500	4	6	8	30
6	30	6	0	......	......		8 600	5	0	9	40
	50	7	0	......	......		9 500	5	0	9	40
8	30	6	0	......	......		10 500	5	0	10	50
	50	7	0	......	......		12 000	5	0	10	50
10	30	7	0	......	......		13 000	5	0	10	50
	50	8	0	......	......		14 500	5	0	10	50

Floor Loads. Live Loads in lb per sq ft on floors of the mill-building class should not be assumed at less than the following:

Mold lofts, pattern and template shops	60
Machine shops, light machinery	120
Machine shops, heavy machinery	150 to 200
Factories	100 to 250
Foundries, charging floor	300 to 800
Power houses	200

It is important that industrial processes be understood sufficiently to make proper provision for their loads and stresses. Provision should also be made for the support of all engines, boilers, tanks or other concentrated loads carried by the steel construction.

The Working Stresses given in Art. 2 may be used in proportioning members.

Wind Stresses. For buildings not more than 25 ft to the eave line a horizontal wind pressure should be assumed at not less than 15 lb per sq ft on the sides and the corresponding normal component on the roof according to the Duchemin formula for wind pressure on inclined surfaces. For buildings more than 25 ft to the eave line the horizontal pressure should be taken at not less than 15 lb for the lower 25 ft and 20 lb for the side surface above 25 ft and the corresponding normal component on the roof. The steel framework should be designed to carry wind pressure to the ground.

The stresses in the framework due to wind acting on the vertical and inclined sides of a building depend to a great extent on the manner of fixing the ends of the columns. If the ends are hinged, that is, free to turn, the stresses thruout the frame are different from those existing when the ends of the columns are fixt. A COLUMN IS FIXT at the end when it is so rigidly held there that the axis of the column cannot change its direction at that point. In the case of a fixt-end column the point of CONTRA-FLEXURE, that is, the point where the direction of curvature changes, is taken halfway between the base of the column and foot of kneebrace. In light buildings the columns should not usually be assumed fixt at the bottom.

The case of an intermediate transverse bent of a kneebraced mill-building will be considered. The example taken will be a bent of 60 ft span, height 18 ft to foot of knee-

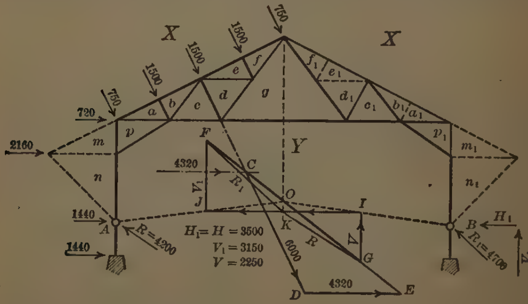


Fig. 14. Load and Reaction Diagram

brace, 24 ft to bottom chord and slope of roof 6 in to 1 ft. Trusses are 16 ft apart c to c. The wind pressure will be taken at 15 lb per sq ft perpendicular to the sides of the building

and at 11.2 lb (Duchemin formula, Table, Art. 1) normal to the roof. The columns are assumed partially fixed at the lower end, with the point of contra-flexure at one-third the distance between the lower end and the foot of the kneebrace; the upper ends are considered supported.

Fig. 14, the load and reaction diagram, shows the arrangement of the forces and the method of finding the reactions at the points of contra-flexure in the columns. The wind

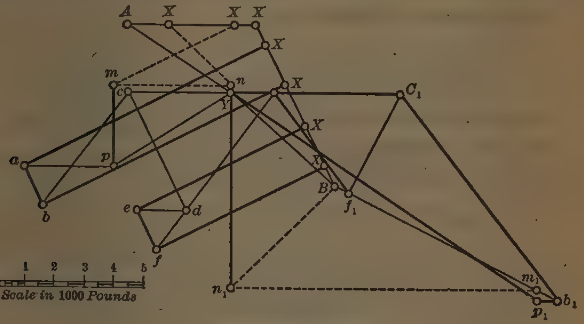


Fig. 15. Stress Diagram

shear is assumed to be divided equally between the two columns, making the horizontal reactions at A and B equal. CE is the resultant of CD and DE, the wind on the roof and the sides. The wind resultant is laid off, FG, so that it is bisected by the mid vertical line at O. The perpendiculars dropped from F and G upon the dotted lines OA and OB give the vertical components of the reactions at B and A respectively. Draw the horizontal line JI intersecting the mid vertical at K. Then $KF = R_1$ and $GK = R$, the reactions at B and A respectively.

Fig. 15 is the stress diagram. The heavy lines indicate compressive and the light lines tensile stresses. Dotted lines indicate stresses in imaginary truss members which are inserted to make the stresses statically determinate. Removing these members will produce bending stresses in the columns but will not affect the stresses in truss members and kneebraces. The stress in any member can also be determined by the method of moments or sections. For instance, let S = required stress in the kneebrace. Take M (Fig. 15a) as center of moments and since the sum of moments of all forces about M = zero; $(4200 \times 15.0) - (4320 \times 9.0) - (S \times 5.0) = 0$ from which $S = 4800$ lb tension. This checks with the stress obtained by the graphical method of Fig. 15.

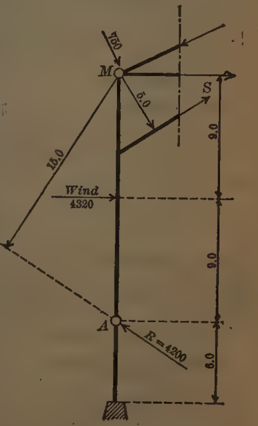


Fig. 15a. Stress in Kneebrace

The stresses in the columns due to wind are as follows: The direct stresses in the portion below the kneebrace are 2250 lb for the windward and 3150 lb for the leeward column, being the vertical reactions V and V_1, scaled from the reaction diagram. The horizontal shear at the point of contra-flexure of each column scaled from the reaction diagram is 3500 lb. The direct stress in the portion of the windward column above kneebrace equals the vertical reaction V plus the vertical component of the kneebrace and in

the leeward column the vertical reaction V_1, minus the vertical component of the kneebrace. The horizontal shear in the portion of the windward column above the kneebrace equals the horizontal component of the stress in the kneebrace (3960)+wind (4320)−H(3500)=4780. The horizontal stress in the portion of the leeward column above the kneebrace equals the horizontal component of the stress in the kneebrace (10 200)−H_1 (3500)=6700.

The bending moments in the columns are:

At foot of leeward column, 3500×6	=21 000 ft lb
At foot of leeward kneebrace, 3500×12	=42 000 ft lb
At foot of windward column (3500×6)+(1440×3)	=25 320 ft lb
At foot of windward kneebrace (3500×12)−(1440×12)	=24 720 ft lb

It is seen that the maximum bending moment is at the foot of the leeward kneebrace.

To Design a Mill-Building Bent proceed as follows: Determine stresses in truss due to a total uniform load over the entire roof surface, as in Art. 1. The total load includes the wind load which may be assumed at 10 lb per sq ft for roofs of 3 in slope and more and at 5 lb for slopes of less than 3 in. Proportion the members for these stresses using the working stresses recommended in Art. 2. Then find the wind stresses due to the normal wind forces by the method of Figs. 14 and 15. If the wind stress in any member from Fig. 15 is greater than the wind stress (obtained by interpolation) due to a vertical load of 10 or 15 lb per sq ft over the entire roof surface, that member is proportioned for the maximum wind plus the stress from the uniform loads other than wind, using working stresses 50% more than in the first calculation, but in no case is a less section to be used than that first obtained. The members bc and b_1c_1 will generally need to be increased; often cd and c_1d_1; occasionally gd, gf, gf_1 and gd_1. Compressive stresses are noted in certain tension members, particularly b_1c_1 and the lower chord. The diagonals bc and b_1c_1 can be made of two angles instead of one as when designed for tension alone. The compressive stresses in the lower chord are overcome by the tensile stresses due to the dead load. The kneebraces have wind stresses only, and are proportioned for the larger working stresses.

The Columns may be designed as follows:

(1) Proportion for the direct stress from the specified total uniform load on the roof surface using the column formula, $S=16\,000-70\,l/r$.

(2) Proportion for combined compressive and bending stresses using the formula of Art. 2,

$$S=\frac{P}{A}+\frac{Mc}{I}\Big/\left(1-\frac{Pl^2}{10\,EI}\right).$$

S in this formula is the actual maximum fibre stress which should not exceed by more than 50% the S obtained from the column formula in (1). The direct stress, P, is that due to the roof loads (other than wind) and the vertical reaction from wind. For M the maximum bending moment due to wind is to be used.

The greater section obtained by (1) or (2) is to be used.

For example, if the roof of Fig. 14 be of corrugated steel, the total direct load on a column is 31 ft × 16 ft × sec 26° 34′ × 40 lb=22 200 lb. The column is proportioned for this load according to (1) above. As with truss members it may be assumed that 10 lb of the 40 lb uniform load is from wind leaving 16 650 lb from loads other than wind. The reaction diagram shows a direct stress in the leeward column of 3150 lb. In proportioning the column according to (2) the value of P in the formula is therefore 16 650+3150=19 800 lb. The value of M is the maximum bending moment, 42 000 ft lb=504 000 in lb. The greater section in this particular case is that obtained by (2) and it should be used.

4. Tall Building Frames

The Steel Frames used in the construction of modern tall buildings such as office buildings carry the entire weight of walls, floors and, in fact, all dead and live loads. To these buildings have been given the name SKELETON CONSTRUCTION.

Columns should always be made of steel, altho in cheap construction cast iron is sometimes used. Fig. 16 shows various common sections employed for columns in buildings. Where two [’s or four L’s are used to form a column they should be connected together by LATTICE BARS, which are flat bars from $1\frac{3}{4}$ to $2\frac{1}{2}$ in width riveted first to one [or pair of L’s and then to the other. Columns made of L’s and plates or Z bars and plates must be riveted together with rivets in the body of the column spaced not over 16 times the thickness of the thinnest outside piece connected, the maximum distance used being 6 in on centers. At ends of such columns the rivet pitch, which is the distance of rivets on centers, should not exceed four diameters of the rivet for a length

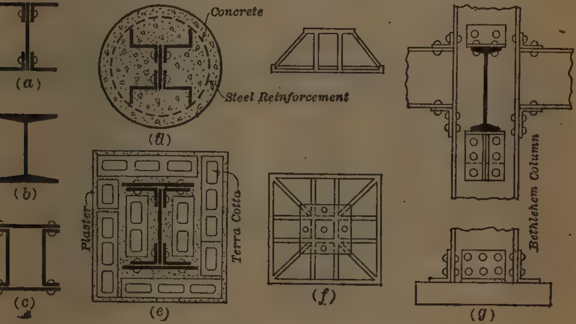


Fig. 16. Columns for Fire-proof Buildings

equal to $1\frac{1}{2}$ times the greatest width of the member. Columns should be continuous over two stories, and splices should be placed just above the floor levels. Cast-iron bases or pedestals are used under columns to distribute the load over the foundations. All columns must be encased in concrete, terra-cotta or other fire-proof materials, Fig. 16, and no pipes should be placed within such casings. Two inches of concrete or 2 to 4 in of terra-cotta is the minimum thickness for casings, and the space between the terra-cotta and the columns should be filled with concrete or other similar filling.

In tall buildings WIND BRACING is a prime requisite. No other feature of their design has called forth a greater variance of opinion. Dependence for bracing is usually placed upon the walls, floors and partitions in buildings less than 100 ft high where the height is not more than twice the minimum horizontal dimension. The columns are strongly spliced and secured to the floor framing. For greater limits the wind stresses should be carried to the ground by the steel frame. This often presents a difficult problem. Kneebraces are excellent when they do not project into rooms and corridors. A system of diagonal bracing, L’s or [’s, as shown in Fig. 17 (a), is ideal but it can seldom be used.

In a system without diagonals the connections of floor girders and beams to columns must be designed to take bending stresses and at the same time come within prescribed architectural limits. Connections that have often been used are shown in Fig. 17 (*b*), (*c*), (*d*). For the lower floor connections details

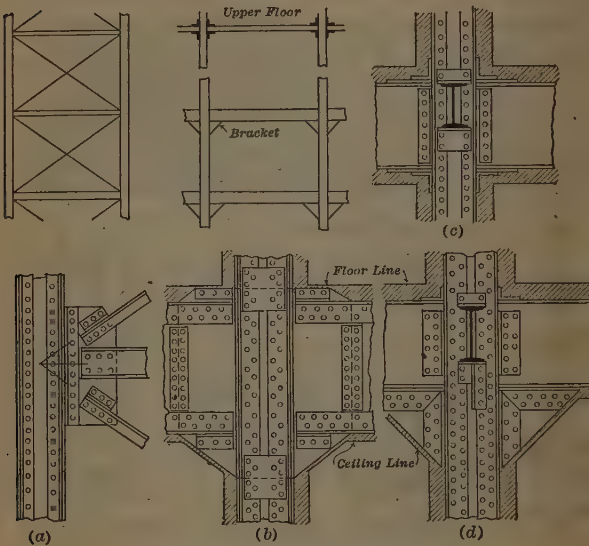


Fig. 17. Types and Details of Wind Bracing

(*b*) or (*d*) are used and for the upper floors where wind stresses are light (*c*) is used.

Live Loads. Tall buildings are usually within the limits of a city and must be designed in accordance with the provisions of a municipal building code. When not governed by a building code the following loads and working stresses are recommended:

Minimum Live Load in Pounds per Square Foot of Floor Area

Dwellings (private residences) first floor	40
Dwellings (private residences) upper floors	30
Apartment houses, first floor	50
Apartment houses, upper floors	40
Hotels, first floor	80
Hotels, upper floors	40
Office buildings, first floor	100
Office buildings, upper floors	50
School buildings, class rooms	50

School buildings, assembly rooms	75
Churches and theatres	75
Places of public assembly where floors are used for drilling or dancing	120
Where not so used	100
Retail stores, ordinary	100
Private garages, pleasure vehicles only	60
Public garages, pleasure vehicles only	90
Garages, motor trucks 1 to 3 tons capacity	150
Garages, motor trucks 3½ to 5 tons capacity	200
Warehouses	200 to 500

Every steel beam in any floor should be capable of sustaining a live load concentrated at its center of not less than 3000 lb if used for business purposes; of 2000 lb if used as a private garage storing pleasure vehicles only; of 3000 lb if used as a public garage storing pleasure vehicles only; of 8000 lb if motor trucks of 1 to 3 tons capacity are stored and of 12 000 lb if motor trucks of 3½ to 5 tons capacity are stored. The kind of material stored in a warehouse will determine the floor load.

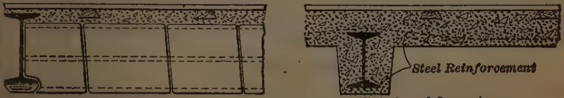
Reduction of Live Load. In computing stresses for columns in such buildings as warehouses and factories which are likely to be fully loaded on all floors simultaneously the full specified live load should be used. In types of buildings such as apartment houses and office buildings the specified floor loads may be reduced in computing column stresses as follows: for the roof the specified live load should be used, for the top floor 90% of the live load, for each succeeding lower floor the live load may be reduced by 5% until 50% of the live load floor loads is reached, when such reduced loads should be used for all remaining floors, except the first or ground floor for which the full specified live load should be used. Girders of office building type carrying more than 200 sq ft of floor may have the specified live load reduced 10%. Dead loads must be computed in all cases.

Wind Pressure should be assumed at not less than 20 lb per sq ft on the sides of building and the corresponding normal component on the roof.

The Working Stresses given in Art. 2 should be used in proportioning members.

5. Fire-proof Floors

Floors for Steel Buildings consist of a wearing surface of wood, cement, marble or tile laid on hollow-tile blocks, brick or concrete arches which are



Terra Cotta Arch (End Construction) *Reinforced Concrete*

Fig. 18. Fire-proof Floor Construction

carried by I beams, called joists or floor beams, spaced from 4 to 8 ft apart, tho usually from 5 or 6 ft, depending on the kind of arches used and the span of the beams. Floor girders, that is, deep I beams or plate girders running from column to column, support the joists, and if between exterior columns, they also support the walls. Hollow-tile blocks made of porous terra-cotta, and used as flat arches, are placed between the I-beam joists and set in cement mortar so as to cover the bottom flanges of the beams, thus affording protection of the beams against fire underneath. A layer of concrete is spread over the blocks, covering the top flanges of beams to a depth of about 2 in, and

on this concrete the wearing surface is placed. CONCRETE FLOORS are either arched or flat. In the latter case they consist of a slab in which steel netting or rods are embedded. The concrete surrounds the steel beams, and a flat ceiling is usually suspended from the beams. Fig. 18 shows details of fire-proof flooring.

Dead or Fixt Loads on floors should be computed for each case. The following are ordinary values in pounds per square foot for fire-proof flooring: wooden wearing surfaces, 4 to 6; screeds or nailing strips, 2; concrete filling, 15 to 30; arch or slab, 20 to 50; steel floor joists and girders, 8 to 12; ceiling construction and plaster, 3 to 10. The total dead weight of fire-proof floors varies from 50 to 100 lb per sq ft. The following table gives weights of hollow-tile floor arches, these being the most common form of construction:

Weight of Hollow Tile Flat Floor Arches

Exclusive of Weight of Steel, Filling or Wearing Surface

End Construction			Side Construction		
Depth of Tile, in	Beam Spacing, ft	Weight, lbs per sq ft	Depth of Tile, in	Beam Spacing, ft	Weight, lbs per sq ft
6	3 to 4	25	6	3.5 to 4.0	27
7	4 to 5	26	7	4.0 to 4.5	29
8	5 to 6	27	8	4.5 to 5.0	32
9	6 to 7	29	9	5.5 to 6.0	36
10	7 to 8	33	10	6.0 to 6.5	39
12	8 to 9	38	12	6.5 to 7.0	44

The Spacing, or distance between centers of floor beams, varies with the type of fire-proofing, the length of span of the beams and character of loading. With floor beams of ordinary spans, 5 to 6 ft, and for long spans 4 to 4½ ft, apart represent good practice. After determining the live load and the type and weight of fire-proofing the total live and dead load can be easily computed and the correct size and spacing of I beams for this loading and span can be taken from tables in steel companies' hand-books; or the spacing may be assumed and the proper beams to be used for this spacing is found by taking from pages 439-449 a size and weight having a section modulus which equals the value of I/c in the following formula:

$$I/c = 3\, dwl^2/2S$$

where I/c is the section modulus in inch units, I being the moment of inertia and c the half depth of beam; d, distance center to center of beams in feet; w, total live and dead load in pounds per square foot; l, span of beam in feet; S, allowable unit fiber stress in pounds per square inch. If a beam cannot be found with a section modulus close to that required, a slight change in the spacing may be advisable. In this case a beam may be chosen and the distance $d = 2\, SI/3\, cwl^2$ computed. The heaviest section of a given depth of I beam should be avoided, as it is not as economical as a lighter beam of a greater depth. The size of a beam required to carry a uniform load may also be chosen by multiplying the load per running foot of beam by the span and selecting from a hand-book a beam corresponding, and in case the load on the beam is concentrated at the center it can be reduced to an equivalent uniform load by multiplying it by two. For loadings not uniform or not con-

centrated at the center of the span the section modulus I/c must be found from the formula $I/c = M/S$, where M is the bending moment in inch-pounds and S the unit stress which is usually taken at 16 000 lb per sq in.

The Deflection of Floor Beams carrying plastered ceilings underneath should not exceed $1/360$ of the span. For I beams carrying uniform loads, simply supported at the ends and strest to 16 000 lb per sq in, this limit of deflection will be exceeded if the ratio of span length to depth of beam exceeds 24.

Compression Flanges of I Beams or girders should be stayed laterally at distances not exceeding 12 times the flange width. If this ratio is exceeded the allowable unit stress should be found by the formula, $S = 19\,000 - 250\,l/b$, where S = allowable unit stress in lb per sq in, l = unsupported length and b = width of flange. A corresponding reduction should be made in the tables of safe loads given in steel companies' hand-books. For example, the tabular allowable uniform load for an 18 in 55-lb beam 20 ft long based on a maximum bending stress of 16 000 lb per sq in is 47 000 lb. The beam length l, is 240 in and the flange width, b, is 6 in. If the top flange is unsupported laterally the allowable unit stress is $19\,000 - 250\ l/b = 9000$ and the safe load is $9/16 \times 47\ 000$ lb $= 26\ 500$ lb.

Tie Rods from ⅝ to ⅞ in in diameter are used to connect the webs of I beams in buildings to assist in construction, in bracing the beams laterally and to take the thrust from the floor arches. For uniformly loaded floor arches the horizontal thrust is given approximately by $T = 3\,wl^2/2r$, and the spacing of the tie rods by $l_2 = 10\,000\ ra/wl^2$, where T = thrust in pounds per linear foot of floor beam; w = total live and dead load per square foot; l = span of arch in feet; r = rise of arch in inches; l_2 = distance between rods in feet; a = net area of tie rods in square inches. For flat arches r is approximately $6/10$ to $8/10$ the depth of the arch.

The Safe Unit Shearing Strength of I beam webs or of plate-girder webs for buildings is given by

$$S_S = 12\,000/(1 + h^2/3000\,t^2)$$

where S_S is in pounds per square inch; d = depth of beam or web of plate girder, t = thickness of web, h = vertical distance between flanges of I beams, and for plate girders horizontal distance between stiffeners or vertical distance between flanges, whichever is the smaller. All dimensions are in inches. To obtain the total safe shearing strength V of a web multiply the above unit stress by the gross cross-sectional area of web, or $V = S_S dt$.

Safe Unit Shearing Strength of I-Beam Webs for Buildings

Pounds per Square Inch

h/t	S_S	h/t	S_S	h/t	S_S	h/t	S_S	h/t	S_S
10	11 610	20	10 590	30	9230	40	7830	50	6550
12	11 450	22	10 330	32	8950	42	7560	52	6310
14	11 260	24	10 070	34	8660	44	7290	54	6090
16	11 060	26	9 790	36	8380	46	7040	56	5870
18	10 830	28	9 510	38	8100	48	6790	58	5660

6. Fire-proof Walls and Partitions

Walls for Steel Mill Buildings are made of brick, concrete, tile, corrugated steel, hy-rib and concrete, metal lath and plaster, wood or metal sash. Two or more kinds of siding are often used in the same building. If corrugated steel is used for the roof the sides are often made of it also but of lighter weight. Less than No. 22 should not be used for roofs nor less than No. 24 for siding. For the roof the usual method of fastening is by straps to [purlins and for

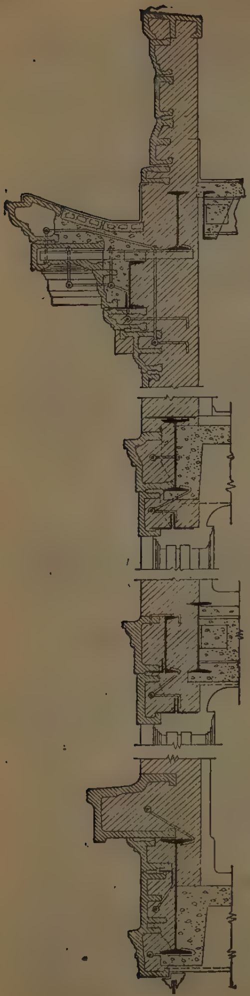


Fig. 19. Spandrel and Cornice Sections

sides by straps or clinch rivets to ⊏ or ⌊ girts. Tho more expensive, galvanized sheets are preferable to painted sheets. Walls between columns are usually made 12 in thick if of brick and 8 in thick if of tile or concrete. A greater thickness may be required when the walls carry all loads or when the columns are set free from the walls.

Where a RECTANGULAR OPENING is to be made in a solid masonry wall, and there are no windows nor other openings in the wall above, one or more I beams placed side by side, and connected together by cast-iron or rolled-steel ⊏ separators fitting between the beams, are used to support the masonry. Assuming that the beams carry a triangular portion of the wall having for its base the span and for its altitude ½ the span, the section modulus of the combined beams $= wtl^3/32\,000$, where w is the weight of the masonry in pounds per cubic foot; t, thickness of wall in feet; l, span of opening in feet. If there are windows above the opening the entire weight vertically above the opening should be taken on the beam. In either case the beams should have a depth of not less than about $1/24$ the span.

The question of lighting has been much discussed in recent years. It is important that natural light should be utilized to the best advantage. The location of windows as well as their area should be considered. The types of sash most used for mill buildings are the ordinary wooden sash and the rolled steel sash. The latter is more satisfactory and is used almost entirely for the best modern buildings.

Walls in Office Buildings having steel frames are made of brick, stone or terra-cotta and are carried by girders or I beams at each floor level, thus separating the wall into parts each one story in height. The thickness of these walls is usually fixt by building laws, 12 in being the minimum. Terra-cotta facing with brick backing for exterior walls is good construction. Cement mortar should be used. Terra-cotta furring blocks, usually 2 to 6 in in thickness, attached to the inside surfaces of exterior walls, are used as a protection against dampness and noise, and serve to hold the plaster. A SPANDREL, which is that part of an outside wall in an office building over a window and under the window vertically above,

is carried on I beams, or on ⊏'s and ⊾'s. The terra-cotta blocks are anchored to the steel. Fig. 19 shows spandrel construction.

Partitions in Fire-proof Buildings, Fig. 20, are made of hollow terra-cotta blocks, plaster, plaster block, brick and of various patented materials. Of these materials terra-cotta hollow blocks are most commonly employed. Partitions of this material are made from 4 to 12 in thick, 4 in being the usual thickness for the blocks, which for the body of the partition are 12 by 12 in in size but for the top and sides are 6 to 8 by 12 in to fit in with the square blocks.

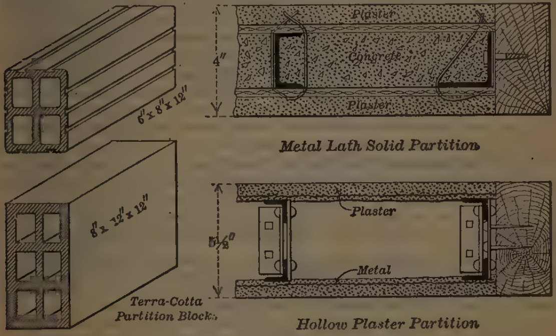


Fig. 20. Fire-resisting Partitions

Terra-cotta should be wet before being laid and also before being plastered, and should be laid in a mortar composed of one part lime-putty, two parts cement and two to three parts sand. PLASTER on both sides of a terra-cotta partition increases the thickness by about 1½ in, that is, ¾ in per side. Twelve ft for 3 in, 16 ft for 4 in, 20 ft for 6 in are the usual heights for the thicknesses of partitions given. Solid plaster partitions are made from 2 to 4 in thick and are supported on wire lath on steel framework. For the 4-in partitions a central body of cinder concrete supported between two such wire nettings is built, then plastered on both sides. HOLLOW PLASTER PARTITIONS consist of a steel framework of ⊏'s to which metal lath is attached on each side, and to this the plaster is applied, leaving a hollow space between the two layers of plaster.

The accompanying table gives weights of various forms of partitions, the weight in each case of terra-cotta and plaster blocks not including the plastered surfaces. For each plastered surface used add 5 lb per sq ft of surface. The weights given are per square foot of partition.

Weights of Partitions

Kind of Partition	Thickness, in	Weight, lb per sq ft	Kind of Partition	Thickness, in	Weight, lb per sq ft
Porous or hard-burned terra-cotta	2	10 to 14	Solid plaster	2	20
	3	11 to 16		4	32
	4	12 to 19	Hollow plaster	4	22
	5	17 to 22	Plaster blocks	2	7
	6	22 to 24		4	12
	8	28 to 33		8	22

7. Trainsheds

Trainsheds are of two general types, those having the roof carried by trusses or arches spanning the entire width of the shed and those having intermediate supports for the roof. Of the first type the Penna. R.R. terminal of Pittsburgh (Fig. 21) is an example, and the second is exemplified in the UMBRELLA CONSTRUCTION of the Union Station in Washington, D. C., and in the modified umbrella type of the D.,L. & W.R.R. station in Hoboken, N. J. (Fig. 22). In the first type the platforms and tracks are covered with high roofs in which are skylights and ventilators, all carried on steel construction. Three-hinged arches are commonly used in these sheds, the longest span being 300 ft, but in a few cases trusses supported at their ends on columns have been used for extremely long spans, as in the Van Buren St. station of the C.,R.I. & P.R.R. in Chicago, which has pin-connected trusses of 207 ft span. In the South Station in Boston the trusses are of the cantilever type, with two side spans each 169 ft 1 in and one center span 228 ft 6 in. The excessive first cost and the difficulty of ventilation and of maintaining the steel work and skylights on the large covered sheds has brought the umbrella sheds into use. These sheds are carried by rows of columns placed between and parallel with the tracks, and the roof extends outward from the columns, thus sheltering the platforms and leaving uncovered the space above the tracks, or else, as in the D ,L. & W.R.R. terminal, a narrow opening is left over the center of each track to allow the locomotive gases to pass out, and the canopies just mentioned are connected at intervals.

The Pittsburgh Trainshed Roof is carried on twenty-four three-hinged riveted arches arranged in twelve pairs, the two arches in each pair being 9 ft apart and the distance one pair to the next being 40 ft 6 in. A 12-in I beam serves as a tie in each arch to connect the lower hinges and take the horizontal component of the thrust. Each arch has one end fixt and one end on rollers. The loads assumed in the design were, roof covering 13 lb per sq ft of roof surface, iron work 16 lb and snow 17 lb per horizontal sq ft, and wind 35 lb per sq ft of elevation.

In the Hoboken Terminal of the D.,L. & W.R.R. the trainshed consists of a low roof made of 2 in of concrete reinforced with expanded metal. Continuous skylights are made of 3/8-in wired glass. Over the center of each track a 2 ft 6 in smoke opening is left which is continuous except at the supporting girders, and along each side of these openings there is a concrete apron extending downward almost to the top of the smokestack of a locomotive.

The roof is supported on steel purlins which are carried by plate girders spanning two tracks and spaced 27 ft apart. These girders are slightly arched, and are supported by cast-iron columns set along the center line of each platform, thus making spans of 43 ft 4½ in for the girders. There are 6 spans of this length and 2 smaller ones. Expansion joints are placed at every fourth bent, that is, 108 ft apart.

Data Concerning Trainsheds

Railway	Location	Type	Span	Width, ft	Length, ft	No. of Tracks
			ft in			
Pennsylvania	Philadelphia	1 Arch	300 8	304	598	16
Pennsylvania	Pittsburgh	1 Arch	255 0⅜	260	555	12
Pennsylvania	Jersey City	1 Arch	252 8	256	653	12
Phil. & Reading	Philadelphia	1 Arch	259 0	266	506	13
South Station	Boston	2 Truss 1 Truss	169 1 228 6	570	602	28
Chicago, R. I. & P.	Chicago	1 Truss	207 0	215	578	11
Union Station	St. Louis	5 Truss	90 to 141 0	601	700	30
Del., Lack. & Western	Hoboken	8 Girders	43 4½	339	607	14

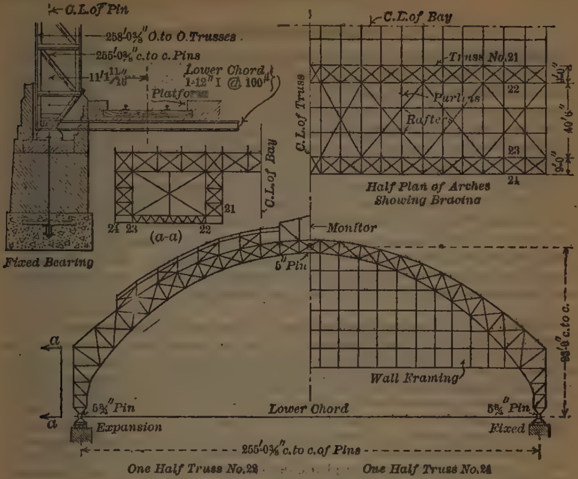


Fig. 21. Pennsylvania R.R. Trainshed, Pittsburgh

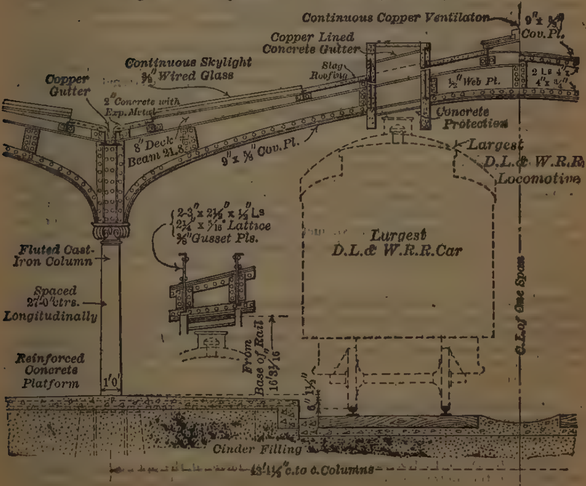


Fig. 22. Part of Trainshed, D.,L. & W.R.R., Hoboken, N. J.

8. Cost Data and Building Laws

The following Costs are average values for buildings. For cost of plain structural steel at the mill see Art. 16. The COST OF SHOP WORK in cents per pound for light building work is as follows: roof trusses 1.00 to 2.50, hip and valley framing 2.50 to 5.00, beam work 0.15 to 0.75, columns from 0.75 for I beams to 1.75 for riveted plate and L sections, plate girders 1.00 to 1.50. Steel mill-building frames, shop work about $35.00 per short ton including drafting. Painting and drafting see Art. 16. ERECTION OF STEEL: bolted work $12.00 to $18.00 per short ton, riveted work $15.00 to $25.00, for office buildings with riveted connections $17.00 per short ton is a common figure. CORRUGATED STEEL $12.00 to $18.00 per square, furnished and laid. ROOFS per square: slate $15.00, to $20.00, three- or four-ply gravel, not including sheathing, $4.00 to $6.00. STRUCTURAL STEEL for buildings costs approximately from 5 to 8 cents per lb in place complete, including painting. Two-inch cement mortar walls on metal fabric $2.00 to $3.00 per sq yd.

Material. The "Standard Specifications for Structural Steel for Buildings" of the American Society for Testing Materials are coming into general use. Among required properties of structural steel mentioned in these specifications are: ultimate tensile strength, 55 000 to 65 000 lb per sq in of cross-section; maximum phosphorus, 0.06%; minimum percent of elongation in 8 in, 1 400 000 divided by the ultimate tensile strength, for material over ¾ in in thickness a deduction of 1 from the above percentage of elongation shall be made for each increase of ⅛ in in thickness above ¾ in to a minimum of 18%, for material under 5/16 in in thickness a deduction of 2.5 from the above percentage of elongation shall be made for each decrease of 1/16 in in thickness below 5/16 in; cold bends without fracture 180 degrees flat for plates, shapes, and bars ¾ in or under in thickness, for thickness over ¾ in and including 1¼ in around a pin the diameter of which is equal to the thickness of the specimen, and for material over 1¼ in in thickness around a pin the diameter of which is equal to twice the thickness of the specimen.

Rivet steel shall have an ultimate tensile strength of 45 000 to 56 000 lb per sq in of cross-section; maximum phosphorus, 0.06%; maximum sulfur, 0.045%; minimum percent of elongation in 8 in, 1 400 000 divided by the ultimate tensile strength; cold bends without fracture 180 degrees flat.

Standard Specification for Steel Castings and for Gray Iron Castings have also been issued by the A. S. T. M.

Workmanship. All workmanship should be first class in every respect and in accordance with practice followed by the best bridge shops. Before being worked material should be thoroughly straightened by methods that will not injure it. Punching should be done accurately, but occasional inaccuracies in matching of holes may be corrected with reamer. The diameter of the punch should not be more than 1/16 in larger, nor that of the die ⅛ in larger than the diameter of the rivet. Rivets should be driven by pressure tools wherever possible. Abutting surfaces of compression members, except where joints are fully spliced, should be planed to even bearing so as to give close contact thruout.

Painting. Red Lead or Graphite Paints are commonly used for painting steel in buildings, one coat being applied at the shop and one after erection. Linseed oil may be used instead of paint for the shop coat. Red-lead paint should be mixt just before using and should be kept thoroly stirred and should be applied with round, not flat brushes. Painting should not be allowed in wet or freezing weather nor on any but clean surfaces. (See Art. 16.)

Cast iron need not be painted until after erection. Steelwork for foundations to be entirely embedded in concrete need not be painted but should be free of dirt, grease or other matter which would impair the bond of the concrete.

Inspection. All inspection and tests are usually made at the option and expense of the purchaser, the contractor furnishing necessary test pieces and the free use of a testing machine.

Erection. Care should be taken that all steelwork be level and plumb before bolting or riveting and that proper provision be made for resisting stresses due to erection operations. Field connections are better when riveted than when bolted but specifications are often needlessly severe in calling for all connections to be riveted. Field connections may be bolted thruout for one story buildings carrying no concentrated loads, shafting or cranes. Other buildings should have connections riveted for column splices; trusses, girders and beams to columns, chord splices of trusses and for bracing. In structures subject to heavy loads and vibrations rivets should be used throughout, except purlins and girts that do not form part of the bracing may be bolted. Drift pins should be used only to bring parts together. Unfair holes should be made to match by reaming.

Building Laws. In the United States there are 60 or 70 cities having a population of over 100 000 each and about 150 cities with a population of from 30 000 to 100 000 each in which are located the great majority of buildings other than mill buildings requiring steel in their construction, the design of which must usually be in accordance with a municipal building code. The engineer should acquaint himself with these regulations before beginning his work. As the data given in hand-books may have been superseded, all information should be obtained from the building code itself. When codes are incomplete, as many are, the engineer must supply lacking data from such authorities as he deems advisable.

Allowable Live Loads for Floors According to Building Laws

Class of Building	Maximum Live Load, pounds per square foot			
	New York	Chicago	Philadelphia	Boston
Dwellings, Apartment Houses, Hotels, Tenement Houses or Lodging Houses	40, 60	40, 50	70	50
Office Buildings, First Floor	....	....	100	100
Office Buildings, above First Floor	60	50	100	100
Schools or Places of Instruction	75	75	....	60–125
Stables or Carriage Houses	....	40* or 100†	....	....
Buildings for Public Assembly	100	100	120	125–200
Buildings for Ordinary Stores, Light Manufacturing and Light Storage	120	100	120	125
Stores for Heavy Materials, Warehouses and Factories	....	....	150	250
Roofs, pitch less than 20°	40	25	30	....
Roofs, pitch more than 20°	40	25	30	....
Sidewalks	300	....	....	....
Public Buildings, except Schools	....	....	....	125

* Stables less than 500 sq ft in area. † Stables over 500 sq ft in area.

Cast-iron Columns to be used in construction of buildings in New York City must not have diameter less than 5 in or thickness of metal less than ¾-in; nor shall they have an unsupported length of more than 70 times their least radius of gyration or 20 times

Allowable Unit Loads for Columns According to Building Laws

Material of Column	Allowable Unit-Load, pounds per square inch			
	New York	Chicago	Philadelphia	Boston
Mild Steel	$16\,000-70\frac{l}{r}$	$16\,000-70\frac{l}{r}$	$\frac{14\,500}{1+\frac{l^2}{13\,500\,r^2}}$	$\frac{16\,000}{1+\frac{l^2}{20\,000\,r^2}}$
Medium Steel	$16\,000-70\frac{l}{r}$	$16\,000-70\frac{l}{r}$	$\frac{16\,250}{1+\frac{l^2}{11\,000\,r^2}}$	$\frac{16\,000}{1+\frac{l^2}{20\,000\,r^2}}$
Wrought Iron	....	$12\,000-60\frac{l}{r}$	$\frac{12\,500}{1+\frac{l^2}{15\,000\,d^2}}$	$\frac{12\,000}{1+\frac{l^2}{20\,000\,r^2}}$
Cast Iron	$9\,000-40\frac{l}{r}$	$10\,000-60\frac{l}{r}$	$\frac{11\,600}{1+\frac{l^2}{400\,d^2}}$	$11\,300-30\frac{l}{r}$

l = length, r = radius of gyration, d = diameter or least side, all in inches.

their least lateral dimension or diameter except by special permission or except as modified by column formula; top and bottom flanges, seats and lugs must be of ample strength reinforced by fillets and brackets, and shall be at least 1 in thick when finished. Column joints secured by not less than 4 bolts of at least ¾-inch diameter. The core of a column below a joint shall be not larger than the core of the column above and the metal shall be tapered down for a distance of at least 6 in, or a joint plate of sufficient strength may be inserted to distribute the load. If the core of a column shifts more than ¼ the thickness of the shell, the strength shall be computed by assuming the thickness of metal all around equal to the thinnest part, and the column will be condemned if this computation shows the strength to be less than that required by the New York building code. Whenever blowholes or imperfections are found which reduce the cross-sectional area of the column more than 10 percent, such column will be condemned. Posts and columns not having one open side or back to be drilled with ⅝-inch hole in shaft to show thickness.

Wind Bracing. Building codes vary greatly in their provisions for wind bracing. The New York code requires that all buildings over 150 ft in height and all buildings or parts of buildings in which the height is more than four times the minimum horizontal dimension, be designed to resist a horizontal wind pressure of 30 lb for every sq ft of exposed surface measured from the ground to the top of the structure. When the stress in any member due to wind does not exceed 50% of the stress due to live and dead loads, it may be neglected. When such stress exceeds 50% of the stress due to live and dead loads, the specified working stresses may be increased by 50% in designing such members to resist the combined stresses. The Chicago code specifies that all buildings shall be designed to resist a horizontal wind pressure of 20 lb per sq ft and allows the same increase of working stresses as the New York code. In Philadelphia a wind pressure of not less than 30 lb per sq ft is called for on all buildings erected in open spaces or on wharves. On tall buildings erected in built-up districts the wind pressure is to be figured for not less than 25 lb at tenth story, 2½ lb less on each succeeding lower story, and 2½ lb additional on each succeeding upper story to a maximum of 35 lb at the fourteenth story and above. In proportioning members subject to stresses due to wind loads the working stresses may be increased 30%. The code of Boston has on the subject only

Allowable Unit Stresses According to Building Laws

Kind of Material	Allowable Unit-Stress, lb per sq in			
	New York	Chicago	Philadelphia	Boston
COMPRESSION: Rolled Steel	16 000	14 000	14 500, 16 250	....
Cast Steel	16 000	14 000	....	....
Wrought Iron	....	10 000	12 500	....
Cast Iron (in Short Blocks)	16 000	10 000	11 600	....
Steel Pins and Rivets (Bearing)	24 000	25 000	....	18 000
Wrought-Iron Pins and Rivets (Bearing)	....	....	....	15 000
TENSION: Rolled Steel	16 000	16 000	14 500, 16 250	16 000
Cast Steel	16 000	16 000	....	....
Wrought Iron	....	12 000	12 500	12 000
Cast Iron	3 000	....	....	....
EXTREME-FIBER STRESS, BENDING:				
Rolled-Steel Beams	16 000	16 000	....	16 000
Rolled-Steel Pins, Rivets and Bolts	20 000	22 500	....	22 500
Riveted Steel Beams (Net Flange Section)	16 000	15 000	....	....
Rolled Wrought-Iron Beams	....	....	....	12 000
Rolled Wrought-Iron Pins, Rivets and Bolts	....	....	....	18 000
Riveted Wrought-Iron Beams (Net Flange Section)	....	....	....	....
Cast Iron,—Compression Side	16 000	10 000	....	16 000
Cast Iron, Tension Side	3 000	3 000	3 750	3 000
Compression in Flanges of Built Beams, Steel	....	....	....	16 000
Compression in Flanges of Built Beams, Wrought Iron	....	....	....	12 000
SHEAR: Steel Web Plates	10 000	10 000	8 750, 10 000	10 000
Steel Shop Rivets and Pins	12 000	12 000	8 750, 10 000	10 000
Steel Field Rivets and Pins	8 000	10 000	8 750, 10 000	10 000
Steel Field Bolts	7 000	....	8 750, 10 000	8 000
Wrought-Iron Web Plates	....	....	7 500	9 000
Wrought-Iron Shop Rivets and Pins	....	....	7 500	9 000
Wrought-Iron Field Rivets	....	....	7 500	9 000
Wrought-Iron Field Bolts	....	....	7 500	7 200
Cast Iron	3 000	....	....	....

Under Philadelphia, the first value is for mild and the second for medium steel.

the ambiguous sentence, " Provision for wind bracing shall be made wherever it is necessary."

A number of provisions besides those for floor and wind loads affect the design of the steelwork. FIREPROOFING AROUND EXTERIOR COLUMNS in New York must be 8 in thick on the outer face while in Chicago 4 in will answer. THICKNESS OF ENCLOSURE WALLS when carried by the steelwork is specified as 12 in for New York, Chicago and many other cities. A large number have a thickness of 12 in for a top section of 75 ft and a greater thickness for sections below. Still other variations are found.

Limitations upon the HEIGHTS OF BUILDINGS are placed in many cities. For fireproof commercial buildings 2½ times the width of the adjoining street is quite common. The absolute limit ranges from 120 ft in Providence to 250 ft in New York, St. Louis and St. Paul. The Boston limit of 125 ft is followed

by a dozen cities. A number of other cities have limits of 150 to 175 ft. Chicago, Indianapolis, Newark, Paterson, place the limit at 200 ft and Milwaukee at 225 ft. SPECIAL REQUIREMENTS must not be overlooked. One such, found in the Atlanta, Newark, Paterson and other codes, is that prohibiting the spacing of floorbeams of fireproof buildings more than 5 ft centers for stores, warehouses and factory buildings and 8 ft centers for other buildings. This eliminates "long-span" construction. Other requirements are confined to the individual code in which they are found. As stated previously, the structural engineer should be acquainted with the municipal code by which the construction of his building is governed.

GENERAL DATA FOR BRIDGES

9. Dead Loads

Simple Bridges include all beam, girder or truss bridges supported at both ends only. CONTINUOUS bridges include those which continue unbroken over two or more spans and which under vertical loads have vertical reactions. A cantilever beam is one that is fixt at one end and unsupported at the other and a CANTILEVER bridge is therefore one having one or more cantilevers. An ARCH bridge may be defined as one which under vertical loads produces inclined pressures on its supports, thus including SUSPENSION bridges which have the end reactions outward as well as the more common forms of arched bridges which have reactions upward and inward.

The first important bridge of metal was a cast-iron arch of 100-ft span built at Coalbrookdale in England in 1779, and the first iron truss bridge in the United States is believed to be the one built at Frankfort, N. Y., in 1840. The largest cast-iron bridge in America was built in 1866, in Chestnut St., Philadelphia, its longest span being an arch of 185 ft. Steel was first used in bridge construction in the arches of the Eads bridge at St. Louis in 1874. The modern steel truss span is the outgrowth of the wooden and the combination truss, the latter being a truss in which some members are of wood and some of iron.

A Deck Bridge is one in which the floor system is supported on or near the top chord of the trusses or girders, and a THRU BRIDGE is one in which the floor is on or near the lower chord. A PONY TRUSS bridge is a thru bridge so shallow that no overhead bracing can be used. Bridges are SQUARE or SKEW, the former including those having the ends of the span at right angles to the length, hence having the trusses parallel and of the same length; while skew spans have one or both ends at oblique angles with the length and the trusses of a span may be of the same or different lengths.

Loads on Bridges include the dead or fixt, the live or movable, impact due to live loads, centrifugal force due to electric cars or trains moving on curved tracks, snow, longitudinal forces due to friction, and forces due to changes of temperature.

The Dead Load of railroad bridges includes the weight of the girders and trusses, of the top, bottom and transverse lateral bracing, and of the floor system which may be of either the open or solid type. The following weights are approximate only but are sufficient for use in preliminary designing and they should be checked from the finished design in every case. For ordinary open floors the weight of track is from 400 to 600 lb per ft for one track including rails, guard timbers, ties and all fastenings. The weight of steel per foot of span in single track railroad bridges of the several types named below, having open floors and designed for Cooper's Class E-40, E-50 and E-60 loadings

(Art. 10) in accordance with the American Railway Engineering Association Specifications, 1910, is approximately exprest by:

Deck plate girders....................	$w=k(13\,l+100)$
Thru plate girders....................	$w=k(15\,l+500)$
Thru pin-connected or riveted trusses....	$w=k(9\,l+700)$

where l = length of span; $k=0.88$ for E–40; $k=1.00$ for E–50 and $k=1.12$ for E–60 loadings.

The weight of steel in a double-track girderbridge is about 100% greater, and in a double-track truss bridge from 80 to 90% greater than for a similar single-track structure.

The Total Weight of Material in one single-track span is found by adding the weight of the track, 400 to 600 lb per ft, to the weight of steel as given by the above formulas and multiplying by the span in feet. A DEAD PANEL LOAD on one truss is the weight on one panel point from one panel length of truss and floor system. For a single-track bridge the panel load on one truss is found by multiplying the total dead load per foot of truss by the panel length. For example, if a single-track thru bridge designed for a Cooper E–50 live load, has a span of 200 ft divided into 8 equal panels of 25 ft the above formula, $w=9\,l+700$ gives 2500 lb per ft as the weight of steel in the bridge. If the track weighs 450 lb per foot of bridge the total load per foot of one truss = $\frac{1}{2}(2500+450)=1475$ lb, and the panel load per truss is $1475\times25=36\,875$ lb. It is customary to assume 2/3 of the panel load on the loaded and 1/3 on the unloaded chord panel points, but a better method is to divide the weight of the truss equally between the upper and lower chords and apply the weight of track and floor after the latter has been obtained from the floor design at the loaded chord.

Highway Bridge Dead Loads include in addition to the weight of the sidewalks, if any, the weight of the roadway and the steel in the stringers, floor beams, trusses and bracing. Highway bridges vary greatly in size and design and no formula will give accurate results for the dead weight. Of the various ones proposed the following by Waddell and Hedrick is commonly used for highway bridges for heavy service:

$$w_1=34+22\,b+0.16\,bl+0.7\,l$$

where w_1 = the total dead load in pounds per linear foot of bridge, including flooring, stringers, trusses, sidewalks and laterals; l = the span in feet; b = clear width of roadway and sidewalks in feet. For computing the loads on highway bridges the following WEIGHTS OF MATERIALS in pounds per cubic foot will be found useful: timber, creosoted, 60; oak, untreated, 54; pine, untreated, 48; concrete, 150; paving brick, 150; granite blocks 160; asphalt, 135; macadam, 130; sand or earth, 100; stone ballast, 100; steel, 490; cast iron, 450. The following weights are in pounds per linear ft: ordinary latticed sidewalk railings, 30; rails and fastenings, street railway tracks, 100 per track; rails and fastenings, railroad tracks, 150 per track.

Snow Loads for railroad bridges are usually negligible, but for highway bridges in cold climates it is necessary to consider the snow load in some cases, especially for drawbridges when open. The maximum allowance is 20 lb per sq ft.

10. Live Loads

Live or Movable Loads ordinarily used for the design of railroad bridges consist of two locomotives followed by a uniform load representing the weight of the heaviest cars. To allow for future increase the weights specified by this

loading should be greater than the heaviest locomotives and cars in service on the road for which the bridge is designed. Wheel spacings and weights of actual rolling stock are so variable that typical loads are largely used, those specified by Cooper being most common. Fig. 23 shows Cooper's Class *E*–50

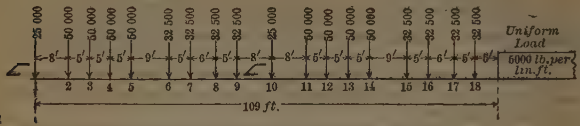


Fig. 23. Cooper's Standard Loading, Class E–50

standard loading, consisting of two consolidation locomotives followed by a uniform load of 5000 lb per linear foot. The loads given are axle loads. Cooper's standards provide for heavier traffic by using Classes E–60 and E–55 and for lighter traffic by using Classes E–45, E–40 and E–30, each of which has the same wheel spacing as Class E–50, but the weights, and hence all functions of these weights such as reactions and stresses, are less, being proportional to the Class number; thus the weights of E–60 are 12/10 those of E–50 and of E–45 are 9/10 those of E–50; E–40, 8/10 of E–50; E–30, 6/10 of E–50.

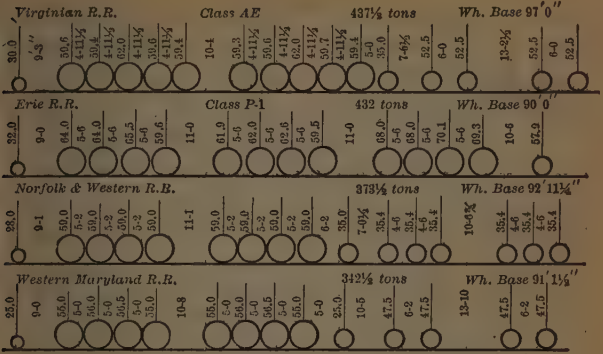
The rapid increase in weight of the engines in actual service has made it necessary to use the heavier Cooper classes and at present, 1918, very few bridges on main lines are designed for less than E–50, while the general tendency is towards the use of E–60 and even higher. An examination of the specified loadings of 50 roads, after reducing each loading to an equivalent based upon the A. R. E. A. specifications, shows that 9 roads use less than E–55; 17 between E–55 and E–60; 19 between E–60 and E–65; and 4 between E–65 and E–75. Several of these roads contemplate increases. The Bessemer and Lake Erie Ry. bridge across the Allegheny river was designed for E–75; and the Paducah and Ill. Central R.R. bridge across the Ohio river at Metropolis for E–90. In the latter case, however, higher unit stresses than those allowed by the A. R. E. A. specifications were used. For stringers and short spaces two axle loads of a heavy passenger engine are used wherever they govern. The Western Maryland Ry. specifies two such axle loads of 72 000 lb each, spaced 7 ft on centers.

Fig. 24 shows four of the heaviest articulated engines of the Mallet type in actual service. They are only slightly heavier than similar engines used on many other roads.

Locomotives of this type produce stresses in spans of from 10 to 150 ft which differ somewhat from those obtained by the use of the Cooper loadings. Thus the Virginian Class A E locomotive produces center moments which exceed those from the E–60 loading by 12% in a 30-ft, 14.7% in a 60 ft, 19.4% in an 80 ft, and 20.2% in an 100-ft span, with lessening percentages in longer spans.

The effect of an overload upon a bridge is to increase maximum stresses in the counters, hip verticals and center web members proportionately more than in the chords and end web members. A well-designed bridge will have initially such excess of material in the former that under the maximum possible overload the specified unit stresses for all members of the truss will be increased in about the same proportion. Such a bridge should carry safely an overload of 50%. Many prominent engineers believe that a bridge properly designed under the A. R. E. A. specifications for E–60 will not be dangerously overloaded for a long time to come, if ever, and before this can happen there are many

other limiting features in railroad and locomotive construction which will have first to be radically changed.



All loads are in thousand pounds per axle. Wheel spacings are in feet and inches.

Fig. 24. Wheel Diagrams of Heavy Locomotives in Service

The ELECTRIC LOCOMOTIVE of the N.Y.C. & H.R.R.R. weighs about 138 000 lb, distributed nearly equally on 4 axles and 131 000 lb on the 4 truck axles, making about 269 000 lb total weight.

Equivalent Uniform Live Loads which will give stresses as close as possible to those computed for actual wheel loads are sometimes used, the most common for computing chord stresses being the uniform load per foot which produces a moment at the center of the span equal to the maximum moment produced on the span by two locomotives followed by a uniform train load. The equivalent uniform load for computing web stresses is the uniform load per foot which produces an end shear equal to that produced by the locomotive loading. Table on p. 850 gives for Cooper's E-50, maximum moments; end, quarter point and center shears; pier reactions for equal spans; and the equivalent uniform loads based upon center moments, end shears and pier reactions.

An Exact Equivalent Uniform Load is one which gives exactly the same stress in a member as the actual wheel loading. Fig. 25 is a chart, constructed by Dr. D. B. Steinman, from which may be obtained the exact equivalent load, based upon Cooper's E-60 loading, for any stress function having a triangular influence line the lengths of whose segments are l_1 and l_2. Since, for end reactions or intermediate shears in a girder without floor beams the influence diagram is a triangle with $l_1 = 0$, instead of extending the chart the values of the corresponding equivalent uniform loads are given in a separate table. Fig. 26, also by Dr. Steinman, gives the governing wheel of a Cooper loading for maximum of any stress having a triangular influence line. The use of these charts will be explained in Arts. 18, 25, 26 and 27 in connection with stresses in girders and trusses.

The Live Loads for a Highway Bridge should be chosen to correspond with the location of the structure, suitable provision being made for future traffic. The specifications most commonly used are those of American Bridge

Maximum Moments, Shears, Floor Beams or Pier Reactions, and Equivalent Uniform Loads for Cooper's Class E-50

For One Rail

Span, ft	Max. Moment, *M*	Max. Shears, *V*			Max. Pier React. *R*	Equivalent Uniform Load		
		End	¼ Pt.	Cent.		For Max. Moment	For Max. End Shear	For Max. Pier React.
10	70.4	37.5	25.0	12.5	50.0	5625	7500	5000
11	82.1	40.9	26.1	13.6	54.5	5430	7450	4960
12	100.0	43.8	27.1	14.6	58.4	5560	7290	4860
13	118.8	46.2	27.9	15.4	61.6	5625	7100	4740
14	137.5	48.2	29.5	16.2	65.2	5610	6900	4660
15	156.3	50.0	31.3	16.6	68.3	5560	6670	4555
16	175.0	53.1	32.9	17.1	71.1	5470	6640	4445
17	193.8	55.9	34.3	17.3	73.5	5360	6580	4325
18	212.5	58.3	35.4	17.4	75.9	5250	6490	4210
19	233.3	60.5	36.5	17.5	78.6	5170	6370	4140
20	257.9	62.5	37.5	17.5	81.9	5160	6250	4100
21	282.5	64.3	39.2	18.1	84.9	5100	6125	4040
22	307.1	65.9	40.9	18.8	87.6	5050	5990	3980
23	331.8	67.4	42.4	19.3	90.2	5000	5860	3920
24	356.5	69.3	43.8	19.8	92.4	4940	5775	3850
25	381.3	71.0	45.0	20.2	94.6	4880	5680	3780
26	406.0	72.6	46.1	20.6	97.1	4820	5585	3735
27	430.8	74.0	47.2	21.1	100.1	4750	5480	3710
28	456.9	75.5	48.2	21.4	102.8	4690	5390	3670
29	485.0	76.9	49.1	21.8	105.4	4625	5300	3635
30	513.0	78.8	50.0	22.1	107.9	4560	5255	3595
31	541.1	80.5	50.9	22.7	110.6	4500	5195	3570
32	569.3	82.1	51.8	23.4	113.7	4450	5130	3555
33	597.4	83.7	52.5	24.0	116.7	4390	5070	3535
34	625.8	85.1	53.5	24.6	119.4	4330	5005	3510
35	653.8	86.5	54.4	25.1	122.0	4275	4945	3485
36	685.8	88.2	55.1	25.8	124.4	4240	4900	3455
37	717.9	89.8	56.0	26.2	126.9	4200	4855	3430
38	750.0	91.4	56.7	26.6	129.7	4170	4810	3410
39	783.3	92.9	57.5	27.1	132.3	4135	4765	3390
40	819.5	94.3	58.5	27.5	135.0	4100	4715	3375
41	855.8	96.0	59.4	27.9	137.6	4070	4680	3355
42	892.0	97.6	60.2	28.3	140.2	4040	4650	3340
43	928.3	99.2	61.1	28.6	142.9	4010	4615	3325
44	964.5	100.7	61.9	29.0	145.6	3980	4580	3310
45	1000.8	102.1	62.6	29.3	148.3	3960	4540	3295
46	1037.3	103.5	63.4	29.6	150.9	3930	4500	3280
47	1073.3	104.9	64.2	29.9	153.4	3900	4465	3265
48	1109.5	106.3	65.1	30.2	156.0	3870	4430	3250
49	1148.5	107.7	66.0	30.6	158.5	3840	4395	3235
50	1188.6	109.0	66.8	31.1	161.0	3810	4360	3220

Maximum Moments, Shears, Floor Beam or Pier Reactions—(Continued)

Span, ft	Max. Moment, *M*	Max. Shears, *V* End	Max. Shears, *V* ¼ Pt.	Max. Shears, *V* Cent.	Max. Pier React. *R*	Equivalent Uniform Load For Max. Moment	Equivalent Uniform Load For Max. End Shear	Equivalent Uniform Load For Max. Pier React.
51	1228.9	110.4	67.6	31.5	163.6	3790	4330	3210
52	1269.0	111.8	68.5	31.9	166.6	3770	4300	3205
53	1309.2	113.1	69.2	32.3	169.6	3750	4270	3200
54	1351.8	114.5	70.1	32.6	172.5	3730	4240	3195
55	1396.1	115.8	71.0	33.0	175.4	3710	4215	3195
56	1440.5	117.2	71.8	33.3	178.5	3690	4185	3195
57	1484.9	118.5	72.7	33.6	181.8	3670	4160	3195
58	1529.2	119.8	73.5	34.0	185.1	3650	4130	3195
59	1576.2	121.2	74.4	34.4	188.4	3630	4110	3195
60	1624.5	122.5	75.2	34.9	191.5	3610	4080	3195
61	1672 9	123.9	76.0	35.2	194.7	3600	4060	3190
62	1721.2	125.2	76.6	35.6	197.7	3585	4040	3190
63	1769.5	126.6	77.4	36.0	200.7	3570	4020	3185
64	1819.4	128.2	78.5	36.4	203.6	3560	4005	3180
65	1871.9	129.7	78.8	36.8	206.7	3550	3990	3180
66	1924.4	131.2	79.5	37.1	209.7	3535	3975	3175
67	1976.9	133.0	80.3	37.5	212 7	3520	3970	3175
68	2029.4	134.8	81.0	37.8	215.6	3510	3965	3170
69	2081.9	136.5	81.7	38.1	218.5	3500	3955	3165
70	2134.4	138.1	82.4	38.4	221.3	3485	3945	3160
71	2186.6	139.8	83.1	38.8	224.1	3475	3940	3155
72	2241.2	141.7	83.8	39.2	226.9	3460	3935	3150
73	2292.4	143.5	84.4	39.6	229.6	3450	3930	3145
74	2349.0	145.3	85.0	40.0	232.4	3440	3925	3140
75	2407.3	147.1	85.7	40.4	235.2	3430	3920	3135
76	2465.0	148.8	86.5	40.8	238.0	3420	3915	3130
77	2523.9	150.5	87.4	41.1	240.7	3410	3910	3125
78	2581.2	152.1	88.2	41.5	243.3	3400	3900	3120
79	2640.4	153.8	88.9	41.7	245.9	3385	3895	3115
80	2700.6	155.3	89.6	42.1	248.6	3375	3885	3110
81	2759.6	157.0	90.4	42.5	251.1	3370	3875	3100
82	2820.9	158.6	91.2	43.0	253.6	3360	3870	3090
83	2883.1	160.3	92.1	43.4	256.1	3350	3860	3085
84	2945.4	161.8	93.0	43.7	258.7	3345	3855	3080
85	3008.6	163.4	93.9	44.1	260.8	3335	3850	3070
86	3074.5	165.1	94.3	44.5	263.0	3325	3840	3060
87	3138.3	166.8	95.7	44.9	265.6	3320	3830	3055
88	3205.3	168.4	96.5	45.2	268.3	3310	3825	3050
89	3269.9	170.0	97.4	45.6	270.8	3300	3820	3040
90	3338.1	171.5	98.4	45.9	273.2	3295	3810	3035
91	3403.7	173.1	99.4	46.2	275.6	3290	3805	3030
92	3470.9	174.7	100.4	46.6	278 0	3280	3800	3020
93	3539.3	176.4	101.2	46.9	280.3	3270	3795	3015
94	3606.6	178.0	102.1	47.3	282.7	3265	3790	3005
95	3674.3	179.5	103.1	47.5	285.1	3260	3780	3000

Maximum Moments, Shears, Floor Beam or Pier Reactions—(Continued)

Span, ft	Max. Moment, M	Max. Shears, V			Max. Pier React. R	Equivalent Uniform Load		
		End	¼ Pt.	Cent.		For Max. Moment	For Max. End Shear	For Max. Pier React.
96	3743.1	181.0	104.1	47.9	287.5	3250	3770	2995
97	3811.2	182.7	105.1	48.1	289.7	3240	3765	2985
98	3883.1	184.3	106.2	48.5	292.0	3235	3760	2980
99	3952.9	186.0	107.2	48.9	294.2	3225	3755	2970
100	4024.9	187.5	108.2	49.2	296.5	3220	3750	2965
101	4097.0	189.0	109.1	49.5	298.6	3220	3745	2955
102	4169.9	190.6	110.1	49.9	300.8	3220	3740	2950
103	4263.3	192.1	111.0	50.1	303.0	3215	3730	2940
104	4344.0	193.6	111.9	50.5	305.3	3215	3725	2935
105	4422.0	195.1	112.7	50.7	307.5	3215	3715	2930
106	4500.4	196.6	113.6	51.1	309.8	3215	3710	2920
107	4583.3	198.1	114.5	51.5	312.0	3215	3700	2915
108	4681.6	199.5	115.5	51.7	314.2	3210	3695	2910
109	4773.0	201.0	116.4	52.0	316.3	3210	3690	2900
110	4858.5	202.5	117.4	52.3	318.5	3210	3680	2895
111	4947.7	204.0	118.2	52.5	320.7	3210	3675	2890
112	5033.6	205.5	119.1	52.7	322.8	3210	3670	2880
113	5123.8	207.0	120.0	53.1	324.9	3210	3665	2875
114	5215.0	208.4	121.0	53.5	327.0	3210	3655	2870
115	5306.2	209.9	121.9	53.9	329.0	3210	3650	2860
116	5398.5	211.3	122.9	54.2	331.1	3210	3645	2855
117	5486.9	212.8	123.7	54.6	333.3	3205	3640	2850
118	5579.7	214.2	124.6	54.9	335.6	3205	3630	2845
119	5673.5	215.7	125.5	55.3	337.8	3205	3625	2840
120	5767.6	217.1	126.4	55.6	340.0	3205	3620	2835
121	5858.1	218.6	127.2	55.9	342.2	3205	3610	2830
122	5953.4	220.0	128.1	56.2	344.5	3205	3605	2825
123	6045.2	221.4	129.0	56.5	346.7	3200	3600	2820
124	6146.7	222.8	130.0	57.0	349.0	3200	3595	2815
125	6245.5	224.2	130.9	57.5	351.2	3200	3590	2810
150	8827.9	259.2	152.2	68.0	406.7	3140	3455	2710
175	11690.6	293.1	172.9	78.2	464.6	3055	3350	2655
200	14841.2	326.3	191.8	88.0	523.8	2965	3265	2620
250	21990.6	391.5	229.6	106.3	644.0	2815	3130	2575

For span lengths L greater than 284 ft, max. moment $= 5/16 L^2 + 2375$.

For span lengths L greater than 100 ft, max. end shear $= 1.25L + 90 - 2750/L$.

For span lengths L each greater than 142 ft, max. pier reaction $= 2.5L + 4750/L$.

Moments are in thousand lb ft; shears are given in thousand pounds; pier reactions are given in thousand pounds and are for piers between two spans each equal to the tabulated span; equivalent loads are in pounds per linear foot.

Co., Cooper, Mass. R.R. Commission, and Waddell. Most of the large cities have their own specifications. In many States the design of highway bridges

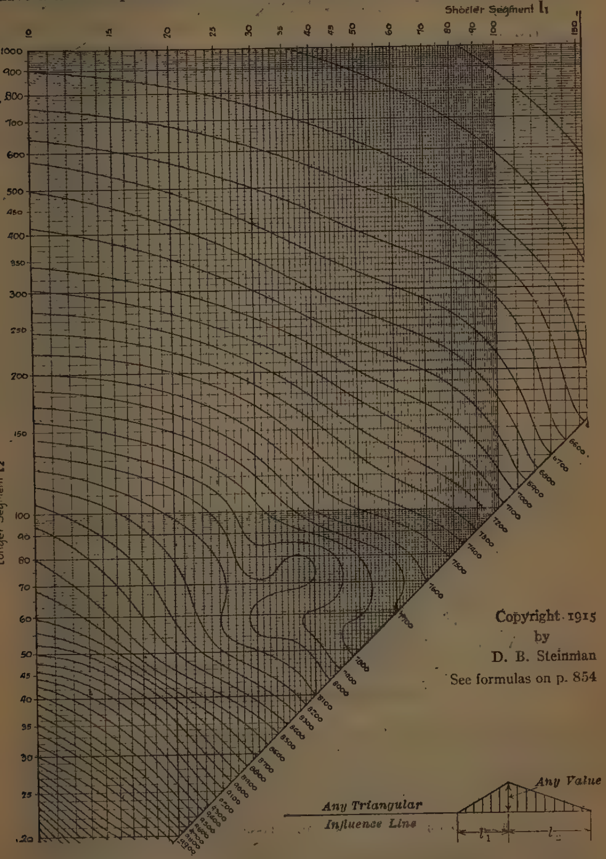


Fig. 25. Equivalent Uniform Loads for Cooper's E60 in Pounds per Linear Foot of Track

must conform with the specifications issued by the Highway Commissions of those States.

The loads are of two kinds: uniform for the trusses, and for the roadway and its supports a uniform load together with wheel loads. On sidewalks the same

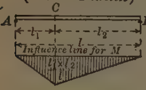
Formulas for Use of Fig. 25

l_1 = Shorter segment of influence triangle.

l_2 = Longer segment.

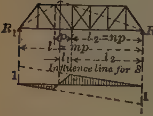
q = Equivalent uniform load, taken from chart for l_1 and l_2.

Bending Moments
(and Chord Stresses)



Max. Moment at C in Span AB. $M = \frac{1}{2}ql_1l_2$.
(l_1 and l_2 = The Two Segments of the Span)

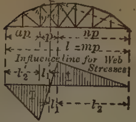
Shears



Max. Shear in Truss Panel: $V = \frac{1}{2}ql_1l_2 \div p$.

$\left(l_1 = \dfrac{l_2}{m-1}\right)$ $(l_2 = np)$.

Web Stresses



$l_1 = \left(\dfrac{m}{ar+b} - 1\right) l_2$ $r = \dfrac{h'}{h}$

For Member shown full:

Max. + Web Stress

$$= \tfrac{1}{2}ql_1l_2\frac{\sec\theta}{rp};$$

Max. — Web Stress

$$= \tfrac{1}{2}q'l'_1l'_2\frac{\sec\theta}{p}.$$

For Member shown dotted:

Max.—Web Stress

$$= \tfrac{1}{2}q'_1l_2\frac{\sec\theta'}{p};$$

Max. + Web Stress

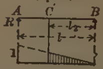
$$= \tfrac{1}{2}q'l'_1l'_2\frac{r.\sec\theta'}{p}.$$

Floorbeam Reactions
Intermediate Pier Reactions

Max. Intermediate Reaction: $R = \frac{1}{2}q(l_1 + l_2)$
(l_1 and l_2 = The two Adjoining Spans.)

Equivalent Uniform Loads for Shears and Reactions

$(l_1 = 0)$



Max. Shear at C: * $V = \frac{1}{2}ql_2\dfrac{l_2}{l}$

Max. Reaction at A: $(l_2 = l)$.
$R = \frac{1}{2}ql$

l_2	q	l_2	q	l_2	q
1000	6418	500	6812	100	9000
975	6429	475	6852	95	9060
950	6440	450	6896	90	9140
925	6451	425	6945	85	9230
900	6463	400	7000	80	9315
875	6476	375	7060	75	9410
850	6490	350	7130	70	9470
825	6504	325	7205	65	9600
800	6519	300	7295	60	9800
775	6535	275	7400	55	10100
750	6552	250	7520	50	10450
725	6571	225	7660	45	10900
700	6591	200	7830	40	11310
675	6612	175	8035	35	11830
650	6634	160	8185	30	12650
625	6658	150	8300	25	13625
600	6683	140	8420	20	15000
575	6711	130	8540	15	16000
550	6741	120	8680	10	18000
525	6775	110	8840	5	24000

* Note: In the formula for "V" the effect of wheel ○ is neglected. To correct for this, subtract

$$30{,}000\left(1 - \frac{l_2 + 8}{l}\right)$$

from the above value of V.

Fig. 25*a*. Influence Diagrams and Equivalent Uniform Loads

uniform load is used as for the roadway. The following is an abstract of loadings adopted by the Mass. Public Service Commission, 1915, for bridges carrying electric railways. They will also apply to bridges without car tracks provided the floor system is designed for both truck and such uniform loading as may reasonably come upon the roadway at the same time.

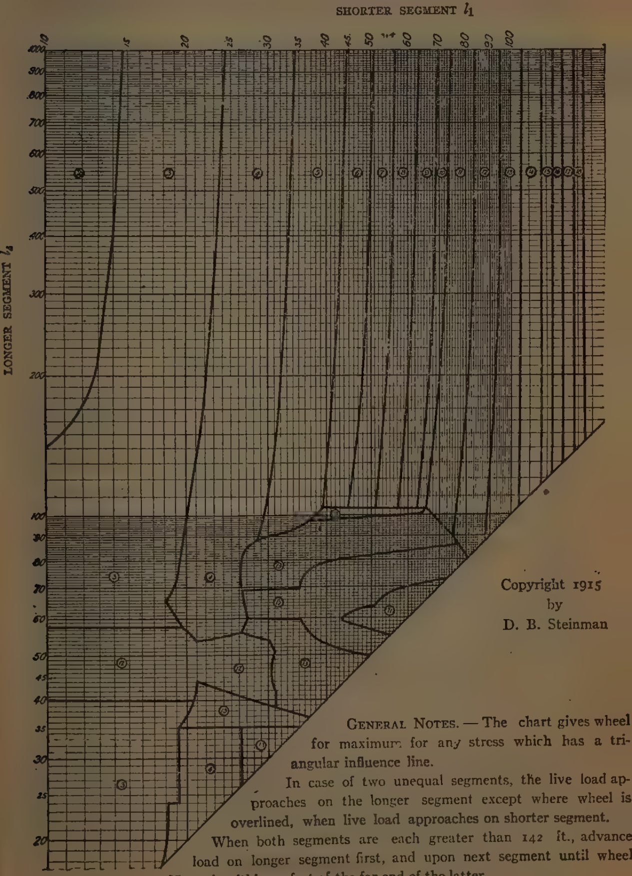
Live Loads. Bridges of the following classes should be designed to carry, in addition to the dead loads, a moving load, either uniform or concentrated, or both, as specified below, placed so as to give the greatest stress in each part of the structure.

For bridges intended for passenger cars:

The floor system and the trusses or girders shall be proportioned to carry on each track a train of two double truck cars coupled together Each car shall be assumed to

weigh, when loaded, 50 tons, and to have a total wheel base of 25 ft. and a wheel base for each truck of 5 ft. The length of each car shall be taken as 40 ft.

For bridges over which it is intended to operate standard steam road freight cars or express or other cars weighing more than 50 tons when loaded:



General Notes. — The chart gives wheel for maximum for any stress which has a triangular influence line.

In case of two unequal segments, the live load approaches on the longer segment except where wheel is overlined, when live load approaches on shorter segment.

When both segments are each greater than 142 ft., advance load on longer segment first, and upon next segment until wheel No. 1 is within 33 feet of the far end of the latter.

Fig. 26. Position of Cooper's Loadings for Maximum Stress

The floor system and the trusses or girders shall be proportioned to carry on each track a train of two double truck cars coupled together, each car weighing, when loaded, 75 tons, the other dimensions being the same as for the passenger car above described.

In highway bridges carrying electric roads the above specifications shall apply with reference to the loads upon the railway track. In addition the following moving loads should be assumed upon the highway floor:

(*a*) For city bridges, subject to heavy loads:

For the floor and its supports, a uniform load of 100 lb per sq ft of surface of the roadway and sidewalks, or the auto truck described under (*d*). In computing the floor beams and supports, the railway load shall be assumed, together with either (1) this uniform load extending up to within 2 ft of the rails, or (2) the auto truck described under (*d*).

For the trusses or girders, 100 lb per sq ft of floor surface for spans of 100 ft or less, 80 lb for spans of 200 ft or over, and proportionally for intermediate spans. This uniform load is to be taken as covering the floor up to within 2 ft of the rails.

(*b*) For suburban or town bridges, or heavy country highway bridges:

For the floor and its supports, a uniform load of 100 lb per sq ft, or the auto truck described under (*d*); these loads to be used as described under (*a*).

For the trusses or girders, 80 lb per sq ft of floor surface for spans of 100 ft or less, and 60 lb for spans of 200 ft or more, and proportionally for intermediate spans; to be used as described under (*a*). (See *d*.)

(*c*) For light country highway bridges:

For the floor and its supports, a uniform load of 80 lb per sq ft, or the auto truck described under (*d*); this load to be used as described under (*a*). (See *d*.)

For the trusses or girders, 80 lb per sq ft of floor surface for spans of 75 ft or less, and 50 lb for spans of 200 ft or more, and proportionally for intermediate spans; to be used as described under (*a*).

(*d*) All parts of the floor of a highway bridge shall be proportioned to carry a 20-ton auto truck having 6 tons on one axle and 14 tons on the other axle, the axles being 12 ft apart and the distance between wheels 6 ft. This truck is assumed to occupy a floor space 32 ft in length and 10 ft in breadth, overhanging all wheels an equal amount.

For the purpose of designing the stringers the following assumptions may be made:

Where plank floors or floors resting on planks are used, each wheel load of the auto truck may be considered to be distributed over a width of floor equal in feet to the thickness in inches of the supporting layer of planking. The width over which the load is distributed shall never, however, be taken as more than 6 ft.

Where solid floors are used each wheel load may be assumed to be distributed over a width of 6 ft.

(*e*) If ties or wooden floor beams are exposed to bending, the weight on one axle shall be considered as distributed equally upon three ties, if the latter are not over 8 in apart in the clear. If they are farther apart, the load on each shall be found by assuming an axle load to be distributed uniformly over a distance of 4 ft.

Impact is the dynamic effect on a bridge due to the moving loads and allowance should be made for it. Two methods are in use; in one the allowable unit stresses for dead and live loads are different, the latter being taken one-half the former. In the other and more common method the allowable dead and live load unit stresses are equal but the live load stress is increased by a certain percent of itself, the increase for railroad bridges being given by

$$I = P\left(\frac{300}{L+300}\right)$$

where I is the impact stress to be added to the live load stress P and L is the loaded length of the bridge in feet that produces the maximum live load stress in the member. For highway bridges the American Bridge Co. specifies that all live load stresses shall be increased 25%. The Mass. Public Service Commission requires an impact percentage to be added to live stresses as follows:

	Percent
For auto truck described under (*d*):	
For stringers, floor beams, hangers, and truss members receiving their whole load from one panel point only	50
For wood flooring and wood stringers, no impact.	

For all other live loads:	Percent
For floor beams and stringers	25
For floor beam hangers	40
For all counters	40

For other members in trusses, and for main girders, the percentage shall be 26⅔ minus $^1/_{12}$ the loaded length in feet, with a maximum of 25 and a minimum of 10 percent.

Wind Loads for railroad bridges are taken as acting horizontally in either direction, (1) At 30 lb per sq ft on exposed surface of all trusses and the floor as seen in elevation: and on the side of a train 10 ft high, beginning at 2½ ft above the base of rail and moving across the bridge. (2) At 50 lb per sq ft on all exposed surfaces of the unloaded structure. The greater calculated stress should be used in proportioning the wind bracing. For determining anchorage for a loaded structure the train should be assumed to weigh 800 lb per linear foot.

For Highway Bridges the above wind loads should be used except that the pressure on the train in (1) should be replaced by 150 lb per linear foot of span and the item regarding anchorage should be omitted.

Traction stresses are those due to the effect of friction of wheels on the rails and are generally negligible except in trestles where the longitudinal bracing in the towers should be designed for a horizontal longitudinal force applied at the rail and equal to 20% of the live load.

The Centrifugal Force due to a weight W moving at a velocity of v ft per second on a radius of r ft is Wv^2/rg, where g is the acceleration due to gravity, 32.2 ft per sec per sec. A speed of $60-3D$ miles per hr is specified by Cooper, D being the degree of curvature. The resulting force is a live load acting 5 ft above the base of rail.

11. Specifications for Loads and Stresses

The General Specifications for Steel Railway Bridges, 1910, of the Am. Railway Engineering Association have been adopted, either in their exact or in some modified form, by the majority of American railways. The essential clauses relating to unit stresses, proportioning, detailing, materials, workmanship and inspection apply and are much used in the design of other classes of steel structures, such as highway bridges and buildings (compare unit stresses for buildings, Art. 2). On account of their importance they are here given nearly in full.

(1) GENERAL

Materials. 1. The material in the superstructure shall be structural steel, except as otherwise specified

Clearances. 2. If the alinement is straight, clearances shall be not less than shown on the diagram. If the alinement is curved, the width of the diagram shall be increased so as to provide the same minimum clearances for a car 80 ft long, 14 ft high, and 60 ft center of trucks, allowance being made for curvature and superelevation of rails. The height of rail shall be assumed as 6 in.

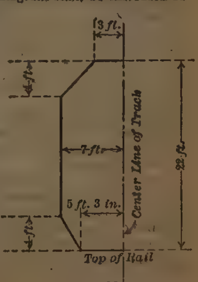


Fig. 26 a

Spacing Trusses. 3. The width center to center of girders and trusses shall not be less than one-twentieth of the effective span, and not less than is necessary to prevent overturning under the assumed lateral loading.

Skew Bridges. 4. In skew bridges without ballasted floors, the ends of girders and beams supporting the track shall be square with the track at the abutments.

Floors. 5. Wooden tie floors shall be secured to the stringers and shall be proportioned to carry the maximum wheel load, with 100 percent impact, distributed over three ties, with fiber stress not to exceed 2000 lb per sq in. Ties shall be not less than 10 ft in length. They shall be spaced with not more than 6-in openings, and shall be secured against bunching.

(2) LOADS

Dead Load. 6. The dead load shall consist of the estimated weight of the entire suspended structure. Timber shall be assumed to weigh 4½ lb per foot B.M.; ballast 100 lb per cu ft, reinforced concrete 150 lb per cu ft, and rails and fastenings, 150 lb per linear foot of track.

Live Load. 7. The live load, for each track, shall consist of two typical engines followed by a uniform load, according to Cooper's series, or a system of loading giving practically equivalent stresses.

Impact. 9. The dynamic increment of the live load shall be added to the maximum computed live load stresses and shall be determined by the formula:

$$I = S\frac{300}{L+300};$$

where I = impact or dynamic increment to be added to live load stresses;
S = computed maximum live-load stress;
L = length of track in feet loaded to produce the maximum stress in the member. For bridges carrying more than one track, the aggregate length of all tracks loaded to produce the stress shall be used.

Impact shall not be added to stresses produced by longitudinal, centrifugal and lateral or wind forces.

Lateral Forces. 10. All spans shall be designed for a lateral force on the loaded chord of 200 lb per linear ft plus 10% of the specified train load on one track, and 200 lb per linear foot on the unloaded chord; these forces being considered as moving.

Wind Force. 11. Viaduct towers shall be designed for that one of the following loads, considered as moving, which gives the greater stress:

(*a*) A force of 50 lb per sq ft applied on one and one-half times the vertical projection of the tower and the portion of the structure which it supports.

(*b*) A force of 30 lb per sq ft, applied on the same surface, plus 400 lb per linear foot of structure applied 7 ft above the top of the rail, for assumed wind force on train, when the structure is loaded, on either one or both tracks, with empty cars weighing 1200 lb per linear foot.

Longitudinal Force. 12. Viaduct towers and similar structures shall be designed for a longitudinal force of 20% of the live load applied at the top of the rail.

13. Structures on curves shall be designed for the centrifugal force of the live load applied at the top of the high rail. The centrifugal force shall be considered as live load and shall be derived from the speed in miles per hour given by the expression $60-2\frac{1}{2}D$, in which "D" = degree of curve.

(3) UNIT STRESSES AND PROPORTION OF PARTS

Unit Stresses. 14. Structures shall be so proportioned that the sum of the maximum stresses produced by the foregoing loads will not exceed the following amounts in pounds per square inch, except as modified in paragraphs 22 to 25:

Tension. 15. Axial tension on net section	16 000
Compression. 16. Axial compression on gross section of columns	$16\,000-70l/r$
with a maximum of	14 000
where l is the length of the member in inches, and r is the least radius of gyration in inches.	
Direct compression on steel castings	16 000
Bending. 17. Bending: on extreme fibers of rolled shapes, built sections, girders and steel castings; net sections	16 000
on extreme fibers of pins	24 000
Shearing. 18. Shearing: shop driven rivets and pins	12 000
field driven rivets and turned bolts	10 000
plate girder webs; gross section	10 000
Bearing. 19. Bearing: shop driven rivets and pins	24 000
field-driven rivets and turned bolts	20 000
expansion rollers; per linear inch	600 d
where d is the diameter of the roller in inches.	
on masonry	600

Limiting Length of Members. 20. The ratio of length to least radius of gyration shall not exceed 100 for main compression members nor 120 for wind and sway bracing.

21. The lengths of riveted tension members in horizontal or inclined positions shall not exceed 200 times their radius of gyration about the horizontal axis. The horizontal projection of the unsupported portion of the member shall be considered its effective length.

Alternate Stresses. 22. Members subject to alternate stresses of tension and compression shall be proportional for the kind of stress requiring the larger section. If the alternate stresses occur in succession during the passage of one train, as in stiff counters, each stress shall be increased by 50% of the smaller. The connections of such members shall in all cases be proportioned for the sum of the stresses so increased.

23. If the live load and dead load stresses are opposite in character, only two-thirds of the dead load stress shall be considered as effective in counteracting the live load stress. This reduction of dead load shall not be made in proportioning members subject to alternate stresses.

Combined Stresses. 24. Members subject to both axial and bending stresses shall be proportioned so that the combined fiber stresses will not exceed the allowed axial stress.

25. Members subject to stresses produced by combinations of lateral, longitudinal. and wind forces with dead load, live load, impact, and centrifugal force, may be proportioned for unit stresses 25% greater than those specified in paragraphs 15 to 19, inclusive; but the section shall be not less than that required for dead load, live load, impact, and centrifugal force.

Net Section at Rivets. 26. In proportioning tension members the diameter of the rivet holes shall be taken 1/8 in larger than the nominal diameter of the rivet.

Rivets. 27. In proportioning rivets the nominal diameter of the rivet shall be used.

Net Section at Pins. 28. The minimum net section through the pin-hole of pin-connected riveted tension members shall be at least 25% in excess of the net section of the body of the member. The minimum net section back of the pin-hole, parallel with the axis of the member, shall be not less than the net section of the body of the member.

Plate Girders. 29. Plate girders shall be proportioned either by the moment of inertia of their net section; or by assuming that the flanges are concentrated at their centers of gravity. In the latter case one-eighth of the gross section of the web, if properly spliced, may be used as flange section. The thickness of web plates shall be not less than 1/160 of the unsupported distance between flange angles (see 38).

Compression Flange. 30. The gross section of the compression flanges of plate girders shall be not less than the gross section of the tension flanges. If the compression flange of any beam or girder consists of angles only or if the cover consists of flat plates the stress per square inch shall not exceed $16\,000-200\,l/b$. If the cover consists of a channel section the stress per square inch shall not exceed $16\,000-150\,l/b$. L represents the length of unsupported flange, and b is the flange width.

Flange Rivets. 31. The flange of plate girders shall be connected to the web with a sufficient number of rivets to transfer in a distance equal to the effective depth of the girder at any given point, the total shear at that point combined with any load that is applied directly on the flange. If the ties rest on the flanges, each wheel load shall be assumed to be distributed over three ties.

Depth Ratios. 32. The depth of trusses shall preferably be not less than one-tenth of the span. The depth of plate girders and rolled beams, used as girders, shall preferably be not less than one-twelfth of the span. If shallower trusses, girders or beams are used, the section shall be increased so that the maximum deflection will not be greater than if the above limiting ratios had not been exceeded.

(4) DETAILS OF DESIGN

GENERAL REQUIREMENTS

Open Sections. 33. Structures shall be so designed that all parts will be accessible for inspection, cleaning and painting.

Pockets. 34. Pockets or depressions which would hold water shall be provided with drain holes, or be filled with waterproof material.

Symmetrical Sections. 35. Main members shall be so designed that the neutral axis will be as nearly as practicable in the center of the section. The neutral axes of intersecting main members of trusses shall meet at a common point.

Counters. 36. Rigid counters are preferred. If subject to reversal of stress the chord connections shall preferably be riveted. Adjustable counters shall have open turnbuckles.

Strength of Connections. 37. The strength of connections shall be sufficient to develop the full capacity of the member for the kind of stress it is to carry even tho the computed stress is less than such capacity.

Minimum Thickness. 38. The minimum thickness of metal shall be ⅜ in, except for fillers.

(Note. 5/16 in may be used in highway bridges and ¼ in in mill buildings.)

Pitch of Rivets. 39. The minimum distance between centers of rivet holes shall be three diameters of the rivet; but the distance shall be preferably not less than 3 in for ⅞-in rivets and 2½ in for ¾-in rivets. The maximum pitch in the line of stress for members composed of plates and shapes shall be 6 in for ⅞-in rivets and 5 in for ¾-in rivets. For angles with two-gage lines and rivets staggered the maximum pitch in each line shall be twice the above. If two or more plates are used in contact, rivets not more than 12 in apart in either direction shall be used to hold the plates together. In tension members composed of two angles in contact, a pitch of 12 in may be used for riveting these angles together.

Edge Distance. 40. The minimum distance from the center of any rivet hole to a sheared edge shall be 1½ in for ⅞-in rivets and 1¼-in for ¾-in rivets, and to a rolled edge 1¼ in and 1⅛ in, respectively. The maximum distance from any edge shall be eight times the thickness of the plate, but shall not exceed 6 in.

Maximum Diameter. 41. The diameter of the rivets in any angle whose size is determined by calculated stress shall not exceed one-fourth of the width of the leg in which they are driven; in angles whose size is not so determined ⅞-in rivets may be used in 3-in legs, and ¾-in rivets in 2½-in legs.

Long Rivets. 42. Rivets carrying calculated stress and whose grip exceeds four diameters shall be increased in number at least 1% for each additional 1/16-in of grip.

Pitch at Ends. 43. The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivets, for a distance equal to one and one-half times the maximum width of member.

Compression Members. 44 In built compression members the metal shall be concentrated in the webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between the lines of rivets, connecting it to the flanges. The thickness of cover plates shall be not less than one-fortieth of the distance between the nearest rivet lines.

Minimum Angles. 45. The minimum thickness of flange angles of girders and built members without cover plates shall be one-twelfth of the width of the outstanding leg.

Tie Plates. 46. The open sides of compression members shall be provided with lacing bars and shall have tie plates as near each end as practicable. Tie plates shall be provided at intermediate points where the lacing is interrupted. In main members the length of end tie plates shall be not less than the distance between the nearest lines of rivets connecting them to the flanges, and the length of intermediate tie plates not less than one-half that distance. Their thickness shall be not less than one-fiftieth of the same distance.

Lacing. 47. The lacing of compression members shall be proportioned to resist the shearing stresses corresponding to the allowance for flexure for uniform load provided in the column formula in paragraph 16 by the term $70\, l/r$. The minimum width of lacing bars shall be 2½ in for ⅞-in rivets, 2¼ in for ¾-in rivets, and 2 in for ⅝-in rivets if used. The thickness shall be not less than one-fortieth of the distance between end rivets for single lacing, and one-sixtieth for double lacing. Shapes of equivalent strength may be used instead of bars.

48. Five-eighths-inch rivets shall be used for lacing flanges less than 2½ in wide, ¾-in rivets for flanges from 2½ to 3½ in wide, and ⅞-in rivets for flanges 3½ in and over in width Lacing bars with at least two rivets in each end shall be used for flanges over 5 in wide.

49. The inclination of lacing bars with the axis of the member shall be not less than 45°. If the distance between rivet lines in the flanges is more than 15 in ,and a single rivet bar is used, the lacing shall be double and riveted at the intersections.

50 Lacing bars shall be so spaced that the portion of the flange included between their connections will be as strong as the member as a whole.

Faced Joints. 51 Abutting joints in compression members faced for bearing shall be sufficiently spliced on four sides to hold the connecting members accurately in place. Other joints in riveted work, whether in tension or compression shall be fully spliced.

Pin Plates. 52 Pin-holes shall be reinforced by plates if necessary, and at least one plate shall be as wide as the flanges will allow, and be on the same side as the angles. Pin plates shall contain sufficient rivets to distribute their portion of the pin pressure to the full cross-section of the member.

Forked Ends. 53. The ends of compression members shall not be forked unless unavoidable; with forked ends, a sufficient number of pin plates shall be provided to give the jaws twice the sectional area of the member. At least one of these plates shall extend to the far edge of the farthest tie plate, and the others to the far edge of the nearest tie plate, but not less than 6 in beyond the near edge of the farthest tie plate.

Pins. 54 Pins shall be long enough to insure a full bearing of all the parts connected upon the turned body of the pin. They shal' be secured by chambered nuts or be provided with washers if solid nuts are used. The screw ends shall be long enough to admit of burring the threads.

55. Pin connected members shall be held against lateral movement on the pins.

Bolts. 56. Where members are connected by bolts, the turned bodies of the bolts shall be long enough to extend thru the metal. A washer at least ¼ in thick shall be used under the nut. Bolts shall not be used in place of rivets except by special permission. Heads and nuts shall be hexagonal.

Indirect Splices. 57. If splice plates are not in direct contact with the parts which they connect, rivets shall be used on each side of the joint in excess of the number required in the case of direct contact to the extent of one-third of that number for each intervening plate.

Fillers. 58. Where rivets carrying stress pass thru fillers the fillers shall be extended beyond the connected member and the extension secured by additional rivets equal in number to 50% of those required to carry the stress.

Expansion. 59. Provision shall be made for expansion and contraction in all bridge structures to the extent of ⅛ in for each 10 ft of length. Means shall be provided to prevent excessive motion at any point.

Expansion Bearings. 60. Spans of 80 ft and over resting on masonry shall have turned rollers or rockers at one end. Spans of less length shall be arranged to slide on smooth surfaces. These expansion bearings shall be designed to permit motion in one direction only.

Fixed Bearings. 61. Fixed bearings shall be firmly anchored to the supports.

Rollers. 62. Expansion rollers shall be not less than 6 in in diameter. They shall be coupled together with substantial side bars, which shall be so arranged that the rollers can be readily cleaned. Segmental rollers shall be geared to the upper and lower plates.

Bolsters. 63 Bolsters or shoes shall be so constructed that the load will be distributed uniformly over the entire bearing. Spans of 80 ft and over shall have hinged bolsters at each end.

Wall Plates. 64. Wall plates may be cast or built up and shall be so designed as to distribute the load uniformly over the entire bearing. They shall be secured against displacement.

Anchorage. 65. Anchor bolts for viaduct towers and similar structures shall be designed to engage a mass of masonry the weight of which is at least one and one-half times the uplift.

Inclined Bearings. 66. The sole plates of bridges on an inclined grade without pin shoes shall be beveled so that the masonry and expansion surfaces will be level.

FLOOR SYSTEMS

Floor Beams, 67. Floor beams shall preferably be square to the girders or trusses. They shall be riveted directly to the girders or trusses or may be placed on top of deck bridges.

Stringers. 68. Stringers shall preferably be riveted to the webs of intermediate floor beams by means of connection angles not less than ½ in thick. Shelf angles or other supports provided to support the stringer during erection shall not be considered as carrying any of the reaction.

Stringer Frames. 69. End floor beams shall be used if possible. Stringers resting on masonry shall be connected at their ends by cross frames. The frames shall be riveted to girders or truss shoes where practicable.

BRACING

Rigid Bracing. 70. Lateral, longitudinal and transverse bracing shall be composed of rigid members.

Portals. 71. The end posts and top chords of the thru truss spans shall be rigidly connected by riveted portal braces. The braces shall be as deep as the clearance will allow.

Transverse Bracing. 72. An intermediate transverse frame shall be used at each panel of thru spans having vertical truss members where the clearance will permit.

End Bracing. 73. Deck spans shall have transverse bracing at each end proportioned to carry the lateral load to the support.

Laterals. 74. The minimum sized angle to be used in lateral bracing shall be 3½ by 3 by ⅜ in. There shall be not less than three rivets at each end connection of the angles.

75. Lateral bracing beneath the track shall be low enough to clear the ties.

Tower Struts. 76. The struts at the base of viaduct towers shall be strong enough to slide the movable shoes when the track is unloaded.

PLATE GIRDERS

Camber. 77. If camber is desired it shall be provided in plate girder spans over 50 ft in length at the rate of 1/16-in per 10 ft of length.

Top Flange Cover. 78. Where flange cover plates are used, one cover plate of the top flange shall extend the whole length of the girder.

Web Stiffeners. 79. There shall be web stiffeners, generally in pairs, over bearings, and at points of concentrated loading. Other web stiffeners shall be used if the width of the unsupported web between flange angles is greater than 60 times its thickness. The distance between stiffeners shall not exceed: (*a*) 6 ft; (*b*) the width of the unsupported web; (*c*) the value of d in the following formula: $d = t(12\,000 - s)/40$. Where d = clear distance between stiffeners of flange angles; t = thickness of web; s = shear per square inch. The stiffeners at the ends and at points of concentrated loading shall be proportioned by the formula of paragraph 16, by assuming the effective length of column equal to one-half the depth of the girder. End stiffeners and those under concentrated loading shall be on fillers. Their outstanding legs shall be as wide as the flange angles will allow and shall fit tightly against them. Intermediate stiffeners may be crimped or on fillers and their outstanding legs shall be not less than 2 in plus one-thirtieth of the depth of the girder.

Stays for Top Flanges. 80. Top flanges of thru plate girders shall be stayed by knee braces or gusset plates at every floor beam, or in solid floor bridges at distances exceeding 12 ft.

TRUSSES

Camber. 81. Truss spans shall be given a camber by so proportioning the length of the members that the tops of the stringers will be in a straight line when the bridge is fully loaded.

Rigid Members. 82. Hip verticals and members performing similar functions and the two end panels of the bottom chords of single track pin-connected trusses shall be rigid.

Eye-Bars. 83. The eye-bars composing a member shall be so arranged that adjacent bars will not be in contact. The bars shall be as nearly parallel to the axis of the truss as possible, the maximum inclination of any bar being 1 in in 16 ft.

Pony Trusses. 84. Pony trusses shall be riveted structures with double webbed chords. The web members shall be laced or otherwise effectively stiffened.

(Note. Light highway pony trusses for short spans may have single webbed chords.)

12. Specifications for Materials and Workmanship

(5) MATERIAL

Steel. 85. Steel shall be made by the open-hearth process.

Properties. 86. The chemical and physical properties shall conform to the following limits:

Elements Considered	Structural Steel	Rivet Steel	Steel Castings
	Percent	Percent	Percent
Phosphorus, max. { Basic	0.04	0.04	0.05
Phosphorus, max. { Acid	0.06	0.04	0.08
Sulfur, maximum	0.05	0.04	0.05
Yield point, minimum, lb per sq in.	30 000	25 000	33 000
Ultimate tensile strength, lb per sq in	Desired. 60 000	Desired 50 000	Not less than 65 000
Elong., min. %, in 8 in, Fig. 26b	1 500 000* / Ult. tensile str'gth	1 500 000 / Ult. tensile str'gth	15 percent
Elong., min. %, in 2 in, Fig. 26c.	22		
Character of Fracture	Silky	Silky	Silky or fine granular
Cold Bends without Fracture.	180° flat†	180° flat‡	90° $d=3t$

* See paragraph 96. † See paragraphs 97, 98, and 99. ‡ See paragraph 100.

87. In order that the ultimate strength of full-sized annealed eye-bars may meet the requirements of paragraph 163, the ultimate strength in test specimens may be determined by the manufacturers. The tests other than those for ultimate strength shall conform to the above requirements.

Allowable Variations. 88. If the ultimate strength varies more than 4000 lb from that desired, a retest shall be made on the same gage, which, to be acceptable, shall be within 5000 lb of the desired ultimate.

Chemical Analyses. 89. Chemical determinations of the percentages of carbon, phosphorus, sulfur and manganese shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt of steel. A copy of each analysis shall be furnished to the Engineer or his inspector. Check analyses shall be made from finished material, if called for by the purchaser, in which case an excess of 25% above the limits specified will be permitted.

Specimens. 90. Plate, shape and bar specimens for tensile and bending tests shall be made by cutting coupons from the finished product. The test specimens shall have both faces rolled and both edges milled either parallel or to the form shown by Fig. 26b; or the specimens may be turned to a diameter of ¾ in for a length of at least 9 in, with enlarged ends.

91. Test specimens of rivet steel shall be cut full size from the rods as rolled.

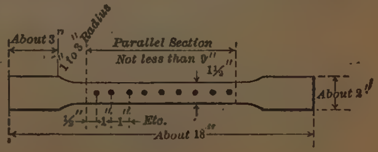


Fig. 26b

. 92. Pin and roller specimens shall be cut from the finished rolled or forged bar, in su manner that the center of the specimen will be 1 in from the surface of the bar. T specimen for tensile test shall be turned to the form shown by Fig. 26c. The specim for bending test shall be 1 in by ½ in in section.

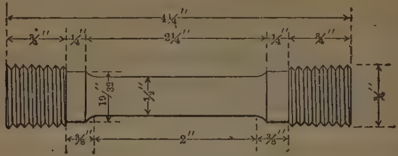


Fig. 26c

93. The number of tests of steel castings shall depend on the character and importan of the castings. Specimens shall be cut cold from coupons molded and cast on sor portion of one or more castings from each melt or from the sink heads, if the heads are sufficient size. The coupon or sink head, so used, shall be annealed with the casting befo it is cut off. Test specimens shall be of the form prescribed for pins and rollers.

93¹. The yield point shall be determined by the drop of beam of the testing machi The beam shall be kept balancing between the upper and lower cross-pieces for some ti preceding the drop. The speed of the machine shall be such that the beam may be ke balanced and, except for the initial tightening of the specimen in the grips, shall not exce ½ in per minute for the standard form of specimen for plates, bars and shapes, and sh not exceed ⅛-in per minute for the standard form of specimen for pins, rollers and st castings. The speed after the yield point shall not exceed 6 in per minute, and the bea shall be kept at balance when the ultimate strength is attained.

Specimens of Rolled Steel. 94. Rolled steel shall be tested in the condition in whi it comes from the rolls.

Number of Tests. 95. At least one tensile and one bending test shall be made fro each melt of steel as rolled. In case steel differing ⅜ in or more in thickness is roll from one melt, tests shall be made from the thickest and the thinnest material rolled.

Modification in Elongation. 96. A deduction of 1% will be allowed from the spe fied percentages of elongation, for each ⅛ in in thickness above ¾ in.

Bending Tests. 97. Bending tests may be made by pressure or by blows. Plat shapes and bars less than 1 in thick shall bend as called for in paragraph 86.

Thick Material. 98. Full-sized material for eye-bars and other steel 1 in thi and over, tested in the same condition as when rolled, shall bend cold 180° around a p the diameter of which is equal to twice the thickness of the bar, without fracture on t outside of the bend.

Bending Angles. 99. Angles ¾ in and less in thickness shall open flat, and angl ½ in and less in thickness shall bend shut, cold, under blows of a hammer, without si of fracture. This test shall be made only when required by the inspector.

Nicked Bends. 100. Rivet steel, when nicked and bent around a bar of the sa diameter as the rivet rod, shall give a gradual break and a fine silky uniform fracture.

Finish. 101. Finished material shall be free from injurious seams, flaws, crac defective edges and other defects, and have a smooth, uniform and workmanlike finis Plates 36 in in width and under shall have rolled edges.

Melt Numbers. 102. Every finished piece of steel shall have the melt number a the name of the manufacturer stamped or rolled upon it. Steel for pins and roll shall be stamped on the end. Rivet and lacing steel and other small parts may be bundl with the above marks on an attached metal tag.

Defective Material. 103. Material which develops weak spots, brittleness, crac or other imperfections, or is found to have injurious defects, may be rejected at the sh and shall be replaced by the manufacturer at his own cost.

Variation in Weight. 104. A variation in cross-section or weight of any piece

steel of more than 2½% from that specified will be sufficient cause for rejection, except that sheared plates shall be subject to the following permissible variations, which shall apply to single plates, when ordered to weight:

105. Plates weighing 12½ lb or more per square foot.

(a) Up to 100 in wide, 2½% more or less than the nominal weight.

(b) One hundred inches wide and over, 5% more or less than the nominal weight.

106. Plates weighing less than 12½ lb per sq ft.

(a) Up to 75 in wide, 2½% more or less than the nominal weight.

(b) Seventy-five inches and up to 100 in wide, 5% more or 3% less than the nominal weight.

(c) One hundred inches wide and over, 10% more or 3% less than the nominal weight.

107. Plates when ordered to gage will be accepted if they measure not more than 0.01 in less than the ordered thickness.

108. An excess over the nominal weight, corresponding to the dimensions on the order, will be allowed for each plate, if not more than that shown in the following table, 1 cu in of rolled steel being assumed to weigh 0.2833 lb:

Thickness Ordered, in	Nominal Weights, lb	Width of Plate			
		Up to 75 in, percent	75 in and up to 100 in, percent	100 in and up to 115 in, percent	Over 115 in, percent
¼	10.20	10	14	18	....
5/16	12.75	8	12	16	....
⅜	15.30	7	10	13	17
7/16	17.85	6	8	10	13
½	20.40	5	7	9	12
9/16	22.95	4½	6½	8½	11
⅝	25.50	4	6	8	10
Over ⅝	....	3½	5	6½	9

Cast-Iron. 109. Castings shall be made of tough gray iron, with sulfur not over 0.10%, except where chilled iron is specified. They shall be true to pattern, out of wind and free from flaws and excessive shrinkage. If tests are required, they shall be made on the "Arbitration Bar" of the American Society for Testing Materials, which is a round bar 1¼ in in diameter and 15 in long. The transverse test shall be made on a clear span of 12 in with the load at the middle. The minimum breaking load so applied shall be 2900 lb, with a deflection of at least 1/10 in before rupture.

Wrought-Iron. 110. Wrought-iron shall be double-rolled, tough, fibrous and uniform in character. It shall be thoroly welded in rolling and free from surface defects. When tested in specimens of the form of Fig. 26 (*b*), or in full-sized pieces of the same length, it shall show an ultimate strength of at least 50 000 lb per sq in, an elongation of at least 18% in 8 in, with fracture wholly fibrous. Specimens shall bend cold, with the fiber thru 135°, without sign of fracture, around a pin the diameter of which is twice the thickness of the piece tested. When nicked and bent, the fracture shall show at least 90% fibrous.

(6) INSPECTION AND TESTING AT THE MILLS

Mill Orders. 111. The purchaser shall be furnished complete copies of mill orders and no material shall be rolled nor work done before the purchaser has been notified where the orders have been placed, so that he may arrange for the inspection.

Facilities for Inspection. 112. The manufacturer shall furnish all facilities for inspecting and testing the weight and quality of all material at the mill where it is manufactured. He shall furnish a suitable testing machine for testing the specimens, as well as prepare the pieces for the machine, free of cost to the purchaser.

Access to Mills. 113. The inspector representing the purchaser at the mills shall have access, at all times, to all parts of the mills where material to be inspected by him is being manufactured.

(7) WORKMANSHIP

General. 114. All parts forming a structure shall be built in accordance with approved drawings. The workmanship and finish shall be equal to the best practice in modern bridge works. Material at the shops shall be kept clear and protected from the weather.

Straightening. 115. Material shall be thoroly straightened in the shop, by methods that will not injure it, before being laid off or worked in any way.

Finish. 116. Shearing and chipping shall be neatly and accurately done and all portions of the work exposed to view neatly finished.

Size of Rivets. 117. The size of rivets, called for on the plans, shall be the actual size of the rivet before heating.

Rivet Holes. 118. If general reaming is not required, the diameter of the punch shall be not more than 1/16-in greater than the diameter of the rivet; nor the diameter of the die more than ⅛-in greater than the diameter of the punch. Material more than ¾ in thick shall be sub-punched and reamed or drilled from the solid.

Punching. 119. Punching shall be accurately done. There shall be no drifting to enlarge unmatched holes. If the holes must be enlarged to admit the rivet, they shall be reamed. Poor matching of holes will be cause for rejection.

Reaming. 120. Where sub-punching and reaming are required, the diameter of the punch used shall be not less than 3/16 in smaller than the nominal diameter of the rivet. The holes shall then be reamed to a diameter not more than 1/16 in greater than the nominal diameter of the rivet. (See 135.)

Reaming After Assembling. 121. [If general reaming is required it shall be done after the pieces forming one built member are assembled and so firmly bolted together that the surfaces are in close contact. If it be necessary to take the pieces apart for shipping and handling, the respective pieces reamed together shall be so marked that they may be reassembled in the same position in the final setting up. No interchange of reamed parts will be permitted.]

122. Reaming shall be done with twist drills without lubricant.

123. The outside burrs on reamed holes shall be removed to the extent of making a 1/16-in fillet.

Assembling. 124. The parts of riveted members shall be well pinned and firmly drawn together with bolts, before riveting is commenced. Contact surfaces shall be painted. (See 152.)

Lacing Bars. 125. The ends of lacing bars shall be neatly rounded, unless otherwise called for.

Web Stiffeners. 126. Stiffeners shall fit neatly between the flanges of girders. Where tight fits are called for, the ends of the stiffeners shall be faced and shall be brought to a true contact bearing with the flange angles.

Splice Plate and Fillers. 127. Web splice plates and fillers under stiffeners shall be cut to fit within ⅛-in of the flange angles.

Web Plates. 128. Web plates of girders, which have no cover plates, shall be flush with the backs of the flange angles or project above them not more than ⅛-in, unless otherwise called for. When web plates are spliced, not more than ¼ in clearance between ends of plates will be allowed.

Floor Beams and Stringers. 129. The main sections of floor beams and stringers shall be milled to exact length after riveting and the connection angles accurately set flush and true to the milled ends. [If required by the purchaser the milling shall be done after the connection angles are riveted in place, the milling to extend over the entire face of the member.] The removal of more than 3/32 in from the thickness of connection angles will be cause for rejection.

Riveting. 130. Rivets shall be uniformly heated to a light cherry red heat in a gas or oil furnace so constructed that it can be adjusted to the proper temperature. They shall be driven by pressure tools wherever possible. Pneumatic hammers shall be used in preference to hand driving.

131. Rivet heads shall be of approved shape, uniform in size, and of neat and finished appearance. They shall be central on the shank and shall grip the assembled pieces firmly. Recupping and caulking will not be allowed. Loose, burned or otherwise defective rivets shall be cut out and replaced. In cutting out rivets, care shall be taken not to injure the adjacent metal. If necessary, they shall be drilled out.

Turned Bolts. 132. Wherever bolts are used in place of rivets which transmit shear, the holes shall be reamed parallel and the bolts shall make a driving fit with the threads entirely outside of the holes. A washer not less than ¼ in thick shall be used under nut.

Members to be Straight. 133. The several pieces forming one built member shall be straight and fit closely together. Finished members shall be free from twists, bends and open joints.

Finish of Joints. 134. Abutting joints shall be cut or drest true and straight and fitted close together, especially where open to view. In compression joints, depending on contact bearing, the surfaces shall be truly faced, so as to provide even bearings after they have been perfectly aligned and riveted complete.

Field Connections. 135. Holes for floor beam and stringer connections shall be sub-punched and reamed to a steel templet not less than 1 in thick. [If required, all other field connections, except those for laterals and sway bracing, shall be assembled in the shop and the unmatched holes reamed; and when so reamed the pieces shall be match-marked before being taken apart.]

Eye-Bars. 136. Eye-bars shall be straight, true to size, and free from twists, folds in the neck or head, and other defects. The heads shall be made by upsetting, rolling or forging. Welding will not be allowed. The form of the heads will be determined by the dies in use at the works where the eye-bars are made, if satisfactory to the Engineer; but the manufacturer shall guarantee the bars to break in the body when tested to rupture. The thickness of the head and neck shall not vary more than 1/16 in from that specified. (See 163.)

Boring Eye-Bars. 137. Before boring, each eye-bar shall be properly annealed and carefully straightened. Pin-holes shall be in the center line of the bar and in the center of the heads. Bars of the same length shall be bored so accurately that, when placed together, pins 1/32 in smaller in diameter than the pin-holes can be passed thru the holes at both ends of the bars at the same time without forcing.

Pin-Holes. 138. Pin-holes shall be bored true to gages, smooth, straight, at right angles to the axis of the member and parallel to each other, unless otherwise called for. The boring shall be done after the member is riveted.

139. The distance center to center of pin-holes shall be correct within 1/32 in, and the diameter of the holes not more than 1/50 in larger than that of the pin, for pins up to 5 in diameter, and 1/32 in for larger pins.

Pins and Rollers. 140. Pins and rollers shall be accurately turned to gages and shall be straight, smooth and free from flaws.

Screw Threads. 141. Screw threads shall make tight fits in the nuts and shall be U. S. standard, except above the diameter of 1⅜ in, when they shall be made with six threads per inch.

Annealing. 142. Steel which has been partially heated shall be properly annealed except where used in minor details.

Steel Castings. 143. Steel castings shall be annealed and free from large or injurious blowholes.

Welds. 144. Welds in steel will not be allowed.

Bed Plates. 145. Expansion bed plates shall be planed true and smooth. Cast wall plates shall be planed top and bottom. The finishing cut of the planing tool shall be fine and parallel with the direction of expansion.

Pilot Nuts. 146. Pilot and driving nuts shall be furnished for each size of pin, in such numbers as may be ordered.

Field Rivets. 147. Field rivets shall be furnished to the amount of 15% plus ten rivets, in excess of the nominal number required for each size.

Shipping Details. 148. Pins, nuts, bolts, rivets and other small parts shall be boxed or crated.

Weight. 149. The scale weight of every piece and box shall be marked on it in plain figures.

Finished Weight. 150. Payment for pound price contracts shall be by scale weight. Not over 2% of the total weight of the structure as computed from the plans will be allowed for excess weight.

(8) SHOP PAINTING

Cleaning. 151. Steel work, before leaving the shop, shall be thoroly cleaned and given one good coating of pure linseed oil, or such paint as may be called for, well worked into all joints and open spaces.

Contact Surfaces. 152. In riveted work, the surfaces coming in contact shall each be painted before being riveted together.

Inaccessible Surfaces. 153. Pieces and parts which will not be accessible for painting after erection, including tops of stringers, eye-bar heads, ends of posts and chords, etc., shall have an additional coat of paint before leaving the shop.

Condition of Surfaces. 154. Painting shall be done only when the surface of the metal is perfectly dry. It shall not be done in wet or freezing weather, unless protected under cover.

Machine-Finished Surfaces. 155. Machine-finished surfaces shall be coated with white lead and tallow before shipment and before being put out into the open air.

(9) INSPECTION AND TESTING AT THE SHOPS

Facilities for Inspection. 156. The manufacturer shall furnish all facilities for inspecting and testing the weight of material and quality of workmanship at the shop where material is manufactured. He shall furnish a suitable testing machine for testing full-sized members, if required.

Starting Work. 157. The purchaser shall be notified well in advance of the start of the work in the shop, in order that he may have a representative on hand to inspect material and workmanship.

Access to Shop. 158. The inspector shall have access, at all times, to all parts of the shop where material to be inspected by him is being manufactured.

Accepting Material. 159. The inspector shall stamp each piece accepted with a private mark. Any piece not so marked may be rejected at any time and at any stage of the work. If the inspector, thru on oversight or otherwise, has accepted material or work which is defective or contrary to the specifications, this material, no matter in what stage of completion, may be rejected by the purchaser.

Shop Plans. 160. The purchaser shall be furnished complete shop plans.

Shipping Invoices. 161. Complete copies of all shipping invoices shall be furnished to the purchaser with each shipment. Shipping invoices shall show the scale weights of individual pieces.

(10) FULL-SIZED TESTS

Eye-Bar Tests. 162. Full-sized tests of eye-bars and similar members, to prove the workmanship, shall be made at the manufacturer's expense, and the members so tested shall be paid for by the purchaser at contract price, if the tests are satisfactory. If the tests are not satisfactory, the members represented by them will be rejected.

163. In eye-bar tests, the minimum ultimate strength shall be 55 000 lb per sq in. The elongation in 10 ft, including the fracture, shall be not less than 15%. Bars shall generally break in the body and the fracture shall be silky or fine granular. The elastic limit as indicated by the drop of the mercury shall be recorded. Should a bar break in the head and develop the specified elongation, ultimate strength and character of fracture, it shall not be cause for rejection, provided not more than one-third of the total number of bars tested break in the head (see 136).

13. Highway Bridge Floors

A Highway Bridge Floor consists of flooring, stringers and floor beam. (Fig. 27.) Flooring is the timber planking forming the wearing surfaces. Stringers are beams placed lengthwise of the bridge which carry the loads from flooring to floor beams, the latter being transverse to the length of the bridge and connected to the trusses or girders at opposite panel points. Sidewalks are usually of plank supported on stringers which are in turn carried either by brackets at the panel points of the trusses or by extensions of the floor beams. The hand railing at the outer edge of the sidewalk is supported by the brackets, and when the panels are long, an intermediate railing support is also provided at the middle of each panel. The economic panel length for light bridges having plank flooring is from 13 to 15 ft, while for heavy traffic it is from 15 to 20 ft and even as much as 25 ft for heavy structures of long spans.

The Roadway of Highway Bridges is of planks, wooden blocks, asphalt, brick, granite blocks, or concrete. One or two layers of planking may be used; one layer is usually 3 inches thick, laid transversely to the length of the roadway; and for two layers the lower is 2 to 4 inches thick and laid transversely, while the 2-inch upper planking is either transverse or diagonal. Planks 6 to 12 inches wide are used, and are spiked to wooden stringers or attached to I-beam stringers as in Fig. 27. For the lower layer yellow pine or oak should be used, and for the upper, spruce or yellow pine. Wooden stringers and the lower layer last from 6 to 12 years, while the upper layer lasts only from 1 to 5 years. Wooden paving blocks, 3 to 4 inches deep, if well prepared make a good floor. They are laid on 3 or 4 inch planking with a one-inch sand cushion or a layer of tar paper between the blocks and planking. Reinforced concrete floors resting upon and encasing I-beam stringers (Fig. 27) have been used in a few instances with and without asphalt wearing surfaces.

Buckled Plates. (Fig. 27) made by pressing dome-shaped buckles in flat steel plates, are used for supporting brick, asphalt, or granite blocks. These pavements are laid on one inch sand spread on concrete which is laid on the buckled plates. Plates are ¼, 5/16, ⅜, 7/16 inch thick, 5/16 being most common for roadways and ¼ for sidewalks; and should be riveted, not bolted, to the stringer flanges with ⅝ or ¾ inch rivets spaced not over 6 inches. Buckles are from 2 to 3½ inch deep, from 2.0 to 5½ ft on a side, and from 1 to 15 buckles are used in one long plate. Plates are stronger with the buckles turned down.

Weights of Floors. For 3-inch yellow pine planking on 3 by 12 inch yellow pine stringers, spaced 2 ft on centers, the total weight of planking and stringers is 21 lb per sq ft; 2-inch spruce on 3-inch pine with 3 by 12 inch stringers, 28 lb per sq ft; 3-inch yellow pine on 4 by 14 inch stringers spaced 2½ ft, 23 lb; 3-inch pine on 4 by 14 inch stringers spaced 3 ft, 22 lb; 2-inch spruce on 3-inch pine with 6 by 14 inch stringers spaced 3 ft, 32 lb per sq ft. Asphalt laid 2 inches thick on 1 inch of sand and 4 inches of concrete on 5/16 buckled plates gives a weight of 85 to 90 lb per sq ft not including stringers.

Wooden Stringers should be yellow pine or oak, and the spacing in feet center to center for ordinary bridges should not exceed the thickness of the main plank in inches; for heavy traffic this rule gives spacings too large. For roadways the minimum width of stringer is 3 in, or ¼ the depth; for sidewalks 2 in is the minimum. Wooden stringers set on the top of the floor beams with ½ in air space between adjacent stringers, or rest on stiffened shelf steel angles riveted to the web of floor beam. They are often placed on short shelf angles without stiffeners underneath, but this is dangerous construction unless the thickness of the shelf is at least equal to that given

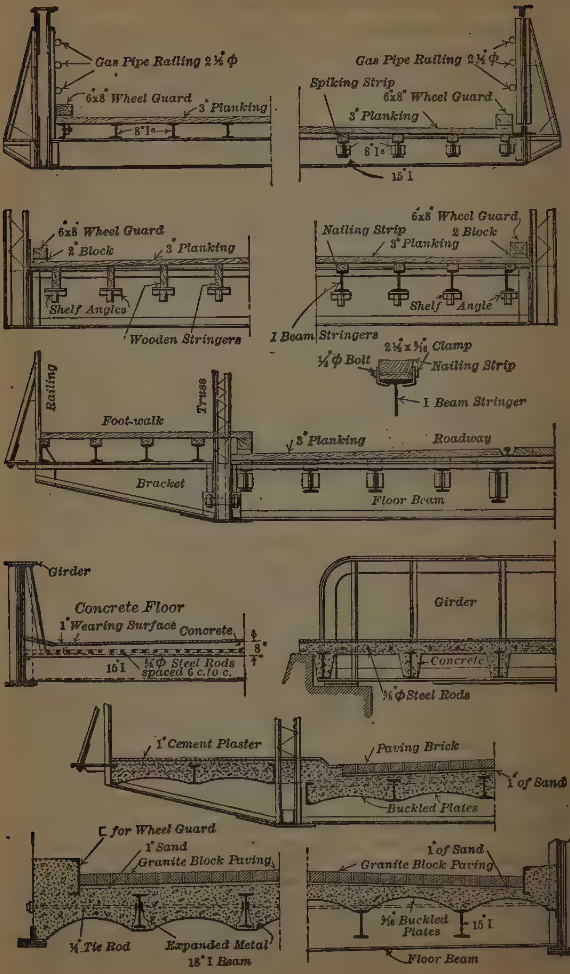


Fig. 27. Floors for Highway Bridges.

by the formula $t = (3\,Rb/Sl)^{1/2}$, where t = the required thickness of L, R = maximum end reaction on stringer, b = width of outstanding leg of L, l = length of L, S = allowable unit stress, say 12 000 lb per sq in. To proportion the cross-section of a wooden stringer compute the maximum bending moment M in inch-pounds, assume the depth h of stringer, and apply the formula $B = 6\,M/Sh^2$, in which S is the allowable unit stress and B the required breadth of the beam. All dimensions should be in inches.

Steel Stringers are usually made of I beams which are set on the top flanges of the floor beams or connected to the floor-beam webs by hitch or connecting L's. To find the size of I beam compute the required section modulus M/S, and look up in a table of properties of I beams the size and weight of beam corresponding. The letters M and S have the same meaning as for wooden stringers, but S for wooden beams is 1200 lb per sq in if no impact allowance be made, and 1500 if such be made, and S for steel = 12 000 and 16 000 respectively.

Each stringer should have two hitch L's at each end to connect stringer to floor-beam web. The number of rivets connecting the two L's to stringer web equals the maximum end shear on one stringer divided by the value of one shop rivet. The rivets thru the floor-beam web, connecting hitch L's from a stringer on each side of the floor beam, are field-driven, and their number is obtained by (1) dividing the maximum end shear on one stringer by either the single shearing value of the rivet or the bearing on one thickness of hitch L whichever is the smaller, or (2) by dividing the maximum load going thru the connection from both sides of the floor beam by the bearing value of a field rivet on the floor-beam web, or on two thicknesses of hitch L if smaller. The greater number of rivets from (1) and (2) should be used. Some engineers claim that double the number just found should be used to allow for bending.

In Designing Stringers the entire weight of one wheel or, if the panel length is great enough, two wheels, should be assumed on one stringer, no allowance being made for distribution of the load to adjacent stringers by the planking. But IN REVIEWING an existing bridge this allowance may be made, altho the axle load should not be assumed equally distributed over the several stringers. With 3 in continuous plank on five wooden stringers spaced 30 in apart with one wheel over the second and one over the fourth stringer, the distribution of the axle load is as follows; 7 % of axle load on first stringer, 30 % on second, 26 % on third, 30 % on fourth, 7 % on fifth.

Floor Beams are made of I beams or of built-up beams. I beams require little shop work and should be used if possible. A built-up beam is a plate girder (Arts. 20 and 21) and is designed as such. In any case the maximum moments and maximum shear must be computed. The following formulas are applicable to floor beams where there is a sidewalk on each side of the roadway and where the total dead load per foot of floor beam and sidewalk brackets is the same. The live load per running foot of floor beam is assumed equal in magnitude to that on the walks and is properly placed to produce the maximum effects.

Maximum moment on floor beam $= \frac{1}{8}(w + w_1)\,d^2 - \frac{1}{2}\,w_1e^2$.

Maximum moment on sidewalk bracket $= \frac{1}{2}(w + w_1)\,e^2$.

Maximum reaction at one truss $= w_1(\frac{1}{2}\,d + e) + w\,(d + e)^2/2\,d$.

Maximum shear on floor beam $= \frac{1}{2}(w + w_1)\,d + we^2/2\,d$, in which w is the live and w_1 the dead load in pounds per lin ft of floor beam and sidewalk brackets, d = distance center to center trusses in ft, e = width of each of two sidewalks in ft. Moments are in lb-ft and reaction and shear in lb.

The method of placing the loads on the floor beam and of computing the maximum moment on the same for a bridge having one roadway and one sidewalk is as follows. Data are: panel length 15 ft, clear width of roadway 28 ft, clear width of the sidewalk 6 ft, width of each truss 2 ft, live load 100 lb per sq ft of roadway and walk, also one car

of 100 000 lb on four axles having a total wheel base of 25 ft and a 5-ft base for each truck, dead load 40 lb per sq ft, center of track 8 ft from center of the truss not carrying the sidewalk. The live load is assumed to cover the roadway between the sidewalk truss and to within 2 ft of the railway track. Fig. 28 shows the loads acting on the floor beam. Each 20 800 lb load is from a track stringer, and the live load of 1500 lb per running foot and dead of 600 lb per foot are taken as uniform loads with no appreciable error. The left reaction is found to be 46 600 lb and the moment is a maximum where the shear changes sign, namely, at the right track-stringer load, which is 10.5 ft from the left end of floor beam. The moment at this point is most easily found by considering the forces to the left, and equals 352 200 lb-ft. Built-up floor beams should have a depth equal to about 1/12 the distance between centers of trusses.

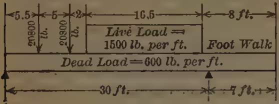


Fig. 28

14. Railroad Bridge Floors

In Open Floors of thru bridges (Fig. 29) the ties rest directly on steel stringers, one or two of these being under each rail. The stringers are supported by the floor beams. In deck bridges the ties rest on the top flanges of the girders. In solid or ballasted floors the ties are embedded in stone ballast carried either by trough floors or by solid floors of buckled or flat plates, or reinforced concrete slabs which are supported on I beams. The I beams for short spans run lengthwise of the track and rest on the abutments, while for long spans they are placed transversely and rest on girders or trusses. On tracks the guard timbers should be notched 1 in over each tie, should be bolted to every fourth tie and at splices, and spaced at least 20 in outside of the main rail. Ties should be not less than 8 by 8 in by 10 ft in length, should be spaced with not more than 6-in openings, notched over the stringers 1 in and secured to the same by hook bolts every third tie. Ties should be designed to carry the maximum axle load, with 100% impact, assumed as distributed over three ties with allowable unit stress not over 2000 lb per sq in. Steel guard rails are spaced 10 in clear inside of each main rail and extend across the bridge 50 ft beyond the abutments and around curves if any exist at the bridge. They should be spiked to every tie on the approaches and at least to every other tie on the bridge, and should be provided with a metal cap at the outer ends where they meet.

For Solid Floors troughs were first used to hold the ballast, but more recent bridges have the ballast carried by flat plates 3/8 or 7/16 inch thick riveted to I beams laid transversely to the track. The I beams are spaced 14 to 24 inches on centers, and in thru bridges are connected to the girder webs and in deck spans rest on the top flanges of the girders with fascia girders or deep L's along their ends to retain the ballast. The ties are spaced from 18 to 24 inches on centers and the guard rails and timbers are usually omitted. Buckled plates have been used instead of the flat plates; they are stronger and the I beams can be spaced 3 ft apart, thus diminishing their number. Floors have been built by supporting the flat or buckled plates on the lower flanges of the beams and embedding the ties in ballast between the beams, but in this construction the I beams are almost entirely covered and cannot be inspected or painted without removing the track and ballast, so that best practise places the plates on the top of the I beams. Ties are usually 6 inches deep and have 6 to 8 inches of ballast underneath. Reinforced concrete is also used in place of the floor plates.

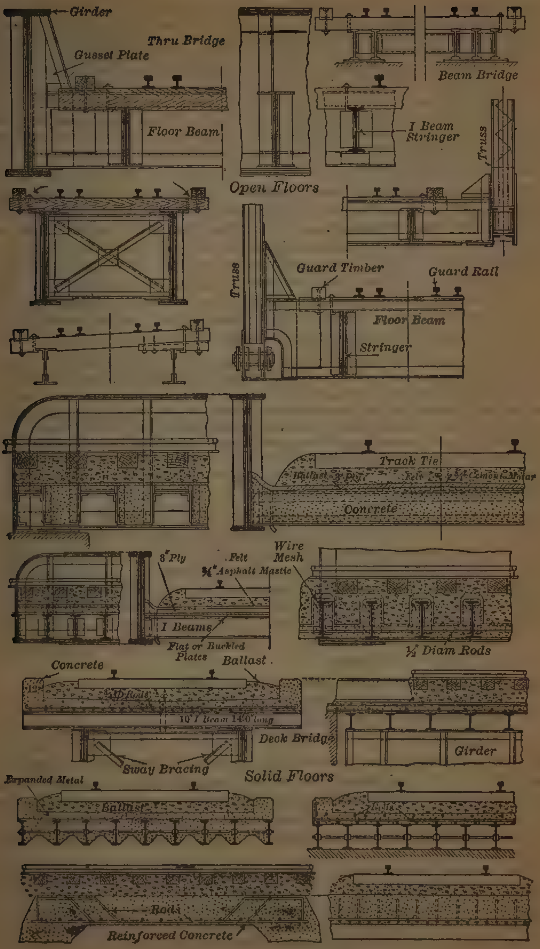


Fig. 29. Floors for Railroad Bridges

A Preservative should be placed between the ballast and steel of ballasted floors. A layer of asphalt-concrete not less than 2 in thick is sufficient, and the upper surface of the coating should be sloped to allow for drainage. The asphalt should cover all steel parts which otherwise would be in contact with the ballast, and it should be applied only to clean, dry steel surfaces, and if the steel is cold or damp a layer of hot sand should be placed on it and swept off just before the asphalt is applied. The best results are obtained by first coating the steel with hot asphalt, then adding a layer of at least 1 in of a mixture of one part hot asphalt and four parts hot sand and finally applying a ¼-in coating of hot asphalt sprinkled with sand. The layers are each tamped or ironed. Examination of steel surfaces thus treated has been made after 11 years' service and the steel found in good condition.

Stringers for thru bridges are I beams or built-up beams; if the latter the depth should be approximately ⅛ the panel length. When only one line is used per rail the stringers should be not less than 6 ft 6 in nor more than 8 ft on centers, and if two lines are used per rail they should be spaced symmetrically about the rail and from 9 to 15 in from its center. Sometimes the main stringer is placed under or near the center of the rail and an additional stringer of smaller size is placed outside near the ends of the ties. This outer is called a safety stringer. Deck plate girder bridges generally have no stringers, the ties resting on the top flanges.

Built-up Stringers are designed in the same manner as deck plate girders except that flange plates are not generally needed. Where flange plates are used the ties must be notched to fit over the rivet heads or else the rivets in the top flange plate must be countersunk. Unless the stringer is directly under the rail, flange ∟'s with unequal legs should be used and the long leg should be against the web, or else two rows of rivets must be used in the vertical leg. Placing the long leg horizontal increases the effective depth but also increases the bending due to the deflection of the ties. If the ratio of the length of top flange to its width exceeds 12, cross frames should be added to reduce the unsupported length.

In Proportioning the Cross-section of I-beam stringers or flanges of built-up stringers the ABSOLUTE MAXIMUM MOMENT (Art. 18) for the live load should be used and to it should be added the dead load moment at the middle. This is not exact but is on the side of safety and the error is very small. The following formulas are useful in computing the absolute live moments on stringers. They apply for equal loads spaced 5 ft on centers, and since consolidation locomotives are generally used in design and since more than four wheels would rarely be on a stringer at a time the formulas cover most panel lengths. There are four cases:

With one wheel on the stringer, $M = \frac{1}{4} Wl$,
With two wheels on the stringer, $M = W(2l - 5)^2/8l$,
With three wheels on the stringer, $M = W(\frac{3}{4} l - 5)$,
With four wheels on the stringer, $M = W(2l - 5)^2/4l - 5W$,

where W = the weight on one wheel in pounds, l = length of stringer in feet, M = absolute maximum live bending moment in lb-ft per rail. With one or with three wheels on the stringer the absolute maximum moment occurs at the middle; with two or with four wheels the absolute maximum is at a wheel 1¼ ft from the middle when the wheel spacings are 5 ft as above assumed.

Floor Beams are ordinarily built-up beams riveted between the webs of girders or the posts of trusses. In connecting them to pin-connected trusses it is advantageous to place the beams opposite the pins even tho this necessitates cutting away the lower corners of the beams (Fig. 29). In addition to its own weight, which is practically uniformly distributed, a floor beam is subjected to a concentrated load at each line of stringers. The distance

between centers of girders varies, for different roads and for different girder depths, from 12 to 16 ft on straight track. For double tracks the stringers are frequently placed 6 ft on centers and the two girders 28 to 30 ft on centers. The economic depth of floor beams is much larger than for stringers, but the economic depth usually cannot be used since in thru bridges the floor must be kept as shallow as possible and the floor beams are frequently made only slightly deeper than the stringers. In thru plate girder bridges a portion of each end of each floor-beam web plate is replaced by a gusset plate of the same thickness but having greater depth at the girder. These gussets extend to the top flanges of the girders to support the flanges laterally, Fig. 29. The gusset and web should be spliced with plates on each side of the web and with sufficient rivets to transmit the shear at the splice and to carry that portion of the moment that the web is allowed to carry. For deep girders the inclined edge of each gusset should be stiffened with two L's. The lower flange of the floor beam should rest on the lateral connecting plate, which in turn should rest on and be riveted to the outstanding leg of the lower flange L of the girder, but there must be sufficient rivets in the hitch L's to carry all the floor-beam reaction into the girder web. The stringers should either rest on the lower flange of the floor beam or else on a small shelf L to aid in erection, but neither the flange nor the shelf should be counted in computing the rivets required to connect stringers to floor beam.

If there is a flange plate on a symmetrical floor beam of a single-track bridge with two lines of stringers the length of the plate can be determined by applying the expression $2\,ba/A + c + 2$, which gives the total length of the plate in feet; b being the distance from girder to stringer and c the distance between stringers, both in feet, a net area of plate in sq in, A net area of bottom flange at point of maximum section plus $\frac{1}{8}$ gross section of web in sq in. If there are two plates on each flange the above expression also holds for the length of the plate next to the L's if the a is made to include the net area of the second plate as well as of the first.

For a uniform live and dead load the maximum load on a floor beam at a stringer connection is twice the maximum live and dead end shear on one stringer, the panels being all equal, but with a series of wheel loads this is not true. For wheel loads the position of the wheels to give maximum load on floor beam is the same as will give the maximum bending moment at the corresponding point on a span whose length equals the sum of the two adjacent panels. For an end floor beam the maximum load on the floor beam at a stringer connection is equal to the maximum live and dead end shear on one stringer.

15. Transportation and Erection

In Loading on Cars care must be taken not to get the steel so high that it will strike overhead obstructions. In some sections of this country the CLEAR HEAD ROOM above top of rail is as low as 14 or 15 ft, but generally more room is available. Posts, chords and other similar members when piled on cars should be braced and bolted in position to prevent displacement. Wherever possible the length of a piece should be such as to overhang one car length by just a few feet.

Long Plate Girders require special attention in loading so that they will not be too high and will be free to ride around curves. They are loaded on flat cars, generally with their webs vertical, and supported at two points only. If a girder is longer than two cars one or more idler or spacer cars carrying no load are used between the end cars on which the whole weight of the girder is carried. Each support consists of a transverse bolster or sill made of one or two timbers laid on the floor of each end car and bolted to the same with one large bolt 2 or 3 in in diameter, passing thru the center of the

sill and countersunk in its top. Under the center and each end of the sill i a steel track plate upon which the sill at one support can rotate and b means of a slotted pin hole can rotate or move longitudinally at the othe support. The top flange of the girder is braced to the ends of the sills b inclined wooden braces, 6 or 8 inches square, connected together at the to by yoke plates passing over the top flange and connected to the ends of th sills by bent plates and bolts. The ends of the sills are tied to the bottor flange of the girder by diagonal rods, two or four rods being used at eac support. For the heaviest girders steel gun-truck cars of 100 000 lb capacit are used and the distance from top of a vertical girder to top of rail is abou 5½ ft more than the depth of the girder. Girders as long as 125 to 130 and from 9 to 12 ft deep have been shipped as above described or wit slight modifications.

Fabricated Structural Work, that is, structural shapes on which some shop wor has been done, is given a fifth-class freight when shipped in car-load lots, and a fourtl class rate in less than car-load lots. Steel in the rough is given a lower rate.

In Designing, the following items should be observed to facilitate erectior Avoid details which necessitate telescoping one member into another, as i top chords, connections of floor beams to girders and stringers to floor beams swinging a member vertically or rotating it horizontally into place is muc easier. Place shelf L's or the equivalent on floor beams under all stringe connections where two stringers connect at the same point on opposite side of the beam. All thru spans should be designed so that either trusses an girders or the floor system may be erected first in final position. Avoi notching ties to clear rivet heads and laterals. Ship lateral plates loose o bolt them to members so they will not be bent or broken. Deep gusse plates on floor beams should be stiffened with L's. Tack "loose" filler with countersunk rivets. Allow clearances for drilling anchor-bolt holes i masonry and for setting the bolts after the steel is all in place. Top chor sections nearest the center should contain at least two panel points. Provid a separate bed plate for each shoe. In riveted bridges the gusset plate at th top of the end post should be shop-riveted to the end post. At pin-connecte joints, tie plates should be kept far enough away from the joints and enoug rivets should be countersunk to allow the members to swing in place.

The Erection of a Bridge consists in assembling its various parts t complete the whole. A DERRICK CAR is a car upon which a derrick or simila device is placed and operated, usually by power carried on the car. FALSE WORK is the temporary steel or wooden supports upon which the final stee work is erected. A TRAVELER is a steel or wooden frame with which th parts of a structure are handled during erection, and which is supported b and moves back and forth upon either the false work as in ordinary trus bridges, or upon a portion of the finished structure as in cantilever erection.

Girders and Small Trusses are Erected with gin poles, derrick cars derricks or gallows frames. Where derrick cars are available they shoul be used, for with them a girder of ordinary size and weight can be take from the cars, moved to the proper position and lowered upon its supports A GALLOWS FRAME consists of a transverse wooden bent guyed at the to and of such shape that a train can pass thru it. A frame of this kind i placed at each end of a span and the cars carrying the girder to be erecte is run out on false work, or on an old bridge as the case may be, and th girder is raised from the cars with two tackles at each frame. The car are then removed and the girder is lowered into position or is placed along side of its final position and later slid laterally into place. A GIN POLE is

solid or framed pole guyed at the top with at least four guys and with its lower end resting on a support. They are set slightly inclined so that the upper end overhangs the load, which is lifted by hoist lines passing over a sheave at the top. Gin poles are useful for erecting simple girders, beams or small riveted bridge or roof trusses. A framed gin pole 146 ft long was used in erection of a tower at Manitowoc, Wis., but ordinarily they are single masts 30 to 60 ft long. Girders are sometimes put in place by blocking and jacks.

False Work of wood consists of pile or trestle bents spaced from 10 to 50 ft apart, tho the distance apart should generally be used equal to the panel length of the truss to be erected. Where piles can be used they make the best false work and four or more are placed in a bent, the bents being braced transversely and longitudinally. If the height above water is only 10 to 20 ft the piles are sawed off just below that height and capped, but for greater heights framed bents are built upon the piles, which are sawed off just above water level. Posts in framed bents should be 10 by 10 or 12 by 12 inches and bracings 3 by 10 or 3 by 12. In swift water the bents should be braced in pairs to form towers and the long openings between towers spanned with trusses. If piles cannot be driven the bents must be framed and rest on sills or small timber cribs to distribute the pressure over the bottom. Wooden stringers are bolted to the bents to carry the steel and the tracks for the traveler. Stringers should be placed about one foot below level of steel to allow for blocking.

Travelers consist of two, three or four bents of wood or steel braced together longitudinally and of such shape that they can surround the steel work to be erected. At the foot of each leg of each bent is a wheel or truck which runs upon the track on the false work, and above the bents are stringers to which hoisting blocks may be attached. Travelers sometimes are made to run inside between the trusses. To erect an ordinary truss span the floor beams and stringers and lower chord members are put in position on the false work. The traveler then sets the four posts for the center panels of the two trusses and places the center diagonals in position. The top-chord sections over these panels are then lowered and the pins driven, or if the trusses are of the riveted type temporary bolts are used. The top laterals are then connected and the traveler moved to the next panel and thus successively erects the several panels to the end, after which it is returned to the center and erects the other half of the span in the same manner. The blocking under the trusses is removed and the rivets driven. Light highway bridges and steel buildings are erected by derricks resting on the false work or upon the steel frame respectively. For steel-frame buildings traveling or tower derricks are used. Roof trusses are assembled on the ground and then hoisted in place by gin poles or derricks.

Pilot and Driving Nuts of cast steel are placed on pins to protect the threads while driving. Trusses or parts of trusses should not be allowed to stand without being well braced, and roof trusses must be carefully guyed to avoid overturning, and in light truss work care must be taken to avoid dropping pieces on the lower chords.

Where False Work cannot be used, simple trusses are sometimes erected as cantilevers by building out from either end, but in such cases the members must be specially designed to allow this. Bridges are sometimes erected along the shore and then floated out into position on barges.

The Time Required for Erection varies greatly. An example of rapid bridge erection is that of the Cairo bridge over the Ohio River in southern Illinois in which one span weighing about 2 000 000 lb and 518 ft 6 in long was erected in six days, and to erect two such spans required only seventy-five men for one month and three days. For erecting steel frames of office buildings two tiers of beams per week is an average,

and work has been done at the rapid rate of 20 stories in 26 days, 18 stories in 2 months and 3 days.

16. Cost of Bridges

The Total Cost of a bridge structure is made up of the cost of abutments, piers, flooring and floor system, bracing and trusses or girders, and can be approximately stated by the formula

$$C=a+n(x-1)+p_1L+pbL^2/x$$

in which a=the cost of two abutments, L=length of bridge in feet, x=the number of spans, p_1=the cost of flooring and floor system per linear foot, p=the cost of steel in trusses and laterals per pound, n=the cost of one pier, b=a factor depending on the kind of bridge, 8 to 10 for railroad bridges. The total cost is a minimum when the cost of one pier is equal to the cost of steel in the trusses and laterals of one span or when $n=pbL^2/x^2$ or when $x^2=pbL^2/n$.

Contracts for the Steelwork of bridges are based on a pound price or on a lump sum, and should require furnishing the material, fabrication, transportation, erection and painting, the erection generally including the furnishing of necessary tools, false work and labor. Steel bridges furnished, erected and painted cost, in 1910, from 3.5 to 5.0 cents per pound and in 1918, about double these amounts.

The Costs of Items entering into the costs of bridge superstructures are subject to variations of time and place. Since 1914 prices have been subject to such constant and startling changes, mainly upward, no reliable nor useful set of costs can be inserted in this edition. The following costs were approximately correct in 1910 and even in 1914 and are retained for their relative value. For 1918 they should be raised from 50 to 100%. Structural steel shapes, however, have been fixed by the U. S. Government and take the Government base of 2.90 cents per lb for steel bars, 3.00 for shapes, 3.25 for plates and 5.00 for checkered plates, all f.o.b. Pittsburgh. The scale of extras is very complex and voluminous.

Approximate Costs in 1914 (reinserted from the Third Edition).

Structural Steel Shapes, F. O. B. Pittsburgh, in cents per pound. Angles, I beams, channels and Z bars 1.50 to 1.70, T's 1.55 to 1.75, deck beams and bulb L's 1.90 to 2.10, plates 1.60 to 1.70 with additional charges for cutting to size or for large sizes varying from 0.10 to 1.55, checkered plates 2.50, round forgings, 3.75, eye-bar flats 1.60 to 2.10, rivet rods 1.60, for punching holes at the mill from 0 15 to 0.35, for fittings such as small L's riveted to beams 1.55 in addition to the cost of fittings.

Cast Iron and Cast Steel, F. O. B. point of manufacture, in cents per pound. Cast-iron railing posts 3.5 up, column bases 2.5, name plates, 4.0; cast-steel bed plates 4.0.

Drafting $1.00 to $2.00 per ton if the details are left to the template shop and $2.00 to $4 00 if all parts are detailed on the drawings.

Shop Costs in cents per pound. Plate girders for highway bridges, 0.75 to 1.00 for railroad bridges 0.75 to 1.50, riveted trusses for highway bridges 0.75 to 1.15, for railroad bridges 0.75 to 1.50, pin-connected trusses for highway bridges 0.85 to 1.25, for railroad bridges 0.85 to 1.50.

Erection. Piles 25 to 50 cts per lin ft, false-work timber $25.00 to $30.00 per 1000 ft BM, placing false-work timber $8.00 to $12.00 per 1000 ft BM, the salvage or proceeds of sale of false work being 25 to 30 percent of first cost of lumber. The cost of placing, bolting and riveting steel truss spans varies from $10,00 to $20.00 per short ton exclusive of painting, being larger per ton for short spans then for long ones.

Timber in place per 1000 ft BM, including framing and spiking. Yellow pine for highway bridge floors $30.00 to $45.00, oak for same $40.00 to $55.00.

Railings in place per linear foot of railing. Gas-pipe railings with three lines of gas pipe on pipe posts $1.00, railings with cast-iron newel posts and gas-pipe hand rail and with some scroll work $1.75 to $2.50, lattice railings of flat bars and light L's $1.50.

INSPECTION of structural steel at the rolling mill and bridge shop costs from 50 to 75 cts per short ton.

Paint for Steel Bridges. Parts of bridge members in contact should be given one coat of red-lead paint before being riveted together in the shop. All other surfaces should have one coat of raw linseed oil or one coat of red-lead paint before leaving the shop. After erection two coats of different colors should be applied, the last after the first is dry. Red-lead paint should be mixt 20 to 25 lb red lead to one gallon raw linseed oil, altho some engineers specify as much as 35 lb per gal of oil. Graphite paint, made by grinding 12.5 lb graphite in one gallon linseed oil, is good for either the first or priming coat or for subsequent coats. For one short ton of structural material allow ½ gallon of paint for the first and ⅜ for the second coat. The following costs were approximately correct in 1910. Paints for steel from $1.00 to $2.50 per gal. Red lead at 7 cts per lb and linseed oil at 50 cts per gallon would make a paint costing approximately $1.50 per gal. For furnishing and applying one coat of linseed oil or metallic paints such as red or brown iron oxide, allow $1.00 per short ton, for one coat of red lead or good quality of graphite $2.00 per short ton. Cleaning new work by means of the sand blast costs from $1.00 to $1.75 per short ton. Pickling is the process of cleaning steel by soaking it in acid and then removing the acid.

The Life of Steel Bridges depends on the maintenance, on the loads, the locality and the kind of structure. Railroad bridges, which are generally properly cared for, are replaced on account of increase in weight of rolling stock; in such cases the average life is about twenty years. For highway bridges not subjected to electric railway traffic and properly maintained the life is probably very much more than the above figure, but in many cases highway bridges need extensive repairs or renewal of the steel after fifteen years.

BEAM AND GIRDER BRIDGES

17. General Arrangement

Beam Bridges are used for short spans up to 25 or 30 ft in length both for railroad or highway and electric railway structures. They consist of I beams running parallel with the railroad or highway, resting at their ends on abutments and carrying open or solid floors in railroad bridges and plank or paved roadways for highways. The beams must be computed from their section moduli, but their depth for railroad bridges should not be less than $1/_{12}$ the span, for electric railway bridges not less than $1/_{20}$ and for ordinary highway traffic not less than $1/_{20}$ to $1/_{24}$. The following spacing of beams is desirable for RAILROAD SPANS with open floors, that is, ties resting directly on the beams: two I beams under each rail, 12 inches between webs and symmetrically under the rail with a safety stringer 2 ft from each outermost track stringer. At each end and at center of the span channel braces should be riveted between the beams, and at intervals of 5 ft channel separators should connect the two beams under each rail. For spans over 10 ft diagonal braces should also be used. For double-track bridges there should be one safety stringer between tracks. For beam bridges with solid floors the I beams should be spaced apart 18 to 30 in, covered with flat steel plates and ballast, and should have braces at ends and center of span. For HIGHWAY BRIDGES the beams are spaced apart 2 to 3 ft and should be connected together at their ends with L's or [’s.

A Plate Girder is a beam made by riveting a flange, consisting of L's alone or L's and plates, along the upper and lower edges of a solid vertical plate called the WEB. Upright pairs of L's called STIFFENERS are usually

riveted to the web at the supports and at certain intermediate points. Plat girders are most suitable for spans between 25 and 100 ft, and even for span as high as 125 or 130 ft they make excellent bridges, expensive in first cos but low in maintenance charges.

Tubular Bridges were built by Robert Stevenson, who in 1850 completed the Br tannia bridge in Wales and in 1859 the tubular bridge at Montreal, the latter bein replaced in 1898 by a truss bridge. The Britannia bridge consists of two parallel inde pendent wrought-iron tubes thru which the trains run, each tube having two vertic solid webs connecting to top and bottom chords formed by a series of cells made c plates and L's. Each tube is 15 ft wide and from 23 ft deep at the ends to 30 ft at th center. There are two 460-ft spans and two 230-ft spans.

Deck Plate Girder Bridges usually consist of two girders under eac track connected by two systems of lateral bracing, one in the plane of th top and the other in the plane of the bottom flanges, and by transverse swa frames at each end and at intermediate points from 15 to 20 ft apart. Fo double-track structures the two inner girders should be connected togethe at the panel points of the top lateral systems by angle struts only without an transverse or lateral bracing between these inner girders. The two girder in single-track deck bridges are spaced from 5 to 9 ft on centers, with 6½ f the best distance for spans up to 60 ft. A good rule for spans over 60 ft i to make the distance on centers generally equal $^1/_{10}$ the span but not less tha the depth of the girders. The deeper the girders the greater should be th spacing. For double-track spans having four girders, the tracks being fror 12 to 13 ft on centers, the inner girders should be 5½ to 6½ ft apart respectivel if the two girders under a track are 6½ ft apart. Deck spans having floo beams and stringers can be well arranged for single track by placing the girder 9 to 10 ft apart and connecting the floor beams to the girders at such an ele vation that the tops of stringers and girders are in one horizontal plane, th stringers resting on top of the floor beams and spaced 5 or 6 ft apart. The tie are supported by stringers and girders, the latter serving as safety stringer in case of a derailment, and the stringers are held laterally to the girders b diaphragm plates, that is, vertical plates placed over the floor beams and be tween the stringers and girders. One lateral bracing system is used at th level of the lower flanges of floor beams.

Thru Plate Girder Bridges generally cost more under the same con ditions than deck, but they are used where head room under the bridge i essential as in the case of grade-crossing elimination. While for short span the distance between centers of girders on single track may be as small a 12 ft, this distance is ordinarily 13 to 15 ft. For clearances see Art. 11 Double-track bridges may have 2 or 3 girders, the center girder carryin weight from both tracks. Panel lengths vary from 9 to 15 ft, the smaller valu being used to secure shallow floor systems. Plate girders for deck span have the upper corners square, but for long thru railroad, and especially fo thru highway bridges, it is desirable to have them rounded to prevent acci dents in the one case and for appearance in the other. The radius of the curv is usually made equal to the length of the bed plate.

Highway Plate Girder Bridges should have two girders for ordinar thru spans, and for heavy construction with two sidewalks and one roadwa three girders, and in extreme widths even more are used. Where two girder are used in thru spans they should be placed at the curbs to avoid interfer ence with vehicular traffic, and for this same reason deck spans should b used if possible where the width of roadway necessitates three or more girder The central girder of thru bridges projecting up into a roadway interfere with traffic, and where two street railway tracks cross over the bridge th

distance between track centers must be increased from the usual 9 or 10 ft to 13 to 15 ft, thus making undesirable curves in the tracks. For deck spans the tops of floor beams are placed near the top of the girders and the stringers are framed between the floor beams. The SIDEWALKS are carried on brackets riveted to the outside girder webs, and since the upper flanges of the brackets are in tension some provision other than tension on rivets must be made to transfer the stress into the floor beam upper flange. This is best done by connecting top of bracket to top of floor beam with a tie plate passing either thru or over the main girder.

The Top Lateral System for deck plate girder railroad bridges should be a Warren truss with panel lengths as nearly equal to the distance between the girders as possible and with an angle strut at each panel point, thus making the slope of the diagonals about 45 degrees. At alternate panel points the L strut is the top bar of the cross frame, and this spaces the cross frames at the ends and at intermediate points at distances apart equal to twice the distance between girders. No L's less than 3½ by 3 by ⅜ should be used for bracing in railroad, nor less than 2½ by 2½ by 5/16 for highway bridges. For intermediate cross frames of deck bridges two horizontal and two diagonal L's 3½ by 3½ by ⅜ are commonly used, while the end cross frames should be computed to carry the end reaction of the top lateral system. These end frames should have two horizontal L's at top and two at bottom, with two diagonal L's or C's in each frame. All frames must be riveted to the horizontal lateral bracing plates and to vertical connecting plates which are riveted to stiffeners on the girders. The lower bracing for deck spans should consist of a Warren truss, the L struts being omitted except where they constitute the lower members of the cross frames. For thru single-track spans the lateral system should consist of two diagonal L's in each panel riveted to plates which are connected to lower flanges of girders and floor beams, and if there are no end floor beams a strut should connect the stringers and girders together at each end of the span. The laterals are generally connected to the bottom flanges of the stringers. For thru double-track railroad or thru highway bridges the laterals sometimes run over two panels instead of one.

Long Spans for American Plate Girders

Location	Railway or Highway	Length, feet	Height, feet	Weight of one girder, short tons
St. Catherine's, Can.	G. T. Ry.	105.7	9.0	64.5
Newark, N. J.	C. R.R. of N. J.	109.5	10.0	77.4
Lyons, N. Y.	N. Y. C. & H. R.	110.7	11.2	96.0
Janesville, Wis.	C. M. & St. P.	114.5	9.5	42.0
Near Jordan, N. Y.	N. Y. C. & H. R.	116.0	10.0	77.5
Albany, N. Y.	N. Y. C. & H. R.	116.5	10.0	85.0
Bradford Division	Erie R.R.	128.0	9.0	49.0
Towanda, Pa.	L. V. R.R.	129.5	10.0	56.0
Hubbard, O.	Erie R.R.	131.3	9.6	66.9
Terre Haute, Ind.	Highway	121.5	9.8	30.3
Sixth St., Phila.	Highway	123.6	9.5	50.0
Cincinnati, Ohio*	Highway	213.0	16.0	......
Worcester, Mass.	B. & A. R.R.	122.5	11.0	170.0
Chicago, Ill.	N. Y. C. & St. L. R.R.	131.75	9.7	98.0
Chicago, Ill.	N. Y., C. & St. L. R.R.	125.7	10.8	130.5
New Durham, N. J.	West Shore, N. Y. C. R.R.	131.50	10.0	105.0

* Built 1870.

Lateral Bracing is frequently made too light. This is probably due to the fact that the lateral force has been considered as wind and not as wind and vibration or train combined. For ordinary railroad spans the lateral force on the loaded chord should be

taken at 200 lb per lin ft plus 10% of the live loading on one track, and on the unloaded chord at 200 lb per linear foot. These forces to be considered as moving loads and the allowable tension placed at 16 000 lb per sq in and compression at $16\,000-70\,l/r$, where l is the length and r the least radius of gyration of the member, l/r should not exceed 120. If the track is on a curve the stresses due to centrifugal force should be added to those from wind and lateral vibration. If a member of a lateral system is subjected to alternate tension and compression from wind, or wind and centrifugal force combined, it should be proportioned for the stress giving the largest section. Impact is not added to stresses produced by longitudinal, centrifugal and lateral or wind forces. For thru spans the laterals usually may be designed in tension to take the entire lateral shear in the panel.

18. Girders without Floor Beams

Notation. V=shear at section of girder, V_0=maximum positiv live shear in end panel, M=moment at any section, M_c=moment at center of span, l=length of span, a=distance from section to right end of span, g=distance from center of span to any section, w=uniform dead load per unit of length, w_1=uniform live load per unit of length, W=total load on span, D=increase in shear due to loads coming on span, p=panel length, n=number of panels to right of a given panel, m=number of panels in span, x=distance from resultant load on span to right support.

The Shear on a section is the algebraic sum of all outer forces or components of same acting on one side of and parallel with the given section; it is positiv when the resultant force on the left is upward and on the right downward. BENDING MOMENT, or moment, at a section of a beam is the algebraic sum of the moments of all forces acting on either side of the section. It is positiv when the resultant moment on left of section is clockwise and on the right is anti-clockwise, causing the beam to bend convex downward and hence causing compression in upper and tension in lower portion of beam.

DEAD LOAD SHEARS. With uniform load over entire span the shear at any section distant a from right end is $V=w(a-\frac{1}{2}l)$. This is the equation of a straight line having for its ordinate at the left end of span the maximum positiv shear of $wl/2$, and for the right end an ordinate of equal amount which is the maximum negativ shear. Shear at center is zero. The above equation shows that the shear at any section is also equal to the amount of load between the section and the center of span. If in addition to the uniform load there are one or more concentrated fixt loads the shear at any section is found by subtracting from either reaction the loads, uniform and concentrated, between the reaction and the section, and the moment is a maximum where the shear passes thru zero.

DEAD LOAD MOMENTS. With uniform load over entire span the moment at any section distant a from the right support is $M=\frac{1}{2}wa(l-a)$. This is the equation of a parabola with axis vertical and vertex over the center of the span. The moment is positiv at all points and is maximum when $a=l/2$, in which case the maximum moment is $M_c=wl^2/8$; if w is in pounds per foot and the span l in feet M is in lb ft and where necessary to reduce this to in-lb multiply by 12, not by 144. If M_c is the moment at center and g is the distance from center to any point, the moment at that point equals $M=M_c-4\,M_cg^2/l^2$.

LIVE LOAD SHEARS AND MOMENTS. With a uniform load the maximum shear at either end occurs for full loading and may be found by use of above formulas for dead load shear. The maximum positiv live shear at any point is $V=w_1a^2/2\,l$, and occurs when the live load covers the portion of the girder to right of section This is the equation of a parabola with axis vertical and vertex at right support. The maximum negativ live shear at any section occurs when the left portion of girder is loaded, and may be found by calling a the distance from left instead of from right support. The maximum moment at any point occurs for full loading, and may be found by use of formulas for dead load moments.

LIVE LOAD SHEARS. With a single concentrated load the maximum positiv shear at a given section occurs when the load is just to right of section and is equal to the left

reaction; the maximum negativ occurs when the load is just to the left of section and is equal to the right reaction. With a system of concentrated loads the maximum positiv shear at any section occurs when one of the heavy loads is just to the right and most of the remaining loads are also to the right of the section. The problem of finding the greatest shear at a section A (Fig. 30) involves therefore first the location of the loads on the span to produce the maximum. Starting with first wheel at A the shear will then be the left reaction, and if all the loads are moved to the left a distance d_1 until the second wheel is at A the shear is increased if $Wd_1/l+D>P_1$, see Fig. 30. W is the total load on span at beginning of movement, d_1 the distance loads are moved, D is left reaction due to loads that may come on the right of span during the movement. This same relation holds for any other similar movement and for any section provided no loads leave the span during the operation. To apply

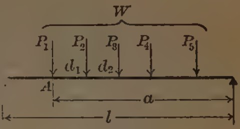


Fig. 30

to the end of span W must include all loads on except the one which is at the end and which therefore leaves the span during the movement. If $d_1>l-a$, P_1 will leave the span when P_2 is moved to A, Fig. 30, in which case the shear at A is greater when P_2 is there if $(W-P_1)d_1/l+D<P_1a/l$. There is one more case to be considered. Suppose P_2 at A and P_1 on the span and that when P_3 is moved to A, P_1 leaves the span. Then the shear at A will be increased if

$$(W-P_1)d_2/l+D+P_1(l-a-d_1)/l>P_2.$$

Maximum Shear at End of Span. Let it be required to find the maximum shear at the left end of a 40-ft span with Cooper's E-50 loading (Fig. 23). Placing wheel 1 at the left end the total load on the span exclusive of 1 is 132.5, and moving the wheels 8 ft to the left until wheel 2 lies at the end the shear at the end is increased, since $132.5\times8/40+D>12.5$. Moving the loads 5 ft further to the left the third wheel lies at the end, and the shear is decreased during this second movement because $140\times5/40+D<25$, the 140 being total load on span exclusive of wheel 2 when wheel 2 is at the end. Hence with wheel 2 at the end the maximum shear exists there and equals the left reaction or $65\times8/40+100\times32.5/40=94.25$ per rail. This shear may be obtained also with the aid of the moment diagram (Fig. 34) as follows. The moment of all wheels to the left of the right support about that support is 4370, and deducting $12.5\times48=600$, there remains the moment about right support of all wheels that are on the span or 3770 thousandths of ft-lb. Dividing this by the span length, 40, the left reaction or shear at left end is 94 250 lb. In the case of a Cooper or similar loading (where a light wheel precedes a series of heavy ones) it is unnecessary to apply the criterion for end shear, since P_1 never gives a maximum and P_2 usually does, the exceptions being unimportant.

Live Load Moments. For a single concentrated load P the maximum moment at a given section will occur when the load is at that section. With a system of concentrated wheel loads the maximum moment will occur at a given fixt section when there are as many loads on the span as is possible consistent with the heavy loads being near, and one of the loads at the section; and this load must be such that when it is just to the right of the section the average load on the left must be less, and when just to the left the average on the left must be greater than the average load on the entire span. Usually several positions of the loads satisfy this criterion in which case the greatest moment must be determined by inspection or by computing all cases.

The Absolute Maximum Moment is the greatest moment that can occur on a given span under a system of concentrated wheel loads. It will always occur at a wheel which lies at or near the center of the span and the criterion for its determination is as follows. When the center of the span is halfway between the center of gravity of loads on the span and one of the wheels near the center of gravity, then the moment under the wheel where the shear changes sign is a maximum and is the greatest which this combination of loads can produce on the span. When the center of gravity of loads on the span coincides with one of the loads the absolute maximum wil occur at the center of span. The method of procedure is then: Assume some loads on the span so as to bring the heavy loads near the center and as many loads on the span as possible. Find the center of gravity of these

loads and place the center of span midway between the center of gravity and the nearest load. If all the loads assumed on are now really on the span and the shear changes sign at this load compute the moment at the nearest load just mentioned. If when the above assumed loads are placed in the position just cited some of the loads assumed on are really off the span, it is then necessary to try again, using a new resultant. Unless one can tell by inspection it is necessary to compute in this manner the moment at several of the wheels until the greatest possible moment is found. In Fig. 31 the maximum occurs at P_2 when the center of span bisects b and therefore $x = \frac{1}{2}(l - b)$ and the expression for the moment at P_2 is $M_2 = W_{1-4}(l - b)^2/4l - \Sigma Pd$ where W_{1-4} is the resultant of loads on span and ΣPd is the sum of the moments of loads to left of P_2 about P_2. If now the dead load is to be taken into account with the wheel loading, the value of $x = \frac{1}{2}l - b(wl + W_{1-4})/(wl + 2W_{1-4})$ and the maximum moment occurs at P_2. It is however sufficiently accurate to compute the absolute maximum due to live load only as above stated and add the dead moment at the center.

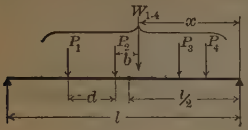


Fig. 31

The Exact Equivalent Uniform Live Load, obtained from Art. 10 may be employed conveniently to obtain maximum moments and shears due to a Cooper loading. For maximum moment at a section of a girder, the equivalent uniform load q may be obtained from the chart, Fig. 25, by taking $l_2 = a$ and $l_1 = l - a$, the two segments into which the section divides the span length. The bending moment $M = \frac{1}{2}ql_1l_2$.

For maximum positiv shear $l_1 = o$ and $l_2 = a$. Instead of extending the chart to include values of q for $l_1 = o$, the latter are given in a separate table, Fig. 25a for the corresponding values of l_2. q as thus obtained, is correct for end reactions, and $R = \frac{1}{2}ql$; but for intermediate sections wheel 2 is presupposed to be placed at the section considered and the effect of wheel 1 is disregarded. The maximum shear at a section, upon the above assumption, is $V = \frac{1}{2}ql_2^2/l$. To correct for the effect of wheel 1 subtract 30 000 $(1 - (l_2 + 8)/l)$ from the above value of V. This correction and the values of q in Art. 10 are based upon loads per track. For loads per rail divide by two.

For example, to find the maximum shears in an 80-ft plate girder due to E-60 loading per rail; at the end, $l_2 = 80$, $q = 4657$, $R = \frac{1}{2}ql_2 = 166\,300$; at the $\frac{1}{4}$ point, $l_2 = 60$, $q = 4900$, $V = \frac{1}{2}ql_2\frac{3}{4} = 110\,250$; at the $\frac{1}{2}$ point, $l_2 = 40$, $q = 5655$, $V = \frac{1}{2}ql_2\frac{1}{2} = 56\,550$. To correct for the effect of wheel 1 subtract $\frac{1}{2} \times 30\,000(1 - (l_2 + 8)/80)$ from the last two values of V or 2250 and 6000 lb respectively.

19. Girders with Floor Beams

For Notation see beginning of Art. 18

Dead Load Shears. The uniform load here assumed is that due to the floor system only or, in the case of a truss bridge, due to floor system and weight of trusses, in other words, the dead load is assumed to act at the panel points only. For the shears and moments due to the dead weight of a plate girder itself the formulas in Art. 18 may be used. The panels are of equal length unless otherwise stated. The shear in any panel is

$$V = wp(n - \tfrac{1}{2}m + \tfrac{1}{2})$$

where n is the number of panels to the right of the one in question. The maximum dead positiv shear is in the left end panel, that is when $n = m - 1$, and is $V_0 = wp(m - 1)/2$. This is of course equal to the gross left reaction minus the $\frac{1}{2}$ panel load at the support. From this maximum positiv value the shear decreases by wp for every panel towards the right, reaching a maximum negativ shear in the right end panel.

Dead Load Moments. The moments at panel points are equal to the moments

existing at the corresponding points on a girder without floor beams, and the moments between floor beams are slightly less than without floor beams. Without floor beams all ordinates are those of a parabola, while with floor beams only the ordinates at floor beams fall on a parabola.

Live Load Shears. The neutral point in a panel is that point at which if a single concentrated load be placed, and no other loads act on the span, the shear in the panel is zero. Every panel has a neutral point and its distance from the right end of the panel $=np/(m-1)$. For maximum positiv live shear the load must extend from the neutral point to right end of span, and for maximum negativ shear from the neutral point to left end of span. The exact maximum positiv live shear in any panel is $V=V_0n^2/(m-1)^2=w_1pn^2/2(m-1)$. The exact equivalent uniform live load for Cooper's loadings for maximum positiv live load shear in any panel may be obtained from the chart, Fig. 25, (Art. 10), by letting $l_1=$the distance from the neutral point to the right end of the panel and $l_2=a=$the distance from the end of the panel to the right end of the span. The maximum negativ shear for a given panel is the same as the maximum positiv shear in the corresponding panel on the opposite side of center of span. The above is the exact method for finding live shears, but the approximate method is mostly used for uniform loads; this assumes that the full live panel load is applied to each panel point to the right of a panel to produce maximum positiv live shear and that no load acts to the left of the panel, thus neglecting the one-half panel load which acts at the left end of the panel. By this method the maximum positiv shear is $V=w_1pn(n+1)/2m$.

Live Load Shears. For maximum positiv shear in any panel under a system of wheel loads most if not all of the loads should lie to the right of the panel and one of the wheels should be on the floor beam at the right end of the panel and this must be such a wheel that when it is just to the right of the panel the total load on the span must be more than m times the load on the panel, and when this wheel is placed just within the right end of the panel the total load on the bridge must be less than m times the load on the panel. If two or more positions satisfy this criterion the one giving the greatest shear may be determined by inspection, by computing the shear for all the positions that satisfy the criterion, or by computing the change in shear due to a movement of the load system. Thus in Fig. 32 to find the change in shear due to a slight movement suppose that the above criterion for panel A is satisfied by either wheels 1 or 2. Place P_1 at right end of panel A and move the loads till P_2 comes to that point; then if $d_1<p$ the shear in A is greater for position 2 than 1 if $Wd_1/l+D>P_1d_1/p$. If $d_1>p$ but P_1 does not leave the span while the movement is taking place then the shear is increased if $Wd_1/l+D>P_1$. When P_2 is at right end of A and P_3 is then moved to that point the shear is increased if $Wd_2/l+D>(P_1+P_2)d_2/p$, where $d_1+d_2<p$; and is increased if $Wd_2/l+D>P_2d_2/p+P_1(p-d_1)/p$, where $d_1+d_2>p$ but $d_1<p$ and $d_2<p$. These expressions do not hold if a load leaves the span during the movement, but similar ones may be written for this case.

Live Load Moments. The criterion for determining the position of loads to give a maximum moment at a panel point is the same as if there were no floor beams, see Art. 18. Clearly the maximum live moment at a panel point next the end as B, Fig. 32,

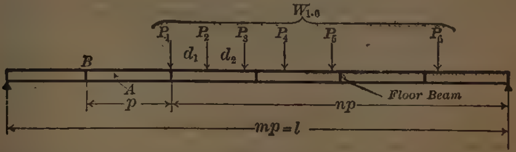


Fig. 32

equals V_0p, and for any point in the end panel the maximum live moment is the maximum live shear in that panel times the distance from the end of span to the point in question. The criterion for maximum moment for a point within a panel such as C (Fig. 33) is $W_1+W_2q/p=Wa_1/l$, where W_1 is the total load on panel to left of, and W_2 the total load on the panel in question.

The Moment Diagram (Fig. 34) aids in computing shears and moments. Its essential parts are weights and wheel spacings for a concentrated load

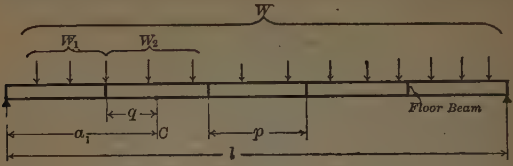


Fig. 33

system, distance of each wheel from the first, sum of all loads from the first to each succeeding wheel inclusive, sum of moments about each wheel of all

Wheel	Moments	Sum of Loads	Wheel Loads	Sum of Distances	Distances
1		12.50	12.5		8.0
2	100.00	37.50	25.0	8	5.0
3	287.50	62.50	25.0	13	6.0
4	600.00	87.50	25.0	18	5.0
5	1037.50	112.50	25.0	23	9.0
6	2050.00	128.75	16.25	32	5.0
7	2693.75	145.00	16.25	37	6.0
8	3563.75	161.25	16.25	43	5.0
9	4370.00	177.50	16.25	48	8.0
10	[illegible]	190.00	12.5	56	8.0
11	7310.00	215.00	25.0	64	5.0
12	8385.00	240.00	25.0	69	5.0
13	9585.00	265.00	25.0	74	5.0
14	10910.00	290.00	25.0	79	9.0
15	13520.00	306.25	16.25	88	5.0
16	15051.25	322.50	16.25	93	6.0
17	16986.25	338.75	16.25	99	5.0
18	18680.00	355.00	16.25	104	5.0
	20455.00	355.00	2.5 per linear foot	109	

Loads are in thousands of pounds per rail

Fig. 34. Moment Diagram for Wheel Loads for Cooper's E-50 Loading

loads to the left of that wheel. The wheel spacings should be plotted to scale and the span lengths to the same scale.

To illustrate let it be required to compute maximum moment at center of a 40-ft span, using the diagram in Fig. 34. The fourth wheel satisfies the criterion in Art. 18. and by inspection gives the greatest moment. To compute this moment find the moment of left reaction about center and deduct the sum of moments of wheels 1, 2 and 3. To do this find the moment of all wheels on the span about right support, 2693.75 + 145 (1) = 2838.75 in thousands of lb-ft, and divide this by the span, 40 ft, and multiply the quotient by the half span, 20 ft; whence 1419.375 is the moment of left reaction about center. Subtract 600, the moment of wheels 1, 2 and 3 about wheel 4, and 819 375 lb-ft is the moment at center of span.

20. Plate Girder Webs

Notation. M = bending moment, t = thickness of web, h = depth of web, V = vertical shear on web, S = allowable tensile unit stress, S_s = allowable shearing unit stress, S_b = bearing unit stress on rivets, S_c = allowable compressive unit stress, A = gross area of cross-section of web plate, d = horizontal distance between stiffeners, H = horizontal force on outside rivet in a vertical row, t_1 and h_1 = thickness and depth of web splice plate, M_r = bending moment carried by one row of rivets in web splice, M_w = net moment carried by web, p = pitch in inches, N = number of rivets in a row. Forces in pounds, dimensions in inches, moments in inch-pounds, stresses in lb per sq in.

The Function of the Web of a girder is to resist the shearing forces acting on the various sections of the girder and also to carry some of the bending moment. The depth and thickness of a web plate must be such that its cross-sectional area will withstand the shear and buckling and at the same time give the girder an economic depth that is, a depth requiring the least material in the girder. Girders are sometimes made with curved upper flanges, giving greater depths at the center than at ends.

The Economic Depth of a Girder varies with the kind and amount of loads, the length of span and the allowable unit stresses. A common rule for girders up to about 40 ft span is that the depth should be such as to make the weight of flanges equal to the total weight of web with its splice plates and stiffening angles and in proportion as the span exceeds 40 ft to make the weight of flanges greater than that of the web and its details. Unless limited by such considerations as head room, girders having constant cross-section should be given a depth equal to ⅛ or 1/9 their length, and girders of variable sections 1/10 to 1/12 their length. For short built-up girders used as stringers the depth must generally be deeper, more nearly ⅙ their length.

The Thickness of Webs must be such as to resist the shear at various sections and also to prevent buckling. The gross cross-sectional area of the web at any section is found by dividing the total shear by the allowable unit shearing stress. The required thickness for shear is $t = V/S_sh$.

Buckling of the Web. The horizontal and vertical shears acting in a web plate produce a tension and a compression of equal intensities on planes making 45 degrees with the neutral axis. This compression tends to buckle the web plate, and the web must be thick enough to withstand the buckling or else stiffeners must be riveted to the web to increase its compressive strength or, what amounts to the same thing, its shearing strength. Stiffeners are used over bearings, at points of concentrated loading and at other points where the width of the unsupported web between flange angles is greater than 60 times its thickness. These stiffeners are usually made of L's riveted vertically to the web. Having found the required thickness for shearing from the formula just given, the proper DISTANCE BETWEEN STIFFENERS must then be determined; and if the distance in the clear between stiffeners, d, comes equal to or greater than the clear vertical distance between horizontal angles the stiffeners are spaced equal to this vertical distance, with 6 ft as a maximum. If d comes out very small so that stiffeners are required too close together, a thicker web should be chosen to reduce the number of stiffeners. The horizontal distance in the clear between stiffeners is given by $d = (12\,000 - s)t/40$, in which s is the average shear per sq in in the web at the section where d is desired. The following table, based on the above formula will be found useful in spacing stiffeners. To obtain the spacing near a given section of the girder find V/A = average shear per sq in at that section, seek in the proper t column the nearest number to V/A and the required spacing d is given in the left column.

The Minimum Thickness for Webs of plate girders in buildings is ¼ in, for highway bridges 5/16 and preferably ⅜ in, and for railroad bridges not less than ⅜ for lightest traffic and 7/16 for heavy traffic. The thickness near the ends of heavy girders is sometimes greater than near the center, and the increase may be made either by riveting reinforcing plates on the sides of the main web plate or by using a thicker plate near each end. In the latter case thin fillers must be used between the flanges and the thin web plates. Webs as thick as 1 inch have been used, but ordinarily they are from 7/16 to ¾ for railroad bridges, 5/16 to 7/16 for highway and from ¼ to ⅜ for buildings. Sheared plates may be obtained as wide as 10 ft, with lengths of 10 or 11 ft, and universal mill plates up to 4 ft wide by 70 to 80 ft long. Plates of great width or length are subject to special prices.

Intermediate Stiffeners, Fig. 44, used to stiffen the web of a railway plate girder

V/A = Shearing Stress for Webs in Pounds per Square Inch

d, inches	Thickness of Web in inches						
	$t = 1/4$	5/16	3/8	7/16	1/2	9/16	5/8
10	10 400	10 720	10 933	11 086	11 200	11 289	11 360
11	10 240	10 592	10 827	10 994	11 120	11 218	11 296
12	10 080	10 464	10 720	10 903	11 040	11 147	11 232
13	9 920	10 336	10 613	10 811	10 960	11 076	11 168
14	9 760	10 208	10 507	10 720	10 880	11 005	11 104
15	9 600	10 080	10 400	10 629	10 800	10 933	11 040
16	9 440	9 952	10 293	10 537	10 720	10 862	10 976
17	9 280	9 824	10 187	10 446	10 640	10 791	10 912
18	9 120	9 696	10 080	10 354	10 560	10 720	10 848
19	8 960	9 568	9 973	10 263	10 480	10 649	10 784
20	8 800	9 440	9 867	10 171	10 400	10 578	10 720
22	8 480	9 184	9 654	9 989	10 240	10 436	10 592
24	8 160	8 928	9 441	9 806	10 080	10 293	10 464
26	7 840	8 672	9 228	9 623	9 920	10 151	10 336
28	7 520	8 416	9 014	9 440	9 760	10 009	10 208
30	7 200	8 160	8 801	9 257	9 600	9 867	10 080
32	6 880	7 904	8 588	9 074	9 440	9 725	9 952
34	6 560	7 648	8 374	8 891	9 280	9 582	9 824
36	6 240	7 392	8 161	8 709	9 120	9 440	9 696
38	5 920	7 136	7 948	8 526	8 960	9 298	9 568
40	5 600	6 880	7 734	8 343	8 800	9 156	9 440
44	4 960	6 368	7 307	7 977	8 480	8 871	9 184
48	4 320	5 856	6 880	7 611	8 160	8 587	8 928
52	3 680	5 344	6 454	7 246	7 840	8 302	8 672
56	3 040	4 832	6 028	6 880	7 520	8 018	8 416
60	2 400	4 320	5 600	6 514	7 200	7 733	8 160
64	1 760	3 808	5 174	6 149	6 880	7 449	7 904
68	1 120	3 296	4 747	5 783	6 560	7 165	7 648
72	480	2 784	4 320	5 417	6 240	6 880	7 392

vary from $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in for spans under 50 ft to $6 \times 4 \times \frac{3}{8}$ in for 100-ft spans. By the A. R. E. A. specifications the outstanding legs should not be less than 1/30 of the depth of the girder plus 2 in. For girders in buildings and highway bridges they are frequently made lighter. They should be used in pairs, one on each side of the web. In deck bridges intermediate stiffeners should be proportioned to carry the greatest wheel load in compression, should be connected to the web with sufficient rivets to transmit this load, should not be crimped, that is, not bent around the vertical legs of the flange L's and should have outstanding legs as wide as possible and not extend over the edge of the horizontal leg of flange L's. For thru spans the connecting L's at floor beams should have outstanding legs of at least 4 in, with rivets arranged so that floor beams may be placed without spreading girders.

End Stiffeners used to connect one girder web to another, as at junction of stringer to floor beam, must be capable of carrying the full end shear on a vertical section thru the line of rivets. Where a girder rests upon a support as at abutments the end stiffeners (Fig. 44) usually consist of two pairs of L's, one pair at the extreme end of girder and the other over the bed plate and about 1 inch from its inner edge. For girders with bearings arranged on rockers each pair of L's should be proportioned for ½ the maximum end reactions, but for those resting on flat bearings each pair should be assumed to carry at least ¾ of the reaction. Stiffeners at the ends and at points of concentrated

loading should be proportioned as long columns, using the unit stress obtained by the formula, $S = 16\,000 - 70\,l/r$; in which l is taken equal to one-half the depth of the girder in inches and r is the radius of gyration of the pair of stiffeners about their gravity axis lying in the central plane of the web plate. The outstanding legs should be as wide as the flange angles will allow and should fit tightly against them.

Thus, given an end shear of 319 500 lb, depth of girder 80½ in, thickness of web plate $^7/_{16}$ in, and thickness of fillers between stiffeners and web ¾ in. Assume two pairs of $5 \times 3\frac{1}{2} \times {}^{11}/_{16}$ in L's with the 5-in legs outstanding. The 3½-in legs are separated by the $^7/_{16}$ in web and the two ¾-in fillers, or $1^{15}/_{16}$ in in all. r for their combined section about the axis in the web is 3.12 in. $S = 16\,000 - 70\,l/r = 16\,000 - 70 \times 40.25/3.12 = 15\,000$ (nearly). Required area $= 319\,500 \div 15\,000 = 21.4$ sq in. The actual area of the 4 L's is 21.5 sq in. The number of rivets in double shear required to transfer the end shear from the angles to the tight fillers is $319\,500 \div 14\,430 = 23$. The number used is 30, not counting those thru the flange angles. The number of rivets in bearing on the web required to transfer the end shear from fillers to web is $319\,500 \div 9190 = 35$. The number used is 48, making the filler plates a rigid part of the web plate.

For intermediate stiffeners, $^1/_{30}$ of the depth of girder plus 2 in requires the outstanding leg to be 5 in. Accordingly each pair of intermediate stiffeners will consist of $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in L's. The spacing should not exceed 6 ft nor be greater than $t(12\,000 - S)/40$, where $t = {}^7/_{16}$ in and S = total shear at section ÷ gross area of web. The exact position of the several pairs of stiffeners is governed by the location of the cross frames.

End Bearings. Expansion and contraction longitudinally due to changes in temperature must be allowed at one end of all spans, a range of 150 degrees Fahr. being the amount generally used. Taking the coefficient of linear expansion of steel as 0.000007 per degree Fahr. and a range of 150 degrees, the expansion amounts to ⅛ inch for 10 ft of length. Spans less than 80 ft may have planed sliding bearings at one end, Fig. 44. For spans over 80 ft long turned rollers not less than 6 in in diameter should be used and each end should rest on a rocker or bolster arranged with a pin to fix the point of application of the reaction. Allowable PRESSURE PER LINEAR INCH OF ROLLERS is variously taken at from $600\,D$ to $300\,D$ depending on whether impact is or is not allowed; D is diameter of roller in inches. BEARING ON MASONRY may be safely taken at from 300 to 600 lb per sq in. SEGMENTAL ROLLERS as used for long spans are made by removing portions of cylindrical rollers as shown in Fig. 35. The dimensions x, y and w must be given by the following formulas to prevent binding when the bridge expands or contracts a distance of e inches: $x \geq t/\cos\theta$, $y \geq (d \sin\theta)/2$, $w \leq (d \cos\theta)/2$, $m \geq e/2$; $\theta = 180\,e/d\pi$.

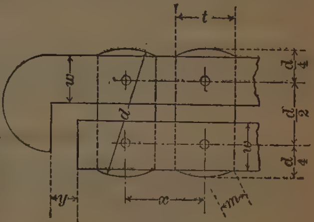


Fig. 35

Web Splices. Girders over 40 to 50 ft in length usually have their webs in two or more pieces; the web plates being spliced generally with one plate on each side of the web at each joint, and sometimes in addition thereto a plate riveted at each joint to the vertical leg of each of the four flange angles. Web plate splices should always be designed for moment, and when shear acts simultaneously it should likewise be considered, altho generally when a splice is sufficient to carry the full bending moment of the net section of the web the shear may be neglected. To design a web splice consisting of two plates (Fig. 36), one on each side of the web. (1) Make the two splice plates

as deep as the vertical distance in the clear between flange L's, and then compute their thickness so that the combined net moment of resistance of the two plates equals that of the web, but no plate less than ⅜ in should be used. Thickness of each splice plate is $t_1 = th^3/2\,h_1^3$. (2) Next space the rivets in one vertical row uniformly if possible; if any uneven spaces are required place the larger spaces near the neutral axis of the girder. The pitch in a row should be not less than 3 and not over 4 in. (3) Compute the bending moment that one vertical row of rivets in the splice plates can carry, using $M_r = (2\,H/y_2)\Sigma_0^{y_1} y^2$, where H is the value of the outermost flange rivet in bearing on the web (Fig. 36) and $\Sigma_0^{y_1} y^2$ is the sum of the squares of the distances from neutral axis to the rivets on one side of this axis only. (4) Compute the net moment of resistance of the web $= M_w = Sth^2/8$, S being the allowed unit stress in the tension flange. (5) The number of rows of rivets on each side of the joint is M_w/M_r. If the number of rows thus found involves a fraction of a row, as 2½ rows, the rivet spacing in each row may be changed accordingly, or better, use only the full rows for each side of joint, two in this case, and place the splice at a point where the flanges have sufficient excess area to carry a moment equal to the difference between the net moment of the web and the moment of the two full rows. In case the splice cannot be so placed, then additional plates having a net area equal to that just mentioned should be placed on each flange.

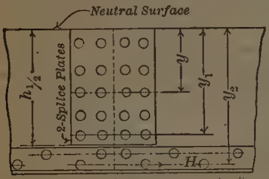


Fig. 36

With a uniform pitch, the bending moment a row of rivets will carry is given by $M_r = HpN(N+1)/6$, where H is the horizontal force on the rivet farthest from the neutral surface and which is included as carrying moment. If H is pounds, p inches, and N the number of rivets in one full row, then M_r is inch-pounds.

21. Plate Girder Flanges

Notation. M = bending moment in inch-pounds, d_1 = distance in inches between center of gravity of flanges, S = allowable unit tensile or compressive stress in lb per sq in, A = net area of cross-section of tension flange in sq in, t = thickness and h = depth of web, l = unsupported length and w = width of compression flange, p = pitch of rivets all in inches, V = vertical shearing force on girder, r = value of rivet in pounds, v = resultant shear per running inch of flange in pounds, W = maximum wheel load in pounds, v_1 = exact longitudinal shear per running inch of flange in pounds, m = statical moment in inch units, I = moment of inertia of entire cross-section of girder in inch units.

The Flanges of a beam or plate girder carry the greater part of the bending moment acting on the girder, and as they receive stress from the web they serve to prevent its deformation. One flange, usually the lower, is in tension and the other is in compression. The latter is a horizontal column receiving its stress from the web at points thruout its length, and held in a vertical direction by the web, and in a horizontal direction, usually tho not always, by bracing. The function of the flanges of a girder, then, is to carry moment and they must be designed for this purpose.

The Composition of Flanges varies in different girders and also usually at different sections of the same girder. For light loads or for short spans each flange may be made of two L's alone (Fig. 37) or for heavier work two

.'s with one or more horizontal flange or cover plates riveted to the out-tanding legs of the L's (Fig. 38). To secure larger cross-sections vertical ange plates between the L's and web are used, and these plates are run the ntire length of the span, while the horizontal flange plates are made of dif-erent lengths. For deck girder bridges the varying thickness of the upper ange causes difficulty in dapping, that is, notching the ties, and Fig. 39 and ig. 40 show sections designed to reduce this difficulty to a minimum; in one

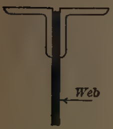


Fig. 37

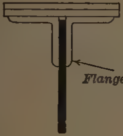


Fig. 38

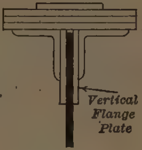


Fig. 39

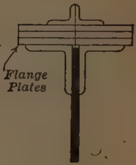


Fig. 40

Fig. 41

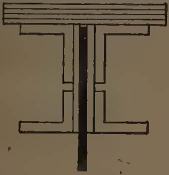


Fig. 42

case the ties rest on and are notched over the narrow upper plate and in the other the two small L's serve to hold and support the ties. In each case the plate and the L's extend the entire length of the girder and have filler plates underneath where the main flange plates are cut off. Fig. 41 is also suitable for the upper flange of a deck girder. For heavy sections an arrangement such as is shown in Fig. 42 ay be used for the top flange. In this case the lower flange is generally ven the more usual section consisting of two L's with cover and vertical lates (Fig. 39). Railroad bridge girders with over 150 sq in of gross cross-ction in each flange have been built.

Proportioning Tension Flanges. The design of a flange (Fig. 44) con-sts in finding the required cross-section at the point of maximum bend-g moment, selecting proper sizes of L's and plates and then finding the ngths of the plates. If the web could carry no moment the compression one flange and the tension in the other would resist the full moment and ould form a couple the arm d_1 of which is the distance between centers of avity of the two flanges; and each flange stress would be M/d_1. The flange ress divided by the allowable tensile unit stress S would give the required t area for the tension flange, that is M/d_1S. But since the web does carry moment $= Sth^2/6$ if there is no vertical row of rivets at or near the section nsidered, and approximately $Sth^2/8$ if there is, then the equivalent area, hich if placed in each flange will carry the same moment as the web can rry, is $th/6$ or $th/8$ respectively. The correct required net area of the ten-on flange is usually based on the latter value and is $A = M/d_1S - th/8$. In oplying this formula care must be taken to use the proper numerical values. hus if d_1, t and h are inches and S is lb per sq in, then M is inch-pounds, and the sult A will be square inches.

With following data, to proportion the cross-section of a tension flange: maximum in ft-lb = 1 500 000 live, 360 000 dead, 1 210 000 impact, total 3 070 000; web 84 by

½ in, distance back to back of flange L's, 84½ in, maximum tensile stress, 16 000 lb per sq in. Assume the distance center to center of gravity of flanges, called the effective depth, as 83.0 in, then the required net flange area $= 3\,070\,000\ (12)/83.0\ (16\,000) - 84/2(8) = 22.5$. Two 6 by 6 by ⅝ L's with gross area of 14.2 sq in have a net area of $14.2 - 4\ (1)⅝ = 11.7$ sq in, which deducted from 22.5 leaves 10.8 as the required net area of plates. Assuming plates 14 inches wide with two ⅞ rivets in one section the net width is 12 in. Required thickness for plates $= 10.8/12 = 0.9$ in. Hence one plate 14 by ½ and one 14 by 7/16 and two 6 by 6 by ⅝ L's are to be used. In computing the net area of the two L's the two holes for ⅞ rivets in each L are taken as ⅛ larger in diameter than the rivet, namely 1 in; similarly in the plates. The position of the center of gravity of this flange must be computed to check the effective depth, and if the assumed effective depth is in appreciable error the design must be revised.

The Compression Flange should not be of less and is generally of the same gross section as the tension flange. It is usually supported laterally at points distance apart not exceeding 12 times its width. When the compression flange consists of angles, or of angles and flat cover plates, it is proportioned by the column formula, $S = 16\,000 - 200\ l/b$, where S = allowable unit stress on the gross section of the flange, l = unsupported length and b = flange width. When the cover of the flange is a channel the stress per square inch should not exceed $S = 16\,000 - 150\ l/b$.

Lengths of Flange Plates. Following is a graphic solution for a single-track thru railroad girder bridge of 60 ft span divided into 6 equal panels. By means of above formula for A compute the required net flange area at each of the panel points, 1, 2, 3 (Fig. 43), these being the points at which the live and dead moments are previously

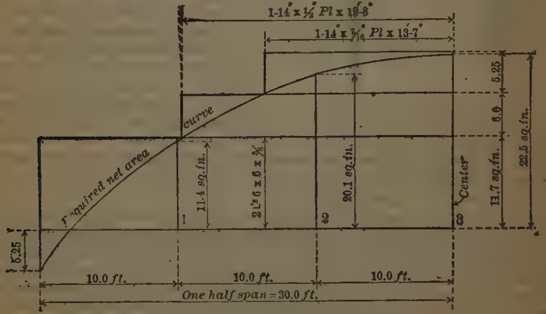


Fig. 43

computed; suppose their net areas to be 11.4, 20.1 and 22.5 sq in respectively. Plot these values at points 1, 2, 3 to any convenient scale, such as ⅛ in = 1 sq in. For the horizontal scale ¼ in = 1 ft is convenient. At the end lay off on opposite side of the horizontal axis the web equivalent, that is, $th/8$, here assumed 5.25 sq in, and then draw a smooth curve thru the four points located. Proportion bottom flange at point of maximum moment, finding the sizes of L's and plates as described above, and, beginning at the horizontal axis, lay off first the net area of the two L's, 11.7 sq in, and then of each succeeding plate, placing the thickest plate next to the L's. The half lengths of plates may now be scaled from the diagram, and to each half length should be added 1 ft or enough to get in at least two rows of rivets. The plates on the top flange may be made the same length as on the bottom except that the one next to the L's should run the entire length of girder.

When the bending moment M can be exprest algebraically the lengths of plates can be computed by reversing the above formula for proportioning flanges and expressing therein the moment in terms of known loads and an unknown quantity representing the position of end of plate. Thus, $M = (A + \frac{1}{8}\,th)\,d_1 S$, in which A is the flange area exclusive of the plate the length of which is being determined and exclusive of all plates outside of this plate. If the girder depth is variable, d_1 and h must be exprest in terms of the unknown quantity.

With a uniform load over span L and a uniform depth of girder the length x of any plate is $x = L\sqrt{a/A_1}$, where A_1 is the total area of the flange at the maximum section plus the web equivalent, and a is the area of the plate being cut plus the areas of all plates outside of it. If there are several plates, a is first the area of outside plate; second, this plus the area of the next plate; third, these two plus the area of third plate.

The Pitch of Flange Rivets (Fig. 44) is the distance from the center of one rivet to the center of the next whether in the same row or not. In bottom flanges of deck girders and in both flanges of thru girders the rivets connecting the flanges to web plates transmit longitudinal shear only, whereas these rivets in the top flanges of deck girders or stringers carry, in addition to the longitudinal, a vertical shear due to the pressure from the wheel loads and track. The required pitch for horizontal flange rivets which connect the flange L's to the web in girders without loads on the top flange is, according to the usual approximate method, $p = rd_1/V$, where r is the value of the rivet, namely, the bearing value on web plate or bearing on two L's or double shear whichever is the least. If the rivets carry vertical loads in addition to the longitudinal shear, as in top flanges of stringers or deck girders, the resultant shear per running inch is $v = \sqrt{(V/d_1)^2 + (W/36)^2}$, in which W is the maximum wheel load, here assumed distributed over 36 inches of flange length. And the pitch is $p = r/v$. This approximate method for finding the pitch of horizontal rivets in flanges gives spacing slightly smaller than the true value and is on the safe side. The exact LONGITUDINAL SHEAR existing between the flange and web or between the flange plates and L's is $v_1 = Vm/I$, in which m is the statical moment about the neutral axis of that part of the flange outside of the section on which the horizontal shear is desired, in other words, that part of the flange section which is connected to the remainder of the flange or to the web by means of the rivets the pitch of which is required.

In Spacing Rivets in Flanges the pitch is computed at intervals of from 2 to 5 ft for stringers and deck girders, and the spacing is varied in accordance with those computed values. For thru girders the pitch thruout any one panel is made constant since the shear is constant except for dead weight of girder. Vertical rivets which connect flange plates to flange L's are generally given the same pitch as the horizontal rivets. The pitch can be computed by the above exact formula.

A Good Method for Finding Pitch of flange rivets is illustrated by the following modification of the above approximate method. Required the pitch of horizontal flange rivets at the end of a thru girder having a web 60 by ½ in; at the end each flange consists of two 6 by 6 by ½ L's and one cover plate 14 by ½. Rivets ⅞ in diameter; bearing 24 000 lb per sq in; shearing 12 000 lb per sq in. End shear including live, impact and dead, 200 000 lb. Effective depth 58.6 in. The gross area of two L's and one cover plate is 11.5 + 7.0 = 18.5 sq in and including the web equivalent is 18.5 + 3.75 = 22.25 sq in, hence the longitudinal shear per running inch between flange L's and web is (18.5/22.25) 200 000/58.6 = 2840 lb. The least value of the rivet is ⅞ × ½ × 24 000 = 10 500 lb, and the required pitch at end of girder is 10 500/2840 = 3¾ in.

23. Riveted Joints

Notation. S_c, S_s and S_t are the allowable unit bearing, shearing and tensile stresses in lb per sq in; d, diameter of rivet; l, end distance; t, thickness, and w width of main plate; all dimensions in inches; n, number of rivets on each side of joint.

Before being driven a rivet consists of a shank, or cylindrical portion, and a head which may be either conical or nearly hemispherical, the latter form being called button-heads. The process of riveting consists of heating the rivet and while it is hot placing

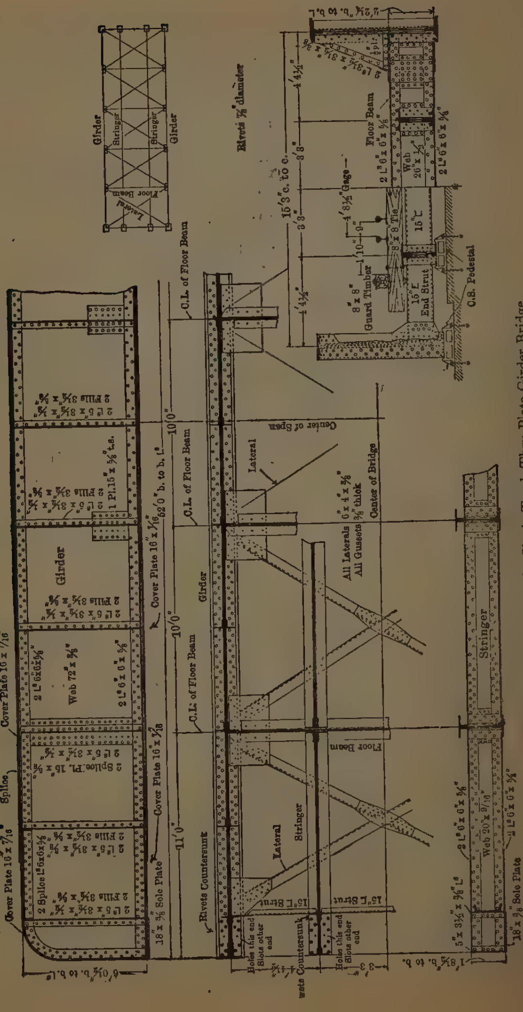


Fig. 44. Design of a Single Track Thru Plate Girder Bridge

it in the hole and upsetting the shank end to form a second head. A countersunk rivet is used where it is desired that the head of the rivet must be flush with the surface of the part into which it is driven, but to secure a surface free from obstruction the countersunk head must be slightly chipped.

Lap Riveted Joints are shown in Figs. 45, 46, the first being single and the second double riveted. Butt joints are shown in Figs. 47–50, those with two cover plates being better because of less bending in the rivets. Butt

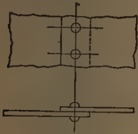


Fig. 45

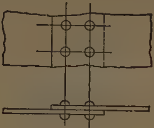


Fig. 46

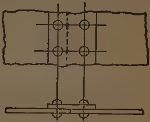


Fig. 47

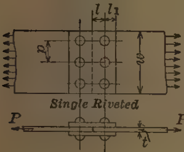


Fig. 48

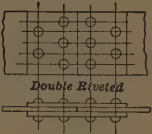


Fig. 49

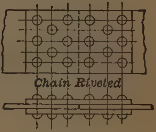


Fig. 50

joints require twice as many rivets as lap joints to transfer a stress from the one main plate into the other. In lap joints, and in butt joints with a single strap or cover, the rivets are in single shear, while in butt joints with double covers all rivets are in double shear.

Design of Riveted Joints. In Fig. 48 the main plate has a tension of P and is spliced with two covers each of which must have a thickness of at least $t/2$, altho usually each would be made from $2/3\,t$ to $\frac{3}{4}\,t$. There must be sufficient net area in the main plate on its cross-section thru the row of rivets to carry the force P in tension, that is, $(w-nd)\,tS_t = P$. The number of rivets on each side of the joint must transmit the force P in double shear, that is, $n = 2\,P/\pi d^2 S_s$; or in bearing on the main plate, $n = P/tdS_c$. The distances l and l_1, Fig. 48, must be sufficient to prevent shearing or tearing of the main plate and two covers respectively. For bridge work, rivets of $\frac{3}{4}$ and $\frac{7}{8}$ inch diameter are commonly used; for buildings, generally $\frac{3}{4}$ in. The minimum pitch used for structural work is $3\,d$, and the minimum $l = 1.5\,d$. Distance between rows in Fig. 49 should not be less than $2\,d$. Bearing value of one rivet on plate having thickness $t = tdS_c$. Single shearing value of one rivet is $S_s\pi d^2/4$.

The Efficiency of a Joint is the ratio of the strength of a given width of joint to that of a solid plate of equal width, and is usually exprest as per-cent; for example, single-riveted lap joints show efficiencies of from 50 to 70% and double-riveted lap joints 60 to 80%. Joints with drilled or reamed holes show higher efficiencies than those with punched holes. Values of from 60 to 70% are what may usually be expected. By careful design the rivets may be arranged so that higher efficiencies may be obtained. Tests on full-size angles show that tensile strength on the net area varies from 75 to 90% depending on the arrangement of rivets and on whether or not both legs of the angles are connected to the end plates by which stress is transmitted into the angles. Both legs of angles should be riveted at con-

nections. FRICTION between the main and splice plates in butt joints and between the two main plates of lap joints should not be relied upon, for altho under quiescent loads it may continue to exist, it is uncertain and for movable loads it may be destroyed altogether.

23. Tables for Rivets

The American Bridge Co's Standards show for button-head rivets: diameter of head 1½ times diameter of shank plus ⅛ in; height of head = 0.425 times diameter of head; for countersunk heads the slope is 30 degrees from axis of shank and the depth of head is ½ the diameter of shank. CAMBRIA STEEL COMPANY'S STANDARDS give for button-heads: height of head = 6/10 times diameter of shank, radius of head = ¾ times

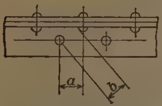


Fig. 51

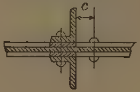


Fig. 52

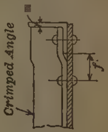


Fig. 53

Weight in Pounds for One Head and Shank of 100 Steel Rivets

Shank, Inches	Diameter of Rivet in Inches							
	3/8	½	⅝	¾	⅞	1	1⅛	1¼
1¼	5.[illegible]	12.8	22.0	29.3	43.9	66.6	93.3	127
1½	6.3	14.2	24.1	32.4	48.2	72.1	100	136
1¾	7.[illegible]	15.5	26.3	35.5	52.5	77.7	107	145
2	7.[illegible]	16.9	28.5	38.7	56.7	83.3	114	153
2¼	8.7	18.3	30.7	41.8	61.0	88.8	121	162
2½	9.4	19.7	32.8	44.9	65.2	94.4	128	171
2¾	10.2	21.1	35.0	48.0	69.5	100	136	179
3	[illegible]1.[illegible]	22.5	37.2	51.1	73.7	105	143	188
3¼	[illegible]1.7	23.9	39.3	54.3	78.0	111	150	197
3½	12.6	25.3	41.5	57.4	82.3	116	157	205
3¾	13.4	26.7	43.7	60.5	86.5	122	164	214
4	14.[illegible]	28.1	45.9	63.6	90.8	128	170	223
4¼	14.[illegible]	29.4	48.0	66.7	95.0	134	177	231
4½	15.7	30.8	50.2	69.9	99.3	139	185	240
4¾	16.5	32.2	52.4	73.0	104	145	192	249
5	17.[illegible]	33.6	54.5	76.1	108	150	199	258
5¼	18.1	35.0	56.7	79.2	112	156	206	266
5½	1[illegible].[illegible]	36.4	58.9	82.3	116	161	213	275
5¾	19.6	37.8	61.1	85.5	120	166	220	284
6	20.4	39.2	63.2	88.6	124	172	227	292
6½	21.9	42.0	67.6	95.1	133	184	241	310
7	23.5	44.7	71.9	101	142	195	255	327
7½	25.1	47.5	76.1	108	150	206	269	345
8	26.6	50.3	80.6	114	159	217	284	362
8½	28.2	53.1	85.0	120	167	227	298	379
9	29.8	55.9	89.3	126	176	239	312	397
9½	31.3	58.7	93.7	133	185	250	325	414
10	32.8	61.4	98.0	139	193	261	340	431
Weight of 100 Heads	1.8	5.8	11.1	13.6	22.6	39.0	58.0	83.5

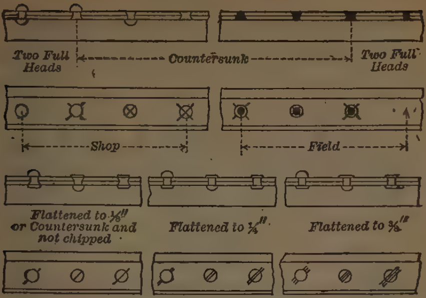


Fig. 54. Conventional Signs for Rivets

diameter of shank + 1/16 in; for countersunk heads the diameter is the same as for button-heads and the slope is 30 degrees. For required clearances see Figs. 51, 52 and 53. In Fig. 51, a must be sufficient to make b at least 1⅛ inch for ¾ or ⅞ rivets. In Fig. 52, minimum c is 1⅛ and 1¼ for ¾ or ⅞ rivets respectively. In Fig. 53 f is 1½ in plus 2 e but never less than 2 in.

Lengths of Rivets in Inches

Fig. 55

Grip, Inches	Diameter in Inches					Diameter in Inches					Grip, Inches
	½	⅝	¾	⅞	1	½	⅝	¾	⅞	1	
½	1½	1¾	1⅞	2	2⅛	1⅛	1¼	1¼	1⅜	1⅜	½
⅝	1⅝	1⅞	2	2⅛	2¼	1¼	1⅜	1⅜	1½	1½	⅝
¾	1¾	2	2⅛	2¼	2⅜	1⅜	1½	1½	1⅝	1⅝	¾
⅞	1⅞	2⅛	2¼	2⅜	2½	1½	1⅝	1⅝	1¾	1¾	⅞
1	2	2¼	2⅜	2½	2⅝	1⅝	1¾	1¾	1⅞	1⅞	1
1⅛	2⅛	2⅜	2½	2⅝	2¾	1¾	1⅞	1⅞	2	2	1⅛
1¼	2¼	2½	2⅝	2¾	2⅞	1⅞	2	2	2⅛	2⅛	1¼
1⅜	2⅜	2⅝	2¾	2⅞	3	2	2⅛	2⅛	2¼	2¼	1⅜
1½	2⅝	2⅞	3	3⅛	3¼	2⅛	2¼	2⅜	2⅜	2½	1½
1⅝	2¾	3	3⅛	3¼	3⅜	2¼	2⅜	2½	2½	2⅝	1⅝
1¾	2⅞	3⅛	3¼	3⅜	3½	2⅜	2½	2⅝	2⅝	2¾	1¾
1⅞	3	3¼	3⅜	3½	3⅝	2½	2⅝	2¾	2¾	2⅞	1⅞
2	3⅛	3⅜	3½	3⅝	3¾	2⅝	2¾	2⅞	2⅞	3	2
2½	3⅝	3⅞	4	4⅛	4¼	3⅛	3¼	3⅜	3⅜	3½	2½
2⅞	4	4¼	4⅜	4½	4⅝	3½	3⅝	3¾	3¾	3⅞	2⅞
3	4¼	4½	4⅝	4¾	4⅞	3¾	3⅞	3⅞	4	4⅛	3
3½	4¾	5	5⅛	5¼	5⅜	4¼	4⅜	4⅜	4½	4⅝	3½
3⅞	5⅛	5⅜	5½	5⅝	5¾	4⅝	4¾	4¾	4⅞	5	3⅞

Sizes of Rivet Heads. American Bridge Company

All Dimensions in Inches

Diameter of Shank	Button-head			Countersunk	
	Height	Diameter	Radius	Depth	Diameter
3/8	19/64	11/16	7/16	3/16	19/32
1/2	3/8	7/8	9/16	1/4	25/32
5/8	29/64	1 1/16	43/64	5/16	1
3/4	17/32	1 1/4	51/64	3/8	1 3/16
7/8	39/64	1 7/16	59/64	7/16	1 3/8
1	11/16	1 5/8	1 1/32	1/2	1 9/16

Shearing and Bearing Values of Rivets, in Pounds

Stress Lb per sq in	Diam. Rivet Inches	Value for Single Shear	Bearing Value, for plate thickness in inches								
			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
6 000 for Shearing 12 000 for Bearing	3/8	660	1130	1410	1690						
	1/2	1180	1500	1880	2250	2 630					
	5/8	1840	1880	2340	2810	3 280	3 750	4 220	4 690		
	3/4	2650	2250	2810	3380	3 940	4 500	5 160	5 630	6 190	6 750
	7/8	3610	2630	3280	3940	4 590	5 250	5 910	6 560	7 220	7 880
	1	4710	3000	3750	4500	5 250	6 000	6 750	7 500	8 250	9 000
7 500 for Shearing 15 000 for Bearing	3/8	830	1410	1760	2110						
	1/2	1470	1880	2340	2810	3 280	3 750				
	5/8	2300	2340	2930	3520	4 100	4 690	5 280	5 860		
	3/4	3310	2810	3520	4220	4 920	5 630	6 330	7 030	7 720	8 440
	7/8	4510	3280	4100	4920	5 740	6 560	7 380	8 200	9 030	9 850
	1	5890	3750	4690	5620	6 560	7 500	8 440	9 380	10 310	11 250
10 000 for Shearing 20 000 for Bearing	3/8	1100	1880	2340	2810						
	1/2	1960	2500	3130	3750	4 380	5 000				
	5/8	3070	3130	3910	4690	5 470	6 250	7 030	7 810		
	3/4	4420	3750	4690	5630	6 560	7 500	8 440	9 380	10 310	11 250
	7/8	6010	4380	5470	6570	7 660	8 750	9 840	10 940	12 030	13 130
	1	7850	5000	6250	7500	8 750	10 000	11 250	12 500	13 750	15 000
12 000 for Shearing 25 000 for Bearing	3/8	1320	2350	2930	3520						
	1/2	2360	3130	3910	4690	5 470	6 250				
	5/8	3680	3910	4880	5860	6 840	7 810	8 790	9 770		
	3/4	5300	4690	5860	7030	8 210	9 380	10 550	11 720	12 890	14 060
	7/8	7220	5470	6840	8210	9 580	10 940	12 310	13 670	15 040	16 410
	1	9430	6250	7820	9380	10 940	12 500	14 060	15 630	17 190	18 750

SIMPLE TRUSS BRIDGES

24. Types of Trusses

Historical. The following are important steps in the development of American metallic truss bridges. First iron truss bridge built in 1840 over the Erie Canal by Earl Trumbull, some members were cast and some were wrought iron. In 1840 Squire Whipple built the first bow-string truss and in 1847 published the first work on stresses in bridge trusses; in 1852 53 he built a railroad span of 146 ft on the Rensselaer and Saratoga R.R. which had the first double intersection trusses. In 1845 first iron truss railroad bridge


Fig. 55. Fink


Fig. 56. Bollman


Fig. 57. Whipple

Fig. 58. Thru Howe

Fig. 59. Thru Pratt


Fig. 60. Thru Warren


Fig. 61. Deck Howe


Fig. 62. Deck Pratt


Fig. 63. Deck Warren


Fig. 64.


Fig. 65


Fig. 66


Fig. 67


Fig. 68. Baltimore

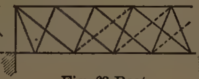


Fig. 69. Post


Fig. 70. K Truss


Fig. 71. Bowstring


Fig. 72. Parker

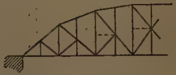


Fig. 73. Pennsylvania

was built on the Reading R.R. by R. B. Osborne. In 1844 Thomas and Caleb Pratt patented the Pratt combination truss of wood and iron, and by 1850 Pratt trusses were made entirely of metal. First Fink truss in 1852 at Fairmont, W. Va., had three spans each 205 ft, with cast chord and wrought-iron diagonals. Longest Fink truss at St. Charles, Mo., built 1871, span 306½ ft. First long Bollman truss built in 1852 at Harper's Ferry, Va.; span 124 ft. In 1858–59 John W. Murphy built for Lehigh Valley R.R. a Whipple truss with pin connections thruout, and in 1863 he first used all wrought iron compression and tension members in a pin-connected bridge with cast-iron chord blocks and pedestals. Howard Carroll began building riveted lattice trusses entirely of wrought iron on New York Central R.R. in 1859. In 1861 J. H. Linville first used wide forged eye-bars and wrought-iron web posts, the upper chords being cast iron. First Baltimore or Pettit truss 1871 at Mt. Union, Pa., designed by bridge department of Pennsylvania R.R. The first Post truss 1865; built by S. S. Post for Erie R.R. About 1870 C. H. Parker modified the Pratt truss, using inclined upper chords, thus originating the Parker truss. Steel first used 1874 in Ead's arch bridge at St. Louis. In 1879 first bridge built entirely of steel at Glasgow over Missouri River. Pegram truss introduced by C. H. Pegram, 1887. The longest simple truss bridge is one of 720-ft span built over the Ohio River at Metropolis, Ill., in 1917, it has Pennsylvania trusses.

Long Span American Simple Bridges

From Merriman and Jacoby's Roofs and Bridges, Part I.

Span center to center of end pins		Kind of truss	Over what river	Location	Railroad tracks		Date of completion
					Single	Double	
ft	in						
720	0	Pennsylvania	Ohio	Metropolis, Ill.		*	1917
668	0	“	Mississippi	St. Louis (Municipal)		*	†19—
640	0	“	St. Lawrence	Quebec		*	1917
586	0	“	Great Miami	Elizabethtown, Ohio			†1906
552	0	“	Ohio	Metropolis, Ill.		*	1917
546	6	“	Ohio	Louisville and Jeffersonville	*		1894
542	6	“	Ohio	Cincinnati and Covington		*	†1889
533	0	“	Delaware	Philadelphia		*	1896
531	0	“	Allegheny	Pittsburgh			†1914
523	0	“	Ohio	Pittsburgh (Brunot's I).	*		1890
522	0	“	Ohio	Wheeling	*		1885
521	11¾	Warren sub-verts	Ohio	Henderson, Ky.	*		
519	2½	Pennsylvania	Ohio	Wheeling			†1892
518	1¾	Whipple	Ohio	Cairo	*		1889
518	0	Pennsylvania	Ohio	Kenova, W. Va.	*		1913
518	0	“	Susquehanna	Havre de Grace, Md.		*	1909
517	8½	“	Monongahela	Glenwood, Pa.			†1895
517	6	“	Mississippi	St. Louis (Merchant's)		*	1890
517	6	“	Mississippi	St. Louis (McKinley)			†1910
517	6	Baltimore	Allegeny	Denny Sta. (Bessemer)	*		1897
515	0	Whipple	Ohio	Cincinnati	*		1877
515	0	Pennsylvania	Monongahela	West Braddock, Pa.			†1897
515	0	“	Monongahela	Webster to Donora, Pa.			†1909
506	8	“	Ohio	Newport and Cincinnati	*		†1896
500	0	“	Missouri	Sioux City	*		†1895
498	0	“	Monongahela	Clairton, Pa.		*	1903
495	8⅛	“	Monongahela	Rankin, Pa.		*	1900
489	3	“	Monongahela	West Braddock, Pa.			†1897
484	6	“	Ohio	Cincinnati and Covington		*	†1889
465	0¼	Parker	Miami	New Baltimore, Ohio			†1901
453	10	Pennsylvania	Monongahela	Pittsburgh (So 10th St.)			†1904
450	0	“	Brazos	Waco, Tex.	*		†1902
447	0	“	Allegheny	Mossgrove, Pa.	*		1899
442	10½	“	Ohio	Beaver, Pa.	*		1889
440	0	Baltimore	Missouri	Bellefontaine, Mo		*	1893
439	3	Pennsylvania	Allegheny	Pittsburgh (Sixth Street)			†1893
435	10	Baltimore	Miami	Hamilton, Ohio			†1895
430	0	Pennsylvania	Mississippi	Red Wing, Minn			†1896
428	0	“	Missouri	Kansas City		*	†191–
425	0	“	Missouri	Mobridge, S. Dakota	*		1907
416	6	“	Columbia	Near Rock Island, Wash.	*		1893
416	0¼	“	Missouri	St. Charles, Mo.			†1904
416	0	“	Ohio	Pittsburgh (Brunot's Id.)	*		1890
415	6	Whipple	Ohio	Point Pleasant, W. Va.	*		1885
413	0	Pennsylvania	Allegheny	New Kensington, Pa.			†1901
410	0	“	Kentucky	Frankfort, Ky			†1904
408	0	“	Massena, Can.	Massena, N. Y.			†1901
407	[illegible]	Parker	Monongahela	Port Perry, Pa.		*	1903

† Highway traffic.

Types of Bridge Trusses are the Pratt (Fig. 59), Parker (Fig. 72), Baltimore (Fig. 68), and Pennsylvania (Fig. 73). The Howe truss while ordiarily built of wood was also made of iron during early stages of truss develop-nent. Post, Bollman and Whipple trusses are no longer built, and the Fink is sed only for roof trusses in a modified form. Warren trusses are serviceable or short spans. The Pratt, Howe, Whipple, Warren and Post types have two orizontal chords connected by the web system composed of diagonals or diagnals and verticals. In the Pratt the verticals are in compression except he end suspender or HIP VERTICAL, *s* in Fig. 59, which is a tension member; nd the diagonals are in tension except the two end ones called end posts. Pratt trusses are sometimes made with vertical end posts even in thru bridges, n which case the diagonal in each end panel slopes downward towards the enter and is in tension. The dotted bars in Fig. 59 are COUNTERS, or OUNTER DIAGONALS, and they come into action when the live load shear n the panel is greater than and of opposite kind from the dead shear in that anel. For greater rigidity it is better practice to omit the counter diagonals nd to make the main diagonals in the panels requiring counter bracing, comression members. In Fig. 60 the dotted lines represent SUB-VERTICALS vhich are frequently used to reduce the panel length, and the short verticals nd diagonals of the Baltimore truss (Fig. 68) are for the same purpose. The OLLISION STRUT, *c* in Fig. 58, is used to brace the end post. For spans larger han about 250 ft, for railroad, and about 200 ft, for highway bridges, the inlined or curved chord trusses are most economical, and the Parker (Fig. 72) nd Pennsylvania (Fig. 73) types are the best forms, In Fig. 73 the dotted orizontal members are not essential for stability but serve to brace the long erticals. MULTIPLE-WEB SYSTEMS (Figs. 57, 64–67) were commonly used n the Whipple, Post and Warren trusses till 1880. They gave short panels, ence small stringers and floor beams, and as the web members had small tresses their connections to the chords were easily made. The most recent ype, suitable for long spans, is the K truss. It has been used for the 230-ft pan built by the Atchison, Topeka & Santa Fe Ry. across the Arkansas river t Pueblo, Colo. The webbing is arranged as shown in Fig. 70. Trusses of his type are economical only for spans of 300 ft or over. The most notable se of this type of webbing is in the cantilever and anchor arms of the new Quebec bridge. The chief superiority of the K type over the Pennsylvania is n the lower secondary stresses and in the avoidance of the use of the short orizontal struts required, in the Pennsylvania truss, to brace the vertical osts.

The Economic Depth of a Truss is that which makes the material in he bridge a minimum. It varies from $1/5$ to $1/8$ of the span depending on the orm of truss, the length and number of panels and the allowable unit stress. For short span thru bridges the minimum depth is fixt by the required clear ead room. As the depth of a truss increases the weights of the chords become ess and the weights of the web members greater, hence there must be a depth or each bridge for which the weight of material is the least. Ordinary spans f 250 ft or less usually have depths of about $1/6$ the span. Longer spans are enerally relatively shallower in order to avoid excessively long web members nd the increased cost of handling and erecting the deeper truss. The greater he live load and the wider the bridge the greater may the relative depth be nade. In the 523-ft spans of the Ohio Connecting bridge the center depth is 1 ft or $1/_{5.75}$ of the span; in the 552-ft spans of the Metropolis bridge it is 3 ft or $1/_{6.65}$ of the span; in the 720-ft span of the same bridge it is 110 ft or $/_{6.54}$ of the span; and in the 640-ft suspended span of the Quebec bridge it s 110 ft or $1/_{6.82}$ of the span. End depth of trusses is fixt by the clearance

plus depth of portal, which latter increases with the length of span. The parabola give a satisfactory curve for the chord.

The Economic Length of Panel is more or less independent of truss depth since in the longer spans, subdivided panels may be used, while for the shorter trusses without subdivided panels there is a practical advantage in the use of long panels, in as much as the heavier floor system is stiffer and freer from vibration. The inferior limit of span length for trusses with subdivided panels varies between 240 ft. for highway to about 300 ft for railway bridges. Panel lengths for ordinary highway bridges vary between 15 and 25 ft; for railway bridges between 25 and 40 ft. Panels are generally kept of uniform length. In the Municipal bridge at St. Louis the panel lengths vary from 30 ft near the end to 48 ft at the center of the span. By varying the panel length greater freedom in truss design is gained at the expense of greater shop cost of the floor system.

25. Stresses in Pratt Trusses

Notation. V = shear on a given panel, θ = angle between diagonal and vertical, W = live plus dead panel load, W_1 = live panel load, p = panel length, m = number of panels in span, n = number of panels to right of a given panel, M_n and M_{n+1} = moments at right and left end of a panel respectively, q = equivalent uniform live load, l = length of a bar, A = area of cross-section, E = modulus of elasticity.

Stresses in Bridge Trusses are determined graphically (Sect. 12) or algebraically; for the latter there are two general methods: that of moments and that of resolution of forces. The second method may be subdivided into method of joints and method of sections. The METHOD OF MOMENTS consists in passing a section thru the truss so as to cut the bar in which the stress is to be found, then writing an equation of moments for all forces on one side of the section and solving this equation for the unknown stress. The METHOD OF JOINTS consists in passing a section around each joint and finding the forces acting at each joint successively. In the METHOD OF SECTIONS a section is passed thru the bar in question, and by applying to one side of the section either or both of the conditions that the algebraic sum of horizontal forces must equal zero or that the algebraic sum of the vertical forces must equal zero, the unknown stresses in the bars cut by the section may be found. To illustrate: consider the single-track thru Pratt truss bridge in Fig. 74 having a dead load of 400 lb per ft for the track and $1000+10L = 1000+10(120) = 2200$ lb per ft for the steel in the bridge; total 2600 lb per ft of bridge or $1300 \times 20 = 26\,000$ lb per panel per truss. Assume all on loaded chord. Live load 3000 lb per ft per truss or 60 000 lb per panel per truss. Find maximum stress in bar 8 by method of moments; bar 2 by joints; and bar 5 by sections. The loads in Fig. 74 and the following computations are exprest in units of 1000 lb.

METHOD OF MOMENTS. For bar 8 pass a section vertically thru 8 and consider forces on left of section. The origin of moments is at intersection of other two main bars cut, namely, at lower panel point 3. Maximum moment at 3 and therefore maximum stress in 8 occur for full live and dead load. $R_1 = 215$. Assume 8 in tension and place algebraic sum of moments of all forces on left of section a–a equal to zero; $215\,(60) - 86(20+40) + 24\,P_8 = 0$. $P_8 = -322.5$. The minus sign denotes that the stress is opposite from that assumed, and is compression.

METHOD OF JOINTS. For bar 2 pass a section around lower end of 2 and consider the vertical forces acting upon the part below the section. Since the panel load is the only vertical force other than the stress P_2 acting on this part of truss the stress P_2 must be equal to this force; hence P_2 is maximum for full panel load and equals 86. It is tension since it acts away from the lower joint.

Method of Sections. For bar 5 pass a section vertically thru 5. Imagine P_5 resolved into horizontal and vertical components, and since the algebraic sum of the outer forces on left of a–a, that is, the shear on the panel, together with the vertical component of P_5 equals zero, it follows that $P_5 = V \sec \theta$. For a truss having horizontal chords the vertical component of a diagonal is equal to the shear in the panel. When shear is positiv P_5 is tension, and its maximum value occurs for maximum positiv shear, which by the approximate method, Art. 19 paragraph 3, occurs when all panel points to right of the panel in question are loaded with live load and dead load covers whole span. Under this loading the positiv shear due to 86 on points, 1, 2, 3 is $86\ (1+2+3)/6 = 86$, and the negativ shear due to 26 on 4 and 5 is $26(1+2)/6 = 13$, hence maximum positiv shear $= 86 - 13 = 73$ and maximum $P_5 = 73 \times 31.24/24 = 95.0$ tension. If negativ shear occurs in panel 3–4, P_5 will be in compression, or if it cannot stand compression a counter as shown by the dotted line in panel 3–4 will be needed, and for symmetry one will also

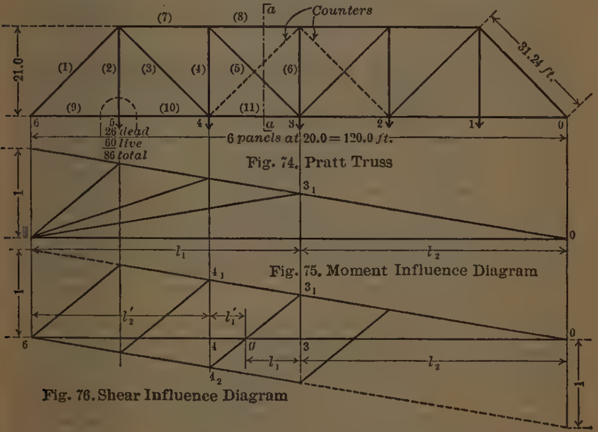


Fig. 74. Pratt Truss

Fig. 75. Moment Influence Diagram

Fig. 76. Shear Influence Diagram

be needed in panel 2–3. For maximum negativ shear place live load on points 4 and 5 only, then negativ shear on panel $3\text{–}4 = 17$, and P_5 will be $17 \sec \theta =$ compression, or else a counter must be used, in which case maximum stress in counter $= 17 \sec \theta =$ tension.

For Concentrated Wheel Loadings the computation of stresses is the same in principle as with uniform or panel loads and the determination of the various moments and shears is simplified with the aid of the moment diagram (Art. 19). For trusses with parallel chords the maximum stress in a diagonal is the maximum shear on the panel containing the diagonal, multiplied by $\sec \theta$, and the maximum chord stress in a bar is the maximum moment about its origin of moments divided by the lever arm of the bar. For the verticals of trusses with parallel chords the maximum stress is the greatest shear on an inclined plane cutting the vertical and not cutting any diagonal in action for the loading producing the shear.

Let it be required to find maximum live load tension in diagonal 5 (Fig. 74) using Cooper's loading E–50 (Fig. 34). To produce the greatest shear the loads must lie partly on panel 3–4 and principally to right of 3, and the total load on bridge must equal m (here 6) times the load on panel 3–4. Wheel 1 being small, place wheel 2 just to right of joint 3, then load on span is 215.0 which is greater than $6 \times 12.5 = 75.0$; now with wheel 2 just to left of joint 3, 215.0 is less than $6 \times 37.5 = 225.0$. Hence wheel 2 at joint 3

satisfies the criterion for maximum shear in panel 3-4. With wheel 3 at joint 3 the criterion is also satisfied. Starting with wheel 2 at joint 3 and moving up the loads till wheel 3 is at joint 3 the shear is increased because $215\times5/120+25\times4/120>37.5\times5/20$, hence position 3 gives more shear than position 2.

Fig. 76 gives the influence lines for shear in the several panels of the truss of Fig. 74. An INFLUENCE LINE shows the variation in a stress function, such as a moment, shear, floorbeam load or stress as a single load rolls across the structure. In Fig. 76 the triangles o-3_1-g and g-4_2-6 are the influence diagrams for the positiv and negativ shears, respectively, in the panel 3-4. g is the neutral point in the panel; l_1 l_2 and l'_1, l'_2 are the segments of the positiv and negativ triangles, respectively. The position of the live load for maximum tension in the diagonal 5 may be obtained from the chart, Fig. 26 (Art. 10). The distance of the neutral point from joint $3=np/(m-1)=3\times20/5=12$ (Art. 19). Entering the chart for $l_1=12$ and $l_2=np=60$, it is found that wheel 3 should stand at 3.

The Shear in Panel 3-4 (Fig. 74) with wheel 3 at joint 3 = left reaction minus floor beam reaction at 4. Using the moment diagram the shear $=(8385+240\times4)/120-287.5/20=63.5$. Maximum tension in diagonal $5=63\,500$ sec $\theta=63\,500\times31.24/24=82\,700$ lb. In this manner live load stresses in all diagonals may be found, and after adding impact and dead load stresses the diagonals can then be designed. The maximum stress in vertical 4 equals the maximum shear on the inclined section cutting bars 7-4-11. This occurs for the same loading, and is of the same magnitude as for maximum shear just found, hence compression in bar 4 = vertical component of diagonal 5 or 63 500 lb.

The shear in panel 3-4 can also be found by multiplying the corresponding triangular influence area, Fig. 76, by the exact equivalent uniform load obtained from Fig. 25 (Art. 10). This shear may be exprest by $V=\frac{1}{2}\,ql_1l_2/p$. Thus for maximum positiv shear in panel 3-4, $l_1=12$, $l_2=60$. From Fig. 25 (Art. 10) the exact equivalent uniform load $q=3600$ for E-50, and $V=\frac{1}{2}\times3600\times12\times60/20=64\,800$. The small error is due to wheel 1, being 1 ft in the negativ area. Similarly the maximum negativ shear $=\frac{1}{2}\times q'l'_1l'_2/p$, where q' is obtained from Fig. 25 for $l'_1=8$ and $l'_2=40$. Actual stresses are obtained by multiplying the shear by sec θ.

THE HIP-VERTICAL bar 2 has a maximum tension equal to the floor beam reaction and hence receives its stress only for loads on the two adjacent panels 4-5 and 5-6. The position of the loads must be the same as will cause maximum bending moment at center of a span of length 4 6, an l after the loads are so placed the stress in bar $2=(M_4-2M_5)/p$, where M_4 is the moment about 4 of all wheels on distance 4-6 and M_5 the moment about 5 of loads on 5-6. This position can be obtained from the chart, Fig. 26 (Art. 10) by taking l_1 and l_2 each equal to 20 ft, the adjacent panel lengths. It is found that wheel 3 should stand at joint 5. The live load tension in the hip-vertical can be taken directly from the table in Art. 10. It is the maximum pier (or floor beam) reaction for two adjacent 20-ft spans and equals 81 900 lb.

The Moment Influence Lines are shown in Fig. 75, that for member 8 being o-3_1-6, with the apex at 3_1 under the center of moments at joint 3. $l_1=60$, $l_2=60$. Fig. 26 shows that wheel 11 at joint 3 is the governing position for maximum moment. From Fig. 25 is obtained the equivalent uniform load $q=3210$ for E-50. The bending moment for a triangular influence diagram is $M=\frac{1}{2}\,ql_1l_2$. Whence $P_8=\frac{1}{2}\times3210\times60\times60/24=240\,750$.

The Moment at Joint, due to any load P standing anywhere on the span is equal to the product (called the load product) of the load and the influence ordinate at P_1 multiplied by the distance of the center of moments from the left end of the span. If several wheels stand on the span the moment is equal to the algebraic sum of all of these products. For any series of wheel loads the position of the loading producing a maximum moment can be found by trial and is the one which makes the algebraic sum of the load products a maximum. The stress is equal to the sum of the load products multiplied by e/h, where e = the distance of the center of moments from the left support and h = the lever arm of the stress with reference to the center of moments.

26. Warren and Baltimore Trusses

Notation the same as that at beginning of Art. 25

To Compute Maximum Stresses for all bars of the Warren truss in Fig. 77, let dead panel load on each bottom chord joint be 20 (thousands of pounds) and live panel load 40. Place dead load of 20 at each bottom joint, and beginning at center and using method of joints write on each web member and then on each chord member a coefficient from which the true stresses for uniform full loading may be found. Since truss is symmetrical each diagonal meeting at panel point 2 has a vertical component of 10.0 and this component or coefficient is written on the bar as shown. Then the joint at upper end of this diagonal is considered and the vertical component or coefficient for the next diagonal (3) obtained, 10.0 in this case, and so on to the end where the vertical component of end post should equal the reaction. By applying method of joints to point 4 the horizontal component of P_7 is found to be 30.0 tan θ, so the coefficient 30.0 is written on the lower chord member; and similarly at joint 3 the actual tension $P=(30.0+30.0+10.0)$ tan $\theta=70.0$ tan θ and the coefficient 70.0 is written on the bar. All diagonal coefficients must be multiplied by sec $\theta=22.36/20$; all chord coefficients by tan $\theta=10/20$. Thus dead load stress in end post $=30\times22.36/20=33.5$ compression; and $P_7=30.0\times10/20=15.0$ tension. For live load chord stresses multiply dead chord stresses by ratio of live panel load to dead panel load. For live web stresses find maximum live shears by method of sections, Art. 25, and multiply these shears by sec θ. Thus for panel 3-4 maximum shear $=40(1+2+3)/4=60$; for panel 2-3, maximum positiv shear $=40(1+2)/4=30$ and maximum negativ shear $=40(1)/4=10$. The following table shows the final stresses.

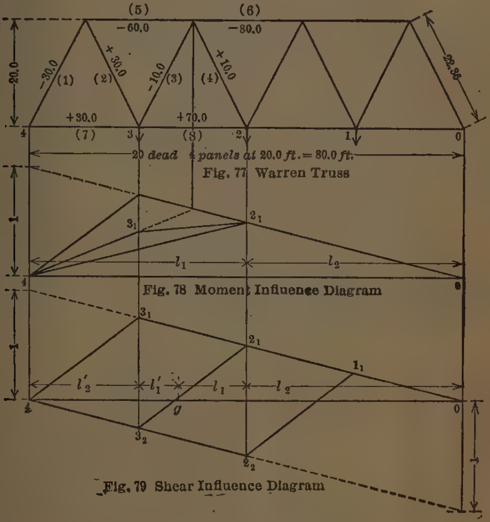


Fig. 77 Warren Truss
Fig. 78 Moment Influence Diagram
Fig. 79 Shear Influence Diagram

Stresses for Truss in Fig. 77 in Thousands of Pounds

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Dead...	− 33.5	+ 33.5	−11.2	+11.2	−30.0	− 40.0	+15.0	+ 35.0
Live...	− 67.1	+ 67.1	{ +11.2 −33.5	−11.2 +33.5 }	−60.0	− 80.0	+30.0	+ 70.0
Max...	−100.6	+100.6	−44.7	+44.7	−90.0	−120.0	+45.0	+105.0
Min....	− 33.5	+ 33.5	0.0	0.0	−30.0	− 40.0	+15.0	+ 35.0

−denotes compression. +denotes tension.

The influence lines for shears, Fig. 79, and for moments at the loaded joints, Fig. 78, for the Warren truss, Fig. 77, are the same as for a Pratt truss of equal length and number of panels. For example, that for the moment for P_6 is $0{-}2_1{-}4_1$, Fig. 78; l_1 and l_2 are the segments of the influence triangles; maximum moment $M = \frac{1}{2}ql_1l_2$; maximum shear in a panel, $V = \frac{1}{2}\,ql_1l_2/p$. The influence line for moments about the unloaded joint opposite P_8 is $0{-}2_1{-}3_1{-}4$ (Fig. 78). P_6 and P_8 may be found by multiplying the respective influence areas by e/h, where h = depth of truss; e = horizontal distance from the center of moments to the left support ($= 2p$ for P_6 and $1\frac{1}{2}p$ for P_8). The charts, Figs. 25 and 26, apply exactly only when the influence diagram is a triangle. When the latter is not a triangle, as in the case of $0-2_1-3_1-4$ for P_8, Fig. 78, the charts may still be applied with sufficient accuracy by assuming a triangular influence diagram coinciding as nearly as possible with and having the same area as the actual influence diagram. l_1 and l_2 to be used in the charts are then the two segments of this triangle, and the governing wheel obtained from Fig. 26 stands at the loaded joint nearest the apex of the triangle.

The Baltimore Truss (Fig. 68) may have its sub-verticals and sub-diagonals arranged as in Fig. 80 or in Fig. 81 which represent panels without counters on left of center of truss. In Fig. 80 maximum $P_1 = Wp\sec\theta_1/(p+p_1)$ and is compression; if $p_1 = p$ (as is usual) then $P_1 = (W\sec\theta)/2$; in other words the vertical component of $P_1 = \frac{1}{2}W$, as may be easily seen by studying the forces below the dotted section and taking moments at a. In Fig. 81, for members in action as shown, $P_4 = (W\sec\theta)/2$ and is tension. Maximum tension in P_2 (Fig. 80) occurs when live load extends from right end of span up to and including b, and if $p_1 = p$, $P_2 = (V - W/2)\sec\theta$. Maximum tension P_3 (Fig. 81) occurs when the live load extends from right end of span up to and including c and $P_3 = (V + W/2)\sec\theta$. If n = number of panels to right of b (Fig. 80) or to right of c (Fig. 81), and m = number of panels in the span, all equal, then maximum live load tension in P_2 and also in P_3 is given by the formula $\frac{1}{2}W[n(n+1)/m - 1]$.

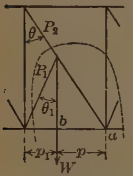


Fig. 80

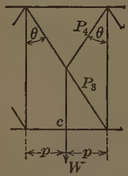


Fig. 81

Influence lines for shears in the main web members of the Baltimore truss, Fig. 82, are shown in Fig. 83, and for moments, in Fig. 84. Maximum live load shear in the upper diagonals (of type P_2, P_5), and in the vertical posts (of type P_0) $= V = \frac{1}{2}ql_1l_2/2p$; that in the lower half of the main diagonals (of type P_1, P_6) $= V = \frac{1}{2}ql_1l_2/p$. For actual stress multiply by sec θ.

The influence line for moments about joint 8, for P_3, is $0-8_1-12$, Fig. 84. That for P_4 is $0-6_1-7_1-12$. Maximum live load stresses may be found by multiplying these areas by the equivalent uniform load q and by e/h; where e = distance from center of

moments to the left support; h=depth of truss. $e=4p$ for either P_3 or P_4. $e/h=\sec\theta$ for all web members of a truss with horizontal chords. l_1 and l_2, for use in the chart,

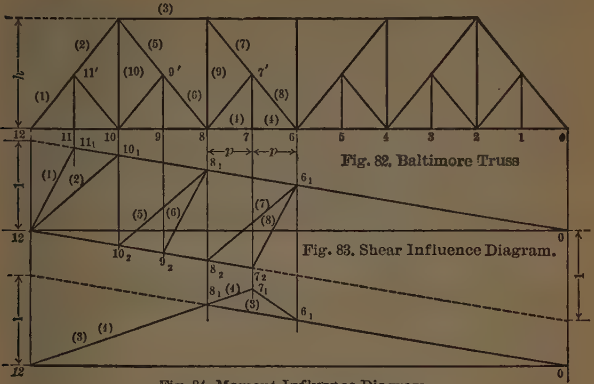


Fig. 82. Baltimore Truss

Fig. 83. Shear Influence Diagram.

Fig. 84. Moment Influence Diagram.

Fig. 25 (Art. 10), are the lengths of the two segments of the triangular influence diagram, or of the triangle approximating in outline and area the actual influence diagram (see also Warren truss, above). For all triangular moment influence diagrams $M=\frac{1}{2}ql_1l_2$.

27. Other Types of Trusses

Notation the same as that given at beginning of Art. 25

Stresses in Trusses having Inclined Chords are found by methods of moments. Maximum chord stresses occur for full live and dead loads over entire span. Referring to Fig. 85, showing a thru Parker truss, the maximum live and dead load stresses in upper and lower chord bars P_1 and P_2 are given by the following formulas

$$P_1=\frac{M_n}{h_n}\,\frac{l_1}{p}=\frac{Wl_1n(m-n)}{2\,h_n}$$

$$P_2=\frac{M_{n+1}}{h_{n+1}}=\frac{Wp(m-n-1)(n+1)}{2h_{n+1}}$$

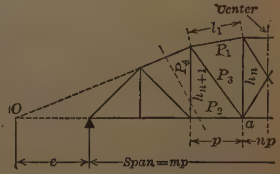


Fig. 85

Maximum tension in P_3 occurs for live panel loads on all panels from right end of span up to and including a, with dead load over whole span, and

$$P_3=\left(\frac{M_n}{h_n}-\frac{M_{n+1}}{h_{n+1}}\right)\frac{l_3}{p}$$

where l_3 is length of P_3, and M_n and M_{n+1} are simultaneous moments at right and left ends of panel respectively. Maximum compression in P_4 occurs

for same loading as for maximum tension in P_3 and is found by methods of moments using all forces to left of dotted section with origin of moments at O. Some verticals in inclined chord bridges are in tension for certain positions of live load and these stresses must be computed.

Chord Stresses due to Wheel Loads are found by method of moments. For example, in Fig. 85 the maximum chord stress P_1 would be found by dividing the maximum moment at the origin of moments a by the lever arm of the bar P_1, that is, by the perpendicular distance from a to the bar. Instead of using the lever arm of the bar however it is often more convenient to use the arm of one of the components of the stress and then find the stress from the component. Thus in the case of P_1 its horizontal component $= M_a/h_n$ and its stress $= M_a l_1/p h_n$. The criterion for determining the position of loads to secure maximum moments is given in Art. 19.

Influence for Moments in the Parker truss, Fig. 86, are similar to those for a Pratt truss and are not here shown. The influence line for a web member, Fig. 87, is con-

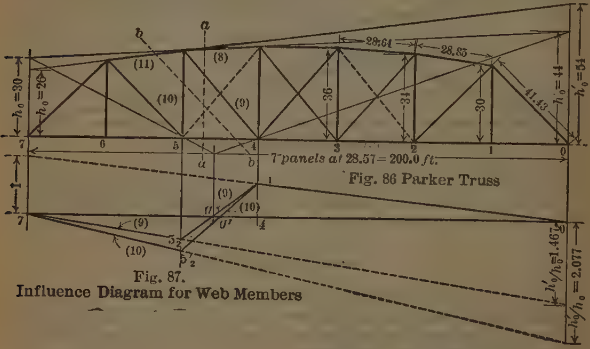


Fig. 86 Parker Truss

Fig. 87. Influence Diagram for Web Members

structed by laying off h'_0/h_0 downwardly from 0; where h'_0 and h_0 are the vertical distances at the right and left supports, respectively, to the continuation of the top chord member cut by the section passing thru the web member. For example, to find the stress in the diagonal 9 due to Cooper's E-60 a vertical section $a-a$ thru the diagonal also cuts the upper chord member 8, the continuation of which gives the vertical intercepts $h'_0=44$ ft and $h_0=30$ ft; $h'_0/h_0=44/30=1.467$. The influence line for P_9 is $0-4_1-5_2-7$. g is the neutral point in the panel. The distance $g-4=l_1=np/(n+h'_0/h_0(m-n-1))$ $=4\times28.57/(4+1.467\times2)=16.5$. $l_2=4\times28.57=114.28$. From Fig. 25 (Art. 10), $q=4020$. The area of the triangle $0-4_1-g-0=\frac{1}{2}\times4/7\times130.78=37.37$. The center of moments for P_9 lies 428.55 ft to the left of the left support; the lever arm for P_9 about the center of moments $=393.7$ ft. $e/h=428.55/393.7=1.088$. $P_9=37.37\times4020\times1.088$ $=164\ 000$.

For the vertical 10, the section $b-b$ cuts the top chord member 11, and $h'_0/h=54/26$ 2.077 which is laid off downwardly from 0, Fig. 87, to furnish the influence line $0-4_1-5'_2-7$. The distance from the neutral point g' to the right end of the panel $=np/(n+h'_0/h_0(m-n-1))=4\times28.57/(4+2.077\times2)=15.6=l_1$. $l_2=114.28$: $q=4030$. The area of the triangle $0-4_1-g'-0=\frac{1}{2}\times4/7\times129.88=37.11$. $e/h=214.28/271.42$ $=0.79$. $P_{10}=37.11\times4030\times0.79=-118\ 000$.

Statically Indeterminate Trusses are those in which the stresses cannot be found by the principles of statics. Some trusses are statically deter-

minate as regards the outer forces but are statically indeterminate as regards the inner forces or stresses. A REDUNDANT MEMBER in a truss is one that is not necessary for the stability of the truss. The condition that a truss is statically determinate is $b=2j-3$ where b is total number of necessary bars, j the number of joints in the truss. The equation applies to the whole or any part of the truss. If $b>2j-3$ the system is statically indeterminate, if $b<2j-3$ it is unstable. Stresses in statically indeterminate trusses are found either by making the assumption that the truss is divided into its separate systems and that each system acts independently, or by the principle of least work.

The Double-system Warren Truss in Fig. 88, also called a lattice truss, is statically indeterminate because 26 bars$>2(14)-3=25$. Assume the web to be divided into two systems as shown by heavy and light lines and let it be required to compute the maximum tension in P_1 under a dead panel load of 6 and a live panel load of 12 thousands of pounds on the lower chord. Maximum positiv shear on panel 4-6 of web with heavy lines occurs for live panel load on points 2 and 4 and stress $P_1=[(2+4)18/6]\sec\theta$.

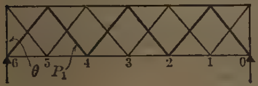


Fig. 88

Fig. 89

The Principle of Least Work requires that the total internal work performed by the stresses in the members of a truss must be a minimum; hence an outer load acting on a statically indeterminate truss will be distributed on the different systems of the truss in such a way that the total internal work done by the stresses will be a minimum. WORK is the product of force times its displacement. The work performed by a stress P on a bar having length l, area A, and modulus of elasticity E is $\frac{1}{2}P^2l/AE$ and total internal work in truss is $\frac{1}{2}\Sigma P^2l/AE$. The truss in Fig. 89 carrying the load W at the center can be separated into two trusses as in Figs. 90 and 91, one with the force kW and the other with a

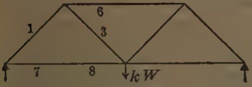


Fig. 90

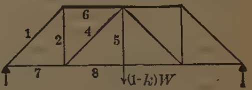


Fig. 91

force $(1-k)W$ at the center respectively. The value of k must be such as to make the total internal work a minimum. The stresses in various bars of Fig. 90 and Fig. 91 in terms of the unknown loads kW and $(1-k)W$ respectively can be found and substituted in the formula. The differential coefficient of the work with respect to k is then placed equal to zero and solved for k. The final expression for k involves the dimensions of the truss and the areas of the bars. This process can be applied for a load of unity at each of the panel points in succession and a table of stresses prepared from which the stresses for any panel load may be determined. Since the areas of the bars must be known before the principle of least work can be applied, the process of design is a tentative one.

28. Design of Tension Members

Notation. S=allowable tensile unit stress, P=direct tension in a member in pounds, S_1=maximum stress in pounds per square inch in eye-bar due to combined tension and bending, t=thickness, l=length and h=width of eye-bar; all inches.

Pin-connected Trusses were generally used in America till about 1890 for spans of all lengths. Since that date the RIVETED TRUSS has come to be

the standard for railroad bridges having spans less than 150 to 250 ft. The railroad bridge over the Missouri River at Kansas City has riveted trusses of 425 ft 6½-in span. The longest riveted simple truss is the 720-ft channel span of the Metropolis bridge. The 552-ft spans of this same bridge are likewise riveted. For highway bridges the pin-connected type is often used for shorter lengths. Short pony trusses for country highway bridges are now invariably riveted, for greater lateral stiffness. The riveted structure is the stiffer form and the individual members thereof are more nearly free from objectionable vibration and rattling. The pin-connected truss usually is somewhat lighter than the riveted and can be more easily and more quickly erected. The ideal truss has hinged joints without friction. Neither of the types used fulfills this requirement, for as the truss deflects the rigidity of the riveted joints and the friction on the pins of a pin joint cause bending or secondary stresses in the truss members. The most complete analysis of secondary stresses that has as yet been made for any riveted truss is given in the paper "Stress Measurements on the Hell Gate Arch Bridge," by Dr. D. B. Steinman, Proc. Am. Soc. of Civil Engineers, Oct., 1917.

Tension Members may be eye-bars or built-up sections. An EYE-BAR has a uniform rectangular cross-section thruout the greater part of its length and at each end has a head or enlarged portion thru which the pin passes. It is the most efficient means of transmitting tension and is economical of material but deficient in rigidity. Eye-bars should not be used where the minimum stresses are light or where compression is possible, as in the first two lower chord panels, for hip verticals, or for diagonals subject to counter stress. Counters are sometimes used in light highway bridges and are composed of one or two adjustable eye-bars or square rods with looped ends, but whenever possible adjustable bars should be avoided.

Built-up or Riveted Tension Members may be made in any of the forms suitable for compression members but less emphasis need be placed upon lateral stiffness. For light construction one L, or two L's placed with their backs together, or separated to form a built channel (Fig. 92), or four L's (Fig. 93), are used. In the last two cases the L's may be separated by tie plates, but lacing or a solid web plate should be used for good work. For heavier construction some form of box section composed of rolled or built channels connected by lacing and tie plates (Fig. 94, 95) is used. The sections may be further enlarged by the addition of one or more side plates riveted to the web of each channel. In riveted trusses the flanges are often turned in to permit the member to go inside of the connecting joint gusset plates. In built-up tension members the rivets should be so placed as to reduce the section as little as possible. The net section will generally occur at the first row of rivets in a splice, end gusset or pin plate. In computing net sections the diameter of the rivet holes should be taken ⅛ in larger than the nominal diameter of rivet.

Eye-Bars vary in size from 2 in in width having a minimum thickness of ⅝ in to 10 in in width for usual construction, and even more for the largest structures; 16-in bars have been used in several of the largest bridges, the maximum size used in the 668-ft span of the Municipal bridge at St. Louis being $16 \times 2^1/_{16}$ in. For wide bars the thickness rarely exceeds 2½ in. Generally the ratio of width of bar to thickness lies between the limits of 4 and 7. If bars are made too thin their heads may buckle, due to the compression at the pins, and if made too thick the bending moments on the pins are increased unnecessarily. The dimensions of eye-bar heads as given in steel companies' handbooks are such that the bars break in the body when tested to destruction. Eye-bar heads are forged or upset in a die, and the net cross-sectional area thru the pin hole is from 33 to 40% larger than that of the body. The heads are finished the same thickness as the body.

To Proportion tension members subjected to direct stress the required net area of the cross-section is P/S. For an eye-bar under the tension P the maximum unit stress S_1 on the bottom fibers due to combined tension and bending caused by the weight of the bar is

$$S_1 = \frac{P}{th} + \frac{4\ 700\ 000\ h}{P/th + 22\ 400\ 000 (h/l)^2}$$

29. Design of Compression Members

Notation. S = maximum or allowable unit stress, A = cross-section of a member, P = direct stress in a member, M = bending moment, c = distance from gravity axis to most stressed fiber, I = moment of inertia, l = length of column, r = radius of gyration of column, e = distance from center of gravity of cross-section to point of application of eccentric load, E = modulus of elasticity, w = weight of compression member per unit of length, I_D and I_c = moments of inertia of one ⊏ about gravity axis perpendicular and parallel to web respectively, A_1 = area of one ⊏, x = distance from back of ⊏ to center of gravity.

Compression Members should be proportioned, whenever possible, to be of equal strength about the two principal axes A-A and B-B (Fig. 94), if there is only direct compression in the member; and if there is transverse bending in addition, the strength about the axis around which bending takes

Fig. 92 Fig. 93

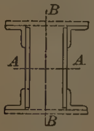


Fig. 94


Fig. 95


Fig. 96

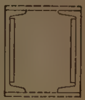


Fig. 97

place should be the greater. The cross-sectional area should be as far removed from the center of gravity of the section as possible consistent with proper thickness of materials and proper bracing of the different segments. Figs. 92 to 97 show typical sections of web compression members and Figs. 98 to 101 of compression chords and end posts for bridge trusses, the smaller ones being suitable for light highway or railroad construction and the larger ones

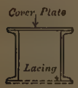


Fig. 98


Fig. 99

Fig. 100

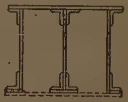


Fig. 101

for heavy work. All of the sections shown except the H section in Fig. 96 require lacing or lattice bars to connect the segments together. In Fig. 99 and Fig. 101, flat bars are riveted to the lower ∟'s to keep the center of gravity of the section at or near the mid-height axis. Chord sections as in Fig. 99 are called BOX SECTIONS, the vertical plates are called WEBS and the upper horizontal plate is the COVER PLATE. See also Art. 38. Compression members should not have l/r exceed 100 for main members nor 120 for laterals.

Short Columns are those having l/r less than 30; if the load is concentric, that is, applied at the centers of gravity of the end cross-sections or uniformly over these sections, the stress is one of simple compression distributed uniformly over the various cross-sections of the column, and the unit compression is $S = P/A$. If the load is eccentric, that is, not applied at the axis of the column, the maximum unit stress is $S = P/A + Mc/I$ and the minimum unit stress on the same section may be found from the same formula by changing the + to a − sign.

The strength of a short column depends primarily upon the elastic limit of the steel and also in large measure upon the effectiveness with which its segments are tied together. The ultimate strength of a column never exceeds and is often considerably less than the yield point of the steel. The results of extensive tests made at the Bureau of Standards under the direction of the Special Committee on Steel Columns and Struts of the Am. Soc. of Civil Engineers (Final Report, Procs. Am. Soc. of Civil Engineers, Dec. 1917) indicate that the elastic limit of columns made of carbon steel having a tensile strength of 60 000 lb per sq in may be as low as 19 500 lb per sq. in. This committee recommends 12 000 lb per sq in as the maximum unit stress to be used when $l/r < 60$. The A. R. E. A. specifications permit the use of 14 000 up to $l/r = 29$.

Long Columns are additionally weakened by their tendency to buckle in the plane of the maximum l/r. Their strength, for ordinary values of l/r, is affected by too many accidental and uncertain conditions to permit of its expression by a rational formula. The committee of the A. S. C. E. after a study of the tests of columns having $l/r = 120$, recommends a unit stress of 8000 lb when $l/r = 120$ and an interpolation between 12 000 and 8000 when l/r lies between 60 and 120. By the A. R. E. A. specifications $S = 16\,000 - 70\,l/r$, with a maximum value of 14 000 for $l/r < 29$. On account of the deflection of structures and the distortions at the joints all columns are regarded as pin-ended. By the A. R. E. A. formula $S = 7600$, when $l/r = 120$. For values of $l/r < 70$ the A. R. E. A. formula gives unit stresses greater than that recommended by the committee of the A. S. C. E.; for values of $l/r > 70$ it gives slightly smaller stresses.

For columns subjected to combined compression and bending, and for tension members under combined tension and bending, the maximum unit compression or tension is

$$S = \frac{P}{A} + \frac{Mc}{I \mp Pl^2/a_2E}$$

in which a_2 is a constant depending on the condition of the ends and the kind of loading causing bending moment M. For members having hinged ends $a_2 = 9.6$ for uniformly distributed loading and 12 for a concentrated load at the middle; for members having fixt ends $a_2 = 32$ for uniform loading and 24 for a concentrated load at middle. The minus sign in the denominator must be used when P is compression and the plus sign when P is tension. This formula may also be used for members having eccentric loads, in which case M is the bending moment due to the eccentric loads and a_2 should be taken for a uniformly distributed loading.

Proportioning Top Chord. Let it be required to proportion the top chord of the 7 panel, 200 ft, typical pin-connected truss of the American Bridge Co., shown in Fig. 109. The maximum compressions from dead, live and impact stresses are: $U_1U_2 = 870\,000$; $U_2U_3 = 980\,000$; $U_3U_4 = 978\,000$. The lengths are 28.85, 28.64 and 28.57 ft respectively. The section for U_2U_3 will also answer for U_3U_4. Allowable unit stress $= 16\,000 - 70l/r$.

Design of U_1U_2. Let the chord be of box section as in Fig. 102. The least radius of gyration is approximately 4/10 the depth of the section which is here assumed to be 25 in, or $r = 10$. With $l = 346$ and a trial r of 10 the allowable average unit stress is 13 580 lb per sq in. A trial section must have $870\,000/13\,580 = 64$ sq in. The section shown

in Fig. 102 with 66.44 sq in as determined below has its gravity axis A–A above the mid-height axis B–B a distance $d = 169/66.44 = 2.53$ in, 169 being the statical moment about axis B–B. $I_{A\text{-}A} = 6602 - 66.44 \times (2.53)^2 = 6177$ and radius of gyration about gravity axis A–$A = (6177/66.44)^{1/2} = 9.65$ and allowable average unit stress is 13 480 lb per sq in. The column has an actual average stress of $870\,000/66.44 = 13\,100$ lb per

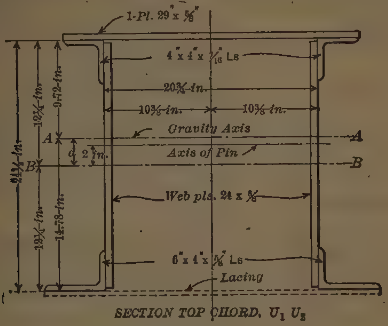


Fig. 102 Section Top Chord U_1U_2

sq in and is therefore of sufficient size for the compression. The statical moment and moment of inertia about mid-height axis B–B are found as follows:

	A	Stat. Mom.	$I_{B\text{-}B}$
1 Pl $29 \times 5/8 =$	$18.10 \times 12.56 =$	$227.5 \times 12.56 =$	2862
2 L's $4 \times 4 \times 7/16 =$	$6.62 \times 11.09 =$	$73.5 \times 11.09 =$	815
2 L's $6 \times 4 \times 5/8 =$	$11.72 \times 11.22 =$	$-132.0 \times 11.22 =$	1485
2 Pls $24 \times 5/8 =$	30.00	00.0	1440
	66.44	169.0	6602

In this computation the moments of inertia of the L's and horizontal plates about their gravity axes have been omitted. The radius of gyration about axis A–A is less than that about the vertical gravity axis, and since A–A is a principal axis, the radius used, 9.65 is the least radius of the section.

The center of the pin $= 2.53 - 2.0 = 0.53$ in below axis A–A. The upward moment due to eccentricity $= 870\,000 \times 0.53 = -461\,000$ in-lbs; the downward moment due to the weight of the member $= wl^2/8 =$ about 302 000 in-lbs. The resultant moment $= -159\,000$ in lbs and the additional compressive stress in the lowest fibre of the section is $S = Mc/I = 159\,000 \times 14.78/6177 = 380$ lbs per sq in. The required sectional area $= 870\,000/(13\,480 - 380) = 66.3$ sq in. Actual area $= 66.4$ sq in.

Design of U_2U_3 and U_3U_4. The same section as for U_1U_2 will be used except that the web plates will be 13/16 in thick instead of 5/8 in. By the method of computation employed above the following data are obtained: $I_{A\text{-}A} = 6534$; $r = 9.3$; $l/r = 37$; $S = 16\,000 - 70 \times 37 = 13\,410$; required area $= 980\,000/13\,410 = 73$ sq in; actual area $= 75.48$ sq in; $d = 2.24$ in and the center of the pin $= 2.24 - 2.0 = 0.24$ in below axis A-A. The moment from the weight of the member is 348 000 in-lb and the resultant moment $= 348\,000 - 980\,000 \times 0.24 = 103\,000$ in-lb. The additional compressive stress on the upper fiber of the section is $S = Mc/I = 103\,000 \times 10.63/6534 = 170$ lb per sq in. The required sectional area $= 980\,000/(13\,410 - 170) = 74.0$ sq in. Actual area $= 75.48$ sq in. By the specifications the thickness of the cover plate must be at least 1/50 the distance between the rivets connecting it to the flanges $= 1/50 \times 25.25 = 0.50$ in; and the thickness of the web plate must be at least 1/40 the distance between the rows of rivets $= 1/40 \times 20 = 0.50$ in. The sections chosen satisfy these requirements.

Proportioning Compression Vertical. Let it be required to proportion the vertical U_2L_2 composed of two [’s (Fig. 103), carrying a maximum concentric dead, live and impact stress of 221 000 lb. The center of the pins is in the axis D D. Assume two [’s 15 in 40 lb having a combined gross area of 23.52 sq in and a radius of gyration for the two [’s about a gravity axis perpendicular to the web of 5.4 in. $l/r=408/5.4=76$; $S=16\,000-70\times76=10\,680$ lb per sq in. Required area $=221\,000/10\,680=20.7$ sq in. The [’s are spaced 12.5 in back to back. To make r about axis B-B equal that about D-D (Fig. 103), the distance z must be $x+\sqrt{(I_D-I_C)/A_1}$ which in this case is less than 12 in. If the flanges of the [’s are turned outward z is the same as above except that x is negativ. z may also be obtained directly from the Steel Co. handbooks. Since the floor-beam and the top strut reduce the unsupported length of the column for bending about the axis B-B, z may be less than this value for equal values of l/r. Axis D-D therefore decidedly governs in this present case. With flanges turned in as in Fig. 103 the distance in the clear between flanges should not be less than 2½ in if lattice bars are to be hand riveted nor less than 4 in if machine riveted.

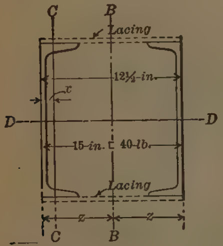


Fig. 103. Section Vertical Post U_1U_2

Inclined End-posts of Thru Bridges are subjected to a direct compression from live, dead and impact stresses and also, at times, to a slight increased direct compression due to wind and to a considerable transverse bending due to wind in a plane perpendicular to the plane of the truss, these wind stresses being due to the action of the wind on the upper portion of the truss. The end-post is a part of the portal system and as such carries to the supports the wind pressure brought to the hip joint by the top lateral system. The cross-sectional area of the post must be such that the actual average unit stress P/A, due to live, dead and impact stresses shall not exceed that given by the column formula $P/A=16\,000-70\ l/r$, where l is the length in inches between pin centers and r the radius of gyration of cross-section about the gravity axis parallel to the cover plate, as axis A-A, Fig. 102. The maximum unit stress on the outermost fiber due to live, dead, impact and wind stresses may exceed by not more than 25% the allowed axial unit stress, that is, for railroad bridges the maximum combined unit stress, S in formula below, should not exceed $16\,000+25\%$ of $16\,000=20\,000$ lb. This maximum fiber stress due to axial compression, the tendency to buckle in the plane of the truss and the bending moment in the plane of the portal due to transverse wind forces is given with sufficient accuracy by

$$S=\frac{P}{A}+70\frac{l}{r}+\frac{Mc}{I}$$

In this formula P must include the direct compression due to all loads, including wind; c is ½ the extreme width of end post, I is moment of inertia of section about gravity axis perpendicular to cover plate, l_1 is length of end post from bottom of knee-brace to lower bearing of end-post; $M=Wl_1/4$ or $Wl_1/2$ depending on whether the bottom of the post is fixt or is not fixt, l/r has the value used in the column formula for direction compression. The post may be considered as fixt at lower end if the end floor beam is rigidly connected to it, or if the moment due to one-half the direct compression in post acting with a lever arm equal to the distance between bearings on the pin is greater than $Wl_1/2$. W is the total wind load in pounds acting at the hip joint including that brought to that joint by the top lateral system. If the weight of the member is also

considered S will contain another Mc/I term in which M is the bending movement due to the weight of the post, c is the distance from the gravity axis parallel to the cover plate to the uppermost fiber, and I is the moment of inertia about the same axis.

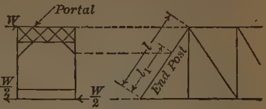


Fig. 104

Design of L_0U_1. The section of U_1U_2 will be used except that the web plates will be increased to ¾ in. $P=873\ 000$; $l=41.4$ ft; $h=31.4$ ft. By the preceding method of computation it is found that $I_{A-A}=6501$; $r=9.5$; $l/r=52.5$; $S=16\ 000-70\times52.1=12\ 325$; required area $=873\ 000/12\ 325=70.7$ sq in; actual area $=72.44$ sq in. $d=2.31$ in and the center of the pin $=2.31-2.0=0.31$ in below the gravity axis A-A. The negativ moment from eccentricity $=873\ 000\times0.31=-270\ 630$; the positiv moment from its own weight $=$ about 370 000, and the resultant moment $=$ about 100 000 in-lb. The additional compression on the upper fibre $=Mc/I=100\ 000\times10.88/6501=170$ lb per sq in. The required sectional area $=873\ 000/(12\ 325-170)=72.0$ sq in, which is less than the actual area.

It is necessary to test the section when the wind stresses are included. The wind load along the top chord is 200 lb per lin ft. $W=200\times3\times28.57=17\ 200$; $Wh/4=17\ 200\times31.4\times12/4=1\ 630\ 000$ in lb. The direct compression from dead and live load is 610 000 lb and from wind 26 400 lb; the distance between bearings on the shoe $=21$ in and the available moment of restraint at the shoe $=636\ 400\times21/2=6\ 700\ 000$. Since this exceeds the moment from the wind the end may be considered fixt and the point of contraflexure taken at the mid-point between pin and foot of knee-brace. The moment at the knee-brace likewise $=1\ 630\ 000$ in-lb. Further it is found that I_{c-c} (about the axis perpendicular to the coverplate) $=7600$; c (to the extreme fiber of the bottom flange ∟'s) $=16.37$ in; l/r (in the plane of the truss) $=52.5$; $70\ l/r=3675$. The maximum unit stress upon the outer fiber of the lower flange ∟ (neglecting effect of weight of member and eccentricity, which is slight) is

$$S=\frac{873\ 000+26\ 400}{72.44}+3675+\frac{1\ 630\ 000\times16.37}{7600}=19\ 485 \text{ lb. per sq in.}$$

Since this is less than the allowable $16\ 000+25\%=20\ 000$ the section does not need to be increased on account of the wind stresses.

30. Design of Details

Notation. $D=$ diameter of pin, $S_b=$ allowable unit fiber stress for bending, Sc for bearing on pins, St for tension, $Ss=$ actual average unit shearing stress on pins, $t=$ required thickness for bearing of a built-up member, $w=$ width of the widest eye-bar on a pin, $P=$ direct tension or compression in a member.


Fig. 105


Fig. 106


Fig. 107

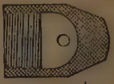


Fig. 108

In Pin-connected Bridges (Fig. 109) there is at each joint a cylindrical steel pin which has at each end screw-threads upon which hexagonal nuts are placed, the Lomas nut, Fig. 106, being most common form. Cotter pins, Fig. 105, are seldom used. Pilot nuts, Fig. 107, and driving nuts, Fig. 108, are placed on pins to protect threads during erection. For ordinary trusses, pins with Lomas nuts vary from 2 to 9 in in diameter, but pins of

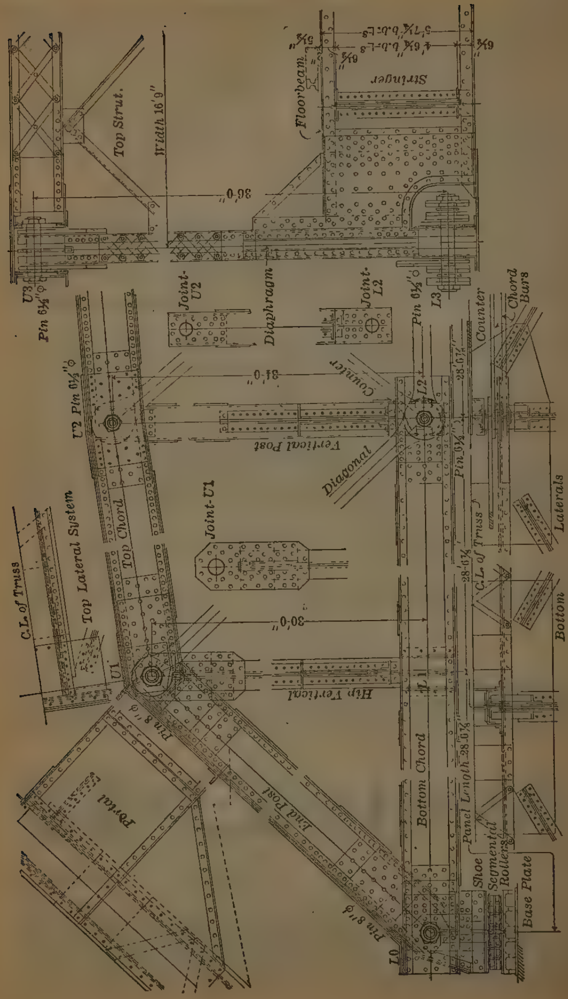


Fig. 109. 200-ft. Thru Pin Connected Span.

greater diameters are used for large trusses. In PACKING, that is, arranging bars on a pin, the bars should be packed as closely as possible, allowing clearances of 1/16 in between two eye-bars and from 1/8 to 1/4 in for built-up members; bars having horizontal components should be close together, similarly those having vertical components; bars should run parallel to plane of truss if possible and maximum deviation should not exceed 1/8 in per foot, better 1/16 in per foot; generally a bar with small stress should be on the outside; bars running in same direction should have a space of at least 1 in between them.

To Determine Diameter of Pin after all joints of the truss are packed, assume the diameter, then compute thicknesses of reinforcing or pin plates for built-up members, and then, taking the forces as concentrated at the centers of the eye-bars and at the centers of the bearings of built-up members, compute the diameter of pin to withstand the maximum bending moment. If this diameter agrees with the assumed diameter the maximum shear on the pin should be investigated; if not, the computation should be revised. Not more than one or two diameters should be used in a truss. The trial and final diameter of pin must be equal to or larger than $(S_t/S_c)\,w$ or otherwise the bearing on the eye-bar is excessive. The value of S_t/S_c is often taken at 3/4. In designing pins to carry moments it is best to assume forces concentrated at centers of bearings, but in investigating the strength of an existing structure this assumption may in some cases lead to a resulting extreme fiber stress larger than the ultimate strength of the material in bending and yet the structure stands. In such a case if an assumption can be made which results in safe unit stresses thruout, the joint is safe. The total required thickness of a built-up member in bearing on a pin is $t = P/S_cD$.

The MAXIMUM BENDING MOMENT, M, at a section of a pin is the resultant of the moment at that section due to horizontal forces, M_h, and that due to vertical forces, M_v, or $M = \sqrt{M_h^2 + M_v^2}$. And the diameter of pin required to withstand this moment is $D = \sqrt[3]{10.2\ M/S_b}$. Tables showing bending moments of pins of various diameters are given in structural hand-books, so that the value of D can be easily taken from the tables as soon as M is known. The MAXIMUM SHEAR, V, at a section of a pin is the resultant of the shear at that section due to horizontal forces, V_h, and that due to vertical forces, V_v, or $V = \sqrt{V_h^2 + V_v^2}$. The shear is usually assumed distributed, in which case $S_s = 1.27\ V/D^2$, but the intensity of maximum shearing stress on the neutral plane is $1.70\ V/D^2$.

In **Riveted Trusses** (Fig. 114) the members are connected together at the joints by gusset plates which must be of sufficient size and thickness to transmit the stresses, and there must be a sufficient number of rivets in each member to develop its full strength. The stress is assumed to be equally distributed over the various rivets hence they should be spaced with this in view. GUSSET PLATES are frequently made too thin. They must be computed to carry the compression and tensile stresses brought on them by the main truss members, and as these stresses are frequently applied eccentrically to the gussets the eccentricity must be allowed for. In such cases as the lower chord joints of Pratt trusses the gussets must have sufficient net area along the horizontal row of rivets connecting the plates to the chord to carry the shearing force on that section and similarly at the vertical row of rivets connecting gusset plates to the vertical. Ordinarily two gusset plates are used at each joint, tho in light trusses only one is used. Center of gravity lines of all members should meet at a point, and the center of gravity of rivets connecting a member to a gusset plate should coincide with the center of gravity of the member.

Lattice Bars must be of sufficient strength to brace the segment of a compression member so that the segments will act as a unit and in doing this they are subjected to tension or compression due to shear resulting from the bending of the column. The ordinary specifications for lattice bars and tie plates are given in Art. 12. The exact determination of stresses in lattice bars is unknown, but the following formula given by C. C. Schneider in the

report of Royal Commission, Quebec Bridge Inquiry, 1908, p. 196, gives the required cross-section in square inches, A_1, of one bar where the column consists of two segments connected by a single system of lacing bars, and is based on the assumption that the elastic line of the column is a parabola

$$A_1 = (8\,car/kd)\ \sec\theta$$

where k is the allowable unit tensile or compression stress in pounds per square inch depending on whether the bar is tension or compression; if the latter, k must be that given by a column formula, a may generally be taken as the actual cross-section of column in square inches; r, radius of gyration in inches of cross-section of column about an axis perpendicular to plane of lacing; d, width of column in inches in plane of lacing; c, constant from column formula (such as 70 in $16\,000 - 70\,l/r$); θ, angle between lattice bar and a line perpendicular to segments connected by the bar. If the segments are connected by two systems of lacing, the required area of one bar, A_1, is one-half that given above.

31. Deflection, Camber, and Bracing

Notation. P = direct tension or compression in a member, l = length of member, A = area of cross-section of member, E = modulus of elasticity, u = stress in member due to load of unity, L = span of truss, p = panel length, p_1 = increase in panel length, H = depth and R = radius of truss, c = camber of truss.

Deflection of a Truss may be due to elastic deformation of the members caused by stresses within them, to imperfect workmanship such as the play in pin holes of a pin-connected truss, and to deformation of the members caused by changes of temperature. The deflection in any direction of any joint of a truss due to any cause may be computed by the following method provided the change in length of each bar can be determined. Place a load of unity at the joint and acting in the direction in which the deflection is wanted and compute the stress in all bars due to this load; call this stress u; then the deflection is given by $\Delta = \Sigma u l_1$, where Δ denotes the deflection of the joint in the direction desired, and Σ denotes the summation of the various products of the stress u due to load unity by the change in length l_1 for each bar successively. The change of length l_1, in any bar due to a series of loads upon a truss is Pl/AE, hence the deflection for this case is given by $\Delta = \Sigma Pul/AE$. In applying this formula, tensile stresses should be given the positiv sign and compressive stresses negativ, both for stresses due to the loads on the truss and for the imaginary load of unity. If all the bars are of the same material the E may be placed on the outside of the summation sign. A is usually the gross area of the cross-section, even for tension bars. To find the ABSOLUTE DEFLECTION of a joint with respect to its original position it is necessary first to find the vertical movement by placing the load of unity vertically at the joint, then the horizontal movement by placing it horizontally at the same joint, and the resultant of these two component movements will give the magnitude of the true deflection.

The **Camber of a Truss** is the distance the center of a chord is raised above the ends to allow for the deflection of the truss. It is usually exprest in inches and varies from 1/1000 to 1/2000 of the span length. Camber is used so that when the maximum load is on the bridge the truss will have the form assumed in the design, and is usually obtained by making the panel lengths of the top chords, or their horizontal projections, longer than the corresponding panels of lower chord in the proportion of 1/8 in in 10 ft. Camber is sometimes made equal to the deflection due to full live and dead loading by increasing or decreasing the lengths of the truss members by amounts equal to their corresponding deformations. When the camber is made equal to this maximum deflection the radius, R, to which the lower chord of the truss is curved and the increase, p_1, in panel length of the upper chord are given with sufficient accuracy by the following formulas respectively

$$R = L^2/8\,c \quad \text{and} \quad p_1 = 8\,cpH/L^2$$

Bracing for Deck Bridges usually consists of a horizontal truss in the plane of the top and also of the bottom chords, together with a vertical transverse system at each panel point. The transverse bracing is also called sway bracing, and the horizontal trusses are called the top and bottom lateral systems. The chords of the main trusses constitute the chords of the lateral system, and the floor beams usually act as the struts of the top system. With a sway bracing at each panel point the bottom laterals are not really necessary, altho desirable, and sometimes are omitted in all or in alternate panels, in which case the sway bracing must be designed to carry the wind loads from bottom to top panel points. When both lateral systems are used, the function of the sway bracing is to equalize the deflection of the trusses when the latter are subjected to unequal loading as when only one track of a double track bridge is loaded. Since the wind loads on top and bottom panel points are unequal the sway bracing receives stress due to unequal lateral deflection of these points. The sway or portal bracing at each end of the bridge must be heavier than at intermediate points.

Thru Bridges have top and bottom lateral systems and also have sway bracing at each top panel point extending down as low as the required headroom allows. When the end posts are inclined the end sway bracing or PORTAL is placed in the inclined plane of the end posts. In the best con-

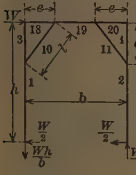


Fig. 110

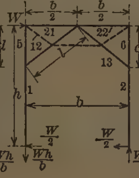


Fig. 111

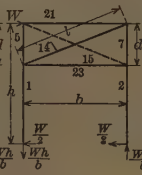


Fig. 112

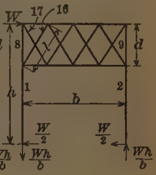


Fig. 113

struction all members of the bracing systems are rigid, that is, made of angles or other similar shapes. Simple portals are shown in Figs. 110, 111, 112 and 113, and the following formulas give the stresses due to wind load W when the posts are hinged at bottom.

For P_1 tension $= Wh/b$, shear $= W/2$, moment $= W(h-d)/2$. For P_2 compression $= Wh/b$, shear $= W/2$, moment $= W(h-d)/2$. For P_3 compression $= Wh(½e - 1/b)$ shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_4 tension $= Wh(½e - 1/b)$, shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_5 and P_6 direct stress $=$ zero, shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_7 compression $= Wh/b$, shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_8 tension $= Wh/2b$, shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_9 compression $= Wh/2b$, shear $= W(h-d)/2d$, moment $= W(h-d)/2$. For P_{10} tension $= Whl/2de$. For P_{11} compression $= Whl/2de$. For P_{12} tension $= Whl/bd$. For P_{13} compression $= Whl/bd$. For P_{14} tension $= Whl/bd$. For P_{15} assumed zero. For P_{16} tension $= Whl/2bd$. For P_{17} compression $= Whl/2bd$. For P_{18} compression $= W(h+d)/2d$, shear $= Wh(½e - 1/b)$, moment $= Wh(½ - e/b)$. For P_{19} compression $= W/2$, shear $= Wh/b$, moment $= Wh(½ - e/b)$. For P_{20} tension $= W(h-d)/2d$, shear $= Wh(½e - 1/b)$, moment $= Wh(½ - e/b)$. For P_{21} compression $= W(h+d)/2d$. For P_{22} tension $= W(h-d)/2d$. For P_{23} compression $= Wh/2d$. Chord stresses in Fig. 113 may be obtained by use of moments. Dotted bars in Fig. 111 have no stress. If posts are fixt at lower ends, h in above formulas should be replaced by distance from top of post to point of contraflexure, which is usually $h/2 + d/2$.

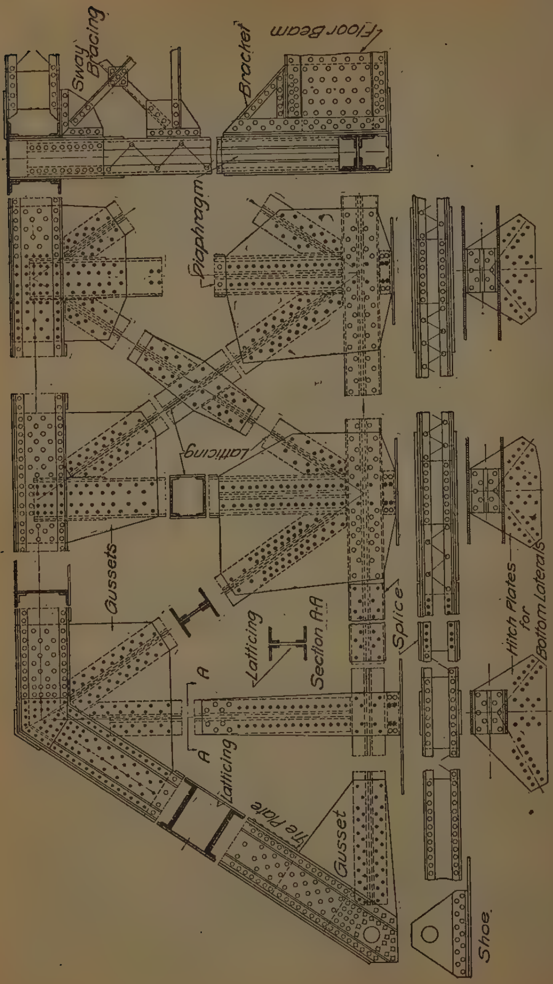


Fig. 114. Partial Detail Drawing of a Thru Riveted Railroad Bridge

MOVABLE AND CANTILEVER BRIDGES

32. Types of Movable Bridges

A Draw or Movable Bridge is one which can be moved from its normal position to allow the unobstructed passage of vessels. Movable bridges are classified into the five following types, swing, bascule, retractile, lift, and ferry bridges. Those of the SWING TYPE revolve horizontally about a vertical axis, which may be at the center, at one end or at an intermediate point of the span. BASCULE BRIDGES include all movable bridges which move in a vertical plane, rolling back from the opening or rotating about a horizontal axis. RETRACTILE DRAWBRIDGES are those which are supported on wheels resting on tracks on shore, and which are rolled horizontally backward from the opening either along the approach of the bridge or at an angle thereto. LIFT BRIDGES can be raised vertically to a sufficient height to allow vessels to pass underneath. FERRY BRIDGES consist of a movable car which is suspended from, and which moves back and forth under an overhead fixt bridge supported on towers, the overhead portion being so high above water level as to afford proper clearance for navigation.

Swing Bridges generally consist of two or more plate girders or trusses supported on a center or pivot pier in such a manner that they can be revolved on a circular track or pivot resting upon and attached to the pier. When the bridge is open there is a passageway for vessels on either side of the pier and the draw, and the trusses act as cantilevers overhanging the middle support. To prevent the open bridge from being struck by a vessel approaching or passing thru the channel, a FENDER PIER of piles and timber is built around the supporting pier, thus protecting the open draw span and guiding the vessel into the channel. The fender pier extends lengthwise of the stream and must be longer and wider than the open draw. Sometimes the pivot is placed on one side of the center between the center and the end of the draw, and the bridge then has two arms of unequal length, the shorter arm being counterweighted to balance the longer. Unequal arms may be used where the stream is too narrow for a central pier; the short counterweighted arm hanging over the shore and the long arm over the water when the bridge is closed. DOUBLE SWING BRIDGES have been used in a few instances. They consist of two swing spans each supported by a pier on each side of the stream and having their outer ends connected at the center of the channel when the bridges are closed. Swing bridges are the simplest and most economical of the various types and where there is ample room for operation they should generally be used. In locations where the pivot pier takes up too much space in the stream, or where room is not available for turning, the swing bridge cannot be used and some form of bascule bridge is preferable.

The **Jack-knife or Folding Drawbridge** is a swing bridge used only for railroad traffic, and as usually built has one Howe truss or steel truss under each rail, the rail being fastened directly to the upper chord of the truss. The trusses are connected together by small pivoted bars every few feet apart along the top chords, and each truss revolves horizontally about a pivot at one end, while the movable end is supported during operation by a needle beam which is guyed to the top of a tower or bent located on the shore at the pivot end of the span. When the bridge is open the trusses are parallel with the bank and are folded closely together. Since jack-knife draws cannot have floors, such bridges should not be used.

Bascule Bridges of various designs are in use, some being patented. Their merits lie in the speed with which they are operated and in the small amount of room required for their erection and operation. One to two minutes is the time often required for opening or closing a bridge altho many structures

cannot be made ready and opened in this time. Bascule bridges having their trusses or girders hinged at one end and the outer ends attached to cables passing over pulleys at the top of a tower are called TRUNNION BASCULE, OR HINGED LIFT BRIDGES. The cables are attached to counterweights which are of such size and move in such paths that the power required for opening or closing the draw is only that necessary to overcome inertia, frictional resistance, wind and snow. The more common forms of bascules have no cables but are counterweighted, and are operated by racks and pinions or other similar arrangements. The Page and the Strauss types are trunnion bridges. The Scherzer rolling lift bridges, Fig. 117, are of the bascule type,

Fig. 115

Fig. 116

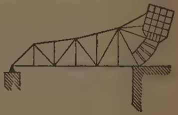


Fig. 117

but instead of rotating on trunnions they roll back on a horizontal track, the ends of the main girders or trusses being curved for this purpose. As the curved portion rolls backward on the track the outer portion or bascule leaf rises, leaving the channel clear. Bascule bridges may have one or two leaves and are used for either highway or railroad traffic.

Retractile, Direct Lift and Ferry Bridges are not common types. Retractile bridges may have one or two leaves overhanging the stream when closed and extending back over the abutments far enough to serve as a counterweight. This type is applicable to short spans only. The direct lift bridge consists of a simple truss structure which when closed rests on the abutments similar to any fixt span, but it is opened by being raised vertically by means of cables or chains passing over sheaves at the top of a tower at either end of the span. Counterweights are used to reduce the work of moving the bridge. The only ferry bridge in the United States is that across the ship canal at Duluth. It has two riveted steel trusses of about 390 ft clear span supported on a steel tower on either side of the canal. The clear height is about 135 ft above the water level. These two fixt trusses carry the track from which are suspended a steel frame and a ferry car, both of which travel back and forth across the canal. The retractile, direct lift and ferry bridges are used for highway traffic only.

Swing Bridges, which are generally supported at the center of the span are either center-bearing, Fig. 115, or rim-bearing, Fig. 116. While in motion a CENTER-BEARING STRUCTURE is supported at the pivot pier on a turntable consisting of some transverse girders which carry the dead weight of the bridge to a center pivot or casting about which the structure revolves. When the bridge is closed wedges are inserted under the trusses at the center so as to carry no dead load but to carry all the live load reaction on the center pier. Trailing wheels placed under the turntable steady the bridge while swinging but receive no load otherwise. From 4 to 8 wheels varying in diameter from 12 to 20 in are used. The CENTER PIVOT consists of an upper and a lower casting between which conical rollers, spherical balls or phosphor-bronze disks are placed, the latter being considered best. RIM-BEARING swing bridges have their trusses supported at the center pier directly on the drum or on distributing girders from which the load is carried to the drum. The DRUM is a circular girder resting on conical wheels which in turn are supported on a circular track of the pier.

Swing Bridges may be continuous, partially continuous or discontinuous. A CONTINUOUS BRIDGE is one in which the trusses are supported at three or more points and continue unbroken over the intermediate supports; shears and moments are transferred from one span into the others. A PARTIALLY CONTINUOUS TRUSS has four supports and the diagonals in the panel between the two central supports are omitted, thus allowing moment but no shear to be transmitted across this panel. A DISCONTINUOUS DRAWBRIDGE is one having either three or four supports, but is so arranged that when closed the ends are raised, or the center lowered, sufficiently to render some central top chord bars incapable of carrying stress, thus converting each truss into two simple spans, one on either side of the center pier. This is usually accomplished by slotting the pin holes of each of the top chord members adjacent to the tower or central panel or by placing a flexible joint at the middle of these members so that when the ends of the trusses are raised the bars are thrown out of stress. The discontinuous truss has the advantage of being more rigid than the continuous types, and where openings for vessels are infrequent, or where the time allowed for opening may be comparatively great, the discontinuous draw should be used. If frequent openings are necessary, much more power is required to raise the ends of the discontin-

Long American Movable Bridges

From Merriman and Jacoby's Roofs and Bridges, Part IV.

Type	Location	Nature of crossing	Span ft	Span in	Railroad or highway	Built
Swing	St. John's, Ore.	Willamette R.	521	0	R.R.	1908
"	East Omaha, Neb.	Missouri R.	520	0	R.R.	1893
"	East Omaha, Neb.	Missouri R.	519	4⅛	R.R.	1903
"	New London, Conn	Thames R	497	7	R.R.	1889
"	Staten Island	Arthur Kill	496	6	R.R.	1888
"	Duluth, Minn.	St. Louis Bay	485	7½	R.R. Hy.	1897
"	C.M. & N.R.R., Chicago	Drainage Can.	474	3½	R.R.	1899
"	Sioux City, Iowa	Missouri R.	469	11⅜	R.R. Hy.	1895
"	Middletown, Conn.	Connecticut R.	447	0	Hy.	1896
Scherzer lift	Terminal Ry., Chicago	Chicago R.	275	0	R.R.	1901
"	B. & O. R.R., Cleveland	Cuyahoga R.	230	0	R.R.	1907
"	22d St., Chicago	Chicago R	216	0	Hy.	1906
Trunnion bascule	Sault Ste Marie	Ship Canal	336	0	R.R.	1914
" "	Chattanooga, Tenn.	Tennessee R.	310	0	Hy.	1917
" "	16th St., Chicago	Chicago R	260	0	R.R.	1918
" "	Lake St., Chicago	So. Chicago R.	245	0	Hy.Elev.	1916
" "	So. Chicago	Calumet R.	235	0	R.R.	1914
" "	Grand Island	Sacramento R.	226	0	Hy.	1916
" "	Northwestern Ave., Chicago.	Chicago R.	205	7	Hy.	1904
" "	Seattle, Wash.	Salmon Bay	200	0	R.R.	1914
" "	Fullerton Ave., Chicago	Chicago R.	186	0	R.R.	1916
" "	West Division St., Chicago	Chicago R.	172	8	Hy.	1904
" "	North Ave., Chicago	Chicago R.	172	8	Hy.	1907
Vertical lift	Fratt B., Kansas City	Missouri R.	428	0	R.R.	
" "	G. N. R.R., Montana	Missouri R.	296	0	R.R.	
" "	Portland, Ore.	Columbia R.	275	0	Hy.	1916
" "	Pa. R.R., Chicago	Chicago R.	272	10	R.R.	
" "	Pine Bluff, Ark.	Arkansas R.	239	4	R.R.Hy.	1916
" "	Portland, Ore.	Willamette R.	220	0	R.R.	

uous type, and they cannot generally be operated as quickly as the continuous bridge. The Charlestown bridge in Boston, having a draw span 240 ft 6 in long and 100 ft wide, is discontinuous when closed, and during the year ending Dec. 31, 1908, the draw was opened 1409 times for the passage of 6218 vessels. The average delay to traffic for each opening was 8 min 30 sec.

Drawbridge Trusses whether center or rim-bearing may have their ends simply supported, latched, or raised. The ENDS are SIMPLY SUPPORTED when the end supports are so placed that the ends of the trusses will just come to a bearing without producing any dead load reactions and the entire dead weight is carried by the center pier either when the bridge is open or closed. The live load can thus produce upward end reactions only, and when it is in such a position as to cause a downward end reaction the truss will rise from the bearing, and, in such cases, when the live load passes over the bridge the ends hammer on the bearings. This is not a good arrangement. With ENDS LATCHED OR LOCKED to the abutments when the bridge is closed, there are no dead load end reactions and the live load end reactions may be either up or down depending on the position of the live load, hence the truss is a true continuous truss for live loads and simply two overhanging beams or cantilevers as far as dead loads are concerned. If when the bridge is closed the ENDS ARE RAISED by suitable machinery, the dead load causes end reactions, the magnitude of which depends on the amount the ends are raised, and the live load reactions are given by the laws of continuous trusses. If the ends are raised just the required amount the dead load reactions follow those laws, but since it is difficult to raise the ends the proper amount the dead load stresses are uncertain if the ends are raised.

33. Continuous Swing Bridges

Notation. M_1, M_2, M_3 = moments at three consecutive supports; k = ratio of distance of load from support to span length ; n = ratio of length of center to length of end span; E = modulus of elasticity of material; I = moment of inertia of cross-section of girder or truss; P = stress in member of truss due to load of unity acting on an intermediate joint; u = stress in member of truss due to load of unity acting at point and in direction in which deflection is desired; L = length of truss member; A = gross cross-sectional area of truss member.

Reactions. There are two methods of computing reactions on a continuous swing bridge when the bridge is closed. In one the truss or girder is assumed to be a continuous structure with a constant moment of inertia and the reactions are computed by the THEOREM OF THREE-MOMENTS. This is an approximate method of sufficient accuracy for a preliminary design, but for a careful determination of the stresses the method of deflections should be used after the preliminary design is made. The three-moment equation gives the relation between the bending moments at three consecutive supports of a continuous beam whose moment of inertia is constant, hence for a given case as many equations may be written as there are unknown quantities, and from these equations the bending moments at the supports may be computed, the reactions found, and then the stresses in the bars computed in the same manner as for simple trusses. The equation for a beam having supports at different levels h_1, h_2, h_3, Fig. 118, above a datum plane is

$$M_1l_1 + 2\,M_2(l_1 + l_2) + M_3l_2 = -\Sigma W_1l_1^2(k - k^3) - \Sigma W_2l_2^2(2\,k - 3\,k^2 + k^3)$$
$$- 6\,EI\left(\frac{h_2 - h_1}{l_1} + \frac{h_2 - h_3}{l_2}\right)$$

When the SUPPORTS ARE AT THE SAME LEVEL the above equation becomes

$$M_1l_1 + 2\,M_2(l_1 + l_2) + M_3l_2 = -\Sigma W_1l_1^2(k - k^3) - \Sigma W_2l_2^2(2\,k - 3\,k^2 + k^3)$$

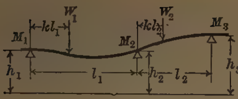


Fig. 118

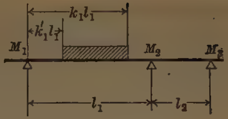


Fig. 119

Vith a uniform load w_1 per unit of length on a portion of the left span only s in Fig. 119, the equation of three-moments for level supports is

$$M_1 l_1 + 2\,M_2(l_1 + l_2) + M_3 l_2 = -\int_{k_1'}^{k_1} w_1 l_1^3 (k - k^3)\,dk$$

or a uniform load of w_1 per unit of length over the entire left span and w_2 er unit over entire right span the equation for level supports is

$$M_1 l_1 + 2\,M_2(l_1 + l_2) + M_3 l_2 = -\tfrac{1}{4}\,w_1 l_1^3 - \tfrac{1}{4}\,w_2 l_2^3$$

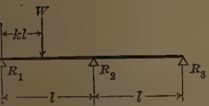


Fig. 120

Trusses with Two Equal Spans. The reactions for a load of W in the left span, Fig. 120, are as follows:

$$R_1 = W(1 - k) - \tfrac{1}{4}W(k - k^3)$$
$$R_2 = Wk + \tfrac{1}{2}W(k - k^3)$$
$$R_3 = -\tfrac{1}{4}W(k - k^3)$$

Reactions, R_1, R_2, R_3, due to a load of unity at successive panel points in the ft span are given in the accompanying table for trusses with two equal oans, the panels in the two spans being all equal. For a load in the left oan, R_1 and R_2 are upward and R_3 is downward, whereas for a load in the ght span R_2 and R_3 are upward and R_1 is downward. Also for a load in e right span the numerical values of the left reaction are those for R_3 in the ble and for the right reaction those for R_1 in the table.

Reactions R_1, R_2, R_3, Fig. 106, for Load of Unity in Left Span

Number of Panels in each Span	k	$+R_1$	$+R_2$	$-R_3$
2	1/2	0.406	0.688	0.094
3	1/3	0.593	0.481	0.074
	2/3	0.241	0.852	0.093
4	1/4	0.691	0.367	0.058
	2/4	0.406	0.688	0.094
	3/4	0.168	0.914	0.082
5	1/5	0.752	0.296	0.048
	2/5	0.516	0.568	0.084
	3/5	0.304	0.792	0.096
	4/5	0.128	0.944	0.072
6	1/6	0.793	0.248	0.041
	2/6	0.593	0.481	0.074
	3/6	0.406	0.688	0.094
	4/6	0.241	0.852	0.093
	5/6	0.103	0.961	0.064

Number of Panels in each Span	k	$+R_1$	$+R_2$	$-R_3$
7	1/7	0.822	0.213	0.035
	2/7	0.649	0.417	0.066
	3/7	0.484	0.603	0.087
	4/7	0.332	0.764	0.096
	5/7	0.198	0.889	0.087
	6/7	0.086	0.971	0.057
8	1/8	0.844	0.187	0.031
	2/8	0.691	0.367	0.058
	3/8	0.545	0.536	0.081
	4/8	0.406	0.688	0.094
	5/8	0.280	0.815	0.095
	6/8	0.168	0.914	0.082
	7/8	0.074	0.977	0.051

+ denotes an upward and − denotes a downward reaction.

Trusses with Three Spans. For swing bridge trusses having four points of supports, Fig. 121, the end spans are equal and the center panel is usually, tho not always, equal in length to the panel of the end spans. The reactions for the closed bridge with a load of W in left span are as follows:

$$R_1 = W(1 - k) - \frac{W(2 + 2n)(k - k^3)}{4(1 + n)^2 - n^2}$$

$$R_2 = Wk + \frac{W(2 + 5n + 2n^2)(k - k^3)}{4n(1 + n)^2 - n^3}$$

$$R_3 = -\frac{W(2 + 3n + n^2)(k - k^3)}{4n(1 + n)^2 - n^3} \qquad R_4 = \frac{Wn(k - k^3)}{4(1 + n)^2 - n^2}$$

With these formulas the reactions for any given truss may be found for a unit load placed successively at each panel point of the left span and a table prepared similar to the preceding table for trusses having three points of support. For a load in the left span R_1, R_2 and R_4 are upward and R_3 is downward, while for a load in the right span R_2 acts downward and R_1, R_3 and R_4 act upward. For the truss shown in Fig. 122, if the downward live reaction at R_3 for live loads on points 1 to 4 inclusive were greater than the upward dead reaction at R_3 when the bridge is closed, the truss will rise from its bearing at R_3 unless it is anchored there. If the points R_2 and R_3 are properly anchored, or if the dead upward reactions are greater than the live downward reactions at those points, the truss will not rise and the reactions

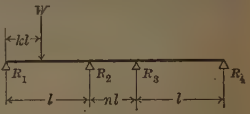


Fig. 121

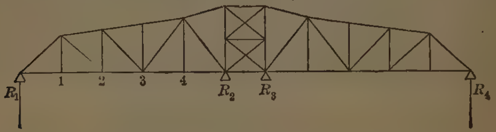


Fig. 122

are all given in accordance with the preceding formulas. If, however, the ends at R_1 and R_4 are latched down and the truss rises from its bearings at R_2 when the right span is loaded with the live load, the structure then becomes a continuous truss supported at R_1, R_3 and R_4 and the formulas for reactions for a truss of unequal spans on three supports must be used. Such trusses as Fig. 108 are frequently computed as partially continuous and the diagonals in the central panel made very small or omitted altogether.

Computing Stresses. With truss in Fig. 123 and the above table for six equal panels in each span, let it be required to compute the maximum tension and maximum compression in the diagonal in panel 2–3 for a dead panel load of 20 thousands of pounds and a

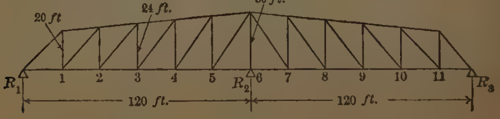


Fig. 123

live panel load of 30 thousands of pounds. The ends are assumed to be latched down, hence the live load causes reactions in accordance with the table and the dead loads cause no end reactions. The dead load stresses in all members are the same for bridge open and closed. It is here assumed that the dead panel load at the end lower joint is 15. The dead shear in panel 2–3 is 20 + 20 + 15 = 55, which is carried by the vertical components of the diagonal and top chord. The latter is (20 × 20 + 15 × 40)/(22 × 10) = 4.55 and the vertical component of diagonal is then 55.00 − 4.55 = 50.45, hence dead load tension is 50.45 × 31.24/24 = 65.67. For maximum live load tension, load joints 1, 2, 7, 8, 9, 10, 11. R_1 = 1.020 for a load of unity at each of these joints, as may be found from the preceding table, and for the live load R_1 = 1.020 × 30 = 30.60. Live tension in the diagonal = 41.97. For maximum compression in diagonal, load joints 3, 4, 5, giving R_1 = 0.750 × 30 = 22.50, and the live compression is 23.97. The diagonal in question has therefore a maximum tension of 65.67 + 41.97 = 107.64, and as the maximum live compression is less than the dead tension the minimum tension is 65.67 − 23.97 = 41.70. The unit used here is 1000 lb. Let it be required to find the maximum tension and maximum compression in lower chord member 2–3, Fig. 123, the ends of the truss being assumed simply supported. The stress in bar 2–3 is equal to the bending moment at the top chord joint above joint 3 divided by the lever arm of the bar, namely 24 ft. The dead load stress is the same for bridge open and closed and is (20 × 20 + 20 × 40 + 15 × 60)/24 = 87.50 compression. For maximum live compression, load joints 7, 8, 9, 10, 11, and from the preceding table R_1 for a load of unity at these joints is 0.366 and for the panel load of 30 is 0.366 × 30 = 10.98 downward. The live compression in bar 2–3 = 10.98 × 60/24 = 27. 5. For maximum live tension load joints 1, 2, 3, 4, 5, in which case R_1 = 2.136 × 30 = 64.08 upward, and the live tension in bar 2–3 = [64.08 × 60 − 30 (20 + 40)]/24 = 85.20. Hence maximum compression is 87.50 + 27. 5 = 114.95 and minimum compression = 87.50 − 85.20 = 2.30.

For Ends Raised sufficiently to make the dead load reactions follow the laws of continuous trusses, let it be required to find the maximum compression in the end post of truss in Fig. 123. The live load must be placed to secure the greatest upward reaction at R_1, that is, placed on joints 1 2, 3, 4, 5, giving a live R_1 = 2.136 × 30 = 64.08 upward. The effective dead load R_1 = 20 (2.136 − 0.366) = 35.40 upward, altho the total dead R_1 is 15.00 more than 35.40. The maximum compression in end post is (64.08 + 35.40) 28.28/20 = 140.66 thousands of pounds. With the live load on joints 7, 8, 9, 10, 11 the live R_1 is 10.98 downward, and as this is less than the dead R_1 of 50.40 acting upward, the ends of the truss cannot rise, and hence the end post has only compression when the bridge is closed. With bridge open the dead load tension in the post is 15.00 × 28.28/20 = 21.21.

Plate Girders are often used instead of trusses for swing bridges, the longest being on the Louisville and Nashville R.R., near Mobile, Ala., which has a span of 196 ft between centers of end bearings. For short spans plate girders are better than trusses. Turntables for turning locomotives are swing bridges with center bearings, and are generally arranged so that the ends just clear the supports when no live loads are on the bridge. Trailing wheels, which are usually placed at the ends, serve to prevent the turntable from rocking on the center support.

True Reactions on Swing Bridges. Reactions found by the preceding formulas, and hence the stresses dependent thereon, are approximate, because the effect of the web members is omitted, and because in many cases the moment of inertia of the truss is not constant. Reactions may be found more accurately by deflections after the sizes of the truss members are determined from the approximate stresses, then the stresses and the sizes may be revised and a new set of reactions and stresses found, the process being repeated till sufficient accuracy is reached. Assuming the gross cross-sectional areas of the members known in Fig. 123, the true reactions due to a load of unity on any joint such as 2 may be found as follows. If the center support is considered as removed, the downward deflection at the center due to the load of unity at 2 is expressed by the formula

$$D = \Sigma PuL/AE$$

where the Σ sign denotes the summation of the quantities PuL/AE for all members of the truss. P is the stress in each of the members due to the load of unity at 2 and u is the stress in each of the members due to a load of unity acting downward at the center. Tension should be given the plus and compression the minus sign. With the truss supported at the ends and with the center support still considered removed, if a single load of unity be applied at the center and acting upward the deflection at the center is

$$d = \Sigma u^2 L/AE$$

where u has the same numerical value as above, but since the load here acts up instead of down as above, the sign of u is different in the two cases; however, since u is squared in the second formula its sign in that formula is of no consequence. The Σ sign here denotes the summation of the quantities u^2L/AE for all members of the truss. Then the center reaction due to a load of unity acting at 2 is.

$$R_2 = \frac{D}{d} = \Sigma \frac{PuL}{AE} \Big/ \Sigma \frac{u^2L}{AE}$$

If all bars are of the same material E may be omitted from this equation. In the above manner R_2, Fig. 123, may be computed for a load of unity at joints 1, 2, 3, 4, 5 successively and then R_1 and R_3 found and a table of reactions, due to a load of unity, prepared similar to the preceding table of this article.

34. Partially Continuous Swing Bridges

For notation see Art. 33

Partially Continuous Swing Bridges. By the omission of the diagonals in the central panel, Fig. 124, no shear can be transmitted across that panel when the upper and lower chords are horizontal, and the truss is no longer a true continuous truss. The theorem of three-moments does not apply, but the reactions may be found by other means if the moment of inertia of the truss be assumed constant. For a load in the left span R_1, R_2 and R_3 are upward and R_4 is downward. With horizontal chords in the center panel the moment at R_2 is equal to that at R_3. The formulas given for this case do not apply if either of the chords in the center panel is inclined.

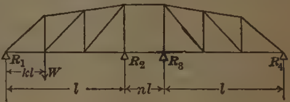


Fig. 124

$$R_1 = W(1 - k) - \frac{W(k - k^3)}{4 + 6n} \qquad R_2 = Wk + \frac{W(k - k^3)}{4 + 6n}$$

$$R_3 = \frac{W(k - k^3)}{4 + 6n} \qquad R_4 = -R_3$$

Twin or Double-swing Bridges. The reactions on a double-swing bridge, Fig. 125, locked at the center C and having the two portions alike, are:

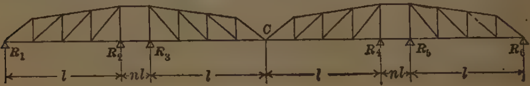


Fig. 125

With load of W on left span distant kl from R_1

$$R_1 = W(1 - k) - \frac{W(k - k^3)}{8 + 12n} \qquad R_2 = W - R_1$$

$$R_3 = -R_4 = -R_5 = R_6 = W(1 - k) - R_1$$

With load of W on the portion between R_3 and C and distant kl from C

$$R_1 = -\frac{W(1 - k)}{2} - \frac{W(k - k^3)}{8 + 12n} \qquad R_2 = -R_1$$

$$R_3 = Wk - R_1 \qquad R_4 = R_5 = -R_6 = W(1 - k) + R_1$$

35. Center Supports for Swing Bridges

Methods of Support. The weight of the bridge which is carried to the center pier should be distributed by girders to the TURNTABLE OF THE RIM-BEARING or to the PIVOT OF THE CENTER-BEARING structure, whence it should be distributed uniformly over the masonry pier. In many heavy bridges, combinations of center- and rim-bearings are used, so that the greater part of the weight is carried on the turntable and the lesser part on the center pivot. With this arrangement the pivot is held in place and the drum is less likely to become distorted. Fig. 126 shows one arrangement for carrying the loads to eight points on the circular drum of the rim-bearing support,

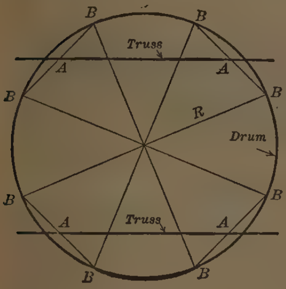


Fig. 126

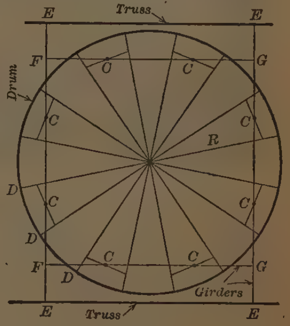


Fig. 127

and Fig. 127 shows a COMBINED CENTER- AND RIM-BEARING SUPPORT in which a portion of the weight is carried to sixteen points on the drum and the remaining portion to the center casting. In Fig. 126 the two trusses are supported at points A on four girders which carry the loads to the drum at eight points B. The girders BB may either rest on top of or be framed into the inside of the drum at points B; and at A rockers should be placed between the bottom of truss and top of girder. The RADIAL STRUTS R are used to brace the drum and are connected to the upper movable part of the center casting about which the turntable revolves. Where the distance between trusses is too great to allow the trusses to rest directly on girders BB, two transverse girders supported at points A may be used, and the trusses be connected to their outer ends as is shown in Fig. 127, where the two trusses

rest at E on the ends of transverse plate or box girders which are supported at points C on short girders which are connected to the radial girders R. The distributing girders FG are riveted between the transverse girders and are supported at points C. The radial girders distribute the loads on sixteen points D of the drum and also carry a portion of the weight to the center casting. For proper distribution of the weight on the wheels the drum should be loaded in at least eight points.

The Turntable for a rim-bearing swing bridge includes the drum, the radial struts or girders, the center casting to which these struts are connected, and the wheels and track upon which the drum revolves. The DRUM is usually a single curved plate girder, tho double drums consisting of two concentric girders have been used, and the strength and depth of the girder should be sufficient to distribute the loads uniformly over the wheels. Many drums are too light to do this and unequal wearing of the wheels and track results, thus offering greater resistance to turning. To be stiff enough to distribute the loading, drums should have a depth equal to from 4/10 to 1/2 the distance between the loading points. Fig. 128 shows details of drum, radial strut, wheels, track and center casting for a rim-bearing turntable. The drum has a cast-steel tread riveted to its bottom flange, the tread being

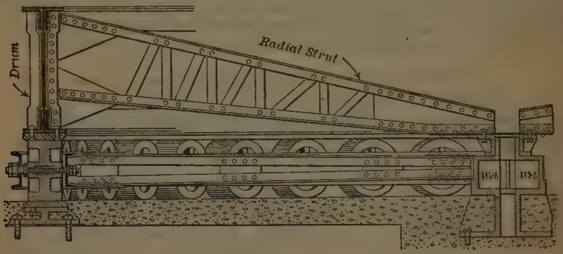


Fig. 128. Rim Bearing Turntable

cast in short lengths, riveted to the drum, and while the drum is temporarily erected at the shop the tread should be machined. The upper tread may be level or beveled, but it is simpler to machine the tread if it is level. Wheels should be made of cast steel and must be conical in form and must be held between two circular channel rings, the axles for the wheels passing thru the channels and provided with nuts, washers and oil holes. Between the wheels the inner and outer channel rings are connected by tie plates. A common but poor form of arranging the wheels is to have each wheel connected to the movable part of the center casting by an adjustable rod which passes thru the wheel, thus serving as an axle and at the same time as a spacing bar to hold the wheel at the proper distance from the center of revolution. The inner channel ring should be connected to the movable part of the center casting by rigid struts, not by rods. Where the center pivot carries no weight both its upper and lower parts may be made of cast iron, but when a portion of the load is taken on this pivot it should consist of an upper movable and a lower fixt part, both of which may be cast iron (tho better of cast steel), but between these two should be placed two cast steel disks or lenses separated by a disk of phosphor-bronze. Oil wells should be provided for the

center casting. The fixt portion of the center pivot is sometimes imbedded in concrete to hold it more firmly. The lower track and rack should be of cast steel in segments about 5 ft long, and must be anchored to the masonry.

Center Bearings, also sometimes called center-bearing turntables, should carry the entire weight of the draw while swinging, and when the draw is closed wedges or other bearings should be inserted under the main trusses or girders so that the live load will not be borne by the center castings. The weight of the swinging bridge is usually carried to the pivot by means of two transverse girders, which are so placed as to either allow the pivot to go between them or else they rest on top of the pivot. In some cases additional longitudinal girders are necessary to get the load to the center. The PIVOT consists of an upper movable casting, preferably of cast steel, upon or suspended from which the weight is carried, and a lower fixt casting attached to the pier. Between these two castings are placed conical rollers (as in the Sellers center), steel balls or disks. The pivot should be so arranged that the disks or rollers can be inspected or replaced without interfering with traffic. Conical rollers are apt to wear unevenly, and even when set in guide rings they are apt to bind in the bearings. Ball bearings have been used very little. The best form of center bearing is that which has a phosphor-bronze disk between two hardened steel disks of oil-tempered tool steel. TRAILING WHEELS should be of cast steel and should have a circular steel track, an ordinary rail being used for light bridges. All trailing wheels and pivots should be adjustable so that the revolving portion of the drawbridge may be slightly raised or lowered. Fig. 129 shows one type of center bearing suitable for a plate girder drawbridge of the swing type. Center bearings may be

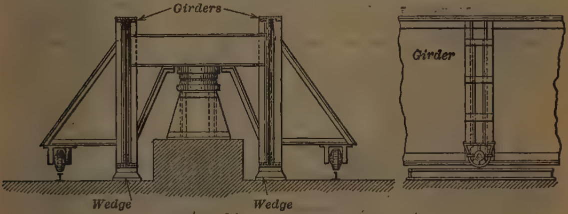


Fig. 129. Center Bearing Turntable

used for long-span single-track railroad bridges, but for wide highway or double-track structures they are not as suitable on account of the heavy girders necessary to transfer the weight from trusses to center pivot.

Allowable Unit Stresses used in good practice are as follows: ROLLERS, in pounds per linear inch; at rest, cast iron 400 d, cast steel 800 d; in motion, cast iron 200 d, cast steel 400 d, where d is diameter of roller in inches. BEARING; of hardened tool steel on phosphor-bronze disks in center-bearing turntables, 3500 lb per sq in; of cast-iron wedges on cast iron or structural steel, 500 lb per sq in; of axle steel on phosphor-bronze trunnion bearings for bascule bridges, 2000 lb per sq in.

36. Machinery for Swing Bridges

Notation. W = total uplift of swing bridge at both ends in pounds, HP = total horse-power required for raising ends of swing bridge, h = total lift in feet, t = time for lifting in seconds, R = radius of center line of track in feet, t_1 = time for opening in seconds, W_1 = weight on rollers under drum of rim-bearing or total revolving weight on pivot of center-bearing bridge in pounds, r = radius of pivot in feet, b = width and l = length of bridge in feet.

Machinery for Operating a Swing Bridge includes that necessary for turning the bridge and for lifting and lowering the ends of the truss or girders. All swing bridges should be so arranged that they can be operated by hand power, and all but the smallest that are opened infrequently should be provided with motive power. The motor may be a steam engine, an electric motor or a gasoline engine. Where electric current is available the electric motor is the simplest and most economical, for it takes up little room, is easily installed and the cost of maintenance is low. For the larger bridges where electric current is not available the steam engine gives the most satisfactory power, especially if the bridge is to be opened frequently, but with light spans and infrequent openings the gasoline engine is best.

Turning Machinery. Whatever form of power is used for turning the bridge it is usually applied thru a vertical shaft which is attached to the drum of a rim-bearing turntable and to some girder or floor beam of the center-bearing type. On the lower end of the shaft is a pinion engaging in the circular rack which is attached to the pivot pier or to the lower track on which the turntable revolves. The rotation of the shaft causes the bridge to revolve. Electric motors are usually set on top of the drum or on a platform projecting from the side of the drum, and they are usually of the railway type, series wound and waterproof. The armature speed varies in different bridges from about 450 to 650 rev per min and generally should not exceed 600 rev per min. The controllers for the motors should be placed in the operator's house, together with the switch-board and necessary switches, meters, circuit-breakers and fuses. Friction brakes are used for most movable bridges,

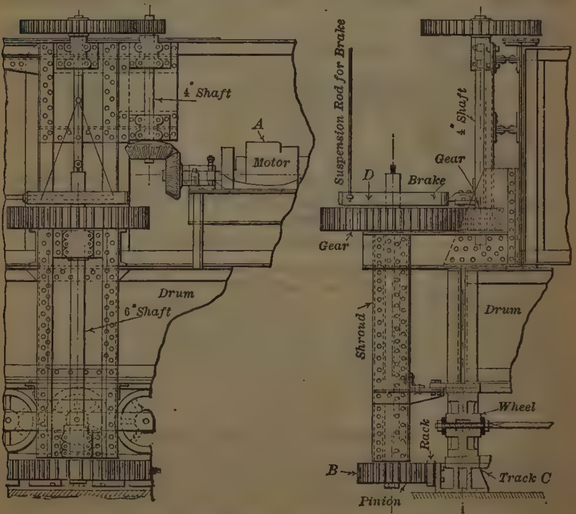


Fig. 130. Turning Mechanism, Charlestown Bridge, Boston, Mass.

altho where the motor is close to the rack of swing bridges they are not absolutely necessary but even here are desirable. When two motors are used, as in most heavy bridges, they are placed on opposite sides of the drum. Fig. 130 shows the turning mechanism for the swing span of the Charlestown bridge in Boston, Mass., which has two trains of gears driven by electric motors, one of which is shown. The motor *A* is attached to a steel supporting frame and drives the reduction gearing which transmits the power thru the pinion shaft to pinion *B* engaging the circular rack bolted to the base casting or track *C*. A brake *D* is attached to each pinion shaft and is operated by comprest air.

End-Lifting Mechanisms. Swing bridges require some form of end-lifting device which will lift the ends of the trusses or girders when the bridge is closed and which lowers them when the bridge is to be opened. Lifting may be done in small bridges by allowing the ends to run up on wheels which are mounted on the fixt piers, but for ordinary cases some better arrangement, such as TOGGLE JOINTS or HYDRAULIC JACKS, is necessary for raising the ends sufficiently to allow WEDGES or other similar supports to be inserted. No end bearings should be used where the support of the truss while the bridge is closed is dependent upon the machinery, for the lifting mechanism is merely to lift and lower the ends and not to support them. Wedges having a bevel of 1 to 5 or 6 are frequently used for bearings, and when in place they should be locked so that they will not work out. End locks or latches must be provided both for centering the draw span when closed and for holding the ends down in case the maximum live downward reaction exceeds the dead load upward reaction. Where electric motors are used for operating the end-lifting mechanisms one motor may be placed at each end of the bridge, thus doing away with long lines of shafting from the center to the ends of the draw.

The end lifting and supporting mechanism for the highway swing bridge at New Bedford, Mass., is shown in Fig. 131. This drawbridge has two trusses carrying a roadway and two sidewalks, the length of the trusses being 282 ft 8 in center to center of end bearings. The trusses are made discontinuous when closed by raising the end sufficiently to render the top chord bars adjacent to the center tower incapable of carrying stress.

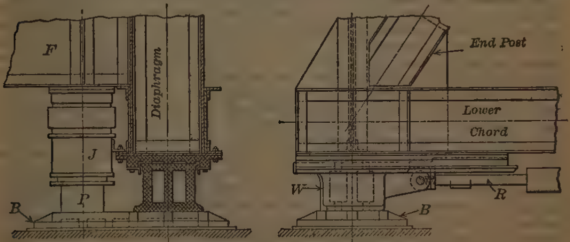


Fig. 131. End Support of Bridge at New Bedford, Mass.

Each end of each truss is raised with a hydraulic jack, placed close to the truss and attached to the under side of the lower flange of the end floor beam *F*. To raise the truss the plunger *P*, which has a maximum stroke of 6 in, is forced down till it bears against the base plate *B*, and then as the pressure is continued the end of the bridge is raised 5½ in, thus bringing the lower surface of the wedge *W* ½ in above the top of *B*. The wedge is then pushed into place over the base plate, the pressure in the jacks decreased and the trusses lowered ½ in till the wedge bears on the base plate. The wedges thus

serve as supports for the trusses when the bridge is closed. The longitudinal motion of the wedge is approximately 2 ft 6 in and is accomplished by a piston rod R operated from a cylinder subjected to a pressure of "zero oil" at from 12 500 to 25 000 lb per sq in, which is the same as in the jacks. Wedges as ordinarily used are beveled, but in this bridge the upper and lower surfaces are both horizontal. The uplift at each jack including the weight of snow at 10 lb per sq ft of roadway and sidewalks is 200 000 lb.

Rail-lifts for railroad bridges are generally provided to raise the ends of the rails on the swing span so that as the bridge revolves they will be above the rails on the adjoining fixt spans. This is usually done by means of shafting connected with the end-lifting device, and there should be automatic home and distant SIGNALS on each side of the bridge interlocked with the end-lifting mechanism and rail lifts so that the signals can be set at safety only when the ends of the bridge and the rails are in their proper positions. Rail-lifts are objectionable because the rails are not properly held during the passage of trains, and those forms of RAIL-LOCKS which insure proper alinement of rails and allow the rails to be securely fastened to the track are better. END LATCHES to bring the bridge into proper alinement when closed should be arranged to work automatically but only when the ends of the bridge are slowly approaching the supports.

The Power Required for operating a swing bridge is that for lifting the ends and for turning the bridge. These two operations are not performed at the same time, so that the maximum power necessary is the greater of the two, not their sum. To LIFT THE ENDS the resistance varies from zero at the beginning to a maximum at the end of the operation, and the total horse-power required, excluding the frictional resistances of shafting and all other movable parts of the end-lifting mechanism, is HP = $Wh/550\,t$. In this formula W must be taken as the total end uplift; for example, if the uplift at each end of a truss is 30 000 lb and there are two trusses, W would be 120 000 lb. The frictional resistances to be overcome in lifting the ends vary greatly and are estimated af from 100 to 250 % of the above value of P. If friction at 100 % is allowed, then the horse-power required for lifting and overcoming frictional resistances in the lifting mechanism is $Wh/275\,t$.

In Turning the Bridge the resistances to be overcome are those due to friction, unequal wind pressure and inertia of the mass. The frictional resistance in a rim-bearing bridge is caused by friction between the wheels and the upper and lower tracks, also at the axles of the wheels and at the center pivot. The COEFFICIENT OF THE TOTAL FRICTION for rim-bearing bridges was found by C. Shaler Smith to be from 0.004 to 0.008 of the load on the wheels; by Boller and Schumacher to be 0.0035 for the Thames River bridge in Connecticut, and by Theodore Cooper to be 0.0038 for the Second Avenue bridge in New York. In center-bearing structures the friction is principally at the center pivot, and the coefficient of frictional resistance at the circumference of the pivot was found by C. Shaler Smith to be 0.09 of the weight turned; and by C. C. Schneider to be 0.067 at the start and 0.045 to maintain motion at uniform speed for a bridge with a center-bearing of hardened steel and phosphor-bronze disks carrying a pressure of 3000 lb per sq in. The last-named engineer also found the highest coefficient of total friction on new bridges including that of shafts and gearing for hand operation, to be 0.115 for starting and 0.08 for maintaining motion. The resistance due to an UNBALANCED WIND PRESSURE of from 4 to 5 lb per sq ft of surface of one arm of the draw is sometimes allowed for in determining the power required, and sometimes this is provided for by using larger coefficients of friction than above given. To overcome the inertia of the bridge and to accelerate the motion the power required depends on the time allowed for opening the bridge. The motion may be accelerated during the first half and retarded during the last half of the movement, as is usual; or it may be accelerated during the first third, then maintained at a uniform velocity for the second third, and retarded for the last third.

The Horse-power required for overcoming all frictional and wind resistances of a rim-bearing bridge while turning is given approximately by $W\,R/11671\,t_1$,

in which the coefficient of resistance is assumed to be 0.015. And for a center-bearing bridge while turning with an assumed coefficient of resistance of 0.15 the horse-power is $W_1 r / 1167\, t_1$. The horse-power required for overcoming inertia is approximately $W_1(b^2 + l^2)/10\ 767\, t_1^3$. In the three formulas just given it is assumed that the motion is accelerated during the first half and retarded during the last half of the time of swinging; and in the first two formulas the force is applied at the center of the track and in the last it is applied at the center of gyration.

37. Cantilever Trusses

Historical. Bridges which have cantilever, that is, projecting, arms are called cantilever bridges and the principle involved in their construction is very old. The FIRST EUROPEAN CANTILEVER structure of note was a highway bridge of 124 ft span designed and built by Gerber in 1867 over the river Main at Hassfurt, and the first cantilever railroad bridge was a span of 148 ft completed in 1876 over the Warthe at Posen. IN AMERICA THE FIRST IRON CANTILEVER bridge was the Kentucky River bridge of the Cincinnati Southern R.R., built in 1876–7 by C. Shaler Smith of the Baltimore Bridge Co. The

Longest American Cantilever Bridges

Span	Crossing	Location	Railroad tracks	Highway	Deck or thru	Date of completion
ft in						
1800 0	St. Lawrence River.	Quebec, Ont.	2	......	Thru	1917
1182 0	East River........	*Blackwell's Id., N. Y ...	......	*	"	1909
812 0	Monongahela River	Pittsburgh, Pa..........	2	......	"	1904
790 5	Mississippi River .	Memphis, Tenn.........	1	*	"	1892
790 5	Mississippi River..	Memphis, Tenn.........	2	*	"	1915
775 0	Ohio River.......	†Sciotoville, O.........	2	......	"	1918
769 0	Ohio River.......	Beaver, Pa.............	2	......	"	19—
750 0	Ohio River.......	Sewickley, Pa	......	*	"	1910
700 0	Ohio River.......	Mingo Junction, Ohio ..	2	......	"	1904
671 0	Mississippi River .	Thebes, Ill.............	2	......	"	1905
660 0	Colorado River....	Red Rock, Cal..........	1	......	"	1890
650 0	Ohio River.......	Marietta, O...........	......	*	"	1903
555 9	Ottawa River.....	Ottawa, Ont...........	1	*	"	1900
525 0	Long Lake........	Hamilton Co., N. Y.....	......	*	"	1901
523 0	Hudson River.....	Poughkeepsie, N. Y......	2	......	Deck	1888
520 3	Allegheny River...	†Bessemer, L. Erie R.R..	2	......	"	1918
520 0	Kentucky River...	Tyrone, Ky	1	......	"	1889
520 0	Ohio River.......	Cincinnati and Newport..	......	*	Thru	1891
483 0	Ohio River.......	Louisville, Ky..........	1	*	"	1886
480 0	Mississippi River..	Burlington, Ia..........	......	*	"	1917
480 0	Kanawha River...	Point Pleasant, W. Va...	1	......	"	1888
477 0	St. John River....	St. John, N. B..........	1	......	"	1885
470 0	Niagara River. ...	Niagara Falls, N. Y.....	2	......	Deck	1883
450 0	Allegheny River...	Highland Park, Pittsburgh	......	*	Thru	1900
442 0	Mississippi River..	Muscatine, Ia	......	*	"	1889
420 0	Mississippi River..	Clinton, Iowa	......	*	"	1892
420 0	St Lawrence River.	Cornwall, Ont..........	1	......	"	1899
413 0	Allegheny River...	Bet. Reno and Oil City, Pa.	......	*	"	1902

* Known as Queensboro bridge. † Continuous bridge.

Michigan Central R.R. bridge over Niagara River was the second important American cantilever and was completed in 1883. The bridge of this type having the LONGEST SPAN in the world (1918) is the new Quebec bridge, which has a span of 1800 ft center to center of river piers. The Quebec bridge which collapsed Aug. 29, 1907, during construction also was being built with a span of 1800 ft. Until the successful construction of the present Quebec bridge, the largest bridge of the cantilever type and the one having the longest span was the Forth bridge in Scotland, which has among others two spans each of 1700 ft center to center of bearings.

The Modern Cantilever Truss is the outgrowth of the continuous truss the change being brought about by the use of hinges which fix the points of contraflexure in the cantilever truss, thus removing some of the uncertainties in the calculations. The ordinary cantilever bridge, Fig. 132, of three spans on four supports consists of two ANCHOR ARMS, one at each shore, two CANTILEVER ARMS, and one SUSPENDED SPAN and the suspended span is supported at the outer ends of the cantilever arms, whereas for bridges of great lengths, one or more intermediate spans, Fig. 134, are used in combination with some of the parts just mentioned. An INTERMEDIATE SPAN is one which extends unbroken between two piers and beyond which it projects to form a cantilever arm in each of the adjoining spans. A HINGE is a junction between a cantilever arm and a suspended span where the bending moment is zero after the bridge is complete, and for trusses is usually made by connecting the two portions by a pin at one chord and either omitting the opposite chord bar of the truss or arranging it so that it cannot carry stress after the bridge is self-supporting. Hinges can transmit shear but no moment after the bridge is complete, but during erection they are arranged to carry both. In PLATE GIRDER CANTILEVER bridges links connecting the suspended span to the cantilever arm have been used as hinges. Cantilever bridges have the advantage over simple trusses that the span containing the cantilever arm and suspended span can be erected without falseworks by being built out from the adjacent spans piece by piece, and the advantage over continuous trusses that the stresses are more accurately determined and are not altered by any reasonable settlement of the foundations.

Typical Trusses of five large American cantilever structures are shown in Figs. 132-136. Fig. 132 shows one half side elevation of the QUEBEC bridge which collapsed on account

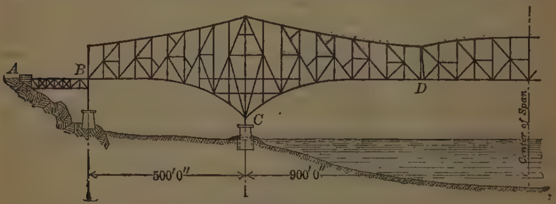


Fig 132. Quebec Bridge, 1907

of the failure of the lower chord in the anchor arm, while being built in 1907 over the St. Lawrence River about 6 miles above Quebec. It consisted of two trusses 67 ft between centers, each having two anchor arms *BC*, two cantilever arms *CD* and one suspended span hung from the cantilever arms at *D*. At each end was a short approach span *AB*. The structure was designed to carry two railroad tracks, two electric railway tracks, two highways and two sidewalks. Clearance above highest tide 150 ft; maximum depth of water about 180 ft. Main trusses were pin-connected with pins as large as 12 in in

diameter and eye-bars as wide as 18 in. Depth of truss: suspended span, at center 130 ft, at hinge D about 97 ft; anchor arm, at end B about 97 ft, at pier C 315 ft. Panel lengths: anchor arm, 10 at 50 ft; cantilever arm, 10 at 56 ft 3 in; suspended span, 12 at 56 ft. 3 in.

The New Quebec Bridge, Fig. 133, is built at the same site and has the same channel span, 1800 ft center to center of river piers, as the bridge which collapsed. The length

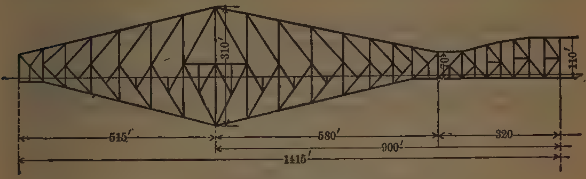


Fig. 133. Quebec Bridge, 1917

of the main structure is 2830 ft and the total length, face to face of abutments is 3239 ft. The depth of truss at the river pier is 310 ft; at the ends of the anchor and cantilever arms, 70 ft; at the center of the suspended span, 110 ft. The main structure consists of two trusses of the K type, 88 ft between centers. It carries two railroad tracks and two 3-ft walks, one on the side of each track. It is designed for a live load consisting of two E-60 engines and 5000 lb per lin ft on each track. Main trusses are pin connected with the pins varying from 8 in to 30 in in diameter and maximum width of eye-bars 16 in. There are thirty-two 16 in by $2^3/_{16}$ in eye-bars, with a net cross-sectional area of 1120 sq in, in the top chord panel adjoining the vertical post over the river pier.

The Thebes Cantilever Bridge, Fig. 134, crosses the Mississippi River at Thebes, Ill., and consists of a steel structure of five spans with several approach spans of concrete arches at each shore. One-half the steel structure is shown in Fig. 134. The suspended

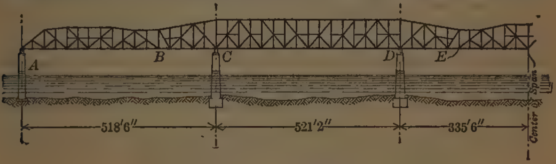


Fig. 134. Thebes Bridge

span AB is supported on pier A and on the cantilever arm BC. Span CD is an intermediate span and DE is another cantilever arm which supports the central suspended span. Clearance above high water 65 ft, above low water 103 ft. There are two lines of trusses 32 ft on centers carrying two railroad tracks. Each of the three suspended spans is 366 ft long, each of the four cantilever arms 152 ft 6 in long and each of the two intermediate spans is 521 ft 2 in center to center of bearings. Depth of truss: suspended span, at center 55 ft; cantilever arms at B and E 50 ft; intermediate spans, 27 ft thruout.

The New Memphis Bridge is located about 200 ft upstream from the structure built in 1892. It has the same length of channel span as the older bridge, 790 ft 5 in. In arrangement of spans, outline and type of truss it is very similar to the portion of the Thebes bridge shown in Fig. 134. But whereas the latter is symmetrical about the center of the channel span, the Memphis bridge terminates with a 186 ft anchor arm on the right of the channel span. The span lengths are, beginning at the left, Fig. 134; suspended span, 417 ft 9¾ in; cantilever arm, 186 ft 3¾ in; fixed span, 621 ft; cantilever arm, 186 ft 3¾ in; suspended span, 417 ft 9¾ in; cantilever arm, 186 ft 3¾ in; anchor

arm, 186 ft 3¾ in. Total length of main structure, not including a 345 ft simple approach deck span, is 2201 ft 10½ in. It consists of two lines of trusses, 32 ft between centers, carrying two railroad tracks and two 14-ft cantilever highways. Superstructure is 76 ft in the clear above high water level. Depths of truss are: at center of suspended spans, 67 ft; at ends of cantilever arms, 55 ft; thruout fixt span, 88 ft. It was designed for Cooper's E-50 loading on each of the railroad tracks, and a 17½-ton road roller and 100 lb per sq ft on the highways. Maximum diameter of pin is 20 in and width of eye-bars is 16 in.

The Wabash R.R. Bridge over Monongahela River at Pittsburgh, Pa., is a cantilever bridge 1504 ft long exclusive of the steel viaduct approach on each shore. Fig. 135 shows one-half of the main structure which consists of two trusses 32 ft on centers carrying two railroad tracks spaced 13 ft on centers. The anchor arm AB is 346 ft long, the cantilever arm BC 226 ft, and the suspended span, which is supported at C, is 360 ft in length. Depth of truss: at portals, 60 ft; suspended span, 60 ft thruout; at main piers, 126 ft 6 in.

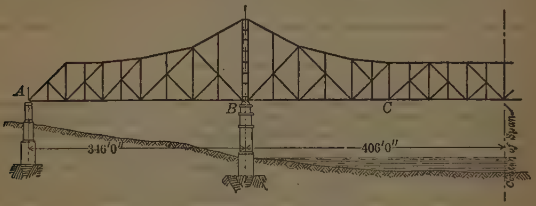


Fig. 135. Wabash RR. Bridge, Pittsburgh, Pa.

The tracks are on a 1% grade, so that the clearance under the central span near one pier is 70 ft, and near the other is 77.86 ft above " full pool " level, which is below extreme high water. Panel lengths vary from 30 to 40 ft. Eye-bars are from 12 to 14 in wide, the latter having heads 33 in in diameter. Pins 12 and 14 in in diameter.

Queensboro or Blackwell's Island Bridge spans the East River and Blackwell's Island at New York and is a cantilever bridge on six masonry piers, two on each shore and two on the island. Fig. 136 shows the outline of that part of truss from the center of the island span at A to the end E in Borough of Queens. The total length of the cantilever portion of the bridge is 3724 ft 6 in, divided into an intermediate span of 630 ft on

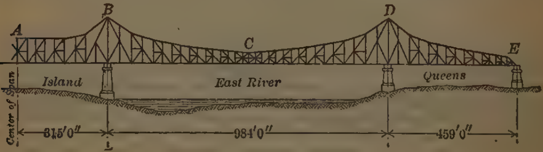


Fig. 136. Queensboro Bridge

the island, two cantilever arms each of 492 ft on the east, two cantilever arms each of 591 ft on the west, one east anchor arm of 459 ft and one west anchor arm of 469 ft 6 in. Longest span 1182 ft. Depth of truss over each of the four intermediate piers such as B and D, 185 ft. There are two trusses 60 ft on centers, made partially continuous by having the two cantilever arms joined together by rockers at C, thus making the structure statically indeterminate.

38. General Deta for Cantilevers

Live Loads on Large Bridges. FIRTH OF FORTH: 2240 lb per lin ft for each of two tracks. MEMPHIS: 4000 lb per lin ft per track; single track. NEW MEMPHIS: Cooper's E-50 loading per track, 17½-ton road roller and 100 lb per sq ft on highways. WABASH R.R. over Monongahela River at Pittsburgh: two consolidation locomotives followed by 4500 lb per lin ft for each of two tracks. THEBES: for floor system, one concentrated load of 50 000 lb followed by 5000 lb per lin ft for each of two tracks; for trusses 20% less. FIRST QUEBEC: trusses in general, (a) on each steam railway track 3000 lb per lin ft for unlimited length, or (b) on each steam railway track two Cooper's Class E-33 locomotives followed by a train of 3300 lb per lin ft, total length of train 900 ft, or (c) on each steam railway track one Cooper's Class E-40 locomotive followed by a train of 1000 lb per lin ft, total length of train 550 ft; no loading on electric railway tracks, roadways or sidewalks; hangers, sub-diagonals and floor system, (a) on each steam railway track two Cooper's Class E-40 locomotives followed by a train of 4000 lb per lin ft, and (b) on each electric railway track 50 000 lb on two axles 10 ft apart with 20 ft to leading axle of car following, and (c) on each roadway 24 000 lb on two axles 10 ft apart. NEW QUEBEC: two Cooper's E 60 engines and 5000 lb per lin ft on each track. QUEENSBORO: main trusses, original specifications for "congested" load of 12 600 lb per lin ft of bridge as follows, two elevated railway tracks at 1700 lb per lin ft each, four trolley tracks at 1000 lb per lin ft each, 35.5-ft roadway at 100 lb per sq ft or 3550 lb per lin ft of bridge, two 11-ft sidewalks at 75 lb per sq ft or 1650 lb per lin ft of bridge, and a "regular" live load of 6300 lb per lin ft; two elevated railway tracks added later increased original truss live loading from 12 600 to 16 000 and from 6300 to 8000 lb per lin ft of bridge for "congested" and "regular" loadings respectively; floor systems and secondary trusses, on each elevated railway track cars of four axles spaced 6-10 6 ft with 26 000 lb per axle, on each street car track cars of two axles spaced 10 ft with axle load of 26 000 lb or 1800 lb per lin ft of track, on any part of roadway 48 000 lb on two axles 10 ft apart and 5 ft gage covering a space 12 ft by 30 ft and on the remaining roadway surface 100 lb per sq ft, on the sidewalks 100 lb per sq ft. When nearly completed the Queensboro bridge was found to be too weak to carry the above loads, so that it is not being used as originally planned.

Unit Stresses in Pounds per Square Inch specified for some large cantilever bridges. FIRTH OF FORTH: maximum compression 17 000; maximum tension 16 350; ultimate tensile strength of steel 67 000 to 74 000 and ultimate compressive strength of 76 000 to 83 000. MONONGAHELA River at Pittsburgh: dead load compression 21 000 where $l/r < 40$; dead load tension 22 000; live load stresses ½ these values; ultimate tensile strength for eye-bar steel 63 000 to 73 000 and for plates and angles 60 000 to 70 000; shearing on rivets 10 000 for rivet steel of ultimate tensile strength of 52 000 to 63 000. QUEENSBORO: compression for structural steel under ordinary loads $20\,000 - 90\,l/r$, and for congested loads $24\,000 - 100\,l/r$; tension for nickel steel eye-bars under ordinary loads 30 000 and under congested loads 39 000; tension for structural steel in main members of trusses, towers and bracing 20 000 for ordinary and 24 000 for congested loads respectively; shear on shop rivets 13 000 and 16 000 for ordinary and congested loads; full sized annealed nickel steel eye-bars up to a maximum size of 16 in × 2.5 in were required to show a minimum elastic limit of 48 000, a minimum ultimate tensile strength of 85 000 and an elongation of 9% in 18 ft; open-hearth structural steel eye-bars, minimum elastic limit 28 000 and minimum ultimate strength of 56 000 and elongation of 10% in body of bar. FIRST QUEBEC: compression, ordinary, 12 000 (1+min/max), extreme 24 000, both for $l/r < 50$; tension, ordinary 12 000 (1+min/max), extreme 24 000; structural steel with ultimate tensile strength of 62 000 to 70 000.

Compression Members of large cantilever bridges in America are latticed box sections as shown in Figs. 137, 138 and 139. The Forth bridge members are hollow cylinders some of which are 12 ft in diameter and made of plates, 1¼ in in thickness, riveted together and stiffened by inside stiffening frames. Fig. 137 shows a section of a lower chord member of the ill-fated Quebec bridge, Fig. 138 of the Queensboro, and Fig. 139 of the Monongahela River bridge. These are typical sections for the three bridges respectively. The segments of the Quebec chord were connected by latticing only, and except at splices there were no diaphragms, while the segments of the Queensboro

and the Monongahela chords are braced by both latticing and diaphragms. The latticing and diaphragms are not completely shown here. The largest

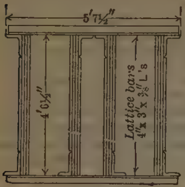


Fig. 137

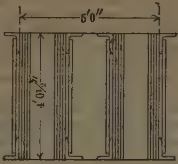


Fig. 138

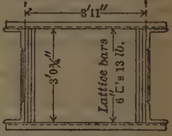


Fig. 139

cross-sectional area of any compression chord in the Queensboro bridge is 1072.1 sq in.

Typical Compression Chords of Large Cantilever Bridges

Bridge	Area of cross-section	Sectional Area of Lattice*	Sectional Area of Lattice Rivets†	Length of Chord l	Radius of Gyration r‡	l/r
	sq in	sq in	sq in	ft in	in	
Quebec (first)	781	10	4.8	57 09/32	19.7	35
Memphis (old)	213	10	4.8	28 2¾	14.8	23
Queensboro	852	25	4.8	31 6⅛	22	17
Thebes	189	11¼	4.8	30 65/32	20.1	18
Monongahela	262	14½	7.2	30 61/32	25.4	14

* Total area (measured at right angles to axis of lattice bar) of lattice bars cut by cross-section of chord. † Area of rivets connecting all lattice bars cut by a cross-section chord to outside web, one end only. ‡ Axis normal to latticing; not necessarily the least radius of gyration.

The Lower Chord, L13-L14 of the new Quebec bridge, Fig. 140, adjacent to the main pier, has a cross-sectional area of 1902 sq in, being about 42 ft 11 in long and 10 ft 3¼ in wide. The depth of the lower chord diminishes in accordance with a regular taper of 1/32 in in 12 in, from about 7 ft 2 in at the main shoe to 4 ft 1¾ in at the end of the anchor arm. The chord section has four vertical ribs each consisting (L13–L14) of 4 web plates with an aggregate thickness of 3¾ in; 4 flange L's, 8 in×8 in×1 in; 2 cover plates, 20 in×1⅛ in; 2 cover plates, 20 in×13/16 in. Each outer pair of ribs is connected at top and bottom flanges by 8½ in×1 in lattice bars, and at mid-height by a horizontal diaphragm consisting of 1 web plate, 33 in×11/16 in, and 4 flange L's, 8 in×8 in×⅝ in. The areas of these longitudinal diaphragms are included in the cross-section of the member. The webs are also strengthened transversely by cross (vertical) diaphragms about 15 ft apart, having 10 in×16 in manholes. The 8½ in×1 in lattice bars have 2 rows of 3 rivets each in each end and 4 rivets at the intersection. The inner pairs of ribs are tied together by vertical and mid horizontal diaphragms and by tie plates in the planes of the flanges. The weight of L13–L14 is about 400 short tons. In order to facilitate shipment and erection this member was

spliced at the center and each of the two sections divided longitudinally. The Main Vertical Post is 310 ft between centers of end pins. Its unsupported length is 145 ft. It is composed of four separate columns latticed together, its outside dimensions being about 9 ft by 10 ft. Its cross-sectional area is 1903 sq in; its weight 1500 short tons and it was shipped in 26 pieces.

The New Memphis Bridge compression chords also have four webs 42 in deep reinforced with side plates, and 8 flange ∟'s 8 in ×8 in. The maximum cross-sectional area is 445 sq in.

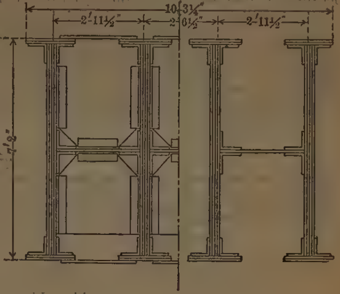


Fig. 140

The Length of the Cantilever Arm for a given location may be varied considerably, but the location of the abutments and piers is generally limited by local conditions. In such a bridge as the Monongahela, Fig. 135, the central span between piers is determined by the width of the waterway, and the abutments are frequently located by other similar requirements. If the abutments are fixt, the position of the piers should be such as to make the total cost of the bridge a minimum, which will practically be when the material in the trusses is a minimum. On account of the varying live and dead loads, different shapes of trusses and different unit stresses, the relation between the length of anchor arm and the entire length of bridge between abutments can be exprest only approximately. With the general arrangement shown in Fig. 135, the anchor arm is usually about 0.20 L, the cantilever arm about 0.17 L and the suspended span about 0.26 L, where L is the total length between abutments. These values are average values only, and many cantilever structures have been built in which the ratios of lengths do not agree with the fractions given.

Anchorages. Most cantilevers with a central suspended span require anchoring at each shore end of the bridge to produce a downward reaction on the anchor-arm truss. These anchorages usually consist of a series of eye-bars attached to the end pin of the truss and extending down into the masonry where they are connected to girders. The anchorage for each end of each anchor-arm truss of the first Quebec bridge consisted of sixteen 10 in by $2^1/_{16}$ in eye-bars connected by pins to a series of eight plate girders each 6 ft deep and 17 ft 6 in long. Above these girders was another layer of girders 8 ft 6 in deep and 36 ft long and above this a layer consisting of twelve 15-in I beams 22 ft long. The eye-bars and the grillage of beams and girders were embedded in the masonry and grouted. In several of the large cantilevers the anchorages are placed in inspection galleries.

39. Reactions for Cantilevers

Reactions may be determined by ordinary methods when the structure is statically determinate as regards the outer forces, and by deflections when statically indeterminate. A cantilever bridge having the points of application of all reactions known and having all bearing points but one arranged so that they can exert only vertical reactions on the truss is STATICALLY DETERMINATE as regards outer forces when the number of equations given

by the manner of construction is equal to $n-2$ where n is number of supports. Thus the reactions on truss in Fig. 141 may be found by statics because the number of equations given by the method of construction, that is, by the insertion of a hinge at C and D, is equal to $n-2=2$. Similarly the truss in Fig. 142 is statically determinate because the two hinges, P and T, and the omission of diagonals in panel LM and the corresponding panel on right of center give the $n-2=4$ equations required. Each hinge makes the bending moment at the section where it is inserted equal to zero. The dotted members in Fig. 141 and Fig. 142 receive stress only during erection.

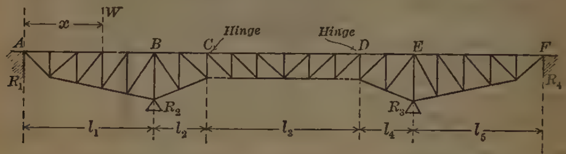


Fig. 141

Reactions for Fig. 141, having a central suspended span and four supports. For a load on the anchor arm AB, R_1 and R_2 are upward; for a load on the cantilever arm BC, R_1 is downward and R_2 upward; in either case R_3 and R_4 are each zero and R_1 and R_2 are as follows:

$$x \leq l_1+l_2 \qquad R_1=W(l_1-x)/l_1 \qquad R_2=Wx/l_1$$

The suspended span is a simple truss supported at C and D, and for a load on that span the pressure on cantilever arm BC at $C=W(l_1+l_2+l_3-x)/l_3$ and on cantilever arm DE at $D=W(x-l_1-l_2)/l_3$; in this case R_1 and R_4 are downward, R_2 and R_3 are upward, and their values are as follows:

for $x>l_1+l_2$ for $x<l_1+l_2+l_3$

$$R_1=-\frac{W(l_1+l_2+l_3-x)l_2}{l_3l_1} \qquad R_2=\frac{W(l_1+l_2+l_3-x)(l_1+l_2)}{l_3l_1}$$

$$R_3=\frac{W(x-l_1-l_2)(l_4+l_5)}{l_3l_5} \qquad R_4=-\frac{W(x-l_1-l_2)l_4}{l_3l_5}$$

With a load on the portion DF, Fig. 141, there are reactions at R_3 and R_4 only, and if x be taken as the distance from load to F, their values are the same as for R_2 and R_1 respectively when the load is on the portion AC.

With a UNIFORMLY DISTRIBUTED LIVE LOAD the maximum downward R_1 (Fig. 141) and maximum downward R_4 occur for live load covering the central span B to E; maximum upward R_1 and R_4, load spans l_1 and l_5 respectively; maximum R_2, load spans l_1, l_2 and l_3; maximum R_3, load spans l_3, l_4 and l_5.

Reactions for Fig. 142, having a central suspended span and six supports, only three of which are shown. Hinges at P and T make the bending moments at those points zero; and the omission of diagonals in panel LM makes the shear in that panel zero; when the top and bottom chords in panel LM are horizontal, and therefore $M_2=M_3$. If one of the chords in panel LM were inclined, the shear in that panel would be equal to vertical component of the inclined chord, and $M_2/h_2=M_3/h_3$, where M_2 and M_3, h_2 and h_3 are the bending moments and heights at supports 2 and 3 respectively. For a con-

centrated load on the anchor arm GL reactions exist at R_1 and R_2 only, and this arm is a simple truss supported at R_1 and R_2. With the load on the

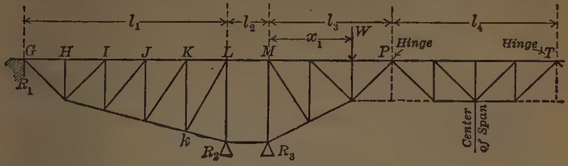


Fig. 142

cantilever arm MP distant x_1 from M there are reactions at R_1, R_2 and R_3; R_1 being downward, R_2 and R_3 upward, and their values are

$$x_1 \leq l_3 \qquad R_1 = -\frac{Wx_1}{l_1} \qquad R_2 = \frac{Wx_1}{l_1} \qquad R_3 = W$$

For a load on the suspended span PT, reactions exist at the six supports, the end ones (at the two abutments) being downward and the four central ones (at the piers) being upward.

40. Stresses in Cantilevers

Stresses in Truss Members may be computed directly or found with aid of influence lines. Let it be required to compute maximum tension, P_{L-M}, in top chord member LM in Fig. 142, for a dead panel load of 10, all on top chord, and a live panel load of 20 thousands of pounds. Let the panel length be 20 ft; depth of truss at L and M 40 ft, and at H 20 ft. Pass a vertical section thru LM, consider forces on right of the section and write moment equation about the origin of moments, namely R_3. The live load should be placed on all top chord panel points between M and T. The moment equation for dead loads is as follows:

$$10(20+40)+2.5\times10\times60-40P_{L-M}=0,$$

and P_{L-M} for dead loads is 52.5 tension. Live $P_{L-M} = 2\times52.5 = 105.0$ thousands of pounds tension.

Maximum Stresses in Top Chord Member IJ (Fig. 142). Pass vertical section thru IJ, consider left side of section and take origin of moments at lower chord joint 25 ft under I. Effective dead load $R_1 = -10(20+40)/100-2.5(10)\,60/100+2(10) = -1.0$, the minus sign denoting a downward reaction. Moment equation for dead loads about origin of moments is

$$25P_{I-J}-10\times20-1\times40=0$$

from which P_{I-J} for dead loads is 9.6 tension. For live tension, load all panel points between M and T, and for live compression load joints H, I, J, K.

$$\text{Live tension, } P_{I-J} = \frac{[20(20+40)+20\times2.5\times60]40}{100\times25}=67.2$$

$$\text{Live compression, } P_{I-J}=\frac{40\times40-20\times20}{25}=48.0$$

Combining the above live and dead stresses: maximum tension in $IJ=76.8$ and maximum compression $=38.4$ thousands of pounds.

Maximum Stresses in Vertical Kk (Fig. 142). Origin of moments 60 ft to left of R_1. Effective dead load R_1 is 1.0 downward, as previously found, hence dead load causes compression in Kk. For live load in any position on the bridge there is compression in Kk. Dividing the moment above the origin by the lever arm of the bar 140 ft,

$$\text{Dead compression, } P_{K-k} = \frac{1 \times 60 + 10(80+100+120+140)}{140} = 31.86$$

Live compression $= 2 \times 31.86 = 63.72$, and the maximum compression is 95.58. There can be no tension in Kk.

The Maximum Uplift at Abutment G (Fig. 142) occurs when the live load covers the central span from M to T. Under dead loads alone the effective reaction R_1 is 1.0 downward, but the total dead R_1 is really upward and is $5.0 - 1.0 = 4.0$ if the dead load on the joint G be 5.0. The maximum live uplift, that is, the maximum downward live R_1, is $[20(20+40) + 20 \times 2.5 \times 60/100 = 42.0]$, hence the true maximum uplift is $42.0 - 4.0 = 38.0$.

41. Influence Lines for Cantilevers

Definition of Influence Line. If a load of unity be allowed to pass over a structure, and at each position of the load there is plotted the shear, moment,

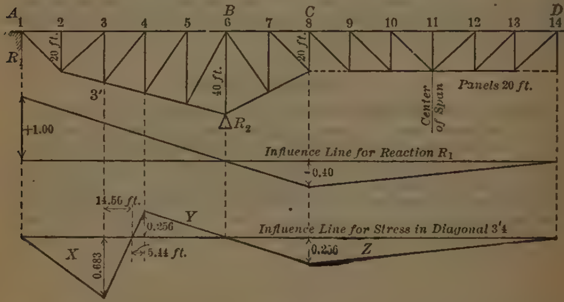


Fig. 143

reaction or stress which exists at some fixt section of the structure, the resulting curve is called an influence line. These lines are useful in determining what portion of a structure should be loaded with a live load to produce the maximum or the minimum effect, and in determining the amount of the effect. For example, Fig. 143 shows the influence line for reaction R_1 and the influence line for stress in diagonal $3'$-4.

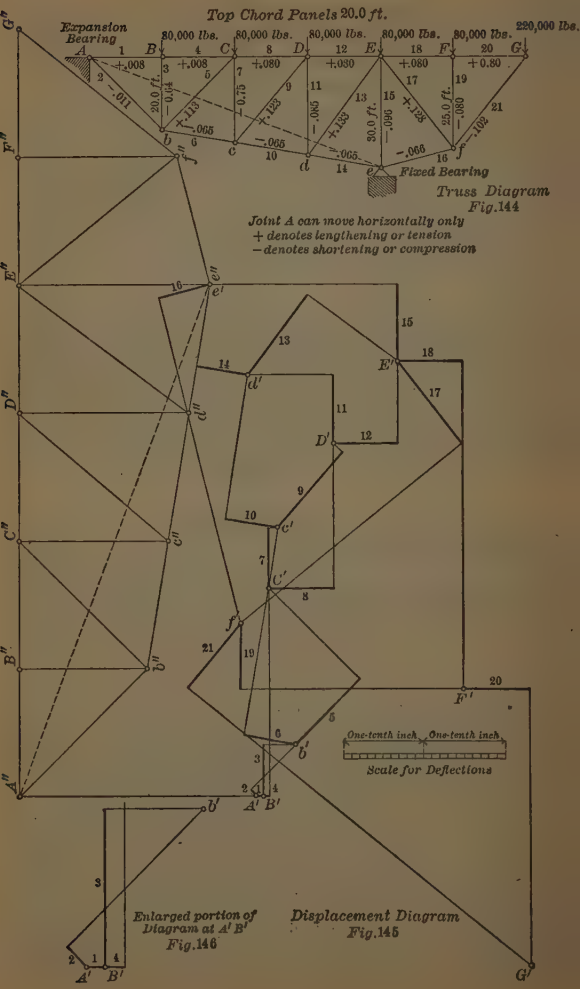
Influence Line for Reaction R_1 (Fig. 143). Let the load of unity approach from the right. There is no reaction at R_1 till the load is to the left of hinge D, and as the load moves from D to C, R_1 gradually increases till the load gets to C, when its value is $-1 \times 40/100 = -0.40$. As the load moves from C to A, R_1 gradually changes from a downward force of 0.40 to an upward one of 1.00, passing thru zero at B. Upward reactions are called positiv and are plotted above the horizontal axis in the diagram. With a uniform dead load the value of R_1 is found from the influence line by multiplying the load per foot by the algebraic sum of the areas of the positiv and negativ triangles, and with a uniform live load the maximum upward or downward live R_1 is found multiplying the live load per foot by the area of the positiv and negativ triangles respectively.

Influence Line for Stress in Diagonal 3′-4 (Fig. 143). As the load of unity approaches from the right the stress in the diagonal is zero till the load passes D, when the stress gradually increases till the load is at C. With load at C the stress is most easily found by considering forces to the left of a vertical section thru the diagonal 3′-4 and taking moments about the origin, which is 60 ft to the left of R_1. This stress is $0.40 \times 60 \times 32.02/120 \times 25 = 0.256$ tension, and is plotted at the load, namely at C. As the load moves from C to joint 4 the stress changes uniformly from tension to compression, and when it is at 4 the compression in the diagonal is 0.256 which is plotted at the load at 4. When the load crosses the panel 4-3 the stress changes from compression to tension and during the remainder of the movement to A there is tension in the bar 3′-4 as shown. Given a dead and a live load of 1000 and 2000 lb per ft per truss respectively, let it be required to find the maximum tension, P'_{3-4}, in bar 3′-4. The area of triangle X, Fig 143, is $54.55 \times 0.683/2 = 18.63$, of Y 5.82 and of Z 20.48. The units used are the foot and the pound. For tension in 3′-4 the live load must cover the distances represented by the bases of the triangles X and Z, hence live $P_{3'-4} = 2000(18.63 + 20.48) = 78\,220$ lb tension; dead $P_{3'-4} = 1000(18\,63 + 20.48 - 5.82) = 33\,290$ lb tension; total tension, $P_{3'-4} = 111\,510$ lb.

Concentrated Loadings are sometimes used for trusses of long spans, but generally a uniform load and one or two excesses are employed instead, each excess representing the additional weight of the locomotive over the uniform train load. With the influence line the effect of an excess load is easily found by placing the excess at the largest ordinate and multiplying the excess load by the ordinate in question. For example, with an excess load of 20 000 lb at joint 3 (Fig. 143), the effect on diagonal 3′-4 is to produce a tension therein of $0.683 \times 20\,000 = 13\,660$ lb. With a system of concentrated loads the stress in a bar is obtained by placing the loads in the position to produce the kind of stress desired and then summing up the products of each load by the ordinate at the point where the load lies. In Fig. 143 the maximum negativ R_1 occurs for the live loads on portion BD, and the position of the loads to give the greatest result is the same position that will cause the maximum moment at C on a span of length BD.

42. Deflection of Cantilevers

Displacement or Williot Diagram (Figs. 144, 145, 146). This is a graphical determination of the exact displacement of all points of a truss for a given loading. It is applicable to all cases where the changes in lengths of the truss members can be found. There are three steps in the solution as illustrated by the determination of deflections of all joints of truss in Fig. 144, which shows the diagram of truss drawn to scale, the joints lettered, the bars numbered in order in which their deformations are laid off in Fig. 145 and the changes in lengths of the various bars due to the loading shown, these changes due to stress being found by dividing the actual unit stress for each bar by the modulus of elasticity. The second step is to draw the displacement diagram by beginning at any joint and assuming this joint fixt in position and one of the bars at the joint fixt in direction. Usually start with a bar which appears to have a small movement. On lines parallel to the bars in the truss diagram lay off the changes in length for the bars and erect perpendiculars to these deformations. Thus, in Fig. 145, starting with joint A as fixt in position and bar AB fixt in direction lay off to scale and to the right from A' the change +.008 in for the bar AB, thus locating $A'B'$; then from A' lay off −.011 in for bar Ab, marked 2 in Fig. 145, and erect a perpendicular to this line, and similarly with bar Bb, or number 3, thus locating b'. Continuing in this way the displacement diagram is finally ended by locating G'. Care must be taken to lay off the deformations in proper directions; thus, bar Ab is in compression and its deformation is laid upward and to the left from A'. This diagram has been drawn on the assumption that A is fixt, whereas e is really fixt and A can and does move horizontally. The third step then is to rotate the truss about e, the real fixt joint, till all bars in the truss in Fig. 145 are at



Figs. 144, 145. 146. Displacements for a Cantilever

right angles to the corresponding bars in Fig. 144. This is here done by moving A' horizontally till $A''e'$ is at right angles to Ae, then completing truss $A''G''e''$. The true displacement of any joint is shown in Fig. 145, for example, joint A has moved to the right a distance $A''A'$; E has moved to the right and downward from E'' to E'. Fig. 146 shows a part of Fig. 145 on an enlarged scale, and is here made necessary on account of the reduction in printing.

The displacement diagram not only shows the final resultant motion of each joint but it also shows the horizontal and vertical motion as well. Thus, while joint A has moved only horizontally to the right a distance equal to $A''\ A'$, joint B has moved vertically a distance equal to the vertical projection of $B''B'$ and horizontally a distance equal to the horizontal projection of $B''B'$.

Two scales are necessary, one of distances for the truss diagram in Fig. 144 and the other for the deflections in Fig. 145.

ARCH AND SUSPENSION BRIDGES

43. Types of Arches

Classification. An arch is a structure which under any and all loads produces inclined reactions at the supports. Arches are classified as to the number of hinges used in one rib, there being three-hinged, two-hinged, one-hinged and hingeless arches. When THREE HINGES are used there is one at each support and one at the crown; when only TWO HINGES are used, one is placed at each support; and if an arch has only ONE HINGE it is located at the crown. HINGELESS ARCHES are also called fixt or continuous. Structures with only one hinge are seldom built, but the other types are common. As to the arrangement of the ribs arches are also classified into two types, namely, those having solid webs extending between and connected to the flanges and those in which the webs are open and consisting of web members connecting the upper and lower chords in the same manner as in ordinary trusses. In ribs having SOLID WEBS both flanges are usually (tho not always) curved, thus making the rib a curved plate girder. Ribs with OPEN WEBS may be either spandrel-braced, in which case the upper chord is horizontal and the lower is curved, or they may have the two chords curved, either parallel or lune-shaped, and be connected by the web members, thus forming what are sometimes called trussed-arch ribs There are, then, SPANDREL-BRACED, TRUSSED-ARCH, and SOLID RIBS, and any one of these may be hinged or fixt.

Historical and Descriptive. The EARLIEST METALLIC ARCH of note in the United States is the cast iron hingeless arch bridge carrying Chestnut St over the Schuylkill River in Philadelphia This bridge was completed in 1866, and has spans of 185 ft, and the ribs, bracing and floor plates are all of cast-iron. The Eads bridge in St Louis having a span of 519 ft 9⅜ in and the Kaiser Wilhelm bridge over the Wupper River in Germany with a span of approximately 558 ft are the longest hingeless arches in the United States and Europe respectively. LONGEST THREE-HINGED ARCH bridge in the United States is (1918) the 592-ft span at Topock, Ariz. LONGEST ARCH SPAN in the world is the two-hinged railroad structure at Hell Gate, N. Y. with a span of 977 ft 6 in. Longest three-hinged arches for supporting roofs were used in the Manufactures and Liberal Arts Building, Columbian Exposition, Chicago, 1893, the span being 368 feet.

44. Two-Hinged Arches

Notation. Δx and Δy, horizontal and vertical movements respectively of origin of coordinates of a curved beam: $\Delta\phi$, angular movement of axis of curved beam at origin of coordinates. M, bending moment of vertical forces at point on arch axis (gravity axis),

Important American Arch Bridges

From Merriman and Jacoby's Roofs and Bridges, Part IV.

Span ft	Span in	Nature of crossing	Location	R.R. or Hy.	Number of hinges	Date of completion
977	6	East River	Hell Gate, N. Y	R.R.	2	1917
840	0	Niagara River	Niagara Falls, N. Y.	Hy.	2	1898
592	0	Colorado River	Topock, Ariz.	Hy.	3	1916
591	0	Cuyahoga River	Cleveland, O.	Hy.	3	1917
550	0	Niagara River	Niagara Falls, N. Y.	R.R.	2	1897
540	0	Connecticut River	North Walpole, N. H.	Hy.	3	1905
519	9⅜	Mississippi River	St. Louis, Mo.	R.R. Hy.	0	1874
508	9⅝	Harlem River	Washington Bridge, N. Y.	Hy.	2	1889
501	8⅛	Mississippi River	St. Louis, Mo	R.R. Hy.	0	1874
456	0	Mississippi River	Minneapolis, Minn.	Hy.	3	1889
448	8¼	Rio Grande River	Costa Rica	R.R.	2	1902
440	0	Pittsburgh Jn. R.R	Oakland, Pittsburgh. Pa.	Hy.	0	1907
428	0	Genesee River	Driving Park Ave, Rochester	Hy.	3	1890
400	0	Whitewater River	Richmond, Indiana	Hy.	3	1886
360	0	Magdalena River	Honda, U. S. of Colombia	R.R.	3	1884
360	0	Gulley and 3 streets	Hawk St, Albany, N. Y.	Hy.	3	1890
360	0	Panther Hollow	Schenly Park, Pittsburgh	Hy.	3	1898
355	0	Oak Orchard Creek	Main St, Waterport, N. Y.	Hy.	2	1900
340	0	Fraser River	Lillooet, B. C.	Hy.	3	1888
336	0	Stony Creek	Near Bear Creek Sta., B. C.	R.R.	3	1893
327	0	Papolopen Creek	Palisade Interstate Park, N. Y.	Hy.	2	1916
290	4½	Surprise Creek	Near Bear Creek Sta., B. C.	R.R.	3	1897
270	0	Salmon River	Near Keefers Sta, B. C.	R.R.	3	1893
258	0	Mississippi River	Hennepin Ave., Minneapolis	Hy.	3	1888
258	0	Mississippi River	Hennepin Ave., Minneapolis	Hy.	2	1891
240	5	Spokane River	Post St, Spokane, Wash.	Hy.	3	1893
240	0	Spokane River	Post St., Spokane, Wash.	R.R.	3	1903
240	0	Canyon	Near Skagway, Alaska	R.R.	3	1901
234	0	Six Mile Creek	Stewart Ave., Ithaca, N. Y.	Hy.	3	1896
216	6	Salmon River	Pulaski, N. Y.	Hy.	3	1888
210	6⅞	Mahoning River	Youngstown, Ohio	Hy.	2	1899
207	0	Menominee River	Near Iron Mt., Mich.	R.R.	3	1903
200	0	Rock Creek	Penna. Ave., Washington	Hy.	0	1858
200	0	Croton River	New Croton Dam, N. Y.	Hy.	0	1905
200	0	Turkey River	Claremont, Iowa	Hy.	3	1881
200	0	Schuylkill River	Fairmont Park, Phila.	Hy.	3	1898

x and y, coordinates of a point on arch axis. ds, elementary length of arch axis. E, modulus of elasticity of material. I, moment of inertia of cross-section of rib. I_0, moment of inertia of cross-section of rib at crown. c, coefficient of linear expansion. t, change in temperature in degrees Fahr. from normal. S, average value of compressive unit stress. L, length. A, area of cross-section of bar. U, stress in bar due to load of unity acting horizontally at one hinge of braced arch. P, stress in bar of braced arch due to vertical loads and vertical reactions.

Arches with Two Hinges have less deflection at the crown due to changes in temperature, and are more rigid than those having three hinges, but the temperature stresses are large and any horizontal movement of one support produces stress thruout the arch. The two hinges are placed at the supports, and the arch may be spandrel-braced or may have two or more

ribs with solid or open webs. The two arches over Niagara River are typical structures, the Grand Trunk Railway bridge being a deck spandrel-braced structure with span of 550 ft carrying two railroad tracks on the upper deck and a highway immediately under this deck. The Niagara and Clifton bridge is near the Grand Trunk bridge and carries a highway on the upper deck, the highway being supported on vertical columns which are carried by the curved trussed ribs.

Deflection of a Curved Beam. If beam OA (Fig. 147) is fixt at A and is free at O, the horizontal, vertical and angular movements of the axis of the beam at the origin of coordinates O are

$$\Delta x = \int_0^A \frac{Myd_s}{EI} \qquad \Delta y = \int_0^A \frac{Mxd_s}{EI} \qquad \Delta \phi = \int_0^A \frac{Md_s}{EI}$$

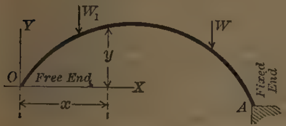


Fig. 147

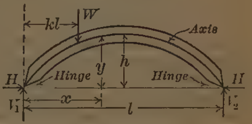


Fig. 148

Reactions on Arch with a Solid Web (Fig. 148). An arch with two hinges is statically indeterminate as regards the outer forces, one condition or equation being required in addition to those of statics. This additional equation may be obtained by the principle of least work or by deflections.

$$V_1 = W(1 - k) \qquad V_2 = Wk \qquad H = \int \frac{Myd_s}{EI} \Big/ \int \frac{y^2 d_s}{EI}$$

This equation for H can be integrated only when the equation of arch axis is known, in which case M, d_s and I must be expressed in terms of the coordinates x and y. The integration extends over entire length of rib. If the material and the cross-section of rib are uniform for whole length the value of H becomes $H = \int Myd_s \Big/ \int y^2 d_s$. In case the arch axis is not a regular curve so that the integration cannot be performed, the value of H for an arch of uniform material and cross-section may be determined approximately by dividing the axis into a number of equal parts and letting M be the bending moment due to vertical loads and reactions and y the ordinate at the center of each division, $H = \Sigma My/\Sigma y^2$, where ΣMy denotes the summation of the various products of M and y for the center of each division and Σy^2 the summation of the squares of the several ordinates to these centers. In all of the above equations for H the effect of shear and axial force is neglected and only the effect of the moment is considered.

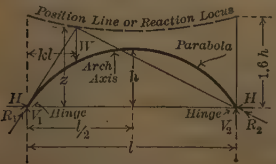


Fig. 149

Parabolic Arch Rib with Solid Web (Fig. 149). A parabolic arch rib is one which has a parabola for an axis, the vertex being at the crown. The formula for H gives its value for bending only and assumes a rib of uniform material but with a moment

of inertia varying from a minimum at the crown to a maximum at the skew-back hinges; the moment of inertia at any section equals that at the crown times the secant of the angle the arch axis makes with the horizontal at that section.

$$V_1 = W(1 - k) \qquad V_2 = Wk$$

$$H = \frac{5\,Wl}{8\,h}(k - 2\,k^3 + k^4) \qquad z = \frac{1.6\,h}{1 + k - k^2}$$

Effect of Temperature on Arch Ribs with Solid Webs. A rise in the temperature tends to increase the span length and a fall in temperature to decrease it. If the span cannot change, a horizontal reaction is exerted on the arch at each support, and temperature stresses are produced within the rib. For the general case, Fig. 148, the horizontal reaction is

$$H = EI_0 clt \Big/ \int_0^l y^2 dx$$

For a parabolic rib, Fig. 149, with solid web this equation for horizontal reaction becomes $H = 15\,EI_0 ct/8\,h^2$. The temperature range t is frequently taken as $\pm\ 75°$ Fahr. from the mean temperature of $50°$ Fahr. For a rise in temperature above normal, t is positiv and the reactions H act inwardly and for a fall in temperature below normal, t is negativ and the reactions H act outwardly. The bending moment at any section due to temperature is Hy, being positiv for a fall and negativ for a rise in temperature. I is here assumed to increase from the crown towards the abutments.

Rib Shortening in Arch Ribs with Solid Webs. The compression of the rib due to the thrust acting along the arch axis shortens the arch axis and produces outward horizontal reactions at the supports. The horizontal reaction is

For general case (Fig. 148) $\quad H = -\,SI_0 l \Big/ \int_0^l y^2 dx$

For the parabolic rib (Fig. 149) $H = -\,15\,SI_0/8\,h^2$.

Rib shortening is similar in effect to a fall in temperature. The moments are all positiv. For a rib having a small rise the effect of rib shortening should not be neglected.

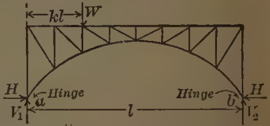


Fig. 150

Reactions on a Braced Arch due to Vertical Loads. If in Fig. 150 the arch were free to move horizontally at hinge b the horizontal movement of b due to the load W acting on the arch would be $\Sigma PUL/AE$. The horizontal movement of b due to a single horizontal load of unity acting at b would be $\Sigma U^2L/AE$ and

$$H = \sum \frac{PUL}{AE} \Big/ \sum \frac{U^2L}{AE} \qquad V_1 = W(1 - k) \qquad V_2 = Wk$$

In formula for H, P is the stress in each bar assuming the arch to act like a truss, that is, with vertical reactions V_1 and V_2 only. Compressive stresses should be given the minus and tensile stresses the plus sign.

45. Three-hinged Arches

Three-hinged Arches are used for highway and railroad bridges as well as for roofs. They may have open or solid webs, but for long-span roofs the

trussed-arch rib having curved chords is most suitable, while for bridges the spandrel-braced rib with a horizontal upper chord and a curved or bent lower chord is commonly used. A three-hinged arch is statically determinate as regards the outer forces, hence reactions are easily found, and the stresses in the members are only slightly affected by small settlements of the foundations or by changes in temperature. For an increase in TEMPERATURE the crown hinge rises and the horizontal component of the reaction is lessened, and for a decrease in temperature the crown hinge is lowered and the horizontal component is increased. As the vertical movement of the crown is small as compared with the total height of the arch rib, the effect on the horizontal thrust is also small and hence the influence of temperature on stresses is usually negligible.

Reactions for Vertical Loads as on bridges may be easily found algebraically, while for inclined loads, as wind loads on roof arches, graphical methods are better. For Fig. 151 the vertical components are

$$V_1 = Wx/l \qquad V_2 = W(l - x)/l$$

With a single load on right of center, $H = V_1 l/2h$, and with a single load on left of center $H = V_2 l/2h$, or

$$\text{when } x \leq l/2 \quad H = Wx/2h \qquad \text{when } x \geq l/2 \quad H = W(l - x)/2h$$

With uniform load of w per unit of length over right half span

$$V_1 = \tfrac{1}{8} wl \qquad V_2 = \tfrac{3}{8} wl \qquad H = wl^2/16h$$

With uniform load of w per unit of length over entire span

$$V_1 = V_2 = \tfrac{1}{2} wl \qquad H = wl^2/8h$$

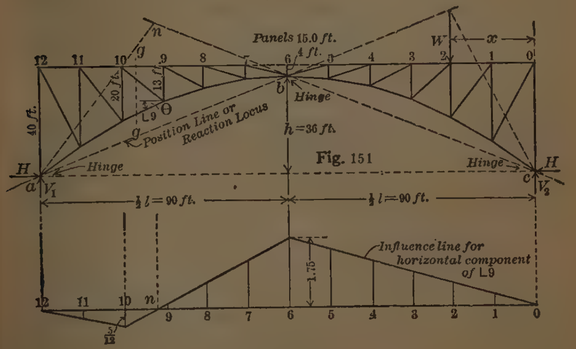


Fig. 152.

Graphical Determination of Reactions and Stresses in Roof Arches. Let it be required to find the reactions for wind loads shown in Fig. 153. Draw the force polygon 012345678 and assume pole O, then draw the equilibrium polygon, beginning by passing the first string $O0$ thru a and ending where the last string $O8$ cuts the position line bc, which is the line of action of R_2 for loading on the left. Draw Ok in the force polygon parallel to the

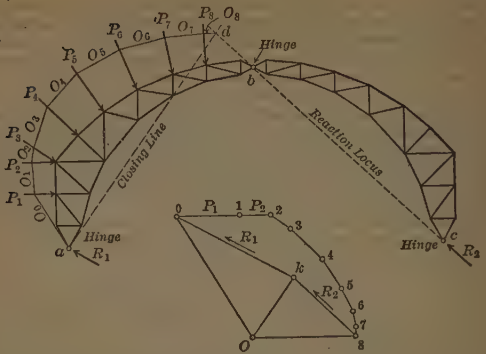


Fig. 153

closing line ad till it intersects $8k$ which is parallel to R_2. $8k$ and ko give the magnitude and direction of R_2 and R_1 respectively. With the reactions known, a stress polygon showing all stresses in both portions of the rib is drawn in the usual way by beginning at joint a. These stresses may be checked algebraically by method of moments, using the true equilibrium polygon; this polygon being started thru a, using the first force polygon with k as the pole, the reaction $R_1 = ko$ being drawn thru a till it intersects P_1, then the string $k1$ till it meets P_2, and so on. The true polygon finally passes thru the three hinges. By method of moments the stress in a bar is equal to moment at the origin of moments divided by the lever arm.

Computation of Reactions and Stresses for Spandrel-braced Arch. With a dead panel load of 20 and a live panel load of 40 thousands of pounds per truss, required the maximum stresses in lower chord L_9, Fig. 151. Passing a vertical section g-g thru L_9, taking origin of moments at 10 and drawing the line an thru 10, it is seen that for a load to right of n the bar L_9 is in compression, for a load at n L_9 is zero, and when the load is to left of n L_9 is in tension. For maximum compression, load joints 1 to 9 inclusive, and for maximum tension joints 10 and 11. Loads at 0 and 12 produce stresses in end-posts only. Consider first the dead load. For this loading the thrust at crown hinge is horizontal, and by taking moments about a for left half of rib is found to be $6 \times 20 \times 3 \times 15/36 = 150$. With origin at 10 and considering the part of rib between b and section g-g the dead load stress in $L_9 = \sec\theta\,[150 \times 4 + 20\ (15 + 30 + 45) + 10 \times 60]/20 = 165.53 =$ compression; where θ is angle L_9 makes with horizontal. For live load tension, loads on joints 10 and 11 cause $V_2 = 40(1+2)/12 = 10$ and $H = V290/36 = 25$, and considering the part of structure between c and g-g with origin at 10, the moment equation is $20\ L_9/\sec\ \theta - 10 \times 150 + 25 \times 40 = 0$; $L_9 = 27.59 =$ tension. The live load compression in L_9 is most easily found as follows. Since ratio of live to dead panel load is 2, the compression in L_9 for full live load only $= 2 \times 165.53 = 331.06$, and compression for live loads on joints 1 to 9 inclusive is full live load compression plus live load tension due to live loads on 10 and 11, or maximum live compression $L_9 = 331.06 + 27.59 = 358.65$. Results: maximum compression $= 524.18$ and minimum compression $= 137.94$.

Influence Line for horizontal component of stress in L_9 (Fig. 151) is shown in Fig. 152. This influence line consists of three straight lines the ordinates to which represent the horizontal components of stress in L_9 due to a load of unity on the portions 0-6, 6-10, and 10-12, respectively. It is only neces-

sary to compute the ordinate at 6 and 10. With load at 6 the moment at 10, Fig. 151, divided by 20 ft gives the horizontal component of $L_9 = 1.75$ compression and this is plotted at joint 6, Fig. 152; similarly with load of unity at 10 the ordinate at 10 is $5/12$ tension. The horizontal component of L_9 may now be found from the diagram either by multiplying the areas of the triangles by the load per foot, which gives exact results, or by multiplying the ordinates to the influence line at the panel points by the panel load giving approximate results that are slightly larger than the exact. For example, using dead and live panel loads of 20 and 40 respectively, the following results verify those obtained in the preceding paragraph, which are computed by the usual approximate methods. Live tension in $L_9 = [40(1 + \frac{1}{2})5/12] \sec \theta = 27.59$. Live compression in $L_9 = [40(\frac{1}{6} + \frac{2}{3} + 29/24) + 40(1 + 5/6 + 4/3 + 3/6 + 2/6 + 1/6)\ 1.75] \sec \theta = 358.65$.

The Position of Live Load for Maximum Stress in a given bar must be determined for each case. For example, the point n, Fig. 151, is the neutral point for bar L_9, and if a load be placed on the right of n as at joint 8, the left reaction acts thru a and thru the intersection of position line bc with the vertical at 8, and since the left reaction is the only force acting on left at section g-g, the moment at origin of moments 10 is negativ, hence L_9 must cause a positiv moment about 10 and is compression. Similarly for a load to left of n, P_9 is tension. With a system of CONCENTRATED LOADS the influence line is useful. Referring to Fig. 152, the part of the influence line from O to n is the same shape as an influence line for bending moments at joint 6 on a simple span of length O-n, hence the position of loads for maximum compression in L_9 is the same as the position of loads that will cause maximum moment at 6 on span O-n, and the maximum tension in L_9 is caused by the same position of loads as will cause maximum moment at 10 on span 12-n.

46. Fixt Arches

Notation. y_1 and y_2 = ordinates of points of application of reactions at left and right ends respectively, d_s = elementary length of arch axis, E = modulus of elasticity of material, I = moment of inertia of cross-section of rib, I_0 = moment of inertia of cross-section of rib at crown, M_L = moment at any point on left half of arch axis of all outer loads between that point and the crown, M_R = moment at any point on right half of arch axis of all loads between that point and the crown, m = number of divisions into which each half of arch axis is divided; H_c, V_c, M_c, respectively, are the thrust, shear and moment at crown, z = ordinate of the reaction locus, M_1 = moment at each support, c = coefficient of linear expansion, t = change in temperature in degrees Fahr. from normal, S = average compressive unit stress.

A Fixt or Continuous Arch has no hinges, and it is so rigidly held at its ends that the arch axis cannot change its direction at those points. Arches without hinges are more rigid than those having hinges, have greater temperature stresses and are more affected by settlements of the supports.

Reactions for Fixt Arches. With vertical load W as shown in Fig. 153 there are five unknown quantities V_1, V_2, H, y_1, y_2, to be determined to completely find the reactions. The following five equations give relations between these unknowns:

$$V_1 + V_2 - W = 0 \qquad Wkl - V_2 l + H(y_1 + y_2) = 0$$

$$\int_0^l \frac{Myd_s}{EI} = 0 \qquad \int_0^l \frac{Mxd_s}{EI} = 0 \qquad \int_0^l \frac{Md_s}{EI} = 0$$

The first two of the above equations are from statics and the last three from the conditions that the horizontal, vertical and angular movement of the

arch axis at the origin of coordinates A is each equal to zero. Moment at A is Hy_1, at B is Hy_2, and at any point D is Hu.

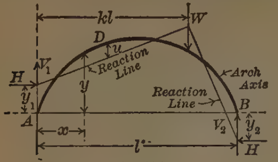


Fig. 153

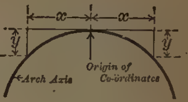


Fig. 154

The Thrust, Shear and Moment at Crown of a fixt arch (Fig. 154) having a shape such that the integration of the preceding paragraph cannot be performed, are given approximately by the following formulas, which assume the arch axis to be divided into short lengths such that the ratio of the length of each division, ds, to the moment of inertia, I, at center of division is a constant.

$$H_c = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R \Sigma y - \Sigma M_L \Sigma y}{2[m\Sigma y^2 - (\Sigma y)^2]}$$

$$V_c = \frac{\Sigma M_L x - \Sigma M_R x}{2\Sigma x^2} \qquad M_c = \frac{\Sigma M_R + \Sigma M_L - 2H_c \Sigma y}{2m}$$

In these equations numerical values of M_R, M_L, x, and y are positiv; the summations cover one-half the length of the span only; a positiv value of M_c denotes a positiv moment at the crown and a negativ value denotes a negative moment there; for a positiv value of V_c the line of pressure at the crown slopes upward towards the left and for a negativ value upwards towards the right.

Reactions for a Parabolic Arch Rib having fixt ends and a variable moment of inertia of cross section, Fig. 155. The modulus of elasticity is

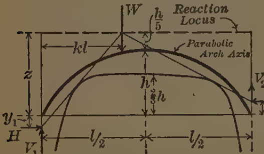


Fig. 155

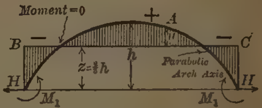


Fig. 156

assumed constant and the moment of inertia assumed to vary from the crown to the abutments; at any section being equal to the moment of inertia at the crown times the secant of angle of inclination of arch axis at that section,

$$V_1 = W(1+2k)(1-k)^2 \qquad V_2 = W(3k^2 - 2k^3)$$

$$H = \frac{15\,Plk^2}{4h}(1-k)^2 \qquad y_1 = \frac{10k-4}{15k}h$$

$$z = \frac{6h}{5} \qquad y_2 = \frac{6-10k}{15(1-k)}h$$

The lines of reactions are tangent to a curve which consists of parts of two hyperbolas.

Temperature Stresses in a Parabolic Rib, Fig. 156.

$$H = \pm\frac{45EI_0ct}{4h^2} \qquad z = \frac{2h}{3} \qquad M_1 = \pm\frac{2Hh}{3}$$

For a fall in temperature the moment at each support is negativ and the reaction H acts outward as shown in Fig. 156; moment at any point such as A is Hu, and is negativ at all points below line BC and positiv at all points above BC. For a rise in temperature the reaction H and the moments are opposite to those in Fig. 156.

Rib Shortening for a Parabolic Rib causes outward horizontal reactions H and bending moments like those for a fall in temperature.

$$H = -45SI_0/4h^2 \qquad z = \tfrac{2}{3}h \qquad M_1 = -\tfrac{2}{3}Hh$$

47. Suspension Systems

Historical and Descriptive. A suspension bridge consists of a floor suspended from cables or chains which are supported by passing over towers and by being anchored to the earth at their ends. In the early suspension bridges chains were used, but they were soon superseded by wire cables. To prevent undue oscillation or deflection the cables are provided with stays or stiffening trusses or else are trussed. When the cables are trussed they are usually made of eye-bars, and the system then becomes an inverted arch suspended between the tops of the towers and having the inward pull from the main

American Suspension Bridges

From Merriman and Jacoby's Roofs and Bridges, Part IV.

Span ft	Span in	Crossing	Location	Number of cables	Date of completion
1600	0	East River	New York (Williamsburgh Bridge).	4	1904
1595	[illegible]	East River	New York (Brooklyn Bridge)	4	1883
1470	0	East River	New York (Manhattan Bridge)	4	19—
1057	[illegible]	Ohio River	Cincinnati, Ohio	4	1867*
1030	[illegible]	Ojuela River	Mampimi, Mexico	2	1900
1010	[illegible]	Ohio River	Wheeling, W. Va.	4	1849†
940	[illegible]	Cauca River	Occidente, Antioquia, Colombia	4	1894
800	[illegible]	Monongahela River.	Pittsburgh, Pa. (Point Bridge)	2	1877‡
800	[illegible]	Ohio River	Rochester, Pa.	2	1897
800	[illegible]	Niagara River	Lewiston, N. Y.; Queenston, Ont.	2	1899
705	0	Ohio River	East Liverpool, Ohio	2	1896
700	[illegible]	Ohio River	Steubenville, Ohio	2	1905
640	0	St. John River	St. John, N. B.	1[illegible]	1852‖
608	0	Monongahela River.	Morgantown, W. Va.	2	1855
510	[illegible]	New River	Capertown, W. Va.	2	1903
500	5	Allegheny River	Oil City, Pa.	2	1876§
470	0	Brazos River	Waco, Texas	2	1870
470	[illegible]	Allegheny River	Warren, Pa.	2	1871
450	[illegible]	Guyan River	Guyandotte, W. Va.	4	1848
400	0	Railroad tracks	Grand Avenue, St. Louis, Mo.	2	1891
400	[illegible]	Kennebec River	Waterville, Maine	2	1903

* Rebuilt 1895–98, two new cables being added above the two old ones. † Rebuilt 1854 and 1862. Before the reconstruction in 1862 the bridge had twelve cables arranged in two groups of six each, placed side by side. ‡ Rebuilt 1904. ‖ Rebuilt 1857. § Rebuilt 1884 and 1905.

arch span resisted by an outward pull from the back stays extending from the top of each tower to the anchorage in the earth. The FIRST SUSPENSION BRIDGE of note in the United States was built in 1801 by James Finley at Greensburg, Pa., and had a span of 70 ft carried by chains. The Chain Bridge near Newburyport, Mass., was built in 1810 by John Templeman on Finley's design, was strengthened in 1900 and again in 1910. The FIRST WIRE SUSPENSION BRIDGE in America was a foot bridge of 408 ft span built in 1816 over the Schuylkill River; in Europe the first one was built in 1834 at Freiburg. In 1855 John A. Roebling completed the Niagara suspension bridge, which had a span of 821 ft 4 in and carried a railroad on the upper and a highway on the lower deck; wooden stiffening trusses 16 ft deep were used. In 1880 iron trusses replaced the wooden ones in this structure, in 1886 wrought-iron towers replaced the stone towers, and in 1897 the structure was replaced by a two-hinged spandrel-braced arch. The largest suspension bridges in the world are in New York City.

48. Cables

Notation. w_1 = intensity of load at point having coordinates x and y, H = horizontal component of tension in cable, T = tension in cable at any point, w = uniform load per horizontal unit of length, l = horizontal length of span of cable, h = sag or vertical distance from lowest point of cable to its support at top of tower, l_1 = length of cable between towers.

Cables for small bridges are made of wire rope, and for large structures they are made by compacting a number of steel wires into cylindrical form and clamping them in this shape by means of steel clamps placed above the panel points of the stiffening trusses thus serving as saddles for the suspenders. Each of the four main cables of the Manhattan bridge in New York is composed of 9472 parallel wires arranged in 37 strands, each wire being 0.192 inch in diameter before and not over 0.197 inch in diameter after galvanizing. These cables are approximately 20¾ inches in diameter, and are the largest yet built (1918), and are wrapt with galvanized-iron wire and painted. The Williamsburgh bridge in New York has four main cables, each 18¾ inches in diameter and each composed of 7696 wires arranged in 37 strands clamped together and covered between clamps with cotton duck soaked in oxidized linseed oil and varnish gum composition and with a sheet-iron shell. Cables are cradled when the horizontal distance between the two cables at a pier is greater than that at the center of the span.

Linear Arch. A linear arch for a given set of forces is the true equilibrium polygon for those forces, and hence it may be defined as an arch in which there is only direct tension or compression, and is incapable of carrying bending moment. The general equation of the linear arch is $d^2y/dx^2 = w_1/H$, where the left side of the equation denotes the second differential. A cable of uniform cross-section and material suspended at its ends and subjected only to its own weight assumes the form of a catenary.

Cable Carrying a Uniform Load. Fig. 157 shows one-half of such a cable, and for this loading the curve of the cable becomes a parabola with axis vertical and vertex at lowest point of curve, C, and its equation is

$$y = 4\,hx^2/l^2 \quad \text{or} \quad y = wx^2/2\,H$$

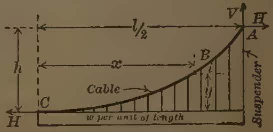


Fig. 157

The TENSION IN THE CABLE at the lowest point C is the horizontal component

H, this component being constant thruout the length of the span, and is $H = wl^2/8\,h$. Tension at any point such as B is given by the following:

$$T_B = \sqrt{H^2 + w^2x^2}\ \frac{w}{8\,h}\sqrt{l^4 + 64\,h^2x^2}$$

Maximum tension exists at the tower A and its value is

$$T_A = \frac{w}{8h}\sqrt{l^4 + 16\,h^2l^2}$$

Tangent of angle of slope ($\tan i$) at any point B and at the upper A is $8\,hx/l^2$ and $4\,h/l$ respectively. Length of cable between supports, that is, twice the length of curve AC, Fig. 157, is

$$l_1 = 2\int_0^{\frac{l}{2}}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = l\left(1 + \frac{8}{3}\,\frac{h^2}{l^2} - \frac{32\,h^4}{5\,l^4} + \cdots\right)$$

The **Sag Ratio or Dip Ratio** is the ratio of the sag of the cable, h in Fig. **157**, to the span l, and this ratio varies from about 1/8 to 1/15. The term versine is also used to denote the sag h, but this is not a proper use since the curve is not a circle. The greater the sag the smaller the tension in the cables but the larger the towers. Sag ratio for Williamsburgh bridge is 1/9, for Brooklyn bridge, about 1/12.5.

Steel Wires for Cables, Suspenders and Hand Ropes of Manhattan bridge were required to be made in an open-hearth furnace lined with silica. Maximum percentages for chemical requirements: C, 0.85%; Mn, 0.55%; Si, 0.20%; P, 0.04%; S, 0.035%; Cu, 0.02%. Strength: 215 000 and 200 000 lb per sq in before and after galvanizing respectively; minimum elongation in 12 in, 2% and 4% before and after galvanizing. For the Williamsburgh bridge the following were the maximum chemical requirements: P, 0.04%; S, 0.03%; Mn, 0.05%; Si, 0.1%; Cu, 0.02%. Minimum ultimate strength, 200 000 lb per sq in; elongation of 2½% in 5 ft and of 5% in 8 inches. The wire reached an ultimate strength of 225 000 lb per sq in.

49. Stiffening Trusses

Notation. w = uniform live load per unit of length, w_1 and w_2 = live loads per unit of length borne by the cable and stiffening truss respectively, h = sag of cable, d = deflection at center of span, A = cross-sectional area of cable, E = modulus of elasticity of material in cable and truss, I = moment of inertia of truss, V = shear on truss, M = bending moment on truss.

Stiffening Trusses are used to distribute a concentrated load or a partial uniform load over the whole or a part of the length of the cable and they are usually steel riveted trusses, tho in small bridges are sometimes made of wood. Where riveted trusses are used they are of the Warren type with one, two or three web systems. For highway bridges, where permissible deflections are large and the loads light, the stiffening trusses are light, while for railroad bridges, which are subjected to heavy and unequal loads and where the allowable deflection is small, the stiffening system must be heavy. The trusses are usually suspended from the cables by hangers or suspenders, which are wire rope or rods attached to the trusses at their panel points. These trusses may be continuous from anchorage to anchorage, may extend unbroken from tower to tower only or may extend from tower to tower with a hinge at the center and at each tower.

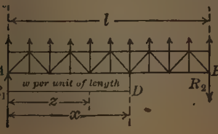


Fig. 158

Stiffening Trusses without a Center Hinge. Under a uniformly distributed load covering the entire span there are no moments and shears in the truss. With a uniformly distributed live load extending from the left tower, A (Fig.

158) a distance x, the following approximate functions of this load exist. Upward pull on truss per unit of length is wx/l.

$$R_1 = \tfrac{1}{2} wx(l-x)/l, \qquad R_2 = -\tfrac{1}{2} wx(l-x)/l$$

The shear at any section distant z from A is

$$z < x, \qquad V = \frac{w}{2l}(xl - x^2 - 2zl + 2zx)$$

$$z > x, \qquad V = \frac{w}{2l}(-xl - x^2 + 2zx)$$

When $z = 0$, x, or l, $V = wx(l-x)/2l$, being positiv at the towers A and B and negativ at D. The GREATEST SHEAR possible occurs when the half span is loaded, that is, when $x = l/2$ and maximum $V = \frac{1}{8} wl$. This maximum shear occurs at the towers and at the center of span. The moment at any section distant z from A, Fig. 158, is

$$z < x, \qquad M = \frac{wz}{2l}(xl - x^2 - zl + xz)$$

this moment being positiv with a maximum $wx^2(l-x)/8l$ when $z = x/2$;

$$z > x, \qquad M = \frac{wx}{2l}(xl - zl - xz + z^2)$$

and these are negative moments with a maximum value of $M = -wx(x-l)^2/8$ when $z = (l+x)/2$. The GREATEST POSITIV MOMENT occurs at the center of the loaded part when the live load extends ⅔ across the span from A and is $M = wl^2/54$; the GREATEST NEGATIV MOMENT has this same value at the center of the unloaded part when the load covers the left third of the span.

More Accurate Methods for Determining Stresses in cables and stiffening trusses make use of deflections; for example, with a stiffening truss without a center hinge the deflections of the cable and of the truss at any vertical section such as the center are equal. The DEFLECTION OF THE CABLE at the center of a span is approximately

$$d = \frac{w_1 l^2 (3l^2 + 16h^2)}{128 h^2 AE}$$

And the deflection of the stiffening truss which is supported at the towers is

$$d = \frac{5}{384}\frac{w_2 l^4}{EI}$$

The ratio of the load borne by the cable to that borne by the truss is

$$\frac{w_1}{w_2} = \frac{5 l^2 h^2 A}{3I(3l^2 + 16h^2)}$$

The total load $w_1 + w_2$ is here assumed to cover the entire span.

Stiffening Trusses Having a Center Hinge (Fig. 159). Shear but no moment can be transmitted across the hinge. For a uniform live load of w per unit of length maximum $R_1 = wl/6$ and acts upward; maximum $R_2 = wl/8$ and acts downward. Maximum positiv and negativ moments are respectively $0.019\, wl^2$ and $0.016\, wl^2$. Maximum shear is $wl/6$ or equal to maximum R_1.

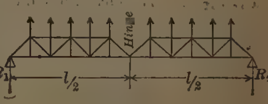


Fig. 159

Trussed or Stiffened Cables are also used to prevent excessive deformation of the cables, the stiffening trusses being omitted. The cables are made

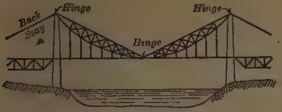


Fig. 160

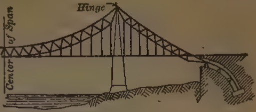


Fig. 161

of eye-bars or links, and serve as members of the trusses from which the roadway is suspended as in Figs. 160 and 161. These structures are inverted arches, the first being three-hinged and the second two-hinged.

50. Stays and Anchorages

A Back Stay is that portion of the main cable which extends from the anchorage to the top of the tower. It may be a continuation of the main cable, as is usual, or it may be a separate member, in which case the main cable extends from tower to tower only. That part of the roadway under the back stay is sometimes suspended from the back stay as in the Manhattan and Brooklyn bridges, or it may be carried on separate trusses as in the Point bridge in Pittsburgh, Pa., or on the stiffening trusses as in the Williamsburgh bridge. When the main cable passes over a movable saddle on the tower the maximum horizontal components of the stresses in the main cable and back stay are equal, and the maximum stresses are equal if the two parts are inclined at equal angles. The friction between the saddle and the tower produces a horizontal force on the tower equal to the vertical weight on the saddle times the coefficient of friction of the roller bearing.

Anchorages for the back stays are provided by attaching them to eye-bar chains and girders or castings which are set in masonry at each end of the bridge. The chain of eye-bars may be inclined in a straight line or in a series of broken lines, and they may be placed in an inspection tunnel as in the Williamsburgh bridge or may be completely embedded in concrete. The eye-bars of the anchor chains in the Point bridge in Pittsburgh were embedded in a poor quality of masonry, and when examined after 27 years of service were found to be in good condition except that they were pitted considerably for 5 or 6 ft near the surface of the ground.

Hangers or Suspenders are vertical wire cables or members built of structural shapes which connect the stiffening trusses to the main cables. When the main cables are made of wire the suspenders are also of wire rope, as in the Manhattan bridge where the suspenders consist of four wire ropes 1¾ in in diameter at each panel point of each stiffening truss, that is, two loops over the main cable. These ropes rest in two grooves in the cast-steel cable bands which clasp the main cables, and the stiffening trusses are suspended from their lower ends. In the Budapest suspension bridge the cables are made of links of eye-bars connected by pins, and the stiffening trusses are suspended from these chain cables by suspenders consisting of two channels. Stay-cables are inclined cables extending from the tops of the towers to the stiffening trusses, where they are attached at the suspender connections. They were used in the main and side spans of the Brooklyn bridge and in the main span of Roebling's bridge at Niagara Falls, but in recent suspension bridges they have not been used.

Saddles are castings resting on the top of the towers to support the main cables at these points. They usually consist of a steel casting on each tower, and each cable is laid in a groove in the casting, the groove and cable being covered with a cast or rolled steel cover. In the Manhattan bridge the tower saddles are rigidly connected to the towers, whereas in the Williamsburgh bridge each saddle rests on a nest of forty cylindrical steel rollers each 2¼ in in diameter and 7 ft 6 in long. In the latter bridge each saddle is in one piece, weighs about 72 000 lb and is 7 ft 8 in wide and 19 ft long. The chain cables of the Budapest bridge are connected to the tops of the steel towers by links and pins and the towers are hinged at their bases.

Piers. There are usually two piers in a suspension bridge, one at either shore. They may be either of masonry or of steel. Masonry gives a better appearance, but is more costly and takes up greater space. A steel tower or pier may be a single bent of two supports hinged at the bottom or a framed tower with four or more supports. The forces on the pier depend on the arrangement of the cables. It was proposed by Morison, Trans. Am. Soc. C. E., Dec., 1896, to connect the cable of the main span and the back stay of the end span in two separate sockets at the top of the tower. In this case there would at times be great inequalities in the horizontal components of the stresses in the main cable and the back stay, and provision would have to be made for these differences in the design of the pier. When saddles on rollers are used to support the cables the maximum longitudinal force which can act on the top of a tower may be taken approximately at 1/100 the total load on the rollers. This assumes the rollers to be in good condition and free from excessive rust. The horizontal force in this case acts just before and until the rollers move. The movement equalizes the horizontal components of the stresses on the two sides of the saddle and the pressure is then vertical on the pier, except in so far as wind pressure and cradling of the cables produce horizontal forces.

MISCELLANEOUS STRUCTURES

51. Viaducts

A Viaduct is a bridge consisting of a series of spans supported on towers, the spans being usually plate girders, tho sometimes trusses are used. Viaducts are used to carry railroads or highways over valleys or rivers. They are also called trestles. Each tower usually has four legs or columns, tho six or two may be used. When two are used they are connected by transverse bracing only, thus forming a ROCKER TOWER; and when four or more legs are

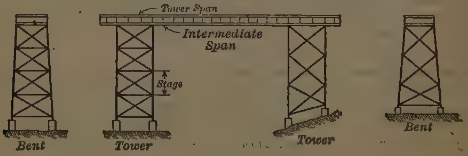


Fig. 162. Viaduct Towers

used in one tower they are connected by transverse and longitudinal bracing. A STAGE or story in a tower is that portion of the tower between the level of one horizontal brace and the level of the one above it. Fig. 162 shows the two usual types of viaduct towers.

Length of Span. For a given viaduct the tower span is usually made constant, 30 ft for low and 45 ft for high structures being common. The intermediate spans should increase in length as the heights of the adjacent towers increase, but to simplify construc-

High Railroad Viaducts

Viaduct	Railroad	Length, ft	Maximum height, ft	No. of tracks	Weight of metal. Short tons	Tower spans, ft	Ordinary intermediate spans, ft
Boone........	Chic. & N.W..	2685	185	2	5680	45	75
Gokteik*......	Burmah State.	2260	335	1	4310	40	60–120
Pecos.........	So. Pacific....	2180	321	1	1820	35	65
Kinzua	Erie.........	2053	301	1	3352	38.5	61
Panther Creek..	Wil. B. & E...	1650	161	1	830	30	40–65
Loa†	Antofagasta...	800	336.5	1	1115	32	80
Spokane River.	Ore. Wash. & Nav.......	3003	187	1	3822	40	80
Snake River...	Ore. Wash. & Nav.......	3920	300	1	8100	60	80

* Burmah. † Chile.

tion not more than two or three different lengths should be used in any one structure. The intermediate span should be approximately twice as long as the tower span, 60 ft for intermediate spans being common practice for ordinary viaducts. The Boone viaduct has alternate spans of 75 and 45 ft thruout its length except for the 300-ft truss span near the center. In the Gokteik structure all tower spans are 40 ft, with intermediate truss spans of 120 ft, except those near the ends, which are plate girder spans of 60 ft. The girders may all be of the same depth, even tho the spans are of various lengths as in the Boone viaduct or of different depths as in the second Kinzua viaduct built in 1900. The length of the tower span should be sufficient to prevent an upward pull on the anchorages due to traction.

Lateral Stability. A tower consisting of two transverse vertical bents has in each bent two inclined columns which are connected by bracing. The width of the bent at the top varies from 8 to 11 ft for single-track railroad structures, and the two lines of girders are usually spaced the same distance apart as the columns. The double-track Boone viaduct has four lines of girders spaced with three equal spaces of 6.5 ft, and the outer girders are vertically over the tops of the columns. The batter of columns is frequently made such that the width of the bent at the bottom is sufficient to prevent an upward pull on the foundation when the structure is loaded with the lightest vertical load consistent with the greatest wind pressure. The batter varies from 1.5 to 3 in per foot.

Bracing. The main girders should be connected together with transverse sway frames and with top and bottom horizontal lateral bracing. The diagonals and horizontal members of the transverse and of the longitudinal bracing of the towers may be made of latticed angles or channels connected to the tower legs with gusset plates which are usually shop-riveted to the columns. Rods and adjustable members should not be used for laterals. The second Kinzua viaduct has ordinary longitudinal bracing, but the transverse bracing in the bents consists of horizontal portal girders at intervals of about 30 ft without any diagonals.

Connections of Longitudinal Girders to the bents are usually made in single-track viaducts by resting them directly upon the horizontal caps of the inclined columns, tho in some cases, the second Kinzua viaduct for example, the tower and intermediate span girders are riveted to the vertical faces of the columns, except at expansion joints, where one end of the intermediate

girder rests in an expansion bearing or pocket which is attached to the vertical side of the column. The expansion joints in this structure are from 199 to 260 ft apart. The longitudinal girders may be connected to a transverse girder which is riveted between the columns of each bent. Fig. 163 shows the design used in the Boone viaduct where the tower girders rest upon

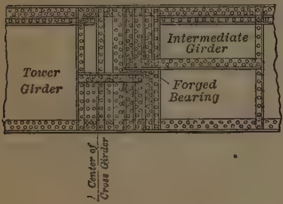


Fig. 163

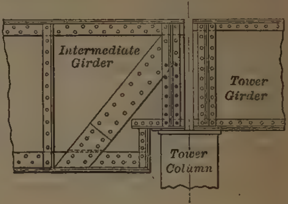


Fig. 164

the upper flanges of transverse girders and are notched to receive the ends of the intermediate girders which have drop ends. The tower and intermediate girders here have a uniform depth of 7 ft. A forged-steel bearing between the lower and upper girders allows for deflection and temperature displacements. In Fig. 164 is shown another form of drop end where the girders have different depths.

Column Bases. In high towers provision should be made for expansion at the bottom of the towers in a direction at right angles to the length of the viaduct. In the Boone viaduct this is accomplished by bolting the base of one column in a bent to the cast pedestal which is anchored to the masonry, while the other column in the same bent has slotted holes for the anchor bolts in its base so that it may move on a phosphor-bronze sliding plate which is inserted between column base and pedestal. The Kinzua viaduct has a nest of rollers under one column in each bent.

Live Loads. First Kinzua viaduct was designed and built in 1882 for locomotives weighing 161 340 lb on a wheel base of 54.25 ft; maximum wind pressure 30 lb per sq ft. Second Kinzua: built in 1900; two locomotives each of 274 000 lb followed by 4000 lb per linear ft; longitudinal traction of 0.2 of the maximum live load; temperature stresses due to 150° variation; wind pressure for the loaded structure of 30 lb per sq ft of vertical surface of train, floor and girders and 100 lb per vertical foot of bent, and for the unloaded structure 50 lb per sq ft of vertical surface of floor and girders and 160 lb per vertical foot of bent. Boone viaduct: built 1900–01; 75-ft spans, 6100 lb live and 1400 lb dead per linear ft; 45-ft spans, 7600 lb live and 1250 lb dead per linear ft. Fort Dodge viaduct: two locomotives each of 308 000 lb followed by 4000 lb per linear ft.

Stresses in Towers due to vertical loads. The vertical loads consist of the weights of the locomotives and train, and the dead loads. In Fig. 165 let W be the total live and dead load acting at the top of each post and W_1, W_2, and W_3 the dead loads at the joints. Stress in bar 1 is designated by P_1 and so on. θ is angle between any bar and the vertical.

$$P_1 = P_2 = W \sec\theta \qquad P_3 = P_4 = (W + W_1)\sec\theta$$
$$P_5 = P_6 = (W + W_1 + W_2)\sec\theta$$
$$P_7 = W\tan\theta \qquad P_9 = W_1\tan\theta$$
$$P_{11} = W_2\tan\theta \qquad P_{13} = (W + W_1 + W_2)\tan\theta$$

The diagonals have no stress under vertical loading. All of the above stresses are compression except P_{13}.

Stresses in Towers due to Wind. In Fig. 165 the horizontal forces shown are those due to wind, H being the wind pressure on the train and H_1, H_2, H_3 the pressures on the structure. + denotes tension, − compression. The diagonals are tension members and the dotted bars are not in action for the forces shown. The stresses are

$$P_1 = Hh \sec\theta / b \qquad P_2 = -[H(h + h_1) + H_1h_1] \sec\theta / b_1$$
$$P_3 = [H(h + h_1) + H_1h_1] \sec\theta / b_1$$
$$P_4 = -[H(h + h_1 + h_2) + H_1(h_1 + h_2) + H_2h_2] \sec\theta / b_2 \qquad P_5 = -P_4$$
$$P_6 = -[H(h + h_1 + h_2 + h_3) + H_1(h_1 + h_2 + h_3) + H_2(h_2 + h_3) + H_3h_3] \sec\theta / b_3$$

For the horizontal and diagonal members the stresses are most easily found by taking moments about the origin, which is at the intersection of the columns produced. The stresses due to traction and temperature should be found.

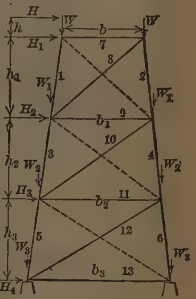


Fig. 165

52. Elevated Railway Structures

Arrangement. Steel structures for supporting elevated railroad tracks, such as are used for passenger traffic in city streets, consist of longitudinal deck girders or trusses upon which the tracks are laid, transverse girders which support those running longitudinally, and columns by which the transverse girders are supported. There are usually two longitudinal girders for each track, and these are connected by a system of lateral bracing in the plane of the top flanges and also at frequent intervals by sway frames. This same general arrangement is applicable to elevated structures for steam railroads. The structures should be built as low as the required head room will allow; a clearance for highway traffic of from 14 to 16 ft being necessary in city streets, with 14 ft as the usual minimum. The only means, other than rigid columns, of stiffening the bents transversely is with brackets or knee-braces which are riveted to the bottom flanges of the transverse girders and to inner faces of the columns.

Floor Systems are of two types: ordinary open floors having wooden ties 6 in × 8 in or 7 in × 8 in spaced from 14 to 16 in on centers and provided with inner and outer guard timbers for each main rail; or solid floors having the ties embedded in stone ballast carried on a steel floor. Footwalks should be provided, and may be supported by extending every fourth tie where the open floor is used. No effective system has been designed for eliminating noise but the ballasted floor is better than the open floor. Solid floors of steel plates having ties laid directly on the steel do not reduce noise.

Longitudinal Girders or stringers are spaced either 5 or 6 ft on centers and each track is carried on two longitudinals. Plate girders are simpler and require less maintenance than riveted lattice girders, but the latter admit slightly more light to the street below. In earlier structures plate girders were not used as much as lattice girders, but in the more recent cases plate girder construction has been more generally adopted. Plate girders without flange plates are preferable, and the top of the top flange angles should be slightly above the top flanges of the transverse girders so that the ties may be spaced over the latter. For example, the longitudinals of the elevated portion of the New York Rapid Transit Railway have their webs notched to avoid

interference with the top flanges of the transverse girders, and the top flange angles of the longitudinal pass over those of the transverse girders. Fig. 166 shows details of a typical transverse plate girder and column base.

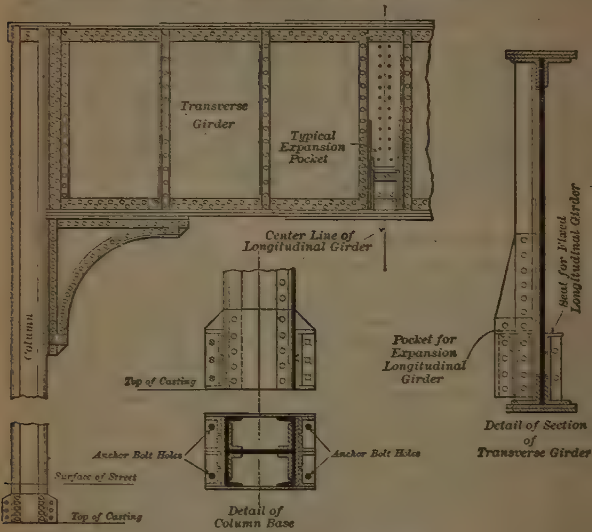


Fig. 166. Details of Elevated Railway

Transverse Girders may be plate girders or riveted trusses and should be riveted to the sides of the columns, not set on top thereof. In wide streets each of the two tracks may be supported on a line of columns placed at each curb, thus resulting in two independent structures each having the center line of a track directly over the center of the column. Another arrangement is to place the two lines of columns 12 ft on centers in the street and have the two structures connected together at the bents, forming two-column bents. In these cases the longitudinals are connected to cantilevers which are attached to the sides of the columns, and brackets are provided for stiffness. In narrow streets the simplest construction has the two longitudinals for each track spaced 5 or 6 ft on centers and the two tracks 12 ft on centers with the longitudinals riveted to the webs of the transverse girders, the columns being placed on the sidewalks near the curbs. When the outer girders are in line with the columns, brackets may be used under these longitudinals as well as under the transverse girders.

Columns should be fixt at the bottom by proper anchor bolts. Four bolts 1½ in in diameter anchored to castings in the concrete piers and attached to the sides of the column with stiffened shelf angles are commonly used. Bases of columns located in streets should be protected by being encased in concrete and by cast-iron wheel fenders, the latter made in two parts bolted together with bolts inside the castings. A common cross-section for columns consists of two 15-in channels with flanges turned in, the two channels being connected by a 15-in I beam riveted to their webs and by tie plates riveted to their flanges. The standard columns of the Boston Elevated Railway structure are made of two 15-in channels of special shape having rounded instead of square corners, the channels being

connected by a web plate and four angles. The standard of the New York Rapid Transit Railway consists of a web plate to which two pairs of special angles are riveted, each angle having one leg finished as a bulb.

Expansion Joints should be provided at intervals of about 150 ft, altho 250-ft spaces have been used. These joints are made by setting the ends of the longitudinal girders in expansion-pockets in which they are held laterally but are unrestrained longitudinally. The longitudinal expansion and contraction rarely exceeds ½ inch in 100 ft in climates like that of New York.

Live Load used in designing Northwestern Elevated and Union Loop Passenger Elevated Railroads in Chicago: a train of one motor car having a loaded weight of 60 000 lb followed by trailers each having a loaded weight of 40 000 lb; each motor car and trailer with the weight equally distributed on eight wheels; wheel spacing for both 5–20–5 ft, with 10 ft between centers of rear wheel and forward wheel of adjacent car. Loads for Boston Passenger Elevated (1909): live load, a train of cars each weighing 100 000 lb loaded and distributed equally on eight wheels, wheel spacing 5.5–26–5.5 ft with 9.0 ft between centers of rear wheel and forward wheel of adjacent car; impact to be added to live load stresses $= 300\, S/(L + 300)$, where S = calculated maximum live load stress, L = length of loaded distance in feet which produces maximum stress in member; dead weight of track system and laterals 220 lb per linear ft per girder for ordinary open-floor track construction; longitudinal force neglected on tangents, but on sharp curves or breaks taken as 5 to 10 % of live load over portion of structure affected; wind pressure 450 lb per linear ft of structure acting horizontally at base of rail; centrifugal force in pounds = $0.02\, WD$ for curvature up to 5 degrees, where W is weight of train in pounds, D degree of curvature, the coefficient 0.02 reduced 0.001 for every degree of curvature above 5 degrees. For Electric Rapid Transit Railway in Berlin, Germany, completed 1902, the longitudinal force was taken at 1/7 of the weight of the train.

Allowable Unit Stresses in pounds per square inch. Boston Elevated: medium steel; tension 17 000; compression 17 000 reduced as follows, $P = 17\,000/(1 + l^2/11\,000\, r^2)$, where P = allowable compression per square inch, l = length of member in inches, r = least radius of gyration in inches; shop rivets, in shearing 11 000, and in bearing 22 000; field rivets, increase the computed shop rivets by 25% if hand driven or 10 % if power driven.

Cost of Elevated Railroad Structures for passenger traffic varies with the time and place. Approximate costs in New York in 1910, including two stations per mile: 2-track elevated road per linear foot complete, $100.00; 3-track, $125.00; 4-track, $150.00, foundations from $100.00 to $200.00 each, track $5.00 per linear foot of single track in Boston in 1902 for electrically operated double-track tangent, 12 ft on centers, about $415 000 per mile exclusive of stations made up as follows: foundations, $42 500; steel structure in place ($75.00 per ton) $264 000, track system, $45 000, electrical accessories (third rail, etc.) $12 000, feeder cables, $11 000, signal system, $17 000, telephone and telegraph, $2500, painting 3 coats, $21 000. The following data are from the Trans. Am. Soc. C. E. 1897, Vol. 37. Mr. J. A. L. Waddell's estimates based on designs: weight of four-track structure on tangents 1565 to 1750 lb per linear ft, on curves 1765 to 1950; weight of two-track structures on tangents 775 to 1142 lb per linear ft, on curves 875 to 1182; cost of line per linear foot $65.10 to $67.60 for four-track structures and $32.30 to $42.00 for two-track structures; cost per mile including four stations in a mile, $383 728 to $396 928 for four-track structures and $210 000 to $267 760 for two-track structures; these figures were based on metal erected and painted, 2.5 cents per pound, vulcanized timber in place $38.00 per M B. M., rails with details and fastenings in place (three 80-lb rails per track) $1.80 per linear foot per track; concrete $7.00 per cu yd; excavation including back filling, 50 cts per cu yd. O. F. Nichols' costs of actual construction of Brooklyn Elevated were as follows:

	Old Brooklyn Lines	Later Union Lines	Sea-Side Lines	Fulton Avenue Extension
Average length of spans	44.7 ft	50.9 ft	52.5 ft	....
Weight of structure per linear foot	870 lbs	1050 lbs	1130 lbs	950 lbs
Cost of structure per linear foot, including foundations	$34.70	$36.50	$37.40	$32.00
Cost per mile, including passenger stations	....	$308 000.00	$286 950.00	$213 900.00
Percentage of cost of bents to cost of spans....................	75%	69%	65%	....

T. C. Clarke gave average cost of repairs on 7.4 miles of Manhattan Second Avenue Elevated structure in New York exclusive of track and painting as $319 per mile per year for 17 years ending 1897; or cost of these repairs per ton of structural iron in 17 years as $1.39 or about 2% of original cost.

53. Stand-pipes

Notation. H = height in feet from a point to surface of water in stand-pipe, D = internal diameter of stand-pipe in feet, t = thickness of side plate in inches, S = allowable tensile unit stress in pounds per square inch, e = efficiency of vertical riveted joint, b = circumferential distance in inches between centers of anchor bolts, W = weight of metal in stand-pipe above a given horizontal joint in pounds, p = pitch of rivets in horizontal joint in inches, M = moment in foot-pounds of wind pressure at a horizontal joint, V = allowable stress on a rivet in pounds.

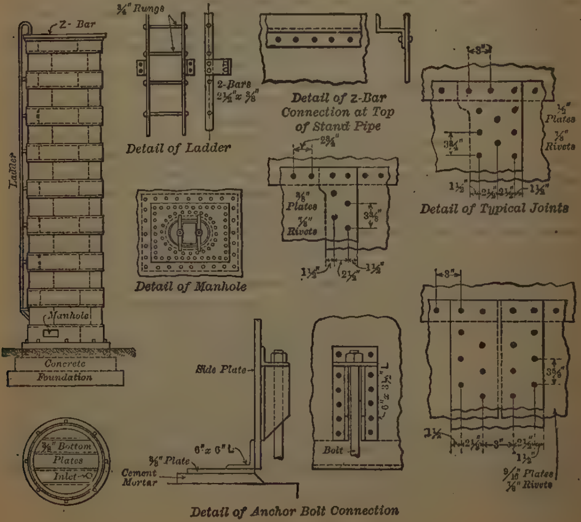


Fig. 167. Stand-pipe, with Details

A Stand-pipe is a metallic tank, usually of cylindrical form, with a flat bottom resting directly upon a masonry or sand foundation and used for the storage of liquids, usually water or oil. For storing water stand-pipes vary in diameter from 12 to 30 ft and in height from 35 to 100 ft, altho there has been constructed a stand-pipe with a diameter of 150 ft and a height of 20 ft and also one with a diameter of 4 ft and a height of 210 ft; the latter being surrounded by a masonry tower. Stand-pipes for the storage of water should generally have no roofs in cold climates but in warm climates should be covered. One pipe may be used to serve as an inlet as well as an outlet and this may connect with the stand-pipe in the horizontal base plate or in the side near the base. Failures of stand-pipes have been caused by defective foundations, by wind or ice pressure, or by hydrostatic pressure. Fig. 167 shows details of stand-pipe construction.

The Thickness of Side Plates must be computed to withstand the tension due to water pressure, proper allowance being made for weakening of plates at the vertical seams or joints. Side plates less than ¼ in in thickness should not be used. The maximum thickness required seldom exceeds 1⅛ in. Tension in side plates at a point H distance down from water surface = 2.6 HD per vertical linear inch, and the thickness of plate required at that depth for the sides of a cylindrical stand-pipe containing water is $t = 2.6\,HD/Se$, in which S should be taken at from 10 000 to 12 000 lb per sq in for structural steel. The efficiency e depends on the design of the vertical joints, which should be so made as to develop an efficiency of from 60% to 75%. With a depth of water of 60 ft, an internal diameter of 18 ft, allowable tension of 12 000 lb per sq in, and efficiency of 66⅔%, the thickness of plate required at that depth = $2.6 \times 60 \times 18/12\,000 \times .66⅔ = .35$ in, or practically ⅜ in.

Side Plates in cylindrical stand-pipes are arranged in horizontal rings or courses about 5 feet deep, with adjacent courses of different diameters so that the courses lap over each other. Plates should be bent by cold-rolling, then punched and finally planed to a bevel along all edges for calking. Rivet holes in plates having thickness of 9/16 or less should be punched 1/16 in larger than the diameter of the cold rivet; those in ⅝ to ¾ in inclusive should be sub-punched 3/16 in less in diameter than the cold rivet and then reamed to 1/16 in larger than the rivet; those in plates over ¾ in in thickness should be drilled 1/16 in larger than the rivet. Plates should be open-hearth "structural" steel and rivets of "rivet" steel or wrought iron.

Bottom Plates for flat-bottom stand-pipes should be not less than 5/16 in in thickness when laid on concrete foundations; when laid on sand as in the case of oil-tanks of large diameters bottom plates are frequently made ¼ in. If on a concrete foundation they should be completely riveted and then laid on a 2-in layer of portland cement mortar before the mortar hardens, or else they should be bedded by pouring cement grout thru 1½ in temporary iron pipes screwed into holes in the plates, the pipes being replaced with screw plugs. Side and bottom plates are connected by one or two L's having beveled edges for calking. When only one L is used its thickness should be approximately equal to that of the lowest side plate.

A Steel Ladder, extending from the top to within 8 or 9 ft of the ground should be attached to the outside of the stand-pipe. The rungs may be made 15 in long, ¾ in in diameter, from 12 to 14 in apart and attached to two vertical bars 2½ in by ⅜ in. A manhole is necessary near the bottom, the side plates being reinforced around this opening by plates or steel forgings. Openings for supply pipes, whether in the bottom or side, must be similarly reinforced.

Wind Pressure may generally be taken at 30 lb per sq ft on vertical surfaces, except in the most exposed locations, where 40 lb per sq ft should be used.

On a cylindrical surface it should be taken as ⅔ that on a plane surface equal to the diameter of cylinder multiplied by its height, and this pressure is assumed to act horizontally in any direction. Unless anchored to the foundation an empty stand-pipe will overturn when the overturning moment of wind pressure taken about the base, that is, 10 H^2D ft-lb for above pressure of 30 lbs per sq ft, is greater than the moment of the weight of the empty stand-pipe taken about the outer edge of the base. ANCHOR BOLTS should always be used and should be computed for a tensile unit stress of not more than 15 000 lb per sq in; minimum diameter 1¼ in, and they should be attached to anchor plates embedded in the foundation at sufficient depth to take at least 1½ times the computed tension in the bolts. Maximum tension in the anchor bolts may be found by regarding the stand-pipe as a cantilever subjected to the overturning moment of the wind and the righting moment of the weight of the empty tank, about the neutral axis. This neutral axis can best be found by trial. On its windward side all of the tension is taken by the bolts; on its leeward side all of the compression is taken by the portion of the base plate immediately under the edge of the tank. It is assumed that the anchor bolt nuts are just brought to a bearing.

In Fig. 168, $N\text{–}A$ is the neutral axis taken close to the leeward edge. A_B = area of bolts on the right of $N\text{–}A$; A_T = area of base angle on the left of $N\text{–}A$, $A = A_B + A_T$, $G\text{–}G$ = axis thru center of gravity of the two areas A_B and A_T; c_1 and c_2 = distances from $G\text{–}G$ to extreme points in tension and compression respectively; g = distance from $G\text{–}G$ to the center of the tank; I = moment of inertia of A, the section under stress, about $G\text{–}G$; S_B = maximum unit tension at extreme point on the windward side; S_c = maximum unit compression at extreme point on the leeward side. $S_B = (M - Wg)c_1/I - W/A$. $S_c = (M - Wg)c_2/I + W/A$. If all dimensions are taken in feet, S_B and S_c will be pounds per sq ft. By proportion, from the extreme fibre stresses S_B and S_c, the correct position of the neutral axis can be found. If it does not agree with the trial one a new position of the line $N\text{–}A$ should be taken and the process repeated until the assumed and the computed positions of $N\text{–}A$ agree closely. S_B may be found with sufficient accuracy by assuming the gravity axis $G\text{–}G$ to coincide with the tangent $G'\text{–}G'$ at the leeward edge of the rim circle, and I to be the moment of inertia of the bolt areas about $G'\text{–}G'$ = sum of the products of the areas of the bolts into the square of their distances to $G'\text{–}G'$. The W/A term, being small, may be neglected. If included, $A = A_B +$ the bearing area of about ⅛ of the rim base angle. Then $S_B = (M - Wr)D/I$.

Fig. 168. Section at Base of Tank.

For example let H = 80 ft; D = 20 ft; W = 75 000 lb; wind = 30 lb per sq ft. M = ⅔ × 20 × 80 × 30 × 40 = 1 280 000 ft-lb. Wr = 75 000 × 10 = 750 000. $M - Wr$ = 530 000 ft-lb. Assume eight 1¼ in anchor bolts upset to 1⅝ in. Area of each bolt = 1.227 sq in = 0.0085

sq ft. $I=2\times0.00854\,(\frac{1}{2}\times2^2+1.7^2+1^2+0.3^2)\,10^2=10.2$. $S_B=530\,000\times20/10.2=1\,040\,000$ lb per sq ft $=7230$ lb per sq in. The area of $\frac{1}{8}$ of the rim bearing, assuming the effective bearing width of base angle to be 3 in $=282.6$ sq in. $A=1.227\times7+282.6=291.2$ sq in. $W/A=75\,000/291.2=260$ lb per sq in. $S_B=7230-260=6970$ lb per sq in. Total stress S in extreme windward bolt $=6970\times1.227=8550$ lb. The total tension in the bolts from $M-Wr=7230\times1.227(20+17+10+3)/20=22\,200$ lb. The average compression over $\frac{1}{8}$ the area of the rim from $M-Wr=22\,200/282.6=80$ lb. Approximate maximum compression on the masonry $=260+80=340$ lb per sq in. The actual maximum is slightly greater than this but well within the safe bearing strength of the concrete. Assume it to be 500 lb. By proportion the neutral axis lies $500\times20/(6970+500)=1.4$ ft from the tangent $G'-G'$. The assumed position of $N-A$ was 0.8 ft from $G'-G'$ indicating that a small portion only of the rim circle takes all of the compression and justifying the approximate assumptions made.

The Pitch of Rivets connecting the base angle to the rim of the tank should also be computed. Assuming the maximum compression to be 500 lb per sq in, the compression per lineal in of angle $=500\times3=1500$ lb. Safe single shearing strength of a $\frac{3}{4}$-in rivet $=4400$. $p=\frac{4400}{1500}=3$ in.

Anchor bolts must be connected to the sides, not to the bottom plate of the structure. To prevent buckling of the side plates under wind pressure a Z-bar or a heavy L must be riveted around the upper edge. In addition L's are sometimes riveted circumferentially on the outside at different elevations.

The Stress on a Rivet in a horizontal joint in pounds due to the weight of the stand-pipe above that joint $=0.027\ pW/D$; and due to wind pressure acting on the cylindrical surface above the joint $=0.016\ pM/D^2$. With a pressure of 30 lb per sq ft this stress on a rivet due to wind becomes $1.06\ pH^2/D$. Or the pitch of these rivets in inches required to withstand the combined stresses due to weight and wind $=DV/(.027\ W+1.06\ H^2)$. The stresses due to ice are indeterminate. All joints must be calked after riveting. Allowable stresses per square inch on steel field rivets: shearing 9000, bearing 18 000. For $\frac{1}{4}$ in plates $\frac{5}{8}$-in rivets should be used; for $\frac{5}{16}$ in plates, $\frac{3}{4}$ in rivets; for $\frac{3}{8}$ to $\frac{7}{8}$ in plates, $\frac{7}{8}$ in rivets; for $\frac{15}{16}$ to $1\frac{1}{8}$ in plates, 1 in rivets. For horizontal joints use lap-joints, usually single riveted. For vertical joints use double-riveted lap-joints for $\frac{1}{4}$, $\frac{5}{16}$ and $\frac{3}{8}$ in plates; triple-riveted lap-joints for $\frac{7}{16}$ and $\frac{1}{2}$ in plates; double-riveted butt-joints for $\frac{9}{16}$ to $\frac{3}{4}$ in plates inclusive; triple-riveted butt-joints for $\frac{13}{16}$ to 1 in plates inclusive.

54. Water Tanks on Towers

Notation. W_1 = weight of water in pounds vertically above and below a horizontal section together with the weight of the conical or hemispherical bottom below the section, W = weight of water in pounds in the tank together with the weight of bottom, θ = angle between element of conical bottom, or between tangent to spherical bottom, and horizontal, D = diameter of cylindrical portion of tank in feet, d = diameter of a horizontal joint in a conical bottom in feet, h = distance in feet from surface of water to any point in a radial joint of a conical bottom, l = unsupported length and r = least radius of gyration of cross-section of column, both in inches, M = moment at a horizontal plane due to wind pressure on structure above that plane, r_1 = radius of circle passing thru column centers.

A Water Tank on a Tower consists of a steel tank, usually cylindrical, supported on a steel tower which may have four or more columns connected to the tank at or near the bottom and connected to each other by horizontal and diagonal braces. The bottom of the tank may be flat, conical or hemispherical; the latter is the best and is riveted directly to the lowest cylindrical

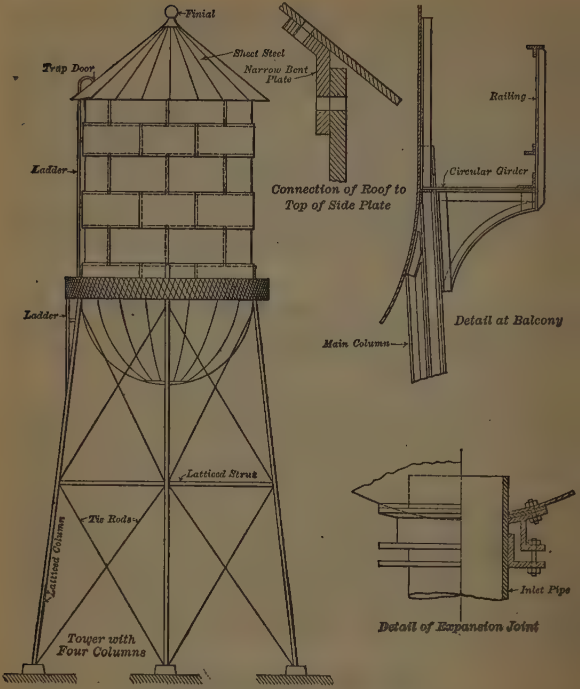


Fig. 169. Water Tank on a Tower

side plates at intersection of center line of column with side plate. Elevated tanks are more economical and of more pleasing appearance than stand-pipes. Ice pressure causes fewer failures of elevated tanks than of stand-pipes, and the tanks are usually covered with conical or curved roofs. Fig. 169 shows typical details of construction of a water tank and tower. The cylindrical part is similar in action to the corresponding portion of a stand-pipe.

The Riser or Inlet Pipe is the supply pipe, which usually also serves as an outlet; enters the tank at the center of the bottom, at which place the tank must be reinforced. An EXPANSION JOINT is placed in the riser at or near the junction with the tank. Risers should be braced laterally at each panel point of the tower and in cold climates should be covered with a FROST-PROOF CASING consisting of three thicknesses of planking and two of tarred paper with air spaces between the thicknesses of planking.

A Conical Roof made of 1/8-inch sheet steel is self-supporting for diameters of 20 ft or less if the roof plates are riveted to the upper edge of the top side plate; for diameters greater than 20 ft L rafters must be used. Iron ladders extend from a trap-door in the

lower part of the roof down the outside and inside of the tank, ending on the balcony on the outside. Another ladder, attached to one of the tower columns, should run from the balcony to within 8 or 9 ft of the ground. The balcony should be 3 ft wide and its floor should be a horizontal plate girder placed at intersection of bottom and side plates.

Conical Bottoms. Tension in pounds per linear inch of any horizontal joint or section = $0.0265\ W_1/d \sin\theta$; at intersection of bottom and side plates this tension becomes $0.0265\ W/D \sin\theta$. Tension at any point of a radial joint in pounds per linear inch, that is, of a joint coinciding with an element of cone = $2.6\ dh/\sin\theta$. A horizontal outside girder must be used at the intersection of side plates with the bottom to resist the annular compression caused by the inward pull from the bottom. Horizontal component of inward pull from bottom = $0.0265\ W/D \tan\theta$ pounds per circumferential inch; annular compression = $0.0132\ W/\tan\theta$; and vertical component = $0.0265\ W/D$ pounds per circumferential inch. This vertical component is esisted by a vertical circular girder.

Spherical Bottoms. Tension in pounds per linear inch of any horizontal joint or section = $0.0265\ W_1/D \sin^2\theta$. The spherical bottom exerts only a vertical pull on the cylindrical side, amounting to $0.0265\ W/D$ pounds per circumferential inch. Best design requires the thickness of spherical bottoms to be equal to that of the lowest side plate in the cylindrical part. The cylindrical side plates must be reinforced at column connections.

Allowable Unit Stresses in pounds per square inch. Tension: in tank plates, 12 000; other parts, 16 000. Compression, $16\ 000 - 70\ l/r$; the ratio l/r should in no case exceed 120. Shear: on field rivets in tank, and bolts, 9000; on shop rivets and pins, 12 000. Bearing: field rivets in tank, and bolts, 18 000; shop rivets and pins, 24 000. Bending in pins, 24 000. For combined wind and other loads the above unit stresses may be increased 25%. All steel except rivets to be of "structural" grade; rivets of "rivet" steel.

Stresses in Columns are due to weight of structure, weight of water in tank, and overturning effect of the wind. The vertical component of stress in any column due to the first two forces is equal to the total weight divided by the number of columns. The maximum vertical component of stress in any column at any horizontal plane due to wind is: for 8-column towers, $M/4\ r_1$; for 6-column towers, $M/3\ r_1$; for 4-column towers, $M/2\ r_1$. For high towers the batter of the columns when uniform is about 1 to 12; in some towers the inclination of the column is changed at each panel point.

55. Coal, Ore, and Grain Bins

A Steel Bin consists of a tank supported on columns and is used for handling or storing materials such as coal, ashes, ore, broken stone, sand and grain. Bins for coal, ashes or other similar materials are usually shallow, and admit of a variety of designs, while grain bins are deep and of cylindrical form. There are three types of bins, the hopper bin, the suspension bin or bunker and the cylindrical bin. For handling coal and ore the construction is such that these materials are dumped into the top of the bins from cars or conveyors above and when needed are taken from the bottom or side by means of gates and chutes.

Hopper Bins (Fig. 170) are generally arranged in units, each unit consisting of a square or rectangular box with four vertical sides and having an inverted frustum of a pyramid for a bottom. This bin is mounted on columns, which may be braced longitudinally with diagonal bracing, and transversely with knee-braces or diagonals. The gate thru which the bin is emptied is placed at the bottom of each hopper, and the inclination of the sides of the hopper should be such that the bin can be emptied by gravity alone.

Suspension Bins (Fig. 171) are steel bunkers supported on steel columns, the bunker being made of plates suspended from two parallel girders which

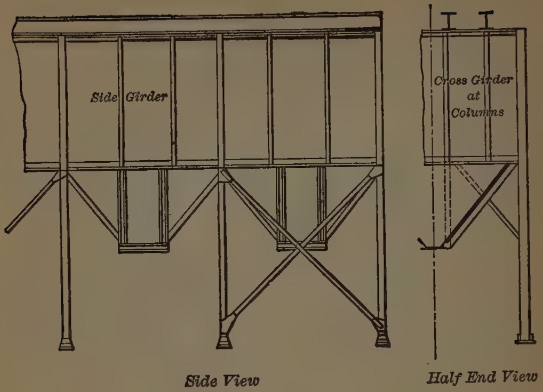


Fig. 170. Hopper Bin

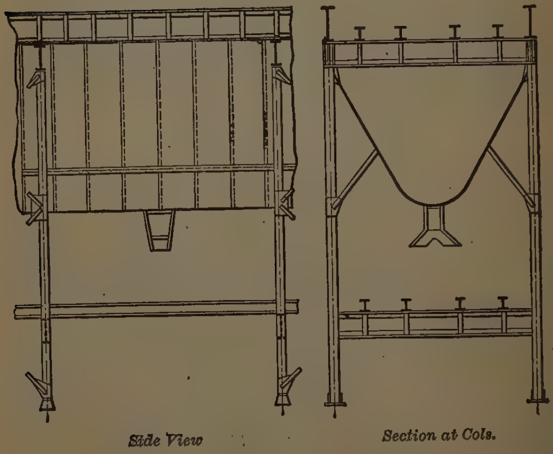


Fig. 171. Suspension Bin

are supported by the columns. The plates are subjected to tension only, and the side girders should be placed with their webs in the plane of this tension, or if placed vertically some provision must be made for resisting the horizontal component of pull from the plates. The ends of the bunker are made of plates stiffened by I beams or Ľ's. To protect the steel against action of materials stored in the bunker, the inside is covered with a layer of concrete reinforced with expanded metal or other similar fabric.

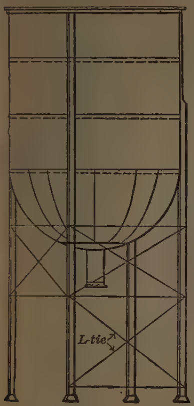


Fig. 172

Circular Bins (Fig. 172) are cylindrical, with hemispherical bottoms, and with supporting columns riveted to the cylinder at the junction of the hemisphere. They are used principally for the storage or handling of ore, the outlet gate being placed at the center of the bottom. The construction is similar to that of water tanks on towers, except that the columns are vertical, and no balcony, with its accompanying horizontal circular girder, is necessary.

Steel Grain Bins are usually cylindrical, with either flat or hemispherical bottoms, and for large grain elevators are arranged in groups. The Great Northern elevator at Buffalo, N. Y., has thirty bins 38 ft in diameter and 85 ft deep set in three rows of ten bins each, and between these are eighteen smaller ones each 15.5 ft in diameter arranged in two rows. All of these have hemispherical bottoms tapering into cones and are supported on circular girders resting on columns.

Weight and Angle of Repose of Dry Materials

Material	Weight. Lb per cu ft	Angle of Repose. Degrees	Material	Weight. Lb per cu ft	Angle of Repose. Degrees
Anthracite coal ..	52	27	Wheat.........	50	28
Bituminous coal .	47 to 55	35	Barley.........	39	27
Ashes..........	40	40	Oats...........	28	28
Sand...........	90 to 115	34	Corn...........	44	28

56. Catenary Bridges

A Catenary Bridge, Fig. 173, is a light truss bridge which crosses the track of an electrified railroad, and whose main function is to support an overhead wire over the center of each track. It is supported at the sides of the outside tracks, and occasionally at an intermediate point when many tracks have to be crossed, by latticed posts or towers to which it is securely riveted. The end towers extend above the bridge to support cross arms which carry the feed, signal and grounding wires.

The trolley wire is supported by a system of suspension cables, called the CATENARY, which carries the weight of the trolley and maintains the latter in a plane parallel to the tracks. The types of catenaries and their supporting bridges described in this Article are those in use on the N. Y., N. H. & H. R. R. The two systems of catenaries are known as the single and the compound.

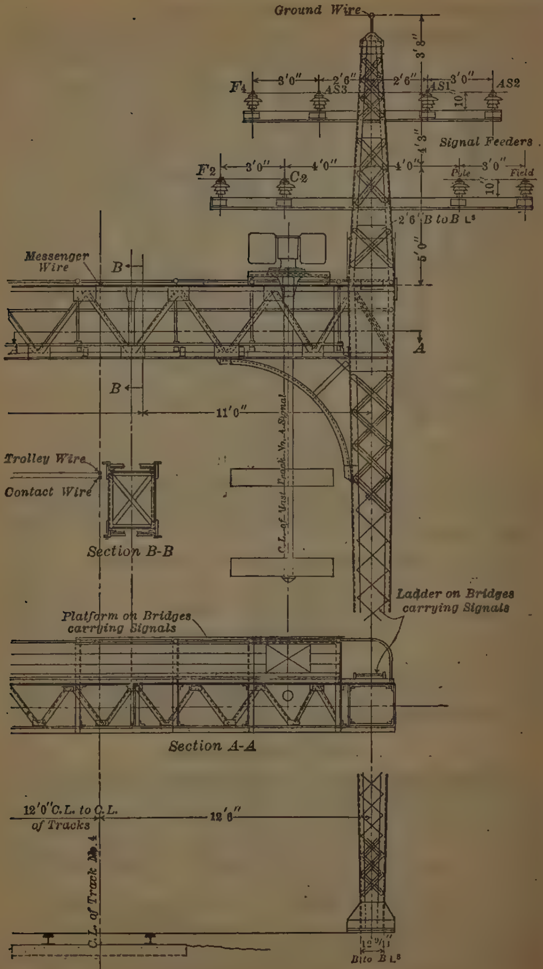


Fig. 173. One End and Tower, Four Track Catenary Bridge

The single consists of a main MESSENGER CABLE which supports, by hangers and clips, the trolley or contact wires. The latter consists of two wires, the top one of copper (to secure proper conductivity), the bottom one of steel for contact purposes, it having been found that copper wire is too rapidly abraided. The copper feeds the steel wire at intervals of ten feet. The entire system is alive and is suspended below the bridges from which it is insulated. The advantages of this system are its lightness, ease of erection and permanency of position on curves. It weighs about 1200 lb per track for spans of 280 to 300 ft between bridges. The disadvantage is the non-protection of the track wire, in electrical storms, by an overhead wire. The compound system consists of a main messenger cable, supported by saddles on top of the bridges, Fig. 173, from which, at the quarter points of the cable span, is supported and insulated from it a single catenary system. In this type the main cable is dead. The advantage of this system is the protection afforded by the main cable by being grounded where it passes over each bridge. The disadvantages are its weight, which is about 2400 lb per track for spans of 300 ft, the additional cost of erection, the effect of temperature in changing alignment on curves, and the greater maintenance cost.

The Economical Spacing of the bridges on the New Haven road is from 150 to 315 ft. In addition to carrying the catenary system a secondary function of the catenary bridges is to support signals where they are required by the necessities of the block system, sidings, bridges, etc., and the platforms needed for the maintenance of the signals. Visibility requires that the signals should project downward and frequent changes in tracks, location of cross overs, etc., necessitating the relocation of signals make it advisable to design each bridge with sufficient strength to carry the signal loadings. The approximate weight of one automatic signal and its platform is 2400 lb.

The Loads for which the bridges are designed are (1) a signal over each track, (2) the vertical and horizontal loads from the catenary over each track, (3) the weight of the bridge, (4) the vertical and horizontal loads from the feed, signal and grounding wires on each cross arm, (5) the wind on the bridge surface, acting either along the track or laterally, and (6) the wind acting laterally upon the wires regarded either as bare or covered with ice.

The Catenaries are strung at definite tensions which depend upon the sag and accordingly vary with the temperature (Art. 48). When the catenaries are taken around curves the messenger cable is pulled away from the track in order to maintain the trolley wire over the center line. The catenary then lies in an inclined surface and the hangers resist the tendency of the trolley wire to straighten out. The lateral force required to maintain this condition is the CURVE PULL which has to be supported by the catenary bridge as a horizontal load. The curve pull is very approximately equal to the cable tension multiplied by the span length and divided by the radius of the curve. Wind is assumed at 30 lb per sq ft on the full windward and half the leeward surfaces of bridges, and upon ⅔ the projected areas of all bare wires. It is taken at 8 lb per sq ft upon ⅔ the projected areas of wires coated with ½ in of ice all around. A higher wind does not permit the ice to form, or will break it off if already formed. The wind blowing across the tracks on the catenaries and cross-arm wires constitutes a lateral load upon the catenary bridges.

Stresses in trusses and towers of the usual two-post type may be found approximately by the method employed for mill-building bents (Art. 3). The towers may be regarded as hinged or as partially fixt at the base according to the degree of efficiency of the anchorage. Equal horizontal reactions may

be assumed at the feet of the towers (or at the points of contraflexure when the feet are regarded as partially fixt).

The Allowable Unit Stresses adapted by the New Haven road are as follows: tension from vertical loads, 20 000 lb per sq in; tension from combined wind and vertical loads, 25 000 lb per sq in; compression from all conditions of loading, $21\,000 - 70\ l/r$. Open hearth steel with an ultimate strength of 60 000 lb per sq in is used.

MATHEMATICAL TABLES

APPENDIX TO SECTION I

26. Six-Place Logarithms of Numbers

The small table on page 1005 gives logarithms of the natural numbers from 1 to 100 with their characteristics.

The extended table on pages 979 to 1005 gives the logarithms of the natural numbers from 100 to 10 000 without their characteristics which are to be supplied by the rules of Art. 24, Sect. 1. For example

$$\log 112 = 2.049218 \qquad \log 1126 = 3.051538$$

Logarithms of numbers with more than four figures are to be found by help of the last column giving differences and the proportional parts at the foot of each page. For example when the number 17 456 is given first find log 1745 = 241795 and the difference 249. Then the proportional part corresponding to 6 in the last place is 149 and log 17456 = 241795 + 149 = 241944 to which the characteristic is to be prefixed; hence log 17456 = 4.241944. Similarly log 174567 = 5.241795 + 149 + 17 = 5.242061.

When a logarithm is given to find the corresponding number: If log is 2.932220 the number is 855.5. When the logarithm is not found exactly in the table take the next smaller one and find the difference between it and the given logarithms; then divide that difference by the number in the column Diff. and add the quotient to the smaller logarithms. For example, given 2.932260, the next smaller logarithm is 2.932220 corresponding to the number 855.5; the logarithm of this is 40 units less than the given one and the Diff. is 51 then 40/51 = 8 so that the number corresponding to the logarithm 2.932260 is 855.58. The table of proportional parts at the foot of the page shows 8 immediately as the figure to be added.

26. Logarithms of Numbers

No. 100 L. 000.] [No. 109 L. 040.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998	397
	04										

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5

26. Logarithms of Numbers

No. 110 L. 041.] [No. 119 L. 078.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.7	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

26. Logarithms of Numbers

No. 120 L. 079.] [No. 134 L. 130.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904	0266	0626	0987	1347	1707	2067	2426	360
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	0012	323
	13										

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

26. Logarithms of Numbers

No. 135 L. 130.] [No. 149 L. 175.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879										
		0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835								
				0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868					
							0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	251.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
202	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.3
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4

26. Logarithms of Numbers

No. 150 L. 176.] [No. 169 L. 230.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8669	289
1	8977	9264	9552	9839							
					0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771		
										0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755						
						0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783									
			0051	0319	0586	0853	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938		
	23									0193	256

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
285	28.5	57.0	85.5	114.0	142.5	171.0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141.5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112.4	140.5	168.6	196.7	224.8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	217.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231.3
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5

26. Logarithms of Numbers

No. 170 L. 230.] [No. 189 L. 278.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550*	9800	0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4

26. Logarithms of Numbers

No. 190 L. 278.] [No. 214 L. 332.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

26. Logarithms of Numbers

No. 215 L. 332.] [No. 239 L. 380.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849			
									0047	0246	199
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860		
										0054	194
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9643	190
9	9835										
		0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958						
						0143	0328	0513	0698	0883	185
5	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849		
	38									0030	181

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19 7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58 8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117 0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37 4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164 7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36 0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.1

26. Logarithms of Numbers

No. 240 L. 380.] [No. 269 L. 431.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752 43	9914	0075	0236	0398	0559	0720	0881	1042	1203	161

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	145.8
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9

26. Logarithms of Numbers

No. 270 L. 431.] [No. 299 L. 476.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752	158
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095	154
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	0146	0296	0447	0597	0748	151
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145	147
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0

26. Logarithms of Numbers

No. 300 L. 477.] [No. 339 L. 531.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959	0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

26. Logarithms of Numbers

No. 340 L. 531.] [No. 379 L. 579.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204	125
7	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	[illegible]8	419[illegible]	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106	123
5	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426	117
2	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	8104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	12.1	24.2	36.3	48.4	60.5	72.6	84.7	96.8	108.9
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	C319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47 2	59.0	70.8	82 6	94.4	106.2
117	11.7	23.4	35.1	46 8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34 8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45 6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44 0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
105	10.5	21.0	81.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6

26. Logarithms of Numbers

No. 415 L. 618.] [No. 459 L. 662.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
103	10.3	20.6	30.9	41.2	51.5	61.8	72.1	82.4	92.7
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
101	10.1	20.2	30.3	40.4	50.5	60.6	70.7	80.8	90.9
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1

26. Logarithms of Numbers

No. 460 L. 662.] [No. 499 L. 698.

N	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967	0060	0153	93
8	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	0063	0154	0245	
9	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9	9309	9398	9486	9575	9664	9753	9841	9930	0019	0107	
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

26. Logarithms of Numbers

No. 500 L. 698.] [No. 544 L. 736.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924	0011	0098	0184	0271	0358	0444	0531	0617	
2	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	0033	
3	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	0077	
5	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6

26. Logarithms of Numbers

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968	0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968	0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
[illegible]70	5875	59[illegible]1	[illegible]27	[illegible]1[illegible]	6180	[illegible]256	63[illegible]	6[illegible]08	64[illegible]	[illegible]	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970	0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	[illegible]	[illegible]
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
[illegible]	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
[illegible]	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	0245	
3	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
[illegible]	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
[illegible]	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
[illegible]	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
[illegible]	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
[illegible]	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
[illegible]	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
[illegible]	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
[illegible]	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.1

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	[illegible]	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5453	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6

26. Logarithms of Numbers

No. 675 L. 829.] No. 719 L. 857.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	
6	9947	0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	0043	
2	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.
63	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.
61	6.1	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.
60	6.0	12.0	18 0	24.0	30.0	36.0	42.0	48.0	54.

26. Logarithms of Numbers

No. 720 L. 857.] [No. 764 L. 883.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3114	3204	3263	
[illegible]0	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
[illegible]	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
[illegible]	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
[illegible]	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
[illegible]	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
[illegible]	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
7[illegible]	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
[illegible]	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
[illegible]	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
[illegible]	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
[illegible]	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
[illegible]	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
[illegible]	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

26. Logarithms of Numbers

No. 765 L. 883.] [No. 809 L. 908.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	991[illegible]	9974	0030	0086	0141	0197	0253	0309	0365	
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	927[illegible]	93[illegible]	[illegible]	9437	[illegible]	9547	[illegible]602	[illegible]	[illegible]711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

26. Logarithms of Numbers

No. 810 L. 908.] [No. 854 L. 931.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
[illegible]0	908485	[illegible]39	8592	8646	8699	8753	8807	8860	8914	[illegible]67	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	51
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

26. Logarithms of Numbers

No. 855 L. 931.] [No. 899 L. 954.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
5	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.

26. Logarithms of Numbers

No 900 L. 954.] [No. 944 L. 975.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

26. Logarithms of Numbers

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7

26. Logarithms of Numbers

No. 990 L. 995.] [No. 999 L. 999.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

Logarithms of Numbers from 1 to 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

	Value at 0°.	Sign in 1st Quad.	Value at 90°.	Sign in 2d Quad.	Value at 180°.	Sign in 3d Quad.	Value at 270°	Sign in 4th Quad.	Value at 360°.
Sin........	O	+	R	+	O	−	R	−	O
Tan.......	O	+	∞	−	O	+	∞	−	O
Sec	R	+	∞	−	R	−	∞	+	R
Versin....	O	+	R	+	2 R	+	R	+	O
Cos	R	+	O	−	R	−	O	+	R
Cot.... ...	∞	+	O	−	∞	+	O	−	∞
Cosec.....	∞	+	R	+	∞	−	R	−	∞

R signifies equal to rad; ∞ signifies infinite; O signifies evanescent.

27. Six-Place Logarithmic Sines, Cosines, Tangents and Cotangents

In the following pages, 1007 to 1051, characteristics of logarithms are increased by 10 in order to avoid negative ones. Thus log sin 1° 10′=8.308794 or $\bar{2}$.308794, log cot 1° 10′=11.691116 or 1.691116, log tan 0° 30′=7.940858 or $\bar{3}$.940858.

Pages 1007–1051 give values of these functions to six decimal places for every minute of the first and second quadrants. The degrees are at the top and bottom of the pages and the minutes at the sides below or above the degrees. For example, on page 1017 the angles 10° 26′ and 169° 34′ have log sin=9.257898, while 79° 20′ and 100° 40′ have log cot=9.274964.

The columns headed D. 1″ enable interpolation to be made for seconds; thus for 10° 26′ 15″ the D. 1″ is 11.42 for log sin, whence 11.42×15=171 and log sin for this angle is 9.257898+171=9.258069. Also for 163° 38′ 15″ the log tan is 9.467880−117=9.467763. The computed difference is to be added or subtracted according as the tabular values of the function increase or decrease with an increase in the angle.

The columns of D. 1″ are omitted on pages 1007 and 1008, except for log cos; while other columns are added which enable intermediate values of the other functions to be found for small angles more accurately than can be done by interpolation. Thus to find log sin A and log tan A, when A contains seconds, the equations

$$\log \sin A = S + \log A'', \qquad \log \tan A = T + \log A'',$$

are to be used, A'' signifying the number of seconds in the angle A. For example, let the angle A be 1° 6′ 33″ or 3993″; for 1° 6′ the value of S is taken from the fourth column on page 1008 and log 3993 from Table 26. Then

For 1° 6′	S=4.685548
log 3993	=3.601299
log sin 1° 6′ 33″=8.286847	

Similarly for 0° 54′ 12″ or 3252″ the log tan is found as follows:

For 0° 54′	T=4.685611
log 3252	=3.512151
log tan 0° 54′ 12″=8.197762	

To find log cot for a small angle the equation log cot A = C−log A'' is to be used where C is taken from the eighth column. For example, for 1° 0′ 16″ or 3616″ the value of C is 15.314381 and that of log 3616 is 3.558228, whence log cot 1° 0′ 16″=11.756153.

To find the angle from a given logarithmic function, the eye must run along the table until the tabular value nearest to it is found. Thus, when log tan is given as 9.516910 this is found on page 1025 and the angle is either 18° 12′ or 161° 48′. Again, when log tan is given as 9.526004, this is found to lie between 9.525778 and 9.526197; to the first value corresponds the angle 18° 33′ and the D. 1″ is 6.98; the difference 9.526004−9.525778 is 226 and 226/6.98=32.4″, so that the required angle is 18° 33′ 32″.4.

When the given function falls on page 1007 or 1008, the number of seconds is found by the equations

$$\log A'' = \log \sin A - S, \qquad \log A'' = \log \tan A - T, \qquad \log A'' = C - \log \cot A.$$

For example, given log tan A as 8.465371 for which T is 4.685700; then log A'' = 8.465371 − 4.685700 = 3.779671 from which by Table 26 there is found A'' = 6021″ and hence A=1° 40′ 21″.

27. Logarithmic Sines, etc.

0°		Sine.	q − l		Tang.	Cotang.	q + l	D1″	Cosine.	179°
″	′		4.685				15.314			′
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		ten	60
60	1	6.463726	575	575	6.463726	13.536274	425		ten	59
120	2	.764756	575	575	.764756	.235244	425		ten	58
180	3	6.940847	575	575	6.940847	13.059153	425		ten	57
240	4	7.065786	575	575	7.065786	12.934214	425		ten	56
300	5	.162696	575	575	.162696	.837304	425		ten	55
360	6	.241877	575	575	.241878	.758122	425	.02	9.999999	54
420	7	.308824	575	575	.308825	.691175	425	.00	.999999	53
480	8	.366816	574	576	.366817	.633183	424	.00	.999999	52
540	9	.417968	574	576	.417970	.582030	424	.00	.999999	51
600	10	.463726	574	576	.463727	.536273	424	.02	.999998	50
660	11	7.505118	574	576	7.505120	12.494880	424	.00	9.999998	49
720	12	.542906	574	577	.542909	.457091	423	.02	.999997	48
780	13	.577668	574	577	.577672	.422328	423	.00	.999997	47
840	14	.609853	574	577	.609857	.390143	423	.02	.999996	46
900	15	.639816	573	578	.639820	.360180	422	.00	.999996	45
960	16	.667845	573	578	.667849	.332151	422	.02	.999995	44
1020	17	.694173	573	578	.694179	.305821	422	.00	.999995	43
1080	18	.718997	573	579	.719003	.280997	421	.02	.999994	42
1140	19	.742478	573	579	.742484	.257516	421	.02	.999993	41
1200	20	.764754	572	580	.764761	.235239	420	.00	.999993	40
1260	21	7.785943	572	580	7.785951	12.214049	420	.02	9.999992	39
1320	22	.806146	572	581	.806155	.193845	419	.02	.999991	38
1380	23	.825451	572	581	.825460	.174540	419	.02	.999990	37
1440	24	.842934	571	582	.843944	.156056	418	.02	.999989	36
1500	25	.861662	571	583	.861674	.138326	417	.00	.999989	35
1560	26	.878695	571	583	.878708	.121292	417	.02	.999988	34
1620	27	.895085	570	584	.895099	.104901	416	.02	.999987	33
1680	28	.910879	570	584	.910894	.089106	416	.02	.999986	32
1740	29	.926119	570	585	.926134	.073866	415	.02	.999985	31
1800	30	.940842	569	586	.940858	.059142	414	.03	.999983	30
1860	31	7.955082	569	587	7.955100	12.044900	413	.02	9.999982	29
1920	32	.968870	569	587	.968889	.031111	413	.02	.999981	28
1980	33	.982233	568	588	.982253	.017747	412	.02	.999980	27
2040	34	7.995198	568	589	7.995219	12.004781	411	.02	.999979	26
2100	35	8.007787	567	590	8.007809	11.992191	410	.03	.999977	25
2160	36	.020021	567	591	.020044	.979956	409	.02	.999976	24
2220	37	.031919	566	592	.031945	.968055	408	.02	.999975	23
2280	38	.043501	566	593	.043527	.956473	407	.03	.999973	22
2340	39	.054781	566	593	.054809	.945191	407	.02	.999972	21
2400	40	.065776	565	594	.065806	.934194	406	.02	.999971	20
2460	41	8.076500	565	595	8.076531	11.923469	405	.03	9.999969	19
2520	42	.086965	564	596	.086997	.913003	404	.02	.999968	18
2580	43	.097183	564	598	.097217	.902783	402	.03	.999966	17
2640	44	.107167	563	599	.107203	.892797	401	.03	.999964	16
2700	45	.116926	562	600	.116963	.883037	400	.02	.999963	15
2760	46	.126471	562	601	.126510	.873490	399	.03	.999961	14
2820	47	.135810	561	602	.135851	.864149	398	.03	.999959	13
2880	48	.144953	561	603	.144996	.855004	397	.02	.999958	12
2940	49	.153907	560	604	.153952	.846048	396	.03	.999956	11
3000	50	.162681	560	605	.162727	.837273	395	.03	.999954	10
3060	51	8.171280	559	607	8.171328	11.828672	393	.03	9.999952	9
3120	52	.179713	558	608	.179763	.820237	392	.03	.999950	8
3180	53	.187985	558	609	.188036	.811964	391	.03	.999948	7
3240	54	.196102	557	611	.196156	.803844	389	.03	.999946	6
3300	55	.204070	556	612	.204126	.795874	388	.03	.999944	5
3360	56	.211895	556	613	.211953	.788047	387	.03	.999942	4
3420	57	.219581	555	615	.219641	.780359	385	.03	.999940	3
3480	58	.227134	554	616	.227195	.772805	384	.03	.999938	2
3540	59	.234557	554	618	.234621	.765379	382	.03	.999936	1
3600	60	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	0
″	′		4.685				15.314			′
90°		Cosine.	q − l		Cotang.	Tang.	q + l	D1″	Sine.	89°

27. Logarithmic Sines,

1°		Sine.	$q-l$		Tang.	Cotang.	$q+l$	$D1''$	Cosine.	178°
″	′		4.685				15.314			′
3600	0	8.241855	553	619	8.241921	11.758079	381		9.999934	60
3660	1	.249033	552	620	.249102	.750898	380	.03	.999932	59
3720	2	.256094	551	622	.256165	.743835	378	.05	.999929	58
3780	3	.263042	551	623	.263115	.736885	377	.03	.999927	57
3840	4	.269881	550	625	.269956	.730044	375	.03	.999925	56
3900	5	.276614	549	627	.276691	.723309	373	.05	.999922	55
3960	6	.283243	548	628	.283323	.716677	372	.03	.999920	54
4020	7	.289773	547	630	.289856	.710144	370	.03	.999918	53
4080	8	.296207	546	632	.296292	.703708	368	.05	.999915	52
4140	9	.302546	546	633	.302634	.697366	367	.03	.999913	51
4200	10	.308794	545	635	.308884	.691116	365	.05	.999910	50
4260	11	8.314954	544	637	8.315046	11.684954	363	.05	9.999907	49
4320	12	.321027	543	638	.321122	.678878	362	.03	.999905	48
4380	13	.327016	542	640	.327114	.672886	360	.05	.999902	47
4440	14	.332924	541	642	.333025	.666975	358	.05	.999899	46
4500	15	.338753	540	644	.338856	.661144	356	.03	.999897	45
4560	16	.344504	539	646	.344610	.655390	354	.05	.999894	44
4620	17	.350181	539	648	.350289	.649711	352	.05	.999891	43
4680	18	.355783	538	649	.355895	.644105	351	.05	.999888	42
4740	19	.361315	537	651	.361430	.638570	349	.05	.999885	41
4800	20	.366777	536	653	.366895	.633105	347	.05	.999882	40
4860	21	8.372171	535	655	8.372292	11.627708	345	.05	9.999879	39
4920	22	.377499	534	657	.377622	.622378	343	.05	.999876	38
4980	23	.382762	533	659	.382889	.617111	341	.05	.999873	37
5040	24	.387962	532	661	.388092	.611908	339	.05	.999870	36
5100	25	.393101	531	663	.393234	.606766	337	.05	.999867	35
5160	26	.398179	530	666	.398315	.601685	334	.05	.999864	34
5220	27	.403199	529	668	.403338	.596662	332	.05	.999861	33
5280	28	.408161	527	670	.408304	.591696	330	.05	.999858	32
5340	29	.413068	526	672	.413213	.586787	328	.07	.999854	31
5400	30	.417919	525	674	.418068	.581932	326	.05	.999851	30
5460	31	8.422717	524	676	8.422869	11.577131	324	.05	9.999848	29
5520	32	.427462	523	679	.427618	.572382	321	.07	.999844	28
5580	33	.432156	522	681	.432315	.567685	319	.05	.999841	27
5640	34	.436800	521	683	.436962	.563038	317	.05	.999838	26
5700	35	.441394	520	685	.441560	.558440	315	.07	.999834	25
5760	36	.445941	518	688	.446110	.553890	312	.05	.999831	24
5820	37	.450440	517	690	.450613	.549387	310	.07	.999827	23
5880	38	.454893	516	693	.455070	.544930	307	.05	.999824	22
5940	39	.459301	515	695	.459481	.540519	305	.07	.999820	21
6000	40	.463665	514	697	.463849	.536151	303	.07	.999816	20
6060	41	8.467985	512	700	8.468172	11.531828	300	.05	9.999813	19
6120	42	.472263	511	702	.472454	.527546	298	.07	.999809	18
6180	43	.476498	510	705	.476693	.523307	295	.07	.999805	17
6240	44	.480693	509	707	.480892	.519108	293	.07	.999801	16
6300	45	.484848	507	710	.485050	.514950	290	.07	.999797	15
6360	46	.488963	506	713	.489170	.510830	287	.05	.999794	14
6420	47	.493040	505	715	.493250	.5 6750	285	.07	.999790	13
6480	48	.497078	503	718	.497293	.502707	282	.07	.999786	12
6540	49	.501030	502	720	.501298	.498702	280	.07	.999782	11
6600	50	.505045	501	723	.505267	.494733	277	.07	.999778	10
6660	51	8.508974	499	726	8.509200	11.490800	274	.07	9.999774	9
6720	52	.512867	498	729	.513098	.486902	271	.08	.999769	8
6780	53	.516726	497	731	.516961	.483039	269	.07	.999765	7
6840	54	.520551	495	734	.520790	.479210	266	.07	.999761	6
6900	55	.524343	494	737	.524586	.475414	263	.07	.999757	5
6960	56	.528102	492	740	.528349	.471651	260	.07	.999753	4
7020	57	.531828	491	743	.532080	.467920	257	.08	.999748	3
7080	58	.535523	490	745	.535779	.464221	255	.07	.999744	2
7140	59	.539186	488	748	.539447	.460553	252	.07	.999740	1
7200	60	8.542819	487	751	8.543084	11.456916	249	.08	9.999735	0
″	′		4.685				15.314			′
91°		Cosine.	$q-l$		Cotang.	Tang.	$q+l$	$D1''$	Sine.	88°

Cosines, Tangents, and Cotangents,

2°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	177°
0′	8.542819	60.05	9.999735	.07	8.543084	60.12	11.456916	60′
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.72	.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.83	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.43	.999689	.07	.577877	55.52	.422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.40	.999665	.08	.594283	53.48	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	.609734	51.48	.999640	.08	.610094	51.58	.389906	40
21	8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.10	.625352	49.80	.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	.639680	48.05	.999586	.08	.640093	48.15	.359907	30
31	8.642563	47.75	9.999581	.10	8.642982	47.85	11.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	.653911	46.52	.999558	.08	.654352	46.62	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	.667689	45.07	.999529	.08	.668160	45.17	.331840	20
41	8.670393	44.78	9.999524	.10	8.670870	44.88	11.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.12	.681544	43.80	.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.53	.305471	10
51	8.696543	42.17	9.999463	.12	8.697081	42.27	11.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	.706577	41.20	.999437	.10	.707140	41.30	.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.12	.714534	40.63	.285466	2
59	.716388	40.28	.999411	.12	.716972	40.40	.283028	1
60′	8.718800		9.999404		8.719396		11.280604	0′
92°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	87°

27. Logarithmic Sines,

3°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	176
0′	8.718800	40.07	9.999404	.10	8.719396	40.17	11.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37.77	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36.18	.234754	40
21	8.76667[illegible]	35.88	9.999257	.12	8.767417	36.02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	.780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9.999181	.12	8.788554	34.32	11.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	.792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.55	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	.812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821343	31.62	.999044	.13	.822298	31.78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.13	.826103	31.48	.173897	10
51	8.827011	31.22	9.999019	.15	8.827992	31.37	11.172008	9
52	.828884	31.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.13	.835471	30.83	.164529	5
56	.836297	30.55	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30.30	.998958	.13	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.15	.842825	30.32	.157175	1
60′	8.843585		9.998941		8.844644		11.155356	0
93°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	86°

Cosines, Tangents, and Cotangents

4°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	175°
0′	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60′
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29.00	.137567	50
11	8.863014	28 73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998[illegible]23	.17	.867632	28.65	132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	998708	.15	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	894643	26.72	.998659	.17	.895984	26.87	.1[illegible]6	30
31	8 896246	26 60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26.50	998639	.17	[illegible]	26.67	.100797	28
33	.899432	26 42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	998619	.17	902398	26.48	.097602	26
35	.902596	26 22	998609	.17	.903987	26.38	.096013	25
36	.904169	26 12	.998599	.17	.905570	26.28	.094430	24
37	905736	26.02	998589	.18	.907147	26.20	.092853	23
38	907297	25 93	.998578	.17	908719	26.10	091281	22
39	908853	25 85	.998568	.17	910285	26 02	.089715	21
40	.910404	25.75	998558	.17	.911846	25.92	.088154	20
41	8 911949	25 65	9 998548	.18	8 913401	25 83	11.086599	19
42	.913488	25 57	998537	.17	.914951	25.73	085049	18
43	915022	25 47	998527	.18	.916495	25.65	.083505	17
44	.916550	25 38	998516	.17	918034	25.57	.081966	16
45	.918073	25 30	.998506	.18	919568	25.47	.080432	15
46	919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25 12	.998485	.18	.922619	25 28	.077381	13
48	.922610	25.03	.998474	.17	924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25 12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	[illegible].998442	.18	[illegible]	24.95	11.071342	9
52	928587	24.68	.998431	.17	.930155	24 87	.069845	8
53	.930068	24 60	.998421	.18	931647	24.78	.[illegible]	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24 43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.[illegible]	.059506	1
60′	8.940296		9.998344		[illegible]		11.058048	0′
94°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	85°

27. Logarithmic Sines,

5°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	174°
0'	8.940296	24.03	9.998344	.18	8.941952	24.20	11.058048	60'
1	.941738	23.93	.998333	.18	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23.98	.053705	57
4	.946034	23.70	.998300	.18	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.20	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	.953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23.52	.045144	51
10	.954499	23.25	.998232	.20	.956267	23.45	.043733	50
11	8.955894	23.17	9.998220	.18	8.957674	23.35	11.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.20	.961866	23.15	.038134	46
15	.961429	22.87	.998174	.18	.963255	23.07	.036745	45
16	.962801	22.82	.998163	.20	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.92	.033981	43
18	.965534	22.65	.998139	.18	.967394	22.87	.032606	42
19	.966893	22.60	.998128	.20	.968766	22.78	.031234	41
20	.968249	22.52	.998116	.20	.970133	22.72	.029867	40
21	8.969600	22.45	9.998104	.20	8.971496	22.65	11.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22.23	.998068	.20	.975560	22.43	.024440	36
25	.974962	22.18	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.03	.016423	30
31	8.982883	21.77	9.997984	.20	8.984899	21.97	11.015101	29
32	.984189	21.70	997972	.22	.986217	21.92	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21.57	997947	.20	.988842	21.78	.011158	26
35	.988083	21.52	997935	.22	.990149	21.70	.009851	25
36	.989374	21.43	997922	.20	.991451	21.65	.008549	24
37	.990660	21.38	997910	.22	.992750	21.58	.007250	23
38	.991943	21.32	.997897	.20	.994045	21.53	.005955	22
39	.993222	21.25	997885	.22	.995337	21.45	.004663	21
40	.994497	21.18	.997872	.20	.996624	21.40	.003376	20
41	8.995768	21.13	9.997860	.22	8.997908	21.33	11.002092	19
42	.997036	21.05	.997847	.20	8.999188	21.28	11.000812	18
43	.998299	21.02	997835	.22	9.000465	21.22	10.999535	17
44	8.999560	20.93	.997822	.22	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.20	.003007	21.08	.996993	15
46	.002069	20.82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20.75	.997784	.22	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	.007044	20.57	.997745	.22	.009298	20.80	.990702	10
51	9.008278	20.53	9.997732	.22	9.010546	20.73	10.989454	9
52	.009510	20.45	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20.57	.985732	6
55	.013182	20.30	.997680	.22	.015502	20.50	.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60'	9.019235		9.997614		9.021620		10.978380	0
95°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	84°

Cosines, Tangents, and Cotangents

6°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	173°
0′	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60′
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19 52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19 40	9.997466	.23	9.034791	19.63	10.965209	49
12	033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19 32	.997439	.23	.037144	19.53	.962856	47
14	035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19 05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9 043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18 83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18 08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.23	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	079676	17.38	.996843	.27	.082833	17.63	.917167	6
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60′	9.085894		9.996751		9.089144		10.910856	0′
96°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	83°

27. Logarithmic Sines,

7°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	172°
0'	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60'
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	.129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.[illegible]	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	[illegible]	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	[illegible]	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	[illegible]	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	[illegible]	15.00	.995771	.30	.146885	15.30	.853115	1
60'	9.143555		9.995753		9.147803		10.852197	0'
97°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	82°

Cosines, Tangents, and Cotangents

8°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	171°
0'	9.143555	14.97	9.995753	.30	9.147803	15.25	10.852197	60'
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	.148026	14.82	.995664	.30	.152363	15.10	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	.152451	14.65	.995573	.30	.156877	14.97	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	.156830	14.50	.995482	.30	.161347	14.82	.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	.161164	14.35	.995390	.30	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.32	9.166654	14.63	10.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.32	.170157	14.53	.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.08	.995203	.32	.174499	14.38	.825501	30
31	9.170547	14.03	9.995184	.32	9.175362	14.37	10.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.93	.995108	.32	.178799	14.27	.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	.178072	13.80	.995013	.33	.183059	14.13	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.08	10.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	14.00	.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.87	.808538	10
51	9.187092	13.52	9.994798	.32	9.192294	13.83	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	.190325	13.42	.994720	.33	.195606	13.73	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.65	.801106	1
60'	9.194332		9.994620		9.199713		10.800287	0
98°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	81°

27. Logarithmic Sines,

9°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	170°
0'	9.194332	13.28	9.994620	.33	9.199713	13.60	10.800287	60'
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13.52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9.994398	.35	9.208619	13.35	10.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.35	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.35	.214989	13.18	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13.13	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13.07	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.65	.994087	.35	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12.95	.777170	31
30	.217609	12.57	.994003	.35	.223607	12.92	.776393	30
31	9.218363	12.55	9.993982	.37	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.35	.226700	12.85	.77330[illegible]	26
35	.221367	12.47	.993897	.37	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.35	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.35	.231302	12.72	.768698	20
41	9.225833	12.33	9.993768	.37	9.232065	12.68	10.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.35	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60'	9.239670		9.993351		9.246319		10.753681	0'
99°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	80°

Cosines, Tangents, and Cotangents

10°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	169°
0′	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60′
1	.240386	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.23	.750736	56
5	.243237	11.83	.993240	.38	.249998	12.20	.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.37	.252920	12.13	.747080	51
10	.246775	11.72	.993127	.38	.253648	12.10	.746352	50
11	9.247478	11.72	9.993104	.38	9.254374	12.10	10.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.02	.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11.57	.992921	.38	.260146	11.95	.739854	41
20	.253761	11.53	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11.48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	.257211	11.45	.992783	.40	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.38	.265138	11.82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	.38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.40	.267261	11.77	.732739	31
30	.260633	11.35	.992666	.38	.267967	11.73	.732033	30
31	9.261314	11.33	9.992643	.40	9.268671	11.73	10.731329	29
32	.261994	11.32	.992619	.38	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.40	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.38	.270779	11.67	.729221	26
35	.264027	11.27	.992549	.40	.271479	11.65	.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.38	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274269	11.58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11.57	.725036	20
41	9.268065	11.15	9.992406	.40	9.275658	11.55	10.724342	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11.40	.718826	11
50	.274049	10 98	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	.277337	10.90	.992069	.42	.285268	11.32	.714732	5
56	.277991	10.90	.992044	.40	.285947	11.28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60′	9.280599		9.991947		9.288652		10.711348	0′
100°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	79°

27. Logarithmic Sines,

11°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	168°
0′	9.280599	10.82	9.991947	.42	9.288652	11.23	10.711348	60′
1	.281248	10.82	.991922	.42	.289326	11.22	.710674	59
2	.281897	10.78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.42	.290671	11.18	.709329	57
4	.283190	10.77	.991848	.42	.291342	11.18	.708658	56
5	.283836	10.73	.991823	.40	.292013	11.15	.707987	55
6	.284480	10.73	.991799	.42	.292682	11.13	.707318	54
7	.285124	10.70	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.70	.991749	.42	.294017	11.12	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.08	.705316	51
10	.287048	10.67	.991699	.42	.295349	11.07	.704651	50
11	9.287688	10.63	9.991674	.42	9.296013	11.07	10.703987	49
12	.288326	10.63	.991649	.42	.296677	11.03	.703323	48
13	.288964	10.60	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.60	.991599	.42	.298001	11.02	.701999	46
15	.290236	10.57	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.57	.991549	.42	.299322	10.97	.700678	44
17	.291504	10.55	.991524	.43	.299980	10.97	.700020	43
18	.292137	10.52	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.52	.991473	.42	.301295	10.93	.698705	41
20	.293399	10.50	.991448	.43	.301951	10.93	.698049	40
21	9.294029	10.48	9.991422	.42	9.302607	10.90	10.697393	39
22	.294658	10.47	.991397	.42	.303261	10.88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.88	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.85	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.85	.694782	35
26	.297164	10.40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10.82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.35	.991218	.42	.307816	10.78	.692184	31
30	.299655	10.35	.991193	.43	.308463	10.77	.691537	30
31	9.300276	10.32	9.991167	.43	9.309109	10.75	10.690891	29
32	.300895	10.32	.991141	.43	.309754	10.75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10.72	.689601	27
34	.302132	10.27	.991090	.43	.311042	10.72	.688958	26
35	.302748	10.27	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.63	.685753	21
40	.305819	10.18	.990934	.43	.314885	10.63	.685115	20
41	9.306430	10.18	9.990908	.43	9.315523	10.60	10.684477	19
42	.307041	10.15	.990882	.45	.316159	10.60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.43	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.43	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10.08	.990750	.43	.319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	.680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	.311893	10.03	.990671	.43	.321222	10.48	.678778	10
51	9.312495	10.03	9.990645	.45	9.321851	10.47	10.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9.98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	.314897	9.97	.990538	.45	.324358	10.42	.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10.40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60′	9.317879		9.990404		9.327475		10.672525	0′
101°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	78°

Cosines, Tangents, and Cotangents

12°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	167°
0'	9.317879	9.90	9.990404	.43	9.327475	10.33	10.672525	60'
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	990161	.45	.333033	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9.68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9.60	.989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9.72	.642434	10
51	9.347134	9.22	9.988985	.48	9.358149	9.70	10.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.63	.637213	1
60'	9.352088		9.988724		9.363364		10.636636	0'
102°	Cosine.	D. 1".	Sine,	D. 1".	Cotang.	D. 1".	Tang.	77°

27. Logarithmic Sines,

13°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	166°
0′	9.352088	9.12	9.988724	.48	9.363364	9.60	10.636636	60′
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	.50	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.48	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.48	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.50	.366237	9.55	.633763	55
6	.355358	9.05	.988548	.48	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9.52	.632047	52
9	.356984	9.00	.988460	.50	.368524	9.50	.631476	51
10	.357524	9.00	.988430	.48	.369094	9.48	.630906	50
11	9.358064	8.98	9.988401	.50	9.369663	9.48	10.630337	49
12	.358603	8.97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8.95	.988342	.50	.370799	9.47	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.43	.628633	46
15	.360215	8.95	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.48	.372499	9.42	.627501	44
17	.361287	8.92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.88	.988163	.50	.374193	9.38	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	.363954	8.85	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.27	.619646	30
31	9.368711	8.75	9.987801	.50	9.380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.72	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8.70	.987679	.50	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10.613562	19
42	.374452	8.63	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.60	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.12	.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.57	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
50	.378577	8.53	.987217	.52	.391360	9.05	.608640	10
51	9.379089	8.53	9.987186	.52	9.391903	9.07	10.608097	9
52	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	.381134	8.48	.987061	.52	.394073	9.02	.605927	5
56	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
60′	9.383675		9.986904		9.396771		10.603229	0′
103°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	76°

Cosines, Tangents, and Cotangents

14°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	165°
0′	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60′
1	.384182	3.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	.412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.53	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.53	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.48	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60′	9.412996		9.984944		9.428052		10.571948	0′
104°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	75°

27. Logarithmic Sines,

15°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	164°
0′	9.412996	7.85	9.984944	.57	9.428052	8.43	10.571948	60′
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.80	.984774	.57	.430573	8.37	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	.417684	7.77	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.33	10.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.28	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.57	9.438554	8.23	10.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.22	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.58	.983911	.60	.442988	8.18	557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.15	10.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	.429170	7.55	.983735	.58	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.58	.447870	8.10	.552130	20
41	9.431879	7.50	9.983523	.60	9.448356	8.08	10.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	.433675	7.45	.983381	.60	.450294	8.05	.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.43	.983273	.58	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	.435908	7.42	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.42	9.983166	.60	9.453187	8.02	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60′	9.440338		9.982842		9.457496		10.542504	0′
105°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	74°

Cosines, Tangents, and Cotangents

16°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	163°
0′	9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60′
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	.442535	7.30	.982660	.60	.459875	7.90	.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	.444720	7.25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	.446893	7.22	.982294	.62	.464599	7.83	.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.13	.981924	.63	.469280	7.77	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7.70	.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7.60	.519199	10
51	9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60′	9.465935		9.980596		9.485339		10.514661	0′
106°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	73°

27. Logarithmic Sines,

17°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	162°
0′	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60′
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.22	.492973	11
50	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60′	9.489982		9.978206		9.511776		10.488224	0
107°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	72°

Cosines, Tangents, and Cotangents

18°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	161°
0'	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.183943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.37	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	[illegible]
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	[illegible]
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6 95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.[illegible]2911[illegible]	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60'	9.512642		9.975670		9.536972		10.463028	0'
108°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	71°

27. Logarithmic Sines,

19°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	160°
0′	9.512642	6.12	9.975670	.72	9.536972	6.83	10.463028	60′
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.82	.460980	55
6	.514837	6.08	.975408	.72	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	.516294	6.05	.975233	.73	.541061	6.78	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6.75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.73	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	.519911	6.00	.974792	.73	.545119	6.75	.454881	40
21	9.520271	6.00	9.974748	.75	9.545524	6.73	10.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.75	.547138	6.70	.452862	35
26	.522066	5.97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6.65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6.63	10.446452	19
42	.527753	5.87	.973807	.77	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5.85	.973671	.77	.555139	6.62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
50	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
51	9.530915	5.83	9.973398	.77	9.557517	6.60	10.442483	9
52	.531265	5.82	.973352	.75	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.77	.558703	6.57	.441297	6
55	.532312	5.82	.973215	.77	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.75	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6.57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
60′	9.534052		9.972986		9.561066		10.438934	0′
109°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	70°

Cosines, Tangents, and Cotangents

20°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	159°
0'	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60'
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	.535783	5.77	.972755	.77	.563028	6.52	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.78	9.565373	6.50	10.434627	49
12	.538194	5.73	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	.539223	5.70	.972291	.77	.566932	6.47	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.68	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.47	.431127	40
21	9.541272	5.68	9.972011	.78	9.569261	6.45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
41	9.548024	5.58	9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	.549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.60	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552 18	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.28	.416200	1
60'	9.554329		9.970152		9.584177		10.415823	0'
110°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	69°

27. Logarithmic Sines,

21°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 158°	
0'	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60'
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	
60'	9.573575		9.967166		9.606410		10.393590	
111° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	6[illegible]

Cosines, Tangents, and Cotangents

22°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	157°
0′	9.573575	5.22	9.967166	.85	9.606410	6.05	10.393590	60′
1	.573888	5.20	.967115	.85	.606773	6.07	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5.20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.85	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6.03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6.03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6.00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6.02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6.00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5.15	.966292	.87	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.87	.612921	6.00	.387079	42
19	.579470	5.12	.966188	.87	.613281	6.00	.386719	41
20	.579777	5.13	.966136	.85	.613641	5.98	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5.10	.965720	.87	.616509	5.97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5.95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9.617582	5.95	10.382418	29
32	.583449	5.08	.965511	.88	.617939	5.93	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.95	.381705	27
34	.584058	5.05	.965406	.88	.618652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.07	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.03	.965195	.87	.620076	5.93	.379924	22
39	.585574	5.05	.965143	.88	.620432	5.92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.02	.964984	.88	.621497	5.92	.378503	18
43	.586783	5.03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.03	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.02	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5.90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5.88	.376377	12
49	.588590	5.00	.964613	.88	.623976	5.90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5.88	.375670	10
51	9.589190	4.98	9.964507	.88	9.624683	5.88	10.375317	9
52	.589489	5.00	.964454	.90	.625036	5.87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5.87	.373203	8
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	.591580	4.97	.964080	.90	.627501	5.85	.372499	1
60′	9.591878		9.964026		9.627852		10.372148	0
112°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	67°

27. Logarithmic Sines,

23°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	156°
0′	9.591878	4.97	9.964026	.90	9.627852	5.85	10.372148	60′
1	.592176	4.95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593362	4.93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9.963434	.92	9.631704	5.82	10.368296	49
12	.595432	4.92	.963379	.90	.632053	5.82	.367947	48
13	.595727	4.90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.82	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.90	.963163	.92	.633447	5.80	.366553	44
17	.596903	4.88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4.90	.963054	.92	.634143	5.78	.365857	42
19	.597490	4.88	.962999	.90	.634490	5.80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4.88	9.962890	.90	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.92	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.90	.635879	5.78	.364121	37
24	.598952	4.87	.962727	.92	.636226	5.77	.363774	36
25	.599244	4.87	.962672	.92	.636572	5.78	.363428	35
26	.599536	4.85	.962617	.92	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.90	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.92	.637611	5.75	.362389	32
29	.600409	4.85	.962453	.92	.637956	5.77	.362044	31
30	.600700	4.83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4.83	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4.82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4.82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.80	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4.80	9.961791	.93	9.642091	5.72	10.357909	19
42	.604170	4.78	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.80	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.72	.356537	15
46	.605319	4.78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4.77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.77	.961290	.92	.645174	5.70	.354826	10
51	9.606751	4.75	9.961235	.93	9.645516	5.68	10.354484	9
52	.607036	4.77	.961179	.93	.645857	5.70	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.68	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.75	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.73	.960955	.93	.647222	5.67	.352778	4
57	.608461	4.73	.960899	.93	.647562	5.68	.352438	3
58	.608745	4.73	.960843	.95	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.93	.648243	5.67	.351757	1
60′	9.609313		9.960730		9.648583		10.351417	0′
113°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	66°

Cosines, Tangents, and Cotangents

24°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	155°
0'	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60'
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.93	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	[illegible]	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60'	9.625948		9.957276		9.668673		10.331327	0'
114°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	65°

27. Logarithmic Sines,

25°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 154
0'	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327
1	.626219	4.52	.957217	.98	.669002	5.50	.330998
2	.626490	4.50	.957158	.98	.669332	5.48	.330668
3	.626760	4.50	.957099	.98	.669661	5.50	.330339
4	.627030	4.50	.957040	.98	.669991	5.48	.330009
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680
6	.627570	4.50	.956921	.98	.670649	5.47	.329351
7	.627840	4 48	.956862	.98	.670977	5.48	.329023
8	.628109	4.48	.956803	.98	.671306	5.48	.328694
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365
10	.628647	4.48	.956684	.98	.671963	5.47	.328037
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381
13	.629453	4.47	.956506	.98	.672947	5.45	.327053
14	.629721	4.47	.956447	1.00	.673274	5 47	.326726
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398
16	.630257	4.45	.956327	.98	.673929	5.47	.326071
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416
19	.631059	4.45	.956148	.98	.674911	5.43	.325089
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480
28	.633454	4.42	.955609	.98	.677846	5.42	.322154
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529
34	.635042	4.40	.955247	1.02	.679795	5 42	.320205
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232
38	.636097	4 38	.955005	1.02	.681092	5.40	.318908
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644
46	.638197	4 35	.954518	1.02	.683679	5.37	.316321
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032
51	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745
55	.640544	4.33	.953968	1.03	.686577	5.35	.313423
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139
60'	9.641842		9.953660		9.688182		10.311818
115°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.

Cosines, Tangents, and Cotangents

26°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	153°
0'	9.641842	4.32	9.953660	1.02	9.688182	5.33	10.311818	60'
1	.642101	4.32	.953599	1.03	.688502	5.35	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5.33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.02	.689463	5.33	.310537	56
5	[illegible]	4.30	.953352	1.03	.689783	5.33	.310217	55
6	[illegible]	4.28	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.30	.953228	1.03	.690423	5.32	.309577	53
8	.643908	4.28	.953166	1.03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1.03	.691062	5.32	.308938	51
10	.644423	4.28	.953042	1.03	.691381	5.32	.308619	50
11	9.644680	4.27	9.952980	1.03	9.691700	5.32	10.308300	49
12	.644936	4.28	.952918	1.05	.692019	5.32	.307981	48
13	.645193	4.28	.952855	1.03	.692338	5.30	.307662	47
14	.645450	4.27	.952793	1.03	.692656	5.32	.307344	46
15	.645706	4.27	.952731	1.03	.692975	5.30	.307025	45
16	.645962	4.27	.952669	1.05	.693293	5.32	.306707	44
17	.646218	4.27	.952606	1.03	.693612	5.30	.306388	43
18	.646474	4.25	.952544	1.05	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.03	.694248	5.30	.305752	41
20	[illegible]	4.27	.952419	1.05	.694566	5.28	.305434	40
21	9.647240	4.23	9.952356	1.03	9.694883	5.30	10.305117	39
22	[illegible]	4.25	.952294	1.05	.695201	5.28	.304799	38
23	.647749	4.25	.952231	1.05	.695518	5.30	.304482	37
24	.648004	4.23	.952168	1.03	.695836	5.28	.304164	36
25	[illegible]	4.23	.952106	1.05	.696153	5.28	.303847	35
26	[illegible]	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.27	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	.649527	4.23	.951791	1.05	.697736	5.28	.302264	30
31	9.649781	4.22	9.951728	1.05	9.698053	5.27	10.301947	29
32	[illegible]	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.20	.95160[illegible]	1.05	.698685	5.27	.301315	27
34	.650539	4.22	.951539	1.05	.699001	5.25	.300999	26
35	.650792	4.20	.951476	1.07	.699316	5.27	.300684	25
36	.651044	4.22	.951412	1.05	.699632	5.25	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.27	.300053	23
38	.651549	4.18	.951286	1.07	.700263	5.25	.299737	22
39	.651800	4.20	.951222	1.05	.700578	5.25	.299422	21
40	[illegible]	4.20	.951159	1.05	.700893	5.25	.299107	20
41	9.652304	4.18	9.951096	1.07	9.701208	5.25	10.298792	19
42	.652555	4.18	.951032	1.07	.701523	5.23	.298477	18
43	[illegible]	4.18	.950968	1.05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5.23	.297848	16
45	.653308	4.17	.950841	1.05	.702466	5.25	.297534	15
46	.653558	4.17	.950778	1.07	.702781	5.23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5.23	.296278	11
50	.654558	4.17	.950522	1.07	.704036	5.23	.295964	10
51	9.654808	4.17	9.950458	1.07	9.704350	5.22	10.295650	9
52	[illegible]	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.23	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.13	.950138	1.07	.705916	5.20	.294084	4
57	.656302	4.15	.950074	1.07	.706228	5.22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60'	9.657047		9.949881		9.707166		10.292834	0'
116°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	63°

27. Logarithmic Sines,

27°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	152°
0'	9.657047	4.13	9.949881	1.08	9.707166	5.20	10.292834	60'
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.07	.708726	5.18	.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	.660746	4.08	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1 08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	.661970	4.07	.948584	1.08	.713386	5.17	.286614	40
21	9.662214	4.08	9.948519	1.08	9.713696	5.15	10.286304	
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	
23	.662703	4 05	.948388	1.08	.714314	5.17	.285686	
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	
25	.663190	4.05	.948257	1.08	.714933	5.15	.285067	
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	
28	.663920	4.05	.948060	1.08	.715860	5 13	.284140	
29	.664163	4.05	.947995	1.10	.716168	5 15	.283832	
30	.664406	4.03	.947929	1.10	.716477	5.13	.283523	
31	9.664648	4.05	9.947863	1.10	9.716785	5.13	10.283215	
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	
33	.665133	4 03	.947731	1.10	.717401	5 13	.282599	
34	.665375	4.03	.947665	1.08	.717709	5 13	.282291	
35	.665617	4 03	.947600	1.12	.718017	5 13	.281983	
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	
37	.666100	4 03	.947467	1 10	.718633	5.12	.281367	
38	.666342	4 02	.947401	1.10	.718940	5.13	.281060	
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	
40	.666824	4.02	.947269	1.10	.719555	5.12	.280445	
41	9.667065	4.00	9.947203	1.12	9.719862	5 12	10.280138	
42	.667305	4.02	.947136	1 10	.720169	5 12	.279831	
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	
44	667786	4.02	.947004	1.12	.720783	5.10	.279217	
45	.668027	4 00	.946937	1.10	.721089	5.12	.278911	
46	.668267	3.98	.946871	1.12	.721396	5 10	.278604	
47	.668506	4.00	.946804	1 10	.721702	5.12	.278298	
48	.668746	4 00	.946738	1 12	.722009	5.10	.277991	
49	.668986	3.98	.946671	1 12	.722315	5.10	.277685	
50	.669225	3.98	.946604	1.10	.722621	5.10	.277379	
51	9.669464	3 98	9 946538	1.12	9.722927	5.08	10.277073	
52	669703	3.98	.946471	1.12	.723232	5 10	.276768	
53	.669942	3 98	.946404	1.12	.723538	5 10	.276462	
54	670181	3.97	.946337	1 12	.723844	5.08	.276156	
55	.670419	3 98	.946270	1.12	724149	5.08	.275851	
56	.670658	3 97	.946203	1.12	724454	5 10	.275546	
57	.670896	3 97	.946136	1.12	.724760	5.08	275240	
58	.671134	3 97	.946069	1.12	.725065	5.08	.274935	
59	.671372	3.95	946002	1.12	725370	5.07	.274630	
60'	9.671609		9 945935		9 725674		10.274326	
117°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

Cosines, Tangents, and Cotangents

28°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	151°
0'	9.671609	3.97	9.945935	1.12	9.725674	5.08	10.274326	60'
1	.671847	3.95	.945868	1.13	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.13	.726892	5.08	.273108	56
5	[illegible]	3 95	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.93	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.95	.945464	1.13	.727805	5.07	.272195	53
8	.673505	3.93	.945396	1.13	.728109	5.05	.271891	52
9	.673741	3.93	.945328	1.12	.728412	5.07	.271588	51
10	[illegible]	3.93	.945261	1.13	.728716	5.07	.271284	50
11	9.674213	3.92	9.945193	1.13	9.729020	5.05	10.270980	49
12	.674448	3 93	.945125	1.12	.729323	5.05	.270677	48
13	.674684	3.92	945058	1.13	.729626	5.05	.270374	47
14	.674919	3.93	.944990	1.13	.729929	5.07	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.03	.269767	45
16	.675390	3.90	.944854	1.13	.730535	5.05	.269465	44
17	675624	3.92	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3 92	.944718	1.13	.731141	5.05	.268859	42
19	.676094	3 90	.944650	1.13	.731444	5.03	.268556	41
20	.676328	3.90	.944582	1.13	.731746	5.03	.268254	40
21	9.676562	3.90	9.944514	1.13	9.732048	5.05	10.267952	39
22	.676796	3.90	.944446	1.15	.732351	5.03	.267649	38
23	.677030	3 90	.944377	1.13	.732653	5.03	.267347	37
24	.677264	3.90	.944309	1.13	.732955	5.03	.267045	36
25	.677498	3 88	.944241	1.15	.733257	5.02	.266743	35
26	.677731	3 88	.944172	1.13	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.13	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.15	.734162	5.02	.265838	32
29	678430	3 88	.943967	1.13	.734463	5.02	.265537	31
30	.678663	3.87	.943899	1.15	.734764	5.03	.265236	30
31	9.678895	3 88	9 943830	1.15	9.735066	5 02	10.264934	29
32	.679128	3.87	.943761	1.13	.735367	5.02	.264633	28
33	.679360	3 87	.943693	1.15	.735668	5.02	.264332	27
34	.679592	3.87	943624	1.15	.735969	5.00	.264031	26
35	.679824	3.87	.943555	1.15	.736269	5.02	.263731	25
36	.680056	3.87	.943486	1.15	.736570	5.00	.263430	24
37	.680288	3 85	.943417	1.15	.736870	5.02	.263130	23
38	.680519	3 85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.87	.943279	1.15	.737471	5.00	.262529	21
40	.680982	3.85	.943210	1.15	.737771	5.00	.262229	20
41	9.681213	3.83	9.943141	1.15	9.738071	5.00	10.261929	19
42	.681443	3 85	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3 85	943003	1 15	.738671	5 00	.261329	17
44	.681905	3.83	942934	1.17	.738971	5.00	.261029	16
45	.682135	3 83	.942864	1.15	.739271	4 98	.260729	15
46	.682365	3.83	.942795	1.15	.739570	5.00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3.83	.942656	1.15	.740169	4.98	.259831	12
49	.683055	3 82	.942587	1.17	.740468	4.98	.259532	11
50	.683284	3.83	.942517	1.15	.740767	4.98	.259233	10
51	9.683514	3 82	9.942448	1.17	9.741066	4.98	10.258934	9
52	.683743	3.82	.942378	1.17	.741365	4.98	.258635	8
53	.683972	3 82	.942308	1.15	.741664	4.97	.258336	7
54	.684201	3.82	.942239	1.17	.741962	4.98	.258038	6
55	.684430	3 80	.942169	1.17	.742261	4.97	.257739	5
56	.684658	3.82	.942099	1.17	.742559	4.98	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60'	9.685571		9.941819		9.743752		10.256248	0'
118°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	61°

27. Logarithmic Sines,

29°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	150°
0'	9.685571	3.80	9.941819	1.17	9.743752	4.97	10.256248	60'
1	.685799	3.80	.941749	1.17	.744050	4.97	.255950	59
2	.686027	3.78	.941679	1.17	.744348	4.95	.255652	58
3	.686254	3.80	.941609	1.17	.744645	4.97	.255355	57
4	.686482	3.78	.941539	1.17	.744943	4.95	.255057	56
5	.686709	3 78	.941469	1.18	.745240	4.97	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.77	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.18	.746132	4.95	.253868	52
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571	51
10	.687843	3.77	.941117	1.18	.746726	4.95	.253274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.93	10.252977	49
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681	48
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384	47
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087	46
15	.688972	3.77	.940763	1.17	.748209	4.93	.251791	45
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311	40
21	9.690323	3.75	9.940338	1.18	9.749985	4.93	10.250015	39
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719	38
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424	37
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.20	.751167	4.92	.248833	35
26	.691444	3 73	.939982	1.18	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243	33
28	.691892	3 72	.939840	1.20	.752052	4.92	.247948	32
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653	31
30	.692339	3.72	.939697	1.20	.752642	4.92	.247358	30
31	9.692562	3.72	9.939625	1.18	9.752937	4.90	10.247063	29
32	.692785	3 72	.939554	1.20	.753231	4 92	.246769	2
33	.693008	3.72	.939482	1 20	.753526	4.90	.246474	2
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180	2
35	.693453	3.72	.939339	1.20	.754115	4.90	.245885	2
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	2
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	2
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003	2
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	2
40	.694564	3.70	.938980	1.20	.755585	4.88	.244415	2
41	9.694786	3 68	9.938908	1.20	9.755878	4.90	10.244122	1
42	.695007	3 70	.938836	1.22	.756172	4.88	.243828	1
43	.695229	3 68	.938763	1.20	.756465	4.90	.243535	1
44	695450	3 68	.938691	1.20	.756759	4.88	.243241	1
45	.695671	3.68	.938619	1.20	.757052	4.88	.242948	1
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	1
47	.696113	3 68	.938475	1.22	.757638	4 88	.242362	1
48	.696334	3 67	.938402	1.20	.757931	4.88	.242069	1
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776	1
50	.696775	3.67	.938258	1.22	.758517	4.88	.241483	1
51	9.696995	3.67	9.938185	1.20	9.758810	4.87	10.241190	
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898	
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605	
54	.697654	3 67	.937967	1.20	.759687	4.87	.240313	
55	.697874	3 67	.937895	1.22	.759979	4.88	.240021	
56	.698094	3 65	.937822	1.22	.760272	4.87	.239728	
57	.698313	3 65	.937749	1.22	.760564	4.87	.239436	
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144	
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852	
60'	9.698970		9.937531		9.761439		10.238561	
119° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	6

Cosines, Tangents, and Cotangents

30°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	149°
0'	9.698970	3.65	9.937531	1.22	9.761439	4.87	10.238561	60'
1	.699189	3.63	.937458	1.22	.761731	4.87	.238269	59
2	.699407	3.65	.937385	1.22	.762023	4.85	.237977	58
3	.699626	3.63	.937312	1.23	.762314	4.87	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.62	.936946	1.23	.763770	4.85	.236230	52
9	.700933	3.63	.936872	1.22	.764061	4.85	.235939	51
10	.701151	3.62	.936799	1.23	.764352	4.85	.235648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.83	10.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.85	.235067	48
13	.701802	3.62	.936578	1.22	.765224	4.83	.234776	47
14	.702019	3.62	.936505	1.23	.765514	4.85	.234486	46
15	.702236	3.60	.936431	1.23	.765805	4.83	.234195	45
16	.702452	3.62	.936357	1.22	.766095	4.83	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	.703317	3.60	.936062	1.23	.767255	4.83	.232745	40
21	9.703533	3.60	9.935988	1.23	9.767545	4.82	10.232455	39
22	.703749	3.58	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.58	.935840	1.23	.768124	4.83	.231876	37
24	.704179	3.60	.935766	1.23	.768414	4.82	.231586	36
25	.704395	3.58	.935692	1.23	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.25	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.23	.769281	4.83	.230719	33
28	.705040	3.57	.935469	1.23	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.25	.769860	4.80	.230140	31
30	.705469	3.57	.935320	1.23	.770148	4.82	.229852	30
31	9.705683	3.58	9.935246	1.25	9.770437	4.82	10.229563	29
32	.705898	3.57	.935171	1.23	.770726	4.82	.229274	28
33	.706112	3.57	.935097	1.25	.771015	4.80	.228985	27
34	.706326	3.55	.935022	1.23	.771303	4.82	.228697	26
35	.706539	3.57	.934948	1.25	.771592	4.80	.228408	25
36	.706753	3.57	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.55	.934798	1.25	.772168	4.82	.227832	23
38	.707180	3.55	.934723	1.23	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	.707606	3.55	.934574	1.25	.773033	4.80	.226967	20
41	9.707819	3.55	9.934499	1.25	9.773321	4.78	10.226679	19
42	.708032	3.55	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.55	.934349	1.25	.773896	4.80	.226104	17
44	.708458	3.53	.934274	1.25	.774184	4.78	.225816	16
45	.708670	3.53	.934199	1.27	.774471	4.80	.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.78	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.78	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.80	.224667	12
49	.709518	3.53	.933898	1.27	.775621	4.78	.224379	11
50	.709730	3.52	.933822	1.25	.775908	4.78	.224092	10
51	9.709941	3.53	9.933747	1.27	9.776195	4.78	10.223805	9
52	.710153	3.52	.933671	1.25	.776482	4.77	.223518	8
53	.710364	3.52	.933596	1.27	.776768	4.78	.223232	7
54	.710575	3.52	.933520	1.25	.777055	4.78	.222945	6
55	.710786	3.52	.933445	1.27	.777342	4.77	.222658	5
56	.710997	3.52	.933369	1.27	.777628	4.78	.222372	4
57	.711208	3.52	.933293	1.27	.777915	4.77	.222085	3
58	.711419	3.50	.933217	1.27	.778201	4.78	.221799	2
59	.711629	3.50	.933141	1.25	.778488	4.77	.221512	1
60'	9.711839		9.933066		9.778774		10.221226	0'
120°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	59°

27. Logarithmic Sines,

[illegible]°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 148°	
0'	9.711839	3.52	9.933066	1.27	9.778774	4.77	10.221226	60'
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	.712889	3.48	.932685	1.27	.780203	4.77	.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713308	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.47	9.932228	1.28	9.781916	4.75	10.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.27	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	.716017	3.45	.931537	1.28	.784479	4.75	.215521	40
21	9.716224	3.47	9.931460	1.28	9.784764	4.73	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	.717053	3.43	.931152	1.28	.785900	4.73	.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	.718085	3.43	.930766	1.30	.787319	4.73	.212681	30
31	9.718291	3.43	9.930688	1.28	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	.719114	3.43	.930378	1.30	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	[illegible]
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	[illegible]
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	[illegible]
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	[illegible]
40	.720140	3.42	.929989	1.30	.790151	4.72	.209849	[illegible]
41	9.720345	3.40	9.929911	1.30	9.790434	4.70	10.209566	[illegible]
42	.720549	3.42	.929833	1.3[illegible]	.790716	4.72	.209284	[illegible]
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	[illegible]
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	[illegible]
45	.721162	3.40	.929599	1.30	.791563	4.72	.208437	[illegible]
46	.721366	3.40	.929521	1.32	.791846	4.70	.208[illegible]	[illegible]
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	[illegible]
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	[illegible]
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	[illegible]
50	.722181	3.40	.929207	1.30	.792974	4.70	.207026	[illegible]
51	9.722385	3.38	9.929129	1.32	9.793256	4.70	10.206744	
52	.722588	3.38	.929050	1.30	.793538	4.68	.206462	
53	.722791	3.38	.928972	1.32	.793819	4.70	.206181	
54	.722994	3.38	.928893	1.30	.794101	4.70	.205899	
55	.723197	3.38	.928815	1.32	.794383	4.68	.205617	
56	.723400	3.38	.928736	1.32	.794664	4.70	.205336	
57	.723603	3.37	.928657	1.32	.794946	4.68	.205054	
58	.723805	3.37	.928578	1.32	.795227	4.68	.204773	
59	.724007	3.38	.928499	1.32	.795508	4.68	.204492	
60'	9.724210		9.928420		9.795789		10.204211	
121°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	[illegible]

Cosines, Tangents, and Cotangents

32°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	147°
0'	9.724210	3.37	9.928420	1.30	9.795789	4.68	10.204211	60'
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.35	.928025	1.32	.797194	4.67	.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.33	.798596	4.68	.201404	50
11	9.726426	3.33	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	.727228	3.33	.927231	1.33	.799997	4.67	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.65	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.67	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	.729223	3.32	.926431	1.33	.802792	4.67	.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	.730217	3.30	.926029	1.33	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.35	9.804466	4.65	10.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4.65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	.731206	3.30	.925626	1.35	.805580	4.65	.194420	25
36	.731404	3.30	.925545	1.33	.805859	4.63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1.37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.62	.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924491	1.37	.809471	4.62	.190529	11
50	.734157	3.27	.924409	1.35	.809748	4.62	.190252	10
51	9.734353	3.27	9.924328	1.37	9.810025	4.62	10.189975	9
52	.734549	3.25	.924246	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924164	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924083	1.37	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.37	.811134	4.60	.188866	5
56	.735330	3.25	.923919	1.37	.811410	4.62	.188590	4
57	.735525	3.23	.923837	1.37	.811687	4.62	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.62	.188036	2
59	.735914	3.25	.923673	1.37	.812241	4.60	.187759	1
60'	9.736109		9.923591		9.812517		10.187483	0'
122°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	57°

27. Logarithmic Sines,

33°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	146°
0'	9.736109	3.23	9.923591	1.37	9.812517	4.62	10.187483	60'
1	.736303	3.25	.923509	1.37	.812794	4.60	.187206	59
2	.736498	3.23	.923427	1.37	.813070	4.62	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.38	.813899	4.62	.186101	55
6	.737274	3.22	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.38	.814452	4.6[illegible]	.185548	53
8	.737661	3.23	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	.738048	3.22	.922768	1.37	.815280	4.58	.184720	50
11	9.738241	3.22	9.922686	1.38	9.815555	4.60	10.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.60	.184169	48
13	.738627	3.22	.922520	1.37	.816107	4.58	.183893	47
14	.738820	3.22	.922438	1.38	.816382	4.60	.183618	46
15	.739013	3.22	.922355	1.38	.816658	4.58	.183342	45
16	.739206	3.20	.922272	1.38	.816933	4.60	.183067	44
17	.739398	3.20	.922189	1.38	.817209	4.58	.182791	43
18	.739590	3.22	.922106	1.38	.817484	4.58	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.60	.182241	41
20	.739975	3.20	.921940	1.38	.818035	4.58	.181965	40
21	9.740167	3.20	9.921857	1.38	9.818310	4.58	10.181690	39
22	.740359	3.18	.921774	1.38	.818585	4.58	.181415	38
23	.740550	3.20	.921691	1.40	.818860	4.58	.181140	37
24	.740742	3.20	.921607	1.38	.819135	4.58	.180865	36
25	.740934	3.18	.921524	1.38	.819410	4.57	.180590	35
26	.741125	3.18	.921441	1.40	.819684	4.58	.180316	34
27	.741316	3.20	.921357	1.38	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.40	.820234	4.57	.179766	32
29	.741699	3.17	.921190	1.38	.820508	4.58	.179492	31
30	.741889	3.18	.921107	1.40	.820783	4.57	.179217	30
31	9.742080	3.18	9.921023	1.40	9.821057	4.58	10.178943	2
32	.742271	3.18	.920939	1.38	.821332	4.57	.178668	2
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	2
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	2
35	.742842	3.18	.920688	1.40	.822154	4.58	.177846	2
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	2
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	2
38	.743413	3.15	.920436	1.40	.822977	4.57	.177023	2
39	.743602	3.17	.920352	1.40	.823251	4.55	.176749	2
40	.743792	3.17	.920268	1.40	.823524	4.57	.176476	2
41	9.743982	3.15	9.920184	1.42	9.823798	4.57	10.176202	1
42	.744171	3.17	.920099	1.40	.824072	4.55	.175928	1
43	.744361	3.15	.920015	1.40	.824345	4.57	.175655	1
44	.744550	3.15	.919931	1.42	.824619	4.57	.175381	1
45	.744739	3.15	.919846	1.40	.824893	4.55	.175107	1
46	.744928	3.15	.919762	1.42	.825166	4.55	.174834	1
47	.745117	3.15	.919677	1.40	.825439	4.57	.174561	1
48	.745306	3.13	.919593	1.42	.825713	4.55	.174287	1
49	.745494	3.15	.919508	1.40	.825986	4.55	.174014	1
50	.745683	3.13	.919424	1.42	.826259	4.55	.173741	1
51	9.745871	3.15	9.919339	1.42	9.826532	4.55	10.173468	
52	.746060	3.13	.919254	1.42	.826805	4.55	.173195	
53	.746248	3.13	.919169	1.40	.827078	4.55	.172922	
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	
56	.746812	3.12	.918915	1.42	.827897	4.55	.172103	
57	.746999	3.13	.918830	1.42	.828170	4.53	.171830	
58	.747187	3.12	.918745	1.43	.828442	4.55	.171558	
59	.747374	3.13	.918659	1.42	.828715	4.53	.171285	
60'	9.747562		9.918574		9.828987		10.171013	
123° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	5

Cosines, Tangents, and Cotangents

Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	145°
9.747562	3.12	9.918574	1.42	9.828987	4.55	10.171013	60'
.747749	3.12	.918489	1.42	.829260	4.53	.170740	59
.747936	3.12	.918404	1.43	.829532	4.55	.170468	58
.748123	3.12	.918318	1.42	.829805	4.53	.170195	57
.748310	3.12	.918233	1.43	.830077	4.53	.169923	56
.748497	3.10	.918147	1.42	.830349	4.53	.169651	55
.748683	3.12	.918062	1.43	.830621	4.53	.169379	54
.748870	3.10	.917976	1.42	.830893	4.53	.169107	53
.749056	3.12	.917891	1.43	.831165	4.53	.168835	52
.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
.749429	3.10	.917719	1.42	.831709	4.53	.168291	50
9.749615	3.10	9.917634	1.43	9.831981	4.53	10.168019	49
.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
.749987	3.08	.917462	1.43	.832525	4.52	.167475	47
.750172	3.10	.917376	1.43	.832796	4.53	.167204	46
.750358	3.08	.917290	1.43	.833068	4.52	.166932	45
.750543	3.10	.917204	1.43	.833339	4.53	.166661	44
.750729	3.08	.917118	1.43	.833611	4.52	.166389	43
.750914	3.08	.917032	1.43	.833882	4.53	.166118	42
.751099	3.08	.916946	1.45	.834154	4.52	.165846	41
.751284	3.08	.916859	1.43	.834425	4.52	.165575	40
9.751469	3.08	9.916773	1.43	9.834696	4.52	10.165304	39
.751654	3.08	.916687	1.45	.834967	4.52	.165033	38
.751839	3.07	.916600	1.43	.835238	4.52	.164762	37
.752023	3.08	.916514	1.45	.835509	4.52	.164491	36
.752208	3.07	.916427	1.43	.835780	4.52	.164220	35
.752392	3.07	.916341	1.45	.836051	4.52	.163949	34
.752576	3.07	.916254	1.45	.836322	4.52	.163678	33
.752760	3.07	.916167	1.43	.836593	4.52	.163407	32
.752944	3.07	.916081	1.45	.836864	4.50	.163136	31
.753128	3.07	.915994	1.45	.837134	4.52	.162866	30
9.753312	3.05	9.915907	1.45	9.837405	4.50	10.162595	29
.753495	3.07	.915820	1.45	.837675	4.52	.162325	28
.753679	3.05	.915733	1.45	.837946	4.50	.162054	27
.753862	3.07	.915646	1.45	.838216	4.52	.161784	26
.754046	3.05	.915559	1.45	.838487	4.50	.161513	25
.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
.754412	3.05	.915385	1.47	.839027	4.50	.160973	23
.754595	3.05	.915297	1.45	.839297	4.52	.160703	22
.754778	3.03	.915210	1.45	.839568	4.50	.160432	21
.754960	3.05	.915123	1.47	.839838	4.50	.160162	20
9.755143	3.05	9.915035	1.45	9.840108	4.50	10.159892	19
.755326	3.03	.914948	1.47	.840378	4.50	.159622	18
.755508	3.03	.914860	1.45	.840648	4.48	.159352	17
.755690	3.03	.914773	1.47	.840917	4.50	.159083	16
.755872	3.03	.914685	1.45	.841187	4.50	.158813	15
.756054	3.03	.914598	1.47	.841457	4.50	.158543	14
.756236	3.03	.914510	1.47	.841727	4.48	.158273	13
.756418	3.03	.914422	1.47	.841996	4.50	.158004	12
.756600	3.03	.914334	1.47	.842266	4.48	.157734	11
.756782	3.02	.914246	1.47	.842535	4.50	.157465	10
9.756963	3.02	9.914158	1.47	9.842805	4.48	10.157195	9
.757144	3.03	.914070	1.47	.843074	4.48	156926	8
.757326	3.02	.913982	1.47	.843343	4.48	.156657	7
.757507	3.02	.913894	1.47	.843612	4.50	.156388	6
.757688	3.02	.913806	1.47	.843882	4.48	.156118	5
.757869	3.02	.913718	1.47	.844151	4.48	.155849	4
.758050	3.00	.913630	1.48	.844420	4.48	.155580	3
.758230	3.02	913541	1.47	.844689	4.48	.155311	2
.758411	3.00	.913453	1.47	.844958	4.48	.155042	1
9.758591		9.913365		9.845227		10.154773	0'
Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	55°

27. Logarithmic Sines,

35°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	144°
0'	9.758591	3.02	9.913365	1.48	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.47	.154504	59
2	.758952	3.00	.913187	1.47	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.47	.846302	4.47	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	3.00	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.98	.912744	1.48	.847108	4.47	.152892	53
8	.760031	3.00	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.98	.912566	1.48	.847644	4.48	.152356	51
10	.760390	2.98	.912477	1.48	.847913	4.47	.152087	50
11	9.760569	2.98	9.912388	1.48	9.848181	4.47	10.151819	
12	.760748	2.98	.912299	1.48	.848449	4.47	.151551	
13	.760927	2.98	.912210	1.48	.848717	4.48	.151283	
14	.761106	2.98	.912121	1.50	.848986	4.47	.151014	
15	.761285	2.98	.912031	1.48	.849254	4.47	.150746	
16	.761464	2.97	.911942	1.48	.849522	4.47	.150478	
17	.761642	2.98	.911853	1.50	.849790	4.45	.150210	
18	.761821	2.97	.911763	1.48	.850057	4.47	.149943	
19	.761999	2.97	.911674	1.50	.850325	4.47	.149675	
20	.762177	2.98	.911584	1.48	.850593	4.47	.149407	
21	9.762356	2.97	9.911495	1.50	9.850861	4.47	10.149139	
22	.762534	2.97	.911405	1.50	.851129	4.45	.148871	
23	.762712	2 95	.911315	1.48	.851396	4.47	.148604	
24	.762889	2.97	.911226	1.50	.851664	4.45	.148336	
25	.763067	2.97	.911136	1.50	.851931	4.47	.148069	
26	.763245	2.95	.911046	1.50	.852199	4.45	.147801	
27	.763422	2.97	.910956	1.50	.852466	4.45	.147534	
28	.763600	2.95	.910866	1.50	.852733	4.47	.147267	
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	
30	.763954	2.95	.910686	1.50	.853268	4.45	.146732	
31	9.764131	2.95	9.910596	1.50	9.853535	4.45	10.146465	
32	.764308	2.95	.910506	1.52	.853802	4.45	.146198	
33	.764485	2.95	.910415	1.50	.854069	4.45	.145931	
34	.764662	2.93	.910325	1.50	.854336	4.45	.145664	
35	.764838	2.95	.910235	1.52	.854603	4.45	.145397	
36	.765015	2.93	.910144	1.50	.854870	4.45	.145130	
37	.765191	2.93	.910054	1.52	.855137	4.45	.144863	
38	.765367	2.95	.909963	1.50	.855404	4.45	.144596	
39	.765544	2.93	.909873	1.52	.855671	4.45	.144329	
40	.765720	2.93	.909782	1.52	.855938	4.43	.144062	
41	9.765896	2.93	9.909691	1.50	9.856204	4.45	10.143796	
42	.766072	2.92	.909601	1.52	.856471	4.43	.143529	
43	.766247	2.93	.909510	1.5[illegible]	.856737	4.45	.143263	
44	.766423	2.92	.909419	1.52	.857004	4.43	.142996	
45	.766598	2.93	.909328	1.52	.857270	4.45	.142730	
46	.766774	2.92	.909237	1.52	.857537	4.43	.142463	
47	.766949	2.92	.909146	1.52	.857803	4.43	.142197	
48	.767124	2.93	.909055	1.52	.858069	4.45	.141931	
49	.767300	2.92	.908964	1.52	.858336	4.43	.141664	
50	.767475	2.90	.908873	1.53	.858602	4.43	.141398	
51	9.767649	2.92	9.908781	1.52	9.858868	4.43	10.141132	
52	[illegible]	2.92	.908690	1.52	.859134	4.43	.140866	
53	.767999	2.90	.908599	1.53	.859400	4.43	.140600	
54	.768173	2.92	.908507	1.52	.859666	4.43	.140334	
55	.768348	2.90	.908416	1.53	.859932	4.43	.140068	
56	.768522	2.92	.908324	1.52	.860198	4.43	.139802	
57	.768697	2 90	.908233	1.53	.860464	4.43	.139536	
58	.768871	2.90	.908141	1.53	.860730	4.42	.139270	
59	.769045	2.90	.908049	1.52	.860995	4.43	.139005	
60'	9.769219		9.907958		9.861261		10.138739	
	125° Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

Cosines, Tangents, and Cotangents

36°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	143°
0'	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60'
1	.769393	2.88	.907866	1.53	.861527	4.42	.138473	59
2	.769566	2.90	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.88	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.90	.907590	1.53	.862323	4.43	.137677	56
5	.770087	2 88	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.53	.863119	4.43	.136881	53
8	.770606	2.88	.907222	1.55	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.53	.863650	4 42	.136350	51
10	.770952	2.88	.907037	1.53	.863915	4.42	.136085	50
11	9.771125	2.88	9.906945	1.55	9.864180	4 42	10.135820	49
12	.771298	2.87	.906852	1.53	.864445	4.42	.135555	48
13	.771470	2.88	.906760	1.55	.864710	4.42	.135290	47
14	.771643	2 87	.906667	1.53	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.55	.865240	4.42	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.42	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.42	.134230	43
18	.772331	2.87	.906296	1.53	.866035	4.42	.133965	42
19	.772503	2.87	.906204	1.55	.866300	4.40	.133700	41
20	.772675	2.87	.906111	1.55	.866564	4.42	.133436	40
21	9 772847	2.85	9.906018	1.55	9.866829	4.42	10.133171	39
22	.773018	2.87	.905925	1.55	.867094	4.40	.132906	38
23	.773190	2.85	.905832	1.55	.867358	4.42	.132642	37
24	.773361	2.87	.905739	1.57	.867623	4.40	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.42	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2 85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.57	.868680	4 42	.131320	32
29	.774217	2.85	.905272	1.55	.868945	4.40	.131055	31
30	.774388	2.83	.905179	1.57	.869209	4.40	.130791	30
31	9.774558	2.85	9.905085	1.55	9.869473	4.40	10.130527	29
32	.774729	2.83	.904992	1.57	869737	4.40	.130263	28
33	.774899	2.85	.904898	1.57	.870001	4.40	.129999	27
34	.775070	2.83	.904804	1.55	.870265	4.40	.129735	26
35	.775240	2.88	.904711	1.57	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.57	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2 88	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	.776090	2.82	.904241	1.57	.871849	4.38	.128151	20
41	9.776259	2.83	9.904147	1.57	9.872112	4.40	10.127888	19
42	.776429	2 82	.904053	1.57	.872376	4.40	.127624	18
43	.776598	2.83	.903959	1 58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.40	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.38	.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2.82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4.40	.125780	11
50	.777781	2.82	.903298	1.58	.874484	4.38	.125516	10
51	9.777950	2.82	9.903203	1.58	9.874747	4.38	10.125253	9
52	.778119	2.80	.903108	1.57	.875010	4.38	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4.40	.124727	7
54	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.78	.902539	1:58	.876589	4.38	.123411	2
59	.779295	2.80	.902444	1.58	.876852	4.37	.123148	1
60'	9.779463		9.902349		9.877114		10.122886	0'
126°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	53°

27. Logarithmic Sines,

37°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 142°	
0'	9.779463	2.80	9.902349	1.60	9.877114	4.38	10.122886	60'
1	.779631	2.78	.902253	1.58	.877377	4.38	.122623	59
2	.779798	2.80	.902158	1.58	.877640	4.38	.122360	58
3	.779966	2.78	.902063	1.60	.877903	4.37	.122097	57
4	.780133	2.78	.901967	1.58	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.60	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.58	.878691	4.37	.121309	54
7	.780634	2.78	.901681	1.60	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.58	.879216	4.37	.120784	52
9	.780968	2.77	.901490	1.60	.879478	4.38	.120522	51
10	.781134	2.78	.901394	1.60	.879741	4.37	.120259	50
11	9.781301	2.78	9.901298	1.60	9.880003	4.37	10.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.38	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	
16	.782132	2.77	.900818	1.60	.881314	4.38	.118686	
17	.782298	2.77	.900722	1.60	.881577	4.37	.118423	
18	.782164	2.77	.900626	1.62	.881839	4.37	.118161	
19	.782630	2.77	.900529	1.60	.882101	4.37	.117899	
20	.782796	2.75	.900433	1.60	.882363	4.37	.117637	
21	9.782961	2.77	9.900337	1.62	9.882625	4.37	10.117375	
22	.783127	2.75	.900240	1.60	.882887	4.35	.117113	
23	.783292	2.77	.900144	1.62	.883148	4.37	.116852	
24	.783458	2.75	.900047	1.60	.883410	4.37	.116590	
25	.783623	2.75	.899951	1.62	.883672	4.37	.116328	
26	.783788	2.75	.899854	1.62	.883934	4.37	.116066	
27	.783953	2.75	.899757	1.62	.884196	4.35	.115804	
28	.784118	2.73	.899660	1.60	.884457	4.37	.115543	
29	.784282	2.75	.899564	1.62	.884719	4.35	.115281	
30	.784447	2.75	.899467	1.62	.884980	4.37	.115020	
31	9.784612	2.73	9.899370	1.62	9.885242	4.37	10.114758	
32	.784776	2.75	.899273	1.62	.885504	4.35	.114496	
33	.784941	2.73	.899176	1.63	.885765	4.35	.114235	
34	.785105	2.73	.899078	1.62	.886026	4.37	.113974	
35	.785269	2.73	.898981	1.62	.886288	4.35	.113712	
36	.785433	2.73	.898884	1.62	.886549	4.37	.113451	
37	.785597	2.73	.898787	1.63	.886811	4.35	.113189	
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	
39	.785925	2.73	.898592	1.63	.887333	4.35	.112667	
40	.786089	2.72	.898494	1.62	.887594	4.35	.112406	
41	9.786252	2.73	9.898397	1.63	9.887855	4.35	10.112145	
42	.786416	2.72	.898299	1.62	.888116	4.37	.111884	
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	
44	.786742	2.73	.898104	1.63	.888639	4.35	.111361	
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	
46	.787069	2.72	.897908	1.63	.889161	4.33	.110839	
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318	
49	.787557	2.72	.897614	1.63	.889943	4.35	.110057	
50	.787720	2.72	.897516	1.63	.890204	4.35	.109796	
51	9.787883	2.70	9.897418	1.63	9.890465	4.33	10.109535	
52	.788045	2.72	.897320	1.63	.890725	4.35	.109275	
53	.788208	2.70	.897222	1.65	.890986	4.35	.109014	
54	.788370	2.70	.897123	1.63	.891247	4.33	.108753	
55	.788532	2.70	.897025	1.65	.891507	4.35	.108493	
56	.788694	2.70	.896926	1.63	.891768	4.33	.108232	
57	.788856	2.70	.896828	1.65	.892028	4.35	.107972	
58	.789018	2.70	.896729	1.63	.892289	4.33	.107711	
59	.789180	2.70	.896631	1.65	.892549	4.35	.107451	
60'	9.789342		9.896532		9.892810		10.107190	
127°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

Cosines, Tangents, and Cotangents

38°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	141°
0′	9.789342	2.70	9.896532	1.65	9.892810	4.33	10.107190	60′
1	.789504	2.68	.896433	1.63	.893070	4.35	.106930	59
2	.789665	2.70	.896335	1.65	.893331	4.33	.106669	58
3	.789827	2.68	.896236	1.65	.893591	4.33	.106409	57
4	.789988	2.68	.896137	1.65	.893851	4.33	.106149	56
5	.790149	2.68	.896038	1.65	.894111	4.35	.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.33	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.63	.895741	1.67	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	.790954	2.68	.895542	1.65	.895412	4.33	.104588	50
11	9.791115	2.67	9.895443	1.67	9.895672	4.33	10.104328	49
12	.791275	2.68	.895343	1.65	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.65	.896192	4.33	.103808	47
14	.791596	2.68	.895145	1.67	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.67	.896712	4.32	.103288	45
16	.791917	2.67	.894945	1.65	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.67	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.67	.897491	4.33	.102509	42
19	.792397	2.67	.894646	1.67	.897751	4.32	.102249	41
20	.792557	2.65	.894546	1.67	.898010	4.33	.101990	40
21	9.792716	2.67	9.894446	1.67	9.898270	4.33	10.101730	39
22	.792876	2.65	.894346	1.67	.898530	4.32	.101470	38
23	.793035	2.67	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	.793354	2.67	.894046	1.67	.899308	4.33	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.68	.899827	4.33	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.68	.900346	4.32	.099654	31
30	.794150	2.63	.893544	1.67	.900605	4.32	.099395	30
31	9.794308	2.65	9.893444	1.68	9.900864	4.33	10.099136	29
32	.794467	2.65	.893343	1.67	.901124	4.32	.098876	28
33	.794626	2.63	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.63	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.65	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.63	.892940	1.68	.902160	4.33	.097840	24
37	.795259	2.63	.892839	1.67	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.70	.902938	4.32	.097062	21
40	.795733	2.63	.892536	1.68	.903197	4.32	.096803	20
41	9.795891	2.63	9.892435	1.68	9.903456	4.30	10.096544	19
42	.796049	2.62	.892334	1.68	.903714	4.32	.096286	18
43	.796206	2.63	.892233	1.68	.903973	4.32	.096027	17
44	.796364	2.62	.892132	1.70	.904232	4.32	.095768	16
45	.796521	2.63	.892030	1.68	.904491	4.32	.095509	15
46	.796679	2.62	.891929	1.70	.904750	4.30	.095250	14
47	.796836	2.62	.891827	1.68	.905008	4.32	.094992	13
48	.796993	2.62	.891726	1.70	.905267	4.32	.094733	12
49	.797150	2.62	.891624	1.68	.905526	4.32	.094474	11
50	.797307	2.62	.891523	1.70	.905785	4.30	.094215	10
51	9.797464	2.62	9.891421	1.70	9.906043	4.32	10.093957	9
52	.797621	2.60	.891319	1.70	.906302	4.30	.093698	8
53	.797777	2.62	.891217	1.70	.906560	4.32	.093440	7
54	.797934	2.62	.891115	1.70	.906819	4.30	.093181	6
55	.798091	2.60	.891013	1.70	.907077	4.32	.092923	5
56	.798247	2.60	.890911	1.70	.907336	4.30	.092664	4
57	.798403	2.62	.890809	1.70	.907594	4.22	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.30	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60′	9.798872		9.890503		9.908369		10.091631	0′
128°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	51°

27. Logarithmic Sines,

39°	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	140°
0′	9.798872	2.60	9.890503	1.72	9.908369	4.32	10.091631	60′
1	.799028	2.60	.890400	1.70	.908628	4.30	.091372	59
2	.799184	2.58	.890298	1.72	.908886	4.30	.091114	58
3	.799339	2.60	.890195	1.70	.909144	4.30	.090856	57
4	.799495	2.60	.890093	1.72	.909402	4.30	.090598	56
5	.799651	2.58	.889990	1.70	.909660	4.30	.090340	55
6	.799806	2.60	.889888	1.72	.909918	4.32	.090082	54
7	.799962	2.58	.889785	1.72	.910177	4.30	.089823	53
8	.800117	2.58	.889682	1.72	.910435	4.30	.089565	52
9	.800272	2.58	.889579	1.70	.910693	4.30	.089307	51
10	.800427	2.58	.889477	1.72	.910951	4.30	.089049	50
11	9.800582	2.58	9.889374	1.72	9.911209	4.30	10.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.73	.911725	4.28	.088275	47
14	.801047	2.57	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.58	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.73	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.28	.086986	42
19	.801819	2.57	.888548	1.73	.913271	4.30	.086729	41
20	.801973	2.58	.888444	1.72	.913529	4.30	.086471	40
21	9.802128	2.57	9.888341	1.73	9.913787	4.28	10.086213	39
22	.802282	2.57	.888237	1.72	.914044	4.30	.085956	38
23	.802436	2.55	.888134	1.73	.914302	4.30	.085698	37
24	.802589	2.57	.888030	1.73	.914560	4.28	.085440	36
25	.802743	2.57	.887926	1.73	.914817	4.30	.085183	35
26	.802897	2.55	.887822	1.73	.915075	4.28	.084925	34
27	.803050	2.57	.887718	1.73	.915332	4.30	.084668	33
28	.803204	2.55	.887614	1.73	.915590	4.28	.084410	32
29	.803357	2.57	.887510	1.73	.915847	4.28	.084153	31
30	.803511	2.55	.887406	1.73	.916104	4.30	.083896	30
31	9.803664	2.55	9.887302	1.73	9.916362	4.28	10.083638	29
32	.803817	2.55	.887198	1.75	.916619	4.30	.083381	28
33	.803970	2.55	.887093	1.73	.916877	4.28	.083123	27
34	.804123	2.55	.886989	1.73	.917134	4.28	.082866	26
35	.804276	2.55	.886885	1.75	.917391	4.28	.082609	25
36	.804428	2.55	.886780	1.73	.917648	4.30	.082352	24
37	.804581	2.55	.886676	1.75	.917906	4.28	.082094	23
38	.804734	2.53	.886571	1.75	.918163	4.28	.081837	22
39	.804886	2.55	.886466	1.73	.918420	4.28	.081580	21
40	.805039	2.53	.886362	1.75	.918677	4.28	.081323	20
41	9.805191	2.53	9.886257	1.75	9.918934	4.28	10.081066	19
42	.805343	2.53	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.52	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.77	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.75	.920990	4.28	.079010	11
50	.806557	2.53	.885311	1.77	.921247	4.27	.078753	10
51	9.806709	2.52	9.885205	1.75	9.921503	4.28	10.078497	
52	.806860	2.52	.885100	1.77	.921760	4.28	.078240	
53	.807011	2.53	.884994	1.75	.922017	4.28	.077983	
54	.807163	2.52	.884889	1.77	.922274	4.27	.077726	
55	.807314	2.52	.884783	1.77	.922530	4.28	.077470	
56	.807465	2.50	.884677	1.75	.922787	4.28	.077213	
57	.807615	2.52	.884572	1.77	.923044	4.27	.076956	
58	.807766	2.52	.884466	1.77	.923300	4.28	.076700	
59	.807917	2.50	.884360	1.77	.923557	4.28	.076443	
60′	9.808067		9.884254		9.923814		10.076186	
129°	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	5

Cosines, Tangents, and Cotangents

40°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	139°
0'	9.808067	2.52	9.884254	1.77	9.923814	4.27	10.076186	60'
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	.809569	2.48	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.50	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	43
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	.071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	.811061	2.48	.882121	1.78	.928940	4.27	.071060	40
21	9.811210	2.47	9.882014	1.78	9.929196	4.27	10.070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.78	.930220	4.25	.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	.812544	2.47	.881046	1.80	.931499	4.27	.068501	30
31	9.812692	2.47	9.880938	1.80	9.931755	4.25	10.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	.813283	2.45	.880505	1.80	.932778	4.25	.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	.066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	.814019	2.45	.879963	1.80	.934056	4.25	.065944	20
41	9.814166	2.45	9.879855	1.82	9.934311	4.27	10.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	.814753	2.45	.879420	1.82	.935333	4.27	.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	.064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	.064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	.815485	2.45	.878875	1.82	.936611	4.25	.063389	10
51	9.815632	2.43	9.878766	1.83	9.936866	4.25	10.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60'	9.816943		9.877780		9.939163		10.060837	0'
130°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	49°

27. Logarithmic Sines,

41°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	138°
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60′
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817	56
5	.817668	2.42	.877230	1.83	.940439	4.25	.059561	55
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306	54
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796	52
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541	51
10	.818392	2.40	.876678	1.83	.941713	4.25	.058287	50
11	9.818536	2.42	9.876568	1.85	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.40	.875571	1.87	.944262	4.25	.055738	40
21	9.819976	2.40	9.875459	1.85	9.944517	4.23	10.055483	39
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229	38
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974	37
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719	36
25	.820550	2.38	.875014	1.85	.945535	4.25	.054465	35
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955	33
28	.820979	2.38	.874680	1.87	.946299	4.25	.053701	32
29	.821122	2.38	.874568	1.87	.946554	4.23	.053446	31
30	.821265	2.37	.874456	1.87	.946808	4.25	.053192	30
31	9.821407	2.38	9.874344	1.87	9.947063	4.25	10.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428	27
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173	26
35	.821977	2.38	.873896	1.87	.948081	4.23	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156	22
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901	21
40	.822688	2.37	.873335	1.87	.949353	4.25	.050647	20
41	9.822830	2.37	9.873223	1.88	9.949608	4.23	10.050392	1[illegible]
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138	1[illegible]
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884	1[illegible]
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629	1[illegible]
45	.823397	2.37	.872772	1.88	.950625	4.23	.049375	1[illegible]
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121	1[illegible]
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867	1[illegible]
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612	1[illegible]
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358	1[illegible]
50	.824104	2.35	.872208	1.88	.951896	4.23	.048104	1[illegible]
51	9.824245	2.35	9.872095	1.90	9.952150	4.25	10.047850	
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595	
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341	
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087	
55	.824808	2.35	.871641	1.88	.953107	4.23	.046833	
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579	
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325	
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071	
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817	
60′	9.825511		9.871073		9.954437		10.045563	
131° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	4[illegible]

Cosines, Tangents, and Cotangents

42°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 137°	
0'	9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60'
1	.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
8	.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
11	9.827049	2.33	9.869818	1.90	9.957231	4.23	10.042769	49
12	.827189	[illegible]	.869704	1.92	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
14	.827467	[illegible]	.869474	1.90	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
16	.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
17	.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
21	9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
24	.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
28	.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
30	.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
31	9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
33	.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
35	.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
37	.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
38	.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
40	.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
41	9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
43	.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
44	.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
46	.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
48	[illegible]	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9.832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
55	.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
56	.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
57	.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
58	.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
59	.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
60'	9.833783		9.864127		9.969656		10.030344	0'
132°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	47°

27. Logarithmic Sines,

43°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang. 136°	
0′	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60′
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60′	9.841771		9.856934		9.984837		10.015163	0
133° Cosine.		D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	46°

Cosines, Tangents, and Cotangents

44°	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	135°
0′	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2.12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60′	9.849485		9.849485		10.000000		10.000000	0′
134°	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	45°

28. Natural Sines and Cosines

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

28. Natural Sines and Cosines

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

28. Natural Sines and Cosines

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

28. Natural Sines and Cosines

′	15° Sine	15° Cosin	16° Sine	16° Cosin	17° Sine	17° Cosin	18° Sine	18° Cosin	19° Sine	19° Cosin	′
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
37	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

28. Natural Sines and Cosines

′	20°		21°		22°		23°		24°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

28. Natural Sines and Cosines

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

28. Natural Sines and Cosines

′	30° Sine	30° Cosin	31° Sine	31° Cosin	32° Sine	32° Cosin	33° Sine	33° Cosin	34° Sine	34° Cosin	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85421	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

28. Natural Sines and Cosines

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	38
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.82310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.7917[illegible]	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.61145	.76717	6
55	.58661	.80987	.69065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.777[illegible]	.641[illegible]	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

28. Natural Sines and Cosines

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

29. Natural Tangents and Cotangents

′	0° Tang	0° Cotang	1° Tang	1° Cotang	2° Tang	2° Cotang	3° Tang	3° Cotang	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

29. Natural Tangents and Cotangents

′	4° Tang	4° Cotang	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.23190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

29. Natural Tangents and Cotangents

′	8° Tang	8° Cotang	9° Tang	9° Cotang	10° Tang	10° Cotang	11° Tang	11° Cotang	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	81° Cotang	81° Tang	80° Cotang	80° Tang	79° Cotang	79° Tang	78° Cotang	78° Tang	′

29. Natural Tangents and Cotangents

′	12° Tang	12° Cotang	13° Tang	13° Cotang	14° Tang	14° Cotang	15° Tang	15° Cotang	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.20159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	77° Cotang	77° Tang	76° Cotang	76° Tang	75° Cotang	75° Tang	74° Cotang	74° Tang	′

29. Natural Tangents and Cotangents

′	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.09447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.18027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.22460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

29. Natural Tangents and Cotangents

′	20° Tang	20° Cotang	21° Tang	21° Cotang	22° Tang	22° Cotang	23° Tang	23° Cotang	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	69°		68°		67°		66°		

29. Natural Tangents and Cotangents

′	24° Tang	24° Cotang	25° Tang	25° Cotang	26° Tang	26° Cotang	27° Tang	27° Cotang	′
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.48459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91552	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91414	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91288	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91142	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91017	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.899[illegible]	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

29. Natural Tangents and Cotangents

′	28° Tang	28° Cotang	29° Tang	29° Cotang	30° Tang	30° Cotang	31° Tang	31° Cotang	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62187	1.60033	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	61°		60°		59°		58°		

29. Natural Tangents and Cotangents

′	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	35° Tang	35° Cotang	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70170	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	57° Cotang	57° Tang	56° Cotang	56° Tang	55° Cotang	55° Tang	54° Cotang	54° Tang	′

29. Natural Tangents and Cotangents

′	36° Tang	36° Cotang	37° Tang	37° Cotang	38° Tang	38° Cotang	39° Tang	39° Cotang	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	53°		52°		51°		50°		

29. Natural Tangents and Cotangents

′	40° Tang	40° Cotang	41° Tang	41° Cotang	42° Tang	42° Cotang	43° Tang	43° Cotang	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12657	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

29. Natural Tangents and Cotangents

′	44° Tang	44° Cotang	′
0	.96569	1.03553	60
1	.96625	1.03493	59
2	.96681	1.03433	58
3	.96738	1.03372	57
4	.96794	1.03312	56
5	.96850	1.03252	55
6	.96907	1.03192	54
7	.96963	1.03132	53
8	.97020	1.03072	52
9	.97076	1.03012	51
10	.97133	1.02952	50
11	.97189	1.02892	49
12	.97246	1.02832	48
13	.97302	1.02772	47
14	.97359	1.02713	46
15	.97416	1.02653	45
16	.97472	1.02593	44
17	.97529	1.02533	43
18	.97586	1.02474	42
19	.97643	1.02414	41
20	.97700	1.02355	40
′	45° Cotang	45° Tang	′

′	44° Tang	44° Cotang	′
20	.97700	1.02355	40
21	.97756	1.02295	39
22	.97813	1.02236	38
23	.97870	1.02176	37
24	.97927	1.02117	36
25	.97984	1.02057	35
26	.98041	1.01998	34
27	.98098	1.01939	33
28	.98155	1.01879	32
29	.98213	1.01820	31
30	.98270	1.01761	30
31	.98327	1.01702	29
32	.98384	1.01642	28
33	.98441	1.01583	27
34	.98499	1.01524	26
35	.98556	1.01465	25
36	.98613	1.01406	24
37	.98671	1.01347	23
38	.98728	1.01288	22
39	.98786	1.01229	21
40	.98843	1.01170	20
′	45° Cotang	45° Tang	′

′	44° Tang	44° Cotang
40	.98843	1.01170
41	.98901	1.01112
42	.98958	1.01053
43	.99016	1.00994
44	.99073	1.00935
45	.99131	1.00876
46	.99189	1.00818
47	.99247	1.00759
48	.99304	1.00701
49	.99362	1.00642
50	.99420	1.00583
51	.99478	1.00525
52	.99536	1.00467
53	.99594	1.00408
54	.99652	1.00350
55	.99710	1.00291
56	.99768	1.00233
57	.99826	1.00175
58	.99884	1.00116
59	.99942	1.00058
60	1.00000	1.00000
′	45° Cotang	45° Tang

SECTION 9

HYDRAULICS, PUMPING, WATER POWER

BY

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HYDROMECHANICS

LOW IN PIPES AND CHANNELS

MEASUREMENT OF WATER

TURBINES AND WATER WHEELS

PUMPS AND PUMPING

WATER POWER

PRINCIPLES OF HYDROMECHANICS

1. Definitions

Hydromechanics deals with the equilibrium and motion of fluids and of bodies in or surrounding them. FLUIDS are those bodies which offer very small resistance to deformation. LIQUIDS are those fluids whose particles tend to cling together. GASES are those fluids whose particles tend to separate from each other.

Hydromechanics is divided into Hydrostatics, which treats of fluids at rest, and Hydrodynamics, which treats of fluids in motion. Hydraulics is that part of Hydrodynamics which treats of water in motion.

Pressure is the force with which one body presses or acts upon another. The pressure acting on any surface is understood to be the sum of all the normal pressures and normal components of pressures acting on that surface. INTENSITY OF PRESSURE at any point is the pressure exerted upon a unit area at that point.

Pressure Head, at any point in a fluid at rest, is the vertical distance of the point from the free surface of the fluid. $h = p/w$, where h is the head, p is the pressure in pounds per square foot, and w is the weight per cubic foot of the fluid.

Velocity Head. If, thru a small area, the velocity of a fluid is v feet per second, the velocity head is $v^2/2g$, g being the acceleration due to gravity in feet per second per second.

Head of Elevation is the height of a point above any assumed datum. If any point is z feet above any convenient datum level, the head of elevation at that point above the given datum is said to be z feet.

Friction Head is the head absorbed in a fluid in motion, among its own particles and between itself and its bounding surfaces.

Density is the number of units of mass in unit volume of a substance. RELATIVE DENSITY is the ratio of a given density to some density taken as a standard. When pure water at its maximum density is taken as a standard, the relative density becomes the specific gravity.

One Atmosphere gives a pressure equal to 14.7 lb per sq in. This is equal to the pressure at the base of a column of water 34.0 ft high or of a column of mercury 30 inches high.

2. Properties of Water

Compressibility of water is very slight and varies greatly with the temperature and amount of air in solution. M. Grassi obtained the following results with distilled water:

Temp. Fahr.	Max. pres., atmospheres	Mean compressibility per atmos.	Temp. Fahr.	Max. pres., atmospheres	Mean compressibility per atmos
32°	7.4	0.0000502	56°	8.4	0.0000476
35°	10.0	0.0000515	79°	7.2	0.0000455
51°	5.1	0.0000480	128°	6.3	0.0000444

At a pressure of 65 000 pounds per square inch, water is said to be comprest over 10%. (Eng. News, Oct. 4, 1900.)

Impurities in water slightly affect the specific gravity, as shown by the following values from various authorities:

Rivers: Garonne, France	1.000149
Thames, England	1.0003
Mississippi, U. S. (filtered)	1.00025
Springs	1.0003 to 1.006
Pacific Ocean	1.0265
Dead Sea	1.172

Air ordinarily dissolved in water varies with the temperature. The amounts of oxygen and of air dissolved at different temperatures under a pressure of one atmosphere, as determined at the Lawrence Experimental Station, are as follows:

Oxygen and Air Dissolved in Water under a Pressure of One Atmosphere

In parts per 100 000 by weight.

Temp. Fahr.	Oxygen	Air	Temp. Fahr.	Oxygen	Air	Temp. Fahr.	Oxygen	Air
32°	1.470	6.38	51°	1.117	4.84	70°	0.899	3.91
33	1.445	6.28	52	1.103	4.78	71	0.889	3.87
34	1.422	6.19	53	1.089	4.72	72	0.880	3.83
35	1.400	6.08	54	1.076	4.67	73	0.871	3.79
36	1.379	5.99	55	1.063	4.62	74	0.862	3.75
37°	1.358	5.89	56°	1.050	4.56	75°	0.853	3.71
38	1.338	5.80	57	1.038	4.51	76	0.844	3.67
39	1.318	5.72	58	1.026	4.46	77	0.835	3.63
40	1.299	5.64	59	1.014	4.41	78	0.826	3.59
41	1.280	5.56	60	1.003	4.36	79	0.817	3.55
42°	1.262	5.49	61°	0.992	4.31	80°	0.808	3.51
43	1.244	5.41	62	0.981	4.26	81	0.800	3.48
44	1.227	5.33	63	0.970	4.21	82	0.792	3.44
45	1.210	5.26	64	0.959	4.16	83	0.784	3.41
46	1.193	5.19	65	0.949	4.12	84	0.776	3.38
47°	1.177	5.12	66°	0.939	4.08	85°	0.768	3.34
48	1.161	5.05	67	0.929	4.04	86	0.760	3.30
49	1.146	4.98	68	0.919	3.99	......	......	......
50	1.131	4.91	69	0.909	3.95	......	......	......

Effect of Temperature. The following table gives relative density or specific gravity and the weight of a cubic foot of water at various temperatures.

Specific Gravity and Weight of Water

Temp. Fahr.	Specific gravity	Pounds per cu ft	Log of weight per cu. ft, lbs	Temp. Fahr.	Specific gravity	Pounds per cu ft	Log. of weight per cu. ft, lbs
15°	0.99831	....	....	80°	.99669	62.217	1.79391
20	.99898	....	....	85	.99592	62.169	1.79357
25	.99947	....	....	90	.99510	62.118	1.79322
30	.99979	....	....	95	.99418	62.061	1.79282
32	.99987	62.416	1.79529	100	.99318	61.998	1.79238
35°	.99996	62.421	1.79533	110°	.99105	61.865	1.79144
39.3	1.	62.424	1.79535	120	.98870	61.719	1.79042
40	0.99999	62.423	1.79534	130	.98608	61.555	1.78926
45	.99992	62.419	1.79532	140	.98338	61.386	1.78807
50	.99975	62.408	1.79524	150	.98043	61.203	1.78673
55°	.99946	62.390	1.79511	160°	.97729	61.006	1.78537
60	.99907	62.366	1.79495	170	.97397	60.799	1.78390
65	.99859	62.336	1.79474	180	.97056	60.586	1.78237
70	.99802	62.300	1.79449	200	.96333	60.135	1.77913
75	.99739	62.261	1.79422	212	.95865	59.843	1.77701

3. Hydrostatics

Laws of Perfect Liquids. (1) At any point in a liquid at rest the hydrostatic pressure is the same in all directions. (2) At all points in a horizontal plane in a liquid at rest the pressure is the same. (3) The pressure of water at rest, upon the surface of the vessel containing it, is normal to that surface at every point of it. (4) If a body be immersed in a liquid, the pressure upon it will be normal at every point of the surface of the body. (5) Every external pressure upon a liquid in a vessel will be transmitted with equal intensity to all parts of the liquid and of the containing surface.

Fig. 1

The Hydrostatic Press, often called the hydraulic press, is used to exert heavy pressures generated in accordance with law (5). Let P_1 be a force applied thru the small area a; its intensity per unit of area is P_1/a, and this acts with equal intensity upon all parts of the area A of a large piston in a strong cylinder. Thus the total pressure P_2 on the large piston is $P_2 = P_1\, A/a$. If $A = 1000a$, then $P_2 = 1000\, P_1$. The work of the resisting force P_2 cannot, however, be greater than the work of the applied force P_1.

Head and Pressure. In still water the unit pressure at any point is directly proportional to its distance below the surface. One ft head gives a pressure of 0.4335 lb per sq in; one lb per sq in pressure is produced by a head of 2.3068 ft. These values are for fresh water at its maximum density (39.°3 F.)

The Center of Pressure for any plane surface acted upon by a fluid is the point of action of the resultant pressure acting upon the surface. The resultant pressure P on any submerged plane area $= Az_g w$, where A is the area, z_g is the vertical distance from the free surface of the fluid to the center of gravity of the area, and w is the weight of unit volume of the fluid.

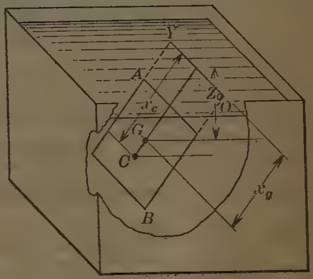

Fig. 2

To find the location of the center of pressure of a plane area, extend the plane until it cuts the free surface of the liquid and call the trace OY then

$$x_c = I_y/Ax_g = r^2_{gy}/x_g$$

where x_c is the perpendicular distance from the center of pressure to OY, I_y is the moment of inertia of the area about OY, A is the area, x_g is the perpendicular distance from the center of gravity of the area to OY, and r_{gy} is the radius of gyration of the surface about the axis Y, and $r^2_{gy} = x_g^2 + r_g^2$. Values of x_g and r_g^2 are found from Section 4, for common plane areas, x_g being equal to the distance from the center of gravity to the upper edge plus the distance from that edge to the axis OY, and r_g is radius of gyration corresponding to axis thru G. For example, let a triangle of base b and altitude d be immersed vertically in water with its vertex at a distance $2\,d$ below the surface and its base parallel to the surface; then $x_g = 2\,d + \tfrac{2}{3}\,d$, and $r_g^2 = {}^1/_{18}d^2$, whence $x_c = {}^{129}/_{48}d$.

Flotation. When a body floats in a fluid, the surface of the body in contact with the fluid is subject to hydrostatic pressures, the intensity of pressure on any element of the surface depending upon its depth below the surface. The resultant of the vertical components of these hydrostatic pressures is called buoyancy.

Archimedes' Principle. The resultant pressure of a fluid on a body immersed in it acts vertically upward thru the center of gravity of the displaced fluid and is equal to the weight of the fluid displaced.

Center of Buoyancy is the center of gravity of the displaced fluid and is the point of application of the resultant of all the upward forces acting on the body.

Stability of Floating Bodies. If the weight of the solid is not equal to the weight of the displaced fluid, that is, to the buoyancy effort, or if its center of gravity does not lie in the same vertical line with the center of buoyancy, the two forces form an unbalanced system and motion begins. If, when the forces are equal, the center of gravity of the solid lies in the same vertical line as the center of buoyancy and underneath the latter, the equilibrium is stable, while if above, the equilibrium is unstable, and if the two centers coincide, the equilibrium is indifferent.

Viscosity is the evidence of cohesion between the particles of a fluid and is that physical property of a fluid which causes it to offer a resistance, analogous to friction, to the relative sliding motion of two adjacent particles. This property, chiefly noticeable when the fluid is in motion, is the cause of all so-called fluid friction.

Surface Tension is caused by the cohesion of the molecules of a fluid and gives it the appearance of having an elastic skin at its surface of separation from a gas or any other fluid.

The Specific Gravity of a solid or liquid is the ratio of its weight to the weight of an equal volume of water. Thus, the specific gravity of a stone which weighs 150 lb per cu ft is $150/62.5 = 2.4$. For a body heavier than water the specific gravity is also the ratio of its weight to the loss of weight in water when entirely submerged. Thus, if W be the weight in air, and W' the weight in water, then specific gravity $= W/(W-W')$; for example, if a piece of lead weighing 6.45 lb in air weighs 5.88 lb when submerged, then the specific gravity of lead is $6.45/0.57 = 11.3$. For a body lighter than water, sink it by means of a heavier body and then deduct the weight of the latter. For porous substances and for liquids special methods are used; see p. 483 for cement.

4. Hydrodynamic Laws

Torricelli's Theorem. The velocity of a jet of liquid discharging under a head H is the same as that acquired by a body falling thru the same height.

Bernoulli's Theorem. In steady flow the sum of the velocity head, pressure head, and head of elevation at any point is equal to the sum of the corresponding heads at any other point $\pm$ the losses of head due to friction; $+$ if the last point is downstream from the first point, $-$ if upstream. This may be exprest by the following equation, known as Bernoulli's Theorem. Let two points be 1 and 2, 2 being downstream from 1; p_1 and p_2 be the fluid pressures at 1 and 2; v_1 and v_2 be the velocities at 1 and 2; h_{e_1} and h_{e_2} be the elevations of 1 and 2 above any convenient datum level; w be the weight of a cubic unit of the fluid; g be the acceleration due to gravity; h_f be the loss of head due to friction between 1 and 2. Then

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + h_{e1} = \frac{v_2^2}{2g} + \frac{p_2}{w} + h_{e2} + h_f$$

Conservation of Energy. In steady flow the sum of kinetic and potential energies at any point in a stream is equal to the sum of the kinetic and potential

energies at any other point in the stream ± the loss of energy due to friction between the two points; + if the second point is downstream from the first point, − if upstream. If both sides of the equation of Bernoulli are multi-

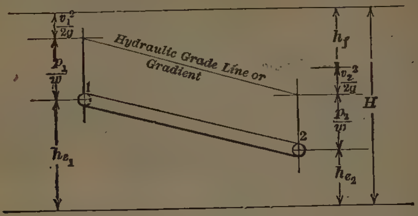

Fig. 3. Bernoulli's Theorem

plied by the mass passing either of the two points in unit time, the two sides can be reduced to the form of the summation of kinetic and potential energies.

Charles' or Gay-Lussac's Law. The pressure remaining constant, the volume of a given mass of gas varies directly with the absolute temperature. If the pressure is constant, the heaviness (and therefore the specific gravity) varies inversely with the absolute temperature. The volume and heaviness remaining constant, the pressure of a given mass of gas varies directly with the absolute temperature.

Boyle's or Mariotte's Law. The temperature remaining constant, the pressure of a given mass of gas varies inversely with the volume and directly with the heaviness.

Dalton's Laws. In a mixture of different gases when there is equilibrium, each gas behaves as a vacuum to all the rest. If one or more liquids are introduced into a space of given volume, the amount of vapor given off by each depends only upon the temperature and pressure and is independent of any other gas or vapor present, and therefore the total pressures of the gases and vapors contained in the space is the sum of the separate pressures, which each would exert if it were the only one present. The law does not hold good when the respective liquids and gases act chemically with one another.

Avogadro's Law. In equal volumes of gases having the same temperature and pressure the numbers of molecules are the same.

5. Jets and Vortices

Impact and Reaction. When a stream of a fluid impinges on a solid surface, it presses on the surface with a force equal and opposite to that by which the velocity and direction of motion of the fluid are changed. Generally in problems on the impact of fluids it is necessary to neglect the effect of friction between the fluid and the surface on which it moves and in the jet itself.

Reaction of Jet upon a vessel from which it issues. Let A be the area of the orifice, h be the head on the orifice, w be the weight of unit volume of the fluid. Then the reaction of jet is $P = 2\,Ahw$.

If the orifice is in a thin plate with a coefficient of velocity $= 0.96$ and a coefficient of contraction 0.64, then $P = 0.96 \times 0.64 \times 2\,Ahw$, or $P = 1.23\,Ahw$.

Impulse of a Jet on a Fixt Curved Vane with borders. Let θ be the angle thru which the jet is turned, Q be the volume of discharge of the jet

per unit of time, U be the absolute mean velocity of the jet. Taking the X axis as the axis of the jet before deflection and the Y axis at right angles to that in the plane of the jet, then

$$P_x = \frac{Q_w}{g} U\,(1 - \cos\theta) \qquad P_y = \frac{Q_w}{g} U \sin\theta$$

$$P = \frac{Q_w}{g} U \sqrt{2\,(1 - \cos\theta)}$$

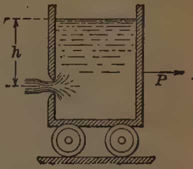

Fig. 4. Reaction of Jet

If ψ is the angle the force P makes with the X axis, $\tan\psi = \sin\theta/(1 - \cos\theta)$. For a flat vane perpendicular to axis of jet, $P = P_x = Q_w U/g$.

Impulse of a Jet on a Fixt Solid of Revolution whose axis coincides with the axis of the jet is the same as in the preceding case except that $P_y = 0$. Then $P = P_x = Q_w U/g\,(1 - \cos\theta)$. The direction of the force is along the axis of the jet.

Impulse of a Jet upon a curved vane with borders moving with a uniform velocity in the direction of the jet. Let V be the absolute velocity of the vane, A the area of the jet; then

Force with which vane is moved is $P_x = \frac{Aw}{g}(U - V)^2(1 - \cos\theta)$.

Work done on the vane, $W = \frac{Aw}{g}(U - V)^2\, V(1 - \cos\theta)$.

Maximum efficiency is obtained when $\theta = 180°$ and $V = U/3$, and is $16/27$, or 59.3%.

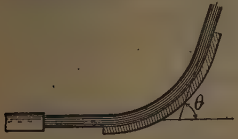

Fig. 5 Impulse upon Curved Vane

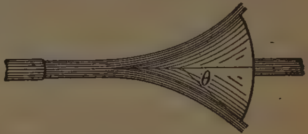

Fig. 6. Solid of Revolution

Impulse of a Jet upon a solid of revolution, whose axis coincides with the axis of the jet, moving with a uniform velocity in the direction of the jet, is the same as in above case of a moving curved vane.

Impulse of a jet upon a series of curved vanes with borders, or upon a series of surfaces of revolution, whose axes coincide with the axis of the jet; moving in the direction of the jet. The force and work are

$$P = \frac{Aw}{g} U\,(U - V)\,(1 - \cos\theta) \qquad W = \frac{Aw}{g} U\,(U - V)\,V\,(1 - \cos\theta)$$

Maximum efficiency is obtained when $\theta = 180°$ and $V = U/2$, and is unity.

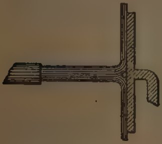
Fig. 7. Flat Plate

Impulse of a Jet upon a flat vane without borders, moving in the direction of the jet. Force and work are

$$P = \frac{Aw}{g}(U - V)^2 \sin^2\theta \qquad W = \frac{Aw}{g}(U - V)^2\, V \sin^2\theta$$

In this case θ is the minimum angle between the vane and the jet, and is 90° when vane is normal to axis of jet. The maximum efficiency is obtained when $\theta = 90°$ and $V = U/3$, and is 29.6%.

Impulse of a Jet upon a series of flat vanes without borders moving in the direction of the jet. Force and work are

$$P = \frac{Aw}{g}(U - V) \sin^2\theta \qquad W = \frac{Aw}{g}(U - V)\, V \sin^2\theta$$

The maximum efficiency is obtained when $\theta = 90°$ and $V = U/2$, and is 50%.

Radiating Current. If water flows from a center outwardly between two parallel plates, and friction be neglected, the pressure head at any point distant R_2 from the center is

$$\frac{P_2}{w} = H - \frac{R_1^2 V_1^2}{2\,gR_2^2} \quad \text{and} \quad R_1V_1 = R_2V_2$$

where V_1 is the velocity at radius R_1 and V_2 that at R_2. If the discharge from between the plates is into air, the pressure at the outlet is atmospheric, and as the pressures decrease toward the center the pressure on the top of the upper plate is greater than that on the bottom of it. This explains the phenomenon of the so-called "ball nozzle."

Vortices. A Free Circular Vortex is a revolving mass of water in which the stream lines are concentric circles and in which the total head for each stream line is the same, or $R_1V_1 = R_2V_2$ as in a radiating current.

A Free Spiral Vortex is a revolving mass of water having a radiating flow, combined with a circular flow, in both of which

$$R_1V_1 = R_2V_2$$

and the same equation holds for the spiral motion in which the direction of the current will make a constant angle with the radius to its axis, or when the path of the current is a logarithmic spiral. The centrifugal pump delivers water into its shell in a free vortex of this kind, the velocity diminishing and the pressure increasing as it flows outward.

A Forced Vortex is a revolving mass of water in which, by the application of some force or forces, the law of velocity variation is caused to be different from the above. The simple case is that in which all the particles have equal angular velocity. Then if ω = angular velocity and $v = R\omega$, the pressure head at any point is

$$\frac{p}{w} = \frac{R^2\omega^2}{2\,g} + \text{a constant}$$

and the relations at any two points, 1 and 2, in the same horizontal plane are

$$\frac{P_2 - P_1}{w} = \frac{\omega^2}{2\,g}(R_2^2 - R_1^2) = \frac{v_2^2 - V_1^2}{2\,g}$$

This is the equation of the surface assumed by the water in a revolving vessel, and indicates that radial planes will cut the surface in parabolas with vertices downward and at the center. The height z_x of the surface at any point x above a tangent to the parabola at its vertex o is

$$z_x = \frac{P_x - P_o}{w} = \frac{\omega^2 R_x^2}{2\,g} = \frac{v_x^2}{2\,g}$$

Rectilinear Translation and Acceleration. When a mass of water is enclosed in a vessel and moves in a straight line with uniform velocity, the pressure upon each surface along horizontal planes will be uniform, but if the motion be uniformly accelerated, then the pressure varies from the front to the rear of the vessel for positiv acceleration and vice versa for negativ acceleration or retardation.

If points 1 and 2 are distant x apart horizontally, and j is the acceleration,

$$(P_1 - P_2)/w = z_1 - z_2 = xj/g$$

and the surface has an angle of inclination ϕ whose tangent is j/g.

6. Hydraulic Computations

Units of Measure used in this Section are the second, the foot, and the pound-weight. For some minor quantities such as pressures and diameters the inch is employed instead of the foot, but the latter is to be used in all hydraulic formulas. (Art. 9 employs metric units.)

Powers of Numbers, Velocities, Velocity Heads

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{0.8}$	$n^{1.87}$	$n^{0.535}$	$\sqrt{2gn}$	$n^2/2g$
0.01	0.0010	0.0464	0.0032	0.0251	0.0002	0.0851	0.802	......
0.02	0.0028	0.0737	0.0075	0.0437	0.0007	0.1233	1.134	......
0.03	0.0052	0.0965	0.0125	0.0605	0.0014	0.1532	1.389	......
0.04	0.0080	0.1170	0.0179	0.0761	0.0024	0.1787	1.604	......
0.05	0.0112	0.1357	0.0236	0.0910	0.0037	0.2013	1.793	......
0.06	0.0147	0.1533	0.0297	0.1053	0.0052	0.2219	1.965	......
0.07	0.0185	0.1699	0.0360	0.1191	0.0069	0.2410	2.122	......
0.08	0.0226	0.1857	0.0425	0.1326	0.0089	0.2589	2.269	......
0.09	0.0270	0.2008	0.0493	0.1457	0.0111	0.2757	2.406	0.0001
0.10	0.0316	0.2155	0.0562	0.1585	0.0135	0.2917	2.537	0.0002
0.2	0.0894	0.3420	0.1338	0.2759	0.0493	0.4227	3.587	0.0006
0.3	0.1643	0.4481	0.2220	0.3817	0.1052	0.5251	4.393	0.0014
0.4	0.2530	0.5429	0.3181	0.4804	0.1802	0.6125	5.073	0.0025
0.5	0.3536	0.6300	0.4205	0.5743	0.2736	0.6902	5.671	0.0039
0.6	0.4648	0.7114	0.5281	0.6645	0.3847	0.7609	6.213	0.0056
0.7	0.5857	0.7884	0.6403	0.7518	0.5133	0.8263	6.710	0.0076
0.8	0.7155	0.8618	0.7566	0.8365	0.6588	0.8875	7.174	0.0099
0.9	0.8538	0.9322	0.8766	0.9192	0.8212	0.9452	7.609	0.0126
1.0	1.000	1.000	1.000	1.000	1.000	1.000	8.021	0.0155
1.1	1.154	1.066	1.127	1.079	1.195	1.052	8.412	0.0188
1.2	1.315	1.129	1.256	1.157	1.406	1.102	8.786	0.0224
1.3	1.482	1.191	1.388	1.233	1.633	1.151	9.145	0.0263
1.4	1.657	1.251	1.523	1.309	1.876	1.197	9.490	0.0305
1.5	1.837	1.310	1.660	1.383	2.135	1.242	9.823	0.0350
1.6	2.024	1.368	1.799	1.456	2.408	1.286	10.15	0.0398
1.7	2.217	1.424	1.941	1.529	2.697	1.328	10.46	0.0449
1.8	2.415	1.480	2.085	1.600	3.002	1.370	10.76	0.0504
1.9	2.619	1.534	2.231	1.671	3.321	1.410	11.06	0.0561
2.0	2.828	1.587	2.378	1.741	3.655	1.449	11.34	0.0622
2.1	3.043	1.640	2.528	1.810	4.005	1.487	11.62	0.0686
2.2	3.263	1.692	2.679	1.879	4.369	1.524	11.90	0.0752
2.3	3.488	1.742	2.832	1.947	4.747	1.561	12.16	0.0822
2.4	3.718	1.793	2.987	2.015	5.140	1.597	12.43	0.0895
2.5	3.953	1.842	3.144	2.081	5.548	1.633	12.68	0.0972
2.6	4.192	1.891	3.302	2.148	5.970	1.667	12.93	0.1051
2.7	4.437	1.939	3.461	2.214	6.407	1.701	13.18	0.1133
2.8	4.685	1.987	3.622	2.279	6.858	1.734	13.42	0.1219
2.9	4.939	2.034	3.784	2.344	7.323	1.768	13.66	0.1307
3.0	5.196	2.080	3.948	2.408	7.802	1.800	13.89	0.1399
3.1	5.458	2.126	4.113	2.472	8.296	1.832	14.12	0.1494
3.2	5.724	2.172	4.280	2.536	8.803	1.863	14.35	0.1592
3.3	5.995	2.217	4.448	2.599	9.324	1.894	14.57	0.1693
3.4	6.269	2.261	4.617	2.662	9.860	1.924	14.79	0.1797

The last column contains values of the velocity head $n^2/2g$ for the values of the velocity n and the last column but one gives values of the theoretic velocity due to a head n. The value of g used in these columns is 32.162 ft per sec per sec.

Powers of Numbers, Velocities, Velocity Heads (Continued)

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{1.8}$	$n^{1.87}$	$n^{0.535}$	$\sqrt{2gn}$	$n^2/2g$
3.5	6.548	2.305	4.787	2.724	10.41	1.955	15.01	0.1904
3.6	6.831	2.349	4.959	2.786	10.97	1.984	15.22	0.2015
3.7	7.117	2.392	5.132	2.848	11.55	2.014	15.43	0.2128
3.8	7.408	2.435	5.306	2.910	12.14	2.043	15.64	0.2245
3.9	7.702	2.478	5.481	2.971	12.74	2.071	15.84	0.2364
4.0	8.000	2.520	5.657	3.031	13.36	2.099	16.04	0.2487
4.1	8.302	2.562	5.834	3.092	13.99	2.127	16.24	0.2613
4.2	8.607	2.603	6.013	3.152	14.64	2.155	16.44	0.2742
4.3	8.916	2.644	6.192	3.212	15.30	2.182	16.63	0.2874
4.4	9.229	2.685	6.373	3.272	15.97	2.209	16.82	0.3009
4.5	9.546	2.726	6.554	3.331	16.65	2.236	17.01	0.3148
4.6	9.866	2.766	6.737	3.390	17.35	2.262	17.20	0.3289
4.7	10.19	2.806	6.920	3.449	18.07	2.289	17.39	0.3434
4.8	10.51	2.846	7.105	3.507	18.79	2.315	17.57	0.3582
4.9	10.85	2.885	7.290	3.566	19.53	2.340	17.75	0.3732
5.0	11.18	2.924	7.477	3.624	20.28	2.366	17.93	0.3886
5.1	11.52	2.963	7.664	3.682	21.05	2.391	18.11	0.4043
5.2	11.86	3.001	7.852	3.739	21.82	2.416	18.29	0.4203
5.3	12.20	3.040	8.042	3.797	22.62	2.441	18.46	0.4367
5.4	12.55	3.078	8.232	3.854	23.42	2.465	18.64	0.4533
5.5	12.90	3.116	8.423	3.911	24.24	2.489	18.81	0.4702
5.6	13.25	3.153	8.615	3.968	25.07	2.514	18.98	0.4875
5.7	13.61	3.191	8.807	4.024	25.91	2.537	19.15	0.5051
5.8	13.97	3.228	9.001	4.081	26.77	2.561	19.32	0.5231
5.9	14.33	3.265	9.195	4.137	27.64	2.585	19.48	0.5411
6.0	14.70	3.302	9.391	4.193	28.53	2.608	19.65	0.5596
6.1	15.07	3.338	9.587	4.249	29.41	2.631	19.81	0.5784
6.2	15.44	3.375	9.783	4.304	30.32	2.654	19.97	0.5975
6.3	15.81	3.411	9.981	4.360	31.24	2.677	20.13	0.6170
6.4	16.19	3.447	10.18	4.415	32.18	2.700	20.29	0.6367
6.5	16.57	3.483	10.38	4.470	33.12	2.722	20.45	0.6568
6.6	16.96	3.519	10.58	4.525	34.08	2.744	20.61	0.6771
6.7	17.34	3.554	10.78	4.580	35.06	2.767	20.76	0.6978
6.8	17.73	3.589	10.98	4.635	36.04	2.789	20.92	0.7188
6.9	18.13	3.624	11.18	4.689	37.04	2.811	21.07	0.7401
7.0	18.52	3.659	11.39	4.743	38.05	2.832	21.22	0.7617
7.1	18.92	3.694	11.59	4.797	39.07	2.854	21.37	0.7836
7.2	19.32	3.729	11.79	4.851	40.10	2.875	21.52	0.8058
7.3	19.72	3.763	12.00	4.905	41.15	2.896	21.67	0.8284
7.4	20.13	3.797	12.21	4.959	42.21	2.918	21.82	0.8512
7.5	20.54	3.832	12.41	5.012	43.29	2.939	21.97	0.8744
7.6	20.95	3.866	12.62	5.066	44.37	2.960	22.11	0.8979
7.7	21.36	3.899	12.83	5.119	45.47	2.980	22.26	0.9217
7.8	21.79	3.933	13.04	5.172	46.58	3.001	22.40	0.9458
7.9	22.20	3.967	13.24	5.225	47.70	3.022	22.54	0.9702

These tables give values of three-halves powers and two-thirds powers of numbers which are useful in weir computations, and also other powers which occur in exponential formulas for flow in pipes and channels. The last

Powers of Numbers, Velocities, Velocity Heads (Continued)

n	$n^{3/2}$	$n^{2/3}$	$n^{1.25}$	$n^{0.8}$	$n^{1.87}$	$n^{0.535}$	$\sqrt{2\,gn}$	$n^2/2\,g$
8.0	22.63	4.000	13.45	5.278	48.84	3.042	22.69	0.9949
8.1	23.05	4.033	13.67	5.331	49.99	3.062	22.83	1.020
8.2	23.48	4.066	13.88	5.383	51.15	3.082	22.97	1.045
8.3	23.91	4.099	14.09	5.436	52.32	3.102	23.11	1.071
8.4	24.34	4.132	14.30	5.488	53.51	3.122	23.25	1.097
8.5	24.78	4.165	14.51	5.540	54.70	3.142	23.38	1.123
8.6	25.22	4.198	14.73	5.592	55.91	3.162	23.52	1.150
8.7	25.66	4.230	14.94	5.644	57.14	3.182	23.66	1.177
8.8	26.10	4.262	15.16	5.696	58.37	3.201	23.79	1.204
8.9	26.55	4.295	15.37	5.748	59.62	3.221	23.93	1.231
9.0	27.00	4.327	15.59	5.800	60.87	3.240	24.06	1.259
9.1	27.45	4.359	15.81	5.851	62.14	3.259	24.20	1.287
9.2	27.91	4.391	16.02	5.902	63.43	3.278	24.33	1.316
9.3	28.36	4.422	16.24	5.954	64.72	3.297	24.46	1.344
9.4	28.82	4.454	16.46	6.005	66.03	3.316	24.59	1.374
9.5	29.28	4.486	16.68	6.056	67.35	3.335	24.72	1.403
9.6	29.74	4.517	16.90	6.107	68.68	3.354	24.85	1.433
9.7	30.21	4.548	17.12	6.158	70.03	3.372	24.98	1.463
9.8	30.68	4.580	17.34	6.209	71.38	3.391	25.11	1.493
9.9	31.15	4.611	17.56	6.259	72.75	3.409	25.24	1.524
10.0	31.62	4.642	17.78	6.310	74.13	3.428	25.36	1.554
10.5	34.02	4.795	18.90	6.561	81.21	3.518	25.99	1.714
11.0	36.48	4.946	20.03	6.810	88.59	3.607	26.60	1.881
11.5	38.99	5.095	21.18	7.056	96.27	3.694	27.20	2.056
12.0	41.57	5.241	22.34	7.300	104.25	3.779	27.78	2.238
12.5	44.19	5.386	23.50	7.543	112.52	3.862	28.36	2.429
13.0	46.87	5.529	24.68	7.783	121.08	3.944	28.92	2.627
13.5	49.60	5.670	25.88	8.022	129.93	4.024	29.47	2.833
14.0	52.38	5.809	27.08	8.259	139.08	4.103	30.01	3.047
14.5	55.21	5.946	28.29	8.494	148.51	4.181	30.54	3.268
15.0	58.09	6.082	29.52	8.727	158.23	4.258	31.06	3.498
15.5	61.02	6.217	30.75	8.959	168.24	4.333	31.58	3.735
16.0	64.00	6.350	32.00	9.190	178.53	4.407	32.08	3.979
16.5	67.02	6.481	33.25	9.419	189.10	4.481	32.58	4.232
17.0	70.09	6.611	34.52	9.647	199.96	4.553	33.07	4.492
17.5	73.21	6.741	35.79	9.873	211.10	4.624	33.55	4.761
18.0	76.37	6.868	37.08	10.098	222.52	4.694	34.03	5.037
18.5	79.57	6.995	38.37	10.321	234.21	4.764	34.50	5.320
19.0	82.82	7.120	39.67	10.544	246.19	4.832	34.96	5.612
19.5	86.11	7.245	40.97	10.765	258.44	4.900	35.42	5.911
20.0	89.44	7.368	42.29	10.986	270.97	4.967	35.87	6.218

column contains values of the velocity-head $n^2/2\,g$ for various values of n, and the last column but one gives values of the theoretic velocity $\sqrt{2\,gn}$ for a head n. The value of g used for these columns is 32.162 ft per sec per sec.

See Sect. 1, Art. 13, for a more extended table of three-halves powers.

FLOW IN PIPES AND CHANNELS

7. Orifices and Short Pipes

Small Orifice in a Thin Plate. When efflux takes place thru an orifice in a thin plate whose diameter is not greater than ⅛ the head, a contracted vein, or "vena contracta," is formed, the filaments of water not becoming parallel until reaching a plane at right angles to the axis of the jet and about ½ its diameter from the orifice; and not until reaching this plane does the internal fluid pressure become the same as the pressure of the surrounding medium. If the vessel were large enough so that the velocity at the surface were zero, the theoretical velocity at the place of greatest contraction would be $v = \sqrt{2\,gh}$, where h is the head at its center of pressure and g is the acceleration due to gravity. For orifices where the diameter is not greater than ⅛ h, the center of pressure and center of gravity are assumed to coincide, and, in the case of orifices in a vertical wall, the centers of gravity of the orifice and of the contracted vein are assumed at the same elevation. Practically, however, the velocity $v = C_v\sqrt{2\,gh}$, where C_v, the coefficient of velocity, is determined experimentally. The discharge then would be $Q = C_cAv$, where A is the area of the orifice, C_c the coefficient of contraction, and v the actual velocity at the place of greatest contraction. Then:

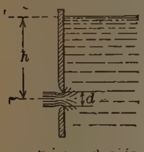

Fig. 8. Small Orifice in Thin Plate

$$Q = C_cC_vA\sqrt{2\,gh} = CA\sqrt{2\,gh}$$

where C is the coefficient of discharge.

For ordinary work very satisfactory results are obtained by using C_v=0.97 and C_c = 0.62. C then $= C_cC_v$ =0.61. The values of C generally range from 0.63 to 0.60, the lower values occurring with the higher heads. Square and rectangular orifices have slightly higher discharges than circular ones, the increases being between 1% and 1½%.

Submerged Orifices. The theoretical discharge $Q = A\sqrt{2\,gh}$, where h is the difference in level of the free surfaces on the two sides of the orifice. Coefficients in this case are, according to Weisbach, 1/75 less than for free discharge into the air.

Short Tube. If an external tube of the same diameter as the orifice and at least 2½ diameters long be attached to it, the effluent stream, after forming a "vena contracta," re-expands and fills the tube and the discharge is increased. To attain this result, however, the tube must be full of water before the outer end is unstopt, and must not be oily; nor must the head h be greater than about 40′ for efflux into the air. The coefficient of velocity is then the coefficient of discharge, since the coefficient of contraction is unity. C = 0.815 for tubes from 2½ to 3 diameters long. For other lengths the coefficients for tubes and short pipes are:

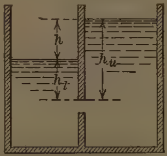

Fig. 9. Submerged Orifice

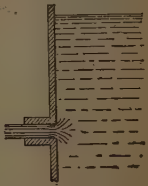

Fig. 10. Short Tube

Length in diameters....	1	3	5	10	25	50	75	100
Coefficient...........	0.62	0.815	0.79	0.77	0.71	0.64	0.59	0.55

Borda's Tube is a short tube, of the same diameter as the orifice, attached to the orifice on the inside of the wall. If this tube is short enough so that the water is discharged without filling the tube, the theoretical coefficient C is 0.5, which is very nearly that obtained by experiment.

Fig. 11. Borda's Tube

Converging Mouthpieces. The discharge thru an orifice is increased as the filaments of water are made to approach it in a more nearly normal direction. Hence, if walls of an orifice be inclined downstream, the flow is increased until it reaches a maximum when the angle between two opposite sides is about 13½°, and C then is about 0.94. For other angles the value of C is

For angle	0°	10°	13.5°	20°	30°	40°	60°
C	0.84	0.93	0.94	0.93	0.90	0.87	0.82

Rounded Mouthpiece. Rounding the entrance further reduces the contraction and when the curve approximates to the "vena contracta," $C \doteq$ unity and $Q = C_v A \sqrt{2\,gh}$. Experimentally $C = C_v$ varies from 0.96 to 0.99.

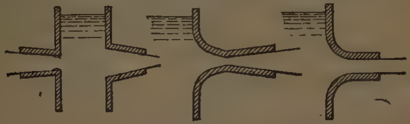

Fig. 12. Divergent, Convergent, Compound and Rounded Mouthpieces.

Divergent Mouthpieces, if they flow full and are more than 5 diameters in length, will increase the value of C above that for the orifice at the inlet and in some cases to as much as 1.4.

Compound Mouthpieces. If a divergent tube, whose angle of divergence between opposite sides is about 10°, be attached to a rounded mouthpiece and caused to flow full, the head at the throat becomes less than the atmospheric pressure and may practically fall as low as -24 feet of water. The total head producing flow thru the throat then is $h + 24$ feet, and $v = C\sqrt{2\,g(h+24)}$, where C is for the rounded orifice less a small allowance for friction in the diverging tube. Theoretically it would be possible to have C as high as 8.0 or 9.0 for heads less than 1 foot, but experimentally 2.43 is the highest value obtained.

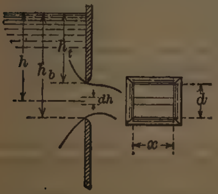

Fig. 13. Large Orifices

Large Orifices. This case differs from the discharge of "small orifices" in that the difference of head between the top and bottom of the orifice is so great that it must be taken into account. Let h_t be the distance from the free surface of the water to the top of the orifice; h_b be the distance from the free surface of the water to the bottom of the orifice; x be the width and $d = h_b - h_t$ the depth of the orifice; h be the distance from the free surface of the water to any horizontal layer thru the orifice of dh depth and width x. Then the velocity of water thru this thin

layer is $\sqrt{2\,gh}$, the small discharge dQ for this area is $\sqrt{2\,gh}\cdot xdh$, and the total theoretical discharge is

$$Q = \int_{h_b}^{h_t} \sqrt{2\,gh}.xdh$$

The width x may be either constant or variable, and if the latter, must be exprest in terms of h before integrating.

For a rectangle with x = constant, $Q = \tfrac{2}{3}\,x\sqrt{2\,g}\left(h_b^{3/2} - h_t^{3/2}\right)$.

For a vertical triangle, base x at the top and horizontal,

$$Q = \tfrac{4}{15}\,\frac{x\sqrt{2\,g}}{h_b - h_t}\left[2\,h_b^{5/2} - 5\,h_b h_t^{3/2} + 3h_t^{5/2}\right]$$

These equations are not exact theoretically, because the head is completely changed into velocity head at the "vena contracta" and not at the orifice as was assumed; because the assumption is that the velocity is constant for any area $x\cdot dh$, when it is known that at the edges the velocity is materially decreased; and because it is assumed that the water flows in plain layers at the orifice, which is not the case. A correct result could be more nearly approached were a point taken at the "vena contracta" instead of at the orifice, but accurate measurement is there impossible. However, by the introduction of an experimental coefficient, which is less variable than the C in the preceding articles, these equations answer every purpose when correction for the velocity v_a with which the water approaches the orifice is made by adding to the observed head the head to which the velocity of approach is due, $=v_a^2/2\,g=h_v$. For heads above two feet the value of C for square orifices varies from 0.60 to 0.62 and for circular orifices from 0.59 to 0.61.

Submerged Rectangular Orifices. Let h_u be the depth of the top of the orifice on the incoming or upper side; h_l be the depth of the top of the orifice on the discharge or lower side; A be the area of the orifice; $h_v = v_a^2/2\,g$ the equivalent head of the velocity of approach. Then (Fig. 9):

$$Q = CA\sqrt{2g(hu+hv-hl)}$$

The value of C for a submerged orifice may be taken as about 1 % less than its value for the same orifice under the same effective head when discharging freely into the air.

Partially Submerged Orifices. Let h_t and h_b be as before, b the width of the orifice, h the difference in elevation of water on the two sides of the orifice. Then values of C are closely the same as for submerged orifices, and

$$Q = C\sqrt{2\,g}\;b\left[\tfrac{2}{3}\left(h^{3/2} - h_t^{3/2}\right) + (h_b - h)\,h^{1/2}\right]$$

Incomplete Contraction. The foregoing matter on orifices is based upon the assumption that the contraction is complete. Suppression of contraction is accomplished by adding an internal projection at the edge of the orifice extending over the entire perimeter. If the projection does not extend over the entire perimeter, or if it is a short distance from the orifice, partial contraction results. Suppressing the contraction increases the discharge, but decreases the energy of the jet. If k is the ratio of the periphery of the orifice with a border to the whole periphery, the coefficient of discharge for partial contraction is

For rectangular orifices, $C_s = C\,(1+0.152\,k)$
For circular orifices, $C_s = C\,(1+0.128\,k)$

If the sides of a vessel are less than 2.7 times the corresponding width of the orifice distant from any side of the orifice, their influence is felt in partially reducing the contraction.

Two Orifices Adjacent, separated by a narrow bar, discharge more than the two considered separately because of mutual velocity influences. When the width of bar is less than the least dimension of the orifices the discharge will nearly equal that thru an orifice of area and form like orifices and bar combined less that of an orifice of area and form equal to the bar with contractions supprest next the orifices.

8. Long Pipes

Resistances. When water flows thru a pipe of such length that the particles come in contact with the wall, resistance is produced by what is commonly termed fluid friction. For short pipes this is inconsiderable and is provided for in the coefficients of discharge, but when a pipe is more than 50 diameters in length this element becomes important, and as the length increases it makes up the major part of the loss encountered.

Hydraulic Grade Line. When water flows from rest into a channel of any sort, an amount of head h_v equal to $v^2/2g$ is absorbed in creating the velocity with which it discharges, and a small amount more is used up in frictional resistances. As the water passes thru the channel, an amount of head h_f is used up in overcoming the resistance to flow. If the pipe expands the velocity is reduced and a part of the h_v is returned and appears as pressure head h_p. If the pipe contracts, some of the h_p is converted into h_v. The pressure head measures the height of a column of water which would exert the same pressure as does the water against the walls of the channel. The Hydraulic Grade Line, or Hydraulic Gradient, is the locus of the tops of all such columns of water that can be placed along the channel, and the distance from the grade line to the center of the pipe at any point measures the pressure head at the latter. In an open channel the surface of the stream is the Hydraulic Grade Line. The total head producing flow is equal to the sum of the velocity head $v^2/2g$ at discharge and all losses of head along the line.

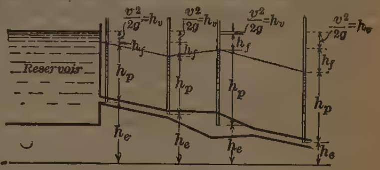

Fig. 14. Hydraulic Grade Line

Conditions of Flow. Normal flow is a flow such that the components of the velocities in the direction of flow at any section vary in approximate accordance with the ordinates of a cylinder with an ellipsoidal end, such that the maximum ordinate is approximately 1.19 times the mean, the lengths of the cylinder being the component of the velocity in the direction of flow at the surface of contact and the lengths of the ellipsoidal end being the difference between the components of the velocities in the direction of flow at the axis of the pipe and at the surface of contact. This condition of flow is disturbed by contraction, expansion or curvature of the stream, and the water requires a length of about 40 diameters of the pipe to readjust itself after passing a contraction, about 50 to 60 diameters after passing an expansion, and from 100 to 300 diameters after passing a curve. In any region of abnormal flow the pressure indicated by an orifice in the pipe wall will be different for the same mean velocity than that indicated in case of normal flow. In the cases of contractions and expansions, the indication of pressure will usually be greater than that for normal flow, and after a curve, if the pressure be read on the concave side of the stream, the indication will be high, while if read on the opposite side it will be low, as also to a less degree if read at right angles to the direction of curvature. It is, therefore, essential, when making accurate measurements of loss of head, to take observations only at points where the flow is known to be normal. In very rough pipes, on account of the retardation of velocity at the wall, the ratio between the mean and the

maximum is reduced and may possibly be as low as 0.60. After a contraction, on the other hand, the ratio may be increased to 0.95, and in the case of a jet from a tapering nozzle to 0.995.

Under normal flow the absorption of head in overcoming the resistances to the motion of the water, called the friction head, h_f, varies as a power of the mean velocity between the 1.7 and the square.

Low Velocity Flow. When water flows in capillary tubes or between plates close together, the friction head has been found to vary as the first power of the velocity, and at very low velocities in larger pipes the same law holds. The velocity below which this form of flow occurs is called the CRITICAL VELOCITY. The critical velocity occupies a considerable range depending upon whether the flow is changing from sub-critical velocity to super-critical velocity flow, or vice versa. In either case the existing condition of flow tends to continue itself, so that for an increasing velocity the critical velocity will be higher than for a decreasing one. Experiments by means of coloring matter in the water have shown that as long as the friction head varies as the first power of the velocity the flow is taking place in straight-line filaments, but as soon as the critical velocity is past the flow has become involved in eddies or vortices, and the exponent of v is at once increased to above 1.70.

9. Capillary Tubes and Soils

For Capillary Tubes let T_c = temperature centigrade, s = slope, D = diameter of tube, v = velocity. The formula deduced by Poiseuille is

$$v = 52.500\,(1 + 0.03368\,T_c + 0.000221\,T_c^2)\,D^2 s$$

which has been approximately confirmed by other experimenters, and may be assumed to apply to sub-critical velocity flow. The critical velocity varies inversely as some power of the diameter. Experiments (Trans. Am. Soc. C. E., Dec., 1903, vol. 51, p. 298) have given

$$\text{Critical Velocity} = 0.088\,/D^{0.784}$$

as an approximate formula, for a temperature of 70° Fahr.

From the above it is evident that sub-critical velocity flow will be encountered in practise only in small pipes, but it appears to extend considerably farther at low than at high temperatures. In a 2-inch pipe with a temperature of 40° Fahr. or 4.4 Centigrade, the critical velocity appeared to be about 0.35 foot per second.

Flow in Soils. For the flow of water thru soils, let T_F = temperature Fahrenheit, s = slope, d_s = effective size of the sand in millimeters, V = velocity thru the soil in meters per day, considering the entire area of the bed as effective in carrying water, and C = a coefficient depending upon the uniformity coefficient of the sand, the shape of the grains, the chemical composition and the closeness and packing of the material, and ranging in value for sands from 370 to 1200, varying inversely as the uniformity coefficients. The Lawrence Experiment Station has derived the formula

$$V = C d_s^2 \left(\frac{T_F + 10^\circ}{60^\circ}\right) s$$

This is seen to be similar to the formula for small pipes in that temperature plays an important part and that h or s varies as the first power of v.

The Effective Size of a sand or gravel is the size of grain such that 10% of the particles by weight are smaller and 90% greater. The size of a sand grain is determined by the diameter of a sphere of equal volume. THE

UNIFORMITY COEFFICIENT of a sand or gravel is the ratio of that size of grain than which 60 % of the sample is finer, to the effective size.

Evidently a high uniformity coefficient indicates a large variation in size of grains, while a coefficient of unity would indicate that substantially all the grains were of one size.

Temperature. In small pipes and in the flow thru soils the temperature of the water is a very important factor. For smooth pipes 2 inches diameter and under, the loss of head has been found to increase about 4 % for each 10° fall of temperature from 70° to 40° Fahr. (Trans. Am. Soc. C. E., 1903, vol. 51, p. 290.) Other experiments have shown that near the boiling point the loss of head increases with the temperature. In large pipes the effect of temperature may be safely overlookt, except in cases where great refinement is necessary.

In some weir experiments where the discharge was about 200 cubic feet per second it has appeared that the effect of a change from 76° to 33° Fahr. was to decrease the discharge about ¾ of 1 percent. With small weirs, laboratory experiments have shown that the influence of temperature between 32° and 45° Fahr. is greater per degree than higher up the scale. Accurate data covering these points are, however, wanting.

10. Formulas for Long Pipes

The Chezy Formula. If v is the velocity in the pipe, C a coefficient dependent upon roughness, density, velocity, and diameter, r the Hydraulic Radius, namely the cross-sectional area divided by the wetted perimeter, h_f the frictional loss of head in a length L, and if h_f/L be designated by s, the inclination or slope, then

$$v = C\sqrt{rh_f/L} \quad \text{or} \quad v = C\sqrt{rs}$$

in which for new pipes C ranges from 95 to 152 and for old pipes from 60 to 120, the value increasing both with the diameter and the velocity, as shown in the following tables.

Values of C in Chezy Formula for Cast-iron Pipes

Diameter of pipe, inches	Velocities in feet per second							
	For new pipes				For old pipes			
	1	3	6	10	1	3	6	10
3	95	98	100	102	63	68	71	73
6	96	101	104	106	69	74	77	79
9	98	105	109	112	73	78	80	84
12	100	108	112	117	77	82	85	88
15	102	110	117	122	81	86	89	91
18	105	112	119	125	86	91	94	97
24	111	120	126	131	92	98	101	104
30	118	126	131	136	98	103	106	109
36	124	131	136	140	103	108	111	114
42	130	136	140	144	105	111	114	117
48	135	141	145	148	106	112	115	118
60	142	147	150	152	.....	.....	.....	.....

For steel riveted pipes see next page. Chezy's formula is also used for conduits and streams but the coefficient C for such cases is generally exprest in terms of r and s (see Art. 15 for formulas of Bazin and Kutter).

Darcy's Formula. The original form of Darcy's equation was $rs = av + bv^2$, where a and b were coefficients. This Darcy later reduced to $rs = Cv^2$, where $C = c_1 + c_2/r$, where c_1 and c_2 are constants. For new cast-iron and for

Values of C in Chezy Formula for Steel Riveted Pipes

Diameter of pipes, inches	Velocity in ft per second 1	3	5	10
3	81	86	89	92
11	92	102	107	115
11	93	99	102	105
15	109	112	114	117
38	113	113	113	113
42	102	106	108	111
48	105	105	105	105
72	110	110	111	111
72	93	101	105	110
103	114	109	106	104

Values of C in Darcy's Formula $CV^2 = Ds$

Diameter, inches	Rough pipes	Smooth pipes
3	0.00080	0.00040
4	0.00076	0.00038
6	0.00072	0.00036
8	0.00068	0.00034
10	0.00066	0.00033
12	0.00066	0.00033
14	0.00065	0.000325
16	0.00064	0.00032
24	0.00064	0.00032
30	0.00063	0.000315
36	0.00062	0.00031
48	0.00062	0.00031

wrought-iron pipes of the same roughness, Darcy's values of these constants are $c_1 = 0.0000773$ and $c_2 = 0.00000162$. The formula then reduces to

$$h_f = 0.00000642 \frac{(12\,D+1)}{D} \frac{v^2 L}{r} \quad \text{or} \quad v = 394\sqrt{\frac{D}{12\,D+1}}\sqrt{rs}$$

where D is the diameter of the pipe in feet. For rough pipe Darcy reduced the velocity one-half. Darcy's formula may be transposed to $CV^2 = Ds$, in which case C has an average value of 0.00032 for clean pipes of diameters from 8 to 48 inches inclusive, the variation being only 3 % from the mean for all except the 8-inch. The preceding table gives more accurate values of C for Darcy's formula in the last form.

Fanning's formula for flow in pipes is

$$h_f = \frac{4fL}{D}\frac{v^2}{2g} \quad \text{or} \quad v = \sqrt{\frac{2gDh_f}{4fL}}$$

where f is a coefficient which ranges from 0.0071 to 0.0028 for new pipes and from 0.0152 to 0.0046 for old ones, the value decreasing as diameter and velocity increase. The other notation is the same as that at the beginning of this article.

Values of f in Fanning's Formula for Cast-iron Pipes

Diameter of pipe, inches	Velocity in feet per second: For new pipes 1	3	6	10	For old pipes 1	3	6	10
3	.0071	.0067	.0064	.0062	.0152	.0139	.0128	.0122
6	.007	.0063	.006	.0057	.0135	.0117	.0108	.0103
9	.0067	.0058	.0055	.0051	.0122	.0105	.010	.0092
12	.0064	.0056	.0051	.0048	.0108	.0096	.0089	.0084
15	.0062	.0053	.0048	.0043	.0099	.0087	.0081	.0078
18	.0058	.0051	.0045	.0041	.0087	.0078	.0073	.0069
24	.0053	.0045	.0040	.0037	.0076	.0067	.0063	.0060
30	.0046	.0040	.0037	.0035	.0067	.0061	.0057	.0055
36	.0042	.0037	.0035	.0033	.0061	.0056	.0052	.0050
42	.0038	.0035	.0033	.0031	.0058	.0052	.005	.0048
48	.0036	.0032	.0031	.0029	.0057	.0051	.0049	.0046
60	.0032	.0030	.0029	.0028	......	......	......	......

Tables for Long Pipes are given on the three following pages. The friction factors $4f$ used in computing them differ slightly from those at the foot of the preceding page and are the same as those given in Merriman's "Treatise on Hydraulics." These tables apply to new, clean, straight cast-iron and wrought-iron pipes, either smooth or coated with coal tar, and laid with close joints. A pipe is said to be long when its length is such that the error in computing v by the last formula does not exceed five percent; this will usually be the case when the length of the pipe is greater than 1000 diameters.

The discharges given in the tables are accurate in the last figure for the given velocities. Thus for a velocity of 3.4 ft per sec the discharges for pipes 6 and 16 inch in diameter are 40.1 and 285 cu ft per min. The friction head, given in the second column under each size of pipe, is however liable to an error of one or two units in the second figure; thus for 3.4 ft per sec, in the 6-inch pipe the head 0.88 per 100 ft may actually range from 0.86 to 0.90 for new, clean pipes.

Velocities and Discharges for a given pipe may be found from the tables when the friction head is known. For example, let a pipe 3500 ft long and 6 in in diameter have a total head of 37.8 ft. Here the friction head per 100 ft is $100 \times 3.78/3500 = 1.08$, whence from the table, velocity $= 3.8$ ft per sec, and discharge $= 44.8$ cu ft per min. Again, let an 8-in pipe 6075 ft long be under a head of 112.5 ft, then friction head per 100 ft is $100 \times 112.5/6075 = 1.85$, whence velocity $= 6.1$ ft per sec, and discharge $= 128$ cu ft per min. These are for new, clean, straight iron pipes. Curves influence results but little unless they are very sharp.

For Old Pipes the actual heads should be multiplied by the following numbers before using the table to obtain velocities and discharges:

For diameter,	3	6	12	16	24	30	36 in
Multiplier....	0.50	0.55	0.60	0.62	0.64	0.65	0.66

For example, let an old pipe 3500 ft long and 6 in diameter be under an actual head of 37.8 ft or 1.08 ft per 100 ft; the true friction head is $0.55 \times 1.08 = 0.59$ ft per 100 ft, whence from the table velocity $= 2.8$ ft per sec and discharge $= 33$ cu ft per min. Similarly, for given velocities the true friction heads are found approximately by multiplying the tabular values by the following numbers:

For diameter,	3	6	12	16	24	30	36 in
Multiplier....	2.00	1.81	1.67	1.61	1.56	1.54	1.52

For example, for a velocity of 6.0 ft per sec the friction head in an old pipe of 12 in diameter is 1.87 instead of 1.12 ft per 100 ft. The term old pipe is a vague one, and refers to the amount of corrosion and incrustation rather than to the actual life in years.

The Required Diameter for a pipe to furnish a given discharge under a given head may also be roughly found from the tables. For example, to find diameter to furnish 100 cu ft per min under a head of 1.2 ft per 100 ft: for a new, clean pipe the tables give 8 inches as required diameter, for an old pipe, assume multiplier as 0.5, then head becomes 0.60 ft per 100 ft, and the table shows that a 10-in pipe is somewhat too large.

The formula for computing the diameter of a long pipe is: $D = 0.479\ (4flq^2/h)^{\frac{1}{5}}$, in which q = discharge in cu ft per sec, h = head in ft, l = length of pipe in ft, D = diameter of pipe in ft, and a rough mean value of f being taken as 0.005 for new pipe. After D is computed the velocity is found by $v = q/\frac{1}{4}\pi D^2$ and thus a better value of f may be obtained from the table at foot of page 1090. Then a new diameter may be re-computed if the change in f seems to warrant it.

For example, to find the diameter of a new pipe to deliver 67 cu ft per sec under a head of 24 ft, its length being 4500 ft. Using $4f$ as 0.020, the formula gives $D \doteq 3.35$ ft, whence $v = 7.6$ ft per sec. Then from the table at foot of page 1090, a closer value of f is found to be 0.0032, or $4f = 0.013$. A second computation now gives $D = 3.06$ ft so that a 36-inch pipe should be used. With the same data the rough value of $4f$ for an old pipe is 0.035 and D is about 42 inches.

Discharge and Friction Heads for New, Clean Iron Pipes

Velocity Feet per Second	1-inch Pipe		1½-inch Pipe		2-inch Pipe		3-inch Pipe		4-inch Pipe	
	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft
1.0	0.327	0.7	0.736	0.45	1.31	0.33	2.95	0.26	5.24	0.14
1.2	.393	1.0	.884	.64	1.57	.46	3.53	.28	6.28	.20
1.4	.458	1.4	1.03	.83	1.83	.60	4.12	.37	7.33	.27
1.6	.527	1.7	1.18	1.04	2.09	.77	4.71	.47	3.38	.34
1.8	.589	2.1	1.33	1.25	2.36	.93	5.30	.57	9.42	.41
2.0	0.654	2.5	1.47	1.5	2.62	1.1	5.89	0.69	10.5	0.50
2.2	.719	3.0	1.62	1.8	2.88	1.3	6.48	.82	11.5	.60
2.4	.785	3.5	1.77	2.2	3.14	1.6	7.07	.97	12.6	.71
2.6	.851	4.0	1.91	2.5	3.40	1.8	7.66	1.12	13.6	.82
2.8	.916	4.5	2.06	2.8	3.67	2.0	8.25	1.26	14.7	.94
3.0	0.982	5.2	2.21	3.2	3.93	2.3	8.84	1.5	15.7	1.07
3.2	1.05	5.9	2.36	3.7	4.19	2.6	9.42	1.7	16.7	1.2
3.4	1.11	6.5	2.50	4.1	4.45	2.9	10.0	1.8	18.7	1.3
3.6	1.18	7.1	2.65	4.4	4.71	3.2	10.6	2.0	18.8	1.4
3.8	1.24	7.9	2.80	4.9	4.97	3.6	11.2	2.3	19.9	1.6
4.0	1.31	8.7	2.95	5.5	5.24	4.0	11.8	2.5	20.9	1.8
4.2	1.37	9.3	3.09	5.8	5.50	4.3	12.4	2.7	22.0	1.9
4.4	1.44	10.2	3.24	6.4	5.76	4.7	13.0	3.0	23.0	2.1
4.6	1.51	11.1	3.39	7.0	6.02	5.1	13.5	3.2	24.1	2.3
4.8	1.57	12.0	3.53	7.6	6.28	5.6	14.1	3.5	25.1	2.5
5.0	1.64	13	3.68	8.1	6.54	6.0	14.7	3.8	26.2	2.7
5.2	1.70	14	3.83	8.7	6.81	6.4	15.3	4.0	27.2	2.9
5.4	1.77	15	3.98	9.3	7.07	6.8	15.9	4.4	28.3	3.1
5.6	1.83	16	4.12	10.1	7.33	7.3	16.5	4.7	29.3	3.4
5.8	1.90	17	4.27	10.6	7.59	7.8	17.1	5.0	30.4	3.6
6.0	1.96	18	4.42	11	7.85	8.4	17.7	5.4	31.4	3.9
6.2	2.03	19	4.57	12	8.12	8.9	18.3	5.7	32.5	4.1
6.4	2.09	20	4.71	13	8.38	9.5	18.8	6.1	33.5	4.4
6.6	2.16	22	4.86	14	8.64	10.0	19.4	6.4	34.6	4.6
6.8	2.23	23	5.01	15	8.90	10.5	20.0	6.7	35.6	4.9
7.0	2.29	24	5.15	15	9.16	11.0	20.6	7.1	36.6	5.1
7.2	2.36	25	5.30	16	9.42	11.6	21.2	7.4	37.7	5.4
7.4	2.42	27	5.45	17	9.69	12.2	21.8	7.9	38.7	5.7
7.6	2.49	28	5.59	18	9.95	12.9	22.4	8.4	39.8	6.0
7.8	2.55	29	5.74	19	10.2	13.4	23.0	8.8	40.8	6.3
8.0	2.62	31	5.89	20	10.5	14.2	23.6	9.2	41.9	6.6
8.2	2.68	32	6.04	20	10.7	14.9	24.2	9.6	42.9	6.9
8.4	2.75	34	6.18	21	11.0	15.5	24.7	10.0	44.0	7.2
8.6	2.81	35	6.33	22	11.3	16.4	25.3	10.4	45.0	7.5
8.8	2.88	37	6.48	23	11.5	16.7	25.9	10.8	46.1	7.8
9.0	2.95	38	6.63	24	11.8	17	26.5	11.3	47.1	8.1
9.2	3.01	40	6.77	25	12.0	18	27.1	11.8	48.2	8.5
9.4	3.08	41	6.92	26	12.3	19	27.7	12.2	49.2	8.8
9.6	3.14	43	7.07	27	12.6	20	28.3	12.7	50.3	9.1
9.8	3.21	45	7.22	28	12.8	21	28.9	13.2	51.3	9.4

To reduce discharge to million gallons per 24 hours, multiply by 0.01077195.

Discharges and Friction Heads for New, Clean Iron Pipes (Continued)

Velocity Feet per Second	6-inch Pipe		8-inch Pipe		10-inch Pipe		12-inch Pipe		16-inch Pipe	
	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft
1.0	11.8	0.087	20.9	0.062	32.7	0.047	47.1	0.039	83.8	0.027
1.2	14.1	.13	25.1	.088	39.3	.068	56.5	.056	101	.039
1.4	16.5	.17	29.3	.119	45.8	.091	66.0	.075	117	.053
1.6	18.8	.21	33.5	.15	52.4	.12	75.4	.100	134	.068
1.8	21.2	.27	37.7	.19	58.9	.15	84.8	.12	151	.085
2.0	23.6	0.34	41.9	0.23	65.4	0.18	94.2	0.15	168	0.10
2.2	25.9	.39	46.1	.28	72.0	.22	104	.18	184	.13
2.4	28.3	.46	50.3	.34	78.5	.26	113	.21	201	.15
2.6	30.6	.53	54.5	.38	85.1	.30	123	.25	218	.17
2.8	33.0	.60	58.6	.43	91.6	.34	132	.28	235	.20
3.0	35.3	0.70	62.8	0.50	98.2	0.40	141	0.32	251	0.23
3.2	37.7	.79	67.0	.57	105	.45	151	.36	268	.26
3.4	40.1	.88	71.2	.63	111	.50	160	.41	285	.28
3.6	42.4	.97	75.4	.69	118	.55	170	.45	302	.31
3.8	44.8	1.08	79.6	.77	124	.61	179	.50	318	.35
4.0	47.1	1.2	83.8	0.85	131	0.66	188	0.55	335	0.38
4.2	49.5	1.3	88.0	.91	137	.71	198	.60	352	.41
4.4	51.8	1.4	92.0	1.00	144	.79	207	.65	369	.45
4.6	54.2	1.5	96.3	1.09	151	.86	217	.70	385	.49
4.8	56.5	1.6	101	1.18	157	.93	226	.76	402	.52
5.0	58.9	1.8	105	1.3	164	1.00	236	0.82	419	0.56
5.2	61.3	1.9	109	1.4	170	1.08	245	.87	436	.61
5.4	63.6	2.0	113	1.5	177	1.15	254	.93	452	.65
5.6	65.9	2.2	117	1.6	183	1.23	264	.99	469	.70
5.8	68.2	2.3	121	1.7	190	1.30	273	1.05	486	.74
6.0	70.7	2.4	126	1.8	196	1.38	283	1.12	503	0.79
6.2	73.0	2.6	130	1.9	203	1.47	292	1.19	519	.83
6.4	75.4	2.8	134	2.0	209	1.56	302	1.26	536	.88
6.6	77.8	2.9	138	2.1	216	1.65	311	1.33	553	.93
6.8	80.1	3.1	143	2.2	223	1.74	320	1.4	570	.98
7.0	82.5	3.2	147	2.3	229	1.82	330	1.5	586	1.03
7.2	84.8	3.4	151	2.4	236	1.91	339	1.6	603	1.06
7.4	87.2	3.6	155	2.6	242	2.02	349	1.7	620	1.12
7.6	89.5	3.8	159	2.7	249	2.13	358	1.8	637	1.18
7.8	91.9	3.9	163	2.8	255	2.23	367	1.9	653	1.25
8.0	94.2	4.1	168	3.0	262	2.4	377	1.9	670	1.32
8.2	96.7	4.3	172	3.1	268	2.5	386	2.0	687	1.37
8.4	99.0	4.5	176	3.2	275	2.6	396	2.1	704	1.43
8.6	101	4.7	180	3.4	281	2.7	405	2.1	720	1.48
8.8	104	4.9	184	3.5	288	2.8	415	2.2	737	1.54
9.0	106	5.1	188	3.7	295	2.9	424	2.3	754	1.60
9.2	108	5.3	193	3.8	301	3.0	434	2.4	771	1.66
9.4	111	5.5	197	3.9	308	3.1	443	2.5	788	1.71
9.6	113	5.7	201	4.1	314	3.2	452	2.6	804	1.77
9.8	115	6.0	205	4.2	321	3.3	462	2.7	821	1.84

To reduce discharge to million gallons per 24 hours, multiply by 0.01077195.

Discharges and Friction Heads for New, Clean Iron Pipes (Continued)

Velocity Feet per Second	20-inch Pipe		24-inch Pipe		30-inch Pipe		36-inch Pipe		42-inch Pipe	
	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft	Discharge Cu ft per min	Head Ft per 100 ft
1.0	131	0.021	188	0.016	295	0.013	424	0.010	577	0.008
1.2	157	.030	226	.024	353	.018	509	.014	693	.012
1.4	183	.040	264	.031	412	.024	594	.019	808	.015
1.6	209	.051	302	.041	471	.031	679	.024	924	.020
1.8	236	.063	339	.050	530	.038	763	.030	1039	.025
2.0	262	0.077	377	0.062	589	0.047	848	0.037	1155	0.030
2.2	288	.093	415	.074	648	.056	933	.044	1270	.036
2.4	314	.110	452	.086	707	.066	1018	.051	1385	.042
2.6	340	.13	490	.099	766	.076	1103	.059	1501	.048
2.8	367	.14	528	.112	825	.086	1188	.066	1616	.055
3.0	393	0.16	565	0.13	884	0.098	1272	0.076	1732	0.062
3.2	419	.19	603	.15	942	.117	1357	.086	1847	.070
3.4	445	.21	641	.16	1001	.12	1442	.096	1963	.078
3.6	471	.23	679	.18	1060	.13	1527	.104	2078	.086
3.8	497	.27	716	.20	1119	.15	1612	.115	2194	.095
4.0	524	0.28	754	0.22	1178	0.16	1696	0.12	2309	0.104
4.2	550	.31	792	.24	1237	.18	1781	.14	2425	.113
4.4	576	.33	829	.26	1296	.20	1866	.15	2540	.12
4.6	602	.35	867	.28	1355	.21	1951	.16	2655	.13
4.8	628	.38	905	.30	1414	.23	2036	.18	2771	.14
5.0	654	0.41	942	0.33	1472	0.24	2121	0.19	2886	0.15
5.2	681	.44	980	.35	1532	.26	2205	.20	3002	.16
5.4	707	.47	1018	.37	1590	.28	2290	.21	3117	.17
5.6	733	.50	1056	.40	1649	.30	2375	.23	3233	.19
5.8	759	.53	1093	.42	1708	.31	2460	.24	3348	.20
6.0	785	0.57	1131	0.45	1767	0.34	2545	0.26	3464	0.21
6.2	812	.60	1169	.47	1826	.35	2630	.28	3579	.22
6.4	838	.64	1206	.50	1885	.38	2714	.29	3695	.23
6.6	864	.68	1244	.52	1944	.40	2799	.31	3810	.25
6.8	890	.72	1282	.55	2003	.42	2884	.33	3925	.26
7.0	916	0.75	1319	0.57	2062	0.44	2969	0.34	4041	0.27
7.2	942	.79	1357	.60	2121	.46	3054	.36	4156	.29
7.4	969	.83	1395	.63	2179	.49	3138	.38	4272	.30
7.6	995	.87	1433	.66	2238	.52	3223	.40	4387	.32
7.8	1021	.92	1470	.69	2297	.54	3308	.42	4503	.34
8.0	1047	0.96	1508	0.73	2356	0.56	3393	0.44	4618	0.35
8.2	1073	1.00	1546	.76	2415	.58	3478	.47	4733	.37
8.4	1100	1.04	1583	.79	2474	.60	3563	.49	4849	.39
8.6	1126	1.08	1621	.82	2533	.62	3647	.51	4965	.40
8.8	1152	1.12	1659	.85	2592	.65	3752	.53	5080	.41
9.0	1178	1.17	1696	0.88	2651	0.68	3817	0.55	5195	0.43
9.2	1204	1.21	1734	.92	2710	.72	3902	.58	5311	.45
9.4	1230	1.24	1772	96	2769	.73	3987	.60	5426	.47
9.6	1257	1.28	1810	1.00	2827	.76	4072	.62	5542	.49
9.8	1283	1.34	1847	1.04	2886	.79	4156	.65	5657	.51

To reduce discharge to million gallons per 24 hours, multiply by 0.01077195.

Kutter. The Kutter formula was designed for open channels and will be treated under that head. It is sometimes used for pipes, but the results from it, since the coefficients, like those of the Chezy and Fanning formulas, change with the velocity in the same pipe, are usually erroneous, except for a very small range of velocity. For this reason it is not to be recommended for general use in computations for pipes.

11. Exponential Formula for Pipes

One form of an exponential formula for flow of water in pipes is

$$h_f = Kv^N L/D^{1.25}$$

where the notation is the same as that at beginning of Art. 10, but where the coefficient K and exponent N may vary with the kind and condition of the pipe. To avoid zeros in the coefficient a unit length of 1000 feet may be taken, when the formula becomes

$$h_f = \frac{Kv^N}{D^{1.25}} \cdot \frac{L}{1000}$$

in which N has a mean value of 1.87 and K ranges from 0.28 to 0.48 with an average value of 0.38 for ordinarily clean pipes. For rough or tuberculated pipes K may become as high as 0.70. The advantage of the exponential formula is that the coefficient for the same pipe is nearly constant and, if the exponent, its range being from 1.70 to 2.00, be properly selected, absolutely so, and the variation in all cases is much less than with other formulas, so that with a few average coefficients for different classes of channels, all hydraulic flow problems may be solved with reasonable accuracy without reference to any tables of coefficients. The foregoing formula with the coefficient 0.38 may be expected to give results within 20 % of accuracy for any pipes likely to be encountered which have diameters from one inch to fifteen feet, except those extremely tuberculated, and with velocities from 1 foot to 20 feet per second. For ordinary cast-iron or riveted pipe with any diameter and velocity, the results may be expected to be within 6 % of accuracy.

The exponential formula is derived from experiment in the following manner: Any plane curve passing thru the origin of coordinates can be represented by an equation of the form $y = mx^N$, in which m and N may be either constant or variable. If the curve be one of single curvature such that the change of inclination of its tangents either continuously increases or continuously decreases, both m and N become constants. All curves which are loci of equations expressing the relation betweeen velocity and loss of head in flowing water are of this latter class, and consequently $h_f = mv^N$ is a general expression for the loss of head in either a pipe or an open channel. If for any pipe line the values of m and N be determined, the equation of flow in that line is established. Expressing the above equation for logarithmic computation, it becomes

$$\log h_f = \log m + N \log v$$

and considering the logarithms as mere quantities, this is at once seen to be the equation of a straight line in which log m is the intercept on the h_f axis and N is the tangent of the angle which the line makes with the v axis. Both m and N may be found by determining two points in the line representing the plottings of the logarithms. If it be desired to draw the straight line which most nearly coincides with a number of points, it must pass thru their center of gravity and also thru the centers of gravity of the two groups into which the center of gravity of the whole divides them. Having the last two points, the equation of the line is readily determined.

The following example will serve to illustrate the process (Fig. 15). The data are from observations on a 12-inch cast-iron water main very carefully laid in a tangent some 3500 feet long, the loss in 1000 feet of which was measured.

C = center of gravity or mean point of the whole group
A = center of gravity of part of group above C
B = center of gravity of part of group below C
$C_h = h_f$ coordinate of C, $C_v = v$ coordinate of C
$A_h = h_f$ coordinate of A, $A_v = v$ coordinate of A
$B_h = h_f$ coordinate of B, $B_v = v$ coordinate of B

Observed Data			Logarithms			
No.	v Ft per sec	h_f Ft of water	log v		log h_f	
1	4.794	6.515	0.6807	Sum = 3.1667	0.8139	Sum = 3.4641
2	4.667	5.577	.6690	mean = 0.63334	.7464	mean = 0.69282
3	4.155	5.100	.6186	$= A_v$	.7076	$= A_h$
4	3.998	4.002	.6018		.6023	
5	3.950	3.926	.5966		.5939	
6	3.519	3.566	.5464	Sum = 2.3715	.5522	Sum = 2.0046
7	3.252	2.888	.5122	mean = 0.47430	.4606	mean = 0.40092
8	3.208	2.942	.5062	$= B_v$	.4686	$= B_h$
9	2.943	2.374	.4688		.3755	
10	2.177	1.405	.3379		.1477	
			Sum = 5.5382		Sum = 5.4687	
			mean = 0.55382 = C_v		mean = 0.54687 = C_h	

$A_v - C_v = 0.63334 - 0.55382 = 0.07952$ $A_h - C_h = 0.69282 - 0.54687 = 0.14595$
$C_v - B_v = 0.55382 - 0.47430 = 0.07952$ $C_h - B_h = 0.54687 - 0.40092 = 0.14595$

Since $A_v - C_v = C_v - B_v$ and $A_h - C_h = C_h - B_h$, the three points A, C, and B are in a straight line, which fact checks the accuracy of the work.

N = tangent of inclination of line ACB

$$= \frac{A_h - C_h}{A_v - C_v} = \frac{C_h - B_h}{C_v - B_v} = \frac{A_h - B_h}{A_v - B_v} = \frac{0.14595}{0.07952} = 1.835$$

Since $\log m = \log h_f - N \log v$, using the coordinates of C

$\log m = 0.54687 - 1.835 \times 0.55382$
$= 0.54687 - 1.01626 = 9.53061 = \log 0.3393$, and $m = 0.3393$

The equation for this 12-inch pipe is therefore $h_f = 0.3393\, v^{1.835}\, L/1000$.

Remark: Evidently the v coordinates must be divided between the same pair of observations as the h_f coordinates. The mathematical determination of what group should include a point whose v coordinate is on one side of C and whose h_f coordinate is on the other, depends on whether the point itself is above or below a normal to the line ACB thru C. This can usually be established by plotting the logarithms on ordinary cross-section paper, or the observations on logarithmic paper.

To introduce the diameter into the equation, a series of values of m and N for pipes of different diameters must be obtained. The range of N is relatively small, the limits for all reliable pipe experiments on record being from 1.70 to 2.08, and if the pipes are of the same character of surface and alinement the value of N will be constant. It is, therefore, only necessary to consider the variation of m, which depends upon the area of the cross-section or upon D and upon the roughness. Evidently m varies inversely as some power of the diameter, and for $1/D = 0$, $m = 0$ for any velocity, so the curve representing the relation between m and D will be $m = KD^{-x}$. Proceeding in the same manner as before, an average value of $x = 1.25$ will be obtained, and the formula for pipe of the same character as that in the above experiment is

$$h_f = \frac{Kv^{1.835}}{D^{1.25}} \cdot \frac{L}{1000} \quad \text{or} \quad h_f = \frac{0.38\, v^{1.87}}{D^{1.25}} \cdot \frac{L}{1000}$$

for average conditions, and this may be transposed to

$$v = 76.54\, D^{0.668} s^{0.535} \quad \text{or} \quad v = 185\, r^{0.668} s^{0.535}$$

12. Variations in Diameter, Material and Conditions

Relation of Diameter of Pipe to Quantity Discharged. In terms of C for Chezy's formula

$$Q = \pi/8\, C\sqrt{sD^5}$$

in terms of Fanning's coefficient f, $Q = \pi/4\sqrt{\frac{2\,g}{4\,f}\cdot\frac{h_f}{L}\cdot D^5}$

Approximately for rough pipe $Q = 1000\, D^{5/2}\, s^{1/2}$

and for smooth pipe $Q = 2 \times 1000\, D^{5/2}\, s^{1/2}$

Material. Most pipes experimented upon previous to 1900 were of cast iron. Experiments since that date and others previous thereto indicate that the exponential formulas (p. 1095) may be safely used for wooden stave pipe, and that for riveted steel pipe the effect of rivets and joints is to reduce the discharge from 10 to 12 per cent below that of cast iron when pipes are new.

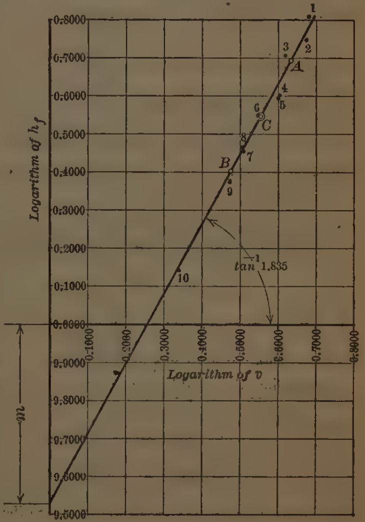

Fig. 15. · Logarithmic Plotting

Roughness. The effect of roughness in a water pipe is in general to retard the flow or increase the loss in head. This is accomplished by reducing the velocity of the water at the surface of contact, thus producing a general reduction in velocity and also causing cross currents or eddies which use up the energy in the stream. Roughness decreases C and increases f and K in the foregoing formulas. It also increases N somewhat in the exponential formula.

Curvature. The effect of curvature is to increase the loss of head. This increased loss is partly due to the cross currents and eddies set up in the bend, but also to the changes of velocity along the stream lines and increased friction along the walls of the channels due to increased velocities over part of the circumference. The loss of head due to a curve may be stated in terms of the velocity head h_v or, better, in terms of the equivalent length of straight pipe which would give the same loss as the curve.

The loss in a curve should be measured as the excess over that in an equal length of straight pipe. An examination of the experiments upon curve resistance by W. E. Fuller, Mem. Am. Soc. C.E. (Jour. N.E.W.W.A., Dec., 1913) has led to the following conclusions:

1. The excess loss of head due to bends is greater for large than for small pipes.

2. For large pipes a 6-ft radius has given the minimum excess resistance except where very long radii are used.

3. If the radius can be made very long, the least excess resistance will be from the curve of longest radius.

4. For small pipes, with long radii, the total loss of head may be less than that in a straight pipe of length equal to the sum of the two tangents to the curve.

5. Approximate formulas:

For 90° bends of a radius between 1.5 ft and 10 ft the excess loss of head due to the curve $h_b = \frac{1}{4}\left(\frac{V^2}{2g}\right)$

For tees = bends of zero radii, $h_b = 1\frac{1}{4}\left(\frac{V^2}{2g}\right)$

For 90° bends of 6-in radius $h_b = \frac{1}{2}\left(\frac{V^2}{2g}\right)$

For loss in 45° bends use $\frac{3}{4}$ of loss in similar 90° bends.

For loss in $22\frac{1}{2}$° bends use $\frac{1}{2}$ of loss in similar 90° bends.

For loss in a Y-branch use $\frac{3}{4}$ of loss in similar tee.

Expansions when sudden always produce eddies which increase the loss of head. Consider two sections of a pipe, 1 and 2; 1 to be taken at a point where normal condition of flow exists before expansion and 2 after expansion. If v_1 and v_2 are the velocities and A_1 and A_2 the areas at the two sections then the loss of head due to this sudden enlargement

$$h_{fe} = \frac{(v_1 - v_2)^2}{2g} \quad \text{or} \quad h_{fe} = \left[\frac{A_2}{A_1} - 1\right]^2 \frac{v_2^2}{2g}$$

According to St. Venant, this quantity should be increased by $v_2/18g$, but this correction is so small as a rule that it can be neglected, and more recent experiments indicate that the formula is as likely to give results in excess as otherwise.

Contraction when sudden produces an effect upon a stream very similar to a sharp orifice; that is, just beyond the contraction occurs the point of minimum cross-section of the stream or the "vena contrata." There result not only the loss of head due to the contraction of the stream, but also that due to the reenlargement of it after passing the "vena contracta." If v is the velocity under conditions of normal flow in the pipe after passing the contraction and C is the coefficient of contraction, the same in this case as for a sharp orifice, then the loss of head due to the contraction is

$$h_{fc} = \frac{v^2}{2g}\left[\frac{1}{C} - 1\right]^2$$

According to St. Venant this quantity should be increased by $v^2/18g$. Also it may be written $h_{fc} = C_c v^2/2g$, where C_c varies from 0.42 to 0.53. A fair assumption to make is $C_c = 0.5$. This may also be taken as the loss of head due to sharp-edged entrance into a pipe. The value of C is probably too high for small pipes and too low for large pipes.

Obstructions. If the sectional area of a pipe be gradually decreased and then gradually increased as in the case of a Venturi meter, the loss of head for moderate velocities is not much increased over that due to normal flow.

When the obstruction causes a sudden contraction or expansion of the stream or there is discontinuity of the pipe wall, the loss of head is increased.

Valves. The losses due to valves in pipe lines have been investigated with accuracy in only a few instances. From these experiments it appears that a fully open gate valve in a pipe causes a loss of head corresponding to about six diameters of length of the pipe.

Siphons. A siphon is a pipe whose center line rises above the hydraulic grade line at some portion of its course and hence operates thru a partial vacuum.

The laws governing flow thru siphons are the same as those for other pipes, but if the siphon be short, allowance must be made for losses at entry and due to curvature. Unless the velocity is maintained at about or above 2 ft per second air will accumulate at the summit and may materially retard or absolutely check the flow. On this account the siphon should be carefully graded up to a high point at which air can be conveniently removed by an ejector or thru a filling chamber from which point the drop should be quite abrupt to the outlet. On account of the tendency of air to expand and separate from the water at low pressure the lift of the siphon should be as low as circumstances will permit, and not above 20 ft unless an ejector or a high velocity is used.

A siphon installed by Geo. S. Pierson, M. Am. Soc. C. E., of 24-in diameter cast-iron pipe about 2900 ft long with a 20-ft lift has been in use for many years connecting a distant well with one near the pumping station at Kalamazoo, Michigan.

A 24-in diameter spiral riveted pipe siphon installed by the writer at Taughannock Falls, N. Y., in 1904, has an initial lift of 9 ft, a run of 411 ft with a rise of 6 in and a drop of 88 ft to a water wheel which is supplied thru it, and drives a lighting plant furnishing current to the village of Trumansburg. The wheel has a draught tube giving a further drop of 14 ft and no difficulty has been encountered from accumulation of air when running, the operation head being from 90 to 93 ft.

13. Oil, Sand, and Air

Oil Pipe Lines. With heavy viscous oils the loss of head becomes so large that transportation by pipe lines is difficult. Raising the temperature of the oil increases its fluidity, but this is satisfactory only for short lines, because to maintain a sufficient fluidity thruout a long line the initial temperature must be raised to such a point that disintegration is likely to result. Experiments have been tried with oil mixt with water, but cross currents and eddies in the pipe line produce at the discharge an emulsion, from which it is very difficult to separate the water. It is, however, possible by mixing about 10 % of water with the oil and forcing it thru a pipe line having rifling grooves or guides along its walls, to facilitate the transportation. The rotation of the liquid by the rifling causes the water to form a thin film or sheet between the oil and the pipe. Emulsification does not take place, and practically all the water can be separated by allowing the mixture to stand for a short time in a tank.

The rifling of an 8-inch line used by the Southern Pacific Railway Company consists of six helical grooves in the circumference, making a complete revolution every ten feet of axial length. This line was laid with a valley every 400 feet of length, the depth being equal to the diameter of the pipe, in order to facilitate starting the flow after the line had been out of service and the expedient seems to have served its purpose.

The results of tests made by the Southern Pacific Railroad Company give the following values of K for the formula $h_f = Kv^2L/1000\,D$. (Engr. News, June 7, 1906, and Eng. Record, May 23, 1908.)

8-inch plain pipe carrying oil only	$K = 134$
8-inch plain pipe carrying 90% oil and 10% water	$K = 79$
8-inch rifled pipe carrying 90% oil and 10% water	$K = 0.95$
3-inch plain pipe carrying oil only	$K = 284$
3-inch rifled pipe carrying 90% oil and 10% water	$K = 0.64$
Mean of above pipes for water only	$K = 0.35$

Transportation of Solids. The percentage of solid matter that can be transported in flowing water in a pipe after a velocity has been attained sufficient to cause the suspension of the solids, is independent of the velocity. At the lower velocities the solids are rolled or dragged along the bottom with a relatively large loss of head; as the velocity increases they are picked up by the water and retained in suspension. The velocity at which the material becomes suspended is dependent upon the size of the grains and their weight. The velocity at which the material becomes approximately all suspended is the one at which the minimum loss of head will occur, and when the cost of pressure is great compared with the cost of the channel, this will be the most economical velocity to use. If the material varies in the size of its grains the condition of minimum resistance may occur before the heavier particles are in suspension. Ordinarily, however, pressure is of less importance than the line cost, and hence higher velocities will be used in order that the delivery per unit of time may be as great as possible.

The accurate experiments bearing on the subject have been made with sands having an effective size varying from 0.16 mm. to 0.75 mm. For a one-inch pipe the velocity of suspension for such material is about 3.5 feet per second, for a three-inch pipe about 4 feet, and for a 32-inch pipe about 9 feet. The loss of head due to the transportation of these sands may be taken as about 25% of the loss due to water alone at the velocity of suspension, for each 1% of sand added to the water. For the higher velocities with fine sands, the resistance appears to increase somewhat more rapidly than with water alone, while with the coarser material the increase is slightly less rapid. For average results the loss of head for the mixture may be taken to vary from the velocity of suspension as v^2, until more complete investigations shall discover the true law. As much as 52% of sand has been transported thru a 4-inch pipe. (Trans. Am. Soc. C. E., 1906, vol. 57, p. 400.)

Sand Ejectors. The success of sand transportation in pipes depends largely upon the ejector used to load the sand. This apparatus is situated at the bottom of a hopper into which the sand is delivered and in which its fluidity is best maintained by jets directed upwards, and is similar to the water ejector, but a gradual taper beyond the throat is of more importance. A slope of 1 in 32 is desirable. The pressure in the discharge and the per cent of mixture are both largely controlled by the diameter of the throat and its distance from the nozzle, the former decreasing and the latter increasing as the throat is enlarged and moved away from the nozzle. The throat of sand ejectors wears away very rapidly and hence should be designed to facilitate removal and replacement. (Trans. Am. Soc. C. E., 1906, vol. 57, p. 339.)

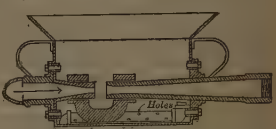

Fig. 16. Portable Sand Ejector

Flow of Compressible Fluids in Pipes. When air flows along a pipe there is necessarily a fall of pressure due to the resistance of the pipe, and, consequently, the density of the air decreases and its velocity increases along

the pipe in the direction of motion. The effect of the resistance is to create eddying motions, which, as they subside, give back to the air the heat equivalent of the work expended in producing them. The result is that, apart from conduction thru the walls of the pipe, the flow is isothermal.

Flow of Gas in Pipes under Small Differences of Pressure. Let p_1 and p_2 be the absolute unit pressures at the inlet and outlet of the pipe, w_g the weight of the gas in pounds per cubic foot, h_{fg} = head lost in feet of the gas, h_f = head lost in feet of water, and f = coefficient. If $p_1 - p_2$ be small so that the change in density may be neglected, then

$$p_1 - p_2 = w_g h_{fg} = w h_f$$

The law of flow may be exprest by the Fanning formula

$$h_f = \frac{4 f L}{D} \frac{v^2}{2 g}$$

and f has an average value of $0.0044 \frac{w_g}{w} \left(1 + \frac{1}{7 D}\right)$.

Head Lost in an Inclined Gas Main. Let h_{p1} and h_{p2} be the heights of water columns at points 1 and 2, z_1 and z_2 be the elevations of the points, w_a the unit weight of air and let the flow be from 1 to 2, then

$$h_{fg} = \frac{1}{w} \left\{ w(h_{p_1} - h_{p_2}) - w_a (z_1 - z_2) \right\} + z_1 - z_2$$

or

$$h_f = (h_{p_1} - h_{p_2}) - \frac{w_a - w_g}{w} (z_1 - z_2)$$

When Variation in Density is taken into account, the flow of air in a long uniform pipe is modified. Let v_1 be the initial velocity in the pipe whose diameter remains uniform; then an approximate formula for ordinary temperature is

$$v_1 = \left(1.132 - 0.726 \frac{p_2}{p_1}\right) \sqrt{222\,900 \frac{D w_a}{f L w}}$$

The distribution of velocity in an air main is very similar to that in a water main; that is, it approximates to a cylinder surmounted by an ellipsoid. The ratio of the mean to the velocity at the center is given as 0.873.

14. Flow over Weirs

Sharp-edged Weirs. When an obstruction is placed in an open channel, so that water is caused to flow over it, it is called a dam or weir. If the top of the weir be a thin straight edge, the conditions of flow are similar to those that would exist in an orifice in a thin wall if the side contractions were supprest and the head fell so low that the water did not fill the orifice to its top. If the portions of the dam near the walls of the channel are raised above the level of the rest so that water does not flow over them, the overflowing jet is contracted at the sides as in the case of an orifice. The general expression for the discharge of water over a weir is

Fig. 17. Sharp-edged Weir

$$Q = \tfrac{2}{3} C L H \sqrt{2 g H}$$

wherein H is the height above the crest of the weir to the level of still water and L is the length of the crest over which the water flows. Practically it is not possible to measure H, but a head h may be observed to the surface of the stream above the curve of depression caused by the weir, and to this the velocity head h_v due to the velocity v_a with which the water approaches the weir, may be added when the result is approximately equal to H. If the velocity of approach be small, h as observed may be treated as equal to H. C is a coefficient which depends upon the height and form of the weir, whether or not there be end contractions, the character of the weir surface and the condition of the water on the downstream side. In weir formulas it is customary to combine one or more of the factors $\frac{2}{3}$, C, and $2g$ into a single coefficient.

Four Recognized Formulas for the discharge of weirs are as follows, but the first and the fourth are the most important.

The Francis Formula

$$Q = 3.33\ LH^{3/2} \quad \text{or} \quad Q = 3.33\,L\left[(h+h_v)^{3/2} - h_v^{3/2}\right]$$

The Fteley and Stearns Formula

$$Q = 3.31\ LH^{3/2} + 0.007\,L \quad \text{or} \quad Q = 3.31\,L\,(h + 1.5\,h_v)^{3/2} + 0.007\,L$$

The Hamilton Smith Formula

$$Q = 3.29\,(L + H/7)\,H^{3/2} \text{ or } Q = 3.29\left(L + \frac{h + 1\frac{1}{3}h_v}{7}\right)(h + 1\tfrac{1}{3}h_v)^{3/2}$$

The Bazin Formula

$$Q = mLh\sqrt{2\,gh}, \quad \text{where } m = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{a+h}\right)^2\right]$$

in which a is the height of the crest of the weir above the bottom of the channel of approach. For weirs with end contractions, Francis concluded that L in the above formulas should be replaced by $L - 0.1\,nH$, where n is the number of full end contractions. This correction has been generally accepted but it is by no means accurate, and for exact work in measuring water a weir without end contractions is to be preferred. These formulas all apply to a weir with a vertical upstream face, a sharp edge and with free access of air to the under side of the overfalling sheet of water.

Hazen's Formula for sharp-edged weirs without end contraction (Trans American Society of Civil Engineers, Vol. LXXVII, p. 1289 et seq.) is:

$$Q = \left(3.27 + 0.5\frac{h}{a}\right)h^{3/2},$$ and for heads above 0.3 ft gives results within 1%

of those obtained by Francis and Fteley and Stearns.

Triangular or V-shaped Weir. This form of weir, suggested by Prof Thomson of Dublin, possesses the peculiarity that, whatever the heads, th sections of the stream are similar, and hence it may be expected to have a coef ficient more nearly constant than the ordinary weir and be particularly we adapted to the measurement of water where the flow varies thru a considerabl range. The coefficient will vary for different inclinations of the sides of th notch. For a sharp-edged weir in which the sides make an angle of 90° wit each other, since $L = 2\,h$, the discharge is $Q = 2.6\,h^{5/2}$.

Prof. H. W. King (Michigan Technic, October, 1916), gives $Q = 2.52\,h^{2.}$ deduced from a series of experiments covering a range of h from 0.15 ft to 1.8 f

Trapezoidal or Cippoletti Weir. By combining the rectangular notcl

or weir with end contractions, with the two halves of a triangular weir, it is possible to so proportion the triangular portions that they shall compensate for the effect of contraction and the result is approximately obtained when the inclination of the ends is one horizontal to four vertical. The formula $Q = 3.367\ Lh^{3/2}$ is commonly used for such a weir without velocity of approach. Professor H. W. King recommends

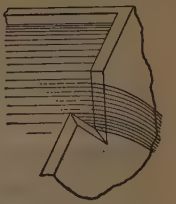

Fig. 18. Triangular Weir

$$Q = 3.34\ Lh^{1.47}\left[1 + 0.56\left(\frac{h}{a+h}\right)^2\right]$$

which takes account of velocity approach.

No experiments have been made upon weirs of this type when other than sharp edged with vertical faces, but the effects of inclination and rounding may be expected to affect them similarly to rectangular weirs.

Rounding the Upstream Corner of the crest of a weir increases the discharge. With flat-crested weirs Bazin found this effect to amount to as much as 13% where the radius of the rounding was 4 inches and the breadth of crest 6.56 feet. Fteley and Stearns, with weirs up to one inch in breadth, found the rounding to be equivalent to increasing the head by $h_R = 0.7\ R$, where R is the radius of the rounding.

Inclining the Upstream Face away from the current decreases the contraction and increases the discharge as much as 10% when the slope is one of 45°. If the inclination be in the opposite direction, the contraction is increased and the discharge decreased. With a 45° slope, the decrease may be as much as 7%. Inclining the DOWNSTREAM FACE does not materially alter the discharge until the slope becomes at least 3 horizontal to 1 vertical, when the discharge is reduced.

Rounding the Entire Crest reduces the discharge for low heads, but increases it for those wherein the curve of the crest approaches the curve of the natural under side of the sheet. By a combination of a rounded crest and an inclined upstream slope, the discharge may be increased 20% above that of the sharp-edged weir.

Flat Crests decrease the discharge until the head becomes so high that the sheet jumps clear of the downstream corner, when they have no effect. A broad flat crest may reduce the discharge 25% below that of the sharp edge.

The Sheet of Water adhering to the downstream face of a vertical sharp-edged weir has increased the discharge about 28%. The sheet being wetted, that is deprest and the space between it and the weir filled with water, due to the formation there of a partial vacuum, has increased the discharge about 15%. The sheet being deprest, but the space only partially filled with water, has increased the discharge about 6%.

Submerged Weirs. When water on the downstream side of the weir rises above the level of the crest, the weir is said to be submerged. If h_u is the head observed on the upstream side and h is the difference of head on the two sides, the usual formula for the discharge of a submerged weir is

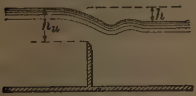

Fig. 19. Submerged Weir

$$Q = CL\sqrt{2\,gh}\,(h_u - h/3)$$

where C for a sharp edge varies from 0.58 to 0.63. On account of the difficulty of measuring h, the head in the lower pool, because of the turbulence

there, accurate results with this formula are impossible. Experiment show that so long as the water flowing over the weir plunges to the bottom of th channel below or dives under that in the lower pool, the discharge of the wei is not decreased more than 10 % by the submergence. In rounded weirs it i possible to submerge the crests to fully 30 % of h_u without varying the dis charge from that for a free weir under the head h_u more than the above per cent; and for submergences of less than 10 % of h_u the discharge is likely t be increased by the exclusion of air behind the sheet.

15. Open Channels

Hydraulic Radius or Hydraulic Mean Depth. The Hydraulic Radiu also called the Hydraulic Mean Depth because it is the depth of a rectang whose area is the same as that of the section under consideration and who width is equal to the wetted perimeter of the latter, is equal to the area divide by the wetted perimeter and is represented by r. In open channels it tak the place of D in the pipe formulas, and for a circular pipe flowing full half full is equal to $D/4$. The cross-sections of natural channels, partic larly in soft materials, approximate to one or to two parabolic segmen It may therefore be useful to utilize the relation of the area to the arc of th parabola in approximating the hydraulic radius. Let y_1 = semi-width channel if symmetrical about a vertical, or the distance from the bank to the vertical of greatest depth, d = the greatest depth, S_1 = length of the arc from the surface to the foot of the deepest vertical. The area of the semiparabola $A_1 = 2/3\, y_1 d$, and the length of the arc when $d < 0.25\, y_1$ is approximately $S_1 = y_1 + 2/3\, d^2/y_1$. If the channel be symmetrical, $r = A_1/S_1$, but if the other part be made up of a different parabola, then a similar expressi may be written for A_2 and S_2 and $r = (A_1 + A_2)/(S_1 + S_2)$.

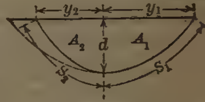

Fig. 20

Long Channels. In the term "open channel" are included all rive artificial canals, aqueducts and conduits and, in addition, sewers and pip of whatever section which run partially full. The force producing flow cann now be provided for by any external head, but is solely due to the slope gradient of the channel.

Normal Flow. When the magnitude and distribution of velocities various sections over a considerable length of the stream are the same, t flow is said to be normal. The curve of the distribution of velocities in vertical section parallel to the direction of motion thru any point of an op channel approximates an ellipse, being tangent to the bottom and with its a in or near the surface of the stream. Wave action, eddies, etc., cause the velo ties near the surface to be decreased, and this effect may extend to nearly h the depth. The actual curve then follows an ellipse halfway to the surfa where it leaves the ellipse and becomes somewhat flattened, giving a maxim velocity in the upper two-tenths of the depth. The mean velocity on a vertical is normally about 0.9 the maximum and occurs about 0.577 d fr the surface, where d is the depth of the stream. The loss of head in o channels, probably on account of their greater irregularity, varies as a so what higher power of v than in pipes. It, therefore, is possible to use f mulas involving v^2 with a less variation in the coefficient than in pipes, tho exponent of 1.9 gives more constant coefficients.

The Chezy Formula is $v = C\sqrt{rs}$, where v is the velocity, C a coeffici s the slope and r the hydraulic radius = area ÷ wetted perimeter.

The Bazin Formula is probably one of the best yet devised for the flow of ter in open channels, altho the same careful scrutiny has not been given to roughness factors as to those of Kutter. It is

$$v = \frac{87}{0.552 + \frac{m}{\sqrt{r}}}\sqrt{rs} = C\sqrt{rs}$$

ere r is the hydraulic radius in ft and v is the velocity in ft per sec. Values the roughness factor m are as follows:

Very smooth surfaces of cement and planed boards	0.06
Smooth surfaces of boards, bricks, concrete	0.16
For brick sewers and dirty concrete	0.28
Ashlar or rubble masonry	0.46
Earthen channels, very regular or pitched with stones, tunnels and canals in rock	0.85
Earthen channels in ordinary condition	1.30
Earthen channels presenting an exceptional resistance, the wetted surface being covered with detritus, stones, or weeds	1.75

Bazin's Formula. Values of C in $v = C\sqrt{rs}$

Hydr. rad. r, feet	A $m=0.06$	B $m=0.16$	C $m=0.28$	D $m=0.46$	E $m=0.85$	F $m=1.30$	G $m=1.75$
0.2	127	96	74	55	35	25	19
0.3	131	103	82	63	41	30	23
0.4	135	108	88	68	46	32	26
0.5	137	112	92	72	50	37	29
0.6	139	116	96	76	53	39	31
0.8	141	119	101	82	58	43	35
1.0	142	122	105	86	62	47	38
1.3	144	126	109	91	67	51	42
1.5	145	128	112	94	70	54	44
1.75	146	130	114	97	73	57	46
2.0	147	132	116	99	76	59	49
2.5	148	134	119	103	80	64	53
3.0	149	136	122	107	84	67	56
4.0	150	138	126	111	89	72	61
5.0	151	140	129	115	94	77	65
6.0	151	142	131	118	98	80	69
8.0	152	144	134	122	102	86	74
10.0	153	145	136	125	106	90	79
12.0	....	......	......	......	109	94	82
15.0	....	......	......	......	113	98	87
20.0	....	......	......	......	117	103	92
30.0	....	......	......	......	123	110	100
50.0	....	......	......	......	129	119	108

A. For very smooth cement and planed boards.
B. For smooth boards, brick, concrete, glazed earthenware pipes.
C. For smooth but dirty brick or concrete.
D. For ashlar masonry.
E. For earth canals in very good condition and canals pitched with stones.
F. For earth canals in ordinary condition.
G. For earth canals exceptionally rough.

The Kutter Formula, tho largely used, is probably not entirely satisfactory large streams with slight slopes. It depends for its accuracy almost entirely on the experimental determination of its coefficient of roughness n, which

changes for different velocities in the same channel, and great care must therefore be exercised in extending computations beyond the limits of actual experiment. Let r = hydraulic radius, s = slope, and v = mean velocity; then the formula is, for the English system of measures,

$$v = \left[\frac{41.6 + \frac{1.811}{n} + \frac{0.00281}{s}}{1 + \left(41.6 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}} \right] \sqrt{rs} = C\sqrt{rs}$$

where the coefficient of roughness n has the following values:

Rectangular wooden flume (very smooth)	0.0098
Neat cement, glazed pipes and very smooth iron pipes	.010
Plaster, 1 : 3 mixture, iron pipes in best order	.011
Unplaned timber, ordinary iron pipe	.012
Brick washt with cement, basket-shaped sewer,	
6 by 6.7 ft, nearly new	.0130
6 by 6.7 ft, one year old	.0148
6 by 6.7 ft, four years old	.0152
Brick washt with cement, 9 ft diameter	
Nearly new	.0116
Four years old	.0133
Old Croton aqueduct, brick lined	.015
Sudbury aqueduct	.01
Glasgow aqueduct, cement lined	.0124
Steel pipe, riveted, clean. 1897 (mean)	.0144
Steel pipe, riveted, clean, 1899 (mean)	.0155
Rough brickwork, incrusted or tuberculated iron	.015
Brickwork or ashlar in bad condition, rubble in cement in good order	.017
Rough rubble in cement, stone pitching, very firm gravel	.020
Earth of tolerably uniform cross-section, slope and direction, in moderately good order and regimen, and free from stones and weeds; or stone pitching in bad condition	.025
Earth, having stones and weeds occasionally	.030
Gravel in bad condition, earth in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus	.035

Biel's Formula, for the mean velocity in pipes and channels was published in 1907. For English measures it is $v = -M + \sqrt{N + M^2}$, in which

$$M = \frac{4.1\,k}{(0.0661\sqrt{r} + f)(100f + 2)} \qquad N = \frac{1891\,r^{3/2}s}{0.0661\sqrt{r} + f}$$

where k is a viscosity factor, which for water is 0.0179 at 32° F, 0.0135 at 50° and 0.01 at 68° F, while f is a roughness factor having the following values:

For planed boards and wrought-iron pipes	f = 0.018
For new cast-iron pipes and smooth cement pipes	0.036
For rough boards and smooth brickwork	0.054
For regular masonry and common brick channels	0.072
For rough masonry and earth canals with brick walls	0.29
For canals in earth and regular streams	0.50
For canals and rivers with stones and weeds	0.75
For canals and rivers in bad condition	1.06

Biel gives the following as showing the relation between his f and Kutter's n:

Kutter's	n = 0.010	0.012	0.013	0.020	0.025	0.030	0.035
Biel's	f = 0.018	0.054	0.072	0.29	0.50	0.75	1.06

He also asserts that his formula applies to oil, gases, and air, k being the ra[tio] of the viscosity factor of the liquid or gas to its specific gravity, while the val[ues] of f are the same as given above; for air at 50° F, the value of k is 0.1

Kutter's Formula. Values of Coefficient C

Hydr. rad. r, feet	Coefficient n						Coefficient n					
	.010	.015	.020	.025	.030	.040	.010	.015	.020	.025	.030	.040
	Slope = 0.000025						Slope = 0.00005					
0.1	57	33	23	17	14	10	67	39	26	20	16	11
0.2	75	45	31	24	19	14	87	51	35	26	21	15
0.4	97	59	42	32	26	19	109	66	46	35	28	20
0.6	112	69	49	38	31	22	122	76	53	41	33	24
1.0	131	83	60	47	38	28	140	89	64	49	40	29
1.5	148	95	69	55	45	33	154	99	72	57	47	34
2.0	160	104	77	61	50	37	164	107	79	62	51	38
3.28	181	121	90	72	60	45	181	121	90	72	60	45
6.0	206	142	108	88	74	57	199	137	105	85	72	56
10.0	225	159	124	102	87	68	212	149	116	96	82	64
16.0	242	174	138	115	100	79	223	160	126	106	91	73
30.0	261	193	157	133	117	95	236	172	139	118	103	84
50.0	274	207	170	147	130	107	245	181	148	127	112	93
	Slope = 0.0001						Slope = 0.0002					
0.1	78	44	30	22	17	12	85	48	32	24	18	12
0.2	98	57	39	29	23	16	105	61	42	31	25	17
0.4	119	72	50	38	31	22	125	76	53	40	32	23
0.6	131	81	57	44	35	25	138	85	60	46	37	26
1.0	147	98	67	52	42	31	151	96	69	54	44	32
1.5	159	103	75	59	48	35	162	105	77	60	49	36
2.0	168	109	81	64	53	39	170	111	82	64	54	40
3.28	181	121	90	72	60	45	181	121	90	72	60	45
6.0	195	134	102	84	71	54	193	132	100	82	69	53
10.0	205	143	111	92	78	62	201	140	108	89	76	60
15.0	212	150	118	98	85	68	207	145	113	95	82	65
30.0	222	160	128	108	95	77	215	154	122	103	89	73
50.0	227	166	134	114	100	83	220	158	126	108	94	78
	Slope = 0.0004						Slope = 0.001					
0.1	89	50	34	25	19	13	94	54	36	27	21	14
0.2	110	65	44	32	25	18	113	66	45	34	27	18
0.4	129	79	55	42	33	23	131	80	56	43	34	24
0.6	140	87	62	47	38	27	142	88	63	48	39	27
1.0	154	98	70	55	45	32	155	99	71	56	45	33
1.5	164	106	78	61	50	37	165	108	78	62	50	37
2.0	170	112	83	65	54	40	171	112	83	66	54	40
4.0	184	124	94	76	63	48	184	124	93	75	63	48
6.0	191	130	99	81	69	53	190	130	99	81	68	52
10.0	199	138	107	88	75	59	197	136	105	87	74	58
20.0	207	146	115	96	83	66	205	144	113	94	81	65
50.0	215	154	123	104	91	75	212	151	120	101	89	72
	Slope = 0.01											
0.1	95	54	36	27	21	14						
0.2	114	67	46	34	27	19						
0.4	133	82	57	44	35	24						
0.6	143	90	64	49	39	28						
1.0	156	99	72	56	45	33						
1.5	165	107	79	62	51	37						
2.0	171	112	83	66	55	40						
3.28	181	121	90	72	60	45						
6.0	190	129	99	81	68	52						
10.0	196	136	105	86	74	58						
20.0	204	143	112	93	80	64						
50.0	210	150	119	100	87	71						

For slopes steeper than 1 foot per 100 feet, the coefficient C remains practically constant for a given n and r and has the values given in the adjacent columns for $S = 0.01$.

When $r = 3.28$ feet the value of C is independent of the slope and its value is $1.811/n$.

According to Biel's formula a rise in temperature increases the mean velocity.

In the formula on p. 1106 M is usually small, so that its square may be often neglected, and then $v = -M + \sqrt{N}$. For river channels $v = \sqrt{N}$ is close.

An Exponential Formula for velocity in open channels is

$v = Cr^{0.67}s^{0.54}$ (English measures)

For very smooth channels	$C = 205$ to 185
For ordinary unplaned plank	$C = 165$ to 155
For ordinary sewer crock	$C = 155$ to 125
For ordinary brick sewers	$C = 155$ to 120
For ordinary earth channels	$C = 105$ to 75
For rough natural channels	$C = 75$ to 45

Prof. S. T. Harding, Assoc. M. Am. Soc. C. E. has adapted this formula to the introduction of Kutter's n, as a factor of roughness by transposing to the form:

$$V = \frac{1.70}{n - 0.002} r^{0.67} s^{0.54},$$

the reduction being based on the experiments recorded in the paper by Mr. George Henry Ellis, Assoc. M. Am. Soc. C. E. (Trans. Am. Soc. C. E., Vol. LXXX)

16. Variations in Sections

Roughness. In general, the effects of roughness in a stream bed have been taken care of by coefficients. The action in streams is similar to that in pipes, causing eddies and cross currents which absorb head and so reduce velocity.

Curvature disarranges the distribution of velocities and prevents the continuance of normal conditions of flow. In general, the tendency is for the velocity on the concave side of the stream to decrease, thus causing some of the material in suspension to be deposited. This decrease in velocity on the concave side is accompanied by an increase in velocity on the convex side, giving rise to erosion. In general terms, the conditions of scour will prevail where the hydraulic axis is convergent to the bank, fill where it diverges from the bank, and no action where the axis and bank are parallel; the hydraulic axis being taken as the locus of the center of gravity of the section of the stream.

Principles which govern a stream freely forming its own channel in an alluvial plane, are: (1) A plastic mass moving in a resisting medium will assume the form in which it encounters least resistance. (2) The transverse section of a body adjusted to the form of least resistance will have the ratio of area to perimeter a maximum. (3) The form of a fluid mass will vary whenever the direction of movement is changed, by virtue of the unequal inertia of particles in different parts of the mass. (4) The form being unsymmetrical and the direction of movement not a straight line, if the mass is variable the path described by the center of gravity, or the hydraulic axis will be variable also in position and length. (5) With given limits of mass variation, the vagation of the paths will lie within a zone of certain width.

Thus a river traversing a homogeneous soil will form a bed whose width and depth will be largely determined by the variations in volume, being wider and shoaler as the vagation of the hydraulic axis is greater; narrower and deeper as the volume becomes constant and the vagation of the hydraulic axis becomes less.

Obstructions. The loss in velocity, due to obstructions, arises mainly in the loss in expansion from the contracted section to the normal section of the stream. Wherever an obstruction occurs, the velocity is increased and

rface level generally lowered. After the obstruction is past, the velocity creases and the surface rises to normal.

Change of Section. (See also General Laws under Curvature.) In genal a change of section is accompanied by a change in the distribution of locity. In a stream thru a homogeneous alluvial soil change in the disibution of velocity is accompanied by either erosion or deposition of material.

17. Transportation of Sediment

Sediment can be transported by a stream in two ways, by being dragged rolled along the bottom or by being carried along in suspension. The eight of particles which will be dragged or rolled along by a stream is pposed to vary as the sixth power of the velocity. The experimental data ually quoted to determine the velocity at which different materials are oved were made before the beginning of the last century. These with some ore added by various observers since give a limiting bottom velocity for ability of material as follows:

Material	Bottom velocity, ft per sec
Soft earth	0.25
Soft clay	0.50
Sand	1.00
Gravel	2.00
Sea pebbles (1.06 inches diameter)	2.20
Brickbats (4.76 cu in)	2.25 to 2.50
Slate (9.06 cu in)	2.75 to 3.00
Broken stone	4.00

bservations made in India show that for any section of channel and character silt there is a certain critical velocity v_s at which silt is neither picked up nor eposited. If the velocity be increased, scour results; and if it be decreased, lting occurs. This critical velocity is found to depend upon the depth nd to be given by the equation $v_s = md^{.64}$, where d is the depth of the channel nd m is a coefficient whose values are given as follows:

For fine light sandy silt of northern India,	$m = 0.82$
For somewhat coarser light sandy silt,	$m = 0.90$
For a sandy loam,	$m = 0.99$
For a rather coarse silt, as débris of hard soils,	$m = 1.07$

he silt-transporting power of a stream is said to vary as $v^{2.56}$. (Kennedy, roc. Inst. C. E., 1894–5, vol. 119.)

Velocities of Descent of certain fine materials thru still water are:

Materials	Velocity, ft per sec
Brick clay (mixt with water and allowed to settle half an hour)	0.009
Fresh-water sand	0.166
Sea sand	0.196
Rounded pebbles (size of peas)	1.000

he heads corresponding to these velocities, h_v, may be taken to measure the mount of pressure head h_p acting upward that is necessary to keep them in spension. If the change of velocity in any vertical is such as to cause the xcess of h_p at the lower level over that next above to be equal to or greater an the h_v obtained above for the particular material, there will be no deposion. As the rate of change of velocity is greatest near the bottom, the greatest

amount of suspended matter will be found in that region, because material which may settle thru the upper layers will have its descent stopt by the more rapidly increasing pressures.

The flow of ground water into a river thru its bed aids in suspension by giving an upward velocity to the water near the bottom; and obstructions in the bottom which direct the current upward aid in suspension temporarily; but the main factor in the transmission of suspended matter is the increase of pressure from point to point on a vertical as the velocity decreases. Whenever due to a disturbance of any sort the velocities in a vertical are equalized, the pressures become equalized also and the material deposits.

The Phenomena of Scour, in the case of beds along which the water passes tangentially, are also due to differences of pressure caused by variations of velocity. The velocity in the semi-fluid layers of sediment next the bottom is relatively very low, while that in the water just above may be considerable. The pressure in the water is then less than that in the saturated material, and the excess in the latter lifts the particles of it up into the current until the velocity of the latter is reduced to a point where equilibrium is established, or else all the loose materials carried away, and in either case the scour ceases.

The proportion of silt in various waters, in parts per 100 000 by weight, is as follows, the amounts for the Rhine, Vistula, and Rhone being very exceptional.

Mississippi, near mouth, in flood	175
Mississippi, near mouth, ordinary	67
Mississippi, above Ohio, ordinary	20 to 30
Ohio, at Louisville, ordinary	35
Ohio, at Cincinnati, ordinary	23
Allegheny, at Pittsburgh, ordinary	5
Interior rivers in Illinois, ordinary	1 to 8
Rhine, Germany, in flood	1000
Vistula, Germany, in flood	2000
Maas, Holland, ordinary to flood	1 to 30
Danube, Austria, ordinary	33
Rhone, France, in flood	2000
Po, Italy, in flood	330
Nile, Egypt, in flood	150
Ganges, India, in flood	130 to 800
Indus, India, in flood	420
Sutlej, India, in flood	300 to 900
Roorkee canal, India, in flood	3000
Indian rivers, average for year	61

Bank Slopes. The angle ϕ at which materials stand when submerge varies considerably. Very light material, as might be expected, assum the steepest slope when the current impinges against the bank in flowir past. The following are given as results of observations in India.

Alluvial soil, soft rock, and very firm gravel; slope ½ hor. to 1 vert., or $\phi = 63° 20'$.

Stiff earth or clay, or alluvial banks with stone pitching; slope, 1 to 1, or $\phi = 45°$.

Ordinary earth; slope 1½ hor. to 1 vert., or $\phi = 33° 40'$.

Loose earth and soft slippery clay; slopes, 2 hor. to 1 vert. to 3 hor. to 1 vert., or ϕ 26° 30′ to 18° 20′.

18. Backwater

Formulas are frequently given for computing the extent and rise backwater caused by an obstruction in a stream, but in order that formula may be applicable to such cases as occur in practise it must extremely complicated. Those usually given only consider a change in dep without the necessarily corresponding change in width. For practical appli tion, except in the extremely rare case of backwater in an artificial chan

with vertical sides, such formulas are valueless. The practical method of olving the problem is to divide the channel in which the backwater occurs nto reaches such that both the velocity and the hydraulic radius will be ubstantially constant thruout each reach, and to compute for each reach, eginning with that next the obstruction, for a mean v and r, the h_f. This h_f when added to elevation of the surface at the downstream end gives the levation at the beginning of the next reach. By making the several sections ufficiently short, any desired degree of accuracy may be obtained within the imits of the formula used for loss of head.

Bridge Piers. An experimental investigation on models of bridge piers aving widths of 6 in and lengths from 18 to 33 in in a flume 26 in wide, the depth f water being about 2 ft, by Floyd A. Nagler, Jun. Am. Soc. C. E. (Trans. Am. Soc. C. E., Vol. LXXXII), has developed the following formula for backwater due to such piers:

Let C = a coefficient;
d_1 = depth of water at point of measurement of velocity of approach above piers in feet;
d_2 = depth of water at point of measurement of velocity of retreat below piers in feet;
h = observed difference of head between water surfaces above and below piers = backwater effect of piers in feet;
K = a coefficient;
Q = flow in channel in cubic feet per second;
V_1 = velocity of approach above piers in feet per second;
V_2 = velocity of retreat below piers in feet per second;
W = unobstructed width of channel at piers in feet.

Then assuming the slope of the channel thruout the area considered to be zero:

$$h = \frac{Q^2}{2g\left[CW\left(d^2 - \frac{0.3V_2^2}{2g}\right)\right]^2} - K\frac{V_1^2}{2}.$$

n which C varies from 0.86 for a rectangular pier to 0.94 for a pier with boat-haped nose and fish-shaped tail.

If there be a perceptible fall in the channel from the point of measurement f d_1 to that of d_2 the value of d_2 must be decreased by this fall, and if there be a rise the value of d_2 must be increased by the rise.

The coefficient K varies with the percentage of the original channel that is bstructed, approximately as follows:

Percent obstructed	0	5	10	15	20	30
Coefficient K	1.00	1.04	1.27	1.50	1.70	1.93
Percent obstructed	40	50	60	70	80	90
Coefficient K	2.00	2.03	2.04	2.05	2.06	2.07

Example. Let Q = 19 500 c.f.s.
Area of unobstructed stream = 3680 sq ft
Area of obstruction = 351 sq ft = 9.5%
Area of clear waterway = 3329 sq ft
Depth of channel d_2 = 16.4 ft

$$V_1 = \frac{19\,500}{3680} = 5.3 \text{ ft per sec}$$

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} = 0.4362 \text{ ft and } 0.3\frac{V_2^2}{2g} = 0.131 \text{ ft}$$

Assume
$C = .894$ from shape of obstruction
$K = 1.25$ from area of obstruction.

Substituting in formula there results:

$$h = \frac{19\,500^2}{64.4 \times 0.894^2 \times 203^2\,(16.4 - 0.13)^2} - 1.25 \times 0.4362$$
$$= 0.677 - 0.545 \text{ ft} = 0.132 \text{ ft}$$

Jump. When an obstruction is present in a stream, and the slope is so steep that the velocity v_1 before the obstruction is greater than $\sqrt{gd_1}$, where d_1 is the depth before the obstruction, a jump is likely to occur, producing a depth d_2, and if the velocity head $v_1^2/2g$ be called h, the value of d_2 is slightly less than that given by

$$d_2 = \tfrac{1}{2}h + \sqrt{h\,(d_1 + \tfrac{1}{4}h)}$$

and the height of the jump is then $d_2 - d_1$.

Investigations by Prof. S. M. Woodward of the State University of Iowa (Miami Conservancy District Technical Reports, Part III, Dayton, Ohio, 1917) have developed the following formula for the hydraulic jump which is in close accord with the experimental results:

Let d_1 = depth of water above jump;
d_2 = depth of water below jump;
V_1 = velocity of stream before jump occurs;

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2\,V_1^2\,d_1}{g}}$$

Waves. When the water in a stream passes under some obstruction an is then thrown upward, a standing wave is produced. When a stream o high velocity discharges into a large body of water moving with a slowe velocity, the curve of heading up does not always extend back till it reache the free surface of the water, but ends abruptly at the point of change c velocity and a standing wave may be there produced. Under these cond tions very little of the velocity head is converted into head of elevation, bu is absorbed in impact and wave formation.

Whirlpools, eddies, and waves all indicate a useless expenditure of force and are likel to cause damage to banks and bed. Where a minimum of loss of head is desired the should be as far as possible eliminated, which may usually be accomplished by smoot ing the channel and removing abrupt changes of section.

MEASUREMENT OF WATER

19. Weight and Volume

The methods of measuring water may be classed as absolute and inferenti The absolute methods are two, by weight and by volume. The inferent methods embrace the nozzle, the Venturi meter, the orifice, the weir, t float, the current meter, the water wheel, the hydrometric pendulum, colori matter and chemicals, as well as variations of these methods. In some ca the method may appear to occupy an intermediate position between the t classes, but generally will fall in one or the other.

By Weight. The most accurate method of measuring water is by weig and when the weight of a quantity of water and its temperature are knov

s volume can be readily computed. This method is necessarily limited by ne capacity of weighing apparatus to moderate or small quantities. The ollowing table gives the volume of 100 pounds of water at various temperatures.

Volume in Cubic Feet of 100 Pounds of Water

Temperature	Volume	Temperature	Volume
32° Fahr.	1.6021	90° Fahr.	1.6098
35	1.6020	100	1.6129
39.3	1.6019	120	1.6202
45	1.6020	140	1.6290
50	1.6024	160	1.6392
60	1.6035	180	1.6505
70	1.6051	200	1.6629
80	1.6073	212	1.6709

By Volume. The method ranking second in accuracy is by volume. The volume that can be so handled is limited by the capacity of the measuring essel or tank, which may vary from a pipette to a reservoir or a natural ake. The accuracy of the observation depends upon the accuracy with which he areas and change of level of the surface are determined. If $V =$ the olume, $L =$ its length, A_1, A_2, and $A_c =$ the areas respectively of the two nds and the mid-section of the body, and $A_m =$ the mean area, then the volume s given by the prismoidal formula $V = \frac{1}{6} L (A_1 + 4 A_c + A_2)$, and the mean rea perpendicular to L is $A_m = \frac{1}{6} (A_1 + 4 A_c + A_m)$. These formulas apply o the sphere, the hemisphere, any segment of a sphere, a ring or torus, a cone, pyramid, a wedge, a frustum, a paraboloid, an ellipsoid or any segment of ne, and to practically all solids having parallel ends and bounded by plane urfaces.

When measurements are made in tanks with irregular bottoms, the bottom should be overed with water and the difference of elevation of the water surfaces, before and after lling, be measured by means of a hook gage, a point gage, a tube gage, or a scale, depending n the accuracy desired; the devices being here enumerated in the order of their accuracy.

Approximate Volumetric Measurements. The methods under this head nclude the various water meters in which a liquid fills a space and is then xpelled thru ports or valves by a piston, as in the water cylinder of a pumping ngine, which latter falls in this class. On account of leakage thru the valves nd past the piston and the possibility of the space not being completely filled vhen operating at high speeds, the volume of the liquid does not exactly oincide with that of the chamber, and where exactness is required the relation etween the two must be experimentally determined. The amount of liquid lischarged may vary from the volume of the chamber by from one to twenty ercent of the latter. A device used for measuring water at Lowell early in he last century belongs in this class; it consisted of a large paddle wheel rranged with its paddles closely fitting a specially prepared channel thru vhich the water to be measured was caused to flow. The paddle, completely bstructing the channel, was driven ahead by the water. The wheel therefore ssumed the velocity of the water, and so long as the paddle was in close conact with the sides and bottom of the channel the volume swept thru by it elow the plane of the water surface measured the quantity of water passing n that time, subject to a correction for leakage past the paddle, which relaively to the total volume could be made very small. A modification of this levice has been utilized in Europe at some of the turbine-testing plants, vhere a very easy-running car, traveling on a track over a channel, carries screen which can be lowered into the water and which fills the cross-section

of the channel. The car is then driven forward at practically the velocity of the current, and the volume swept thru by the screen below the water surface measures the water discharged, with a small correction for leakage around the edges. With a sufficiently long channel, the possibilities of accuracy in the apparatus considerably exceed those of any of the inferential or of the other semi-inferential devices.

To this class belong the various automatic flush tanks which are designed to discharge their contents when the water reaches a certain elevation, either by causing the tank to turn on its axis, or by bringing a siphon into action, or by operating valves with the aid of a float. These are sometimes designated as low-pressure meters, because they use up or dissipate the major part of the pressure energy of the stream. This class also includes those of the pressure water meters in which there is a definite space filled by the water, whether it be that formed by the displacement of a reciprocating piston, a vibrating disk, or a revolving chamber, in all of which the moving part is essentially a piston. All such apparatus must be calibrated before its indications can be relied upon in cases where exact determinations are essential.

20. Nozzles and Venturi Meters

By the Nozzle. The measurement of water by means of a nozzle requires first an accurate determination of the area of the outlet and of the section at which the pressure is read, and thereafter the observation of the pressure at some point as close as possible to the inlet of the nozzle. This pressure should be observed as communicated thru at least four orifices situated 90° apart in a plane perpendicular to the axis of the nozzle. The orifices should be cut with their walls normal to the axis of the pipe where the elements of the interior surface are parallel, or at least the tangents to the elements at the plane of the orifice should be parallel to each other and to the axis of the channel. The edges of the orifices should be smooth without projecting slivers or burs, and the several orifices should be united to a common equalizing chamber to which the pressure gage is connected. Frequently this chamber is an annular space surrounding the pipe, being made in a special casting designed for the purpose. Fairly good results may be obtained by connecting the gage to a single orifice at the base of the nozzle, but in this case it is important to remove to a considerable distance upstream, ten diameters of the pipe at least, any curvature or obstruction causing an unsymmetrical disturbance of the velocities. In any event a curve should not be within five diameters of the place where the pressure is read, unless a screen or baffle is used to equalize the velocity distribution before the gage is reached. Let A_g = area at the gage section, A_n = area at the nozzle outlet, h_p = pressure head at gage section, h_{vg} = velocity head at gage section, h_{vn} = velocity head at nozzle outlet, Q = quantity of water discharged, v_g = velocity at gage section, v_n = velocity at nozzle outlet, C = coefficient of discharge. When the nozzle is horizontal,

$$h_{vn} = \frac{h_p}{1 - (A_n/A_g)^2}$$

When the center at the gage section is a vertical distance z below the center of the nozzle outlet, h_p in the above equation must be decreased by the distance z, and if the gage section be above the outlet by the distance z, h_p must be increased by z. Then

$$Q = CA_nv_n = CA_n\sqrt{2\,gh_{vn}} = CA_n\sqrt{2\,g\,\frac{h_p \pm z}{1-(A_n/A_g)^2}}$$

With a smooth tapering nozzle, 2 inches or more in diameter at outlet, the angle of convergence between the two sides being from 10° to 15°, and a waterway leading to it of such form as to prevent swirls and eddies, and a

arefully made piezometer connection having at least four orifices as above lescribed, connected to an accurate gage for measuring the head, the coeffi- ient C may be taken at 0.995 and the resulting measurement may be within ¼ of 1 percent of the actual discharge. In ordinary work, however, a oefficient of 0.98 is as high as it is safe to rely upon, and the discharges so omputed, with careful observation, should be within 1 percent of correctness. n smaller nozzles the coefficient will be somewhat lower than in large ones, nd may be expected to increase for very large sizes.

If the nozzle outlet be submerged, h_p in the foregoing expressions must be decreased y the depth of water over the center of the nozzle outlet, and in the event of discharge ıto a vacuum, h_p must be increased by the height of water column equivalent to the meas- re of the vacuum. (Trans. Am. Soc. C. E., 1891, vol. 23. p. 492.)

By the Venturi Meter. This device consists of a short truncated con- erging cone united by a short cylindrical or gorge section, called the throat, o a diverging truncated cone, the latter being much longer than the former and having, therefore, a more gradual taper. The pressure head is read thru piezometer connections at the inlet of the upstream cone and at the throat, the difference of pressure head at the two points representing the increase in velocity head at the smaller section over that at the larger and the frictional loss

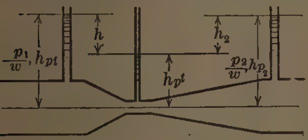

Fig. 21. Venturi Meter

etween the two. As the upper cone is short, usually not over two diam- ters in length, and may be made quite smooth, the latter element is ordinarily egligible. The discharge is computed by the same formula as that of the ıozzle when h_p is replaced by the difference of the pressures read at the inlet ınd the throat. Let A_i = area at inlet to meter, A_t = area at throat, h_{pi} = ›ressure head at inlet and h_{pt} = pressure head at throat. Then

$$Q = CA_t\sqrt{2\,g\,\frac{h_{pi}-h_{pt}}{1-(A_t/A_i)^2}}$$

The coefficient C has an average value of 0.96 to 0.98 and is increased when he inlet is near a curve over the condition when following a straight pipe. Ordinarily the accuracy of measurement should be within 3 percent. The oss of head in passing thru the meter is usually from 10 to 15 percent of the lifference of pressure head between inlet and throat. A special recording levice is furnished with the meter castings by the manufacturers. The commercial size of the meter is that of the inlet, or the same as that of the pipe in which it is to be set. (Trans. Am. Soc. C. E., 1887, vol. 17, p. 228.)

Let d and D be diameters of the throat and inlet pipe in inches, and h the so-called "head on the meter," or $h_{pi} - h_{pt}$, in feet. Then the discharge, in U. S. gallons per 24 hours, is

$$Q = 28276\,C\,\frac{d^2\sqrt{h}}{\sqrt{1-(d/D)^4}}$$

In Engr. News, July 31, 1913, Allen Hazen concludes that 0.99 is the usual value of C, and J. W. Ledoux gives 0.97 as derived from tests on 2- and 8-in throats for high velocities. In Engr. News, Oct. 2, 1913, A. T. Safford gives 0.97 for small discharges and nearly 1.00 for large ones. Tests at Cornell Univ. by E. W. Schoder in 1915 on a 12 by 5-in meter gave about 0.99 for C when the discharge was about 9000 gals per minute.

Fire Streams

Size of nozzle	Pressure at base of play pipe	Best fire stream		Gallons per min	Required hydrant pressure for 2½-in best rubber-lined fire hose of lengths as indicated						
		Height of jet	Reach of jet		Feet						
					50	100	200	300	500	800	1000
In	Lb	Ft	Ft		Lb	Lb	Lb	Lb	Lb	Lb	Lb
¾	10	17	19	52	10	11	11	12	13	15	16
	20	33	29	73	21	22	23	24	26	30	32
	30	48	37	90	31	32	34	36	40	45	49
	40	60	44	104	42	43	46	48	53	60	65
	50	67	50	116	52	54	57	60	66	75	81
	60	72	54	127	63	65	68	72	79	90	97
	70	76	58	137	73	75	80	84	92	105	114
	80	79	62	147	84	86	91	96	106	120	130
	100	83	68	164	105	108	114	120	132	150	163
⅞	10	18	21	71	11	11	13	14	16	19	22
	20	34	33	100	22	23	25	27	32	39	43
	30	49	42	123	33	34	38	41	48	58	65
	40	62	49	142	43	46	50	55	64	78	87
	50	71	55	159	54	57	63	69	80	97	108
	60	77	61	174	65	69	75	82	96	116	130
	70	81	66	188	76	80	88	96	112	136	152
	80	85	70	201	87	91	101	110	128	155	173
	100	90	76	224	109	114	126	137	160	194	216
1	10	18	21	93	12	12	14	16	20	26	30
	20	35	37	132	23	25	29	33	41	52	60
	30	51	47	161	34	37	43	49	61	79	90
	40	64	55	186	46	50	58	66	81	105	120
	50	73	61	208	57	62	72	82	102	131	151
	60	79	67	228	69	75	87	98	122	157	181
	70	85	72	246	80	87	101	115	142	183	211
	80	89	76	263	92	100	115	131	162	209	241
	100	96	83	295	115	125	144	164	203		
1⅛	10	18	22	119	12	14	17	20	27	36	43
	20	36	38	168	25	28	34	41	54	73	85
	30	52	50	206	37	42	52	61	80	109	128
	40	65	59	238	50	56	69	81	107	145	171
	50	75	66	266	62	70	86	102	134	181	213
	60	83	72	291	74	84	103	122	160	218	256
	70	88	77	314	87	98	120	143	187	254	
	80	92	81	336	99	112	138	163	214		
	100	99	89	376	124	140	172	204			
1¼	10	19	22	148	14	16	21	26	36	51	61
	20	37	40	209	27	32	42	52	72	101	121
	30	53	54	256	41	49	63	78	108	152	182
	40	67	63	296	55	65	84	104	144	203	243
	50	77	70	331	68	81	106	130	180	254	
	60	85	76	363	82	97	127	156	216		
	70	91	81	392	96	113	148	182	252		
	80	95	85	419	110	129	169	208			
	100	101	93	468	137	162	211	261			
1⅜	10	20	23	182	16	19	27	34	49	71	86
	20	38	42	257	31	39	53	68	98	143	173
	30	55	56	315	47	58	80	103	147	214	259
	40	69	66	363	62	77	107	137	196		
	50	79	73	406	78	96	134	171	245		
	60	87	79	445	93	116	160	205			
	70	92	84	480	109	135	187	239			
	80	97	88	514	124	154	214				
	100	103	96	574	156	193					

Fire Streams. The preceding table gives the characteristics of fire stream from smooth conical nozzles, as established by the experiments of John R. Freeman, Mem. Am. Soc. C. E. (Trans. A.S.C.E., Vol. XXI). The pressures indicated are those existing while the stream is flowing. For ring nozzles the characteristics correspond to those of the next size smaller smooth nozzle.

21. Orifices

For Small Orifices one inch in height or less, the coefficient 0.61 for the discharge from circular orifices and 0.62 for square and rectangular ones, if the head be more than eight times the vertical dimension, will give results sufficiently accurate for ordinary cases. If more accurate gagings are required, the orifices should be calibrated in place by weighing or volumetrically measuring the water discharged under different heads. When the vertical dimension of the orifice is more than one-sixth the head over the top, the variation of pressure between the top and the bottom causes the center of pressure on the orifice to be appreciably below the center of gravity, and the center of pressure at the vena contracta is below that at the plane of the orifice. Let the notation be

A = area of orifice, A_c = area of jet at vena contracta,
C_v = coefficient of velocity, C_c = coefficient of contraction,
C or C_vC_c = coefficient of discharge,
h_b = head above bottom of orifice, h_{bc} = head above bottom of vena contracta,
h_g = head above center of gravity of orifice, h_t = head above top of orifice,
h_{tc} = head above top of vena contracta, h_v = head due to velocity of approach,
L = length of sill of rectangular orifice, v = velocity thru orifice,
v_c = velocity thru vena contracta, Q = quantity discharged,
R = radius of a circular orifice,
x = width of orifice at any point, x_c = width of vena contracta at any point,
z = height of water surface above any point of the orifice,
z_c = height of water surface above any point in the vena contracta.

The General Formula for discharge per second is

$$Q = Av = A_cv_c = C_v\sqrt{2g}\int_{h_{tc}+h_v}^{h_{bc}+h_v} x_cz^{1/2}dz \qquad (1)$$

in which, if x_c be not constant, it must be exprest as a function of z. The application of this formula requires the determination of the dimensions of the vena contracta, which is a difficult operation, and, therefore, the dimensions of the orifices may be substituted for those of the vena contracta and the coefficient of contraction C_c be introduced, when the above equation becomes, since $C_vC_c = C$,

$$Q = Av = C\sqrt{2g}\int_{h_t+h_v}^{h_b+h_v} xz^{1/2}dz \qquad (2)$$

If the orifice be horizontal $h_b = h_t$ and z is constant. For a rectangular orifice in which $x = L$, after integrating,

$$Q = Av = \tfrac{2}{3}CL\sqrt{2g}\left[(h_b + h_v)^{3/2} - (h_t + h_v)^{3/2}\right] \qquad (3)$$

The best experiments show the value of the coefficient with full contraction on all sides to be practically constant and for square orifices $C = 0.604$, for circular orifices $C = 0.597$.

If contraction be supprest on one side by bringing the wall of the channel of approach into coincidence with the plane of the side of the orifice, the contraction on the opposite side is increased somewhat, but not by an amount equal to the original contraction on the side suppress.

For a rectangular orifice 0.656 foot high and 2.624 feet wide with end contractions

supprest, Bazin's experiments give the coefficient C in equation (3) as 0.624, showing about 3 per cent increase of discharge above that for full contraction on all sides.

For square orifices the experiments of Lebros give: Contraction supprest on one side, C is increased 2.9%. Contraction supprest on two sides, C is increased 5.25%. Contraction supprest on bottom, C is increased 3.25%. Contraction supprest on bottom and one side, C is increased 7.25%. Contraction suppr̃est on bottom and two sides, C is increased 11.5%.

For circular orifices Bidone deduced the effect of full suppression as increasing C or the discharge 12.8 percent and partial suppression correspondingly. It is to be noted that suppression at the top would have less effect than at the bottom, and may be taken as increasing the discharge of a square 2.6%. From this it may be expected that the suppression of contraction over the lower half of a circular orifice would increase the discharge about 8%, over the upper half about 5%, and over one side half about 6%.

For the discharge of sluice gates where the bottom and sides coincide with those of the channel, the coefficient for the corresponding orifice may be used provided there is no apron beyond the gate and the discharge is allowed to fall away freely. If there be an apron or shoot or continuation of the channel, the friction along it will reduce the coefficient, particularly at low heads. When hg was about 2½ times the height of the opening, the discharge has been found to be as much as 12 percent less than for the orifice with full contraction.

22. Weirs

The Weir affords the most commonly used method of measuring water in moderately large quantities. The standard weir, or sharp-edged weir, consists of a vertical partition across a channel with its top edge horizontal, sharp cornered and narrow enough so that at the heads used the overflowing sheet jumps from the upstream edge clear of the downstream corner. Such weirs may be either with or without end contractions. A weir with end contractions is one whose crest extends only part way across the channel and is terminated by partitions in its plane, with their vertical edges rising above the level of the water on the upstream side. Such a weir may be compared to a rectangular orifice upon which the head has fallen below the top. A weir without end contractions is one which extends entirely across the channel. If a = height of crest of weir above bottom of channel of approach, A_w = area of stream in the plane of the weir, H = height above the crest of the surface of still water upstream from the weir, h = head above crest as observed, v_w = velocity in and perpendicular to the plane of the weir, then the formula for the discharge is similar to that for the orifice and is

$$Q = A_w v_w = \tfrac{2}{3}\, CL\sqrt{2g}\cdot H^{3/2} = \tfrac{2}{3}\, CL\sqrt{2g}\,(h + h_v)^{3/2}.$$

Since $LH = L\,(h + h_v)$ = area of the stream above the crest level at the plane of still water and CLH is the area in the plane of the crest, the total head producing flow is $\tfrac{2}{3}\sqrt{2g\,(h + h_v)}$. ($h_v$ = head due to velocity of approach.)

The Francis Formula. The coefficient C in this formula was determined experimentally by James B. Francis as about 0.62, and by combining this with $\tfrac{2}{3}\sqrt{2g}$, the well-known coefficient of the Francis Formula 3.33 is obtained. This formula was considered by its inventor to be reliable between heads of 0.5 foot and 2.00 feet. Later investigators have modified it into the form:

$$Q = 3.33\, L\,(h + 1.4\, h_v)^{3/2}$$

In applying this formula the process is as follows: Having measured the head h at a point above the surface curve to the weir, compute an approximate value of the discharge by the equation $Q_1 = 3.33\, Lh^{3/2}$. Find the approximate velocity at the plane where the head is observed by the equation

$v = Q_1 \div L(h + a)$ and the velocity head by $h_v = v^2/2g$. Then Q is obtained by substitution in formula on p. 1118, and should be within 3 to 4 percent of correctness if the head is not more than 30 percent of a and has beeen properly measured, and the sheet is fully aerated underneath.

For weirs with end contractions Francis recommended reducing the length L in the above formula by 0.1 H for each full end contraction. This correction is only an approximation and, for accurate gagings, weirs with end contractions should not be used.

The Machinery Builders Society in October, 1917, in connection with the Testing Code for Hydraulic Turbines, adopted the following coefficients for use with the Francis formula in the form $Q = CLh^{3/2}$ which include the effect of velocity of approach.

Table of Values of C for Various Heads and Heights of Crest a

Head	Height of crest a, in feet									
h	4	5	6	7	8	9	10	12	14	16	20
1.0	3.376	3.356	3.344	3.335	3.329	3.325	3.322	3.317	3.314	3.311	3.308
1.2	3.391	3.366	3.350	3.339	3.332	3.326	3.322	3.316	3.311	3.308	3.305
1.4	3.409	3.378	3.359	3.346	3.336	3.330	3.324	3.316	3.311	3.307	3.303
1.6	3.429	3.392	3.370	3.354	3.343	3.334	3.328	3.319	3.312	3.308	3.302
1.8	3.450	3.408	3.382	3.363	3.350	3.340	3.333	3.322	3.315	3.309	3.303
2.0	......	3.425	3.394	3.373	3.358	3.347	3.338	3.325	3.317	3.311	3.304

The above coefficients are the averages of values computed by the following three formulas (on p. 1102):

(1) Bazin,

$$Q = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\frac{h^2}{(a+h)^2}\right]\sqrt{2g}\,Lh^{3/2}$$

(2) Rehbock,

$$Q = \left[0.605 + \frac{1}{320h\text{-}3} + 0.08\frac{h}{a}\right]\frac{2}{3}\sqrt{2g}\,Lh^{3/2}$$

(3) Fteley-Stearns,

$$Q = 3.31\,L\,(h + 1.5h_v)^{3/2} + 0.007\,L, \text{ in which}$$

h_v = head due to velocity of approach.

The formula of Rehbock is based upon rather small scale laboratory experiments, and in the writer's opinion is given undue weight in this connection.

The Bazin Formula is the most accurate one for wide ranges of head, and it may be safely applied between heads of 0.2 foot and 6 feet, and does not require a correction for velocity of approach, as it is based upon the observed head h. It applies only to weirs without end contractions and is

$$Q = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{a+h}\right)^2\right]Lh\sqrt{2gh}$$

and the following tables give values of Q for a weir one foot long and for various values of h and a. The value of g used in computing these tables is 32.17 feet per second per second.

Discharge in Cubic Feet per Second per Foot of Length over Sharp-edged Vertical Weirs without End Contractions

Computed by Bazin's Formula.

Head h, feet	Height in feet of crest of weir above bottom of channel of approach						
	$a=2$	$a=3$	$a=4$	$a=5$	$a=6$	$a=7$	$a=8$
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
0.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.4	0.88	0.88	0.88	0.87	0.87	0.87	0.87
0.5	1.23	1.21	1.21	1.21	1.21	1.21	1.21
0.6	1.62	1.59	1.59	1.58	1.58	1.58	1.58
0.7	2.04	2.01	1.99	1.98	1.98	1.98	1.98
0.8	2.50	2.45	2.43	2.42	2.41	2.41	2.41
0.9	3.00	2.93	2.90	2.88	2.88	2.87	2.86
1.0	3.53	3.44	3.40	3.38	3.36	3.36	3.35
1.2	4.68	4.55	4.48	4.47	4.42	4.41	4.40
1.4	5.99	5.78	5.68	5.62	5.58	5.56	5.54
1.5	6.68	6.44	6.30	6.23	6.20	6.18	6.16
1.6	7.40	7.12	6.97	6.89	6.84	6.80	6.78
1.8	8.93	8.56	8.37	8.25	8.18	8.13	8.09
2.0	10.58	10.12	9.87	9.72	9.62	9.55	9.51
2.2	12.34	11.77	11.46	11.27	11.14	11.06	10.99
2.4	14.20	13.53	13.15	12.91	12.75	12.64	12.56
2.5	15.17	14.45	14.03	13.76	13.59	13.47	13.38
2.6	16.16	15.38	14.92	14.63	14.44	14.30	14.20
2.8	18.23	17.32	16.79	16.44	16.21	16.04	15.92
3.0	20.39	19.36	18.74	18.33	18.06	17.86	17.71
3.2	22.64	21.48	20.77	20.31	19.98	19.75	19.58
3.4	24.98	23.70	22.89	22.36	21.99	21.72	21.52
3.5	26.20	24.83	24.00	23.43	23.01	22.73	22.48
3.6	27.41	25.99	25.09	24.49	24.06	23.75	23.52
3.8	29.94	28.38	27.38	26.70	26.22	25.87	25.60
4.0	32.54	30.84	29.74	28.99	28.45	28.05	27.74
4.2	35.22	33.39	32.18	31.35	30.75	30.30	29.96
4.4	37.99	36.01	34.70	33.78	33.12	32.62	32.24
4.6	40.83	38.71	37.29	36.29	35.56	35.01	34.58
4.8	43.75	41.49	39.96	38.87	38.07	37.46	37.00
5.0	46.71	44.31	42.67	41.49	40.62	39.96	39.44
5.2	49.81	47.27	45.50	44.23	43.29	42.57	42.01
5.4	52.94	50.23	48.38	47.02	46.00	45.22	44.60
5.6	56.15	53.33	51.34	49.88	48.79	47.94	47.28
5.8	59.42	56.45	54.34	52.79	51.62	50.71	49.99
6.0	62.77	59.65	56.43	55.78	54.53	53.55	52.78

When the weir is so high that the velocity of the approaching water is practically zero Bazin's formula reduces to

$$Q=\left(0.405+\frac{0.00984}{h}\right)Lh\sqrt{2gh}$$

At low heads, less than 0.2 of a foot, Bazin's Formula gives discharges somewhat too high and the formula proposed by Fteley and Stearns is recommended, which is:

$$Q=3.31\,LH^{3/2}+0.007\,L.$$

The results by this formula are within 4 to 6 percent of the experimental values for heads ranging from 0.2 to 0.007 ft, and the actual discharges were generally in excess of those

Discharge in Cubic Feet per Second per Foot of Length over Sharp-edged Vertical Weirs without End Contractions — Continued

Computed by Bazin's Formula.

Head h, feet	Height in feet of crest of weir above bottom of channel of approach						
	$a=9$	$a=10$	$a=12$	$a=16$	$a=20$	$a=25$	$a=30$
0.2	0.33	0.33	0.33	0.33	0.33	0.33	0.33
0.3	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.4	0.87	0.87	0.87	0.87	0.87	0.87	0.87
0.5	1.21	1.21	1.21	1.21	1.20	1.20	1.20
0.6	1.57	1.57	1.57	1.57	1.57	1.57	1.57
0.7	1.97	1.97	1.97	1.97	1.97	1.97	1.97
0.8	2.40	2.40	2.40	2.40	2.40	2.40	2.40
0.9	2.86	2.86	2.86	2.86	2.85	2.85	2.85
1.0	3.35	3.34	3.34	3.33	3.33	3.33	3.33
1.2	4.39	4.38	4.38	4.37	4.36	4.36	4.36
1.4	5.53	5.52	5.51	5.49	5.49	5.48	5.48
1.5	6.14	6.13	6.12	6.11	6.10	6.09	6.09
1.6	6.76	6.74	6.73	6.71	6.69	6.69	6.69
1.8	8.07	8.05	8.02	7.99	7.98	7.97	7.96
2.0	9.47	9.44	9.40	9.36	9.34	9.33	9.32
2.2	10.95	10.91	10.86	10.81	10.78	10.76	10.75
2.4	12.50	12.45	12.39	12.32	12.28	12.25	12.24
2.5	13.31	13.26	13.18	13.10	13.06	13.03	13.01
2.6	14.13	14.07	13.99	13.90	13.85	13.82	13.80
2.8	15.83	15.76	15.66	15.54	15.48	15.44	15.42
3.0	17.60	17.52	17.39	17.25	17.18	17.13	17.10
3.2	19.45	19.34	19.19	19.02	18.93	18.87	18.83
3.4	21.36	21.24	21.06	20.86	20.75	20.68	20.63
3.5	22.38	22.22	22.00	21.83	21.69	21.62	21.60
3.6	23.34	23.20	22.99	22.75	22.62	22.53	22.48
3.8	25.39	25.23	24.99	24.71	24.56	24.45	24.39
4.0	27.51	27.32	27.05	26.72	26.55	26.42	26.35
4.2	29.69	29.48	29.17	28.79	28.59	28.45	28.36
4.4	31.94	31.70	31.34	30.92	30.66	30.52	30.42
4.6	34.25	33.98	33.58	33.10	32.84	32.65	32.53
4.8	36.62	36.33	35.88	35.35	35.05	34.83	34.70
5.0	39.03	38.70	38.21	37.61	37.28	37.03	36.88
5.2	41.56	41.20	40.65	39.99	39.61	39.33	39.17
5.4	44.11	43.71	43.12	42.38	41.96	41.66	41.47
5.6	46.74	46.31	45.65	44.84	44.38	44.04	43.83
5.8	49.41	48.94	48.22	47.33	46.83	46.45	46.22
6.0	52.15	51.64	50.86	49.90	49.34	48.92	48.67

given by the formula. It holds only so long as the sheet jumps free of the crest and the space behind it is fully aerated.

The Flow over Irregular Crests may be computed by multiplying the discharge of a standard weir of the same height and length and at the same head by a factor depending on the form of the crests. The following tables give the multipliers for various forms of weirs (Fig. 22) as determined from experiments upon full-size models at the Hydraulic Laboratory of Cornell University:

Multipliers for Flat-topped Weirs. Fig. 22A

Head h, feet	Width of flat crest in feet							
	b = 0.48	b = 0.93	b = 1.65	b = 3.17	b = 5.84	b = 8.98	b = 12.24	b = 16.30
0.5	0.902	0.830	0.795	0.790	0.785	0.783	0.783	0.783
1.0	0.972	0.904	0.810	0.797	0.800	0.798	0.795	0.792
1.5	1.000	0.957	0.875	0.797	0.807	0.803	0.802	0.793
2.0	1.000	0.989	0.930	0.815	0.805	0.800	0.798	0.791
2.5	1.000	1.000	0.970	0.842	0.800	0.795	0.792	0.790
3.0	1.000	1.000	1.000	0.870	0.796	0.791	0.787	0.789
3.5	1.000	1.000	1.000	0.896	0.793	0.787	0.783	0.787
4.0	1.000	1.000	1.000	0.925	0.790	0.783	0.780	0.785

Multipliers (m) for Triangular Weirs. Fig. 22B

Head h in feet,	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
For b = 6.65 ft, m =	1.060	1.079	1.091	1.086	1.076	1.067	1.060	1.054
For b = 11.25 ft, m =	1.060	1.079	1.092	1.097	1.096	1.095	1.094	1.093

Multipliers for Compound Weirs. Fig. 22

Head h, feet	Type F	Type G	Type H	Type I	Type J	Type K	Type L
0.5	0.964	0.932	0.934	0.968	0.971	0.971	0.971
1.0	1.026	0.982	1.000	1.008	1.040	1.040	0.983
1.5	1.064	1.015	1.040	1.030	1.083	1.092	1.022
2.0	1.066	1.031	1.061	1.034	1.113	1.126	1.040
2.5	1.025	1.038	1.073	1.038	1.118	1.146	1.057
3.0	0.992	1.044	1.082	1.042	1.120	1.163	1.072
3.5	0.966	1.049	1.090	1.046	1.122	1.177	1.085
4.0	0.944	1.053	1.097	1.050	1.125	1.190	1.097

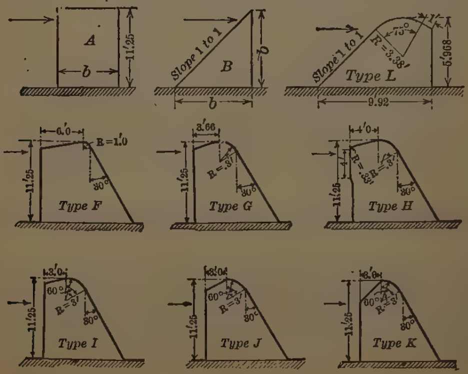

Fig. 22. Types of Weirs and Dams

Measurement of Head. When the head on a weir exceeds 1 ft the means used for observing the head may cause an appreciable variation in the computed discharge.

The following table gives the percentage variation of heads observed in various ways, from those indicated by a device *A* similar to that used by Bazin:

Variations of Head Observations on Weirs

(Experimental)

A. Head observed by hook gage in pit connected at bottom of channel of approach by 4-in diameter pipe flush with wall, 29.88 ft upstream from weir 6.55 ft high.

B. Head observed by tape and bob over center of 16 ft wide channel 14 ft upstream from weir 6.55 ft high.

Head observed by 1-in glass tube gage connected to ¼-in diameter orifice 6 ft upstream from weir 6.55 ft high:

C. In brass plate flush with channel wall 2¼ in below level of crest.

D. In iron cap flush with timber channel wall 1 ft above bottom of channel.

E. In brass plug flush with face of 2 in by 12 in by 18 ft plank on center line of channel, 3 in above bottom.

F. Head observed by 1-in glass tube gage connected to 1-in pipe with ⅛-in orifices, 2 in apart on under side, set transversely across channel 1 ft above bottom and 10.2 ft upstream from weir 5.85 ft high.

G. Head observed by hook gage in barrel connected to five ⅝-in orifices in weir bulkhead 9⅝ in below crest of weir 6 ft high and 65 ft long.

H. Head observed by 1-in glass tube gage connected to 1-in pipe with ¼-in orifices, 6 in apart on underside set transversely across channel 8 in above bottom and 37 ft upstream from weir 5.2 ft high.

Variation of Observed Head in Percent of *A*

Head *A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Feet	%	%	%	%	%	%	%
0.0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	−0.68	−0.4
1.0	0	0	0	0	−0.28	−1.97	−0.8
1.5	0	+0.40	+1.10	+1.30	−0.50	−2.41	−1.2
2.0	0	+0.33	+1.25	+1.28	−0.82	....	−1.75
2.5	0	−0.08	+1.22	+0.80	....	....	−2.48
3.0	−0.7	−0.40	+1.10	+0.08	....	....	−3.17
3.5	−2.0	−0.57	+0.92	−0.35	....	....	−3.85
4.0	....	....	....	....	....	....	−4.62
4.5	....	....	....	....	....	....	−5.23
5.0	....	....	....	....	....	....	−6.10
5.5	....	....	....	....	....	....	−6.74

23. Current Observations

By Floats. In shallow streams approximate discharges may be obtained by surface floats, the mean of whose velocity when multiplied by 0.80 will give the average velocity probably within 10 percent. For more accurate gagings the ROD FLOAT may be used, and when the conditions of the channel are ideal the accuracy of this method places it next to the weir. Rod floats may consist of tin tubes or bamboo poles loaded with shot, or of wooden rods wrapt at their bottom with sheet lead or otherwise weighted. They should be so loaded as to stand vertically in the water, and the point to which they sink should be marked by a ring of paint. Wooden floats, if to be used in accurate work for more than a very few minutes at a time, should be thoroly

oiled, or coated with a waterproof varnish, to prevent absorption of water and the consequent change of buoyancy. In starting the floats they should be immersed to the submergence mark with the bottom inclined slightly against the current and allowed to swing to a vertical position under the influence of the current while held with the thumb and finger as trunnions whose axis is perpendicular to the current and parallel to the surface. As soon as the float comes to a vertical position it should be released without disturbing its submergence or its motion, and allowed to float far enough to take up the velocity of the stream before passing the point at which the upstream observation is taken. The course selected should be in a straight reach of as nearly uniform depth and section as possible, free from weeds and having a straight channel leading to it.

The cross-section should be accurately determined and a length for the course laid off on the bank parallel to the axis of the current. If the stream be narrow, two cords or wires situated in a vertical plane a foot or more apart should be stretched across at the upper and lower ends of the course, and either one of the wires or a separate line should be tagged at intervals to assist in locating the float. The observer should take the time as the float passes the plane of the wires and the recorder or the starter its location along the tagged line. If the stream be wide, the floats should be observed with a transit whose telescope is at right angles to the course and the position determined by means of the vertical angle to the water surface, the elevation of the telescope axis above the water being measured. Where less accurate results are required a single line or a pair of ranges may be used to locate the upper and lower ends of the course and the position may be estimated. If but a single observer and one or two floats are employed, the time may be best taken by means of a stop watch, but for rapid work a large number of floats may be used and with an observer using an ordinary watch and a recorder, at each end of the course, fully as accurate work may be done. The two watches must be set together and frequently compared during the observations. The floats should be so started as to cover as far as possible the entire cross-section of the channel thruout the course, and where the depth of the cross-section varies, floats of different lengths should be used. It is customary to consider the course of the float as that indicated by the mean of its positions at the upper and lower wires and that its velocity is uniform thruout the run and that the area of the stream is the mean area thruout the course.

The floats should be made to pass as close to the bottom as possible without dragging or striking on obstructions, but it is never possible to have them run closer than several inches in ordinary streams. The velocity of the float will therefore be greater than the mean velocity of the water in the whole vertical in which the float travels. The float gives practically the mean velocity in that part of the vertical thru which its submergence extends, and may be taken to represent the velocities horizontally for half the distance each way to the next float. When d = depth of water, i = immersion of float, u = mean velocity of water, v_f = velocity of float, then

$$u = v_f \left\{ 1.0 - 0.116 \left[\frac{d-i}{d} - 0.1 \right] \right\} \text{ and } Q = Au$$

In a rectangular channel 10 feet deep and 16 feet wide it was found that float discharges agreed with weir discharges within 2 percent for ranges of immersion from 60 to 98 percent of the depth. Double floats and subsurface floats are of little value for measuring water.

By Screen or Diaphragm. This is a special form of float and is mounted on a truck running on rails along a straight and uniform section of the channel and is so devised that it can be quickly lowered into the water and made to occupy nearly the whole area of the channel during the run and be lifted out at the end. A proper distance being allowed for starting and stopping and the

screen occupying practically the whole cross-section of the channel, its velocity corresponds very nearly to that of the water. If, however, the screen does not approximately fill the cross-section of the channel the Francis Float Formula of the preceding paragraph may be applied. As the screen is usually run in a rectangular channel of considerable depth it may be desirable to make a second further application of the formula to cover the spaces between the screen and the side walls by considering the distance from the center of the channel to the edge of the screen as i, half the width of the channel as d and the first computed u as v_f for each half of the channel. The depth of water then becomes the width. The sum of the discharge as computed for each half then will be the true discharge.

By Current Meter. The current meter consists usually of a revolving device driven by the current, whose revolutions bear some relation to the velocity of the water, and which are transmitted to some form of a recording or sounding apparatus whereby the number occurring in a given time may be observed.

Two types of meters may be distinguished, the screw or direct acting-meter, and cup or differential meter. The Haskell, Fteley-Stearns and Ott meters are examples of screw or direct acting meters. The Price meter is the best example of a cup meter, and has been adopted as a standard by the U. S. Geological Survey.

The meter should be rated to determine the ratio between its revolutions and the velocity of the passing water which rating is usually accomplished by drawing the meter at known speeds thru still water, either attached to a truck running on rails over a channel or to a boat propelled at a uniform speed, usually by a person walking along the shore. In gaging, the meter is attached to a line and sunk by a weight, or in shallow water attached to a rod, and held at various points in a vertical until sufficient observations are obtained to give the mean velocity in the vertical with the required degree of precision. The meter is then moved to another vertical and the mean velocity determined there. Since it is not possible to operate the meter close to the bottom, the mean of the velocities indicated by it in any vertical will be greater than the mean velocity of the water in that vertical, as in the case of the float, and the same formula may be used for reducing the observations, where v_f becomes the mean velocity determined by the meter; or the vertical curve may be drawn from the observations and extended to the bottom by the eye, when the mean velocity of the water can be found directly with the aid of a planimeter or any integrating or area-measuring process.

As the meter wheel may easily become clogged by weeds attaching themselves to it or by floating matter getting into the bearings, it should be examined from time to time during a gaging. As soon as a meter is rated, what is known as the hand test should be applied. That is, the instrument is held firmly in the position in which it stands when in operation and the wheel is spun by a single effort of the hand, the length of time that elapses before it ceases its revolutions being observed and recorded. This should be repeated several times, and thereafter before starting a gaging and whenever the meter is taken from the water the hand test should be applied and the result recorded. In this way it will be possible to detect both the existence and time of a change in the rating of the instrument.

Comparisons between meter and weir gagings show them to agree under most favorable conditions for the former within from 3 to 4 percent. It is to be noted that during an observation the meter should be stationary in the current or only moving as impelled thereby. The method of gaging sometimes used in which the meter is moved continuously about the cross-section is entirely untrustworthy.

Cup meters may over-register in considerably perturbed water 25 percent.

Screw meters may under-register in considerably perturbed water 10 percent.

Resistance of Meters, Cables and Rods to Dragging Thru Still Water

(Experiments made for Author at Univ. of Mich. Naval Tank)

Vel., ft	Meters only. No weights or cables included		Cables per foot		Rods per foot	
	Ritchie-Haskell direction current meter 8-in wheel, weight 24.7 lb	Haskell current meter 8-in wheel weight 10.3 lb	3/8-in steel cable d = 0.400 to 0.392 in	1/4-in steel cable d = 0.264 to 0.259 in	3/8-in steel Rod d = 0.382 in	1/4-in steel Rod d = 0.267 in
	Resistance, lb	Resistance, lb	Resistance, lb	Resistance, lb	Resistance, lb	Resistance, lb
3.0	1.70	0.45	0.40	0.26	0.36	0.29
4.0	3.75	2.40	0.72	0.47	0.64	0.52
5.0	5.75	4.15	1.12	0.75	0.99	0.78
6.0	8.40	6.00	1.52	1.04	1.40	1.07
7.0	11.60	8.05	1.91	1.37	1.88	1.38

The 3/8-in cable used, 10 ft 1/2 in long weighed 1 lb 7 1/2 oz in air and 1 lb 2 oz in water and had a 3-strand rope center 3/16 in diameter, covered by 6 strands 0.121 in diameter, each composed of 6 strands of 0.037 in diameter wires on a 1/16-in rope center.

The 1/4-in cable used, 11 ft long weighed 1 lb 2 oz in air and 15 oz in water, and had an insulated electric wire 0.038 in diameter for a center covered by 6 strands 0.086 in diameter composed of 7 wires 0.030 in diameter.

By a Water Wheel. Since practically all submerged water wheels are composed of a series of orifices or nozzles, they may be used for measuring water whenever the discharge thru them for any head and number of revolutions is known, as the velocity, and hence the discharge for constant gate, increases as $\sqrt{2gh}$, and the speed of the wheel for similar relative discharges must vary as that of the water.

By the Hydrometric Pendulum. This apparatus consists of a metal ball suspended upon a cord. When the ball is lowered into a current the cord is inclined away from the vertical in some relation to the force of the current. With a suitable scale for measuring the inclination, this becomes a cheap and crude current meter.

A method somewhat akin to the hydrometric pendulum is that of the torsion disk. A disk or plate is attached by its vertical edge rigidly to a small rod which is supported in a vertical position by guides and carries a pointer at the top which travels over a graduated arc. A zero reading is taken with the disk out of the water and at right angles to the direction of the current. The disk is then immersed parallel to the current and revolved by means of an arm on the rod above the pointer until the disk is again perpendicular to the current. The variation of the pointer from its first position measures the torsion of the rod, which is itself a measure of the moment of the force of the current on the disk.

By Coloring Matter, Bran, Sawdust, and Chemicals. These materials are deposited in the water and partake of the nature of floats which disperse themselves thruout the whole body of the stream. The various anilines or even ordinary bluing may be used as colors, and are available not only in open channels and pipes but also in subterranean streams. The observations are ocular. In the case of chemicals, as lithium or common salt, the

observations depend usually upon chemical analyses. The latter method is often valuable in determining the amount of water from a particular source that occurs in a mixt stream, as knowing the dilution at inlet and at outlet the added untreated water can be estimated. The use of chemicals for the measurement of water passing thru turbine wheels has been very fully developed by B. F. Groat, M. Am. Soc. C. E. (Trans. Am Soc. C. E., Vol. LXXX). The essentials for accuracy in this process are the thoro mixing of a known solution with the influent water and a sufficiently distributed sampling of the effluent, care being taken that eddy and back currents do not dissipate the chemical into portions of the water that is not utilized in the measurement.

By a Cord. The measurement of surface velocities by the curve formed by a loose cord attached to the two banks and floating in the stream is possible.

By Ripple Formation. Surface velocities may be measured by the ripples formed by two pins allowed to scratch the surface at known distances apart in a line at right angles to its axis. The distance from this line to the intersection of the ripples varies with the velocity.

24. The Pitot Tube

The Pitot Tube is an instrument having two orifices or two sets of orifices one of which may be so directed as to receive the impact of a stream, and the

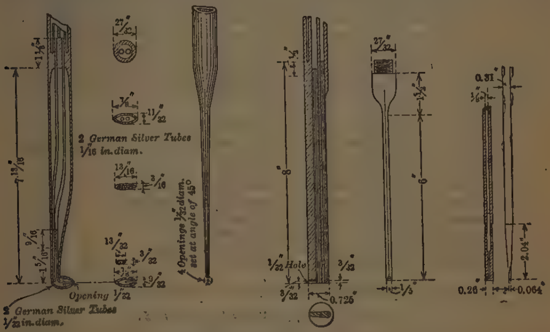

Fig. 23. Details of Pitot Tube

other opening either at right angles thereto, or at some other angle so as to receive only the pressure energy of the water. These orifices are connected to a gage composed of two parallel glass tubes with a scale between or behind them, so that the difference of height of the two columns of liquid supported by the pressures on the orifices may be observed. This difference is then a function of the velocity head of the flowing water. The impact opening may be a hole bored in a piece of small-diametered pipe, the bottom of which is closed, and which is inserted in the stream of water flowing thru a conduit or other channel, and the pressure orifice may be an opening in the channel wall normal to its interior surface. Experiment shows that for ranges of velocity up to 10 feet per second and for pipes of diameters up to 8 inches the indication of

a water column gage, when the openings are as last described, gives the velocity head within one or two percent. In large open channels this statement does not appear to hold true and for work in the latter it is necessary that the instrument be rated. This rating may be accomplished with fair accuracy in a manner similar to that of rating current meters. A better rating can be obtained by measuring with the instrument the flow thru a pipe, the discharge of which is simultaneously determined volumetrically or gravimetrically. The method of gaging is to locate the impact opening of the instrument successively at different points in the cross-section of the channel and read the indication of the gage, from which the velocity at the several points may be computed and thereby the discharge of the pipe or channel be determined. Since in the case of normal flow in a circular pipe flowing full the central velocity is about 1.19 times the mean velocity, if the existence of normal flow at the point of gaging be established, the impact orifice of the instrument may be placed at the center of the pipe and the observed velocity divided by 1.19, or multiplied by its reciprocal 0.84, to get the mean velocity of the water. Such a determination when the area of the cross-section is accurately known and the condition of normal flow established should be correct within 3 percent. If normal flow does not exist but the distortion of the velocity curve is symmetrical about the axis of the pipe, the ratio of the mean to the maximum velocity, sometimes called the coefficient of the section, may be obtained by observations across a diameter and the mean velocity thereafter obtained by multiplying the observed central velocity by the so obtained coefficient. The accuracy of the results will depend upon the accuracy with which the coefficient has been established, but with careful work should be within 5 percent. Observations with the Pitot tube in closed channels where the velocity distribution is distorted unsymmetrically as by a curve or gate cannot be relied upon for establishing the mean velocity unless they cover several diameters of the pipe.

The formula for reducing Pitot tube observations is usually misunderstood. It may be deduced as follows for the case of the pressure orifice in the wall of a circular pipe:

h_d = observed difference of head by the gage,
p = pressure head at any point in the cross-section of the pipe,
p_c = pressure head at the center of the cross-section of the pipe,
p_w = pressure head at the wall of the cross-section of the pipe,
v = velocity at any point in the cross-section of the pipe,
v_c = velocity at the center of the cross-section of the pipe,
v_w = velocity at the wall of the cross-section of the pipe.

By Bernoulli's Theorem for all points in a horizontal diameter of the section $p+v^2/2g$ is constant and

$$p_c + v_c^2/2g = p_w + v_w^2/2g = p + v^2/2g.$$

The impact opening of the tube receives and transmits the pressure of the stream at that point, plus twice the head to which the velocity is due by the theory of impact, or its gage column represents $p+v^2/g$ for the point at which the orifice is situated. The opening in the wall transmits to its gage column the pressure p_w. Then the difference of the columns of the gage is

$$h_d = p + v^2/g - p_w$$

and for the point at the center of the pipe

$$h_d = p_c + v_c^2/g - p_w$$

But by Bernoulli's Theorem

$$p_w = p_c + v_c^2/2g - v_w^2/2g$$

and the gage difference at the center then is

$$h_d = p_c + v_c^2/g - (p_c + v_c^2/2g - v_w^2/2g) = \frac{v_c^2 + v_w^2}{2g}$$

When $v_w = v_c/5$, then $h_d = 1.04\, v_c^2/2g$; and when $v_w = v_c/10$, then $h_d = 1.01\, v_c^2/2g$.

From this it appears that the observed result of the tube with the pressure orifice in the wall of the pipe giving the correct mean velocity on the theory that $h_d = v^2/2g$ indicates the actual wall velocity to be about or less than one-tenth of that at the center. As the closest measurement to the wall yet recorded was at a point about 3/5 of 1 percent of the diameter from the wall in a 30-inch pipe, at which point the velocity was about ½ that at the center, it is not difficult to conceive the necessary reduction of velocity between this point and the wall to bring the wall velocity within the above limits.

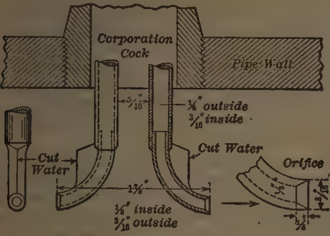

Fig. 24. Cole-Flad Pitometer

When the instrument carries its own pressure opening it does not seem possible to deduce its coefficient theoretically for the reason that the pressure orifice is always so situated that the velocity of the water next the instrument is considerably retarded, and hence the pressure increased, in passing from the impact to the pressure opening. Consequently the pressure transmitted is not that corresponding to the velocity to which the impact opening is subjected. The head indicated on the gage will therefore always be less than v^2/g, and as the velocity past the pressure opening is never less than that at the pipe wall, the head on the gage will always be greater than $v^2/2g$. Therefore when, as is commonly the practice, h_d is assumed to represent $v^2/2g$, the indication must be multiplied by a coefficient less than unity, usually between 0.7 and 0.9. (Trans. Am. Soc. C. E., vol. 47.)

A Pitot Tube of special form combined with a photographic recording apparatus is known commercially as the Cole-Flad Pitometer. In this instrument the pressure opening points downstream or opposite to the impact opening.

25. Fountain Jets

When water discharges over the horizontal circumference of a vertical pipe, the conditions partake in part of the nature of the flow over a weir and in part of the discharge thru a nozzle, at low heads the former and at higher heads the latter predominating, with a range of transition from one condition to the other between. The head on such a device may be most satisfactorily read by an orifice at the center of the pipe directed against the current and located a short distance below the outlet. The head to be used is that measured by a water column attached to the orifice, less the height of the column of water from the orifice to the discharging lip of the pipe.

From a series of experiments made by Lawrence and Braunworth at the Hydraulic Laboratory of Cornell University (see Trans. Am. Soc. C. E., Dec., 1906, vol. 57, p. 265) in which the pipes had sharp edges tapered on the outside and the head was read as above, the diameters used being 2, 4, 6, 9, and 12 inches, the following formulas were deduced, which may be expected to give average results within 5 percent of the truth:

For Weir Flow, $Q = 8.8\,D^{1.29}h^{1.29}$, which is applicable for heads below $h = 0.028\,D^{1.04}$ with the jet falling free from the pipe; that is, aerated underneath.

For Jet Flow, $Q = 5.84\,D^{2.025}h^{0.53}$, which is applicable for heads above $h = 0.107\,D^{1.04}$. For the range between these two heads the result lies between those given by the above formulas whose lines intersect at $h = 0.045\,D^{1.04}$.

For ordinary computations these formulas, with the same limitations, may be used as:

For the weir, $Q = 8.8\,D^{1.25}h^{1.25}$

For the jet, $Q = 5.84\,D^2h^{\frac{1}{2}}$

26. Observations of Heads

In Open Channels the most common method of reading head is by a graduated scale upon the face of which the water rises. If this is situated in running water the sides should be tapered to their edges to prevent a depression of the surface due to an eddy at the upstream corner. On account of capillarity the reading will be higher than the true surface of the water and the observer may be expected to read to about one-half the smallest division that is clearly visible. The United States Geological Survey uses for its stream-gaging stations the BALL AND CHAIN GAGE, which consists of a chain running over a pulley and carrying a ball at its lower end which may be read by allowing the ball to scratch the surface in running water or to be just immersed in still water. The scale is usually horizontal, and a pointer on the chain indicates thereon the position of the ball.

A more accurate apparatus and one which may be readily provided in the field, consists of a plumb bob suspended from a steel tape which is past over a block cut at the edge to an arc of about 2 or 3 inches radius, having on the top a mark against which the reading of the tape is taken. By fastening an ordinary leveling rod target to the block and causing the graduated edge of the tape to lie next the vernier, very accurate readings of the position of the bob may be obtained. In running water the bob should be allowed to scratch the surface and cause a fine ripple. When the surface undulates, readings of the high and low of the vibration should be taken. In still water the bob may be swung until it barely cuts the surface, or it may be slowly lowered until contact with the water is indicated by the rising of the water to the point due to capillarity. The POINT GAGE consists of a point attached to a graduated rod sliding past a vernier. It is read similarly to the plumb bob, and is only second in accuracy to the hook gage. The ordinary leveling rod with its upper section fastened to a post and the lower carrying a metal point makes an excellent and easily obtained point gage.

The Hook Gage, an apparatus invented by Uriah Boyden, consists of a hook attached to a graduated rod sliding past a vernier, or to a vernier sliding past a fixt scale. The hook is lowered below the surface of the water and slowly raised until its point comes in contact with the surface film of the water, when the latter is distorted, and if the point of contact be in the reflection of a light a black spot appears; if in diffused light, a bright spot. In still water the delicacy of the instrument is such that variations of the surface of $\frac{1}{10000}$ of a foot may be readily observed. For best results the point of the hook should be a cone with a vertex angle of about 90 degrees.

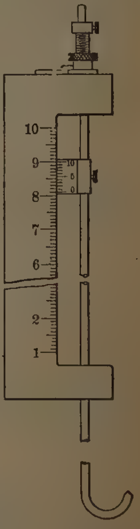

Fig. 25. Hook Gage

The Water Column which rises in a glass tube connected to the channel may be utilized for reading head, but in this case the attraction of the walls of the tube for the water causes the liquid to rise higher than the height due to pressure alone. If the tube be less than one-half inch in internal diameter this effect must be allowed for in accurate work. Under no circumstances should the height of the column be measured to the intersection of the water and the glass, but a line tangent to the extreme edge of the curve of the meniscus at the center

of the tube should be taken. Except on a vibrating column during the rise of the liquid the water meniscus is always concave upwards.

The Mercury Column, owing to the lack of affinity between the glass and the mercury, has its meniscus convex upwards and should be read on a tangent to the top of the curve. As the specific gravity of mercury is 13.58 at 60° Fahrenheit, one foot of mercury is the equivalent of 13.58 feet of water.

The Bourdon Pressure Gage, tho sometimes used in hydraulic work, is not sufficiently delicate to give satisfactory results except where the pressure to be measured exceeds five pounds, and is unsuited for determining small differences of pressure.

The Differential Gage is used for measuring losses of head. This consists of two vertical glass tubes connected at one end, with a scale between or behind them, the free ends being connected respectively by hose or pipes to the upstream and the downstream end of the channel in which the loss is to be observed. For measuring large differences of head the tubes are connected at the bottom and are then filled about half full of *mercury*. The difference of height of the mercury columns, when multiplied by the specific gravity of mercury minus one, gives the height of the water column that would measure the difference in pressure head at the two connections to the channel.

For smaller differences of head the tubes should be connected at the top and the water allowed to rise in the tubes entrapping air above it. Unless the pressure is so great as to compress the air sufficiently to cause its weight to be appreciable, the difference in height of the water columns measures the difference of pressure heads at the connections.

For very small differences the air in the upper portion of the gage may be replaced with oil. If G_o is the specific gravity of the oil and h_u and h_l the height of the upstream and downstream water columns, the difference of pressure head indicated will be $c\ (1 - G_o)\ (h_u - h_l)$, where c is a coefficient less than unity depending on the kind of oil. When a gage of this kind is used it should be calibrated by comparison with a water-column gage to determine the value of c, or better the value of one unit of the scale. For kerosene oil with $G_o = 0.7879$ at a temperature of 60° Fahrenheit, the reading of the gage has been found to be 4.895 times the head; with gasolene having $G_o = 0.7159$ at the same temperature, the multiplication was 3.52; and with ordinary sperm oil it was 9.05. (Trans. Am. Soc. C. E., vol. 47, pp. 78–90, April, 1902.)

TURBINES AND WATER WHEELS

27. Classifications

Turbines are of two types, reaction or pressure turbines and tangential or impulse turbines. In the former the wheel is completely filled with water, which acts by its pressure, and in the latter the head is converted into velocity and the wheel should never be filled.

A reaction or pressure turbine is an assemblage of curved channels or nozzles revolving uniformly about a fixt axis, usually either horizontal or vertical, with suitable stationary or movable guides to direct the flow into the revolving channels. The revolving part of such an assemblage is called the runner or wheel.

Runners are designated as Right Hand or Left Hand, according to the direction of their rotation. When the observer stands in the line of the shaft of the wheel with the inlet end toward him, or is looking in the direction along the shaft in which the water flows to enter or leave the wheel, a wheel which appears to revolve in a clockwise direction is Right-handed, and one revolving in a counter clockwise direction is Left-handed.

Turbines are classed according to the direction in which the water passes thru them, as: (1) Outward flow, or Fourneyron wheels. (2) Inward flow, or Francis wheels. (3) Parallel or axial flow, or Jonval wheels. (4) Mixed flow, or American wheels. In

the last named class the flow is first inward, then axial. When standing still the laws of flow thru nozzles apply to this apparatus without modification, but when revolving the centrifugal force of the water inclosed in the wheel complicates the equations.

Nomenclature

b = barometric column of water.
d_i = depth of wheel at inlet.
d_o = depth of wheel at outlet.
e = efficiency of turbine.
e_h = hydraulic efficiency.
h_c = centrifugal head.
h_{ei} = height of center of inlet above tail-water.
h_{eo} = height of center of outlet above tail-water.
h_{pi} = pressure head at inlet.
h_{po} = pressure head at outlet.
H = total head acting.
H_n = head corresponding to a runaway speed of n revolutions.
k_1 = power constant.
k_2 = capacity constant.
k_3 = speed constant.
K = wheel characteristic.
n = number of revolutions per minute.
Q = quantity of discharge per unit of time.
R_i = radius of wheel at inlet.
R_o = radius of wheel at outlet.
u_i = radial component of velocity at inlet of wheel.
u_o = radial component of velocity at outlet of wheel.
U_i = absolute velocity of water at inlet of wheel.
U_o = absolute velocity of water at outlet of wheel.
U_t = absolute velocity of water at outlet of draft tube.
v_i = velocity of wheel at inlet.
v_o = velocity of wheel at outlet.
V_i = velocity of whirl at inlet.
V_o = velocity of whirl at outlet.
V_{ri} = relative velocity of water at inlet.
V_{ro} = relative velocity of water at outlet.
w = weight of one cubic unit of water.
W = power.
ϕ = speed coefficient.

Fundamental Relations. Bernoulli's equations for the flow thru a turbine:

At inlet,
$$H - h_{ei} + b = h_{pi} + \frac{U_i^2}{2g}$$

Between inlet and outlet the wheel impresses upon the water a head due to centrifugal force $= (v_1^2 - v_o^2)/2g$, so that within the wheel:

$$h_{ei} + h_{pi} + \frac{V_{ri}^2}{2g} = h_{po} + \frac{V_{ro}^2}{2g} + \frac{v_1^2 - v_o^2}{2g} + h_{eo}$$

and between outlet and tail-water:

$$h_{po} = \frac{U_o^2}{2g} + h_{eo} = b + \frac{U_t^2}{2g}$$

To relate V_{ri} and V_{ro} to U_i, U_o, v_i and v_o, referring to Fig. 26, where

$AB = U_i$ $ab = U_o$ $AC = v_i$ $ac = v_o$ $AD = V_i$ $ad = u_o$ $BC = V_{ri}$ $bc = V_{ro}$

$BD = u_i$ $bd = V_o$ R_i = radius at inlet. R_o = radius at outlet.

α = angle between absolute velocity of water and tangent to periphery of wheel at inlet.
β_i = angle between tangent to inlet of bucket and tangent to periphery of wheel.
β_o = angle between tangent to outlet of bucket and tangent to interior of bucket circle.
θ = angle thru which the jet is diverted or angle between U_i and U_o.

$$V_{ri}^2 = (v_i - U_i \cos\alpha)^2 + (U_i \sin\alpha)^2$$
$$V_{ro}^2 = [v_o - U_o \cos(\alpha + \theta)]^2 + [U_o \sin(\alpha + \theta)]^2$$

Or otherwise, since the flow is radial, the product of the radial velocity and the area of the circumferential sections of the channels must be constant, whence:
$$\pi R_i d_i U_i \sin\alpha = \pi R_i d_i u_i = \pi R_o d_o u_o$$
or
$$u_o = R_i d_i u_i / R_o d_o$$

The total work done by the water in passing thru the turbine is measured by the change in kinetic energy and equals $W = Qw(U_i^2 - U_t^2)/2g$, and the

energy carried away in the water is $QwU_t^2/2g$. The work is also equal to $W = Qw\,(V_i v_i - V_0 v_0)/g$, and by the aid of these equations all the factors may be interrelated.

When U_0 is radial or axial, as it should be for maximum efficiency, the relations are materially simplified, as U_0 is then equal to u_0.

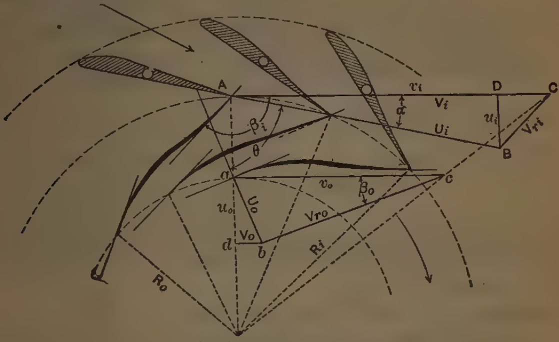

Fig. 26. Turbine Diagram

28. Definitions

The theoretical power of a fall of water is the force, or weight of the water, multiplied by the space passed thru, or the height of the fall; when H = the height thru which the water falls, Q = the discharge in cubic feet per second, w = the weight of a cubic foot of water, the theoretical horse-power is $= QwH/550$. A turbine wheel converts a part of this theoretical power into effective energy, and the mechanical horse-power of the wheel $= Wh_p = cQwH/550$, where c is the efficiency of the wheel.

The Efficiency of the wheel is the ratio of its output in power to the total power of the water and head used. As c is usually about 80%, the horse-power of the wheel $= QH/11$ approximately, or 11 cubic feet of water per second falling thru 1 foot will yield 1 horse-power at the turbine shaft.

The Hydraulic Efficiency of a turbine is the quotient obtained by dividing the change in kinetic energy by the total potential energy of the fall utilized and is

$$e_h = \frac{Qw}{2g}\,\frac{(U_t^2 - U_i^2)}{QwH} = \frac{U_t^2 - U_i^2}{2gH}$$

This differs from the actual efficiency, and is what would be its limiting value if there were no frictional or impact losses either in the water itself or in the mechanism. The actual efficiency, or the efficiency of turbines as shown at the Holyoke Testing Flume, ranges from 70% to 93%.

The Discharge Q of a wheel varies as the velocity and is therefore proportional to $\sqrt{H}$.

The Mechanical Power of a given wheel varies as the head H and also as the discharge Q, which in turn varies as $\sqrt{H}$. Therefore, the power varies as $H\sqrt{H} = H^{3/2}$.

Unit Power is the power at 1 foot head = horse-power$_1$ = $\dfrac{\text{horse-power}}{H\sqrt{H}}$.

The Speed at which a wheel will give its maximum efficiency under a given head depends upon the radius of the inlet circle and the form of the buckets or vanes. For best effect the receiving edge of the bucket must be parallel to the direction of the velocity of the inflowing water relative to the moving vane, and at discharge such that the water leaves either axially or radially.

In Fig. 27, if AB represent in direction and magnitude the absolute velocity of the water entering the wheel and AC that of the bucket, then CB is the relative velocity of the water on the bucket, and the inlet edge should be tangent to it. If the velocity of the water be increased to AD, then for the same bucket the velocity of the wheel must be increased to AE, to keep ED parallel to CB, so that the water may still glide upon the bucket without shock. It follows, then, that for any given wheel a certain fixt ratio must exist between its speed and that of the water, for maximum efficiency. The speed of the water is evidently a function of the head, and hence for every head there is some speed at which a given wheel will yield the maximum efficiency. The best speeds are therefore proportional to $\sqrt{H}$.

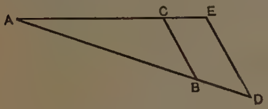

Fig. 27

Unit Speed is the speed of a wheel at 1-foot head = $n_1 = \dfrac{n}{\sqrt{H}}$.

The Speed Coefficient, usually designated ϕ, is the ratio of the linear velocity of the runner periphery to the spouting velocity of the water under the given head;

$$\phi = \frac{\pi D_1 n}{60 \times \sqrt{2gH} \times 12}$$

where D_1 = outside diameter of runner in inches, measured at center line of inlet.

The Discharging Capacity, and hence the POWER of a wheel, is dependent upon the area of the cross-section of the water passages thru it, which are themselves a function of the diameter of the wheel. Therefore in similar wheels the power at a given head will vary as the square of the diameters.

When the performance of a given wheel has been determined for a particular head as to horse-power, discharge and efficiency, the performance at any other head may be computed, and the performance of any similar wheel at any head may be determined with reasonable accuracy by the foregoing relations. By similar wheel is meant one in which all dimensions are in the same ratio to those of the first wheel, all angles being the same.

By changing the form of the vanes or buckets the speed at which the wheel gives its best efficiency may be varied. Starting with a normal speed equal to the tangential component of the velocity of the inflowing water in which case the inlet edge of the bucket would be radial, the speed may be either increased or decreased twenty-five percent without appreciably affecting the efficiency. If the speed be increased the bucket tends to become convex toward the incoming water, and is inclined backward toward the outlet; if the speed be decreased the bucket becomes more concave toward the jet and has less backward inclination toward the outlet. The power of a given sized wheel is decreased by varying its speed from the normal, by reason of the resulting contraction of the waterways.

29. Selection of Runners

Type Characteristic. Turbine runners of the same general class may differ widely in their characteristics of speed and capacity, or power, depending upon the size and shape of the blades. In order that runners of different types, or of different manufacture, may be compared on a common basis, a type char-

acteristic has been adopted. *The type characteristic is defined as the speed in R.P.M. at which a wheel would run if it were reduced proportionally in all dimensions so as to develop 1 horse-power under 1 foot head.* This characteristic is designated K and is made up of:

(*a*) A power constant, k_1 = horse-power/QH, which is the power of a similar runner using 1 cubic foot of water under a 1-foot head.

(*b*) A capacity constant, $k_2 = Q/D_i^2\sqrt{H}$, which is the discharge of a similar runner of 1-foot diameter under a head of 1 foot, where D_i = diameter of 1 foot.

(*c*) A speed constant, $k_3 = V_i/\sqrt{H} = \pi D_i n/60\sqrt{H}$, which is the speed of the runner itself under a head of 1 foot. Then

$$K = \frac{60\,k_3\sqrt{k_1\,k_2}}{\pi} \quad \text{or} \quad K = \frac{n\sqrt{\text{horse-power}}}{H^{5/4}}$$

For 1-foot head $K = n_1\sqrt{HP_1}$ where n_1 = unit speed $= \frac{n}{\sqrt{H}}$, HP_1 = unit horse-power $= \frac{HP}{H^{3/2}}$.

Obviously this characteristic combines the considerations of power and speed and enables a complete comparison to be made between wheels of known efficiency.

The values of K for American, Francis or pressure type turbines vary from 12 to 160, as shown by the table on p. 1136.* For impulse wheels the maximum value is about 6, where a single jet is used. The interval between 6 and 12 is not covered by either the Francis type or a true impulse wheel, tho the old Girard turbine would apply between these limits.

The turbines included in the following table are all of recent design and high efficiencies. There are other modern high efficiency wheels made and the manufacturers listed make other types than the ones shown, but these are all on which data have been furnished to the writer.

In general, high values of K indicate relatively "high-speed" wheels, or those adapted for operation under low heads of water. Experience has shown that for any given head there is a limiting value of K beyond which it is impracticable to go, as the resulting speeds would cause pitting of the runner blades and develop excessive centrifugal forces. The limiting values of K as determined by best American practise are approximately as follows:

For heads above 600 ft,	K = 12 to 25
For heads from 100 to 600 ft,	K = 25 to 55
For heads below 100 ft,	K = 55 to 150

Fig 27*a* shows graphically the relation between K and H, the curve being an average of the practise of several leading turbine manufacturers.

In using this characteristic K, its value is computed from the requirements of the plant under consideration. If this value falls within the allowable limit for the given head as shown by Fig. 27*a*, the turbine best suited to the place will be the one having a value of K nearest that computed. If the value of K, computed from the plant requirements, considerably exceed the limiting values, it will be necessary to reduce the power per runner, or reduce the speed,

* The *speed* value used in computing K is that at which the best efficiency is attained by the runner, usually referred to as "best speed" or "normal speed." The *horse-power*, for the sake of definiteness and uniformity in practise should preferably be taken at the actual best efficiency point or gate-opening. However, as this can only be arrived at by a plotting of the test, it has become the practise of most manufacturers to figure K from the full gate horse-power at the best efficiency speed. The former method is of course the more conservative, but the latter the more readily applied.

Table of Type Characteristics

Manufacturer	Name of runner	*K* *
Allis-Chalmers Mfg. Co.	Type A	13.55
	Type B	20.3
	Type C	29.4
	Type D	40.7
	Type E	51.7 to 60.5
	Type F	72
	Type G	82
	Type H	92.5
	Type I	105
	Type Nx_1	130
	Type Nx_2	160
Hydraulic Turbine Corp.	Camden, Type 4	53.2
	Camden, Type 8	71.3
	Camden, Type 11	83.5
J. and W. Jolly	McCormack Special	68.2
James Leffel & Co.	Samson-Standard	73.0
	Leffel-Sparks, Type F	81.0
	Leffel, Type Z	105.0
Platt Iron Works Co.	Victor, Type 1	84
	Victor, Type 2	75
	Victor, Type 3	60
	Victor, Type 4	45
	Victor, Type 5	32
	Victor, Type 6	20
	Victor, Type 7	14
S. Morgan Smith Co.	Type E	25.1
	Type F	33.8
	Type G	46.1
	Type H	52.0
	Type K	60.0
	Type N	71.3
	Type O	76.6
	Type R	86.0
	Type S	95.5
Wellman-Seaver-Morgan Co.	Runner No. 42	84.5
	Runner No. 48	71.5
	Runner No. 40	65.5
	Runner No. 11	50
	Runner No. 18	27.2

* *K* computed using horse-power values at full gate instead of at best efficiency point.

in case this be permissible and *K* may then be recomputed, using a value of the horse-power equal to the total power desired, divided by the number of wheels assumed, and the runner selected as before. This will be illustrated by several typical examples.

Example 1. "It is desired to install a 60-cycle, 2300-volt, hydro-electric unit, with 450 H.P. available under 25-foot head."

For this low head installation a high-speed turbine (one having the highest permissible value of *K*) should be used. From the curve (Fig. 27*a*) $K_{max.}$ for 25-foot head is about 95.

$HP_1 = \frac{HP}{H^{\frac{3}{2}}} = \frac{450}{125} = 3.6, \quad n_1 = K/\sqrt{HP_1} = 95/\sqrt{3.6} = 50, \quad n = 50 \times \sqrt{25} = 250$ R.P.M. As this is not a synchronous speed, 240 will be assumed.

Then, $n_1 = \frac{240}{\sqrt{25}} = 48$ and $K = 48\sqrt{3.6} = 91.1$.

At least four of the wheels shown in Table (page 1136) have a K above 91.

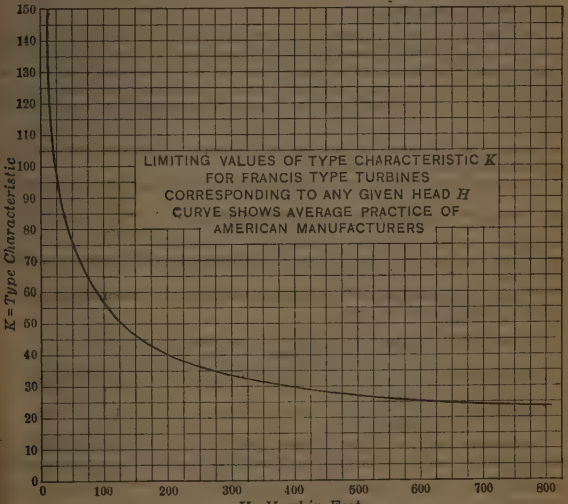

Fig. 27a. Limiting Values of Type Characteristic K

The performance of a wheel of given size being known, it is possible to determine the approximate size required to fulfil the assumed conditions, remembering that speed varies inversely with D and power varies as D^2. Or, from such tables as are given on pages 1140 to 1145 the size under any type that comes nearest to the required conditions may readily be determined.

Thus, from Table (page 1141) a type of ($K = 92.5$) Allis-Chalmers Co. turbine shows the following performance.

Dia.	H.P.$_1$	n_1	Q_1	H.P.	R.P.M.	Q
30"	3.40	50.0	37.4	425	250	187
34	4.36	44.2	48.6	545	221	240

Obviously the required diameter is between 30 and 34 inches.

Example 2. Required to develop 6000 H.P. at 360 R.P.M. under a head of 125 feet.

$$\text{H.P.}_1 = \frac{6000}{125\sqrt{125}} = 4.3, \quad n_1 = \frac{360}{\sqrt{125}} = 32.2, \quad K = 32.2\sqrt{4.3} = 66.8.$$

The allowable limit for K at 125-foot head is only about 50 from Fig. (27a). Therefore, two units will be assumed developing 3000 horse-power each.

Then $\text{H.P.}_1 = \dfrac{3000}{125\sqrt{125}} = 2.145$ and $K = 32.2 \times \sqrt{2.145} = 47.2$, whic falls below the limiting value.

Referring to the table on pages 1143 and 1144 we find that a 42-inch Victo type 4(K = 45) turbine develops 3026 horse-power at 342 R.P.M. und 125-foot head, and a 42-inch Camden type 4(K = 51) develops 3400 H.P. 365 R.P.M. Both of these wheels very closely approximate the assumed co ditions and by a slight modification of the design, would be able to meet the exactly.

Example 3. Required to develop 3000 horse-power with one hydro-electr unit installed under 900-foot head.

For economical generator construction, the speed for a unit of this size shou not exceed about 450 R.P.M. On this assumption, $n_1 = \dfrac{450}{\sqrt{900}} = 15$ H.P.$_1$.

$\dfrac{3000}{900\sqrt{900}} = 0.111$, $K = 15\sqrt{0.111} = 5$. As this value of K falls below t minimum of 12 recommended for Francis turbines, the pressureless type, impulse wheel, should be used.

Assuming 80% efficiency, $Q = \dfrac{3000 \times 11}{900} = 36.6$ c.f.s.

Velocity of free jet $= .97\sqrt{2\,gh} = .97\sqrt{64.4 \times 900} = 233.5$ ft per sec.

Area of jet $= \dfrac{Q}{V} = \dfrac{36.6}{233.5} = .157$ sq ft $= 22.55$ sq in. Dia. jet $= 5^3/_8$ in

For maximum efficiency, the peripheral velocity of the runner buckets sho theoretically be one-half the spouting velocity of the jet. In actual practi however, this ratio is taken at about $.47\sqrt{2\,gh}$. $.47\sqrt{64.4 \times 900} =$ ft per sec.

At 450 R.P.M., $D = \dfrac{113 \times 60}{450 \times \pi} = 4.8$ ft dia or 57.6 in.

$\dfrac{57.6}{5.375} = \dfrac{10.7}{1}$ = ratio of wheel diameter to diameter of jet.

This is about the minimum ratio allowable in present practise and it is c sidered better to increase it to 16 or 18. Assuming 16:

$D = 5.375 \times 16 = 86$ in. A 7-ft wheel would probably be used. T R.P.M. $= \dfrac{113 \times 60}{7 \times \pi} = 308$, and the nearest synchronous speed would be R.P.M.

30 Turbine Performances

Variation of Discharge. The quantity of water discharged for an inw flow turbine is greatest when the wheel is still, and on account of centrif force it decreases as the speed increases.

* It will be necessary, then, to use more than one runner. Where it is desire install only one unit, a multiple-runner turbine may be used. This expedient is, how being resorted to less and less frequently in best practise and should be avoided wher possible.

For the other types the discharge is less when still than at a velocity near that giving aximum efficiency, on account of the loss due to shock at entrance, and increases until e wheel gets up to speed; from the maximum it decreases, on account of the inter-rence of the revolving buckets with the stream flowing thru, to a theoretical zero at a eed of infinity.

Gates. The quantity of water passing thru a turbine, and hence its power, ay be regulated by opening or closing the gates. In the older turbines, e gate was a cylinder sliding parallel to the axis and reducing the height the inlet openings. This form is objectionable, as it does not permit the ucket to be uniformly filled at part gate openings, and the more generally lopted type with recent wheels is the so-called wicket gate, which swings 1 a pivot set parallel to the axis of the wheel, and forms not only the gate ut the guide for directing the water upon the wheel.

Constant Speed. In the operation of electrical machinery a constant eed is necessary, and hence if the head fall below the normal the turbine ust be run at a higher speed than that for which it was designed, in which ise the power and efficiency is rapidly reduced and the outlet edges of the uckets are liable to corrosion.

If, on the other hand, the head increases, the wheel must run at a lower eed than that for which it was designed, which, while not reducing power nd efficiency nearly so rapidly as overspeeding, nevertheless renders the

Performances of McCormack Wheels at Over and Under Speeds

Percent of original head	Percent of original power at best speed	Percent of original power at original speed	Percent of best power at original speed	Percent of best speed at original speed	Approximate percent of best efficiency at original speed
150	183.7	180	97.5	81.6	96.5
145	174.6	172	98.4	83.0	97.1
140	165.7	164	99.0	84.5	97.6
135	156.9	156	99.4	86.1	98.0
130	148.2	148	99.7	87.7	98.4
125	139.8	140	100.1	89.4	98.7
120	131.5	132	100.2	91.3	99.0
115	123.4	124	100.3	93.3	99.2
110	115.4	116	100.5	95.3	99.4
105	107.6	108	100.4	97.6	99.7
100	100	100	100	100	100
95	92.6	92	99.3	102.6	99.3
90	85.4	84	98.3	105.4	98.4
85	78.4	76	96.9	108.5	97.0
80	71.5	68	95.2	111.8	94.5
75	64.9	60	92.4	115.5	92.5
70	58.6	52	88.8	119.5	89.0
65	52.4	44	84.0	124.1	85.0
60	46.5	36	77.5	129.0	78.5
55	40.8	28	68.5	134.8	69.5
50	35.4	20	56.5	141.4	57.5
45	30.2	12	39.8	149.0	41.0
40	25.3	4	15.8	158.2	17.0
37.5	22.9	0	0.0	163.4	0.0

buckets liable to be cut out by the water, particularly if it carry sand or grit. Let c = a coefficient, H = the head acting on the wheel, H_n = the head corresponding to a runaway speed of n revolutions per minute. Then the power to be obtained at over and under speeds may be computed from the equation $W = c(H - H_n)$ if the runaway speed of the wheel at any head is known, and its most efficient speed, and performance at that speed, for any head is known, since the runaway speed varies as $\sqrt{H}$ and the power at best efficiency varies as $H^{3/2}$, and the best efficiency is practically constant.

By Runaway Speed is meant the speed at which the wheel would rotate under any given head, with no load; that is, its highest speed under that head. From this it follows that a wheel with a runaway speed, that is relatively high as compared with the best speed, will fall off in power rapidly as it is overspeeded, and vice versa. For low-speed turbines the runaway speed varies between 1.6 to 1.7 times the best speed and for high-speed turbines is about 1.5 times the best speed. Hence, high-speed turbines decrease less rapidly in power when overspeeded than do low-speed ones.

The table on p. 1139 shows the performance of a pair of McCormack wheels with K = 68.2 computed from tests. Other American wheels will give results varying slightly from these figures which may however be safely used in approximate calculations.

31. Manufacturer's Tables

Manufacturer's Tables. The following tables give the performances at 1-ft head of various turbines commonly met with. The values have been taken from the makers' catalogs, both old and new, and do not necessarily show the best performances or latest types developed. They are partly intended for reference in connection with old installations and wherever possible, the date of the catalog from which the tables were taken is shown, to indicate the age of the design.

All values have been computed for 1-ft head. At any head H:

To obtain horse-power of wheel multiply its tabular horse-power by $H^{3/2}$

To obtain discharge of wheel multiply its tabular discharge by $H^{1/2}$

To obtain speed of wheel multiply its tabular speed by $H^{1/2}$

Turbine Performances at One-foot Head

Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin

(From Tables Furnished by Maker in 1916)

	Type A K = 13.55 ϕ = 0.585			Type B K = 20.3 ϕ = 0.625			Type C K = 29.4 ϕ = 0.665			Type D K = 40.[illegible] ϕ' = 0.70		
Din.	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$
15	0.0358	0.394	71.7	0.0705	0.776	76.6	0.150	1.43	81.4	0.226	2.49	85.7
21	0.0705	0.776	51.2	0.138	1.525	54.7	0.225	2.48	58.2	0.442	4.86	61.3
27	0.116	1.276	39.8	0.229	2.52	42.5	0.423	4.65	45.2	0.731	8.04	47.6
34	0.184	2.024	31.6	0.363	3.99	33.8	0.668	7.35	35.9	1.158	12.74	37.8
42	0.280	3.08	25.6	0.551	6.06	27.4	1.016	11.18	29.1	1.765	19.4	30.6
50	0.398	4.38	21.5	0.79	8.69	23.0	1.450	15.95	24.4	2.50	27.5	25.[illegible]
60	0.573	6.30	17.9	1.13	12.43	19.1	2.08	22.88	20.4	3.61	39.7	21.[illegible]
70	0.785	8.64	15.4	1.53	16.83	16.4	2.82	31.00	17.5	4.90	53.9	18.[illegible]

Turbine Performances at One-foot Head. From Makers' Catalogs.

Allis-Chalmers Mfg. Co.—*Continued*

Dia.	Type E $K = 51.7$–60.5 $\phi = 0.75$			Type F $K = 72$ $\phi = 0.80$			Type G $K = 82$ $\phi = 0.77$			Type H $K = 92.5$ $\phi = 0.815$		
	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$
14	0.277	3.05	98.4	0.47	5.17	105.0	0.66	7.26	101.0	0.74	8.15	107.0
18	0.471	5.18	76.5	0.775	8.52	82.0	1.09	12.0	78.5	1.22	13.4	83.5
22	0.731	8.04	62.6	1.15	12.65	67.0	1.63	17.9	64.0	1.82	20.0	68.5
26	1.055	11.60	53.0	1.62	17.8	56.5	2.27	25.0	54.5	2.55	28.1	58.0
30	1.436	15.80	46.0	2.15	23.6	49.0	3.02	33.2	47.0	3.4	37.4	50.0
34	1.89	20.80	40.5	2.76	30.4	43.5	3.88	42.7	41.5	4.36	48.0	44.2
38	2.42	26.60	36.3	3.44	37.8	39.0	4.85	53.3	37.0	5.45	60.0	39.5
42½	3.09	34.0	32.4	4.32	47.5	34.5	6.06	66.6	33.3	6.8	74.8	36.3
47½	4.01	44.1	29.0	5.4	59.4	31.0	7.6	83.6	29.7	8.5	93.5	31.6
52½	4.95	54.5	26.3	6.6	72.5	28.0	9.28	102.0	27.0	10.6	116.5	28.3
57½	6.10	67.1	24.0	7.9	87.0	25.6	11.1	122.0	24.5	11.8	130.0	26.8
64	7.63	83.9	21.6	9.75	107.0	23.0	13.75	151.5	22.0	15.4	169.5	23.5
72	9.58	105.4	19.2	12.3	135.0	20.4	17.4	191.5	19.6	19.5	214.5	20.8
80	12.30	135.3	17.3	15.3	168.0	18.4	21.5	237.0	17.6	24.1	265.0	18.8
90	......	......	....	......	......	.....	27.2	299.0	15.7	30.5	336.0	16.7
100	......	......	....	......	......	.....	33.6	370.0	14.1	37.7	415.0	15.0

All values of the discharge Q_1 are calculated from Unit Horse-power HP_1, using an efficiency of 80 percent.

Jas. Leffel & Co., Springfield, Ohio
From 1916 Catalog

Type	Dia.	$H.P._2$	Q_2 c.f.s.	$R.P.M._2$
Standard Samson (Double buckets) $K = 73$	17″	0.616	6.71	92.8
	20	.808	8.8	81.4
	23	1.064	11.63	70.8
	26	1.368	14.87	62.6
	30	1.816	19.79	54.2
	35	2.464	26.83	46.4
	40	3.232	35.19	40.6
	45	4.088	44.54	36.2
	50	5.048	54.99	32.4
	56	6.328	68.97	29.
	62	7.76	84.55	26.2
	68	9.336	101.7	24.
Z (Single buckets) $K = 105$	12″	0.66	7.21	125
	15	1.042	11.29	100
	18	1.512	16.35	83.3
	24	2.763	29.45	62.5
	30	4.415	46.45	50
	36	6.36	66.85	41.6
	42	8.67	91.1	35.75
	48	11.31	118.7	31.25
	54	14.31	150.3	27.75
	60	17.67	186.0	25.0

Trump Mfg. Co., Springfield, Ohio
From 1914 Catalog

Type	Dia.	$H.P._2$	Q_2 c.f.s.	$R.P.M._2$
Trump Standard $K = 64$	17″	0.57	6.28	79.2
	23	1.21	13.31	58.6
	30	2.135	22.61	44.4
	40	3.646	40.1	33.6
	48	5.247	57.7	28.
	56	7.136	78.6	24.
	66	10.38	114.3	20.
Small Capacity For high Heads $K = 33$	10″	.0607	.669	137.8
	13	.1008	1.113	104.0
	15	.1314	1.446	90.4
	17	.1771	1.950	78.8
	20	.2276	2.505	68.9
	23	.3024	3.340	59.8

Turbine Performances at One-foot Head. From Makers' Catalogs.

Dayton Globe Iron Works, Dayton, O. From 1909 Catalog

Type	Dia.	H.P.$_2$	Q_2 c.f.s.	R.P.M.$_2$
New American $K = 54.1$	13″	0.264	2.96	99.
	19	.64	7.09	67.8
	25	1.048	11.63	51.4
	30	1.648	18.2	42.8
	36	2.488	27.5	35.8
	42	3.024	33.4	30.6
	48	4.08	46.1	26.8
	54	5.256	57.9	23.8
	60	6.68	73.7	22.0
Improved New American $K = 79$	16″	0.616	6.76	102.
	19	.808	8.82	87.
	22	1.064	11.63	75.
	25	1.36	14.87	66.8
	29	1.808	19.74	59.
	34	2.464	26.83	49.8
	39	3.232	35.21	43.6
	44	4.104	44.63	38.8
	49	5.072	55.21	34.8
	54	6.344	69.08	31.6
	60	7.784	84.66	29.
	66	9.344	101.8	26.2

Stillwell-Bierce Co., Platt Iron Works, Dayton, Ohio From 1904 Catalog

Type	Dia.	H.P.$_2$	Q_2 c.f.s.	R.P.M.$_2$
Victor A Obsolete $K = 63.5$	12″	0.296	3.26	117.4
	18	.666	7.34	78.6
	24	1.183	13.04	58.6
	30	1.849	20.39	47.
	36	2.662	29.36	39.
	42	3.624	39.97	33.6
	48	4.741	52.2	29.
	54	5.99	66.1	25.6
	60	7.39	81.6	23.
Victor Increased Capacity $K = 66.6$	12″	0.325	3.59	117.4
	18	.732	8.07	78.6
	24	1.292	14.35	58.6
	30	2.036	22.43	47.
	36	2.928	32.30	39.
	42	3.986	43.96	33.6
	48	5.206	57.42	29.
	54	6.59	72.68	25.6
	60	8.13	89.72	23.

Risdon-Alcott Turbine Co., Mt. Holly, N. J.

Type	Dia.	H.P.$_2$	Q_2 c.f.s.	R.P.M.$_2$
Alcott High Duty Special $K = 46.7$	13″	0.257	2.84	94.2
	18	.456	5.08	70.2
	24	.812	9.02	52.
	30	1.269	14.1	42.
	42	2.483	27.61	30.
Risdon Double Capacity $K = 43.8$	16″	0.225	2.34	78.4
	20	.385	4.07	66.
	25	.51	6.78	54.4
	30	.802	11.	47.2
	40	1.824	18.93	35.2
	50	2.829	31.2	26.8
	60	4.563	47.3	22.4
	72	6.566	72.4	18.8
Leviathan $K = 74.1$	18″	0.608	6.67	95.
	21	.824	9.08	81.4
	24	1.08	11.86	71.2
	27	1.368	15.01	63.4
	30	1.688	18.53	57.
	36	2.424	26.68	47.6
	42	3.304	36.32	40.8
	48	4.312	47.43	35.6
	54	5.456	60.03	31.6
	60	6.736	74.11	28.6
	66	8.152	89.67	26.
	72	9.696	106.72	23.8

S. Morgan Smith Co., York, Pa. From 1894 and 1910 Catalogs

Type	Dia.	H.P.$_2$	Q_2 c.f.s.	R.P.M.$_2$
McCormack (1894) $K = 51.4$	18″	0.56	6.17	64.4
	24	1.046	11.53	50.6
	30	1.595	17.59	41.6
	36	2.24	24.71	35.4
	42	3.233	35.67	30.
	48	3.885	42.85	24.
	57	5.843	64.45	22.
New Success (1894) $K = 55$	18″	0.543	5.99	70.8
	24	1.014	11.19	55.6
	30	1.547	17.06	45.6
	36	2.173	23.97	38.8
	42	3.136	34.6	33.
	48	3.768	41.57	28.6
	54	5.051	55.12	25.
	60	7.34	81.03	22.6
	72	11.18	123.37	18.6
Smith (1910) $K = 80.6$	18″	0.76	8.28	92.[illegible]
	24	1.353	14.73	69.[illegible]
	30	2.113	23.01	55.[illegible]
	36	3.047	33.19	46.[illegible]
	42	4.143	45.1	39.[illegible]
	48	5.362	58.43	34.[illegible]
	54	6.85	74.9	30.[illegible]
	60	8.45	92.1	27.[illegible]
	72	12.18	132.6	23.

Turbine Performances at One-foot Head

Victor Turbines, Platt Iron Works, Dayton, Ohio

Tables Submitted by Maker in 1918

Dia.	Type 1 $K = 84$ $\phi = 0.88$ Max. $H = 40$			Type 2 $K = 75$ $\phi = 0.84$ Max. $H = 60$			Type 3 $K = 60$ $\phi = 0.76$ Max. $H = 125$			Type 4 $K = 45$ $\phi = 0.70$ Max. $H = 200$		
	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$
15	.....	......	.....	.531	5.84	103.0	.414	4.56	93.3	.276	3.04	85.7
18	.....	......	.....	.762	8.38	85.9	.593	6.53	77.8	.396	4.36	71.5
24	.....	......	.....	1.36	14.95	64.3	1.060	11.65	58.3	.705	7.75	53.6
30	2.42	26.70	54.0	2.12	23.30	51.5	1.640	18.05	46.7	1.100	12.10	42.9
36	3.48	38.40	45.0	3.06	33.70	42.8	2.375	26.15	38.9	1.585	17.45	35.7
42	4.74	52.20	38.6	4.16	45.70	36.8	3.24	35.65	33.3	2.160	23.80	30.6
48	6.20	68.30	33.7	5.43	59.70	32.2	4.22	46.40	29.2	2.810	30.90	26.8
54	.....	......	.....	6.87	75.60	28.6	5.36	59.00	25.9	3.570	39.30	23.8
60	.....	......	.....	8.50	93.50	25.7	6.58	72.40	23.4	4.42	48.60	21.4
68	.....	......	.....	10.89	120.00	22.7	8.48	93.30	20.6	5.67	62.40	18.9
76	.....	......	.....	13.60	149.50	20.3	10.65	117.00	18.4	7.09	78.00	16.9

Dia.	Type 5 $K = 32$ $\phi = 0.66$ Max. $H = 400$			Type 6 $K = 20$ $\phi = 0.62$ Max. $H = 600$			Type 7 $K = 12$ $\phi = 0.59$ Max. $H = 800$		
	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$
15	.157	1.73	80.9	.0693	.836	76.0	.0274	.302	72.4
18	.226	2.49	67.4	.0996	1.10	63.3	.0396	.436	60.3
24	.402	4.43	50.5	.1770	1.95	47.5	.0704	.775	45.2
30	.625	6.87	40.5	.277	3.05	38.0	.1110	1.22	36.2
36	.901	9.91	33.7	.398	4.38	31.7	.1575	1.74	30.2
42	1.220	13.40	28.9	.544	5.99	27.1	.2160	2.38	25.8
48	1.700	18.70	25.3	.706	7.77	23.8	.2820	3.10	22.6
54	2.020	22.20	22.5	.897	9.87	21.1	.3560	3.92	20.1
60	2.505	27.60	20.2	1.105	12.20	19.0	.4400	4.84	18.1
68	3.230	35.50	17.8	1.415	15.60	16.8	.5630	6.20	16.0
76	4.050	44.50	15.9	1.770	19.50	15.0	.7150	7.86	14.2

Turbine Performances at One-foot Head

Camden Turbines, Hydraulic Turbine Corporation, Camden, N. J.

Table Submitted by Maker in 1918

Dia.	Type 4, $K = 53.2$, $\phi = .755$, Max. power 9% above tabulated H.P.			Type 8, $K = 71.3$, $\phi = .75$, Max. power 17% above tabulated H.P.			Type 11, $K = 83.5$, $\phi = .72$, Max. power 12% above tabulated H.P.		
	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$	$H.P._1$	Q_1 c.f.s.	$R.P.M._1$
15	.311	3.05	91.4	.51	5.10	92.0	.82	8.20	87.4
18	.448	4.39	76.2	.73	7.35	76.7	1.180	11.80	72.8
21	.610	5.97	65.2	1.00	9.73	65.7	1.607	16.07	62.4
24	.796	7.80	57.1	1.31	13.00	57.5	2.099	20.99	54.6
30	1.25	12.20	45.7	2.05	20.40	46.0	3.280	32.80	43.7
36	1.80	17.56	38.0	2.45	29.29	38.3	4.723	47.23	36.4
42	2.44	23.91	32.6	4.01	39.81	32.8	6.428	64.28	31.2
48	3.19	31.63	28.5	5.24	52.03	28.7	8.396	83.96	27.3
54	4.03	39.52	25.3	6.64	65.63	25.5	10.627	106.27	24.2
60	4.98	48.80	22.8	8.20	81.60	23.0	13.120	131.20	21.8
66	6.03	58.04	20.7	9.92	98.50	20.9	15.875	158.75	19.8
72	7.17	70.27	19.0	11.80	117.17	19.1	18.892	188.92	18.2
75	7.78	76.25	18.2	12.81	127.20	18.4	20.400	204.00	17.4

Turbine Performances at One-foot Head

S. Morgan Smith Company, York, Pa.

(From Data Submitted by Maker in 1918)

The following table shows the performance of the various types of S. Morgan Smith Company turbines of 30 in diameter at 1-ft head, and the maximum heads under which they are designed to operate. Except for special conditions, sizes are made at intervals of 3 in, as 27 in, 30 in, 33 in, etc. Also for special conditions turbines are designed with characteristics differing from those tabled, but the characteristics must be within certain limits of power and speed, as indicated in the table, according to the head under which the turbines are to be placed.

Type	$H.P._2$	Q_2c.f.s.	$R.P.M._2$	Max. head	K
E	0.375	3.89	41	750′	25.1
F	0.600	6.22	43.6	300	33.8
G	1.08	11.2	44.3	150	46.1
H	1.2	12.45	47.5	120	52.0
K	1.6	16.60	47.5	90	60.0
N	2.25	23.35	47.5	65	71.3
O	2.45	25.40	49.0	55	76.6
R	3.08	32.00	49.0	40	86.0
S	3.65	37.90	50.0	30	95.5

Turbine Performances at One-foot Head

J. and W. Jolly, Holyoke, Mass.

From Catalog issued about 1910

Type	Dia.	H.P.₁	Q₁c.f.s.	R.P.M.₁
McCormack	12″	0.240	2.65	99.6
Holyoke	18	0.559	6.17	64.3
$K = 54$	24	1.045	11.53	50.6
	30	1.595	17.59	41.7
	36	2.241	24.71	35.5
	42	3.234	35.67	30.0
	48	3.885	42.85	24.5
	54	4.959	54.69	22.8
	60	6.122	67.52	20.6
	66	7.407	81.70	18.7

On p. 1146 is shown a more complete table of the Standard Samson wheel by which computations from the foregoing tables may be approximately checked.

Turbine Windage and Friction. The approximate power absorbed by windage and friction in modern turbines in vertical settings may be computed by the following formula:

$$\text{Windage and friction in horse-powers} = 0.000154\, B\, D^4\, n^3.$$

Where B = height of gate or guide vane opening in feet

D = diameter of wheel at middle of guide vane opening, in feet

n = revolutions per minute

32. Tangential or Impulse Turbines

The Girard Turbine may be either radial inward or outward flow, or axial flow. The buckets are so shaped that on leaving the wheel the water has a relative velocity in the direction opposite to that of motion, it being received at inlet from a series of nozzles pointed as nearly as may be in the direction of motion. Air is admitted to the wheel to prevent spraying of the jets, and the turbine cannot run submerged. Regulation of speed and power is accomplished by closing part of the inlet nozzles. This motor is very satisfactory for high heads, but its speed cannot be varied appreciably without loss of efficiency unless the head also changes. The efficiencies range from 70 to 80 percent.

The Pelton Turbine is usually mounted on a horizontal shaft and consists of a series of cups which successively receive a jet from one or more nozzles. The jet on striking the cup is divided and turned thru nearly 180° and is then discharged with a low absolute velocity. Regulation is accomplished by closing or deflecting the nozzle. The maximum efficiency in this type of turbine occurs when the velocity of the cup V is about one-half that of the jet and when $\theta = 180°$, in accordance with the law of curved vanes, and varies from 70 to 85 percent.

Wheels of the Pelton type are frequently equipped with the Doble nozzle which is closed by a pin or needle moving longitudinally in the axis of the jet from the inside and which may be adjusted to give a symmetrical jet of any effective area from that due to the full nozzle opening to zero. This apparatus affords opportunity for excellent control of the discharge, and hence greatly facilitates regulation without causing a waste of either water or energy.

Performances of Samson Wheels. From 1908 Catalogs.

Size in ins		Heads in feet												
		4	6	8	10	12	14	16	20	25	30	35	40	50
17	Power	4.9	9.1	13.9	19.5	25.5	32.3	39.5	55.0	77.0	101	128	156	218
	Water	13.4	16.4	19.0	21.3	23.2	25.1	26.8	30.0	33.6	36.8	39.7	42.4	47.4
	Speed	186	228	264	294	322	348	372	416	464	510	550	588	657
20	Power	6.4	11.9	18.3	25.5	33.6	42.3	51.5	72.2	101	133	167	204	285
	Water	17.6	21.6	24.9	27.8	30.5	32.9	35.2	39.3	44.0	48.2	52.1	55.6	62.2
	Speed	162	199	230	257	282	304	325	364	407	445	481	514	575
23	Power	8.5	15.7	24.2	33.8	44.4	55.9	68.3	95.5	133	175	221	270	377
	Water	23.2	28.5	32.9	36.8	40.3	43.5	46.5	52.0	58.1	63.7	68.8	73.6	82.3
	Speed	141	173	200	224	245	265	283	316	354	387	418	447	500
26	Power	10.9	20.1	30.9	43.2	56.7	71.5	87.3	121	171	224	283	345	482
	Water	29.7	36.4	42.1	47.0	51.5	55.6	59.5	65.3	74.3	81.4	88.0	94.0	105
	Speed	125	153	177	198	217	234	250	280	313	343	370	396	442
30	Power	14.5	26.7	41.1	57.5	75.5	95.2	116	162	227	299	376	460	642
	Water	39.6	48.5	56.0	62.6	68.6	74.1	79.2	88.5	99.0	108	117	125	140
	Speed	108	132	153	171	188	203	217	242	271	297	321	343	381
35	Power	19.7	36.2	55.7	77.9	102	129	158	220	308	405	510	623	....
	Water	53.7	65.1	75.9	84.9	93.0	100	107	120	134	147	159	170	....
	Speed	93	114	132	147	161	174	186	208	232	255	275	294	....
40	Power	25.8	47.5	73.1	102	134	169	207	289	404	531	668	817	....
	Water	70.4	86.2	99.5	111	122	131	141	157	176	193	208	223	....
	Speed	81	100	115	129	141	152	163	182	203	223	240	257	....
45	Power	32.7	60.1	92.5	129	170	214	262	366	511	672	847	1034	....
	Water	89.0	109.1	126	141	154	167	178	199	223	244	263	282	....
	Speed	72	88	102	114	125	135	145	162	181	198	214	229	....
50	Power	40.5	74.2	114	160	210	264	324	451	631	829	1045	....	....
	Water	110	134.7	156	174	190	206	220	245	275	301	325	....	....
	Speed	65	80	92	103	113	122	130	145	162	178	192	....	....
56	Power	50.6	93	143	200	263	332	405	566	791	1040	1314	....	....
	Water	138	169	195	218	239	258	276	308	345	378	408	....	....
	Speed	58	71	82	92	101	109	116	130	145	159	172	....	....
62	Power	62.1	114	176	245	323	407	497	694	970	1275	1634	....	....
	Water	169	207	239	267	293	316	338	378	423	463	502	....	....
	Speed	52	64	74	83	91	98	105	117	131	144	155	....	....
68	Power	74.7	137	211	295	388	489	597	835	1167	1494	....	....	....
	Water	203	249	288	322	352	381	407	455	509	557	....	....	....
	Speed	48	59	68	76	83	89	96	107	120	130	....	....	....
74	Power	88.5	162	250	350	460	579	708	992	1382	1805	....	....	....
	Water	242.1	795	341	381	417	451	482	539	602	659	....	....	....
	Speed	44	54	62	70	76	82	88	99	110	120	....	....	....

For each size the first line gives the horse-power, the second the discharge in cu ft per sec, and the third the speed in revolutions per minute.

Water Wheels. This term is applied to the overshot, the breast and the undershot wheels, all of which have become practically obsolete in America. The efficiencies of overshot and breast wheels ranged from 60 to 88 percent. In these wheels the work was done principally by the weight of the water. The efficiencies of straight-bladed undershots were from 25 to 35 percent, and for the curved-bladed or Poncelet wheels from 50 to 65 percent. In these wheels the work was done by the impulse of the stream.

Spiral Wheels are similar to axial-flow turbines except that the runne

blades are helical surfaces and they are usually mounted on a horizontal or slightly inclined shaft. For low heads and large quantities of water this form is very satisfactory, efficiencies as high as from 80 to 85 percent being reported.

PUMPS AND PUMPING

33. Centrifugal Pumps

The Work in foot-pounds done by a pump is the product of the weight in pounds of the liquid pumped and the height in feet thru or against which it is lifted. The POWER of a pump is the work done in unit time, and the horsepower is the work per second in foot-pounds divided by 550, or the work per minute divided by 33 000.

The Centrifugal Pump is essentially an inward-flow turbine reversed, the power being applied to rotate the runner and the water being admitted at the center or eye and discharged at the outer periphery. The earlier centrifugal pumps consisted of a spider made up of three or more arms, either straight or curved, which rotated in a concentric chamber to which water was admitted at the center and discharged at some point in the circumference. The efficiency of this apparatus rarely exceeded 30 percent, and it was only used for low lifts. By shifting the enclosing chamber so that the area outside the runner increases toward the outlet in proportion to the quantity of water that must pass the several sections, and by encasing the blades in a revolving shell, thus reducing the friction of the revolving water on the fixt shell, and by more rational proportioning of the blades and casing, the pump has been greatly improved until its efficiency approaches 80 percent, and by passing the water successively thru a series of runners any desired lift may be accomplished. As the apparatus contains no valves or parts, it is particularly adapted to the handling of water containing sand or gravel, and of viscous liquids and sewage; and as its discharge is continuous it has an advantage over reciprocating pumps in freedom from water hammer in the suction and discharge pipes.

In operation it is necessary that the pump, when set above the level of its supply, be primed before it will start, and in such cases a foot valve is advantageous. It does not begin to deliver water until a certain speed of revolution is attained depending upon the height of the lift. When delivery has commenced the speed may be lowered somewhat below this point before it will cease on account of the inertia of the flow when once established, but the conditions of flow are unstable in this region.

As the water passes thru the pump, the impeller impresses upon it a head h_c due to centrifugal force; this is equal to the statical head that the pump would maintain when running at its normal speed, but at which it would deliver no water were it not that the rotation of the water in the eye of the wheel and in the outer casing has the effect of increasing h_c from 1 to 12 percent. This increase is greater with vanes having radial tips than with those which are turned back, and is raised when the wheel is surrounded by a large vortex chamber.

If the head be dropt slightly below the true value of h_c, the discharge will commence, and will continue so long as it remains below this amount. The discharge cannot, however, be reduced below that corresponding to the head h_c after flow has started without danger of its stopping altogether, so the statement is generally made that a centrifugal pump will begin to deliver against a certain head at a certain speed and cannot deliver less than a certain quantity.

Lift for Single-stage Centrifugals. A single-stage centrifugal pump has been made to lift against a head of 936 ft and such pumps with a lift as high as 350 ft are in service, but it is usually considered that 100 ft is the

practical limit without a second stage. For higher lifts a series of pumps is used on the same shaft, delivering water from one pump to the next, by which any desired head may be overcome.

The following table gives the approximate average performance of a centrifugal pump so designed that the discharge ceases when the head becomes 50 percent greater than that for which the pump is speeded.

Average Performance of a Centrifugal Pump

At constant speed					At constant head
Percent of designed head	Percent of designed discharge	Percent of designed brake horse-power or input	Percent of designed water horse-power or output	Percent of best efficiency	Percent of best speed
150	....	....	0	0	81.6
140	31.5	91	44	48.2	84.5
130	55	94.5	71.5	75.6	87.6
120	74	97	89	91.6	91.3
110	88	99	97	98	95.4
100	100	100	100	100	100
90	109	101	98.5	97.4	105.2
80	116	101.5	92.6	91.4	112
70	123	102	86.1	85	119.5
60	129	102.5	77.5	75.6	129
50	134.5	103	67.5	65.5	141
40	140	103.3	56.	54.1	158
30	144	103.4	43.2	41.6	182
20	148	103.5	29.6	28.6	222
10	151	103.5	15.1	14.1	316
0	....	....	0	0	....

The Suction Lift of a pump depends on the pressure at which the entrained air in the water separates. This separation occurs at lower pressures in rapidly moving than in slowly moving water. The lift also depends upon the velocity in the suction pipe, being less for high and greater for low velocities. The head at which the air separates is usually between 24 and 28 feet below atmospheric pressure, and 26 feet may be taken as an average safe value.

Power and Efficiency. Let Q = volume of water lifted per second, w = weight of a cubic unit of water, g = acceleration of gravity, v_0 = velocity of outer circumference of pump wheel, β_0 = angle which vane makes with outer circumference, u_0 = radial component of velocity at outer circumference, H = total head = height water is raised plus all frictional heads. Then

$$\text{Power required to drive pump} = \frac{Qw}{g} v_0 (v_0 - u_0 \cos \beta_0)$$

$$\text{Hydraulic efficiency of pump} = gH / v_0 (v_0 - u_0 \cos \beta_0)$$

The speed v_0 required to pump against a given head must increase as β_0 decreases, and it becomes a minimum when $\beta_0 = 90°$. In this case the vanes are radial at the outer circumference, and for perfect efficiency $v_0^2 = 2\,gH$. As the efficiency decreases v_0 must increase so that $v_0^2 = k \cdot 2\,gH$, where k is a coefficient whose value ranges from 1.2 to 1.8.

Performances at Best Speeds of Allis-Chalmers Co.'s Standard Centrifugal Pumps

Size in ins		Lifts in feet										
		5	10	15	20	25	30	40	50	60	80	100
3	Power	0.25	0.75	1.00	1.5	2.25	3	4.5	6	8	12	17
	Water	76	107	131	152	170	185	214	240	262	303	339
	Speed	447	632	774	893	998	1095	1 265	1415	1550	1790	2000
5	Power	.60	1.75	2.75	4.5	6	8	12	17	24	33	46
	Water	205	290	356	411	460	504	582	650	712	823	920
	Speed	268	380	465	536	600	658	760	850	930	1075	1200
6	Power	1.00	3	5	8	11	14	21.5	30	40	61	....
	Water	415	586	718	830	926	1015	1 170	1310	1435	1660	....
	Speed	190	270	330	381	426	467	539	602	660	762	....
8	Power	1.75	4.5	8	12	17	22	34	47	62	96	....
	Water	650	918	1125	1300	1450	1590	1 835	2050	2250	2600	....
	Speed	161	227	278	322	359	394	455	508	556	643	....
10	Power	2	5.5	9.5	15	21	27	42	59	77	....	....
	Water	850	1200	1400	1700	1900	2080	2 400	2680	2940	....	....
	Speed	143	202	247	285	318	350	403	450	494	....	....
12	Power	3	7.5	13.5	21	28.5	38	58	80	106	....	....
	Water	1165	1645	2030	2330	2600	2850	3 300	3680	4030	....	....
	Speed	128	178	222	256	286	314	363	405	444	....	....
14	Power	4	9.5	17.5	27	37.5	50	76	106	140	....	....
	Water	1595	2250	2760	3190	3560	3910	4 510	5040	5520	....	....
	Speed	122	173	212	245	274	300	346	387	424	....	....
16	Power	4.5	12	22	34	47	62	95	132	....	....	....
	Water	2085	2950	3610	4170	4660	5110	5 900	6600	....	....	....
	Speed	117	165	202	233	261	286	330	368	....	....	....
18	Power	6	16	29	44	62	81	125	....	....	....	....
	Water	2830	4070	4900	5660	6330	6925	8 000	....	....	....	....
	Speed	112	158	193	223	250	274	316	....	....	....	....
18 Special	Power	7.5	20.5	37	57	80	105	162	....	....	....	....
	Water	3675	5200	6360	7350	8220	8980	10 380	....	....	....	....
	Speed	112	158	193.	223	250	274	316	....	....	....	....

Power = brake horse-power to drive pump shaft. Water = gallons per minute delivered. Speed = revolutions per minute.

34. Reciprocating Piston Pumps

Types. Reciprocating piston pumps may be classed as to operation as either single or double acting. The former takes water on one stroke and discharges it on the other, while the double-acting pump takes and discharges during both strokes. As to construction they are divided into inside-packed and outside-packed pumps, an example of the latter being shown in Fig. 28.

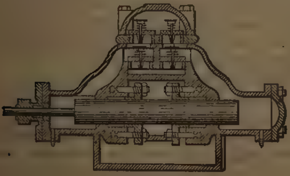

Fig. 28

A continuous discharge from a single-acting pump may be obtained by either of the devices shown in Fig. 29, in which when suction takes place on the upstroke the chamber below the piston is filled with water and at the same time the

volume of water in the discharge chamber above the piston is delivered. The volume of the discharge chamber being less than that of the suction chamber, as the piston descends part of the water in the suction chamber is delivered thru the discharge chamber and part of it remains there to be delivered on the suction stroke.

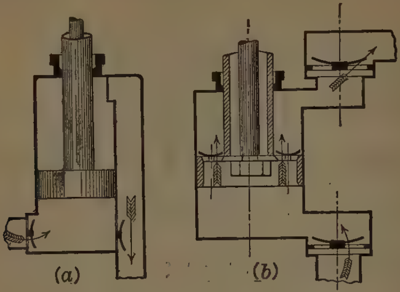

Fig. 29

Displacement Curves. If the piston velocities as ordinates be plotted to time, or to space past thru by a crank moving at uniform velocity, as abscissas, the area enclosed between the curve, the axis of abscissas, and a vertical ordinate will represent the displacement of the piston at any time. For uniform effort and uniform flow this displacement should be uniform, but this can only be approximated in a reciprocating pump.

Long Suctions. If the suction of a reciprocating pump be long, even tho the lift be low, on account of the variable velocity, a considerable portion of the suction head is absorbed in accelerating the water at the beginning of the stroke. A part of this force is recovered at the end of the stroke as pressure forcing the piston ahead, but unless the head h_j be available at the beginning of the stroke the pump will not lift. If the speed of the piston is such that it moves more rapidly than the inertia of the water can be overcome, the water separates from the piston and may ultimately overtake it, as the latter is retarded in the second half of its stroke, causing severe shock or hammer

The head at inlet of the suction for a maximum is equal to the depth of the suction opening below the water surface in the suction well plus the height of the water barometer, and the head at outlet is that corresponding to a perfect vacuum less the effective lift of the pump. When, as is usually the case, the pump is located above the level of water in the suction well the head available for producing flow is reduced.

Separation. The suction lift of a pump is dependent upon the pressure at which the entrained air in the water separates. The head at which the air separates is usually taken at about 26 ft below atmospheric pressure. Separation also may occur on the discharge side of the piston during the second half of the stroke when the piston is being retarded.

Air Chambers. To prevent separation an air chamber may be installed on either the suction or the discharge pipe and should be located as near as possible to the pump. The value of the air chamber increases with its size, cross-section being more effective than height. It should be so located that the water is drawn as low as possible without actually emptying the chamber at each stroke.

Flywheel pumps, kind	Ratio of change of air volume in air chamber to volume of	
	1 stroke of piston	1 revolution of pump
Single-acting	0.557	0.557
Double-acting	0.211	0.106
2 double-acting 90 degrees apart	0.048	0.012
Three-throw single-acting 120 degrees apart	0.0032	0.0011
Three-throw double-acting 120 degrees apart	0.0032	0.0011

The duplex pump without a flywheel, if the piston were accelerated uniformly, would give a uniform delivery; in practise, however, the variation is about the same as that of a three-throw flywheel pump.

35. Pumping Engines

Types. Pumping engines are a combination of a pump with a steam engine in a single machine, and the term is usually restricted to machines of at least one million gallons daily capacity. There are four general types:

(a) The Common Duplex Pumping Engine, consisting of two parallel double-acting pumps directly connected to two steam engines, and taking strokes alternately or 90 degrees apart, each engine operating the valves of the other. In this type the length of stroke is variable, and the steam must follow thruout the full stroke, and can be used expansively only by adding a low-pressure cylinder into which it expands continuously during the discharge stroke of the high-pressure cylinder.

(b) The Duplex Pumping Engine with the Worthington High-Duty Attachment, which is a duplex pump having attached to each piston rod a pair of auxiliary cylinders filled with oil and mounted on trunnions thru which they are connected to the delivery main. These cylinders rotate in such a manner that at the beginning of the stroke they oppose the movement of the piston and thus prevent racing while the steam follows at full pressure. At midstroke just after cut-off they neutralize each other and toward the end of the stroke assist the movement of the piston when the steam is working expansively. This device enables the steam to be expanded in one, two, or three cylinders, and thereby increases the economy of operation.

(c) The D'Auria Pumping Engine, in which the use of steam expansively is accomplished by circulating thru the frame of the pump a body of water which is acted upon by an auxiliary piston and accelerated during the first half of the stroke, and then acts upon this piston during its retardation in the second half of the stroke.

(d) The Crank and Flywheel Pumping Engine, in which the expansive use of steam is provided for by the inertia of the flywheel.

Performance. The performance of pumping engines is usually stated as DUTY, which was originally defined as the number of foot-pounds of work delivered for each 100 pounds of coal burned under the boilers. Owing to the variation in evaporative power of coal, and also of boiler efficiency, the coal basis for computing duty is more or less unsatisfactory for accurate work, and a new basis was adopted using an assumed evaporation of 10 pounds of water per pound of coal, so that duty is also defined as the number of foot-pounds of work delivered for each 1000 pounds of dry steam, this being determined by the amount of water evaporated from and at 212 degrees Fahrenheit with corrections for moisture in the steam delivered to the engine. On account of the varying dryness of the steam and the condensation between boiler and engine it has been suggested that duty be based upon each 1 000 000 British thermal units in the steam delivered to the engine. This would correspond to the preceding if the steam were supplied to the engine at an abso-

lute pressure of about 5 pounds, corresponding to a temperature of about 162 degrees Fahrenheit. It follows that the duty on the last-named basis will in practise be less than that on the steam basis and usually less than that on the coal basis.

The duty of pumping engines and steam pumps ranges from about three to eight million foot-pounds in the single-cylinder boiler-feed pumps to 160 million foot-pounds in the best high-duty, direct-acting and flywheel pumping engines, and in turbo-driven centrifugals, all on the 1000 pounds of dry steam basis. The coal consumed per horse-power per hour on a basis of 10 pounds of water evaporated per pound of coal may be obtained by dividing 198 by the duty in million foot-pounds.

Duty and Coal Consumption of Steam Pumping Machinery

Kind of pumping machinery	Duty in 1 000 000 ft-lb per 1000 lb of dry steam	Pounds of coal per horse-power per hour
Single-cylinder boiler-feed pumps and air pumps	3 to 5	66 to 40
Steam fire engines:		
Silsby rotary	6.5 to 8	30 to 25
Amoskeag	6 to 9	33 to 22
Clapp and Jones	7.5 to 14	27 to 14
High-pressure duplex pumps	15 to 20	13 to 10
Duplex compound non-condensing pumps	25 to 40	8 to 5
Duplex compound condensing engines	30 to 45	6.6 to 4.4
Duplex triple condensing engines	40 to 60	5 to 3.3
Cornish flywheel engines	60 to 70	3.3 to 2.8
Walking-beam compound condensing engines	60 to 85	3.3 to 2.3
Steam turbines and centrifugal pumps	80 to 90	2.5 to 2.2
Worthington high-duty duplex engines	100 to 160	2.0 to 1.25
Compound condensing flywheel engines	100 to 130	2.0 to 1.5
Triple-expansion flywheel engines	110 to 160	1.8 to 1.25
Steam-turbine driven centrifugals (large)	100 to 160	2.0 to 1.25

Direct-acting Water Motor. When the steam end of a duplex pump is replaced by a similar end with large valve openings, which may be connected to a supply of water under pressure, the device is called a direct-acting water motor or pump. On account of the high frictional losses the device is not so efficient as a turbine connected to a centrifugal or a reciprocating pump. As the apparatus requires little attention, it may be advantageously used as a continuous relay at the end of a long line of pressure pipe to increase the pressure in a following line where a smaller amount of water is needed.

36. Jet Pump and Hydraulic Ram

The Jet Pump. In this apparatus (Fig. 30) a small jet of water at high velocity is discharged thru a throat the upstream end of which connects to a suction pipe and the downstream end to the discharge. The velocity of the jet reduces its pressure below that of the atmosphere and thus creates a partial vacuum at the throat into which water from the pump well rises and is then mingled with the jet and carried along at reducing velocity by it. The efficiency of this apparatus is greatest with a high suction lift

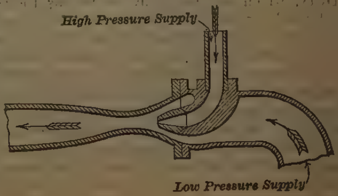

Fig. 30

and low pressure on the discharge, when it may reach 25 or 30 percent. When a steam jet replaces the water jet, the ordinary steam injector is obtained in which the vacuum is formed not only by the velocity of the jet but by the condensation of the steam.

The water-jet pump may be advantageously used to produce a large discharge thru a nozzle at a low pressure for a fountain, or for fire fighting, by means of the apparatus shown in Fig. 31, which is sometimes called an injector nozzle. (Proc. Inst. Mech. Eng. of Great Britain, 1879.)

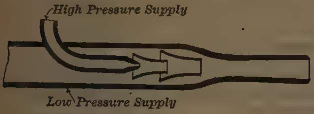

Fig. 31

The following table gives the quantity of water at a head of 700 pounds per square inch required to deliver thru a one-inch nozzle a jet of 150 gallons per minute, the head at the nozzle being 100 feet.

Low-pressure supply. Pressure, lb per sq in.	60	50	40	30	20	10
High-pressure supply at 700 lb per sq in. Gallons per minute.	3.7	10.9	18.1	25.2	32.4	39.6

The Hydraulic Ram (Fig. 32) consists of a drive pipe delivering into a chamber whence the water escapes to waste until the waste valve is closed by

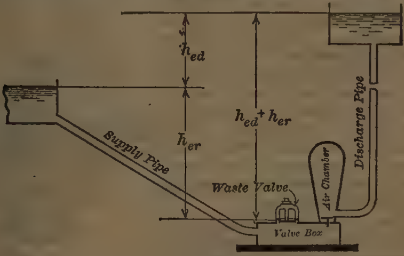

Fig. 32

the discharging water, and then into the air chamber, in which it compresses the air until the flow stops, when the air expands, closing the valve at the chamber inlet and forcing water up the delivery pipe. When the flow is stopt there is a rebound of pressure toward the inlet. This creates a reduction of pressure and causes the waste valve to drop open and flow is started again. The apparatus thus intermittently utilizes the momentum in the drive pipe, and the longer this pipe the more work the machine will do.

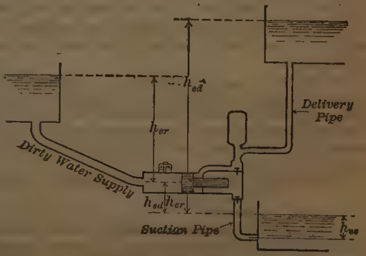

Fig. 33

If h_{er} is the height of the supply reservoir above the

ram, the efficiency is $e = Q_d(h_{ed} + h_{er})/Q_s h_{er}$ when a portion of the water used to drive is pumped, the inlet of the drive pipe being taken as the suction inlet. When a separate supply is pumped, as may be done with the machine shown in Fig. 33, $e = Q_d h_{ed}/Q_s h_{er}$. The efficiency decreases as the ratio of h_{ed}/h_{er} increases, and becomes very small for high lifts. The value of e in the above expressions may be as high as 70 percent. Rankine gives $e = 1.12 - 0.2\sqrt{(h_{de} + h_{er})/h_{er}}$.

37. Pulsometer and Air Lift

The Pulsometer (Fig. 34) has a chamber with two parts, each connected to a suction pipe at the bottom and to a steam pipe at the top with a discharge at the side near the bottom. Steam enters and fills one chamber and then condenses when the steam inlet valve shifts, allowing steam to enter the other chamber. As the steam condenses, a vacuum is formed which lifts water into the first chamber. When the second chamber is filled with steam, the valve shifts again and steam enters the first chamber, forcing out the water thru the discharge valves and then condensing again, making the process continuous. Tests of a pulsometer reported by DeVolson Wood, Trans. Am. Soc. Mech. Engs., 1892, vol. 13, p. 211, gave a duty ranging from 9 to 13 million foot-pounds. The apparatus is practically adapted to pumping out cofferdams, on account of the ease with which it can be set up, as it may be simply suspended from a ring at the top and connected with steam, suction and discharge hose lines.

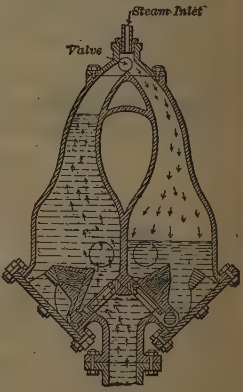

Fig. 34

The Air Lift. This device involves the discharge at the bottom of the well, or at least a considerable distance below the water surface, of air into the mouth of the delivery tube. The air as small bubbles mixes with the water, and the specific gravity of the mixture is so reduced that the pressure of the water outside the delivery tube causes the mixture to overflow at the top. Evidently the greater the length of the pipe below the surface the greater the difference between the weight of the columns within and without the tube and the higher the water can be lifted. Generally the depth of submersion is made 1.5 to 2 times the lift.

The air lift is especially adapted to raising liquids from great depths, and is more efficient in handling hot liquids on account of the greater expansion of the air. It is particularly easy to install when the well has sufficient depth below the water surface. The air at entrance should be distributed over the area of the pipe rather than concentrated at the center, and small bubbles give better efficiencies than those which fill the entire delivery tube.

Experimentally the following relations are found to give the best results:

[illegible] feet	10	20	30	50	100
Q_a/Q_w	1.0	1.5	2.0	2.5	3.0

Of the indicated horse-power of the air chamber of the compressor the lift under favorable conditions may return as much as 45 percent, but ordinarily not more than 25 and frequently only 10 or 12 percent. The duty of air compressors ranges usually from 6

to 40 million foot-pounds, and the duty of the whole plant may therefore range from less than one million to perhaps 20 million foot-pounds as an upper limit. (See Proc. Inst. Civ. Engs. of Gt. Britain, 1900, vol. 140, p. 323, and 1906, vol. 163, p. 353.)

During the operation of the air lift the following equations apply:

a_p = area of eduction pipe in sq ft above air inlet.
c_e = coefficient of entrance = 0.97. (See Art. 7 of this Section.)
c_p = coefficient due to pipe friction and average slip varying from 0.70 to 0.99.
d = diameter of eduction pipe in inches.
h_l = lift in feet.
h_s = submergence of air discharge in feet.
p_b = barometric pressure on water in well and at upper end of discharge pipe in ft of water.
p_i = absolute pressure at inlet in foot piece in ft of water.
q_b = discharge of air at pressure p_b in cu ft per sec.
q_w = discharge of liquid in cu ft per sec.
u_w = density of liquid pumped.
v_i = velocity of liquid in eduction pipe below air inlet.
W = work output in ft gals per sec.

$$h_s - \frac{p_i - p_b}{u_w} = \frac{v_i^2}{2g}(1 + c_e) \quad (1)$$

$$\frac{q_b\, p_b}{q_w\, u_w} \log_e \frac{p_i}{p_b} = h_l + \left[\frac{\left(1 + c_p\frac{W}{d}\right)(q_b + q_w)^2 + c_e q_w^2}{2\, g\, a_p^2}\right] \quad (2)$$

$$\frac{p_b}{q_w\, u_w} \log_e \frac{p_i}{p_b} = \frac{1 + c_p\frac{W}{d}}{2\, g\, a_p^2}(q_b + q_w) \quad (3)$$

$$\left(1 + c_p\frac{W}{d}\right)(q_b^2 - q_w^2) = 2\, g h_l\, a_p^2 + c_e q_w^2 \quad (4)$$

Example. To determine the air required to pump water at a rate of 2 cu ft per sec from a well 10 in in diameter and 330 ft deep, in which the air pipe inlet is submerged 64.33 ft, and the lift is 25.67 ft, the air pipe being 2½ in diameter and passing thru the 10-in pipe. From which:

a_p = area of 10-in pipe − area of outside of 2½-in pipe = 0.5 sq ft.
c_e = 0.97.
c_p = 0.86.
d = 10 in − 3 in = 7 in = net diameter of 10-in pipe enclosing 2½-in pipe.
h_l = 25.67 ft.
h_s = 64.33 ft.
p_b = 14.7 lb = 33.96 ft.
p_i = 64.33 + 33.96 = 98.29 ft = pressure of water over air outlet + atmospheric pressure.
q_w = 2.0 cu ft per sec.
q_b = air required in c.f.s. atmospheric pressure.
u_w = density of mixture of air and water $= \dfrac{62.5q_w + 0.08q_b}{62.5\,(q_w + q_b)}$.
v_i $= \dfrac{2}{0.545} = 3.67$ = discharge in cu ft ÷ area of 10-in discharge pipe.
W = 2 × 7.48 × 25.67 = 384.8 ft gal = product of gal pumped and lift.

Applying equation (4):

$$\left(1 + c_p\frac{W}{d}\right)(q_b^2 - q_w^2) = 2\,gh_l\,a_p^2 + C_e q_w^2$$

and substituting therein:

$$\left(1 + 0.80\,\frac{384.8}{7}\right)(q_b^2 - 2^2) = 64.4 \times 25.67 \times 0.5^2 + 0.97 \times 2^2$$

or $$44.97\,q_b^2 = 179.88 + 413.2 + 3.88$$

and $$q_b^2 = \frac{596.6}{44.97} = 13.25 \quad \therefore \quad q_b = \sqrt{13.25} = 3.64 \text{ c.f.s.}$$

If the submergence were not given its proper amount could be computed from equation (3) by determining the value of p_l, the only unknown therein, which can then be introduced in equation (1) to obtain h_s, as follows:

Equation (3) $$\frac{p_b}{q_w\,u_w}\log_e\frac{p_l}{p_b} = \frac{\left(1 + c_p\frac{W}{d}\right)(q_b + q_w)}{2\,g\,a_p^2}$$

Substituting values:

$$\frac{33.96}{2 \times \left(\dfrac{62.4 \times 2 + 0.08 \times 3.64}{62.4 \times 5.64}\right)}\log_e\frac{p_l}{33.96} = \frac{44.97\,(3.64 + 2)}{64.4 \times 0.5^2}$$

or $$\frac{33.96}{0.706}\log_e\frac{p_l}{33.96} = \frac{254.5}{16.1}$$

$$\therefore \log_e\frac{p_l}{33.96} = \frac{0.706}{33.96} \times \frac{254.5}{16.1} = \frac{180.5}{546.76} = 0.33$$

0.33 being the hyperbolic or natural logarithm of 1.38

$$\frac{p_l}{33.96} = 1.38, \quad \text{and} \quad p_l = 1.38 \times 33.96 = 46.86 \text{ ft.}$$

By equation (1) $$h_s - \frac{p_l - p_b}{u_w} = \frac{v_l^2}{2\,g}(1 + c_e),$$

and substituting: $$h_s - \frac{46.86 - 33.96}{0.353} = \frac{(3.67)^2}{64.4}(1 + 0.97)$$

or $$h_s = \frac{12.90}{0.353} + \frac{13.47 \times 1.97}{64.4} = 36.54 + 0.49 = 37.03 \text{ ft.}$$

This submergence being that giving the highest efficiency is about 60 percent of the total length of eduction, or sum of submergence and lift, and indicates the submergence used in the apparatus as specified was too great for best results.

Davis and Weidner (see Bull. 450 Univ. of Wisconsin) conclude:

1. The central air tube pump has the greatest theoretical capacity for a given size of well.

2. The coefficient of pipe friction and slip decreases as the discharge increases, and decreases as the ratio of volume of air to volume of water increases.

3. The coefficient of pipe friction and slip varies with the length of pump, but seems to be independent of the percentage of submergence and of the lift.

4. The length of pump, the percentage of submergence, and therefore, the lift remaining constant, there is a definite quantity of air causing the maximum discharge. This quantity of air for maximum discharge, as also the ratio of volume of air to volume of water, differs for different percentages of submergence and lift, the length of the pump remaining constant.

5. The length of pump remaining constant, the maximum output (e.g., foot-gallons) occurs at about the same percentage of submergence for all rates of air consumption, being at from 61 to 65 percent for the pump used in the Wisconsin experiments. At other submergences the output varies as the ordinates of a parabola having a vertical axis. Under these conditions the lift does not remain constant as the percentage of submergence varies.

6. The length of pump and percentage of submergence remaining constant, and therefore constant lift, the efficiency increases as the input decreases, that is, the highest efficiencies are obtained at the lowest rates of pumping.

7. By varying the percentage of submergence, and therefore the lift, the length of pump remaining constant, the maximum efficiency is obtained at approximately 63 percent submergence for all rates of input or discharge.

8. The lift remaining constant, the efficiency increases as the percentage of submergence increases, for all rates of input and all practical percentages of submergence.

9. With the same size and type of pump, the percentage of submergence remaining constant, the efficiency increased as the lift increased for the small lifts experimented on, that is, up to about 24 ft. From a theoretical study, however, the indications are that a point will be reached from which the efficiency will decrease as the lift increases.

10. Other conditions remaining constant, there is no advantage to be gained by introducing compressed air above the surface of the water in the well.

11. The type of the foot-piece has very little effect on the efficiency of the pump, so long as the air is introduced in an efficient manner and the full cross sectional area of the eduction pipe is realized for the passage of the liquid. Anything in the shape of a nozzle to increase the kinetic energy of the air is detrimental.

12. A diverging outlet which will conserve the kinetic energy of the velocity head increases the efficiency.

The Hydraulic Air Compressor. This apparatus (Fig. 35) is practically the reverse of the air lift. Water is caused to pass vertically downward thru a tube or shaft at a moderately high velocity, and by means of small pipes or an open surface at the top opportunity is given for the water to absorb as much air as possible. At the bottom the water is discharged against a considerable head and is made to pass underneath a collecting hood which is connected with a chamber for storing air. The reduction of velocity at outlet causes the water to release a portion of the entrained air, which rises thru the collecting hood and passes to the air chamber under a pressure equal to h_{es} the head of water above the free surface underneath the hood.

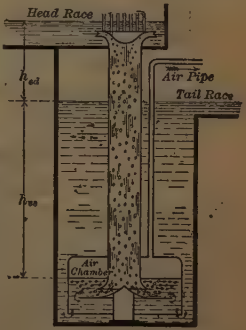

Fig. 35

The velocity at which bubbles of air separate themselves from running water is about 0.75 foot per second, and the flow down the tube or shaft must exceed this. Experimentally a velocity between 12 and 16 feet per second gives the best results. (Eng. and Min. Jour., Jan. 19, 1907.)

38. Miscellaneous Pumps

The Archimedean Screw has been largely used in Holland against heads not exceeding 10 ft. It consists of an inclined shaft carrying one or more helices of considerable diameter, which are rotated with little clearance in a circular or semicircular channel connecting head and tail water. The angle of inclination of the channel should be less than the angle of the helix so that the water always tends to run down the helix and up the channel. The best angle for the helix is found to be between 30° and 40°, and an efficiency of 75 percent has been reported.

The Screw Pump is the name applied to a spiral acting on a vertical axis in a throat smaller than the inlet and discharge pipe. These pumps have efficiencies from 75 to 85 percent.

The Scoop Wheel is used for drainage pumping, and consists of a series of flat vanes revolving in a curved channel, and is practically a reversed undershot water wheel. It may be used for lifts as high as 6 ft and has been constructed to deliver 160 cu ft per sec. An efficiency of 70 percent is claimed under favorable circumstances.

The Positiv Rotary Pump (Fig. 36) is intermediate between the centrifugal and the reciprocating piston type. It has long been familiar in the Silsby fire engine, where both the steam and water ends are of this class. The same construction is used in the ordinary rotary blowers for delivering free air. The chief objection to this pump is the difficulty of keeping the revolving pistons tight. Like the centrifugal it requires no valves, and like the reciprocating pump is positiv in action. The discharge of such a pump per revolution is $Q = \pi d(R_i^2 + R_0^2)$, where d = axial length of the piston, or distance between crowns, R_i = radius of piston hub, and R_0 = radius of piston tip. The discharge as computed from this formula must be reduced for leakage past the pistons.

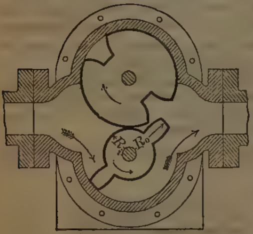

Fig. 36

WATER POWER

39. Power, Slope, and Discharge

The Power of a Stream is the product of the fall H and the weight Qw of water flowing per second, Q being the discharge and w the weight of a cubic unit of water. When H is feet, Q cubic feet per second, and w pounds per cubic foot, then work per second in foot-pounds $= QwH$ and horse-power $= QwH/550$. Assuming a plant to deliver 80 percent of the power of the stream the horse-power developed at the wheel shaft is simply $QH/11$. Assuming an efficiency of generating apparatus of 90 per cent, since 1 kilowatt = 1.34 horse-power, the electrical output at the station switchboard is $QH/16$ in kilowatts. Since a stream of water flows continuously, its power should be stated on the basis of 24 hours per day and 365 days in a year, and a horse-power year is $24 \times 365 = 8760$ horse-power hours, or 525 600 horse-power minutes, or 31 536 000 horse-power seconds.

Conversion Table

Head-discharge to Horse-powers and Kilowatts

Head × discharge Q.H.	Theoretical Horse-power T.H.P.	Mechanical Horse-power at Wheelshaft. Eff. 80% M.H.P.	Electrical Kilowatts at Switchboard. Eff. of Gen. 92% K.W. at S.B.
100	11.3456	9.0765	6.2269
125	14.1820	11.3456	7.7836
150	17.0184	13.6147	9.3403
175	19.8548	15.8838	10.8970
200	22.6912	18.1530	12.4538
225	25.5276	20.4221	14.0105
250	28.3640	22.6912	15.5672
275	31.2004	24.9603	17.1239
300	34.0369	27.2295	18.6806
325	36.8733	29.4986	20.2374
350	39.7097	31.7678	21.7941
375	42.5461	34.0369	23.3508
400	45.3825	36.3060	24.9075
425	48.2189	38.5751	26.4642
450	51.0553	40.8442	28.0210
475	53.8917	43.1134	29.5777
500	56.7281	45.3825	31.1344
525	59.5645	47.6516	32.6911
550	62.4009	49.9207	34.2478
575	65.2373	52.1898	35.8046
600	68.0738	54.4590	37.3613
625	70.9102	56.7282	38.9180
650	73.7466	58.9973	40.4747
675	76.5830	61.2664	42.0314
700	79.4194	63.5355	43.5882
725	82.2558	65.8046	45.1449
750	85.0922	68.0738	46.7016
775	87.9286	70.3429	48.2583
800	90.7650	72.6120	49.8150
825	93.6014	74.8811	51.3718
850	96.4378	77.1502	52.9285
875	99.2742	79.4194	54.4852
900	102.1107	81.6886	56.0419
925	104.9471	83.9577	57.5986
950	107.7835	86.2268	59.1554
975	110.6199	88.4959	60.7121
1000	113.4563	90.7650	62.2688
	$= QH \times .113456$	$= QH \times .090765$ $=$ T.H.P. $\times .80$	$= QH \times .062269$ $=$ T.H.P. $\times .54884$ $=$ M.H.P. $\times .68604$

In ordinary industrial operations it is usually estimated that a plant will operate during 310 days in a year and from 10 to 16 hours per day, depending on the character of the industry. The number of horse-power hours in a year required for each horse power capacity is therefore from 3100 to 4960. If the water which would flow during the remaining 14 to 8 hours and the remaining 55 days can be stored for use during the time the plant runs, the capacity of the plant per hour for the running time will be from 8760/3100 to 8760/4960 or from 2.83 to 1.76 horse-powers per horse-power of average stream capacity. In using the term horse-power in connection with water-power development the unit of time upon which it is based should be stated, and when no unit is designated 24-hour 365-day power should be understood. Evidently a stream will yield twice as much power on a 12-hour basis as it will on a 24-hour basis if the flow of the other 12 hours can be stored.

The Slope. The distance thru which the water falls vertically per unit of horizontal distance traveled is sometimes called the slope per unit of distance, and is for uniform fall the tangent of the inclination of the water surface from the horizontal. Since the slope of natural streams is ordinarily small, the inclined length and the horizontal length practically coincide.

In ordinary natural streams, except in the vicinity of an obstruction, the low-water slope is greater than the flood slope. For streams with nearly vertical side walls the reverse will ordinary be true. Upstream from and in the vicinity of a dam over which the water flows, the flood slope will be greater than the low-water slope, except in channels which enlarge their cross-section very rapidly by overflowing extensive flats as the water rises. Generally for the estimation of power the natural slope may be taken as practically the same at all times unless the location is upstream from an obstruction which may cause the water to back up from below against the base of the dam without causing a similar rise above its crest.

Backwater. The distance upstream to which the backwater effect of a dam or other obstruction extends in an ordinary stream will be greater at low water than in time of flood.

The Discharge or Run-off of all streams varies from day to day, month to month, and year to year, and bears some relation to the rainfall on the drainage area. This relation is by no means a simple one, and stream flow measurements are essential to the reliable determination of the flow, unless a comparison can be made with the known discharge from a similar water shed similarly located; and even then great care must be taken in estimating to allow for the influence of a range of hills if one exist between the two basins considered, as greater precipitation and hence greater run-off may be expected on the windward side of a summit of land than on the leeward side.

The water that falls on the surface of the earth is partly evaporated by the direct rays of the sun and partly by the aid of vegetation, partly absorbed by the soil to reach the streams thru their banks and beds, and partly flows away over the surface. The quantity of discharge of a stream is therefore dependent upon both the rainfall and the evaporation, and ordinarily amounts to the surface run-off plus the subsurface flow, and of this discharge the continuous as distinguished from the intermittent flood flows, when corrected for the effect of visible storage in ponds and lakes, represents the subsurface supply, or the percolation, which generally ranges from 25 to 60 percent of the total supply, being greater on flat, sandy, and porous drainages than on steep, impervious ones.

Occasionally it happens that percolation from an area whose surface is tributary to one stream is delivered by underground strata into another, but such conditions are relatively rare. It will be found that while the percentage of rainfall appearing as run-off is greater for precipitous and rocky drainages, the amount of the run-off that can be utilized for power will be greater from flat, sandy areas.

Run-off Data. The most useful as well as most accurate data available for American conditions are those of Desmond FitzGerald, obtained at Boston

on the Sudbury and Cochituate watersheds. (Trans. Am. Soc. C. E., 1892. vol. 27, p. 253.)

Run-offs in Percentages of the Rainfall

Period	Sudbury, 1875 to 1890			Cochituate 1863 to 1891
	Mean	Max.	Min.	
January	49.1	88.8	7.6	53.1
February	78.2	206.9	42.9	71.9
March	109.6	261.7	74.0	84.6
April	109.1	188.3	48.5	83.8
May	62.3	260.2	39.9	47.9
June	29.1	54.7	14.0	27.9
July	8.9	20.9	3.6	13.1
August	13.0	61.2	4.1	17.7
September	14.2	30.9	6.1	23.5
October	23.1	71.4	4.8	23.6
November	39.5	174.7	11.3	34.0
December	52.5	127.3	9.6	49.9
Year	49.5	62.2	31.9	43.8

The Sudbury watershed has an area of 75.199 square miles, is hilly, with steep slopes having some large swamps within its borders, and the material is mostly unmodified drift. The Cochituate watershed has an area of 18.87 square miles, with flat, sandy slopes and the surface mostly modified drift. The two are adjacent to each other.

The records of the Sudbury watershed are given in more detail than those of the Cochituate, and the more important data are in the following table:

Sudbury River Watershed, Record of 16 Years

Period	Rainfall in inches			Run-off in cu ft per sec per sq mile			Evaporation from water surface, ins			Mean temperature, Fahr.
	Mean	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	
Jan.	4.179	6.36	1.83	1.779	4.305	0.159	0.96	......	......	27°
Feb.	4.062	6.55	0.74	3.023	7.428	1.469	1.05	......	......	28
Mar.	4.578	8.36	1.07	4.354	7.448	2.071	1.70	......	......	34
Apr.	3.320	5.79	1.82	3.247	5.094	1.342	2.97	3.12	2.78	44
May	3.203	5.21	0.96	1.732	2.526	0.796	4.46	5.89	3.35	57
June	2.985	6.24	1.47	0.779	1.346	0.271	5.54	7.01	3.94	67
July	3.784	9.13	1.41	0.292	0.980	0.096	5.98	7.50	4.82	71
Aug.	4.227	7.18	0.74	0.478	2.216	0.086	5.50	7.41	4.25	69
Sept.	3.232	8.74	0.32	0.412	1.786	0.068	4.12	5.13	3.08	62
Oct.	4.413	10.51	0.81	0.882	3.515	0.109	3.16	4.13	2.51	52
Nov.	4.107	7.23	1.15	1.453	4.267	0.271	2.25	3.00	0.66	40
Dec.	3.710	6.37	0.87	1.689	4.708	0.271	1.51	......	......	30
Year	45.800	53.00	32.78	1.669	2.626	0.824	39.20	41.51	34.05	48.4

When the rainfall is high, a greater portion appears as run-off and when low a less percentage than the mean, since the vegetation takes its share before the water gets away, except in severe storms, and when there is little rain during the growing season vegetation takes nearly all. With rainfall records as a basis and the data of FitzGerald's experiments, it is possible to estimate with a fair degree of accuracy the annual run-off of streams in the humid portions of the United States, and also to approximate the low-water flow, particularly if cognizance is taken of the character of the surface and the influence of the vegetation on the evaporation.

In working from rainfall to run-off it is to be noted that a given amount

of precipitation concentrated in a few sharp showers will appear to a larger extent as run-off than the same amount falling during the same period in a more continuous but mild storm. Also when the soil, of a clayey nature, is baked by a drouth and vegetation is drooping, a larger proportion of the precipitation will run off than when the soil is slightly moist and vegetation thriving. Rain falling at night contributes more to the run-off than that falling in the daytime by reason of the direct evaporation in the latter case by the sun's rays, and a mild shower falling on a hot surface may be almost wholly evaporated without the aid of vegetation. Rain falling on frozen ground appears almost wholly as run-off. The water which runs off over the surface reaches the streams within a few hours, but the subsurface flow is likely to continue for at least two weeks under ordinary conditions before it all appears in the open channels.

40. Rainfall in the United States

The table on p. 1163 compiled from the records of the U. S. Weather Bureau gives the normal monthly and annual precipitation in inches at 175 places in the United States, as established from records prior to 1906. In the arid regions, the minimum annual precipitation may be as low as 25 percent of the normal, but in the humid portions, while records as low as 40 percent occur, an annual minimum less than 50 percent of the normal is rare. The terms Rainfall and Precipitation are usually synonymous, meaning the vertical depth of rain and melted snow.

41. Percolation and Run-off

Percolation, or subsurface run-off, varies greatly with the character of the soils and their covering. The following data, from both European and American sources, give the percolation in percentages of the rainfall:

Sand, bare	65 to 85
Ordinary ground, bare	29 and 29.2
Ordinary ground, cultivated, no crops	36
Loam, bare	33
Peat, bare	44
Sand with grass growing 6 months	14
Loam with grass growing 6 months	1.3
Peat with grass growing 6 months	14.6 and 8.7
Ordinary soil with sod	33
Bare ground	45
Bare ground covered with 1 cm sand	82
Bare ground covered with 5 cm straw	94
Bare ground covered with 5 cm dry leaves	93
Bare ground with grass growing	−34*
Meadow grass, short	0 to 1.15
Meadow grass, long	0 to − 100*
Beans and peas	5
Clover	− 12*
Indian corn	40 to − 50*
Wheat	25
Rye	40 to 18
Oats	38
Potatoes	40
Ash forest	37
Alder forest	43
Beech forest	44
Elm forest	50
Maple forest	57
Oak forest	58 to 70
Vineyards	66
Mixt forest	74
Norway spruce forest	91
Pine forest	93
Fir forest	94

* Water in excess of rainfall taken from subsurface run-off of adjacent areas.

The above data so far as it deals with crops and forests must be accepted as giving only relative relations. It is, however, safe to say that the percolating run-off from evergreen forests will be about twice that from deciduous forests and three times that from cropt areas during the growing season.

Normal Precipitation in United States to 1906

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Alabama:													
Anniston	5.31	4.65	5.78	3.63	3.09	3.88	4.73	4.48	3.53	2.34	3.40	4.54	49.36
Birmingham	5.32	4.75	5.76	3.67	3.09	3.88	4.70	4.48	3.50	2.34	3.39	4.60	49.48
Mobile	4.85	5.36	7.17	4.35	4.00	5.95	7.04	6.81	5.02	3.18	3.74	4.57	62.04
Montgomery	5.11	5.52	6.38	4.25	3.82	4.21	4.68	4.24	2.88	2.44	3.13	4.50	51.16
Arizona:													
Phœnix	1.17	0.69	0.49	0.43	0.03	0.12	1.07	0.96	1.01	0.35	0.96	0.59	7.87
Yuma	0.43	0.60	0.35	0.10	0.03	0.00	0.12	0.35	0.16	0.19	0.32	0.45	3.10
Arkansas:													
Fort Smith	2.51	2.74	3.62	3.90	4.91	4.00	3.84	3.66	3.17	2.83	3.28	2.88	41.34
Little Rock	4.79	4.18	4.94	4.51	5.10	4.09	3.99	3.65	3.26	2.55	4.59	4.24	49.89
California:													
Eureka	7.63	7.03	6.97	3.93	2.54	1.06	0.11	0.10	1.11	2.65	5.67	7.25	46.05
Fresno	1.60	1.36	1.76	0.71	0.65	0.10	0.00	0.00	0.27	0.72	1.03	1.53	9.73
Independence	1.56	1.27	1.79	0.66	0.65	0.09	0.00	0.00	0.24	0.69	1.03	1.55	9.53
Los Angeles	2.84	2.91	3.00	1.13	0.48	0.07	0.00	0.00	0.06	0.77	1.48	2.90	15.64
Mt. Tamalpais	4.31	3.76	3.26	1.95	0.91	0.17	0.01	0.00	0.46	1.29	2.43	4.25	22.80
Red Bluff	3.94	3.62	3.76	1.85	1.32	0.46	0.00	0.02	0.80	1.58	3.19	4.49	25.03
Sacramento	3.69	3.14	3.01	2.00	0.98	0.15	0.00	0.01	0.39	1.04	2.15	3.53	20.09
San Diego	2.00	1.96	1.70	0.74	0.41	0.03	0.00	0.00	0.06	0.46	0.83	1.82	10.01
San Francisco	4.33	3.70	3.14	1.82	0.81	0.17	0.01	0.00	0.29	1.29	2.47	4.24	22.27
San Jose	4.28	3.71	3.12	1.82	0.81	0.17	0.00	0.00	0.29	1.30	2.47	4.23	22.20
San Luis Obispo	4.72	3.58	3.98	1.48	0.88	0.10	0.01	0.04	0.45	1.33	1.70	2.34	20.61
Colorado:													
Denver	0.42	0.49	1.00	2.17	2.54	1.47	1.62	1.34	0.89	0.96	0.52	0.60	14.02
Pueblo	0.35	0.47	0.86	1.43	1.68	1.47	1.97	1.57	0.62	0.70	0.37	0.46	11.95
Connecticut:													
Hartford	3.83	3.55	4.32	3.57	3.54	3.08	4.11	4.56	3.50	3.86	3.82	3.57	45.31
New Haven	3.91	3.75	4.45	3.56	3.64	3.17	4.78	4.99	3.79	3.92	3.58	3.65	47.19
Dist. of Col.													
Washington	3.37	3.42	3.85	3.25	3.83	4.18	4.65	4.40	3.59	3.09	2.71	3.16	43.50
Florida:													
Jacksonville	3.12	3.43	3.52	2.72	4.25	5.53	6.20	6.21	8.03	5.06	2.19	2.99	53.25
Jupiter	3.58	3.05	3.12	2.63	4.76	6.93	5.37	5.85	9.56	9.48	3.05	2.87	60.25
Key West	1.98	1.64	1.48	1.30	3.36	4.25	3.59	4.69	6.79	5.38	2.36	1.84	38.66
Pensacola	4.04	4.49	5.36	3.16	2.68	4.87	7.27	7.16	5.23	4.08	3.74	4.17	56.25
Tampa	2.80	3.27	2.81	1.85	2.92	8.34	8.43	8.59	7.41	2.97	1.72	2.02	53.13
Georgia:													
Atlanta	5.31	4.65	5.78	3.63	3.09	3.88	4.73	4.48	3.53	2.34	3.40	4.54	49.36
Augusta	4.15	4.38	4.85	3.50	3.23	4.53	5.29	5.57	3.71	2.33	2.92	3.43	47.89
Macon	5.18	4.58	5.48	3.40	2.88	3.58	4.63	4.24	3.40	2.12	3.10	4.41	47.00
Savannah	3.13	3.28	3.65	2.99	3.00	6.03	6.18	7.50	5.56	3.55	2.37	3.10	50.34
Thomasville	4.13	4.48	5.09	3.65	4.01	4.72	5.32	5.03	4.25	3.46	2.64	3.69	50.47
Idaho:													
Boise	1.89	1.42	1.44	1.18	1.29	0.88	0.18	0.16	0.41	1.28	0.86	1.72	12.71
Lewiston	1.58	1.34	1.28	1.13	1.63	1.04	0.42	0.37	0.65	1.20	1.32	1.52	13.48
Pocatello	0.66	0.85	1.75	2.02	2.20	0.99	0.63	0.56	0.88	0.98	0.55	0.86	12.93
Illinois:													
Cairo	3.82	3.33	4.02	3.57	3.83	4.31	3.45	2.93	2.47	2.63	4.02	3.33	41.71
Chicago	2.00	2.16	2.55	2.88	3.37	3.66	3.64	2.88	3.02	2.55	2.50	2.07	33.28
La Salle	2.16	2.44	2.82	3.10	3.92	3.99	3.27	2.86	3.20	2.58	2.64	2.28	35.26
Peoria	2.20	2.69	2.96	3.28	4.26	4.30	2.97	2.93	3.12	2.57	2.64	2.37	36.29
Springfield	2.25	2.77	3.07	3.31	4.49	4.31	2.90	2.80	3.37	2.60	2.66	2.43	36.96
Indiana:													
Evansville	3.69	3.06	4.60	3.46	3.43	4.17	3.81	3.24	2.66	3.10	4.11	3.83	43.16
Indianapolis	2.81	3.08	4.01	3.47	3.94	4.31	4.13	3.33	3.05	2.79	3.52	3.04	41.48

Normal Precipitation in United States to 1906 (Continued)

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Iowa:													
Charles City	0.97	0.97	1.92	2.83	4.94	5.10	3.58	3.42	2.80	2.03	1.39	1.28	31.23
Davenport	1.57	1.56	2.21	2.88	4.19	4.11	3.55	3.64	3.14	2.39	1.76	1.69	32.69
Des Moines	1.21	1.08	1.65	2.98	4.56	4.96	3.86	3.61	3.07	2.68	1.48	1.31	32.45
Dubuque	1.49	1.38	2.21	2.92	4.32	4.55	4.30	3.04	3.59	2.68	1.81	1.72	34.01
Keokuk	1.69	1.62	2.37	3.34	4.25	4.35	4.03	3.24	3.97	2.49	1.85	1.87	35.07
Sioux City	0.55	0.55	1.26	2.77	4.37	3.88	3.61	3.00	2.47	1.79	0.97	0.74	25.96
Kansas:													
Concordia	0.72	0.75	1.48	2.42	4.70	4.97	3.62	2.81	2.58	2.00	0.94	0.48	27.47
Dodge City	0.47	0.71	0.88	1.87	3.34	3.32	3.38	2.59	1.77	1.40	0.55	0.56	20.84
Topeka	0.93	1.38	2.29	2.72	4.88	4.81	4.85	4.29	3.56	1.91	1.26	0.88	33.76
Wichita	0.77	1.07	2.25	2.69	4.95	4.75	3.62	3.11	3.13	2.30	1.18	0.79	30.61
Kentucky:													
Lexington	3.83	3.23	4.72	3.34	3.52	3.99	4.44	3.57	2.42	2.22	3.48	3.32	42.08
Louisville	3.90	3.75	4.32	4.07	3.64	4.23	3.74	3.51	2.63	2.63	4.18	3.73	44.33
Louisiana:													
New Orleans	4.63	4.47	5.30	4.91	3.88	6.16	6.47	5.61	4.81	2.93	3.79	4.46	57.42
Shreveport	4.42	3.61	4.52	4.58	4.16	3.58	3.72	2.24	3.22	3.18	4.08	4.37	45.68
Maine:													
Eastport	3.84	3.62	4.28	2.94	3.80	3.24	3.42	3.26	2.97	3.85	4.08	3.97	43.27
Portland	3.81	3.65	3.75	3.11	3.67	3.36	3.25	3.57	3.20	3.66	3.80	3.68	42.51
Maryland:													
Baltimore	3.22	3.51	3.88	3.27	3.56	3.84	4.82	4.21	3.85	3.02	2.92	3.08	43.18
Mass:													
Boston	3.82	3.44	4.08	3.55	3.51	3.03	3.36	4.03	3.19	3.86	4.10	3.41	43.38
Nantucket	3.42	3.11	3.98	2.64	2.67	2.41	2.68	3.05	2.73	3.39	3.28	3.64	37.00
Michigan:													
Alpena	2.20	1.79	2.04	2.20	3.33	3.55	3.06	3.34	3.48	3.43	2.56	2.22	33.20
Detroit	1.98	2.19	2.37	2.33	3.27	3.89	3.48	2.77	2.48	2.38	2.63	2.39	32.16
Escanaba	1.55	1.34	1.95	2.07	3.42	3.60	3.34	3.61	3.58	3.09	2.26	1.70	31.51
Grand Haven	2.80	1.91	2.51	2.44	3.34	2.51	2.58	2.58	3.17	2.49	2.53	2.51	31.37
Grand Rapids	2.78	1.91	2.52	2.45	3.34	2.52	2.63	2.59	3.12	2.54	2.53	2.54	31.47
Houghton	2.04	1.71	2.10	2.03	3.29	3.49	3.10	2.87	3.54	3.18	2.80	2.47	32.62
Marquette	2.04	1.72	2.08	1.99	3.32	3.51	3.10	2.86	3.51	3.19	2.79	2.52	32.63
Port Huron	1.89	2.15	2.44	2.07	3.24	3.24	2.74	2.63	2.68	2.73	2.67	2.17	30.65
Sault Ste. Marie	2.17	1.41	1.85	2.07	3.25	2.77	2.75	3.14	3.46	3.26	2.92	2.33	31.38
Minnesota:													
Duluth	0.98	0.99	1.55	2.14	3.47	4.53	3.65	3.53	3.55	2.74	1.58	1.22	29.93
Minneapolis	0.69	0.73	1.65	2.44	3.92	4.01	3.81	3.69	3.66	2.58	1.18	0.95	29.31
Moorhead	0.71	0.73	1.14	2.33	2.95	4.13	3.74	3.10	2.30	2.07	0.98	0.74	24.92
St. Paul	0.90	0.84	1.60	2.33	3.62	4.41	3.40	3.46	3.42	2.34	1.30	1.06	28.68
Mississippi:													
Meridian	5.54	4.90	5.61	5.04	3.92	4.62	4.50	3.55	3.47	2.70	4.14	5.21	53.20
Vicksburg	5.67	4.61	6.25	5.16	4.26	4.49	4.42	3.53	3.34	2.80	4.19	5.02	53.74
Missouri:													
Columbia	2.23	1.13	3.03	3.70	4.86	4.38	3.65	3.04	2.85	2.42	2.31	2.01	36.61
Hannibal	2.20	2.60	2.76	3.25	5.00	3.52	3.75	3.38	3.56	1.64	1.95	1.65	34.26
Kansas City	1.13	1.50	2.82	3.29	5.11	4.66	4.84	4.75	3.76	2.21	1.85	1.36	37.28
St. Louis	2.27	2.75	3.43	3.52	4.24	4.47	3.43	2.66	2.91	2.41	2.88	2.23	37.20
Springfield	2.66	2.27	4.07	3.86	5.55	5.19	4.79	4.31	3.76	2.80	2.64	2.67	44.57
Montana:													
Havre	0.69	0.47	0.48	1.01	2.09	2.82	1.92	1.26	1.03	0.50	0.77	0.63	13.67
Helena	0.93	0.74	0.74	1.14	1.95	2.11	1.07	0.69	1.06	0.82	0.72	0.80	12.77
Kalispell	1.59	1.46	1.08	1.06	2.03	1.74	0.84	0.89	1.33	1.17	1.90	1.85	16.94
Miles City	0.62	0.56	0.76	1.18	1.98	2.77	1.37	1.03	0.92	0.78	0.60	0.62	13.17

Normal Precipitation in United States to 1906 (Continued)

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Nebraska:													
Lincoln	0.62	0.70	1.33	2.77	4.25	4.32	3.83	3.71	2.64	1.82	0.85	0.67	27.51
North Platte	0.47	0.40	0.87	2.15	3.06	3.25	2.68	2.46	1.50	1.15	0.40	0.47	18.86
Omaha	0.65	0.76	1.39	3.01	4.50	5.05	4.33	3.62	3.03	2.35	1.06	0.91	30.66
Valentine	0.52	0.60	1.20	2.32	3.33	3.81	3.40	2.82	1.79	1.32	0.73	0.62	22.46
Nevada:													
Reno	1.95	1.79	1.22	0.61	0.79	0.24	0.14	0.25	0.27	0.41	1.09	1.67	10.43
Winnemucca	1.04	0.93	0.95	0.88	1.03	0.64	0.17	0.17	0.34	0.52	0.74	0.99	8.40
N. H.:													
Concord	3.34	3.28	3.40	2.79	3.24	3.34	3.79	3.74	3.21	3.24	3.39	3.35	40.11
New Jersey:													
Atlantic City	3.40	3.27	3.73	2.99	3.00	3.03	3.78	4.30	3.05	3.30	3.23	3.74	40.82
Cape May	3.37	3.29	3.73	2.99	2.99	3.04	3.78	4.26	3.00	3.30	3.22	3.78	40.75
New Mexico:													
Santa Fe	0.59	0.84	0.73	0.86	1.11	1.04	2.71	2.36	1.64	1.07	0.78	0.76	14.49
New York:													
Albany	2.59	2.52	2.74	2.39	2.98	3.76	3.90	3.96	3.18	2.99	2.80	2.57	36.38
Binghamton	1.98	1.90	2.64	2.25	3.09	3.59	3.54	3.35	2.77	3.12	2.27	2.44	32.94
Buffalo	3.30	2.85	2.62	2.45	3.10	3.14	3.40	2.99	3.18	3.53	3.35	3.37	37.28
Canton	3.16	2.57	2.84	2.26	2.85	3.43	3.23	2.69	2.81	3.34	3.41	3.59	36.18
Ithaca	2.16	1.87	2.44	2.29	3.43	3.88	3.75	3.24	2.83	3.17	2.53	2.64	34.23
New York	3.79	3.74	4.10	3.30	3.18	3.26	4.54	4.53	3.59	3.71	3.44	3.45	44.63
Oswego	3.16	2.57	2.84	2.26	2.85	3.43	3.23	2.69	2.81	3.34	3.41	3.59	36.18
Rochester	3.13	2.83	2.93	2.44	2.94	3.13	3.09	2.96	2.32	2.86	2.76	2.88	34.27
Syracuse	2.14	1.80	2.38	2.31	3.39	3.89	3.68	3.33	2.82	3.21	2.70	2.65	34.30
N. Carolina:													
Asheville	4.67	4.65	5.08	4.04	3.78	4.35	4.86	4.79	3.04	2.94	3.30	4.06	49.56
Charlotte	4.29	4.39	4.57	3.44	3.92	4.46	5.49	5.55	3.22	3.15	2.86	3.86	49.20
Hatteras	4.94	4.48	5.47	4.40	4.14	4.33	6.13	5.84	5.33	6.01	4.67	5.11	60.85
Raleigh	3.55	4.34	4.26	3.47	4.89	4.72	6.11	5.90	3.34	3.50	2.35	3.17	49.60
Wilmington	3.50	3.39	3.59	2.86	4.03	5.62	6.97	6.51	5.27	3.74	2.45	3.12	51.05
N. Dakota:													
Bismarck	0.54	0.50	1.04	1.88	2.50	3.54	2.14	1.98	1.19	1.03	0.68	0.62	17.64
Devil's Lake	0.60	0.53	1.01	2.03	2.20	3.53	3.78	2.76	1.39	1.23	0.71	0.39	20.16
Williston	0.58	0.47	0.68	1.23	2.26	3.57	2.03	1.31	0.91	0.77	0.60	0.66	15.07
Ohio:													
Cincinnati	3.36	3.24	3.64	2.95	2.52	3.98	3.54	3.33	2.31	2.32	3.21	2.93	37.33
Cleveland	2.45	2.61	2.79	2.31	3.22	3.68	3.55	3.15	3.22	2.73	2.75	2.58	35.04
Columbus	2.95	3.07	3.21	2.87	3.72	3.49	3.65	3.22	2.52	2.35	3.11	2.76	36.92
Sandusky	2.10	2.39	2.55	2.55	3.25	3.82	3.79	3.37	2.68	2.43	2.74	2.35	34.02
Toledo	1.92	1.96	2.28	2.28	3.27	3.38	3.24	2.70	2.36	2.26	2.65	2.30	30.62
Oklahoma:													
Oklahoma	1.34	0.98	2.38	2.80	5.75	3.07	3.65	3.17	2.75	1.81	2.25	1.74	31.69
Oregon:													
Baker City	1.34	1.34	1.44	0.94	1.73	1.21	0.43	0.39	0.76	0.91	1.18	1.53	13.20
Portland	6.50	5.73	5.18	3.05	2.36	1.78	0.54	0.65	1.84	3.69	6.47	7.34	45.13
Roseburg	5.70	4.56	3.98	2.48	2.05	1.07	0.32	0.33	1.04	2.61	4.37	5.92	34.43
Pennsylvania:													
Erie	3.03	2.85	2.66	2.40	3.43	3.75	3.21	3.26	3.49	3.80	3.61	3.06	38.55
Harrisburg	2.82	2.70	3.12	2.49	3.67	3.55	3.87	4.25	2.85	2.95	2.35	2.65	37.27
Philadelphia	3.41	3.38	3.45	2.91	3.20	3.30	4.33	4.61	3.38	3.10	3.06	3.04	41.17
Pittsburgh	2.87	2.66	3.01	2.90	3.30	3.89	4.42	3.18	2.48	2.36	2.55	2.73	36.35
Scranton	2.80	2.72	3.12	2.65	3.44	3.57	3.83	4.25	2.86	2.91	2.29	2.61	37.05
Rhode Island:													
Block Island	3.86	4.29	4.37	3.62	3.75	2.87	3.31	3.48	3.00	4.11	3.88	3.82	44.36
Providence	3.82	3.44	4.08	3.55	3.51	3.03	3.36	4.03	3.19	3.86	4.10	3.41	43.38

Normal Precipitation in United States to 1906 (Concluded)

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
S. Carolina:													
Charleston	3.45	3.41	3.72	2.99	3.47	5.39	7.26	6.97	5.46	3.93	2.87	3.15	52.07
Columbia	3.29	4.59	3.70	2.86	3.19	4.18	6.06	6.78	3.45	2.85	2.23	2.90	46.08
S. Dakota:													
Huron	0.51	0.44	0.99	2.65	2.92	3.78	2.94	2.64	1.69	1.34	0.58	0.62	21.10
Pierre	0.46	0.44	1.33	1.98	2.13	3.08	2.35	2.01	1.11	0.81	0.43	0.50	16.63
Rapid City	0.44	0.46	1.05	2.30	2.91	3.59	2.55	2.11	1.26	1.10	0.46	0.46	18.69
Yankton	0.56	0.58	1.14	2.80	3.90	4.27	3.52	3.12	2.45	1.57	0.79	0.73	25.43
Tennessee:													
Chattanooga	5.56	4.94	6.21	4.39	3.62	4.31	3.87	3.75	3.25	2.84	3.58	4.36	50.68
Knoxville	4.97	4.90	5.58	4.64	3.70	4.17	4.21	4.00	2.81	2.61	3.60	4.16	49.35
Memphis	5.21	4.35	5.77	4.83	4.34	4.37	3.51	3.20	3.05	2.74	4.59	4.38	50.34
Nashville	4.85	4.32	5.44	4.36	3.50	4.37	4.35	3.47	3.68	2.48	3.85	3.82	48.49
Texas:													
Abilene	0.90	1.08	1.38	2.28	3.72	3.17	2.40	1.97	3.14	2.33	1.24	1.13	24.74
Amarillo	0.60	0.88	0.65	1.72	3.67	2.99	3.17	2.81	2.36	1.71	1.16	0.83	22.55
Corpus Christi	2.25	2.04	1.85	1.80	2.78	2.74	1.66	2.28	4.00	2.05	2.41	1.32	27.18
El Paso	0.51	0.46	0.38	0.23	0.35	0.55	2.13	1.72	1.45	0.95	0.59	0.52	9.84
Fort Worth	0.93	1.27	1.76	2.65	4.15	2.97	3.04	1.87	2.95	2.51	1.57	1.22	26.89
Galveston	3.62	3.10	2.90	3.13	3.23	4.75	3.98	5.01	5.41	4.18	4.02	3.73	47.06
Palestine	3.87	3.39	3.47	4.08	4.89	4.00	3.00	2.22	3.20	3.60	3.63	3.67	43.02
San Antonio	1.68	1.78	1.68	2.94	2.96	3.11	2.22	2.69	2.94	1.49	1.78	1.56	26.83
Taylor	2.82	2.58	2.62	3.97	4.01	3.52	2.62	2.47	3.04	2.56	2.64	2.62	35.47
Utah:													
Modena	0.74	0.77	1.37	1.70	1.38	0.44	0.31	0.39	0.50	0.83	0.91	0.81	10.15
Salt Lake City	1.35	1.38	2.00	2.26	1.95	0.77	0.54	0.78	0.85	1.40	1.42	1.33	16.03
Vermont:													
Burlington	1.83	1.37	1.83	1.87	2.83	3.26	3.78	4.01	3.35	3.16	2.58	1.69	31.56
Northfield	2.49	2.26	2.79	2.09	2.80	3.23	3.70	3.93	2.76	2.48	2.60	2.71	33.84
Virginia:													
Cape Henry	3.35	3.77	4.32	3.92	4.03	4.42	5.85	6.10	4.08	3.88	2.74	3.43	49.89
Lynchburg	3.72	3.49	3.81	3.17	3.99	3.89	4.03	4.25	3.63	3.38	2.79	3.27	43.42
Mt. Weather	3.30	3.16	3.98	3.07	3.75	4.80	4.66	3.62	2.85	2.42	2.86	3.09	41.56
Norfolk	3.37	3.75	4.28	3.79	4.07	4.33	5.80	5.97	4.06	3.91	2.72	3.49	49.54
Richmond	3.02	3.10	3.72	3.45	3.85	3.52	4.42	4.41	3.43	3.32	2.39	3.00	41.63
Wytheville	4.30	4.09	4.45	3.66	3.91	4.11	4.44	4.54	3.29	3.14	3.04	3.74	46.7[illegible]
Washington:													
North Head	5.68	5.79	5.25	3.22	2.39	1.77	0.54	0.56	1.85	3.91	6.33	7.48	45.7[illegible]
Port Crescent	5.53	4.71	3.92	2.52	2.31	1.83	0.61	0.70	2.30	3.47	7.45	6.88	42.2[illegible]
Seattle	4.52	3.86	3.60	2.68	2.32	1.72	0.69	0.49	1.93	2.88	5.86	6.04	36.5[illegible]
Spokane	2.30	1.93	1.51	1.29	1.62	1.61	0.67	0.48	1.01	1.51	2.30	2.62	18.8[illegible]
Tacoma	5.78	5.13	3.98	2.76	2.54	2.13	0.66	0.70	2.47	3.40	8.53	7.33	45.4[illegible]
Tatoosh Is.	12.2	8.76	8.57	6.31	4.09	4.18	1.78	2.11	6.14	8.00	12.1	14.6	88.7[illegible]
Walla Walla	2.01	1.58	1.89	1.70	1.83	1.19	0.39	0.45	0.93	1.47	2.13	2.10	17.6[illegible]
W. Virginia:													
Elkins	3.34	3.17	4.07	3.29	3.98	5.04	4.64	3.65	2.87	2.42	2.86	3.42	42.[illegible]
Parkersburg	3.19	3.24	3.82	2.91	3.46	4.65	4.66	3.53	2.72	2.44	2.83	2.77	40.[illegible]
Wisconsin:													
Green Bay	1.69	1.60	2.40	2.44	3.57	3.55	3.51	3.10	3.12	2.37	1.96	1.81	31.[illegible]
La Crosse	1.08	1.06	1.65	2.29	3.75	4.43	4.07	3.41	4.12	2.46	1.52	1.33	31.[illegible]
Madison	1.56	1.47	2.21	2.38	3.62	4.10	3.99	3.21	3.18	2.42	1.80	1.77	31.[illegible]
Milwaukee	2.01	1.89	2.67	2.70	3.42	3.67	3.01	2.82	2.92	2.39	1.98	1.92	31.[illegible]
Wyoming:													
Cheyenne	0.40	0.56	0.95	1.85	2.43	1.57	1.99	1.47	0.94	0.72	0.41	0.31	13.[illegible]
Lander	0.43	0.65	1.60	2.46	2.90	1.11	0.86	0.54	1.02	1.05	0.60	0.70	13.[illegible]
Yellowstone Pk.	2.24	1.84	2.18	1.38	1.91	1.64	1.18	1.02	1.01	1.14	1.44	1.82	18.[illegible]

Run-off Data of Various American Rivers

River and Station	Area, sq mi	Years	Mean yearly: Rainfall, ins	Mean yearly: Run-off: Per cent rainfall	Mean yearly: Run-off: Cu ft per sec per sq mi	Year of min. flow: Rainfall, ins	Year of min. flow: Run-off: Per cent rainfall	Year of min. flow: Run-off: Cu ft per sec per sq mi	Min. m'thly flow, cu ft per sec per sq mi
Sudbury:									
Boston, Mass.......	75.2	1875–97	45.77	48.6	1.637	32.78	34.1	0.825	......
Cochituate:									
Boston, Mass.......	18.9	1863–96	47.08	43.2	1.498	31.20	31.3	0.719	......
Mystic:									
Boston, Mass.......	26.9	1878–96	43.79	45.6	1.471	31.22	29.8	0.687	......
Connecticut:									
Hartford, Conn......	10 234	1871–85	44.69	56.5	1.860	40.02	45.6	1.345	......
Housatonic:									
Gaylordsville, Conn..	1 020	1900–05	47.86	61.6	2.170	39.77	59.8	1.750	.354
Croton:									
Old Croton Dam....	338	1870–94	48.38	50.8	1.810	38.52	37.8	1.071	......
Upper Hudson:									
Mechanicsville, N. Y.	4 500	1888–96	39.7	59.0	1.720	33.49	52.2	1.286	......
Genesee:									
Mount Morris, N. Y.	1 060	1894–96	39.82	32.5	0.955	31.00	21.5	0.492	......
Passaic:									
Dundee Dam, N. J...	822	1877–93	47.08	54.0	1.875	35.64	42.7	1.122	......
Perkiomen:									
Philadelphia, Pa.....	152	1884–97	47.98	49.2	1.741	38.67	40.4	1.154	......
Tohickon:									
Philadelphia, Pa.....	102	1884–97	50.17	56.7	2.095	38.34	49.0	1.381	......
Neshaminy:									
Philadelphia, Pa.....	139	1884–97	47.88	48.5	1.712	36.30	44.3	1.192	......
Susquehanna:									
Harrisburg, Pa......	28 030	1891–05	39.38	53.6	1.553	31.62	51.7	1.203	.141
Wilkes-Barre, Pa. ...	9 810	1899–05	39.85	58.2	1.708	31.77	47.7	1.116	.106
Williamsport, Pa....	5 640	1895–05	40.02	55.6	1.640	37.46	44.3	1.221	.133
Ohio:									
Wheeling, W. Va. ...	23 820	1884–05	41.71	54.4	1.670	33.47	48.7	1.200	.106
Potomac:									
Pt. of Rocks, Md....	9 650	1895–05	36.86	38.6	1.047	37.25	21.9	0.602	.124
Shenandoah:									
Millville, W. Va.....	3 000	1895–05	38.33	35.6	1.005	30.47	25.8	0.579	.177
James:									
Cartersville, Va.	6 230	1898–05	42.98	42.4	1.342	30.58	35.0	0.788	.186
Buchanan, Va.......	2 060	1895–05	41.17	41.1	1.246	30.45	37.6	0.844	.159
North:									
Glasgow, Va........	830	1895–05	40.76	39.2	1.177	36.49	33.3	0.896	.150
Appomattox:									
Mattoax, Va........	745	1900–05	42.98	38.4	1.214	30.80	35.5	0.805	.239
Roanoke:									
Roanoke, Va........	390	1896–05	42.68	41.5	1.303	35.21	25.2	0.654	.194
Randolph, Va.......	3 080	1900–05	43.80	42.6	1.375	34.00	32.4	0.810	.265

Run-off Data of Various American Rivers (Concluded)

River and Stations	Area, sq mi	Years	Mean Yearly: Rainfall, ins	Mean Yearly: Run-off: Per cent rainfall	Mean Yearly: Run-off: Cu ft per sec per sq mi	Year of min. flow: Rainfall, ins	Year of min. flow: Run-off: Per cent rainfall	Year of min. flow: Run-off: Cu ft per sec per sq mi	Min. m'thly flow, cu ft per sec per sq mi
Savannah: Augusta, Ga.	7 294	1884–91	45.41	48.9	1.635	43.10	37.7	1.197	......
Des Plaines: Riverside, Ill.	630	1889 *	30.56	21.6	0.487	32.38	9.9	0.235	......
Huron: Geddes, Mich.	757	1905–08	34.02	32.5	0.813	32.15	21.8	0.517	.107
Grand: Grand Rapids, Mich.	4 900	1899 †	28.85	40.8	0.867	31.35	26.8	0.618	.255
Upper Mississippi: Pokegama Falls	3 265	1885–99	26.57	18.4	0.361	22.86	7.1	0.119	......
Tennessee: Chattanooga, Tenn.	21 400	1874–06	.....	....	1.860	.....	....	1.272	......
Colorado: Yuma, Ariz.	225 000	1879–06	.....	....	0.045	.....	....	0.012	......
Arkansas: Canyon, Colo.	3 060	1888–92	.....	....	0.266	.....	....	0.141	.059
Rio Grande: Del Norte, Colo.	1 400	1890 ‡	.....	....	0.644	.....	....	0.311	.049
Embudo, N. Mex.	10 900	1889 §	.....	....	0.107	.....	....	0.036	.012
El Paso, Tex.	30 000	1890 ‖	.....	....	0.035	.....	....	0.002	.000
Bear River: Battle Creek, Idaho	4 500	1890 ¶	.....	....	0.283	.....	....	0.163	.060

* Also 1893–95. † Also 1902–1906 and 1908. ‡ Also 1891–93 and 1901–02. § Al 1890–92 and 1901–03. ‖ Also 1891–92 and 1901–03. ¶ Also 1891–93 and 1901–02.

Evaporation and Reservoirs. The final run-off of a stream is also affect considerably by the extent of water surface from which evaporation may ta place. The evaporation from water surfaces in the United States varies fro 18 to over 100 inches annually, being greatest in the arid regions and lea at high altitudes in the north. For water-power calculations in the hum portions, the evaporation during the summer months is usually all that nee to be considered, as at other times it is insignificant, or else a portion of t flow is necessarily wasted. During summer in the northern states this eva ration may amount to about 60 percent of the annual rainfall in open count but where the surrounding shores are forested and the water surfaces narr so as to be little exposed to wind the evaporation is much less.

The evaporation, particularly from water surfaces, is considerably influen by the wind, as shown by the following observations:

Wind velocity in miles per hour	0	5	10	15	20	25	30
Relative evaporation	1.0	2.2	3.8	4.9	5.7	6.1	6.

The aquatic plants exercise a considerable influence upon the evaporation from w surfaces. From experiments made under the direction of the author, it appears that

bulrushes, like long grass, exhale much more moisture than would be evaporated from open water, while the pond lily exhales considerably less, the common arrow-leaf occupying an intermediate position.

Run-off from Typical Streams in various parts of the United States is shown in the table on pp. 1167 and 1168 with other appurtenant data.

In applying the data of this table to cases not therein included, cognizance must be taken of both the rainfall and the topographical conditions of the watershed to which it is applied.

Run-off from Great Lakes Drainage Basins

FROM LOCAL DRAINAGE

By Local Stream Gaging within Period from 1892 to 1913

Month	Huron-Michigan, c.f.s. per sq mi	Erie-St. Clair, c.f.s. per sq mi	Ontario, c.f.s. per sq mi
January	0.806	0.987	1.512
February	0.849	0.748	1.402
March	1.380	2.073	2.682
April	1.763	1.438	3.328
May	1.195	0.867	2.019
June	0.910	0.639	1.223
July	0.642	0.306	0.804
August	0.539	0.219	0.681
September	0.534	0.258	0.638
October	0.619	0.242	0.902
November	0.689	0.292	1.058
December	0.666	0.598	1.259
Full year	0.883	0.722	1.459
April to November, inclusive	0.861	0.533	1.332
June to November, inclusive	0.655	0.326	0.884
Percent of drainage area covered by gagings	32	33	37

BY U. S. LAKE SURVEY GAGINGS OF OUTLETS

Average of 1900 to 1907, Inclusive

River	From Total drainage		From Local drainage	
	c.f.s.	c.f.s. per sq mi	c.f.s.	c.f.s. per sq mi
St. Lawrence	247 762	0.84	47 690	1.45
Niagara	200 072	0.785	6 544	0.16
St. Clair	193 528	0.905	115 328	0.84
St. Marys	78 200	1.028	78 200	1.028

42. Flood Discharge of Rivers

The flood discharge of a stream will be greater per square mile for small drainage areas than for large ones on account of the greater intensity of precipitation on the former in time of storm. While many flood formulas for

computing the run-off per square mile in time of storm have been suggested, the data in the following table of observed flood flows in rivers of the United States as established by the Hydrographic Branch of the U. S. Geological Survey will be found, for those rivers, much more valuable and useful than any formula can ever be.

In using this table to estimate flood flows the character of the drainage area must be carefully considered together with the rainfall conditions. The ordinary storm rate of precipitation in the humid regions is less than 0.4 inch per hour. Excessive rates are from 0.4 to 0.7 inch per hour, but on rare occasions extreme rates of 2 inches are recorded for periods of about an hour over areas of 20 to 100 square miles. In the arid regions, as much as 17 inches in 24 hours has been recorded over a considerable area. The average slope of the watershed and that of the stream itself also influence the maximum discharge, as also whether the ground be frozen or not.

Maximum Rate of Discharge of Streams in the United States

Stream and place	Drainage area, sq mi	Date	Cu ft per sec per sq mi
Beacon Brook, near Fishkill, N. Y.	0.25	1897	3.200
Budlong Creek, Utica, N. Y.	1.13	1904	120.40
Sylvan Glen Creek, New Hartford, N. Y.	1.18	1904	56.58
Pequest River, Hunts Pond, N. J.	1.70	1904	25.30
Starch Factory Creek, New Hartford, N. Y.	3.40	1904	109.62
Starch Factory Creek, near New Hartford, N. Y.	3.40	1905	209.00
Reels Creek, Deerfield, N. Y.	4.40	1904	48.36
Mad Brook, Sherburne, N. Y.	5 00	1905	262.00
Skinner Creek, Mannsville, N. Y.	6.40	1891	124 20
Coldspring Brook, Mass.	6.43	1886	48.40
Croton River, South Branch, N. Y.	7.80	1869	73.90
Woodhull Reservoir, Herkimer, N. Y.	9.40	1869	77.80
Mill Brook, Edmeston, N. Y.	9.40	1905	241.00
Stony Brook, Boston Mass.	12.7	....	121.00
Great River, Westfield, Mass.	14 0	....	71.40
Smartswood Lake, N. J.	16 0	....	68 00
Williamstown River, Williamstown, N. Y.	16.5	....	34.00
Croton River, West Branch, N. Y.	20 5	1874	54.4
Beaverdam Creek, Altmar, N. Y.	20 7	....	111 0
Trout Brook, Centerville, N. Y.	23.0	....	50 6
Pequonnock River, Bridgeport, Conn.	25 0	1905	157.0
Wantuppa Lake, Fall River, Mass.	28.5	1875	72.0
Pequest River, Huntsville, N. J.	31.4	....	19.3
Sawkill, near mouth, N. J.	35.0	....	228.6
Whippany River, Whippany, N. J.	38.0	1896	84.2
Cuyadutta Creek, Johnstown, N. Y.	40.0	1896	72.4
Six Mile Creek, Ithaca, N. Y.	46.0	1905	185.0
Dog River, Northfield, Vt.	47 0	1913	72.2
West Canada Creek, Motts Dam, N. Y.	47.[illegible]	....	34 1
Sauquoit Creek, New York Mills, N. Y.	51.5	....	53.4
Rockaway River, Dover, N. J.	52.5	....	43.0
Oneida Creek, Kenwood, N. Y.	59.0	1890	41.
Flat River, R. I.	61.0	1843	120.
Camden Creek, Camden, N. Y.	61.4	1889	24.
Nine Mile Creek, Stittville, N. Y.	62.6	1898	124.
Wissahickon Creek, Philadelphia, Pa.	64.6	1898	43.
Sandy Creek, Allendale, N. Y.	68 4	1891	87.

* Average flow for day of maximum discharge.

Maximum Rate of Discharge of Streams in the United States (Continued)

Stream and place	Drainage area, sq mi	Date	Cu ft per sec per sq mi
Rock Creek, Washington, D. C.	77.5		126.30
Sudbury River, Farmington, Mass.	78.0	1897	41.38
Pequanock River, Pompton, N. J.	78.0	1902	55.78
Hockanum River, Conn.	79.0		78.10
Nashua River, Mass.	84.5	1850	71.04
Independence Creek, Crandall, N. Y.	93.2	1869	66.50
Passaic River, Chatham, N. J.	100	1903	17.20
Deer River, Deer River, N. Y.	101	1869	78.10
Wanaque River, N. J.	101	1882	66.00
Tohicken Creek, Mount Pleasant, Pa.	102	1894	138.30
Fisk Creek, East Branch, Point Rocks, N. Y.	104	1897	80.50
Onondaga Creek, Syracuse, N. Y.	108.0	1913	30.00
Nashua River, Mass.	109	1848	104.53
Sandy Creek, North Branch, Adams, N. Y.	110	1897	67.30
Scantic River, North Branch, Conn.	118		51.80
Ramapo River, Mahawah, N. J.	118	1903	105.09
Rockaway River, Boonton, N. J.	125	1902	22.24
Patuxent River, Laurel, Md.	127	1915	40.02
Neshaminy Creek, below forks, Pa.	139	1894	97.60
Oriskany Creek, Colemans, N. Y.	141	1888	55.80
Oriskany Creek, Oriskany, N. Y.	144	1904	29.00
Perkiomen Creek, Frederick, Pa.	152	1894	115.80
Mohawk River, Ridge Mills, N. Y.	153		46.40
Mohawk River, State dam, Rome, N Y.	158	1904	27.34
Ramapo River, Pompton, N. J.	160	1882	65.88
Fish Creek, E. Branch, Taberg, N. Y.	169	1913	65.09
Pawtucket River, Providence, R. I.	190	1867	56.85
Catskill Creek, S. Catro, N. Y.	210	1901	100.00
Fish Creek, W. Branch, McConnellsville, N. Y.	187	1885	32.70
Unadilla River, New Berlin, N. Y.	204	1905	40.00
Salmon River, Altmar, N. Y.	221		27.60
Black River, Forestport, N. Y.	268		39.00
Piscataquis River, Foxcroft, Me.	286	1909	77.62
Antietam Creek, Sharpsburg, Md.	295	1902	23.17
Croton River, Croton Dam, N. Y.	338.8		74.87
Great River, Westfield, Mass.	350	1878	151.40
East Canada Creek, Dolgeville, N. Y.	356	1898	24.70
West Canada Creek, Hinckley, N. Y.	372	1869	104.57
Pompton River, Two Bridges, N. J.	380	1903	61.60
Moose River, Ayers Mill, N. Y.	407		31.00
Stony Creek, Johnstown, Pa.	428		70.00
Ausable River, Ausable Forks, N. Y.	444	1913	56.30
Missisquoi River, below Richford, Vt.	445	1913	23.00
Deerfield River, Shelburne Falls, Mass.	501	1909	42.51
West Canada Creek, Middleville, N. Y.	518	1898	24.90
Farmington River, Conn.	584		41.70
Hoosic River, Johnsonville, N. Y.	605	1913	38.01
Monocacy River, near Frederick, Md.	660	1902	31.00
Passaic River, Little Falls, N. J.	773	1882	24.20
North River, Port Republic, Va.	804	1896	29.80
Passaic River, Dundee, N. J.	823	1903	43.56
North River, Glasgow, Va.	831	1896	44.80
Raritan River, Boundbrook, N. J.	806	1882	64.52
Potomac, North Branch, Cumberland, Md.	891	1897	22.80
Black River, Lyons Falls, N. Y.	897	1869	46.00
Schoharie Creek, Fort Hunter, N. Y.	909.3	1913	44.56

Maximum Rate of Discharge of Streams in the United States (Continued)

Stream and place	Drainage area, sq mi	Date,	Cu ft per sec per sq mi
Winooski River, Richmond, Vt.	985	1904	29.80
Genesee River, Mount Morris, N. Y.	1 070	1894–1896	39 20
Penobscot River, E. Branch, Grindstone, Me.	1 100	1909	23 36
Mohawk River Little Falls, N. Y.	1 306	1913	26 25
Youghiogheny River, Connellsville, Pa.	1 320	1907	41 20
Greenbrier River, Alderson, W. Va.	1 344	1913	44 76
Black River, Carthage, N. Y.	1 812	1869	21 20
Schuylkill River, Fairmount, Pa.	1 915	1898	12 20
Chemung River, Elmira, N. Y.	2 055	1889	67.10
James River, Buchanan, Va.	2 058	1896	15 60
Androscoggin River, Rumford, Me.	2 220	1869	25.00
Susquehanna River, Binghamton, N. Y.	2 350	1902	26.60
Genesee River, Rochester, N. Y.	2 365	1865	17.00
Kennebec River, betw. Forks and Waterville, Me.	2 700	1901	48 56
Hudson River, Fort Edward, N. Y.	2 825	1900	15.60
Shenandoah River, Millville, W. Va.	2 995	1896	46.65
Connecticut River, Fairlee, Vt.	3 100	1913	18.50
Mohawk River, Rexford, N. Y.	3 384	1892	23.10
Mohawk River, Cohoes, N Y.	3 472	1913	28.50
Merrimac River, Lowell, Mass.	4 085	......	19 80
Kennebec River, Waterville, Me.	4 270	1901	35 36
Susquehanna, W. Branch, Williamsport, Pa.	4 500	......	11.60
Hudson River, Mechanicsville, N. Y.	4 500	1913	26.67
Merrimac River, Lawrence, Mass.	4 553	......	23 40
Potomac River, Dam No. 5, Md.	4 640	......	22 20
Delaware River, Lambertville, N. J.	6 500	......	53 80
Delaware River, N. J.	6 750	......	50.00
Delaware River, Stockton, N. J.	6 790	1841	37.59
Susquehanna River, Northumberland, Pa.	6 800	1889	17 50
Juniata River, Newport, Pa.	3 380	1889	53.80
Connecticut River, Holyoke, Mass.	8 660	1854	21.10
Potomac River, Point of Rocks, Md.	9 654	1889	48 90
Connecticut River, Hartford, Conn.	10 234	1854	20 00
Potomac River, Md.	11 043	......	42 60
Allegheny River, Freeport, Pa.	11 400	1891	26.60
Potomac River, Great Falls, Md.	11 427	1889	41.15
Potomac River, Chain Bridge, D. C.	11 545	1893	17.20
Susquehanna River, Harrisburg, Pa.	24 030	1889	30 60
Camp Brach River, Ensley, Ala.	7 43	1909	68 77
Cane Creek, Bakersville, N. C.	22	1901	1341 00
Bear Grass Creek, Louisville, Ky.	27.5	1908	100 00
Elkhorn Creek, Keystone, W. Va.	44	1901	1363.00
Tocca River, Blueridge, Ga.	231	1901	53.20
Middle Oconee River; Athens, Ga.	395	1902	49 52
Pacolet River, Spartanburg, S. C.	400	1903	88.90
Tygart Valley River, Belington, W. Va.	403	1907	40 88
Hiwassee River, Murphy, N. C.	410	1899	54 54
Coosawattee River, Carters, Ga.	532	1901	31.86
Occoquan Creek, Occoquan, Va.	546	1915	38 30
Tugaloo River, Madison, S. C.	593	1905	36 86
Etowah River, Canton, Ga.	604	1895	31 50
Tuckasegee River, Bryson, N. C.	662	1899	58.23
Little Tennessee River, Judson, N. C.	675	1901	85.30
Broad River, Carlton, Ga.	762	1902	38.22

Maximum Rate of Discharge of Streams in the United States (Continued)

Stream and place	Drainage area, sq mi	Date	Cu ft per sec per sq mi
Holston River, S. Fork, Bluff City, Tenn.	828	1902	39.80
Shenandoah River, N. Fork, Riverton, Va.	1 037	1901	20.86
Saluda River, Waterloo, S. C.	1 056	1903	18.00
Flint River, near Woodbury, Ga.	1 090	1913	32.35
Greenbriar River, Alderson, W. Va.	1 344	1913	44.76
Catawba River, Catawba, N. C.	1 535	1901	61.89
Chattahoochee River, Oakdale, Ga.	1 560	1899	27.92
Shenandoah River, S. Fork, Front Royal, Va.	1 570	1902	48.92
Ocmulgee River, Macon, Ga.	2 425	1902	20.97
New River, Radford, Va.	2 725	1900	63.78
Catawba River, near Rock Hill, S. C.	2 987	1901	50.50
Shenandoah River, Millville, W. Va.	2 995	1896	46.65
Chattahoochee River, West Point, Ga.	3 300	1901	26.86
Yadkin River, Salisbury, N. C.	3 399	1899	38.30
Tallapoosa River, Milstead, Ala.	3 840	1901	18.23
Coosa River, Rome, Ga.	4 001	1901	16.04
Broad River, Alston, S. C.	4 609	1901	28.44
Black Warrior River, Tuscaloosa, Ala.	4 900	1895	38.80
New River, Fayette, W. Va.	6 200	1899	17.83
Coosa River, Riverside, Ala.	6 850	1898	10.53
Savannah River, Augusta, Ga.	7 294	1884	42.50*
Roanoke River, Old Gaston, N. C.	8 350	1877	32.90
Kanawha River, Charleston, W. Va.	8 900	1875	13.50
Yazoo River, Miss.	13 850	....	10.04
Tennessee River, Chattanooga, Tenn.	21 382	1867	34.37
Ohio River, Wheeling, W. Va.	23 800	1884	20.80
Tennessee River, Florence, Ala.	30 800	1897	16.20
Ohio River, Louisville, Ky.	90 600	1913	8.49†
Mississippi River, Columbus, Ky.	930 540	1858	1.59
Mississippi River, Miss.	1 244 000	....	1.19
Cherryvale Creek, Cherryvale, Kan.	2	....	930.00
Loramie Res., Outlet, O.	72	1913	97.22
Devils Creek, Viele, Ia.	143	1905	1300.00
Whiteface River, below Meadowlands, Minn.	446	1916	13.15
Little Wolf River, Royalton, Wis.	485	1914	11.00
Olentangy River, Columbus, O.	520	1913	115.30
Silver Creek, near Lebanon, Ill.	535	1908	15.64
Des Plaines River, Riverside, Ill.	630	1889	20.80*
Milwaukee River, Milwaukee, Wis.	661	1915	7.98
Kettle River, near Sandstone, Minn.	825	1912	7.15
Scioto River, Columbus, O.	1 047	1913	80.82
Elkhorn River, near Norfolk, Neb.	2 470	1903	3.24
Sangamon River, Riverton, Ill.	2 560	1911	7.50
Saline River, Beverly, Kan.	2 730	1896	5.86
Verdigris River, Liberty, Kans.	3 067	1904	16.45
Wabash River, Logansport, Ind.	3 163	1904	17.99
Neosho River, Iola, Kans.	3 670	1904	20.30
Grand River, Grand Rapids, Mich.	4 900	1905	10.00
St. Croix River, Minn.	5 950	1883	6.00
Fox River, Rapide Croche Dam, Wis.	6 200	1895	2.49
Cedar River, Cedar Rapids, Ia.	6 320	1917	9.00
Chippewa River, Eau Claire, Wis.	6 740	1905	9.00
Smoky Hill River, Ellsworth, Kans.	7 980	1903	1.43*
Blue River, Manhattan, Kans.	9 490	1903	9.13

* Average flow for day of maximum discharge. † Maximum daily.

Maximum Rate of Discharge of Streams in the United States (Continued)

Stream and place	Drainage area, sq mi	Date	Cu ft per sec per sq mi
Illinois River, Peoria, Ill	13 480	1904	5.94
Loup River, Columbus, Neb	13 540	1896	5.17
Republican River, Junction, Kans	25 837	1903	1.80*
Mississippi River, St. Paul, Minn	36 085	1881	13.32
Mississippi River, Prescott, Wis	44 070	1881	2.50
Platte River, Columbus, Neb	56 900	1905	0.83
Kansas River, Lecompton, Kans	58 550	1903	3.98
Kansas River, Lawrence, Kan	59 841	1903	3.80
Mississippi River, Grafton, Ill	171 570	1883	2.10
Missouri River, Sioux City, Ia	323 462	1881	1.64
Baker Creek, Baker, Nev	10	1914	17.00
Willow Creek, Heppner, Ore	20	1903	1 800.00
Pinal Creek, Globe, Ariz	25	1904	560.00
Chalk River, Fillmore, Utah	38	1914	12.85
Gallinas River, Las Vegas, N. Mex	90	1904	129.10
Asay Creek, Hatch, Utah	96	1913	16.70
Ohanapecosh River, near Lewis, Wash	116	1909	64.70
Rio Mora, bclpw Mora, N. Mex	159	1904	139.70
Sapello River, Los Alamos, N. Mex	221	1904	36.70
Miller Creek, Lovella, Ore	270	1907	24.93
St. Regis River, St. Regis, Mont	278	1913	22.40
Rapid Creek, Rapid, S. Dak	320	1904	2.85
Carson River, E. Fork, Gardnerville, Nev	381	1904	8 69
Rio Mora, Weber, N. Mex	422	1904	65.70
Grand River, N. Branch, Haley, N. D	500	1913	11.60
Yakima River, Cle Elum, Wash	500	1915	51.20
Price River, Helper, Utah	530	1913	8.50
Moyie River, Snyder, Id	717	1913	11.15
Purgatory River, Trinidad, Colo	742	1904	61.20
Clearwater River, S. Fork, Grangeville, Id	940	1912	10 46
Cut Bank Creek, Cut Bank, Mont	971	1908	9 07
Redwater River, Belle Fourche, S. D	1 006	1904	8.00
Virgin River, Virgin, Utah	1 010	1912	11.90
Truckee River, Reno, Nev	1 070	1913	7.02
Cowlitz River, Mossy Rock, Wash	1 170	1906	43.50
Wenetchee River, Dryden, Wash	1 200	1913	20.10
Heart River, Richardton, N. D	1 250	1906	6.40
Hondo River, reservoir, N. Mex	1 387	1904	4 56
San Juan River, Arboles, Colo	1 390	1911	28.80
Canadian River, French, N. Mex	1 478	1904	105.56 †
Rogue River, Tolo, Ore	2 020	1909	23.90
Truckee River, Clark, Nev	1 740	1911	3.00
Yellowstone River, Corwin Springs, Mont	2 630	1911	8.67
Pecos River, Santa Rosa, N. Mex	2 649	1904	17.56
Canadian River, Taylor, N. Mex	2 832	1904	32.11 ‡
Salt Creek, at mouth, N. Mex	3 052	1904	4.10
Spokane River, Spokane, Wash	4 000	1894	8.80
White River, near Interior, S. D	4 090	1905	4.03
Clearwater River, Kamiah, Id	4 850	1913	15.80
Willamette River, Albany, Ore	4 860	1861	62.20
Guadalupe River, near Cuero, Tex	5 020	1903	14.20
Yakima River, Kiona, Wash	5 520	1906	11.50
Salt River, Roosevelt, Ariz	5 756	1893	36.0
Verde River, McDowell, Ariz	6 000	1893	24.05 §

* Average flow for day of maximum discharge. † Rate for 0.5 hour. ‡ Rate for 7 hours. § Rate for 24 hours.

Maximum Rate of Discharge of Streams in the United States (Concluded)

Stream and Place	Drainage area, sq mi	Date	Cu ft per sec per sq mi
Pecos River, Fort Sumner, N. Mex.	6 191	1904	7.29
Gunnison River, Whitewater, Colo.	7 863	1905	3.67
Rio Grande, Rio Grande, N. Mex.	11 250	1904	2.75
Canadian River, Logan, N. Mex.	11 440	1904	12.29 *
Salt River, Ariz.	12 000	1891	24.69
Yellowstone River, Huntley, Mont.	12 000	1907	4.03
Salmon River, Whitebird, Id.	13 600	1913	5.97
Humboldt River, Oreana, Nev.	13 800	1897	0.22
Pecos River, Roswell, N. Mex.	14 840	1904	3.75
Grand River, Fruita, Colo.	16 800	1909	3.81
Gila River, Florence, Ariz.	17 750	1891	7.50
Missouri River, Cascade, Mont.	18 300	1908	2.70
Big Horn River, Hardin, Wyo.	20 700	1908	1.97
Red River, Grand Forks, N. D.	25 000	1897	1.70
Clark Fork River, Metaline Falls, Wash.	25 600	1913	4.33
Colorado River, Austin, Tex.	34 200	1900	3.57
Red River, Ark.	97 000	....	2.32
Arkansas and White Rivers, Ark.	189 000	....	0.84
Colorado River, Yuma, Ariz.	225 000	1909	0.67
Columbia River, The Dalles, Ore.	237 000	1894	4.89
Missouri River, St. Charles, Mo.	530 810	1883	1.13
Mississippi River, St. Louis, Mo.	702 380	1883	1.28
Switzer Canyon, San Diego, Cal.	3.55	1916	188.00
Grand Central River, below Forks, Alaska.	14.6	1906	100.00
Arroyo Seco River, Pasadena, Cal.	16.4	1916	192.00
Yuba River, Bowman Dam, Cal.	19	....	31.60
Sweetwater River, Descanso, Cal.	43.7	1916	226.00
San Vicente Creek, Foster, Cal.	74.9	1916	248.00
Kruzgamepa River, Outlet Salmon Alaska.	84	1902	51.20
Otay River, Lower Otay Reservoir, Cal.	98.6	1916	379.00
San Jacinto River, San Jacinto, Cal.	108	1916	278.00
Sweetwater River, Jamacho, Cal.	172	1916	250.00
Santa Ana River, Mentone, Cal.	182	1914	46.70
Sweetwater River, Sweetwater Dam, Cal.	186	1916	177.00
Santa Ynez River, Santa Barbara, Cal.	207	1907	45.65
San Gabriel River, Azusa, Cal.	222	1916	180.00
Calaveras River, Jenny Lind, Cal.	395	1911	176.20
Chatanika River, below Poker Creek, Alaska.	456	1911	7.62
San Luis Rey River, Oceanside, Cal.	565	1916	169.00
Stony Creek, Fruto, Cal.	760	1904	29.21 †
Yuba River, near Smartsville, Cal.	1 220	1909	90.91
Tuolumne River, Lagrange, Cal.	1 501	....	30.60
San Joaquin River, Hamptonville, Cal.	1 637	1881	36.51 †
King River, State Point, Cal.	1 742	1901	25.22
American River, Fair Oaks, Cal.	1 910	1907	62.20
Birch Creek, Fourteen Mile House, Alaska.	2 150	1911	6.90
Kern River, Rio Bravo, Cal.	2 345	1897	2.3 †
Feather River, Oroville, Cal.	3 640	1907	51.37 †
Sacramento River, Iron Canyon, Cal.	9 295	1904	23.47 †
Sacramento River, Red Bluff, Cal.	10 400	1909	24.42
Yukon River, Eagle, Alaska.	122 000	1911	2.08
Chagres River, Alhajuela, Panama.	427	1909	398.10
Chagres River, Bohio, Panama.	779	1909	115.50
Chagres River, Gatum, Panama.	1 320	1909	93.90

* Rate for 12 hours. †Mean for day when discharge was a maximum.

43. Storage of Water

The Mass Diagram. To determine either the amount of storage necessary to utilize at an average or variable rate of consumption the entire flow of a stream or any part of it, or the amount of water that can be continuously utilized from a stream, the flow for a considerable period being known, the most simple process is by the aid of the so-called mass diagram. The reliability of this process depends upon the completeness and accuracy of the information as to the run-off of the stream. A comparison of the run-off data of the Ohio, Tennessee, Sudbury, and Colorado rivers by John C. Hoyt in Eng. News, April 23, 1908, vol. 59, p. 459, shows that a single year's record is entirely insufficient for reliable prognostication and that five-year periods in the humid sections usually give mean results within 10 percent of the normal average, but may be in error as much as 30 percent, and in the arid region in error from 4 to 88 percent. Ten-year averages in the humid region are usually accurate within less than 10 percent, but on the Colorado the error ranges from 1 to 48 percent. It may therefore be concluded that at least a five-year record should be secured or a much longer one constructed from rainfall data, to obtain results of reasonable accuracy in the humid regions, and that a prediction of run-off in the arid region should be adopted with extreme caution.

Having the desired record, the beginning for purposes of computation may advantageously be taken at the last of the latest low-water month of the earliest year, but such a starting point is not essential. The diagram is usually constructed with a time unit of one month, tho when only a part of the flow is to be stored a weekly unit may be preferred. The record should be arranged chronologically in a table, and after each item the sum of it and all the preceding items should be entered. These sums then represent the total quantity of water that has been discharged by the stream at the end of any period since the beginning of the tabulated observations, and may be exprest in cubic feet per second, or any other convenient unit. With unit of time as the abscissa and total discharge as the ordinate, the mass curve is plotted, and if the lowest possible line be drawn tangent to the curve at two points an ordinate to the line will represent the maximum quantity of water that could have been used from the stream for the period between the ordinate considered and the intersection of the tangent with the axis of abscissas, assuming the quantity to be used uniformly during the period and a similar use to continue indefinitely. This ordinate divided by the time above included will then give the quantity to be so used per unit of time. The amount of storage required to equalize this flow will then be represented by the maximum partial ordinate between the tangent and the curve.

Fig. 37 shows a mass curve for a river in Michigan. In this case, for the sake of distinctness, the mass data has been plotted as a stept curve rather than as a smooth one, discharge being in cubic feet per second per month. When this is done it must be remembered that the ordinates considered are always those to the right-hand end of the steps. The tangent line *bd* shows the average daily available yield to be, by the ordinate *d*, 59 532 cu ft per second divided by the time from the last of December, 1903, to January, 1909, inclusive, about 61.2 months, or 974 cu ft per second. The partial ordinate between the point *c* on the tangent and the right end of the step of the mass curve for April, 1906, gives 5605 cu ft per second for one month, or $5605 \times 60 \times 60 \times 24 \times 30 = 14\ 528\ 16$[illegible] cubic feet as the storage required to provide a constant flow of 974 cubic feet per second.

If instead of a constant use a variable one is desired, a mass curve of the variable flow should be plotted in the same manner as the original diagram, but this variable flow curve must be of such dimensions that it falls either below or is tangent to the original mass curve. Such curves will usually have an opposite phase to the mass curve and must fall below or tangent to the line *bd*, as shown by the lower sinuous curve of the f[illegible]

The storage required to provide a supply of the nature represented by this curve is shown by the partial ordinate at f between the consumption and the mass curves to be 7400 cubic feet per second for one month, or 19 180 800 000 cubic feet.

If it be desired to determine the amount of storage necessary to develop continuously a less quantity than the total flow of the stream, draw from the low points of the mass curve a line having an inclination parallel to the mass consumption line for the quantity desired. In the figure the lines *dg*, *ch*, *bk*, etc., represent such consumption lines, and the maximum storage required for the corresponding uniform consumption is represented by the partial ordinate for May, 1904, between the consumption line *bk* and the mass curve. The intersection of the consumption line, with the vertical of the mass curve, indicates the time when storage should have begun, in the particular year, to yield the consumption represented by the line, and the ordinates between consumption and mass curves represent the amount of water in

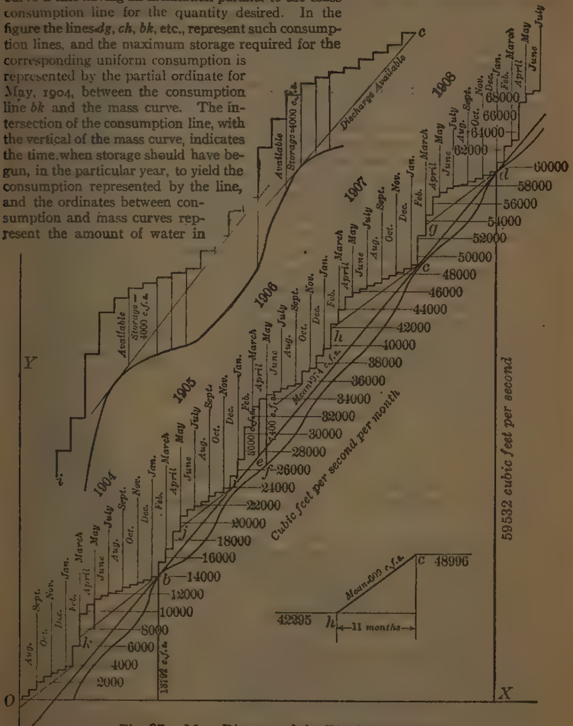

Fig. 37. Mass Diagram of the Tittabawasse River

storage at the time shown by each ordinate. The triangle in the lower right of the figure shows the rate represented by the lines *dg*, etc.

To determine the equalizing value upon the flow of the stream of a given amount of storage, plot downwards from the mass curve for the six high months of each year the magnitude of the storage in the unit of the diagram. Join the points so located in each year by a curve as in the upper diagram of the figure, in which *ic* is an enlarged representation of the mass curve from *i* to *c*, and from each of the succeeding low points of the mass curve draw a tangent to the curve just located. The minimum inclination of such tan-

gents which do not cut the mass curve will give the rate of consumption that may be uniformly utilized with the storage in question. This method fails when the storage considered is so great as to exceed the maximum partial ordinate between the tangent *bd* and the mass curve, as in that case the storage could not be filled by the stream between dry periods.

Corrections for Evaporation. When from the amount of storage needed as shown by the mass diagram and a study of the topography of the watershed the area of water surface of the required reservoir is determined, the evaporation may be compiled and the net yield of the stream determined. A corrected mass diagram may then be constructed for more accurate computations.

Bank Storage. The visible capacity of reservoirs is very materially augmented in sandy or gravelly districts by the water held in the adjacent ground between the high and low water planes. The volume of such storage is dependent on the slope of the natural water table and on the proportion of voids in the soil. The former can frequently be determined by observations on wells in the territory affected, and the latter for the most porous soils may amount to 40 percent of the total volume, but for ordinary sands and gravels may be safely taken at about 20 percent. For clay it is inconsiderable, probably about 3 or 4 percent, and for loam about 10 percent. Sandstone rocks take up about 15 percent of their volume of water, limestones 5 percent, shales 4 percent, and granites 1 percent. The difficulty of estimating the value of bank storage lies in the uncertainty as to the actual character of the strata below the surface. The water stored in the ground is given out much less rapidly than that from an open reservoir, and hence is of little value for sudden fluctuations of draft, but in a long gradual lowering of the surface of the reservoir will add materially to the quantity available.

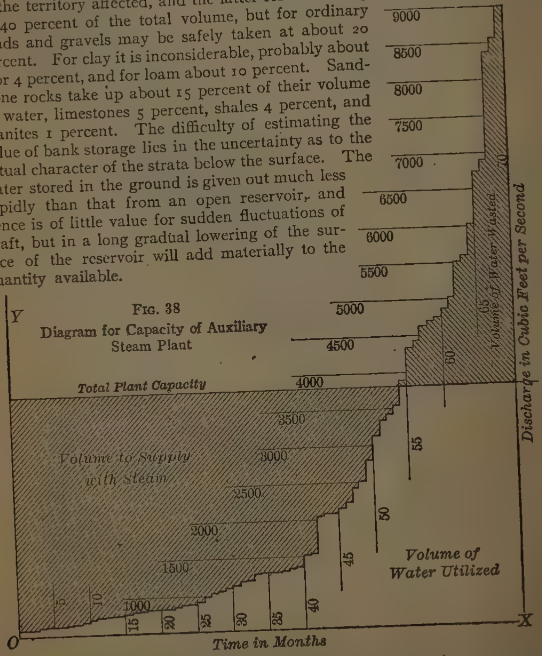

Fig. 38
Diagram for Capacity of Auxiliary Steam Plant

Emptying of Reservoirs. The time required to empty a pond or reser[voir] between known levels thru a certain opening, or over a weir of known dim[en]sions, is obtained from the general formula

$$t = \int_{H_2}^{H_1} \frac{A_r}{CA_0\sqrt{2\,gh}}\,dh \qquad (1)$$

in which A_r is the area of the reservoir at any elevation and A_0 the area of the orifice thru which discharge takes place, while H_1 is the elevation of the water surface, H_2 a lower elevation, h the height of the water surface above the center of the orifice, and C the coefficient of discharge. If A_r be variable it must be exprest in terms of h. When A_r is constant and $H_2 = 0$ this becomes $t = 2\,A_r\sqrt{H_1}/CA_0\sqrt{2\,g}$, which is twice the time required for the same quantity to be discharged under a constant head H_1. For a weir, the expression for weir discharge is substituted for that of the orifice in the denominator of (1).

(a) In practise the area of reservoirs will be variable, and where definite information as to the relation between A_r and h is not available, the reservoir volume may be assumed as semi-parabolic in longitudinal section and parabolic in cross-section and plan. With X as the length, Y as the half width, and Z as the depth, the area of the surface is $^4/_3\,XY$ and the area at any elevation is $A_r = {}^4/_3\,X_1Y_1Z/Z_1$. If a be the height of the center of the orifice or of the crest of the weir above the reservoir bottom $Z = h + a$ and $A_r = {}^4/_3\,X_1Y_1(h+a)/Z_1$, which may be substituted in (1) before integration. (b) Another assumption which may be made is that X is directly proportional to Z, which is equivalent to saying that the reservoir has a uniform slope lengthwise. In this case $A_r = {}^4/_3\,X_1Y_1\,(h+a)^{3/2}/Z_1^{3/2}$. (c) If it be assumed that Y also varies directly with Z, or that the sides of the reservoir slope uniformly to the center which is, however, a condition never met with in natural ponds then $A_r = {}^4/_3X_1Y_1(h+a)^2/Z_1^2$. Of the above assumptions that under (b) is the most likely to conforn to natural conditions, except where the reservoir has silted considerably, when assumption (a) will be more accurate. It may be added that since $^4/_3\,X_1Y_1$ in cases (a), (b), and (c) gives the area of the pond surface, if this be known it may be substituted at once for A_r, when the only assumption involved is that of the form of the vertical sections.

44. Auxiliary Steam Plants

Desirability. When sufficient storage cannot be economically secured to sufficiently equalize the flow to yield the required quantity of power, the deficiency may be made up by a steam plant. It is to be remembered that a steam plant depreciates about as fast when idle as when running, and that the interest charges are to be carried during the whole year. When operation, interest, depreciation, and maintenance of a steam auxiliary are capitalized it will often be discovered that storage would have been a better investment.

Capacity. The capacity of a steam plant necessary to make up the deficiency of flow is best determined by the use of a diagram of graded discharge, as in Fig. 38, in which, with the time unit as the abscissa, the discharges, either natural or equalized by storage as the plan may be, as ordinates, are plotted, not chronologically but in the order of their magnitudes from least to greatest. A horizontal line drawn thru the diagram, whose ordinates represent the desired total capacity, limits between itself and the curve the total amount of water that must have been replaced by auxiliary power, and for each month the amount is given by its individual partial ordinate between the capacity line and the curve.

Operation of Combined Steam and Water Plants. When a steam auxiliary is used, it will usually be advantageous to run it continuously during the periods when it must run at all and use the water power to provide for the fluctuations, if the size of the pond at the plant permits of this without wasting water. For the reason that a steam plant when idle is radiating heat, and hence not

only wasting interest and depreciation charges but also a part of the fuel cost, it is better to let the water power lie idle while the pond fills up, as by such procedure nothing is lost, the full value of the water being utilized. On this account the importance of a large pond at the station is apparent.

45. Design of Power Plant

Power House. The Power House should be so located as to be protected from floating ice and other debris. When situated in a gorge, the position should be such as to admit of a maximum of sunlight reaching the building. The intakes, whether direct to the wheel pits or to a closed flume or pipe line, should be so located that debris will readily pass by them. If water be flowing over the spillway or thru the waste gate, these structures may be advantageously put near by, in order that they may be controlled to keep the intakes clear.

To avoid ice troubles the intake to the wheelpits or closed flumes may best communicate directly with the pond, rather than with a canal or head race, and a curtain wall may wisely be carried a few feet below the water surface at the entrance. To facilitate inspection the entrance to the wheelpits should be provided with easily operated gates.

See paper by Benj. F. Groat in Trans. Am. Soc. C. E., Vol. LXXXII, for special constructions to divert ice.

Wheel Pit. For low and moderate heads, up to 40 or 50 feet, the open wheel pit is to be preferred to the closed flume or pipe line. Of this head 20 feet may be covered by the draft tube. Vertical wheel and umbrella type generators give higher efficiencies than horizontal shaft installations, due mainly to the reduction of bearings. The passages leading to the wheels should be so designed that the water is never retarded in its velocity as it passes toward the wheels but if possible is gradually accelerated. The losses between head and tail water in the best-designed plants, excluding the turbine loss, is about 4 percent of the total output for plants with heads below 20 feet, and may frequently amount to 12 percent. For higher heads the percentage is less, as there is little change in the rack losses and friction in flume, wheel pit, and draft tubes; while the power increases with the head. For approximate calculations a uniform loss of 1 foot may be taken for all heads.

The Draft Tube should expand gradually from the outlet of the wheel until an area of outlet is attained such as to require a velocity thru it a little greater than that of the water into which it discharges. If the draft tube have a turn, as great expansion should be secured before it is reached as is consistent with maintaining the angle of departure of the sides from the center line less than 8° when under a pressure below atmospheric, and less than 11° when under atmospheric or greater pressure. Immediately after passing into the curve it is well to contract the section in the direction of the radius of the curve and enlarge it at right angles thereto.

The proper design of the draft tube is one of the most important features of a hydraulic power plant as it may affect the efficiency and output of the turbines from 5 to 10 percent on low heads. A special form of draft tube, known as the hydracone, has been invented by W. M. White, Mem. Am. Soc. Mech. Engr., and is an application of the principle of the ball nozzle to the jet issuing from a short draft tube, the ball being replaced by a plate or a cone of rigid material, located at a particular distance from the outlet. Fig. 39.

This device utilizes to some extent the centrifugal force of the discharging

water and on tests has increased the efficiency about 3 percent over the be obtainable with the ordinary form of tube.

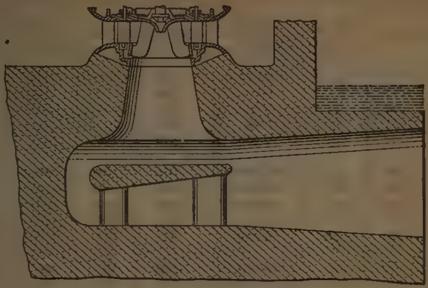

Fig. 39. White's Hydracone

Head or Fall Increasers are devices connected with the draft tube to increase the discharge by reducing the tail-water pressure. The most effective device of this kind appears to be that designed by Clemens Herschel, which is an application of the principle of the Venturi tube thru the throat of which water is passed from the pond drawing into itself thru apertures in the throat, water discharged by the wheel. (See Eng. News, vol. 59, June, 1908, p. 635.)

Another device has involved the surrounding of the discharge from the wheel in the vertical part of its course with a stream from headwater.

By the introduction of a jet from headwater at the bottom of a curved draft tube the author has increased the output of a plant in flood time between 5 and 6 percent and by a utilization of the flow over the dam, if adjacent to the power-house, a considerable further advantage can be secured.

The Generating Unit. In the earlier installations horizontal shaft generators were largely used, either belted or geared to the turbines, or driven directly from a shaft on which several turbines were mounted. Tests of such installations have shown them to be considerably less efficient than the vertical-direct-connected units, and the multiple wheel unit, whether vertical or horizontal is not now looked upon with favor except under very special conditions.

The direct-connected unit requires that both turbine and generator be adapted to the same speed of rotation.

In selecting machinery the best speed of rotation is one slightly less than one-half the so-called spouting velocity, or velocity due to the head, but by specially designed wheels this can be increased as much as 50 percent, which gives a fairly wide range of generator speeds from which to select.

The table on p. 1182 gives data as to the vertical water wheel generators, developed by the Allis-Chalmers Manufacturing Company, and may be taken as fairly representative of the machines of other manufacturers.

Current. Except where current is to be used in the immediate vicinity of the plant or for arc lighting and electric railway service, direct-current generators are not commonly used in the United States. For incandescent lighting, 3-phase 60-cycle alternating-current generators have a preference, while for power work 25 or 30 cycle machinery is considered more satisfactory. The

Vertical Water Wheel Generators
Allis-Chalmers Manufacturing Company
60 Cycles, 2 300 Volts or Less, 2 or 3 Phase

K.V.A.	R.P.M.	Outside dia in in	Approx. total wt	K.V.A.	R.P.M.	Outside dia. in in	Approx. total wt
65	277	73.5	9 000	500	100	148	45 000
75	277	73.5	9 100	500	90	185	65 000
85	277	73.5	9 850	500	60	225	90 000
100	277	73.5	10 000	525	300	95.5	22 000
110	277	73.5	12 500	530	150	137	36 500
125	277	73.5	13 000	600	300	95.5	24 000
150	720	48	5 500	625	150	137	42 000
150	150	107	20 000	625	120	148	50 000
150	120	128	24 000	625	100	148	55 000
187½	720	48	5 600	675	180	138	38 000
187½	150	107	21 000	750	150	137	44 000
187½	120	128	24 500	750	120	148	53 000
200	150	107	22 500	750	100	148	57 000
200	120	128	28 500	800	100	195	63 000
250	150	107	24 500	800	180	138	40 000
250	120	128	29 500	850	128	194	67 000
250	90	151	34 000	850	82	195	83 000
250	64	195	48 000	940	180	171	58 000
300	90	151	35 000	940	100	195	70 000
300	64	195	50 000	1 000	82	195	85 000
312	300	95.5	19 000	1 080	150	192	72 000
312	150	107	28 000	1 080	128	194	77 000
312	120	128	31 000	1 250	180	171	68 000
312	100	148	35 000	1 250	150	192	75 000
325	180	138	28 000	1 250	128	194	80 000
375	300	95.5	19 500	1 250	100	195	90 000
375	180	138	30 000	1 750	150	192	102 000
375	150	137	33 000	1 875	164	194	90 000
375	120	128	33 000	2 000	150	192	105 000
375	100	148	35 000	2 200	164	194	92 000
425	120	148	38 000	2 400	150	192	116 000
425	100	148	45 000	2 500	164	194	100 000
425	90	185	60 000	2 800	164	194	105 000
425	60	225	87 000	2 800	150	192	120 000
437	180	138	32 000	3 120	150	214	132 000
450	150	137	35 000	3 750	150	214	135 000
450	300	95.5	22 000	5 000	150	214	220 000
500	180	138	33 500	6 250	150	214	230 000
500	120	148	40 000	8 500	144	242	280 000
				10 000	144	242	280 000

current is frequently generated at about 2300 volts pressure, and is transformed upward to the line voltage for transmission. The line voltage may be anything between 110 and 120 000 volts; the higher the pressure the more power that can be delivered over a given size of conductor, and consequently the low voltages are only used in local distribution or inside buildings.

Bearings. The bearings should be accessible at all times, and in the case of a vertical shaft installation the entire load may be wisely concentrated on a single thrust bearing of special design. Such bearings may be placed either above or below the generator, and according to the loads to be carried may be of the simple plate type, roller bearings, the Kingsbury or the Spring bearing, or for very heavy loads the oil pressure bearing.

Of these the PLATE BEARING consists of two plates, the lower of which is supported in a fixed position while the upper is fastened to the shaft and revolves with it, resting upon a film of oil between it and the lower plate, both plates being immersed in a bath of oil. Such bearings are satisfactory for speeds encountered in low head plants.

The ROLLER BEARING consists of two plates as above with the film of oil replaced by a cage of rollers, either conical and of considerable length, or cylindrical and of length not greater than their diameter. The plates and rollers of this bearing are all immersed in an oil bath.

The KINGSBURY BEARING replaces the cage of rollers with a series of segments or shoes supported by a pivoting surface near one end and slightly beyond the center of gravity in the direction of rotation. This causes the space between the segment and the upper plate to be more open at one end than the other so that a wedge shaped film of oil is drawn in by the revolving disk. The parts of this bearing are immersed in an oil bath and the bearing operates satisfactorily under surface pressures of 400 to 500 lbs per square inch in high speed turbine installations.

The SPRING BEARING introduces a series of spiral springs fixed on the lower plate and carrying above them a rubbing plate which takes the load of the upper plate on a film of oil. On account of the ready adjustment of the springs to irregularity of load the bearing can be made of smaller diameter than the others and hence occupies less space. Like the preceding the parts are immersed in a bath of oil.

The OIL THRUST BEARING provides for the introduction of oil under a high pressure to a chamber in the plates near the shaft from which it flows out between the plates insuring complete lubrication. The loads carried depend upon the pressure of the oil which is circulated by a positiv piston pump. These bearings are used for the heaviest loads and the highest speeds, and are particularly adapted to steam turbine work.

The Exciter may be most economically driven from the main turbine shaft, but in many plants an independent exciter driven by a separate wheel is installed. After the plant is once started, and as long as it continues in operation, the excitation may be provided by a motor generator or rotary converter transforming the alternating current of the main generators into direct current for their excitation. The economy of this is slightly less than that of the separate exciter so far as operation is concerned, but when interest is considered may prove more economical, so that a single separate exciter may be provided to excite the first machine of a series and the rest be operated from the main current as above indicated. The exciting power usually ranges from 0.5 to 3 percent of the power at full load, being nearly constant in amount whether the main generator is loaded to full capacity or not.

The Switchboard should be so located as to enable the operator to command a view of the machines which he controls, and should have the following instruments: (a) On generator panel, 1 voltmeter, 1 ammeter for each phase, 1 frequency meter and 1 field ammeter. (b) On feeder panel, 1 wattmeter. (c) On exciter panel, 1 voltmeter and 1 ammeter. (d) On a bracket

1 synchroscope when more than one generator is installed. To which are added the several switches and connections, and for studying the station output an integrating wattmeter should be placed on the main circuits.

The Governor for turbines operates upon the gates and must be capable of exerting a large amount of power at times. This is usually accomplished by a piston driven by oil which is maintained at a high pressure in a reservoir with comprest air by the aid of a force pump operated from the turbine shaft. The action of the piston is controlled by valves moved by the governing mechanism proper. For less rapid operation governors depending on simple mechanical friction are utilized, but these governors are not sufficiently delicate for combined lighting and power loads. In governing impulse wheels, less power is needed, and the nozzle is frequently deflected away from the bucket without altering the flow. In the Doble Nozzle a tapered pin is pushed forward thru the center to close and pulled back to open, either operation requiring a relatively small amount of power. The governor should be so located as to be under the eye of the attendant at the switchboard, unless a second attendant is available to look after it.

The usually accepted equation for determining the torque T of the regulating shaft resulting from the operation of the gates, is:

$$T = \frac{C \times \text{horse-power of turbines}}{\sqrt{\text{head}}}$$

Where C has a maximum value of 50 and a minimum of 25.

For low head open flume plants the percentage temporary change in speed for load thrown off is given by:

$$d = 81\,000\,000 \frac{(\text{H.P.}) \times t}{WR^2 \times (\text{R.P.M.})^2};$$

d = percentage change of speed;
(H.P.) = maximum horse-power of turbine;
t = time in seconds occupied by governor in moving gates through their range;
W = weight of rotating parts;
R = radius of gyration of rotating parts;
R.P.M. = normal revolutions per minute.

Crane. As this piece of apparatus is only used occasionally, it may wisely be of the hand-operated type where first cost is an item of importance. Cranes operating with chains are preferable to those using cable for power-house work.

The Racks. The loss of head thru the racks is of considerable importance, particularly in low-head plants, and may be considerably reduced by using lenticular or fish-shaped rack bars. A platform from which to rake debris from the racks and a chute or other device for removing it is important.

46. Operation of Plant

The Load Factor of a plant is the ratio of the average to the maximum load carried. For economical operation this should be brought as near unity as possible, and it becomes good policy to sell the output at a low rate during the low-demand hours to stimulate consumption at such times.

In ordinary plants supplying municipal, domestic, manufacturing and commercial service the load factor is usually between 40 and 50 percent, which means that for a large part of the time the plant runs at part capacity, while late in the afternoon it has for 1 or 2 hours a peak load of nearly the maximum amount, the actual maximum covering only a few minutes. The result of this is that such a plant must be designed to deliver very much more than the average capacity of the water utilized. Usually the machinery installed is made capable of delivering the entire daily output in from 6 to 8 hours.

The Plant or Capacity Factor is the ratio of the average load to the rated capacity of the plant.

The Diversity Factor is a term used to indicate the diversity of uses in which the output is absorbed and may be defined as the ratio of the sum of maximum demands of the subdivisions of the load to the maximum demand of the whole system, measured at the point of supply.

The Connected Load is the combined continuous rating of all the receiving apparatus on the customer's premises connected to the system.

The Demand Factor is the ratio of the maximum demand to the total connected load.

The Power Factor of a plant is the ratio of the effective power in watts to the volt-amperes, all measured at the switchboard. The existence of a power factor, which is peculiar to alternating-current installations, is due to the fact that there usually exists in a line an induced current which neutralizes in part the power current. The effect of the induced current may be reduced by special expedients, but with ordinary motor loads is likely to cause the power factor to drop to 0.85, and in some distributions it may fall as low as 0.30. The power factor is determined by the load and not by the generating apparatus, and so far as the latter is concerned should be taken as unity unless otherwise specified.

Except as the efficiency of the generator may be affected by a change in power factor, there is no effect upon the input to the generator, hence the turbine requirement is always based on the kilowatts delivered and not on the volt-amperes.

Regulation. If the speed of the generator changes, the voltage or pressure changes and the brilliancy of illumination and power of the output varies. Reducing speed reduces voltage, and vice versa. To maintain constant speed with a varying load requires a reduction or increase of gate opening, and where the wheels are supplied thru a pipe line, a consequent change in velocity and pressure thru the conduit.

47. Testing Code for Hydraulic Turbines. (Abstract)

(Approved by Machinery Builders' Society, Oct. 11, 1917)

Introduction. This code is intended to apply to acceptance tests of hydraulic machinery.

Efficiency of the plant takes account of all losses of energy between head water and tail water outside the plant and as far as, or through the switchboard.

Efficiency of the machinery takes account of all losses of energy between the water in the wheelpit and tail water and as far as the generator terminals or switchboard.

Efficiency of the turbine takes account of all losses of energy between the water in the wheel pit and tail water and as far as the coupling on the turbine shaft. Losses through racks, in intake to penstock and in penstock shall not be charged against the turbine, nor shall the head necessary to discharge the water from the end of the draft tube. The net or effective head acting on the turbine shall be measured from a point near the intake to the turbine casing in turbines equipped with casings, or from a point immediately over the turbine in turbines having an open flume setting, to a point in the tail-race, and a correction for the velocity head required to discharge the water into the tail-race shall be added to the tail-water elevation; and a similar correction applied at the intake to encased turbines. The power developed by the turbine shall be taken as the mechanical power delivered on the turbine shaft and transmitted by the turbine shaft to the generator or other driven machine or system.

General. When the contract calls for the performance of the guide vanes, runner, draft chest and draft tube only, the velocity head at entrance to the casing, shall be excluded, and the pressure shall be measured by piezometers so connected to the casing as to avoid velocity effects, instead of at entrance to the casing.

Apparatus shall be inspected before, during and after tests.

Testing apparatus must not interfere with operation of unit under test, which shall have been operated under load for at least 3 days prior to test.

Leakage of air into wheel or draft tube or of water out of wheel pit or penstock must be prevented or measured and allowed for.

Tests should not be made when variations of load exceed 3 percent; of head 2 percent, and of speed 1 percent, above or below the average of each.

Important instruments shall be installed in duplicate and calibrated before and after test.

Power Output. Power output may be measured by the electrical generator, or by a Prony Brake or Dynamometer. If by a generator, the latter must be tested either in the shop or in the field and its efficiency curve established for the range covered by the tests, in accordance with the Standardization Rules of the Am. Inst. E. E. of September, 1916. The output of the wheel then is the kilowatt output of the generator divided by its efficiency and reduced to horse-power, the exciting current not being charged against the turbine. If the exciter be driven from the turbine then its output similarly computed shall be added to that of the main generator.

When a dynamometer, either of the Prony brake, friction disk or other type is used, the dynamometer is to be so arranged as to avoid imposing either end thrust or side thrust on the turbine shaft and bearings, or to avoid adding any friction load which is not measured, and the brake must be capable of operating with the weighing beam floating free of the stops during the entire duration of a run.

Power Input or Water Horse-power. The turbine shall be tested, if possible, under the effective head stated in the contract, and at the speed specified in the contract.

Variation of 10 percent in head may be allowed if the speed be adjusted to correspond according to the law that

$$\text{Speed varies as } h^{1/2}$$

$$\text{Power varies as } h^{3/2}$$

Errors in adjustments of speed to head may be corrected for up to 2 percent, on the basis of test curves of the same or a similar turbine. The hydraulic equivalent of the speed is equal to the specified speed multiplied by the square root of the ratio of the effective head existing during the test to the specified effective head. The hydraulic equivalent of the horse-power is equal to the specified horse-power, multiplied by the three-halves power of the ratio of the effective head existing during the test to the specified effective head. Tests shall not be run when the head differs from the specification by more than 10 percent, or when the total draft head approaches within 5 feet of the height of the barometeric water column.

For turbines having closed casings the head is to be measured by at least two and when possible not less than four piezometers connected to a straight portion of the penstock near the turbine casing intake, and by two or more board, ro

or float gages in the tail-race, placed at points reasonably free from local disturbances.

The conditions of measurement, including velocity distribution, length of straight run of penstock, and conditions of piezometer orifices shall be such that no piezometer shall vary in its readings by more than 20 percent of the velocity head from the average of all the piezometers in the section of measurement. The piezometer orifices shall be flush with the surface of the penstock wall, the passages shall be normal to the wall, and the wall shall be smooth and parallel with the flow in the vicinity of the orifices. The piezometer orifices shall be approximately ¼ in in diameter.

The effective head on the turbine is taken as the difference between the elevation corresponding to the pressure in the penstock near the entrance to the turbine casing, and the elevation of the tail water at the highest point attained by the discharge from the unit under test, the above difference being corrected by adding the velocity head in the penstock at the point of measurement and subtracting the residual velocity head at the end of the draft tube. The velocity head in the penstock shall be taken as the square of the mean velocity at the point of measurement, divided by $2g$; the mean velocity being equal to the quantity of water flowing in cubic feet per second, divided by the cross-sectional area of the penstock at the point of measurement in square feet. The residual velocity head at the end of the draft tube shall be taken as the square of the mean velocity at the end of the draft tube, divided by $2g$; the mean velocity being equal to the quantity flowing in cubic feet per second, divided by the final cross-sectional discharge area of the closed or submerged portion of the draft tube in square feet.

For turbines set in open flume, the head is to be measured by board, rod or float gages, located above the center of the turbine, and by board, rod or float gages in the tail-race, all gages being placed at points reasonably free from local disturbances, and not less than two gages being installed in the flume and not less than two in the tail-race.

Such gages are to be free of velocity effects, and if this is not obtainable when the gages are set in the open channel, they shall be placed in properly arranged stilling boxes.

The effective head on the turbine is to be taken as the difference between the elevation of the free water surface immediately above the center of the turbine, and the elevation of the tail water at the highest point attained by the discharge from the unit under test, the above difference being corrected by subtracting the residual velocity head at the end of the draft tube, computed as in the previous case.

Quantity of Water. The quantity of water discharged from the turbine is to be measured by weir, current meter, Pitot tube, screen or diaphragm, or by the chemical method, in accordance with the methods described in Articles. 22, 23 and 24 of this section.

When the quantity of water is measured by weir, weirs with supprest end contractions shall be used.

The weir or weirs shall if possible be located on the tail-race side of the turbine, and care shall be taken that smooth flow, free from eddies, surface disturbances or the presence of considerable quantities of air in suspension, exists in the channel of approach. To insure this condition the weir should not be located too close to the end of the draft tube, and stilling racks and booms should be used when required. The channel of approach should be straight, of uniform cross-section and should be unobstructed by racks and booms, for a length of

at least 25 feet from the crest. The racks should be arranged to give approximately uniform velocity across the channel of approach. The uniformity of velocity should be verified by current meter or otherwise.

When the discharge is measured by current meter, observations shall be taken by two different types of meter, one type having preferably such characteristics that it will slightly over-register under conditions of turbulent or oblique flow, and the other type having characteristics such that it will under-register under similar conditions. The true velocity obtained by reducing the meter readings on the basis of their still-water ratings may then be taken as a weighted mean between the two series of observations.

The point method of observation shall be used and sufficient points shall be obtained to enable both vertical and horizontal velocity curves to be plotted for all portions of the section of measurement. The average velocity shall be determined from these curves by planimeter.

When the Pitot tube method is used, the Pitot tube shall be located in a straight run of penstock or conduit, at a distance equal to at least ten pipe diameters from any upstream bend and at least five diameters from a downstream bend. When the observation is made in a circular pipe or penstock, at least two Pitot tubes shall be arranged to traverse two relatively perpendicular diameters, but in the case of very large penstocks or those having unsymmetrical flow. Pitot tubes shall be arranged to traverse completely or partially the intermediate diameters, giving traverses at 45° intervals.

When the screen method is used, the length of run of the screen shall be sufficiently in excess of the portion used for measurement to provide ample space for starting and stopping the screen so as to insure uniform conditions over the measured portion of the run. In determining the discharge the velocity of the screen shall be multiplied by an area intermediate between the net immersed area of the moving screen and the average area of stream cross-section of the portion of the channel traversed. The variation of the level in the flume shall be observed during the course of the run and the average elevation shall be used in determining the area.

When the chemical method is used, samples shall be taken from points distributed over the entire sampling section.

SECTION 10

WATER SUPPLY, SEWERAGE

BY

ALLEN HAZEN

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

George C. Whipple, M. Am. Soc. C. E., assisted in writing the second, fourth, and fifth chapters.

No revision of costs is attempted: Figures refer to conditions before the war.

COLLECTION OF WATER

1. Intakes

Intakes are structures built out into a body of water for the purpose of drawing water for use. The position of intakes is often affected by considerations of local pollution when sewage is allowed to flow into the same body of water from which the supply is taken. This is commonly the case in cities located upon rivers and great lakes. The depth of intake is frequently a matter of importance where water of different qualities is to be obtained at different levels. There are three types of intakes: (1) Unprotected intakes, (2) Submerged intakes, (3) Exposed or tower cribs.

Unprotected intakes are used for small supplies. The pipe is allowed to terminate at the desired point, sometimes being protected by a coarse screen. A fine screen is not permissible because it will be clogged by matters carried by the water.

A Submerged Crib is a structure built on the bottom of the lake or river from the interior of which the water is taken. It serves the purpose of roughly screening the water and also of protecting the end of the intake pipe from damage. Exposed or TOWER CRIBS are structures built on the bottom of the river or lake and extending above high water. They are frequently provided at different levels with ports controlled by gates, and screens may be located in their interiors. Tower cribs have many advantages for large supplies. The ports may be closed and the water pumped out of the intake pipe and everything inspected for tightness and condition. Screens in them may be reached for cleaning and repairs. Tower cribs require excellent foundations and they must be built strong enough to withstand ice pressures. In cold climates they are only used for large supplies. In warmer climates where ice pressure is not effective they are also used for small supplies.

Intake pipes or conduits are the connecting channels between the intakes and the shore. Intake pipes up to 36 inches in diameter are generally of cast iron. From 48 to 72 inches in diameter the most common material is riveted steel pipe. For larger intakes tunnels driven under the bed of the lake reaching from the shore to the bottom of an exposed crib are most common.

Long intakes in lakes should be of ample size to avoid friction and excessive suction on the pumps. Short intakes in sediment-carrying rivers should be smaller with velocities to prevent them from filling up with silt. Where pipe is laid on the bottom of a lake or river a channel should be dredged for it so that the top of the pipe is well below the natural bottom, but such dredging may be omitted in lakes deeper than about 30 ft.

Cast-iron Pipe for intakes is commonly of a special type. From four to eight 12-foot lengths of ordinary pipe are joined in the ordinary way and then a special joint is used (Figs. 1 and 2) which is made flexible with a turned spherical casting moving in a bed of lead which has been run inside of an enlarged bell.

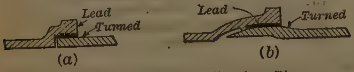

Fig. 1. Special Joints, Cast-iron Pipe

Such joints, well made, give a considerable amount of flexibility while remaining nearly water-tight. The pipe is put together above water and gradually lowered to position, the flexible joints making this possible.

Where the water is not too deep to allow a diver to work comfortably, the flexible joints are sometimes omitted and flange joints bolted together by a diver are used. Numerous other kinds of joints are also used.

Trouble from leaks in intake pipes has frequently been experienced where the water thru which they past was subject to pollution. Where the intake passes under highly polluted water it is safer to have it below the bed of the river or lake, well covered with sand or other protecting material.

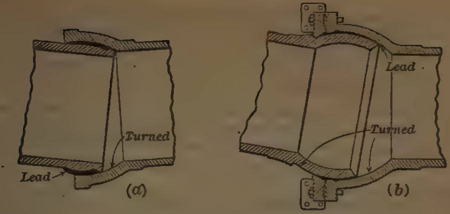

Fig. 2. Special Joints, Cast-iron Pipe

Steel Pipes for intakes are laid in much the same way as cast-iron pipes. Flexible joints are riveted to the ends of the steel pipes where required (Figs. 3 and 4), but the length of steel pipe between such joints may be greater, as steel pipe is stronger and more rigid than cast-iron pipe. Steel pipe is frequently designed to fit closely the contour of the bottom and it can then be put together with ordinary flange joints bolted up by a diver.

The difficulty of making such joints with divers increases rapidly with the depth. Up to 30 or 40 feet there is but little difficulty. Beyond this depth the difficulty increases and the joints practically become impossible before 100 feet of water is reached.

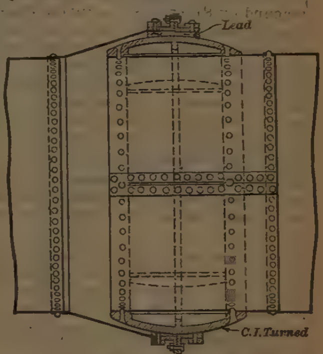

Fig. 3. Flexible Joint of Rochester Steel Pipe

Tunnels under lake or river bottoms are most safely and cheaply driven in rock. Intakes under the lakes at Cleveland and Chicago have been driven in clay. The greatest difficulty has been from marsh gas contained in the clay, which comes into the tunnels during construction forming an explosive mixture. There has been great loss of life and property from accidental explosions. Where clay is free from marsh gas it is

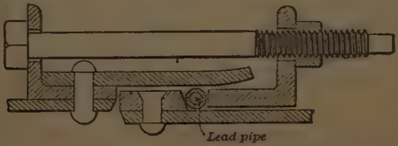

Fig. 4. Flexible Joint of Erie Steel Pipe

a good material to tunnel thru. Tunnels may be driven thru sand and gravel, but with greater difficulty and expense. Air pressure must always be used

corresponding to the distance that the tunnel is below the surface of the water. The length of time that men can work under pressure decreases rapidly as the pressure increases, so that such tunnels cannot be driven at too great a depth. Sixty or eighty feet represents the ordinary limit and about 100 ft is the extreme limit.

Anchor Ice is troublesome in northern climates in the case of intakes taking their supply near the surface or in shallow water. Much trouble has been experienced from this cause. A steam boiler furnishing steam to be taken to the seat of trouble thru a hose is the most effective means of maintaining the supply.

Pumping Stations are commonly at the shore end of an intake pipe or tunnel. The pumps must be so placed as to be able to take water at the lowest water level and they must also be protected from water during the highest floods. In the case of rivers having a wide range of water level the protective structures are frequently very expensive.

2. Data of Yield and Storage

Impounding Reservoirs are artificial lakes built on upland streams for the purpose of storing freshet flows for use during those times when the natural flow of the stream is insufficient to maintain the supply. Impounding reservoirs are formed by the construction of a dam, preferably across a narrow valley at a point where there is an enlargement of the valley above to give large storage capacity. A reservoir on a tributary is substantially as useful in maintaining the supply as a reservoir on the main stream, up to the point where the reservoir on the tributary can be filled from bottom to top by the run-off of an ordinarily dry year.

Storage and Yield. To estimate the yield of an actual reservoir, or of a reservoir to be built, or to estimate the size of reservoir required to develop a given output, the best starting point is a record of the flow of the stream feeding it.

From the flow record make a mass diagram by the method described in Section 9, Art. 43. Assume a rate of draft and find the maximum depletion that would have occurred at that rate during each year. These maximum annual depletions form a series. The series may be examined by probability methods, and the amount of storage that will be exceeded on 5% of the years may be accepted as practically sufficient. The calculation is repeated for other rates of draft until enough points are found to make a storage diagram which will serve as a basis for the calculation.

95% Dry Year Used. The 95% dry year is used as a basis for municipal water supplies. It is defined as the year of such dryness that 5% of the years are dryer and 95% of the years are wetter than it. The dryer years are the years that require more storage. For the 5% years that are dryer than the normal less than full supply will be available. On Eastern data about 94% of the full supply on an average will be available in these dryer years.

For extra conservative estimates, a 98% dry year may be used. This in a general way will call for 14% more storage. If the 99% dry year were used, 25% more storage would be required.

American cities have experienced moderate water shortages at intervals, and building storage that will be used less than once in twenty years is only warranted where the cost of storage capacity is unusually low.

Probable Error. The probable error (due to variation in annual quantities) in the mean flow of a stream is found by the formula

$$\text{Per cent of probable error} = \frac{67.5 \text{ Coefficient of Variation}}{\sqrt{T}}$$

T being the number of years in the record. Thus, for a 30-year record on an Eastern stream having a coefficient of variation of 0.25, the probable error is 3%; and for a Western stream a 10-year record with a c.v. of 1.10, the probable error is 23.5%. The errors in gaging must also be considered. No estimate can be more accurate than the data on which it rests, and available data do not often permit precision.

Data for Other Streams. If the record of the stream for which an estimate is to be made were sufficiently long and accurate, no other data would be needed for computing storage; but in most cases records are short or absent. A good short-term record is useful but the probable error is large. It is best to give weight to available records of flow and storage of other streams as similar as may be to the one for which an estimate is required.

Rainfall. Runoff comes from rainfall and follows it. Rainfall data are more abundant than runoff data. With meager runoff data there is a temptation to try to find relations between rainfall and runoff and to build up theoretical runoff tables and use them as a basis of estimate. Something can be done but great skill is required, for the relations are exceedingly complex and depend upon unknown and only partly known factors. Years ago when runoff data were less common there was more need of such expedients Now, the records of the U. S. Geological Survey are numerous and widely distributed. These and other gagings will nearly always serve as a safer starting point.

Cycles. A close study of rainfall data discloses cycles. Corresponding cycles in runoff are to be expected. As a practical matter not much attention need be given to them. But if a short term record is used, get the record of the nearest stream that has been long continued, and from it find whether the short term period has been unusually dry or wet. Numerical corrections for such differences are difficult, but a general idea of relative conditions is obtained.

Daily and Annual Storage. The required storage is made up of two parts: (1) daily storage required to balance fluctuations of flow within a single year; and (2) annual storage required to carry over surplus of wet years and make it available in dry years.

Methods of collecting, combining and stating the results are given, T. A. S. C. E., 77; p. 1539; 1914; but since that publication more data and some improvements in methods have become available and are used in preparing the tables that follow.

Practical Procedure. The mean flow and other data to be described are obtained for each stream flow from its record. With these data at hand, an estimate of the mean flow is made for the given stream. The required storage or the available output may then be computed with the aid of the tables that follow.

The mean annual flow is conveniently exprest in inches of runoff. To find the mean flow in gallons, multiply the area in square miles by inches of runoff and the product by 47 581 to obtain the mean flow in gallons per day, or by 17 380 000 to obtain the mean flow in gallons per annum.

Owing to the variations in flow and the irregularity of the variations, the whole of the mean flow of a stream can never be utilized. Some of it will be lost in flood flows. The more storage provided, the higher the percentage of the mean flow that can be utilized.

Three tables are given showing the storages required to yield various percentages of the mean flow for different parts of the country and with different

degrees of development. These are followed by Tables 4 and 5, giving statistics of actual water supplies and stream flows for comparison and to aid in selecting the proper starting points for the estimate.

Table 1 is for streams north of the Potomac and east of the Alleghanies, and for low developments with not more than 50% of the mean flow utilized. In this table the second line of classification is according to ground storage.

Ground Storage. Ground storage is an important element in stream flow. Streams having extensive deposits of sand and gravel on their catchment areas have better maintained flows than where the materials are impervious. Ground storage may amount to several inches of runoff. Where ground storage exists, artificial storage may be less, and vice versa.

Ground storage is best exprest as day's supply at the rate of draft and represents the reduction in needed artificial storage below the amount otherwise required. Table 4 shows the computed ground storage in days for a number of actual supplies. In Table 1 four grades of ground storage are shown. In judging of the amount of ground storage in the absence of definite data, a stream that goes dry in droughts has no ground storage; one that has very well maintained flows in droughts has abundant ground storage.

Table 1 may also be used for streams on the western slope of the Alleghanies and on the South Atlantic Coast, but with less confidence, and greater variations from it must be expected. It should not be used for any other part of the country. For corresponding low developments in the West, data are not available to permit general statement, and local data must be studied by the mass diagram method.

Table 2 is for high developments, where more than 50% of the mean flow is made available, and when the reservoir will not refill in dry years and stored water must be carried over from year to year. In this table the second line of classification is the relative variation in annual flow. Some streams vary from year to year much more than others, and the amount of storage is dependent more upon this characteristic than any other. The coefficient of variation is the best index of this variation and is used as a basis for classification.

Coefficient of Variation. To compute the coefficient of variation, make a table of annual flows in any convenient units and find the mean. The difference between each term and the mean, without regard to sign, is the variation. Each variation is squared and the sum of the squares is found. Divide this by the number of terms less 1. The square root of the quotient is the *standard variation.* The *coefficient of variation* (c.v.) of annual flows is the standard variation divided by the mean.

The coefficient of variation of annual flows should be calculated for each record of ten years and over that is to be used. It is well to take into account records of neighboring streams, and it will be safer to use the indications of good long-term records of neighboring streams than a value derived from a short-term record of the stream under consideration.

For streams having coefficients of variation under 0.50, use Table 2. This applies to all that part of the United States east of the Mississippi River and also to Oregon and Washington. It also applies to eastern Canada. It is not recommended that a value below 0.20 should be used for any estimate; and the table shows no lower values. North of the Potomac and east of the Alleghanies values from 0.20 to 0.25 prevail. South of the Potomac the values are somewhat higher. West of the Alleghanies they increase more rapidly.

If ground-water storage is indicated, deductions from the storage shown in the table may be made in the same amounts that would be used in classifying

the stream under Table 1. The last column shows the deduction for thirty days' ground storage. Other amounts are in proportion.

Table 3 is for streams having coefficients of variation above 0.50. These are most common in that part of the United States west of the Mississippi River excepting Oregon and Washington. Storage equal to ninety days has been added in this table to cover the seasonal distribution of rainfall and runoff which is usual in the western part of the country.

Table 4 shows data of mean flow, coefficients of variation in annual flows and ground storage for selected streams.

In making estimates, all local data should be examined. Unless local data are very good and records of 20 years and upward are available, it will not pay to make mass diagrams. More reliable estimates will be reached by the shorter process of mean flows and coefficients of variation and by the aid of Tables 1 to 3.

Table 5 shows statistics of runoff and storage for some of the more important municipal water supplies in the United States.

Example. Compute the storage required to develop 35 million gallons of water per day from an area of 48 square miles of mountain country with little ground storage, the mean runoff being assumed from data for neighboring areas to be 25 in and the coefficient of variation in mean annual flow to be 0.20.

The mean annual flow is $48 \times 25 \times 17.379 = 20\,855$ mil gals, or 57.1 mil gals per day. The required delivery, 35 mgd, is 61% of the mean flow. As this is more than 50%, Table 2 is used. Under c.v. = 0.20, 60% and 65% of mean flow available call for storages of 0.31 and 0.35. By interpolation 61% calls for 0.318 of the mean annual flow. $0.318 \times 20\,855 = 6632$ mil gals. This is the required storage. To this must be added an allowance to cover loss by evaporation, by a method to be explained below.

From the same area for a first installment of 20 mgd. equal to 35% of the mean flow, Table 1 is used. No ground storage assumed. Storage required, 0.128 of the mean flow, or 2669 mil gals. An allowance for evaporation must be made.

Evaporation. The loss of water by evaporation from the surface of the water in the reservoir must be allowed for, except that when the records of flow represent conditions after reservoir construction, no correction is necessary.

Two methods for allowing for evaporation are recommended.

(1) Where the average rainfall is greater than the evaporation from a water surface, a deduction from available storage must be made to cover the loss of water by evaporation during a hot, dry summer, representing a 95% dry year. A deduction equal to the water held in the top foot of the reservoir is usually sufficient. In calculating the required size of reservoir, the allowance may be made by putting the flow line 1 ft higher than computed. The allowance of 1 ft is arbitrary even for the Eastern States. Another rule is to use in place of the 1 ft, the estimated mean annual evaporation from a water surface, less two-thirds of the mean annual rainfall.

(2) Where the evaporation from a water surface is greater than the rainfall, compute the yield for the full capacity of the reservoir and deduct the net loss by evaporation, computed as a draft, which is the amount by which evaporation from water area is greater than evaporation from land area. The net loss in

runoff is equal to the evaporation from water surface plus runoff from land surface less rainfall. The net loss in inches is applied to the average water area, and this for practical purposes may be taken as 0.9 of the area at the flow line of the reservoir. The amount so found is to be deducted from the computed daily capacity of the source.

Example. What will be the average daily loss by evaporation from a reservoir of 850 acres, where the rainfall is 28 in, the runoff is 8 in, and the evaporation from the water surface is 40 in?

The net loss in inches is $40 + 8 - 28 = 20$. 20 in in depth on 0.9 of 850 acres is $\frac{20}{12} \times 0.9 \times 850 \times 325\,851 = 415$ mil gals per annum or 1.14 mil gals per day.

Economical Development. The economical development of a catchment area is reached when the dam has been built so high that the cost of making it higher is more than the value of the additional water secured. The height of dam corresponding to economical development may increase with the demand for and value of water, and this often leads to raising dams. On the other hand, full probable economic development of any supply that is utilized should be taken into account to the end that partial developments first made may be arranged to be increased without too much difficulty.

In a general way, in the northeastern states the economical limit is reached by the storage of an amount equal to at least half the mean annual flow, and not greater than the mean annual flow, and when between 75% and 90% of the mean flow is made available for use. If water is low in value and conditions of storage difficult, the limit will be lower. Where water is especially valuable and the site is favorable for cheap storage, additional amounts may be advantageous.

In the western states more storage is required and the proportion of the mean flow that can be used is less. Storages, of three times the mean annual flow are not uncommon.

(1) Northeastern States North of the Potomac and East of the Alleghanies

PARTIAL DEVELOPMENT. RESERVOIR FILLING EACH WINTER: GROUND STORAGE USED FOR SECOND LINE OF CLASSIFICATION

Percent of mean flow used	Storage in Terms of Mean Annual Flow			
	Impervious soils. No ground storage	Average soils. 30 days' ground storage	Deep gravel and sand. 60 days' ground storage	Greatest natural storage. 90 days' ground storage
50	0.229	0.188	0.147	0.106
45	0.192	0.155	0.118	0.081
40	0.159	0.126	0.093	0.060
35	0.128	0.099	0.070	0.042
30	0.098	0.073	0.049	0.024
25	0.072	0.052	0.031	0.010
20	0.048	0.032	0.015	0
15	0.029	0.017	0.004	0
10	0.014	0.006	0	0

HIGH AND COMPLETE DEVELOPMENTS: RESERVOIR NOT REFILLING EACH WINTER. RELATIVE VARIATION IN ANNUAL FLOWS MEASURED BY THE COEFFICIENT OF VARIATION USED AS THE SECOND LINE OF CLASSIFICATION.

(2) East of the Mississippi River: Also Oregon and Washington

Per cent of mean flow available	Storage in Terms of Mean Annual Flow									Deduction for 30 days' ground storage*
	C.V. = 0.20	C.V. = 0.22	C.V. = 0.24	C.V. = 0.26	C.V. = 0.28	C.V. = 0.30	C.V. = 0.35	C.V. = 0.40	C.V. = 0.45	
95	1.21	1.33	1.46	1.60	1.74	1.90	2.30	2.70	3.10	0.078
90	0.85	0.92	1.00	1.09	1.20	1.31	1.60	1.88	2.20	0.074
85	0.66	0.71	0.77	0.83	0.91	1.00	1.23	1.47	1.70	0.070
80	0.54	0.57	0.61	0.66	0.71	0.78	0.97	1.19	1.39	0.066
75	0.45	0.47	0.50	0.53	0.57	0.62	0.77	0.95	1.13	0.062
70	0.39	0.40	0.41	0.44	0.47	0.50	0.62	0.76	0.92	0.058
65	0.35	0.35	0.35	0.37	0.39	0.41	0.50	0.61	0.74	0.053
60	0.31	0.31	0.31	0.32	0.33	0.34	0.40	0.49	0.60	0.049
55	0.27	0.27	0.27	0.27	0.28	0.28	0.33	0.39	0.49	0.045
50	0.23	0.23	0.23	0.23	0.23	0.24	0.26	0.32	0.39	0.041

* See tables 1 and 4 for classification and data regarding ground storage. For larger or smaller amounts the deductions are in proportion to the number of days' storage.

(3) West of the Mississippi River, Except Washington and Oregon

Per cent of mean flow available	Storage in Terms of Mean Annual Flow								
	C.V. = 0.50	C.V. = 0.60	C.V. = 0.70	C.V. = 0.80	C.V. = 0.90	C.V. = 1.00	C.V. = 1.10	C.V. = 1.20	C.V. = 1.50
90	3.00	3.80	4.70	5.60	6.40	......	......	......	......
85	2.30	3.00	3.70	4.50	5.30	6.10	7.00	......	......
80	1.85	2.40	3.10	3.70	4.40	5.10	5.90	6.70	9.30
75	1.55	2.00	2.60	3.15	3.70	4.40	5.00	5.70	8.10
70	1.28	1.70	2.20	2.70	3.20	3.80	4.40	5.00	7.20
65	1.05	1.44	1.85	2.30	2.85	3.40	3.90	4.50	6.50
60	0.89	1.21	1.60	2.00	2.50	3.00	3.50	4.00	6.00
55	0.74	1.02	1.35	1.75	2.20	2.65	3.10	3.60	5.50
50	0.61	0.86	1.15	1.50	1.90	2.35	2.80	3.25	5.00
45	0.51	0.72	0.98	1.30	1.70	2.10	2.50	2.90	4.40
40	0.42	0.61	0.84	1.12	1.45	1.80	2.15	2.50	3.80
35	0.34	0.51	0.72	0.96	1.22	1.50	1.80	2.15	3.30
30	0.27	0.42	0.61	0.80	1.00	1.25	1.50	1.80	2.75

(4) Statistics of Flow for a Few Selected Streams

(In any particular case look for local data and recent data, bringing old records up to date before using them.)

River	Place of measurement or use	Number of years in record	Last year of record	Area in square miles	Mean annual flow in inches	Coefficient of Variation in annual flow	Days to be deducted for ground storage
Hudson	Mechanicsville	24	1911	4 500	23.9	0.16	73
Susquehanna	Harrisburgh	20	1911	24 000	20.6	0.16	......
Ohio	Wheeling	21	1905	23 800	22.7	0.19	......
Columbia	Dalles. Ore.	32	1910	237 000	13.4	0.20	......
Pequannock	Newark, W. W.	20	1911	62	30.0	0.21	......
Manhan	Holyoke, W. W.	21	1917	13	26.2	0.21	25
Wachusett	Boston, W. W.	22	1918	109	22.2	0.21	61
Perkiomen	Philadelphia	25	1909	152	22.7	0.22	33
Neshaminy	Philadelphia	25	1909	139	22.8	0.23	10
Merrimack	Lawrence	36	1915	4 634	20.1	0.23	66
Croton	New York, W. W.	45	1912	375	23.0	0.24	23
Tohicken	Philadelphia	25	1909	102	27.5	0.25	2
Willamett	Albany, Ore.	17	1910	4 860	38.4	0.26	......
Sudbury	Boston, W. W.	44	1918	75	20.5	0.27	25
Gunpowder	Baltimore, W.W.	29	1911	308	19.2	0.31	91
Potomac	Pt. of Rocks	15	1911	9 650	14.5	0.33	......
Tuolumne	LaGrange, Cala.	19	1914	1 500	26.2	0.41	......
Mississippi	Pekegama Falls	30	1914	3 265	6.8	0.45	......
Colorado	Austin, Tex.	20	1917	37 000	0.75	0.57	......
Cheesman	Denver, W. W.	15	1914	1 796	1.30	0.58	......
Rio Grande	Elephant Butte	20	1914	32 000	0.66	0.58	......
Crystal Spgs.	San Fr'so, W. W.	25	1914	36	10.8	0.61	......
Alameda Creek	San Fr'so., W. W.	25	1914	620	4.75	0.63	......
Bear Creek	Denver, W. W.	14	1914	172	4.95	0.66	......
Sweetwater	San Diego	31	1918	186	1.64	1.90	......

3. Effects of Storage

Storage in an impounding reservoir may have important effects on the quality of the water, both for good and for harm. By reason of the opportunity given for sedimentation, storage tends to remove suspended mineral matter such as silt and clay and thus clarifies the water. Bacteria are also reduced in number by natural death, by the destructive action of sunlight and by the effect of other organisms, so that the sanitary quality of the water is improved. Colored waters stored for long periods in impounding reservoirs tend to become bleached. On the other hand, storage offers exceptional opportunities for the growth of algæ and protozoa which give rise to objectionable odors and tend to increase the turbidity and sediment present in the water. At the bottom of large reservoirs also decomposition of organic matter may take place, with the production of foul gases and of carbonic

(5) **Statistics of Some of the Larger Storage Systems for Municipal Supply in the United States, January 1, 1919**

Data in *italics* are estimated from the best available data.

System	Area in sq miles tributary to reservoirs	Mean runoff in inches	Coefficient of variation in annual flows	Storage in terms of mean annual flow.	No. of reservoirs	Storage in billions of gals including reservoirs now building	Actual net available storage in use Jan, 1919
New York City, Croton, Ashokan, Kensico and Schoharie	968	25.7	0.22	0.65	17	281	261
Boston-Met W. W.	203	21.3	0.24	0.96	9	72	72
San Francisco S, V. W. Co.	171	11.0	0.60	2.47	4	81	34
Denver-Cheesman	1796	1.3	0.58	0.61	1	25	25
Los Angeles	2740	2.6	0.65	0.19	4	23	23
San Diego	219	*2.0*	*1.50*	*4.00*	2	31	15
Troy, N. Y.	67	*20.0*	*0.20*	*0.51*	1	12	12
Hartford, Conn.	44	23.0	*0.22*	0.66	7	11.6	11.6
Wilkes Barre, Pa.	152	*20.0*	*0.20*	*0.18*	8	9.4	9.4
Jersey City, N. J.	121	*25.0*	*0.22*	*0.16*	1	8.6	8.6
Bridgeport	86	*22.0*	*0.22*	*0.24*	....	8.0	8.0
Newark	62	30.0	0.21	0.19	3	6.1	6.1
St. Paul	138	4.4	0.45	0.53	7	5.7	5.7
Seattle	79	90.0	....	0.04	1	4.9	4.9
Oakland-East Bay Water Co.	77	8.0	0.65	1.73	2	18.0	4.8
Lynn, Mass.	....	*20.0*	*0.22*	....	....	4.1	4.1
New Haven, Conn.	88	*22.0*	*0.22*	*0.11*	....	3.6	3.6
Worcester, Mass.	22	*22.0*	0.20	0.40	3	3.4	3.4
Cambridge, Mass.	25	*20.0*	*0.24*	0.36	4	3.1	3.1
Springfield, Mass.	48	27.0	*0.20*	0.11	1	2.5	2.5
Holyoke, Mass.	9	27.0	0.21	0.54	4	2.3	2.3

acid. One result of this is that the water is more liable to attack lead pipes.

Period of Storage. The normal period of storage of large reservoirs is exprest in days and is taken to be the capacity of the reservoir divided by the average daily flow thru it. This expression is useful for comparing different reservoirs, but it means little as far as indicating the length of time that the water is actually stored on account of the variations in volume of the water entering the reservoirs. For example, in a reservoir that has a normal period of storage of thirty days there may be one or more times each year when the stream flow is sufficient to displace all the water in the reservoirs in two or three days. Furthermore, the water entering a reservoir at one end reaches the outlet at the other end not always after the displacement of all the waters, for, by reason of difference in the density of water due to difference in temperature, the action of the wind, etc., part of it may cross the reservoir

in a period much shorter than that required for displacement; so that even in a reservoir that has a normal storage period of thirty days the actual time of transit of water from the inlet to the outlet may be reduced to a period of a few hours. For this reason storage reservoirs cannot be depended upon alone to protect the water supply from a sanitary standpoint.

Horizontal Currents Induced by the Wind. Wind blowing over the surface of a body of water causes a movement of the surface water in the same direction. The velocity of such induced currents of water varies according to the intensity and duration of the wind. Experiments made at Lake Erie have shown that for winds blowing in one direction for at least ten hours the velocity of the surface water is from 3 to 7 percent of that of the wind. For winds blowing long from one direction this percent increases. The depths of such induced currents or the velocity of currents below the surface are not well known, but the former are doubtless 10 to 20 ft.

Stagnation. The lower portions of deep reservoirs are not affected by wind action and are stagnant. The water there becomes foul in taste and odor, due to putrefaction of organic matter. With intakes located comparatively near the surface the amount of this water drawn is ordinarily small, but there is always some, due to currents. Once in the spring and once in the fall, however, there takes place a thoro mixing of the top and bottom waters, called OVERTURNING, at which times the water is materially affected in increased color and in tastes and odors. This overturning is due to change in temperature. During the warm weather the top water is warmer and lighter than the bottom water; during cold weather the reverse is true. As the maximum density of water is at a temperature of about 39° Fahr., the time comes when this overturning takes place.

Effect of Stagnation. If there are deposits of organic matter at the bottom of a reservoir bacterial putrefaction will take place at the bottom during the period of stagnation. The oxygen will become exhausted and the water impregnated with carbonic acid, sulfureted hydrogen, carbureted hydrogen, and compounds of ferrous iron and organic matter. Crenothrix and fungi may develop. During the following periods of circulation this stagnant water becomes mixt with the rest of the water in the reservoir and may temporarily increase its color. The ferrous iron becoming oxidized tends to act as a coagulant and thus to exert a purifying effect and facilitate subsequent filtration and decolorization.

During the period of circulation the spores of various algæ and other organisms are distributed thru the water together with food for their growth, and under the influence of sunlight in the upper layers they grow and remain near the surface by reason of gases evolved during the process of growth. Diatoms, in particular, are likely to be abundant in large impounding reservoirs after the spring and fall "overturns."

Algæ Growths. Some of the most troublesome forms of algæ, such as Anabæna, occur only during hot weather. They seldom develop when the temperature is less than 65° or 70°, altho in the fall they may linger in the water even tho the temperature is lower than that. It is because of the lower summer temperature that algæ growths are less prevalent in England than in America.

Soil Stripping. In order to prevent the growths of algæ reservoir sites are sometimes cleaned by removing the vegetation and top soil. This practise has been followed extensively in Massachusetts reservoirs. It results in a temporary benefit, but ultimately deposits of material occur on the bottom that contain as much organic matter as that found in the soil, and the advantage of stripping is lost. The organic matter that has the greatest effect on the quality of impounding waters is derived from grass, weeds and other

vegetation on the reservoir site. This should be removed by cutting and burning just before the reservoir is filled. For the sake of appearance and to prevent growths of water weeds and filamentous algæ the shores of the reservoir from 2 to 5 feet vertically above the high-water mark and for 10 to 20 feet or more below, according to circumstances, should be cleared of stumps and roots. Elsewhere the stumps should be cut to 12 inches or less above the mean surface of the ground.

Drainage of Swamps. The color of a stored water may be sometimes reduced by draining swamps on the catchment area. This also tends to reduce the danger of the reservoir becoming seeded with algæ. The advantage is less where the water is to be purified.

Penetration of Sunlight into Water. On account of the rapid absorption of the sun's rays by water, the disinfecting effect of sunlight and its bleaching action on the coloring matter are limited to a shallow layer near the surface. In clear water there is little bleaching effect below a depth of five feet, and in turbid waters the effect of the sunlight may be felt only a few inches. For this reason growths of organisms are much less likely to occur in turbid, silt-bearing waters than in clear waters.

4. Ground-Water Supplies

Ground Water is that part of the rainfall that has accumulated in the ground, either in soil or in rock. Its upper surface is called the water table, or the ground-water level. The water is actually present in the pores of the soil or granular rock or in fissures, crevices and seams. These may be considered as underground storage reservoirs.

Classification. The water in the ground, above an impermeable stratum, and relatively near the surface, is termed the upper ground water, and wells for obtaining it are called shallow wells. Water taken from beneath an impervious stratum is termed artesian, or deep-seated water, and wells for taking it are artesian wells, or deep wells. Originally the term "artesian" was applied only to deep-seated water that was under a sufficient head to cause it to flow naturally to the surface without pumping. All ground water is derived primarily from the rainfall. Geologically there are three principal classes of ground water, namely, those occurring in underlying stratified porous rock covering large areas, those occurring in old lake or river beds, and those occurring in deposits of sand and gravel, that is, in the drift.

Ground-water supplies are obtained from (1) sand and gravel deposits; (2) sandstone rock; (3) limestone rock. In the first two cases the water is present in the pores of a more or less homogeneous material. In the last it is present in caverns and fissures, usually extending indefinitely into the limestone rock, and replenished by surface waters that flow into them or by ground waters from sand and gravel deposits above the limestone. Water from either sand or sandstone is well filtered and of good hygienic quality. Water from limestone is often surface water that has flowed simply thru a limestone cave and may be of inferior hygienic quality.

Percolation is the downward flow of rain water thru the ground under the influence of gravitation and capillarity. It varies in amount according to the rainfall, porosity of the soil, temperature, etc. Under specially favorable conditions, illustrated by the sands of Long Island and the sand dunes of Holland, percolation may amount to from 30 to 60 percent of the rainfall. With less pervious material, such as is found in the South and middle West, percolation may be as low as 10 to 20 percent.

Velocity and Direction of Subterranean Flow. The slope of the water table can be determined by measuring the elevation of the water in a series

of bore holes, enough measurements being made to determine not only the amount of slope but also the direction of greatest declivity. The direction of flow is the direction of greatest declivity. The velocity may be estimated approximately from the slope for clean sands and gravels where samples can be secured, by making mechanical analyses and determining the average effective size of the material; but such calculations, based upon conditions largely unknown, are to be used with caution. The formula for the flow of water thru sand is given in Art. 11.

Measurement of the Velocity of Flow of Ground Water. The actual velocity of ground water may be measured by introducing a solution of salt into one of two wells and noting the time required for its flow to the second well, the time being determined by making frequent analyses of the water. A more accurate method is that of Prof. Slichter, who used an electrical device by which the conductivity of the water in the lower well and in the ground between the two wells could be determined. Ammonium chloride or some other electrolyte was placed in the upper well and the time required for it to flow to the lower well was determined by noting the increase in conductivity of water. Ground water in sandy soils often has a velocity of 5 to 10 ft per day, altho velocities of 50 ft or more per day are not unknown. The deep-seated water found in underlying rock often moves with extreme slowness. In some cases calculations have shown the velocity to be as small as 10 ft per year.

Quantity of Underground Flow. The quantity of underground flow is obtained by multiplying the velocity by the area of the cross-section under consideration and by the percent of voids in the material. These usually are from 30 to 40 percent in natural gravels and somewhat less in sandstones.

Flow of Water into Wells. If a well is sunk into the ground water and pumped, the surface of the ground water adjacent to the well will be deprest and assume a form similar to that shown in Fig. 5. The shaded area is commonly called the cone of the depression, and the area within which the water table is appreciably affected is termed the circle of influence. If pumping were continued and the ground water received no acquisitions, the circle of influence would widen without limit. Usually a ground water has a natural slope and a flow in a definite direction, so that, when the circle of influence has broadened until the ground water flow tributary to the area equals the amount of water pumped, a condition of equilibrium is obtained. The curve assumed by the water table near the well is parabolic in form. Formulas have been worked out to show the relations existing between the various elements of the problem, but they are valuable chiefly as indicating relative conditions caused by drawing down the water table to different depths. They cannot be depended upon to determine with any degree of accuracy the capacity of a well or the radius of the circle of influence, except in special cases where unusual attempts have been made to determine the values of the constants involved.

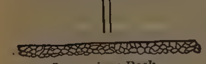

Fig. 5. Ground Water

Interference with Wells. Wells placed in the same stratum near together mutually interfere according to the size and spacing of the wells, the radius of the circle of influence and the lowering of the ground-water table. As an illustration of this interference Slichter has given the following example.

If in place of a 6-inch well that has a radius of the circle of influence of 600 feet, and a lowering of 10 feet in the water table by pumping, there were two 6-inch wells 200 feet apart, the yield of each would be 85 percent as much as a single well, while if there were a number of wells 100 feet apart the yield of each would be only about one-third as much as that of a single well of same size.

Tests of Wells are frequently carried out by putting down test wells and pumping them for a limited period. Such tests have little value beyond showing the perviousness of the material about the test well. If there is water in the underground reservoir, water can be pumped at any rate at which the well is capable of taking it from the reservoir as long as the supply lasts. This may be a day, a week, a year, or permanently, according to the supply. Actual use thru dry periods is usually the best test. Wells extending below sea level, as on Long Island and in New Jersey, may draw fresh water in excess of their permanent capacity for several years, while sea water is gradually filling the voids previously occupied by the fresh water.

Yield of Ground Water. The yield of ground-water-collecting works depends upon the area of the catchment area, the rainfall, percolation, evaporation, etc., as well as upon the nature and efficiency of devices employed for utilizing the ground-water flow. The average safe yield is the amount of water that can be obtained by complete development under conditions of normal rainfall. It is usually exprest in gallons per square mile per day. For example: On Long Island the average yield of the original catchment area supplying Brooklyn has been estimated at 900, 000 gallons per square mile per day. This corresponds to 18.9 inches on the catchment area, or 43 % of 44 inches rainfall. During a long dry period the safe delivery is 20 % to 25 % less. The soil of Long Island, on account of its sandy nature, serves as an extended underground storage basin, causing a more uniform flow of ground water than would otherwise be obtained and carrying the effect of the rainfall over from one year to another.

Conditions on Long Island are exceptionally favorable for the collection of ground water. In other localities where the rainfall is the same the safe yield might not amount to 500 000 gallons per day per square mile. In some parts of the country the safe yield is as low as 100 000 gallons per day per square mile. It is seldom that more than about 3 million gallons per day can be economically obtained from sand or sandstone at any one place, and often the limit is as low as one million gallons or less. When larger quantities are required it is customary to have several stations with pumps operated separately or by power transmitted from a central station.

Temperature of Ground Water. One of the advantages of a ground water supply is its cool and equable temperature. In the case of ground water found in the drift the ordinary temperature of the water is usually not far from the average temperature of the air in the same place.

5. Wells and Pumping

Ground water is made available for public water supplies by large dug wells, driven wells, infiltration galleries, and springs. There are three principal types of driven wells, that is, shallow, tubular wells; deep, bored wells in soft material; and rock wells.

Wells of Large Diameter. The chief advantages of the well of large diameter are, the storage that it affords and the possibility of placing the pumps at a low level and short suction pipes. The effect of the size of the well on the yield is comparatively small. Large wells are useful where the pumping is variable and especially in cases where the ground water flows thru fine material with low velocity. Dug wells avoid the clogging that occurs in driven wells located in iron-bearing sands. As the cost of large wells increases rapidly with the depth, they are seldom made more than 50 ft. Large wells vary in diameter up to 50 or 100 ft. They are commonly lined with brick, concrete, or masonry, openings being left for the entrance of water. They

are covered to exclude light and dirt. The capacity of large wells is sometimes increased by driving small wells or galleries horizontally into the ground, or by sinking vertical wells in the bottom of the large well.

Shallow, Tubular Wells. The ordinary driven well consists of a wrought-iron or steel tube, 2 to 8 inches diameter, with a strainer near the bottom. It is forced into the ground by a hammer or by the use of a falling weight or with the aid of a jet of water carried thru a small pipe to loosen the material in advance of the point. The strainers may be merely holes or slots in pieces of brass pipe, or larger holes in the pipe, covered with brass gauze. Porcelain strainers and tile wells also are used. The use of different metals in a strainer is objectionable, as it gives opportunity for galvanic action, causing corrosion and clogging. The size of the openings must be adjusted to the texture of the soil. They must be small enough to prevent the entrance of any large quantity of sand but large enough to reduce the entrance velocity to a point where the friction will not be excessive, that is, less than about 0.2 ft per second.

Deep, Bored Wells. Deep wells in soft material are bored. The loosened material is brought up by the use of a water jet or sand bucket; such wells are usually cased. Wells are bored in rock by the use of a cutting drill which is raised, revolved, and let fall with a blow. Where strict artesian conditions prevail a casing is put down to the impervious stratum.

Infiltration Galleries. Infiltration galleries are closed conduits of masonry, wood, iron, brick or vitrified pipe, laid with numerous small openings to allow the inflow of water. They serve the double purpose of collecting the ground water and conveying it to the pump well. To prevent the entrance of fine material thru the openings the latter are surrounded on the outside with graded gravel and sand. Infiltration galleries are placed at right angles to the general line of flow of the ground water, altho sometimes they are placed near the shores of streams. In the latter case, as the bed of the stream is generally covered with silt, the greater proportion of water is derived from the land side. The attempt to use the bank of the stream as a filter and collect the stream water in a filter-gallery beside it has been frequently attended with failure.

Pumping is required to utilize most ground waters. It is possible to lift water by a suction pump not more than 25 ft, and special effort should be made to locate the works so that water can be obtained without more than this amount of suction.

An air pump on the suction pipe frequently facilitates pumping with high lifts and long suction lines. An air chamber is placed on the suction just before connection is made with the pump, and the air that leaks into the system is separated and removed by an air pump, so that the main pump always draws water without air.

A Deep-well Pump is a small pump of peculiar construction lowered thru the pipe to the bottom of the well and operated by a connecting rod reaching to an engine above the surface of the ground.

A Screw Pump is a screw propeller, or a series of them, placed in the pipe and lifting the water in the well by being revolved rapidly, motive power being applied to the shaft above the surface of the ground.

The Air Lift is a process of taking water out of wells by the pressure of comprest air. The well is extended in depth to a considerable distance below the water level. Comprest air is carried to the bottom of the well thru a small pipe and allowed to escape. The air mixes with the water in the discharge pipe, which must not be too large in diameter. The column of mixt material is lighter than solid water, and it becomes higher in proportion until water mixt with air overflows at the surface into a separating device.

All three of these means of getting water out of deep wells are of very low efficiency from a mechanical standpoint. The expense of lifting water from wells 30 to 50 feet deep frequently exceeds the cost of pumping it afterward against a much higher lift.

Hardness is common in ground waters. Waters obtained from sands and gravels free from lime, as in New England, Long Island, and parts of New Jersey, are soft. The glacial drift from central New York westward contains lime, and waters obtained from it are hard. The hardness depends more upon the fertility of the overlying soil than upon the amount of lime in the sand, because rich soil produces carbonic acid, which is taken up by the water as it passes thru the soil and this promotes the solution of lime. Waters from limestone rock are always hard.

6. Tests for Potable Water

Sanitary Inspection. No surface water is entirely without danger of infection. The closer the proximity of the sources of pollution to the intake of the waterworks, the greater is the danger, altho it is a question of time of flow rather than distance of flow. The population on catchment areas may be classed as, urban (population above 4000), village (population between 4000 and 1000), and rural (population below 1000). The figures are best exprest in population per square mile of drainage area as above.

Sanitary regulations for the prevention of contamination of water supplies are in force in many states. The guiding principle should be; to make the sewage pass thru the ground and not over the ground, and if this cannot be done, to remove it from the catchment area or purify and disinfect it. In considering the danger of privies located in proximity to watercourses, the character of the soil and the slope of the surface should be taken into account, and also the method of cleaning. Cases of typhoid fever or dysentery existing on a catchment area should receive special consideration.

Water Analysis. A complete water analysis consists of four sections, physical, chemical, bacteriological, and microscopical. The standard methods of analysis are given in "Standard Methods for the Examination of Water and Sewage," American Public Health Association. (Third Edition may be obtained from the Secretary of the A. P. H. A., Boston, Mass.) The standard method of expressing the results of chemical analysis is in parts per million by weight, which is practically equivalent to milligrams per liter. To change such results to "parts per 100 000," divide by 10; to "grains per U. S. gallon," divide by 17.1; to "grains per Imperial gallon," divide by 14.3. Bacteria and microscopic organisms are exprest in "number per cubic centimeter." The expression "degrees of hardness" usually refers to "grains per gallon." Degrees of hardness on Clark's scale are "grains per Imperial gallon." On account of the variety of conditions there are no accepted standards of purity to either chemical or bacteriological analysis.

Samples of Water for physical, chemical, and microscopical analysis should be collected in clean, glass-stoppered bottles holding two quarts or a gallon. Bacteriological examinations demand special sterilized, glass-stoppered bottles holding at least two ounces. Bottles may be sterilized by heating in an oven for one hour at 160° C. Bacteriological examinations must be ordinarily made within six hours after collection, and the sample must be packed in ice for transportation. Tests for gases, such as dissolved oxygen and carbonic acid, should be made at the time of collection of the sample. Chemists usually prefer to furnish their own bottles.

Turbidity is caused by clay, silt, microscopic organisms and other fine particles. A perfectly clear water has a turbidity of zero. Turbidities higher than 2 to 5 parts per million are noticeable in drinking water and are objectionable. Turbidities as high as 50 or 100 make a water "muddy." In the Mississippi River and elsewhere in the South and middle West, below

the region of the glacial drift, turbidities of 2000 or 3000 are not uncommon. The turbidity of a river varies more or less directly with the stream flow.

The Standard of Turbidity is a water which contains 100 parts per million of silica in such a state of fineness that a bright platinum wire 1 mm in diameter can just be seen when the center of the wire is 100 mm below the surface of the water and the eye of the observer is 1.2 meters above the wire, the observation being made in the middle of the day in the open air, but not in sunlight, and in a vessel so large that the sides do not shut out the light so as to influence the results. The turbidity of such a water is 100. For actual use a standard suspension is prepared from a diatomaceous earth, to contain 1 gram of silica per liter and have a turbidity of 1000. From it silica standards are prepared by dilution with distilled water. For turbidity readings below 20, gallon bottles of clear whiteglass are used and smaller bottles for higher turbidities. The turbidity of a sample of water is determined by comparing it with these standards, looking thru the bottles sidewise and noting the distinctness of some object, such as a series of ruled parallel lines, seen thru them, or looking at a black surface while standing with back to the light.

For field use the United States Geological Survey turbidity rod consists of a rod with a platinum wire, 1 mm in diameter near the end, projecting at right angles about one inch. At the other end of the rod, at a distance of 1.2 meters (about 4 feet), is placed a wire ring thru which the observer looks when making the examination. The rod is graduated, so that the distance from the wire to any mark indicates the turbidity when that mark is at the water surface and the wire just disappears from view. This graduation is determined by experiment.

The Color of water is caused chiefly by organic matter derived from decayed vegetation. Swamp waters are usually high colored. Ground waters are usually colorless. The color of the water in large lakes is usually below 10. River waters are colored in proportion to the area of swamps on the catchment area. If the color is as high as 20, water will look unsightly in a porcelain bathtub or in a glass on a white tablecloth. Higher colors are objectionable and warrant purification of the water.

Color is Measured by comparing the sample of water with artificial standards by dissolving platinum and cobalt chlorides in distilled water. The unit of color is that produced by one part per million of metallic platinum. In the field the glass disk method of the U. S. Geological Survey is used. Disks of colored glass, standardized against the platinum solution, are placed at the end of a metallic tube, comparison being made with the sample placed in a similar tube.

Odor. Fishy, grassy, and aromatic odors are caused by microscopic organisms such as Asterionella, Anabæna, Synura, and are due to oily secretions. Moldy and musty odors are due to decomposing organic matter. Peaty odors are due to the same substances that give water its color. Ground waters sometimes have sulfurous odors, due to dissolved gases.

Hardness is caused by carbonates, sulfates, and chlorides of calcium and magnesium. Hard water destroys soap. It has been estimated that the loss due to waste of soap for domestic purposes amounts to ten cents per million gallons for each part per million of hardness. This is based on average conditions and a use of 100 gallons per capita daily. A hardness of ten parts per million is practically unnoticeable, and it requires a hardness of 20 or 30 parts per million to produce curdling with soap. Water may be called hard if the hardness is above 100; very hard, if above 200; and excessively hard, if above 300. Hardness is due to the solvent action that water containing carbonic acid has on soil containing lime. Sewage pollution increases hardness. (See Table on page 1207.) For temporary hardness see Art. 7.

Chlorine in water represents salt and may be due to sewage pollution, or to proximity to the sea, or, in the case of deep wells, to deposits of salt. The normal chlorine in waters decreases from the seacoast inland, and normal chlorine maps have been prepared by the U. S. Geological Survey for New England, New York, and New Jersey. Sewage pollution is indicated by

Hardness of Water Supplied in American Cities
(Parts per Million)

Albany	70	Newark	25
Baltimore	56	New Haven	25
Birmingham	37	* New Orleans	119 to 66
Boston	12	New York, Croton	38
Bridgeport	8	Ashokan	10
		Oakland, Alverado	205
Buffalo	109	San Leandro	142
Cambridge	29		
Chicago	120	Omaha	230
Cincinnati	93	Paterson	45
Cleveland	110	Philadelphia, Schuylkill	95
		Delaware	55
* Columbus	279 to 100		
Dayton	300	Pittsburgh	50
Denver	165	Portland, Ore	10
Detroit	90	Providence	30
Fall River	7	Richmond	50
		Rochester	71
* Grand Rapids	249 to 100		
Indianapolis	300	* St. Louis	190 to 102
Jersey City	50	St. Paul	180
Kansas City	230	San Francisco	160
		Seattle	14
Los Angeles, Old supply	250	Spokane	154
Owens river	128		
Louisville	95	Syracuse	98
Lowell	25	Toledo	224
Milwaukee	115	Washington	80
Minneapolis	158	Worcester	18

* Softening plants. The first figure is the raw water. The second the water as delivered.

excess above the normal. If the chlorine exceeds 15 or 20 parts per million it is apt to cause corrosion in boilers and plumbing fixtures. Chlorine is not removed by filtration or by any artificial process.

Iron in water is apt to cause trouble if present in quantities of more than 0.3 to 0.5 part per million. It is usually present as carbonate or hydrate, occasionally as sulfate, and often in organic combination. Manganese causes similar trouble but is less common.

Dissolved Gases. Carbonic acid is exceedingly soluble in water, but is easily reduced in amount by exposure to the atmosphere. Oxygen also dissolves readily in water, but the amount that can be present depends upon temperature and pressure. In winter waters normally contain nearly twice as much dissolved oxygen as in summer. (See Art. 33.)

Microscopical Examinations are required in studying the algæ and protozoa in connection with the subject of odors. The method is in general that of concentration by filtration thru a small sand filter in a glass funnel and examination of the concentrate with a microscope that magnifies about 100 diameters. Special cells are required, but the operations are not difficult.

Bacteriological Examinations require special methods and the use of sterilizers, incubators, and delicate pieces of apparatus. Several days are required for making the tests. The reports usually give the number of bacteria per cubic centimeters and state the presence or absence of the colon bacillus (B. coli) in 0.1, 1.0, and 10.0 cu cm of the sample.

Ground waters contain very few bacteria, seldom more than 100 per cu cm. Surface waters often contain bacteria in large numbers, several hundred and sometimes many

thousands per cc. In silt-bearing streams the numbers vary approximately with the turbidity. Pollution generally causes an increase in the number of bacteria. Lake waters generally contain fewer bacteria than river waters. Most of the bacteria found in water are probably harmless. High numbers, however, are objectionable, because among them some objectionable species are more likely to be present. Filtered waters contain few bacteria, and the reduction in the number of bacteria by filtration is commonly taken as an indication of the efficiency of the process. The germs of typhoid fever, Asiatic cholera, dysentery, etc., may exist in water, but there are no reliable methods for detecting their presence, altho certain recent methods are promising. The bacillus coli communis, commonly referred to as B. coli, is a constant inhabitant of the intestines of man and warm-blooded animals. It can be detected in water with comparative ease, and its presence is often taken as an indication of fecal contamination.

The Typhoid Fever Death-rate of a community is the number of deaths per year from typhoid fever per hundred thousand. In cities possessing satisfactory water supplies the rate is not often above 10, but typhoid fever is carried in other ways, and higher rates with good water sometimes occur. The substitution of a filtered water, or other pure supply, for a polluted water usually reduces the typhoid fever death-rate.

7. Water for Boilers

A Steam Boiler requires good water as much as it does good coal. Bad boiler waters cause corrosion, scale, foaming, overheating, and leaks, resulting in loss of heat, increased labor of attendance, increased cost of operation and repairs, a shortened life of the boiler, and increased danger of explosion. All natural waters are more or less corrosive. Magnesium chloride and other salts cause corrosion, especially when concentrated in a boiler. Galvanic action sometimes causes corrosion. Local corrosion is termed "pitting" or "grooving."

Boiler Scale is formed by the precipitation from the water of the carbonates and sulfates of calcium and magnesium, together with smaller amounts of other salts and suspended matter. Calcium carbonate is quite insoluble after its extra molecule of carbonic acid has been driven off by heat; calcium sulfate becomes almost insoluble above 250° F.; magnesium carbonate is changed to magnesium hydrate and precipitated. Besides these, boiler scale often contains iron, silica, alumina, organic matter, etc. Carbonates often separate as soft mud which is comparatively unobjectionable; they may also form a hard scale. Sulfates always form a hard scale. Calcium sulfate precipitates in a compact, crystalline form, removed by hammering and chipping. It may happen that different kinds of scale occur in the same boiler, due to the different temperatures of the sheet in different parts and to the circulation of the water. The scale in the tubes is often different from that on the sheets.

Hard Waters invariably form scale, and comparatively soft waters may also do so if the boiler is used too long without being emptied. Concentrated soft waters are almost as bad in their effects as waters naturally hard. The greater the hardness, however, the more troublesome the water. FOAMING is caused chiefly by an excess of alkaline salts, which cause the water to form suds, as if soap had been added. This makes a boiler unmanageable and affects the quality of the steam. If grease is present in the water the sludge or scale may become very sticky. In this condition it adheres tenaciously to the plates and causes overheating, which usually occurs in spots.

The **care of a boiler** has very much to do with the effects of hard waters. The frequent blowing off of a boiler tends to reduce the amount of sludge, and to that extent is advantageous, but in the process of blowing off only a part of the water is removed. Better results are obtained by allowing the boiler to cool, emptying it and cleaning it if necessary. Corrosion due to gases can be eliminated to some extent by allowing a thin scale to form in the boiler.

Boiler Compounds are often employed as a remedy and have their legitimate use. In serious cases of acid corrosion, lime or caustic soda may be used with advantage. Nothing will effectually prevent the precipitation of calcium carbonate, but the use of soda will cause some of the calcium sulfate to settle as carbonate instead of sulfate, thus making the character of the scale less objectionable. To prevent adherence of the scale to the boiler shell many substances have been used, such as potatoes, kerosene, and all sorts of nostrums, organic and mineral. Most of these are practically worthless. Of the boiler compounds more commonly sold none have given more general satisfaction than those which have soda and some form of tannic acid as primary constituents. Tannic acid has a slight action on the iron of the boiler, and is reasonably efficient in preventing scale from sticking, while if properly used its action on the iron is not serious.

One gallon of hemlock extract with two gallons of water and three pounds of soda ash forms a good compound. Hemlock extract costs from 3 to 5 cents a pound in barrel lots and soda ash costs less than 2 cents a pound. Tri-sodium phosphate is also used with excellent results with many waters.

Chemicals for Water Softening. The following table gives the number of pounds of commercial lime (85% available) and soda ash (58% Na_2O) which are required to remove each part per million of the substances mentioned in the first column from one million gallons of water.

	Lime	Soda ash
Free carbonic acid	12.5	
Free sulfuric acid (as H_2SO_4)		9.03
Alkalinity (in terms of $CaCO_3$)	5.5	
Incrustants (in terms of $CaCO_3$)		8.85
Magnesium	22.9	

Temporary Hardness is that part of hardness (Art. 6) due to carbonates. It produces scale in boilers, but is partly removed by boiling, so that treating the water before it goes to the boiler reduces the amount. In most waters the temporary hardness is equivalent to the "alkalinity." Sulfates and chloride of lime and magnesia produce hard scale in boilers and are not removed by boiling. They comprise the permanent hardness, or incrustants, and ordinarily are equal to the difference between the alkalinity and the total hardness. For purposes of water softening it is necessary to know the total hardness, the alkalinity or acidity, the amount of magnesia, and the free carbonic acid.

PURIFICATION OF WATER

8. Auxiliary Processes

Water Purification is frequently spoken of as "filtration," but the processes employed are much broader than are properly covered by the word "filtration," and include such auxiliary processes as aeration, straining, coagulation, disinfection, and sedimentation.

The Capacity of purification works must be sufficient to meet the requirements at the maximum rate of use, and it is usually necessary to provide reserve parts so that the full supply may be maintained while cleanings, repairs, etc., are carried out. As a general rule the capacities of purification plants should be 50% greater than the greatest expected annual rate of use, but the ratio varies according to circumstances.

Aeration consists in bringing water into intimate or violent contact with air for the double purpose of introducing oxygen and of removing objectionable gases. Aeration is used as a preliminary treatment in all cases where oxygen is deficient in the raw water. Oxygen is very easily introduced.

Water falling in drops thru a height of two or three feet will take up more than half the quantity of oxygen that it will take up by the fullest exposure. The fall of water over a dam, or the play thru a jet of a fountain, will serve to fully aerate it as far as introducing oxygen is concerned.

More vigorous aeration is required to remove the gases of substances that produce tastes and odors resulting from the growth and decay of organisms in quiet water in the light. Sufficient aeration greatly reduces or removes these tastes and odors. Perforated trays have frequently been used, thru the bottoms of which the water drops in small streams. Allowing water to fall over steps is a good method of aeration where circumstances permit. Fountains furnish one of the best means of aeration where a head of five or ten feet or more is available. Numerous small jets are better than one large one, and they should be controlled separately or in groups to permit flexibility of operation. The best dispersion of the water in the air is obtained when the water is revolving in the pipe as it approaches the jet. Thin sheet-iron guides may be introduced to make the water revolve.

Screens are used for removing dead leaves, sticks, etc. Stationary inclined screens raked off at intervals are most commonly used. Revolving screens of various types are used for closer screening. Screening thru brass wire cloth with as many as 60 meshes per inch is used in paper mills. Screening as a preliminary to filtration is advantageous within certain limits, but close screening is unnecessary.

Coagulation consists in the addition of some substance to the water that reacts with substances in the water, producing a flocculent precipitate which surrounds minute suspended particles in the water and draws them together into aggregates that can be removed by subsequent processes which would not serve to remove the individual particles. Coagulation is an essential part of the treatment of all waters containing (1) large amounts of very finely divided mineral matter, or turbidity, and (2) all waters highly colored by vegetable stain. It is also frequently used for other waters, but is not necessarily essential.

Sulfate of Alumina, commonly called alum, is the most widely used coagulant. It is clean to handle, easily dissolved, and efficient in its action. It requires lime or other alkalinity to decompose it. In most cases sufficient lime is present in the water treated. Otherwise lime or soda ash must be added. The approximate quantities of sulfate of alumina (17 % Al_2O_3) required to coagulate waters of various degrees of turbidity and color, and the amounts of alkalinity required to react with them, are as follows:

Turbidity	Alum		Alkalinity necessary for reaction (parts per mill.)
	Grains per gallon	Pounds per mill gals	
0	0.50	72	5
50	1.00	143	9
100	1.35	193	12
200	1.78	254	16
300	2.05	293	18
500	2.50	357	22
700	2.87	413	25
1000	3.32	474	30
1500	3.90	557	34
2000	4.40	628	38
Color			
50	1.00	143	9
100	2.00	286	17
200	4.00	572	35

Different kinds of turbidity and color vary considerably in the amounts of coagulant that they require, so that figures varying considerably from the above average will be found. Deep reservoir waters containing iron in solution are more easily coagulated. It is well to keep the alkalinity of the treated water as high as 10 or 12 parts per million to guard against corrosion of metals.

At some large filter plants alum is manufactured from bauxite.

Ferric Salts have sometimes been used as coagulants. They are nearly equivalent in action to sulfate of alumina but less convenient to apply and have been used much less widely.

Ferrous Sulfate, or Copperas, is extensively used as a coagulant. It is cheaper than sulfate of alumina and equally efficient in removing turbidity, but requires a more alkaline water. For this reason it is always necessary to use lime in connection with it, and the amount of lime must be closely adjusted to the chemical condition of the water.

Copperas cannot be successfully used except with adequate chemical supervision. The process is therefore preferred principally in large plants where continued close expert supervision can be given.

By a judicious regulation of the amount of lime it is possible to partially soften hard waters by this treatment. Lime as a coagulant is used for softening hard waters and is not usually to be otherwise classed as a coagulant. The softening results from the application of lime (calcium hydrate) in quantity to combine with the free carbonic acid in the water, and with the carbonic acid half combined with the lime already present in the water. The amount must be so adjusted that there is neither free lime nor free carbonic acid left. Under these conditions the lime is precipitated as carbonate and the water is softened. It is not all removed, because calcium carbonate is slightly soluble in water, because the reaction takes place slowly at the last, and because the adjustment of the lime to the carbonic acid is at best only approximate. The calcium carbonate is crystalline and not flocculent, and has but little coagulating value, but in applying the process to river waters containing magnesium the magnesium hydrate is thrown down with the calcium carbonate and this has a considerable coagulating value.

Disinfection consists in the addition to the water of some substance to kill objectionable organisms in it, which substance must be so adjusted as not to be injurious in the other uses to which the water is put.

Ozone from a theoretical standpoint is one of the best disinfectants. It is produced by the discharge of high-tension electricity thru dried and cooled air in special apparatus. The air is then taken upward thru towers thru which the water is falling, or otherwise brought in contact with the water. As ozone is another form of oxygen, which in any case soon returns to its normal condition, nothing objectionable is added. It has been proposed to use ozone on filter effluents. When applied to the raw water too much of the ozone is used up in destroying the organic matter. To kill the bacteria in a well-filtered effluent the ozone required is 0.1 or 0.2 part per million.

The quantity of electricity required to produce ozone in any apparatus thus far proposed has been so large and the operation has been so uncertain that the process has not received wide application. On account of the insolubility of ozone it is hard to secure an intimate mixture with the water.

Sulfate of Copper has been extensively and successfully used to kill organisms in reservoirs. It is applied in a crude way by putting in a coarse bag and dragging through the water. The quantity required depends upon the organism present. It should be used prior to filtration, with the idea that most of the copper will be stopped in the filter. Copper sulfate is also used to disinfect the water of swimming pools.

Chloride of Lime, more correctly hypochlorite of lime, commonly called bleaching powder, is chlorine absorbed by lime and is an effective disinfectant. When used prior to filtration, from 8 to 15 lbs per million gallons is an average dose; when used after filtration the average dose may be smaller.

It is added to waters used without filtration in the pump suction or at other convenient point. Some bacteria resist the action of the chlorine, but with a sufficient dose a great majority of them are killed. Sometimes the raw water

contains substances that use up the chlorine in chemical reactions, and where this occurs the dose must be correspondingly increased.

The application of an excessive dose to water going immediately to a sand filter may interfere with the establishment of the normal process of purification in the filter. It would therefore seem better to apply it some-time before it goes to the sand filter, or preferably to the effluent.

Chloride of lime seems to do everything as a disinfectant that is done by ozone and with greater certainty and at much less expense. The chlorine constantly liberated is very destructive to all metal structures in its neighborhood, and the apparatus should be designed and placed to avoid such troubles as far as possible. About one third of the weight of chloride of lime is in the form of "available chlorine."

Because of its general availability and ease of application bleaching powder is well adapted to emergency treatment.

Liquid Chlorine. Chlorine gas, liquefied and furnished in steel cylinders is also extensively used for disinfecting waters. Its advantages over bleaching powder are smaller weight, more compact apparatus, less constant attention, better admixture with the water and no resulting sludge; its disadvantages are danger to operators (gas is very poisonous) and the delicate character of feeding apparatus resulting in occasional interruption of service. Liquid chlorine may be applied to the water directly as a gas; but a better way is to first dissolve it in a small stream of water. Practically 100% of the liquid chlorine is effective. For clear waters the quantity required is about 3 lbs per million gallons; for waters which contain suspended and organic matter, larger quantities are required. One part per million by weight = 8.3 lbs per million gallons. One pound per million gallons = 0.12 part per million.

Water after being mixed with the chlorine should be kept by itself, as in a pipe or in a small baffled compartment of a reservoir for a few minutes during which the action takes place. If the treated water is at once discharged into a larger reservoir, the chlorine is diluted before full action and a much larger dose is required to be effective.

Ultraviolet Light is also a powerful germicide, but methods of use, though steadily improving, have not yet reached the practical stage.

9. Sedimentation

The Sedimentation Process of purification consists in taking water thru basins in which the velocity of the flow is reduced and hence the heavier suspended matters settle to the bottom by gravity. Sedimentation is widely used as a preliminary process and is the cheapest way of removing those relatively large particles which settle out in a moderately short length of time.

Sedimentation Basins are usually large open basins with masonry floors and walls holding from 8 to 48 hours' supply. Natural basins much larger in size without special means of cleaning also serve as sedimentation basins. COAGULATING BASINS are sedimentation basins in which water that has received a coagulant is settled. They are often smaller, sometimes holding less than an hour's supply. BAFFLES, consisting of light dividing walls separating one basin into parts thru which the water passes successively, increase the efficiency by preventing the partially cleared water from mixing with uncleared water. Too many baffles increase the length of the horizontal courses and the velocity to a point where deposition of the finest particles is prevented. CLEANING is usually accomplished hydraulically, by opening a gate and flushing out the sediment. To facilitate this, drains are built, and the whole bottom slopes to them. Jets of water from hose are used to facilitate the movement.

In the Intermittent System of operation a basin is filled and allowed to stand and then drawn off. In the CONTINUOUS SYSTEM of operation water flows constantly in at one end and out at the other.

The Efficiency of a Basin depends upon its area and upon the system of baffling employed. A deep basin is not more efficient than a shallow one of the same area, but a certain depth is necessary in order to hold an accumulation of sediment and to prevent the velocity of flow thru the basin from becoming too great to permit full deposition. As a rough rule the depth may be one-sixtieth of the average course that the water will follow when baffled.

Settlement of Particles in Still Water at 50° F.

Kind of material	Diameter of particles in mm	Rate of settlement, mm per second
Coarse sand..	1	100
	0.20	21
Fine sand....	0.10	8
	0.06	3.8
	0.04	2.1
	0.02	0.6
Silt.........	0.01	0.15
Coarse clay..	0.001	0.0015
Fine clay....	0.0001	0.000015

Bacteria and other organisms settle more slowly if at all because their specific gravity is so near to that of water. The rate of settling is greater as the temperature is higher. Twice as much water can be past thru the basin with the corresponding results in summer as in winter. The limit of size of particles removed by settling basins can be computed approximately (Trans. Am. Soc. C. E., vol. 53, p. 45) as follows:

The Diameter of Particles in millimeters, such that 75 percent will be removed with continuance of operation, may be computed by

$$d = 0.0027 f \sqrt{\frac{\text{million gallons daily}}{\text{area of basin in acres}}} \sqrt{\frac{60}{t+10}}$$

in which f is a factor depending upon the arrangement of basins and baffling. Use 1.73 for a basin with one inlet and one outlet well separated; 1.41 for two basins thru which the water passes successively; 1.22 for a well-baffled basin or other specially good arrangement. $f = 1.00$ is a theoretical limit not reached in practise. In the last term t is temperature in degrees Fahrenheit. For comparisons use $t = 50$ in all cases. The rule does not apply for separations above 0.05 millimeter. It is not precise, but it affords a convenient basis for comparing sedimentation and coagulating basins.

In a general way basins holding six hours' supply, well baffled, in connection with mechanical filters, remove particles more than 0.02 mm in diameter. Sedimentation basins for sand filters, 24 hours' supply, remove particles more than 0.007 mm. At Washington., D. C., in a succession of three reservoirs holding a week's supply, particles larger than 0.003 mm are removed.

The Amount of Sediment removed in settling basins at St. Louis amounts to 12 cubic yards per million gallons; with less turbid waters it is much less. For the Hudson at Albany only about 0.15 cubic yard is removed.

Scrubbers, or preliminary filters, are rapid coarse-grained filters or their equivalent. Substantially they take the place of sedimentation basins, doing the same work but doing it more quickly and in less space, tho usually at greater cost.

It is easy to design a scrubber that is efficient in operation. It is difficult to design one that can also be economically cleaned and kept continuously in efficient working order. From the standpoint of design and construction the cleaning devices are the most important parts of a scrubber. The object of scrubbers is to lighten the work of the filters at which the water is subsequently applied. It is not believed that any improvement in the quality of the ultimate effluent results from their use. This is because only the coarser particles are removed. The finer particles, which test the resisting power of the filters, cannot be

removed by them, and the separation of the coarser particles does not materially affect the final action of the smaller particles in the filter.

The size of particles in millimeters, such that 75 percent will be removed by a scrubber or preliminary filter of sand or gravel 40 inches deep, may be computed by

$$d = 0.0003\sqrt{\begin{Bmatrix}\text{effective size of}\\ \text{sand or gravel}\end{Bmatrix} \times \begin{Bmatrix}\text{rate of filtration, mill}\\ \text{gallons per acre daily}\end{Bmatrix}}\sqrt{\frac{60}{t+10}}$$

The scrubbers and preliminary filters that have given most satisfactory results are cleaned and operated practically as mechanical filters, but with simpler arrangements.

10. Sand for Filters

A Filter consists of a horizontal layer of sand thru which water is past to underdrains beneath, together with the containing structure and all auxiliaries. Filters act primarily as strainers, the interstices between the sand grains being small and serving to stop all particles too large to pass thru them. They also serve to purify the water in other ways. Filters are classified as mechanical filters, sand filters, intermittent filters, and special kinds of filters according to the construction, rates at which they are operated, and the methods used for cleaning.

The Sand for filtering purposes is best clean quartz sand, free from gravel and large particles, and also free from excessive quantities of fine particles and dirt of every description. The presence of a small amount of fine material often aids the action of filter sand. For filtering river waters and any waters carrying carbonic acid, filter sand should be free from lime, as otherwise the water will be hardened. Waters containing less carbonic acid, such as lake waters, will not dissolve lime and sand containing lime may be used.

The Size of Sand Grains is determined by sifting thru a set of rated sieves. About 110 grams of moist sand are put in a small iron dish and dried over a lamp. After cooling, 100 grams are put in the coarsest of a set of sieves, and the sieves are put in a mechanical shaker. A definite number of turns found by experience to be sufficient, is given. The shaking is not continued until no more passes, but only until the amount passing is small, so that doubling the number of shakes would not greatly change the result. The sieves are then taken apart, the material that has past all the sieves is first put upon the pan of the scale and weighed, then the material remaining on the finest sieve is added to it and again weighed. The process is repeated until all the material is on the scale, when it should equal the original weight. The percentages finer than the sizes corresponding to the several sieves are then plotted on a diagram, from which the required data are taken.

The Effective Size of sand is that size such that 10% of the sand grains by weight are finer than it. The size of a sand grain is always taken as the diameter of a sphere of equal volume.

The Uniformity Coefficient is the ratio between the effective size and that size such that 60% of the sand is finer than it.

In rating sieves an ordinary sand is put upon them and the shaking is performed with the usual number of revolutions. The sieves are then taken apart, each sieve is taken separately, and is given a further slight shaking. A small additional amount of sand passes. The grains so passing are substantially larger than all the grains that have previously past and smaller than those that remain. This small quantity of sand represents the size of separation of the sieve. A certain number of sand grains are counted out and weighed on an assay balance and the average weight is obtained. The diameter is obtained by the formula

$$D \text{ in mm} = 0.9\sqrt[3]{w} = \sqrt[3]{\frac{6}{\text{Sp. Gr.} \times \pi}}\sqrt[3]{w}. \quad w = \text{weight in milligrams.}$$

There is but little difference in the results of using round-grained and sharp-grained sands, and between grains of different shapes, but the rating is best carried out with various representative sands. Rating of sieves once made does not change appreciably with use until some openings become enlarged or some wires become broken. When this happens the sieves should be at once replaced. When a set of sieves is rated a lithographed sheet is made for plotting the results, in which one line represents each sieve in the set. The best results are with a mixt plotting, partly logarithmic and partly natural, laid out so that normal sands plot as nearly straight lines. On this system accuracy is obtained with a smaller number of sieves. Enough sieves so that each has a size of separation not more than twice as great as the next below will suffice, except where the uniformity coefficient is under 2. In that event one intermediate sieve should be used between each two.

The following represents approximately the relation between commercial brass wire cloth and the sizes of separation:

Meshes per inch........	200	140	100	50	40	30	20
Separation in mm.......	0.10	0.13	0.17	0.33	0.48	0.63	0.95

Owing to variations in weaving, individual sieves will vary 15% either way, and no sieve should be used without rating. In selecting wire cloth make sure that the spacing of the wires is even and that the number of wires per inch in one direction is not more than 10% greater than the number in the other direction. Coarser sieves are best of brass plates perforated with round holes and rated in the same way as wire cloth sieves.

Turbidity of Sand is the measure of clay in it. Put 10 grams of moist sand into a glass vessel holding one liter and fill with clear water. Agitate vigorously until all fine matter is in suspension. Allow to settle one minute, take the turbidity with a rod. Multiply the turbidity so found by 100 to obtain the parts per million of turbidity in the sand. The weight of clay is from ½ to ⅔ the turbidity, depending upon the size of the clay particles. Turbidity of sand prepared from stock containing clay should always be taken, but when there is no clay in the stock it is unnecessary to do it. At the Washington filtration plant the turbidity of the filter sand was not allowed to exceed 4000 in parts per million, or 0.4%, corresponding to about 0.2% actual clay.

11. Use of Filter Sand

Loss of Head is the frictional resistance of the sand to the passage of water. It is measured by the vertical distance between the level of the raw water over the sand and the level of the water in a small standpipe connected with the drains below the sand. Filters are provided with loss-of-head indicators. For any given condition of the filter bed the loss of head is directly proportional to the rate of filtration. The loss of head is made up of two parts, the frictional resistance of the clean sand and the resistance due the accumulation of dirt on the surface of the filter. The initial loss of head is that at the beginning of a run, but even this is more than for clean sand because there is some dirt on the surface from the very start.

The Frictional Resistance of sand to water when closely packed, with the pores completely filled with water, and in the entire absence of clogging, is indicated by

$$v = cd^2 \frac{h}{l} \left(\frac{t^\circ + 10^\circ}{60} \right)$$

where v is the velocity of the water in meters daily in a solid column of the same area as that of the sand, or approximately in million gallons per acre daily; c is a factor depending upon the uniformity coefficient, the shape of the sand grain, the chemical composition, the cleanness, and closeness of packing, commonly varying from 600 to 1200 for new sand and from 400 to 800 for old sand; d is the effective size in millimeters; h is the loss of head; l is the thickness of sand thru which the water passes; t° is the temperature (Fahr.).

Accuracy depends more upon good work in determining d, the effective size in millimeters than upon any other matter. The formula applies to filter sands with uniformity coefficients less than 3, but also less closely to those with uniformity coefficients to 6, and even 10. It does not apply to coarse gravels in which the viscosity of water is no longer controlling.

Required Head in Feet for Clean Sand One Foot Thick., $c=800$, $t=50°$ F

The adjacent table covers the ranges most commonly used in filtration. It applies to clean sand with no surface clogging. For thicker sand layers the head is greater in direct proportion.

Rate of filtration, million gallons per acre daily	Effective size of sand, millimeters			
	0.20	0.30	0.40	0.50
3	0.09	0.04	0.02	0.01
5	0.15	0.06	0.04	0.02
10	0.29	0.13	0.07	0.05
20	0.59	0.26	0.15	0.09
60	1.76	0.78	0.44	0.28
125	....	1.62	0.91	0.59

Preparation of Filter Sand. Occasionally natural sand is found suitable for use in filters, especially sea sand. Most bank and river sands require treatment. Coarse particles are removed by screening. Fine particles are removed by washing. If the stock contains clay it must first go thru a pug mill or otherwise be agitated to break up all the lumps of clay and loosen the particles from the sand grains. It is then past thru a washing box in which the sand moves horizontally and the water vertically at a rate corresponding to the rate of settlement of sand grains of the size of the required separation. The washt sand is drawn off with but little water. The box should have one square foot of area for each cubic yard per hour to be handled.

Filter sand can be prepared from a great variety of raw stocks. The essential requirement is that the stock contain a sufficient proportion of sand grains of the right size. To find the amount of filter sand that can be prepared from a given raw stock make a mechanical analysis of it and find the percentage finer than the desired effective size and the percentage finer than the 60% line. The difference between these percentages multiplied by 2 is the theoretical amount of filter sand that can be obtained. Actually from 75 to 95 percent of this amount should be obtained by a suitable plant. It will not often pay to work stock from which more than 10% of fine material must be removed because of the difficulty of washing, nor stock that has more than 50% of gravel to be excluded, nor stock where the size of separation of gravel to maintain the required results is lower than about 3 mm. For method see Trans. Am. Soc. C. E, vol. 57, p. 327.

The **size of separation** in sand washing depends principally upon the area of the boxes containing the mixt sand and water to which the mixture is coming, and from which the sand is drawn to the bottom, while the dirty water overflows, and upon the volume of water, taken always as the volume of the waste that overflows at the top. The approximate size of separation such that 75% of the particles of that size will be retained may be computed by

$$D \text{ in mm.} = 0.0065 f \frac{\text{gallons per minute of water overflowing}}{\text{square feet of box area}}$$

in which f ranges from 3.0 for ordinary single boxes to 1.5 for specially designed boxes with water entering steadily at the bottom and overflowing at well-distributed points at the top. The rule does not apply for sand grains less than 0.10 mm. in diameter.

Voids in filter sand range from 35 to 45 percent, according to the uniformity coefficient and the method of packing. Close packing is obtained with sand either perfectly dry or saturated with water. Sand packed moist always has more voids and settles from 4 to 8 percent when it is filled with water. Voids in sand are determined by driving a cylinder of sheet iron into the sand in the filter, carefully cutting away the adjoining material with a mason's trowel, and taking out sand equal to the exact contents of the cylinder. The sand is dried, weighed, and the volume of the solid particles computed, taking into account the specific gravity which for nearly all filter sands is 2.65. This is compared with the volume of the cylinder.

Results obtained by filling voids of dry sand with water are invariably too low because all the air is not driven out, even when filled in the most careful manner, from below.

Gravel is required in filter construction, usually in several sizes, prepared by screening. Crusht rock is used where gravel is not available. Gravel, especially in finer grades, is often obtained as a by-product from the preparation of the filter sand. Gravel should always be washt free from sand, clay, and fine particles. Where the water contains carbonic acid it is preferable free from limestone.

Washing and Handling Sand. Filter sand is commonly handled and washt hydraulically. Water thru a jet under a hundred pounds pressure passes thru an open space and enters a throat a little greater in diameter than the jet and carries with it sand loosened by water which surrounds the space. The mixture of sand and water will flow as a liquid thru pipes to the washers and from the washers to the points where it is stored.

At the Washington filtration plant, with dirty sand, one volume of water is required to take up one volume of sand to make slush, which slush has about 60% of the solid sand and a specific gravity of 1.59.

12. Sand Ejectors

Sand Ejectors and Flow of the Water and Sand in Discharge Piping

Pounds pressure feed water	Diameter jet, inches	Best diameter for throat, inches	Percent sand in discharge by volume	Cu yds sand per hour	Pressure of discharge in feet	Friction in ft per 1000 in discharge piping			
						2½″	3″	4″	5″
60	0.5	0.87	20	5.0	28	150	140	.....	....
	0.5	1.01	25	7.2	20	176	150	.....	....
	0.5	1.21	30	10.0	14	206	168	.....	....
	0.6	1.04	20	7.2	28	178	124	90	....
	0.6	1.21	25	10.3	20	222	144	110	....
	0.6	1.46	30	14.3	14	275	170	120	....
	0.7	1.06	15	6.5	39	200	114	70	....
	0.7	1.21	20	9.7	28	250	140	88	....
	0.7	1.41	25	14.0	20	325	175	102	....
	0.8	1.21	15	8.5	39	298	145	71	....
	0.8	1.39	20	12.7	28	370	180	89	73
	0.8	1.61	25	18.4	20	480	240	109	82
	0.9	1.20	10	6.5	52	350	160	61	42
	0.9	1.37	15	10.8	39	435	202	80	56
	0.9	1.56	20	16.1	28	540	250	102	71
80	0.5	0.87	20	5.7	38	154	130	.....	....
	0.5	1.01	25	8.3	27	186	145	.....	....
	0.5	1.21	30	11.5	18	225	160	.....	....
	0.6	1.04	20	8.3	38	208	128	90	....
	0.6	1.21	25	11.9	27	260	153	107	....
	0.6	1.46	30	16.5	18	320	187	117	....
	0.7	1.06	15	7.5	52	245	127	70	....
	0.7	1.21	20	11.2	38	305	158	86	....
	0.7	1.41	25	16.2	27	400	204	104	71
	0.8	1.21	15	9.8	52	370	176	76	56
	0.8	1.39	20	14.7	38	465	222	95	71
	0.8	1.61	25	21.2	27	600	290	115	84
	0.9	1.20	10	7.5	70	450	200	70	43
	0.9	1.36	15	12.4	52	550	248	91	58
	0.9	1.56	20	18.6	38	680	315	114	73

The Resistance to be overcome in the discharge piping is made up of actual lift and friction. For the lift, multiply the actual lift in feet by the specific gravity of the mixture. For friction of sand and water in 3 and 4 inch pipes, compute the friction for water alone, and add 3.5 feet per thousand for each percent of sand in the mixture. For 6-inch or larger pipe add 2.5 ft and for 2.5-inch hose, add 4.5 ft per thousand. These figures will be close enough for velocities 5 ft per second or over. Sand and water mixtures will flow well at all velocities above 5 ft per second and fairly well from 4 to 5 ft per second. Between 3 and 4 ft per second there will be more friction than calculated and some stoppages; and below 3 ft per second sand and water mixtures will not flow.

$$\left.\begin{array}{l}\text{Velocity in}\\ \text{ft per second}\end{array}\right\} = \frac{137.5 \text{ cu yds per hour}}{\text{Per cent of sand in discharge} \times (\text{diameter of pipe in inches})^2}$$

$$\text{Cubic yards per hour} = \frac{\text{Per cent sand} \times \text{velocity} \times \text{diameter}^2}{137.5}$$

Sand Ejectors and Flow of the Water and Sand in Discharge Piping

Pounds pressure feed water	Diameter jet, inches	Best diameter for throat, inches	Percent sand in discharge by volume	Cu yds sand per hour	Pressure of discharge in feet	Friction in ft per 1000 in discharge piping			
						2½″	3″	4″	5″
100	0.5	0.87	20	6.4	47	162	126	.....	....
	0.5	1.01	25	9.2	34	200	143	.....	....
	0.5	1.21	30	12.9	23	250	164	120	....
	0.6	1.04	20	9.2	47	230	133	87	....
	0.6	1.21	25	13.3	34	300	165	103	....
	0.6	1.46	30	18.5	23	380	210	118	105
	0.7	1.06	15	8.4	65	292	144	71	....
	0.7	1.21	20	12.6	47	360	180	88	71
	0.7	1.41	25	18.1	34	470	235	108	83
	0.8	1.21	15	11.0	65	450	208	82	56
	0.8	1.39	20	16.4	47	550	260	104	71
	0.8	1.61	25	23.7	34	.....	340	130	86
	0.9	1.20	10	8.3	87	.....	240	78	45
	0.9	1.36	15	13.9	65	.....	300	100	60
	0.9	1.56	20	20.8	47	.....	370	128	76
150	0.5	0.87	20	7.8	71	195	124	90	....
	0.5	1.01	25	11.4	51	240	150	110	....
	0.5	1.21	30	15.8	35	305	180	118	....
	0.6	1.04	20	11.3	71	310	160	86	75
	0.6	1.21	25	16.3	51	400	205	104	83
	0.6	1.46	30	22.7	35	520	260	125	100
	0.7	1.06	15	10.3	98	410	190	78	56
	0.7	1.21	20	15.4	71	500	236	98	71
	0.7	1.41	25	22.2	51	640	310	120	85
	0.8	1.21	15	13.5	98	.....	285	99	59
	0.8	1.39	20	20.0	71	.....	350	123	75
	0.8	1.61	25	29.0	51	.....	450	160	93
	0.9	1.20	10	10.2	131	.....	335	100	50
	0.9	1.36	15	17.0	98	.....	410	129	67
	0.9	1.56	20	25.5	71	.....	510	163	85

Hydraulics of Ejecto.. The best form of throat is one approaching the shape of a venturi meter. The calculations on page 1220 are for a throat of this shape but somewhat worn by use and not quite in the best condition. With a carefully turned new throat the results may be higher and with worn throats lower. With the best size and shape of throat for any given condition, the tables on pages 1217 and 1218 show the approximate relations.

Fig. 6 shows the method of removing dirty sand from the filters at Washington. It is shoveled to a movable ejector which throws the sand thru a

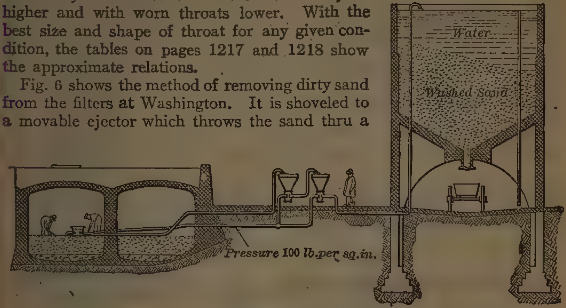

Fig. 6. Sand Ejector, Washington, D.C.

line of hose and pipe to the sand washer outside. The washt sand is thrown by another ejector thru pipe to an elevated sand bin. From the sand bin the sand was formerly replaced in the filters by carts drawn by horses, but

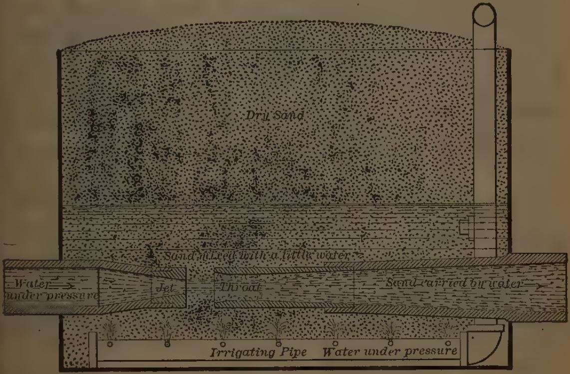

Fig. 7. Action of Sand Ejector

it is now replaced hydraulically, another ejector serving to throw it from the bin to the place where it is to be deposited. Fig. 7 shows the working parts of a sand ejector and indicates the way in which it operates.

Percent of sand in water thrown by vol..	5	10	15	20	25	30
Specific gravity of mixture............	1.05	1.10	1.15	1.20	1.25	1.30
Percent slush by volume..............	8.3	16.7	25.0	33.3	41.7	50.0
Percent nozzle water by volume........	91.7	83.3	75.0	66.7	58.3	50.0
Weight of slush per part water from nozzle..........................	0.15	0.32	0.53	0.80	1.14	1.59
Q = ratio total weight of discharge to weight of jet water................	1.15	1.32	1.53	1.80	2.14	2.59
P = proportion of jet pressure developed in discharge......................	0.50	0.37	0.28	0.20	0.14	0.10
T = ratio of diameter of throat to diameter of jet........................	1.18	1.33	1.51	1.73	2.02	2.43
V = ratio of velocity in throat to velocity in jet..........................	0.79	0.68	0.59	0.50	0.42	0.35

The formula $(P+V)\,T^{1.5} = 1.65$ applies to well-shaped venturi throat ejectors throwing sand. The best results are obtained when $QV = 0.9$ with 5% or 10% variation either way, and within this approximate range $PQ^2 = 0.65$. This is the most convenient equation for comparing efficiencies.

13. Mechanical Filters

Mechanical filters are filters operating at a high rate with mechanical appliances for cleaning the sand without removing it from the filter. The filter tank containing the sand was formerly of wood or sometimes of steel, but reinforced concrete is now used almost exclusively. It is essential that the filter tank should be air tight as well as water tight, to prevent the entrance of air from the outside in the last portion of the run.

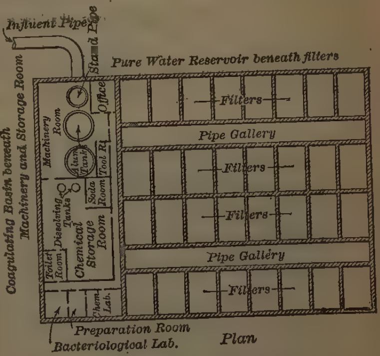

Fig. 8. Layout of a Mechanical Filter

The Sand is commonly from 18 to 36 inches deep, and with effective sizes ranging from 0.35 to 0.50, commonly about 0.40 mm. The uniformity coefficient should be as low as possible and must not exceed 1.6. Sand with high uniformity coefficients is separated into coarse and fine parts in the act of washing, and full filtering cannot then be obtained. The sand is supported at the bottom by layers of gravel, which must be heavy enough and open enough to allow the wash water to pass without lifting. In some designs the gravel is held down by wire cloth on top, but with carefully prepared gravel this is not necessary.

The Strainer System serves the double purpose of introducing wash water to wash the sand with a reverse current and of carrying off the effluent. In order to serve its purpose of carrying off the effluent the frictional resistance of the system must not exceed one-fourth the frictional resistance of the sand at the beginning of the run. Otherwise the near parts of the filter would operate at higher rates than the remote parts.

The fundamental requirement of the strainer system when used for washing is that it shall give an equal dispersion of wash water over the whole area of the filter. There is a tendency for the sand to lift at one place. When this happens the pressure on the strainers at that place is reduced and there is a tendency for all the wash water to go out at one place. This is controlled by putting restrictions on the outlets having resistance at the rate of use greater than the resistance offered by the sand, so that these throats control the position of overflow of the water. The throats must be small enough to give the desired rate of wash with a frictional resistance that will insure the even distribution of the wash water, and large enough so that it does not exceed the economical head applicable to this purpose. The area of main drains should equal twice the area of all the throats supplied by them, and lateral piping should have 2½ times the area of all the throats supplied.

The throats are necessarily small in size and the full area of each is needed to carry off the effluent in the operation of the filter. It is therefore necessary to protect the throats

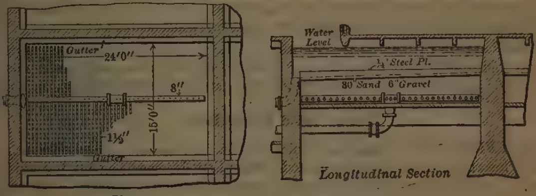

Fig. 9. One Unit of a Mechanical Filter

so that the gravel will not come over them and reduce their area in the operation of filtration. This is commonly done by putting strainers above them but in some cases simply surrounding them with gravel is relied upon.

Mud Balls are accumulations of dirty sand on the bottom of the filter, resulting from the settling of masses of dirt at the top at the beginning of the wash. Mud balls are prevented in three ways: (1) By a mechanical agitation of the sand by a revolving rake, most used in the early designs. (2) By air blown thru the bottom of the filter during washing to agitate the sand, used in a large part of recent designs. (3) By pulling them out with wire nets on poles by the attendant while the filter is being washt. This is not to be recommended, as it tends to disturb the gravel.

In some recent designs the wash water is used at a higher velocity, and this alone is depended upon to prevent the formation of mud balls. In this case the filter sand must be coarser than would otherwise be necessary.

The Rate of Washing is commonly from 9 to 24 vertical inches per minute, 12 inches being the commonest figure. GUTTERS to carry off the wash water should not be less than 12 nor more than 18 inches at their edges above the surface of the sand. The eddying action of the water entering them reduces velocity, and the size of the gutters must be larger than is computed by the usual hydraulic formulas.

Regulating Apparatus, or rate controllers, is automatic apparatus on the outlet for controlling the rate of filtration. There are two general types, those operated by floats, which are incapable of acting submerged, and those operating by the difference in pressure on two sides of a diaphragm or piston, which are capable of acting submerged.

Loss of head ranges from 2 ft or less at the beginning of a run to the limit that is allowed, which may be anything from 4 to 10 feet. With the loss of

head limited to 4 ft, no suction exists in the filter, and the whole operation said to be performed by a positiv head. With loss of head going to 6 and over the outlet pipe acts as a draft tube and produces suction in filter in the latter part of the run. This is known as negativ head.

Negativ head is efficient in making a filter operate, but practically its action is limi because with it air is extracted from the passing water which fills the voids in the sa and soon cuts off the suction. The first foot of negativ head is only a little less us than the last foot of positiv head, but each additional foot utilized yields a smaller ret than the last. The full effect of negativ head may be utilized by putting an air cham on the outlet pipe and pumping the air from it in the latter part of the run, thereby mak the full negativ head available.

The Frequency of Washing depends upon the character of the wa and especially on the size and manner of operation of the coagulating bas and commonly ranges from 8 to 48 hours. Washing requires about minutes and the auxiliary operations more than as much more. From two five percent of the water filtered is required for wash water.

The Rate of Filtration is generally 2 gallons per square foot per minu equal to 125 million gallons per acre, or 117 cubic meters per square me per day. 347 sq ft of filter pass water at the rate of one mill gals per 24 hou

14. Sand Filter Beds

Sand Filters differ from mechanical filters in that the rate of filtratio much lower and the filter is cleaned when dirty by scraping off the surf layer of dirty sand, which is washt and ultimately replaced. The s should have an effective size between 0.25 and 0.35 and a uniformity co cient not exceeding 3.0. Many old filters had sands with uniformity coeffici up to 5, but such sand is much more difficult to keep in good order. S suitable for sand filters is more cheaply obtained than sand with a low formity coefficient required for mechanical filters.

The Gravel is commonly placed in three layers. The lowest layer 7 inc thick and with ¾-inch to 2-inch screens having an effective size of about 20 m the second layer 3 inches deep, ⅜ to ¾ inch screens with an effective siz about 8 mm.; and the top layer 2 inches deep, with an effective size of f 2 to 3 mm.

Underdrains must be designed so that, at the proposed rate of filtrat the frictional resistance of the whole system will not be more than about the frictional resistance of the sand when clean. Otherwise when the filt started that part of the filter near the outlet would do most of the work.

In very large filters compensating orifices are used, artificially utilizing resistan equalize the rates in different parts and prevent undue increase in the size of drains.

Underdrains for Sand Filters (no compensating orifices used).

Rate of filtration, million gallons per acre daily..	5	6	8	10
Average resistance of clean sand in feet........	0.150	0.180	0.240	0.300
Total allowable friction and velocity head in underdrainage system....................	0.037	0.045	0.060	0.075
Approximate ratio of filter area to area of main drain..................................	5100	4700	4200	3800
Approximate maximum velocity in main drain (varying somewhat with size)...............	0.90	1.00	1.18	1.34
Approximate maximum velocity in laterals (varying somewhat with size)..................	0.55	0.61	0.72	0.82

Masonry Covers for sand filters are required where the winters are se in general for all of the United States north of Washington, Cincinnati,

St. Louis. Open filters, sometimes built north of this line, are operated with lecreased efficiency and at increased expense during periods of ice, but for special service such as removal of iron, or tastes and odors from summer growths, they are sufficient. The net height inside the masonry structure s commonly 12 feet, to give convenient head room for cleaning operations.

Filters are built in units, ranging from an acre in the largest plants to half an acre and ess in small plants. There should always be enough units so that the supply can be naintained with one unit out of service, in small places, and with two or more out of ervice in large plants.

Maximum Areas of Filter Beds Drained in Square Feet

Diam. of drain, inches	Shape and kind of drain	Rate of filtration, million gallons per day				
		5	6	8	10	15
4	Round lateral....	264	245	218	200	168
5	Round lateral....	420	390	345	316	266
6	Round lateral....	610	570	500	460	390
8	Split lateral	520	490	430	400	320
10	Split lateral.....	830	770	680	630	530
12	Split lateral	1 200	1 120	1 000	910	770
10	Round main.....	2 700	2 500	2 200	2 000	1 700
12	Round main.....	3 900	3 600	3 200	2 900	2 400
15	Round main.....	6 200	5 800	5 100	4 600	3 900
18	Round main.....	9 000	8 300	7 400	6 700	5 600
21	Round main.....	12 300	11 400	10 000	9 100	7 600
24	Round main.....	16 100	14 900	13 200	12 000	10 000
36	Round main.....	37 000	34 000	30 000	27 000	22 000

Automatic controllers are sometimes used on the outlets of sand filters, but as the hange in loss of head is slow, hand control in connection with adequate devices for indicating at all times the rate of filtration and loss of head is sufficient.

The Period of a sand filter is the time between cleanings exprest in nillions of gallons per acre. That is to say, if a filter operates 20 days at a 5-million rate the period is 100. The average period for a plant is found by dividing the number of million gallons filtered in one year by the total number of acres of filter surface cleaned.

Periods are increased by drawing the water off when the loss of head has reached the llowed limit and raking the surface of the sand and then proceeding. It does not usually pay to rake more than once in one period. Thoro sedimentation lengthens the period. The application of coagulant lengthens it if applied sufficiently long before filtration, but with a short period of settling after application it shortens it. Preliminary filters engthen the period. Periods commonly range from 50 to 200. If they average less than o there is something wrong with the arrangements.

The Loss of Head in sand filters is commonly limited to 4 ft, but this s arbitrary and losses up to 5 or 6 ft are sometimes permitted. The RATES employed with sand filters filtering river waters without preliminary chemical reatment are about 3 million gallons per acre daily. For lake and reservoir waters rates twice as high are used. For filtering river waters with preliminary chemical treatment and settling when the water requires it, corresponding rates can be used.

Cleaning. At the end of a period the dirty sand to a depth of from ½ to 1½ inches is removed. It is best shoveled to movable ejectors throwing the water thru the piping system to sand washers outside. Scrapings are repeated for a considerable period, averaging 2 years, when washt sand is restored

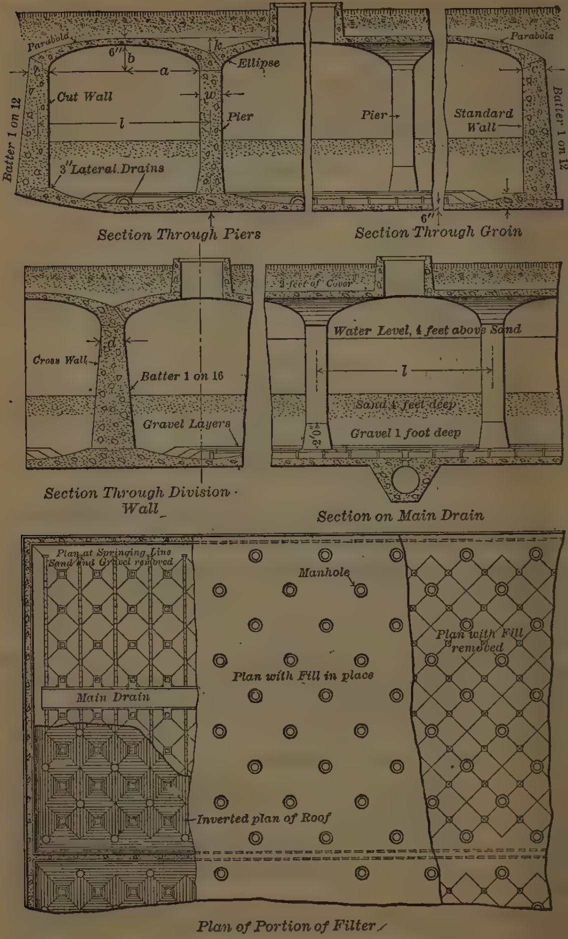

Fig. 10. Slow Sand Filter with Groined Roof

bringing it back to the original level. Cleanings in very cold weather may be done with the dirty sand piled up on the surface of the filter, where it remains until spring. The quantity of water under pressure required for handling and washing sand is commonly from 0.4 % to 1.0 % of the quantity filtered.

An old filter is commonly more efficient than a new one, and the water filtered during the last part of the period is frequently better than in the first part. It has sometimes been recommended that the first water from sand filters be wasted. It is better practise, however, to start the filter at a lower rate, at which the first effluent will be good, and gradually increase the rate after processes of purification become established.

15. Results of Purification

The Cost of installing filters may range from $10 000 to $50 000 per million gallons daily capacity, figures between $20 000 and $30 000 being most common. As the filters must usually be 50 % greater in capacity than the average output, the cost based on the average output will be 50 % greater. Operating costs range from $1 per million gallons and less for sand filters treating clear waters to $5 or $6 per million gallons for mechanical filters with highly turbid and colored waters. The whole cost of filtration, including both operating expenses and interest and depreciation charges on the capital, commonly ranges from $7 to $15 per million gallons.

Sand filters are often more expensive to build but cheaper to operate than mechanical filters. Lake waters are cheaper to purify than river waters, and reservoir waters are intermediate. Large amounts of color and turbidity add greatly to the cost of treatment. Softening is more expensive than ordinary filtration.

In filtering river waters 99 % removal of bacteria is commonly obtained. The efficiency is usually somewhat higher in summer than in winter. Growths of bacteria in the underdrains occasionally occur. These have no hygienic significance but reduce the percentage apparent removal. These are most likely to occur in filters operating at low rates. Purification as practised does not absolutely remove infection, but it reduces the risk to one so extremely small that it may be considered negligible. Experience with filtration extending over half a century has shown that well-designed, well-constructed, and well-operated filter plants can be relied upon to furnish a safe and acceptable supply of water.

Intermittent Filters are sand filters in which the oxidizing action of the air is more important than the straining action of the sand. They are built of coarse sand with large underdrains freely open to the outlet and with no controlling or throttling devices. Water is applied generally once a day and generally for a period of not more than 12 hours. It is essential that the water on the filter should draw down, leaving the surface entirely exposed for some time before the next application of water. This fills the pores of the filtering material with air and the water next filtered is brought in contact with this air. The oxidizing action of an intermittent filter is similar to the oxidizing action of an intermittent filter or sprinkling filter which is used for treating sewage.

Intermittent filters are used for treating badly smelling waters, especially from shallow reservoirs and reservoirs that have at times high temperatures, as in the tropics. Such waters may contain large quantities of organic matter and be incapable of being filtered satisfactorily by either mechanical filters or sand filters of the ordinary type. The rate of filtration depends upon the amount of organic matter in the water, and may be from 3 to 6 million gallons per day. Cleaning the sand is done by the same methods as those used for sand filters.

DISTRIBUTION OF WATER

16. Systems of Distribution

A Waterworks System must secure its supply when and where it can be gotten, and must deliver it when and where required by its customers, or "takers." Waterworks structures are required to collect the water; to hold it from times when it is available until it is required; to pump it to a higher elevation; to convey it from the point where it is available to the points where it is required, and to allow the water to be measured and controlled at all points. Water is required by the takers at very unequal rates, following the requirements and emergencies that arise in their business, and the fundamental requirement controlling the design of works is to secure the ability to supply water wherever and whenever required and in whatever reasonable amounts may be needed.

The Distribution System includes all the main pipes and lateral pipes, the standpipes and distributing reservoirs, gates, meters, all services and connections as far as owned by the water department within and near the area that is actually served with water. The piping in a distribution system must be designed so that water can be supplied to any point at any time at the greatest rate at which water may be fairly demanded at that place.

Gridiron System. This is a system in which all pipes are connected with all other pipes at street intersections, so that in case of a fire at any point water comes to that point thru pipes from all directions. This arrangement is more advantageous in supplying water for fire protection than the branching system, which would be sufficient and often best for supplying water for all purposes except fire service. The gridiron system is practically universal in American cities.

An economical system for the distribution of water for routine uses only would consist of a system of branching pipes, each branch being made sufficiently large to supply the water to the territory served by it at the time of day when use is greatest.

High-service Systems. Where there are considerable parts of a city where the ground is high, so that the service and pressure used in most of the area do not suffice to supply them properly, it is common to cut them off and make special high-service systems for them. Each high-service system has its own distribution and distributing reservoir, (or else direct pumping), and it is to be considered as if it were an entirely separate system of waterworks. Water is commonly taken from the low-service mains, and those mains must be large enough to supply the high-service water in addition to their other functions. In all other respects they may be treated as if the high service did not exist. In nearly all cases water is pumped specially into the high-service pipes.

The Direct Pumping System is used in the middle states and in general wherever the ground is flat. With it there is no distributing reservoir, but water is pumped into the pipes at all times as needed. Pumps of special design are required for this service. When but little water is used the pump must go slowly and automatic regulators must close the throttle so that it will not go too fast and increase the pressure. When water is to be used rapidly the pump must increase its speed and deliver the increased volume of water at full pressure.

Pumps operating on this system are not as efficient as pumps operating at constant speed with a distributing reservoir connected with the system. The pumps must have much greater capacity than where there is a distributing reservoir. The capacity of the pumps must always be equal to three times the average annual rate of consumption, and in small works the capacity of the pumps must be four or five or more times as great. In general the capacity of pumps must be sufficient to maintain the fire service

and the domestic service at the same time. It is further good business as far as possible to have one pumping unit in reserve beyond this capacity.

Reservoirs with Direct Pumping. Wherever the water is filtered or ground water obtained there is a limit to the rate at which pure water can be provided in emergency, and it is commonly necessary to have water during large fires at a greater rate than can be furnished by the filters or wells. To meet this condition pure-water reservoirs at the pumping stations are provided. The size of reservoir is to be determined by the same considerations that determine the size of an elevated distributing reservoir. With a reservoir of this kind kept ordinarily full or nearly full the supply works may operate at a continuous rate thruout the twenty-four hours, and there will always be a reserve of pure water that can be pumped as fast as the pumps can take it in case of fire or other emergency. Such reservoirs are always covered.

Auxiliary Fire Service Systems consist of high-pressure pumping station and pipes laid in central portions of cities where the greatest values of property subject to fire hazard are concentrated. These pipes are built to withstand pressures up to 300 lbs, and are connected with hydrants of special design and painted some other color than the hydrants connected with the ordinary pipes. The pumps may be operated by gas engines, electricity, or by other power. Such systems are used only in case of fire. In case of need the pumps are started up and the pressure is raised on the system to such a point that efficient fire streams are obtained by attaching lines of hose to the hydrants. No steam fire engines are used.

Salt Water is sometimes used and is believed to be more efficient in extinguishing fires than fresh water. It also results in more damage to goods and property not destroyed by fire with which it comes in contact. Otherwise water is taken from the nearest lake, river, or reservoir or from the ordinary water mains. Where auxiliary fire systems are used there is obviously less demand upon the regular supply system for fire service, and its capacity for rendering fire service may therefore be reduced somewhat. Frequently more money is spent on auxiliary fire service systems than would suffice to reinforce the regular waterworks system to the extent that good service could be obtained from it.

Increased Pressure for Fire Service is obtained in some direct pumping systems by having additional boilers kept under steam and turning on more steam when a fire alarm is sounded. By this means the pressure thruout the city is increased and hose streams are obtained from hydrants without the use of fire engines. This system is much used with direct pumping in the middle states. It is best adapted to small places. The expense of the system and the disturbance that comes to the distribution from increasing the pressure in the whole city are too great in large cities where fire alarms are sounded frequently.

17. Water Consumption

Per Capita Consumption is the amount of water used per day for each person living in the city of area supplied on the basis of the annual average figures. In other words, it is the whole quantity of water supplied in gallons in one year, divided by 365 and divided by the total population of the district supplied with water.

Maximum Monthly Rate of Consumption. During that month in the year when the consumption is highest, from 15 to 25 % more water is used than the average for the year. In some cases 40 % more water is used.

High monthly rates of consumption are usually associated with either a very dry period, with more than the usual sprinkling of streets and lawns, or an exceptionally cold month, with a continued draft of water thru many services to keep exposed and imperfectly protected pipes from freezing. Where services are metered the excess consumption in cold weather largely disappears. It is cheaper to cover the pipes or otherwise to protect them from freezing than to pay for the water that it is necessary to allow to run in order to protect them.

Place	Year	Estimated Population	Approximate total daily consumption, mill gals	Consumption per capita, gals daily	Percent consumption metered	Percent services metered
Boston, Met. Dist.	1917	1,215,840	110	90	....	72
Buffalo	1917–18	505,000	167	331	29	5
Cambridge	1916–17	114,000	9	79	....	36
Cincinnati	1916	425,000	52	126	50	69
Columbus	1915	211,000	17	80	64	95
Detroit	1915–16	781,133	128	163	53	36
Fall River	1917	124,799	6	50	73	98
Hartford	1917–18	157,000	12	76	81	99
Haverhill	1916	50,534	6	116	22	32
Kansas City	1917–18	391,000	37	95	....	71
Louisville	1917	250,000	28	112	....	....
Lowell	1918	125,000	8	60	58	86
Memphis	1916	151,877	12	78	43	70
Milwaukee	1917	475,000	60	125	71	99
Minneapolis	1918	386,000	31	80	....	100
Newark	1916	412,800	47	115	57	65
New Bedford	1917	115,000	9	81	85	96
New Orleans	1917	384,000	30	77	66	100
Philadelphia	1916	1,720,500	321	187	....	16
Providence	1917	289,700	19	66	70	94
Reading	1915	106,780	13	125	38	32
Rochester	1918	260,000	28	108	60	98
St. Louis	1917–18	765,000	104	135	30	7
St. Paul	1917	276,000	16	58	64	91
Springfield	1917	109,089	12	109	73	98
San Francisco	1918	555,000	37	67	....	100
Toledo	1915	244,242	16	63	85	78
Washington	1917	359,997	54	149	....	....
Wilmington	1914–15	95,000	10	102	64	93
Worcester	1915	169,599	13	75	72	....

Maximum Weekly and Daily Rates. There will be some weeks and some days when the quantities will considerably exceed the average for the maximum month. Generally a maximum daily consumption of 10 or 15 gallons per capita in excess of the average for the maximum month must be expected.

Hourly Fluctuations in Flow. Water is required primarily for domestic and manufacturing purposes, and for these purposes is required in quantities that are fairly well determined and at times that do not vary very much from day to day. The greatest normal use of water is in the morning hours. The afternoon use is a little less. The night use of water is comparatively small.

The hourly fluctuation is found to be more nearly in proportion to the population supplied than in proportion to the quantity of water supplied. This is because the waste of water is in general at a nearly constant rate thruout the twenty-four hours, while the use of water is mainly concentrated during the hours of daylight. For example: If the actual use is at rates ranging from 40 to 80 gallons per capita daily, with no waste, then the range in actual rate would be from 40 to 80 gallons per capita daily, or 33% each way from an average of 60 gallons. With 100 gallons per capita daily of waste and the same proportions in use, then the range in rate of supply will be from 140 to 180 gallons per capita daily, or 12 percent each way from an average of 160 gallons.

The Ordinary Range in Rate is from 20 gallons per capita daily below to 20 gallons per capita daily above the average per capita rate. This will cover the fluctuations for most of the time. Less numerous fluctuations extend from 30 to 40 gallons per capita daily above, and still less frequent fluctuations to 60 and 70 gallons per capita daily above the per capita consumption. The last may be taken as an extreme figure for American conditions. If the pipes are capable of supplying water at the rate of the average annual consumption, and 70 gallons per capita in addition, they will serve to take care of all the fluctuations that are to be reasonably anticipated, except those occasioned by fire service.

Fluctuations above 50 gallons per capita are comparatively rare, but the additional cost of providing pipe capacity for 70 gallons per capita daily not being very great, and such fluctuations being occasionally experienced, it is better to use the larger figure for design.

Waste of Water. The amount of water that must be supplied at the source is always greater than that actually needed for use. There is leakage from the main pipes, and from the service pipes, and from a considerable percentage of the plumbing fixtures in the houses, and water is allowed to run uselessly from many openings. In many American cities four times as much water is supplied as is actually used. The greater part is wasted without benefit. It is mainly for this reason that there are such great differences in the per capita consumption of American cities.

Waste from Service Pipes. Bad plumbing and the unnecessary draft of water from fixtures are best checked by putting meters on the services and adopting a schedule of rates by which the payments of takers are dependent upon the amount of water passing thru the meters. If the water that is wasted is paid for at a fair price, then the water department has no occasion to object to the waste. The waste beyond the meter becomes entirely a matter for the taker to consider.

Loss of Water by Leakage from Mains is best detected by dividing the system by closing the valves into comparatively small sections and by supplying water to each small section in rotation thru a meter. Meters may be put on the mains temporarily or permanently for this purpose.

It is often possible to shut all the gates on a section and maintain the supply to it thru a line of fire hose, connecting one hydrant inside the area with another hydrant outside, a meter being placed in the line of hose.

By opening and closing gates, a number of adjoining sections may be tested with the same equipment and in the course of a short time. Tests of this kind are best made between midnight and five o'clock in the morning, when the normal use of water is at a minimum and there are practically no fluctuations in the rate of draft. Such tests show the relative tightness of different parts of the system, and the lengths of pipe where the greatest leakage occurs are then to be dug up and repaired or replaced with pipes that do not leak.

Amount of Growth to be Anticipated. In designing pipe lines, it is necessary to anticipate growth to a certain extent in order to avoid the necessity of duplicating the lines at an early date. On the other hand, anticipating future growth to an unreasonable extent results in burdening present takers with the cost of facilities provided for the future to an unreasonable extent. In general, all new pipe lines should be designed to serve a population 50% greater than the present population, and in cases of special difficulty, where an additional line would be specially difficult or expensive, a greater growth than this should be anticipated.

Increasing the diameter of the pipe 1% increases the carrying capacity 2.63%, and increases the cost of the pipe from 1% to 1.5%, according to the

size and class of pipe and the conditions under which it is laid. On this basis adding 1 % to the investment adds from 1.75 % to 2.63 % to the carrying capacity, $100 invested now in increasing the size of the pipe adds as much to the capacity as from $175 to $263 invested in a new pipe line at some time in the future if the new line is of the same size as the present one. $100 invested now at 5 % will amount to $175 in 11 years and $263 in 20 years; at 4 % the increase will be reached in 14 years and 25 years respectively. In general these represent economical limits of time to be anticipated.

As a general rule design should be made for ten or fifteen years only, where the growth is over 3% per annum or where money is hard to get, and design for twenty or twenty-five years where growth is under 2% per annum or where money is obtainable at a low rate, and also in all cases where pipe is less than 12 inches in diameter or where pressure is light. There are many exceptions to this rule under peculiar conditions and it must be applied with caution.

18. Fire Protection

The Requirements of Fire Service vary greatly. In European cities, with fire-proof buildings, but little water is required for the extinguishment of fire. In tropical countries, where buildings are widely separated and represent but small value, and often in wet climates, it does not pay to furnish fire service. It is better to let buildings burn now and then than to provide long and larger pipes and other equipment that would be required for fire service. In American cities wooden construction is common and wooden floors are used in many buildings having brick walls. A large pipe capacity is required to provide the water which is required for extinguishing fires in such buildings.

The Amount of Water required for extinguishing fires is not very large in the aggregate, but when fires occur it is wanted at a high rate, and pipes must therefore be provided of large capacity to meet this demand. Pipe sizes required for fire protection in American cities are always larger than those required for other uses, and the size of pipe to be selected within the area of the distribution system, and between it and the distributing reservoir or pumping station where direct pumping is used, is mainly controlled by questions of fire protection.

Water Required for Fire Service. The amount of water to be provided for fire service depends upon many matters; among others, the size of buildings; the materials and methods of construction; upon how near the buildings are together; the pressure at which the water is available; upon whether auxiliary fire systems are available; upon how great a loss of life and property might result from a bad fire, and upon the cost of making a given quantity of water available and the financial ability of the system or community to pay for doing it.

For Average American Conditions, take the square root of the population in thousands and this indicates the rate in millions of gallons of water per day at which water should be provided for fire service.

For example: If the population is 9 thousand allow water at a rate of 3 million gallons per day for fire service. If the population is 25 thousand allow 5 million gallons per day, and if 100 thousand allow 10 million gallons of water per day.

The pipes must be designed large enough so that the quantity of water for fire service will be available even tho the fire occurs at a time when water is being used at a high rate for other purposes. It is not necessary to assume the extreme maximum rate of draft for other purposes; some chances can be taken. To find the required capacity add, first, the average annual rate of consumption; second, 20 gallons per capita to cover ordinary fluctuations;

third, the amount of water allowed for fire protection. If the fluctuations are unusually great, take 30 or 40 gallons per capita in place of 20.

Concentration of Water for Fire Service. In the case of cities up to 100 000 inhabitants it is generally necessary to provide pipe capacity so that the whole amount of water provided for fire protection can be delivered with some loss of pressure in the neighborhood of the closest, largest, highest and most valuable buildings, and at each of such points if there are several; elsewhere piping capable of delivering smaller quantities varying with the kind and value of construction and the proximity of the various buildings.

Population that can be Supplied by Pipes of Various Sizes

Based on an average use of one hundred gallons per capita daily

Diameter of one pipe line, inches	For two or more pipes, sum of areas in sq ins Sectional area of pipe sq ins	With an average amount of fire service *			With no fire service. Maximum draft 170 gallons per capita daily		
		Flat slopes and long lines $V = 2$ †	Average conditions $V = 3$	Steep slopes and short lines $V = 4$	Flat slopes and long lines $V = 2$	Average conditions $V = 3$	Steep slopes and short lines $V = 4$
4	13	12	27	48	660	990	1 530
6	28	61	132	228	1 490	2 240	2 950
8	50	182	392	666	2 650	3 980	5 320
10	79	425	900	1 500	4 150	6 190	8 280
12	113	835	1 720	2 850	5 950	8 950	12 000
16	201	2 320	4 620	7 400	10 600	15 900	21 300
20	314	4 940	9 520	14 900	16 500	24 800	33 200
24	452	8 900	16 700	25 500	23 900	35 800	47 800
30	707	17 200	32 000	48 000	37 400	56 100	74 800
36	1018	30 300	53 300	78 200	53 800	80 500	108 000
42	1385	46 600	80 400	117 000	73 200	110 000	146 000
48	1810	67 100	114 000	163 000	95 300	142 000	190 000
54	2290	91 600	153 000	219 000	121 000	181 000	242 000
60	2827	120 000	200 000	282 000	148 000	224 000	299 000

* Gallons daily = 120 pop. + 1 000 000 $\sqrt{\text{Pop. in thousands.}}$ † V = velocity, ft per sec.

This table may be used as a very general guide. With high per capita consumption and bad fire conditions the sizes should be increased. Under opposite conditions they may be reduced. It will often pay to make pipe sizes a little smaller in the distribution and larger in the supply mains without changing the total capacity of the system.

A Standard Fire Stream is one flowing 250 gallons per minute thru a smooth nozzle 1⅛ inches in diameter, with a pressure at the base of the tip of 45 pounds. Such a stream is effective to a height of 70 feet above the ground or with a horizontal carry not exceeding 63 feet. When fed thru the best quality 2½-inch rubber-lined hose the hydrant pressure required to throw such a stream taken while the stream is running is as follows:

Feet of hose =	50	100	200	400	600
Lb per sq in =	56	63	77	106	135

The hydrant pressure is less during the fire than at other times, because more head is lost in friction in the pipes, and the ordinary pressure must be greater to insure standard conditions during fire. The best hydrant pressure for general use is considered to be from 80 to 100 lbs, but as other conditions are frequently controlling, fire service must be largely adapted to what is available.

The best statement of the hydraulics of fire streams and nozzles is in a paper by John R. Freeman, Trans. Am. Soc. C. E., 1889, vol. 21, p 303.

Pressure for Domestic Service Only. At the street line 20 lb per sq in will raise water to the upper floor of three-story residences and allow a fair service, but generally 40 lb per sq in is the least allowance for fair domestic service. For business blocks and higher buildings higher pressures are needed; 60 or 70 lb per sq in is not too much to give fair service in mills and business blocks that are not especially high. High steel buildings generally pump their own water and no effort is made to supply them without such pumping.

Pressure for Fire Service. If steam fire engines are used and depended upon as in many American cities, the only requirement for pressure is that during fires and with the heaviest draft the pipes shall have sufficient capacity to supply water to the steam fire engines and at the same time retain as much pressure as is needed for domestic service. If the pressure is higher, hose streams can be obtained from the hydrants without the use of the fire engines. The additional pressure to permit this to be done is very desirable. 70 lbs during fires is the lowest pressure that permits effective hose streams to be obtained for use on buildings of moderate size. If only residences are involved, 50 or 60 lbs will give fair streams. In business districts with large buildings better hose streams are obtained with higher pressures, and in general the higher the pressure the better the fire service. 100 lbs gives a good working service without steam fire engines. Higher pressures up to 150 lbs and more are available in many cities.

19. Reservoirs and Standpipes

Distributing Reservoirs are connected immediately with the distribution system and as near as possible to the center of population supplied. Their function is to take water when it comes and to make it available when it is needed. They are especially to maintain the service at times of fire and on other occasions when water is drawn rapidly. Frequently they also serve the purpose of allowing the pumps supplying the service to be shut down during certain hours of the day or at night, thereby economizing labor. This is especially the case in small plants. OPEN RESERVOIRS with earth embankments or masonry walls have been frequently used. Ground waters and filtered waters always deteriorate in quality in such reservoirs, owing to the growth of certain organisms in the sunlight. COVERED RESERVOIRS are always to be preferred for distributing reservoirs. Roofs are sometimes used to exclude the light and keep the water from deteriorating. A light roof not necessarily water-tight serves this purpose.

Masonry Covers for distributing reservoirs are often used. At Washington, D. C., Springfield, Mass., and elsewhere, groined arch construction has been used. Floors to carry the weight of the roof and distribute it over the whole base are built as inverted groined arches. The piers are of concrete, as thick as 12% of the span on centers, and not more than 12 times as high as thick. If the reservoir is deep, large piers will be required to meet this condition, and the span of the arches is increased to correspond. The roof is of groined arch vaulting without reinforcing. The outside walls, with a minimum thickness of about 12% of their height at top and 16% at bottom, are braced at the bottom by the floor blocks and at the top by the roof blocks, and are calculated as reinforced beams, with breaking moment at about 43% of the distance from the bottom to the top, equal to $4\,h^3$, h being the height of the wall in feet. In deep reservoirs economy is secured by carrying the floor on a slope of about 1 in 6 to the raised base of the walls, thereby reducing the height of the walls. The masonry is backed up by solid earth embankment, and two feet of soil is placed over the top to keep frost from the masonry. Ventilators are provided to allow the passage of air as water rises and falls in the reservoir. The top is covered with grass and shrubbery, but trees or any plants with strong heavy roots should not be planted.

Data for Steel Standpipes (Cont'd on p. 1234)

Dia. in feet	Height in feet	Capacity in thousand gallons	Required thickness of lowest plate with stress not exceeding 10 000 lbs, in	Thickness of bottom, inches	Approximate weight in net tons	Approximate relative costs: Tank at 5 cents per lb	Approximate relative costs: Foundation 5 feet deep at $7.00 per cu yd	Approximate relative costs: Total with 10% added for appurtenances and connections	Approximate relative costs: Per thousand gals
20	20	47	1/4	1/4	9	$880	$540	$ 1 560	$33
	30	71	1/4	1/4	12	1 237	540	1 960	28
	40	94	1/4	1/4	16	1 590	540	2 350	25
	50	118	5/16	1/4	20	2 040	540	2 840	24
25	20	74	1/4	1/4	12	1 155	800	2 150	29
	30	110	1/4	1/4	16	1 595	800	2 640	24
	40	147	5/16	1/4	21	2 090	800	3 180	22
	50	184	3/8	1/4	26	2 640	800	3 780	21
	60	221	7/16	3/8	34	3 390	800	4 600	21
	70	258	1/2	3/8	42	4 210	800	5 510	21
30	20	105	1/4	1/4	15	1 450	1110	2 820	27
	30	158	1/4	1/4	20	1 980	1110	3 400	22
	40	211	5/16	1/4	26	2 640	1110	4 130	20
	50	264	7/16	3/8	37	3 700	1110	5 300	20
	60	316	1/2	3/8	47	4 680	1110	6 370	20
	70	369	9/16	3/8	59	5 870	1110	7 680	21
	80	422	5/8	3/8	72	7 200	1110	9 150	22
	90	475	3/4	1/2	89	8 920	1110	11 350	24
	100	528	13/16	1/2	105	10 530	1110	12 830	24
	110	580	7/8	1/2	124	12 400	1110	14 890	26
	120	633	15/16	1/2	144	14 400	1110	17 090	27
35	20	144	1/4	1/4	18	1 770	1480	3 580	25
	30	215	5/16	1/4	25	2 470	1480	4 350	20
	40	287	3/8	3/8	37	3 680	1480	5 670	20
	50	359	1/2	3/8	48	4 830	1480	6 950	19
	60	431	9/16	3/8	61	6 130	1480	8 380	19
	70	502	11/16	1/2	80	8 010	1480	10 450	21
	80	574	3/4	1/2	98	9 770	1480	12 400	22
	90	646	7/8	1/2	118	11 850	1480	14 700	23
	100	718	15/16	1/2	141	14 100	1480	17 100	24
	110	790	1 1/16	1/2	166	16 580	1480	19 900	25
	120	862	1 1/8	5/8	195	19 530	1480	23 100	27
	130	934	1 3/16	5/8	225	22 500	1480	26 400	28
40	20	188	1/4	1/4	21	2 150	1900	4 450	24
	30	282	5/16	1/4	30	2 990	1900	5 490	19
	40	376	7/16	3/8	45	4 500	1900	7 040	19
	50	470	9/16	3/8	60	5 980	1900	8 670	19
	60	564	5/8	3/8	77	7 750	1900	10 620	19
	70	658	3/4	1/2	101	10 120	1900	13 220	20
	80	751	7/8	1/2	125	12 500	1900	15 850	21
	90	846	15/16	1/2	151	15 140	1900	18 750	22
	100	941	1 1/16	5/8	184	18 400	1900	22 350	24
	110	1035	1 1/8	5/8	216	21 600	1900	25 850	25
	120	1130	1 1/4	5/8	251	25 100	1990	29 700	26

Data for Steel Standpipes — Continued

Dia. in feet	Height in feet	Capacity in thousand gallons	Required thickness of lowest plate with stress not exceeding 10 000 lbs, in	Thickness of bottom, inches	Approximate weight in net tons	Approximate relative costs: Tank at 5 cents per lb	Foundation 5 feet deep at $7.00 per cu yd	Total with 10% added for appurtenances and connections	Per thousand gals
45	20	238	¼	¼	25	2 480	2350	$5 320	23
	30	357	⅜	⅜	40	4 020	2350	7 030	20
	40	476	½	⅜	55	5 500	2350	8 650	18
	50	594	⅝	⅜	74	7 380	2350	10 700	18
	60	713	¾	½	101	10 100	2350	13 700	19
	70	833	⅞	½	128	12 780	2350	16 640	20
	80	951	15/16	½	156	15 640	2350	19 800	21
	90	1070	1 1/16	⅝	193	19 360	2350	23 000	22
	100	1190	1 3/16	⅝	230	23 020	2350	27 900	23
50	20	293	5/16	¼	30	2 970	2870	6 430	22
	30	440	7/16	⅜	50	4 950	2870	8 600	20
	40	586	9/16	⅜	68	6 820	2870	10 660	18
	50	733	11/16	½	97	9 680	2870	13 800	19
	60	880	13/16	½	124	12 430	2870	16 820	19
	70	1025	15/16	½	156	15 620	2870	20 300	20
	80	1170	1 1/16	⅝	198	19 800	2870	24 900	21
	90	1320	1 3/16	⅝	238	23 870	2870	29 400	22
60	20	423	5/16	¼	38	3 830	4050	8 670	20
	30	633	½	⅜	66	6 600	4050	11 720	19
	40	846	⅝	⅜	91	9 120	4050	14 500	17
	50	1060	13/16	½	132	13 200	4050	19 000	18
	60	1270	15/16	½	170	17 030	4050	23 200	18
	70	1480	1⅛	⅝	224	22 440	4050	29 200	20
	80	1690	1¼	⅝	276	27 600	4050	33 700	20
70	20	574	⅜	⅜	61	6 140	5400	12 700	22
	30	861	9/16	⅜	87	8 760	5400	15 600	18
	40	1150	¾	½	133	13 370	5400	20 600	18
	50	1438	15/16	½	178	17 850	5400	25 600	18
	60	1725	1⅛	⅝	241	24 160	5400	32 500	19

Overflows should invariably be provided for distributing reservoirs and should have sufficient capacity to discharge all the water that the pipes or pumps are capable of bringing to them. Many reservoirs have been lost and great damage done by failure to provide sufficient overflow capacity.

The Cost of Covered Masonry Reservoirs ranges from $15 000 to $20 000 per million gallons capacity for small reservoirs and reservoirs on difficult sites to $7000 or $8000 for large reservoirs and reservoirs on favorable sites; $10 000 to $12 000 are common figures. Open distributing reservoirs commonly cost from $2500 to $10 000 per million gallons, $4000 being a common figure for a medium-sized reservoir on a favorable site.

Capacity of Distributing Reservoirs. With the water coming to the city thru a gravity line at a steady rate, a storage of about 0.15 of one day's supply serves to balance all the fluctuations in hourly consumption, not including fire drafts. With a pumping system where all the pumping is done during the day a reservoir should hold at least one day's supply. Considerations of fire service frequently determine the size of reservoirs, so that

in addition to meeting the ordinary service, drafts amounting to not more than one-quarter or one-half of a day's supply can be made to maintain full fire service for a period of two, four, eight, or some other number of hours, depending upon the size of the city and the amount of property that is to be protected.

The Elevation of distributing reservoirs is a matter of great importance, as it controls the pressure of water in the entire distribution system. Reservoirs have been generally built at levels to give necessary pressures at the time that they were built and in the district to be served. As time has gone on larger buildings and higher buildings and buildings upon higher ground have been erected, and there is demand for, and need of more pressure than is available. This sometimes involves the abandonment of old reservoirs and the construction of newer ones at higher levels. Abandonment of distributing reservoirs because of insufficient elevation has been common.

Standpipes are elevated reservoirs built of sheet steel entirely above the surface of the ground, and are commonly used where the desired water level is a considerable distance above the surface of the ground. The limitations of steel construction do not in general allow standpipes to be used in large works. Roofs should be provided on all standpipes holding waters deteriorating in the sunshine, that is, in general, for ground waters and filtered waters.

Reinforced concrete standpipes have been used with satisfactory results. It does not appear that any very large financial saving has been made by their use. Towers of masonry are frequently built about standpipes for ornamental purposes, and to protect them from wind pressure, and to make very tall standpipes small in diameter safe.

Elevated Steel Tanks supported on steel trestles are used in place of standpipes where the quantities of water to be stored are not large and the elevation above the surface of the ground is considerable. The East Providence tank holding 1 000 000 gallons, from 135 to 205 feet above the ground, was erected in 1904 at a cost of about $100 000. (N. E. W. W. A., vol. 19, p. 55). WOODEN TANKS are frequently used in railway supplies and in industrial operations, but are seldom to be recommended for public water supply.

20. Cast-iron Pipes

Designs and specifications have been adopted by the New England Water Works Association for cast-iron pipe, and most foundries are prepared to furnish pipe as specified. The American Water Works Association has adopted standards which differ slightly from those of the New England Water Works Association. BRACKETT'S FORMULA for the thickness in inches of cast-iron pipe is

$$T = \frac{(P + P')r}{3300} + 0.25$$

in which P is the maximum static pressure in pounds per square inch for which the pipe is designed, P' is the allowance made for water ram, and r is the radius in inches. This includes a large factor of safety to cover inequalities of the castings, water ram, strains brought on the pipe from other causes than the water pressure, and to give sufficient thickness to insure the pipe against excessive breakage in shipping and laying. The amount of allowance depends upon the character of the service. For ordinary waterworks conditions for the larger pipes, that is, from 42 to 60 inches inclusive, 70 lb per sq in is enough for P', but for smaller pipe Brackett allows the following values:

Diam. pipe, inches =	36	30	24	20	16	12	10 to 3
P', lb per sq in =	75	80	85	90	100	110	120

The thickness calculated in this way may be used for average conditions; for city work where great damage would be caused by breakage and for single lines with no reserve where an interruption of the supply would be a very serious matter the pipes may be made thicker than computed by this formula. For reverse conditions some engineers use thinner pipe.

Coating. Cast-iron pipe is always coated by dipping in coal tar at a temperature of about 300° F. Redistilled coal tar of good quality is to be used. With continued heating it loses its more volatile parts and becomes brittle. To prevent this a heavy oil obtained in the distillation of the tar is added to the dipping tank from time to time.

Depth of Cover. In the latitude of New York and Philadelphia pipe is laid with a cover of about four feet of earth, but large pipe, that is, pipe two feet in diameter and over, may be safely laid with a little less cover than smaller pipe. In the latitude of Boston, Albany and Chicago the depth of cover is increased to 4½ or 5 ft. Further north still greater depths of cover are required. In the latitude of North Carolina and south it is only necessary to give the pipes sufficient cover so that they will not become exposed in the street, or, in other words, to protect them from physical injury aside from frost. In countries where there is no frost main pipes outside the city are sometimes laid on top of the surface of the ground. In other cases they are just covered with soil. The former arrangement leaves them open to inspection for leakage. The latter arrangement protects them against excessive heat. The cost of pipe laying is greater as the cover is greater.

Cost of Cast-iron Pipe. Cast-iron pipe is sold by the net ton. It was formerly sold, and is still occasionally sold, by the long ton. The maker guarantees the weight. Pipes underrunning the standard weight by more than a certain percentage (the percentage depending upon the diameter and class of pipe) are rejected, and pipes overrunning the standard weight by more than a certain percentage are accepted, but the weight beyond the allowed excess is not paid for. From 1875 to 1909 the price of cast-iron pipe in the northeastern states has ranged from $17 to $35 per net ton. The average price has been not far from $25, and this was approximately the selling price in 1909. Prices of over $30 per ton ruled in 1875, 1880, 1883, 1884, and 1907. Prices under $20 ruled from 1895 to 1898 inclusive.

Maximum Bends in Cast-iron Pipe Joints

Size of pipe in inches	Bend in one joint	Deflection of 12-ft length, in inches	Approximate radius in feet of curve produced by succession of joints
4	4°	10.0	170
12	3°	7.5	230
16	2° 41′	6.8	260
20	2° 9′	5.4	320
24	1° 47′	4.5	390
30	1° 26′	3.6	480
36	1° 12′	3.0	570
42	1° 2′	2.6	660
48	0° 55′	2.3	750

Special bends are used for shorter curves than those in the table. SPECIALS are all castings other than straight pipe in 12-ft lengths. Bends, reducers, increasers, crosses, tees, branches, are some of the common forms of specials. Standard designs and weights for all the more frequently used specials have been adopted by the New England Water Works Association, and the American Water Works Association.

An average price for specials is $60 per net ton. The weight of specials in long lines of supply pipe in the country is less than 1% of the weight of the pipe. In the distribution system in villages and small cities the weight of the specials will average 2½% of the weight of the pipe. In city work, with short blocks and many obstacles to be past specials will average 4% of the weight of the pipe.

The following table gives dimensions and weights for lead joints of cast iron pipes. An average price for lead is 5 cents per pound.*

* For year 1910.

Amount of Lead Required for Joints

Dimensions in inches				Lead required, pounds per joint		
Dia. of pipe	Approx. diam. lead ring	Depth of lead	Thickness of lead	Theoretical weight including bead	Range in weights reported for various works	To be used for estimate
4	5.2	2.00	0.4	6.5	4–7	7
6	7.3	2.00	0.4	9.0	6–10	10
8	9.5	2.00	0.4	11.7	8–12	12
10	11.8	2.00	0.4	14.6	10–16	16
12	13.9	2.25	0.5	19.1	13–20	20
16	18.3	2.25	0.5	30.3	24–32	32
20	22.4	2.25	0.5	37.0	30–40	40
24	26.6	2.50	0.5	48.5	38–50	50
30	32.9	2.50	0.5	59.6	56–75	62
36	39.2	2.75	0.5	77.8	68–115	80
42	45.6	2.75	0.5	90.5	78–130	95
48	51.9	3.00	0.5	111.0	89–150	120

Data for Cast-iron Pipe (Cont'd on p. 1238)

Diameter, inch	Class, N.E.W.W.	Thickness, inches, standard	Approximate pounds per lineal foot,	Weight per 12-ft length, lbs	Working pressure		Relative cost per lin ft for average conditions: 4-ft cover, 3% rock, pipe at $25.00 per net ton, labor at $2.00 per day		
					Lb per sq in	Feet head	Country work average conditions	Village and suburban work macadam	City conditions, brick and asphalt
4	C	0.36	18	215	65	150	$0.53	$0.74	$1.20
	E	0.39	19	230	108	250	0.55	0.75	1.21
	G	0.42	21	250	152	350	0.57	0.78	1.24
	I	0.45	22	265	195	450	0.58	0.79	1.25
	K	0.48	23	280	238	550	0.60	0.81	1.27
6	C	0.42	29	350	65	150	0.72	0.95	1.40
	E	0.46	32	380	108	250	0.76	0.98	1.43
	G	0.50	35	420	152	350	0.80	1.02	1.48
	I	0.54	37	445	194	450	0.83	1.05	1.51
8	C	0.48	44	530	65	150	0.96	1.20	1.67
	E	0.53	48	575	108	250	1.00	1.25	1.72
	G	0.58	53	640	152	350	1.08	1.32	1.79
	I	0.63	57	690	195	450	1.13	1.38	1.85
10	C	0.53	60	720	65	150	1.21	1.47	1.97
	D	0.56	63	760	87	200	1.25	1.51	2.02
	E	0.60	68	810	108	250	1.31	1.57	2.07
	F	0.63	71	850	130	300	1.35	1.61	2.12
	G	0.67	74	890	152	350	1.39	1.66	2.16
	H	0.70	78	935	174	400	1.44	1.71	2.21
12	C	0.57	76	910	65	150	1.47	1.74	2.28
	D	0.61	81	970	87	200	1.53	1.81	2.34
	E	0.65	87	1 040	108	250	1.61	1.88	2.42
	F	0.69	92	1 100	130	300	1.67	1.95	2.49
	G	0.73	97	1 160	152	350	1.73	2.02	2.56
	H	0.77	102	1 220	174	400	1.80	2.08	2.63

Joints are made Deeper where the pressure is high, and not so deep where it is light. Workmen often make the joints heavier or lighter than specified. Some lead is oxidized in melting and pouring, and otherwise wasted and lost. Variations in the diameter of bell and spigot make a difference in the amount of lead. A small fraction of an inch variation makes a large percentage difference in the amount of lead required. For good management find out how much lead is required, and require inspectors to hold the dimensions of bells and spigots to the design, and the workmen in the trench to the depth of joint required, and investigate closely any large discrepancy between the computed and required amounts of lead.

Leakage in pipe should always be tested under full working pressure before the trench is backfilled, and all joints showing leakage calked until they are tight. It is impossible to keep lines of cast-iron pipes permanently tight. The expansion and contraction from temperature changes are accompanied by a slight slipping of the lead on the iron at each joint; settlements cause movements in the joints.

Data for Cast-iron Pipe (Cont'd from p. 1237)

Diameter, inch	Class, N.E.W.W.	Thickness, inches, standard	Approximate pounds per lineal foot	Weight per 12-ft length, lbs	Working pressure		Relative cost per lin ft for average conditions: 4-ft cover, 3 % rock, pipe at $25.00 per net ton, labor at $2.00 per day		
					Lb per sq in	Feet head	Country work average conditions	Village and suburban work macadam	City conditions, brick and asphalt
16	C	0.65	116	1 390	65	150	2.10	2.42	3.03
	D	0.70	124	1 490	87	200	2.21	2.53	3.15
	E	0.75	134	1 610	108	250	2.34	2.66	3.28
	F	0.80	143	1 710	130	300	2.45	2.77	3.40
	G	0.85	151	1 810	152	350	2.55	2.88	3.51
	H	0.90	158	1 900	174	400	2.65	2.98	3.62
20	C	0.72	160	1 920	65	150	2.80	3.16	3.85
	D	0.79	174	2 090	87	200	2.98	3.35	4.05
	E	0.85	188	2 260	108	250	3.16	3.54	4.24
	F	0.92	202	2 420	130	300	3.33	3.72	4.42
24	C	0.80	213	2 550	65	150	3.63	4.03	4.80
	D	0.88	232	2 780	87	200	3.87	4.28	5.06
	E	0.95	250	3 000	108	250	4.11	4.53	5.31
	F	1.03	270	3 240	130	300	4.37	4.79	5.59
30	C	0.91	300	3 600	65	150	4.98	5.45	6.35
	D	1.01	329	3 950	87	200	5.35	5.84	6.75
	E	1.10	362	4 340	108	250	5.76	6.27	7.19
	F	1.20	392	4 700	130	300	6.15	6.66	7.60
36	C	1.02	403	4 840	65	150	6.52	7.10	8.15
	D	1.13	443	5 310	87	200	7.02	7.62	8.69
	E	1.25	492	5 900	108	250	7.65	8.27	9.36
	F	1.37	533	6 400	130	300	8.19	8.83	9.93
42	C	1.13	523	6 270	65	150	8.34	9.01	10.19
	D	1.27	581	6 970	87	200	9.09	9.78	10.99
	E	1.40	643	7 720	108	250	9.89	10.61	11.84
	F	1.53	697	8 360	130	300	10.57	11.31	12.57
48	C	1.25	660	7 920	65	150	10.42	11.22	12.80
	D	1.40	732	8 780	87	200	11.33	12.18	13.78
	E	1.55	812	9 740	108	250	12.36	13.23	14.88
	F	1.70	883	10 500	130	300	13.27	14.18	15.86

With well-tested work under average conditions in a waterworks system a leakage of three gallons per 24 hours per lineal foot of lead joint under a pressure of 100 lb per sq in may be anticipated. On this basis the leakage per thousand feet of pipe in gallons per 24 hours will be as follows:

Ins. diam.	6	8	10	12	16	20	24	30	36	48
Gallons	500	650	775	900	1200	1500	1800	2200	2600	3400

Lines of pipe when first laid should have less leakage, and those that have been down for many years may be expected to leak more. It will pay to dig up pipe and calk joints whenever and as far as the value of water saved during an assumed period of five years exceeds the cost of locating and stopping the leakage.

Sixty-inch Cast-iron Pipe is sometimes used, but often when more capacity is needed than is furnished by one 48-inch pipe, two or more such lines are laid. Larger sizes of pipe are most frequently made of steel.

Cost of Cast-iron Pipe Laid. The table on p. 1237 shows the approximate cost of pipe of all the sizes and thicknesses in most common use, laid under average conditions. These prices include the cost of the pipe, specials, lead, and all costs of laying, assuming 3% of the trench rock excavation; pipe $25 per ton and labor $2 per day. The column for country work may be used for outside pipe lines where there is no special difficulty. The column for village and suburban work includes taking up and replacing macadam pavements. The column for city conditions includes taking up and replacing brick or asphalt pavements.

In Congested District of cities where there are many existing pipes, conduits, sewers, and other structures to be met and past, the cost per foot may be increased very greatly above the figures given, which are intended to cover only ordinary city conditions. Many local conditions affect the cost of the work, and prices fluctuate, and the costs will often be found differing considerably from those given, which are intended to represent average conditions, and to show approximately the relative cost of pipe of different sizes and the thicknesses under ordinary conditions of use.

21. Riveted Steel Pipes

Steel pipe is used principally in long supply mains, leading from the source of supply to the city. It is not generally adapted to use in city streets or in the distribution system.

Steel pipe is generally cheaper than cast-iron pipe in sizes 36 inches in diameter and upward. It has occasionally been used for 30-inch pipe and is seldom to be recommended for smaller sizes.

Riveted Pipe. The older lines of steel pipe were all riveted. Generally the sheets of steel are seven or eight feet wide, and are bent so that one sheet goes entirely around, forming one section of pipe. Four of these sheets are riveted together in the shop, making a length of pipe 28 to 30 feet long. This is tested for tightness and dipped in protective coating, and then shipped to the place where it is to be used.

The circular seams may be single riveted. The longitudinal seams which alone are required to carry the stress due to the pressure of the water are double riveted, in all cases, except where the pressure is very low.

IN-AND-OUT COURSES are used, alternate rings being larger and smaller. TAPER LENGTHS are also used in which one end of each pipe is smaller than the other end and will slip into the large end of the next length. Pipes have also been made with all the lengths the same size fastened together with butt straps on the outside, but as this is a more expensive method it has not been often used.

Lead Joints have been used in Australia on long lines at Coolgardie and Sydney. Cast-iron or steel sleeves are used in making them. The lead acts

as an expansion joint and each length of pipe is free to expand and contra with the temperature changes, the steel slipping over the lead to permit thi

Place	Year installed	Diam., inches	Thickness, inches	Length, feet	Kind
Rochester, N. Y.	1873	36	3/16	51 000	Riveted wrought iro
Rochester, N. Y.	1873	24	3/16–1/4	15 400	Riveted wrought iro
E. Jersey Water Co.	1891	48	1/4–3/8	116 000	Riveted steel
E. Jersey Water Co	1891	36	1/4	26 000	" "
Rochester, N. Y.	1893	38	1/4–3/8	140 000	" "
Cambridge, Mass.	1895	40	1/4–5/16	24 000	" "
New Bedford, Mass.	1896	48	5/16	42 500	" "
Allegheny City, Pa.	1896	60	....	50 000	" "
Brooklyn, N. Y.	1896	66	....	79 200	" "
E. Jersey Water Co.	1896	48	....	26 400	" "
E. Jersey Water Co.	1896	42	....	89 700	" "
Adelaide, Australia	1897	15–24–26	1/4	63 472	Lock-bar steel
Minneapolis, Minn.	1897	50	5/16–3/8	34 400	Riveted steel
Passaic Water Co., N.J.	1897	42	....	21 000	" "
Albany, N. Y.	1898	48	5/16	7 900	" "
E. Jersey Water Co.	1899	51	....	47 500	" "
Atlantic City, N. J.	1901	30	1/4	(?)27 000	" "
Pittsburgh, Pa.	1901	51–42	....	26 000	" "
Jersey City, N. J.	1902	72	5/16–7/16	91 000	" "
Coolgardie, Australia	1903	30	1/4–5/16	1 848 000	Lock-bar steel
Newark, N. J.	1903	60	....	39 300	Riveted steel
Troy, N. Y.	1903	33	....	35 300	" "
Schenectady, N. Y.	1903	36	....	24 000	" "
Lynchburgh, Va.	1905	30	....	15 000	Lock-bar steel
Wilmington, Del.	1905	43–48	....	19 350	" "
Passaic Water Co., N.J.	1905	42–48	....	11 000	" "
Pittsburgh, Pa.	1905	50	....	26 500	" "
Springfield, Mass.	1909	42	1/4–7/16	61 000	" "

Continuous Riveting has been used in nearly all American steel pipe li that is, each length of pipe in the field has been tightly riveted to its neighb Practical experience with this system of construction has been satisfactor

Temperature Stresses. Under conditions in the northern states the tem ature of water and consequently of the pipe will range from 32 to 75 or 80 or for average conditions the range is about 45°. The pipe must be so that it will not move with expansion and contraction. Under these ditions the change in temperature, which tends to expand or contract the uses all its force in putting it under stress. The stresses thus produced range from nothing at the lower temperature to the maximum amoun compression at the higher temperature, or from nothing at the higher tem ature to the maximum amount in tension at the lower temperature, or may be divided, coming partly in tension at the lower temperature and p in compression at the higher temperature. This depends on the tempera at which the pipe is laid and finally connected up. Under the most unfa able conditions the stresses produced by temperature in this climate are a 9000 lb per sq in, and as this comes well within the strength of the stee difficulty is occasioned by them. Ordinarily they are less than this.

Anchorages are built to keep the pipe from moving at all free ends at all sharp bends. These anchorages are formed by riveting angle to the steel pipe, usually four angles being attached to one 30-foot le

Data for Steel Pipe

Diam. in inches	Thickness of plate in inches	Approximate weight in lbs per lineal foot	Relative cost laid complete under average conditions	Lock-bar pipe: Gross pressure without allowance for water ram and deterioration	Lock-bar pipe: Safe working pressure after allowing for average water ram and corrosion, Lbs per sq in	Lock-bar pipe: Safe working pressure after allowing for average water ram and corrosion, Feet of head	Double-riveted pipe: Gross pressure without allowance for water ram and deterioration	Double-riveted pipe: Safe working pressure after allowing for average water ram and corrosion, Lbs per sq in	Double-riveted pipe: Safe working pressure after allowing for average water ram and corrosion, Feet of head	Greatest allowable depth of cover in feet	Temperature stress in net tons
30	3/16	80	$4.60	180	100	231	144	86	199	5	92
	1/4	104	5.20	240	140	323	192	118	273	8	122
	5/16	127	5.80	300	180	416	240	150	347	12	152
	3/8	151	6.40	360	220	508	288	182	420	18	183
	7/16	174	7.00	420	260	600	331	209	483	25	214
36	1/4	123	6.30	200	113	261	160	97	224	5	147
	5/16	151	7.00	250	147	339	200	123	284	9	183
	3/8	179	7.70	300	180	416	240	150	347	12	220
	7/16	207	8.40	350	213	491	280	177	408	17	257
	1/2	235	9.30	400	247	570	320	203	468	22	294
42	1/4	141	7.40	171	94	217	137	81	187	4	172
	5/16	174	8.20	214	123	284	171	104	240	6	214
	3/8	207	9.00	257	151	349	206	127	293	9	257
	7/16	240	9.80	300	180	416	240	150	347	12	300
	1/2	273	11.00	343	209	482	274	173	400	16	343
48	1/4	160	8.50	150	80	185	120	70	161	3	196
	5/16	198	9.40	187	105	242	150	90	208	5	244
	3/8	235	10.30	225	130	300	180	110	254	7	293
	7/16	273	11.40	262	155	358	210	130	300	9	342
	1/2	310	12.70	300	180	416	240	150	347	12	391
54	5/16	221	10.70	167	91	210	133	79	182	4	275
	3/8	263	11.70	200	113	260	160	97	224	6	330
	7/16	305	13.00	233	135	312	187	115	265	8	386
	1/2	348	14.40	267	158	365	213	132	305	10	441
	5/8	432	17.20	333	202	466	267	168	388	15	551
60	5/16	244	12.20	150	80	185	120	70	161	3	305
	3/8	291	13.30	180	100	231	144	86	201	4	366
	7/16	338	14.60	210	120	277	168	102	235	6	430
	1/2	385	16.20	240	140	323	192	118	273	8	488
	5/8	479	19.40	300	180	416	240	150	347	12	612
72	5/16	291	15.00	125	63	146	100	57	132	2	367
	3/8	347	16.20	150	80	185	[illegible]	70	161	3	440
	7/16	404	17.90	175	97	224	140	83	191	4	515
	1/2	460	19.60	200	113	261	160	97	224	6	588
	5/8	573	23.50	250	147	339	200	123	284	9	735

with a sufficient number of rivets to hold the temperature stresses, and these lengths of pipe are surrounded with concrete reinforced with steel rails of such a shape as to be capable of withstanding the computed amount of stress. In some lines anchorages have been omitted and expansion joints provided at all gates, ends, and other places when continuity of riveted connection cannot be maintained. The temperature push or pull on the anchor-

age in tons of 2000 lbs for steel pipes of various diameters and thicknesses is shown in the last column of the preceding table.

Gates on Steel Pipes. There are two ways of connecting gates in steel pipes: (1) By flange connections, the flanges being riveted to the steel pipe and bolted to flange gates. The gates must have cases heavy enough and strong enough to withstand the temperature stresses in the steel pipe. This is essential. If flange gates of ordinary construction are used the cases are sure to be broken by the expansion and contraction of the pipe. (2) The gates may be connected with the steel pipe thru short pieces of cast-iron pipe and lead joints. In this case it is necessary to build anchorages on the steel pipe on either side of the gates. The two anchorages having equal and opposite temperature strains to hold may be conveniently connected by old steel rails laid in concrete.

Lock-bar Steel Pipe is made by upsetting the edges of the plates and connecting them by a lock bar in the shape of an H going over the opposite edges and being forced down over them by hydraulic pressure. This takes the place of the riveting in the longitudinal joints. The circular joints may be made by riveting or otherwise as for riveted pipe. While double riveting develops only about 72 % of the strength of the steel plate, the lock bar is capable of developing 100%. Owing to occasional defects in material or workmanship on the lock bars, in making calculations it is recommended that only 90 % of the strength of the plate should be used for lock-bar pipe. Two sheets of steel each 30 ft long are joined with two bars at their edges to make a 30-foot length of pipe. The circular joints between these lengths are usually riveted in the field. Thus far lock-bar pipe has been made from plates ranging from ¼ to 7/16 inch in thickness.

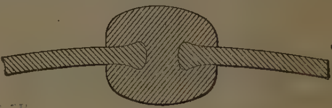

Fig. 11. Lock Bar for Steel Pipe

Carrying Capacity of Steel Pipe. Lock-bar steel pipe has a comparatively smooth interior and has substantially the same carrying capacity as cast-iron pipe. Riveted pipe is less smooth in the interior, the projecting rivets increasing the friction of the flowing water, and also the numerous joints in the plates with either in-and-out joints or with taper joints. For these reasons steel pipe carries from 10 to 15 percent less water than lock-bar pipe or cast-iron pipe. In comparing riveted pipe with lock-bar pipe and cast-iron pipe, a riveted pipe should be taken with a sufficiently greater diameter so that it will carry as much water as the others. This may be accomplished practically by making the riveted pipe 4 % larger.

The Diameter of Steel Pipe, either riveted or lock-bar, can be made of any required diameter. In this respect it differs from cast-iron pipe, which is commonly made only of the sizes for which the foundries have molds. The diameter should always be specified as the smallest diameter of the smallest ring, where the rings are not all of the same size.

Weight of Steel Pipe. The finished weight of steel pipe per lineal foot, either riveted or lock-bar, including the excess weight of plates rolled so that the thinnest points in the plate will be approximately of the nominal thickness, and including the laps, rivets and lock bars, material in the joints and coating, may be found approximately by the formula:

$$\text{Weight in lbs per foot} = (12.5 \times \text{diameter} \times \text{thickness}) + 10 \text{ lbs}$$

in which diameter and thickness are to be taken in inches. The weights of commonly used sizes are given in the table on page 1241.

Thickness of Steel Pipe. The required thickness is computed by

$$\text{Thickness in inches} = \frac{\text{Diameter in inches} \times \text{Pounds pressure}}{2 \times 16\,000 \times \text{efficiency of joint}}$$

To the thickness thus computed it is customary to add something as an allowance for weakening of pipe by corrosion. In other words, make the pipe heavier so that it may rust to a certain extent and still have needed strength. The strength of the metal with open-hearth steel under standard specifications is taken at 16 000 lb per sq in, which allows a factor of safety of about 3½. The efficiency of the joint may be taken at 55 % for single-riveted pipe, 72 % for double-riveted pipe, with the best spacing of rivets, and at 90 % for lock-bar pipe.

In the steel-pipe table, p. 1241, are entered under lock-bar pipe and double-riveted pipe separately, first, the gross pressures, in lbs per sq in, that can be carried by pipe of various dimensions without making deduction for water ram and for deterioration, and second, the fair safe working pressures after making reasonable allowances for water ram and corrosion under average conditions.

Proper allowances for water ram and for corrosion necessarily vary with local conditions. Generally speaking, where great variations in rates of flow are to be expected and where a great deal depends upon the continuous service of a pipe, thicker pipe should be used, while under conditions of steady flow and where pipes are in duplicate or otherwise safeguarded thinner pipe may be used.

Depth of Cover for Steel Pipe must not be excessive or the weight of the earth will flatten and deform it. A slight flattening is not objectionable, as it does not cause the pipe to leak and does not greatly reduce its carrying capacity.

The table, p. 1241, shows the greatest depth that should be placed over various sizes and thicknesses of pipe under average conditions. In bad trenches and slippery material keep the depth of cover somewhat less than indicated. With firm material carefully placed around the pipe and well rammed on the sides a somewhat greater depth of cover for short distances may be permitted. If it is necessary to cover the thin pipe to a greater depth it may be stiffened by angle irons riveted to it at frequent intervals. A more substantial result is obtained by surrounding the pipe with concrete.

Cost of Steel Pipe. The relative cost of steel pipe laid complete under average conditions is shown in the table. The costs represent generally those in the northeastern states in the years 1905–09. They include pipe excavation and average amount of water in trench, and of rock excavation, anchorages, brook crossings, etc., but do not include river crossings or special and unusual obstacles or difficult conditions. Steel pipe has usually been placed under contract at a price per foot laid, including the pipe and laying, but rock, valves, river crossings, and all other auxiliary works are usually paid for separately.

Bends in Steel Pipe are usually made by cutting off some of the plates at the joint. Both horizontal and vertical bends are made in the same way. It is easier to lay out the work if the horizontal and vertical bends are made in separate joints, but in case of need they can be combined.

The amount of bend that can be made in one joint depends upon the size and thickness of the plate. With 5/16-inch plates bends up to 5° in one joint are easily made; with 3/8-inch plates 4°, and with 7/16-inch plates 3°. Sharper bends are made when necessary but it is harder to calk them tight. With crooked lines the lengths of pipe may be cut, one bend made every 15 ft or every 7½ ft. With sharper bends special arrangements are made.

It is better to make all bends in steel pipe of steel plates riveted up, rather than of castings, and in case of sharp bends the pipe should be anchored on both sides, to carry the resultants of the temperature stresses.

Coatings for Steel Pipe are more important than coatings for cast-iron pipe, because the steel is thinner and if unprotected may rust thru more quickly. ASPHALT was used for some of the earlier lines. Asphalt is entirely stable in a dry atmosphere, but it is quickly oxidized and becomes brittle when kept under water. Asphalt coating on steel pipe has actually become brittle and pulverized and has ceased to act as a protective coating in periods usually not exceeding ten years. Coatings made from the residue from the distillation of petroleum are now frequently used; these are often sold as asphalts. REDISTILLED COAL TAR used on cast-iron pipe has usually been more durable than the asphalts and special preparations used on the earlier steel pipes. Part of the Rochester pipe line, 1893, was dipt in equal parts of coal tar and asphalt. At Springfield, the 42-inch pipe line, 1909, was dipt in coal tar alone. A JAPAN COATING has been used in some cases for steel pipe. The protection has been good where the coating remained unbroken, but brittleness and liability to cracking have resulted in many places.

The Life of steel pipe seems to be limited in general by perforations of the plate which start with imperfections in the steel and first show after a considerable number of years.

Wood Stave Pipe. Wood was one of the earliest materials used for water pipes. Old methods limited its usefulness, and other materials have largely superseded it. Recently methods of manufacture have been developed and it is again finding increased use. It has been largely used in cantonments for somewhat temporary services under war conditions. Decay is less rapid in pipe laid in impervious soil and under pressure that tends to keep the wood constantly saturated with moisture. If laid in loose ground, or in any situation which permits the material of the pipe to become dry at times, it will rot.

Wood pipe may be either "machine made," in sections running up to 20 ft or more in length, shipped to the work to lay; or "continuous," built up of staves and banded in the trench. Tightness in the longitudinal joints is secured by grooves and beads on the edges of the staves, pinched into tight contact by the bands. Tightness in transverse joints is obtained by the use of metal plates set into the joints. Properly built and placed, wood stave pipe gives good service at moderate pressure within the elastic limit of the bands.

For small sizes, up to 24 in, machine made pipe is usually used. For larger sizes the pipe is built continuously in the trench.

Wood stave pipe will carry as much water as cast-iron pipe when new and as it is not subject to tuberculation, when old it will carry more than iron or steel pipe of equal age.

22. Gates and Gate Valves

Gate Valves are placed at intervals on all pipe lines of considerable length. In city streets four are placed at each intersection, one on each line of pipe, Outside the city valves are placed less frequently, and are best placed on summits where the pressure is least. The gates serve the purpose of facilitating tests of the pipe and shutting off portions of it for repairs in case of emergency.

Gates Smaller than the Pipe are often used on pipes 30 inches in diameter and over, connection being made by reducers on either side. The cost is less and the smaller gates are operated more quickly and easily. There is a little head lost, and the smaller the gate the more head is lost. This is controlling in determining how much smaller than the pipe it is best to make the gates,

Loss of Head in Gates with Taper Cone Connections

Diam. of gate, inches	Diameter of pipe in inches 30	36	42	48	54	60	72
20	1.57	3.47	6.59	......	......	......	......
24	0.65	1.57	3.07	5.40	8.72	......	......
30	0.15	0.52	1.14	2.09	3.47	5.40	11.45
36	......	0.15	0.44	0.91	1.57	2.51	5.40
42	......	......	0.15	0.40	0.75	1.25	2.83
48	......	......	......	0.15	0.35	0.65	1.57

The figures given are in velocity heads (or in feet, when the velocity in the main pipe is 8.03 ft per sec) They may also be taken as tenths of feet when the velocity in the main pipe is 2.54 ft per sec.

Basis. Loss of head in a gate, 0.15 velocity head. Loss of head in cones, 0.20 of the amount that the velocity head at the throat is greater than the velocity head in the pipe.

The actual amount of head lost in a gate depends upon the form of gate, and considerable variations are to be anticipated with gates of different designs. The amount lost in the cones depends upon the taper design and smoothness of the surfaces, and considerable variations either way are to be anticipated.

Generally 24-inch gates may be used on 30 and 36 inch pipes, 30-inch gates on 42 and 48 inch pipes, and 36-inch gates on 60 and 72 inch pipes; but if head or elevation is very valuable the gate should be one size larger than above indicated.

The Cost of Standard Double Disk Gates with bronze working parts of the best and most permanent design varies with market conditions The following are given as relative representative prices, including gears where needed, but not including by-passes. These prices are not very different from those which ruled in fair-sized contracts in the years 1905-09.

Cost of Double Disk Gates

Size in inches	20 lb per sq in test pressure suitable for filter plants and for working pressures not exceeding 10 lbs per sq in	100 lb per sq in test pressure for working pressures not exceeding 50 lb per sq in	300 lb per sq in test pressure for working pressures not exceeding 150 lb per sq in suitable for nearly all water-works purposes
4	$ 7	$ 7	$ 8
6	[illegible]	10	13
8	15	15	20
10	22	22	27
12	30	30	35
16	50	55	65
20	70	90	125*
24	100	125	190*
30	160	250*	340*
36	250*	375*	530*
42	380*	570*	800*
48	600*	850*	1200*

* Geared.

Connections and Accessories of Gates. Gates are furnished with either flange or bell ends at about the same cost. Bell ends are generally used in pipe lines in street work; flange connections are used in gatehouses, pumping stations, and about filter plants. GATE BOXES are metallic boxes covering the wrench connection and gate, extending to the surface of the ground,

with an expansion joint to protect them from damage by frost and with a removable cover to allow the gate to be opened from the surface with a suitable wrench by removing the cover.

Manholes of masonry are often built about gates of special importance, large gates, and gates operated by gears, especially when located under pavements or in other places not easily accessible. It is not necessary to build such manholes about small gates nor about gates on supply mains outside the city, because such gates can be readily and cheaply dug up in the infrequent cases of access to them being necessary.

Gears are used on large gates and gates under heavy pressure. In general 36-inch gates, 10 lb per sq in working pressure; 30-inch gates, 50 lb per sq in working pressure; and 20-inch gates, 150 lb per sq in working pressure, are the smallest gates to be geared. SPUR GEARS are used on gates set vertically and opening upward, and BEVELED GEARS on gates set horizontally and opening sideways. The latter are to be used wherever the vertical space is not sufficient to put in the spur-geared gates.

By-passes are provided in many cases on large gates operating under heavy pressures. These are built into and form part of the main gate. A small gate on the by-pass is opened to equalize the pressures in the pipe on either side of the gate before the main gate is opened. This allows the main gate to be opened with less effort than would otherwise be required.

Hydraulically Operated Gates in which the screws of the ordinary gate are omitted, have hydraulic cylinders provided with plungers attached directly to the moving parts. A small control valve allows high-pressure water to act on one side or the other of the plunger, opening and closing the gate. The cost is about twice that of ordinary gates. Gates should not be placed where they cannot be inspected and tested and kept in good order. They are especially useful for gates that have to be opened and closed frequently in pumping stations and about filter plants.

Electrically Operated Gates are furnished with electric motors geared to the screws that open and close them. Such gates are used occasionally in pumping stations and about filters where electric current is available.

Sluice Gates are of simpler construction, arranged for being built into masonry of reservoirs and other structures, and for holding water against moderate heads only. There is great variety in the design of sluice gates. They are usually cheaper than standard gates, and for the services to which they are adapted are fully as satisfactory.

23. Auxiliaries

Air Valves are small valves attached to pipes for the purpose of automatically letting out air. They are placed on summits only. Automatic air valves need only to be placed on summits of cast-iron pipe lines where the pressure is light and variable, that is, on summits nearly up to the hydraulic grade line. On all summits where the water is under considerable pressure it is sufficient to put on a petcock or a larger valve to be opened while the pipe is being filled and which can be closed at all other times. As air is more soluble in water under pressure there is no danger of the separation of air at summits under considerable pressure, and should air be accidentally introduced to them it would be slowly dissolved and removed by the passing water. As a general rule air valves with a diameter of one inch for each foot in diameter of the water pipe are sufficient. The air valves must be pro-

tected from frost by specially constructed boxes to insure their being in readiness to act in winter.

For Steel Pipe air valves are also required to let air into the pipe rapidly in case of need, as the pipe is not strong enough to support itself against outside pressure with a vacuum in the inside. A break in a pipe at a low point, allowing the water to run out rapidly, would cause a vacuum in higher parts of the pipe, which would cause the pipe to collapse. Consideration of this feature has led to placing air valves for automatically admitting air on summits of steel pipe. Generally the air valve for this purpose should have a net area equal to one sixty-fourth the area of the pipe.

Air valves are to be insisted upon in all steel-pipe lines, but it must be remembered that they are called upon to act very rarely indeed, and for this reason a defective valve or arrangement may be used without the discovery being made that it is defective, and the fact that a simpler or cheaper type of air valve has been used in certain cases where there have been no breaks and consequently no demand that has taxed its capacity is not to be taken as an indication of the sufficiency of that particular design.

Blow-offs are small pipes attached at low points for the purpose of drawing off and wasting the water contained in the pipe during times of inspection and repair. Blow-offs are usually much smaller in diameter than the main pipe. The necessity of blow-offs depends upon the character of the water and the service of the pipe.

Manholes consisting of saddles attached to the pipe and removable covers capable of being bolted securely to the frame are placed on steel pipes at distances ranging from 1000 to 2000 feet apart, to allow the pipe to be entered during construction and afterward for inspection and repair. In some cases manholes have been placed on cast-iron pipes, altho most lines have been built without them.

Twin Lines of pipe are used in places of special danger. Either line will maintain at least a partial supply in case of break in the other. In case twin lines are long, there should be cross connections with gates so that in case of a break in either line a section only can be cut out, the flow at other points continuing thru both lines. With this arrangement, the amount of water flowing thru the system will be more than would flow thru one line only.

The Cost of Twin Lines with cross connections is from 30 to 50 percent greater than the cost of a single line of pipe of the same strength and capacity. Where no other purpose than safety is secured by dividing the flow, it is generally better to spend the added money, or a part of it, in strengthening one line and making it secure beyond question rather than dividing it between two smaller lines. River crossings, lines over coal fields, where there are sure to be settlements, and other points of special hazard are best crost with twin lines. THREE LINES of pipe cost from 60 to 80 percent more than one line of equal strength and capacity.

Tubercles in Cast-iron Pipe. The carrying capacity of cast-iron pipe is reduced in course of time by the growth of tubercles upon the interior of the pipe. In a general way the capacity of the pipe, other things being equal, is reduced from this cause by as much as one percent per annum. In small pipes the deterioration is more rapid. Generally the deterioration is less rapid with clear lake waters and more rapid with turbid river waters, and especially waters carrying organic matter. Filtered river waters act more nearly like lake waters.

Tubercles can be Removed by sending an instrument driven by the water pressure thru the pipe. This instrument is called a "go-devil." Scraping off the tubercles in this way increases the carrying capacity of the pipe. After the pipe has been scraped

tubercles grow more rapidly than before, so that the remedy is a temporary and not a permanent one. When the pipe is once scraped it is usually necessary to scrape it again, and the process becomes an annual one, or the period may be even shorter.

Effect of Cleaning upon the Quality of the Water. The corrosion and tuberculation of iron pipes always adds iron to the water, and this iron gives it a color, tends to deposit, and is objectionable. Scraping the pipes frequently increases the rate of tuberculation and increases whatever objection there may be to the iron in the water from this source.

Hydrants are attached to pipes in a distribution system to allow water to be drawn for fire purposes. Hydrants have 4-inch connections to allow them to be connected with steam fire engines and 2½-inch connections for hose connections. A hydrant having one 4-inch and two 2½-inch connections is a common arrangement. The water is commonly shut off from the hydrant by a 6-inch gate valve at the bottom; closing this shuts off water from all connections. Hydrants are also made in which the different connections can be shut off separately. The cost of hydrants varies from $30 to $100 according to size, type, and the way in which connection is made with the pipe. Hydrants are commonly placed at intervals of two hundred feet where buildings are large, close together, and inflammable, and further apart elsewhere, but in no case more than 500 feet apart.

Domestic Meters are small instruments set on the service pipes of the takers to measure and record all water drawn thru them. The most commonly used meters in the United States are disk meters, consisting of an oscillating disk in a case moved by the water and communicating the motion which represents a certain volume of water to a gear train which indicates the amount on a dial. Rotary displacement meters are also used.

Water meters are least accurate with the smallest flows. It is comparatively easy to make a meter that is accurate with large flows. Meters that are accurate at such flows often fail to register very small flows, especially after they have been some time in use. In testing meters therefore it is necessary to pay particular attention to low rates of flow, as otherwise small leakages which when sufficiently numerous would sap the system may fail to be registered by the meters.

The Cost of Meters varies with market conditions. In a general way the following have been about representative costs in the years 1905–09.

Cost and Discharge of Domestic Meters

Nominal size, inches	Approximate cost of disk meters	Approximate cost of rotary meters	Manufacturers' rating, gallons per minute	Capacity of disk meters with loss of head of about 5 lbs.	
				Gallons per minute	Million cu ft per annum running all the time
⅝	$8	$12	21	7	0.50
¾	12	21	35	12	0.85
1	16	30	62	21	1.50
1½	30	50	100	43	3.00
2	50	65	160	71	5.00
3	90	135	390	142	10.0
4	175	250	570	213	15.00
6	375	500	1000	427	30.00

Disk Meters of different makes vary in frictional resistance. The figures given in the above table should be approximately reached by any standard

make. ROTARY METERS differ much more in frictional resistance than disk meters. In some makes resistance would be from 50 to 100 percent greater than given in the table for disk meters. Other makes have less resistance than shown in the table. INFERENTIAL OR CURRENT METERS have considerably less frictional resistance than disk meters, the results differing widely for different designs.

The Load Factor of a meter is the percentage which the average sales from it are of the capacity shown in the last two columns of preceding table. Thus a 2-inch meter recording 40 000 cubic feet per annum is said to have an 8 percent load factor $\left(\frac{40\,000}{5\,000{,}000} = 0.08\right)$. Meters of the smallest size, ⅝ inch, are large enough to give service under ordinary conditions to practically all private houses, and will normally be used on from 80 to 98 percent of all services. The average load factor on ⅝-inch meters in an average system will be between 1 and 2 percent. Meters ¾ inch and larger used on the remaining 2 to 20 percent of the services, most commonly have load factors between 4 and 8 percent. In some industrial establishments with roof tanks load factors of 20, 40 and very rarely 100 percent are found. Establishments having large services for fire protection and using but little water have low load factors. Meters should not be too large for the service, and when load factors are under 4 percent, inquiry may well be made as to whether a smaller size would not answer. Too large a meter results in loss of unregistered water. PROTECTOR METERS AND COMPOUND METERS are used where large capacity is essential and where ordinary use results in a very low load factor.

Classification of Services has been made by the New England Water Works Association for uniformity in statistics and rates. Three classes are used.

1. Domestic, all using less than 820 gallons per day, equal to 10 000 cu ft per quarter or 300 000 gallons per annum.
2. Intermediate.
3. Wholesale or manufacturing, all using over 8200 gallons per day, equal to 100 000 cu ft per quarter, or 3 000 000 gallons per annum.

Statistics for 33 Completely Metered Systems for 1915

	Percent of number of takers	Average daily sales per service in gallons	Percent of total sales in gallons
1. Domestic	95.17	159	38.4
2. Intermediate	4.37	1 750	19.6
3. Wholesale	0.46	35 700	42.0

Systems vary greatly in sales to takers of the several classes, and especially in sales to large takers.

The Resistance, in all meters, increases nearly as the square of the quantity of water passing. Resistance is added intentionally by makers of meters to prevent them from being used for flows so large that the meter would not be durable.

The use of meters on domestic services is the most efficient means of preventing useless waste of water thru leaky plumbing fixtures. The growth in the use of meters during the last years has been very rapid. Many cities sell water only by the meter and the system promises to become as universal as in the sale of gas and electricity.

Water Not Accounted for. In completely metered systems the total quantity of water registered by meters of the takers is always much less than the total output. The loss is made up of (1) leakage from the street mains; (2) leakage

from service pipes and abandoned services; (3) under registration of meters; (4) loss by seepage and evaporation from service reservoirs; (5) water used for special purposes not metered, such as fires, sewers, flushing, etc. With present American methods there are but few systems so tight and well managed that 80 percent of the water is accounted for. The average is about 70 percent but there are some systems where no more than 60 percent or even 50 percent can be accounted for. The records of 27 completely metered systems for the year 1915 showed an average loss equal to 130 gallons per service daily.

The Venturi Meter, invented by Clemens Herschel, consists of contraction of the pipe thru the throat of which the water flows at an increased velocity. As the velocity increases the pressure decreases. A differential gage connected at its two sides with the water at the entrance and throat of the meter indicates the rate of flow. A mechanical integrating device shows the total amount of water which flows through the pipe. There are several devices for accomplishing this, but the principle of the meter in all cases is the same, see Sect. 9, Art. 20. With a good registering device the results are accurate within one or two percent. Attachments are made by which the rate of flow is shown by a pen on a chart, the paper being replaced daily or weekly, so that a permanent record is made not only of the rate of flow at all times but also of the total discharge since a specified date.

Venturi meters should be placed on all important supply lines to show the quantity of water used. In connection with the pumps they serve to show the amount of water actually pumped, and form a basis for computing the slip of the pumps and to show any falling off in the efficiency. They are especially useful on the outlets of distributing reservoirs and elsewhere, where they show at all times the actual rate of consumption, the fluctuation at different hours of the day, and the rate of flow during the early hours of the morning when there is but a small amount of use and the bulk of the flow usually represents leakage, and they also serve to show when fire drafts commence and end and how much water is used during fires and at what rates.

Service Pipes extend from the main in the street to the limits of the street. The same pipe usually extends within the taker's property, but he pays for and owns all beyond the street line or curb line. A majority of works furnish the service pipe to the property line without cost to the taker. Lead service pipes are permanent and therefore desirable. Their use eliminates cutting pavements to remove less durable pipe when it must be removed. Lead is not strong enough for large sizes or the highest pressures. Lead lined iron pipe is stronger. It is made by forming a thin lead pipe inside an iron pipe. Some very soft or acid waters dissolve lead. Lead poisoning is serious, and lead pipe cannot be used with such waters. In case of doubt, thoro test of the water should be made. Galvanized steel or galvanized iron pipe is used where lead pipes cannot be used. It is cheaper than lead and for many localities it is sufficiently durable. Galvanized pipe should be from ¼ to ½ inch greater diameter than lead for the same service, and at least ¾ inch, as its capacity will be gradually reduced by corrosion while it is still otherwise serviceable. With lead pipe the size of meter may correspond with the diameter of the pipe, but with iron pipe the meter should always be smaller than the pipe. The service cock, tapped into the street main, may be smaller. A flexible connection between the tap and the service pipe, most often a lead goose neck, is necessary to prevent damage from temperature change and settlement.

On an average from 5 to 7 people are served per service (more in very large cities) and there is one service for every 50 feet of main pipe. For statistical purposes the number of services should correspond with the number of live accounts, abandoned or dead services and services placed in anticipation of business not being counted.

24. Electrolysis

Electrolysis in Cast-iron Pipes is caused by stray return currents of electricity from various sources, especially from trolley cars. These stray currents find their way into water pipes thru the soil or thru service pipes or hydrant connections or gas pipes or telephone conduits or any other metallic structures coming in contact with the water pipes or the services connected with them. Such currents flow in the pipes, leaving them at points near the power stations, or go thru other metallic conductors to the power station.

Destruction of a Pipe by electrolysis occurs in two ways: (1) By a current collected by the pipe, following it for a distance and then leaving it in moist soil, the electrolysis occurring at the point where the current leaves the pipe. (2) By the flow of electricity in the pipe, a part of which leaves the pipe at lead joints or other points of extra resistance, coming back into the next length of pipe. These two kinds of electrolysis, while having the same effect on the pipe, are to be sharply distinguished. Electrolysis of the first kind may be corrected in great measure by connecting the pipe system with the negativ poles of the dynamos at all power stations. This has the effect of taking the return current out of the pipes directly thru a copper wire and avoiding the necessity of currents leaving the pipe in moist ground on the return journey. This method of treating the electrolysis question was proposed in the early days of electrolysis and used to a considerable extent. The principal objection to it is that it produces electrolysis of the second kind. This system is openly followed in some works, and is actually followed by unknown and indirect connections in others.

In cast-iron pipe lines the lead joint is a point of high resistance. The temperature of the melted lead is not sufficient to burn off the tar coating, and actual metallic connection is not made in all cases. The electric current goes thru the soil around the joint in sufficient quantity to produce electrolysis on one side of the joint. This takes place in wet soil only. Dry soil is a non-conductor. When electricity makes a passage it goes thru the water contained in the pores of the soil, and not thru the soil particles. Water is a non-conductor, but it becomes a conductor when mineral substances are dissolved in it.

Electrolysis of the interior of pipes is extremely rare, because the water used for public water supply is not sufficiently mineralized to act as a conductor. If the water in the soil outside the pipe were equally pure from an electrolytic standpoint there would presumably be little trouble from this kind of electrolysis.

Electrolysis occurs because the ground water contains mineral matter and salts which increase its conductivity. The mineral matters in ground water may result from many sources, among them: (1) From cesspools and similar sources, which are known to increase the chlorine contents of ground water to from 10 to 100 times the natural amounts, in villages and cities, and to less extent in rural districts, (2) Urine of horses falling on public roads, (3) Sea water brought by the rain, this being a matter of importance only when pipes are not very far from the ocean, (4) Solution of mineral matters from the soil.

Insulation Joints are joints made of some non-conducting material to prevent the flow of electricity in pipes. Such joints have been made by driving wooden wedges between the spigot and bell of the cast-iron pipe in place of the lead. To be effective they must be repeated at short intervals, as the electric current will jump a number of such joints, passing thru the surrounding moist soil and causing electrolysis at each of them.

Electrolysis of Steel Pipe. The riveted joints of steel pipe are almost perfect conductors of electricity. There is no evidence that a current flowing in a steel pipe injures it in any way as long as it does not leave the pipe. An electric current flowing in steel pipe and leaving it results in electrolysis at the point where it leaves the pipe.

SEWERAGE SYSTEMS

25. Definitions and Purposes

A Sewer is an underground channel for carrying off waste waters and liquid filth, and especially those containing a mixture of fecal matters. SEWAGE is waste water, especially water carrying fecal and other polluting matters. A SEWERAGE SYSTEM is a collection of sewers with all their appurtenances, combined so as to operate together and serve a certain district. A DRAIN is a channel for removing waste waters in which there is no admixture of fecal or other polluting matters. A sub-drain is a channel built beneath a sewer for the purpose of gathering ground water and of preventing it from entering the sewer. A STORM SEWER is a large channel built to carry away water falling upon city or other areas rapidly in time of storms, and not ordinarily carrying sewage. A LATERAL SEWER is a small sewer serving one particular street. A TRUNK SEWER is a sewer of larger size to which various laterals are tributary. AN INTERCEPTING SEWER is a large sewer, generally parallel with a stream, collecting the water from a number of trunk sewers and carrying it to a point of discharge. A RELIEF SEWER is a sewer built generally parallel with an old sewer of inadequate capacity and sharing its work.

In the Absence of a Sewerage System some of the most commonly used means of disposing of fecal matters are the following: VAULTS are brick or masonry structures more or less water-tight, in which fecal matters are deposited in a comparatively dry state and from which they are periodically removed. EARTH CLOSETS are vaults of less substantial construction, in which dry earth is frequently thrown to become mixt with the fecal matters. THE PAIL SYSTEM is where water-tight metallic receptacles are used to receive fecal matters, and which are frequently changed. This system allows a most complete sanitary control, and for this reason it has been frequently used where more than ordinary precaution was required. CESSPOOLS are pits dug in the earth, usually lined with loose rubble walls, into which water carrying fecal matter from a single house or small group of houses is discharged. They are of two types, (1) without overflows, where all liquid matters are absorbed by a pervious soil and from which solid matters only are removed at considerable intervals, and (2) with overflows thru which the liquids, or a portion of them, flow by a waste drain to a neighboring watercourse.

The Purposes Served by Sewerage Systems are: (1) Removal of sewage, that is, liquid fecal and household matter. (2) Removal of trade wastes, that is, waste waters from manufacturing establishments carrying organic or inorganic waste products which make, or tend to make, them objectionable. (3) Removal of storm water. (4) Removal of ground water entering the sewers either intentionally to get rid of it or accidentally and unavoidably because the sewers are not water-tight. (5) The removal of run-off from areas beyond the limits of those served but most conveniently carried thru the sewerage system.

A Combined System of Sewerage is one in which all purposes are served by a single set of sewers, so far as they are served at all, and especially where both storm water and sewerage are carried in the same channels.

A Separate System of Sewerage is one in which it is the object to carry only sewage and trade wastes in the sewers and from which ground water, storm water, and all other wastes are excluded as far as possible. Storm sewers, and often other drains or channels, are required for the separate removal of waters that would otherwise be troublesome.

The Capacities of Sewers are fixt to carry off the greatest volumes of the respective liquids that it is proposed that they shall carry, taking into account the maximum rates of discharge from the various sources and the times when the maximum rates are likely to occur. The VOLUME OF SEWAGE

is closely connected with the volume of water supplied by the waterworks system. Some water is used in boilers, is evaporated from sprinkled lawns, and is used for other purposes not contributing to the volume of sewage. In general, however, the volume of sewage is nearly equal to the volume of water supply. The monthly, daily, and hourly fluctuations in the volume of sewage follow closely those in the volume of water supplied, as mentioned in Art. 17. For the purpose of design it will suffice to take the volume of sewage to be provided for as the average per capita consumption of water, with from 70 to 100 gallons per capita added to cover fluctuations. The Volume of Trade Wastes varies with industries and local conditions. Sometimes the trade wastes in a small city will exceed the volume of sewage. In such cases separate and independent disposition of the trade wastes is often better than taking them into the sewers. Where trade wastes are smaller in amount and of a polluting character they are generally best discharged into the sewers.

The Amount of Ground Water entering sewers varies with the character of the soil, the natural level of the ground water with reference to the level at which the sewers are laid, and the tightness of the sewers. If sewers were built entirely water-tight no ground water would enter them. It is not possible to build sewers water-tight. If effort is made to make them as tight as possible and they are below the ground-water level, then, within limits, the amount of leakage will depend upon the excellence of construction, and will be proportional to the diameter and length of the sewer. If, on the other hand, the leakage into the sewers is great enough to serve for the entire removal of the ground water from the drainage area, then all that part of the rainfall not otherwise carried off and evaporated will ultimately find its way into the sewers as ground water. This proportion will be larger with sandy or gravelly soil and in a flat country, and will be smaller with clayey or impervious soils and with steep slopes.

Where the tightness of the work controls, leakages of from 5000 to 50 000 gallons per mile per day may be reasonably anticipated in lateral sewers and larger amounts up to 100 000 gallons and more per mile of trunk sewer, even when every reasonable effort has been made to make them tight.

Where the sewers are not especially tight, the average amount of ground water entering the system thruout the year may range from one million gallons per day per square mile of drainage area, with very sandy soils, to almost nothing with steep slopes and clayey soils. Two-thirds of the maximum amount, or approximately 1000 gallons per day per acre, may be taken as a roughly average amount. In the spring of the year the maximum rate of discharge of ground water may be expected to be two to four times the average rate for the whole year. Outside areas contribute their proportion of ground water as well as of surface water, and with a large area of higher ground adjoining the area sewered the amount of ground water may be very much larger than could have its origin in the area actually served by the sewers.

26. Storm Flows

The Volume of Storm Flows is one of the most important matters entering into the design of sewerage systems. The volume that must be carried depends upon (1) the climate, that is, upon the rapidity with which rain sometimes falls; (2) upon the character of the soil, whether pervious or impervious; (3) upon whether a large part of the area is covered by buildings, pavements, and roads, which are in general impervious and throw off water rapidly and completely; (4) upon the rapidity with which the water reaches the sewers, this in turn depending both upon the average slope of the ground and upon the completeness of the provisions made for carrying waters to the sewers; (5) upon the area tributary to a given point; and (6) upon

the shape of that area, short compact areas bringing the maximum rates of discharge from all points to the outlet nearly at the same time, while the maximum rates of discharge from the lower parts of long narrow areas have past before the water from the more remote portions reaches the same point.

Empirical Formulas have usually been used for computing storm flows, altho these leave out of consideration elements known to be of importance, such as the shape of the area drained, the arrangement of the sewers, and the respective times required for flows from the several parts of the system to reach the outlet. Methods of calculation taking into account these elements have been used by many engineers. McMath's formula for determining maximum quantity of rain water to be removed by a sewer is $Q = CR\sqrt[5]{SA^4}$, in which Q is the quantity in cubic feet per second; C is the proportion of rainfall that will reach the sewers, that is, it makes allowance for loss by evaporation, absorption, and retention; its value for any locality is a matter of judgment, taking into consideration the season at which the heaviest rainfall occurs; the condition of the surface, paved or naked; the soil, porous or impenetrable; the kind of ground, whether urban or suburban, park or lawn; for St. Louis the proportion is three-fourths of the rainfall. R is the number of cubic feet of water falling upon an acre of surface per second during the greatest intensity of rain, and practically it is the same as the rate of rainfall in inches per hour; for St. Louis R is taken as 2.75. S is the mean surface grade in feet in a thousand. A is the area in acres.

The surface slope S is taken by McMath for St. Louis at 15. No precise rule for determining the value of S has been given, and uncertainty of this determination is one of the most unsatisfactory matters connected with the use of this formula. Fortunately, considerable variation in S makes only a relatively small difference in the amount of discharge, so that a roughly approximate value of S is sufficient.

The proportion C of water reaching the sewers has been frequently discussed. Perhaps as accurate results as any may be obtained by taking $C = 0.90$ for all areas covered by roofs and impervious, or nearly impervious, pavings, and $C = 0.10$ for naked areas of sandy or gravelly materials, and $C = 0.20$ for naked areas of clayey or but slightly pervious materials. The areas for which sewers are commonly designed are partly naked and partly covered, and the coefficient for the combined area may be obtained by applying these factors to the parts and adding them up for the total.

McMath's Formula is the one in most general use. The following tables enable approximate computations to be rapidly made.

Values of $CR\sqrt[5]{S}$ in McMath's Formula to be Obtained as a Preliminary to Taking the Run-off from the Following Table by Use of the Identification Letters

R taken as 2.75 inches per hour in all cases.

Percentage of total area covered by roofs and pavements		Value of C	Steep slopes, 58 per 1000	Average slopes, 15 per 1000	Flat slopes, 4 per 1000	Very flat slopes, 1 per 1000
Sandy soil	Clayey soil					
100	100	0.90	5.58 = A	4.25 = B	3.24 = C	2.47 = D
73	70	0.70	4.25 = B	3.24 = C	2.47 = D	1.89 = E
53	46	0.50	3.24 = C	2.47 = D	1.89 = E	1.44 = F
37	28	0.40	2.47 = D	1.89 = E	1.44 = F	1.10 = G
25	15	0.30	1.89 = E	1.44 = F	1.10 = G	0.84 = H
16	5	0.23	1.44 = F	1.10 = G	0.84 = H	0.64 = I
10	....	0.18	1.10 = G	0.84 = H	0.64 = I	0.49 = J
5	....	0.14	0.84 = H	0.64 = I	0.49 = J	0.37 = K
0	....	0.10	0.64 = I	0.49 = J	0.37 = K	0.28 = L

Run-off in Cubic Feet per Second per Acre Corresponding to Data of Preceding Table

Area A in acres	Value of $\sqrt[8]{A}$	Identification letters and corresponding numbers										
		A 5.58	B 4.25	C 3.24	D 2.47	E 1.89	F 1.44	G 1.10	H 0.84	I 0.64	J 0.49	K 0.37
50	2.19	2.55	1.95	1.48	1.13	0.86	0.66	0.50	0.38	0.29	0.22	0.17
70	2.34	2.38	1.82	1.38	1.06	0.81	0.61	0.47	0.36	0.27	0.21	0.16
100	2.51	2.22	1.69	1.29	0.99	0.75	0.57	0.44	0.33	0.25	0.19	0.15
150	2.72	2.05	1.56	1.19	0.91	0.69	0.53	0.40	0.31	0.23	0.18	0.14
200	2.89	1.93	1.47	1.12	0.86	0.65	0.50	0.38	0.29	0.22	0.17	0.13
300	3.13	1.78	1.36	1.04	0.79	0.60	0.46	0.35	0.27	0.20	0.16	0.12
500	3.46	1.61	1.23	0.94	0.71	0.54	0.42	0.32	0.24	0.18	0.14	0.11
700	3.71	1.50	1.15	0.87	0.67	0.51	0.39	0.30	0.23	0.17	0.13	0.10
1000	3.98	1.40	1.07	0.81	0.62	0.47	0.36	0.28	0.21	0.16	0.12	0.09
1 500	4.32	1.29	0.99	0.75	0.57	0.44	0.33	0.25	0.19	0.15	0.11	0.09
2 000	4.57	1.22	0.93	0.71	0.54	0.41	0.31	0.24	0.18	0.14	0.11	0.08
3 000	4.96	1.12	0.86	0.65	0.50	0.38	0.29	0.22	0.17	0.13	0.10	0.07
5 000	5.49	1.02	0.77	0.59	0.45	0.34	0.26	0.20	0.15	0.12	0.09	0.07
7 000	5.88	0.95	0.73	0.55	0.42	0.32	0.25	0.19	0.14	0.11	0.08	0.06
10 000	6.31	0.88	0.67	0.51	0.39	0.30	0.23	0.17	0.13	0.10	0.08	0.06

To use the tables find in the first place the nearest percentages of area to be covered by roofs and impervious surfaces, under sandy soil or clayey soil as the case may be, and opposite this in the first table find a letter in the one of the four columns for steep slopes, average slopes, flat slopes and very flat slopes that is selected to represent the conditions. With this letter go to the second table, and follow under it to find a figure opposite the area most nearly equal to the area under consideration. This figure represents the run-off in cubic feet per second per acre that is to be used, and this is to be multiplied by the number of acres. The product is the quantity of storm water in cubic feet per second to be provided for in the sewer. The result is only roughly approximate and is to be accepted with caution.

At Baltimore Kenneth Allen used values between **A** and **B**. Hering, Gray, and Stearns recommended for Washington quantities approximating **B**. McMath's formula, as adopted by him for St. Louis, is slightly more than **C**. At Winnipeg with very flat slopes values are used approximating **H**. At Chicago with very flat slopes assumed to be 1 in 1000 for developed areas between 100 and 10 000 acres, figures approximating **J** are used, and for undeveloped suburban areas values approximating **K**. Fanning's figure for overflow weirs, based on New England experience, with generally sandy watersheds of 640 acres and over, natural conditions, no buildings, is about **G**. Stony Brook Conduit, report of Francis, Clark and Herschel, for a generally flat and sandy watershed of 8000 acres near Boston, with prospect of development, is on line **F**. Kuichling's study of flood flows in the Mohawk Valley, natural conditions, 10 000 to 100 000 acres, steep slopes, generally impervious material, gives for "floods which occur occasionally," about D; and for "floods which occur rarely" between B and C.

Local Data. McMath's method of analyzing local data was to note every case where a sewer proved inadequate, to compute the drainage area tributary to the sewer at that point and the carrying capacity of the sewer, and to plot these two figures on a large diagram. After a considerable number of such points had been plotted, a line was drawn on the diagram representing sewer capacity enough to go above all the points. The line so found corresponds to the McMath formula as deduced for St. Louis. The same procedure can be carried out for any other city and is one of the best ways of getting a basis for local design.

A More Rapid Method of finding indications of local data may be adopted where time does not permit carrying out the above program. Make a tabu-

lation of the drainage areas of all sewers, the sizes of sewers at outlets, their controlling slopes, which controlling slopes represent the slope between points in the top of the sewer some distance apart and near the outlet, and where that slope is least, and frequently and usually less than the actual physical slope of the lower end of the sewer. The discharging capacity of the sewer in cubic feet per second is then calculated and the capacity in cubic feet per second per acre of drainage area. The character of the soil, the general steepness of the slopes, and the percentage of the area covered by roofs, streets, and other impervious surfaces should also be estimated as closely as possible and tabulated. Special inquiry should be made as to whether each of the sewers has proved sufficient or insufficient. The McMath formula may then be solved backward, the value of C that can be met by existing conditions being obtained for each sewer, or more simply, the letters in the second part of the above table may be found corresponding most nearly to those conditions. Comparing the results so found with the observed sufficiency or insufficiency of the sewers in the different cases, an indication will be obtained of the value of the coefficients or of the line of figures in the second part of the table that will give satisfactory results.

Storms of extreme intensity occur only at very rare intervals. The sewerage system may be designed for a certain rate of discharge and its capacity may not be overtaxed for twenty years. A rain may then occur more severe than any previous ones and the capacity of the system will be overtaxed and trouble caused. In laying out a system the probable amount of damage caused by overflows in such extreme storms at long intervals must be taken into account, and more liberal allowances made in closely built up sections where the resulting damage would be very large, while in outlying districts where overflows would find and flow thru natural channels with but little damage, less liberal allowances may be made.

Run-offs from Outside Areas, where such areas exist, must always be taken into account in designing sewerage systems. This is one of the most troublesome features of design. It often involves designing works to provide drainage for districts entirely outside of the municipality building the sewers. Failure to so provide results in inundation of the sewered area sooner or later at times of extreme rainfall.

Some of the best American discussions on the amount of storm water to be removed by sewers are in Trans. Am. Soc. C. E.: R. Hering, vol. 10, p. 361; R. E. McMath, vol. 16, p. 179; E. Kuichling, vol. 20, p. 1; R. L. Hoxie, vol. 25, p. 70; C. E. Gregory, vol. 58, p. 458; C. E. Grunsky, vol. 65, p. 294.

27. Sizes and Grades of Sewers

The Capacity of Combined Sewers is controlled substantially by the allowances made for storm water, the amount of such flows being greater than the amounts of sewage and of ground water to be ordinarily removed. For separate sewers the capacity is based on the estimated amount of sewage plus the estimated amount of ground water.

Future Growth to a reasonable extent is always anticipated in sewer design. The additional cost of building a sewer larger to provide for increased population is less in proportion than the corresponding increase in the cost of a larger water pipe. Sewers are laid deeper than water pipes, and the expense and inconvenience of taking them up are greater. Therefore in designing sewers it is good business to anticipate growth to a greater extent than in designing waterworks.

In designing separate sewers, especially with laterals and small trunk sewers, it is well to assume the greatest population to be reasonably anticipated ultimately in the areas that are served. This ultimate population may be estimated by experience with other cities in similar country with similar population. Populations of 30 to 70 per acre may be assumed for suburban and outlying areas, and from 100 to 300 per acre for closely

built up down-town districts, but no fixt rules can be given. In designing combined sewers a great increase in buildings and in the extension of paved areas should be assumed.

The Size of Sewers is computed by the ordinary hydraulic formulas, but owing to grease and other foreign matters carried, the walls ordinarily become roughened, the velocities are less, and the required sizes are somewhat greater than they would be for the same volume of clean water. A coefficient of discharge of 100 in the Hazen and Williams formula may be generally used. Velocity head, that is, the head required to produce the velocity, must always be taken into account.

The Elevation of a sewer, unless otherwise specified, always refers to the elevation of its invert; that is, to the center of the bottom on the inside. The SLOPE of a sewer, for the purpose of calculating self-cleaning velocities, is the actual physical slope of the structure, or in general, the slope calculated by dividing the difference in elevation at two points by the distance between. The slope of a sewer, for the purpose of calculating capacity when running full, is the average slope at the top, and when the sewer increases in size between two points, the slope is the difference in elevation at the inverts at the two points, less the increase in vertical diameter between those points, divided by the horizontal distance.

Old sewers built on irregular grades often have short sections where the computed carrying capacity is less than for sections above. Within certain limits sewers will act under pressure at such places, and the carrying capacity of the sewer as a whole will be represented by the average of a section of some length, and may be greater than computed for a short section of less than average capacity.

Near Outlets sewers will carry more than computed by the usual formulas, because a drop in water level at the outlet increases the hydraulic slope for a certain distance back, and with it the velocity and up to a certain point, the increase in velocity more than offsets the decrease in area. A part of the head is required to produce the extra velocity, and only the remainder is available for overcoming increased friction, but in short stretches with large sewers the gain may be considerable.

Submerging the Outlets, due to high water in the stream into which discharge is made, reduces the capacity of the sewers. Sewers discharging to streams that have high flood levels with reference to the districts served are often submerged, and this condition is unavoidable. A sewer that has its outlet submerged frequently or for long intervals becomes filled with deposits and this condition is to be avoided as far as possible.

Flushing consists in the artificial addition of water in large quantities, usually from the waterworks system at intervals, to temporarily increase the velocity and clean the sewers. Sewers with slopes great enough to produce self-cleaning velocities do not need to be flushed. Flushing is used especially in sewers that must be built with slopes so flat that they are not self-cleaning. Flushing may be done by putting temporary dams in the manholes, filling up a section of sewer and manholes above with water from a fire hydrant discharged thru a hose, and then quickly removing the dam and letting the water flow rapidly and flush the sewer below.

Automatic Flush Tanks are masonry tanks built underground to which water is constantly allowed to flow from the water pipes, with automatic siphons that discharge their contents to the sewer whenever the tank becomes full. Automatic flush tanks usually waste a great deal of water, thereby increasing the required capacity of both the waterworks system and the sewerage system. They are difficult to regulate, and commonly discharge much more frequently than needed for flushing purposes. For this reason the other means of flushing are preferred.

A Self-cleaning Velocity is that velocity at which no permanent deposits will be formed. There is no general rule for this. Heavy sand and silt will deposit at a higher velocity than finer materials. Two feet per second may be a self-cleaning velocity for a separate system, while three feet per second is commonly taken as the minimum for the combined system carrying street wash. The slopes of sewers should as far as possible be made great enough to produce self-cleaning velocities which will prevent the formation of deposits.

Minimum Slopes for self-cleaning must produce velocities to prevent deposits when average or minimum quantities are flowing, and the slopes must therefore be steeper than would serve to produce self-cleaning velocities when running full. Generally with the following slopes flushing will not be needed: 6-inch sewer, 10 per thousand; 8-inch, 7; 10-inch, 5; 12-inch, 4; 15-inch, 3; 21-inch, 2; and 36-inch, 1 per thousand. The accuracy with which lines and grades are maintained in laying and the smoothness of finish of the interior surface materially influence the minimum slope that will suffice.

Pills are balls of wood a little smaller in diameter than circular sewers, that are put in at one manhole and allowed to go thru the sewer to the next manhole. Water accumulates behind the pill until sufficient pressure is developed to drive it forward. As the pill tends to float, most of the water discharges underneath and scours out deposits on the bottom of the sewer, thereby cleaning it. Other more elaborate methods of cleaning are used in large sewers with flat grades.

Flat Grades are necessary in many cases where the slope of the ground is slight. In such cases the largest possible grade should be secured, and the work of cleaning and removing such deposits as are unavoidable should then be systematized and made as simple as possible. Pumping must be used to increase the flow where sewage is carried for long distances in flat country, as, for example, to a remote tidal outfall.

28. Materials and Shapes of Sewers

Vitrified Tile Pipe is the commonest material for sewers 24 inches in diameter and under. It is also used occasionally for sewers up to 36 inches in diameter. The pipe is made in lengths 24, 30, and 36 inches, with bell joints which in laying are filled with jute gaskets and cement mortar. The diameters commonly used for sewers are 6, 8, 10, 12, 15, 18, 21, and 24 inches; also less commonly 27, 30, 33, and 36 inches. With pipe having a thickness equal to 1/12 of the diameter, but little trouble will be experienced from breakage in the deepest ordinary trenches. With especially strong pipe and with trenches that are not very deep satisfactory results are sometimes obtained with pipe only as thick as 1/16 of its diameter. CEMENT PIPES are used to a considerable extent for sewers of small size.

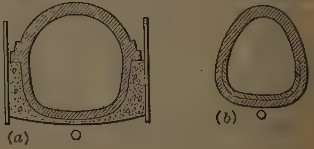

Fig. 12. Brick Sewers at Boston

Brick Sewers (Fig. 12) laid in portland-cement mortar, the brick being especially hard burned, were formerly used for sewers of all sizes, and a majority of the old sewers now in use are of this construction. Brick sewers on slopes that produce velocities of more than from 7 to 10 ft per second wear rapidly, especially on the bottom. Such velocities are produced in 48-inch sewers by slopes of more than 6 per thousand and in 72-inch sewers

by slopes of more than 4 per thousand. Inverts of vitrified paving brick or of concrete well mixt, of good cement and hard durable sand and ballast, wear well under high velocities and probably are durable at any slope at which large sewers are required (Figs. 13 and 14).

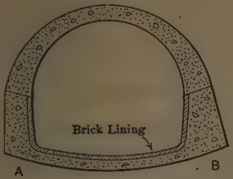

Fig. 13. Plain Concrete Sewer

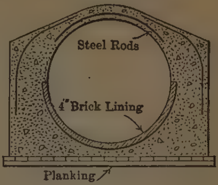

Fig. 14. Reinforced Concrete Sewer

Concrete and Reinforced Concrete are generally both the best and the most economical materials for the construction of large sewers at the present time. These sewers are usually formed in place, but sometimes are made in sections in advance, the sections afterwards being laid in place (Fig. 15).

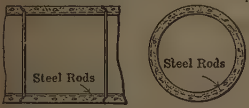

Fig. 15. Reinforced Concrete Pipe

Round Sewers are most commonly used for all sewers up to 24 inches in diameter, and very often for larger sewers. The round sewer is the most economical of material for a given carrying capacity.

Egg-shaped Sewers (Fig. 16) are small at the bottom and large at the top. The small bottom concentrates small flows, thereby giving greater depth, and increases the velocity and reduces the likelihood of deposit. The large top gives increased carrying capacity for storm water. The advantage of egg-shaped sewers is that they are self-cleaning at lower slopes. Egg-shaped sewers are most conveniently rated and spoken of as of a size equivalent in area or carrying capacity, preferably the latter, to circular sewers.

Fig. 16. Egg-shaped Sewer

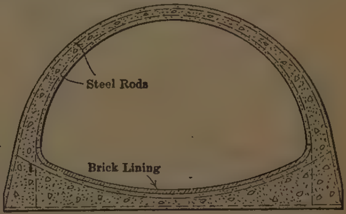

Fig. 17. Reinforced Concrete Horseshoe Sewer

Horseshoe-shaped Sewers (Fig. 17) are often used in large sizes with ample slopes because cheap to build. The sewage with low flows is spread over a wide bottom, and velocities with small quantities are low and the slope for a self-cleaning velocity is greater.

Wide Sewers in proportion to their height are used where head room is desired, and especially where wide fluctuations between high and low water levels of the sewage in them are objectionable.

Sewer Arches with walls following the lines of thrust and slightly arched bottoms are, after circular sewers, the most economical in material. Such sewers, with reasonable cover, can be built so that the cross-section of the concrete does not exceed from 0.75 to 0.90 of the clear area. With reinforced concrete still lighter sections have been used. On bad foundations and in deep cuts larger sections are needed. Taylor and Thompson give the rule that the minimum thickness at the top in inches should equal the diameter in feet plus one.

A sewer with the smallest section of concrete is not always the cheapest, because the shapes reached by purely theoretical considerations are not cheap to build. By approximating the theoretical lines, but simplifying them, the forms can be cheapened and the work done for less money even tho more material is used.

Common types of sewers are shown above. Fig. 12 shows two brick forms used in Boston, Fig. 13 a plain concrete sewer with brick lining used in Baltimore, Fig. 14 a reinforced concrete type used in Brooklyn, Fig. 15 one section of a reinforced concrete pipe, Fig. 16 an egg-shaped section, and Fig. 17 a reinforced concrete horseshoe form which has been used in St. Louis.

29. Sewer Appurtenances

Siphons are pipes or conduits under pressure connecting sections of sewers. They are used to carry sewers under rivers, across valleys, and in other places where the continuous grade cannot be maintained. Siphons have ordinarily smaller areas than the sewers with which they are connected, so as to increase the velocity in them and diminish the tendency to form deposits. At the same time, some extra slope or head must be allowed for siphons, both to give the added velocity and to overcome resistance due to partial silting up of the channels.

Siphons serving large sewers in which the flow is very variable are best made of twin or triplicate pipe lines. Low flows then pass thru one line of pipe, the other lines being brought into service automatically for the flood flows. Siphons should be so designed that the maximum velocities thru them will be sufficient to scour out even considerable deposits of sand. Maximum velocities can sometimes be provided artificially by flushing when they would not occur otherwise. Siphons of considerable length and drop are best made of cast-iron or steel pipe. Small siphons with but little drop may be made of tile pipe, and large siphons with but little drop of reinforced concrete.

Manholes are masonry structures reaching from the channel of a sewer to surface of the ground, closed at the top with a cast-iron frame and cover, and affording access to the sewer for inspection, cleaning and repair. Manholes are usually placed at all street intersections, and frequently at intermediate points with small sewers. Manholes are provided in larger sewers wherever laterals enter, and in general not more than three to five hundred feet apart. If the sewer is large enough so that it can be inspected by passing thru on the inside readily, there is less necessity for frequent manholes.

Inlets for Storm Water are provided for combined sewers or storm drains on each of the four corners at street intersections, and in the middle of long blocks. Rain water is carried to these points by the gutters.

Catch Basins are masonry pits built in connection with, and frequently directly under, the inlets, thru which the storm water flows, and in which the coarser mineral matters from the street wash are deposited. Catch basins require to be cleaned at intervals. They should be used on all sewers except those having very steep slopes and where the discharge of coarse-grained

mineral matter at the outfall is not objectionable. The idea of the catch basin is that it is easier and cheaper to remove heavy matter collected from it than from the bottoms of the sewers.

Outlets are the ends of sewers, where the sewage is discharged into streams or other bodies of water. Outlets for small sewers are most conveniently made of cast-iron pipe of sufficient length to be well anchored into the bank. Larger sewers have outlets of masonry built on stable foundations, and sufficiently massive to resist the action of ice and logs or other material carried by floods.

Sewer outlets are best placed so as to discharge directly to the thread of the stream, so that the sewage will always mix directly with flowing water. With streams having great range in flow and elevation it is often impractical to carry the main outlet to such a point of discharge. In such cases a cast-iron pipe sufficient to carry the low-water flows may be laid under the bed of the stream, reaching the main channel, the pipe being connected with the bottom of the sewer back of a low dam. The low flows will then pass thru the pipe, while storm and other large flows will overtop the dam and find their way directly to the river.

In very cold climates, cold air enters the sewer outlets, chilling the sewers and their connections to an objectionable extent. In some cases sewer outlets are filled with straw in winter to protect them from extreme cold. This is safe because any flood requiring the capacity will force out the straw, making a channel for itself.

Where size is increased the axis of the sewer may be continued thru such increase, or the bottoms may be brought flush, an offset being made at the top, or the tops may be brought flush, the offset being made at the bottom. Where the slope is not steep it is always best to bring the bottoms flush, because this gives the greatest velocity for self-cleaning.

Intercepting Sewers do not carry storm flows. They are often much longer than lateral and trunk sewers, and are most frequently built after other parts of the sewer system and to correct defects, and especially to change the point of discharge or to allow other disposition of sewage to be made. The areas involved are often large, and it would be so difficult as to be almost impossible to build intercepting sewers large enough to carry all sewage and storm flows to the new point of discharge. It usually suffices to take to that point the dry-weather or ordinary flows, and to allow the storm flows with the sewage mixt with them to continue for the most part to discharge as formerly.

The Capacity of Intercepting Sewers is computed as if for the separate system, even tho combined sewers are used in a whole or a part of the area tributary to them. For example, the intercepting sewer is made sufficiently large to carry 100 gallons of actual sewage per capita daily, 100 gallons per capita daily in addition, representing fluctuations in rate of flow, and further 100 gallons per capita daily, representing ground water entering the system, and all other waters, making a total capacity of 300 gallons per capita daily, reckoned on the population to be ultimately provided for in the area under consideration. With an intercepting sewer so designed all of the sewage can be carried except when there are considerable quantities of storm water or melting snow.

Storm Overflows are structures built at the connections of the trunk sewers and the intercepting sewer, arranged so that the low water flows will be diverted to the intercepting sewer, and so that storm flows in excess of the capacity of the intercepting sewer will automatically go thru their old channels.

Storm overflows usually consist of long weirs with their crests a little way above the bottom of the trunk sewer arranged so that the water that does not pass over the weir goes to the intercepting sewer. With this arrangement the storm overflow will not ordinarily operate until the full contribution has been diverted to the intercepting sewer and until that sewer is running full.

Automatic Valves operated by floats have been sometimes installed so that when the intercepting sewer becomes filled the connection with the trunk sewer is shut and no more sewage goes to the interceptor until the level in it falls. Such devices allow the entire flows during storms to go as before.

Pumping Stations are often necessary in connection with intercepting sewers. This happens where the natural slope of the valley is not sufficient to give the requisite fall to the sewer. In such cases the economical slope of the intercepting sewer must be carefully considered. A steep slope means cheaper sewer construction and more expense for pumping, while a flatter slope means a larger sewer and less expense for pumping. If the intercepting sewer is very long it may be cheaper to install two or more pumping stations along its length, as this keeps the flow line near the surface of the ground and the sewer does not have to be built so deep, the cost of construction being thus reduced. Pumping at two stations is more expensive than pumping at one station, so that this procedure must be followed with caution.

Storm Sewers as distinguished from sewers carrying fecal matters are used separately in connection with sewers on the separate system. The principles covering their design are substantially the same as those covering the design of sewers on the combined system, except that with their use it is generally possible to utilize gutters to carry the storm flows for greater distances and therefore to omit the storm sewers in side streets to a considerable extent, using the gutters as part of the system. The total mileage of a system of storm sewers will be materially less than the mileage of a system of sanitary sewers. Storm sewers may also be often built at a higher level and so at less cost.

Where the system of storm sewers is installed it is often possible to utilize natural valleys in their natural condition, or with paving and other improvements to carry storm flows. This can often be done for less money than it costs to build entirely artificial channels. Valleys used for this purpose are best owned by the city for a width sufficient to give capacity for the greatest flood flows. Ordinary flood flows are very much less than the extreme flood flows, and the sides of the valleys, overflowed but seldom, may be parked and may be made attractive features of the plan. The fact that such areas are overflowed at long intervals in exceptional storms does not detract materially from their usefulness in other ways. It is difficult to apply this system to old cities already developed along other lines, but where additions are being developed and laid out it is usually possible to apply it.

30. Maintenance and Operation

Use of Combined and Separate Systems. Most of the older cities were sewered on the combined system. In the last decades there has been a great increase in the use of the separate system. The combined system is more economical in cities compactly built up and near points of discharge where no question is raised as to the discharge at the outlet of all that comes in the sewers from whatever source. The separate system is to be preferred in most cases where the sewage has to be treated before discharge, or where it has to be carried for any considerable distance to an acceptable point of discharge, or where it has to be pumped. The advantage of the separate system in these cases lies in the great reduction in the volume of sewage to be carried, treated, or pumped.

Sewerage systems are seldom laid out and built as units. A little is built by each successive administration, and the additions frequently represent the individual ideas of those in charge at the time that the additions are made. For this reason, sewerage systems, especially old systems that have been long building, seldom represent a comprehensive plan. Portions of combined and separate sewers are found in the same system, and the greatest divergence in carrying capacities is found in different streets.

The Problem Presented in Sewerage Design is seldom to design a new system, but rather to find out what already exists and how that can be best utilized in the development of the system to meet changed and changing needs and new conditions. RECORDS should be kept of all sewers built, showing exact profiles, connections, cross-sections, materials, and all essential facts. When not so kept, changes and additions must be based upon surveys, often made at great expense and then inadequate because not revealing conditions that are covered up, but which could easily have been shown on proper record plans when the sewers were built.

Ventilation of Sewers. It was formerly supposed that various diseases were caused by sewer gas, and the ventilation of sewers was studied from this standpoint and with reference to preventing the air which had been in the sewers from finding its way where it might be breathed. Increased information has not supported these views. Diseases are carried rarely, if at all, by sewer gas, and there is less reason for apprehension than was formerly supposed. Even the men working in sewers and cleaning them are not especially subject to disease.

The ventilation of sewers, therefore, is primarily a question of æsthetics, and not of sanitation. Objectionable smells from sewers are occasionally experienced, especially where sewers are very flat, or where for any reason considerable deposits collect and decompose on their bottoms and sides. Such odors are most likely to occur in sewers on the combined system in long periods of dry weather when natural flushing is absent, or in very cold climates where, during the winter, all the precipitation is in solid form and does not contribute to the flushing of the sewers.

Ventilation thru Manholes. The manhole covers are commonly perforated, and considerable ventilation takes place thru them. When the sewer system is otherwise nearly air-tight cold air will go down thru a certain number of manhole covers and up thru the others. If the sewers are dirty, bad-smelling, objectionable odors will be noticed by those passing over those manholes in which there is an upward current.

Special Ventilators have sometimes been built on the sides of streets, carried to considerable elevations. Such a ventilator will act as a chimney. There will always be an upward flow of warm air thru it, and thruout a considerable distance in the sewerage system all the flow of air thru other openings will be into the system, and consequently no odors will be observed in passing them. Such sewer ventilators have seldom been used in America.

Ventilation thru Soil Pipes. In most Continental and in many American cities the soil pipes of houses are mainly relied on for sewer ventilation. When this is done the traps on the entrances from the houses to the sewers, universally used in England and in many American cities, are omitted. There is then a flow of warm air from the sewer thru each house connection to the soil pipe and thru it to its end above the roof of the house.

In cold climates special protection of the top of the soil pipes must be applied to prevent them from filling up with frost. This is best done by surrounding them with galvanized iron or other metallic structures, with suitable openings, which prevent the top of the soil pipe from becoming chilled. This method of making house connections without traps and of ventilating the sewers thru them has been satisfactory wherever it has been used.

31. Disposal of City Refuse

City Refuse consists of various solid substances incapable of being carried by sewers. It includes ashes, garbage, rubbish, manure, etc. Most of it is collected by householders in cans and removed by the municipality in carts. This gives rise to three interrelated problems, household storage, transportation and disposal. The method selected for final disposal controls the other two,

There are two principal systems, the mixt system and the separate system. With the mixt system the ashes, garbage and rubbish are stored and transported together and disposed of by incineration. This is the method common in Europe. With the separate system the three classes of refuse are kept apart, the ashes being used for filling low land, the rubbish being dumped or burned, and the garbage being fed to animals, reduced or otherwise disposed of. This requires separate household receptacles and separate transportation.

Refuse Incineration consists of burning the mixt refuse of a city instead of disposing of them separately. There are two general types of destructors, (1) the mutual assistance type, having several grates and divided ash pits, the products of combustion commingling above, thus combining several furnaces into one; and (2) the separate-unit type. The minimum temperature of combustion is about 1250° F.; maximum, seldom above 2000° F. The capacity of existing plants is about 1200 to 1500 pounds per day of mixt refuse per square foot of grate surface. One to two pounds of water are evaporated per pound of mixt refuse. The cost of the process is about $1.50 per ton of mixt refuse.

Volume of Refuse in Pounds per Capita per Year, New York City

Borough	Ashes	Garbage	Rubbish	Street sweepings
Manhattan	1283	215	108	318
Bronx	747	118	51	183
Brooklyn	483	143	87	174
Queens	487	160	49	293
Richmond	546	244	33	797

Seasonal Variation in Volume of Refuse, in Percent of Monthly Average New York City

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec
Ashes:												
Richmond	124	125	149	127	118	86	71	67	66	67	93	120
Manhattan and Bronx*	127	140	144	105	86	77	70	73	73	83	101	12[illegible]
Garbage:												
Richmond	71	71	84	82	104	127	132	145	141	88	79	80
Manhattan and Bronx	59	56	69	78	107	123	134	142	128	118	102	88
Rubbish:												
Richmond	75	75	92	93	120	137	112	121	112	92	92	92
Manhattan and Bronx	76	73	93	98	114	116	116	116	102	102	96	83
Street sweepings:												
Richmond	120	73	83	107	79	92	92	93	107	128	123	87

* Includes street sweepings.

Average Weight of Refuse, in Pounds per Cubic Yard, New York City

Borough	Ashes	Garbage	Rubbish	Street sweepings
Manhattan and Bronx	1086	1100	143	1016
Brooklyn	975	1100	154	769
Richmond	1200	932	200	1800

Average Percentage of Constituents of Refuse

Kind of refuse	City	Water	Volatile matter	Ash	Carbon	British thermal units†
Ashes	Milwaukee	15	10	50	25	3700
	Pittsburgh	7	10	58	25	
	New York City	27	8	53	18	
Garbage	Milwaukee	78	....	14	8*	2000
	New York City	69	22	5	4	
	West New Brighton, N. Y.	73	17	5	5	
Rubbish	Milwaukee	15	45	15	25	6000
	New York City	12	40	8	40	
	West New Brighton, N. Y.	6	65	14	15	
Street sweepings	Washington	38	20	42	....	....
	Cincinnati	46	27	27	....	....
	New York City	37	31	32	....	....
	Berlin	40	22	38	....	....
Manure	Milwaukee	40	25	15	20	....

* Percentage of combustible. † Approximate average per pound of refuse.

Garbage Reduction consists of separating from the garbage such substances as glass, tin cans, etc., and submitting the garbage proper to a steam-cooking process, whereby certain by-products, such as grease, fertilizer, ammonia, etc., are recovered. Naphtha is sometimes used in the grease extraction. At Barren Island, N, Y., the garbage was cooked by steam for 8 or 10 hours, at 310° F., pressure 60 to 65 lbs. At Buffalo, N. Y., the percentage of grease amounts to 1 to 3% of the garbage and the marketable tankage is about 10%.

The Cobwell method of garbage reduction avoids some of the disagreeable odors. It involves steam cooking and tankage drying. The garbage is put into steam jacketed extraction tanks and treated with naphtha (or kerosene) and stirred mechanically. The garbage becomes dehydrated, naphtha and water going off together to a separator, and then the fats are dissolved in the solvent and recovered by distillation. The tankage, containing 8% moisture and very little grease, is removed, dried, ground and sold as fertilizer. The naphtha is recovered and used again with but little loss. The largest plant of this type is at Staten Island. It has a nominal capacity of 2000 tons per day and is used for the disposal of the garbage of New York City.

DISPOSAL OF SEWAGE

32. Composition of Sewage

The Constituents of Domestic Sewage are about the same in all places, but the quantities vary with the water supply, diet of the people, the use of soap, etc. The concentration varies according to the per capita consumption of water, ground-water leakage, and storm-water flow. The admission of street wash and trade wastes materially alters the character of sewage. The first washings of a street after a shower are sometimes as foul as domestic sewage, while the wastes from certain industries add materially to the mineral and organic constituents. The age of the sewage modifies its composition and its physical and biological character. The sewage of a separate system is usually more uniform and more concentrated than that of a combined system.

Analysis of Sewage. The methods are practically the same as those for analyzing water. The standard methods are given in "Standard Methods for the Examination of Water and Sewage," third edition, 1917, revised by committees of American Public Health Association, American Chemical Society, etc. The most important determinations are those of suspended matter, organic matter, fats and dissolved oxygen, but in many cases determinations of iron, alkalinity, acidity, incrustants, chlorine, free carbonic acid, etc., are required, besides microscopical and bacteriological tests. Observations of turbidity cannot be as well substituted for determinations of suspended matter as in the case of water analysis. The organic matter is best represented by determinations of total nitrogen and oxygen consumed. Analyses of single samples of sewage are of little value from the standpoint of sewage purification. In order to obtain a fair idea of the average composition of sewage, samples should be collected in series and integrated so as to obtain an average sample, taking into account the time and the volume of flow. On account of the liability to rapid putrefaction it is necessary to have analyses of sewage made very soon after the sample is collected.

Per Capita Constituents of Sewage. In the absence of analyses of properly collected samples the following estimates of the constituents of sewage may be used for the purpose of approximate calculation except where trade wastes play an important part. The effect of trade wastes on the character of the sewage is very great, and the figures given in the table for this class of sewage do not indicate the extreme conditions, but are taken as representing those which generally obtain.

Constituents of Sewage in Grams per Capita Daily

Sewage matter	Total solids	Organic matter	Mineral matter	Chlorine	Nitrogen	Nitrogen as albuminoid ammonia	Nitrogen as free ammonia
Fecal matter	70	50	20	7	8	....	....
Domestic sewage	110	60	50	10	10	1.4	6.4
Domestic sewage plus polluted ground water ...	170	70	100	20	11	1.7	7.0
Domestic sewage plus street wash	220	100	120	25	13	2.0	8.0
Domestic sewage plus manufacturing wastes	220 to 500	100 to 200	120 to 300	25 to 50	13 to 18	2.0 to 4.0	8.0 to 10.0

To change to parts per million multiply these figures by 264 and divide by the volume of sewage in gallons per capita per day. To change to tons per year per 1000 population, divide by 2.5.

For average sewage the amount of suspended matter is about 100 grams per capita daily; fats, 20 grams; bacteria, 300 billion per capita daily.

Sewage, consisting of the water-carried wastes of a city, is disposed of either by dilution, that is, by discharge into a body of water without purification, or is first submitted to some form of purification.

33. Disposal by Dilution

Disposal by dilution consists in allowing sewage to flow into the water of streams, lakes, or harbors. The dissolved oxygen in the water into which the sewage flows is depended upon to oxidize the organic matters in the sewage before they become objectionable. In order to be successful the sewage must be mixt within a short time with a sufficient body of water so that the oxygen in it will do its work. The supply of oxygen is increased by oxygen absorbed

from the atmosphere as the supply in the water is exhausted. The disposal of sewage by dilution is much the most common method. A secondary source of trouble from disposal by dilution in still water is the formation of deposits of sewage mud immediately about the sewer outlets. This difficulty is principally experienced in lakes and in mill ponds upon small streams and in the vicinity of docks where the sewer outfalls are not carried out to the pierheads. It is not likely to happen where there are considerable tidal currents, and in large streams flood flows usually remove such deposits before they accumulate to an objectionable extent.

Pollution of Streams. The disposal of sewage by dilution in streams tends to their pollution. The pollution of streams when it goes too far is objectionable: (1) In causing local nuisance; that is, by causing odors, scums, and deposits objectionable to sight, and making the watercourses and the lands near them less desirable as places of residence, business, or pleasure. (2) In the pollution of public water supplies taken from the watercourses at points below.

Purification of Sewage is generally undertaken solely to prevent local nuisance and in those cases where the local nuisance otherwise produced would be so detrimental to the public interest as to warrant the expense of sewage purification. The conditions arising from local nuisance are not in general injurious to health.

The purification of sewage to prevent the pollution of public water supply has been attempted only in exceptional cases by small towns and cities upon small catchment areas, as, for instance, some of the villages upon the catchment area of Boston's water supply. In these cases the party profiting by the treatment has paid most or all of the expense. To make the process effective it must be bacterially efficient and the treatment must apply to all storm-water flows. These two conditions make the application of sewage purification to cities on large rivers to preserve the purity of the streams for water-supply purposes practically out of the question in the present state of the art, and, in general, the purification of water supplies can be carried out more certainly and at less cost than corresponding results can be reached by purification of sewage.

Natural Agencies of Purification. The method of disposal by dilution makes use of the following natural agencies: (1) The oxidation of organic matter of all kinds by the oxygen in the water or by that taken from the atmosphere, either destroying the organic matters or changing them into stable inoffensive residues. (2) Sedimentation, removing suspended matters wherever the flow is sluggish, thereby purifying the flowing water and forming deposits of sewage mud, which deposits, in rapid streams, will be flushed out at the next flood. (3) Bacterial changes, especially the death of pathogenic bacteria, which in temperate climates are unable to multiply in flowing waters and which gradually disappear under the influence of sunlight, aeration, and the antagonism of other organisms.

Pollution of Harbors. In the case of the disposal of sewage by dilution in harbors there are no questions of public water supplies to be considered, but, on the other hand, there is some danger of the contamination of oysters and other shellfish and from the use of the water for bathing. The problem is often complicated by tidal phenomena which influence the circulation of the water; by the greater specific gravity of salt water, which has an effect upon the dispersion of sewage in water, and by the fact that salt water holds less dissolved oxygen than fresh water.

Dissolved Oxygen in Water. The amount of oxygen dissolved in fresh water saturated with air varies according to the temperature as given on p. 1268. The figures, which are in parts per million by weight, are for sea level, or, more strictly, for a barometric pressure of 760 mm.

Temp. C.	Oxygen	Temp. C.	Oxygen	Temp. C.	Oxygen
0	14.70	10	11.31	20	9.19
1	14.28	11	11.05	21	9.01
2	13.88	12	10.80	22	8.84
3	13.50	13	10.57	23	8.67
4	13.14	14	10.35	24	8.51
5	12.80	15	10.14	25	8.35
6	12.47	16	9.94	26	8.19
7	12.16	17	9.75	27	8.03
8	11.86	18	9.56	28	7.88
9	11.58	19	9.37	29	7.74

Deduct 1% for each 270 ft of elevation above sea level. For sea water deduct 20% and for mixtures of fresh and sea water in direct proportion to the amount of sea water.

Dilution of Sewage Required to Prevent Nuisance. The following formula represents the degree of dilution required to oxidize sewage and prevent nuisance: $D = x/s = fm/o$, in which x = the volume of water, s = the volume of sewage, o = the amount of dissolved oxygen in the water, m = the result of the "oxygen consumed" test, express in parts per million, and f = a factor depending upon the method used for determining the oxygen consumed, and wh'ch is approximately 4 for the five-minute test as made in this country, 6 for the two-minute test, 7 for the four-hour test as made in England, and 12 for the fifteen-minute test as made in England. For example: the amount of "oxygen consumed," as shown by the five-minute test for an average city sewage, is 58 parts per million when the volume of sewage is 100 gallons per capita daily. If the amount of oxygen in the water is 10.14 parts per million, which corresponds to a fully saturated water at 15° C., then $D = x/s = 4 \times 58/10.14 = 23$, that is, the dilution of the sewage required to prevent putrefaction is 23 times the volume of the sewage. This is further equal to 3.5 cu ft per second per 1000 of population.

River waters carry less oxygen in summer, and therefore more dilution is required. Stream flows are ordinarily less in summer than in winter, and in general pollution is greater in summer than at other seasons. The problem of local nuisance is mainly a summer problem which need only be discussed with reference to summer conditions.

Flow Required for Dilution. The Chicago Drainage Canal was designed by Rudolph Hering to provide a flow of at least 3.33 cu ft per sec of water for the dilution of the sewage of each thousand persons. X. H. Goodnough places the limits for Massachusetts conditions between 3.5 and 6 cu ft per sec. Rafter used 4 cu ft per sec for certain New York conditions. No single figure can be applicable in all sewages, but the above figures may be taken as reasonable for those conditions in which no opportunity is offered for absorption of oxygen from the atmosphere, as in sluggish, stagnant streams or bodies of standing water. Where these conditions are favorable for the absorption of atmospheric oxygen, the required dilutions are less. Such conditions are found in rapidly flowing streams with many ripples and waterfalls, and in bodies of standing water subject to strong wave action.

It would be possible to have a channel in which the velocity would prevent deposits, and in which falls would produce aeration, thru which sewage could be discharged continuously without dilution, and without the production of local nuisance, until, if the channel were long enough, the sewage became fully oxidized. This extreme condition is not reached in practise, but some mountain streams approach it, and receive sewage without offense in quantities that would be intolerable in a sluggish stream.

The application of dilution ratios is difficult because the flows of streams are constantly changing. In a given case the natural flow of the stream may be sufficient for dilution for ten months in the year and inadequate for two months in the year, or the flow may be sufficient for dilution at all times during an average year and may fail to give sufficient dilution in a dry year. In general the method is insufficient only when it fails to give reasonably satisfactory results for some considerable length of time.

The required oxidation for dilution is affected by the amount and character of the population on the banks of the stream below the outfall. If there is a large population to be

affected, and if the water is used for boating and bathing, higher standards are properly required than where there is only a scattered rural population and no special use made of the water. In some cases there is no population below for some distance, and it may be then questioned whether a nuisance exists. The amount of discomfort and damage resulting from pollution should be taken on its merits on actual evidence in each case where sewage purification is proposed, and the amount of damage to be prevented by such purification should be reasonably commensurate with the required expenditure in order to justify the works.

34. Screening and Sedimentation

The Processes involved in sewage purification may be classified as follows: (1) Preparatory Processes, including screens, roughing filters, detritus tanks, plain settling tanks, septic tanks, and chemical precipitation tanks. (2) Purification Processes, including sub-surface irrigation, broad irrigation, intermittent filtration, contact beds, and sprinkling filters. (3) Finishing Processes for clarification are sedimentation and coarse filtration, while those for bacterial improvement are land treatment, sand filtration, mechanical filtration, and disinfection.

Processes of sewage purification may be divided in another way into three classes, as follows: (1) Processes separating without destroying the objectionable matters, afterward disposing of them in a concentrated form by burning, burying, or otherwise. (2) Processes which have for their object the oxidation and destruction of the organic matters in sewage by oxygen taken either from the air in the pores of filtering materials or contained in water with which sewage is mixt. (3) Processes for killing objectionable organisms without otherwise purifying the sewage. The first class generally includes all the processes listed above as preparatory processes, and also, in the finishing processes, the final sedimentation, coarse filtration, and the disposal of sludge. The second class covers especially intermittent filtration, broad irrigation, sprinkling filters, and contact beds.

The purification of sewage by any process is often attended by a certain amount of nuisance. For this reason it is customary to locate works at distances of one-quarter to one-half mile and farther if convenient from residential districts.

Screening. Where sewage is to be pumped or purified it is usually screened thru a grating of iron bars or thru a coarse or fine wire cloth. The object of screening is to remove the larger substances that might injure pumps, clog filters, or appear as unsightly litter. Coarse screening plays but little part in the real purification of sewage, but fine screening is an important preliminary process. Where gratings are used the bars are usually flat bars, spaced ½ to 1 inch apart, cleaned either by hand or by automatic cleansing devices. Screening has attained its greatest development in Germany.

In Birmingham, England, the screens are constructed as endless belts carried over two cylinders and operated by an undershot water wheel turned by the sewage. The screenings fall into a trough, where a worm forces them into the cart by which they are removed. Fine screens usually choke badly, but in Reading, Pa., a revolving cylindrical screen is used and kept clean by jets of water playing upon it from the outside. The sewage passes thru the cylinder, which is inclined, and the screenings pass to one end and are carried upward by means of a bucket lift to a room where they are dried in a centrifugal drier.

The material screened from sewage are sometimes prest or dried and burned under a boiler, sometimes treated in a garbage crematory and sometimes buried in land. The volume of screenings varies with the nature of the sewage and fineness of the screens.

Fine screening is attracting increasing attention. Screens may be called fine if the clear opening is less than about half an inch. Often the mesh is much finer than this. Naturally they clog rapidly and for that reason they are movable instead of stationary

Quantity of Screened Sludge and of Water in Same

City or town	Screen	Cubic yards per million gallons of sewage	Percent of water
Boston, Mass.	Iron bars 1 inch apart	0.06	....
Plainfield, N. J.	½ inch clear spaces	0.20	85
Columbus, Ohio	2 screens of 0.5 and 0.375 inch mesh	0.17 (300 lb)	....
Reading, Pa.	Wire cloth, 40 mesh per inch	0.75 to 1.10	85
Hamburg	Band screen, 0.6 inch clear openings	0.34	87
Frankfort	Wing screen, 0.4 inch clear openings	0.7	....
Strassburg	Shovel vane, 0.1 inch clear openings	1.6	89
Dresden	Reinsch-Wurl, 0.08 inch clear openings	0.97	84

and special arrangements are made for cleaning. The collected material is usually scraped off by a continuous process from a part of the screen as it slowly emerges from the sewage. There are at least six types of fine screens: (a) The Band Screen, an endless flexible band of wire or links which passes over upper and lower rollers, and which is inclined in the sewage channel; (b) the Wing Screen, consisting of meshed vanes or paddles on a horizontal axis, and which in the lower positions, are across the path of the sewage; (c) the Shovel-Vane Screens which differ from the preceding in having curved vanes; (d) the Cylindrical Screen, which revolves in an inclined position on an axis nearly parallel with the sewage flow, the sewage flowing through the cylinder, and the screen being kept clean by jets of water playing upon it from the outside, the screenings passing to one end of the cylinder and evacuated by means of a bucket lift; (e) the Drum Screen, a truncated cone of wire mesh or perforated plate, which rotates on a horizontal axis; (f) the Inclined Disk Screen, commonly known as the Reinsch-Wurl Screen, which consists of a perforated disk surmounted by a truncated cone, which moves on an inclined axis.

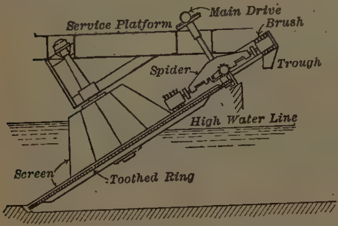

Fig. 18. General Arrangement of Reinsch-Wurl Screen

The openings in the plate are slots commonly about 2 mm wide and 30 mm long, staggered in rows, 6 mm apart, but these dimensions vary. The plate is swept by brushes as it emerges from the sewage. This process forces some of the friable solid matter through the screen, but leaves it in a finely divided state.

Sedimentation is accomplished by checking the velocity of the sewage in tanks so as to permit some of the suspended solids to settle to the bottom. The laws of sedimentation of suspended matter in water apply to sewage, but are modified because of the low specific gravity of the suspended organic matter and are further complicated by the bacterial actions that occur in the liquid and in the accumulated sludge. Four forms of sedimentation are distinguished, namely, grit chambers, plain settling basins, septic tanks, and chemical precipitation.

Grit Chambers, or detritus chambers, are small settling basins in which the sewage remains for only a brief interval, seldom more than an hour and usually not more than a few minutes, and where the velocity is commonly between 10 and 30 inches a minute. Cleaning is required at frequent intervals. Grit

chambers remove especially sand, gravel, and other heavy mineral matters which would otherwise be troublesome in subsequent operations.

Plain Settling Basins are large enough to retain the flow of sewage from 1 to 12 hours. Sludge is removed at frequent intervals in order to prevent too much bacterial action. The velocity of flow is commonly from 0.1 to 0.5 inch per minute. Bacterial action is of minor importance.

Dortmund Tanks (Fig 19) are deep settling tanks with conical bottoms, in which the sewage enters at the bottom and flows out at the top, and in which the sludge accumulates at the bottom and is forced out thru a pipe leading from the bottom and discharging below the elevation of the surface thru the pressure of the liquid in the tank. They may be used for any of the sedimentation processes, and are the cheapest tanks to operate because of the ease with which the sludge may be removed from them.

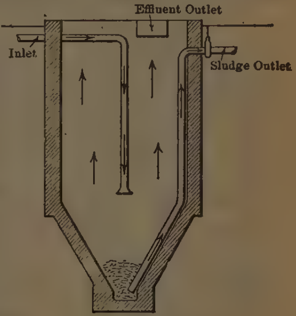

Fig. 19. Dortmund Tank

Removal of Suspended Matter by Sedimentation depends upon the period of retention, the velocity of the liquid in the tank, the presence of baffles, the amount and character of the suspended matter, the frequency of cleaning, temperature, and bacterial action, and varies from 10 to 25 % in grit chambers up to 50 to 85 % in septic tanks. The figures in the following table show the approximate percentages of removal for weak, medium, and strong sewage, corresponding to various periods of retention in well-designed tanks.

Percentage of Suspended Matter Removed by Sedimentation

Period, hours	Weak sewage	Medium sewage	Strong sewage	Remarks
1	10	15	25	Grit, or detritus, tanks
6	25	40	60	Plain sedimentation
12	30	50	75	Plain or septic sedimentation
24	40	65	80	Septic sedimentation
48	50	75	85	Septic sedimentation

Trade Wastes in sewage modify the effect of sedimentation. The presence of iron tends to act as a coagulant, hastening sedimentation, while soap tends to retard it on account of its colloidal character.

Removal of Suspended Matter by Sedimentation

Place	Process	Period of sedimentation in hours	Suspended matter in parts per million			Remarks
			Influent	Effluent	Percentage removed	
Columbus, Ohio	Grit chamber	0.3	209	163	22	Experiment Sta.
Columbus, Ohio	Grit chamber	1.5	209	138	34	Experiment Sta.
Columbus, Ohio	Plain settling	6.0	209	63	63	Experiment Sta.
London, Eng....	Plain settling	6.0	281	125	55	....
Columbus, Ohio	Plain settling	8.0	209	71	66	Experiment Sta.

35. Septic Tanks and Chemical Precipitation

Septic Tanks are settling tanks large enough to retain the flow of sewage for from 8 to 24 hours or longer, the sludge being allowed to remain for a long period in order to give opportunity for bacterial action to occur. The velocity of flow varies from 0.1 to 0.3 or more, inches per minute.

Hydrolytic Tanks are modified septic tanks in which the sludge is separated from the liquid and enters a compartment where septic action takes place, the liquid effluent being only partially septic.

Action of Septic Tanks. The object of septic tanks is to retain the sewage and give time for bacterial action, so that the oxygen in the sewage will be used up, permitting the growth of anaerobic bacteria that tend to act upon solid organic matter and liquefy or gasify it, and thus reduce the amount of sludge in the tank. The process is spoken of as "digestion of the organic matter." It is usually accompanied by the presence of a scum on the surface and by a continual rising and falling of sludge masses thru the liquid. The amount of solid organic matter digested varies from 10 to 40 %, being greatest in strong, domestic sewage and least in the weak sewage containing trade wastes. The following figures illustrate the amount of digestion of solid organic matter in certain typical sewage works in England: Birmingham, 10 %; Manchester, 25 %; Exeter, 25 %; Sheffield, 33 % The suspended matter in sewage consists approximately of one-third mineral matter not alterable in the septic tank, one-third organic matter not alterable and one-third organic matter digested and destroyed or changed in character. Long-continued septic action interferes with sedimentation in the septic tank. For example, in Huddersfield, England, the septic effluent at the beginning of a run contained 66 parts per million of suspended matter and 233 parts after eleven months operation.

Covers for Septic Tanks are of no material advantage so far as the action of the tank is concerned, but they serve to make them more sightly and less of a nuisance. Care must be taken to avoid explosions under covers due to inflammable gases liberated by the septic action. The amount of gas is from 1 to 8 times the volume of sewage.

Removal of Suspended Matter in Septic Tanks

Place	Period of sedimentation, hours	Suspended matter in parts per million			Remarks
		Influent	Effluent	Percentage removed	
Columbus, Ohio	8.0	209	82	61	Experiment Station
Plainfield, N. J.	10.0	118	54	54	
Birmingham, Eng.	10.0	484	138	72	
Exeter, Eng.	11.5	372	125	66	Experiment Station
Boston, Mass.	12.0	135	81	40	" "
Leeds, Eng.	12.0	272	131	52	
Columbus, Ohio	13.0	304	101	67	
Reading, Pa	15.0	165	42	75	
Manchester, Eng.	15.0	350	108	69	
Columbus, Ohio	16.0	209	71	66	Experiment Station
Columbus, Ohio	24.0	209	69	67	" "
Leeds, Eng.	24.0	162	47	71	" "
York, Eng.	26.0	212	53	81	
Guildford, Eng.	36.7	421	159	62	
Accrington, Eng.	42.0	389	194	50	
Leeds, Eng.	48.0	155	42	73	

The Effluent from Septic Tanks is not more easily purified by subsequent processes than that of plain settling tanks. The odor from the effluent in sprinkling filters or other purifying devices may be somewhat greater. The SLUDGE is drier and for that reason less in amount, and more easily and cheaply handled, and has less smell than sludge from either plain sedimentation or from chemical precipitation. The improved condition of the sludge is the chief reason for the use of septic tanks.

Two-story Tanks, also called Imhoff or Emscher tanks, are tanks in which the septic action is confined chiefly to the sludge which settles into a lower sludge chamber from an upper sedimentation chamber through which the sewage flows. The two chambers are separated by inclined baffles so arranged that the sediment may pass downwards altho the gases resulting from the septic action may not ascend to the upper chamber. Separate vents are provided for these gases. By the use of tanks of this type the sludge becomes better digested, less offensive and more compact, while the sewage flowing thru the upper tank does not become charged with the products of decomposition from the sludge.

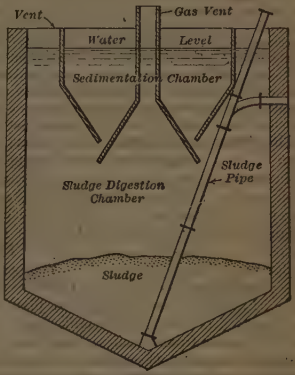

Fig. 20. Typical Cross-section of Imhoff Tank

Two-story tanks are usually deep. They may be rectangular or circular. The flow of sewage may be transverse or radial, i.e., from the center outward. Several tanks may be used in series. Provision is made for drawing off the ripened sludge thru a sludge pipe without interfering with the flow of sewage. The sludge contains from 87 to 93% of water. If well digested it is black, and smells like coal tar. It dries readily. The gas consists chiefly of methane, carbonic acid, nitrogen and hydrogen. The quality of gas is greatly influenced by temperatures. The period of detention of the sewage varies from 1 to 3 hours; the rate of flow from 20 to 100 feet per hour. The slope of the baffle is from 1.2 to 1.5 to 1. The slot is from 6 to 12 inches wide.

Chemical Precipitation is sedimentation hastened and increased by the use of chemicals. Lime, copperas (ferrous sulfate), and alum (aluminum sulfate) are most commonly used and are applied either singly or in combination. The active coagulants are aluminum hydroxide and ferrous hydroxide, formed by the reactions of these chemicals. Frequently the sewage itself contains all the iron necessary and lime only has to be added. The percentage of suspended matter removed by chemical precipitation usually varies from 50 to 75% and in some instances reaches 85%.

Acid sewages require lime. Sulphuric acid is used when the sewage is rich in fats, as, for example, in cities where wool wastes are prominent. Alumino-ferric and ferrozone are crude forms of sulfate of alumina commonly used in Europe. In England, where chemical precipitation is most common, the amount of alumino-ferric applied to domestic sewages varies from 500 to 1500 pounds per million gallons, according to the

strength of the sewage. Somewhat smaller amounts are sufficient if lime is used with the alumino-ferric.

The sewage of London receives about 500 pounds of lime and 120 pounds of copperas per million gallons; Glasgow, 600 pounds of lime and 1000 pounds of copperas; Worcester, Mass., about 1000 pounds of lime and no copperas; Providence. R. I., about 600 pounds of lime and little or no copperas. Sewage at both Worcester and Providence carries an excess of copperas put in by pickling works of wire mills.

In chemical precipitation tanks the flow is usually continuous, the capacity of the tanks being equal to 6 to 12 hours dry-weather flow of sewage. Intermittent sedimentation is sometimes used, the period of quiescence being about two hours.

The amount of dry sludge produced by this process commonly varies from 1 to 3 tons per million gallons of sewage. In Worcester, Mass., in 1905 it was 1.65 tons. In Providence, R. I., in 1903, it was 1.07 tons. In London in 1904 it was 2.30 tons. The percentage of water in the sludge commonly varies from 90 to 95%. In London and Glasgow the volume of wet sludge varies from 40 to 50 cu yds per mill galls of sewage.

Chemical precipitation is sometimes the only method of purification used, as in London, Glasgow, and Dublin. In cities where the sewage contains large amounts of trade wastes chemical precipitation is coming to be used as a preparatory process, the effluent from the chemical tanks being submitted to further purification.

The Amount of Sludge that collects in grit chambers in practise varies from 0.1 to 1.0 cubic yard of wet sludge per million gallons of sewage, in plain settling tanks from 1 to 4 cubic yards, and in septic tanks from 1 to 2 cubic yards. In chemical precipitation tanks the volumes of sludge are much larger, being often 20 to 25 cubic yards per million gallons of sewage. The weight of sludge per cubic yard is from 1750 to 1850 pounds, corresponding to specific gravities of 1.035 to 1.095. The percent of organic matter in dry sludge varies from 25 % to 50 %. The percent of water varies with the period of sedimentation and the time that the sludge is allowed to remain in the tank. Freshly deposited sludge in plain settling tanks commonly contains 90 % to 95 % of water, septic tank sludge, after several months storage, 80 % to 85 %, sludge from chemical precipitation works, 90 % to 92 %.

Sludge Disposal. The methods of sludge disposal may be classified as (1) Preliminary treatment: pressing, lagooning, air-drying, mixing with house refuse, centrifugal drying, and recovery of fats. (2) Final disposal: use as manure, depositing at sea, shallow burial in the ground, spreading on land, and burning.

Sludge Pressing is sometimes done in filter presses and the dry cake used for filling low ground or for fertilizer. The presses are operated by hydraulic power. To facilitate pressing lime is added. Sludge pressing is best adapted to the chemical precipitation process. Septic sludge is more difficult to press. The following data from Worcester, Mass., and Providence, R. I., are illustrative of sludge pressing in connection with the process of chemical precipitation.

	Worcester (1905)	Providence (1904)
Wet sludge prest, cu yd per mil gal of sewage	22.1	19.5
Prest cake, cu yd per mil gal of sewage	5.9	....
Prest cake, tons per mil gal of sewage	5.3	5.08
Percent of water in prest cake	67.6	73.5
Dry solids in prest cake, lb per mil gal of sewage	3420	2700
Lime used for precipitation, lb per mil gal of sewage	999	726
Lime used for pressing, lb per mil gal of sewage	265	137
Cost of pressing, hauling and dumping sludge, per mil gal of sewage	$6.33 *	$3.43 †

* Spread on land. † Dumped at sea.

The manurial value of sewage sludge is just about equal, pound by pound, to that of animal manure.

36. Intermittent Sand Filtration

In this method sewage is applied intermittently to prepared beds of sand at such a rate that it quickly soaks away, leaving the bed bare for a number of hours or days between doses in order to facilitate aeration and give opportunity for oxidation of organic matter. The sand or gravel constituting the filtering material holds a considerable amount of water in its pores by capillarity when thoroly drained. The amount of sewage furnished at one application is less than the amount so held by capillarity. When the dose is applied it increases the amount of water in the filtering material temporarily, but the water previously held commences to flow out and the flow continues until as much old water has been displaced as new water has been added. The sewage is held by capillarity in the filtering material in contact with air. This air must change often, and the application of the doses tends to change it. Many organisms establish themselves on the grains of the filtering material, and these organisms facilitate the oxidation of the organic matters by the air and the change of nitrogenous organic matters to nitrates, called nitrification. This process results in destruction of the greater part of the organic matters. A regular circulation of air in the pores of the filter is essential to success, and is on the whole the most important matter to be secured.

An inadequate supply of air in the pores of the filter bed results in an imperfectly purified effluent. One of the first signs is the presence of ferrous iron which oxidizes and separates on exposure to the air. This condition is at once detected by an experienced operator, and beds showing it should always be rested or treated.

Some organic matters are especially stable and not readily oxidized. These accumulate in the filtering material, especially near the surface. This accumulation increases the amount of water held by capillarity, and makes circulation of air thru the surface layer more difficult. For this reason the amount of sewage that can be applied after a few years' use is less than with new material. If the amount of sewage that is applied thru a term of years does not exceed the amount which can be handled continuously, then these more stable organic matters oxidize slowly, and after a while an equilibrium is reached beyond which there is but little if any further accumulation.

Two Rates of Filtration may be used in connection with a given filtering material: (1) A higher rate which can be used successfully during a year or two, until the surface layer has become partially clogged. (2) A lower rate which may be used thru a long term of years without the continued accumulation of objectionable quantities of organic matter. Generally the second or lower rate will control. Sometimes it is better business to scrape off the dirty sand and replace it with new sand for the sake of using the higher rate. Intermittent filtration is extensively used in New England, where favorable material is frequently found in place, requiring only grading and underdraining.

The Rate of Application of sewage to the beds is usually from 50 000 to 150 000 gallons per acre daily, the population served per acre being from 300 to 1200. The sewage is usually applied with no preliminary treatment except the use of screens and grit chambers, but experiments have shown that with sedimentation higher rates of application may be used and the sewage of 1500 to 2000 persons per acre successfully handled. The filter area is divided into beds by means of earth embankments in which are placed the distributing pipes. Underdrains are used where required. In fine material they are sometimes laid 20 to 30 feet apart, in coarse material as much as 100 feet apart. Sand depth is from 3 to 6 or 8 feet. The effective size of the material used for intermittent sand filters varies from 0.15 to 0.75 mm, but the most favorable size is from 0.30 to 0.50. The nitrogen applied should

not exceed 1 to 2 grams per day for each square yard of filter area. Coarse sands can receive higher nitrogen loads than finer sands.

The sewage is sometimes applied at one point in the bed and allowed to flow naturally over the surface. Sometimes it is distributed by means of wooden troughs or by channels and ditches controlled by hand gates worked by an attendant. The doses are applied at intervals of one-half day to three or four days, the beds meanwhile lying idle. Fine materials do best with large doses at long intervals; coarse materials require small doses at short intervals. Crops are commonly grown. Corn does particularly well, and does not interfere with purification. After repeated application the beds become clogged, requiring raking, harrowing, plowing, and finally a removal of the surface scum. Just before winter the beds are plowed into ridges so that the ice may form above them and permit the distribution of sewage during cold weather in the channels beneath the ice.

The Efficiency of intermittent sand filters is generally higher than that of any other process. Well-operated plants are capable of removing from 95 % to 98 % of the suspended matter, 90 % to 95 % of organic matter, and 95 % to 98 % of bacteria. In practise, however, the efficiency is often less than these figures, as shown by the following table, taken from analyses made by the Massachusetts State Board of Health.

Results of Sewage Purification by Intermittent Filtration in Massachusetts Cities

City or town	Period in years	Area in acres	Number of connected persons per acre	Sewage per acre daily in thousand gallons	Oxygen consumed, parts per million			Suspended matter, parts per million. Raw sewage
					Raw sewage	Filter effluent	Percent removed	
Andover	4	3.8	999	34	87	7	90	333
Brockton	7	21.5	1160	41	43	2	90	139
Clinton	4	23.5	426	33	121	12	91	255
Framingham	5	19.9	376	33	168	3	95	490
Gardner	12	2.7	1390	121	48	8	84	153
Hopedale	3	.....	....	65	29	6	79	55
Leicester	7	2.4	870	83	130	27	78	101
Marlborough	12	11.9	890	98	50	48	90	195
Natick	7	11.1	360	51	24	3	87	54
Pittsfield	1	24.8	690	67	79	4	95	487
Southbridge	4	7.3	303	48	36	3	91	131
Stockbridge	4	3.6	223	21	17	4	79	63
Westborough	4	4.0	770	71	39	7	82	266

Intermittent Sand Filtration of Sewage in Massachusetts

	Minimum	Maximum	Approximate average
Percent of population connected to sewers	14	80	50
Population per sewer connection	5	13	8
Population per mile of sewers	160	934	445
Population per acre of sand filter, basis of total population	580	2 570	1 090
Population per acre of sand filter, basis of sewered population	222	1 390	540
Rate in gallons of sewage per acre daily	20 800	120 800	63 400
Cost of land per acre	$23	$279	$100
Cost of filters per acre (approximate),			
conditions very favorable	$ 500	$1 000	
conditions favorable	1 000	2 500	

	Minimum	Maximum	Approximate average
Cost of filters per acre (approximate)			
conditions unfavorable	\$2 500	\$4 000	
conditions very unfavorable	4 000	7 000	
all conditions (actual)			\$2 715
per 1000 persons connected	1 110	10 300	3 785
Cost of disposal works per 1000 persons connected, gravity plants	3 452	12 150	5 790
Cost of disposal works per 1000 persons connected, pumping plants	4 070	40 700	17 000

The above data relate to 16 sewage-disposal plants in Massachusetts and are taken from the Annual Report of the State Board of Health for 1903.

37. Disposal by Irrigation

Broad Irrigation consists in the application of sewage to land so that it may serve as food for crops, the liquid being spread over the surface by means of ditches and other channels as in the ordinary irrigation of land. The sewage of Berlin, Germany, and of Paris, France, is purified by broad irrigation. The land used in both cases is sandy and is thoroly underdrained and the process differs from intermittent filtration only in degree, the rate of application being lower and more attention being paid to the crops. The purification obtained from a bacterial standpoint is most complete, but the effluents sometimes carry dissolved iron due to an inadequate supply of oxygen in some part of the filtering material, and growths of various organisms take place in the effluent channels. In 1903 the following rates were used:

	Paris	Berlin
Area actually under irrigation in acres	13 100	17 500
Average daily quantity of sewage applied per acre, gallons	12 300	3 530
Number of persons served by each acre	207	112
Gallons of sewage per capita daily	59	32

The sewage farms are attractive, and several villages stand upon them. There is some odor at times, but the health of those living in the neighborhood and working on the farms is not injuriously affected. Crops raised on the Berlin farms more than pay the expenses of operation, not including the cost of pumping the sewage, but are not enough to pay interest on the investment. In arid or semi-arid climates broad irrigation may become profitable.

Irrigation has often been used with clayey or other impervious soils, especially in England. The amount of water that can be absorbed and filtered is much less and the area required correspondingly greater. Clayey soil will not absorb sewage in wet weather. In many works sewage is allowed to flow thru basins or simply over the nearly level surface of fields and flow away without having entered the soil at all at wet times or even at all times. Suspended matters may be removed, and where the sewage flows in a thin layer thru a field of growing grass considerable purification may result. The suspended matters are retained on the surface of the soil and are ultimately absorbed or plowed in.

Broad irrigation is especially adapted to treating sewage where purification is only required in dry, hot weather. At such times almost any land is benefited by a moderate application of sewage, and this is frequently the time when sewage purification is required to prevent local nuisance. With many streams it may be the only time that sewage treatment is needed. Considered in this way broad irrigation may be more attractive than when the purification of sewage thruout the year, wet and dry, hot and cold, is required.

Sub-surface Irrigation. The sewage is discharged thru 3 or 4 inch tile pipes laid in the ground 10 to 18 inches deep, in rows 2.5 or 3 ft apart. In

sandy soils this method is extremely useful for small installations, as all nuisances are avoided, but it cannot be used with compact soils on account of clogging. Under favorable conditions the sewage of 150 to 250 people can be applied to one acre provided that it is first submitted to sedimentation in a septic tank or cesspool. The rate of application is commonly 1 to 2 gal per linear ft, or from 20 000 to 30 000 gal per acre daily.

An automatic siphon is used so that the sewage may be applied rapidly for a short interval, after which there must be a considerable period of rest before another dose. The lines of pipe must further be divided into two or three sections to be used in rotation. A careful application of these principles is essential to success, as a slow, continuous flow of sewage cannot be successfully treated.

38. Contact Beds

Contact Filters consist of water-tight compartments filled with porous material used for the purification of sewage in the following manner: the bed is slowly filled with sewage and allowed to remain full for a certain period, after which it is slowly emptied and allowed to remain empty for a longer period of rest. The applied sewage is usually first subjected to a preparatory treatment, either plain or septic sedimentation. The number of times a day that the bed is filled and emptied depends upon the volume and character of the sewage, the nature of the material in the tank, and other factors. In England it is common to employ three cycles a day, allowing one hour for filling, two hours for contact, one hour for emptying, and four hours for rest. The principal factor involved in this method of purification is sedimentation, but chemical and biological processes are also at work. During the period of contact the suspended matter of the sewage tends to settle on and adhere to the exposed surfaces of the material filling the bed. Some of this washes out when the bed is emptied, but much of it remains behind. While the bed is standing full anaerobic conditions exist and septic action occurs. There is also an absorption of organic matter by the accumulated film. When the bed is being emptied air is drawn into it and in consequence of this there is a partial oxidation of the organic matter adhering to the material in the bed. While the bed is resting aerobic conditions prevail. The result is a partial removal of suspended matter and a slight amount of oxidation of the organic matter, the degree of purification depending upon the character of the sewage and the opportunities permitted for sedimentation and oxidation.

Single Contact Beds are sometimes used, but more often there are two sets of beds, the second receiving the effluent from the first. These are termed double contact beds. Occasionally triple contact is used. As between single contact and double contact, the total area being the same, there is comparatively little to choose, but it is generally believed that it is more economical and efficient to employ double contact.

Materials for Contact Beds are broken stone, clinker, coke, slag, and broken brick. Broken stone has the advantage of permanence, but coke, clinker, and slag possess slight advantages from the standpoint of sedimentation, on account of their rough surfaces. The material used for primary beds is usually coarse, the particles often exceeding two or three inches in diameter. For the secondary beds the material is finer, a common size being from ½ to 1 inch. For the tertiary beds, and sometimes for the secondary beds, material finer than ¼ inch is used. The depth of the material in the contact beds varies from 2½ ft to 6 ft, and usually there is 6 inches of very coarse material at the bottom to serve as drainage. With the depth between these limits

the efficiency of the bed depends more upon the volume of material than upon its depth. The choice of size of material and depth depends, to a large extent, upon the allowed rate of application and the amount of suspended matter in the applied sewage and the admission of air. Beds of fine material are made shallower than those of coarse material in order to provide for air circulation. For sewages containing large amounts of suspended matter coarse material is used and the rate of application reduced. For well-settled or dilute sewages, in which the amount of suspended matter is small, finer material and higher rates may be used.

Area. In order to obtain a sewage effluent that will not putrefy on standing, the amount of nitrogen applied should not exceed 3.5 grams per sq yd for each vertical foot in depth. For ordinary domestic sewage, therefore, contact beds should have about 1 acre foot for each 1000 population connected with the sewers; that is, the population should not exceed 5000 per acre for a bed 5 feet deep, while the rates should be between 300 000 gallons and 800 000 gallons per acre daily.

Permanency. The air space in a newly constructed contact bed amounts to from 40 % to 50 %. With continued use the deposition of suspended matter and other causes reduce this pore space to from 20 to 30 percent and sometimes to even less. The factors that tend to reduce the average capacity of contact bed are: (1) Disintegration of the material; (2) Consolidation of the material; (3) Deposition of suspended matter; (4) Growth of organism; (5) Insufficient rest; (6) Insufficient drainage, etc. Fats, colloidal matter, and fibrous suspended matter tend to clog contact beds.

After periods variously estimated at from one to six or eight years primary contact beds must have their material removed and washt. Secondary beds may run from four to ten or twelve years without washing, but ultimately they too become clogged. In estimating on the cost of this process of purification the item of washing and removing the material is an important one. In 1906 data obtained in England showed that the cost of this varied from 25 to 75 cents per cubic yard.

Rates for Double Contact Beds

From 5th Report of Royal Commission on Sewage Disposal.

Suspended matter, parts per million	Gallons per acre daily for each foot of depth			Frequency of washing, in years	
	Material larger than 3 inches	Material between ½ and 1 inch	Material smaller than ¼ inch	Primary beds	Secondary beds
300 to 500	525 000	....	....	1½ to 2	4 to 6
200 to 300	525 000 to 700 000	....	....	1½ to 2	4 to 6
100 to 150	....	875 000	....	3 to 5	7 to 8
40 to 70	....	1 400 000	....	4 to 5	7 to 8
10 to 40	....	1 750 000 to 2 800 000	1 750 000 to 2 800 000	6 to 8	12 to 15

Automatic Control. An important detail of contact beds is usually the arrangement for automatically diverting the flow of sewage from one bed to another and of emptying a bed after it has remained full for its required period and filling it again at the proper time. In large works these operations are controlled by hand, but in smaller works they are controlled automatically by siphons, of which there are many patterns. In cold climates these have to be protected from freezing, and are usually located in small buildings.

Efficiency. Contact beds properly operated, and receiving sewage after plain or septic sedimentation, should remove from 85 to 90 percent of suspended matter, from 65 to 70 percent of organic matter, and from 80 to 85 percent of bacteria.

39. Sprinkling Filters

Sprinkling filters consist of beds, with or without side walls, filled with porous material and thoroly underdrained with tiles or channels in the concrete floor, the whole so arranged as to provide thoro aeration of both drains and filtering material. The sewage is applied at the surface, by means of fixt or movable sprinklers, in such a manner that it becomes well aerated

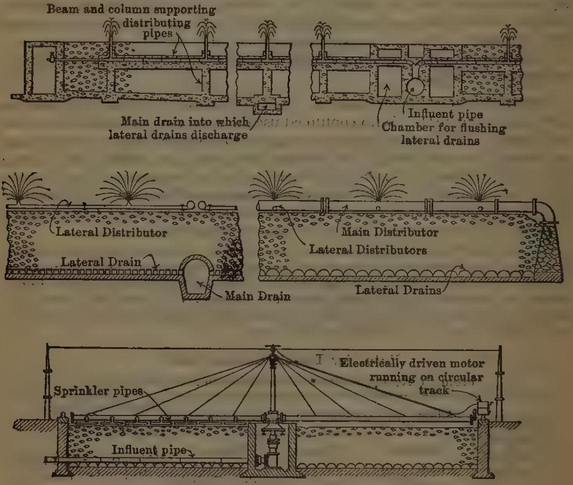

Fig. 21. Three Types of Sprinkling Filters

before it enters the bed. It percolates downward thru the material and flows out of the underdrains. The fundamental idea of sprinkling filters is the greatest possible exposure of the sewage to the air with the oxidation of organic matters and nitrification, and the construction of the filter in such a way that no deposit of suspended matter in it is possible. If there is a deposit of suspended matter in the filter, the filter will become clogged, the air supply shut off, and the process interrupted. The design should therefore permit flushing and cleaning of the lower parts. In the long run the effluent must contain as much suspended matter as the applied sewage. This suspended matter is removed by a final sedimentation or filtration. Three types of sprinkling filters are shown in Fig. 21.

The Material used in sprinkling filters is crusht stone, coke, clinker, slag, etc. In size it varies from particles ¼ inch diameter to particles larger than 3 in. Material from 1 to 2 in is perhaps the most common. Coarse material is used for deep beds where

the applied sewage contains large quantities of suspended matter. Finer material is used in shallower beds for the treatment of sewages that contain comparatively little suspended matter. Beds of coarse material are operated continuously. When fine material is used an intermittent application is preferred in order to provide the necessary aeration. The depth varies from 5 to 12 ft, but depths of from 6 to 8 ft are most common.

Rates for Percolating Filters in Gallons per Acre Daily for Each Foot in Depth

From 5th Report of Royal Commission on Sewage Disposal.

Suspended matter, parts per million	Coarse material larger than 3 in	Medium material between ½ and 1 in	Fine material smaller than ¼ in
300	875 000	437 500	....
100 to 200	1 750 000	....	....
100 to 150	....	875 000	....
40 to 70	1 750 000 to 2 625 000	1 312 500 to 1 750 000	1 312 500 to 1 750 000
10 to 40	2 625 000 to 3 500 000	2 625 000 to 3 500 000	2 625 000 to 3 500 000

The settled sewage is applied to sprinkling filters by means of sprinklers, rarely by trough distribution. Sprinklers may be classified as follows: (1) Traveling, either rotary operated by discharging sewage or by power, or rectilinear, operated by power. (2) Fixt, either upward discharge or downward discharge, the flow for each being constant or intermittent.

Fixt Sprinklers are best adapted to beds of coarse material where the amount of suspended matter in the sewage is large and where the application is constant. Traveling sprinklers are most useful where an intermittent application of sewage is desirable; that is, where the material in the filters is fine and the amount of suspended matter in the sewage small. Rotary sprinklers, used with circular beds, consist of two horizontal arms of pipe perforated on one side, attached to a central vertical shaft. The sewage discharging horizontally in opposite directions in the two arms causes the apparatus to rotate. With this apparatus the sewage is applied uniformly over the bed in intermittent doses, the frequency of which is equal to the speed of rotation of the arms. In beds of large diameter, the rotating arms are sometimes supported by wheels running on a circular track around the edge of the bed and operated by electrical power. In the rectilinear travelers the perforated discharge pipe moves back and forth over the bed, being supported at either end by wheels running on tracks, and being operated by electric power.

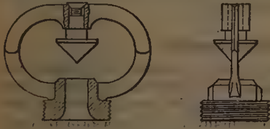

Fig. 22. Columbus Nozzle

When fixt sprinkler nozzles are used the sewage is distributed thru pipes laid either above the filter bed, as in Birmingham, England, or near the bottom of the bed, as in Columbus, Ohio. In the former the sprinkler nozzles are screwed into the distribution pipes; in the latter they are placed at the top of vertical risers that extend up thru the bed from the distributing pipes. Various types of sprinkler nozzles are in use; the types shown in Figs. 22 and 23 are the Columbus nozzle and the Taylor nozzle. They are both intended to distribute the sewage in the form of a circular jet, altho certain forms of the Taylor nozzle are designed to produce a square or hexagonal distribution.

To have the sewage applied uniformly over the surface of the bed it is necessary to vary the head on the nozzle, which is done automatically by means of a siphon which gives an intermittent flow from the nozzle, or by a controlling valve on the discharge pipe in which case the flow may be continuous. The sprinklers are spaced regularly over the beds, either in regular rows or staggered. The distance apart of the sprinklers varies from 10 to 15 feet. The head utilized is from 3 to 6 feet.

In nozzles with downward discharge the sewage falls from an orifice in the bottom of the distributing pipe and drops on a sort of pan that causes it to fly off on all sides as spray. All kinds of sprinkler nozzles give more or less trouble from clogging.

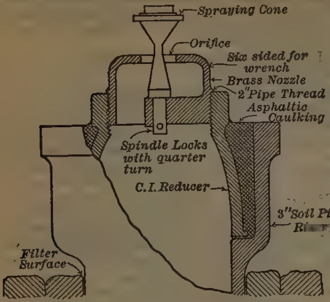

Fig. 23. Taylor Nozzle and Riser

Underdrains. Sprinkling filters require to be thoroly underdrained in order to provide ready ingress and egress of air. The underdrainage system sometime consists of half tile pipes laid in rows side by side and sometimes of circular channels covered with flat tiles in a concrete bottom. Provision should be made for flushing out or otherwise cleaning the underdrains. In large installations spaces are left between beds in order to permit cleaning of the underdrains which are likely to become clogged thru fungi.

Area. The amount of sewage applied to sprinkling filters should be such that the nitrogen should not exceed six grams per square yard per day for each vertical foot in depth. For sewage, therefore, sprinkling filters should have about 0.5 acre foot for each thousand population connected with the sewer; that is, for sprinkling filters 5 feet deep the population should not exceed 10 000 per acre, with the rate of filtration from 0.5 to 1.3 million gallons per acre daily.

Efficiency of Sprinkling Filters. Well-operated sprinkling filters, receiving sewage after plain or septic sedimentation, should remove from 85 to 90 percent of the suspended matter, 65 to 70 percent of organic matter, and 90 to 95 percent of bacteria, and yield an effluent that is non-putrescible. If the filters are overdosed or the sewage not properly applied, the efficiency may be much less than the figures given. Sprinkling filters operate best where the climate is warm, altho experience has shown that they may be operated under winter conditions with some loss of efficiency.

40. Activated Sludge Method

This is a method of treating sewage in tanks by blowing in compress air at the bottom thru porous plates or perforated distributors, the object being to supply oxygen and, by air agitation, to cause the sludge to be brought into intimate contact with the sewage. Fundamentally it is a biological process. The " ripened " sludge, sweeping thru the water, accumulates bacteria, organic matter and colloidal substances to such an extent that subsequent sedimentation results in a rapid precipitation of the suspended matter leaving a clear supernatant liquid. The process may be conducted on the fill and draw principle or in continuous flow tanks. In the former case duplicate tanks are necessary; in the latter case the sewage leaving the air tanks carries out some sludge and must be past thru settling tanks, the sludge, or part of it, being returned to the air tanks. The process is still (1919) in the experimental stage.

The air pressure used is about 0.5 lb per square inch in excess of the pressure of the water over the air outlets. The tank depth varies from 8 to 15 feet. The quantity of air varies from 1.0 to 2.5 cubic feet per gallon of sewage. The resulting sludge contains about 98 to 99 percent of water, which makes

difficult the problem of disposal. To offset this high water content the sludge is rich in fertilizing constituents. The treatment is capable of removing 98 to 99 percent of bacteria from the sewage, the effluent being nearly clear and relatively stable. The cost of the process is not well known, as the engineering details remain to be worked out. It will probably be proven to be an expensive method of treatment, but useful under favorable conditions. Being a biological method it is subject to derangement, as such processes are influenced by extreme changes in temperature, by sudden changes in the composition of the sewage, by too much or too little agitation, by the accumulation of unaërated sludge in the tanks. At the beginning of the process a certain time is required to ripen the sludge.

41. Results of Purification

Finishing Processes. The effluent from sprinkling filters is not clear but contains large quantities of suspended matter. This suspended matter is removed by sedimentation or by filtration thru coarse material such as broken stone, coke, slag, etc. The sludge obtained in this way is more stable and less objectionable that that from plain settling tanks or septic tanks. Altho the effluents from contact beds and sprinkling filters contain fewer bacteria than the applied sewage, they are not bacteriologically pure enough to improve materially the character of a stream from the standpoint of water supply.

The Cost of sewage-purification works, exclusive of land and engineering expenses, was at Columbus, Ohio, in 1907–1909: Septic tanks with a capacity of 8 020 000 gallons cost $66 730, or $8320 per million gallons. Sprinkling filters cost $240 400 for 10 acres, or $24 000 per acre. Settling basins with a capacity of 4 000 000 gallons cost $37 920, or $9480 per million gallons.

Results of Sewage Purification at Columbus, Ohio

(Septic Tank, Sprinkling Filters, Final Settling Basins.)

M'th	Volume pump'd in million gallons per day	Flow in septic tanks. Average period, in hours	Average rate of filtration. Million gal per acre per day	Flow in final settling basin. Average period, in hours	Suspended matter. Parts per million				Final effluent		Bacteria. Millions per c.c.	
					Scr'd s'age	Septic tank effluent	Filter effluent	Settling basin effluent	Dissolved oxygen. Parts per million	Percent of days when effluent was putrescible	Scr'd s'age	Settling basin effluent
Jan.	11.8	13.0	3.9	8.0	304	105	81	50	4.6	[illegible]	2.7	0.8
Feb.	15.2	12.0	3.9	5.0	285	120	....	65	8.0	52	0.9	0.5
Mar.	......	.....	4.1	2.8	178	87	47	39	9.7	41	1.2	0.5
Apr.	12.1	16.9	3.8	5.6	209	78	59	26	6.2	41	1.6	0.7
May	13.7	14.4	4.0	3.4	200	78	118	24	7.6	30	2.2	1.0
June	13.4	10.6	4.3	......	214	49	56	.....	7.3	20	3.6	.....
July.	10.9	7.9	4.3	......	115	74	91	......	6.8	20	1.4	.....
Aug.	......	7.8	4.5	5.1	136	55	75	20	6.4	6	0.9	0.4
Ave.	12.8	11.8	4.1	4.9	208	81	75	37	7.1	35	1.8	0.6

Hygienic Efficiency of Intermittent Sand Filters, Sprinkling Filters, and Contact Beds

From Review of Experiments on the Purification of Sewage at the Lawrence Experiment Station, by H. W. Clark and S. de M. Gage.

Filter number	Years operated	Depth in feet	Material	Average rate, gallons per acre daily	Average number of bacteria per c.c. in effluent	Average percent of bacteria removed
			Intermittent sand filters			
4	21	5	Sand. eff. size = 0.04	22 800	220	99.88
2	21	5	Sand, eff. size = 0.08	35 100	490	99.97
6	21	3⅔	Sand, eff. size = 0.35	55 400	10 900	99.42
1	21	5	Sand, eff. size = 0.48	68 300	35 000	98.14
			Sprinkling filters			
135	9	10.5	Broken stone	1 511 800	101 000	93.77
158	6	5	Gravel, eff. size = 5.1	405 000	242 100	90.40
248	5	8	Broken stone	1 490 800	217 600	84.20
233	5	5	Clinker	1 018 600	305 100	78.10
			Contact beds			
103	7	5	Coke	679 200	158 300	82.10
176	8	5	Coke	532 000	408 200	76.00
251	5	2.5	Coke	614 200	616 200	34.50

Disinfection of Sewage. The best known disinfectant for sewage or sewage effluents is calcium hypochlorite, commonly called chloride of lime or bleaching powder. Hypochlorite of sodium prepared by electrolysis has substantially the same action. The best commercial grades of bleaching powders contain from 30 % to 40 % of available chlorine, cost (1910) $20 to $25 per ton. The chemical is applied in the form of a 1 % to 5 % solution. The quantity required depends upon the nature of the sewage or sewage effluent to be disinfected and upon the efficiency required. By using this chemical in the following quantities from 95% to 99% of the bacteria may be destroyed.

Quantity of Bleaching Powder (Calcium Hypochlorite) Required for Disinfection

Kind of sewage	Available chlorine in parts per million	Approximate quantity of bleaching powder, lbs per mill gals	Approximate time of contact required, hours
Septic sewage	10 to 15	250 to 375	0.5
Crude sewage	5 to 10	125 to 250	0.5 to 0.8
Poor effluents from sprinkling filters or contact beds	3 to 5	75 to 125	0.8 to 1.6
Good effluents from sprinkling filters or contact beds	1 to 3	25 to 75	1.6 to 5.0

42. Sewage Disposal for Small Installations

A cardinal principle in designing sewage disposal plants for small installations is to keep the liquids which contain soapy and fatty matters (sink wastes) apart from the water-closet wastes. The latter can often be satisfactorily disposed of in cess-pools, if not too near a well. Cess-pools are unsafe in limestone countries. Soapy wastes are best disposed of upon the surface of the ground by soakage, by filtration thru sand or by application to an improvised trickling filter made of broken stone, lathes piled cob-house fashion, or even brush.

SECTION 11

DAMS, AQUEDUCTS, CANALS, SHAFTS, TUNNELS

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* Alfred Noble and Silas H. Woodard were associate editors for the first edition. Since the death of Mr. Noble in 1914, changes and revisions have been in entire charge of Mr. Woodard.

† Matter on Panama Canal written by Henry Goldmark.

‡ Written by Frederick C. Noble.

DAMS

1. Masonry Dams

Foundations of high masonry dams should be only on rock, which should be stript and all loose or soft rock removed. The rock below if otherwise sound is made practically impervious by filling all crevices, and if seamy, by excavating a cut-off trench near the upstream toe to be filled with good masonry. It should be thoroly washt and if smooth it is usually roughened by chipping or blasting a fresh surface. Drains may be laid on the bed rock down stream from the cut-off wall and led out to the downstream toe to care for slight seepage that might otherwise produce some uplift.

Rubble Masonry. From the earliest times to the beginning of the present century rubble has been most frequently used for masonry dams. It is probably much better than the more expensive cut stone because of the better bond obtained. In the New Croton dam, one of the best examples of the rubble masonry type, the method of building was as follows. The derrick stones, limestone quarried near site, 15 to 50 cu ft in volume, were picked up by the derrick and while suspended, fins, sharp corners, and edges were sledged off, and the stones thoroly washt with a hose and then swung to place and lowered into a thick bed of mortar. Each stone was then picked up and examined for perfect bedding and at the same time spawls were packed into the mortar bed where it was thick enough. It was placed a second time and men with pinch bars worked it to a firm bed, being careful to move it only horizontally and not to rock it; sometimes the stone was removed and placed several times before the bedding was satisfactory. The vertical joints were then filled with hand rubble, consisting of spawls, well wetted and laid with soft mortar. On account of the irregular shape of the derrick stones the vertical joints averaged about 12 in in width. The cut stone facing was kept about one course above the rubble work, and shearing planes were prevented by allowing the hearting rubble to bond between the courses. The resulting proportions were: derrick stones 50%, spawls 26% and mortar 24%. In the Wachusett the proportions of the granite rubble were: derrick stones 54%, spawls 17%, and mortar 29%.

Cyclopean Masonry is rubble in which concrete is used in the place of mortar. Joints are correspondingly thicker. As portland cement became of better quality and cheaper, and skilled labor became dearer, cyclopean masonry began to replace rubble for masonry dams. In the case of several dams built between 1900 and 1910, the general method of constructing cyclopean masonry was as follows: One bucket or more of soft concrete was dumped, making a bed, say 1½ ft deep, on which large stones from 10 to 50 cu ft in volume were set by derrick as closely as their irregularities would permit. These settled into the bed, and were not usually picked up by the derrick again, as is the practise in bedding large rubble in mortar. Into the spaces between the derrick stones, averaging from 1 to 2 ft in width, soft concrete was dumped and as many spawls as possible, from ¼ to 1 cu ft in volume were forced into it. Care was always taken to prevent the formation of shearing planes by keeping a large part of the derrick stones projecting above the general level to form a bond with future work. In Ashokan dams 25% and in Kensico dam 27% of the total mass was composed of derrick stones.

Concrete Masonry. A variety of methods have been used in the construction of concrete dams. There are many advantages in the practise of building the dam in a series of detached piers, filling the intermediate spaces later. This is especially true with low dams, where each pier may be built without stopping. In the McCalls Ferry dam, built by this method,

the concrete was mixt wet, and contained stone graded from ¾ in to about 5 in, and large derrick stones were put in as plums. The mixing plant was large, and very large quantities of masonry could be placed without stopping work, thus avoiding the dangers of a weak section between two days' work. The largest day's work was 2057 cu yds in 11 hours and the largest month's work was 37 000 cu yds. Where the body of concrete is large the practise of placing large stones, called plums, in the concrete as it is being deposited has many advantages. It is usually specified that the plums shall not be over 15 to 25% of the whole volume, depending on the thickness of the walls and that they shall not touch one another. The Connellsville dam and others were built up in layers, depending on neat cement wash and plums to bond old layers with new. The San Mateo and the Pedlar River dams were built up of blocks containing 200 to 300 cu yds of concrete molded in place on the dam and so constructed as to dovetail with one another. This was also intended to reduce strains from the shrinkage of setting concrete.

Temperature Cracks. It is observed that all straight dams and most curved dams in localities having considerable range of temperature have developed cracks which open in winter and close in summer. The question of temperature cracks has received much study, but the total knowledge derived is still meager. It seems to be fairly well established that temperature cracks in large masses of masonry are widest at the surface, becoming gradually narrower with penetration; that in the latitude of New York they disappear from 10 to 20 ft depth depending on exposure, character of masonry, and whether the masonry was laid in warm or cold weather, so that if the dam is less than 20 or 30 ft thick the cracks extend thru. At the New Croton dam liquid dye was run into cracks and later the surrounding masonry was removed. All cracks were found to have the same general profile at their bottoms, as indicated in Fig. 1. In general the cracks formed a vertical plane located without regard to joints, passing, in some cases, thru headers within two or three inches of their edges. Thermophones were placed in the masonry of Boonton and Kensico dams as they were built and observations made for a period of several years in each case. In the former many instruments became unreliable so that complete laws could not be deduced; in the latter the readings for nearly all positions were maintained for a period of four years. It was observed that the temperature at any point began to rise as soon as the concrete was placed and rose various amounts depending on the temperatures of the air and foundations, the mass of the concrete, the rate of placing the masonry, the distance to the nearest exposed face, the exposure of that face, and possibly upon the brand and content of cement used and the wetness of the mix. The time required to reach the maximum temperature depended on the same factors. The time required for dissipation of the heat generated by the reactions of the cement and water depended primarily upon the distance from an exposed face, the exposute of that face and the temperature difference inside and outside the dam.

Fig. 1

The measurements at Boonton dam indicated the conclusion that a range of 130° F. in the atmosphere reduces to a range of about 70° F. 2 to 4 ft below the surface of the masonry with a gradual but slower decrease of range with greater depths, also a reduction in range of 10° F. per 1¾ ft depth.

The last measurements in Kensico dam made in 1917 when the heat of setting practically had been dissipated, indicate that with a range in mean atmospheric

temperature of 73° F. the range in temperature R in the masonry for depths in the masonry from 0 to 40 ft may be approximately exprest by the formula $R = 48 - 12.3 \log_e D$ where D = distance from nearer face; which face had a sunny exposure. Measurements were taken at elevations where thickness of masonry ranged from 45 to 90 ft and where D was always equal to or less than ½ such thickness.

Expansion Joints. Contraction joints were provided in several dams recently constructed, notably in Ashokan and Kensico dams. Fig. 2 shows the joint in Kensico dam. $ABCDE$ is the trace of vertical planes of weak bond, made so by building first one side of precast concrete blocks, coating it with soft pitch, and later pouring the adjacent block against it. At B a sheet of copper $^1/_{16}$ in thick was placed across the joint with its two edges embedded in the concrete on either side in such a way that it will be bent but not ruptured by the movements of the concrete. At Ashokan the joint is the same except that there is no copper strip. CD is an inspection well.

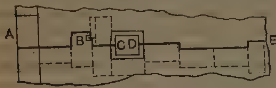


Fig. 2. Expansion Joint

Openings in Ashokan and Kensico due to temperature changes have for the most part been confined to the joints provided. Leakage in the latter has been prevented, and in the former it has been slight, not more than 300 gallons per minute at the maximum.

Waterproofing and Drainage Galleries. Water-tightness in dams is desirable because water in penetrating increases the stress and may reduce the strength of the masonry. To construct a nearly vertical wall over a hundred feet high perfectly water-tight is a matter of great difficulty. The method most commonly depended upon is to make the water face of cut stone and point its joints very carefully with rich portland cement mortar, so that if the stone is impervious the leakage may be reduced and the downstream face will be practically dry. The following notes taken from published comments give the extent of leakage in some of the dams treated in this manner. The Lake Cheesman dam under a head of 212 ft shows no sign of leakage and is entirely free from the appearance of seepage. The Sodom dam shows a few damp spots on humid days only. The Furens dam in France, under a head of 154 ft, showed only a few damp spots. The Bear Valley dam, which for the greater part of its height is less than 8 ft thick, showed only sweating under a head of about 50 ft. There is a growing practise of providing a drainage system in the masonry a short distance from the upstream face. In the Ashokan and Kensico dams there are drainage galleries 5 ft by 7.5 ft, large enough for inspection, one near the top and the other near the base of the dam, connected by inclined drainage holes 16 in in diameter. These drainage holes are formed in blocks of porous concrete 3 ft by 3 ft in plan set about 15 ft from the upstream face and 12 ft apart along the axis of the dam. The water face of the Mouche dam was given 3 coats of hot pitch and then whitewashed. The upstream face of the Urft dam was waterproofed with a plaster coat of cement 1 in thick, on which a coat of asphalt was applied; against this to hold it in place a 3-ft wall of masonry was built, backed by an earth embankment, and drainage pipes were placed in the masonry downstream from the asphalt coating. The upstream face of the Sand River dam, South Africa, was treated as follows: soap and alum washes were applied to the faces with a flat brush, the soap solution being at 212° F. and the alum at 60° to 70° F. The washes contained 1 lb soap to 1.6 gal water and 1 lb alum to 9.6 gal water. Three coats of each were used.

Essential Details of notable precedents are given in the following condensed statements, where the numbers in parentheses refer to corresponding numbers within small circles on the cross-sections in the figures. Of all structures dams are probably most governed by precedents in their design and method of construction.

Straight Masonry Dams without Overflow. (Fig. 3)

(1) Gros-Bois, France. 1830–38. 1805 ft long, 21.3 ft wide at top. Founded on soft rock. When filled slid 2 in. Reinforced by 11 buttresses 37 ft thick at bottom and 13 ft thick at top. 96 ft high; 52.5 ft wide.

(2) Lozoya, Spain. About 1850. 238 ft long; 105 ft high; 128 ft wide. Wall of cut stone backed by rubble masonry, partly covered on back face by sloping bank of gravel. Top width 21.98 ft.

(3) Habra, Algiers. 1856–71. Rubble masonry. 1082 ft long. Top width 14.1 ft. Rock poor and porous. Mortar, natural hydraulic lime and fine clayey sand. When filled leaked badly, later leakage practically ceased. Failed 1881 during flood which overtopt dam; 300 ft carried out down to base. About 400 lives lost. Repaired 1883–87. Profile was much changed and strengthened. 124.2 ft high; 89.2 ft wide.

(4) Cagliari, Sardinia. 1866. Granite, rubble, hydraulic mortar. 345 ft long. Top width 16.4 ft. Height 70.5 ft. Width 52.4 ft.

(5) Boyd's Corner. New York City Water Supply. 1866–72. Cut-stone face, concrete heart (large plums in lower half). Concrete weighed 133.25 lbs per cu ft. Length 670 ft. Top width 8.6 ft. Earth embankment upstream side 20 ft wide on top, 4 : 1 slope. 27 000 cu yds masonry. Height 78 ft. Width 57 ft.

(6) Poona, India. 1868. Uncoursed rubble founded on rock. Length 5136 ft, of which 1453 ft is waste-weir, 11 ft below top of dam. Alinement is several tangents. Reinforced by heavy buttresses at intersections. At first masonry showed signs of weakness. Reinforced by earth bank on lower face. Top width of bank 60 ft, height 30 ft. 360 000 cu yds masonry. Cost $630 000. Height of masonry dam 100 ft. Width 60.7 ft.

(7) Tlelat, Algiers. 1869. 325 ft long. Top width 13.12 ft. Height 68.9 ft. Width 40.3 ft.

(8) Djidionia, Algiers. 1873–75. Resultant said to be outside middle third. Top width 13.12 ft. Height 83.7 ft. Width 52.4 ft.

(9) Bouzey, France. 1878–81. 1700 ft long. Red-sandstone foundation pervious. When filled dam slid 1 ft. Reinforcement at toe added 1888. Failed 1895, upper 33 ft overturning for length of 558 ft. 150 lives lost. Height 84.5 ft. Width 57.3 ft.

(10) Gran-Cheurfas, Algiers. 1882–84. Rubble. 508.4 ft long. Top width 13.12 ft. Partially failed when filled 1885; immediately repaired. Height 131.2 ft. Width 134.5 ft.

(11) Lagolungo, Italy. Genoa Water Supply. 1883. Raised 10 ft 1903. Top width 8.2 ft. Height 145.9 ft. Width 140 ft.

(12) Vingeanne, France. 1885. Top width 11.48 ft. Height 113.8 ft. Width 80.1 ft.

(13) Hamiz, Algiers. 1885. Rubble. 532 ft long. Top width 16.4 ft. Height 124.6 ft. Width 91.2 ft.

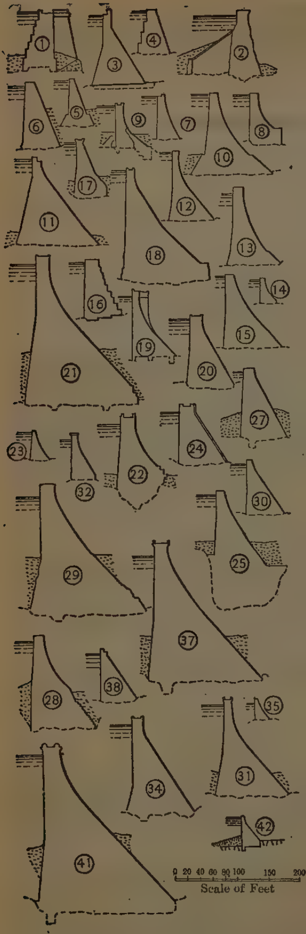
(14) Bridgeport, Conn. 1886. Gneiss rubble. Rosendale 1 : 2 mortar. 640 ft long. Top width 8.0 ft. Base length 50 ft. Height 45 ft. Width 29.5 ft.

(15) Tansa. Bombay Water Supply. 1886 91. Total length 8800 ft. 1650 ft of dam is waste-weir, 3 ft below top of dam. Uncoursed rubble masonry. Stones, hard trap or greenstone. Cement, hydraulic lime burnt from kunkur nodules. Masonry 408 520 cu yds. Reported to be water-tight. Cost $988 000 by contract. Designed strong enough to permit height being increased 17 ft. Height 118 ft. Width 99.8 ft.

(16) Tytam, China. 1887. Granite ashlar and concrete. Intended for 20 ft higher. Present crest width 21 ft. Height 95 ft. Width 63.4 ft.

(17) Sodom. New York Water Supply. 1888–93. Coursed rubble on solid rock. Faces cut stone; beds perpendicular to faces. Mortar, 1 portland cement to 2 sand, 500 ft long. Top width 12 ft. 35 887 cu yds. Cost $366 499. Max. progress 3000 cu yds per mo. No leakage except slight sweating at joints. Height 94.7 ft. Width 53 ft.

(18) Periyar, India. 1888–97. Faces uncoursed syenite rubble; heart concrete; hydraulic lime. Situated in jungle. Unskilled labor. 1231 ft long. Top width 12 ft. 185 000 cu yds masonry. Upstream face plastered with hydraulic lime and sand 1 : 1. Height 183 ft. Width 135.5 ft.



(19) Mouche, France. 1885–90. Rubble, 134.2 lbs per cu ft. 1346 ft long. Top width 11.5 ft. Given three coats hot pitch on upstream face, then whitewashed. Height 101.5 ft. Width 66.5 ft.

(20) Titicus. New York Water Supply. 1890–95. Rubble rough coursed, cut stone faces; beds perpendicular to faces. Mortar, 1 portland or natural cement to 2 sand. 534 ft long. Top width 20.7 ft. Max. progress 5700 cu yds monthly. Cost $933 065. Height 109 ft. Width 75.2 ft.

(21) New Croton. New York Water Supply. 1892–1907. Rubble faced with ashlar. Part cyclopean. Length 1200 ft. Top width 18 ft. Rock in greater part of dam 185 lbs per cu ft. 855 000 cu yds masonry. Max. progress 5700 cu yds monthly. Height 238 ft. Width 185 ft.

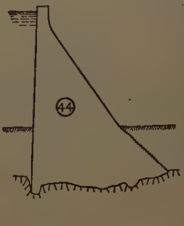
(22) Burrator, England. 1893–96. Granite blocks bedded in rich concrete. Cut stone face. Joints pointed and calked with neat cement mortar. Length 361 ft. Top width about 18 ft. Cost $495 700. Height 77 ft. Width 63 ft.

(23) Indian River, New York. 1898. Top width, 7 ft. Length 207 ft. Height 47 ft. Width 33 ft.

(24) Assuan, Egypt. 1898–1902. Granite rubble laid in 1 : 4 portland cement mortar. Length 6400 ft, 1800 ft solid, remainder has 180 sluices each 6.56 ft wide. Top width 17.8 ft. Masonry 704 000 cu yds. Cost $11 907 000. Height 95 ft. Width 80.3 ft.

(25) Coolgardie, Australia. 1900–02. Rubble concrete. Length 755 ft. Top width 10 ft. Height 119.9 ft. Width 88 ft.

(26) Blackbrook, England, 1900–05. Length 525 feet. Top width 14 ft. Height 108 ft. Width 65 ft.



(27) Boonton, New Jersey. 1900–1906. Cyclopean syenite masonry. Top width 17 ft. Length 2150 ft. Masonry 166 lbs per cu ft. 255 000 yds. Max. progress 21 000 cu yds per month. Height 103 ft. Width 77 ft.

Fig. 3. Straight Masonry Dams without Overflow

(28) Spier Falls, New York. 1900–05. Rubble. Length 552 ft. Top width 17 ft. 180 000 cu yds masonry. Height 150 ft. Width 107 ft.

(29) Wachusett, Mass. 1900–06. Granite, rubble, cut stone faces. Top width 22.5 ft. Length 971 ft. ¾ of heart is laid in natural cement. Volume 266 663 cu yds. Cost $2 270 117. Height 205 ft. Width 187 ft.

(30) La Jalpa, Mexico. 1902. Body of dam rough limestone rubble laid in mortar of native hydraulic lime and sand. Clay embankment, slope 2 : 1 on upstream face. Length 1800 ft. Top width 9.1 ft. Contents 92 000 cu yds. Cost $500 000 gold. Height 87.2 ft. Width 65.6 ft.

(31) Cataract, Australia. 1902–08. Cyclopean masonry. Sandstone blocks in cement mortar packed with concrete; blocks about 65% of mass. Faces, concrete downstream, concrete blocks upstream. Length 811 ft. Top width 16.5 ft. Rectangular conduits filled with broken stone and earthen pipes for drains. 146 242 cu yds. Height 154 ft. Width 120 ft.

(32) Pedlar River, Virginia. 1904. Concrete with large stones embedded. Length 415 ft. Width at crest 10 ft. Concrete laid as large blocks about 10 × 10 × 15 ft and dovetailed. Portland cement on face, natural in heart. Contract price $103 708. Height 73.5 ft. Width 39.2 ft.

(33) Swansea, Wales. 1905. Cyclopean masonry, brick facing. In heart concrete 1 : 2 : 5. Lower base and upper 6 ft of water face 1 cement : 2 sand : 3.4 parts of fine crusht rock. Length 1250 ft. Max. height 144 ft. Thickness at base 107 ft. Max. depth water 100 ft.

(34) Cross River, New York. 1905–07. Cyclopean masonry, concrete block facing. Length 772 ft. Top width 23 ft. 158 000 cu yds. Max. progress 18 400 cu yds per month. Contract price $1 246 212. Height 155 ft. Width 125.3 ft.

(35) Connellsville, Pa. 1906. Concrete, bowlders used as plums. Ashlar facing. Length 650 ft. Top width 6 ft. Bowlders about 25% of masonry. Height 39 ft. Width 26 ft.

(36) Sand River, South Africa. 1906. Concrete 1 : 1.3 : 5.5 with large quartzite rocks, iron rods and rails embedded. Length 398 ft. Height 55 ft. Base width 38 ft. Foundation hard shale. Contents 9000 cu yds. Cost about $140 000. Masonry 150 lb per cu ft.

(37) Ashokan. Olive Bridge dam, New York. 1908-1913. Cyclopean masonry. Length 1000 ft. Top width 26.3 ft. Facing concrete blocks. Height 220 ft. Width 190.2 ft. Contraction joints, wells and galleries for inspection and drainage, and vertical drains provided. (See Fig. 51, page 717.)

(38) Blackwater, Scotland. Cyclopean concrete. Top width 10 ft. Height 84 ft. Width 58 ft.

(39) Esperanza, Mexico. Rubble masonry. Length 580 ft. Top width 19.7 ft. Base width 75.7 ft. Max. height 137.5 ft. 55 000 cu yds. Mortar of lime somewhat hydraulic. Leakage estimated 0.4 cu ft per sec.

(40) Barker, Col. 1909. Masonry 1 : 3 : 5 concrete with plums. Expansion joints in upper 145 ft, 48 ft apart. Length 625 ft. Top width 16 ft. Height 175 ft. Max. base width 124 ft.

(41) Kensico, New York. Cyclopean masonry; concrete blocks on upstream face, cut stone on downstream face. Length 1830 ft. Top width about 27.75 ft. Base width 227.7 max. Expansion joints, vertical drains and inspection galleries. Height 250 ft.

(42) Austin, Pa. 1909. Max. height 50.5 ft. Width at base 30 ft. Top width 2.5 ft. Length 554 ft. Foundation, sandstone in layers 8 in to 3 ft thick with beds of shale and disintegrated sandstone between the layers. When filled in January, 1910, a section 90 ft long slid 18 in at bottom and 31 in at top Dam then repaired but not strengthened. In September, 1911, it failed suddenly with loss of eighty lives and $3 000 000 of property.

(43) Medina, Texas. 1911–1912. 1 : 3½ : 6½ concrete with 9.9% plums. Length 1580 ft. Top width 25 ft. Max. height 164 ft. Base width 128 ft. Total volume 300 000 cu yds.

(44) Elephant Butte, N. M. 1910–1916. Cyclopean concrete. Length 1200 ft. Top width 18 ft. Max. height 304.5 ft. Base width 212.58 ft. Volume 611 400 cu yds. Drainage gallery, vertical drains and expansion joints.

(45) Brisbane, Australia. Completed, 1916. Heart 1 : 2.5 : 6.5 concrete with 25% plums. Upper 30 ft and faces to toe and heel 1 : 2 : 4 concrete. Length 580 ft

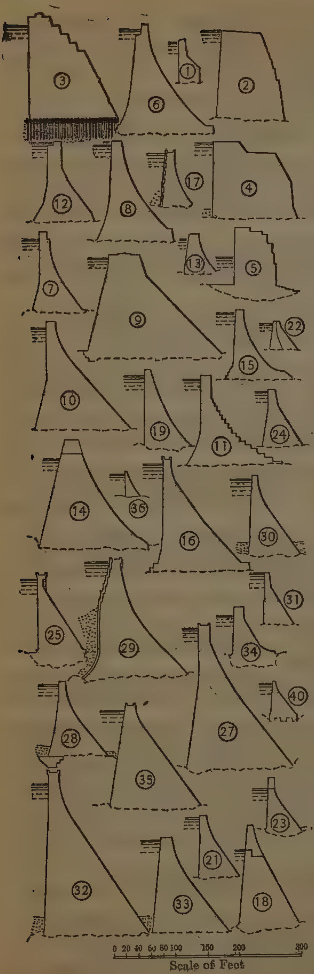


Fig. 4. Curved Masonry Dams with Gravity Sections without Overflow

Top width 10 ft. Max. height 125 ft. Volume 58 400 cu yds.

Curved Masonry Dams with Gravity Sections without Overflow. (Fig. 4)

(1) Almanza, Spain. Built before 1586 Radius 86 ft. Rubble cut stone face. Oldest existing masonry dam. Top 9.84 ft thick. Height 67.8 ft. Width 33.7 ft.

(2) Alicante, Spain. 1579–84. Rubble, cut stone facing. Radius at top 351 ft. Height 134.5 ft. Width 110.6 ft.

(3) Puentes, Spain. 1785–91. Polygonal in plan, arched upstream; 925 ft long. Rubble, cut stone facing. Mostly founded on rock; on piles over deep pocket. Held 82 ft depth for 10 years. 1802 water rose to 154 ft depth and pile foundations washt out, leaving dam bridging deep pocket. 608 lives lost. Height 164.2. Width 145.2 ft.

(4) Val de Infierno, Spain. 1785–91. Polygonal arch. Reservoir silted to top of dam. Built on rock. Height 110.3 ft. Width 136.4 ft.

(5) Nijar, Spain. 1843–50. Rubble, cut stone facing. Height, 90.5 ft. Width 67.6 ft.

(6) Furens, France. 1862–66. Rubble of mica schist with cut stone front face. Length about 330 ft. Top width 9.91 ft. Radius 828.4 ft. Built in narrow gorge. No leakage. Few damp spots. 52 300 cu yds. Cost $318 000. Height 170.6 ft. Width 161 ft.

(7) Ternay, France. 1865–8. Granite rubble. Top width 13.12 ft. Radius 1312 ft. Height 124.6 ft. Width 81.7 ft.

(8) Ban, France. 1867–70. Rubble, Length 512 ft. Top width 16.4 ft. Radius 1325 ft. Built on rock. Cost $190 000. Height 156.8 ft. Width at base 126.9 ft.

(9) Gillepe, Belgium 1870–75 Sandstone rubble. Top width 49.2 ft. Radius 1640 ft. Length top 771 ft. 325 000 cu yds. Height 156.5 ft. Width 215.89 ft.

(10) Villar, Spain. 1870–78. Rubble. Top width 14.75 ft. Length 349 ft. Radius 440 ft. Cost about $390 000. Height 170.3 ft. Width 154.6 ft.

(11) Hijar, Spain. 1880. Top width 16.4 ft. Length 236 ft. Radius 210 ft. Height 141.1 ft. Width 146.9 ft.

(12) Gorzente, Italy. Genoa Water Supply. 1880–83. Serpentine rubble with casale lime mortar. 492 ft long. Top width 22.96 ft. Height 126.3 ft. Width 99.4 ft.

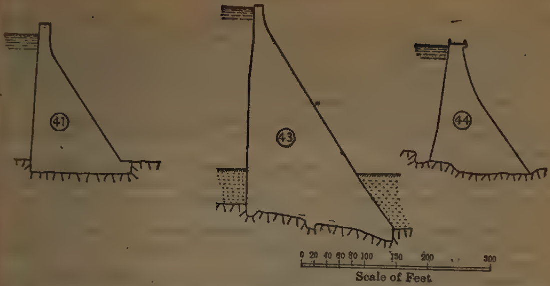


Fig. 4 (continued). Curved Masonry Dams with Gravity Sections without Overflow

(13) Thirlmere, England. 1886–93. Top width 18.5 ft. Plan is reverse curve to follow bed rock. Radius 100 ft. Height 63 ft. Width 51.7 ft.

(14) San Mateo, California. 1887–88. Concrete molded into dovetailed blocks on dam. Planned for max. height 170 ft. Stopt at 146 ft. Top width 25 ft and top length 680 ft at 170 ft height. Radius 637 ft. Volume 139 000 cu yds. A few damp spots on face only sign of leakage. No cracks visible. Concrete 1 : 2 : 6.5. Width at base 176 ft.

(15) Beetaloo, Australia. 1888–90. Concrete. Top width 14 ft. Length 580 ft. Radius 1414 ft. 60 000 cu yds masonry. Founded on rock. Cost $573 300. Height 110 ft. Width 110 ft.

(16) Chartrain or Tache, France. 1888–92. Rubble or porphyritic rock and hydraulic mortar. Wt per cu ft 150 lbs. Top width 13.12 ft. Radius 1312 ft. Upstream face coated to 33 ft below coping with artificial cement of slaked lime, 1¼ in thick, made of equal parts of cement and sand. Some leakage. Height 174.7 ft. Width 15[illegible].9 ft.

(17) Remscheid, Germany. 1889–92. Rubble Linneite slate in trass mortar. Water face plastered with cement and asphalt, covered by brick wall. Top width 13.1 ft. Radius 410 ft. 22 886 cu yds. Height 82 ft. Width 4[illegible].2 ft.

(18) Hemet, California. 1890–95. Granite rubble and concrete. Designed with max. height 160 ft; only carried to 122.5 ft. Top width 10 ft. Length 260 ft including 50 ft spillway in center. Radius 225.4 ft. 31 105 cu yds at present height. In time of flood spillway too small. Base width 100 ft.

(19) Bhatgur, India. About 1891. Uncoursed rubble. Upper portion concrete and rubble blocks. Faces coursed rubble. Top width 12 ft. Length 3257 ft. Irregular curve to follow outcrop of bed rock. Height 127 ft. Width 73.7 ft.

(20) Butte City, Montana. 1892. Concrete, granite facing. Top width 10 ft. Max. height 120 ft. Bottom width 83 ft. Length of top 350 ft. Radius 350 ft.

(21) Lauchensee, Germany. 1892–95. Cyclopean masonry. Trass mortar, 1 lime, 1 trass, 2½ sand made by crushing sandstone. Earth bank to protect from sun. Top width 13 ft. Length top 840 ft. Radius 2950 ft. Cu yds 37 400 of which 65% stone and 35% mortar. On sandstone. Total cost $243 750. Height 98 ft. Width 65.3 ft.

(22) Lennep, Germany. Old dam, 1893, length 416 ft. Radius 460 ft. Dam was increased in height from 37.7 ft to 48.4 ft and buttresses built to take added thrust. Width at base 50.8 ft.

(23) Wigwam, Conn. 1893–96. Designed for 90 ft height, built only 75 ft high. Width 12 ft at designed height and length 600 ft. 14 887 cu yds in completed portion. Radius 600 ft. Width at base 62 ft. Completed to designed height in 1903.

(24) Einsiedel, Germany. 1894. Rubble. Top width 13.12 ft. Length 590 ft. Radius 1310 ft. 31 600 cu yds. Height 93.6 ft. Width 65.5 ft.

(25) Echapre, France. 1894–98. Top width 17 ft. Length 541 ft. Radius 1148.2 ft. Hydraulic mortar coat on upstream face, on which coat of hydraulic cement was later placed. Max. depth of water 121.4 ft. Width at base 88.6 ft.

(26) Ondenon, France. 1900–04. Top width 15.4 ft. Length 420 ft. Radius 984 ft. Max. height 123 ft. Base width 94 ft.

(27) Lake Cheesman, Colorado. 1900–04. Top width 18 ft. Length 710 ft on crest,

30 ft at base. Granite rubble, coursed rough-pointed facing. Radius 400 ft. 103 000 cu yds. Total cost about $1 000 000. Height 227 ft Width 176 ft.

(28) Komotau, Austria. 1901–04 Cyclopean masonry. Gneiss in portland cement concrete. Top width 13 ft. Length 508.5 ft at top, 170 6 ft at bottom. Radius 820 ft. 53 600 cu yds. Asphaltum and tar coat held in place by concrete dovetailed into upstream face. Height 118.9 ft. Width 98.4 ft.

(29) Urft, Germany. 1901 04. Slate and trap masonry, earth embankment to 77 ft. below crest, slope 2 : 1, rock paving. Length 741 ft. Top width 18 ft. Radius 656 ft. Height 190 ft. Width 165.5 ft.

(30) Mercedes, Mexico, 1901–05. Rubble, cut stone facing. Top width 11.48 ft. Length at crest 535 ft, also 98-ft spillway; 13 ft at base. Straight for half the length, rest 196.8 ft radius 28 000 Cu yds. Cost $200 000 Mexican currency. Height 135.6 ft. Width 84.5 ft.

(31) Granite Springs Wyoming 1903–04. Uncoursed rubble. Wt per cu ft 165 lbs. Top width 10 ft. Length 410 ft on top, 10 ft at base. Radius 300 ft. 14 422 cu yds. Cost $109 194. Height 96 ft. Width 56 ft.

(32) Roosevelt, Arizona. 1905 1911. Cyclopean masonry, range rubble facing. Sandstone. Top width 16 ft. Length 680 ft. Radius 400 ft. About 340 000 cu yds. Height 260 ft. Width 158 ft.

(33) Marklissa, Germany. 1905. Gneiss rubble. Upstream face has 2 in mortar layer coated with siderosthen. Has interior drainage system. Length 427 ft. Radius 410 ft. 83 700 cu yds. Height 147.7 ft. Width 124.8 ft.

(34) Marquina, Manila, P. I. 1906. Rubble, hard crystalline limestone marble. Length 400 ft. Radius 500 ft. Height 75 ft. Width 68 7 ft.

(35) Cher, France. Top width 15.4 ft. Radius 656 ft Height 154 ft. Width 141 ft.

(36) Burraga, Australia. Cyclopean concrete in heart T rails bedded near top. Top width 2 ft. Length 285.6 ft. Radius 539.8 ft. Cost $44 500. Height 41 ft. Width 25.3 ft.

(37) San Jose, Mexico. Rubble. Top width 15 ft. Max. height 151 ft. Base width 128 ft. Length 592 ft including 2 spillways 95 ft and 86 ft. Radius 6560 ft.

(38) Murrumbidgee River, Australia. Concrete, large stones embedded. Max. height 232 ft. Length 910 ft. Radius 940.5 ft. Max. base width 160.33 ft. Crest width 18 ft.

(39) Mochne. Germany. Length 1312 ft. Max. height 98.4 ft 353 160 cu yds.

(40) Griffin, Pa. Concrete laid in 12-in courses usually from end to end during one working day. Length 284 ft. Radius 400 ft. Top width 4 ft. Spillway near center 80 ft long. Concrete 8000 cu yds. At first leaks appeared near ends. Height 62 ft. Width at base 44 ft.

(41) Barren Jack, New So. Wales. 1909–13. Cyclopean concrete. Length 784 ft. Top width 18 ft. Base width 163 ft. Height 240 ft. Radius 1200 ft.

(42) La Boquila, Chihuahua, Mexico. Cyclopean masonry with 10 ton blocks of limestone Length 840 ft. Top width 19 ft. Base width 200 ft. Radius 866 ft.

(43) Arrowrock, Boise, Idaho. 1912–13. 1 : 2.5 : 5 concrete with large rock and cobblestone. Length 1050 ft. Top width 15.5 ft. Height 351 ft. Base width 238 ft. Radius 662 ft. Inspection gallery. Contraction joints every 150 ft.

(44) Mauer, Silesia, Germany. Completed in 1912. Largest dam in Europe. Stone masonry. Length 918 ft. Top width 23.6 ft. Height 203 ft. Base width 165 ft. 332 000 cu yds. Radius 820 ft. Drainage and inspection tunnels.

Curved Masonry Dams Depending on Arch Action without Overflow. (Fig. 5)

(1) Meer-Allum, India. About 1800. Large arch made up of 21 small arches with buttresses between. Spans between buttresses 70 to 147 ft in clear. Spillway provided, but at times water flows few inches deep over entire dam. Length about 2640 ft.

(2) Zola Dam, France. 1843. Rubble masonry. Length 205 ft. Top width 19 ft. Radius at top 158 ft.

(3) Bear Valley dam, Cal. 1884. Radius 335 ft. Rough granite ashlar facing and rubble hearting. 300 ft long. Thickness 2.5 ft to 3 ft at top and 8 5 ft 48 ft below crest. Inaccessible location. Haulage of cement to site cost $10 per bbl. 3400 cu yds. In 1911, in order to store more water, a new reinforced concrete dam 30 ft higher than the old was built just below this dam, submerging it.

(4) Sweetwater, Cal. 1887–88. Rubble. Weight per cu ft. 164 lbs. Length about

380 ft. Radius 222 ft. 19 269 cu yds. Top width 12 ft. Mortar used mostly 1 portland to 3 sand, but near upstream face mortar was 1 : 2. Cost about $234 000. 1895 dam was overtopt by 22 in of water for 40 hours without damage. In 1911 the dam was increased 20 ft in height and a new section, reinforced with old rails, was built on the downstream side, 76 ft wide at the base and reducing to nothing at the elevation of the top of the old dam. Care was taken to bond the old and the new concrete.

(5) Belubula, Australia. 1898. Concrete and brick. Lower part up to 23 ft height is concrete, above this brick arches and buttresses built 36.7 ft higher. Series of buttresses

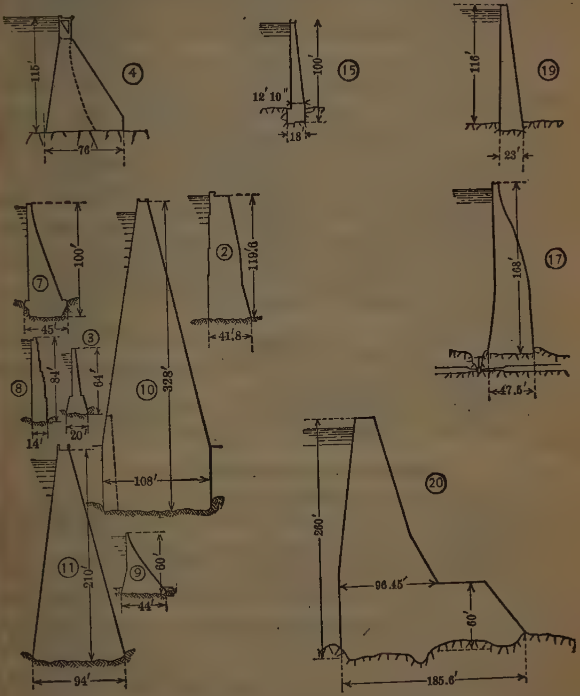


Fig. 5. Arched Masonry Dams

28 ft. c to c, elliptical arches between, axes inclined 30° to vertical. Arches 4 ft. thick at bottom, 1 ft 7 in at top. Buttresses 40 ft. long, 12 ft wide at wall and 5 ft at outer end. Each buttress forms a segment of circle 36 ft 2 in radius and diminishes in thickness from 8½ ft at center to 4 ft at outer circumference.

(6) Johannesburg, So. Africa. 1898. Rubble masonry, hard blue quartzite, concrete foundations reinforced with rails. Top width 7 ft. Length 585 ft. 30 000 cu yds. Arch and tangent; part on tangent, gravity section. Radius of arch section 275 ft at crest, decreasing to 206 ft 75 ft below to meet lines of gravity section.

(7) Barossa, South Australia 1899-1903. Concrete with plums, reinforced at top with steel rails. Length 472 ft. Radius 200 ft. Cu yds 17 975, of which 12.3% large stones. Top width 4.6 ft. Range of temperature during construction was from 30° to 168° F. Total cost $827 000.

(8) Upper Otay, Cal. 1900. Masonry reinforced with steel plates and cables. Top width 4 ft. Length 350 ft at crest, 20 ft at base. Radius 359 ft. In a rock gorge.

(9) Geelong, Australia. Sandstone concrete. Top width 2.5 ft. Radius 300 ft.

(10) Shoshone, Wyoming. 1903 10. Concrete. Top width 10 ft. Length 200 ft. Radius 150 ft. 75 000 cu yds. In narrow solid granite gorge. Contract price $515 730.

(11) Pathfinder, Wyoming. 1905-1909. Cyclopean masonry. Radius 150 ft. 54 000 cu yds. Length on top approximately 425 ft, at base 80 ft. In narrow granite gorge. Contract price $482 000.

(12) Sand River, Port Elizabeth Waterworks, So. Africa. 76 ft high. Radius 300 ft. Length 360 ft. Cyclopean masonry faced with concrete blocks. Soap and alum washes on faces for waterproofing. Dam reinforced with steel rails.

(13) Dam at Hume, Cal. Concrete reinforced by railroad iron and cable. Concrete approximately 1 : 2 : 4. crusher run of granite being mixt with sand. Series of buttresses parallel to stream with arches between; 12 arches each of 50-ft span. Water face and parts of downstream face plastered. Water face two coats cement mortar 1 to 1½ and wash coat of neat cement on bases of middle arches. 2207 cu yds.

(14) Salmon River dam, Idaho. Cyclopean masonry. 220 ft high. 463 ft long. Top width 15 ft. Base width 119 ft. Radius upstream face 225 ft.

(15) Huacal, Mex. 1911-1912. Crest length 140 ft. Radius 76 ft. Max. height 100 ft. Base width 12 ft 10 in. 80 ft ± below top. (See (15) in Fig. 5.)

(16) Goodwin, Cal. 1911 1912. Concrete 2 spans of 135 ft radius abutting on central pier. Length 466 ft. Width at top 8 ft and bottom 16 ft. Max. height 78 ft.

(17) Salmon Creek, Juneau, Alaska. 1913 1914. Concrete. Constant angle arch type. 52 000 cu yds Height 168 ft. Width at top 6 ft. Base 47.5 ft. Radius at top 331 ft at bottom 147.5 ft.

(18) Eagles Nest, N. M. 1916-1918. Concrete. Length 300 ft. Top width 8 ft. Bottom width 46 ft. Max. height 140 ft. Radius 155 ft.

(19) Corfina, Italy. 1915. Concrete. Top width 5 ft. Bottom width 23 ft. Max. height 116 ft Radius 75 ft. Built in 65 days

(20) Spaulding, Cal. Built to height of 225 ft in 1913 and raised to 260 ft in 1916. Estimated yardage 169 000. Width at 260 ft level 14 ft. Planned ultimately to be 305 ft high. 304 000 cu yds. Vertical drains and inspection gallery. Lower 60 ft of gravity section.

Straight Masonry Overflow Dams. (Fig. 6)

(1) Vir Weir, India. Uncoursed rubble. 2340 ft long 43 5 ft high. Top width 9 ft.

(2) Henares Weir, Spain. Concrete with ashlar facing. 390 ft long. Maximum height 23 ft. Base width 45.8 ft.

(3) Vyrnwy England. 1882–89. Cyclopean masonry. Length 1164 ft.

(4) Lynx Creek, Arizona. Intended for ultimate height of 50 ft. Height was about 31 ft. In 1891 flood overtopt and destroyed dam.

(5) Folsom, Cal. 1886–91. Rough granite ashlar blocks. 48 590 cu yds. Top width 24 ft.

(6) Betwa, India. 1888. Length 3296 ft made in three tangents between islands. Top width 15 ft. Rubble masonry laid in native hydraulic line. Ashlar coping 18 in thick in portland cement mortar. Cost $160 000. Has had 16 4 ft head on crest.

(7) Austin, Texas. 1891-92. Hard line stone rubble, granite facing and coping Length about 1100 ft. 88 000 cu yds. Cost $608 000. Failed under head of 11.07 ft on crest. Previously stood 9 8 ft head. About 300 ft slid forward No overturning.

(8) New Holyoke, Mass. 1897. Rubble with drest stone cap and facings and concrete toe. Length 1020 ft. Upper part of downstream face is parabolic, lower part is cycloidal

(9) McCall's Ferry, Pa. 1904. Concrete with plums. 2350 ft long

(10) Great Northern Power Co., Minn. 1907. Concrete. 365 ft long, 38 ft high 42 ft wide at base. curved crest.

(11) Connecticut River dam. Concrete with plums (1 : 2.5 : 5 except in deep foundations where 1 : 3 : 6 was used). Length 600 ft.

(12) Escanaba. Concrete with earth banks at end. Length 810 ft. Concrete i

heart 1 : 3 : 6 on sides and bottom 1 : 2 : 4. Facing 1 ft thick upstream and 1 5 ft downstream. Founded on stratified limestone 8-in drain pipes at 10 ft intervals from cut-off to toe of dam.

(13) Hales Bar, Tenn. 1905–1913. Cyclopean concrete. Foundation limestone badly fissured. Fissures closed in part by grouting and in addition, 26 reinforced concrete caissons 30 × 32 to 54 × 72 were sunk by pneumatic process thru fissured rock to solid foundation. Height 65 ft. Length 1200 ft. Width 72 ft. Curved crest.

(14) Holter, Wolfcreek, Mont. 1910. Concrete with plums. Length 1362, 504 ft.

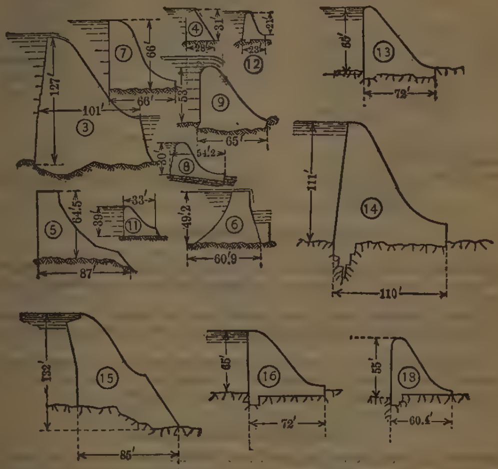


Fig. 6. Straight Overflow Dams

overflow section. Height 111 ft. Width 110 ft plus 70 ft of apron. Cut-off trench 44 ft, deep.

(15) Hauserlake, Montana. Completed, 1911. Replacing steel dam which failed by undermining. Length 490 ft, 85 000 cu yds. Max. height 132 ft. Width 85 ft. Ogee type. Part of foundations placed by pneumatic caissons.

(16) Coosa River, Lock 12, Alabama. 1912–1914. Cyclopean masonry. Length of spillway 930 ft. Height 65 ft. Width 72 ft. Parabolic crest.

(17) Keokuk, Ia. 1-3-5 concrete. Length 4278 ft. Height 32 ft. Width 42 ft. Ogee type.

(18) Youngstown, Ohio. 1916– . Concrete. Length 638 ft. Height 55 ft. Width 60 ft 5 in. Earth embankment at ends.

Curved Masonry Overflow Dams. (Fig. 7)

(1) Elche Spain. 16th century. Rubble, cut stone facing. Length 230 ft. Radius 205.38 ft. 1836 breach made during flood.

(2) Verdun, France. 1866-70. Rubble, cut stone facing downstream. Concrete foundation. Riprap in front of wall. Length 131.23 ft. Radius 108 83 ft. Width of top 14.17 ft. Designed for depth 16.4 ft water on crest.

(3) La Grange or Turlock, Cal. 1891 94. Rubble. Length 320 ft on crest and 80 ft at base. 39 500 cu yds. Radius 300 ft. Has had about 15 ft depth of water on crest,

(4) Cornell University dam, New York, 1897 Concrete of 4 parts argillaceous shale. 2 parts of sand, 1 part of cement Length 153 ft. Rad. 166 5 ft. Max. height 30 ft.

(5) Walnut Canyon, Airzona. Rubble faced with ashlar. Length about 268 ft. Radius 400 ft. 6 986 cu yds.

(6) Seligman, Arizona. 1897-98. Concrete foundation. Rough rubble above with drest ashlar facings Top width 1.75 ft. Total length on crest 643 ft. The center section, 340 ft long, is built as an overflow dam. Length at base 145 ft. Radius 800 ft.

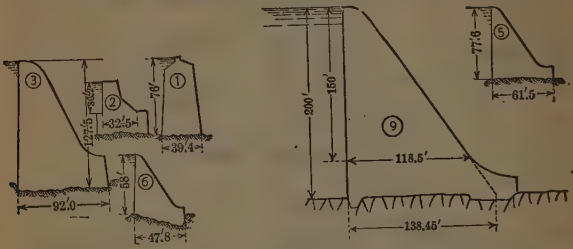


Fig. 7. Arched Overflow Dams

(7) Espanola, Ontario. Concrete with plums. Built for 10 ft overflow. Radius in plan 187 ft.

(8) Tallulah, Georgia. 1914. Crest length 444 ft. Radius 900 ft Automatic flashboards of the Stauwerke type installed between piers on the 280 ft spillway. Height 135 ft.

(9) Yadkin Narrows, N. C. 1917. Cyclopean concrete 525 000 cu yds with 25% plums. Length 1400 ft. Radius 1678 ft. Height 200 ft. Designed for 10 ft overflow. Taintor gates on crest. Drainage and inspection galleries.

2. Reinforced Concrete Dams

Reinforced concrete dams are usually of cellular construction, in some cases containing power houses. If properly designed they are admirably adapted to many situations. The general practice is to build a series of piers or buttresses 12 to 18 ft apart and cover with a flat deck of concrete reinforced between the different bays as a beam. In the multiple arch type, the buttresses are spaced 30 to 40 ft apart and support a deck of concrete arches springing between the buttresses.

Unit stresses in such dams should be much lower than would be safe for ordinary building construction for an incipient failure in the latter case usually means only an unsightly crack, while for a dam it might mean a catastrophe. The following data refer to Fig. 8.

(1) Schuylerville New York 1904. Rollway 250 ft long. Designed for 5 ft head on crest. Founded on Hudson River shale 5 × 3 cut-off wall. Passageway thru dam from shore to shore Average height 25 ft with base width 52 ft.

(2) Ellsworth, Maine. 1907-08. Buttresses 15 ft centers, 1 ft thick at top, 2 ft at bottom and braced by 12 in × 18 in reinforced concrete beams Deck 1 ft 2 in thick at top, 3 ft 1 in at base. Total length 500 ft. Concrete, 8000 cu yds. Flashboards provided. Designed for flood 6.5 ft on crest. Vents to prevent vacuum on apron.

(3) Pittsfield, Mass. 1907-08. 42 ft high. 465 ft. long. 3950 cu yds concrete Founded on gravel underlain at depth of 12 ft by dense yellow clay. Cut-offs extend 3 ft into clay. Work carried on continuously when temperature was 12° below zero, by

heating materials and using "salamanders." Concrete laid in winter and in moderate weather said to be uniform in quality.

(4) La Prele, Wyoming. 1908. Buttresses 18 ft centers, 1 ft thick at top, 4 ft 2 in at bottom, braced by 18 in × 24 in reinforced concrete beams. Deck 1 ft thick at top, 4 ft 6 in at base. About 15 000 cu yds concrete. Cut-offs 15 ft to 18 ft deep Concrete in buttresses 1 : 3 : 6, deck 1 : 2 : 4.

(5) Dam at Dansville, New York. 368 ft long including earth embankment. Gravel concrete used. Footing course on water bearing gravel and bowlders. Bed of seamy blue stone under gravel. Cut-off around footing course. At height of 10 ft base width 14 ft. Failed by undermining when water stood about 14 in against flashboards. Deck slab snapt where reinforcing bars came together. Bars were not bent after failure.

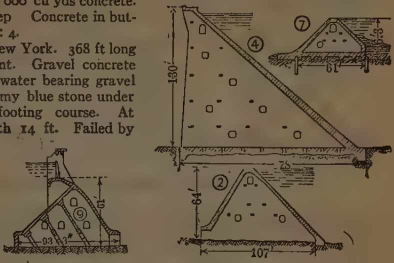


Fig. 8. Reinforced Concrete Dams

(6) D. L. & W. R. R., Scranton, Pa. 262 ft long. 16.5 ft high, base width 11.5 ft. Uniform slab thickness 10 in. Buttresses 7½ ft apart in center, increasing to 10½ ft at end. Soil under dam partly sand and partly yellow clay. ½-in cup bars 12 in c to c. 1½ in from face, to prevent temperature cracks. Clay bank 8 ft high at center to prevent percolation under dam.

(7) Uncas Power Co., Conn. Spillway 80 ft long. Foundation very compact gravel. Buttresses 10 ft centers, 1 ft 6 in thick at bottom, 1 ft at top, stiffened by 12 in × 15 in beams. Upstream deck 1 ft 6 in to 1 ft 8 in thick, downstream 1 ft 8 in.

(8) Bear Valley, Cal. 1910–11. Multiple arch of Eastwood type. Ten arches between buttresses. 33 ft 6 in centers. Radius of extrados 16 ft axis of arches vertical for 14 ft height at top below which are on batter of 4 on 3. Buttresses 1 ft 6 in thick at top increasing by .36 in per ft to bottom. Arch rings 12 in thick at top increasing by .17 in per ft in radial thickness. Downstream edge of buttress 2 : 1 slope. Buttresses stiffened by reinforced concrete struts. Max. height dam 92 ft length 363 ft. 4684 cu yds concrete.

(9) Austin, Texas. 1911-1915. 560 ft long. Replaces portion of straight masonry overflow section which failed in 1900. Buttress 20 ft centers. Height 61 ft to crest, which is provided with automatic gates 14 ft high.

(10) Stony River, West Va. 1912–1913. Buttresses 15 ft centers. 12-in thick at top and 18 in at bottom. Braced with reinforced concrete struts and a curtain wall. Six months after erection, foundation for length of 7 spans was washed out and buttresses and deck dropped into the hole. Reconstructed 1914–15 by deepening cut off wall, increasing the safety against sliding and adding 3 ft flashboards. Max height 51.17 ft.

3. Steel Dams

Steel dams are not very numerous. In many cases they may be built much more cheaply than other types, but to keep them safe requires frequent and careful inspection, and the question of their maintenance and length of life should always be taken into account when considering their relative economy. The following data refer to Fig. 9.

(1) Ash Fork, Arizona. 1898. Steel portion 184 ft long and 46 ft max height. 24 triangular bents, vertical downstream side and 1 : 1 upstream, 8 ft apart c to c braced laterally in pairs. Batter posts 20 in I beams, 65 lb per ft and reinforced on under side with plate ½ in thick and 18 in wide. Series of ⅜ in curved plates on upstream face concave upstream with radius 7½ ft. No leakage. Total weight of steel 478 704 lbs. Framed and erected at $55.78 per 2000 lbs.

(2) Redridge, Michigan. 1900–1. Steel portion 464 ft long. Max height of dam 74 ft. Concrete base thruout. "A" frame bents 8 ft apart, ⅜ in face plates concave upstream radius 7 ft 6 in, riveted to I beams 15 to 24 in deep. Batter upstream face 2 : 3. To cut off percolation under dam, line of drill holes 2 in diam and 10 ft long, 7 in apart

drilled and filled with grout under heavy air pressure. Rock floor covered with concrete and then by bank of puddle clay.

(3) Hauser Lake, Montana. 1907. 630 ft long. Maximum height 81 ft. Inclination upstream face 3 : 2. Portion of dam founded on solid rock. About 300 ft founded on steel sheet piles in gravel. Bents 10 ft apart connected in groups of four. Actual head

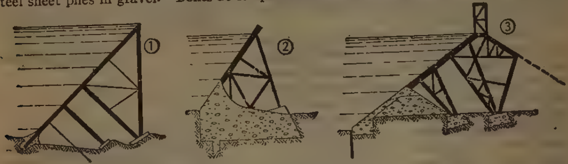


Fig. 9. Three Types of Steel Dams

to top of flashboards 69 ft. Working head about 66 ft. About 1700 tons of steel. Blanket of volcanic ash above dam to prevent seepage. Downstream face had flat plates on slope 7 : 11, connecting with plank apron carried on stone-filled cribs. Dam undermined where founded on gravel. Concrete placed over top of steel piling, said to have been placed in depth of 10 ft or more water so quality may have been very poor. Part of dam on rock remained standing. Replaced by concrete dam in 1911.

4. Timber Dams

The Rafter and Strut-Framed Dam (Fig. 10) is a structure which may be proportioned with reference to its stresses, but requires careful design, sawn timber and skilled workmen and is proportionally more expensive.

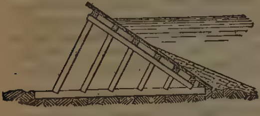


Fig. 10. Framed Timber Dam

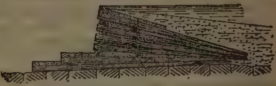


Fig. 12. Beaver Type Dam

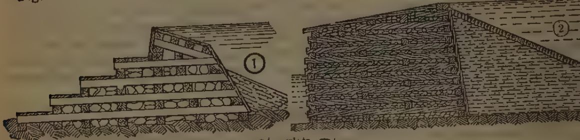


Fig. 11. Crib Dams

Crib Dams, (Fig. 11), have been extensively used where timber was cheap and masonry dear. They must be designed largely by judgment and following precedent. They are not subject to a very rigid analysis of stresses. The whole crib filled with loose rock may be considered as a unit adapted to resisting ordinary stresses of compression, tension and shear. The compressive stresses are resisted largely by the bearing of the timbers, one on the other; the tensile stresses resisted by the bolting and sometimes dovetailing of the timbers and the shear largely by the friction in the rock fill. Considered from this angle the crib dam can be designed with some reference to stresses. Another way of looking at it is to give about the width required for a rock fil

dam and consider the crib acting somewhat as a binder and mainly as an attachment for a water-tight timber sheeting to cut off leakage on the upstream side and to take wear and impact of the overflow on the top and downstream sides. The width of the crib-work is seldom less than one and one-third times the height and may be two or three times where long aprons are provided for overflow as in (1) Fig. 11. The highest dam of this type which has not failed was the Butte dam, about 70 ft high. No. (2) Fig. 11 is a cross section of a crib dam in the Schuylkill River subject to floods 11 ft high over crest. If crib dams are subject to heavy floods provision should be made for admitting air to the downstream side underneath the sheet of overflowing water.

The Beaver Type of timber dam is adapted to comparatively small heights. It is built as indicated in Fig. 12. The longitudinals are all laid with butts downstream. The crevices are filled with gravel, and the whole is covered with brush tightly packed and a bank of the most impervious earth obtainable. A plank flooring is carried from the crest well under the earth fill. Most failures of timber dams have been due to leakage underneath or to undermining by the overfall

5. Earth Dams

Foundations. One of the reasons for building a dam of earth is usually that a rock foundation cannot be had, tho several of these dams have been built on the best of bed rock. The foundation, whether earth or rock, should be thoroly examined to learn if it is impervious. If not impervious, cut-offs should be constructed, if possible down to an impervious stratum. Imperviousness is a relative condition. No earthy material is absolutely impervious, and if the width of the dam is so extended that the loss of head of water flowing thru the shortest course to an exit is so great that the velocity is so slight that water will not move the earth or reduce its cohesion at the exit, then the cut-offs under the dam may be unnecessary. In any case the site is usually stript and shallow trenches are dug across it. If the foundation is rock the creeping of water along its surface is guarded against usually by building a masonry wall entirely across the valley. This may be extended upward to the top of the dam as a core-wall, as in the Titicus dam, or, if the embankment is impervious, it is sometimes only 3 or 4 ft high, as in the Borden Brook dam, Mass. Where the foundation is open sand or gravel, deep cut-off trenches have usually been excavated and filled with concrete or puddle clay. Sheet piling is sometimes used. The character of the ground and other local conditions must determine the kind of cut-off wall best to use. Springs which may be found within the area of the foundation may sometimes be dug out to their source outside the area of the dam, or intercepted outside that area and diverted.

Core-Walls. The dam itself usually consists of three parts: an interior impervious part with an embankment on each side to support it. The impervious part is a masonry, puddle or steel core-wall or a heart of selected impervious material. Occasionally the builders are fortunate enough to have at hand a natural mixture of gravel and clay of which the whole dam may be built, thus combining the three parts in one, but usually the impervious material available is either too expensive or is unstable at a practical slope. What is the best type of core-wall is a mooted question. Rubble, concrete, puddle clay, sheet steel and wood have been used. It is argued that masonry, whether rubble or concrete, must have a solid rock foundation, and that even then it is in danger of being cracked by settlement of the embankment or by temperature changes. On the other hand, it is argued that slight

cracks will not be made larger by water but will be silted tight, and that if embankments are well built and the masonry core wall is 4 to 6 ft thick at top with sides battered at about 1 to 10, it will not crack from settlement of the embankments, and also when outlets pass thru or under the dam, that they are much safer when masonry core-walls are used.

In regard to clay puddle it is argued that its integrity depends entirely upon great care and much intelligence and skill in mixing and placing, and that if once punctured its entire integrity is destroyed. On the other hand, it is argued that settlement does not rupture it, and that it makes a better union with the rest of the embankment.

At the Boston Water Works the endeavor was made to overcome all objectionable features and preserve the merits of both by building a masonry core-wall and placing a selected clayey material next the upstream side of the masonry.

Riveted steel plate cores coated with asphalt have been used. Timber sheeting should not be used, as it is likely to decay. Whether the core-wall should be placed in the middle or near the upstream side of the embankment will depend upon the material used and local conditions. If the core-wall is to be of masonry it should be vertical, and this usually requires it to be under the crest of the dam. If the core-wall is of clay it is usually placed vertically under the crest, but sometimes on the upstream face, in which case it should be protected by additional filling. The slope must be very flat or else there must be considerable filling outside it to prevent slipping; also provision must be made against burrowing animals, if the clay is near the surface. Small stones or furnace slag mixt with the puddle are frequently used for that purpose.

The Placing of Embankment when not done by the hydraulic process is usually done by carts or cars. It is usually specified that the earth shall be spread in from 4 to 8 in layers, rolled with a heavy steam roller, and that, before placing the next layer, the last one shall be sprinkled and harrowed to insure bonding. Variations in details of construction on different dams have been the use of a drove of horses and sometimes goats to trample the earth, also heavy four-horse teams for hauling, runways being avoided. Rollers having grooves or projections on their faces are used on many dams.

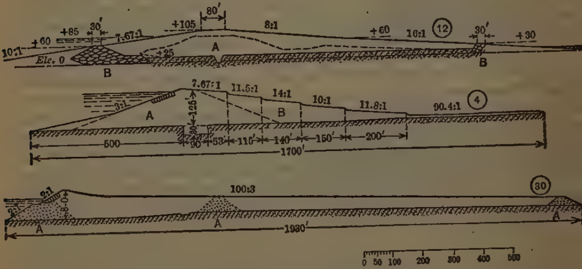


Fig. 13. Earth Dams

The embankment is usually kept four or five feet higher on the outer edg than in the middle.

Slopes must be determined by the engineer for each individual case. Prece dents may be studied in the following table. If it were simply a questio of the stability of the face the slope for the same material should be flatte on the upstream side than on the downstream side, for the angle of repos

Data of Earth Dams

Numbers in parentheses refer to Figs. 13 and 14 and to the following notes.

Number and name	Max height, ft	Top width, ft	Slope up-stream	Slope down-stream	Material	Core-wall
(1) Goose Creek, Idaho	145	16	3:1	2:1	Sand, clay & gravel	C
(2) Idaho Irrigation Co	135	40	3:1	5:2	Earth and rock	O
(3) Patillas, Porto Rico	135	20	3:1 2:1	2:1	Gravel, boulders and clay	O
(4) San Leandro, Cal	125	28	3:1	5:2		O
(5) Lahontan, Nev	124	20	3:1	2:1	Gravel and silt	O
(6) Tabeaud, Cal	123	20	5:2 3:1	5:2	Red gravelly clay	
(7) New Croton, N. Y.	120	30	2:1	1:1		R
(8) Costilla, Col	120	20	3:1	2.5:1	Clay and gravel	O
(9) Druid Lake, Md	119	60	4:1	2:1		P
(10) Bell Fourche, S. Dak.	115	20	2:1 3:1	7:4 2:1	Heavy clay and gumbo	O
(11) Dodder, Ireland	115	22	7:2	3:1		
(12) Gatun, Panama	115	80	7.67:1 10:1	8:1 16:1	Clayey sand and rock fill	
(13) Standley Lake, Col	113	20	2:1 3:1	2:1	Blue clay	P
(14) Titicus, N. Y.	110	30	2.4:1	5:2		R
(15) Mudduk Tank, India	108	......	3:1	5:2		
(16) Ashokan Dikes, N. Y.	......	34	2:1 5:2 11:4	2:1 11:4 3:1	Clay, sand, gravel and small stones	C
(17) Somerset, Vermont	106	......	3:1	2.5:1	Clay, sand, gravel and small stones	O
(18) Temescal, Cal	105 115	18	3:1 3:1	5:2 5:1		
(19) Carite, Porto Rico	105	20	2.75:1	2:1	Clay and rock	O
(20) Cummum Tank, India	102	......	3:1	1:1		
(21) Yarrow, England	100	24	3:1	2:1		P
(22) Morris, Conn	100	20	3:1	2:1		C
(23) Pilarcitos, Cal	95	24	2:1	2:1		P
(24) Dale Dyke, Eng	95	12	5:2	5:2		P
(25) San Andreas, Cal	93	25	7:2	3:1		P
(26) South Haiwee, Cal	91	20	2.5:1	2.5:1	Tufa, shale and clay	P
(27) Forest Park, Md	87	15	......	5:2	Rock and earth	O
(28) Dry River, N. Y.	85	20	2.5:1	2:1	Earth	C
(29) Sherburne Lakes, Mont	83	22	3:1	......	Clay, sand and gravel	
(30) Wachusett, N. Dike, Mass	82	......	2:1	100:3	Soil sand gravel	O
(31) Cold Springs, Ore	98 ½	20	3:1	2:1	Gravel and earth	O
(32) Borden Brook, Mass.	.....	24	3:2 2:1	3:2 2:1		P
(33) Talla, Scotland	78	20	4:1	3:1	Clayey and open materials	P
(34) Throttle, N. M.	77	......	2.5:1	1.5:1	Earth and course rock	P
(35) Seros, Spain	75	13	3:1	2:1	Earth, clay and gravel	O

P = Puddle. C = Concrete. R = Rubble. O = No core-wall.

Data of Earth Dams—*Continued*

Numbers in parentheses refer to Figs. 13 and 14 and to the following notes.

Number and name	Max height, ft	Top width, ft	Slope up-stream	Slope down-stream	Material	Core-wall
(36) Las Vegas, N. M.	75	20	3:1	2:1	Heavy clay, sand gravel	C
(37) Mammoth, Utah	70	......	......	......	Clay and fine gravel	C
(38) Johnstown, Penna.	70	10	2:1	3.2	Selected earth with rock	O
(39) Keechelus Lake, Wash	70	20	3:1	2:1	Gravel and earth	O
(40) Phelps Brook, Conn	68	15	2:1	2 5:1	Gravelly material	C
(41) Youngstown, Ohio	66½	20	2:1	2:1	Clay and gravel	O
(42) Kachess, Wash	65	20	3:1	2:1	Earth and gravel	C
(43) Glenwild, N. Y.	63	13	2:1	5:2		R
(44) Hatchtown, Utah	63	20	2:1	2.5:1	Loamy sand	P
(45) Bog Brook, New York	60	25	2:1	5:2		R
	25	12	2:1	2:1		
(46) Ashti, India	58	6	3:2 3:1	3:2 2:1	Black and brown soil and "Murrum"	
(47) Hebron, N. M.	56½	12	3:1	1.5:1		O
(48) Horse Creek, Col.	55½	16	1.5:1	1.5:1 2.5:1	Sandy loam and clay	O

P = Puddle. C = Concrete. R = Rubble. O = No core-wall.

especially of very fine material, is less in water than when not saturated bu slightly damp, which is usually its condition on the downstream side. If th dam is without a core-wall or other impervious hearting and subject to seepag either under or thru the embankmen it may be necessary to make th downstream slope very flat. Th upstream face is pitched or other wise protected against wash waves. The downstream slope sodded to protect it from wash rains. Berms with paved drai are used frequently in addition sodding.

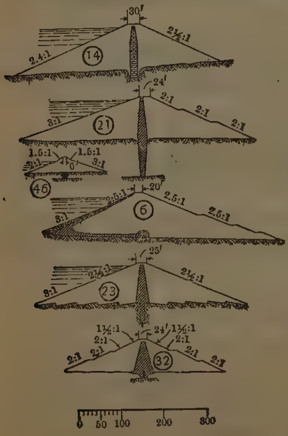


Fig. 14. Earth Dams

Top Width and Height abo Water. The formula $W = 1/5$ + 5, in which h = height of da in feet, has been suggested as gi ing safe widths in feet, for tops earth dams. The top should be high and wide that no burrowi animals can reach the heart of t dam at water level. It shou also be high enough above the hig est water level to be beyond the rea of all waves. Stephenson's form for height of waves is commo used,

$$X = 1.5\sqrt{F} + (2.5 - \sqrt[4]{F})$$

in which X is the height in feet, F the fetch in miles or sweep of the wind in the longest straight line which may be drawn from the dam on the water surface of the reservoir.

(1) Goose Creek, Idaho. 1910-13. Upstream face covered with 3 ft of loose rock, and loose rock dumped at toes. Core-wall 3 ft thick to 10 ft above natural surface and 2 ft thick from this point to top. Length 1050 ft.

(3) Patillas, Porto Rico. 1914 Length 1020 ft. Contains 970 000 cu yds. First 50 ft in height made by dumping from two trestles near either toe. Successive lifts then made by shifting track. Outer slopes of gravel and boulders. Middle of dam about 25 per cent clay washed into position and allowed to settle in pool maintained by the side banks. Total settlement 0.56 ft in three years.

(4) San Leandro, Cal. 1874-76. Portion "A" in cut built in 1874 of earth in layers 1 ft thick. No rollers used, but material dumped and sprinkled and band of horses used to compact. In 1898 portion "B" was placed by the hydraulic process. Shaken by San Francisco earthquake 1906. No damage. Puddle trench 30 ft below ground.

(5) Lahontan, Nev. 1913. Length 1359 ft. Vol. 710 000 cu yds. Upstream portion of gravel and silt, mechanically mixed and rolled in 4-in layers and faced with riprap and gravel. Downstream portion of pit run gravel. Foundations a red sand stone. Puddled cut-off wall. Grouted rock foundation.

(6) Tabeaud, Cal. 1900-02. Built in 6-in layers first 60 ft. Not over 8-in layers above. Top kept basin-shaped. Clay puddle on bottom and part of upstream face. 1 berm upstream, 3 downstream. Foundation hardpan and rock.

(7) New Croton, N. Y. Dam partly constructed then replaced by masonry structure. 2 berms downstream.

(8) Costilla, Col. 1917. Length 600 ft. Upstream slope riprapped. Cut-off wall 30 ft into clay and gravel foundation and 5 ft above natural ground surface. Built by dumping earth in ridges about 5 ft high and partly filling trench between with water. Then filling the trenches by dumping from wagons, then building new ridge over former trench and repeating process.

(9) Druid Lake, Md. 1864-70. Upstream face covered with puddle. Stone wall in bottom of core-wall. Embankment each side of puddle core-wall well rolled. Part of the material in dam placed in basins filled with water. Surface soil stripped, and where foundation is sand, core carried to impervious stratum.

(10) Belle Fourche, So. Dakota. 6-in layers. Some sheet piling near upstream toe. $819 164 contract price.

(12) Gatun, Panama. Portion A in cut is hydraulic fill, B is rock fill and the remainder is dry fill. There is a layer of 3 ft of rock paving 170 ft wide on the upstream slope to resist wave action.

(13) Standley Lake, Col. 1908-12. Length 6630 ft Material lumpy, blue clay which slacked very readily and became a slimy ooze on being saturated. Upstream face protected by loosely dumped riprap. In 1912 a small slide of the downstream face occurred and in 1914 there was a much larger slide of the same face. In 1916, following a rapid draining of the reservoir for irrigation purposes, about 88 000 cu yd slid from the inner face.

(17) Somerset, Vt. 1911-13. Length 2100 ft. Contains 1 000 000 cu yds. Material was coarse gravel and sand with some clay. Excavated by steam shovel, hauled to site and dumped from trestles on the upstream and downstream slopes. From there the material was sluiced down about a 2 1 slope toward the middle of the dam, the object being to leave coarse material on the faces and gradually grading to a fine impervious heart. Length 2100 ft.

(18) Temescal, Cal. 1866-68. Originally built in layers with carts and scrapers, with top width 18 ft at maximum height of 105 ft and slopes 3 : 1 upstream and 5 : 2 downstream. In 1869 downstream slope flattened by sluicing. In 1886 height increased to 115 ft by sluicing. As finished downstream slope 5 : 1

(19) Carite, Porto Rico. 1914. Length 520 ft. Upstream face paved. Down-stream slope, lower half paved. 200 000 cu yds. Fill dumped from trestle and spread by scrapers and wheelbarrows. Very little sprinkling done. Settled 0.96 ft. in three years.

(21) Yarrow, Eng. Max depth excavation 97 ft. Puddle core-wall on concrete foundation in rock. 2 berms on downstream slope.

(22) Morris, Conn. 1910. Upstream slope 3 : 1 for a height of 30 ft from toe, 2.5 : 1 above, faced with 1 ft 6 in of paving stones. 4 berms 8 ft wide on downstream slope, and

slope sodded. Core wall carried to rock 12 ft wide at base and 2 ft 5 in at top. Leng
1100 ft.

(23) Pilarcitos, Cal. 1864–66. Puddle core-wall 24 ft thick, keyed to rock with co
crete wall 3 ft by 6 ft. Length of dam 640 ft. 1¾ miles from fault line of San Francis
earthquake of 1906.

(24) Dale Dyke, Eng. Failed; opinions as to cause varied. Said to be because ou
let pipes laid naked or because dam above core-wall was a sort of rock fill, bringing
static pressure against core-wall.

(25) San Andreas, Cal. 1868–70. Dam raised in 1875. Puddle core-wall keyed
rock with concrete wall 3 ft by 5 ft. Length 800 ft. Fault line of San Francisco ear
quake of 1906 passes across east end. Crack 2 to 3 inches wide along axis of dam.

(26) South Haiwee, Cal. 1912. Length 1523 ft. 559 750 cu yds. Upstream sl
paved with 3-in concrete. Cut-off trench carried down to rock, in some cases to a de
of 120 ft.

(27) Forest Park, Md. Clay puddle on upstream face and carried to bed rock i
trench near upstream toe.

(28) Dry River, Watervliet, N. Y. 1913–14. 1 berm 8 ft wide, 40 ft below top
both faces. Material rolled in 6-in layers. Upstream face partly paved.

(29) Sherburne Lakes, Mont. 1918. Length 900 ft. 250 000 cu yds. Upstre
slope has 18-in paving of field stone on a 12-in layer of gravel, and concrete parapet
3 ft high at top. Downstream slope covered with 12 in of gravel. Unique feature
permeable core-wall of screened gravel 5 ft thick at crest, increasing with 3.5 in
batter, with system of pipe drains running from base of core to downstream toe.
cut-off trenches on upstream side, 1 on downstream. Material hauled in dump wag
spread in 6-in layers rolled with 20-ton roller.

(30) Wachusett, N. Dike, Mass. 1900–05. This dam is about 10 000 ft long. 5 500
cu yds. There are three embankments of sand and gravel shown at "A" in the
The remainder was built largely of fine loamy soil stripped from the reservoir site. M
rial spread in 1-ft layers and rolled by steam road roller and harrowed before adding
layer. Foundation pervious sand and gravel. For a portion of the length a trench
dug 60 ft deep in this gravel, and triple lap tongued and grooved sheet piling built u
2-in plank was jetted down from 20 to 60 ft. On Apr. 11, 1907, a portion of the upstr
face 675 ft long, of a thickness of 35 ft normal to the slope, slid down the bank, cau
little damage but indicating that 1 vertical to 2 horizontal was too steep for such
material under water.

(31) Cold Springs, Ore. No core-wall. Cut-off trench.

(33) Talla, Scotland. 1897–04. The clayey or adhesive material in middle
placed in 9-in layers. Outside this is stony or open material in 18-in layers.

(34) Throttle, N. M. 1914. Length 1060. Concrete core wall 600 ft long with
corrugated iron extending from cut-off wall to crest. Upstream slope paved. C
portion of dam well puddled. Downstream slope faced with selected rock, hand
(35) Seros, Spain. 1913–14. Dam No. 3 is the highest of 7 earth dams in hy
electric development near Barcelona. Length 1312 ft. Vol. 497 250 cu yd. Upst
face riprapped. Puddled cut-off trench. 1 berm downstream slope.

(36) Las Vegas, N. M. 1917. Upstream slope riprapped 9 in thick. 1 berm on d
stream face. Length 1400 ft. Material hauled by dump cars and sluiced into
with water jet.

(37) Mammouth, Utah 1908 14. Proposed height 125 ft. Concrete core-wall
buttresses. Earth placed by wagons and scrapers and rolled. In June, 1917, dar
at a height of 70 ft with water stored to nearly the same height. Washout occ
at flume carrying water across the top. One-half billion cu ft water released cuttin
to foundation causing great damage to structures below.

(38) Johnstown, Pa. Lower face entirely stone, 4 ft thick at top, 20 ft at bo
and backed by slate rock 3 ft thick at top and 30 ft at bottom. Heart of selected
Upstream face of dry rubble 15 in thick. Failed: Flood overtopped dam.

(39) Keechelus Lake, Wash 1914. Length 6500 ft. 522 000 cu yds. Ups
slope faced with riprap. Concrete cut-off wall.

(40) Phelps Brook, Conn. 1917. Length 1200 ft. Upstream slope 3 : 1 below
surface, 2 : 1 above, faced with 12-in or riprap. 1 berm on downstream slope.
crete core-wall to rock.

(41) Youngstown, Ohio. 1913–16. Length 2202 ft. Concrete cut-off. Up
slope faced with 9 in to 18 in of concrete.

(43) Glenwild, N. Y. 1902. Core-wall is broken bowlders grouted with 1 : 3 cement mortar. Dam curved upstream with radius of 708.6 ft. Contract cost $47 360.

(44) Hatchtown, Utah. 1908. Failed in May, 1914, probably due to leakage along the outer surface of the outlet culvert and leakage thru foundations of black clay sand and gravel. Length, 780. 126 000 cu yds.

(45) Bog Brook, N. Y. Masonry core-wall 10 ft thick at base and 6 ft at top. Upstream face has 12-in paving on 6-in broken stone. Berm on downstream side.

(47) Hebron, N. M. 1914. Length 3700 ft. Puddled cut-off trench. In May, 1914, a section 200 ft long washed out.

(48) Horse Creek, Col. 1911-12. Length 5150. Vol. 714 000 cu yds. Downstream slope 1.5 : 1 for distance of 16 ft from crest, and 2.5 : 1 for balance. Upstream slope 1.5 : 1 face paved with 5 in of concrete. Earth placed by dump wagons and spread with scrapers. No sprinkling and no packing of material done. In January, 1914, 200 ft of the dam above the concrete culvert outlet washed out. Immediately after reservoir emptied, 3600 ft concrete paving slid down slope.

Failures of Earth Dams. In a record of 30 failures of earth dams, which may be assumed to be representative, 11 were due to overflowing, 10 to leaks in outlet pipes; 3 to undermining, and 1 was attributed to burrowing animals. The causes of the other five were somewhat doubtful, but several causes were assigned, such as poor foundations, slopes too steep, lax construction methods and carelessness in preparing the site.

6. Hydraulic Fill Dams

Sluicing is a method of building earth dams by which, especially in mountainous regions, the material often may be excavated, conveyed to the site and placed much more cheaply than by any other means. The ideal material

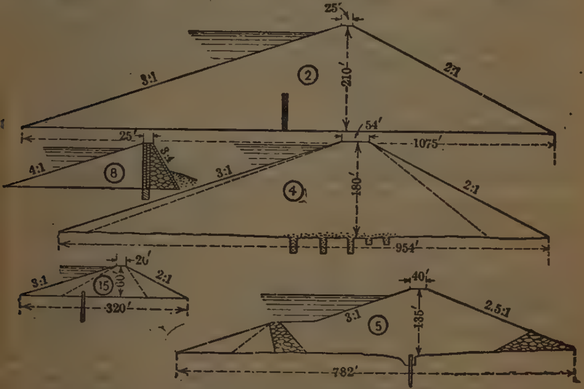


Fig. 15. Hydraulic Fill Dams

is a mixture of sand, clay and gravel with plenty of small rock. The development of the art has evolved a number of excellent properties of such dams, so many, in fact, that a number have been built by hauling dirt with cars to the site and sluicing it into place. The practise is for the sluices to discharge near the outside edges of the embankment. The coarse material drops there, and the fine flows with the water toward the middle of the dam, depositing

as it goes, the coarser first, the medium next, most of the clay going to the pond of still water in the center, where it is precipitated, forming a wide clayey core-wall. The coarse material remaining on the outer edge is most useful in maintaining the slopes due to its greater angle of repose. In practise, however, the danger is that the material will not be properly graded, either too fine, making the faces unstable, or too coarse, making the core permeable. Even if the material is properly graded the sluices may not deliver a uniform mixture, resulting in permeable layers across the dam. This danger is believed to be avoided by keeping men continuously spading and kneading the bottom of the pond in the middle of the dam. As the work progresses, a few men keep the edges built up just enough to keep the discharge from the sluices always flowing toward the center. The pond is sometimes allowed to drain by filtering thru the coarser outside material, and sometimes by a vertical waste pipe built up in short lengths of from 6 to 15 inches, changing its direction at the foot to a horizontal leading to the downstream face.

Data of Hydraulic Fill Dams

Numbers in parenthesis refer to following notes and to Fig. 15.

No. and location	Date	L'gth, feet	Max height, feet	Top width, feet	Slope up-stream	Slope down-stream	Total volume, cu yds	Water used, cu ft per sec
(1) Calaveras, Cal.	1918	1260	240	25	3 1	2 5.1	....	....
(2) Terrace, Col.	1905–9	605	210	25	3.1	2 1	500 000	26
(3) Little Bear Val, Cal.	......	....	200	20	5.2	2:1	....	......
(4) Necaxa, Mexico	1909	1220	180	54	3 1	2.1	2 000 000	15–30
(5) Magic Res, Idaho	......	.....	135	40	3.1	5:2	....	......
(6) San Leandro, Cal	1874–6	500	125	28	3 1	5:2	542 700	10–15
(7) Crane Valley, Cal.	......	720	100	20	2.1	3.2	....	15
(8) Waialua, Hawaii	1904–6	460	98	25	4:1	......	141 000	8
(9) Santa Maria, Colo.	1912	1300	95	20	3:1	2.1	310 000	.....
(10) Quemahoning, Johnston, Pa.	1913	950	95	20	4 1 3.1	4 1 3 1	600 000	.....
(11) Lake Frances Cal.	1899 1901–2	992 1300	50 77	16	3 1	2 3	80 265 280 700	.....
(12) Snake Ravine, Cal	......	294	64	12	Bet 3.2	and 2.1	....	.....
(13) Santo Amaro, Brazil	1907	5300	63	33	3 1	2:1	......	8
(14) Croton, Mich.	1906–7	200	60	.....	5 2 to 6 1	2:1 and 4:1	104 000	.....
(15) Conconully, Wash	......	1030	60	20	3:1	2.1	351 000	.....
(16) Silver Lake, Cal.	......	900	56	....	......	......	146 000	2 5
(17) Yorba, Cal.	1907	800	47	16	7.2	2.1	100 000	3–4
(18) Tyler, Texas	1894	575	32	......	3 1	2.1	24 000	1 4

(1) Calaveras, Cal. Material is clay sand and gravel with dry rock fill at toes. Material sluiced from borrow pits down open channel 5 to 7% grade into concrete sumps an raised to dam with 12-in centrifugal pumps Designed to contain 3 000 000 cu yd After 2 800 000 cu yds placed, upstream face for length of 700 ft slid. Aggregate to 50% clay.

(2) Terrace, Col. Material sluiced from a clayey bank to form heart of dam, and fro a separate bank clean rock and gravel were sluiced to the two slopes in flume on 7 grade using 26 sec ft of water Concrete core-wall about 65 ft high in bottom of canyo

(3) Little Bear Valley Cal. Disintegrated granite and clay deposited from spoil trai on toes of dam and finer materials were washt to the center. Concrete core-wall.

(4) Necaxa, Mexico. Broken limestone and yellow clay. Concrete core-wall, abo

6 5 ft above stript surface. Sluicing flumes 20 in diameter, 4 ft wide, rect section with V-shaped bottom, grade 5% and 8%. During construction a slide of the upstream face occurred which was attributed to the use of unsuitable material. Dam since completed and now in use.

(5) Magic Reservoir, Idaho. Cut-off walls in sides of canyon. Cut-off trench and sheet piling under dam Constructed by combination of methods. Excavated by steam shovels; spoil trains delivered material close to site, thence it was sluiced to place.

(6) San Leandro, Cal. 160 000 cu yds sluiced. Sluicing flume grade 4% to 6% See earth dams.

(7) Crane Valley, Cal. Sluicing flume 12 in by 10 in. Flumes on 6% grade. Ground sluicing was used, that is, material was dumped into flume after loosening.

(8) Waialua, Hawaii. Rock fill portion: base width 80 ft, crest 11 5 ft, downstream batter ¼ to 1, upstream face vertical, volume 26 000 cu yds. Wooden diaphragm embedded at bottom in concrete wall. Earth fill 141 000 cu yds. Soil dumped into flowing stream of 8 sec ft in ditch 1300 ft long with bottom grade 4%. The soil was a decomposed lava of a cohesive and unctuous character, very free from grit. Earth fill 11 cents per cu yd Leaked during first year. 1905 material obtained by enlarging spillway sluiced against lower toe.

(9) Santa Maria, Colo. Two cut-off trenches to hardpan sheeted and puddled. Dirt and boulders conveyed by rectangular sluices 3 × 3 ft.

(10) Quemahoning, Johnston, Pa. Slopes 4 : 1 for lower half and increasing 3 : 1 for the upper half Concrete cut-off wall extends 10 to 27 ft into the shale rock also puddled cut-off. Rectangular flumes 24 in wide and 18 in deep, grade 6%. Material a mixture of clay and shale rock.

(11) Lake Frances, Cal. Old earth dam failed, about 20% being washt away. Repaired and enlarged by hydraulic process. 182 937 cu yds of new dam sluiced in 1.76 sec ft of water used at first; later 4.5 to 7 sec ft. Sluicing flume 22 in diameter, minimum grade 2 2% Most of material sluiced was clay. Brush used to maintain slopes.

(12) Snake Ravine, Cal. Built of fine silt and clay, slid as a body 1000 ft down ravine at rate 6 to 10 ft per sec Failure attributed to lack of coarse material on slopes and fact that fine material did not drain when building.

(13) Santo Amaro Brazil. Clay and disintegrated granite. Slip of blanket over core-wall of section east of hydraulic fill occurred during construction Excess of clay in blanket did not allow good drainage outward. Sluicing flume 2000 ft long, grade 3%.

(14) Croton Mich. Fine yellow sand. Sluicing flume 30 in in diameter semi-circular, grade 8% to 9%, length 800 ft Cost 6 8 cents per cu yd, including plant, materials, labor, power, etc.

(15) Conconully, Wash. Loose rock, gravel, sand and silt. Sluicing flume, sloping side 2 ¾ ft apart at top No 10 steel curved bottom bent to 12-in radius, clear depth being 2¼ ft. Lateral flumes trapezoidal 12 in wide at bottom. Sheet piling across valley 70 ft upstream from center line driven 33 ft and projecting 3 ft above valley bottom. Flume grade 4%

(16) Silver Lake, Cal. Heavy sandy loam containing considerable clay which was carefully separated and placed in center. Drains to prevent slipping during construction Concrete core-wall under dam and 3 to 6 ft above original surface. Sluicing flume 8 in diameter, 2 5 sec ft water used in later stages. Flume length 4000 ft.

(17) Yorba, Cal. About 80% by hydraulic ground sluicing process. Adobe clay soil, sand and gravel. No core-wall; puddle trench thru top soil. Material supplied to parts too high for gravity flume by pumping thru an 8-in pipe a maximum distance of 800 ft. Cost 8 cents per cu yd for ground sluicing Flume grade 4 to 7%.

(18) Tyler, Texas Sluicing flume 13 in in diameter. Material 65% sand and 35% clay. Cost 4¾ cents per yard, including everything

7. Rock-fill Dams

Dams of Loose Rock were designed to meet the requirements of miners in western America, where the great cost of hauling cement made its use prohibitive Since most of these structures are located in deep rock canyons, it was a simple matter to quarry the material at a higher elevation than the dam and transport it by gravity to the site or even to throw the material directly to place by blasting Dams of this type are made watertight by a variety of methods In some cases the upstream face is covered with planking spiked to studding buried in the rock-fill, or with steel plates fastened to I

beams set in the face, or with rubble masonry walls or reinforced concrete facing. Steel core-walls are also used. A later development is the combination earth and rock-fill dam in which the earth is either dumped or sluiced into place on the upstream face.

Most of the failures of rock-fill dams have been due to insufficient spillway, causing overtopping. Some have been caused by poor design in making the slopes steeper than the natural angle of repose of the loose fill and trusting to dry rubble facing walls to hold the mass in position.

The following numbers correspond to those on the cross-sections of Fig. 16:

(1) Escondido, Cal. 1895. Faced with redwood plank. Timber face backed with concrete. Contains 6000 cu yds dry rubble and 31 157 cu yds rock fill. Leakage when full 450 000 gal per day. Cost $110 000. Stones laid by hand on inner face to form dry wall 15 ft thick at bottom and 5 ft at top. Bed-rock trench at upper toe filled with rubble masonry, in which plank facing is embedded. 76 ft high. 380 ft long on top and 100 ft on bottom. 140 ft wide at base and 10 ft on top. Slopes are 1:2 on water face; 1:1 on back for half the height and 5:4 from mid height to base.

(2) Castlewood, Col. 1890. Lower slope covered in steps of 2 ft with large blocks of stone in cement mortar. General slope 1:1. Upper face wall rough rubble masonry 4 ft thick, on batter 1:10. Walls joined at top with coping. Crest width 8 ft. Dam founded on clay. Front face wall carried 6 ft to 22 ft into clay. Lower slope wall also founded on clay at depth 10 ft below surface. Length 600 ft. Height above reservoir floor 70 ft, above foundation of face wall 92 ft. Rock pavement 25 ft wide, 200 ft long, 3 ft to 6 ft deep heavily grouted at top with cement mortar to prevent scouring. Dam leaked. Earth embankment added to upstream face, 8 ft wide at crest with 3:1 slope faced with 12 in riprap.

(3) Lake Avalon, New Mexico. Length 1050 ft. Earth slope covered with revetment of loose stone 2 ft to 3 ft thick for wave protection. Failed 1893, water passing over top, washing out 300 ft length to bed rock. Immediately repaired and built 5 ft higher. Second failure 1904. Stated that water did not pass over top but forced way thru the dam. Failure attributed to burrowing animals, or faulty construction where dam connected with bank. Reconstructed with top width 43 ft. Substantial addition made to earth fill and core-wall built of concrete heavily reinforced. Part of core-wall on concrete base and part on sheet piling driven to rock.

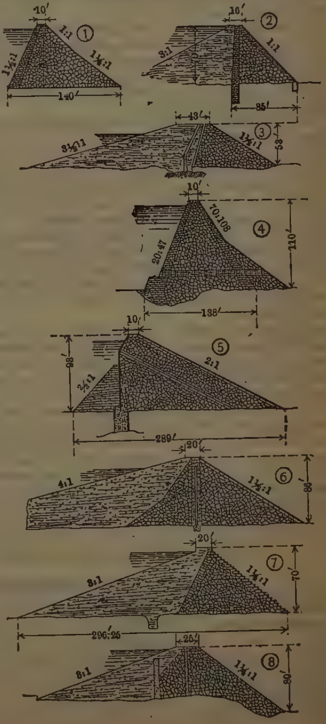


Fig. 16. Rock-fill Dams

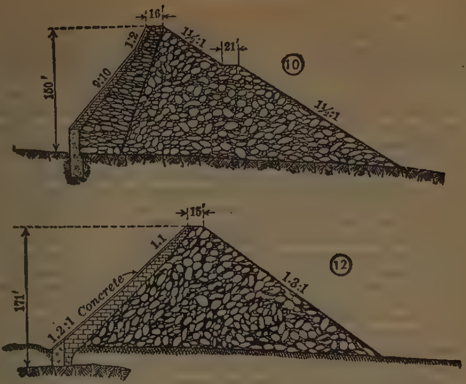


Fig. 16 (continued). Rock Fill Dams

(4) Walnut Grove, Arizona. 1887–88. Destroyed by flood 1890 with loss of 129 lives. Length 400 ft. Slopes much steeper than natural angle of repose of loose rock. Faces laid up as dry walls. With 70 ft of water above bed rock, dam leaked 3.75 cu ft per sec.

(5) East Canyon Creek, Utah. 1899. Originally built 68 ft high. Length 100 ft. Concrete wall 15 ft thick, carried down thru gravel to bed rock and steel plates for core anchored in center. Rock thrown on wall by blasting sides of canyon. Open cut made in pile of rocks thrown down and steel core-wall built from anchor plates. Asphalt concrete 4 in each side of plate. Settlement of wall caused asphalt to draw away from steel an extreme distance of 5 ft at top. Dam cost $60 200 and required 23 000 cu yds rock, 810 cu yds cement concrete, 183 cu yds asphalt concrete, 69 800 lbs of steel and 50 500 ft B.M. of lumber. Enlarged 1900–01 by addition to downstream side. Max. height 93 ft. Additional work required 16 000 cu yds rock, 410 cu yds of masonry and 370 cu yds concrete, 62 000 lbs steel and 20 000 ft B.M. lumber. Length 173 ft. Crest width 10 ft. Steel core-wall continued to top, protected by concrete.

(6) Milner, Idaho. 1903–05. Three dams form one main structure. Rock fill with core of wood. Earth embankment sluiced into voids of rock above core. Main Channel Dam: Length 340 ft. Max. height 86 ft. Crest width rock fill 10 ft. Downstream slope rock fill 3:2; upstream 3:4. Crest width earth fill 10 ft; slope 4:1. Rock fill 39 650 cu yds. Earth fill 58 000 cu yds. Middle Dam: 335 ft long. 81 ft max. height. 42 800 cu yds rock, 62 850 cu yds earth. South Dam: 560 ft long. Max. height 66 ft. 34 700 cu yds rock; 48 000 cu yds earth. Total length of 3 dams, spillway and reg. gates, considered as one structure, 2100 ft.

(7) Zuni, New Mexico. Combination rock and hydraulic fill. Rock fill built as dry wall. Rock fill 720 ft long. Total volume dry wall 40 160 cu yds. Indian labor. Cost rock work $2.50 per cu yd. For sluicing, steam pumping plant with capacity 2.5 sec ft at 90 to 95 lbs per sq in pressure. Total volume earth in dam 60 120 cu yds, about 40 000 yds sluiced in. Total cost for sluicing 12 cents yd. All earth, either very fine sand or clay, no attempt being made to separate them. Undermined 1909 by passage of water under cap of lava rock which flanked dam and extended beneath spillway. Spillway, south abutment and extreme south end of dam undermined. Considerable portion of spillway dropt 7 ft and settlement of 9 ft at junction of spillway and abutment. Small amount of fill at south end washt out and there was 5 ft settlement about 30 ft long in earth fill at north end. Main part of dam appears uninjured.

(8) Minidoka, Idaho. Rock fill, earth facing and concrete core-wall. Length 625 ft. Max. height 80 ft above bed rock and about 60 ft above original bed of stream. Crest

width 25 ft. Base averages about 300 ft. Total volume 191 000 cu yds. Concrete core on solid rock. Top of wall 44 ft below crest in central portions and about 11 ft near end. Earth and rock-fill banks built up and core-wall built in trough between them. Fill then carried up on both sides over core-wall to required height. Cost $425 923.

(9) Lower Otay, Cal. 1897. Originally planned for masonry dam and masonry carried up 8 ft above river bottom. Core of steel plates anchored to masonry and carried up to crest, protected with asphaltum and burlap and 1 ft thickness of concrete either side. Failed January 27, 1916, by flood overtopping the dam. About 30 lives lost.

(10) Morena, Cal., 1896–1909. Length 520 ft. 306 000 cu yds. Built in canyon about 130 ft wide at stream bed with narrow fissure 4 to 16 ft wide, extending 112 ft below. In 1896 a rubble concrete wall 36 ft thick at bottom and 12 ft thick at top was built to seal fissure, and carried up 30 ft above stream bed, when work was suspended in 1898. Construction resumed 1908. Upstream face for width of 7 ft built of 6 to 10 ton granite blocks, set in cement mortar. Behind this for width of 50 ft at bottom, reducing to 16 ft at top, rock was placed by hand and derricks. Remainder of rock-fill dumped by 2 cableways. Reinforced concrete slab 1 ft thick on upstream face.

(11) Swift, Montana. 1914. Upstream face covered with a layer of hand-placed rock, 4 to 6 ft thick, with face plastered with cement mortar. On top of this was placed a concrete slab heavily reinforced, 6 in thick at top of dam and 2 ft at toe. Dam built on curve of 1276 ft radius. Foundation of sand, gravel and boulders.

(12) Strawberry, Cal. 1914–1916. Upstream slope faced with 9 in to 18 in of concrete carried down to concrete cut-off wall and backed by hand-laid rock. Main portion of rock dropped from cableways. River bed a cemented gravel. Dam arched in upstream direction. Radius 1880 ft. Contents 400 000 cu yds. Length 612 ft.

8. Overflow Dams on Earth Foundation

Three Conditions influence the design of overflow dams on earth foundations. First, the character of the earth, ranging from loose fine sand to firm, well cemented hardpan; second, the height of the fall; and third, the volume of water to be past. Designs for dams to meet various combinations of degrees of severity of these conditions may best be studied from precedents. The engineers of India have developed a type for their rivers in alluvial beds subject to great floods. The type has been adopted by the U. S. Reclamation Service for similar conditions, the Laguna being a case in point. One form of the India dam is essentially a rock fill with very flat slopes downstream protected by heavy pitching or concrete-slab construction and containing one or more masonry core or cut-off walls. This type has never been used except for very moderate heights of from 6 to 15 or 20 ft. They have, however, been built on sand foundations and have past great floods. The greatest trouble is usually experienced at the toe where the structure is likely to be undermined. The Avignonet dam is a precedent for a high dam on an earth foundation, but there the earth foundation is compact gravel, fine sand, and bowlders. Following numbers in parentheses refer to Fig. 17.

(1) Narora Weir, India. Brick. Base 8 ft thick. Founded on blocks or wells 10 ft square and 18 in thick sunk 7 ft below river bed. Downstream from weir floor is 40 ft wide, 5 ft thick, consisting of 3 ft 3 in concrete, 9 in brick and 12 in ashlar. Beyond floor talus is 100 ft wide. Crest covered with ashlar. First crest covering destroyed and replaced with masonry set with wider joints. Head on crest 8 ft. End of talus 14 ft below crest.

(2) Dauleshwiram, India. 1840. 16.8 ft head on crest. Length 12 000 ft. Crest 11 ft above normal bed level. Width 230 ft. Crest covered with ashlar masonry.

(3) Srivakantham, India. Length 1380 ft. Discharge over weir 123 000 sec ft. Weir wall and curtain walls on clay which lies under 3 ft bed of sand.

(4) Pelandorai, India. Length 860 ft. Head on crest 13.3 ft. Crest 9 ft above the river bed above weir.

(5) Burma Crib Weir, India. Boards were placed on top at first pending grouting of weir body with silt, and final settlement, with idea of replacing by rubble.

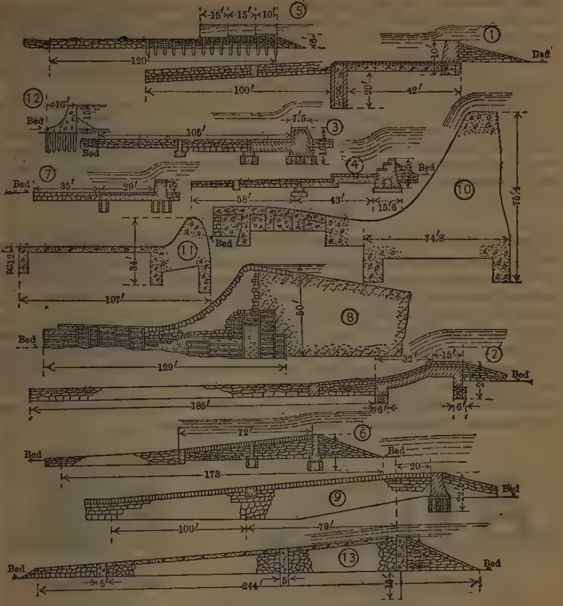


Fig. 17. Overflow Dams on Earth Foundations

(6) Mahanuddee Weir. 6400 ft long. Folding shutter 3 ft high on crest. Calculated to discharge about 900 000 sec ft. Crest 13 ft above average summer water level of river. Face of weir protected by rubble-stone packing with base varying from 20 to 40 ft. Breach 1886 at site of center sluices; about half sluices and portion of weir carried out and deep hole scoured on line of weir and below it. Cause never explained.

(7) Upper Colerum, India. 1836. Length 2929 ft. Head on crest 8.6 ft. Crest 7.4 ft above river bed. River bed, fine sand. Founded on double row of wells sunk 6 ft into river bed on upstream side and single row on downstream side. Masonry floor 27 ft broad, 3 ft thick, 2 ft being brick masonry. Season after completion about 240 ft of weir swept away. Caused by leakage undermining foundation. Weir repaired.

(8) Old Croton. 1837–42. 180 ft long. Mud and bowlders cleaned from river bottom and crib cofferdams built enclosing space where masonry was to be laid. These cribs were left in the foundation. Space between cofferdams excavated to hardpan and filled with concrete and masonry. Earth bank 5 : 1 slope paved near top and with bottom width 275 ft against upstream face. Dam has past 8 ft depth on crest.

(9) Kistna Weir, India. 1854–55. Length 3000 ft weir proper; nearly 4000 ft including under sluices and piers. Crest 20 to 25 ft above original river bed. Flood velocity over weir said to be 16 ft per sec and depth on crest 20 ft. Deep holes that had been scoured in river bed by floods were filled with sand and wells sunk for foundation of weir.

(10) Avignonet, France. 1902. Concrete. Length 197 ft. Curved radius 656 ft. Height 75.5 ft. Built in narrow canyon on bowlders, sand and gravel. 2 cut-offs, 13 ft deep and 8 ft thick. Passes over 35 000 sec ft during flood.

(11) Granite Reef, Arizona. 1906–08 Partly founded on gravel and bowlders 1000 ft long. Three curtain walls. Openings in the wall under lower toe 6 in sq 5 ft apart and about 6 ft above bottom of wall to allow drainage of seepage. About 3-in joints in apron to allow escape of water.

(12) Spooner, Wisconsin. Concrete structure 72 ft long. Foundation sandy soil covered with thin layer of gravel. Pile foundation.

(13) Laguna, Arizona. Begun 1905. Crest 19 ft above natural river bed. Three concrete walls 4800 ft long and 57 and 93 ft apart. Upper wall on row of sheet piling 12 to 20 ft long. Between walls is rock fill. Apron of derrick stone extends 40 ft beyond lower wall. Rock fill between walls covered by 18-in concrete. Approximate quantities: rock fill 350 000 cu yds; concrete paving 37 000 cu yds; concrete core-walls 28 000 cu yds, sheet piling 90 000 lin ft. Discharge from 4000 to 100 000 sec ft.

9. Movable Dams

Needle, Wicket and Curtain Dams form a group of movable dams having many common and numerous interchangeable features. Common to all are upright pieces, the lower ends of which bear against a sill on the river bottom, the upper ends of which bear against some form of bridge. The bridge may be a stationary structure or a collapsible structure. In case needles are used (Fig. 18) they form the closure. In case wickets are used (Fig. 19) the closure is formed by sliding stop gates down grooves in the upright pieces. With curtain dams (Fig. 20) an overhead bridge is generally used, and a screen rolling on the upright pieces is let down from above. When a draw or fixt bridge is used, the upright pieces are hinged at the upper end and are swung up out of the water with suitable machinery.

Fig. 18. Needle Dam

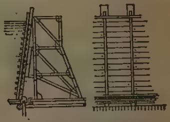


Fig. 19. Wicket Dam

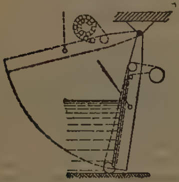


Fig. 20. Curtain Dam

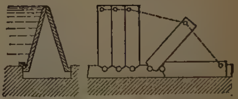


Fig. 21. Thomas A Frame Dam

The Thomas A Frame Dam (Fig. 21) consists of a series of *A* frames set side by side and pivoted at the bottom to a sill. The plates which fasten the two legs of each section together at the top form a footpath. One o these dams 120 ft long and 1 ft high was built as part o Dam No. 6 in the Ohio River

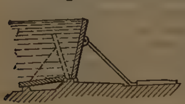


Fig. 22. Thénard Shutter Dam

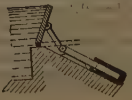


Fig. 23. Girard Shutter

The Thénard Shutter Dar (Fig. 22) consists of leave hinged to a sill which extend

across the channel. The leaves are raised and supported by struts hinged to the leaf and at the lower end fitting into a socket from which they may be tript and the leaf lowered. Above is a counter shutter which is raised by the current as a temporary dam until the main dam can be raised by hand. A dam of this type, built in France and finished in 1843, has shutters 5.6 ft high and 3.9 ft wide. Others have been built in India to close flushing sluices, and some of them support heads of nearly 10 ft. The counter shutter is held in position by chains, which on account of the severe shocks occurring when the gate swings up, often break and are objectionable.

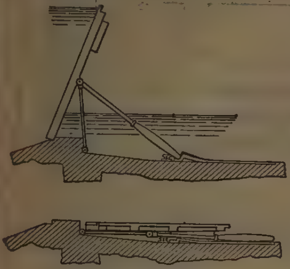


Fig. 24. Chanoine Wicket

The Girard Shutter (Fig. 23), invented as a substitute for the Thénard, does away with the counter shutter, and the main dam is raised directly by hydraulic pressure. Seven of these were erected at Auxerre, the shutters of which were 11½ ft wide and 6½ ft high.

The Chanoine Wicket (Fig. 24) was invented in 1852. The shutter is pivoted a little below its center point to a collapsible horse held in place by a long prop. It is adapted to large rivers where the flood rises are very sudden.

The Rolling Dam (Fig. 25) consists of a large steel cylinder placed across the current between piers or abutments and arranged to be rolled up entirely clear of the current on an inclined rack track. They are operated by cables wrapt around one or both ends of the cylinder and passing to a winch. They have been built for openings up to 115 by 6½ ft and 60 by 14 ft and are said to operate very quickly and pass ice readily.

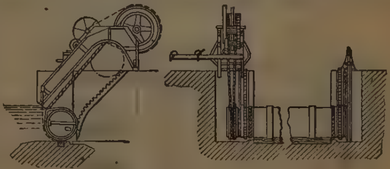


Fig. 25. Rolling Dam

Stoney Sluice Gates (Fig. 26) consist of a gate bearing at its ends against a train of rollers. The gate is usually hung to chains at either end, which pass over sheaves to a counter-weight. The roller train is also hung by a cable and counter-weighted. They are usually operated by gearing on the supporting sheaves turned by hand cranks or traveling electric motors. These gates have given excellent satisfaction and are widely used. Power for operating has usually been underestimated. Fig. 26 is an example of Stoney sluice gate at the lake regulating works, Sault Ste Marie, Canada. Fig. 27*a* shows the detail of the end bearing Fig. 27*b* shows the end bearing of the Stoney gates on the Manchester Canal, Eng.

On the New York Barge Canal several movable dams of the bridge type have been used (Fig. 65*a*). These dams have abutments, piers and superstructures

like ordinary bridges: from the downstream sides hang steel frames resting against shoes in concrete sills that stretch across the stream between the abut-

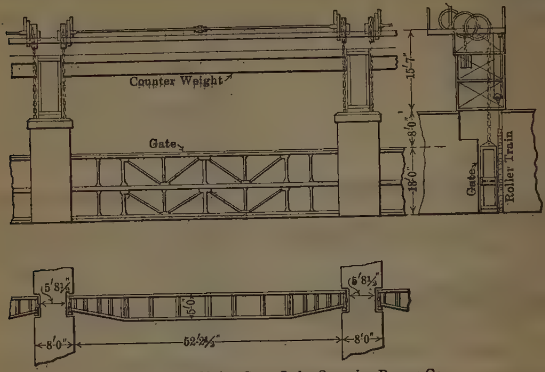


Fig. 26. Stoney Sluice Gate, Lake Superior Power Co.

ment and pier. Against the upright frames are placed "Boule" gates formin the dam proper. Electric winches raise the gates and uprights by chain The spans vary from 150 ft to 240 ft.

The Bear Trap Dam (Fig. 28), of which there have been many mod fications, consists of two rectangular leaves extending over the full width the opening, an when in their low position the u stream leaf overla the downstream lea The gate is rais by admitting wat from the upper po to a chamber und the leaves. A da built at Davis Isla on the Ohio Riv in 1889 was of th type. The ga were of wood a 52 ft long. In 19 this was replaced

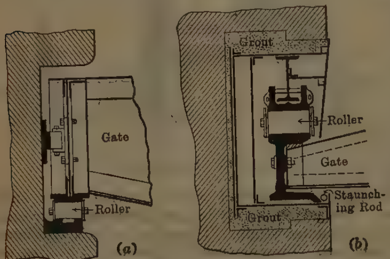


Fig. 27. Ends of Stoney Gates

a new bear trap with wooden leaves heavily bound with steel. The n leaves were much stronger than the old ones and were proportioned so th they would just float without the need of air forced to the underside. At t bear trap on the Marne in France a counter shutter was provided, as in t Thénard shutter dam, as an assistance to raising the main dam.

The Bear Trap on the Chicago Drainage Canal (Fig. 29) is of somewh different design. The two leaves do not overlap, but are hinged togeth

at the apex. The downstream edge of the dam is hinged to the foundation, while the upstream edge carries a roller and moves up and down on the masonry face. The gate is counterweighted and its movements up and down are controlled by hydraulic cylinders. The dam is 160 ft long, the downstream leaf 35 ft wide and the upstream leaf 21 feet wide.

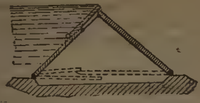


Fig. 28. Bear Trap Dam

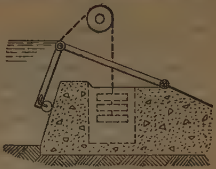


Fig. 29

The Parker Gate (Fig. 30) modifies the original bear trap by hinging the two leaves at the apex and introducing a hinge at a little above the mid-point of the upstream leaf; this does away with the sliding friction between the leaves. An auxiliary leaf, called the idler, is introduced above the upstream leaf to protect the moving parts from drift and ice. A dam for the Muscle Shoals Canal in Tennessee was built in 1892, 49 ft wide and 8.5 ft high.

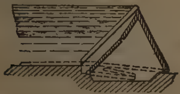


Fig. 30. Parker Gate

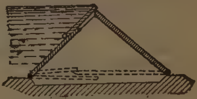


Fig. 31. Lang Gate

The Lang Gate (Fig. 31) is a modification of the Parker gate, in which the section of the upstream leaf above the hinge is removed and rods or chains take its place. The auxiliary leaf, or idler, is hinged to the downstream leaf, and its lower edge either slides or rolls on the upstream leaf. Several Lang gates, reaching in size to 80 ft long and 14 ft high, have been built in America.

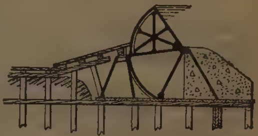


Fig. 32. Chittenden-Drum Dam

The Chittenden-Drum Dam (Fig. 32) consists of a gate in the form of a sector of a circle, with a central angle of 67½°, hinged at its center, and a water-tight chamber into which it fits snugly when down. Water admitted to this chamber from the upper pool raises the dam. A dam of this type was constructed on the Osage River, Missouri, in 1901, with a difference in head of 16 ft between the upper and lower pools.

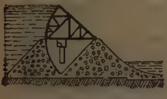


Fig. 33. Drum Dam, Chicago Drainage Canal

The Drum Dam of the Chicago Drainage Canal, Fig. 33, consists of a 45° sector of a cylinder 52 ft in diameter, having its face covered with steel plates carried by radial frames pivoted at the center on the downstream side and a water-tight chamber into which it fits when down. Dam raised or lowered by varying the head of water in the chamber, for which special valves are provided. This dam differs from earlier forms of sector dams in being pivoted on the downstream side. The vertical range of move-

ment is 18 ft. A drum dam similar to that noted in Fig. 33 has been built across the Genesee River at Rochester with a vertical range of 9.4 ft and each section 53 ft 11 in wide.

The Taintor Gate is similar to the drum dam (Fig. 32) except that the axis is higher and on the downstream side, and to open it, it is usually lifted clear of the water by cables and winches. It is often used on power dams. The U. S. Government built at Sterling, Ill., gates 21 ft wide and 13½ ft high. On the New York Barge Canal there are several Taintor gates, the largest having a lift of 14.3 ft and a width of 48 ft.

Data of Needle Dams

Name or location	Date	Height trestle, feet	Width of base, feet	Distance c to c frames, feet	Lift, feet	Weight of each trestle, lbs	Weight of needles, lbs
Original Poiree..	1834	6.25	4.92	3.25	3.25	220	5
Belgian Meuse...	......	13.17	8.33	3.92	8.25	800	55
Louisa Ky. Pass.	1896	15.17	9.85	4.00	13.00*	1145	263
Louisa Ky. Weir.	1896	9.67	6.42	8.00	7.00	920	80
Klecan Moldau, Bohemia......	1900	12.10 to 15.40	8.25	4.10	13.00	....	46 to 72

* = On Sill.

Stoney Sluice Gates

Name and location	Date	Number gates	Height gates, feet	Clear width, feet	Clear lift opening, feet
Belleek, Ireland *	1883	4	14.50	29.17	9.00
Manchester, England †	1892–94	30	13–26	30.00 and 20.00	13–20 13–26
Richmond, England ‡	1892–94	3	12.00	66.00	....
Glasgow, Scotland	......	3	12.00	80.00	....
Hagneck, Switzerland	1895–97	2	21.30	32.80	....
		1	9.80	41.00	....
Beznau	1898–00	6	20.70	52.50	....
Chevres	......	....	27.90	32.80	....
Rheinfelden	1895–98	3	16.40	32.80	....
Lauffenburg	1906	1	55.80	65.60	....
		2	41.00	47.60	
		1	41.00	65.60	
Electrical Works at Zurich	1906	....	26.20	49.20	....
Assouan Dam	1898–03	25	11.50	6.60	....
Assiout	1898–03	111	16.00	16.40	....
Sault Ste. Marie Canal, Mich. §	......	4	26.79	48.00	....
Sault Ste. Marie Regulation Works ‖	......	4	13.00	52.21	14.25
Chicago Drainage Canal ¶	......	8	20.00	30.00	....
Minneapolis	1906	....	18.00	16.30	....

* One man can easily operate. Distance between roller bearings 31 ft.

† Some of gates designed to withstand 26 ft head. Rollers 7½ in diameter.

‡ Weight each gate 32 tons. Water pressure about 100 tons per gate. Can readily be raised to full height by 2 men in 7 min. Guides arranged so as to turn gates horizontally as raised so as to be out of sight under bridge. 15 rollers.

§ 32 rollers per train. 7 in diam. Medium steel.

‖ Rollers 5 in diam. 24 rollers in one train. Medium steel. 2 trains per gate.

¶ Rollers of pin steel with brass bushings. Diam. rollers 6 in. 30 rollers per train.

Chanoine Wicket Dams

		Pass of Port à l'Anglais, Paris, 1870	Passes of upper Seine prior to 1870	Weirs of Belgian Meuse, about 1876	Passes of Kanawha River 1896	Weirs of Kanawha River, 1896	Passes of Ohio River, 1899
Props	Depth on sill, ft	11.83	9.83	7.42	13.00	8.50	13.17
Props	Diam. of prop, ft	0.29	0.29	0.29	0.30	0.25	0.29
Props	Length of prop, ft	11.83	8.83	6.25	12.67	7.83	14.58
	Braces	2 h*	2 h	....	2 h	1 d*	2 d
Horses	Diam. of upper journals, ft c*	0.42	....	0.20	0.46	0.18	0.41
Horses	Diam. of upper journals, ft e*	0.19	0.20	0.20	0.25	0.17	0.23
Horses	Diam. of lower journals, ft c*	0.26	....	....	0.21	0.19	0.24
Horses	Diam. of lower journals, ft e*	0.19	....	0.20	0.21	0.15	0.24
Horses	Bar iron uprights, ft	0.26 by 0.14	0.21 by 0.10	0.20 by 0.13	†2–0.25	0.17 by 0.12	0.25 by 0.12
Wickets	Width of wickets, ft	3.28	....	....	3.75	3.75	3.75
Wickets	No. uprights	2	....	....	2	2	2
Wickets	Width uprights, ft	1.00	....	....	1.00	1.00	1.00
Wickets	Thickness uprights, ft	0.67	....	....	0.75	0.50	0.83
Wickets	Thickness planking, ft	0.17	....	....	0.17	0.12	0.17

* c = center, e = end, d = diagonal, h = horizontal. † Two channels back to back.

AQUEDUCTS

10. Reservoir Outlets

Outlets for all reservoirs are preferably put in the natural formation outside the dam whether earth or masonry. In masonry dams they weaken the section somewhat, changing the distribution of stresses, and if the opening is large the change of section may concentrate shrinkage and temperature cracks. In earth dams the inevitable settlement of the embankment is liable

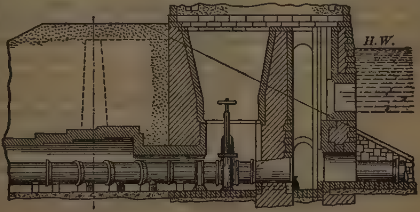


Fig. 34. Gatehouse, Jerome Park Reservoir

to break outlet pipes. They are sometimes carried thru tunnels in the adjacent hillsides, but usually this is impracticable and the common practise is to carry them thru the dam. If the dam is masonry, it is strengthened to compensate. If the dam is earth and high, the outlet may be placed in the natural earth below, where, if properly built, they are reasonably safe from dangers due to settlement. In low earth dams they are usually carried directly thru the

embankment. The fact that many failures of earth dams have been due to leaky outlet pipes demonstrates that the outlet pipes should not be in contact with the earth but placed in a masonry conduit, as in Fig. 34, where they may be inspected frequently and where any leakage will be quickly shown. Where the head of water in the outlet is small it is usually placed directly in the earth. This is the case when the embankment is low or when the gatehouse

Fig. 35. Outlet Borden Brook Reservoir

is above the core-wall and the outlet below is without pressure. Fig. 35 shows one of the latest designs of this type.

Gatehouses. For domestic water supply it is usually desirable to be able to draw water from either the top, bottom or mid-depth of a reservoir. It is desirable to have at each outlet opening a gate or valve which may be quickly operated and in addition a second emergency gate to be used in case the other is out of order, and a fish and trash screen. Fig. 36 shows the gatehouse of the Boonton dam, which is typical of many water-supply reservoirs. Where the dam is of earth the gatehouse is put either in the dam, as in the case of the Jerome Park Reservoir (Fig. 34), or as an intake tower built inside the reservoir, as at the San Mateo dam (Fig. 37). In the San Mateo dam the outlet is thru a natural hill. The former method complicates the construction of the dam, and the latter is more subject to ice pressure. Fig. 35 shows the gate house of the Borden Brook Reservoir, which is placed above the core-wall and protected from ice by thickening the upper part of the dam at that point. The fish and trash screens are usually copper netting having about 6 meshes to

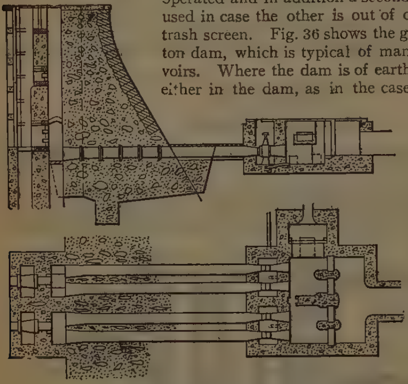


Fig. 36. Gatehouse, Boonton Dam

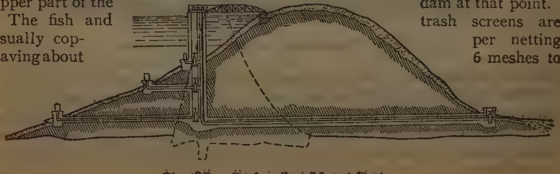


Fig. 37. Outlet, San Mateo Dam

the inch, with wire about $\frac{1}{16}$ in in diameter made up in frames which are slid into grooves and may be easily removed.

Fishways. The laws of many states require a fish-ladder or fishway to be built at dams. Fig. 38 shows the fish ladder recommended by the U. S. Commission of Fish and Fisheries. The cut shows two pools of the ladder which may be multiplied to surmount any desired height. Each pool is 1½ ft higher than the one below, 4 ft wide, 6½ ft long on one side, and 4½ ft on the other. The opening in the bulkheads between pools should be about one foot square at the lower pool, increasing in size toward the upper level to insure a waterfall over the bulkhead; the amount of increase will depend upon the leakage.

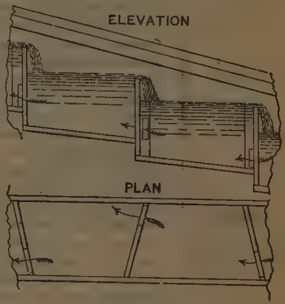


Fig. 38. Fishway

11. Flow Line Masonry Aqueducts

Closed Masonry Aqueducts have been used to convey water from distant sources to great cities for more than 2000 years. The first Roman aqueduct was built in the year 312 B.C. Altho inverted siphons were known to the ancients, the materials available for their construction, principally lead and earthenware, were not such as to admit of great success. Consequently, the first great aqueducts were built of masonry, either stone or concrete, on side hills, following nearly the contour of the ground, in tunnels, where it became necessary to pierce sudden rises, and on masonry arches over depressions, where it was impossible to follow the ruling gradient on the ground surface. Most of the flow line aqueducts constructed since 1900 have been of concrete of horseshoe shape built in open cut and back-filled. Where the detour would be too great to carry the flow line around valleys, inverted siphons of steel pipe or pressure tunnels are introduced. In some cases for short distances the flow line with the horseshoe cross section has been carried across depressions on filled ground Masonry aqueducts constructed on embankments require great care in securing an unyielding foundation, since even a slight settlement may cause cracks and leaks A crack below the water-line may erode the embankment materials and possibly give rise to destruction of a portion of the masonry with serious consequences A small quantity of steel reinforcement in the invert of concrete sections on embankment will increase the ability of the structure to resist the ill-effects of slight settlement where the construction of the embankment type cannot reasonably be avoided. Elevated crossings of valleys and streams are seldom resorted to. The Assabet Bridge 359 ft long on the Wachusett Aqueduct completed in 1898, is the most notable case of a recent bridge crossing. The horse-shoe shape is best adapted to resisting the pressure from the back-fill and to ease of construction, cleaning and maintenance and at the same time has fairly good hydraulic properties.

Contraction Joints. The practise in flow-line conduit construction is to place the concrete continuously in sections with a contraction joint, designed to prevent leakage between sections. The Catskill Aqueduct developed transverse cracks in some cases in those sections where the contraction joints were 75 ft

apart, but none in sections which were 60 ft or less in length. Two types of contraction joints were tried indicated by *A* and *B* in Fig. 39. Type *C* has been tried elsewhere with indifferent success, because of the construction difficulties of properly placing in the masonry of the curved sidewalls the thin bent strips of non-corrosive metal In the Catskill Aqueduct, type *A* was found unsatisfactory because of the practical difficulty of making the tongues and grooves sufficiently exact for sliding, the tongues breaking off in some cases. Type *B*, a steel plate coated with asphaltum, gave in general good results, although the concrete was cracked by the plate in pulling in a relatively small number of joints, due to improper placing of the plate or to other defects of construction. However, all of the joints, of whatever type of construction and condition, were stopped when open the maximum amount at the lower masonry temperatures in the cold season Several kinds of caulking materials were used with more or less satisfactory results, but the most successful treatment was the complete filling of the joint with Portland cement grout. Tests made subsequent to the grouting indicated that the outward leakage from the whole length of aqueduct, when carrying the maximum quantity, would not exceed one-quarter of one per cent of the capacity. For lesser quantities and aqueduct depths the leakage will, of course, be less, and moreover it has been the experience with this aqueduct, as well as with similar structures elsewhere, that the masonry tightens in service with consequent reduction in leakage.

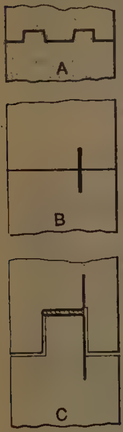


Fig 39

Location and name	Date built	Length, miles	Height, feet	Width feet	Capacity, sec ft	Grade ft per 1000
(1) Boston, Mass. Cochituate	1848	14 60	6 33	5 00	27 9	0 05
(2) Boston Mass, Sudbury	1878	15 90	7 67	9 00	155 0	0 19
(3) Boston, Mass, Wachusett	1898	11 95	10 50	11 50	465 0	0 40
(4) Boston, Mass, Weston	1904	13 44	9 25	10 00	465 0	0 80
	....	......	12 17	13.17		9.20
(5) Brooklyn, N Y.	1859	12 40	6 33	8 17	71 3	0 10
	....	......	8 67	10 00	......	....
(6) Brooklyn, N. Y.	1891	7 40	6 92	9 33	91.3	0 10
	....	......	5 92	7 33	62 8	......
(7) Birmingham, Eng, Elan	...	73 37	8 00	7 66	116 0	......
(8) Glasgow, Scotland	1859	25 75	8 00	8 00	65 1	0 16
(9) Glasgow, Scotland	1894	23 50	9 00	12 00	77 5	0 18
(10) Los Angeles, Cal *	1913	233 76	8 25	7 50	430 0	2 00
	...	....	10 28	9 00	430 0	0 40
(11) Manchester, Eng., Thirlmere	1894	95 88	7.00	7 08	93 0	0 31
(12) New York, N.Y., Old Croton	1842	38 14	8 46	7 42	147.2	0 21
(13) New York, N Y, N Croton	1890	33 25	13 53	13 60	465 0	0 13
(14) New York, N Y., Catskill	1916	110 00	17 00	17 50	775 0	0 37
(15) Rochester Intake, N Y.	......	2 27	6 00	6 00	23.2	0.25
(16) Vienna — 2d	1910	114 00	6 71	6 42	82 0	2.74
	......	....	8 21	6.42	....	.....
(17) Washington, D. C, Potomac	1863	11 00	9 00	9.00	118 6	0.15
(18) Apulian, Italy	1915	151.00	10 3	9.3	....	0.25
	......	....	8.7	5.4	....	...
(19) Winnipeg, Canada	1918	97 00	9 0	10.75	131 0	0 60
	......	....	5.4	6.4	....	....

* Sections given are typical.

Flow-line Masonry Aqueducts.

Numbers in parentheses in the table on page 1322 refer to Fig. 40 and to the accompanying notes.

(1) Lining is brick thruout.

(2) Lining is brick and rubble. Cost per lin ft, \$23.90.

(3) Lining is concrete with brick-lined invert. 2 miles in tunnels, 6.9 miles in cut and cover or embankment, and last three miles in open channel. Cost per lin ft \$22.90.

(4) Lining is concrete with brick-lined invert. 9.14 miles are in cut and cover or embankment, 1.02 miles are in open channel, 2.30 miles are in tunnel and 0.98 mile is in cast-iron pipe siphons. Cost per lin ft \$32.60.

(5) Arch and invert are brick, walls and backing rubble. Invert on concrete cradle.

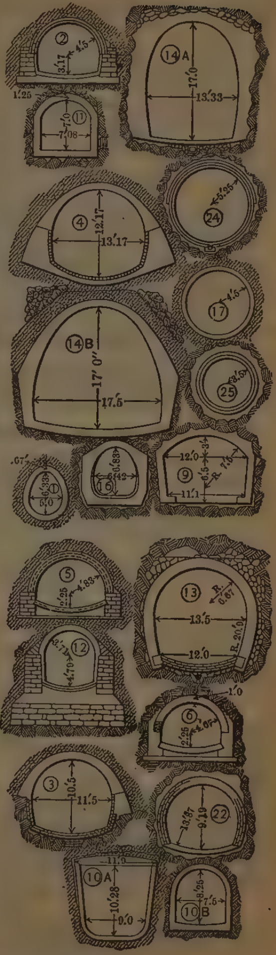
(6) Arch and invert are brick, walls and backing are rubble. Invert placed on concrete cradle. Average depth of cutting 16 ft. Maximum depth of cutting 23.9 ft.

(7) Lining is blue brick. About 11½ miles are in tunnel 25 miles in cut and cover and the remainder in cast-iron pipe siphons.

(8) Part of aqueduct was lined with masonry. About 10 miles are in open channel, 12 miles in tunnel and 4 miles in siphons.

(9) Lining is concrete. 1.42 miles are in cut and cover or on embankment, 18.97 miles in tunnel and 3.10 miles in siphons.

(10) 164 tunnels with combined length of 42.7



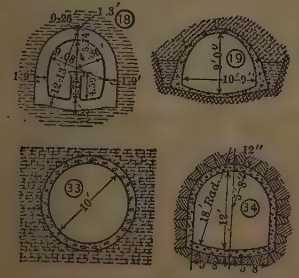


Fig. 40. Flow Line Aqueducts.

miles, not including 9 2 miles of power tunnel. Open unlined canals 21.1 miles Open lined canals 39 56 miles, concrete covered conduit 97.72 miles Siphons, steel and concrete 12.06 miles.

(11) The lining in tunnel and in cut and cover is concrete. 36 75 miles are in cut and cover, 14.12 miles in tunnel and 45.01 miles are in 40-in cast-iron siphons. Cost of whole work per lin ft $42 00

(12) 37 1 miles are in cut and cover or on embankment, the lining being brick backed with rubble, 0 8 mile is in 4-36 in cast-iron pipes, and 0.3 mile is in pipes carried on High Bridge over the Harlem River. Average cost was about $59.00 per lin ft.

(13) Lining is brick backed with rubble. 29 75 miles are in tunnel of which 7.17 miles are siphons, 1.12 miles are in cut and cover and the last 2.38 miles are in cast-iron pipe. Cost about $112.00 per lin ft.

(14) Lining is concrete even in siphons. 54 miles of flow line sections, 14*A* and 14*B* in cut, and cover at grade of 21 ft per thousand. 0 5 mile is in embankment, 13 9 miles are in grade tunnel, 35 miles in pressure tunnel and 6 3 miles in siphons.

(15) Concrete. 1.42 miles are in tunnel and 0.8 mile is in cut and cover.

(16) Concrete is used quite generally. 44 miles are in tunnels, the longest being 17 630 ft; there are also several iron pipe siphons, the longest of which is 6 miles, and a masonry arch bridge carrying a concrete aqueduct.

(17) Lining is brick and rubble masonry. 9.51 miles are in cut and cover or on embankment and 1 49 miles are in tunnel.

(18) Concrete. 99 tunnels of an aggregate length of 60 miles, the longest being 12 miles. 85 bridges.

(19) Concrete conduit in cut and cover or on embankment. Part on pile foundation. River crossing made by reinforced concrete siphons. Supply line from reservoir to city, a 5 ft 6 in diameter reinforced concrete pipe 10 miles in length.

12. Siphon and Pressure Tunnels

Data regarding siphons and pressure tunnels are given below, the numbers in parentheses referring to the notes and to Fig 40 The first pressure tunnel

Data on Siphons and Pressure Tunnels

Name and Aqueduct	Date built	Diam. feet	Length, miles	Equivalent grade in ft per 1000	Max internal head, ft	Max. external head, ft	Capacity sec ft
(20) Harlem River, New Croton, N. Y.	1890	10 5	7 17	2 90	431	304	465
(21) Milwaukee, Wis	1895	7 5	0 60	.....	115	115	.....
(22) Washington Extension, D. C	1902	9 87	3 92	....	175	......	......
(23) Jersey City, N. J.	1904	6.00	10 25	....	230	50	108 5
(24) Torresdale, Phila, Pa.	1904	10 50	2.61	0.75	....	99	465 0
(25) Cincinnati, Ohio	1905	7 0	4 21	0 51	170	95	155 0
(26) Buffalo, N. Y.	1910	......	1 95	....	50	50	......
(27) Rondout, Catskill, N. Y.*	1916	14 5	4 47	0 68	726	487	775
(28) Wallkill, Catskill, N.Y.	1916	14 5	4 43	0 66	544	265	775
(29) Moodna, Catskill, N.Y.	1916	14 2	5 68	0 80	630	240	775
(30) Hudson, Catskill, N. Y.	1916	14 [illegible]			1517	1100	775
(31) Yonkers, Catskill, N. Y.	1916	16 6	2.46	....	155	100	775
(32) City Tunnel, Catskill, N. Y.	1916	11 0-15 0	18.11	....	1005	710	775
(33) Cleveland	1917	10 0	3.00	....	99	99	....
(34) Chicago, Ill.	1918	.....	8 00	....	130	130	....

* Section shown for Rondout typical for all pressure tunnels on the Catskill system.

of note was the Harlem River crossing on the New Croton Aqueduct, and this was lined with brick.

(20) Water Supply, N. Y. City. Rock tunnel lined with brick, backed by rubble masonry. The face of the brick lining was washed with three coats of neat cement mortar.

(21) Milwaukee Water Works, Wisconsin. Hard clay and water-bearing sand and gravel. Compressed air used. No timbering used in hard clay. Progress averaged 6¾ ft per day in clay. Drills and explosives used in hard clay. Circular section lined with 4 rings of brick.

(22) Water Supply. Rock tunnel lined with brick backed by rubble. Length of 500 ft is iron lined. Average cover is 150 ft.

(23) Water Supply, Jersey City, N. J. Steel plate pipe $^5/_{16}$ in to $^7/_{16}$ in thick except at crossing of Hackensack and Passaic Rivers where the thickness is $^{11}/_{16}$ in.

(24) Water Supply. Rock tunnel lined with 13½ in of brick in the rock and 9 in in the invert, backed with a minimum of 6 in of concrete. The maximum cover is 115 ft.

(25) Water Supply, Cincinnati. Rock tunnel lined with two rings of brick backed by a minimum of 6 in of concrete. Average cover over tunnel is 130 ft.

(26) Water Supply. Rock tunnel, concrete lined 1 to 2 ft thick. Extends 6000 ft under Lake Erie, inside dimensions 12 by 11 ft 3 in. Land tunnel 4300 ft long inside dimensions 9 by 8 ft.

(27) Rock tunnel driven from 8 shafts by top heading and bench method, known as American method. Maximum progress (full section) 488½ ft per month. Four 3¼ in Ingersoll-Rand drills on 2 vertical columns and 2 on bench. 22 holes, 8 to 12 ft deep per round in heading. 175 to 200 lbs of 60% dynamite per round. Concrete lining 1 : 2 : 4, 15 to 17 in thick. Steel forms. Approximate cost to city per lin ft based on contract prices: Tunnel \$180, shaft \$285 in rock and \$350 in earth. Minimum rock cover 180. Maximum depth 710 ft.

(28) Rock tunnel driven from 6 shafts in Hudson River shale, by American method. 4 air drills in heading and 2 on bench. 22 to 24 holes per round. Muck hauled by mules and electric locomotives. Maximum monthly advance one face 523 ft. Ventilation by blowers. Concrete lining 15 to 17 in thick.

(29) Tunnel in shale and granite. 7 shafts Concrete lining 13 to 15 in thick.

(30) Tunnel in sound granite, 1100 ft below surface of river. Hydraulic gradient 400 ft above river. Shaft at either end. Concrete lining 13 to 15 in thick. Water bearing fissure sealed by building concrete bulkhead 8 ft thick heavily reinforced with rails and grout, under pressures up to 1060 lbs per sq in forced through pipes set in bulkhead and leading to fissures.

(31) Tunnel in gneiss, driven from 4 shafts. Concrete lining 15 to 17 in thick.

(32) Tunnel under New York City in gneiss, schist and limestone from 200 to 750 ft below surface. Driven from 25 shafts by American method. Electrically operated machinery generally used. Ventilation by blowers. Cost of excavation per lin ft tunnel based on contract prices varied from \$86 for 11 ft diameter tunnel to \$90 per 15 ft diameter tunnel. Lining 11 to 19 in thick of 1 : 1½ : 3 concrete. Cost of lining based on bid prices varied from \$51 for 11 ft diameter tunnel to \$52 for 15 ft diameter.

(33) Waterworks tunnel in stiff clay under Lake Erie, driven by hydraulic shield. Excavation for about half the length of tunnel done by Rotary cutting machine which was abandoned and rest of work done by hand. Maximum monthly advance 886 ft. Considerable trouble with inflammable gas. Tunnel lined with concrete blocks 11 in thick radially by 18 in wide, six segment blocks and keys to the ring, blocks weighing 1200 lbs each.

(34) Water works tunnel in rock, 3 miles under Lake Michigan and 5 miles under Wilson Avenue, Chicago. 4 shafts. The top heading and bench method and bottom heading method were tried and the former proved more economical. Maximum monthly progress one heading 630 ft Excavated rock crushed and screened and made into concrete, mixed in the tunnel and placed by pneumatic process back of steel forms. Tunnel of horseshoe section; 5883 lin ft of lake tunnel, 12 ft 3 in wide and 13 ft high. Rest, 11 ft 4 in wide and 12 ft high. Lining 12 in thick. Cost per lin ft. \$61.48 done by city force account.

There are several cast-iron siphons not included in the above. The aqueduct for the Liverpool and Manchester Water Supply have siphons consisting of from three to five 39 in and 42 in cast-iron pipes laid side by side under heads of 300 to 480 ft. The new Croton aqueduct passes under the 125th Street valley in 8 cast-iron pipes 48 in in diameter

laid side by side. The Catskill aqueduct crossing under the Narrows is a 36 in diameter cast-iron pipe.

13. Reinforced Concrete Conduits

Data regarding reinforced concrete conduits are given below, the numbers in parentheses referring to the following notes and to Fig. 41. In the Salt Lake City conduit the joints at the end of the sections were thoroly grouted in the coldest weather and the concrete was thus thrown into compression, effectually cutting off leakage. In the Jersey City aqueduct fine cracks appeared at the joints, but the leakage was not serious, and eventually they silted up. The Cottonwood and Pinto pressure pipes were built without any provision being made for expansion joints, the ends of section being left rough for the purpose of affording a good bond with the next day's work. Numerous cracks developed which were from time to time repaired, with the result that, one year after their completion, test showed no leakage in the two Cottonwood pipes; in the north Pinto pipe a leakage of 0.002 sec ft, and in the south Pinto pipe 0.03 sec ft, all tests being made under maximum head.

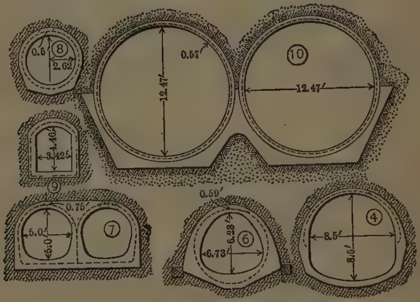


Fig. 41. Reinforced Concrete Conduits

Name and location	Date built	Length, miles	Height, feet	Width, feet	Capacity, cu ft per sec	Grade in ft per 1000	Min. thickness, ins
(1) Champ Isere, France	......	1.30	10.82	10.82	......	......	7.90
(2) Cottonwood, Arizona	1906	0.20	5.25	5.25	250.0	1.05	7.00
(3) Eastern Col. Power Co.	1909	12.00	3.00	3.00	......	4.92	3.00
(4) Jersey City, N. J.	1904	3.59	8.50	8.50	108.5	0.09	5.00
(5) Kensico By-Pass, N. Y.	1916	2.21	11.00	11.00	......	2.56	12.0
(6) City of Mexico, Mexico	1906	17.00	6.23	6.72	77.5	0.30	7.12
(7) Newark, N. J.	......	1.33	5.00	5.00	......	......	....
(8) Pinto, Arizona	1906	0.92	5.25	5.25	250.0	1.23	6.00
(9) Salt Lake City, Utah	1906	7.20	4.46	3.42	48.0	1.20 to 1.50	6.00
(10) Sosa & Ribabona, Spain	1909	0.61	12.50	12.50	1240.0	......	6.89
(11) Umatilla, Oregon	......	0.89	3.92	3.92	......	......	2.50
(12) Albelda, Spain	......	0.45	13.12	13.12	......	......	....

(1) Hydro-electric plant. Hoops and longitudinal rods forming a 4-in mesh used fo[r] reinforcement. Maximum head sustained 65.6 ft.

(2) Power pipe lines. This is a double line siphon with a maximum internal head of 75 ft and an external head not exceeding 10 ft. Reinforced with ⅝-in steel hoops spaced 3 in c to c and 10 longitudinal rods.

(3) Pressure Pipe. In a trench 4 ft wide and back-filled with a minimum cover of 30 in. Used for siphons up to a head of 105 ft. Reinforced with hoops of No. 5 steel wire spaced 12 in c to c. 7 tunnels on the line aggregate in length 1500 ft. Pipes were cast in lengths of 2 ft before being placed in trench.

(4) Flow-line water supply aqueduct. Reinforced with transverse ⅝-in twisted steel rods 12 in c to c and longitudinal ¼-in twisted steel rods 24 in c to c. Portion of the 22.99 miles of aqueduct from Boonton to Jersey City.

(5) By-pass on Catskill Aqueduct at Kensico reservoir. Maximum depth of invert below flow line of reservoir in which by-pass is located is about 53 ft.

(6) Flow-line water supply aqueduct. The reinforcement is expanded metal. Conduit entirely built in cut and cover.

(7) In Cedar Grove Reservoir. Maximum head inside 50 ft and outside 50 ft depending on whether pipe or reservoir is empty. Reinforcement is No. 10 gage expanded metal with three-inch mesh and joints lapt. 4000 ft are single conduit and 1500 ft are double conduit.

(8) Power pipe lines. Double-line siphon with maximum internal head of 31 ft and external head not exceeding 10 ft. Reinforcement is ⅝-in steel hoops placed 6 in c to c and 6 longitudinal rods.

(9) Big Cottonwood Water Supply, flow-line aqueduct. Part of conduit was built in cut and cover, part in fill and part in tunnel. The reinforcement is twisted steel bars ⅝-in to ½-in in diameter, space 6 in to 9 in c to c. Cost was about $9.80 per lin ft.

(10) Consist of a steel tube 0.118 in thick, embedded between an inner mortar coat 0.9 in thick, plastered on wire mesh, and an outer concrete shell 5.9 in thick reinforced by hoops of T iron, all in shallow trench, maximum head 85 ft.

(11) A siphon on the Umatilla irrigation system. The maximum head is 55 ft. The reinforcement is a coil of 5/16-in wire.

(12) Same irrigation system as Sosa and Ribabona. Maximum pressure head 97 ft. Shell of pipe 7.28 in of concrete faced with 0.59 in cement mortar. Reinforcement 124 longitudinal round bars 4 in apart and circumferential T bars. Concrete, 1 Portland cement: 1.28 sand: 2.56 gravel under 1¼ in and 0.58 to 1.00 part of water, all by volume. Lining 1 cement: 1 sand. Seepage loss under full head 0.14 sec ft.

14. Wooden Stave Pipe

Stave Pipe is much used in Western United States on account of the cheapness of lumber and the high cost of steel and cement. It is built in place by assembling the staves for the lower half in a cradle, after which a pipe ring is set inside and the staves for the upper half assembled on it. The staves all break joints with those adjoining and are "driven home" on the end. Bands are placed but not finally tightened until the wood is thoroly saturated. Junctions are made by cutting the last staves a little long and springing them into place.

The Staves vary between 1¼ in and 2½ in in thickness and 6 to 8 in in width and are uniform thruout the length of the pipe, the variation of head being met by variation in spacing of bands. The staves are shaped to true cylindrical surfaces on the inside and to true radial lines on edges. The outside is usually curved, but sometimes flat. A very shallow bead left on the edges crushes and results in greater water-tightness. Thin steel tongues let into the ends of the staves make tight joints and insure a smooth interior surface. The timbers most used are California redwood, Oregon and Washington fir, yellow pine and several kinds of spruces. The timber should be clear, somewhat seasoned and protected from warping.

Bands are from ⅜ in to ¾ in in diameter. They are proportioned for the initial tension on erection, plus the stress from water pressure plus the stress from swelling wood. It is generally agreed that a pressure sufficient to keep

Name and location	Dia. ins	Total length of pipe line, miles	Length of wood stave pipe, miles	Capacity, cu ft per sec	Thickness of staves, ins	Diam. of bands, ins	Band spacing, ins	Max. head ft
(1) Astoria City, Ore.	18	11.6	7.4	6.4	1⅜	7/16	2¼–12	175
(2) Butte, Montana	24	9.0	9.0	....	1 7/16	½	2 7/16–6	202
(3) West Los Angeles	30	7.5	7.5	....	1½	½	3½–12	105
(4) Vancouver, B. C.	30	4.3	4.1	....	2	½	1¾	210
(5) Pikes Peak Plant	30	4.4	....	....	1½	½	2½–8	215
(6) Denver, Col.	30	20.3	16.4	13.0	1⅝	1½	2 3/16–12	185
(7) Johnstown, Pa.	36	6.4	0.6	....	1½	½	2¾–12	176
(8) Atlantic City, N. J.	42	1.9	1.9	18.6	1 3/16	9/16	12	9
(9) Johnstown, Pa.	42	6.4	4.1	....	1½	½	....	....
(10) Johnstown, Pa.	44	6.4	1.7	....	1⅝	½	4⅛–12	63
(11) Bear Valley, Cal.	52	0.4	0.4	....	2	⅝	12	28
					2.6	⅝	8	165
(12) Deep Canyon Pipe	52	......	0.2	120.0	2–2.6	......	2⅛–12	307
(13) Morton Canyon Pipe	52	......	0.1	120.0	2–2.6	......	2⅛–12	158
(14) Warm Spring Pipe	52	......	0.1	120.0	2	......	5½–12	61
(15) Ogden, Utah	72	6.0	5.1	250.0	2	⅝	2⅞–5¼	11
(16) Manchester, N. H.	72	0.1	0.1	100.7	4	......	....	3
(17) Tumwater, Wash.	102	2.2	2.1	500.0	.....	......	....	17
(18) Floriston, Cal.	108	0.3	0.3	....	3¾	¾	4¾–10	5
(19) Madison Canyon	120	1.4	1.4	600.0	.....	⅞	....	2
(20) Madison Canyon	144	1.4	1.4	....	.....	⅞	....	2

(1) 1895. Water Supply and Power. Oregon yellow fir. Stave pipe cost city 90 ce per lin ft. Grade 5.03 ft per 1000 ft. Leakage 0.099 sec ft. At end of 10 years 1 of all staves were replaced and 5½% of the line was reconstructed.

(2) 1892. Redwood. After 14 years service was in very good condition; up to t time no staves were ever replaced on account of rot.

(3) 1896. Redwood.

(4) Water Supply. Mostly fir used, some cedar. Average spacing of bands 3¾ in

(5) Beaver Creek, Pueblo. Redwood. One tunnel on line 1533 ft long.

(6) 1890. Texas pine. Cost of stave pipe, including erecting, $1.365 per lin ft. Tren ing and back filling $0.483 additional.

(7) Little Conemaugh River Water Supply. Washington fir. Average cost of wh line $2.60 per lin ft exclusive of trench and supports. Grade 1 to 2 per 1000.

(8) 1903. Water Supply. Washington fir. Grade 0.16 ft per 1000 ft. Cost $ per lin ft. (9) See note under (7). (10) See note under (7). (11) 1893. Redwood

(12) Santa Ana Canal. Cost of pipe $6.20 per lin ft. Cost complete $11.39 per li

(13) Santa Ana Canal. Cost of pipe $7.46 per lin ft. Cost complete $10.04 per li

(14) Santa Ana Canal. Cost of pipe $4.06 per lin ft. Cost complete $6.51 per li

(15) 1897. Pioneer Power Plant. Douglas fir. Pipe is laid in trench 8.5 ft and covered to a depth of 3 ft on top.

(16) 1874. Power Development. Southern pitch pine. Grade 43.33 ft per 100 Said to be the first wood stave pipe.

(17) Cascade Tunnel Power Plant. 1909. Washington fir. Mostly in rock trenc

(18) 1898. Power for paper mill. Redwood. Mainly laid above ground; bo portion on earth and stone spalls. After 8 years service there was no evidence of r

(19) No. 2 plant of the Madison River Power Co., Norris, Mont. 120-in pipe fini 1906, 144-in pipe finished 1908. Redwood. (20) See note under (19).

the wood saturated, or nearly so, prolongs the life of the staves, and contact with earth charged with decaying vegetable matter and crushin the exterior surface of the wood in adjusting bands shortens it. Examinat of pipes which have been in use for a number of years show that the int surfaces wear smoother and do not become fouled.

15. Flumes

Flumes are used where excavation for a canal is difficult, as on steep hill-sides, where the soil is so loose that an unlined canal is impossible, or where the canals cross rivers or deep depressions and the use of siphons is undesirable. Since much higher velocities can be used than in canals, and the frictional resistance is less, it is possible to use a much smaller area of cross-section. Concrete, iron and wood are the principal materials employed

Reinforced Concrete Flumes of varied design are used by the U. S. Reclamation Service on its irrigation projects. No. 1, Fig. 42 shows the type con-

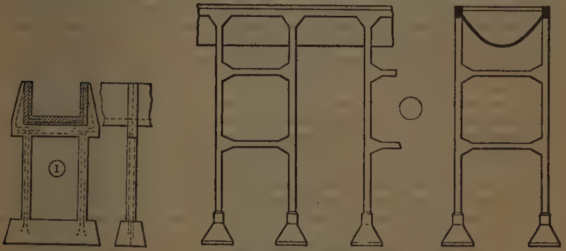


Fig. 42. Reinforced Concrete Flumes

structed in 1908. This design is more or less standard for flumes up to 72 sq ft section, the spans between bents varying from 14 to 30 ft. No. 2, Fig. 42 shows the Brooks Aqueduct, Alberta, consisting of a curved shell of reinforced concrete suspended between two girders on a reinforced concrete trestle, bents 20 ft c to c.

Wooden Flumes are most numerous on account of smaller first cost, tho they have short life. Fig 43 shows 2 by 3 ft wooden flume on trestle. The bents are 15 ft between centers, 4 in by 4 in up to 7 ft high and 6 in by 6 in up to 12 ft high. The stringers are 4 in by 10 in by 16 ft long. The sides and bottom are 2 in by 12 in with ⅝ in by 1 in splines set in edges and quarter round molding nailed in corners; the sides are spiked to the bottom by 20d nails 1 ft

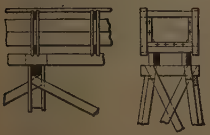


Fig. 43. Wooden Flume

Fig. 44.

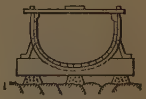


Fig. 45. Wooden Stave Flume

centers; the yoke frames are spaced 4 ft centers. Fig. 44 is a cross-section of the San Diego flume, Cal. Fig. 45 shows the wood stave flume. The greatest difficulty in construction is encountered in making the junction between the flume and the natural earth bed of the canal, since this joint must be absolutely watertight.

Name and location	Length, miles	Height, feet	Width, feet	Thickness of walls, ins	Thickness of floor, ins	Grade, ft per 1000	Capacity, cu ft per sec
(1) Bear River, Utah	0.02	4.00	10.00	......	......	......	......
(2) Conconully, Wash.	3.03	2.00	3.00	......	......	1.50	15
(3) Dulgura	0.94	3.83	4.50	2.0	2.0	0.76	60
(4) Henares, Spain	0.01	6.20	10.17	......	......	......	177
(5) Illinois & Mississippi	0.34	7.00	40.00	9.0	6.0	......	......
(6) Kern River, Cal.	0.50	7.17	8.00	3.0	3.0	1.50	470
(7) Nadrai, Ind.	0.20	7.00	130.00	......	......	......	......
(8) Nebraska-Wyoming	0.04	12.50	34.00	11.5	24.0	......	......
(9) Northern Colorado	0.50	7.00	25.00	......	......	1.00	1184
(10) Northern Colorado	0.19	7.00	36.00	......	......	......	1184
(11) North Poudre, Col.	0.64	6.00	8.00	......	......	2.00	......
(12) North Poudre, Col.	0.14	4.50	12.00	......	......	2.00	......
(13) Pecos Val, New Mex.	0.09	18.00	20.00	24.00	48.0	......	1500
(14) Puget Sound, Wash.	10.20	4.50	8.00	2.50	2.75	1.36	280
(15) San Diego, Cal.	36.00	4.00	5.83	2.00	2.00	......	......
(16) Santa Ana	2.16	5.00	5.83	1.75	1.75	......	240
(17) Tieton, Wash.	......	5.83	8.30	4.00	4.00	......	......

(1) Bear River Canal, Utah. Plate girder bridge of three spans, viz., 60 ft, 45 ft, and 25 ft. Flooring between lower flanges carries canal.

(2) Washington. Temporary flume for water supply used in hydraulic fill dam in 1909. Material used was timber.

(3) Sides and bottom are redwood. Carried on trestle. Frames spaced 4 ft c to c.

(4) Canal in Spain. Steel bridge aqueduct formed by 2 box girders with iron floor between, attached to lower flanges. 62 ft clear span.

(5) Canal. Reinforced concrete bridge on concrete piers. Framework of floor reinforcement is 19 longitudinal I beams 20 in deep and 34 ft 11 in long laid abreast. Side wall reinforcement is 2 I beams 20 in deep, 6 ft apart vertically and trussed.

(6) Kern River Power Plant, Los Angeles. Redwood, seams beveled and calked on sides and flushed with asphalt on the bottom. 1 in by 6 in battens used. Carried on concrete foundations.

(7) Aqueduct of the Lower Ganges Canal. Masonry bridge of 15 spans 60 ft clear.

(8) Canal crossing Spring Canyon. Bridge carried on three arches of reinforced concrete. Walls are reinforced with ¾-in to 1-in steel rods 6 in c to c in 2 rows. Floor reinforcement is 1-in steel rods 6 in c to c.

(9) Platte or Highline Canal. Timber.

(10) Platte or Highline Canal crossing Plum Creek.

(11) Colorado. Timber with joints calked with oakum. Constructed on rock bench.

(12) Colorado. Timber with joints calked with oakum. Constructed on trestle.

(13) New Mexico. Irrigation canal built in 1903. Concrete reinforced with T iron spaced 4 ft c to c. Supported on four concrete arches of 100 ft clear span and 25 ft rise.

(14) Power canal in Washington built in 1903. Timber, carried on trestle from 6 to 80 ft high. The maximum curve is 70° and the total curvature is 10 280°.

(15) Redwood. Carried on timber stringers and mudsills.

(16) Wooden Stave. Redwood. Held by T irons 2¼ in by 2¼ in 4 lbs per lin ft spaced 8 ft c to c and 2 intermediate ⅝ in steel binders. Supported on timber cradles and concrete piers.

(17) Washington. Semicircular reinforced concrete. Transverse reinforcement ⅝-in steel rods 4 in c to c, longitudinal 18 steel rods ¼ in in diameter 12 in c to c. Carried on reinforced concrete trestles.

CANALS

16. Navigation Canals

Canal Prisms (Fig. 46). The cross-section varies with the material past thru, the value of adjacent property, and the facilities to be given to traffic. In earth the side slopes are usually from 2 : 1 to 3 : 1 but there are many examples of steeper and flatter slopes. The Erie and Manchester canals have in places side slopes of 1 : 1, but these are protected by paving for the full height, and have failed in some cases. In cities it has often been found economical, as in the case of the Erie canal, to provide vertical side walls. In rock the sides are usually made vertical, or nearly so. The recent use of channeling machines has facilitated the formation of smooth vertical sides, with great benefit to vessels. Clearance between the keel of the vessel and the bottom of the canal varies with the form and area of cross-section of the vessel, the width of the channel and the speed desired, as the following table shows:

Canal	Depth, ft	Depth, in	Haulage	Speed, miles per hour	Clearance, ft	Clearance, in	Authority
Erie (enlarged)	7	0	Horse	1.67	1	0	N. Y. S. Eng., 1877
Upper Escaut, St. Quentin, Lateral Oise	7	2½	Horse	1.67	1	3½	Lindley
Dortmund & Ems, Ger	8	2½	Tug	3.10	1	7½	Herman
Merwede, Holland	10	0	Tug	4.66	1	6	Int. Engr. Congress
Brussels to Rupel, Belgium	10	6	Tug	3.73	0	4	
Ghent-Terneuzen, Holland	20	8	Steam	5.40	1	5	Int. Engr. Congress
Amsterdam (before last enlargement)	28	3	Steam	5.59	2	0	Int. Engr. Congress
Suez (before last enlargement)	31	1	Steam	6.20	4	1	Panama Report, 1906

The width of the canal should be at least sufficient to permit two boats meeting to pass each other, but on account of the rapidly increasing resistance to the movement of a boat as the cross-section of the canal diminishes, the width of boat canals is generally greater, so that the area of the wet cross-section may be three to five times the area of the cross-section of the boat. On the New York State Barge Canal, the ratio is designed to vary from 4.76 in earth sections to 4.51 in rock sections.

Level Sections are required for a navigation canal, since strong currents are inadmissible. Difference of level between adjacent sections is overcome by locks, inclines or mechanical lifts. Inclines or high mechanical lifts are suitable only in the exceptional cases where the slope of the valley is very steep or where they can be so located as to concentrate a large descent within a short distance. They effect a saving of water as compared with locks. Locks are usually best adapted to meet topographical conditions; in general, lifts of locks vary from 5 or 6 ft to 15 or 20 ft, but some have less than 5 ft and a few have been built with lifts of from 25 ft to 30 ft and upwards. Larger lifts require more water for lockage, reduce the time required by a boat to overcome a given elevation but tend to reduce the ultimate traffic capacity of the canal. Locks are used in most places where an incline or high mechanical lift would be practicable. If the slope of the ground is not too steep, sections of canal are placed between adjacent locks which enables some saving of water to be effected; where the topography does not permit these

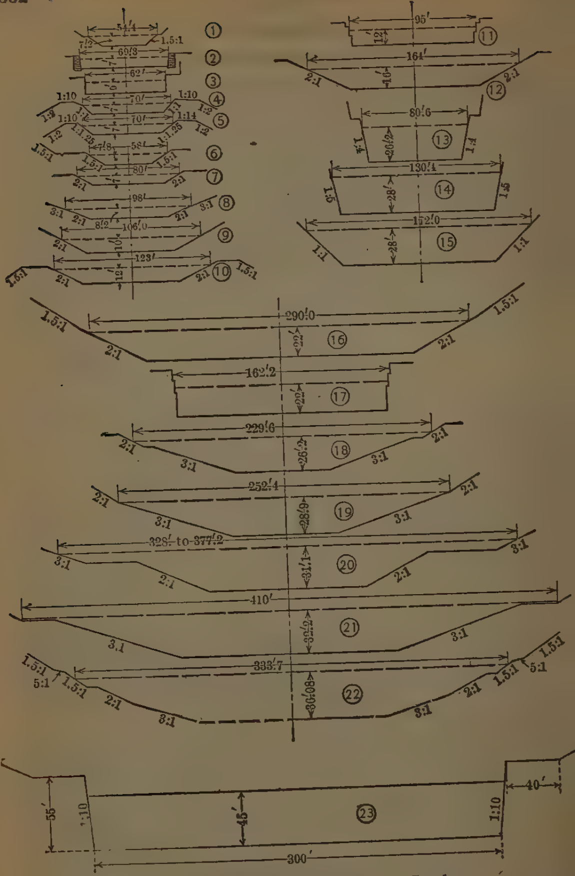


Fig. 46. Cross-sections of Navigation Canals

(1) Marne to Rhine
(2) Erie
(3) Erie
(4) Erie
(5) Erie
(6) Canal du Centre, Belg.
(7) Illinois and Mississippi
(8) Dortmund and Ems
(9) Merwede
(10) N. Y. Barge, Earth sec
(11) N. Y. Barge, Rock sec
(12) Soulanges
(13) Corinth
(14) Manchester, Rock
(15) Manchester, Earth
(16) Chicago Drainage, Eart
(17) Chicago Drainage, Roc
(18) Bruges-Maritime
(19) Ghent to Terneuzen
(20) Suez
(21) Amsterdam
(22) Kiel
(23) Panama

intermediate canal sections the locks are connected into a succession of steps forming a "flight."

Widening Prism on Curves. It has not been usual in the United States to increase the width of canals on curves. In the plans of the New York Barge Canal no increase in width was proposed for curves of 5° or less; for 6° the widening proposed was 5 ft. On the Canal du Centre in Belgium the widening was $400/R$ in meters, R being the radius of the curve. At the International Congress for Inland Navigation held at Vienna in 1886, a rule was adopted that the amount of widening should be twice the versed sine of the arc whose chord is equal to the length of the largest boat traversing the canal. This was adopted for the Dortmund and Ems Canal, the length of the boat being taken at 220 ft.

On the Ship Canal from Ghent to Terneuzen, the widening on curves was determined by the formula $4\,(R-\sqrt{R^2-200^2})$ in feet, R being the radius of the curve. On the Kiel Canal, the widening was determined by the formula $26-0.01\,R$ in meters, or $85.3-0.01\,R$ in feet. On the Kiel Canal it has been found by experience that "it is not so much a large radius that is required as a proper widening of the cross-section." This view is abundantly confirmed in the United States as regards vessels moving under steam power.

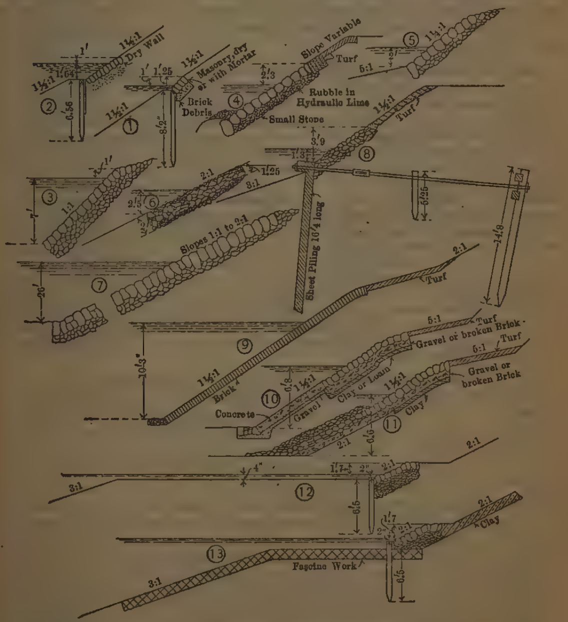


Fig. 47. **Methods of Bank Protection**

(1) Canals in the north of France. (2) Luik Maastrecht Canal. (3) Enlarged Erie Canal, Eastern Division. (4) Canal from Maastrecht to Bois le Duc. (5) Dortmund and Ems Canal. (6) New York Barge Canal. (7) Manchester Canal. (8) Ghent-Terneuzen Canal. (9) (10) (11) Kiel Canal. (12) (13) Amsterdam Canal.

Protection of Side Slopes (Fig. 47). In earth some form of protection is usually advisable, particularly in sandy soils, to protect the slopes against wave action caused by passing boats. The extent required depends upon the character of the traffic. At the Dortmund and Ems Canal, traversed by boats drawing 6′7″ at a speed of 3.1 miles per hour, it has been found sufficient to extend the protection 2 ft under water; one of the several forms used is shown in Fig. 47. On the New York Barge Canal, the protection is extended 2′6″ or more under water. In larger canals, traversed by large ships and at higher speed, the protection is usually extended 5 to 8 feet under water, and terminated at its lower edge on a berm. Of the three figures relating to the Kiel Canal, the last one is designed for use when the canal is filled with water, and the stones cannot be laid with regularity below water level without unwarranted expense. In some cases the protection has been designed to serve the further purpose of maintaining the bank at a steeper slope than it would otherwise sustain, as in the standard section of the Erie Canal and at places in the Manchester Canal. This is decidedly objectionable for propellers, as the slope walls are liable to break the wheels and it is better practise in nearly all cases to give the earth below the belt requiring protection from wave action a slope at which it will stand.

When the protection is of stone it usually consists of a paving of rubble masonry or selected stone laid dry with as close joints as practicable in order to prevent washing thru the joints of the paving, the earth is first covered with a layer of gravel or of small broken stone. A cheaper form of revetment has been used successfully at the Soulanges Canal, and has been adopted for the New York Barge Canal where specifications describe it as consisting of three grades of stone; first one-quarter of the whole volume to be quarry waste, stones ranging in size from 9 in to 15 in in longest dimension; second one-quarter of stones from 4 in to 9 in in longest dimension; and, third, one-half of stones from 1 in to 2 in in diameter. The larger stones are bedded on smaller ones and rammed firmly in place; the remainder of the small stones are used to fill voids and form a smooth surface. No flat stones are permitted. On the eastern section of the Illinois and Mississippi (Hennepin) Canal a trench was cut in face of the embankment, extending from 2 ft above normal water level to 1 ft below it, and filled with rubble laid roughly by hand. On other sections no protection was provided until after the canal was put in use and a berm had been formed by wave-wash; stone was then brought in on barges and deposited in the trench thus formed. Bricks have been used in various forms; also concrete blocks and reinforced concrete. Where water seeps into the canal, revetments of masonry or of concrete require to be back-drained by being laid on sand or gravel or by other means.

Control of Leakage from Canal (Fig. 48). Where canals are formed in sandy soil or other permeable material, the loss of water by percolation or seepage may be so great as to endanger the adequacy of the water supply. If the water supplied to the canal carries silt the seepage will become less as the voids in the sand gradually become filled with fine particles of silt carried by the water, or with silt, sawdust or similar material thrown into the water near the points where leakage is greatest; but in such material it is usually advisable to take means to prevent or reduce seepage during the construction of the canal by interposing less permeable material, and where the canal is carried on an embankment this is usually essential to insure the safety of the work. In such a case the canal should have an impermeable lining, which is usually of clay with sand or gravel well puddled or rammed. In order to prevent shrinkage cracks above the water line and in the sides and bottom when the water is drawn out, and also to protect the puddled material from abrasion by boats, the puddle is usually covered by a layer of ordinary soil.

In the canal from the Marne to the Rhine the water-tight layer contained three parts of sand to two parts of clay, the mixture, after spreading, being sprinkled and rammed to

⅗ of its original volume. In the navigable feeder of the Arroux, Canal du Centre, France, the mixture contained 65 to 70% of clay; the material in the bottom was compacted by grooved rollers drawn by horses, and in the slopes was rammed; the material was spread in 4-in layers and reduced in volume 40% by rolling or ramming. In the Dortmund and Ems Canal, where the canal was in embankment, the material of the embankment was principally marl, and the water-tight lining was of the form shown in the figure, the thickness of the puddled material increasing with the height of the embankment. In the Canal du Centre, Belgium, a lining of concrete has been used successfully, protected on the bottom of the canal by a fill of clayey earth. This is suitable only where the bank is of firm material not liable to irregular settlement, and is liable to injury by boats. In the canal from the Marne to the Rhine concrete in banks was also covered with earth.

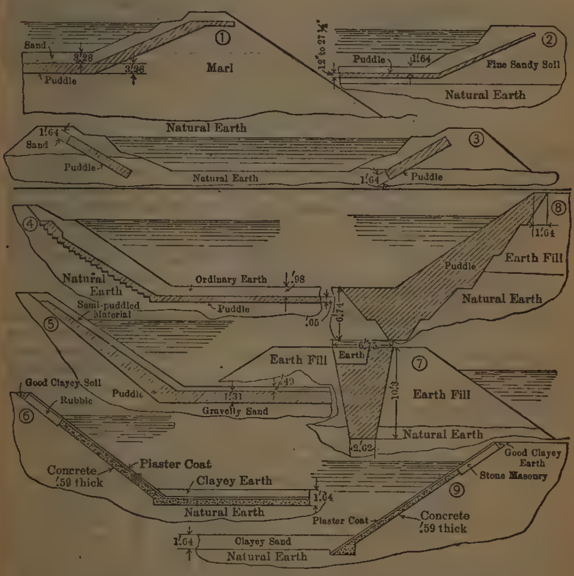


Fig. 48. Methods of Tightening Canal Prisms. (1) (2) (3) Dortmund and Ems. (4) Marne to Rhine. (5) Arroux. (6) (7) (8) (9) Canals du Centre, Belgium.

Where the bottom of the canal is subjected to up-lifting pressure when the canal is laid dry, the water-tight layer is likely to be breached; underdraining is sometimes practicable. The case may arise where the head of the inflowing water is above the bottom of the canal but below the water surface desired; each case of this kind must be studied as it arises. A water-tight lining of the sides and a silt lining of the bottom may be suitable in such a case. Where the bottom of the canal is in excavation and the material sufficiently water-tight, while the sides are embankments, seepage may be prevented by a puddle lining on the slope or by a puddle wall in the middle of the embankment.

When placed on the slope the thickness should be great enough to permit considerabl abrasion by boats without cutting thru it. This is avoided by forming a puddle wall i the middle of the embankment, but the fill on the water side is likely to become saturate and then may contribute little to the stability of the embankment, or may even reduce it Care must be taken to remove all loose or yielding material and to avoid forming a perme able joint between the original ground and the fill.

Drainage of Adjacent Territory. A canal in a river valley is so locate that its banks may be above the highest flood level. Small streams, arti ficial drains, or storm water may be received directly into the canal or pas under it thru culverts (Fig. 49). When circumstances permit, the latter i

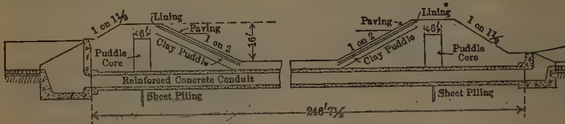


Fig. 49. Dive Culvert, N.Y. Barge Canal

preferable; the principal objections to draining into the canal are the resultin silting, which will involve continual expense, and the variations produce in the water level. Where practicable, water, if silt bearing, should be mac to pass thru a settli basin before entering t canal. If the amount water flowing into t canal is considerable, t resulting current is o jectionable. The surpl water received into t canal is discharged ov WASTE-WEIRS (Fig. 50

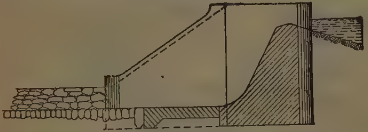


Fig. 50a. Waste-Weir, N. Y. Barge Canal

or thru WASTE-GATES. Where the requisite control can be given by a wei has the advantage of being automatic; but this may at times cause waste, si

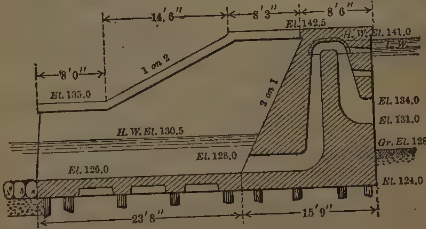


Fig. 50b. Siphon Spillway, N. Y. Barge Canal

a high wind blowing in the direction of the canal, or a succession of b moving in one direction, may cause a rise at one end of a canal level. Waste also result from the difficulty of adjusting the supply to the summit level to

varying demand for lockages; to overflow from short levels; to unequal lift at locks, etc.

In the New York Barge Canal the crests of the waste weirs are at low water elevation. Channels are provided for placing flashboards to raise the crest of the waste weir if required. On this canal a siphon spillway shown by (Fig. 50*b*) has been used.

Waste-gates are usually placed with their sills at canal bottom and are useful for draining the level when desired for repairs.

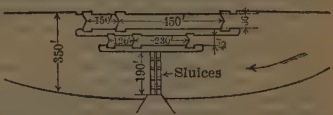


Fig. 51. Irlam Locks, Manchester Ship Canal

At the Manchester Canal, where large amounts are received into the canal at times of floods, each level is controlled by Stoney sluice gates placed in a sluiceway adjacent to and parallel with the locks. The sills are at the same elevation as the sill at the upper end of the locks; the sluice gates are each 30 ft opening, and can be raised about 20 ft. The Irlam sluices (Fig. 51) can discharge 26 000 cu ft per second.

Large streams are usually crossed by aqueducts forming part of the canal, but usually reduced in width to the requirement for a single boat. The superstructure, or "trunk," may be of wood, masonry or iron. The crossing of the Erie Canal over the Mohawk River at Rexford Flats is a good example of a trunk of wood; of the same canal over the Genesee River at Rochester is a good example of a trunk of masonry; both of these structures have been in use for many years. The one at Rexford Flats has been partially removed and the one at Rochester is still in use. The Bridgewater Canal is carried over the Manchester Canal on a swing bridge, the trunk is of wrought iron, has a clear width of 19 ft and a depth below ordinary water surface of 6 ft and a freeboard of 1 ft.

17. Water Required for Operation

Filling the Prism of the Canal. The prism of the canal may be emptied either for repairs or as a result of accident. In dealing with the question of water supply for the proposed New York Barge Canal, E. Kuichling assumed that the prism of the summit level would be laid dry once per year; also, that in addition to the volume of the prism, an equal amount would be required to saturate the adjacent soil to its normal condition, but this additional amount depends on the water-tightness of the wetted contour of the canal prism, and, if this is not well provided for, by the porosity of the adjacent ground. In the case considered by Kuichling it was contemplated to line the canal with clay where formed in permeable ground.

Evaporation and Seepage. For the New York Barge Canal, Kuichling estimated that the evaporation from the water surface would be 0.30 inch per day, to which he added 10% as a liberal provision for consumption by aquatic plants, or a total of 0.33 inch evaporation per day from the surface of the canal.

In the summer of 1900 observations were made to determine the loss by percolation from various sections of the Erie and Champlain canals. In some cases where the canal had been deepened within a short time before the gagings were made the loss was large, running up to 773 cu ft per mile per minute; in other cases, even where percolation thru the banks could be seen, the gagings indicated little loss, and, on the whole, the results were not of great value, but, rejecting the abnormal results, a loss from percolation of 100 to 150 cu ft per mile per minute was indicated.

Kuichling finally estimated that the loss from evaporation and seepage from the Barge Canal would aggregate 4⅓ in in depth per day, equivalent to 169 cu ft per mile per minute, but this was based on the assumption that clay linings would be provided in permeable soil. The thickness of the puddled material was intended to be 3 to 4 ft on the slopes on

embankments and 1½ ft in excavation where the materials were of a nature to permit seepage. The thickness on the bed of the canal was to be 1½ ft in all cases where puddle was deemed necessary. The canal was to be 123 ft wide at the water line, 75 ft at the bottom and 12 ft deep.

G. W. Rafter estimated the probable percolation from the summit level of the proposed deep waterway from the Great Lakes to tidewater at 330 to 440 cu ft per mile per minute. At the divide near Rome, N. Y., the water in the canal would be slightly below the level in the adjacent streams, and about 15 ft above at the ends of the summit level. No puddle lining was intended for the canal.

The excessive loss of 4144 cu ft per mile per minute from the Marne-Saône Canal is said to be due to fissures in the rock in which a section of the canal is excavated. A similar loss, and for the same reason, has been met in the Glens Falls feeder for the Champlain Canal. Such leaks are difficult to close after the water is once admitted to the canal. On the other hand, earth sections of canals become tighter with use. Some of the large losses shown in the table occurred when the canal was new or had been deepened within a short time.

Measurements of Loss of Water from American Canals by Evaporation and Seepage, Including, in Some Cases, Waste at Structures

Canal	When observed	Length of observed section, miles	Width at surface, feet	Width at bottom, feet	Depth, feet	Loss per minute per mile, cu ft
Original Erie Canal:						
Lodi to Little Falls	1841	61.8 20.7	40.0 70.0	28.0 52.5	4.0 7.0	67.5*
Palmyra level	1841	8.3	40.0	28.0	4.0	108.8*
Clyde level	1841	27.7	40.0	28.0	4.0	35.3*
Lockport to Pittsford	1841	69.0	40.0	28.0	4.0	73.0*
Western Division	1841	122.0	40.0	28.0	4.0	74.5
Enlarged Erie Canal:						
Tonawanda to Clyde	1858	126.0	70.0	52.5	7.0	200.0*
Spencerport to Rochester	1877	12.0	72.5	53.7	7.5	190.0
Chenango Canal:						
Summit level	1838	22.0	40.0	28.0	4.0	65.5
Genesee Valley Canal:						
General	1846	......	40.0	28.0	4.0	19.0
General	1846	......	40.0	28.0	4.0	25.0
General	1846	......	40.0	28.0	4.0	30.0
Rochester to Mt. Morris	1854	36.0	40.0	28.0	4.0	37.0
Mt. Morris to Oramel	1854	42.0	40.0	28.0	4.0	153.0
Rochester to Mt. Morris	1855	36.0	40.0	28.0	4.0	21.9
Mt. Morris to Dansville	1855	16.0	40.0	28.0	4.0	23.8
Sonyea to Portage	1855	15.0	40.0	28.0	4.0	25.6
Portage to Oramel	1855	22.0	40.0	28.0	4.0	43.0
Chesapeake and Ohio Canal	1838	40.0	50.0	32.0	6.0	60.0
Chesapeake and Ohio Canal	1838	40.0	50.0	32.0	6.0	64.0
Sandy and Beaver Canal, O.	......	......	38.0	26.0	4.0	13.0
Ohio Canal:						
Fort Wayne summit	1839	54.0	50.0	36.0	5.0	52.0
Fort Wayne summit	1847	54.0	50.0	36.0	5.0	46.0
Various places	1882	......	50.0	36.0	5.0	33.0
Illinois and Mississippi Canal	1894	......	80.0	60.0	8.0	120.0
Illinois and Mississippi Canal	1895	......	80.0	60.0	8.0	60.0
Morris Canal, N. J.:						
Mountain View to Stonehouse Plains	1878	4.8	40.0	25.0	5.0	211.7
	1879	8.0	40.0	25.0	5.0	170.8
	1879	8.0	40.0	25.0	5.0	164.1
	1879	4.7	40.0	25.0	5.0	3.8

* For these cases evaporation, seepage, and waste were measured. For the other cases only evaporation and seepage were measured.

Percolation, infiltration or seepage is usually much greater in amount than evaporation. The combined loss from evaporation, seepage and waste is usually measured together by current observations at two or more cross-sections of the canal, and simultaneous measurements made of waste. The loss by evaporation plus seepage is then given by subtraction and usually reported together. Evaporation can be estimated with reasonable correctness and seepage determined within a small range of uncertainty. The table on p. 1338 is condensed from Kuichling's report on Water Supply for New York Barge Canal.

The loss by evaporation, seepage and leakage from the Illinois and Mississippi Canal is stated to be less than 60 cu ft per mile per minute.

In some of the European canals where the water supply is limited, and particularly where the summit level is maintained by pumping from the lower levels, greater expense is incurred to reduce seepage than is usual in the United States. In the Dortmund-Ems Canal the loss by percolation and evaporation amounted to 32 cu ft per mile per minute of which 9 cu ft was estimated to be due to evaporation and 23 cu ft to percolation. This was rather more than had been anticipated but it is expected that it will be reduced by silting. The water-tightness of the prism was of much importance because the water on the upper levels was supplied in large part by pumping. Lindley states that the mean value of the loss by evaporation and percolation is estimated for the whole of France at about 40 cu ft per mile per minute. In a given permeable soil the amount of seepage varies with the width and depth of the canal, but no satisfactory expression of the relation has been advanced.

Leakage at Aqueducts, Culverts and Waste-Gates. The amount of leakage at these structures depends principally upon the care taken in design, construction, maintenance and operation, and can be limited to a small amount. In a report made in 1837 by W. H. Talcott the leakage of waste-weirs and aqueducts on 22 miles of the Chenango Canal (depth 4 ft) was given at 220 cu ft per minute, or 10 cu ft per mile per minute. Kuichling gives an estimate of 1.6 cu ft per mile per minute on certain old canals in France. The losses will obviously depend on the depth of water and may be assumed to vary with the square root of the depth as an approximation. Kuichling estimates for the New York Barge Canal (depth 12 ft) 12 000 cu ft per day for each culvert and waste-gate, and 96 000 cu ft per day for each aqueduct, which appears to correspond approximately with the observations of Talcott, without allowance for increased depth. With modern methods the older class of masonry should be improved upon.

Leakage at Locks. This will be at the valves for filling and emptying the lock and at the lock-gates. Seepage around the locks should be prevented by suitable construction or reduced to a negligible amount. From observations at the Chenango Canal (in 1839) W. H. Talcott estimated leakage at a canal lock of 11 ft lift at 447 cu ft per minute or 643 680 cu ft per day. Kuichling gives an estimate as low as 14 100 cu ft per day on certain small French canals. Measurements of leakage around the filling valves of the Weitzel lock, St. Marys Falls Canal, made by J. Ripley in 1899, gave a coefficient of 0.49 for use in the formula $v = C\sqrt{2gh}$ or $v = 4\sqrt{h}$ nearly. The valves were of the butterfly type, 8 ft by 10 ft, turning about the 10-ft axis. The mean width of opening around the closed valves was about 3/8 in. The valves had been in use about 18 years, and saving of water was not important. This type of valve is wasteful and should not be used where saving of water is important, altho there should be no difficulty in reducing the leakage to 1/2 the amount observed by Ripley; its advantages are in simplicity, low cost of maintenance, and facility of operating. Leakage at the gates results from imperfect fitting; in the case of the ordinary mitering gates, the gates may be too long, in which case one or both (usually one) will not be in contact with the sill when closed, resulting in an opening, approximately triangular, extending from the miter to or toward the quoin; or the gates may be too short

(usually from wear), in which case an opening of similar form will exist between the ends of the gates, extending from the sill upwards.

For the gates of boat canals, maintained in good condition, it will be sufficient to assume an opening between the sill and one gate ½ in wide at the miter and gradually tapering to nothing at the quoin. For ship canals the maximum width of opening may be doubled. While locks are being operated the head on the opening thru which water is being wasted may vary from nothing, when the filling or emptying of the locks begins, to the full lift of the lock when the operation is completed; but as a safe provision, which covers small leaks between the quoins and the gates and elsewhere, the full head is usually taken.

Using the coefficient for leakage at valves found by J. Ripley, and computing the leakage at gates as just indicated, the leakage at the Weitzel lock (60 ft wide at gates, 20 ft lift) is estimated at 72 cu ft per second, but the valves are not close-fitting. The various Commissions of the old and new French Panama Canal Company allowed 35 to 53 cu ft per second for each lock, the lifts being about 30 ft. Kuichling estimated the leakage at locks from the summit level of the Barge Canal at about 23 cu ft per second, the lock at one end having a lift of 20½ ft and the lock at the other end 16 ft. He followed the method above outlined, assuming ¼ in opening around each valve and an opening of 1 in at the miter between the sill and one gate of a pair.

Loss over Waste-Weirs. It is impracticable to adjust the supply of feed water to the varying requirements of traffic; therefore the crest of the wasteway must be fixt at such a height that the canal will always have the required depth of water, or the crest may be provided with flashboards, permitting a temporary surplus of water to be held until wanted, or the water must be wasted. Waste may also result from the action of the wind blowing toward the weir, from a succession of boats moving toward it, etc. It is customary to provide a limited amount of storage by fixing the elevation of the crest of the wasteway somewhat above the level required for traffic, but it is restricted by the cost of the corresponding increase in height of the canal banks lock walls, etc. In general, the best form of wasteway for ordinary conditions is a simple overflow weir, because its action is automatic; its length mus be fixt with reference to the allowable rise of water in the canal.

Data regarding existing or projected canals vary greatly. For the eight waste-weir on a 22-mile level of the Chenango Canal W. H. Talcott observed (in 1839) a wast of 96 cu ft per minute; for the proposed deep waterway from the Great Lakes to tidewater G. W. Rafter estimated the waste for the summit level at 150 to 200 cu ft per second = 9 000 to 12 000 cu ft per minute, but his large estimate resulted from the provision of waste-weir long enough to pass the floods of the Mohawk which was to be taken into th summit level. For the summit level of the New York Barge Canal, Kuichling assume a waste-weir long enough to control the water level within about 5 in with a water suppl sufficient for normal amount of lockages flowing in but with no lockages, and estimate the discharge over the waste-weir from this and other minor causes of waste to averag 46 cu ft per second (400 000 cu ft per day) from each end of the summit level.

Power for Operating Locks. This is usually developed by water fro the higher level. Power may be required for (A) opening and closin valves; and (B) hauling boats into or out of the lock. (A) For opening an closing of gates and valves, occupying only a small part of the time, accumu lators may be employed. At the Dortmund and Ems Canal, where the lock are 28¼ ft wide with 9.84 ft on the sills and maximum lifts of 20⅓ ft, th lock-gates and sluice valves are moved by power; the greatest amount re quired for any single operation is 7.2 H.P. At the Weitzel lock, St. Mar Falls Canal, there are two turbines of 25 H.P. each; one is sufficient to accumu late power as quickly as necessary, the other is a duplication for safet The largest gates are 39 ft high, the lock 60 ft wide at the gates; two valv for filling the lock and two for discharging, each valve 8 ft by 10 ft. The pow

required being given or estimated, the corresponding water consumption can be approximated in any given case. (B) The power required for haulage at the locks of the Erie Canal was given at about 8 H.P., the immersed cross-section of the boats being 17.5 ft × 6 ft = 105 sq ft. For the Barge Canal, and boats of 25 ft × 10 ft = 250 sq ft immersed cross-section, Kuichling proposed 20 H.P. as sufficient, the power to move a boat at a given speed being assumed to vary with the immersed cross-section.

Power Required for Lighting. For the New York State Barge Canal from 10 to 14 arc lights each of 750 c p. have been used. The United States canal at Sault Ste. Marie has 13 lights, each of 1200 nominal c p. (300 actual) for the duplicate locks, and one similar light each 400 ft, approximately, on each side of the canal. The Canadian canal on the opposite side of the St. Marys River has 8 lights, each 1 200 c p. nominal, at the locks, and one similar light on each side of the canal every 300 ft. The length of each of these canals is approximately 1½ miles. The Kiel Canal is lighted by incandescent lights placed opposite each other on the canal banks, spaced on curves at distances $R/15$ between pairs up to a maximum of 820 ft, on tangents.

Practise is not well defined as to the extent of lighting desirable, and the only practicable course is to prepare a plan for lighting believed to be the best suited to the case in hand, from which the amount of water required can be computed. The fall at the locks is usually the most convenient source of power, in which case the amount of water consumed is an element of the quantity to be supplied to the upper level, but in many cases the electric current may be supplied from a distance, between the reservoir and the canal, requiring no additional water; in other cases it may be found economical to buy current.

Under present conditions arc lighting is estimated to require 0.4 H.P. per 1000 c.p. with efficient plant management, and incandescent lighting with carbon filament 4.2 H.P. per 1000 c.p. These are approximations, and for canal illuminations the power should probably be doubled. For the New York State Barge Canal, Kuichling estimated a consumption from the summit level of 21 cu ft of water per second or 6 H.P. for four arc-lights, each of 1000 c.p.

Water Consumed for Lockages. Let Q = area of lock multiplied by lift of lock, D = displacement of vessel, N = number of locks in flight. There are four possible conditions of lockage affecting the consumption of water. Case 1: A boat ascending after another has descended will draw from the upper level, past the upper gate, $V = NQ + D$. Case 2: A boat descending after another has ascended will force into the upper level, past the upper gate, $V = D$, the draft from the upper level being $-D$. Case 3: A boat ascending following another that has ascended, will draw from the upper level, past the upper gate, $V = Q + D$. Case 4: A boat descending following another which has descended will draw from the upper level, past the upper gate, $V = Q - D$. The ascending boats draw from the summit level their displacements, and the descending boats force their displacement back into the summit level. Ordinarily no material error will be introduced if the displacements be assumed equal, therefore D may be eliminated from the formulas which become:

Case 1, $V = NQ$ (boat ascending); Case 2, $V = Q$ (boat descending);
Case 3, $V = Q$ (boat ascending); Case 4, $V = Q$ (boat descending).

If the summit level is reached by a single lock, and the number of boats passing in one direction is equal to the number passing in the opposite direction the minimum consumption of water would be given by alternating ascending and descending boats, cases 1 and 2; in Case 1, N becomes 1, and the two boats of a pair would consume $Q + O = Q$, or an average per boat of ½ Q. If the boats follow each other the average consumption per boat is Q. The actual consumption will be between these extremes ½ Q and Q since it will frequently be impracticable to delay a boat which is awaiting another from the opposite direction.

If the summit level is reached by a flight of two locks, ascending and descending boats alternating require 2 Q for the ascending boat and O for the descending boat, or an average of Q, and the same average amount if the boats follow in the same direction.

For a flight of three locks two boats alternating will require 3 Q or an average for the two of $\frac{3}{2}$ Q, and the average for all lockages will equal or exceed Q. When the number of locks in a flight exceeds two, economy in use of water for lockages results from duplicating the flights of locks, using one flight for up-bound and the other for down-bound boats, the consumption for each boat being Q.

Water for lockages may be economized by the use of side ponds receiving the water discharged from the lock chamber during the earlier part of the process of emptying and returning it to the lock chamber during the earlier part of the operation of filling. At the Dortmund and Ems Canal two pairs of side ponds are used, at different elevations, two ponds at each side of the lock and about 50% of the lockage water is saved.

Pumping is often necessary to supply the higher levels. The average cost of pumping by water power at the French canals may be taken at 60 cents to $2.90 per million gallons delivered into the canal, or 1 to 2 cents per million gallons raised 1 ft high, according to the length of delivery main. The average cost of pumping by steam power at the French canals is $6.82 to $12.18 per million gallons delivered into the canal, or $0.055 to $0.105 per million gallons raised 1 ft high, depending on the distance. The electric pumping plant for supplying Canal de Bourgogne cost $15 600. Annual cost of working and up-keep $400 or $1.75 per million gallons raised 17.3 ft, or $0.10 per million gallons raised 1 ft.

18. Speed of Canal Boats

Where steam power is used the speed is usually limited by regulations. The power required to move a boat increases rapidly as the ratio (wet cross-section of canal) ÷ (wet cross-section of boat) decreases. On boat canals the boats are usually of the largest dimensions permitted by the locks. On ship canals the tendency to uniformity is not well marked, and is affected or controlled by numerous conditions other than lock dimensions. Following are data for various canals:

Canal	Cross-section of canal			Cross-section of boat		Ratio of cross-section	Power	Speed, miles per hour
	Surface width, feet	Bottom width, feet	Depth, feet	Beam, ft in	Draft, ft in			
Erie, Original	40.0	28.0	4.0	13 6	2 6	4.03	Horse	2.00*
				14 6		3.13	2 Horses	2.00*
Erie, Enlarged	70.0	56.5	7.0	17 5	6 5	4.20	2 Horses	2.00*
Erie, Enlarged	......	......	......	17 0	5 6	4.72	Steam	2.54†
Dortmund-Ems	98.5	59.0	8.2	27 0	6 7	3.82	Steam	2.50‡
Dortmund-Ems	98.5	59.0	8.2	27 0	5 9	4.41	Steam	3.10‡
Ghent-Ter. Belg.	183.7	55.8	21.3	......	......	3.92	Steam	6.21§
Ghent-Ter. Hol.	155.0	56.0	20.7	......	......	......	Steam	5.40‡
Suez	......	......	......	......	......	3.10	Steam	6.21‡
Kiel	......	......	......	......	......	......	Steam	9.32‖
Teltow	101.0	66.0	6.6	......	......	......	Electric	2.5–3.1*
Brussels-Rupel	108.0	49.0	10.5	23 9	10 2	3.43	Steam	3.73
Merwede	107.0	66.0	10.2	34 3	8 6	3.06	Steam	4.75‡
Manchester	200.0	120.0	26.0	......	......	4.00	Steam	6.90¶

* Actual average is somewhat less. † By trial. ‡ Permitted by rules.

§ By regulations; actual speeds allowed on tangents 7.4 miles per hour. Average speed on sharp curves 3.7 miles per hour.

‖ Permitted by rules for small vessels. Large ships cannot attain 6.2 miles per hour.

¶ By regulation; enforced for large ships. Small vessels specially licensed to run 9.2 miles, but attain 11.5 to 15 miles per hour.

The New York Barge Canal with a wet cross-section of 1188 sq ft is intended for steam barges having a wet cross-section of 250 sq ft when loaded, giving a ratio of 4.75. In concluding report on the probable speed in such a traffic, E. Sweet expressed the opinion that a higher speed than 3 miles per hour would be "economically inadmissible."

In France the speed limit on canals is considered to be 1.67 miles per hour; traction by horses. On the Northeastern canals where on account of congested traffic a concession for horse traction was instituted in 1875, the contractor is bound to maintain the speed of 1.24 miles per hour for ordinary service and 1.8 miles per hour for accelerated service. Interesting experiments have been made on the French waterways on the tractive force required by various types of boats at various speeds and in various channels. The types of boats were: "Peniche flamande," — box bow and stern, only slightly rounded where the ends join the sides and bottom; "Flute," cut-away bow, rising in a curve from the keel; "Toue," pointed peaked bow and square box stern; "Prussian barge," pointed bow and stern; "Margotat," punt-shaped. The boats were towed in a river of large section (practically an unlimited cross-section) at a uniform speed of 3.35 miles per hour to determine the influence of the form of boat on tractive force required.

Type of boat	Length, feet	Beam, feet	Draft, feet	Coefficient of displacement	Gross tonnage	Traction force in lbs at 3.35 miles per hour: On boat	Per gross ton
Peniche flamande..	126	16.4	5.25	0.99	305	1530	5.03
Flute..............	126	16.4	5.25	0.95	283	785	2.77
Flute..............	126	16.4	4.27	0.95	250	695	2.78
Toue..............	126	16.4	5.25	0.97	299	590	1.97
Prussian barge.....	113	16.4	4.27	0.94	210	410	1.95
Margotat.........	72	16.4	4.27	0.82	116	310	2.67

Another set of experiments was made in various canals with the "flute" at 4.27 ft draft at various speeds. The coefficient of resistance is the factor to be applied to the tractive force required in an unlimited cross-section in order to obtain the tractive force required in the canal named. The ratio of cross-section of channel to cross-section of boat is represented by r.

	Depth, feet	Area, sq ft	r	Coefficient of resistance for speed in miles per hour: 0.56	1.12	1.67	2.23	2.79
Diversion of the Yonne at Joigny..................	8.20	450	6.39	......	......	1.38	......	......
Canal de Bourgogne.......	7.95	320	4.54	......	......	1.98	......	......
Canal de la Curé (Yonne).	6.85	250	3.57	1.92	2.13	2.38	2.75	3.17
Canal de St. Dizier at Vassy	6.80	207	2.94	......	......	3.32	......	......
Canal du Nivernais........	5.60	207	2.94	......	......	3.82	......	......

The resistance at 1.67 miles per hour in the unlimited section was 172 lbs, and in the Diversion of the Yonne at Joigny with $r = 6.39$ it was 238 lb or 38% greater; but with r reduced to 2.94 it increased 232% in one case and 282% in another, the greater resistance being in the shallower canal, a diminution of 1.2 ft in depth or 48% in clearance under the boat, increasing the resistance 15%.

Experiments to determine influence of form of boat and cross-section of canal on cost of transportation were made in 1898 on the Dortmund and Ems Canal, and in 1906 on models in the government experimental tank in Berlin comparing cross-sections of equal area, one with a greater central depth, also cross-sections of different areas, and two forms of boats moving at different speeds with a view of determining the most economical conditions of transport for assumed amounts of traffic. The effects of draft and speed

were found to be similar to those indicated by the French experiments and for the conditions assumed $r = 4.2$ to 5.3, and boats carrying 667 tons one way and $^1/_5$ as much the other way, with mechanical haulage, a speed of 3 1 miles gave economical transport.

19. Locks and Lock Gates

Locks are usually employed to overcome differences of level. In the earlier canals in the United States they were frequently built of wood, but these were soon replaced with stone masonry, the face being cut accurately to true lines and surfaces, and until within the last 20 years practically all canal locks in the United States were of stone masonry. Recently concrete made with Portland cement has come into use. Brick masonry laid in cement or hydraulic lime has been used largely in European canals, either for the entire mass of masonry or for facing, except at quoins and other exposed places, where heavy stone blocks have been built into the brickwork. Bricks for facing are of specially good quality.

In the locks of the Kiel Canal the floors and lower portion of the side walls up to the floor level are of concrete; above the floor the side walls are of brick masonry built with

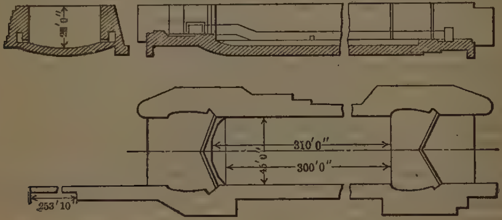


Fig. 52. Barge Canal Lock

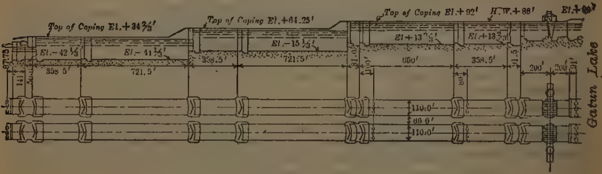


Fig. 53

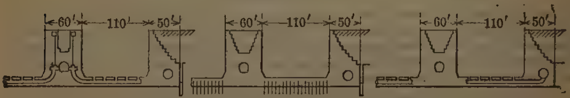


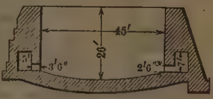
Fig. 54. Gatun Locks, Panama Canal

large openings filled with cheap beton of 1 cement to 8 sand. Fender courses, lock-sills, angles of gate recesses, and guide grooves for the pontoon gates are of granite. The locks of the Manchester Canal are of concrete, with brick facing above low-water

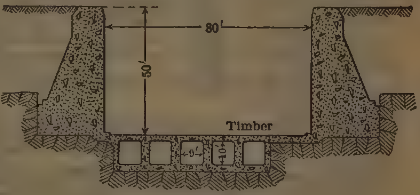
and stone copings, miter sill masonry facing of sandstone, the coping of specially hard quality, but stones in contact with gates (no timber sill) are of granite; brick and stone are used at other exposed places.

The Poe and Weitzel locks at Sault Ste. Marie are of limestone masonry. The new locks are of concrete. Fig. 52 illustrates a lock of the New York Barge Canal. Fig. 53 shows the principal dimensions and Fig. 54 the cross-sections of the Gatun locks of the Panama Canal.

Foundations and Floors of Locks. Where rock foundation is not available, the lock is usually founded on piles, and the entire foundation enclosed within sheet piling and with lines of sheet piling under the sills. In American practise the floors of small locks have usually been built of wood, but with increased width the use of timber is not advantageous. In foreign canals, lock floors are usually of concrete in the form of an inverted arch. The thickness of the floor is usually sufficient to resist the full upward pressure of water when the lock is pumped out, but in some cases weep holes have been formed in the floor to relieve this pressure.



(a) Barge Canal



(b) St. Marys Falls Canal
Fig. 55. Locks of Large Canals

Where founded on rock, the thickness of the floor is more frequently reduced and may be, in great part, omitted if the rock is especially sound and free from fissures, but usually a covering is required, either to protect the rock or to give a smooth surface. In the St. Marys Falls Canal this protection is of timber and concrete; the floors are firmly bolted to the underlying rock. With the increasing price of timber the reduced cost of Portland cement, and the introduction of reinforced concrete, the economy formerly effected by the use of timber is reduced.

Facilities for Filling and Emptying Lock. Small locks are usually filled and discharged thru valves in the gates or thru short culverts around the quoins. This causes the boat to surge, and in large locks this action prevents rapid operation. The large boat canals, and nearly all ship canals, have sluiceways or culverts formed in the side walls of the locks, provided with a valve at each end to open or close connection with either level and with openings leading into the lock chamber near the bottom of the side walls. At the St. Marys Falls Canal, the timber floors have facilitated forming culverts underneath, discharging upward thru many small openings, affording a more uniform distribution of water supply and giving the least possible disturbance to ships.

Valves in gates are usually of the butterfly type turning on a central axis. In culverts or sluices they may be plain slide valves, either of wood or metal, but with large sluices friction makes the operation of such valves difficult or slow. Stoney sluices have been used, for example, at the Manchester Canal. At the St. Marys Falls Canal butterfly valves are used for the 8 ft by 10 ft openings, but are not economical of water. Valves moving on four large wheels have been adopted for the N. Y. Barge Canal (Fig. 62). On this canal one lock at Oswego has no valves for

filling and emptying the lock chamber, it being filled and emptied by means of siphons in the lock wall.

Lock Gates. Mitering gates are in pairs, each of the two "leaves" turning about a vertical axis in a recess in the lock wall. When closed, the two leaves, usually of equal length, meet and support each other about ¼ to 1/5 the width of the lock from a straight line connecting the two axes of rotation. In earlier practise, and particularly in small locks, the gates were made of timber, but steel is coming into more general use; at the Manchester canal, as well as the Liverpool docks, greenheart timber is still used for openings up to 80 ft or more in width, and the engineers claim for it much longer life than for metal gates.

Timber gates (Fig. 56) consist of horizontal girders usually framed into strong end verticals called, respectively, "quoin post" or "heel post" and "miter post" or "toe post"; the gates are planked on the upstream face, where necessary for water-tightness. On the Canadian canals the miter and quoin post and planking are omitted and the gates built solid.

Steel gates (Fig. 57) have a strong vertical girder at each end which is usually furnished with a timber-bearing face. The intermediate framing may consist of horizontal girders carrying stresses directly from the miter to the quoin, or of a heavy girder top and bottom to which the vertical girders carry the water-pressure loads. The upstream or both faces are covered with continuous plating, and in cases of heavy gates an air-tight chamber is formed to facilitate moving the gate.

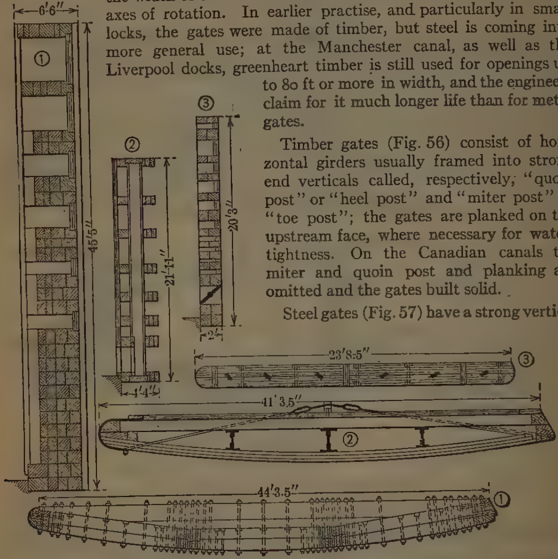


Fig. 56. Wooden Lock Gates
(1) Manchester (2) Kampsville (3) Kanawha

Fig. 58 shows one leaf of a gate for the New York Barge Canal. The gate is supported by a pivot under the quoin post and a steel strap or other device at the top strongly anchored to the side wall of the lock. In the United States and Canada, and in some foreign canals, these arrangements suffice to support the gates in position. In English practise, a heavy gate is usually supported on a roller at the miter end, which moves along a path laid in the lock floor.

Many forms of single-leaf gates are in use. On the Erie Canal the gate at the head of the lock is frequently of the form called "tumble" gate, which turns about the lower edge, falling upstream when opening and lying flat on the bottom of the lock, boats

passing over it (Fig. 59). The span (width of lock) is 18 ft and the depth of water 7 ft. Similar gates are in use at the head of locks on the Oder and Spree Canal, where the span is 29.52 ft and depth of water 8.2 ft. These gates are of wood.

On boat canals, where a limited overhead clearance is admissible, gates lifting vertically have been used, as in the recently built Teltow Canal. A lifting gate on the Coevorder Canal, Holland, has a length of 21.3 ft and a height of 5.6 ft, is of metal, and lifts vertically high enough for boats to pass under. On this canal one lock with a lift of 40.5 ft has a vertical lift gate at the lower end of the lock. Gates turning about a vertical axis at one end and closing the entire width of the lock are in use in some European canals. Following are examples: In the diversion of the Scarpe around Douai the single-leaf gates at the foot of the locks are 21.8 ft

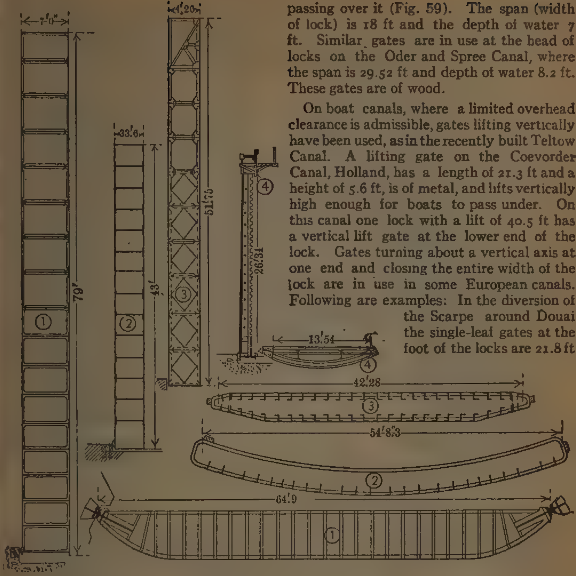


Fig. 57. Steel Lock Gates
(1) Panama (2) Poe (3) Kiel (4) Spree

long and 25.2 ft high; those at the head of the locks are 21.6 ft long and 11.7 ft high; the depth of water over the sills is 8.2 ft and the lift of the lock 13.4 ft; gates are of iron with wood sheathing and contact pieces. At Lichmis on the Dedemsvaart Canal, Holland, single-leaf gates at the head of the lock span 19.7 ft; depth of water on sill 6.6 ft; height of gate 7.3 ft; built of iron. At the Canal St. Denis a single-leaf gate is used at the foot of a lock where the lift is 32.5 ft. As a clear headroom of only 17.2 ft is required, a masonry arch is extended across the lock, and the gate when closed is sup-

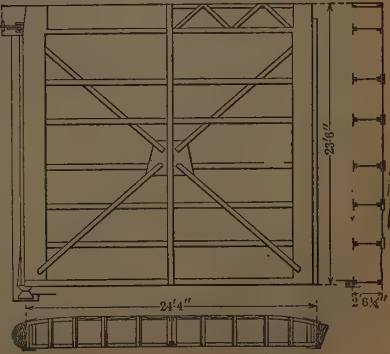


Fig. 58. Barge Canal, Steel Lock Gate

ported on all four sides; the gate is 28.7 ft long and 33.6 ft high; depth of water over sills about 10 ft. It has a frame of iron with wood sheathing and contact pieces.

Rolling gates moving transversely to the canal are in use in many places. In the locks for the Ohio River navigation at Davis Island and elsewhere (Fig. 60) which are 110 ft wide, the gates at the head and foot of the lock are alike; the lift is limited to about 8 ft and the maximum height of gate built up to this time is 18.5 ft. The gates have steel frames with wood sheathing, and are carried on wheels which run on tracks laid in the lock floor. The gates are opened and closed in 1½ to 2 minutes with steam power. In the Canal François, Hungary, the locks (two in flight) near the R. Sisza have a width of 52.5 ft and a depth of 8.2 ft on sills; the upper and middle gates are

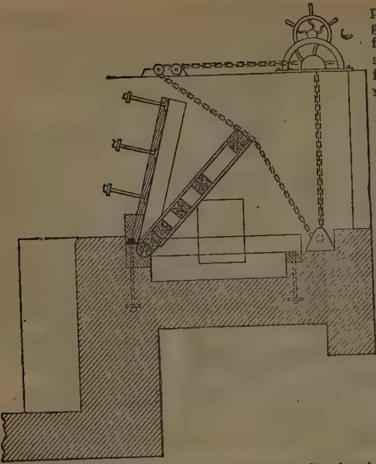


Fig. 59. Tumble Gate, Junction Lock, Erie Canal and New York Barge Canal

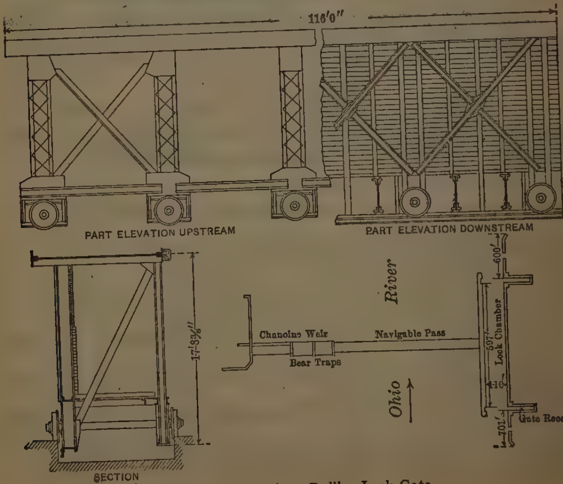


Fig. 60. Ohio River, Rolling Lock Gate

of iron and of the roller type, suspended from trucks running on an overhead bridge. The middle gate is 55.6 ft long, 30.1 ft high, and weighs 150 tons mounted. It has air chambers which lift 44 tons, leaving the net weight to be moved 106 tons; operating machinery is electric. Time to open and close the gate 3 minutes.

Operating Machinery. In the smaller boat canals the gates and valve are operated by hand. The lock gates of the Erie Canal have "balance beams" attached to the top of the gate and extending back over the side walls, and the gate is moved by men pushing against the beam. In some foreign canals a segment of a toothed wheel is attached to the quoin post and extends back over the wall and is operated by means of a windlass with vertical axis carrying a pinion which acts upon the toothed segment. At the Dortmund and Ems Canal the piston rod of a hydraulic cylinder acts upon an arm extending back from the gate, replacing the toothed gear just mentioned. At the larger locks, hydraulic machinery is in most general use. As the machinery is operated for only a small part of the time, accumulators are employed, usually in the form of a vertical cylinder, into which water is pumped, lifting a heavily loaded plunger, and from which the water is drawn when wanted. At many places, as the Manchester Canal and the Liverpool docks, the gate is opened by means of a chain attached to one end of the gate and then led around guides and rove thru two sets of pulleys, one attached to the lock wall and the other to the plunger of a hydraulic cylinder. The effect of this is to multiply the movement of the end attached to the gate when the plunger is operated. A similar arrangement effects the closing of the gate. At the Weitzel lock of the St. Marys Falls Canal a single cylinder is used to open and close each gate, a piston being substituted for the plunger, a long piston rod passing thru both cylinder heads with a set of pulleys at each end (Fig. 61). At other places winding drums are operated by three-cylinder hydraulic engines, as at the Kiel Canal. A simple and efficient machine, consisting of a hydraulic cylinder with its piston rod guided and a connecting rod attached directly to the gate and opening or closing the gate by a single movement of the piston was introduced at the Barry dock at Bristol, England, and is used at several places.

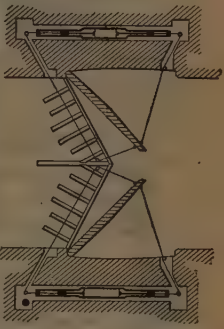


Fig. 61. Method of Operating Gates, St. Marys Falls Canal

In the smaller canals the valves for filling or emptying the locks are operated by hand power. The 8 ft by 10 ft butterfly valves at the locks of the United States canals at St. Marys Falls are opened or closed by a single stroke of a hydraulic engine, the connecting rod being attached directly to the valve—all below the lock floor. At the Canadian Canal, St. Marys Falls, the valves are operated by a long connecting rod attached to a crank arm of the valve stem. On the New York Barge Canal the valves which are counterweighted are operated by electricity (Fig. 62).

In foreign canals the water pressure used in hydraulic machinery at locks is usually 40 to 50 atmospheres. In recent years the use of electric current for operating lock machinery is increasing; two notable examples are the Canadian lock at the St. Marys Falls and the sea lock of the Amsterdam Canal.

Approaches to Locks. In canals where boats are moved by men or horses at low speed the change from canal section is effected abruptly at the ends of the lock walls but where vessels are moved by steam the danger of accidents to lock gates is greatly increased, and a long guide wall should be built at each end of every lock or flight of locks with posts strongly anchored placed at short intervals for use in checking speed by means

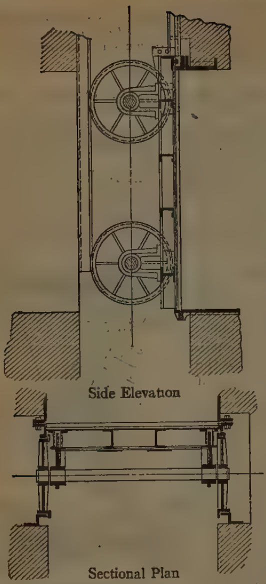


Fig. 62. Lock Valve, New York Barge Canal

of lines. Such arrangements exist a the St. Marys Falls canals, at the Nev York Barge Canal, and at the Panam Canal.

20. Lifts and Inclines

High Lifts. There are severa examples in European and Canadia canals of so-called "hydraulic lifts. This type consists of two tanks each large enough to receive boat, and supported by a plunge working in a cylinder of sufficien length to give the desired lift b one stroke of the plunger. Th cylinders of the two tanks a interconnected, so that the tank balance each other. The boat t be lifted or lowered is placed i one of the tanks, displacing it own weight of water, and thu not affecting the balance. Move ment is effected by admitting small amount of excess water to t tank which is desired to be lowere The tank has a gate at each en a gate is also placed at the end each canal level. Following a some of the details of existi hydraulic lifts:

Data	Anderton, Eng. Trent & Mersey Canal	Fontinettes, France. Neufosse Canal	La Louvieres, Belgium. Canal du Centre	Peter-borough, Canada. Trent Canal	Kirk-field, Canada Trent Canal
When built	1875	1888	1888	1904	1906
Tank, inside length	73′ 9″	132′ 10″	141′ 7″	139′ 0″	139′ 0
Tank, inside width	15′ 3″	17′ 0″	18′ 4″	33′ 0″	33′ 0
Tank, depth of water	4′ 5″	6′ 6¾″	8′ 6″	8′ 0″	8′ 0
Height of lift	50′ 2″	43′ 0″	50′ 6″	65′ 0″	48′ 5
Diameter of ram	2′ 11¼″	6′ 6¾″	6′ 6¾″	7′ 6″	7′ 6
Pressure of ram, lbs per sq in	532	....	500	600	600
Weight to be lifted, tons	250	770	1100	1700	1700
Displacement of boat, tons	120	300	400	800	800
Carrying capacity of boat, tons	80	....	....	....	....
Time to lift boat, minutes	....	....	2¼	1½	....
Time complete operation, minutes	....	....	15	12 *	....
Cost in dollars	230 000	380 000	320 000	500 000	....

* The record time of complete operation was 6 ½ minutes.

The Anderton lift, the first to be built, was operated with little trouble for several years, but finally the plunger became scored to such an extent that it was decided to remove the hydraulic appliances; each tank is now counterweighted separately, the counterweights being connected to the tank by wire cables carried over pulleys supported by an elevated frame; the operation is by electric machinery. This change was carried out in 1908.

The high lift at Henrichenburg, built in 1899 on the Dortmund and Ems Canal, consists of a single tank supported on five water-tight air-filled cylinders immersed in wells which are interconnected and kept full of water. Principal data are:

Depth of wells	98′ 0″	Length of largest boat	220′ 0″
Diameter of wells	30′ 2″	Width of largest boat	27′ 0″
Diameter of cylinder floats	27′ 6″	Draft of largest boat	6′ 7″
Length of cylinder floats	32′ 9½″	Time to lift or lower tank	2½ min.
Length of tank (useful)	225′ 0″	Time for one round trip, lifting one boat and lowering another	25 min.
Width of tank (useful)	28′ 4″	Power supplied	440 H.P.
Depth of water in tank, min.	8′ 3″	Cost, including installation	$630 000
Height of lift	45′ 11″		
Weight to be lifted, tons	3100		
Carrying capacity of boat, tons	950		

The movement of the lift is controlled by guides, and uniformity is secured by four massive vertical screws, interconnected, and working thru nuts fixt to the lift. It is stated that "a lift constructed on the open trough principle should only be adopted where the conditions for it are favorable, and a good foundation can either be found or can be prepared without any great trouble and expenditure. In conclusion, it should be mentioned that when the further extension of the Dortmund and Ems Canal is taken in hand, it is contemplated to construct a flight of locks by the side of the canal lift."

Inclined planes have been used to surmount high lifts, both in the United States and Europe. The following table gives details of several examples:

Canal	No. of inclines	Total lift, feet	Largest boat: Length	Largest boat: Width	Largest boat: Draft	Largest boat: Load, tons	Built	In use or abandoned
Morris, U. S.	23	1449.0	89.0	10′ 6″	4′ 0″	70	1831	In use
Pennsylvania, U. S.	10	2007.0	85.0	13′ 6″	.....	75	1831	Abandoned
Ches. & Ohio, U. S.	1	39.0	90.8	14′ 5″	5′ 0″	115	1876	"
Monkland, Scotland	1	96.0	70.0	14′ 4″	2′ 2″	60	1850	"
Shropshire, Eng.	1	213.0	20.0	6′ 0″	2′ 9″	.....	.....	In use
Shrewsbury, Eng.	1	73.5	20.0	6′ 0″	2′ 3″	.....	.....	"
Chard, Eng.	3	27.5 to 86.0	26.0	......	.....	60	.....	"
Grand Junction, Eng.	1	75.2	75.0	12′ 0″	3′ 4″	75	1900	"
Ourcq, France	1	40.0	92.0	10′ 0″	4′ 0″	70	1888	"
Oberland, Germany	5	66.0 to 80.3	......	......	.....	70	1860	"
Bude	1	......	20.0	5′ 6″	1′ 8″	.....	.....	"

The small boats using the Bude incline have wheels built in, and are drawn up the incline in trains, counterbalanced to some extent by descending trains or boats. On the inclines of the Morris, Pennsylvania, Shropshire, Shrewsbury, Ourcq and Oberland canals, boats are (or were) carried on trucks; on the inclines of the Chard and Monkland and Chesapeake and Ohio canals, the boats are hauled while water-borne, or nearly so, in tanks. In some cases the incline is double, the boats or tanks counterbalancing each other. Boats are usually carried endwise, but on the Foxton incline on the Grand Junction Canal, the boat is carried sidewise and is water-borne; two tanks are used counterbalancing each other.

The incline of the Chesapeake and Ohio Canal (at Georgetown, D. C.) provided for larger boats than any other yet built. In connection with the projected Austrian sys-

tem of canals to connect the Danube with the Ober, the Elbe and the Moldau, inclines have been proposed for boats of 600 to 800 tons capacity. It was announced in 1902 that the Government of Austria had decided to build an experimental incline for boats of 600 tons, investigations having indicated a saving in cost of transport by the use of inclines instead of locks, provided no unforeseen difficulties develop in actual use of inclines constructed on a scale so much exceeding any previously built.

21. Guard Gates, Safety Gates, Stop Gates

Guard Gates are arranged at the St. Marys Falls canals by extending the lock walls at each end sufficiently to receive an additional pair of gates, both opening outwardly. They could be used to protect the ordinary lock gates from approaching vessels, but are not so used, their purpose being to enable the lock to be closed quickly by unworn and perfectly fitting gates, for repairing the lock and its gates and valves. The summit level of the Charleroi Canal (supplied by pumping) is divided into short sections by SAFETY gates so that in case of a break in the canal bank the gates may be closed and the loss of water limited to about 20 000 000 cu ft. STOP gates (Fig. 63) were introduced in the Dortmund and Ems Canal which in places is carried on heavy embankments. These are on the same principle as Taintor gates, and when opened permit the passage of boats underneath. In the New York Barge Canal guard gates (Fig. 64) are placed at least every 10 miles in canal sections; these are lifting gates and in duplicate, each affording a clear opening of 55 ft and a clear head room of 15.5 ft above the water surface at high navigable stage when opened. These gates are steel and carry wheels moving on vertical tracks fixt in the masonry. They are operated by hand power. The last three devices are designed to be operated in moving water.

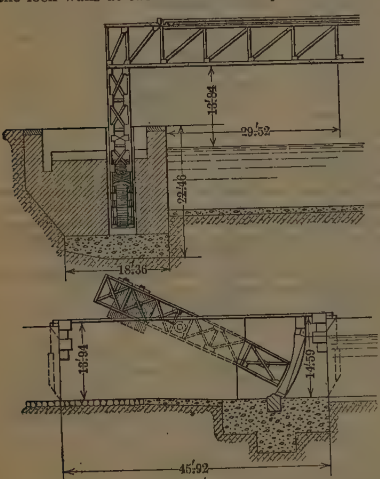


Fig. 63. Guard Gate, Dortmund and Ems Canal.

These devices are impracticable for closing a ship canal where unlimited height is required, and for such canals the safety gates are some form of movable dam. As these are to be manipulated in powerful currents, a subdivision of the structure is considered necessary, and as the emergency requiring their use will seldom arise, and if not in sight they would be more liable to be neglected and unserviceable when needed, they should be above water and readily inspected when not in use. These conditions have been met at the St. Marys Falls canals by swing bridges carrying the sectional dams. Fig. 65 shows the structure in the canal on the Michigan side of the rapids. The canal, which is in rock excavation, is separated by an island into two channels, each 108 ft wide and 25 ft deep. Each channel is closed by 16 wickets which can be lowered

separately. Each wicket will consist of two parts, a frame and a shutter movable in the frame. It is expected that the frames will be lowered first, offering only a partial obstruction to the current, and when all are properly placed, the shutters, guided in the frames,

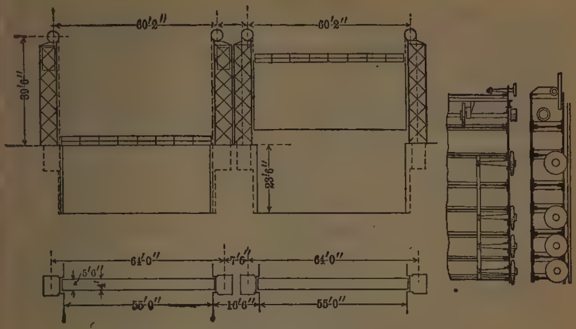


Fig. 64. New York Barge Canal Guard Gates

will be lowered one at a time. This structure is similar to the one at the Canadian Canal which was successfully operated when lock gates were carried away in June, 1909. On

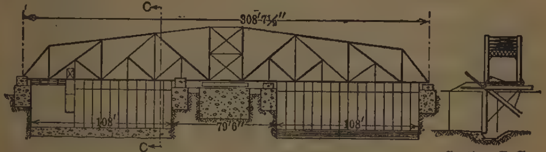


Fig. 65. Movable Dam. United States Ship Canal, Saulte Ste. Marie

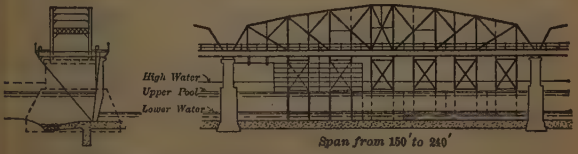


Fig. 65a. Movable Dam. New York Barge Canal

the Panama Canal a safety device is used similar to that shown in Fig. 65. It is described in more detail on p. 1360.

22. Statistics of Canals

United States. Total approximate mileage and cost of construction is

Owned by	Total length, miles	Cost of construction
United States Government	194.49	$42 443 357 *
States †	1 358.98	156 983 538
Private parties	635.58	50 573 160
Total in operation	2 189.05	250 000 055 *
Abandoned canals	2 444.26	81 171 374
Total	4 633.31	$331 171 429

* To June 30, 190
Includes expenditure by prior owners.
† Includes Chicag Sanitary and Shi Canal, built by Sanita District of Chicag Length 32.95 mile Cost $52 697 495.

	Illinois and Mississippi	New York Barge Canal			
	Main Line	Main Line	Oswego Canal	Champlain Canal	Cayuga-Seneca Canal
Length, miles	75**	338.6*	23.8	62.6	92.7§
Width at bottom earth, ft.	52	75	75	75	75
Width at water surface earth, ft.	80	123-171	123-171	123-171	123-171
Width at bottom rock, ft.	....	94	94	94	94
Width at water surface rock, ft.	....	96.4	96.4	96.4	96.4
Width, bottom, in rivers	....	100-200	100-200	100-200	100-200
Depth, ft.	7	12	12	12	12
Number of locks	33	35	7	11	4
Maximum lift	12	40.5	27	19.50	26
Total lift, ft.	292	364.75	118.6	168.3	71
Length of lock chamber, ft.	170	339	339	339	339
Useful length of lock, full width, ft.	....	300	300	300	300
Width of lock, ft.	35	45	45	45	45
Culverts	In masonry at gates	In side walls	In side walls	In side walls	In side walls
No. of inlets to chamber	....	16-28	18-28	16-28	16-28
Gates, type	Miter ††	Miter (†)	Miter	Miter	Miter wood
Gates, material	Wood	Steel	Steel	Steel	Steel
Valves, type	Butterfly	¶	¶	¶	¶
Speed allowed, miles per hour	6	6‡	6	6	6
Motive power	Any	Steam or gasoline	Steam or gasoline	Steam or gasoline	Steam or gasoline
Cost of maintenance per year	175 000	....	....	....	....
Time required for passage, hrs.	20	....	....	....	....
Bank protection	Riprap	Riprap	Riprap	Riprap	Riprap
Tightening	None	Steel piling concrete Clay	Steel piling concrete Clay	Steel piling concrete Clay	Steel piling concrete Clay

* Hudson River to Niagara River. † 1 lift gate Little Falls. ‡ Land lines.
§ Includes Cayuga Lake 31.2 and Seneca Lake, 34.4.
¶ Rectangular steel frame valves on wheels, see Fig. 62.
** Not including 29.3 miles of navigable feeder. †† Tumble gates at head of 14 l

Nearly all the canals built by the United States Government are lateral canals, avoiding rapids in the natural waterways and providing for steamboats ar well as barges. The most important one is the St. Mary's Falls Canal which will be described as one of the great ship canals. The Colbert Shoals Canal is typical of lateral canals along navigable rivers. It avoids Big Tree and Colbert Shoals in the Tennessee River; length 8 miles, depth at low water 7 ft. Width for lower 5½ miles in earth excavation 112 ft at bottom, 140 ft at low-water surface, side slopes 2:1. Width for upper 2½ miles, mainly in rock excavation, 175 ft at bottom, about 190 ft at water surface. Lock at lower end of canal: available length 350 ft, width 80 ft. Lift, maximum at extreme low water, 26 ft. Mitering gates steel, circular in plan. Culverts in side walls for filling and discharging, 8.6 ft by 9 ft, with 11 openings into the chamber from each wall. Valves of the Stoney type. Banks 20 ft wide at top extend 9.5 ft above low-water line and are protected by Bermuda grass above that line. Side drainage received into the canal usually thru culverts. Five waste-weirs, each 100 ft long, are built into the bank on the river side, exclusive of a guide wall 7000 ft long at head of canal serving also as a weir. Crests of weirs 0.5 ft above low-water surface.

Only two canals with summit levels and intended for boat or barge navigation have been undertaken within recent years in the United States. One of these is the Illinois and Mississippi Canal (also called " Hennepin " Canal), and the other is the New York Barge Canal with its branches, the Oswego, Champlain, Cayuga and Seneca canals.

Traffic in the boat canals of the United States has been steadily decreasing with increasing competition of the railroads and this has not been checked materially by the removal of tolls and the assumption of fixt eharges and operating expenses by the state. On the continent of Europe, where the canals are generally owned by the state and free from tolls, the assumption of construction, operation and maintenance by the state enables water transportation to compete with rail, and traffic on the waterways is increasing rapidly.

New York Barge Canal. The Barge Canal is the improvement of four existing canals, and consists of (1) the Erie, stretching across the state from east to west and joining the Hudson river and Lake Erie; (2) the Champlain, running northerly from the eastern Erie terminus to Lake Champlain; (3) the Oswego, starting north midway on the Erie and reaching Lake Ontario; (4) the Cayuga and Seneca, leaving the Erie a little west of the Oswego junction and extending south, first to Cayuga lake and then to Seneca lake.

The Barge Canal is largely a river canalization project, natural water-courses being available for much of its length. With intervening lakes and adjoining rivers there is a total length of a little more than 800 miles in the State's internal waterway system of Barge Canal dimensions.

The main water supply is taken from Lake Erie, from reservoirs in the Adirondack region and from the upper Hudson River. The supply is adequate not only for the 10 000 000 tons seasonal traffic for which the canal was designed, but also for the maximum traffic which the canal is capable of handling, namely, from 18 500 000 to 20 000 000 tons per season.

The locks on all branches of the canal are of standard dimensions. The lifts range from 6 ft to 40½ ft. They are built of concrete thruout and except on rock or hardpan have pile foundations. Within each side wall runs a culvert for filling and emptying the lock. The culverts are connected with ports that open into the chamber at the bottoms of the walls. With a few exceptions the lock gates are of the mitering, girder type, carrying the principal load as beams. They are built of steel, with single skin-plates, have white oak quoin and toe posts, swing on cast-steel pivots set in the concrete, are held at the top by adjustable anchorage and bear against cast-iron quoin plates bolted into the side walls. In general a power station at each lock supplies the energy for operating and lighting the lock. A majority of these installations are hydroelectric, but at locks beside movable dams gasoline-electric stations are employed.

At about fifty cities and villages along the waterways the State has built terminals,

which provide suitable dockage, mechanical devices for handling goods quickly and cheaply, a building for temporary storage and, where possible, railway connections—al available to any shipper or boatman.

France. Practically the whole of the canal system is owned and controlled by the state. In 1905 the total length of canals in use was 3010 miles. The more importan canals have a bottom width of 33 ft or more; depth 6.6 ft to 7.25 ft.

Belgium. The total mileage of the waterways (canals and rivers) is 1345 miles The total length of the more important canals is 334 miles. The total lift at 141 lock (excluding lifts) is 1445 ft The total lift at 4 lifts (1 completed, 4 building) is 217 ft When new work or enlargement is taken up the locks are given greater lifts, up to 14 ft, and water saving effected by side ponds.

Germany. The total length of the waterways on which the traffic is of importance 6200 miles and the total lift at 359 locks (excluding lifts) is 3129 ft. In the canal connecting most directly with the French waterways the width at the water surface is abou 52 ft, at the bottom 33 ft, depth 6.6 ft: useful length of locks 126 ft, width 17 ft, adapte for use of boats on the French and Belgium canals.

Great Britain. The canals were built by independent companies, without assistan from the State. Nearly 1/3 of the total mileage has been acquired by railway companie The total length is 3811 miles (Engr. News, Apr. 16, 1890). The annual traffic on th independent canals is about 33 000 000 tons: on the railway owned canals, about 6 000 00 The greater part of the traffic is short distance, there being no uniformity of canal dime sions, making frequent transshipment necessary on thru business. Excluding the sh canals, only 10% of the total mileage has a depth of 6 ft or more, while 46% has a dep of less than 4 ft. On account of the diverse ownership a material extension or improv ment of the boat canal system is not expected.

Holland. There are 265 canals, with a total length of 2100 miles, of which abo $1/_{20}$ belongs to the State: the remaining canals are under control of provincial or municip authorities, administrative corporations, or private companies. Many of these cana are designed primarily for drainage, and some of the branch canals can be navigated on by small boats poled or towed by manual labor Power boats and tugs are used on t larger canals. On the Merwede Canal recently built, length 43¾ miles, 106 ft wide water surface, 66 ft at bottom, 10.2 ft minimum depth, the tugs tow trains of 12 boa This canal cost $8 480 000 for construction: annual maintenance, $103 000. All t canals owned by the State, and nearly all the others, are free from tolls.

23. Ship Canals

Suez Canal. Connects the Mediterranean and the Red Sea. Shorte the sailing distance from Northern Europe to India about 5000 miles; Australia about 1000 miles. Total length of canal including harbor at P Said and roadsteads at both ends 104.08 miles. Minimum depth 31.1 Width at this depth (excluding harbor and roadsteads) 108.26 ft to 118.11 widened on curves to 131 to 262 ft. There are 23 passing places where t bottom width is increased to 147 63 ft for a length of 2460 ft, reducing at ea end to standard width in a distance of 984 ft. Side slopes vary from 2 : 1 3 : 1 or flatter. The width and depth are being increased gradually. C of the work up to the end of 1908 was $122 275 000. Expenses during 19 were about $8 668 000 and net profits about $12 850 000

On account of the tides and effect of the wind in the Mediterranean and Red s currents are produced in the canal reaching 1⅛ miles per hour between the Bitter La and Port Said, and 3 miles per hour between the Lakes and Suez. The speed of ves is limited to 7¾ miles per hour on the 28-mile tangent beginning at Port Said, an 6 2 miles per hour elsewhere in canal sections. A high speed is permitted in the la The average time actually consumed in passing the canal is from 18 to 19 hours.

Corinth Canal. Connects Gulf of Corinth with Gulf of Ægina. Is at level. Shortens sailing distance from Adriatic ports about 175 miles, from Mediterranean ports about 100 miles. The canal is 3.91 miles lo

The great central cut is 2½ miles long with a maximum depth of 286 ft from surface of ground to bottom of canal. The canal is 26.64 ft deep and has a bottom width of 68.9 ft. The sides have a batter or slope of 1 horizontal to 4 vertical (1 : 4). Unless vessels move very slowly they strike the sides of the cut frequently. The traffic is small.

Manchester Canal. Connects Manchester with the River Mersey at Eastham near Liverpool. It is 35½ miles long; when opened for navigation (1894) its depth was 26 ft; ruling bottom width 120 ft. Side slopes in earth from 1 : 1 to 3 : 1, in rock 1 : 5. For a distance of four miles from Manchester the bottom width was 170 ft. The depth of water on the lock sills was made 28 ft, and the canal was deepened to the same.

The Manchester level is 70 ft above mean tide at Eastham, and is reached in five lifts, varying from 9 ft 6 in to 16 ft 6 in; the smallest lift is at Eastham and overcomes the difference of level between mean tide and mean high water.

At the Eastham terminus there are three parallel locks having the length and width, respectively, of 600 ft by 80 ft; 350 ft by 50 ft and 150 ft by 30 ft; the lengths given are from quoin to quoin. At the other lock sites there are only two parallel locks, respectively, 600 ft by 80 ft and 350 ft by 50 ft.

Each set of locks is paralleled by a sluiceway, with sluice gates of the Stoney type 20 to 30 ft wide (30 ft except at Eastham), and with sills at the same level as the upper sill of the adjacent lock. Each 30-ft gate has a discharging capacity of about 6500 cu ft per second. There are three to five (excluding Eastham) such outlets in each sluiceway, so that in times of flood strong currents may be set up for short periods. The smaller sluiceway at Eastham is supplemented by extensive sluices in the bank of the canal in the level immediately above. The speed permitted is 6 miles per hour for the largest vessels, rising to 12 miles or more for smaller vessels and tugs. The time required for passing thru the canal varies from 5 to 8 hours.

Kiel Canal. Connects Baltic and North seas. It is 61.31 miles long, 36.08 ft deep, 144.32 ft wide at bottom, 333.74 ft and upward at water surface. The width is increased on curves at bottom. The canal has 11 passing places, four of which are 984 ft wide at bottom to serve as turning places; the others are 439 52 ft wide at bottom, length 1968 to 3608 ft. There is a lock at each end of the canal with ebb and flood gates at each end of the lock. Normal water level in the canal is mean tide in the Baltic. There is also in each an intermediate pair of ebb gates and one pair of flood gates, permitting use of a shorter lock chamber. The locks have a useful length of 984 ft, width 147 ft 8 in, depth on sills 45 ft.

The speed allowed by regulations is 9 3 miles per hour, but the largest vessels cannot attain this. The time required to pass thru varies with the draft, from about 8 hours for the smaller vessels to 13 or 14 hours for the larger.

Amsterdam Canal. Connects the North Sea at Ymuiden with Amsterdam and also with Zuyder Zee. Its principal purpose is to make Amsterdam a seaport. Its length from the North Sea locks at Ymuiden to the Zuyder Zee locks is 17.4 miles, and from the North Sea locks to the entrance to Ymuiden Harbor about 2 miles, making a total of 19.4 miles. Depth 32.14 ft below "ordinary water-level;" width at bottom, 164 ft on tangents increased to 196.8 ft on curves. Side slopes generally 3 : 1 with a berm of varying width and depth.

The new great lock at Ymuiden is intended to accommodate vessels of the maximum length of 721.6 ft, width 82 ft, draft 30 18 ft. It has intermediate gates to permit the use of a shorter lock for small vessels, and ebb and flood gates, making 6 pairs in all. The operating machinery is electric thruout, and it is the second ship-canal lock to be so equipped.

The speed permitted varies with the draft of vessel; it is 6.52 miles per hour for vessels drawing 19.68 ft or more, 7.45 miles per hour for vessels drawing 13.12 to 19.68 ft, and 9.32 miles per hour for vessels drawing less than 13.12 ft.

Petrograd and Cronstadt Canal. Connects deep water in the roadstead in the Gulf of Finland with one of the delta mouths of the River Neva at Petrograd, and it is intended to make Petrograd a seaport. It consists of a channel about 17¾ miles long dredged in the open water of the bay and across the Neva bar and protected by dikes for a distance of about 6 miles at the upper end. Its depth was made 28 ft. Bottom width from 213 to 275½ ft in the diked portion and 355½ ft in the portion not diked.

Cape Cod Canal. Sea level canal connecting Cape Cod and Buzzards Bays; shortens the sailing distance from Boston to New York by 66 miles and avoids the dangerous Cape Cod Coast. Highest part of ridge cut through was 29 ft above mean sea level. Material chiefly sand. Length of canal proper 8 miles, 25 ft deep at low water 100 ft wide at bottom side slopes 1 on 2. There are 5 miles of approach channels having bottom widths of 250 ft and 300 ft and side slopes of 1 on 3. Mean range of tide at one end 8.9 at the other end 3 6 ft. This tidal difference produced velocities as great as 5 ft per sec. Completed in 1915.

St. Marys Falls Canals. These are lateral canals avoiding St. Marys Rapids in St. Marys River between Lakes Superior and Huron. There are two canals one on the American side of the river and the other on the Canadian side. The fall in the rapids varies from about 17 to 21 ft with the varying stages of the lakes. and is overcome by one lift in each of the canals.

The American Canal is about 1.6 miles long with four parallel and adjacent locks at the downstream end. The canal consists of two channels, the Poe and Weitzel locks serving the old south channel, and the new third and fourth locks serving the new channel to the north. The Weitzel lock is 515 ft long from quoin to quoin, 60 ft wide at gates 80 ft wide in chamber and has from 15 to 17 ft of water on the sills. The Poe lock is 800 ft long, 100 ft wide, and has 20 to 22 ft of water on the sills. The two new locks are each 1350 ft long, 80 ft wide, and have 24½ to 26 ft water on sills.

Upstream from the Poe and Weitzel locks, is a narrow island on which is located the pivot pier of a swing bridge carrying the wickets and machinery of a movable dam, the purpose of which is to close this portion of the canal in case of a break at the locks permitting a free flow. (See Fig. 65, p. 1353.)

The Canadian canal is 1.4 miles long, 141 ft 8 in wide at bottom, 150 ft at water surface and about 22 ft deep. The single lock is 900 ft long from quoin to quoin, 60 ft wide and has 20 to 22 ft of water on the sills, depending on the varying stages of the lakes.

Canadian Canals from Lake Erie to Tidewater at Montreal

Canal	Width, feet			Locks				Total lift, feet
	Length, miles	Bottom	Water surface	Length, feet	Width, feet	Depth, sills	No.	
Welland	26 75	100.0	164 0	270 0	45.0	14.0	26*	326.7
New Welland §	25 00	200.0	301.25	800.0	80.0	30 0	8*	325.5
Galops	7.33	80.0	144.0	800.0	45.0	14.0	1	15.5
				270.0	45.0	14 0	2*	
Rapide Plat	3.67	80 0	152.0	270.0	45.0	14.0	2	11.5
Farran's Point	1.25	90 0	154 0	800.0	45.0	14.0	1	3.5
Cornwall	11.00	100.0	164.0	270.0	45.0	14.0	6	48 0
Soulanges	14.00	100.0	164.0	280.0	45.0	15.0	5*	84 5
Lachine	8.50	150 0†	‡	270.0	45.0	14 18	5	45.0

* One is a guard lock. † Minimum. ‡ Variable. § Under construction 1919

The two canals actually constitute one system. No charges are made to vessels passing, which usually take the canal having the smaller number of waiting vessels. Hydraulic power is used at the American canal; the Canadian canal is operated electrically, and was the first ship canal to be provided with such operating machinery. A boat can be passed thru the lock in either canal in 8 to 11 minutes, but on account mainly of fleet lockages, the average time for a lockage is about 33 minutes in the American canal. In the Canadian canal, where fleet lockages form a smaller proportion of the total time, the average is about 20 minutes.

The time necessary to pass thru the canal depends largely on the amount of traffic and the congestion resulting, and has varied in the American canal from 4 hrs 58 min in 1895, when only one lock was in operation, to 1 hr 10 min in 1900, when two locks were in use. With increased traffic since that date the time has increased to 2 hrs 40 min.

Panama Canal. Connects the Atlantic Ocean (Carribean Sea) with the Pacific, crossing the Central American Isthmus in Lat. 9° N. and Long. 85° W. at approximately its narrowest and lowest point. It reduces the sailing distance from New York to San Francisco by 7873 miles (5262 miles as compared with 13 135 miles via Magellan Straits) with a similar saving in distance to many ports, in South America and the Orient. The canal is 50 miles long between deep water in the two oceans. The average time of passage is 12 hours, the shortest recorded time is 6 hours 30 minutes.

Its construction was began in 1882, by the French, whose rights were acquired by United States for $40 000 000 in 1904, when a Canal Zone 10 miles wide was secured from the Republic of Panama, for $10 000 000 and a perpetual rental of $250 000 per annum. The canal was completed by the U. S. Government in 1914.

The canal is a lock canal with a summit level normally 85 ft above mean tide in the two oceans, the maximum tidal range being 22 ft at the Pacific and 2 ft at the Atlantic end. Its most distinctive feature is Gatun lake, an artificial body of water, 164 square miles in area, which forms the summit level. The lake provides wide and deep channels which permit vessels to travel at high speed for over half the distance between the two oceans. Its formation involved the building of a huge earth dam for closing the valley of the Chagres at Gatun and of three expensive groups of locks, but on the other hand, the depth of the summit cut at Culebra was reduced decidedly, and a large amount of excavation eliminated so that there was a decided saving in the total time and cost of construction in comparison with a sea level canal. As practically no excavation was required in the deeper portion of Gatun lake the work of building the canal, resolved itself essentially into the excavation of the sea level sections, and of the cut through the Continental Divide, the building of the Gatun and some smaller dams and the construction of the locks and spillways. Some details of the several parts of the work are given below:

Beginning at the Atlantic End, the successive sections of the Canal are the following: (1) A sea-level channel 7 miles long and of 500 ft bottom width, extending to the locks and dam at Gatun. (2) Gatun lock flight with 3 lifts of about 28 ft each. (3) Gatun lake at Elev + 85 forming the summit level which extends across the Continental divide through the Culebra (Gaillard) Cut to the lock at Pedro Miguel.—The total length of channel in the summit level is 32 miles of which 16 miles is 1000 ft wide, this section having a depth of 45 to 75 ft—4½ miles is 800 ft wide, 4 miles is 500 ft wide, and 8 miles (in Culebra Cut) is 300 ft wide. (4) Pedro Miguel lock with a 30 ft single lift. (5) Miraflores Lake (area 1.6 sq miles) at Elev + 55 with a channel 500 ft wide 1½ miles long and 45 ft deep. (6) Miraflores Locks, a flight having two lifts of 22 to 33 ft each, according to the tidal stage, and finally (7) A sea level section 500 ft wide and 7 miles long to deep water in the Pacific.

It will be noted that, except in the locks, the channel has a minimum width of 300 ft and a minimum depth of 45 ft. Where there is a change in the direction of the Canal axis, instead of connecting the tangents by curves, the channel was widened so as to allow vessels to turn readily

Locks. The principal dimensions of the locks are shown in Figs. 53 and 54. They have a clear width of 110 ft, a usable length of 1000 ft, and a minimum depth on the sills of 41 ft 4 in. There are twin chambers, with a central wall 60 ft thick between them, which is extended 1000 to 1200 ft beyond each end of the lock—to act as a guide wall for vessels.

For filling and emptying there is one longitudinal culvert 255 sq ft in cross-section (equal to a circle 18 ft in diameter) in the middle wall and in each sidewall connecting with transverse culverts of elliptical section built in the lock floor, which are each 41 sq ft in area and discharge upwards into the lock chamber through 5 circular outlets of 12 sq ft each. These laterals extend across the lock at intervals of about 35 ft connecting alternately with the culverts in the side and middle walls. The main culverts are controlled by rising stem gate valves of the Stoney type, located close to the upper, intermediate and lower lock gates. They are 8 ft wide and 18 ft high and are placed in pairs separated by a dividing wall. As a safeguard, a duplicate set is provided close to each pair of valves. The valves are, each, opened and closed by a single valve stem, which extends through a horizontal cast-iron bulkhead, placed just above the valve, and fitted with a stuffing box. The stem connects with a guided cross head, which is raised and lowered by 2 non-rising screws driven through reducing gears by a 50 H.P. electric motor. The time of operation is one minute. Each lateral culvert leading from the middle wall is fitted with a cylindrical valve 78 in in diameter, placed vertically, which is opened or closed in 10 seconds by a 7 H.P. motor.

Lockgates. The lockgates are double-skin steel gates of the mitering type with air chambers. The sill angle is 26° 33′ 54″. There are duplicate operating gates at both ends of Pedro Miguel lock and at the upper chambers in the lock flights and an intermediate set in all the chambers except the lower ones at Miraflores. These subdivide the locks into usable lengths of 550 and 278 ft. A reverse guard gate is provided in the lower approach to each log. There are in all 46 gates of 2 leaves each, weighing nearly 60 000 tons. The leaves are uniformly 65 ft long and 7 ft thick and vary in height from 47 to 82 ft and in weight from 395 to 745 tons. The horizontal girders which are spaced from 3 ft 8 in to 5 ft apart have straight parallel sides and curved ends. See Fig. 57. The bearings along the quoin and miter posts and in the hollow quoins are polished steel plates. Their joints have proved to be absolutely watertight. There is a wooden clapping sill with a rubber seal.

The gate-moving machinery is of a novel type developed and first used on the Panama Canal. A horizontal strut is used, hinged at one end to the top of gate, and at the other to a crank pin fixed in a horizontal wheel 19 ft in diameter, (the so-called "bull wheel") which is supported on the lockwall. This is turned by a 25-H.P. motor through a series of reducing gears, the time for opening or closing a leaf being 2 minutes. The special merit of the design is the fact, due to the kinematics of the mechanism, that it gives the leaf a relatively slow motion at both ends of its travel, where the hydraulic resistances are greatest.

Safeguards. The following safeguards are provided in the locks: (1) The duplicat lockgates mentioned above. (2) A novel system of chain fenders placed in the upp and lower approaches to all the locks and also just above the intermediate and low gates in Pedro Miguel lock and in the upper chambers of the other two locks. The consist of 3-inch anchor chains stretched across the locks at the top, which are raise and lowered by vertical hydraulic cylinders in the walls. These chains serve to chec vessels that may strike them, by paying out under strain, the amount of pull bein regulated by relief valves on the cylinders. It has been shown by actual tests that vessel of 18 000 tons displacement, moving at a speed of 2½ miles per hour, can readily brought to rest in a distance of 55 ft, without injury to the vessel or chain. (Electric towing locomotives, traveling on rack railways on the walls on both sides of t lock chambers, which are used in the case of all vessels for moving them during th passage through the entire length of the lock flight. They exert a pull of about 25 o lbs and move at a speed of 1 or 2 miles per hour. (4) Emergency dams at the upp end of each lock for shutting off the flow of water in case of a serious accident to t lockgates, which has resulted in establishing a connection between the upper and low levels. The dams are of the swing bridge type, similar in design to the movable da at the St. Mary's Falls canal, shown in Fig. 65. At Panama, there is an independe dam, (supported on the side wall) for each twin lock flight. The arm spanning the 'c is 164 ft 3 in long, while the other arm, which is 98 ft long, carries a heavy concrete count

weight and the operating machinery. The wicket girders are 60 ft long and spaced 9 ft 2 in apart. There are 5 wickets, in each vertical bay, which roll down along the upstream flanges of the girders. In operation, the flow of water is to be gradually shut off by building up the wickets in horizontal tiers from the bottom. In still water, the dam has been swung, and all the girders and wickets put in place, in 26 minutes.

All the machinery in the locks is operated electrically. The current which is 3 phase at 25 cycles is generated at 2200 and 6600 volts in an hydro-electric station at the spillway in the Gatun dam operating under a head of 77 ft. Its present capacity is 13 140 kw to be ultimately increased to 22 140 kw. There is also a reserve steam turbine plant of 7500 kw capacity near Miraflores. The current is distributed at 6600 volts to transformer rooms in the lockwalls at Gatun, all motors operating at 220 volts. It is carried across the isthmus at 44 000 volts on an overhead transmission line supported on structural steel frames along the line of the Panama Railroad.

The machines for moving the lockgates, valves and chain fenders are located in small rooms in the lockwalls just below the coping level, which are all connected by continuous longitudinal tunnels.

The principal machines in each of the three groups of locks are normally operated from a central control house on the middle wall. This contains a horizontal control board or table, which resembles a miniature ground plan of the lock, with models of the different gates, valves, etc., which open and close when the actual gates are being operated. Local control is also provided in the several machine rooms. The principal gates, lockgates and chain fenders are interlocked so as to guard against errors in operation.

In the operation of the locks, several novel and important hydraulic features have been observed:

(1) A very unequal discharge from the several orifices in a given lateral culvert, the flow being greatest from the outlet farthest from the main culvert. This results in forcing a vessel in the lock strongly towards the wall from which filling is taking place and has made it necessary to use both culverts throughout the process of filling, thus obtaining a very uniform distribution and permitting a maximum rate of changing levels of 7.5 ft per minute with safety.

(2) An "over travel" of the water levels, due to the acceleration of the water in motion through the culverts. Owing to the great length and cross-section of the locks, this action, not taken into account in the ordinary formulæ, is important, as it shortens the time of filling by about two minutes.

The coefficients of flow are approximately:

For filling—using side and middle wall culverts, $C = .65$
using side wall only, " " $C = .82$
For emptying—using side and middle wall culverts $C = .67$
using side wall only, " " $C = .73$

The time for filling (for a length of lock of 900 ft) is 7 to 8 min, using both culverts, and 12.5 to 13.5 min, using the side wall culvert only.

Dams. The Gatun dam is shown in cross-section in Fig. 13, No. 12. It is about 1½ miles long and polygonal in ground plan. The maximum bottom width is 2300 ft. The central portion of the cross-section, except near the top, was hydraulically filled, by pumping a mixture of sand and clay from the bed of the Chagres by 20-in suction pumps, working in two lifts. The hydraulic fill was confined on each side by rock-fill dams, built for the purpose, which form a part of the main dam. The remainder of the cross-section is dry filling, earth and rock, brought partly from the Culebra Cut, partly from borrow pits close to the dam. There is no core wall. The total volume of Gatun Dam is 22 958 000 cu yd, of which 12 229 000 cu yd was dry filling. Its cost, exclusive of the spillway, was $9 871 635. The much smaller dams, at the Pedro Miguel and Miraflores locks, are of a similar character.

Spillways. Lake Gatun is regulated entirely by a concrete spillway, built on a rocky ledge, which divides Gatun dam into two approximately equal parts. The total watershed drained is 1320 sq miles, the annual rainfall 120 in with a runoff of 62%. The yearly evaporation of the lake area is 60 in, while there is no appreciable seepage.

The maximum observed momentary discharge of the river was 175 000 cu ft per sec, the maximum for 33 hours, 137 500 cu ft per sec, and for 48 hours 120 000 cu ft per sec. The spillway has a capacity of 180 000 cu ft per sec, with the lake at +87. It is circular

in plan with 14 clear openings of 45 ft controlled by electrically operated Stoney ga
19 ft high. The crest is at +69.

A similar spillway but straight in plan, with 8 openings of 45 ft, is provided adjac
to Miraflores lock, as a safeguard in case of a serious accident to the Pedro Miguel lo
Its capacity is about 92 000 cu ft per sec.

Capacity of the Canal. The maximum number of complete lockages at both e
of the canal possible with an unlimited water supply is about 48 in 24 hours, co
sponding to the passage through the canal of 60 commercial vessels of average s
With the present provisions for water storage, the practicable number of lockage
much lower, varying with the power demands and the rainfall in different years.
average years 21 lockages will be possible even when operating the power plant at its
projected capacity of 22 140 kw. In very dry years, the number would be still smal
but the capacity of the canal can be largely increased by using the auxiliary steam p
for part of the year, or by providing additional storage by reservoirs on the upper Chag

Excavation. The earth and rock excavation formed the largest single item in
canal construction, its cost amounting to nearly half the total cost of the canal. To J
30, 1918, the total excavation was 267 834 201 cu yd, of which 130 465 874 cu yd
dry excavation, the rest being removed by dredging and hydraulic methods. A
from a portion of the lock excavation and a few short channel sections elsewhere,
dry excavation was confined almost wholly to the 8¾ miles of the Culebra Cut, w
included all the most difficult work. The prism section is shown in Fig. 46. The
cavation below the berm was wholly in rock as was fully 70% of all the excavatio
the cut. The established slope above Elev + 95 was 2 on 3 which was generally m
tained except in the deepest portion close to the Continental divide. This section,
over ⅞ mile long, was the scene of the serious slides, which delayed the opening o
canal and subsequently stopped navigation for many months. These slides have
practically overcome by flattening the slopes to approximately 1 on 7 by dredging
the channel was filled with water in October, 1913. The material removed from
slides to June 30, 1918, was about 56 000 000 cu yd.

The wet excavation, mainly in the sea-level sections, the lock pits and the slides,
carried on by an elaborate plant, comprising suction, ladder and dipper dredges of v
types and great capacity. Hydraulic sluicing was also resorted to on some parts o
work.

Aids to Navigation. In addition to the light houses at the entrances to the c
a complete system of range lights, beacons, and buoys, mark the channel clearly
by day and at night. The tangents in the lake and sea-level channels are define
range lights, vessels going in opposite directions using different ranges giving co
200 to 250 ft apart, a somewhat novel arrangement. In the Culebra Cut, be
placed at frequent intervals on the banks, take the place of range lights. Ther
also numerous gas buoys at the sides of the channel.

Cost of Construction of Panama Canal to June 30, 1918

Prism Excavation	$137 674 924.79
Locks and Dams	92 768 591.66
Breakwaters	9 098 018.41
Aids to Navigation	650 139.87
Power Plants and Transmission System	5 417 309.72
Atlantic Terminals	5 248 350.47
Pacific Terminals	14 394 256.12
Permanent Townsites	2 414 087.09
Buildings and Playgrounds	15 740 443.64
Sanitary Fills, etc.	718 947.64
Water Works and Sewerage Systems	3 285 837.85
Real Estate	2 776 121.35
Rebuilding Panama Railroad	9 800 626.46
Concessions from Republic of Panama	10 000 000.00
Purchase of Canal from New Panama Canal Co.	38 728 484.05
Miscellaneous	154 649.28
Total	$348,870 782.40

Terminals. Both terminals are protected by rock-fill breakwaters, giving ample anchorage grounds and equipped with concrete and steel piers for the transfer of freight. At the Pacific end, there is a concrete drydock with a clear width of 110 ft and a usable length of 1000 ft close to which large repair shops have been built. There are very complete coaling plants at each end of the canal, the one at the Atlantic end having a maximum storage capacity of 100 000 tons of submerged and 350 000 tons of dry coal. The corresponding figures for the Pacific Plant are 44 500 and 167 000 tons.

Total Cost of Canal. The total amount charged to Canal construction to June 30, 1918, is given on bottom of p. 1362; but a considerable additional sum expended since 1914 for dredging out the slides and for many other purposes should properly be charged to capital account. In the figures given, an overhead charge is included, covering general and administrative costs, expenses for hospital and sanitation, etc. An additional sum of $30 000 000 was appropriated for fortifications.

24. Power, Drainage, and Irrigation Canals

Cross-Sections. Unlined canals in earth have usually a large trapezoidal section, at least when first built. The side slope must be fixt with reference to the character of the soil in which the canal lies. It is usually assumed that the slope should be a little flatter than the angle of repose of the earth in water. In very light fine soil the slope may be as great as 3 or 4 horizontal to 1 vertical; in clayey loam or coarse soil or ordinary firm soil 1½ or 2 horizontal to 1 vertical; in firm clayey gravel or hardpan or firm clay 1¼ or 1 horizontal to 1 vertical. Experiment with the earth of the locality may be necessary, to fix the best side slope.

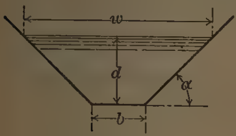


Fig. 66

Trapezoidal Sections. Having the side slope fixt, the relation of depth to breadth may be chosen. The efficiency of canals used as aqueducts, other things being equal, is directly proportional to the square root of the hydraulic radius. Let b, d and w be as in Fig. 66, A the area of cross-section, p the wetted perimeter and r the hydraulic radius. The values of b and d which will furnish a section of the maximum hydraulic radius for a given value of A, are given by the formulas:

$$b = \frac{A}{d} - d \cot \alpha \qquad d^2 = \frac{A \sin \alpha}{2 - \cos \alpha} \qquad p = b + \frac{2d}{\sin \alpha} \qquad r = \frac{A}{p}.$$

The following table, computed by these formulas, gives the values of d, b, p and r in terms of A, which will furnish sections of maximum hydraulic radius, and consequently maximum efficiency, for various slopes in common use.

Slope	α	d	b	p	r
1 on 3	18° 26′	$0.548\sqrt{A}$	$0.178\sqrt{A}$	$3.647\sqrt{A}$	$0.274\sqrt{A}$
1 on 2	26° 34′	$0.636\sqrt{A}$	$0.300\sqrt{A}$	$3.145\sqrt{A}$	$0.318\sqrt{A}$
4 on 7	29° 45′	$0.662\sqrt{A}$	$0.352\sqrt{A}$	$3.021\sqrt{A}$	$0.331\sqrt{A}$
2 on 3	33° 41′	$0.689\sqrt{A}$	$0.417\sqrt{A}$	$2.902\sqrt{A}$	$0.345\sqrt{A}$
4 on 5	38° 40′	$0.716\sqrt{A}$	$0.502\sqrt{A}$	$2.794\sqrt{A}$	$0.358\sqrt{A}$
1 on 1	45° 0′	$0.740\sqrt{A}$	$0.613\sqrt{A}$	$2.704\sqrt{A}$	$0.370\sqrt{A}$
3 on 2	56° 19′	$0.759\sqrt{A}$	$0.812\sqrt{A}$	$2.636\sqrt{A}$	$0.379\sqrt{A}$
7 on 4	60° 15′	$0.760\sqrt{A}$	$0.882\sqrt{A}$	$2.632\sqrt{A}$	$0.380\sqrt{A}$
2 on 1	63° 26′	$0.759\sqrt{A}$	$0.938\sqrt{A}$	$2.635\sqrt{A}$	$0.379\sqrt{A}$
Vertical	90° 0′	$0.707\sqrt{A}$	$1.414\sqrt{A}$	$2.828\sqrt{A}$	$0.354\sqrt{A}$

The Semi-Circular Section has the greatest hydraulic radius for a given area of cross-section, but it is only practicable when lined with concrete or steel. For a given A, the formulas are

$$d = 0.798\sqrt{A} \qquad p = 1.773\sqrt{A} \qquad r = 0.564\sqrt{A}.$$

For small canals the equations above will often fix the best dimensions, but there are numerous considerations which may modify the section. If the canal is very large the excavation may be enough cheaper for a shallower canal, so that the necessary greater sectional area may be removed more economically than the smaller section of equal capacity. This will depend much upon the methods of excavation and the type of machinery used. Again, losses from seepage increase with depth, so that where such losses are liable to become important, or where the bottom of a deep canal would reach an open impervious stratum, it may be wise to adopt a shallower section than would otherwise be economical. On the other hand, for canals on side hills, a still deeper section of greater area may be more economical.

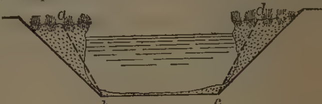


Fig. 67

Small canals are subject to a reduction of cross-section by encroachment of vegetation on the banks, and silting. Jeffreys, in Professional Papers on Indian Engineering, says: "Whatever slope is adopted in construction it is found that this cannot be maintained after the channel has been in use some time. A distributary at the close of an irrigation season invariably assumes the shape seen in Fig. 67. When the time for clearance comes around the engineer in charge, if he is wise, will not attempt to restore the original section. . . . The custom in the Ganges Canal distributaries is to trim off the slope at ½ to as shown by dotted lines a-b and c-d."

The canal may be all in cutting, or all in embankment, or part in each. Each type has its advantages and disadvantages, depending on the local condition. In some localities canals in cutting may be cheaper in maintenance and suffer less loss from seepage; drainage from the surrounding territory may be more easily handled. In other localities there may be danger of reaching an open pervious stratum in cutting which may require lining to prevent excessive loss.

Irrigation Canals all in embankment are sometimes necessary to command the surrounding land or for short lengths to avoid long detours. It is easy with this type, if for any reason it becomes necessary to avoid absorbing the drainage of the country, to pass that drainage across the line of the canal in culverts. The disadvantages are the danger that the banks may break, the higher cost of construction, inspection, and maintenance, and, usually, the greater loss by seepage, which in some cases is further objectionable by forming stagnant pools on the neighboring land. This type of canal generally costs more than either of the other two types.

The advantages of the type in which the section is part cutting and part embankment are that it is usually the cheapest and quickest constructed, that for irrigation it is high enough to command the neighboring land, and that the head of water against the embankments is usually small enough to reduce seepage thru the banks to little or nothing.

Limiting Velocities. Loss of head is to be avoided as much as possible in power canals, while in irrigation and drainage canals the velocity will often be limited only by the ability of the earth of the banks and bed to resist erosion. The velocity should be great enough to prevent the growth of weeds and the deposit of silt, and not so great as to cause erosion. The experiments of Du Buat give velocities in ft per sec next to the bed of the canal which will begin to erode various materials as follows: 0.25 for soft clay, 0.50 fine sand, 0.70 for gravel as large as peas, 2.2 for gravel one inch in diameter, 3.3 for pebbles 1½ inch in diameter, 4.0 for heavy shingle.

Seepage. Sometimes power canals, and usually irrigation canals, are located on hillsides well above ground water level, so that seepage f

unlined canals often amounts to 25%, and sometimes 50%, of the total water entering their headworks. For example, from 20 irrigation canals in operation, taken at random, including canals in France, Belgium, Italy and the United States, the average loss from seepage expressed in depth times area of the water surface of the canal was 3.8 ft depth per day. Omitting two which were between 6 and 7 ft per day, the average was 1.5 ft depth per day. In gravel the loss was from 3 to 7 ft depth per day; in sandy soils 1 to 2 ft; in sandy loam and firm compact alluvial soil 0.2 ft to 1 ft depth per day. These canals varied in depth from 2 to 8 ft.

The common remedies for loss by seepage are silting, puddling and lining with concrete. Excessive losses are likely to be in some short length. Good results are reported to have been obtained by producing stagnant water, which is then made turbid by dumping pulverized clay or other fine material. If the canal water is normally turbid, slack water may be formed by a temporary weir and silting encouraged at the leaky length. A more effective method, but more expensive, is to puddle the bottom and sides with clay when the canal is drained. It may be necessary to protect the puddle from erosion by a layer of coarser material. Where the saving will warrant the expense, the canal may be lined with concrete, as has been done in the case of numerous canals in California.

Canal Linings. Power canals in earth are frequently lined either with timber or concrete to reduce the friction and assure permanency of the cross-section. The timber lining consists of mud sills laid at right angles to the direction of flow and planed plank flooring spiked to the mud sills. The

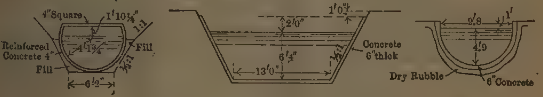


Fig. 68. Three Types of Concrete Lining.

mud sills may rest upon and be held down by bearing piles or be held down by rods to masonry anchors. Fig. 68 shows linings used in the west.

If the ground is very firm, concrete may be laid upon it without other support; in very soft ground bearing piles may be necessary, care being taken, however, that the concrete is in contact with the ground. The concrete should be deposited back of very smooth forms in order to get a smooth surface. For irrigation canals the concrete-lined canal not only costs less to maintain, and prevents loss of water by seepage, but the first cost is often less than for an unlined canal, for the allowable velocities may be from two to four times as great and the section correspondingly smaller.

For the Tieton Canal (left-hand cut of Fig. 68) the reinforced concrete lining was molded in steel forms in sections 2 ft long which were later transported on cars to the canal and set in position by derricks; each section weighing about 1800 lbs was 4 in thick, of gravel concrete reinforced by ⅜ in corrugated rods 4 in apart. Each section was stiffened by a 4 in by 6 in crossbar reinforced by two ⅜ in rods. The molds were removed after about 3 days and the section allowed to harden for at least 30 days, during which time it was frequently sprinkled. The sections were set in place about 1½ in apart and immediately backfilled with earth which on the lower quarter was selected and tamped in from the ends with the greatest care; above this the backfilling was placed from above. The 1½ in space between sections was later filled with rich concrete of fine aggregate which would pass a ½ in ring. The joint was rubbed smooth on the inside before the joining concrete was completely set. The canal has a fall of 8.71 ft per mile and was designed to have a capacity of 300 second feet. A test in the completed canal gave a value of .012 for the coefficient n in Kutter's formula. The gross cost, exclusive of excavation, of 13 000 ft of the concrete lining was found to be $5.80 per foot.

SHAFTS AND BORINGS

25. Timbered Shafts

Timbered shafts are sunk vertically by mining much the same as a heading is driven horizontally. A frame of four waling timbers joined at their ends, forming a rectangle, is laid out on the ground. Outside are driven vertica polings or sheeting; inside, as the sheeting is being driven, the earth is excavatec to a little above or a little below its lower end, depending upon whether the ground is soft or hard. As soon as the excavation is from 2 to 5 ft below one frame a new frame is placed, and so on to the depth of the length of the sheeting; a new frame is then set with clearance for a new set of sheeting. If the shaft is to be shallow it may be started large enough at the top so that the sheeting may be vertical, and the shaft reduced in size by the thickness of the timbering at the bottom of each set of sheeting, which may be from 10 to 20 ft long, as in Fig. 69*a*. If the shaft is to be deep, the sheeting will ordinarily be 5 or 6 ft polings, as in Fig. 69*b*.

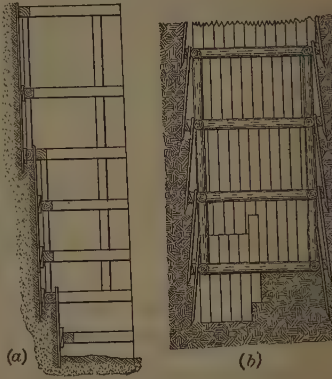


Fig. 69. Timbered Shafts

The main thing to be avoided in shaft sinking is the starting of a general movement of the earth. The great danger is that cavities will be left or formed back of the timbering or that the timbering tho not slack will have to deflect to develop resistance and a small progressive movement of the earth will be started which may extend to the surface. T precautions are, if possible, to allow no cavity to form, securely pack any accidental c and to wedge all timbering tight against the sheeting so that it will develop resistance bef the sheeting begins to move. Perhaps the greatest danger is in water-bearing ear where leakage is liable to bring into the shaft earth in such small amounts as not to noticeable but which may form cavities back of the sheeting in time.

In rock shafts the principal use of timbering is to prevent the falling of loose pie from the sides of the shaft, endangering the workmen below. If the rock is fairly go the timbering is usually kept from 10 to 30 or 40 ft behind the excavation. It is not uncommon practise to place the permanent lining, usually of concrete, in short leng 40 or 50 ft above the bottom of the shaft as the work of excavation progresses.

The method of open excavation and timbering or otherwise lining the shaft as sink progresses is no doubt cheaper, where practicable, than any other method that has b devised, but when quicksand or other soft material is encountered or where rock yield too much water is encountered, the method may be either impossible or impractica For such cases various devices have been used, notably caissons sunk by the aid of c prest air, drop shaft, freezing process, and under-water boring.

Shafts by caissons are frequently used in tunnel construction for railroa The depth of comprest air work is limited by the pressure at which w becomes either two expensive or too dangerous. After a depth of 70 ft be water is reached the cost of labor in comprest air increases very rapidly account of both greater wages and shorter hours. Very little caisson w has been done under heads of over 100 ft.

26. Drop Shafts

Drop shafts are sunk by building a heavy lining, usually circular, upon the surface, and then excavating the earth inside, allowing the structure to settle slowly into the ground. Drop shafts have been used in a great variety of places, from small wells a few feet deep to mine shafts 20 ft in diameter and over 500 ft deep. The shell or lining may be of timber, masonry, brick, reinforced concrete, cast iron or steel.

Several very large drop shafts of moderate depth in water-bearing soil have been built circular of brick masonry provided with a cast-iron shoe or cutting edge at the bottom. A large number of small drop shafts, having steel plate shells, have been sunk for foundation work. Most drop shafts for deep mines have had shells of cast-iron rings with, in some cases, steel shoes for cutting edges. The excavation in some cases has been by hand, but, usually, drop shafts are in water-bearing strata, often in quicksand, and the water is allowed to fill the interior while the earth is removed by under-water excavators, such as chain-bucket dredges, grab buckets, and sand pumps.

In moderate depth shafts the weight of the lining is usually sufficient to overcome friction on the sides. In the case of deep shafts this is not the case, and the lining must be weighted with pig iron or sand or be forced down by jacks. Water or air jets which may be forced up along the exterior surface have been used to reduce friction on the exterior, but it has frequently been necessary to abandon the sinking of the first shell and proceed with a lining of a smaller diameter sunk inside the outer lining, which will be subject to friction only below the depth of the bottom of the exterior lining.

The cost of drop shafts is often less than sinking by comprest air, the freezing process, or the boring method. The method is, however, less sure than any of these. In the drop-shaft method the lining is very liable to be distorted, and in some cases ruptured, due to drawing in material from outside the lining, forming cavities, which results in unequal pressures.

27. Freezing Process for Shafts

About 60 shafts, mostly in northern Europe (France, Germany, and Belgium), were sunk up to 1910 by the aid of the freezing process. The general features of all cases are indicated by Fig. 70. Vertical pipes are sunk, usually forming a circle in plan, completely surrounding the site of the proposed shaft; inside each pipe is placed a smaller pipe, called a circulating tube, opening into the outer pipe at the lower end. These circulating tubes are all joined together at the top by a pipe known as the circulating ring. The freezing pipes are joined together at the top by a similar ring called the collector ring. Cold brine at a temperature of from $-10°$ to $-30°$ Fahr is then pumped into the circulating ring and down the circulating tubes, back up the freezing tubes to the freezing machine.

The freezing pipes are put down by one of the well-known boring methods. Originally the casing of the bore hole was used for the freezing pipe. On account of the difficulty of closing the lower end of the pipe and making all joints of the casing water-tight, in all later work the freezing pipes are put together and tested on the surface and then lowered into the bore hole, after which the casing is withdrawn. The freezing pipes are from 4 to 6 inches in diameter, the circulating pipes from 1 to 1½ inches.

In the center of the shaft an equilibrium pipe is usually sunk. This pipe is kept open, as a vent, for the escape of water forced out of the interior mass by the expansion of the freezing material; it is sometimes necessary to keep this open with a steam pipe, because rock or clay strata will freeze much faster than quicksand and possibly congeal to the equilibrium pipe and freeze it before the lower sand strata are frozen.

It is very difficult to keep deep bore holes for the freezing pipes from departing from the vertical. Long freezing pipes are very likely to spring a leak from contraction at low temperatures. Both these considerations have led to the attempt to do the shaft in shorter lengths, say 150 ft in height. The time required for freezing a solid wall about the shaft site will vary with the character of the strata, size and distance apart of the freezing pipes, and the temperature at which the brine is maintained. The freezing

usually continues from 4 to 10 months or longer. After an ice wall has been formed around the site of the shaft, the shaft may be sunk by hand methods and lined as the excavation progresses or it may be sunk as a drop shaft.

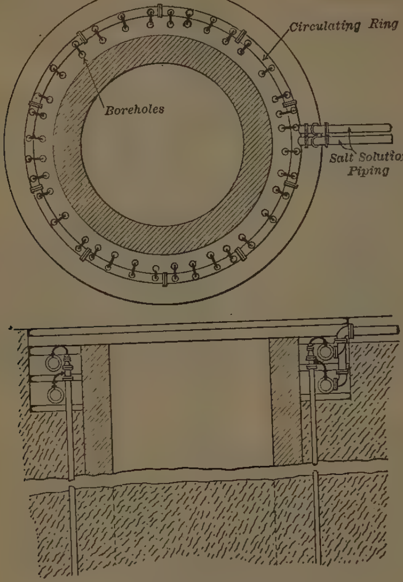


Fig. 70. Freezing Process for Shafts

28. Shaft Sinking by Borings

The Kind-Chaudron System for sinking shafts in water-bearing r
strata is almost identically the same method as ordinary well drilling ex
that it is on a very large scale. In Fig. 71 a very large chopping bit or tre
is seen at *a*, which may be from 10 to 15 ft in diameter, and weighs f
30 000 to 50 000 lbs, and which cuts a hole the full diameter of the sh
b are rods, usually of wood, each about 65 ft long, connected by iron li
c is a bar in a swivel joint and screw connection by which the tool is rot
and lowered slightly between the blows, *d* is a walking beam which a
nately lifts and drops the trepan, and *e* is a steam cylinder which operate
walking beam. The shaft is completely filled with water during sin
A small trepan is used alternately with the large one, and bores a pilot
usually about 1/3 the diameter of the shaft, into which the choppings
the large trepan fall and are removed by a grab bucket.

After the shaft is drilled, sometimes to a depth of 1300 ft, a water-tight cast-iron
or tubbing is then sunk inside the bore and sealed at the bottom. The sealing

bottom is accomplished by building what amounts to a great stuffing-box. The packing used is moss.

In sinking the tubbing, which sometimes weighs several million pounds, the first rings are suspended in the top of the shaft from beams. Rings are added on top until the tubbing, which is provided with a bottom, reaches the water surface; the whole thing is then partially and later completely water-borne, after which it is sunk as the rings are added by admitting water to the interior. On reaching the bottom the great weight bearing upon the moss in the packing rings presses it against the sides of the rock and makes a water-tight joint. There is then an annular space above the moss box between the tubbing and the rock, which must be filled with either grout or concrete. After this is placed and set, the shaft is pumped out, the false bottom removed, and sinking continued in the impervious strata below.

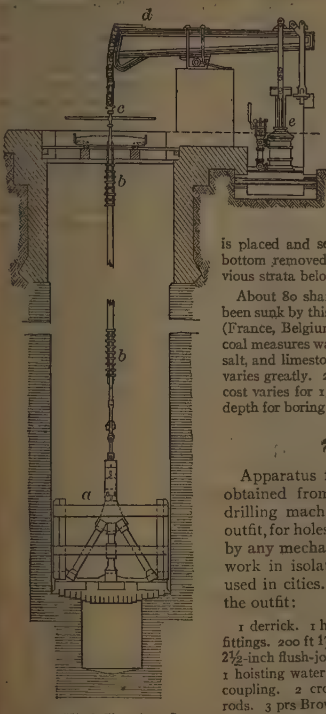


Fig. 71. Kind-Chaudron System

About 80 shafts from 300 to 1300 ft in depth have been sunk by this method, nearly all in northern Europe (France, Belgium, and Germany), where to reach the coal measures water-bearing strata of marl, shale, rock salt, and limestone must be past. The rate of sinking varies greatly. 25 feet per month is a fair average. The cost varies for 15 ft shafts from $500 to $1000 per foot depth for boring and lining with cast iron.

29. Wash Borings

Apparatus for making wash borings can be obtained from companies which make rock-drilling machinery. However, a very efficient outfit, for holes 150 feet deep or less, may be made by any mechanic, which is not only adapted to work in isolated localities but has often been used in cities. The following is an inventory of the outfit:

1 derrick. 1 hand force pump with suction hose and fittings. 200 ft 1⅞-inch drill rods with couplings. 200 ft 2½-inch flush-joint casing. 1 bushing to 2½-inch casing. 1 hoisting water swivel 1⅞ inch. 1 hoisting plug with coupling. 2 cross chopping bits 6 inches long to fit rods. 3 prs Brown's pipe tongs; 2 No. 3, 1 No. 4., 1 pr. No. 3 Brock's chain tongs. 2 Coe's monkey wrenches, 15 and 10 in. 3 Stillson wrenches, 10, 14, and 24 in. 1 15-lb crowbar. 1 pair sister hooks for 1¼-inch rope. 1 16-inch iron hoisting sheave for 1¼-inch rope, with strap and hook. 50 ft 1¼-inch manila rope. 40 ft ¾-inch three-ply water hose. 150 ft 1-inch pipe with couplings. 1 blasting battery with 500 ft No. 20 wire, and supply of exploders and 40 percent dynamite. 1 tool box. 1½-gal oil can. 1 squirt can. 1 pail and cup. 1 pick. 1 spade. 1 ax. 1 hatchet. 3 1¾-inch sand pumps. 2 1¾-inch earth augers to screw into rods. 4 sets reducers 1½-inch to ½-inch. 1 portable forge, small anvil, and small set blacksmith's tools.

The derrick is illustrated by Fig. 72. When set up, the drum and attached wheel are used as a hoisting winch. When folded and brought horizontal, the wheels become a truck on which the whole apparatus is moved to the next hole.

The drill rods are usually 1⅞ inch, exterior diameter, extra heavy pipe, in 5 to 10 lengths, with square female threads on each end and coupling of about 6 inches lo of the same pipe with square male threads at each end. The casi (for shallow holes usually 2½ inch external diameter pipe) may either "flush joint" or "drive cas- ing." The drive casing consists heavy pipe with ordinary sleeve couplings. The flush-joint casing of heavy pipe, each length of which screws into the next, making t joint flush, outside and in. The chopping bit may be screwed in the drill rods, and is sharpened like a ro drill, but is provided with holes to allow t water, forced down the drill rods, to pass o under the bit. A chop- ping bit should never used unless a full force of water is passing thru

The process of sinking holes w flush-joint casing is usually about as f lows: The der- rick is set and a hole is made, usua with a crowbar, to admit a length of casing set a few inc into the ground; into this is serted a 10-ft length of with chopping bit on the lo end and a hoist- ing water sw

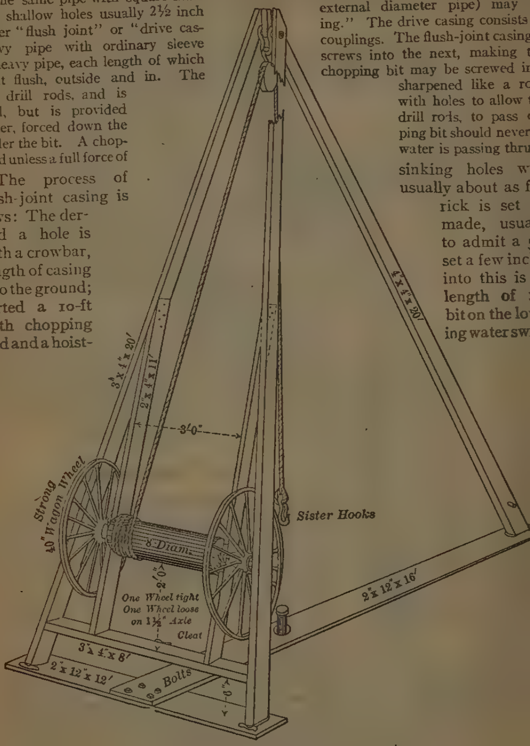


Fig. 72. Derrick for Wash Borings

on the upper end, which is connected to the derrick rope and by hose t force pump. Large pipe tongs are then clamped to both the casing anc Water is turned on, and as the material is washt from inside the casin latter is turned by the pipe tongs and gradually settles into the ground. A length is added to both casing and rods, and then the washing and tu are repeated.

Careful measurement should always be kept so that the relative position of the b of the casing and drill rods is always accurately known. In soft clayey silts and holes may be sunk to a hundred feet or more in depth in this way with very little t

but conditions are seldom so favorable. If hard sand and gravel are encountered, the drill rods are churned as well as turned, by which the material may be broken up, pulverized, and washt out with water and the casing allowed to descend. In general the drill rods work below the bottom of the casing. If loose sand or gravel is encountered, there is likely to be trouble by the binding of the casing. If the sand is a thin stratum it may be possible to wash or even blast out a cavity in it which will allow the casing to pass thru into the next possibly more favorable stratum. Another expedient is to withdraw the rods, cap the top of the casing and attach the water supply to the casing, then to twist or drive, if drive casing is being used, while the water, which is being forced down inside the casing and up the outside, reduces the friction between the sand and outside of the casing. If a casing becomes badly bound so that no further progress downward can be made, it may be necessary to abandon the hole, or having started with a large casing to put a smaller casing inside the first when the latter becomes fast, in which case the friction will only begin at the bottom of the first casing. If the required depth cannot be reached with the second casing, a third and sometimes fourth will be required. When a bowlder or other obstacle is encountered a charge of dynamite is lowered to the bottom of the hole, the casing raised 5 ft or more, depending somewhat on the size of the charge, a good rule being 5 ft for one pound and 2 ft for each additional pound of dynamite used, one-half to two cubic feet of sand is then poured down the hole, the charge exploded, the casing worked down again as quickly as possible, and if the bowlder is a small one it may be found to be shattered so that the casing may be advanced thru it. A single shot often makes no gain when the third or fourth will begin to break the bowlder and show progress, but on bedrock 7 or 8 shots will make practically no progress. If after blasting several times in this way no progress can be made, the conclusion is generally drawn that either bedrock or a very large bowlder has been encountered, and if the purpose of the boring requires it, the hole is continued with a core-boring apparatus. A core boring is the only sure test to determine whether or not bedrock is reached.

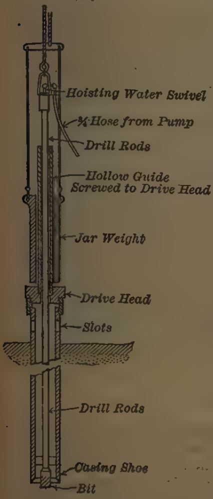


Fig. 73. Drive Casing

For Drive Casing the bottom is provided with a heavy steel shoe or cutting edge, and instead of being simply twisted down as in the case of the flush joint casing, it is driven by using the jar weight (Fig. 73); at the same time the chopping and washing is being done with the drill rods. Working with the drive casing is somewhat slower than with flush-joint casing, but in some cases, such as where considerable depth of loose sand and gravel is encountered, it is preferable. In hard ground the process is to bore or chop with the drill rods a hole some distance ahead of the bottom casing, then withdraw the rods 10 or 12 ft above the bottom and drive the casing down as far as it will go.

Fig. 73 shows one form of apparatus in place, where drive casing is used. There are many possible variations of this; the jar weights are of various sizes and shapes; a second drive head or collar is often screwed to the top of the hollow guides for use in pulling the casing. The jar weight hammers upward against this collar; at the same time the casing is turned with pipe tongs if possible. It is sometimes necessary, however, to clamp a yoke on to the casing itself and hoist on this yoke with jacks. When flush-joint casing is used, the drive head is seldom necessary, and the joints of the casing will not stand much driving.

To Obtain Samples of the material past thru by the drill the wash water is caught in a bucket after it rises to the surface thru the casing, and

the sediment is allowed to settle. Water should not be allowed to continu running into the bucket after it is once full, for it will wash out the clay an leave an unrepresentative sample. Such samples are often misleading a to the true nature of the material in place. A drive sample may be obtaine by attaching to the bottom of the drill rods a sample barrel consisting of pipe open at the bottom and provided with a small vent hole in the side nea the upper end but plugged at the top to prevent the weight of water in th rods from forcing out the sample. This may be lowered and driven int the material which is removed from the sample barrel after being raised t the surface. Samples obtained in this way are much better than wash sample but have the disadvantage that they have been considerably compacted in th taking, and unless the sample barrel is driven only a short distance into th material it may be doubtful from just what elevation the samples com When the material is sand, samples may be taken or the casing cleane out by using an ordinary sand pump which is simply a length of pipe pr vided with a foot valve. This is churned up and down in the hole un filled with sand and water.

An auger may be screwed to the bottom of the drill rod and lowered into the hole a bored into the material at the bottom. If the material is not clean open sand it w remain in the helix of the auger and come to the surface with it. In soft clayey ma rials it is often possible to use drive casing in connection with an auger instead of usi water and the wash method. A hole is bored ahead of the bottom of the casing, wh may then be driven down after the auger has been lifted well above the bottom.

The cost of wash borings is variable, depending on the character of the material pe trated. At present (1919) all costs are unstable and pre-war costs give the best co parison. To these the prevailing factor may be applied to give present day pric For holes up to 200 ft. deep in soft ground, such as silt and clayey sands the c of well-managed work was $0.25 per foot and less under very favorable conditio Where the ground was running sand or hard earth without boulders, the price ran fr $0.50 to $1.00 per foot depth. Where boulders or other obstructions were encounte the cost was from $1.00 in ordinary cases to $4.00 or $5.00 per foot. The above pri were for favorable conditions on the surface and enough work to perfect an organizati If the location were unfavorable, say in a swamp where staging would have to be bu or if water were not available, or if the conditions were otherwise difficult, the cos overcoming the difficulties would have to be added. For jobs involving only a few ho the cost of moving the apparatus to the ground and organizing the work may m than double the above prices. For deep holes, say between 800 and 1,500 ft. deep, cost is much increased. A larger plant with steam or other power must then be use

Subaqueous borings are made upon the ice, or from a platform built on piles, or use of a floating pile driver or from a catamaran.

30. Core Borings

The Diamond Drill consists of a short length of soft iron tube calle "bit," into the lower edge of which black diamonds are set. This bi screwed into the bottom end of another tube called a "core barrel," wh in turn is screwed to the hollow drill rod. A casing is sunk thru the overly earth to the rock surface by wash boring; inside this the drill rods w bit attached are placed and then rotated and prest down by the dril machine; at the same time water is forced down thru the drill rods and and comes to the surface outside the drill rods, carrying with it the r ground to a powder by the diamonds and bit. At intervals the drill are pulled out of the hole, the core removed from the barrel, the bit inspec and the operation repeated. Diamond drilling, and especially the ar setting the diamonds in the bit, can be learned only by experience. All can be done here is to call attention to some of the difficulties and met

of overcoming them. The first care should always be that an ample supply of water is being forced down the drill rods to the bit at all times; the second is to use every precaution to see that the space outside the drill rods does not become clogged by either borings or sand washing in around the top or bottom of the casing.

The difficulty most often encountered arises when seamy rock is met. Sometimes it happens that the water forced down the drill rods flows away thru seams, carrying drillings with it. If drillings are carried into a seam sloping upward, they are likely to run back into the hole and pack around and bind the drill rods when the flow of water is shut off. Small pieces from the wall of the hole of seamy rock are likely to fall in and wedge the drill rods. A single bad seam may often be stopt by pouring sawdust or horse manure into the hole; the escaping water carries this into the seam and clogs it. The second method is to drop balls of soft neat cement mortar down the hole and after it has hardened proceed with the drilling. It is usually wise to treat all seamy rock in this way and drill carefully and remove drill rods relatively often. It is sometimes possible to blast or chop with a chopping bit and sink the casing thru a loose seamy layer of rock. Frequently the hole is reamed out with a special tool and the casing driven down thru the seamy rock.

It is a good rule to let the water run a little time after stopping the drill to clean out the hole and take careful measurement of the position of the bit. In lowering the rods again, if the bit does not reach the original position, the water should be turned on, and if then it will not sink to position without drilling. the bit should be raised and the hole cleaned out with a chopping bit to a smooth clean bottom.

Shot Drilling is done in much the same way as with diamond drills, and about the same rules apply; the difference being that instead of diamonds being set in the bottom of the bit, hardened steel balls are fed down with the water inside of the drill rods, and are rolled around under the edge of the rotating bit and wear out an annular ring the same as cut by diamonds. It has an advantage over diamond drilling because the apparatus is less expensive both to buy and to operate. It takes a much larger core, which is a great advantage in well drilling, and for prospecting usually gives a better sample because the large core is not so liable to break and crumble. On the other hand, shot drilling is usually slower than diamond drilling and it has been mostly used for vertical holes.

The cost of diamond drilling varies greatly with circumstances. In the softer rocks like limestone, for holes 200 ft deep or less, taking 1¼-in cores from 1¾-in holes, the rate of actual drilling will vary from 1 to 3 ft per hour. Delays will average from 50 to 200 percent of time of actual drilling, and moving from hole to hole will usually take one or two days. Pre-war costs varied from $1 to $3 per foot. The present (1919) costs will be more than double these prices. For harder rocks, such as granite, the cost will be 50% greater than for limestone.

31. Test Pits

Test pits are sunk to determine the character of the earth as a preliminary to more expensive operations. If deep, the method of work is the same as already described for timbered shafts. One test pit is usually considerably more expensive than several wash borings, but it is worth much more in the reliability of the information which it gives. A combination of test pits and wash borings often yields the greatest amount of reliable information for a given outlay; the borings may locate the strata with sufficient exactness, while one test pit to every ten or twenty borings will show the character of the strata.

As the earth is excavated from the test pit, the different kinds of material should be placed in separate piles, and the bowlders and smaller stones in each kind separated. It often happens that workmen in digging a test pit will pile the dirt in a single pile and cover up all the bowlders. While the walls of a test pit generally show the character of the material, they do not give a correct idea of the amount of bowlders likely to be encountered.

TUNNELS

32. Tunneling in Earth by Mining

The Elementary Operation of Mining is illustrated in Fig. 74. Starting with the face of earth which is to be mined, sharpened boards P, called polings, supported by the cross timber C, are driven into the earth. Under the protection of the polings the earth is excavated. The excavation is advanced beyond the end of the first polings by erecting another cross roof timber, C_2, and starting new polings and repeating the operation indefinitely. The mining of soft ground is largely a multiplication of this elementary operation. The sides of the excavation may be treated in the same way; small headings may be enlarged to full-sized tunnels by opening the sides by the same process. In principle this elementary operation covers all conceivable requirements of protecting the sides and roof of a working where pressures are within the strength of the timbering. But there is always exposed a face F which must remain unprotected, at least for a short time, and here is where mining troubles begin.

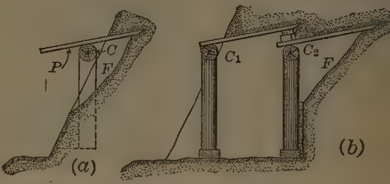


Fig. 74. Mining in Earth

The angle at which an exposed earth face will stand depends: first, upon the nature of the earth, in general, clay standing steeper than sand, and coarse sand steeper than fine sand, and a mixture of clay with coarse sand or gravel being better than either alone. Second, upon the amount of water present. Dampness will increase cohesion, but as complete saturation is approached the angle becomes flatter. If sand besides being completely saturated has an unbalanced head of water, that is, if the head or pressure of water in fine sand exposed to the air is greater than the air pressure, then it is quicksand and will take nearly a horizontal slope. Whatever the material, its thoro draining is of the greatest importance for successful mining. Third, the length of time exposed. Time is an important element in tunneling. Some of the most unstable materials will stand even vertical for a little while, so that a small vertical face may be exposed and a breast board quickly placed. Clayey earth sometimes may move so slowly that the excavation may be made without the protection of the poling, which may later be simply put up against the roof and supported by timbering which will be thoroly loaded later. Fourth, the area of the face exposed. A small area exposed will often stand vertical for a long time when a larger one would cave quickly. Fifth, the distance below the earth surface. By the ordinary accepted theory of earth pressures the horizontal thrust of earth is a function of the vertical pressure or distance below the surface. But when small areas are exposed, dry or only damp earth will arch across the area, so that depth does not usually appreciably affect the angle at which the exposed earth face will stand until some time has elapsed. The arching cannot be depended upon in wet running material. Sixth, the amount of agitation the earth has received. Earth which may be stable in its natural state may be very treacherous after it has been moved. The fear of starting a general movement of the earth makes miners thoroly pack all voids back of supporting timbering and use the greatest care to prevent the starting of a run.

Timbered Headings. In all methods of soft-ground mining the face is opened with a heading varying in cross-section according to the material penetrated from a one-man burrow to full width and 7 or 8 ft high and any length (Fig. 75). The heading timbering consists of posts P, caps C, poling boards p, and breast boards b. If the ground is stiff the face of the heading is cut down straight at nearly the forward end of the polings and new posts and a cap are set. If the ground is soft a small excavation, say two feet

high, under one or two polings is made and a short board is quickly set vertical under the end of the polings and acts as a combined breast board and prop for the poling. This is repeated under all polings. As soon as all short vertical breast boards are placed a new cap is set supported by short posts. The remainder of the face is then worked down, usually with horizontal breast boards, to the bottom of the heading.

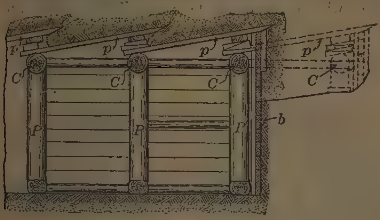


Fig. 75. Timbering Headings

33. Methods of Soft-Ground Tunneling

Small bores are driven by simple headings, large ones are worked in a variety of ways which have been classified in general according to the country in which the method originated, as English, Belgian, German, Austrian, and Italian. Obviously the classification tho widely accepted is far from strict, because each method has many variations, nor has the use of any of the methods been confined to the country for which it is named.

English Method. The entire section is removed in short lengths, usually from 12 to 20 ft, in advance of the permanent lining already built, as shown in Fig. 76. The masons and miners alternate in the possession of the face, and the work of excavation and building the masonry is uninterrupted until each is complete for the length. A small drift in wet material is commonly driven at the bottom of the tunnel prism from end to end of the

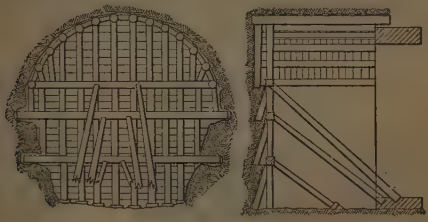


Fig. 76. English Method

tunnel, both for the purpose of furnishing good drainage facilities and for the establishment of the alinement below ground; also it allows the tunnel to be attacked at several points. The main attack on the face, however, always begins at the top of the section with a heading which in stiff material is often taken out without timbering but in softer material is timbered. Two roof bars are then placed in this heading with their forward ends resting on posts and their rear ends supported on the completed masonry lining. Transverse polings are then driven over these bars. The heading is then widened out under the transverse polings for the length of the roof bars, vertical breasting boards placed under the transverse polings and new side bars are placed; when the ground is stiff enough the polings are not driven over the side bars

but placed against the earth roof after excavation as in Fig. 77. This operation is continued down the sides as far as required by the nature and pressur of the ground, sometimes to the bottom, while at the same time the face i securely breasted and back-strutted to the completed lining. The miner now give way to the masons, who construct the length of lining within th completed timbering, and the operation is repeated. If the roof bars ar not too firmly gripped by the overlying material, the lagging is blocked u on the masonry and the roof bars are barred or pulled forward by jacks an used over again.

This method as a whole is best adapted to very firm material, but has been used wit success where the ground was heavy and wet. The Saltwood tunnel on the South eastern Railway between London and Dover was built thru greensand, which whe disturbed was a quicksand and not workable by the English method without being draine A small bottom drift driven thru from end to end drained off the water, care being take to confine the sand, and the main construction proceeded by the usual English metho The advantages of the system are the large open area in which the masonry lining ca be built up in one operation and the facility with which the spoil cars can be handle The disadvantages are that the excavators miss alternate shifts while the masons are work, thus delaying the work, and that the timbering is partly carried on fresh masonr The fact that the full section is excavated at once limits the use of the method to fair firm material.

Belgian Method. The upper half of the tunnel is excavated much th same as in the English method except that the excavation is frequentl carried considerable distance ahead of the masonry, and as the method most often used in firm ground, the transverse polings are frequently n driven over the roof bars, but instead are placed as in Fig. 77, against th earth roof after the excavation has been made for a side roof bar.

A cut is then excavated thru the center half of the tunnel to the invert, leaving a ber on either side to support the arch of the tunnel lining. From this center trench narro cuts are made at intervals to the side and the masonry arch is underpinned; the cuts a widened and the underpinning extended until a complete side wall is built, and final the invert arch is turned. The main advantage of this method lies in the fact that is possible in firm material to push the work from many points of attack, the successive developments of the tunnel, the heading, its enlargement, the masonry arch, the underpinning operations, etc., all being carried on simultaneously at different points along the tunnel. When the work is carried on with this system of successive development the timbering is arranged as in Fig. 77, to permit the ready access of the spoil cars to the furthest workings. This method is adapted to firm material, but may be used, with considerable risk however, in the heavier soils by shortening the timbered section, in which case it loses its chi economy mentioned above. The disadvantage of this method is that the arch is fir carried on earth and then on timbering, both liable to unequal settlement with the co sequent risk of fracture in the key. If it becomes necessary in very stiff material to blasting, danger to the temporarily supported arch is increased.

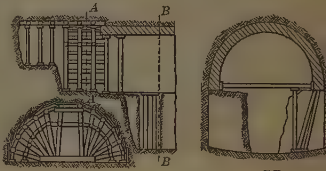


Fig. 77. Belgian Method

German Method (Fig. 78). The invert is put in last, but the rest of t lining is built from the bottom up, but without removing the center core earth. In its most characteristic form the method consists in driving tw bottom headings, one at the foot of each side wall. In these the side wa are built as high as the roof of the heading will allow. On top of these hea ings two others are driven and the side walls brought up to their roofs, a

so on until the two side walls are joined in a center heading at the top. Practically the number of headings one on top of the other is limited to three, with a top center heading which is widened out to the upper side headings. Usually only two headings on each side have been used, which brings the sides up to the springing of the arch; above this the entire section is taken out by widening out a center top heading down to the side heading by the same plan as in the Belgian method.

The most notable example in America of a tunnel done by this method is the Baltimore Belt Line tunnel. There only one heading on each side was driven; the center top heading

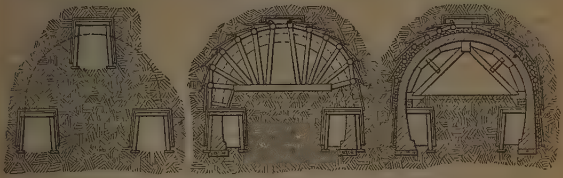


Fig. 78. German Method

was widened out down to the spring line and pits sunk down from this large upper working to the side walls which had been built in the side bottom headings. The sequence of operations is shown in Fig. 78. The chief advantages of the German method are the saving in timber due to large cores of earth being used to brace against; the cheapness with which the central core, a relatively large mass of earth, can be removed; the fact that only small areas are opened, thus increasing safety in soft materials, and the fact that the masonry is built continuously from the bottom upward. The disadvantages are that the restricted area available for removing spoil from the small headings causes a great amount of inconvenience and interference with the masons and timbermen; ventilation in these small passageways is very difficult; and there may be danger in some cases of movement of the center core when used as an abutment for strutting.

Austrian System (Fig. 79). The essential feature is a center cut from top to bottom of the tunnel with timbering as in *A-A*. The cut is widened out

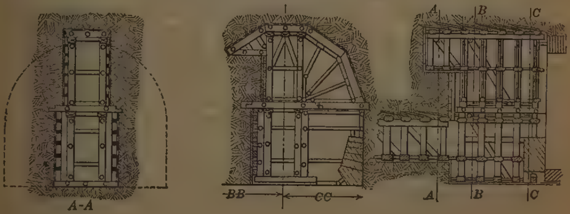


Fig. 79. Austrian System

from the top as indicated in section *BB* and timbered with segmental arch timbering up to the time the sills are placed above the horizontal diameter and is supported by the central frame. The poling boards over the segmental arch timbers are all driven parallel to the axis of the tunnel, and the widening out is done in short lengths, working practically the full face. The variations

in the method, depending largely on the character of the material, are prac cally all in the method of making the central cut. A top heading may driven, which then may be cut down in short lengths to the invert of t tunnel, or a bottom heading may be driven, then the top heading, then the co between the two removed in short lengths, the short posts of the top headi being replaced by long ones resting on the cap timbers of the bottom headi

Mr. Rziha, the eminent Austrian engineer, who did much to develop the system a who is its principal advocate, recommends the bottom heading to be first driven fr end to end of the tunnel, then enlarging this to about half the height of the tunnel a placing in it all of the lower half of the timbering required in the center cut, section C Fig. 79. The upper half of the center cut is then taken out in a top heading and timbers placed as in Fig. 79. The excavation of the remainder of the full section is th done, being kept from 10 to 20 ft ahead of the masonry. The masonry lining is beg at the bottom and carried up continuously, the reaction of the timbers being transfer to the masonry as they are reached. The advantages of the method are that the timb ing is strong, that the timbers are in short lengths and easily handled, and that the w may be attacked at a number of points and each operation carried on continuously. disadvantage is its expense.

The Italian Method (Fig. 80) was especially developed for very s and treacherous ground and is strictly an emergency method. The succ

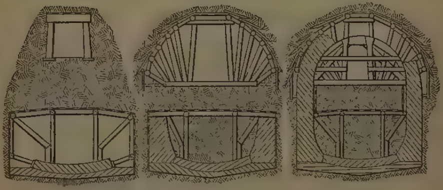


Fig. 80. Italian Method

of the system depends on the fact that only small areas are opened and material can thus be well controlled. A bottom center drift about ⅓ height of the tunnel is started and driven a very short distance, 6 to 10 after which it is enlarged to full width of the tunnel, very heavy and ti timbering being used. The invert and as much as possible of the side w are constructed in the drift, and the open area is backfilled with earth. center top heading is then driven and enlarged to full width and about ⅓ height of the tunnel, leaving a bench the full width of the tunnel and someth less than ⅓ the height. This upper section of the tunnel is very hea timbered and braced; connection between the upper section and the maso walls already built is made by trenching thru the bench, after which the s walls are completed and the arch turned as one operation. The cen bench and the backfilled earth are excavated inside the completed tunnel.

The chief advantage of this method is that the workings are so small as to be rea braced and maintained in very treacherous ground. The disadvantage is the exces cost. Modern shield methods would now be used for all tunnels for which this me was designed, except possibly where only a short length of very treacherous groun encountered.

American Method (Fig. 81). Here the segmental arch timbering and posts only are used and no interior struts. It has been much used in rock; it is specially adapted to fairly firm material. In rock a top head is driven and timbered with posts and caps. A short length is widened

down to the springing line of the arch, the sills are placed and the segmental timber arches erected between the posts and caps, which may then be removed. In earth the face is opened by a top heading, which is poled and timbered in the usual way. In the space between two bents of the heading timber the crown segments of the timber arches are set in position and held in place temporarily by secondary posts or by strips of scantlings spiked to the main posts as shown in Fig. 81. A short length of from 2 to 6 feet, depending on the nature of the ground, section *A-A*, is then widened out without polings or other roof support and the segments adjacent to the crown segments are put in position and held by iron dowels and a short prop. If the segmental timbers are not set close together, lagging is inserted above the timbering and cavities between the lagging and the earth are packed. The widening out for the next timber is done in the same way down to the sill. After the two sills are placed the roof timber is completed and the bench is then removed.

Fig. 81. American Method

During each operation the sill timbers are underpinned by any one of the several different methods. If the material is very firm a longitudinal cut is made in the bench, leaving a berm on which the sill rests, and the counterforts are excavated at intervals under the sill and posts placed. If the material is too soft for this, pits are sunk at intervals, and first short posts and later, if excavation has proceeded, long posts are placed to support the sills. If the material is firm enough, as for example rock, the sills or wall plates are set in niches at about the springing line and no posts are used. The number of segments in the arch varies from 3 to 7 or even more. It can be used successfully in firm material which will stand for a short time without support. This type of timbering has been used in American tunnel practise as a semi-permanent lining, but is usually replaced within 10 to 15 years by masonry. The chief advantages of this method are in the large open area within which the masonry lining can be built continuously from invert to crown and the saving of timber.

34. Tunneling in Rock

Top Heading and Bench Method. Tunneling in rock by the top heading and bench method, shown in Fig. 82, is the prevailing practise in America. The heading usually comprises the upper 7 or 8 ft of the tunnel prism and is driven with compress air drills mounted on vertical columns by one gang, while a second gang with tripod drills works on the bench. In the more recent practise the bench is kept quite close to the heading, so that the mucking of both heading and bench is done together, the blasting being so arranged that considerable of the heading excavation is shoveled into spoil cars by hand, or, if the tunnel is large enough, by air-operated steam shovels. What remains in the heading is handled directly to cars with wheelbarrows and a movable platform at the bench.

Fig. 82. Top Heading and Bench Method

Bottom Heading Methods in Rock Tunnels. There are two general types of bottom heading methods in use in Europe. The first is the method used notably in the Loetschberg and Simplon tunnels, in which a bottom heading is driven in advance; at intervals, in the Simplon every 164 ft, small upraisers or shafts are excavated to the top of the tunnel and top headings driven in both directions and enlarged laterally. The next step is the removal of the bench between the top and bottom headings and finally the side benches. The bottom heading is kept clear for the muck cars by closely timbering its roof. In the second type, a bottom heading is driven in advance and at intervals break-ups are made to the crown of the arch. The material of the full upper portion of the tunnel is shot down upon a heavy timber platform which acts as a roof for the bottom heading.

Outside Pioneer Heading Method. By this method a heading is driven entirely outside the main tunnel which is to be built. From this heading drifts are driven to the center line of the main tunnel and from the ends of these cross drifts, headings are driven along the main tunnel, usually on its centerline. As soon as one of these center headings reaches the working being carried on from the main portal, radial holes are drilled from it to the limits of the cross section of the main tunnel. These holes are loaded and the rock is shot down in front of a steam shovel working inward from the main portal.

The advantage of the method is that the drilling operations do not interfere with the work of the shovel because the drillers and the air mains to the drills all enter from the side heading.

Drilling Diagrams. Fig. 83 shows the method of spacing the drill holes in the headings of several rock tunnels. The practise of setting drill holes normal to the face of the heading is nearly universal in Europe and was used in America prior to the inauguration of the center cut system in the Hoosac tunnel. The depth of holes used in European practise is seldom more than 4.5 ft while in America 6 to 10 or 12 ft depth prevails. In general, the higher speed has been attained in the European practise,

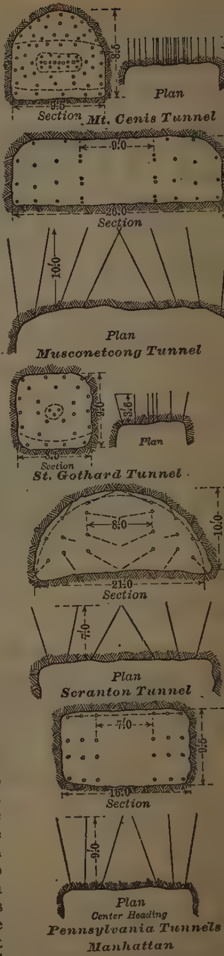


Fig. 83. Drilling Diagrams

with a great number of holes normal to the f ce and from 3 to 4.5 ft deep, but the amount of powder and the cost of driving are greater than where the center cut with deep holes is used. It is not entirely evident, however, that the arrangement of drill holes accounts for the difference, as the higher speeds in the European practise were made where large drills mounted on a heavy c rriage were used instead of the smaller drills mounted on columns as is usual in America.

Mucking is the universal term for removing the blasted rock, called muck, from the tunnel. It is usually done by laborers with shovels. Steel plates are often laid on the floor before the blast; from these the muck is shoveled more easily. Many mechanical muckers have been devised, but none have come into general use except the small steam shovel operated by comprest air.

35. Data Regarding Tunnels

In the following notes the numbers in parentheses refer to the tables on pages 1382 and 1383 and to the numbers within the circles in Fig. 84.

(1) Box, Great Britain. Oölite rock, forest-marble, and Lias marl. Lined with brick 27 in thick at springing line.

(2) Blechingly, Great Britain. Blue clay of Weald. 11 shafts. Maximum progress 792 ft per month, 187 ft per month average for completed work. Hand drills and gunpowder. Lined with brick varying from 22½ to 27 in thick.

(3) Saltwood, Great Britain. Greensand (quicksand before being drained). 12 shafts. Average progress completed work 238 ft per month. Lined with brick in horseshoe section; thickness varies between 22½ and 27 in.

(4) Moncreiffe, Great Britain. Poor rock. Widened and relined in 1902, 88% lined with brick arches 18 in thick on 9-in brick walls.

(5) Lydgate, Great Britain. ¼ of length on curve. Clay, strong shale, rock with veins of shale, limestone, fire clay, and coal. Thickness of arch in shale 2 ft, in rock 1 ft 6 in. 5 shafts. Progress for completed work 52 ft per month. Hand drills used in rock and strong shale; clay and loose shale axed out.

(6) Netherton, Great Britain. Carries a canal. Marl, coarse sand, rock, hard shaly clay, coal, Lias ironstone and fireclay. 17 shafts. Average progress for completed work 294 ft per month. Hand drills used. Lining 22½ in thick in horseshoe section.

(7) Bergen No. 1, Erie Railroad, N. J. Dolerite (very hard trap). Hand drills used. 20% lined with brick arch and stone masonry side walls.

(8) Buckhorn Weston, Great Britain. Kimmeridge clay and veins of loose wet rubble, 5 shafts. Average progress 168 ft per month. Lining 27 to 32 in thick.

(9) No. 6 Union Pacific Railroad, California. Granite. Average progress 1.6 ft finished tunnel per working day, using 4 working faces.

(10) Sand Patch, Pennsylvania. Red sandstone and shale varying from hard to soft. About 50% lined with sandstone masonry.

(11) Mont Cenis. From France to Italy. About ¾ mile on curve. Limestone, calcareous schist, quartz, carbonaceous schist, gneiss and schistose sandstone. Maximum depth below surface 5277 ft. Maximum rock temperature was 80° Fahr. No shafts. Maximum progress one heading 297 ft per month, average for both headings completed 231 ft per month. Cost $272 per lin ft. Hand drilling used for first four years with holes 1.5 ft deep, subsequently 3-in cylinder comprest air drills with holes 3.5 ft deep were used. After first four years 70 to 80 drill holes 1.2 in in diameter in heading averaged 67 lbs gunpowder per round, or about 3.8 per cu yd. Mucking done by hand; horses for hauling. Gas for lighting. Ventilation at first by exhaust from drills, then blowers and bratticing were installed, and finally exhaust bells. 2 rounds were fired in 24 hours. Time of drilling 6 to 8 hours, loading 1.5 to 2 hours, mucking 3 to 5 hours. Lined with side walls of stone and arch which is stone for ½ length and brick for the rest; the thickness is 31½ in.

(12) Baltimore, B. & P. R.R., Maryland. 20% on curve. Soft rock, clay, sand, and earth. 16% tunnel, 84% cut and cover. Hand drills and black powder used. Brick arch 22 in thick; limestone side walls 54 in thick.

(13) Clifton, Great Britain. Mountain limestone, conglomerate, and red sandstone. 2 shafts. Average progress of completed headings 196 ft per month, using 6 faces. 92% by hand drilling, 8% diamond boring machine. Partly lined with 5-ring brick arch on stone masonry walls.

35. Data Regarding Tunnels (Fig. 84)

Number and Name	Single (S) or double (D) track and date	L'gth, miles	Clear height, feet	Clear width, feet. Straight (S) or curved (C)	Material, excavation method	Lining	Cost per lineal foot
(1) Box, G. B.	D 1837–41	1.8	28.0	30.0	Rock E	Brick	$16
(2) Blechingly, G. B.	D 1840–42	0.8	25.0	24.0 S	Clay E	Brick	11
(3) Saltwood, G. B.	D 1842–43	0.5	25.5	24.0 S	Sand E	Brick	19
(4) Moncreiffe, G. B.	D 1845–48	0.7	19.0	26.5 C	Rock	Brick	...
(5) Lydgate, G. B.	D 1854–56	0.8	20.0*	25.0 Q	Mixt E	B & M	4
(6) Netherton, G. B.	 1856–58	1.7	24.3	27.0 S	Mixt E	Brick	6
(7) Bergen, Erie R.R.	D 1855–61	0.8	21.0	28.0 S	Rock H	B & M	18
(8) Buckhorn Weston.	D 1859–63	0.4	24.0	25.0 S	Clay E	Brick	11
(9) No.6 U.P.R.R., Cal.	S 1866–70	0.3	20.2	16.0 S	Rock H	Timber	...
(10) Sand Patch	S D 1854–71	0.9	16.5 19.0	16.0 24.0 S	Rock H	Rubble	8
(11) Mont Cenis	D 1857–72	7.9	24.6	26.2 Q	Rock B	B & M	27
(12) Baltimore, B. & P.	D 1871–73	1.3	18.5	27.0 Q	Mixt V	B & M	14
(13) Clifton, G. B.	D 1871–74	1.0	20.8	26.0 S	Rock P	B & M	..
(14) Church Hill, Va.	D 1872–74	0.7	19.2	27.5 Q	Clay A	B & M	1
(15) Musconetcong	D 1872–75	0.9	21.0	26.0 S	Mixt W	B & M	..
(16) Hoosac, Mass.‡	D 1854–76	4.7	19.8	25.0 S	Rock W	Brick	3
(17) Bergen, D. L. & W.	D 1874–77	0.8	21.0	27.0 S	Rock H	B & M	..
(18) Osakayama, Japan	S 1878–80	0.4	14.0*	14.0	Rock B	Brick	
(19) St. Gothard	D 1872–82	9.3	24.6	26.2 Q	Rock B	B & M	2
(20) Mullan, Mont.	S 1883	0.7	20.0*	15.0 S	Rock	B & M	..
(21) Arlberg	D 1880–83	6.5	25.0*	26.3 S	Rock L	Rubble	1
(22) Severn, G. B.	D 1879–86	4.4	24.5	26.0 Q	Mixt E	Brick	2
(23) Vosburg, Pa.	D 1883–86	0.7	20.7	28.0 Q	Mixt K	B & M	1
(24) Stampede, N.P.R.	D 1886–88	1.9	22.0	16.5	Rock H	B & C	1
(25) Ronco, Italy	D 1882–89	4.8	......	......	Rock X	B & M	..
(26) Balt. Belt Line	D 1890	1.6	22.0	27.0 S	Earth G	Brick	..
(27) Little Tom	S 1888–90	0.4	20.1	14.0	Rock	Brick	..
(28) Cowburn, G. B.§	D 1888–92	2.1	24.5	27.0 S	Rock L	B & M	[illegible]
(29) Totley, G. B.§	D 1888–92	3.5	26.2	27.0 Q	Mixt R	B & M	[illegible]
(30) Niagara Falls Co.	 1890–92	1.3	21.0	19.0 S	Rock K	Brick	..
(31) Busk, Colo.	S 1890	1.7	21.0	15.0	Rock H	Timber	
(32) Panir, India	D 1893	0.6	20.7*	29.5	Mixt B	....	1
(33) East River Gas	 1891–94	......	8.5	10.5 S	Mixt M	....	..
(34) Tequixquiac, Mex.	 1866–95	6.2	14.0	14.0 S	Rock H	B & S	.
(35) Ampthill No.2, G.B.	D 1895	0.4	25.0	25.5	Clay E	Brick	.
(36) Cwm Cerwym, G.B.	S 1897	0.6	....	Q	Mixt	Iron	.
(37) Catesby, G. B.	D 1895–97	1.7	25.5	27.0	Clay E	Brick	.
(38) Boulder, Mont.	S	1.2	21.5	15.7 Q	Rock	B & M	.
(39) Palisades	D	......	....	18.0	Rock H	....	.
(40) Cascade, G.N.R.	S 1897–00	2.6	21.5	16.0 S	Rock K	Concr.	.
(41) Sherman, Wyo.	D 1900–01	0.3	....	....	Rock K	....	.
(42) Graveholtz	S 1895	3.3	....	....	Rock	....	.
(43) Barrientos, Mexico	D 1903	0.1	28.2	37.0 S	Mixt H	Concr.	.
(44) Scranton	S 1904–05	0.9	22.0	17.0 Q	Rock K	TC&M	
(45) Chicago Subway	 1901–	65.0	7.5	6.0 Q	Clay M	Concr.	.
(46) Gallitzin, Pa.	S 1903–05	0.7	23.5	17.5 S	Rock K	C & M	
(47) Alfreton No.2, G.B.	D –1905	0.5	25.5	26.5	Rock E	Brick	.

* Above rails. † Contract price. ‡ Height = 18.5 to 21.0 ft; width 24.0 to 26
|| Excavation and timbering only. § Height = 23.2 to 25.8 ft.

Number and Name	Single (S) or double (D) track and date	L'gth, miles	Clear height, feet	Clear width feet. Straight (S) or curved (C)	Material, excavation method	Lining	Cost per linear foot
(48) Simplon.........	S 1895-06	12.4	18.0*	16.4 Q	Mixt L	Rubble	240
(49) Capitol Hill......	D 1904-07	0.8	20.0	16.0 Q	Earth Z	BC&M	.....
(50) Haversting.......	1902-08	1.4	20.0	15.0 S	Rock	....	.....
(51) Bergen, D. L. & W.	D 1906-09	0.8	26.9	30.0 S	Rock H	Concr.	.....
(52) St. Paul Pass.....	1907-	1.6	....	....	Rock H	Timber	.....
(53) a Penn.Twin,N.Y.	D 1905-09	1.8	18.2	16.3 Q	Rock H	B & C	.....
b Penn. 3 Tr.,N.Y.	1907-09	0.2	18.2	39.7 S	Mixt Y	B & C	.....
(54) Bergen, P.R.R....	D 1905-09	2.2	18.3	19.0 S	Rock H	B & C	.....
(55) Gunnison, Colo...	1905-09	5.8	11.4	10.5 S	Mixt H	Concr.	.....
(56) ArthursPass, N. Z.	S 1908-	5.31	16.75	15 S	Rock R	Concr.	.....
(57) Fu-Chin-ling,Mch.	S 1909-11	0.92	18.0	12.0	Rock P	B & M	75
(58) Loetschberg,Switz.	D 1906-13	9.03	22.3	25.0	Rock L	....	.....
(59) Necaxa,No.1,Mex.	1909-13	2.15	9.83	9.17	Mixt Z	Concr.	165
(60) Piraby, Brazil....	1911-13	5.25	13.2	12.85	Rock	....	.....
(61) Laramie-Poudre,C.	1909-	2.27	7.5	9.5	Rock M	....	.....
(62) Mt. D'Or,F.&Swz.	1910-15	3.8	20.0	26.2	Rock L	Mas'ry	.....
(63) Astoria-Brx.,N.Y.	1910-15	0.88	18.0	16.75	Rock	Concr.	.....
(64) Rogers Pass., B.C.	D 1914 16	5.0	21.12	29.0 S	Rock Z	....	.....
(65) Twin Peaks, Cal..	D 1914-17	2.27	15.0	25.0	Mixt.	Concr.	.....
(66) Roosevelt, Colo...	1907-	4	....	....	Rock M	....	.....
(67) Rove, France.....	....	4.0	40.0	72.0	Rock Z	Concr..	.....
(68) Mt. Royal, Mont'l.	D 1912-18	3.25	19.75	13.0	Rock L	Concr.	.....

* Above rails. For waterworks tunnels see Aqueducts.

S = Straight, C = Curve, Q = Straight and curved, B & M = Brick and stone masonry, B & C = Brick and concrete, B & S = Brick and artificial stone bricks, T C & M = Timber, concrete, and stone masonry, C & M = Concrete and stone masonry, B C & M = Brick, concrete, and stone masonry.

A = American method, B = Belgian method, E = English method, G = German method, H = Top heading and one bench, K = Top heading and two benches, L = Bottom headings, from which small shafts at intervals are driven to top of tunnel and full width top headings are excavated in both directions. The bench is removed between the top and bottom headings and the lower section of the tunnel is enlarged to full width, M = The full tunnel prism is removed as one operation, O = Cut and cover method, P = Top heading and enlargement, no other particulars are available, R = Bottom headings followed successively by their enlargement laterally and upward, V = Cut and cover and English method, W = Top heading and one bench and English method, X = Belgian and English method, Y = Top heading and one bench and cut and cover method, Z = see description of methods in the notes on pages 1387–1389.

Costs are taken from published accounts in which the elements composing the figures are seldom recorded.

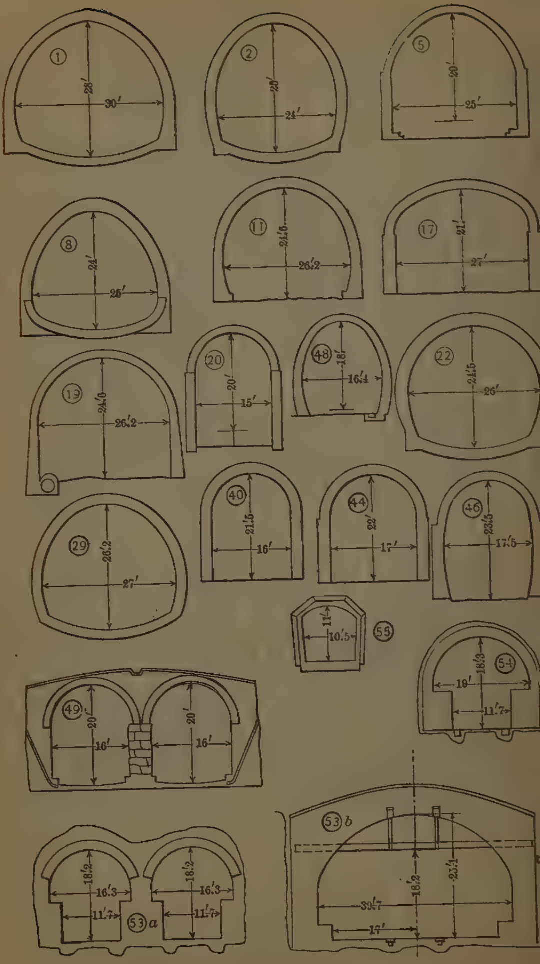


Fig. 84. Sections of Tunnels

(14) Church Hill, Virginia. Miocene clay with infusorial shells. 22-inch brick arch on masonry side walls.

(15) Musconetcong, N. J. Soft ground, limestone and syenitic gneiss. English method in soft ground; top heading and bench method used in rock. 3 shafts. Maximum progress in one heading 144 ft per month, average for whole work 181 ft per month, using 8 faces. 5-in cylinder Ingersoll compress-air drills and dynamite used. 36 drill holes in heading, 10 in bench. Depth of holes 11 ft. 270 lbs explosive per round in heading, 107 in bench. 6 drills in heading, 2 on bench. Spoil loaded into barrows, wheeled to traveling platform at bench, and dumped into chutes over cars. A derrick on the platform handled large bench excavation. Average time of drilling and loading for one round 32 hours. 20% lined with 7 or 8 ring brick arch on masonry walls.

(16) Hoosac, Massachusetts. Hard granitic gneiss, conglomerate, and mica schist. Maximum cover 1800 ft. English method in soft ground. Top heading and bench method in other material. 1 shaft. Maximum progress for 1 heading 184 ft per month, average per month for all faces 194 ft. Holes 2.1 ft deep were drilled by hand during the first 12 years, using 16 lbs of explosive per lin ft of tunnel. During the last 10 years holes 11.5 ft deep were drilled by 12 compress-air drills at each face, and 33 lbs of powder per lin ft of tunnel were used. Spoil loaded into cars, wheeled to traveling platform at bench. Derrick on platform handled large bench excavation. Lined with brick in poor rock.

(17) Bergen No. 1, D. L. & W. R. R., New Jersey. Dolerite (very hard trap). Rend-rock powder used. Brick arch on masonry side walls.

(18) Osakayama, Japan. Soft rock with clay veins. Lining is 2, 4, or 6 ring brick arch on brick side walls. Hand drills. Fans for ventilation.

(19) St. Gothard, Switzerland. Straight except about 500 ft. Gneiss, mica schist, serpentine, and hornblende schist. Maximum depth below surface 5576 ft. Maximum rock temperature 88° Fahr. No shafts. Average progress of single heading 225 ft per month. Sommelier drills used for a few years, then Ferroux 2¼-in cylinder and McKean 4-in cylinder drills with compress-air. Enlargement by hand drills. 6 drills, mounted on a carriage, bored 24 to 26 holes in each heading 2.6 ft to 4.2 ft deep. 29 lbs, 75% dynamite, explosive per round, or about 5 lbs per cu yd, were used in each heading. Horses used in headings, compress-air locomotives in full-size sections. Oil lamps for lighting. Exhaust bells for ventilation. Time of drilling 3 to 5 hours, mucking 3 to 4 hours. Brick arch on rubble masonry walls varying between 17.7 and 29.6 in thick. Altho in rock, was built by the Belgian method.

(20) Mullan, Montana. Treacherous rock. Relined with masonry in 1892. Costs, including engineering superintendence and interest: concrete $8 per cu yd; brick $17 per cu yd. Total relining $50 per lin ft, brick arch 20 in and concrete side walls 30 in thick.

(21) Arlberg, Austria. Mica schist and gneiss more or less rich in quartz. Maximum rock cover 2300 ft. Maximum rock temperature 64° Fahr. No shafts. Maximum progress for 1 heading 641 ft per month, average for single headings 408 ft per month. Ferroux compress-air and Brandt's rotary hydraulic drills used. With Ferroux drills 4.8 lbs of powder per cu yd and with Brandt 4.0 lbs. Lining is mostly between 19.7 and 37.4 in thick, 6% of length not lined.

(22) Severn, Great Britain. Conglomerate, limestone, carboniferous beds, marl, gravel, and sand. Lining of vitrified brick 27 to 36 in thick.

(23) Vosburg, Pennsylvania. 138.5 ft on curve, the rest is on tangent. Red, green, and black shales and red and blue sandstone. Short length earth roof. American system of timbering used. No shafts. Average progress single heading 137 ft per month. In the heading 4 Rand or Ingersoll 3⅛-in cylinder drills bored 12 to 18 holes 9 to 10 ft deep. 3.13 lbs rack-a-rock were used per cu yd excavated. 2 drills and Altas powder were used on the bench. 10% of all the drilling was done by hand. The average amount of explosive used was 1.71 lbs per cu yd of all excavation. No special provision was made for ventilation. Mostly arched with 3 rings of brick. About ⅛ arched with stone 18 to 24 in thick. Side walls of stone 30 to 36 in thick. Stone used is black limestone.

(24) Stampede, Washington. Soft basaltic rock. No shafts. Maximum progress of single heading 274 ft per month, average for two headings 413 ft per month. About 9% with hand drills, the rest with compress-air drills. 31 lbs of Giant and Hercules 45% and 60% dynamite were used per lin ft of tunnel. 20 to 23 drill holes 12 ft deep in heading, 18 in bench. Spoil from heading hauled to chutes at traveling platform at bench by mules. Removed in cars from chutes by small locomotives. Electric lighting. Ventilation by exhaust fans. Concrete side walls and brick arch.

(25) Ronco, Italy. Argillaceous schist with considerable water. 6 shafts. Average progress one heading 231 ft per month, average of completed tunnel 123 ft per month. Brandt rotary hydraulic drills and Ferroux compress-air drills. 0.48 to 0.68 lbs 75% to 78% dynamite used per cu yd. Sheeles system of ventilation. 2 exhausts coupled.

(26) Belt Line, Baltimore. Howard Street Tunnel. Sand with seams of loam, clay, and gravel. 5-ring brick arch.

(27) Little Tom, Norfolk & Western R.R. Seamy gray sandstone disintegrating on contact with air. 1410 ft originally lined with timber. Later relined with brick. Arch made of four rings of brick. Cost of relining complete was $33.50 per lin ft.

(28) Cowburn, Great Britain. Material about ⅓ shale, ⅔ rock intermixt with thin beds of shale. Quite dry. Average progress for single heading, dry shale by hand drilling 299 ft per month, rock by hand drilling 111 ft per month, rock and shale by machine drilling 270 ft per month, rock by machine drilling 199 ft per month. ⅓ excavated with hand drills, rest with Larmuth compress-air drills. 17 lbs gelignite used per lin ft. 2 drills in heading. Horses used in headings and locomotives in finished sections. Ventilation by fan. Arch entirely brick. Side walls ⅔ length stone masonry and ⅓ length brick.

(29) Totley, Great Britain. Hard and soft black shale, coal, fire clay, sandstone, and grit rock. 4 shafts. Average progress machine-drilled heading 242.4 ft per month. 3¼ and 3½ in Schram percussion drills and 3-in Larmuth. Average number drill holes in heading 13.4. Holes 6.2 ft deep. 38 lbs gelignite per round. 2 drills in heading. For clearing heading of fumes and dust, a jet of compress air and spray of water were used. The arch is entirely brick and the walls are brick for 70% of length and coursed masonry for 30% of length.

(30) Niagara Falls Power Co., New York. Limestone and shale with slaty seams extremely wet. Progress 304 ft per month with 5 headings. Rand compress-air drills, 4 to 6 per heading. Dynamite. Electric lighting. Ventilation by exhaust from drills. Lined with from 4 to 6 rings of brick.

(31) Busk, Colorado. Gray granite, disintegrated in places. Maximum progress of single heading 202 ft per month, average completed tunnel 190 ft per month. 3½-in cylinder Ingersoll compress-air drill. Holes 12 ft deep. Electric lighting. Blower for ventilation. 78% lined with timber.

(32) Panir, India. Limestone, clay, and soft sandstone. Average progress single heading 145 ft per month, average whole work 95 ft per month. 4-in cylinder Climax compress-air drill bored 25 holes in heading 3.8 ft deep and 1¾ in in diameter. Dynamite and gelignite were used. The time for drilling one round was 5 hours. 520 ft is one-half lined and 2690 ft is three-quarters lined.

(33) East River Gas, New York. Gneiss, mica schist, veins of decomposed feldspar, and black mud. Maximum progress 101 ft per week on New York side in rock, average 69 ft per week in good rock in each heading. Compress air 15 to 48 lbs per sq in used while in soft material. Good rock not lined; soft material lined with cast iron—see table of cast-iron lining in tunnels, p. 1398.

(34) Tequixquiac, Mexico. Sandstone with lime, soapstone, and conglomerate. Very wet. Progress 18 ft per day in heading. Timbered. Lined with 20-in arch of brick and 18-in walls of artificial stone blocks. Hand drills and dynamite. 6 drill holes in heading 6.5 ft deep. 6 lbs of explosives per round in heading.

(35) Ampthill 2nd, Great Britain. Kimmeridge clay. Progress averaged 6 days to complete 1 length of 12 ft. 1½ lbs tonite per lin ft. Horseshoe section, walls, and arch lined with 33 in of brick; invert 27 in brick.

(36) Cwm Cerwym, Port Talbot R.R. Slight curves at end. Shale, hard clift, fire clay thin veins of coal, and about 500 ft of pennant rock. Originally lined with brick without invert, afterward relined without interrupting heavy traffic. Concrete blocks in invert and cast-iron segments above.

(37) Catesby, Great Britain. Clay. 9 shafts. Average progress for finished work 330 ft per month. Lined with brick 22½ to 31½ in thick.

(38) Boulder, Montana. 150 ft curved at one end. Material penetrated 80% seamy blue trap rock and 20% syenitic bowlders and debris. Originally lined with timber. Later relined with 20-in coarse granite rubble in walls and 4 rings of brick in arch.

(39) Palisades, New Jersey. Hard trap rock with many seams in places. 1 shaft. Average progress 186 ft completed tunnel per month. 24 holes in heading drilled by four Ingersoll-Sergeant 2½-in compress-air drills. 6 drills on bench. A derrick car was used t

handle large stones from the bench. Horses were used at first, dummy locomotives later.

(40) Cascade, Washington. Medium hard gray granite, seamed and wet. Temporary 5-segment timbering used. Maximum progress using two headings 527 ft per month. Maximum progress in single heading 301 ft per month. Average two headings 350 ft per month. Six 3¼-in Rand compress-air drills bored 24 to 28 holes 12 ft deep in heading. 8 drills on bench. Spoil run out in wheelbarrows to a traveling platform at the bench and dumped thru chutes into cars. A six-ton derrick mounted on platform handled large bench excavation. Ventilation by exhaust fan. Electric lighting and electric motor haulage.

(41) Sherman, Wyoming. Solid granite. Top heading and two benches. Heading progress 4.92 ft per day. Drill holes 9 ft deep. 6 drills on 3 columns in heading.

(42) Graveholtz, Norway. Quartzy granite or gneiss. Average progress of finished headings was 263 ft per month. Hand drills used at one end for 2½ years, after which Brandt boring machines and compress-air drills were used. Only about 3% lined.

(43) Barrientos, Mexico. Hard granitic porphyry with clay seams. No shafts. Heading progress 367 ft per month, completed excavation 245 ft per month. 3 Ingersoll Sergeant compress-air drills bored 14 holes in heading 10 to 12 ft deep. 29.3 lbs of 60% dynamite used per lin ft. 3 drills on bench. Small cars were loaded and run to platform at bench, where they were dumped into larger cars. Lined with concrete blocks 28 to 31 in by 60 in deep by 7 ft 8 in long.

(44) Scranton, Pennsylvania. Sandstone and shale. 2 shafts. Maximum progress in single heading 261 ft per month, average all headings 387 ft per month. 3¼-in cylinder compress-air drills and 70% dynamite. 1305 ft not lined, 2717 ft 5-segment timber lining and 725 ft concrete and stone masonry.

(45) Chicago Telephone and Express Subway, Illinois. Stiff blue clay. Full-size excavation. Excavated with spades and draw knives. Compress air used, 9 lbs per sq. in Floor 30 ft below street. Lined with 10 in of 1 : 3 : 5 concrete.

(46) Gallitzin, Pennsylvania. Sandstone and shale, also some limestone, fire clay, and slate. No shafts. Average progress of completed tunnel 164 ft per month. 16 holes in the heading and 11 on the bench 10 feet deep were drilled respectively by 4 and 2 Ingersoll-Sergeant compress-air drills. 2.5 lbs 40% forcite were used per cu yd. An air-operated steam shovel following up at the bench handled the excavated material. Horses were used in the headings and dinky locomotives in the full-size tunnel. Electric lighting. Time of drilling was 8 hours, two rounds being fired per day. Concrete arch 22 in thick and concrete and rubble side walls increasing from 38 in at spring line to 54 in at bottom.

(47) Alfreton 2nd, Great Britain. Gray sandstone, rock, shales, and fire clay with seams of coal. Worked from 5 shafts with no portal headings. The average time of mining a 15-ft length was 19.7 days. Lined with 28 in of brick.

(48) Simplon, Italy to France. Short curves at ends. Mostly very hard gneiss and calcareous rock, also short lengths in green cipolin, disintegrated slate, clay, and mica schist. Maximum depth below surface 9118 ft. No shafts. Maximum progress single heading 685 ft per month, average both headings 440 ft per month. In bottom headings Brandt rotary hydraulic drills, enlargement by hand drills. 83% dynamite and 64% blasting gelatine used in opposite ends of tunnel. 10 to 12 drills holes 4.5 ft deep, 3½ in in diameter, in heading. 2.8 lbs explosive per cu yd. 3 drills in the heading. Time of drilling one round 2.75 hours and the time of loading, firing, and mucking was 6 hours. Steam and compress-air locomotives for hauling. Blowers and water sprays used to keep the tunnel air clean. Both gas and oil for lighting. Lined with coursed masonry of various thicknesses.

(49) Capitol Hill, Washington, D. C. About ⅛ on curve. Gravel and fine sand overlaying hard blue clay. Very wet. 1600 ft in open cut and 2400 ft in tunnel. Separated by lining into twin tunnels. Each tube excavated separately. 2 side drifts at bottom and crown drift excavated in one tunnel. Crown bars 24 ft long placed in top drift and supported by radial struts. As top heading was widened segmental timbering was placed under the crown bars and carried down to posts on concrete footings in the side drifts. The radial struts were removed and the core of earth between the side and crown drifts was excavated with a steam shovel. The lining was placed in this portion of the tunnel and the excavation of the other tube was proceeded with, the timbering for it abutting against the crown bars of the first tube. The arch is brick backed by concrete

of a total thickness of 36 in. The side walls are concrete 72 in thick and the core-wall is stone 48 in thick.

(50) Haversting, Norway. The cross-section is about 270 sq ft. Gneiss with varying amounts of feldspar and quartz. Total progress both ends 114 ft finished tunnel per month. Hand drills. Horses for hauling.

(51) Bergen No. 2, D. L. & W. R.R., New Jersey. Very hard trap rock. Ingersoll-Sergeant compress-air drills. 29 holes in heading 7 to 8 ft deep. 8 holes in bench. Electric lighting. Air-operated steam shovel for mucking. Lined with concrete 24 in thick.

(52) St. Paul Pass, Idaho to Montana. Laminated quartzite with some talc layers. Average progress two headings for four months 576 ft per month. 3½-in cylinder Ingersoll-Rand compress-air drills, 8-in heading and 5 on bench. An air-operated steam shovel used at bench. Excavated material hauled by electric locomotives. Fans for ventilation. Electric lighting. Lined with five-segment timber arch.

(53a) Pennsylvania R.R., twin tunnels under Manhattan from Sixth Avenue to East River. Hudson schist. Two sets of twin tunnels averaging about 4730 ft long. At east end 350 ft excavated as 4 single tunnels and the remainder was excavated as two tunnels from 42 to 52 ft wide and separated by lining into four single tubes in sets of two. 2 shafts for each set of tunnels but no portals. Three methods of excavation used in wide tunnels: double heading, center heading, and full-width heading. Bench was usually 10 to 15 ft behind the face. The maximum progress of a single heading was 206 ft per month, the average for all six headings was 451 ft. The average length of tunnel completed per month was 257 ft. The number of holes drilled varied greatly with the width and location of the headings and averaged between 8 and 9 ft in length. In single headings 4 Ingersoll-Rand 3¼-in drills mounted on 2 columns and in full-width headings 10 drills mounted on 5 columns were used. 1.9 lbs 60% forcite were used per cu yd of all excavation. Air-operated steam shovel followed at the bench. Electric motors for haulage. Blowers for ventilation. Electric lighting. Lined mostly with concrete. Small sections in wet rock have brick arch and concrete side walls. Minimum thickness of lining 22 in.

(53b) Pennsylvania R.R., three-track tunnels under Manhattan. 218 ft long on west end of one set of twin tunnels and 683 ft long on the other. Hudson schist. Sand in the crown at places. Part done in open cut. Lining in open-cut sections concrete thruout with a minimum thickness of 59 in and in tunnel brick arch 56½ in thick on concrete side walls.

(54) Pennsylvania R.R., Bergen Hill Tunnels. Mostly hard trap rock, also 940 ft in decomposed sandstone, shale, feldspar, calcite, and sandstone. The maximum progress of a single heading was 145 ft per month, the average progress was 3.13 ft per day at each face. 8 Rand 3⅝-in drills on 4 columns drilled 32 holes 12 ft deep in the headings. The holes tapered from 2¾ or 3 in at the top to 1¾ or 2¼ in at the bottom. 1 ft of steel was used up for every 10 cu yds of excavation. An average of 5 lin ft of hole was drilled per cu yd at the average rate of 2.66 ft drilled per hour of the machine at the face. An average of 2.9 lbs of 60% forcite was used per cu yd of all excavation. The average time of each attack was 36 hours. Air-operated steam shovels handled the muck. Fan for ventilation. Electric lighting. Lined with concrete with a minimum thickness of 22 in.

(55) Gunnison, Colorado. Irrigation Canal. Mostly black shale and metamorphic granite; other materials penetrated, clay, water-bearing gravel and sand, sandstone, limestone, coal, and marble. 1 working shaft. Maximum progress of single heading 824 ft per month, average length of tunnel completed 566 ft per month. Sullivan 3-in and 2½-in and Leyner 2½-in cylinder compress-air drills were used. Jeffrey coal augers were used in shale. 40% gelatine dynamite used. 18 to 22 drill holes 6 ft to 7 ft deep in the heading. 4 drills in the heading and one on the bench. Blowers for ventilation. It was necessary to drive an inclined shaft for ventilation. Electric lighting. Lined with concrete 16 to 26 in thick.

(56) Arthurs Pass, New Zealand. Badly fissured sandstone and shale, 3% grade, not much timbering. 2 to 3 Ingersoll-Sergeant 3¼ in drills on a 4½ in cross-bar without arms in heading. 10 to 16 holes 4½ ft to 6½ ft deep per round and 4 to 6 lbs of Nobel's gelignite per cu yd in heading. Electric haulage. Side walls of concrete poured in place, arch roof of concrete blocks 12 in radial by 9 in by 18 in along tunnel, mortar joints. Over break of hand-packed stone.

(57) Fu-Chin-ling, Manchuria. Hard limestone. No timbering. Lined with cu

limestone on sides with a 3 ring brick semi-circular arch of 8 ft radius in roof. Cost $75.00 per ft lined and ready for track.

(58) Loetschberg, Switzerland. Limestone, slate, gneiss and granite. Greatest thickness of overlying rock 4692 ft. No shafts. Temperatures 75° to 110° F. Maximum progress of single heading 1013 ft per month. 4 to 5 Ingersoll-Rand 3⅝ in drills on carriages in bottom heading and on columns in top heading. In the headings there were 12 to 14 holes 4 ft to 5 ft deep and 85% dynamite was used. Lighting by individual acetylene lamps. Due to an inrush of water, sand and gravel which filled 5900 ft of the excavation and buried 25 men 0.85 miles of the old working was abandoned and the alignment changed.

(60) Pirahy, Brazil. To divert water from Pirahy river to power company's reservoir in adjacent valley. Mostly solid gneiss. Bottom heading 13 ft wide by 8 ft high. Best month 512 ft. Very little timbering. About 1000 ft of concrete lining, the rest unlined. Five 3¼ in Ingersoll-Rand drills at each face on an 8 in horizontal bar operated by a Carter tunnel carriage. Total construction period less than 23 months working from portals and 4 shafts.

(61) Laramie-Poudre, Colo. Irrigation tunnel. Granite syenite and quartzite. Leyner pneumatic hammer drills. 20 holes 7 ft to 9 ft deep per round. 100% blasting gelatin in the bottom of the cut holes and 50% and 60% dynamite for the rest of the blasting. Holes fired and fuse cut to proper length to discharge them in pre-arranged order. Muck hauled out by hoist and by mules. Average monthly advance 473.7 ft for 19 months.

(62) Mount D'Or France and Switzerland. Limestone and marl. Swiss heading averaged 566 ft advance per month for 25 months. 4 Meyer type air drills mounted on a bar and 12 to 15 five-ft holes per round in heading. Water up to 20,000 c.f. per min. encountered. Lining placed immediately behind the excavation. Compress air locomotives.

(63) Astoria-Bronx, New York. For gas mains under East River. Gneiss and dolomite. About 250 ft below mean sea level. Much trouble with water. Rock seams grouted thru pilot drill holes driven in advance of excavation, grouting under pressures up to 500 lbs per sq in and using a total of 118 000 bags of cement. Working once flooded by leak of 12000 gals per min. Tunnel then bulkheaded and whole heading grouted back of bulkhead. Work then advanced by small heading thro grout with pilot drill holes for grouting as before.

(64) Rogers Pass, B. C. Quartzite and schists. Not much water. No shafts. Thickness of overlying rock 5690 ft. A pioneer tunnel 7 ft by 8 ft was driven from each end parallel to the main tunnel and about 50 ft away. The central mile of this pioneer tunnel was not excavated. 3 and 4 light hammer drills on a horizontal bar per heading and 21 to 28 holes per round. Maximum monthly advance for a single heading 932 ft. From the pioneer bore cross cuts to the main tunnel were made at intervals of 1500 ft to 2000 ft and headings 11 ft wide by 9 ft high carried each way along the tunnel center line. This central heading was then enlarged to full section at one operation working in from each portal. Steam shovels and air locomotives used in mucking. The best monthly advance for one shovel was 1030 ft. Only partly lined with concrete.

(65) Twin Peaks, San Francisco. For 2-track street railway. Clay, sand and sandstone. Heading 8 ft by 8 ft. 65 000 cu yd of reinforced concrete lining placed by pneumatic process through a maximum length of 4,000 ft of pipe. 8 lbs hydrated lime was used per 100 lbs cement.

(66) Roosevelt, Colo. For underdraining Cripple Creek mining district. The tunnel at its inner end is over 1800 ft below the surface and takes the ground water from the surrounding mines. Maximum discharge 17 000 gals per min. Part in granite and remainder in hard volcanic rock. In granite it cost $27.27 per lin ft. Sec about 75 sq ft. In volcanic rock cost $20 to $25 per ft. Sec about 80 sq ft. Two Ingersoll-Leyner drills on a horizontal bar, about 30 holes per round. Muck hauled to shafts by mules.

(67) Rove, France. Rhone-Marseilles canal tunnel. Dolomite and limestone. "Holed through" Feb. 1916. 2 side drifts 9.8 ft by 10.7 ft are driven at bottom, enlarged to give room for track and drainage and small heading at the crown. Side headings enlarged upward and connect with crown heading. After this the lining is placed and the core removed.

36. Shield Tunneling

A Shield (Fig. 85) consists of a circular steel box or ring generally provide with a transverse diaphragm. The forward end of the ring, or cutting edg projects into the earth to be excavated, while the rear end or tail projec backward a little distance over the completed lining, which usually consis of rings of cast iron. To the shield are attached powerful hydraulic jacl

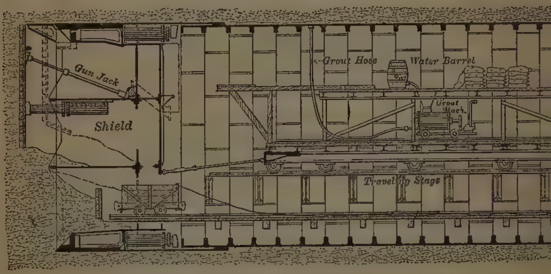


Fig. 85. Shield Tunneling

which react against the completed lining and shove the shield forward as t earth is excavated. As soon as the shield is advanced a short distance t jack plungers are withdrawn, a ring of lining built, and the operation repeate

Methods of Work. In clay or other firm material which will sta alone for a short time the full face is simply excavated to about the length one shove in front of the cutting edge without poling or breasting and t shield quickly shoved forward. If the earth is not hard enough to inju the cutting edge, only the center of the face is excavated for a depth equal length of shove and the cutting edge is allowed to break in the rest. softer clay the cutting edge and working floors may be always buried; o the back of the working chamber in front of the diaphragm is kept clear earth. In still softer material the shield is sometimes shoved against the ea face, part of which is allowed to flow thru the openings in the diaphra of the shield and the rest is pushed aside. In sand or gravel the face is u ally breasted with timber, which is held by struts against the shield. Th struts must be collapsible so that the shield may be shoved up to the fa A favorite form is simply two pieces of pipe which telescope as the shi is shoved, the resistance being regulated by set screws in the larger pi Hydraulic rams attached to the shield have been used for this purpose.

Numerous London shields have been provided with sliding platforms operated hydraulic rams. When driving the shield for the Waterloo and City tunnel, Lond thru sand and gravel pot holes were raked out in front of the cutting edge and filled v soft tempered clay. This formed a continuous ring of soft clay into which the cutt edge was shoved. The full face was then excavated under cover of the shield.

Where the material to be tunneled is part earth and part rock the full-sized tur above the rock is usually poled and excavated and the face breasted as in ordinary mi for a length of one or two shoves in front of the cutting edge, the rock in the same ler is then drilled, blasted and removed, and the rock bottom smoothed off to the form the bottom of the shield with concrete in which steel rails are imbedded; the shiel then advanced for that short length and the operation repeated.

The General Design of the different types of shields used for tube tunnels up to the year 1910 is shown in Figs. 86–89. The working chamber in front of the diaphragm has been small as in Fig. 86 or large as in Figs. 87, 88, 89,

Fig. 86. Shield, Central London Tunnel

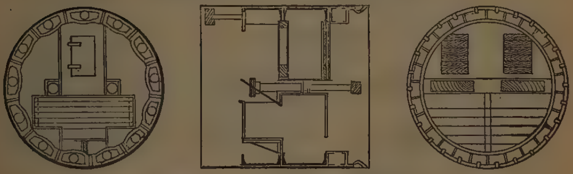


Fig. 87. Shield, Greenwich Tunnel

depending upon the character of the material and the methods of excavation. The diaphragm has openings for the passage of men and materials, and may be designed to be closed by locks or doors or be always open. Under either condition the diaphragm should be designed to lend stiffness to the skin, and since, when closed, it may take a part or even all of the water or earth pressure at the face, a system of transverse girders is introduced as reinforcement. Some of the later shields working in London clay had practically no diaphragm because the material was well known as being uniform, and not water-bearing. The shields for the Blackwall tunnel, Greenwich (Fig. 87), and Pa. R. R. East River tunnels (Fig. 89) were built with a double diaphragm, the doors of which when closed formed air locks between the working chamber and the tail for the passage of men and materials. These locks were never used, and the lower plates in all cases were removed during the course of the work to give easy access from the tunnel to the face.

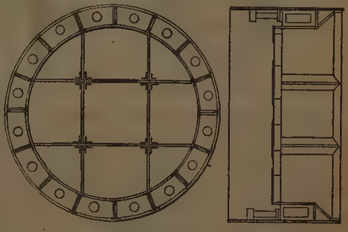


Fig. 88. Shield, Hudson Tunnel

The cutting edge at the extreme forward end of the shield in the earlier shields was the edge of the skin plates stiffened by brackets, but in later practise with few exceptions was a heavy ring made up of cast-steel segments beveled on the forward edge. Costly delays have resulted from damage to cutting edges caused by unexpected bowlders and rock ledges. The reaction of the hydraulic jacks must be transmitted to skin, cutting

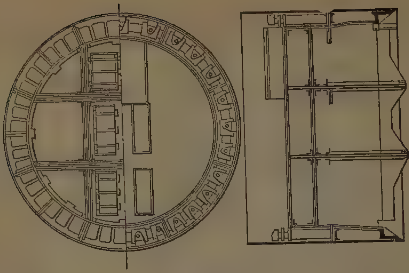


Fig. 89. Shield, East River Tunnel, P. R. R.

edge, and partly to diaphragm. These requirements have developed the circular box girder, a form of which is shown in Fig. 90, attached to the skin, bearing on the cutting edge brackets and carrying the connections of the transverse diaphragm girders. The box girder is provided with cells into which the jacks are placed, the location being convenient for substitution and repairs.

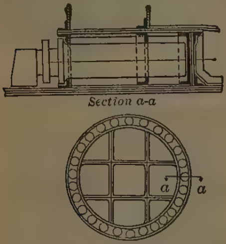


Fig. 90. Circular Box Girder

Fig. 91

Shield jacks are all connected with a battery of valves under the hand of the operator so that he may advance or withdraw any one or any combination of jacks at his will. A number of spare jacks should be provided and they should be designed so that they may be easily and quickly removed as they are sure to need repairs and delays are very expensive.

Most shields for work in sand are provided with a hood or extension of the upper part of the cutting edge. The tail of the shield should be long enough to exend about 6 inches beyond the joint between the second and third rings before the shove, to facilitate repairs to jacks and shield, and, more particularly, so that, should a segment of the tunnel lining be broken during the movement of the shield, it may be replaced at the conclusion of the shove without exposing the earth. The inside of the tail at the extreme end is provided with a bead of narrow steel plate, Fig. 91, which more nearly closes the annular space between the tunnel lining and the skin of the shield and yet allows the shield to be pointed at an angle large enough to pass around curves or regain direction when off line or grade.

37. Data Regarding Tunnel Shields

Elements in the Design of a shield which are known are merely the depth below the ground and water surface. The meager information as to the character of the material to be penetrated, generally obtained from wash borings, and the absolute uncertainty of the nature of the forces involved make its rational design impossible; the only guide is precedent. The following tables give the principal dimensions and important facts about several shields which have been used under a great variety of conditions. The numbers in parentheses in the first column of the first table refer to the notes below where additional data are given.

Number and name of tunnel	No. of sh'lds	External diam. ft	External diam. ins	Skin, ins thick	Length ft	Length ins	No. of j'ks	Inside diam. of jacks, ins.	Max. thrust, tons c	Wt. of sh'ld, tons c	Distance driven, miles
(1) Thames	1	a		.....	9	0	10	.....	.....	134	.....
(2) Tower	.....	.....		½	4	9	6	b	.....	.....	.....
(4) City & South London	18	11	1	½	5	11	6	6½	179	.....	6.20
		11	4½	½	6	6	6	6½			
(5) Mersey	1	10	3	¾	11	7	10	7	770	.....	0.25
(6) St. Clair	2	21	6	1	15	3	24	8	1795	92	1 14
(7) Hudson	2	19	11	1¼	10	6	16	8	1540	92	1.60
(8) Glasgow Harbor	1	17	3	¾	7	0	13	7	250	.....	0.41
	2	17	3	1	8	6					
(9) Glasgow Distr.	16	12	2½	½	6	6	6	6½	100	7	3 70
(11) Blackwall	1	27	8	2½	19	6	28	8	5785	224	0 59
							6	10			
(12) Clichy Siphon	1	8	4½	¾	6	8	5	6½	80	73	0.29
(13) East River Gas	2	11	0¾	⅞	7	2¼	12	5	595	12	d
(15) Waterloo & City	2	13	2	½	7	0	7	7	135		
	2	13	9	½	9	6	7	7	135	.....	2 84
	1	24	10	1	10	0	22	7	845	112	.....
(16) Concorde Siphon	1	6	9	¾	6	8	4	6½	65	.....	0 15
(17) Central London	.....	12	8	½	7	0	6	7	185	.....	13.00
		22	10	1	6	10	22	7	680	.....	
(19) L'Oise Siphon	1	8	7½	9/16	16	2	10	6	430	.....	0.18
(20) Baker Street & Waterloo	1	13	0	1	9	8	14	6	475	33	6.28
(21) Greenwich	1	13	0	1	14	1	13	7	840	84	0.23
(22) Lea	1	12	5	1	11	6	11	6½	.....	.....	0.21
(23) Battery, East R.	6	16	11¼	1⅛	9	6	14	8	1750	55	1 67
(24) E. River, P R R	8	23	6½	2¼	18	0	27	9	7730	240	2 87
(25) N. River, P R R.	4	23	6¼	2⅛	17	3⅜	24	8½	3300	193	2 31
(26) Brackenagh	1	6	2	½	6	9	4	6½	.....	.....	0 12
(27) Rotherhithe	2	30	8	2¼	18	0	40	9	5600	.....	0.69
(28) Kingsway	1	16	2	1	8	9	16	8	.....	.....	0 09
(29) River Dee	1	8	8⅝	½	6	9¾	7	6½	.....	.....	0 06
(30) Charing Cross	.....	12	8	.....	8	9	10	.....	.....	.....	.....
(31) Great Northern Strand	.....	12	8	½	6	11	8	.....	.....	.....	.....
(32) 14th St. E. River.	8	18	5½	2	15	3¾ e	17	8	125 g	116h	1.88
(33) Whitehall St E. R	4	18	6	2¼	16	4 f	17	8	125 g	104h	1.60
(34) Lawrence St., Bk'n	2	18	6	2¼	16	4 f	17	8	125 g	104h	1.00
(35) Old Slip, E R	6	18	0	2¼	16	4 f	17	8	125 g	98h	2 31
(35) 60th St. E. R.	2	18	6⅞	2 7/16	16	0¾ e	20	8	125 g	142h	0 68

a Height 22′ 3″; width 37′ 6″. *b* Hand-operated screw jacks. *c* Tons of 2000 lbs. *d* Driven a very short distance. *e* Includes 2 foot hood. *f* Includes 2 foot 2 in hood. *g* Includes hydraulic equipment which weighs about 40 tons on 60th St shield and 28 tons on other shields. *h* Per jack.

(1) Thames. 1825-42. London clay. Max. progress 2 ft, average 0.5 ft per day.

(2) Tower. 1869. London clay. Maximum progress 9 ft. per day.

(4) City and South London. 1886. Mostly clay, also several lengths of water-bearing sand and gravel. The average progress of each shield was 333 ft per month during ⅓ of work.

(5) Mersey. 1888. Clay, coarse ballast and running sand. Maximum progress was 57 ft per week.

(6) St. Clair River. 1889. Almost entirely in soft damp clay. The maximum progress per month was 382 ft and the average 231 ft per month.

(7) Hudson. 1879 1905. Mostly in silt; at New York side some sand and short length in rock. From May, 1890, to August, 1891, the average progress per month was 126 ft in the first tunnel; in the second tunnel the average progress per day was 15.3 ft.

(8) Glasgow Harbor. 1890 One-third in clay with bowlders, the rest in sand and gravel. The average progress in sand was 2 ft per day.

(9) Glasgow District Subway 1891-95. Clay, sand, gravel, and quarry waste. The average progress was 100 ft per month.

(11) Blackwall. 1892. London clay, sand, and ⅓ in open gravel. The maximum progress was 12½ ft per day. The average progress under the river was 98 ft per month.

(12) Siphon de Clichy. 1892. Open water-bearing sand. The maximum progress was 10 ft per day and the average 6 5 ft per day.

(13) East River Gas. 1892 Short lengths in decomposed schist and black mud.

(15) Waterloo and City. 1893. Clay and ballast. 10 ft per day was the average progress while the work was in full swing.

(16) Siphon de la Concorde. 1895. Clay and sand.

(17) Central London. 1896. London clay and silty sand.

(19) Siphon de l'Oise. 1896. Sand.

(20) Baker St. and Waterloo 1898. About 350 ft was in gravel under the Thames River, the rest was in London clay. The average progress per month was 117 ft in gravel and 137 ft in clay.

(21) Greenwich. 1898. Mostly in clay with the crown in close gray sand; about ⅓ length is in close gray sand and a small part in ballast. The maximum progress was 260 ft per month and the average 148 ft.

(22) Lea. 1901. Mostly peaty clay and open ballast.

(23) Battery, East River. 1901. 500 ft almost entirely in rock; 1500 ft in fine sand with some clay, the rest was coarse sand. Rates of progress follow: In normal air above water line in sand and gravel average 140 ft per month; under river in rock, fine sand and clay, and sand and gravel, 91 ft per month.

(24) East River, Pennsylvania R.R. 1903-09. Land portion in rock, soft top in places. Under river 18% in fine sand stretched with clay, stiff clay, sand and bowlders 35% sand and bowlders, sand and clay and fine sand; 22% all in rock and 25% in rock with sand, gravel, and bowlders in the crown. Rates of progress were: Best day's work 17.5 ft. Average for month in earth 176 ft, in rock and earth 56 ft, and in rock 61 ft.

(25) North River, Pennsylvania R.R. 1903-09. Mostly in silt, also short length in sand and in rock. The maximum month's progress was 545.2 ft. The averag progress in rock was 58 ft per month and in silt 2.39 ft.

(26) Brackenagh. 1903. Water-bearing glacial deposit and running sand.

(27) Rotherhithe. 1904. Clay, conglomerate sand and gravel.

(28) Kingsway. 1904. Clay. Average progress 5.0 ft per day.

(29) River Dee. 1904. Alluvial clay and bowlders. Average progress per da 4.5 ft.

(30) Charing Cross and Hampstead. 1904. London clay. Max. progress 180 ft pe week.

(31) Great Northern and Strand. London clay.

(32) N. Y. Subway. 1916. B. R. T. system from North 7th St, Bklyn to 14th S Manhattan. Shaft at each side of river. 2 land shields, 2 river shields from each sha (8 shields in all). Tunnel part in rock and part in sand and part in clay. Averag progress per month river section, in earth 147 ft—in earth and rock 44 ft. Land sectio in earth 168 ft.

(33) N. Y. Subway. 1914-1919. B. R. T. system from Montague St, Bklyn Whitehall St, Manhattan. Shaft on each side of river. From Brooklyn shaft tw shields driven each way. River shields met rock tunnel driven without shields und river from Manhattan shaft. Material on Brooklyn side sand with some clay. Avera progress per month river section, in earth 192 ft, in rock and earth 80 ft. Land secti in earth 209 ft. Max. progress per month, river section 345 ft, land section 454 ft.

(34) N. Y. Subway. 1914-1919. B. R. T. 2 shields driven under streets of Brookly at a depth of from 43 ft to 75 ft. All above tide water. Extension of the Whiteha Montague street tunnel. Material sand. Compressed air not used.

(35) N. Y. Subway. 1914-1919. I. R. T. Shafts on each side of river. 2 shiel in each direction from Brooklyn shaft; 2 shields toward river from Manhattan sha

Material sand on Brooklyn side. Tunnels part in rock, part in sand on Manhattan side. Average progress for month, river section, in earth 184 ft in earth and rock 74 ft. Land section in earth 167 ft.

(36) N. Y. Subway. 1916–1919. B. R. T. from North Jane St, Queens to East 60th St, Manhattan. One shaft on each side of river and shaft in rock on Blackwells Island. From Queen's Borough shaft, 2 shields were driven under east channel of East River, pushed through rock tunnels, previously excavated under Blackwell's Island and continued through earth under west channel to rock on Manhattan side. Average progress for month, river section in earth 188 ft, in rock 119 ft.

38. Roof Shields

Roof shields for masonry-lined tunnels have been used with success principally in France and to some small extent in America. In use they are peculiarly adapted to the drier soils, usually under streets where open-cut methods are not permissible, tho they have been used, as in the East Boston tunnel, in connection with comprest air for subaqueous tunnels. They are usually semielliptical or semicircular in form and consist of a skin of steel plates riveted to girders which at their lower extremities are tied together with a transverse girder and mounted on rollers. Hydraulic jacks supported by the ribs and bearing against the completed masonry or the arch centers move the shield forward. The two general methods of using roof shields are: first, to excavate the area of cross-section of the shield, build the masonry arch, and subsequently underpin with the side-wall construction; second, to drive timbered headings in advance and construct in them the side walls on which the shield is later supported or rolled.

(1) Commenced 1895. Contract price $55.45 per lin ft. Semielliptical shield. Average progress 14 ft 9 in per day of 24 hours, maximum 29 ft. Material penetrated of loose sandy nature.

(2) Commenced 1897. Average progress 9 ft per day of 24 hours. Material penetrated: 450 ft compacted clay and sand, 100 ft loose sand and gravel. Height = over all.

(3*a*) Commenced 1898. Material penetrated mostly made ground, old masonry walls and foundations. Average progress 3.4 ft in 24 hours, maximum 22 ft.

(3*b*) Material penetrated, see No. 3*a*. Average progress 10 ft in 24 hours.

Number and name of tunnel	No. of sh'lds	Width of shield, ft	ins	Height of shield*, ft	ins	Skin, ins thick	L'gth†, ft	ins	No. of j'ks	Inside diam. of jacks, ins	Max. thrust, short tons	Distance driven, miles
(1) de Clichy	1	23	9	9	8	9/16	17	3	6	9½	616	.78
(2) Boston, Tremont.	1	29	4	8	7¼	1	14	0	10	6	1250	.11
(3) Orleans Railway Extension	1	34	9	12	6	1	19	8	10 10	10 4½	1120 168	.54
	1	32	0	11	8	¾	23	0	8	9¼	1021	.22
Collector de Bievre:												
(4*a*) Chagnaud type..	1	16	2	6	6¾	¾	18	0	6	6¾	228	.11
(4*b*) Chagnaud type..	1	16	2	7	6¾	¾	18	0	6	6¾	228	.18
(4*c*) Dioudonnat type.	1	16	3	10	6*	9/16	13	4	5	6¼	224	.07
Paris Metrop. Ry:												
(5*a*) Champigneul type	4	28	3	8	9½	7/10	23	2	8	9¼	923	.95
(5*b*) Dioudonnat type.	3	28	3	8	7	1	22	2	8	8½	896	.33
(5*c*) Weber type.....	2	28	3	8	7	1	16	1	4 8	5¾ 8½	179 896	.09
(5*d*) Lamarre type...	2	28	3	8	7	1	19	9	9	7½	1149	.19
(6) Boston Harbor..	2	28	10	14	5	1¼	13	0	16	3¼	75	.81

* Height = From springing line to crown unless noted. † Length = Distance over all.

(4*a*) Employed on elliptical section. Commenced 1898. Material penetrated: ma ground, decomposed limestone, marl, and foundation walls. Average progress 8.2 ft day, maximum 101.7 ft per week.

(4*b*) Employed on circular section. Started 1898. Material penetrated, fine sand a gravel. Average progress 9 ft per day.

(4*c*) Employed on circular section. Started 1898. Material penetrated, made grou sand, foundation walls, etc. Average progress 7.2 ft per day. * Height over all.

(5*a*) Two shields of this type employed on 1st section, 1 on 8th section, and 1 on 1 section. Started 1899. Material penetrated, made ground and soft sand, with so marl. Average advance 13 ft per day on 1st section, 10 ft on 8th, and 12 ft on 11th.

(5*b*) One shield of this type employed on 2nd section, two on 3rd section. Star 1899. Material penetrated, sand and gravel, some water-bearing. Maximum adva 29 ft per week.

(5*c*) Two shields employed on 4th section. Started 1899. Material penetrated, s and gravel. Shields proved unsatisfactory.

(5*d*) One shield of this type employed on 6th section, one on 7th section. Mat penetrated, sand, gravel, and marl. Average advance 6.5 ft per day. Shield pr unsatisfactory.

(6) Started 1901. Material penetrated, blue clay and bowlder clay, with some sa and silty permeable clay. Air pressure, 18 to 20 lbs average, 25 lbs maximum; cu ft per man per hour. Average progress 4.8 ft per day.

39. Cast-Iron Tunnel Lining

The Lining for shield-driven tube tunnels has usually been cast i The reasons for its use are that it can be most quickly built; as soon as ere it furnishes a reaction for the shield jacks; it offers greater assurance water-tightness, and being thin for its strength saves excavation. For aqueous work done in comprest air all of these considerations are weig The number of segments forming a ring and the width of the ring de on the economic weight that can be handled. Sometimes in small tur up to 13 or 14 ft in diameter iron lining has been built by hand, but hydr erectors attached to the shields or to traveling platforms are generally u The length of shield which can be conveniently used is another factor go ing the width of segments, the length of the tail being affected correspond by increases in this width. Fig. 92 shows the cast-iron lining used in Pa. R.R. tunnels under the East River, N. Y. There has been bui comprest air at least one shield-driven tube tunnel lined with brick.

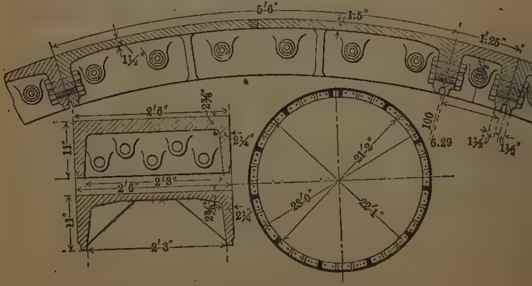


Fig. 92. Cast-Iron Lining, East River Tunnel

the construction of the 39th Street sewer at Chicago, segmental timbering formed a lining inside of which the brick was built. The timber was used to receive the reaction of the shield jacks.

The design of the cast-iron lining depends as much on the requirements of building as upon the stresses involved, and the best guide is a study of precedents. The following table gives principal dimensions and details of iron lining in existing tunnels.

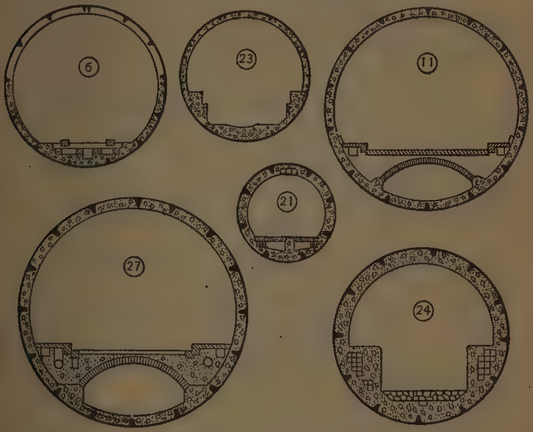


Fig. 93. Cast-Iron Lined Tunnels

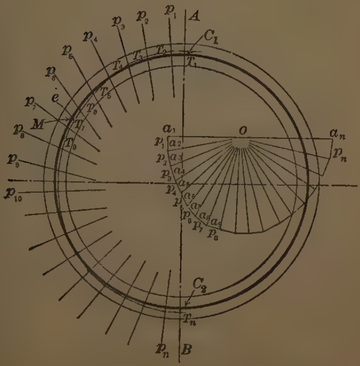


Fig. 94. Stresses in Tunnel Shell

Details of Iron Lining in Tunnels (Fig. 93)

Number and name of tunnel	External diam. of ring, ft	ins	L'gth of ring, ins	Thickness of web, ins	D'pth of fl'ge, ins	No. of segments excl. of key	Weight lbs per lin ft	Bolts per ring C	H	Dia. of bolts, ins
(2) Tower, London	7	1¾	18	⅞	3	3	971	31	..	¾
(3) Antwerp	*		19¾	......	4–6	4	......	14	4	.....
	10	10¾	19	1	4⅝	6	1 880	47	..	1
(4) City & So. London	11	3	20	⅞	4½	6	1 880	47	..	⅞
	22	6	18	1⅛	7¾	12	......	83	..	1⅛
	32	0	18	1½	12	16	......	113	..	1¼
(5) Mersey	10	0	18	1$\frac{11}{16}$	6	10	3 061	28	22	.....
(6) St. Clair	21	0	18¼	2	7	13	9 333	157	56	⅞
	19	6	20	1½	9	11	6 055	66	36	1¼
(7) Hudson	18	9¼	20	2½	8	......	......	66	36	.....
(8) Glasgow Harbor	17	0	18	1	6	13	4 559	66	28	.....
(9) Glasgow District	12	0	18	¾ – 1	6	9	2 241	46	20	1
(10) Kingston	9	0	18	⅞	4	6	1 580	37	14	1
(11) Blackwall	......		.....	1½	10	14	9 408	...	..	.....
	27	0	30	2	12		13 265	70	75	1½
(12) Clichy Siphon	8	2½	19¾	1	4	5	1 453	36	24	1
(13) East River Gas	10	10	16	1¼	4	9	2 400	46	20	1
(14) Mound of Edinburgh	17	6	18	1¾	7	14	7 100	...	..	1⅜
(15) Waterloo & City	13	0	20	⅞	5⅛	7	2 326	57	24	1
	13	7¼	20	⅞	5⅛	7	2 431	57	24	1
	24	6	18	1¼	9	13	......	92	..	.....
(16) Concorde Siphon	6	6½	19¾	⅞	3⅝	4	895	21	15	⅞
(17) Central London	12	6	20	⅞	4⅞	......	......	...	..	⅞
	22	6	18	1⅛	8	12	......	81	39	1⅛
(18) Spree †	13	1½	25⅝	⅜	4	9	1 270	72	36	.....
(20) Baker St. and Waterloo	12	9¾	18	1	4⅞	6	2 643	53	14	⅞
(21) Greenwich	12	9	20	1¼	6	8	3 080	47	..	1⅜
(22) Lea	12	3¾	21	⅞	4⅞	6	......	29	21	⅞
(23) Battery, E't River	16	8½	22	1⅛	7½	8	4 000 4 540 5 130	49	27	1
(24) East River, Pa.R.R.	23	0	30	1	8	11	5 166	67	60	1¼
Land	23	0	30	1¼	9	11	6 776	67	60	1¼
	23	0	30	1½	11	11	9 102	67	60	1½
River	23	0	30	1½	11	11	9 102	67	60	1½
(25) No. River, P.R.R.	23	0	30	2	11	11	12 127	67	60	1¾
	23	0	30	1½	11	11	9 273	67	60	[illegible]
(26) Hilsea Creek	12	6	20	......	....	6	2 240	...	..	...
				2	14	16	16 600	...	..	...
(27) Rotherhithe	30	0	30	1¾	14	16	14 700	85	79	[illegible]
(28) River Dee	8	6	18	1	5	5	1 960	42	18	1

* Height 5′ 7″; width 4′ 11″. † Steel lining. C = Circumferential, H = Ho

Stresses in tunnel lining can only be approximated by theoretical because they originate in earth pressures and the law of action of ea sures is only imperfectly known. While the designer should depend judgment and precedent than upon theoretical analysis, such an anal be helped by indicating the nature of the stresses.

Let C_1, C_2 (Fig. 91) be the line of centers of resistance (locus of centers of shell is homogeneous) of a unit length of tunnel shell which is taken circular b any other form. Let p_1, p_2, p_3, . . . to p_n be all forces acting upon the shell. T

Details of Iron Lining in Tunnels (Cont'd)

Name of Tunnel	External diam. of ring, ft in	L'gth of ring, ins	Thickness of web, ins	D'pth of fl'nge, ins	No. of segments excl. of key	W'ght lbs per lin ft	Bolts per ring C	Bolts per ring H	Dia. of bolts, ins	High water to base, max., feet
(32) 14th St., E. River......	*18-0	26	1⅜	9	9	6323	55	50	1¼§	97
	†17-2	26	1	7	9	3801	55	40	1	115
(33) Whitehall St., E. River...	*18-0	26	1⅜	9	9	6323	55	50	1¼	87
	‡18-0	26	1	7	9	4048	55	40	1	80
	†17-2	26	1	7	9	3801	55	40	1	90
(35) Old Slip, E. River......	*17-6	26	1⅜	9	9	6166	55	50	1¼	89
	‡17-6	26	1	7	9	3949	55	40	1	89
(36) 60th St., E. River......	*18-0	26	1⅜	9	9	6323	55	50	1¼	116
	‡18-0	26	1	7	9	4048	55	40	1	85

* Earth or earth and rock. † Rock without shield. ‡ Rock with shield.
§ Bolts upset ⅛ in for thread; also upset ⅛ in near the head end.

satisfy the condition that the forces on opposite sides of any diameter must be equal and opposite, otherwise there would be movement. For example, the reactions and active forces, if any, on the bottom must balance the loads on top. It is assumed in Fig. 94 that the loading and the shell are symmetrical about the vertical axis A-B, which will usually be the case. The analysis would be similar in any case.

The load diagram a_1, a_2, a_3 . . . is constructed in the usual way by laying off the forces p_1, p_2, p_3 . . . to scale parallel to their direction. The position of the pole o is found by laying off on a_1 a_n the thrusts T_1 and T_n, which are found by taking moments of the horizontal components of p_1, p_2, p_3, . . . about C_1 or C_2. Then a_2, a_3, a_4, etc. are connected with the pole, and the line of thrust T_1 T_n is constructed in the usual way by drawing T_1, T_2, T_3, . . . parallel with oa_1, oa_2, oa_3, . . . But an infinite number of such lines of thrust may be drawn. The correct one may be found by trial from the consideration that as the tunnel shell is a closed ring the sum of the positiv bending moments must be equal to the sum of the negativ moments. The bending moment at any point, say at M, is oa_7 times e. The bending moments at equal small intervals around the ring for any line of thrust may be computed and summed up. A few trials will determine the correct line of thrust.

40. Waterproofing and Grouting

Grummets. Bolt holes being slightly larger than the bolts which are placed in them, permit water to seep into the tunnel unless a tight calking material is carried around the bolt hole or some form of grummet is placed under plate washers at the head and nut. Three principal types of grummets have been used. In the Baker Street and Waterloo, the North River Pennsylvania, the Battery, and for a time in the East River Pennsylvania tunnels rings of hemp saturated in red lead and oil under plate washers were used. In the East River Pennsylvania, where the bolts had rolled threads, the diameter of the threaded portion being larger than the diameter of the shank, greater success was attained by wrapping strands of hemp saturated in red lead and oil around the shank. In the Greenwich and Rotherhithe tunnels lead washers were used, and when the bolts were tightened the lead washers were forced into conical bolt holes.

Calking. The practise since 1890 has been to cast a small depression in the flanges at its inner edge which matches a like depression in the adjoining

plate and forms a calking groove when the plates are bolted together; [illegible] Fig. 92. Into this is calked either soft lead or a rust mixture or sometin[illegible] Portland cement. The grooves have usually been from ¼ to ¾ in wide a[illegible] about 1½ in deep. For lead it is desirable to make the groove much n[illegible] rower, say ⅛ in wide, but this is difficult to cast.

The rust mixture is usually one part by weight of sal ammoniac mixt with 400 pa[illegible] of iron filings. The sal ammoniac is best dissolved in enough water to dampen the [illegible] filings which are then calked into the groove and well compacted either by hand t[illegible] or pneumatic hammer. The finer the iron filings the better. Rust mixture call[illegible] is not successful if there is water coming thru the joint during calking. Even when [illegible] joints are made absolutely water-tight slight movements of the flanges, such as are li[illegible] to occur from contraction in winter or in adjustment of bolts, are liable to cause a s[illegible] amount of leakage. The following table shows the practise in some of the more im[illegible] tant tunnels:

Name	Date	Max. head water, feet	Kind of calking
Antwerp	1879	26	Groove in flange outside of bolts. Rop[illegible] tarred hemp.
City and S. London	1886	75	In joints between rings oakum in gr[illegible] covered with cement. Longitudinal joint[illegible] filled with soft wood packing and the [illegible] filled with cement.
Mersey	1888	54	Face of flange beveled. Portland cement [illegible] tar, just damp, in groove.
St. Clair	1889	78	Canvas coated with resinous compound bet[illegible] rings, creosoted oak packing 5/16 in thic[illegible] longitudinal joints.
Blackwall	1892	80	Rust joint material — 1 lb sal ammoniac t[illegible] lbs iron borings. In wet joints lead [illegible] hammered into groove.
Baker St. & Waterloo	1898	70	Pine packing between rings outside ½ of [illegible] rust joint material inner ½. Calked ar[illegible] bolts. Rust joint material in shallow g[illegible] on longitudinal joints.
Greenwich	1898	70	Rust joint material in grooves into which[illegible] lead had been hammered.
Battery, East River	1901	94	Lead in grooves.
East River, P.R.R.	1903–09	93	20 per cent calked with rust material — [illegible] sal ammoniac to 400 lbs iron borings. [illegible] cent calked with lead and pointed [illegible] portland cement.
North River, P.R.R.	1903–09	98	Sides and invert — 2 lbs sal ammonia[illegible] sulphur, and 250 lbs iron borings; arch[illegible] sal ammoniac, 3 lbs sulphur, and 125 lb[illegible] borings.
Rotherhithe	1904–09	97	Lead hammered into groove and covere[illegible] rust joint material.
East River, N. Y. Public Service	1910–19	116	The caulking for all the cast-iron [illegible] tunnels was done with leadwire over [illegible] a Portland Cement pointing was pla[illegible] fill the groove. The caulking groove was ¼ in wid[illegible] 1¼ in deep. Lead wire elliptical in section (max. [illegible] ½ in, min. diam. ¼ in) has bee[illegible] throughout the entire work.
14th St. / Whitehall St. / Old Slip / Lawrence St. / 60th St.			

Grouting. The shield being larger than the tunnel lining leaves an annular space which should be filled immediately after the shield has past. This is especially necessary at the sides to prevent the lining from spreading. A grout of about equal parts of hydraulic cement and sand is generally used.

Waterproofing. Most subaqueous tunnels have had cast-iron or steel shells which are waterproof of themselves; the exceptions are those that are in nearly impervious clay, which prevents any quantity of water reaching the tunnel lining, and such tunnels as the Severn, which has brick lining and is kept dry by pumping. Numerous tunnels and subways in water-bearing ground are of masonry lining waterproofed by pitch in some form.

The most generally used form is a combination of pitch, usually applied hot, with sheets of some fibrous material used as binders; for this purpose burlap, building paper, and so-called felt have been used. These binder sheets are usually saturated with pitch by the manufacturer, and are applied to the outside of the wall in from 2 to 6 thicknesses, each layer being given a coat of hot pitch as it is placed. The specification for such work in the Pennsylvania R.R. land tunnels in New York call for "straight run coal tar pitch which will soften at 60° Fahr and melt at 100° Fahr, being a grade in which distillate oils, distilled therefrom, shall have a specific gravity of 1.05." Felt was coated and saturated with asphaltic products, and it was required that the wool in the unsaturated felt should be not less than 25% by weight and that the felt should weigh 0.05 to 0.06 lb per sq ft unsaturated and 0.12 to 0.14 lb saturated. Another type of waterproofing consists of an outside course of asphaltic brick laid in soft pitch or a mastic of soft pitch and fine sand.

41. Compress Air

The Shield and Compress Air generally go together, but there have been several exceptions. A portion of the Pennsylvania tunnels in Long Island City were mined, timbered, and lined in compress air without use of shields, and the builders of the telephone tunnels in Chicago used compress air but no shields. The amount of air required is very variable in different tunnels and at different times in the same tunnel. In the East River tunnels of the Pennsylvania R.R. in open sand with a clay blanket 10 to 20 ft deep and 350 ft wide dumped upon the river bed, the capacity of compressors required was about 80 400 cu ft free air per min. The average used was about 3500 cu ft per min per heading. In the Rotherhithe tunnel under the Thames it was specified that the minimum amount of air pumped into the tunnel should be 8000 cu ft of free air per man per hour.

The great difficulty in water-bearing sand or gravel when compress air is used arises from the impossibility of balancing the water pressure at more than one elevation, with the result that if the water pressure is balanced at the bottom of the tunnel there is an excessive escape of air at the top, which is expensive, and there is always danger that a "blow," or sudden outrush of air, will occur at the top, which endangers the workmen and may cause delay by flooding the tunnel. If the water pressure is balanced at the top, the bottom may be so wet that the sand is unstable and difficult to handle. As a compromise the water pressure in such materials is usually balanced by the air pressure at about the center of the shield, and blows are guarded against by breasting and plastering the face with clay and depositing a clay blanket on the river bed above.

42. Subaqueous Tunnels Built from Surface

In cases where the top of the tunnel is desired to be close to the bed of the river thus giving insufficient cover for shield work, or if the tunnel is short making the plant installation required for shield work proportionately excessive, tunnels have been built successfully from the surface.

(1) Detroit River Tunnel, Detroit to Windsor. Trench was dredged to grade in the stiff clay of the river bottom. Twin steel tubes of 3/8 in pl 23 ft 4 in diam and 262 ft

Compress Air Data

Tunnel	High water to invert, max., ft	Min. cover, ft†	Max. air pressure, lbs per sq in	Average air pressure, lbs per sq in	Cubic feet free air supplied
City and South London	34	42	15	......	In water-bearing sand 1660 per min per face; when grouted 1000 to 1300 per min per face.
Blackwall	80	5	37	35	10 000 per min per face in open ballast for some time.
Baker St. and Waterloo	70	18	35	28	In gravel 3300 per min per face; parallel tunnel 1650 per min per face.
Greenwich	70	30	28	20	Average 5000 per man per hour; never less than 4000.
Battery, East River	94	12	42	26	Two working faces, max. 32 000 in sand.
East River, P. R.R.	93	8	42	27	Max. at one face 25 000 per min for 24 hours. Capacity of plant for 8 faces 80 400 per min.
North River, P. R.R.	98	20	37	26	Max. in gravel 10 000 per man per hour. Generally ranged between 1500 and 5000.
14th St., E. River, N. 7th St. Tunnels	97	4*	39½	33	Max. in sand 9000 in one heading for 24 hours.‡
Whitehall St., E. River, Montague St. Tunnels	87	8	34½	30	Max. in sand 12 000 in one heading for 24 hours.‡
Old Slip E. River, Clark St. Tunnels	89	3*	37½	30	Max. in sand heading 10 000 one heading for 24 hours.‡
60th St., E. River, N. Jane St. Tunnels	116	9*	47½	41	Max. in sand 12 000 in one heading for 8 hours.‡

* The shield cut into the permanent clay blanket placed on the river bed. The clay blanket is protected from washing by a rock revetment on the sides and top.

† Does not include clay blankets.

‡ Requirement for all contracts was 10 000 cu ft of free air per min per heading; except in one case where one power house supplied 8 headings, the requirement was reduced to 8000 cu ft of free air per min per heading.

6 in long connected by structural steel diaphragms at 12 ft c to c and provided with bulkheads were sunk in position on prepared foundations. Sections were connected up by divers with special bolting system. A timber form or sheathing was fastened to outer edges of the diaphragms. Concrete was placed by tremie between this sheathing and the tubes—completely encasing the tubes. This concrete was 3 ft thick on the sides and 4 ft 6 in thick top and bottom. Reinforced concrete lining having an interior diam of 20 ft was placed inside the steel tubes. (See Fig. 95). Distance from water surface to top of rail in mid-channel 66 ft. Current in river over 2 miles per hour. Total length of river section 2667 ft. Built 1906–10.

(2) Chicago River Tunnel. Single length of two parallel steel cylinders with a longitudinal dividing wall; lined with reinforced concrete and sunk on prepared foundation in a trench dredged in the river bottom. Tubes of ⅜ in steel plate, stiffened by plate and angle gussets above and below the central wall every 7½ ft. Concrete lining 2 in to 40 in thick. Central wall 3 ft thick. Length of tubes 278 ft, width 41 ft, dep

24 ft. Weight when sunk 8000 tons. Backfill and cover sand. Connection to shore tunnels made by cofferdam. Built 1910–12. (Fig. 96.)

(3) Paris Subway under Seine, Pont Mirabeau crossing. The River portion 644 ft long and made up of 5 sections 116.8 ft to 144.4 ft long. Each section was a pneumatic caisson consisting of a structural steel frame covered inside and out with steel plates and filled with concrete, with a double track tube of similar construction above the working chamber. Over-all width 29.9 ft, over-all height 30.1 ft. These sections were sunk in position 15¾ in apart, this space filled with tremie placed concrete through which an opening of required tunnel section was later excavated. A tunnel having a total caisson length of 1313 ft was built under the Seine by a very similar method in 1906–7.

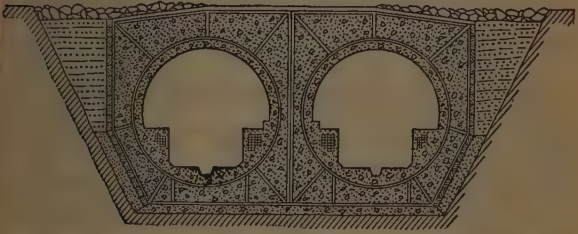


Fig. 95

(4) Harlem River crossing, New York Subway, Lexington Avenue Line. Four track tunnel built very much the same as the Detroit River tunnel noted above. The sections as assembled had an over-all width of 76 ft and height of 24½ ft. Four of the sections were 220 ft long and the remaining one was 200 ft long. Contract price $1500 per lin ft for 4 track structure. Built 1911–12.

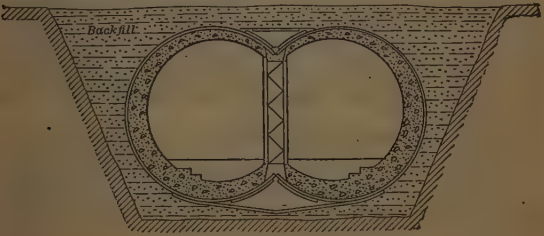


Fig. 96

(5) Harlem River Tunnels, New York Subway, Lenox Avenue Branch. Twin tube sections 610 ft long of cast iron lining, inside diam 15 ft Across the river trench was excavated, bottom of which was a little below the middle of the finished tunnel. Two lines of heavy sheet piling were driven outside the line of the finished tunnel and cut off at the springing line of the arched roof. A floating timber box was then built in which the arched roof of cast-iron segments was erected and covered with concrete with proper internal bracing and bulkhead ends. The floating box was sunk from underneath and the roof left floating like a diving bell. This with steadying tackle was sunk and left resting on the sheet piling and bearing piles previously driven in the trench. After

divers connected this to the section already in place, the lower half of the tunnel built in comprest air. Built 1903–4.

43. City Subways

The Cut-and-Cover Method is usually cheaper than tunneling at dep less than 30 ft and may be the more practicable method when the cove shallow, as often with city subways for rapid transit. Attempts to exca by shield within a few feet of street surfaces have not met with entire suc because of disturbance to pavements and pipes. Where traffic will not pe an open cut, the pavement is taken up, if necessary at night, and replaced a timber deck under cover of which the subway is built. Where wide cut is objectionable, side-walls and interior supports are sometimes built in tre and roofed, after which the core is removed and the bottom placed. W there is groundwater, its level frequently may be lowered by draining to su and pumping carefully. Where quicksand is met at subgrade, tight shee is driven to extra depth and it may be necessary also to excavate for and p the floor quickly in short sections. On the other hand, where the groun dry and hard enough to stand with but little bracing and where decking is required, steam shovels have been successfully used. Intercepted se may be depressed or under-siphoned.

Adjacent foundations are underpinned, either temporarily or permane where required to prevent settlement. Finally, subsurface structures restored or rebuilt and the street is backfilled and repaved.

Types of Sections are shown in Fig. 97. Earlier sections usually of the single arch type if headroom permitted; where close to the surfac roof was of beams with jack-arches between them. Later, beams cam be used also in sidewalls, forming bents, saving in width and facilitating struction. More recent American design has tended toward reinforced crete, requiring somewhat less and cheaper steel and permitting the u large forms, but bents continue in favor because better adapted to requirem of bracing and piecemeal construction. The single arch of concrete, pla reinforced, is still used where conditions of headroom and lateral suppor favorable. In soft or wet ground the floor may be formed as an inve may be reinforced with beams or rods. Waterproofing is commonly fe fabric saturated and laid with tar or asphalt, and in dry situations ma omitted from floor and sides. Stations are painted, plastered or tiled in or light colors.

The New York Subway System, the largest, has 69 miles undergr structure built or nearing completion, mostly 2- or 4-track subway, of nearly all is of type (1) Fig. 97 in the portion first (1900–3) built and mu type (3) in portions now (1918) or recently under construction. Both are bent type, though certain portions built since 1902 are of reinforced con mostly of type (2). The recent type (3), provides a partition betwee middle tracks to promote ventilation by train movement. The roof and necessary the sides and floor are waterproofed generally with felt or b and pitch. The inside tracks are for express, and the outside for local t At express stations the platforms are between local and express tracks, se both, and at local stations at the sides. Excavation was partly in Groundwater and very fine quicksand were frequently met. Methods widely. The system includes several miles of arch tunnels in rock, cast lined tubes and steel viaduct extensions.

The Boston Subway System, the first in America, with 9 miles underg structure, mostly 2-track subway, is principally of bent type (4) and arch ty

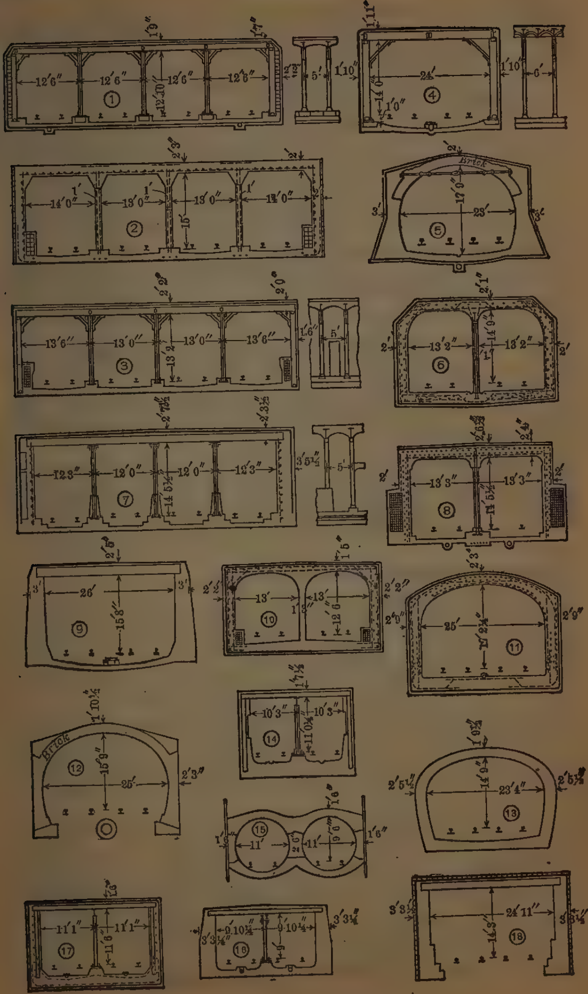


Fig. 97. Subway Sections

in earlier (1895–1908) portions and of reinforced concrete type, (6) in recent (1912–17) portions. Where traffic was dense a short portion somewhat like (5) was tunneled with roof shield. Excavation was nearly all in sand, clay and gravel. The system is for both surface cars and trains. It includes two shield-driven subaqueous tunnels and steel viaduct extensions or connections.

The Philadelphia Subway, in Market Street, built 1903–07, 2¼ miles long, is in about equal parts a 4-track (7) and a 2-track (8) structure. The walls of (7) and the roof and walls of (8) are rod-reinforced. The roof is waterproofed with asphaltic mastic and the sides with asphalted burlap. The outer tracks of (7) are for street cars and the inner tracks, continuous with those of (8), for trains. The columns of (7) are protected against derailment by low reinforced concrete bulkheads. Excavation was largely in gravelly soil and the structure is underdrained.

The Atlantic Avenue Subway, Brooklyn, (9), built 1901–7, 2 miles long, is for electric trains of the Long Island R.R. While not a city subway in a strict sense, it is structurally such. The sides are retaining walls and with the roof are waterproofed with tarred felt. Excavation was mostly in sand and gravel.

The Sixth Avenue Subway, N. Y., (10), built 1906–11, 1 mile long, is for Hudson and Manhattan R.R. trains entering the city by tunnel under the Hudson. Roof and sides and, where in earth, the floor are reinforced. Waterproofing was mostly of asphalted burlap but in a short portion a waterproofing compound was mixed with the concrete. Excavation was partly in rock. Some quicksand was met.

The Cambridge Subway (11), built 1909–12, 2¼ miles long, connects by bridge with the Boston Subway. Roof, sides and floor are reinforced and waterproofed with tarred felt. Where cover is shallow the roof is flat. A portion of similar section, but with less reinforcement and not waterproofed, was tunneled with roof shield. Excavation was mostly in sand and gravel with some clay.

Foreign subways generally are of smaller cross-section.

The London Metropolitan and District Subway, (12), an irregular oval line termed the Inner Circle, was the first city subway for rapid transit. It was built 1860–84 and later extended from its original length of 13 miles, part of which is open cut. The arch and walls are of brickwork. The extrados is waterproofed with asphalt. Where cover is shallow, cast- or wrought-iron roof girders were used. With soft or wet foundation, an invert was added. Excavation was mostly in gravel, sand and clay. Later extensive underground lines are in deep-level tubes, shield driven in London clay.

The Paris underground railway, (13), a network of belt and intersecting lines, begun 1898 and opened 1900, had in 1914 a length of 56 miles, mostly subway and the rest viaduct, tunnel and open cut, in addition to extensions in progress or planned. Short portions first built were tunneled with roof shield or by the Belgian method. Watertightness was secured by grouting. Excavation was mostly in sandy soil and in some parts in soft rock.

The Berlin underground railway, (14), an original east-west trunk, begun 1896 and opened 1902, with later extensions or connections to suburbs, had in 1916 a length of 23 miles, mostly subway and the rest viaduct, tunnel and open cut, in addition to 18 miles of extension, or new lines in progress or planned. The section shown is that of the Schöneberg branch, opened 1910, of the bent type. The longitudinal girders are discontinuous, each supporting 5 roof beams, and rest on two columns over which they cantilever. A feature was the use of small I-beams, driven as piles at each side every few feet, to the inner flanges of which were clipped breast boards, retaining the earth and serving as back forms for the concrete of the sidewalls, in such manner as to permit the final withdrawal of the beams. The beam-piles were braced apart only at the top, leaving a free working cut. An earlier type has thicker walls without beams. A late

type has no columns. Waterproofing is asphalted paper. Excavation was mostly sandy and much of it waterbearing.

The Glasgow District Subway, (15), an irregular oval line, built 1902-6, is 6½ miles long, comprising subway, tunnel and open cut. For the subway, a heavily sheeted trench was excavated deep enough to build the roof which was next waterproofed with layers of asphalt and covered with the restored pavement. The roof, supported by the sheeting, was then undermined with drifts in which were built the walls and floor, completing the structure. A short section in soft ground was built under air pressure. Excavation was in sand, clay, mud and rock. Traction is by cable.

The Budapest Subway, (16), for surface cars, a line 2 miles long, built 1894-6, was the first underground electric railway. The I-beams of the roof, which is close under the pavement, are supported at the middle on a continuous girder of two I-beams resting on columns spaced about 13 feet apart. Waterproofing consists of sheets laid in pitch.

The Hamburg Subway, (17), begun 1907, forms 5 miles of a ring with external branches opened in part in 1912 and since being extended. Excavation was in varied soil, in parts waterbearing. In dry ground, openings are left in the floor under the tracks and only the roof and sides were waterproofed. Beam piles were used at the sides, as in Berlin. Where depth permitted, a single arch section with invert was used.

The Buenos Aires Subway, (18), for surface cars, was begun 1911 and opened 1913 for a length of two miles, with extensions in progress or planned. It is of the earlier German type without steel in side walls. Waterproofing is tar paper and asphalt. Excavation was in hard clay. Electric shovels were used.

The cost of subway work varies widely. The kind and depth of excavation, proportion of temporary decking required for traffic, size and number of sewers, pipes etc., to be supported or changed and the extent of underpinning to protect adjacent buildings, greatly limit methods and thereby influence cost. The following is a table of costs, per lineal foot of equivalent single track, of several New York subway sections built between 1912 and 1919, all of a type resembling (3) of Fig. 97. The figures are based on contract prices which may be unbalanced, and preliminary estimates which may be large. They are affected in some cases by war increases. They include nothing for station finish, track, equipment, engineering or inspection, and are not safe for use in estimates without fuller presentation of conditions than is possible here.

Section No.*	11-1	4-3	29-2	5-4	4-1	4-5	8-5	4-2	5-3	8-4	8-2	48-2
Length, miles	1.1	0.8	1.2	0.7	0.3	0.7	0.7	0.6	0.5	0.7	0.6	0.5
Tracks, No.	4	4	2	4	4	4	2	4	4	2	2	2
Av. depth, ft.	26	24	24	25	25	24	28	28	26	29	28	29
Rock, percent	....	....	....	3	26	34	....	56	....	....	6	....
Stations, No.	3	2	3	1½	1½	3	2	2	1	2	2	2
Date contract	1912	1913	1915	1913	1913	1913	1916	1914	1912	1916	1916	1914
Excavation	$29	$45	$39	$76	$88	$70	$69	$102	$101	$ 91	$119	$135
Structure	54	63	66	48	51	42	81	62	67	125	139	125
Sewers	1	5	12	10	3	35	5	4	12	6	13	15
Pipes, etc	1	2	4	7	12	7	8	14	2	4	18	31
Underpinning	....	8	6	20	8	6	15	11	34	24	1	114
Repaving. etc	6	3	2	4	4	8	7	5	4	7	9	6
Total (per lineal foot of track)	$91	$126	$129	$165	$166	$168	$185	$198	$220	$257	$299	$426

*Section No. 11-1. 4th Av., Brooklyn; 40th to 61st St.—Firm dry soil; steam shovel much open.

4-3. Varick St.; Beach to Commerce St.—Dry; one side decked; steam shovel.

29-2. Nostrand Av. Brooklyn; Erasmus to Flatbush Av.—Dry sandy soil; part decked.

*Section No. 5–4. Broadway, Bleecker St. to Union Sq.—Dry; decked except in square.

4–1. Broadway; Union Sq. to 26th St.—Mostly dry; decked except in square.

4–5. 7th Av.; 17th to 30th St.—Dry; all decked; large sewer rebuilt.

8–5. Metropolitan-Bushwick Ave., Brooklyn; Manhattan Av. to Meserole St.—Dry; mostly decked.

4–2. Broadway; 27th to 38th St.—Much rock; some sliding; one deep cut; all decked.

5–3. Broadway; Howard to Houston St.—Wet fine sand, dried by pumping; all decked.

8–4. N. 7th-Metropolitan Av., Brooklyn; Bedford to Manhattan Av.—Wet fine sand; mostly decked.

8–2. 14th St.; Irving Pl. to Av. B.;—Wet sand one end; all decked.

48–2. William St.; Beekman to Pearl St.—Quicksand; all decked; heavy underpinning.

SECTION 12

MATHEMATICS AND MECHANICS

BY

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ELEMENTARY MATHEMATICS

ELEMENTARY MATHEMATICS

1. Algebra

Notation. Known or constant quantities are usually represented by the first letters of the English alphabet and unknown or variable quantities by the last letters. The Greek letter π is universally used for 3.14159 . . . In this book ϵ is used for 2.71828 . . ., and other Greek letters are rarely used except for angles. Names of Greek letters are

α Alpha	η Eta	ν Nu	τ Tau
β Beta	θ Theta	ξ Xi	υ Upsilon
γ Gamma	ι Iota	o Omicron	ϕ Phi
Δ, δ Delta	κ Kappa	π Pi	χ Chi
ϵ Epsilon	λ Lambda	ρ Rho	ψ Psi
ζ Zeta	μ Mu	Σ, σ Sigma	ω Omega

In this volume the sign / is often used to indicate division; thus, 3/8 is the same as $\frac{3}{8}$ and a/b is the same as $a \div b$. For the product of the first n natural numbers $n!$ is used.

Powers and Roots. The notation a^n denotes the nth power of a, even when n is negativ or fractional. Rules for powers are: $(+a)^n = +a^n$; $(-a)^n = +a^n$ if n is an even integer and $-a^n$ if n is odd; also $(a^m)^n = a^{mn}$; $a^{-n} = 1/a^n$, $a^0 = 1$. For multiplication and division of powers, $a^m \times a^n = a^{m+n}$ and $a^m/a^n = a^{m-n}$.

The notation $a^{1/n}$ or $\sqrt[n]{a}$ denotes the nth root of a. Replacing m and n in the last paragraph by $1/m$ and $1/n$, the rules there given for powers apply also to roots. An even root of a positiv number is positiv or negativ and an odd root is positiv; an odd root of a negativ number is negativ and an even root is impossible or imaginary.

The symbol i denotes $\sqrt{-1}$ which is the simplest imaginary quantity. If a is positiv and n even, then $(-a)^{1/n} = a^{1/n} i$. Powers of i are $i^2 = -1$, $i^3 = -\sqrt{-1}$, $i^4 = +1$. A complex quantity is one partly real and partly imaginary, as $a + bi$. The complex numbers $-\frac{1}{2}(1 + 3^{\frac{1}{2}} i)$ and $-\frac{1}{2}(1 - 3^{\frac{1}{2}} i)$ are two of the cube roots of unity.

Factors of some algebraic expressions are

$$a^2 - b^2 = (a - b)(a + b) \qquad a^2 + b^2 = (a - bi)(a + bi)$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$$
$$a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots - b^{n-1}) \text{ if } n \text{ is even}$$
$$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots + b^{n-1}) \text{ if } n \text{ is odd}$$

When a and b are small compared to 1, then approximately,

$$(1 + a)(1 + b) = 1 + a + b \qquad (1 + a)^m(1 + b)^n = 1 + ma + nb$$
$$1/(1 + a) = 1 - a \qquad 1/(1 - a) = 1 + a \qquad \sqrt{ab} = \tfrac{1}{2}(a + b)$$

Logarithms. When $y = b^x$ and b is a fixt number, x is the logarithm of y to the base b, or $\log_b y = x$. When $b = 10$ the logarithms are in the common system; thus since $100 = 10^2$, and $0.1 = 10^{-1}$, the common logs of 100 and 0.1 are +2 and −1. When $x = 0$, $y = 1$, and hence for all systems the log of 1 is 0. Numbers greater than 1 have positiv logs and those less than 1 negativ logs. When $b = 2.71828$ . . . the logs are in the Naperian system.

Proportion. All rules are included in this, where m, n, p, and q have any values:

$$\text{if} \quad \frac{a}{b} = \frac{c}{d}, \quad \text{then} \quad \frac{ma + nb}{pa + qb} = \frac{mc + nd}{pc + qd}.$$

A Permutation is any arrangement that can be made of several things; thus, for three letters there are six arrangements, abc, acb, bac, bca, cab, cba. The number of permutations of n things taken r at a time is $n(n -$

. . . $(n-r+1)$. For example the number of permutations of seven things taken three at a time is $7 \times 6 \times 5 = 210$. A COMBINATION is a group made by taking things without reference to their order; thus for three letters there is only one combination *abc*. The number of combinations of n things taken in groups of r is $[n(n-1)\ldots(n-r+1)]/r!$. For example, when seven lines radiate from a point the number of combinations of these taken two at a time is $7 \times 6/2 = 21$, and hence there are 42 angles that can be measured.

The Binomial Theorem or formula is a means for writing out the power of a binomial in a series It is

$$(a \pm b)^n = a^n \pm na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \cdots$$

When n is a positiv integer it holds in all cases; when n is a fraction or negativ it holds only if a is greater than b. In the first case the expansion (right-hand member) has $n+1$ terms; in the second the number of terms is infinite. The coefficients of the powers of a and b in the expansion are called binomial coefficients; the general formula for the rth coefficient is $Cr = [n(n-1)(n-2)\ldots(n-r+1)] \div [1 \times 2 \times 3 \times \ldots r]$. Thus when $n = 5$, the coefficients are 1, 5, 10, 10, 5, 1.

If x stands for b/a, then $(a \pm b)^n = a^n(1 \pm x)^n$, and the expansion of a binomial of the type $(a \pm b)$ can be exprest by that of one of the type $(1 \pm x)$. When n is a positiv integer the following hold in all cases, but when n is a fraction or negativ they hold only if x is between 0 and 1.

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{1 \cdot 2}x^2 \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^3 + \cdots$$

$$\frac{1}{1 \pm x} = (1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + x^4 \mp x^5 + \cdots$$

$$\sqrt{1 \pm x} = (1 \pm x)^{1/2} = 1 \pm \frac{1}{2}x - \frac{1 \cdot 1}{2 \cdot 4}x^2 \pm \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \cdots$$

$$\frac{1}{\sqrt{1 \pm x}} = (1 \pm x)^{-1/2} = 1 \mp \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 \mp \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots$$

When x is small compared to 1, the first two or three terms in the four preceding expansions give generally a sufficient approximation in ordinary computations.

A Series is a succession of numbers which proceed according to some fixt law. A converging series is one whose sum, as the number of its terms is indefinitely increased, approaches some finite value as a limit; a diverging series is one whose sum, so taken, increases indefinitely. An ARITHMETIC SERIES, or arithmetic progression, is one in which the differences between successive terms are equal, as in $a, a+d, a+2d, a+3d, \ldots$; the nth term is $a+(n-1)d$, and the sum of the n terms is $\frac{1}{2}n[2a+(n-1)d]$. A GEOMETRIC SERIES, or geometric progression, is one in which the ratios of successive terms are equal, as in $a, ar, ar^2, ar^3, \ldots$; the nth term is ar^{n-1}, and the sum of the n terms is $a(r^n-1)/(r-1)$.

Some special series (see also preceding paragraph for Binomial series):

$$1+2+3+4+\cdots+(n-1)+n = \tfrac{1}{2}n(n+1)$$
$$2+4+6+8+\cdots+(2n-2)+2n = n(n+1)$$
$$1+3+5+7+\cdots+(2n-3)+(2n-1) = n^2$$
$$1+2^2+3^2+4^2+\cdots+(n-1)^2+n^2 = \tfrac{1}{6}n(n+1)(2n+1)$$
$$1+2^3+3^3+4^3+\cdots+(n-1)^3+n^3 = \tfrac{1}{4}n^2(n+1)^2$$

$$a^x = 1 + \frac{x}{1}\log_\varepsilon a + \frac{x^2}{2!}\log_\varepsilon^2 a + \frac{x^3}{3!}\log_\varepsilon^3 a + \cdots \quad (1)$$

$$\varepsilon^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{where } \varepsilon = 2.71828\ldots \quad (2)$$

$$\tfrac{1}{2}\log_\varepsilon x = \frac{x-1}{x+1} + \tfrac{1}{3}\frac{(x-1)^3}{(x+1)^3} + \tfrac{1}{5}\frac{(x-1)^5}{(x+1)^5} + \cdots$$

If $x^2 < 1$, $\log_\varepsilon(1 \pm x) = \pm x - \tfrac{1}{2}x^2 \pm \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 \pm \ldots$ (3)

$$\tfrac{1}{2}\log_\varepsilon\frac{1+x}{1-x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots \quad (4)$$

In right-hand members of the following, x is the angle in radians:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\cdots \quad (5) \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots \quad (6)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{3\cdot 5} + \frac{17x^7}{3^2\cdot 5\cdot 7}\cdots \quad (7)$$

Approximations when x is Small. In series (1) the first two terms, or at most three are generally sufficient when x is small compared to a, and in (2), (3), (4) when x is small compared to 1. In (5), (6), (7), two terms, or even one, may be sufficient when x is less than the arc of 10°; thus, for 8° the value of x is $(8/180)\pi = 0.13963$, and $\sin x = x - \tfrac{1}{6}x^3 = 0.13918$, which is one unit in error in the fifth decimal place. When the angle is less than 5°, the values of x and $\sin x$ do not differ more than one unit in the fourth decimal place. (See Sect. 1, Art. 5.)

2. Solution of Equations.

Algebraic Equations, rational and integral, and of the first, second, third or fourth degrees have been solved algebraically; that is, formulas have been found for the values of the unknown, the roots of the equation.

A Linear Equation, one of the first degree, as $ax + b = 0$, has only one root, namely $x = -b/a$.

A Quadratic Equation, one of the second degree, as $x^2 + 2ax + b = 0$, has two roots, namely $-a \pm (a^2 - b)^{\frac{1}{2}}$. If $a = 0$, the roots are equal and opposite in sign; if $b > a^2$ both roots are imaginary.

A Cubic Equation, or one of the third degree, can be written in the form $x^3 + 3ax^2 + 3bx + 2c = 0$; it has three roots, here called x_1, x_2, and x_3. To determine these compute B and C from $B = -a^2 + b$ and $C = a^3 - \tfrac{3}{2}ab + c$, and s_1 and s_2 from $s_1 = \left(-C + \sqrt{B^3 + C^2}\right)^{\frac{1}{3}}$ and $s_2 = \left(-C - \sqrt{B^3 + C^2}\right)^{\frac{1}{3}}$;

then
$$x_1 = -a + (s_1 + s_2)$$
$$x_2 = -a - \tfrac{1}{2}(s_1 + s_2) + \tfrac{1}{2}\sqrt{-3}\,(s_1 - s_2)$$
$$x_3 = -a - \tfrac{1}{2}(s_1 + s_2) - \tfrac{1}{2}\sqrt{-3}\,(s_1 - s_2)$$

When $B^3 + C^2$ is negativ the numerical solution leads to irreducible imaginary forms tho the three roots are real.

Equations of Degree Higher than the Second can generally be solved best by factoring, by trial, or graphically. In the following $f(x)$ is used to denote any expression containing x, and is read "a function of x," and $f(x) = 0$ may denote any equation containing x; the value of $f(x)$ when $x = a$ is denoted by $f(a)$.

(1) Factoring Method: Factor $f(x)$ and equate each factor to zero; solve each of the new equations for x; these roots are also roots of $f(x) = 0$. Thus, to find the roots of $x^3 - 19x - 30 = 0$, note that $x^3 - 19x - 30 = (x + 3)(x^2 - 3x - 10)$; from

$3 = 0$, $x = -3$ and from $x^2 - 3x - 10 = 0$, $x = +5$ and -2; hence -3, $+5$, and -2 are the roots sought.

(2) Trial Method: Make guesses at the roots until a value is found which nearly satisfies the equation; such a value is an approximate root. In finding it, note that if $f(x) = 0$ designates the equation, and if x_1 and x_2 are numbers such that $f(x_1)$ and $f(x_2)$ are of opposite sign, then there is an odd number of roots of $f(x) = 0$ between x_1 and x_2. Thus, to find a root of $f(x) = x^3 + 4x^2 - 5x - 15 = 0$: trying $x = 1$ and $x = 3$ one gets $f(1) = -15$ and $f(3) = +33$; 1 and 3 are not close values of the root but there is a root between them; trying 2 one gets $f(2) = -1$, a closer value. Trying 2.1, one gets $f(2.1) = +1.4$; the root is between 2 and 2.1.

(3) Plotting Method: The equation to be solved being $f(x) = 0$, plot the graph of $y = f(x)$ (see Art. 3); the abscissas of the intersections of the graph with the x axis are roots of $f(x) = 0$. Or, break up $f(x)$ into the difference of two functions of x as $f_1(x)$ and $f_2(x)$ and plot the graphs of $y = f_1(x)$ and $y = f_2(x)$; then the abscissas of the intersections of the two graphs are roots of $f(x) = 0$. Thus, to solve $x^3 + 4x^2 - 5x - 15 = 0$: the graph of $y = x^3 + 4x^2 - 5x - 15$ is shown by the solid curve of Fig. 1, and the abscissas of its intersections with the x axis are about 2.1, -1.6, and -4.3, which are also the roots sought. Or, again, $x^3 + 4x^2 - 5x - 15$ can be broken up into $(x^3 + 4x^2) - (5x + 15)$; the dashed curve is the graph of $y = x^3 + 4x^2$ and the straight line is the graph of $y = 5x + 15$; the abscissas of the intersections of these two graphs are about 2.1, -1.6 and -4.3, and they are the roots sought.

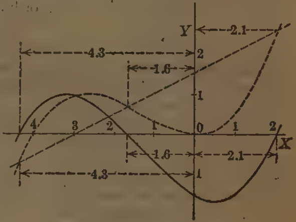


Fig. 1

Transcendental Equations are those involving trigonometry or logarithms; such equations with one unknown can be solved by one of the foregoing methods. For example, to find the roots of $\tan x - 2x = 0$ (x being in radians): plot the graphs of $y = \tan x$ and $y = 2x$, and determine the abscissas of the intersections of the graphs; the abscissas are the roots, one being $x = 1.16$.

To find the horizontal tension t in a catenary cable whose length is 22 ft, span 20 ft, and weight 10 lb per lin ft. The equation to be solved is $e^{10/t} - e^{-10/t} - 22 = 0$. By trial t is found to lie between 12 and 14; trying $t = 13$. the equation reduces to $+0.0298 = 0$; trying $t = 13.1$ it reduces to $-0.0012 = 0$, hence the root is a little less than $t = 13.1$.

An equation which arises in the theory of a column round at one end and fixt at the other is $x - \tan x = 0$, where x is to be in radians. This equation has many roots, the smallest one being $x = 4.49341$.

3. Graphic Representations

Rectangular Coordinates. XX' and YY' (Fig. 2) are two rectangular coordinate axes or x and y axes; and XOY, YOX', $X'OY'$, and $Y'OX$ are the first, second, third, and fourth quadrants respectively; O is the origin of coordinates. The position of any point in the drawing, as P, relative to the axes may be specified by two coordinates, the x coordinate, or abscissa of P, and the y coordinate, or ordinate of P. The abscissa, denoted by x, is the distance of P from the y axis, and the ordinate, denoted by y, is the distance of P from the x axis; the coordinates have signs as follows:

x is positiv when P is to the right of the y axis,
x is negativ when P is to the left of the y axis,
y is positiv when P is above the x axis,
y is negativ when P is below the x axis.

When P is actually plotted by means of its coordinates, the coordinates are also said to have been plotted. If the coordinates of P are x and y, say, P is referred to as the point (x, y).

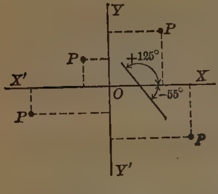


Fig. 2

Graphs. An equation between any two variables as x and y is sometimes written $y = f(x)$, for brevity; it is read "y equals a function of x." The graph of any equation as $y = f(x)$ is a graphical representation of the relation between y and x; to determine the graph compute values of y corresponding to assumed values of x, plot points whose coordinates are the corresponding values of x and y computed, and then draw a smooth curve thru the points.

Logarithmic Plotting. The equation $y = f(x)$ may be represented by a graph obtained by plotting corresponding values of log x and log y, instead of x and y, values of y having been obtained from $y = f(x)$ for assumed values of x; such an one is a logarithmic graph. If values of x and log y, or y and log x be plotted, the graphs are semi-logarithmic. These graphs are more advantageous than the ordinary one in some cases, and can be got as readily by using logarithmic or semi-logarithmic rulings as in Figs. 4 and 6 in which the unequal divisions are like those on an ordinary slide rule, based on the logarithms of numbers, and the figures on the rulings are not logarithms but the corresponding numbers, also as on a slide rule. If $y = f(x)$ be graphed on such ruling by plotting corresponding values of x and y, the graphs are logarithmic or semi-logarithmic, exactly like those first described.

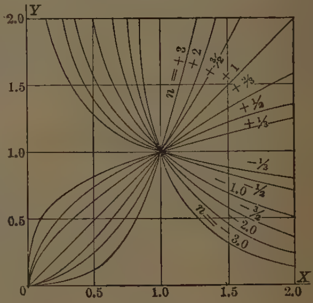


Fig. 3

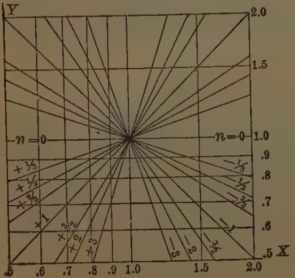


Fig. 4

The Intercept of a graph on the x (or y) axis is the x (or y) coordinate of the intersection of the graph with x (or y) axis. The SLOPE ANGLE of a straight line is the angle between the line and the positiv x axis; the slope angles of all lines may be exprest by angles between 0 and 180°, or $-90°$ and $+90°$ (see Fig. 2). The slope, or gradient, of a graph at any point is the tangent of the slope angle of its tangent line at that point.

The graph of $y = mx + c$, into the form of which any equation between two variables of the first degree can be put, is straight; m = the slope of the graph and c = its intercept

on the y axis. Thus $y = -2x + 3$ has a graph sloping upwards to the left at an angle of 63½° and cutting the y axis 3 units above the origin.

The graph of $y - b = m(x - a)^n + c$ is curved, its shape depending upon the numerical values of m and n; a, b, and c affect only the position of the graph with respect to the coordinate axis. Fig. 3 represents graphs of $y = x^n$ for the different values of n noted. The corresponding graphs of $y = mx^n$ are enlargements in the y direction if $m > 1$ and reductions if $1 > m > 0$ of those shown; if m is negativ, the effect is enlargement or reduction as explained and a rotation of the graphs through 180° about the x axis. The logarithmic graph of $y = x^n$ is straight; Fig. 4 shows these graphs corresponding to those of Fig. 3. The logarithmic graphs of $y = mx^n$ are also straight, but they do not pass thru the point (1, 1).

The graph of $(y - a) = me^{nx-b} + c$, a "logarithmic or compound interest" equation, is curved; its shape depends on the numerical values of m and n; a, b and c affect only the position of the curve relative to the coordinate axes. Fig. 5 represents graphs of $y = \epsilon^{nx}$ for the several values of n noted. In $y = m\epsilon^{nx}$, m affects the graphs shown as explained in the preceding paragraph. The semi-logarithmic graph of $y = \epsilon^{nx}$ is straight; Fig. 6 shows these graphs corresponding to those of Fig. 5. The semi-logarithmic graph of $y = m\epsilon^{nx}$ is also straight, but it does not pass through the point (0, 1).

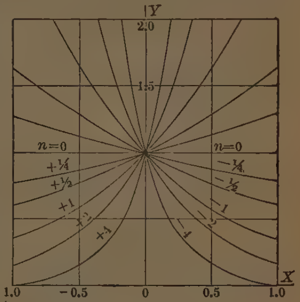


Fig. 5

To Find an Equation for a Given Graph. If the graph, plotted on axes x and y, is straight, the equation is of the first degree; let (x_1, y_1) and (x_2, y_2) be the coordinates of any two points of the graph; the equation is $(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$. If the intersection of the graph with either axis is available, that intersection may well be taken as one of the points; if it passes thru the origin, that may be advantageously chosen. If the graph is curved, there may be no corresponding equation but an equation can usually be found which will approximately fit at least a limited portion of the graph.

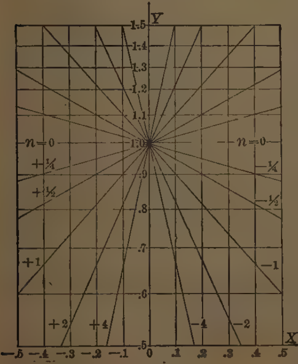


Fig. 6

(1) If the graph resembles a portion of one of the family represented in Fig. 3, plot its logarithmic graph; if this is practically straight, the original graph can be closely represented by $y = mx^n$. To determine m and n: $m = y$ when $x = 1$, and this value of y can be got from either graph, probably more accurately from the logarithmic; then in $\log(y/m) = n \log x$ substitute the coordinates x and y of any point and solve for n.

(2) If the graph resembles a portion of one of the family represented in Fig. 5, plot semi-logarithmically (Fig. 6); if the resulting graph is practically straight, then the original graph can be closely represented by $y = m\epsilon^{nx}$. To determine m and n: the value of

m is found from $m = y$ when $x = 0$, and this value of y can be got from either graph, probably most accurately from the semi-logarithmic graph; then in $\log_e y = \log_e m + nx$ substitute the x and y coordinates of any point on either graph and solve for n.

4. Trigonometry

Functions of Acute Angles. If from a point in either line bounding an angle (Fig. 7) a perpendicular is drawn to the other line, then in the right triangle thus formed the leg adjacent to α is the base, and the leg opposite, the perpendicular; they are denoted by b and p respectively and the hypothenuse by h. The sine, cosine, tangent, cotangent, secant, and cosecant of the angle are defined (and abbreviated) thus:

$$\sin\alpha = p/h \qquad \cos\alpha = b/h \qquad \tan\alpha = p/b$$
$$\csc\alpha = h/p \qquad \sec\alpha = h/b \qquad \cot\alpha = b/p$$

The versed sine, abbreviated vers, is unity minus the cosine; thus vers $\alpha = 1 - \cos\alpha$. The exsecant, abbreviated exsec, is secant minus unity, thus exsec $\alpha = \sec\alpha - 1$. From the foregoing it follows that

$$\sin\alpha/\cos\alpha = \tan\alpha \qquad \cos\alpha/\sin\alpha = \cot\alpha$$
$$\sin\alpha\csc\alpha = \cos\alpha\sec\alpha = \tan\alpha\cot\alpha = 1$$
$$\sin^2\alpha + \cos^2\alpha = \sec^2\alpha - \tan^2\alpha = \csc^2\alpha - \cot^2\alpha = 1$$

If $\alpha + \beta = 90°$, $\sin\alpha = \cos\beta$, $\tan\alpha = \cot\beta$, $\sec\alpha = \csc\beta$. If the arc $A''B'$ (Fig. 7) be struck from O with a radius unity, then in the triangle $OA'B'$ h is unity and in the triangle $OA''B''$ b is unity. Then also

$$\sin\alpha = A'B' \qquad \cos\alpha = OA' \qquad \tan\alpha = A''B''$$
$$\sec\alpha = OB'' \qquad \text{vers}\,\alpha = A'A'' \qquad \text{exsec}\,\alpha = B'B''$$

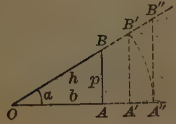


Fig. 7

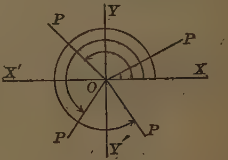


Fig. 8

Functions of any Angle. Let $\alpha = XOP$ (Fig. 8), x and y = the coordinates of P, and $h = OP$. Then whether α is in the first, second, third, or fourth quadrant, the functions of α are defined as follows:

$$\sin\alpha = y/h \qquad \cos\alpha = x/h \qquad \tan\alpha = y/x$$
$$\csc\alpha = h/y \qquad \sec\alpha = h/x \qquad \cot\alpha = x/y$$

Since x and y may be positiv or negativ, the functions may also be; the sign of a function depends on the quadrant of α, thus:

quad	sin	cos	tan	cot	sec	csc
1	+	+	+	+	+	+
2	+	−	−	−	−	+
3	−	−	+	+	−	−
4	−	+	−	−	+	−

The values of the functions for the four cardinal angles are,

angle	sin	cos	tan	cot	sec	csc
0°	0	+1	0	∞	+1	∞
90	+1	0	∞	0	∞	+1
180	0	−1	0	∞	−1	∞
270	−1	0	∞	0	∞	−1
360	0	+1	0	∞	+1	∞

Functions of angles greater than 90° are not given in trigonometric tables generally. These can be obtained from the functions of acute angles by means of the following in which α_2, α_3, and α_4 denote angles in the second, third, and fourth quadrants respectively.

α_2	$180° - \alpha_2$	$\alpha_2 - 90°$
sin =	+ sin =	+ cos
cos =	− cos =	− sin
tan =	− tan =	− cot

α_3	$\alpha_3 - 180°$	$270° - \alpha_3$
sin =	− sin =	− cos
cos =	− cos =	− sin
tan =	+ tan =	+ cot

α_4	$360° - \alpha_4$	$\alpha_4 - 270°$
sin =	− sin =	− cos
cos =	+ cos =	+ sin
tan =	− tan =	− cot

Negativ Angles. Angles in the preceding figures and those referred to in the foregoing were regarded as measured from OX in the counterclockwise direction and considered positiv; an angle regarded as measured clockwise is considered negativ. OP (Fig. 8) may be specified by a positiv or a negativ angle, the arithmetic sum of the two being 360°. If α denotes merely the numerical value of any angle, then

$$\sin(-\alpha) = -\sin\alpha \qquad \cos(-\alpha) = \cos\alpha \qquad \tan(-\alpha) = -\tan\alpha$$
$$\csc(-\alpha) = -\csc\alpha \qquad \sec(-\alpha) = \sec\alpha \qquad \cot(-\alpha) = -\cot\alpha$$

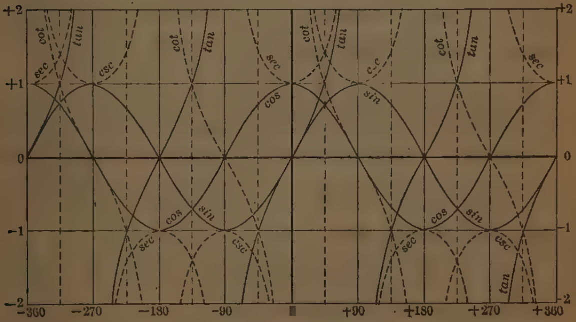


Fig. 9. Graphs of Trigonometric Functions

Graphs of sin α, cos α, tan α, cot α, sec α, and csc α are seen in Fig. 9 for values of α between −360° and +360°; the graphs of the last four functions have infinite branches which are asymptotic to the solid vertical lines adjacent at the upper and lower margins. The figure shows the sign and periodicity of any function of any angle between the limits named and roughly the relative values of the functions of any particular angle.

Inverse, or Anti-Functions. The symbol $\sin^{-1} x$ means the angle whose sine is x, and is read inverse sine of x and anti-sine of x (also arc sine x). Similarly $\cos^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$, $\sec^{-1} x$, $\csc^{-1} x$, $\text{vers}^{-1} x$, the last meaning an angle α such that $(1 - \cos\alpha) = x$. While the direct functions (sine, etc.) are single valued, the indirect are many valued; thus $\sin 30° = 0.5$, but $\sin^{-1} 0.5 = 30°$ or $150°$ . . . .

Trigonometric Relations. Functions of an angle exprest in terms of each of the others; the following abbreviations are used:

$$A = \pm\sqrt{1 - \sin^2\alpha} \qquad B = \pm\sqrt{1 - \cos^2\alpha} \qquad C = \pm\sqrt{1 + \tan^2\alpha}$$

$$F = \pm\sqrt{\csc^2\alpha - 1} \qquad E = \pm\sqrt{\sec^2\alpha - 1} \qquad D = \pm\sqrt{1 + \cot^2\alpha}$$

$$\sin\alpha = B = \tan\alpha/C = 1/D = E/\sec\alpha = 1/\csc\alpha$$

$$\cos\alpha = A = 1/C = \cot\alpha/D = 1/\sec\alpha = F/\csc\alpha$$

$$\tan\alpha = \sin\alpha/A = B/\cos\alpha = 1/\cot\alpha = E = 1/F$$

$$\cot\alpha = A/\sin\alpha = \cos\alpha/B = 1/\tan\alpha = 1/E = F$$

$$\sec\alpha = 1/A = 1/\cos\alpha = C = D/\cot\alpha = \csc\alpha/F$$

$$\csc\alpha = 1/\sin\alpha = 1/B = C/\tan\alpha = D = \sec\alpha/E$$

Functions of the sum and difference of two angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta)/(1 \mp \tan\alpha\tan\beta)$$

$$\cot(\alpha \pm \beta) = (\cot\beta\cot\alpha \mp 1)/(\cot\beta \pm \cot\alpha).$$

If x is small, say 3 or 4°, then the following are close approximations, in which the coefficients x must be exprest in radians ($1° = 0.01745$ radians).

$$\sin(\alpha \pm x) = \sin\alpha \pm x\cos\alpha, \qquad \cos(\alpha \pm x) = \cos\alpha \mp x\sin\alpha.$$

Functions of half and double angles:

$$\sin \tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1 - \cos\alpha)} = \tfrac{1}{2}\sqrt{1 + \sin\alpha} - \tfrac{1}{2}\sqrt{1 - \sin\alpha}$$

$$\cos \tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1 + \cos\alpha)} = \tfrac{1}{2}\sqrt{1 + \sin\alpha} + \tfrac{1}{2}\sqrt{1 - \sin\alpha}$$

$$\tan \tfrac{1}{2}\alpha = \sqrt{(1 - \cos\alpha)/(1 + \cos\alpha)} = (1 - \cos\alpha)/\sin\alpha = \sin\alpha/(1 + \cos\alpha)$$

$$\cot \tfrac{1}{2}\alpha = \sqrt{(1 + \cos\alpha)/(1 - \cos\alpha)} = (1 + \cos\alpha)/\sin\alpha = \sin\alpha/(1 - \cos\alpha)$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha, \qquad \cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha,$$

$$\tan 2\alpha = 2\tan\alpha/(1 - \tan^2\alpha), \qquad \cot 2\alpha = (\cot^2\alpha - 1)/2\cot\alpha.$$

Sums and products of functions:

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$

$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

5. Plane and Spherical Triangles

Plane Triangles. The three angles are denoted by α, β, and γ, and the respective opposite sides by a, b, and c; also s denotes $\frac{1}{2}(a+b+c)$, r radius of inscribed circle, R radius of circumscribed circle, and A area of the triangle. Then, $\alpha+\beta+\gamma=180°$,

$$a/\sin\alpha = b/\sin\beta = c/\sin\gamma \text{ ("law of sines")}$$
$$c^2 = a^2+b^2-2ab\cos\gamma, \quad r=\sqrt{(s-a)(s-b)(s-c)/s}$$
$$A = sr = \tfrac{1}{2}ab\sin\gamma, \qquad R=\tfrac{1}{2}a\csc\alpha.$$

Solution of Right Triangles. If an acute angle and one side or if two sides of a right triangle are given the other elements can be determined. Let h = hypothenuse, α and β acute angles, and a and b the legs opposite them, respectively. The acute angles are complementary, that is, $\alpha+\beta=90°$; the area is $\frac{1}{2}ab$ always. Five cases may be distinguished:

Given h and α; $a = h\sin\alpha$, $b = h\cos\alpha$
Given a and α; $b = a\cot\alpha$, $h = a\csc\alpha$
Given b and α; $a = b\tan\alpha$, $h = b\sec\alpha$
Given a and h; $\alpha = \sin^{-1}a/h$, $b=\sqrt{(h+a)(h-a)}$
Given a and b; $\alpha=\tan^{-1}a/b$, $h=\sqrt{a^2+b^2}$

Solution of Oblique Triangles. If any three of the six elements (three angles and three sides) of a triangle are known, the remaining three can be determined provided one of the given three is a side. Any problem will fall under one of four cases.

Case 1. Given one side and two angles. Then the third angle equals 180° minus the sum of the two given. If the given side be a, then

$$b = a\sin\beta/\sin\alpha \quad \text{and} \quad c = a\sin\gamma/\sin\alpha.$$

Case 2. Given two sides and the included angle (a, b and γ). Then

$$\tfrac{1}{2}(\alpha+\beta) = 90° - \tfrac{1}{2}\gamma,\ \tfrac{1}{2}(\alpha-\beta) = \tan^{-1}[\tan\tfrac{1}{2}(\alpha+\beta)\cdot(a-b)/(a+b)]$$
$$\alpha = \tfrac{1}{2}(\alpha+\beta)+\tfrac{1}{2}(\alpha-\beta) \qquad \beta=\tfrac{1}{2}(\alpha+\beta)-\tfrac{1}{2}(\alpha-\beta)$$
$$c = a\sin\gamma/\sin\alpha.$$

Case 3. Given two sides a and b and the angle α opposite one of them. Then $\sin\beta=(b/a)\sin\alpha$, giving two values of β, one acute and one obtuse unless $\sin\beta>1$, in which case the data are impossible. Calling these two angles β_1 and β_2 respectively, then

corresponding to β_1, $\gamma_1 = 180-(\alpha+\beta_1)$ and $c_1 = a\sin\gamma_1/\sin\alpha$
corresponding to β_2, $\gamma_2 = 180-(\alpha+\beta_2)$ and $c_2 = a\sin\gamma_2/\sin\alpha$.

That is, there are two solutions unless $\gamma_2<0$, when only the first holds. (The meanings of these exceptions, $\sin\beta>1$ and $\gamma_2<0$, will become evident if a geometrical construction of the triangle is attempted.)

Case 4. Given the three sides. Let s denote $\frac{1}{2}(a+b+c)$. Then

$$\cos\tfrac{1}{2}\alpha=\sqrt{s(s-a)/bc} \quad \cos\tfrac{1}{2}\beta=\sqrt{s(s-b)/ca} \quad \cos\tfrac{1}{2}\gamma=\sqrt{s(s-c)/ab}$$

Spherical Triangles. The intersection of the surface of a sphere with a plane thru its center is a great circle of the sphere; an arc of the circle is measured by the angle between the radiuses of the sphere drawn to the ends of the arc. A spherical angle is the angle between two intersecting arcs of great circles; it is measured by the angle between their tangents at the intersection. A spherical triangle is a portion of the surface of a sphere bounded

by arcs of three great circles. Spherical triangles are classified in the sam way as plane ones; thus, isosceles, equilateral, right, etc. The spherica excess of a triangle is the excess of the sum of its angles over 180°. An spherical triangle can be solved if any three of its six elements (three side and three angles) are given; from the following formulas a solution may b made in any given case. The notation is as follows: α, β, and γ represen the three angles, while a, b, and c designate the opposite sides respectivel and e = the spherical excess.

$$\sin a / \sin \alpha = \sin b / \sin \beta = \sin c / \sin \gamma$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma$$

$$\cos \gamma = -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c$$

$$\cot \tfrac{1}{2} e = (\cot \tfrac{1}{2} a \cot \tfrac{1}{2} b + \cos \gamma) / \sin \gamma$$

A spherical right triangle can be solved if any two of its elements, not including th right angle, are given. In the following h is the hypothenuse, a and b the other tw sides, and α and β the angles opposite a and b respectively.

$$\sin a = \sin h \sin \alpha = \tan b \cot \beta \cdot \quad \cos \alpha = \cos a \sin \beta = \cot h \tan b$$

$$\cos h = \cos a \cos b = \cot \alpha \cot \beta$$

6. Geometry and Mensuration

Angles. The common unit of angle is the degree (one-ninetieth of a rig angle); the degree is divided into 60 minutes and the minute into 60 second Another unit is the *radian*, called also the unit in circular measure of angle it is an angle equal to that between two radiuses of a circle which embra an arc equal to the radius. Then

$$1 \text{ radian} = 180 \div \pi \text{ degrees} = 57.296°, \ 1 \text{ degree} = 0.0175 \text{ radian}$$

If an angle α subtends a circular arc whose length and radius are s and respectively, then $\alpha = s/r$ or $s = r\alpha$ provided that s and r are exprest the same unit and α in radians.

Triangles. Let a, b, and c = lengths of sides; α, β, and γ the opposite angl h the altitude to the side b, s denote $\frac{1}{2}(a + b + c)$, and A = area.

$$A = \tfrac{1}{2} bh = \tfrac{1}{2} ba \sin \gamma = \tfrac{1}{2} b^2 \sin \gamma \sin \alpha / \sin \beta = \sqrt{s(s-a)(s-b)(s-c)}$$

Quadrilaterals. Let D_1 and D_2 = lengths of the two diagonals and α the an between them, then $A = \frac{1}{2} D_1 D_2 \sin \alpha$. Parallelogram: let a = one side, b = ba α = angle between a and b, h = altitude; then $A = bh = ab \sin \alpha$. Trapezoid: let b a B = parallel sides, and h = altitude; then $A = \frac{1}{2}(b + B)h$.

Polygons. The area A equals the sum of the areas of the constitue triangles. For a regular polygon (equal sides and equal angles) let R radius of circumscribed circle, r = radius of inscribed circle, a = length o side, n = number of sides, 2α = central angle subtended by one side; the

$$A = \tfrac{1}{4} na^2 \cot \alpha = \tfrac{1}{2} nR^2 \sin 2\alpha = nr^2 \tan \alpha \qquad a = 2R \sin \alpha = 2r \tan \alpha$$

Circle (Fig. 11). Circumference $L = 2\pi r = \pi d$. Area $A = \pi r^2 = \frac{1}{4}\pi$ Chord $C = 2r \sin \frac{1}{2}\theta$. Rise of arc $h = r(1 - \cos \frac{1}{2}\theta)$. Arc subtended b $a = \pi d \times (\theta \text{ in degrees})/360 = r\,(\theta \text{ in radians})$; when h is small compared C, $a = \frac{1}{3}(8c - C)$; this is approximate, but even when $h = r$, the error less than $1\frac{1}{4}$%. Sector $OMPN$: area = area of circle (θ in degrees)/3 Segment $MPNM$: area = area sector $- \frac{1}{2} r^2 \sin \theta$; when h is small compa to C, area = $\frac{2}{3} Ch$, also $h(8c + 6C)/15$, both approximate; when $h = \frac{1}{}$ the first errs about $3\frac{1}{2}$% and the second less than 1%.

To Find the Center of a Given Circle, draw a chord and then a perpendicular to the chord at its middle point; this line extended in both directions to the circumference is a diameter, etc. To draw a circle thru three points: join one of the points with the others, thus getting two chords; at the middle points of the chords erect perpendiculars; the intersection of these is the center of the circle. If, in the preceding problem, the center is inaccessible or the radius is too long for drawing instruments, proceed as follows, the three points being A, O, A', Fig. 10: draw arcs Aa' and $A'a$ with centers at A' and A respectively; extend AO to determine a and $A'O$ to determine a'; divide aA' into equal parts ab, bc, etc; lay off $a'b'$, $b'c'$, etc., equal to ab; join A with any point as b and A' with the corresponding point b'; the intersection P of these joining lines is a point in the circle. If in the preceding problem it is desired to determine points in the circle by ordinates as NP, let r = radius of the circle, h = rise or center ordinate OQ, C = chord AA', y = any ordinate as NP, and $x = QN$ (distance of that ordinate from the center one), then $r = (4\,h^2 + C^2)/8\,h$, and

$$y = \sqrt{r^2 - x^2} - (r - h) = \sqrt{r^2 - x^2} - \sqrt{r^2 - C^2/4}$$

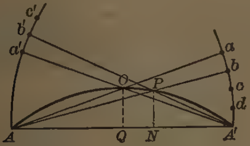


Fig. 10

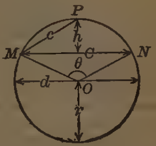


Fig. 11

Ellipse (see also Art. 11). The circumference of an ellipse whose semi-axes are a and b is $\pi(a + b)\,k$ where k is an abbreviation for $(1 + c^2/4 + c^4/64 + c^6/256 + \cdots)$ and $c = (a - b)/(a + b)$. The following table gives k for several values of c:

c	k	c	k	c	k	c	k	c	k
0.05	1.0006	0.25	1.0158	0.45	1.0516	0.65	1.1083	0.85	1.1903
.10	1.0025	.30	1.0215	.50	1.0635	.70	1.1267	.90	1.2154
.15	1.0054	.35	1.0311	.55	1.0768	.75	1.1466	.95	1.2430
.20	1.0100	.40	1.0404	.60	1.0922	.80	1.1677	1.00	1.2732

The area of an ellipse is given by πab; the area of any part as $ABQP$ (Fig. 12) is $xy + ab \sin^{-1} x/a$, in which x and y are the distances of P or Q from the axes.

Hyperbola (see also Art. 11). The area of any symmetrical segment like $PAQP$ (Fig. 13) is $xy - ab \log_e (x/a + y/b)$, where $2\,a$ and $2\,b$ are the lengths of the axes of the hyperbola and x and y the distances of P or Q from the axes.

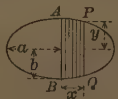


Fig. 12

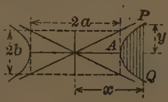


Fig. 13

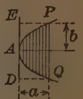


Fig. 14

Parabola (see also Art. 11). The length of any arc beginning at the vertex as AP (Fig. 14) is $(b^2/4\,a)\left[\sqrt{c\,(1 + c)} + \log_e\left(\sqrt{c} + \sqrt{1 + c}\right)\right]$, in which c is an abbreviation for $4\,a^2/b^2$, a and b being distances as marked. The area of a symmetrical segment like $PAQP$ is two-thirds that of the rectangle $PEDQ$.

Irregular Figure (Fig. 15). To find the area divide the figure into an even number of strips of equal width; the more numerous these strips the more

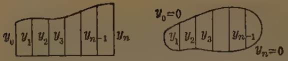


Fig. 15

accurate is the result given by the following formulas. The first of the ordinates bounding the strips is called y_0, the second y_1, etc., n the numbe of strips, w their common width and A the area. The following formulas give A approximately:

$A = w(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n)$, Trapezoidal.

$A = \frac{1}{3}w(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$, Simpson's

$A = w(0.4y_0 + 1.1y_1 + y_2 + y_3 + \cdots + y_{n-2} + 1.1y_{n-1} + 0.4y_n)$, Durand's

Simpson's rule applies only when n is odd; when n is even the area of $n-3$ strip may be computed and then the area of the remaining 3 strips may be found b

$$A = \tfrac{3}{8}w(y_0 + 3y_1 + 3y_2 + y_3), \text{ Cotes's.}$$

Prism. Let V = volume, h = altitude, A = base area; $V = Ah$. Trun cated prism: V = the product of the length of line joining the centers of gravit of the bases and the area of section perpendicular to the line. Truncate triangular prism: V = product of one-third the sum of the parallel edge and the area of section perpendicular to edges.

Cylinder. Let V = volume, A = base area, h = altitude; $V = Ah$. Rigl circular cylinder: Let r = base radius, S = area cylindrical surface; V $\pi r^2 h$, $S = 2\pi rh$, $A = \pi r^2$. Frustum right circular cylinder: H = greatest height, h = least height; $V = \pi r^2 (H + h)/2$, $S = \pi r(H + h)$. Wedge of right circular cylinder (Fig. 16); Let $2C$ = straight edge of base, p = altitude of the base (segment of circle), 2ϕ = central angle of the arc of base in degrees, r = radius of base; $V = [C(3r^2 - C^2) + 3r^2(p - r)\phi\pi/180]\,h/3p$, $S = [C + (p - r)\phi\pi/180]\,2rh/p$. Hollow right circular cylinder: R = outer radius of base, r = inner radius, t = thickness $= R - r$; $V = \pi(R^2 - r^2)h = \pi(2R - t)th = \pi(2r + t)th$.

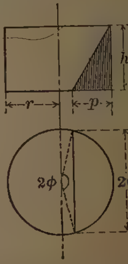


Fig. 16

Pyramid. Let V = volume, A = area base, h = altitude; $V = Ah/3$. Frustum of a pyramid: let A and a = areas of bases; $V = (A + a + \sqrt{Aa})\,h/3$.

Cone. Let V = volume, A = area of base, h = altitude; then $V = Ah/3$. Circular Cone: let r = radius of base; $V = \pi r^2 h/3$. Right circular cone: area of conical surface, $S = \pi r\sqrt{r^2 + h^2}$. Frustum of right circular cone: let R = rad larger and r = radius smaller base; $V = \pi(R^2 + Rr + r^2)\,h/3$; $S = \pi(R +$ $\sqrt{(R - r)^2 + h^2}$.

Sphere. Let V = volume, A = area, r = radius, and d = diameter. Th $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$; $A = 4\pi r^2 = \pi d^2$. Zone: let h = a tude, a = radius larger base, b = radius smaller base; $V = \frac{1}{2}$ $(a^2 + b^2 + h^2/3)$; convex $A = 2\pi rh$; $r^2 = a^2 + [(a^2 - b^2 - h^2)/2$ Segment (zone with one base): make $b = 0$ in formulas zone. Sector with one conical surface (Fig. 17): $V = \frac{2}{3}\pi$ total $A = \pi r(2h + a)$.

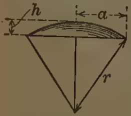


Fig. 17

Ellipsoid. Every section is a circle or an ellipse. a, b, and c = the semi-axes; then the volume = $\frac{4}{3}\pi abc$.

Paraboloid of Revolution generated by revolving a parabola about own axis (Fig. 18). Let r = radius of base and a = altitude of parabol

then volume = $\frac{1}{2}\pi r^2 a$. Volume of a frustum, R and r being radii of bases and h = height (see Fig. 18), is $\frac{1}{2}\pi(R^2 + r^2)\,h$ (approx.)

Surface and Solid of Revolution. The first may be generated by revolving a plane curve about a line in its plane, or else by a straight line about an intersecting line; and the second by revolving a plane area about a line in its plane. The AXIS of the surface or solid is the line about which the revolution takes place. The area and volume can be determined by the principles of Pappus and Guldinus which are:


Fig. 18

(a) For area A: let l = length of generating curve (must not cut the axis of revolution) and x_1 = distance of its center of gravity from the axis; then $A = 2\pi x_1 l$.

(b) For volume V: let a = area of the generating figure (must not be intersected by the axis) and x_2 = distance of its center of gravity from the axis; then $V = 2\pi x_2 a$.

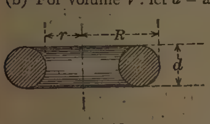


Fig. 19

The revolution of a circle about a line in its plane but not cutting the circle generates a 'tore,' or 'torus' or anchor ring (Fig. 19). The preceding formulas give its area and volume thus: $A = 2\pi \frac{1}{2}(R+r)\,\pi d = \pi^2(R+r)\,d$, and $V = 2\pi\frac{1}{2}(R+r)\frac{1}{4}\pi d^2 = \frac{1}{4}\pi^2(R+r)d^2$.

A Prismoid is a solid with plane faces two being parallel (and called ends) and each of the others containing a part of the perimeter of one end and at least a point in that of the other end. Let V = volume of the prismoid, A_1 and A_2 = areas of the ends, A_m = area of the cross section of the prismoid midway between the ends, and l = length of prismoid (distance between ends); then $V = \frac{1}{6}(A_1 + A_2 + 4A_m)l$.

7. Interest and Sinking Fund

Interest. The notation in the formulas is: p = principal, the sum loaned; i = rate of interest, exprest thus, 0.06 or 6/100 for 6%; n = time, term, or period of the loan in years; a = amount due after n years, that is, the sum of the principal and interest. When interest is paid only on the principal and not also on interest which may be due, the interest is simple; then

$$a = p(1 + in) \qquad i = (a - p)/pn \qquad p = a/(1 + in) \qquad n = (a - p)/pi$$

The adjoining table gives the simple interest on $100 at various rates for one year, one month (1/12 year), and one day (1/30 month, bank practise).

	2%	3%	4%	5%	6%	8%
1 year.....	$2.00	$3.00	$4.00	$5.00	$6.00	$8.00
1 month ...	0.16⅔	0.25	0.33⅓	0.41⅔	0.50	0.66⅔
1 day......	0.0055⅚	0.0083⅓	0.0111⅑	0.0138⅞	0.0166⅔	0.0222⅖

When interest is not paid periodically but regarded as additions to the principal, also to draw interest during the remainder of the time, then the interest is compound. If additions of interest to principal are made annually, then

$$a = p(1 + i)^n, \quad i = \sqrt[n]{(a/p)} - 1, \quad p = a/(1 + i)^n, \quad n = (\log a - \log p)/\log(1 + i)$$

For semiannual compound interest, $a = p(1 + i/2)^{2n}$. The adjoining table gives the amount of one dollar at interest compounded annually at various rates for various periods. Thus, $2000 at 3% in 20 years amounts to 2000 × $1.80611 = $3612.22; the interest earning is $3612.22 − $2000, or $1612.22.

Amount (Principal plus Interest) of $1 at Compound Interest

No. of Years	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000
2	1.04040	1.05062	1.06091	1.07122	1.08160	1.09202	1.10250
3	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15762
4	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551
5	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628
10	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889
15	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893
20	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330
25	1.64061	1.85394	2.09378	2.36324	2.66584	3.00543	3.38635
30	1.81136	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194
35	1.99989	2.37321	2.81386	3.33359	3.94609	4.66735	5.51602
40	2.20804	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999
45	2.43785	3.03790	3.78160	4.70236	5.84118	7.24825	8.98501
50	2.69159	3.43711	4.38391	5.58493	7.10668	9.03264	11.46740

An Annuity is a succession of equal sums paid regularly, generally annuall each payment is also called annuity, but more properly, installment. Th amount of an annuity for a term of years is the sum of the installments pl their interest earnings, interest being generally considered as compoun Let A = amount; i = interest rate express decimally (as 0.05 for 5%); n number of years in the period of the annuity; and I = (annual) insta ment, one at the end of each year. Then $A = I\,[(1 + i)^n - 1]/i$. The fo lowing table gives the amount of an annual annuity of $1 for various perio and various rates of interest. Thus, $200 invested at the end of each ye of a 20-year period at 4% amounts to 200 × $29.77808, or $5955.62, at the e of that period; the interest earned is $5955.62 − (20 × $200) = $1955.62.

Amount (Installments plus Interest Earned) of an Annual Annuity of $

No. of Years	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.000
2	2.02000	2.02500	2.03000	2.03500	2.04000	2.04500	2.050
3	3.06040	3.07562	3.09090	3.10622	3.12160	3.13702	3.152
4	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819	4.310
5	5.20404	5.25633	5.30914	5.36247	5.41632	5.47071	5.525
10	10.94972	11.20338	11.46388	11.73139	12.00611	12.28821	12.57
15	17.29342	17.93193	18.59891	19.29568	20.02359	20.78405	21.57
20	24.29737	25.54466	26.87037	28.27968	29.77808	31.37142	33.06
25	32.03030	34.15776	36.45926	38.94986	41.64591	44.56521	47.72
30	40.56808	43.90270	47.57542	51.62268	56.08494	61 00707	66.43
35	49.99448	54.92821	60.46208	66.67401	73.65222	81.49662	90.32
40	60.40198	67.40255	75.40126	84.55028	95.02552	107.03032	120.79
45	71.89271	81.51613	92.71986	105.78167	121.02939	138.84996	159.70
50	84.57940	97.48435	112.79687	130.99791	152.66708	178.50303	209.34

A Sinking or Amortization Fund is one created to provide a def sum (for canceling a debt for example) at a certain time; it is usually cre by equal and regular contributions or installments and their interest earn compounded, that is, by an annuity and its interest earnings. Let S = amount of a sinking fund; n = number of years for its creation; i = int rate expressed decimally (as 0.05 when the rate is 5%); and I = an

installment required, paid at the end of each year of the period. Then $I = Si/[(1+i)^n - 1]$. The following table gives annual installments required to create a fund of one dollar (equal to I/S) in various periods at various interest rates. Thus the annual installment necessary to accumulate $22 000 in 25 years, at 4 %, is 22 000 × $0.02401, or $528.22; the interest earning is 22 000 − (25 × 528.22), or $8794.50.

Annual Annuity Required to Accumulate $1 (Installments plus Interest Earnings)

No. of Years	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.49505	0.49382	0.49261	0.49140	0.49020	0.48900	0.48780
3	0.32675	0.32514	0.32353	0.32193	0.32035	0.31877	0.31721
4	0.24262	0.24082	0.23902	0.23725	0.23550	0.23374	0.23201
5	0.19218	0.19025	0.18835	0.18648	0.18463	0.18279	0.18098
10	0.09133	0.08926	0.08723	0.08524	0.08329	0.08138	0.07950
15	0.05782	0.05577	0.05380	0.05183	0.04994	0.04811	0.04634
20	0.04116	0.03915	0.03722	0.03536	0.03356	0.03187	0.03024
25	0.03122	0.02928	0.02743	0.02567	0.02401	0.02244	0.02095
30	0.02465	0.02278	0.02102	0.01937	0.01783	0.01639	0.01505
35	0.02000	0.01821	0.01654	0.01499	0.01358	0.01227	0.01107
40	0.01655	0.01484	0.01326	0.01183	0.01052	0.00934	0.00828
45	0.01391	0.01226	0.01080	0.00945	0.00826	0.00720	0.00626
50	0.01182	0.01026	0.00886	0.00763	0.00655	0.00560	0.00478

The Present Worth of a given sum due at a future time is such a sum which if now placed at interest, would amount to the given sum at that time. Let p = present worth, a = given sum due in n years from now, and i = interest rate; then on simple interest basis, $p = a/(1 + in)$, and on annual compound $p = a/(1+i)^n$. The following table gives present worths, computed at various rates of compound interest, of $1 due in various periods. For example, an addition to plant necessary 5 years hence will then entail a cost of $8000; how much can one afford to pay for such addition now, it being without use or expense for 5 years? As much as the present worth of $8000 due in 5 years at the current rate of interest, that is, if the rate is 5 %, 8000 × $0.78353, or $6268.24.

Present Worth of $1 Due at a Future Date

No. of Years	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238
2	.96117	.95181	.94260	.93351	.92456	.91573	.90703
3	.94232	.92860	.91514	.90194	.88900	.87630	.86304
4	.92385	.90595	.88849	.87144	.85480	.83856	.82270
5	.90573	.88385	.86261	.84197	.82193	.80245	.78353
10	.82035	.78120	.74409	.70892	.67556	.64393	.61391
15	.74301	.69047	.64186	.59689	.55526	.51672	.48102
20	.67297	.61027	.55368	.50257	.45639	.41464	.37689
25	.60953	.53939	.47761	.42315	.37512	.33273	.29530
30	.55207	.47674	.41199	.35628	.30832	.26700	.23138
35	.50003	.42137	.35538	.29998	.25342	.21425	.18129
40	.45289	.37243	.30656	.25257	.20829	.17193	.14205
45	.41020	.32917	.26444	.21266	.17120	.13796	.11130
50	.37153	.29094	.22811	.17905	.14071	.11071	.08720

The Present Worth of an Annuity is the sum which now placed at compound interest will reach the same amount as will be reached by the annuity. Let V = the present value of an annuity I; then $V = I[1 - (1+i)^{-n}]/i$. The following table gives values of an annuity of \$1 for various periods and various rates. Thus, the present worth of an annuity of \$1 for 20 years at 4% is \$13.59.

Present Value of an Annuity of \$1

No. of Years	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238
2	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941
3	2.88388	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325
4	3.80773	3.76197	3.71710	3.67308	3.62989	3.58753	3.54595
5	4.71346	4.64582	4.57971	4.51505	4.45182	4.38998	4.32948
10	8.98258	8.75206	8.53020	8.31660	8.11090	7.91272	7.72173
15	12.84926	12.38138	11.93793	11.51741	11.11839	10.73955	10.37966
20	16.35143	15.58916	14.87747	14.21240	13.59033	13.00794	12.4622
25	19.52346	18.42438	17.41315	16.48151	15.62208	14.82821	14.0939
30	22.39646	20.93029	19.60044	18.39204	17.29203	16.28889	15.3724
35	24.29862	23.14516	21.48722	20.00066	18.66461	17.46101	16.3741
40	27.35548	25.10277	23.11477	21.35507	19.79277	18.40158	17.1590
45	29.49016	26.83302	24.51871	22.49545	20.72004	19.15635	17.7740
50	31.42361	28.36231	25.72976	23.45562	21.48218	19.76201	18.2559

Annual Depreciation of a property is the necessary annual installment of the annu which will amount to the first cost of that property at the expiration of its useful l annual depreciation is also exprest in percentage of first cost. Thus, if the first c of a part of a plant is \$10 000 and its life is estimated at 40 years, then if money is wo 3%, the annual depreciation is 10 000 × \$0.01326, or \$132.60, and the rate of depre tion is 1.326%. CAPITALIZATION of a plant is the sum of the first cost C and the cap necessary to earn annually (a) the annual cost of operation O and (b) the annual de ciation D; that is, if i is the interest rate, capitalization equals $C + (O + D)/i$. Th if in the preceding illustration O = \$500, the capitalization equals \$10 000 + (\$50 \$132.06)/0.03 = \$31 068.

Annual Annuity for Various Periods which \$1 will Purchase

No. of Yrs.	Rates of compound interest						
	2%	2½%	3%	3½%	4%	4½%	5%
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.050
2	0.51505	0.51883	0.52261	0.52640	0.53020	0.53400	0.53
3	0.34675	0.35014	0.35353	0.35693	0.36035	0.36377	0.36
4	0.26262	0.26582	0.26903	0.27225	0.27549	0.27874	0.28
5	0.21216	0.21525	0.21835	0.22148	0.22463	0.22779	0.23
10	0.11133	0.11426	0.11723	0.12024	0.12329	0.12638	0.12
15	0.07783	0.08077	0.08377	0.08683	0.08994	0.09311	0.09
20	0.06116	0.06415	0.06722	0.07036	0.07358	0.07688	0.08
25	0.05122	0.05428	0.05743	0.06067	0.06401	0.06744	0.07
30	0.04465	0.04778	0.05102	0.05437	0.05783	0.06139	0.06
35	0.04000	0.04321	0.04654	0.05000	0.05358	0.05727	0.06
40	0.03656	0.03984	0.04326	0.04683	0.05052	0.05434	0.05
45	0.03391	0.03727	0.04079	0.04445	0.04826	0.05220	0.05
50	0.03182	0.03526	0.03887	0.04263	0.04655	0.05060	0.05

The formula for this table is $i/[1 - (1 + i)^{-n}]$.

ADVANCED MATHEMATICS

8. Differential Calculus

Definitions and Symbols. In a discussion of quantities, a constant is one regarded as having the same value thruout, and a variable is one supposed to take different values. When several variables are related, their values being interdependent, each is a function of the others; thus the area and the base of a triangle of constant altitude being definitely related, these quantities are functions of each other. Also any expression containing the symbol of a quantity is a function of that quantity; thus $ax^2 + bx - c$ is a function of x, and $ay^2 + bx$ is a function of x and y. The following abbreviations are used: $f(x)$, $F(x)$, $\phi(x)$, etc., for functions of x; $f(x, y)$, $F(x, y)$, $\phi(xy)$, etc., for functions of x and y, etc. The letters f, F, ϕ, etc., are functional symbols; and $f(x)$ and $f(y)$, or $F(x)$ and $F(y)$, denote the same functions of x and of y; thus if $f(x)$ denotes $x^3 + 6x - 7$, then $f(y)$ denotes $y^3 + 6y - 7$. When $y = f(x)$ as $y = x^3 + 4x$, the equation being solved for y, then y is an explicit function of x; but in $F(x, y) = 0$, as $xy + 4x^2 - y^2 = 0$, the equation not being solved for y, then y is an implicit function of x. If $y = f(x)$, and for each value of x there is only one value of y, then y is a single-valued function of x; if for each value of x there is more than one value of y, then y is a multiple-valued function.

If two variables x and y are related and x is regarded as taking on any values and then y its corresponding values, x is the independent and y the dependent variable. When any variable x changes from a value x_1 to a value x_2, the difference $x_2 - x_1$ is an increment of x; any increment of x is denoted by Δx, also δx. An increment of x may be positiv or negativ; when negativ, the numerical value of the increment is called decrement. A variable regarded as taking on equal increments is an equicrescent variable. If in $y = f(x)$ all equal changes in x produce equal changes in y, then y is a uniform variable with respect to x, and the graph of $y = f(x)$ is straight. The rate of y with respect to x, or the x-rate of y, is the change in y per unit change in x. If $x_2 - x_1$, or Δx (Fig. 20), is any change in x, and $y_2 - y_1$, or Δy, is the corresponding change in y, then the x-rate of y is $(y_2 - y_1)/(x_2 - x_1) = \Delta y/\Delta x$. If in $y = f(x)$ all equal changes in x do not produce equal changes in y, then y varies non-uniformly with respect to x, or at a variable rate; the graph of $y = f(x)$ is a curve. The average rate of y with respect to x, or average x-rate of y, for any change in x, is that constant rate which would give the actual change in y due to the change in x. For the change $x_2 - x_1$ in x (Fig. 21) this constant rate is $(y_2 - y_1)/(x_2 - x_1)$, or $\Delta y/\Delta x$, and is represented by the slope of the chord AB. The actual x-rate of y when $x = x_1$, say, is the value which the average rate $\Delta y/(x_2 - x_1)$ approaches as x_2 is taken closer and closer to x_1; this limiting average rate, or actual rate, is represented by the slope of the tangent at A. As $y = f(x)$, the x-rate of y is also the x-rate of $f(x)$.

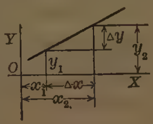


Fig. 20

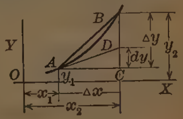


Fig. 21

The Derivative, the differential coefficient, and the derived function of y or $f(x)$, y being equal to $f(x)$, with respect to x are expressions which denote the x-rate of y or of $f(x)$; the first is the most common. There are several standard notations for this quantity: thus, D_xy or $D_xf(x)$, y_x' or $f_x'(x)$ (the subscript is generally omitted), and dy/dx or $d/dx f(x)$. The last two are the most common symbols and have the advantage of suggesting how the rate is obtained in the first instance. If the curve in Fig. 21 is the graph of $y = f(x)$, $dy/dx = \tan\alpha$ (when $x = x_1$), α being the slope angle at A. The derivative dy/dx is essentially one quantity, but may be regarded as a fraction provided that dy and dx

be regarded so that their ratio equals $\tan\alpha$; thus if dx is regarded as a fin increment of x, equal to Δx, then dy must be taken as CD, not equal to Δ So taken, dy is the DIFFERENTIAL of y with respect to x; it is an hypotheti increment of y, equal to the increment which occurs in y for a change in from x_1 to $x_1 + dx$ on the supposition that the x-rate of y remains consta during the change in x and is equal to its actual value when $x = x_1$. Th $dy/dx = \phi(x)$ may also be written $dy = \phi(x) \cdot dx$.

The fluxion of a variable function is its time rate. This term was used by Newton, is now uncommon; a fluxion of a variable as y was denoted by him thus, $\dot{y}$.

Differentiation is the process of finding the differential or derivative o function. The following formulas give differentials (and derivatives division) of some simple functions. Differentials of many other functi can be obtained by combining formulas; thus according to the formula $d(uv)$, $d(\sin x \cdot \cos x) = \sin x \cdot d(\cos x) + \cos x \cdot d(\sin x)$, and as gi $d(\cos x) = -\sin x\, dx$ and $d(\sin x) = \cos x\, dx$; hence $d(\sin x \cdot \cos x) = -$ si $dx + \cos^2 x\, dx$. In the formulas, a and n are constants, e is the base of Naperian system of logarithms, and in each case the differential of the fu tion is given with respect to x.

$$d(a + x) = dx \qquad d(ax) = a\,dx \qquad dx^n = nx^{n-1}\,dx$$
$$de^x = e^x dx \qquad da^x = a^x \log_e a\, dx \qquad d\log_e x = dx/x$$
$$d\sin x = \cos x\, dx \qquad d\csc x = -\csc x \cot x\, dx$$
$$d\cos x = -\sin x\, dx \qquad d\sec x = \sec x \tan x\, dx$$
$$d\tan x = \sec^2 x\, dx \qquad d\cot x = -\csc^2 x\, dx$$
$$d\operatorname{vers} x = \sin x\, dx \qquad d\operatorname{covers} x = -\cos x\, dx$$
$$d\sin^{-1} x = dx/\sqrt{1 - x^2} = -d\cos^{-1} x$$
$$d\tan^{-1} x = dx/(1 + x^2) = -d\cot^{-1} x$$
$$d\sec^{-1} x = dx/x\sqrt{x^2 - 1} = -d\csc^{-1} x$$
$$d\operatorname{vers}^{-1} x = dx/\sqrt{2x - x^2} = -d\operatorname{covers}^{-1} x$$
$$d\sinh x = \cosh x\, dx \qquad d\operatorname{csch} x = -\operatorname{csch} x \coth x\, dx$$
$$d\cosh x = \sinh x\, dx \qquad d\operatorname{sech} x = -\operatorname{sech} x \tanh x\, dx$$
$$d\tanh x = \operatorname{sech}^2 x\, dx \qquad d\coth x = -\operatorname{csch}^2 x\, dx$$
$$d\sinh^{-1} x = dx/\sqrt{x^2 + 1} \qquad d\operatorname{csch}^{-1} x = -dx/x\sqrt{1 + x^2}$$
$$d\cosh^{-1} x = dx/\sqrt{x^2 - 1} \qquad d\operatorname{sech}^{-1} x = -dx/x\sqrt{1 - x^2}$$
$$d\tanh^{-1} x = dx/(1 - x^2) \qquad d\coth^{-1} x = -dx/(x^2 - 1)$$

In the four following, u, v, w, etc., are functions of x and all differentia with respect to x:

$$d(u + v + w + \ldots) = du + dv + dw + \ldots \qquad d(uv) = u\,dv + v\,d$$
$$d(uvw \ldots) = [du/u + dv/v + dw/w + \ldots]\, uvw \ldots$$
$$d(u/v) = (v\,du - u\,dv)/v^2$$

Partial Derivatives and Differentials. The partial derivative of a func two or more independent variables with respect to one of them is the derivation function obtained on the supposition that the others are for the time being con the partial x-derivation of a function u is written $\partial u/\partial x$, also (du/dx); for ex if $u = y^3 + 4xy + x^2 + 2$, $\partial u/\partial x = 4y + 2x$ and $\partial u/\partial y = 3y^2 + 4x$. If u (or function of two independent variables, the partial derivatives have geometrical cance. The equation $z = f(x, y)$ represents a surface; suppose that APa, Fig. 2 intersection with a plane parallel to ZOX, and BPb its intersection with a plane to ZOY: on these curves y and x respectively are constant. PT' and PT'' are tan the curves at P as shown, and the slopes of these tangents represent $\partial z/\partial x$ and

espectively. The partial differential of a function of two or more independent variables vith respect to any one of them is the differential of the function obtained on the upposition that the other variables are, for he time being, constants. Thus if $z = y^3 + 4xy + x^2 + 2$, the x-partial differential of z s $4\,ydx + 2\,xdx$ and the y-partial differenial is $3\,y^2dy + 4\,xdy$. If z is a function of nly two independent variables the partial lifferentials have geometrical significance. f Pm and Pn are taken to represent the lifferentials dx and dy respectively, then mT' nd nT'' represents the x- and the y-partial lifferentials of z. Also these partial differenials are equal to $[\partial z/\partial x]\,dx$ and $[\partial z/\partial y]\,dy$.

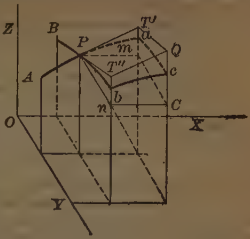


Fig. 22

Total Derivatives and Differentials. The total differential of a function of two or nore variables is the differential obtained n the supposition that all change. It equals he sum of the partial differentials of the unction with respect to the several variables; thus if $u = f(xyz)$,

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

f u is a function of only two variables, du has a geometrical significance. Thus if $z = (x, y)$ and Pm and Pn (Fig. 22) are taken as dx and dy respectively, then CQ represents z. The differential dz then is a hypothetical and not the actual increment, Cc in z ue to changes dx and dy in x and y. The total derivative of a function of two or nore variables all dependent on a single variable is the rate at which the function changes er unit change of that variable; thus if $u = f(x, y, z)$ and x, y, and z are all functions of t,

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

nd similarly for any number of variables. If the independent variable is x say, then

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx} + \frac{\partial u}{\partial z}\frac{dz}{dx}$$

Successive Differentiation is the process of finding derivatives or differentials f derivatives or differentials. The successive derivatives and differentials re called first, second, third, etc., derivatives and differentials, and of the first, econd, third, etc., order; those of the second, third, etc., order are "higher erivatives" and higher differentials. For the successive x-derivatives of $y = f(x)$ the following symbols are used:

first,	$Dy, y', f'(x)$, and dy/dx
second,	$D\,(Dy)$ or $D^2y, y'', f''(x)$, and $d/dx\;dy/dx$ or d^2y/dx^2
third,	$D^3y, y''', f'''(x)$, and d^3y/dx^3

he first x-differential of y is dy, the second d^2y, the third d^3y, etc.

For the successive partial derivatives of $u = f(xy)$ the following symbols are used:

first partial x-derivative $\partial u/\partial x$, first partial y-derivative $\partial u/\partial y$
second partial x-derivative $\partial^2u/\partial x^2$, second partial y-derivative $\partial^2u/\partial y^2$

The first partial x-derivative may be taken with respect to x and the second with respect y; this second is written $\partial^2u/\partial y\partial x$. Similarly $\partial^2u/\partial x\partial y$ means the x-partial derivative the y-partial derivative of u. But the order of differentiation is immaterial, that is, $\partial^2u/\partial y\partial x = \partial^2u/\partial x\partial y$, and this independence is true for any number of successive differntiations with respect to any number of variables.

Maxima and Minima. A maximum value of a function is one which is lgebraically greater than, and a minimum value is one algebraically less

than its adjacent values. Thus, if the curve in Fig. 23 is the graph of $y = f$ y_1, y_2, and y_3 are maximum values of y or $f(x)$ and y_4, y_5, and y_6 are minir values; again, imagine $u = f(x, y)$ graphed, x and y independent vari plotted on horizontal axes and u vertically, thus defining in general an u lating surface; then the valu u corresponding to high and points of the surface are maxi or minimum values of u or $f($ Maximum and minimum v are also known as turning va the values of the independent able or variables correspondi

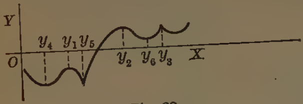


Fig. 23

turning values are critical values. The following formulas refer to ma and minima, like y_1 and y_4, where the tangent line is horizontal and not t like y_3 and y_5 (rare), where the tangent line is not horizontal. At maxim minima of the first kind the gradient dy/dx changes sign gradually, a the peaks the change of sign is sudden. From this property peak val a function can be obtained "by trial" with dy/dx.

(1) Turning value of $y = f(x)$: Solve $f'(x) = 0$; any root obtained, as x_1, is a value if the lowest x-derivative of y which does not vanish for $x = x_1$ is of an even the turning value, $f(x_1)$, is a maximum or minimum according as that lowest x-de is negativ or positiv for $x = x_1$.

(2) Turning value of y in $f(x, y) = 0$: Solve $\partial f(x, y)/\partial x = 0$ and $f(x, y) = 0$ taneously for x and y; the values obtained, x_1 and y_1, are critical and turning respectively if $\partial f(x, y)/\partial y$ does not vanish for $x = x_1$ and $y = y_1$; y_1 is maxir minimum according as $[-\partial^2 f(x, y)/\partial x^2] \div [\partial f(x, y)/\partial y]$ is negativ or positiv for and $y = y_1$.

(3) Turning values of $u = f(x, y)$: Solve $\partial f(x, y)/\partial x = 0$ and $\partial f(x, y)/\partial y =$ taneously for x and y; the values obtained, x_1 and y_1, are critical values if $[\partial^2 f(x,$ $[\partial^2 f(x, y)/\partial y^2] - [\partial^2 f(x, y)/\partial x \partial y]^2$ is positiv for $x = x_1$ and $y = y_1$;

$$\left.\begin{array}{l} u \text{ is a maximum} \\ u \text{ is a minimum} \end{array}\right\} \text{ if } \frac{\partial^2 f(x, y)}{\partial x^2} \text{ and } \frac{\partial^2 f(x, y)}{\partial y^2} \text{ are both } \left\{\begin{array}{l} \text{negativ} \\ \text{positiv} \end{array}\right.$$

Indeterminate Forms. In general a function of a variable has a value for any particular value of the variable; yet the values of som tions for special values of the variable lead to expressions like $0/0$, $0\cdot$ the like, which are so-called indeterminate forms. For example, the f $(1 - \cos x)/x$ for $x = 0$ becomes $0/0$ and $(\frac{1}{2}\pi - x)\tan x$ for $x = \pi/2$ is

(1) The quotient $f(x) \div F(x)$ becomes $0/0$ for a particular value of x as a: terminate form equals the value of $f'(x) \div F'(x)$ when $x = a$; if this ratio i indeterminate form, the original one equals $f''(x) \div F''(x)$ when $x = a$.

(2) The quotient $f(x) \div F(x)$ becomes ∞/∞ when $x = a$: the true value obtained as in case (1).

(3) The product $f(x) \times F(x)$ becomes $0 \cdot \infty$ when $x = a$: the product equal $[1/F(x)]$ which takes the form $0/0$ when $x = a$ and may be evaluated as in (1

(4) The difference $f(x) - F(x)$ becomes $\infty - \infty$ when $x = a$: the differen $\phi(x) \div \psi(x)$, case (1), where $\phi(x) = [1/F(x)] - [1/f(x)]$ and $\psi(x) = 1/[f($

(5) The function $[F(x)]^{f(x)}$ becomes 1^∞, 0^0, or ∞^0 when $x = a$: the functi $e^{f(x)\log F(x)}$ (the logarithm being Naperian), and the value of this form can $f(x) \log F(x)$ can be evaluated; but $f(x) \log F(x)$ falls under case (3).

Maclaurin's Theorem expresses the law for the expansion of a of a variable in a series of ascending powers of the variable, thus.

$$f(x) = f(0) + \frac{x}{1}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

wherein $f(0)$ means the value of $f(x)$ when $x = 0$ and $f'(0)$ is the $f'(x)$ when $x = 0$, etc.

As illustration, $\cos x$ is here expanded by Maclaurin's theorem: $f(x) = \cos x$, $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f''''(x) = \cos x$, etc.; $f(0) = 1$, $f'(0) = 0$, $f''(0) = -1$, $f'''(0) = 0$, $f''''(0) = 1$. Hence $\cos x = 1 - x^2/2! + x^4/4! - \ldots$

Taylor's Theorem expresses the law for the expansion of a function of a ariable plus an increment in a series of ascending powers of the increment; hus h, denoting an increment of x,

$$f(x+h) = f(x) + \frac{h}{1} f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \ldots$$

To expand $\cos(x+h)$ by Taylor's theorem: $f(x) = \cos x$ $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f''''(x) = \cos x$, etc., and $\cos(x+h) = \cos x\,(1 - h^2/2! + h^4/4! - \ldots) - \sin x\,(h - h^3/3! + h^5/5! - \ldots)$.

9. Integral Calculus

Definitions and Notation. Integration is the process of determining function from its derivative or differential; it is the inverse of differentiation. The function is called the integral of the derivative or differential; also anti-derivative and anti-differential. The symbol of integration is $\int$. Thus he x-derivative of $f(x)$ being written $f'(x)$, $\int f'(x) = f(x)$, but integration of he differential instead is generally indicated, as $\int f'(x)\,dx = f(x)$; dx indicates the 'variable of integration,' $f'(x)$ is called integrand, and the result of the integration, that is, the function determined $f(x)$, is the integral of the derivative or differential from which it is found. Strictly $\int f'(x)$ or $\int f'(x)\,dx = f(x) + C$, C being a constant, called constant of integration; $f(x)$ is called the indefinite integral and $f(x) + C$ the general integral. In a general integral the constant of integration is an arbitrary constant, but in a particular problem of integration it has a special value determinable from data of the problem. Thus, suppose in a given case it is known that when $x = a$, $\int f'(x) = A$, then $A = f(a) + C$ or $C = A - f(a)$; and $f(x) + A - f(a)$ is a particular ntegral.

Integral Forms. In the following formulas K is the constant of integration; a, b, c, A, B, m, and n are constants; logarithms indicated are Naperian, and ε is the base of that system (Art. 2); Y is an abbreviation for $a + bx$, Z for $a + bx + cx^2$; and u and v are any functions of x.

$$\int a\,du = a\int du \qquad \int (u+v)\,dx = \int u\,dx + \int v\,dx$$

$$\int u\,dv = uv - \int v\,du \qquad \int Y^n dx = \frac{Y^{n+1}}{(n+1)b} + K$$

$$\int \frac{dx}{Y} = \frac{1}{b}\log Y + K \qquad \int \frac{x\,dx}{Y} = \frac{1}{b^2}(Y - a\log Y) + K$$

$$\int \frac{x\,dx}{Y^2} = \frac{1}{b^2}\left(\log Y + \frac{a}{Y}\right) + K$$

$$\int \frac{x^2\,dx}{Y} = \frac{1}{b^3}\left(\tfrac{1}{2}Y^2 - 2aY + a^2\log Y\right) + K$$

$$\int \frac{x^2\,dx}{Y^2} = \frac{1}{b^3}\left(Y - 2a\log Y - \frac{a^2}{Y}\right) + K$$

$$\int \frac{dx}{xY} = -\frac{1}{a}\log\frac{Y}{x} + K \qquad \int \frac{dx}{xY^2} = \frac{1}{aY} - \frac{1}{a^2}\log\frac{Y}{x} + K$$

$$\int \frac{dx}{x^2Y} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{Y}{x} + K \qquad \int \sqrt{Y}\,dx = \frac{2}{3b}Y^{3/2} + K$$

$$\int x\sqrt{Y}\,dx = -\frac{2(2a-3bx)Y^{3/2}}{15b^2} + K$$

$$\int x^2\sqrt{Y}\,dx = \frac{2(8a^2-12abx+15b^2x^2)Y^{3/2}}{105b^3} + K$$

$$\int \frac{dx}{\sqrt{Y}} = \frac{2\sqrt{Y}}{b} + K \qquad \int \frac{x\,dx}{\sqrt{Y}} = -\frac{2(2a-bx)\sqrt{Y}}{3b^2} + K$$

$$\int \frac{x^2\,dx}{\sqrt{Y}} = \frac{2(8a^2-4abx+3b^2x^2)\sqrt{Y}}{15b^3} + K$$

$$\int \frac{dx}{x\sqrt{Y}} = \frac{1}{\sqrt{a}}\log\frac{\sqrt{Y}-\sqrt{a}}{\sqrt{Y}+\sqrt{a}} + K, \text{ if } a > 0$$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\frac{\sqrt{Y}}{\sqrt{-a}} + K, \text{ if } a < 0$$

$$\int \frac{\sqrt{Y}\,dx}{x} = 2\sqrt{Y} + a\int \frac{dx}{x\sqrt{Y}}$$

$$\int \frac{dx}{x^2\sqrt{Y}} = -\frac{\sqrt{Y}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{Y}}$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\sqrt{\frac{b}{a}}\,x + K, \text{ if } a > 0 \text{ and } b > 0$$

$$\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}}\log\frac{\sqrt{ab}+bx}{\sqrt{ab}-bx} + K, \text{ if } a > 0 \text{ and } b > 0$$

$$= \frac{1}{\sqrt{ab}}\tanh^{-1}\sqrt{\frac{b}{a}}\,x + K, \text{ if } a > 0 \text{ and } b > 0$$

$$\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+bx^2}$$

$$\int \frac{dx}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a+bx^2}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right) + K$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + K$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left(x + \sqrt{x^2 \pm a^2}\right) + K$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + K = -\cos^{-1}\frac{x}{a} + K$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x} + K$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \log \frac{a + \sqrt{a^2 \pm x^2}}{x} + K$$

$$\int \frac{\sqrt{a^2 \pm x^2}}{x}\,dx = \sqrt{a^2 \pm x^2} + a^2 \int \frac{dx}{x\sqrt{a^2 \pm x^2}}$$

$$\int \frac{\sqrt{x^2 - a^2}}{x}\,dx = \sqrt{x^2 - a^2} - a^2 \int \frac{dx}{x\sqrt{x^2 - a^2}}$$

$$\int \frac{x\,dx}{\sqrt{a^2 \pm x^2}} = \pm \sqrt{a^2 \pm x^2} + K \qquad \int \frac{x\,dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2} + K$$

$$\int \frac{dx}{Z} = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{b + 2cx}{\sqrt{4ac - b^2}} + K, \text{ if } 4ac - b^2 > 0$$

$$\int \frac{dx}{Z} = \frac{1}{\sqrt{b^2 - 4ac}} \log \frac{\sqrt{b^2 - 4ac} - b - 2cx}{\sqrt{b^2 - 4ac} + b + 2cx}, \text{ if } b^2 - 4ac > 0$$

$$= \frac{-2}{\sqrt{b^2 - 4ac}} \tanh^{-1} \frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K, \text{ if } b^2 - 4ac > 0$$

$$\int \frac{x\,dx}{Z} = \frac{1}{2c} \log Z - \frac{b}{2c} \int \frac{dx}{Z}$$

$$\int \frac{x^2 dx}{Z} = \frac{x}{c} - \frac{b}{2c^2} \log Z + \frac{b - 2ac}{2c^2} \int \frac{dx}{Z}$$

$$\int \frac{(A + Bx)\,dx}{Z} = A \int \frac{dx}{Z} + B \int \frac{x\,dx}{Z}$$

$$\int \frac{dx}{\sqrt{Z}} = \frac{1}{\sqrt{c}} \log\left(b + 2cx + 2\sqrt{c}\sqrt{Z}\right) + K, \text{ if } c > 0$$

$$= \frac{1}{\sqrt{c}} \sinh^{-1} \frac{b + 2cx}{\sqrt{4ac - b^2}} + K, \text{ if } c > 0 \text{ and } 4ac - b^2 > 0$$

$$= \frac{1}{\sqrt{c}} \cosh^{-1} \frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K, \text{ if } c > 0 \text{ and } b^2 - 4ac > 0$$

$$= \frac{-1}{\sqrt{-c}} \sin^{-1} \frac{b + 2cx}{\sqrt{b^2 - 4ac}} + K, \text{ if } c < 0$$

$$\int \frac{x\,dx}{\sqrt{Z}} = \frac{\sqrt{Z}}{c} - \frac{b}{2c} \int \frac{dx}{\sqrt{Z}}$$

$$\int \frac{(A + Bx)\,dx}{\sqrt{Z}} = A \int \frac{dx}{\sqrt{Z}} + B \int \frac{x\,dx}{\sqrt{Z}}$$

$$\int \frac{x^2 dx}{\sqrt{Z}} = \left(\frac{x}{2c} - \frac{3b}{4c^2}\right)\sqrt{Z} + \frac{3b^2 - 4ac}{8c^2} \int \frac{dx}{\sqrt{Z}}$$

$$\int \sqrt{Z}\,dx = \frac{b + 2cx}{4c}\sqrt{Z} + \frac{4ac - b^2}{8c} \int \frac{dx}{\sqrt{Z}}$$

$$\int x\sqrt{Z}\,dx = \frac{Z\sqrt{Z}}{3c} - \frac{b}{2c} \int \sqrt{Z}\,dx$$

$$\int x^2 \sqrt{Z}\,dx = \left(x - \frac{5b}{6c}\right)\frac{Z\sqrt{Z}}{4c} + \frac{5b^2 - 4ac}{16c^2} \int \sqrt{Z}\,dx$$

$$\int \sin x\,dx = -\cos x + K \qquad \int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + K$$

$$\int \sin^2 x\,dx = -\tfrac{1}{2}\cos x \sin x + \tfrac{1}{2}x + K$$

$$\int \sin^n x\,dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos x\,dx = \sin x + K \qquad \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + K$$

$$\int \cos^2 x\,dx = \tfrac{1}{2}\sin x \cos x + \tfrac{1}{2}x + K$$

$$\int \cos^n x\,dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \sin x \cos x\,dx = \tfrac{1}{2}\sin^2 x + K$$

$$\int \sin ax \cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + K$$

$$\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + K$$

$$\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + K$$

$$\int \cos^n x \sin x\,dx = -\frac{\cos^{n+1} x}{n+1} + K \qquad \int \sin^n x \cos x\,dx = \frac{\sin^{n+1} x}{n+1} + K$$

$$\int \sin^m x \cos^n x\,dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\,dx$$

$$= -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} \cos^n dx$$

$$\int x \sin x\,dx = \sin x - x\cos x + K$$

$$\int x^2 \sin x\,dx = 2x\sin x - (x^2 - 2)\cos x + K$$

$$\int x \cos x\,dx = \cos x + x\sin x + K$$

$$\int x^2 \cos x\,dx = 2x\cos x + (x^2 - 2)\sin x + K$$

$$\int \tan x\,dx = -\log\cos x + K \qquad \int \tan^2 x\,dx = \tan x - x + K$$

$$\int \cot x\,dx = \log\sin x + K \qquad \int \cot^2 x\,dx = -\cot x - x + K$$

$$\int \sec x\,dx = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + K \qquad \int \csc x\,dx = \log\tan\tfrac{1}{2}x + K$$

$$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1 - x^2} + K$$

$$\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1 - x^2} + K$$

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \tfrac{1}{2}\log(1 + x^2) + K$$

$$\int \cot^{-1} x\,dx = x\cot^{-1} x + \tfrac{1}{2}\log(1 + x^2) + K$$

$$\int \text{vers}^{-1} x\,dx = (x - 1)\,\text{vers}^{-1} x + \sqrt{2x - x^2} + K$$

$$\int a^x\,dx = \frac{a^x}{\log a} + K \qquad \int \log x\,dx = x\log x - x + K$$

$$\int (\log x)^n\,dx = x(\log x)^n - n\int (\log x)^{n-1}\,dx$$

$$\int \frac{(\log x)^n}{x}\,dx = \frac{1}{n+1}(\log x)^{n+1} + K$$

$$\int \frac{dx}{x(\log x)^n} = -\frac{1}{n-1}\,\frac{1}{(\log x)^{n-1}} + K$$

$$\int \varepsilon^{ax}\,dx = \frac{\varepsilon^{ax}}{a} + K \qquad \int x\varepsilon^{ax}\,dx = \frac{\varepsilon^{ax}}{a^2}(ax - 1) + K$$

$$\int x^n\varepsilon^{ax}\,dx = \frac{x^n\varepsilon^{ax}}{a} - \frac{n}{a}\int x^{n-1}\varepsilon^{ax}\,dx$$

$$\int \frac{\varepsilon^{ax}}{x^n}\,dx = -\frac{\varepsilon^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\varepsilon^{ax}}{x^{n-1}}\,dx$$

$$\int \varepsilon^{ax}\log x\,dx = \frac{\varepsilon^{ax}\log x}{a} - \frac{1}{a}\int \frac{\varepsilon^{ax}}{x}\,dx$$

$$\int \varepsilon^x \sin x\,dx = \frac{\varepsilon^x(\sin x - \cos x)}{2} + K$$

$$\int \varepsilon^x \cos x\,dx = \frac{\varepsilon^x(\sin x + \cos x)}{2} + K$$

Definite integrals result from integration between "limits"; thus $\int_a^b f'(x)\,dx = [f(x)]_a^b = f(b) - f(a)$; a and b are "lower" and "upper limits" respectively and $(b - a)$ is the "range" of integration. Interchanging the limits in a definite integral changes the sign of the result, and a definite integral can be exprest as the sum of several other integrals; thus

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \quad \text{and} \quad \int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Approximate Integration. An approximate value of $\int f(x)\,dx$ between any limits as x_0 and x_n can be obtained as follows: divide the range $x_n - x_0$ into an even number (n) of equal parts (w) and compute the values of $f(x)$ when $x = x_0, x_0 + w, x_0 + 2w, \ldots, x_n$, and call these values $y_0, y_1, y_2, \ldots, y_n$. Then substitute in an approximate formula for the area of an irregular figure (Art. 6), as Simpson's for example; the value obtained for A is an approximate value of the integral. If the values of x and y be plotted and a smooth curve be drawn thru the points obtained, then the area between the curve, the x axis, and the end ordinates y_0 and y_n represents the integral; and the area according to the proper scale in a given case equals the integral. If the area is cut by the x axis, the parts above and below should be regarded as positiv and negativ respectively.

10. Plane Analytic Geometry

Plane Coordinate Systems. For rectangular coordinate systems, se Art. 3. An oblique coordinate system is like the rectangular except that th coordinate axes are not at right angles, and in the oblique the x and y coord nates of a point are measured parallel to the x and y axes respectively. Re tangular and oblique systems are called Cartesian. Th POLAR COORDINATES of a point P (Fig. 24) are its di tance from some point or pole O and the angle betwe a reference line as OX and the line joining P and the line OP is the radius vector of P. Change or tra formation of coordinates from one system to anoth can be made from $x = r \cos \phi$, $y = r \sin \phi$, or $\sin \phi = y$ $\cos \phi = x/r$, $r = \sqrt{x^2 + y^2}$; the reference line and p of the polar system and the axes of the rectangular system must be related shown in the figure. For example, the equation of the circle (Fig. 25) w respect to axes OX and OY is $x^2 + y^2 = 2\,ay$, a being the radius; its po equation with OX as reference line and O as pole is $r^2 \cos^2 \phi + r^2 \sin^2 \phi$ $2\,ar \sin \phi$, or $r = 2\,a \sin \phi$.

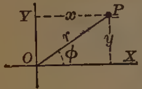


Fig. 24

Transformation from a set of rectangular axes to a parallel set: XOY (Fig. 26) is original set, UQV the new, and (x_1, y_1) the coordinates of the new origin Q with resp to the original set; then P being any point whose coordinates with respect to the two of axes are (x, y) and (u, v) respectively, $x = u + x_1$ and $y = v + y_1$.

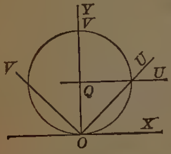


Fig. 25

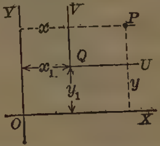


Fig. 26

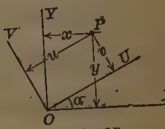


Fig. 27

For example, the equation of the circle (Fig. 25) with respect to the axes XOY b $x^2 + y^2 = 2\,ay$, its equation with respect to UQV is

$$(u + o)^2 + (v + a)^2 = 2\,a\,(v + a), \text{ or } u^2 + v^2 = a^2$$

Transformation from a set of rectangular axes to another set, they having a com origin but different directions: XOY (Fig. 27) is the original set, UOV the new, a denotes the angle through which OX must be turned to bring it into OU, a being p for counter-clockwise turning; then P being any point whose coordinates are as sh $x = u \cos a - v \sin a$ and $y = u \sin a + v \cos a$. For example, the equation o circle (Fig. 25) with respect to XOY being $x^2 + y^2 = 2\,ay$ its equation with resp UOV is $(u \cos 45° - v \sin 45°)^2 + (u \sin 45° + v \cos 45°)^2 = 2\,a\,(u \sin 45° + v \cos$ or $u^2 + v^2 = a\sqrt{2}\,(u + v)$.

Straight Line. When a straight line lies in a coordinate plane (Fig. by its slope-angle is meant the angle thru which XM must be turne bring it into the line, the angle being regarded as positiv or negativ accor as the turning is counter-clockwise or not, and by the slope or gradient o line is meant the tangent of its slope-angle. OM and ON are the "interce cut off by the line on the coordinate axes, and the intercepts are rega as positiv or negativ according as they lie on the positiv or negativ of the coordinate axes. The equation of a straight line is of the first de its general form being $Ax + By + C = o$; written in the form $y = mx$

m is the slope or gradient of the line, and b the intercept on the y axis; written in the form $x/a + y/b = 1$, a and b are the intercepts on the x and y axes respectively. The equation of a line whose slope is m and containing the point (x_1, y_1) is $y - y_1 = m(x - x_1)$; the equation of a line containing the points (x_1, y_1) and (x_2, y_2) is $y - y_1 = (x - x_1)(y_2 - y_1)/(x_2 - x_1)$; the equation of a line whose angle with the x axis is α and distance from the origin is p, is $y \cos \alpha - x \sin \alpha = p$. (See also Art. 8.)

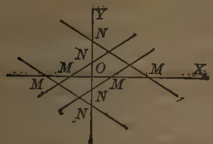


Fig. 28

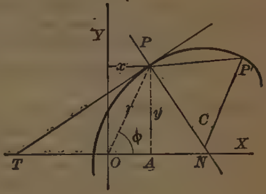


Fig. 29

Equation of a Curve. The equation of a curve referred to any coordinate reference frame is an equation between the coordinates of any and all points on the curve. When rectangular or oblique coordinates x and y are used, it is an equation between x and y; when polar coordinates r and ϕ are used, it is an equation between r and ϕ and is called the polar equation of the curve.

Tangents, Normals, Asymptotes. The TANGENT to a curve at P is the limiting position of a secant PP' (Fig. 29) as P' is taken nearer and nearer to P. The NORMAL at P is a perpendicular to the tangent there, and in the plane of the curve. Generally these are understood to be lines of indefinite length, but when they are referred to as of definite lengths, the parts between the points of tangency and the x axis are meant, PT and PN. The subtangent and the subnormal are the projections of the definite tangent and normal on the x axis, AT and AN respectively. If a curve extends to a point of which one (or both) of the coordinates is infinitely great, then the tangent at that point is an ASYMPTOTE of the curve. The equation of the tangent to a curve at a point P at (x_1, y_1) is $y - y_1 = (dy/dx)(x - x_1)$, and the equation of the normal at that point is $y - y_1 = -(dx/dy)(x - x_1)$. The definite tangent $PT = y_1 ds/dy$; the definite normal $PN = y_1 ds/dx$; the subtangent $AT = y_1 dx/dy$; and the subnormal $AN = y_1 dy/dx$.

Curvature. The total curvature of an arc PP' (Fig. 29) is the angle between the tangents to the curve at the points P and P'. The average curvature of the arc is the ratio of the curvature to the length of the arc; or $\Delta\alpha/\Delta s$, if $\Delta\alpha$ is the total curvature and Δs the length of arc. The curvature at a point P of the curve is the limiting value of the average curvature as P' approaches P, that is, $d\alpha/ds$; also curvature is given by

$$\frac{d\alpha}{ds} = \frac{d^2y}{dx^2} \Big/ \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

When for any arc the tangent lines are only slightly inclined to the x axis, then dy/dx is nearly zero for all points on the curve and the curvature is approximately equal to d^2y/dx^2.

PC and $P'C$ are normals to the curve at P and P'; as P' is taken nearer and nearer to P, the intersection of the normals moves along the normal

at P, approaching a definite point on that normal. This definite point is the center of curvature of the curve for the point P; the line joining the center and P is the radius of curvature of the curve at P, and a circle with that center and radius is the circle of curvature of the curve at P. If R denotes the radius of curvature, then

$$R = \frac{ds}{d\alpha} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \div \frac{d^2y}{dx^2}$$

and if x_1 and y_1 denote the coordinates of the center of curvature corresponding to any point (x, y) on a curve, then

$$x_1 = x - Rdy/ds \qquad \text{and} \qquad y_1 = y + Rdx/ds$$

and for R, dy/ds and dx/ds must be substituted, their values corresponding to the point (x, y).

Evolute and Involute. The line determined by the centers of curvature of a curve is the evolute of the curve, and the curve is an involute of the evolute. Thus C_1, C_2, and C_3 (Fig. 30) being centers of curvature for points on $P_1P_2P_3$, they lie on the evolute of $P_1P_2P_3$. The radii of curvature P_1C_1, P_2C_2 and P_3C_3 are tangent to the evolute; and the free end of a thread fastened at C_3, wound around the evolute and stretched into the position $C_3C_1P_1$, would, when unwound, describe $P_1P_2P_3$, an involute of $C_1C_2C_3$. The free end of a shorter cord, as $C_3C_1Q_1$, would, if unwound, describe $Q_1Q_2Q_3$, another involute of $C_1C_2C_3$. All points of the straight part of the thread describe parallel curves all of which are involutes of $C_1C_2C_3$, and $C_1C_2C_3$ is the evolute of all the parallel curves. The equation of the evolute of a given curve, $f(x, y) = 0$, can be obtained by eliminating x and y from $f(x, y) = 0$ and the equations for the coordinates of the center of curvature; then the final equation (free from x's and y's) is the equation sought, x_1 and y_1 being regarded as the variables in it.

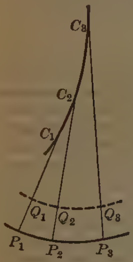


Fig. 30

Gradient, Convexity and Concavity. The slope-angle of a curve at any point of the curve is the angle which the tangent line to the curve makes with the positiv x axis; the gradient or slope of the curve there is the tangent of the slope-angle; it is given by the value of dy/dx for that point of the curve. This derivative is positiv or negativ according as the curve extends upward and to right or left (see Fig. 31 for all possible cases). The second derivative d^2y/dx^2 relates to the bending of the curve. It is positiv or negativ according as the curve is concave or convex upward (see Fig. 31 for all possible cases).

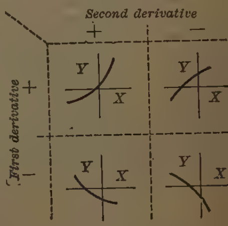


Fig. 31

Envelope. An equation of a curve contains one or more constants, called PARAMETERS; thus in $y = 4x + 2$, the equation of a straight line, 4 and 2 are parameters. The equation $y = mx + 2$ represents an infinitude of straight lines, one for each possible numerical value of m; a literal parameter, as m, regarded as capable of taking on different values is a variable parameter. The diagram representing an equation with a variable parameter is called a FAMILY OF CURVES, and the parameter is the parameter of the family. In general

members of the family intersect; the successive intersections of members lie on some line, and the line containing the successive intersections of all members of a family is the envelope of the family. In general, the envelope of a family touches each member. If $f(x, y, a) = 0$ is the equation of a family of curves, the equation of the envelope may be obtained by eliminating a between $f(x, y, a) = 0$ and $\partial f(x, y, a)/\partial a = 0$.

Singular Points. An inflection point of a curve is one where the curve crosses the tangent line at the point and bends away from the tangent in opposite directions on opposite sides of the point (Fig. 32). (The tangent is an inflectional tangent.) At an inflection point $d^2y/dx^2 = 0$, and to locate an inflection point on a curve (having one) whose equation is $y = f(x)$, get

Fig. 32

Fig. 33

Fig. 34

Fig. 35

$f''(x)$, that is, dy^2/dx^2; equate to zero and solve for x; from that value of x and the equation of the curve get y; in general (x, y) is an inflection point. To make sure that it is, determine whether $f''(x)$ changes sign at the point; if so, it is an inflection point. When two or more branches of a curve intersect, or cross one another (Fig. 33), the point of intersection is a multiple point; if two branches cross, the intersection is a node. At a multiple point, dy/dx has two or more real unequal values and y at least two equal values. A cusp is a point of a curve where two branches have a common tangent; if the two branches stop at the point the cusp is single (Fig. 34); if not it is double (Fig. 35).

11. Conic Sections

A Conic is a curve traced by a point moving in a plane so that the distance of the point from a fixt point is in a constant ratio to its distance from a fixt line, the point and line lying in the plane. The fixt point is the focus, the fixt line the directrix, and the constant ratio the eccentricity of the conic. If the directrix is taken as a y axis, a line perpendicular to it and passing thru the focus as the x axis, d to denote the distance between focus and directrix, and e the eccentricity, then the equation of the conic is $(x - d)^2 + y^2 = e^2x^2$. If

$e > 1$, the equation represents a hyperbola
$e = 1$, the equation represents a parabola
$e < 1$, the equation represents an ellipse
$e = 0$, the equation represents a circle

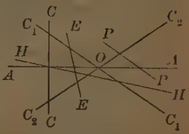


Fig. 36

Hence the circle is a special case of the ellipse, and the parabola is a limiting case for both hyperbola and ellipse.

The general second-degree equation between two coordinates x and y, $Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$, represents a conic. It is an ellipse (or circle), parabola, or hyperbola according as $AB - H^2$ is positiv, zero or negativ; but if $ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$, the conic is a "degenerate," it being a point, two intersecting straight lines, or two parallel straight lines in the three cases respectively.

A conic may be defined with reference to a double right cone with a circular base, whence the name conic. Let AA (Fig. 36) be the axis of such a cone, C_1C_1 and C_2C_2 elements of the surface in the plane of the paper, and let HH, PP, EE, and CC represent planes perpendicular to the paper; then HH, which cuts the elements on opposite sides of the vertex O, cuts a hyperbola from the surface; PP, which is parallel to an element

cuts off a parabola, EE which cuts the elements on the same side of O and is not perpendicular to the axis, cuts off an ellipse, and CC, perpendicular to the axis, cuts off a circle.
For mensuration of the conic sections, see Art. 6.

Circle. The equation of a circle of radius r is, if the center is at the origin of rectangular coordinates, $x^2 + y^2 = r^2$; if the center is at a point (a, b) the equation is $(x - a)^2 + (y - b)^2 = r^2$. Any equation of the form $Ax^2 + Ay^2 + 2Gx + 2Fy + C = 0$, A not being zero, is the equation of a circle.

An Ellipse is a curve such that the sum of the distances of any and every point on it from two fixt points is always the same (Fig. 37). The two fixt points are foci of the ellipse, and the distances of a point on the curve to the foci are focal distances, or focal radii, of the point. An ellipse has two lines of symmetry, called the axes of the ellipse; the longer is the major axis and the shorter the minor axis. The equation of an ellipse with respect to x and y axes coincident with the major and minor axes respectively, is $x^2/a^2 + y^2/b^2 = 1$, a and b denoting semi-major and semi-minor axis respectively. Any line thru the center, terminating in the ellipse, is a diameter. Two diameters are conjugate to each other when either is parallel to tangents to the ellipse at the extremities of the other. The acute angles α and β which two conjugate diameters make with the major axis are related thus: tan α tan $\beta = b^2/a^2$, and if A and B are semi-conjugate diameters, $A^2 + B^2 = a^2 + b^2$.

The definition of ellipse here given is in accordance with the one given in the present article under conic, but the focus and directrix there named are in case of the ellipse double; that is, there are two of each. As there, let e = eccentricity of the ellipse, d = distance from either focus to corresponding directrix, also c = distance from center to either focus; then $a = de/(1 - e^2)$, $b = de/\sqrt{1 - e^2}$, $c = de^2/(1 - e^2)$, $e = 1 - b^2/\ldots$, $c = ae$, $c^2 = a^2 - b^2$. The latus rectum is the length of the focal chord perpendicular to the major axis; denoting it by p, then $p = 2de = 2a(1 - e^2) = 2b^2/a$. The polar equation of the ellipse with F as pole and FA as polar axis (Fig. 37) is $r = p/2(1 - e\cos\phi) = a(1 - e^2)/(1 + e\cos\phi)$.

Geometric Constructions of Ellipse. (1) To determine the foci of any ellipse, axes given: from either end of the minor axis draw an arc whose radius equals the semi-major axis; the intersections of arc and major axis are the foci. (2) To draw a tangent and a normal to the ellipse at any point on it: from the point draw focal radii;

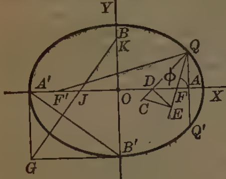


Fig. 37

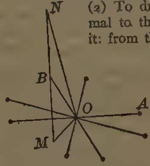


Fig. 38

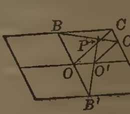


Fig. 39

the bisector of one of the angles between them is the tangent and that of the other angle is the normal. (3) To draw a tangent from a point P without the ellipse: join the point with either focus; on the joining line as diameter construct a circle; on the major axis as diameter construct a circle; the lines connecting the intersections of the two circles with P are tangents. (4) To find the center and the radius of curvature for any point of the ellipse: let Q (Fig. 37) be the point; at Q draw the normal and at its intersection with the major axis erect a perpendicular to the normal; at intersection of this perpendicular with either focal radius from Q erect a perpendicular to that radius; the intersection C of this last line with the normal is the center of curvature and QC is the radius of curvature. This construction fails for a point at either end of the major axis. For such points determine G, and thru G draw a perpendicular to $A'B'$; the intersection J is the center of curvature for A' and K for the point B'. (5) To determine the axes of an ellipse fr

pair of semi-diameters: from one end B (Fig. 38) of the shorter diameter draw a perpendicular to the longer and make BM and $BN = OA$; then the bisectors of the angles between the lines OM and ON are the directions of the axes; their lengths are $ON + OM$ and $ON - OM$. (6) Construction of an ellipse on its axes: draw auxiliary circles on the axes as diameters (Fig. 40); draw any diameter of these circles; thru its intersections b with the small circle draw lines parallel to the major axis; thru the intersections a with the other draw lines parallel to the minor axis; the intersections c of these lines are points on the ellipse. (7) Construction of an ellipse on a pair of coordinate diameters: construct a parallelogram on the diameters as medians (Fig. 39); divide OA and CA into proportional parts, beginning at O and at C, readily done by parallels to OC; join any point of division C' on AC with B and determine the intersection of BC' with the line joining B' and the corresponding division point O' on OA; this intersection P is on the ellipse.

Mechanical Constructions of an ellipse from its axes: (a) First locate the foci; then take an inelastic string and fasten it at the foci so that the length of the loop between the fast points equals that of the major axis; then if the string is pulled out taut into any position, as FPF' (Fig. 40), P is a point on the ellipse, and a moving pencil, or other scribe, pressing against the string at P will describe the ellipse. (b) On a strip of stiff paper or other suitable material, from a point Q (Fig. 40) lay off QM = the semi-major axis and QN = the semi-minor; a pencil, or other scribe, at Q will trace the ellipse when the strip is moved so that M moves along the minor axis and N along the major.

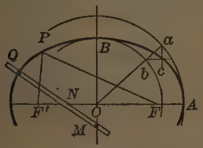


Fig. 40

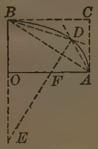


Fig. 41

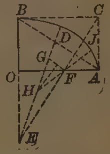


Fig. 42

Approximate Ellipse, consisting of arcs of circles; it is supposed that the axes of the ellipse are given. (1) Semi-ellipse with three centers: OA and OB (Fig. 41) are the given semi-axes; join A and B and bisect the angles CAB and CBA, thus determining D; thru D draw a perpendicular to AB, thus determining F and E; from F with radius FA and from E with radius EB draw arcs; they meet tangentially at D. (2) Semi-ellipse with five centers: OA and OB (Fig. 42) are the given semi-axes; join A and B, and thru C draw a perpendicular to AB, determining F and E, two of the centers; from E with EB as radius draw an arc BD as long as thought suitable, and join D with E; make $DG = AF$; join F and G; at the center of FG draw a perpendicular to FG and note its intersection H (the third center) with DE; from H with radius HD draw an arc to HF extended, and from F with FA as radius complete the curve.

The Hyperbola is a curve such that the difference between the distances of any and every point on the curve from two fixt points is always the same. The two fixt points are foci, the distances are focal distances, focal radiuses, or radius vectors. A hyperbola has two axes of symmetry (Fig. 43), one cut by the curve; the length AA' cut off by the curve is the transverse or real axis of the hyperbola, and the length BB' on the other line equal to $\sqrt{(FF')^2 - (AA')^2}$ is the conjugate or imaginary axis. The intersection of the axes is the center of the hyperbola, and any line thru the center terminating on the hyperbola is a diameter. The equation of the hyperbola referred to the transverse and conjugate axes as x and y coordinate axes is $x^2/a^2 - y^2/b^2 = 1$.

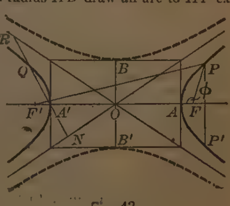


Fig. 43

a and b denoting semi-transverse and conjugate axes respectively. Two diameters are conjugate when each bisects all chords parallel to the other; either of two such diameters is parallel to the tangents at the extremities of the other. The acute angles α and β which two such diameters make with the transverse axis are related thus; $\tan\alpha \tan\beta = b^2/a^2$; if A and B denote semi-conjugate diameters, $A^2 - B^2 = a^2 - b^2$. The two hyperbolas $x^2/a^2 - y^2/b^2 = 1$ and $-x^2/a^2 + y^2/b^2 = 1$ are conjugate hyperbolas; the latter is shown by dotted lines in Fig. 43. They have the same asymptotes.

The definition of hyperbola given above is in agreement with the one given earlier in this article. As there, let e = eccentricity and d = distance between either focus and corresponding directrix; also let c = distance from center to either focus; then

$$a = de/(e^2 - 1) \qquad b = de/\sqrt{e^2 - 1} \qquad c = de^2/(e^2 - 1)$$
$$e^2 = 1 + b^2/a^2 \qquad c = ae \qquad c^2 = a^2 + b^2$$

The latus rectum of a hyperbola is the length of either focal chord perpendicular to the transverse axis; denoting it by p, then $p = 2\,de = 2\,a\,(e^2 - 1) = 2\,b^2/a$. The polar equation of the hyperbola with F as pole and FA as polar axis is

$$r = p/2\,(1 + e\cos\phi) = a\,(e^2 - 1)/(1 + e\cos\phi)$$

A hyperbola whose axes are equal is an equilateral or rectangular hyperbola; the asymptotes are at right angles. The equation of such a hyperbola referred to the axes is $x^2 - y^2 = a^2$, or $-x^2 + y^2 = a^2$; referred to the asymptotes, the equation is $xy = \frac{1}{2}a^2$. The eccentricity is $\sqrt{2}$.

Geometric Constructions of Hyperbola. (1) To determine the foci when the axes are given: make OF and OF' (Fig. 43) equal to AB; then F and F' are the foci. (2) To draw a tangent and a normal at any point of a hyperbola: from the point draw the focal radii; the bisector of one of the angles between the radii is the tangent and that of the other is the normal. (3) To construct the asymptotes to a hyperbola, the axes being given: construct a rectangle on the axes as medians; diagonals of the rectangle extended are the asymptotes. (4) Construction of a hyperbola, its axes AA' and BB' (Fig. 43) being given: (a) Locate the foci F and F', mark any point M on the transverse axis extended but not between the foci; with MA and MA' as radii, strike arcs from F and F' respectively; their intersections are points on the hyperbola. Repeat for other points like M. (b) Draw the asymptotes; through A' draw any oblique line and note its intersections N and R with the nearer and remoter asymptote; lay off from R toward A' a length $RQ = A'N$; then Q is on the hyperbola. Repeat with other oblique lines through A', A, or through any other known point of the curve, as Q.

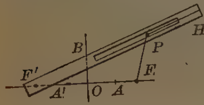


Fig. 44

Mechanical construction of a hyperbola from its axes AA' and BB': Determine the foci F and F' (Fig. 44); pivot one end of a strip of stiff paper or suitable material at F'; fasten an inextensible string HPF at H and F and so that the length HPF is less than HF' by AA'; place a pencil or other scribing point under the string and thru the slot; keep the string taut with the pencil and then rotate the strip about F'; the pencil describes an arc of a hyperbola.

Parabola. This is a curve such that any and every point of it is equidistant from a fixt point and a fixt line. The point is the focus, the line the directrix of the parabola, and the distance of any point of the curve to the focus is a focal radius, or focal distance. A parabola has a line of symmetry called the axis of the parabola (Fig. 45); it contains the focus F and is perpendicular to the directrix. The vertex is the intersection of the axis with the curve; the latus rectum is the length of the focal chord perpendicular to the axis.

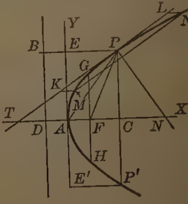


Fig. 45

The equations of the parabola referred to the axis as x axis and the vertex tangent as a y axis is $y^2 = px$ where p = latus rectum; the polar equation referred to F as pole and FA as polar axis is $r = p/2\,(1 + \cos\phi)$.

Geometric Properties and Constructions. The vertex is midway between the focus and the directrix. The length of the latus rectum equals four times the distance between focus and vertex. Any line parallel to the axis of a parabola bisects a system of parallel chords and is therefore a diameter of the parabola; the chords are parallel to the tangent at the end of the diameter terminating in the parabola. The extension of a diameter at P to the directrix equals the focal radius at P ($PB = PF$), and the distances from the focus to the ends of a definite tangent are equal ($FT = FP$). The tangent and the normal at any point bisect the angles between the focal radius and the diameter at that point; a subtangent is bisected by the vertex ($AC = AT$), and all subnormals equal one-half the latus rectum ($CN = ½ GH$). The projection of the radius of curvature R at any point on the axis of the parabola equals twice the focal distance of the point, and the extension of R to the directrix also equals twice the focal distance; R at the vertex equals the latus rectum. To construct a parabola: (a) Given the vertex A (Fig. 46), the axis AX, and a point P. Join A and P, and thru P draw a line parallel to the axis; draw any line as AQ; then perpendicular to the axis a line QR to AP; then parallel to the axis a line RS to AQ; S is a point on the parabola. Repeat for other points Q. Or, divide AB into equal parts and BP into the same number of equal parts; number the points of division, beginning at A on AB and at B on BP; draw lines as shown and note intersections of corresponding lines; they are on the parabola. (b) The vertex A and focus F (Fig. 47) are given. Join A and F and draw a perpendicular to AF at A; draw any line as FB and then BC perpendicular to FB; BC is a tangent to the parabola. Repeat with other lines like FB and BC and thus determine the parabola by means of tangents.

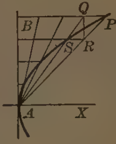


Fig. 46

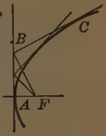


Fig. 47

12. Higher Curves

A Cycloid is the curve generated or traced by any point on the circumference of a circle which rolls on a straight line. Fig. 48 shows one half of one branch of a cycloid OA generated by the point P on the circle PQ rolled on OX; the other half of the branch is symmetrical with respect to AX. Let θ be the angle described by any line of the circle while it rolls from its original position OaY to any other as the one shown, r the radius of the circle, and x and y the coordinates of P relative to the axis shown; then $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$, and the equation of the cycloid is $x = r\cos^{-1}(1 - y/r) \pm\sqrt{(2r - y)\,y}$. A cycloid may be constructed as follows: Draw the circle OaY of the chosen radius r, and make $OX = \pi r$; divide the arc OaY and OX into the same number of equal parts; at two corresponding points of division, as a and b, draw a horizontal and a vertical respectively to their intersection c and make $cd = ae$; then d is a point on the cycloid. Repeat for other points of division. To find the position of the generating circle corresponding to any point of the cycloid as P: make $Pf = gh$, and from f drop a perpendicular to OX; its foot Q is the lowest point of the circle desired. The normal at any point of the cycloid, as P, passes thru the lowest point of the corresponding position of the generating circle; $PQ = 2r\sin ½\theta = \sqrt{2ry}$. The radius of curvature is twice PQ; at the highest point it equals $4r$ and at the lowest 0. The length of any arc as OP beginning at the lowest point is $4r\,(1 - \cos ½\theta)$ =

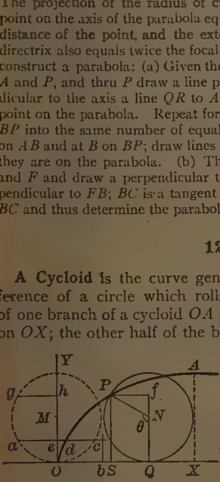


Fig. 48

$4r - 2\sqrt{2r(2r-y)}$, y being the ordinate of P; the length of one complete cycloid or 2 OPA is $8r$. The area of any part as $OPSO$ is $r^2(\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta) = \frac{3}{2}rx - \frac{1}{2}y\sqrt{(2r-y)y}$, x and y being the coordinates of P; the entire area between one cycloid and the track, that is 2 $OAXO$, is $3\pi r^2$.

When a circle rolls upon the outside of a fixt circle, each point of the circumference of the rolling circle describes or traces an epicycloid; and when it rolls upon the inside the curve described is a hypocycloid. A point without or within the rolling circle and fixt to it describes a trochoid, an epitrochoid, or a hypotrochoid according as the circle rolls on a straight line, on the outside of a fixt circle, or on the inside of a fixt circle.

The Spiral of Archimedes is a curve generated by a point moving at a constant speed in a given straight line which rotates at constant speed about a fixt point of the line. Fig. 49 shows such a spiral; O is the fixt point and OA the position of the line when the moving point is at the fixt point. The polar equation of the spiral is $r = (r'/2\pi)\phi$, in which r is the distance of the moving point P from O, ϕ the corresponding angle described by the moving line exprest in radians, and r' is the distance traveled by the moving point along the line while the line turns thru 360°. To construct the spiral having selected r': draw lines OA, OB, OC, etc., so that the successive angles between them is $360/n$, n being a convenient number; make $OB = r'/n$; $OC = 2r'/n$;

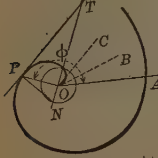


Fig. 49

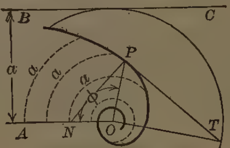


Fig. 50

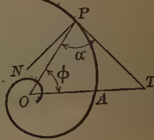


Fig. 51

$OD = 3r'/n$, etc.; B, C, D, etc., are on the spiral. To draw a normal and tangent at any point P: draw a circle with center at O and radius $r'/2\pi$; joi P and O, and draw a perpendicular to OP at O, and note the intersection N as shown; then PN is the normal and PT (perpendicular to PN) the tangen The length of any arc OP is $(r'/4\pi)[\phi\sqrt{1+\phi^2} + \sinh^{-1}\phi]$; for many turns equals approximately $r'\phi^2/4\pi$.

The Hyperbolic Spiral may be constructed as follows: draw a numb of concentric circles and then a radius of the largest circle; from this radiu measure off on the circles equal arcs all on the same side of the radius; th other ends of the arcs lie on the spiral (Fig. 50). The polar equation the spiral is $r\phi = a$, a being the constant length of arc referred to, r the distan of any point of the curve as P from O and ϕ the angle between the radius O and OP. The curve has an asymptote BC distant a from the referen radius. To draw a tangent and a normal at any point P: draw a circ with center at O and radius a, and join P and O; to OP at O draw a perpe dicular, and note its intersection T with the circle on the side shown; th TP is the tangent and PN (perpendicular to PT) is the normal.

The Logarithmic or Equiangular Spiral is a curve such that the ang between its tangents and the corresponding radii drawn to the center of t curve are equal. Its polar equation is $r = a\varepsilon^{m\phi}$; a is the value of r wh $\phi = 0$ and $m = \cot\alpha$, α being the constant angle referred to (Fig. 51). construct a curve for given values of a and α: compute m and then $m\phi$ a number of values of ϕ in radians, as 20° = 0.349 radian, 40° = 0.6

60° = 1.047, etc.; from a table of Naperian logarithms (page 35) find the values of $\epsilon^{m\phi}$ (these are the numbers corresponding to the logarithms $m\phi$); compute values of $a\epsilon^{m\phi}$ or r; then lay off these values of r from a fixt point on radii making the different angles ϕ (20°, 40°, 60°, etc.) with a reference line thru the fixt point. The length of any arc as $PO = r/\cos\alpha$.

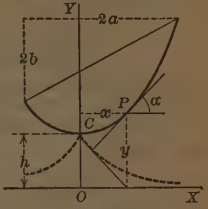


Fig. 52

The Catenary is the curve assumed by a chain suspended from two points. Its equation is $y = \frac{1}{2}h\ (\epsilon^{x/h} + \epsilon^{-x/h}) = h \cosh x/h$, or $x = h \log_\epsilon (y/h \pm \sqrt{(y/h)^2 - 1}) = h \cosh^{-1} y/h$; h is the distance from the lowest point of the curve to the origin (Fig. 52). The gradient at any point P whose coordinates are x and y is given by $\tan\alpha = \sinh x/h$ or $\cos\alpha = h/y$, from which the direction of the tangent line at any point can be computed. The radius of curvature R at any point P is $R = y^2/h = h/\cos^2\alpha$; it also equals the length cut off by the x axis from the normal thru P. The length of any arc, as CP, is given by $s = h \sinh x/h = h \tan\alpha = \sqrt{y^2 - h^2}$, and the abscissa of P is

$$x = h \log_e [s/h + \sqrt{1 + (s/h)^2}] = h \sinh^{-1} s/h$$

To locate the origin of coordinates for a chain of length $2l$, supported from two points whose horizontal distance is $2a$ and vertical distance is $2b$: Let A and B denote the distances from the middle of the line joining the points of suspension to the y and x axis respectively; $A = \psi h$ and $B = l \cot\phi$, in which $h = a/\phi$ and $\psi = \tanh^{-1} b/l$, ϕ to be obtained by trial from $a \sinh\phi = \phi\sqrt{l^2 - b^2}$. The catenary is the evolute of the other curve shown in Fig. 52; its tangents are all equal to h. It is also called "anti-friction" curve, it being the axial section of the surface of a vertical pivot of uniform wear.

13. Hyperbolic Functions

The Hyperbolic Functions are related to an equilateral or rectangular hyperbola much as the circular functions are related to a circle. Let P (Fig. 53) be any point (x, y) on the hyperbola $x^2 - y^2 = a^2$; r = the radius OP always positiv, l = arc AP positiv or negativ according as the arc is above or below the axis OX, and $u = l/r$; then

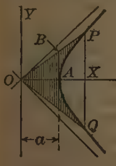


Fig. 53

hyperbolic sine of u, or $\sinh u = y/a$
hyperbolic cosine of u, or $\cosh u = x/a$
hyperbolic tangent of u, or $\tanh u = \sinh u \div \cosh u = y/x$
hyperbolic cotangent of u, or $\coth u = 1/\tanh u = x/y$
hyperbolic secant of u, or $\operatorname{sech} u = 1/\cosh u = a/x$
hyperbolic cosecant of u, or $\operatorname{csch} u = 1/\sinh u = a/y$

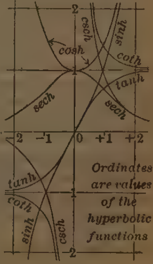


Fig. 54

If a is made 1, then the numerical values of double the area of the hyperbolic sector $OPAO$ equals l/r or u; also $\sinh u = XP$, $\cosh u = OX$; $\tanh u = AB$. See Sect. 1, Art. 22, for tables of hyperbolic functions. Fig. 54, plotted to scale, shows the relations between the functions for values of u from -2 to $+2$; it also indicates the sign of any function of u as dependent on the sign of u. Inverse hyperbolic

sine of n means the l/r or u whose hyperbolic sine equals n; it is written $\sinh^{-1} n$; similarly, inverse hyperbolic cosine, tangent, etc.

The hyperbolic functions of u are closely related to the exponential function ε^u and ε^{-u}, ε being the Naperian base, thus

$$\sinh u = \tfrac{1}{2}(\varepsilon^u - \varepsilon^{-u}), \quad \cosh u = \tfrac{1}{2}(\varepsilon^u + \varepsilon^{-u})$$
$$\tanh u = (\varepsilon^u - \varepsilon^{-u})/(\varepsilon^u + \varepsilon^{-u})$$
$$\sinh u + \cosh u = \varepsilon^u, \quad \cosh u - \sinh u = \varepsilon^{-u}$$

Inverse hyperbolic functions as functions of Naperian logarithms:

$$\sinh^{-1} u = \log_\varepsilon \left(u + \sqrt{u^2 + 1}\right), \quad \cosh^{-1} u = \log_\varepsilon \left(u + \sqrt{u^2 - 1}\right)$$
$$\tanh^{-1} u = \tfrac{1}{2}\log_\varepsilon [(1 + u)/(1 - u)]$$

The hyperbolic functions are related to circular functions, thus $\left(i = \sqrt{-1}\right)$:

$$i \sinh u = \sin iu, \quad \sin u = -i \sinh iu$$
$$\cosh u = \cos iu, \quad \cos u = \cosh iu$$
$$i \tanh u = \tan iu, \quad \tan u = -i \tanh iu$$

The following are some of the relations between hyperbolic functions:

$$\cosh^2 u - \sinh^2 u = 1 \qquad 1 - \tanh^2 u = \operatorname{sech}^2 u$$
$$\coth^2 u - 1 = \operatorname{csch}^2 u$$
$$\sinh(u \pm v) = \sinh u \cosh v \pm \cosh u \sinh v$$
$$\cosh(u \pm v) = \cosh u \cosh v \pm \sinh u \sinh v$$
$$\tanh(u \pm v) = (\tanh u \pm \tanh v)/(1 \pm \tanh u \tanh v)$$
$$\sinh 2u = 2 \sinh u \cosh u$$
$$\cosh 2u = 1 + 2\sinh^2 u = 2\cosh^2 u - 1$$
$$\tanh 2u = 2 \tanh u/(1 + \tanh^2 u)$$
$$\sinh \tfrac{1}{2} u = \sqrt{\tfrac{1}{2}(\cosh u - 1)}$$
$$\cosh \tfrac{1}{2} u = \sqrt{\tfrac{1}{2}(\cosh u + 1)}$$
$$\tanh \tfrac{1}{2} u = \sinh u/(1 + \cosh u) = (\cosh u - 1)/\sinh u$$

14. Probability of Errors

The Probability of an event is the ratio of the number of favorable chances of its occurrence to the total number of chances, favorable and unfavorable. Thus, if there are a white and b black balls in a jar, the probability of drawing a white ball at a single trial is $a/(a + b)$. If the probabilities of two independent events are p_1 and p_2, the probability of their concurrence in any single instance is p_1p_2. Thus, suppose that there are two jars, J_1 and J_2, J_1 containing a_1 white and b_1 black balls, and J_2 containing a_2 white and b_2 black balls; the probability of drawing a white ball from J_1 is $a_1/(a_1 + b_1)$, the probability of drawing a white one from J_2 is $a_2/(a_2 + b_2)$, and that of drawing a pair of white balls, one from each jar, in a single trial is $a_1a_2/(a_1 + b_1)(a_2 + b_2)$.

An Error of an Observation is the true value of a quantity minus the observed value. Errors are accidental or systematic; accidental errors are those which in the long run are as often negativ as positiv, and they affect the mean result but little; systematic errors due to the same cause affect the mean in the same sense, and do not tend to balance each other in the mean. Only accidental errors are referred to in the following. From the theory of probabilities, it has been shown that in a series comprising a great number of observations the relative frequency (proportionate number) of the errors whose values lie between x and $x + \Delta x$ is approximately $(h/\sqrt{\pi})\varepsilon^{-h^2x^2}\Delta x$ and the relative frequency of the errors whose values lie between a and b is $(h/\sqrt{\pi})\int_a^b \varepsilon^{-h^2x^2}dx$; here ε is the Naperian base (Art. 1) and h a constant for any particular series of observations. The graph of $y = (h/\sqrt{\pi})\varepsilon^{-h^2x^2}$ (Fig. 55) is a probability or "error curve"; the shaded area represents the relative frequency of the errors between a and b, and the whole area between

the curve and the x axis is unity, according to the scale used, and represents the frequency of the whole number of errors that is 1 or 100 %. The dotted curve is also an error curve but the constant h for that curve is greater than for the first; also in the second the relative frequency of small errors is greater and that of large errors is smaller than in the first. Thus the constants h serve as a means for comparing the accuracies of several series of observations, and the value of h for any given series is called the measure of precision for that series. Another index of accuracy is the so-called probable error of the series, an error of such value that the number of errors in the series less and greater than it are equal, the signs of the errors being disregarded in the count. It is the error r (Fig. 55) corresponding to the symmetrical ordinates which include one-half the whole area below the error curve. This middle error is more often called the PROBABLE ERROR of a single observation to convey the idea that the error of any subsequent observation in such a series is just as apt to be less as greater than the middle error. The probable error and the measure of precision are always related thus, $rh = 0.4769$. The expression for relative frequency of the errors falling between $-a$ and $+a$ is

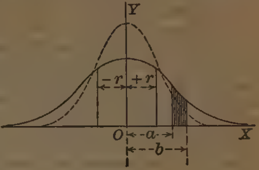


Fig. 55

$$\frac{h}{\sqrt{\pi}}\int_{-a}^{+a} \epsilon^{-h^2x^2}\,dx = \frac{2}{\sqrt{\pi}}\int_{0}^{0.4769\,a/r} \epsilon^{-h^2x^2}\,d\,(hx)$$

The table below gives the values of the expressions for the various values of the arguments a/r given therein. Thus, in a large series of observations, the relative frequency of the errors falling between $a = -r$ and $a = +r$ is .500, or 50 %; between $a = -2r$ and $a = +2r$, 0.823, or 82.3 %; the relative frequency of the errors arithmetically greater than $4r$ is 0.007, or 0.7 %; etc.

Values of the Probability Integral

a/r	frqncy	a/r	frqncy	a/r	frqncy	a/r	frqncy	a/r	frqncy
0.	0.000	1.0	0.500	2.0	0.823	3.0	0.957	4.0	0.993
0.1	0.054	1.1	0.542	2.1	0.843	3.1	0.963	4.1	0.994
0.2	0.107	1.2	0.582	2.2	0.862	3.2	0.969	4.2	0.995
0.3	0.160	1.3	0.619	2.3	0.879	3.3	0.974	4.3	0.996
0.4	0.213	1.4	0.655	2.4	0.895	3.4	0.978	4.4	0.997
0.5	0.264	1.5	0.688	2.5	0.908	3.5	0.982	4.5	0.998
0.6	0.314	1.6	0.719	2.6	0.921	3.6	0.985	4.6	0.998
0.7	0.363	1.7	0.748	2.7	0.931	3.7	0.987	4.7	0.998
0.8	0.411	1.8	0.775	2.8	0.941	3.8	0.990	4.8	0.999
0.9	0.456	1.9	0.800	2.9	0.950	3.9	0.991	4.9	0.999
1.0	0.500	2.0	0.823	3.0	0.957	4.0	0.993	5.0	0.999

The probable error r for n observations of equal precision on the same quantity is computed as follows: (1) find the arithmetic mean by adding the observations and dividing by n, (2) subtract each observation from that mean thus obtaining n residuals v_1, v_2, v_3, etc., (3) square each residual and find their sum Σv^2. Then

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}} \quad \text{and} \quad r_0 = \frac{r}{\sqrt{n}}$$

the first formula giving the probable error of a single observation, while the sec-

ond gives the probable error of the arithmetic mean. For a numerical example, see below.

14½. Method of Least Squares

An Observation is the result of a measurement, thus when 635.74 ft is stated as the measured length of a line, the quantity 635.74 ft is called an observation, it being understood that all constant errors have been removed therefrom; this quantity, however, is still affected by the result of accidental errors. The true value of a quantity cannot be found by measurement, but the best that can be done is to deduce from the observations the most probable value of the quantity. The Method of Least Squares teaches how to obtain the most probable values of observed quantities.

Direct Observations are those which result from measurements made directly upon a single quantity. When only one observation is at hand, it is the most probable value of the quantity. When two or more observations are made with equal precision on the same quantity, their arithmetic mean is the most probable value, and the probable error of this arithmetic mean can be computed by the formulas at the end of Art. 14.

Example: Let a line be measured eight times with equal care by a tape graduated in centimeters and the following results be found, M indicating an observation, v a residual found by subtracting M from the arithmetic mean, and v^2 the square of a residual:

M	789.7	788.1	789.1	789.9	788.3	788.0	788.1	788.8
v	0.95	0.65	0.35	1.15	0.45	0.75	0.65	0.051
v^2	0.902	0.423	0.122	1.323	0.203	0.562	0.423	0.002

Here, the arithmetic mean, found by adding the observations and dividing by 8, is 788.75, which is the most probable length of the line in centimeters. The eight residuals are some positive, some negative, their sum being zero. The sum of the squares of the residuals is $\Sigma v^2 = 3.96$. Then by the formula at foot of last page $r = 0.51$ cm, which is the probable error of a single observation; also $r_0 = 0.18$ cm, which is the probable error of the arithmetic mean. The final result of this series of observations may then be written 788.75 ± 0.18, that is, the true length of the line is just as likely to be between 788.57 cm and 788.93 cm as it is to be outside of those limits.

Weighted Observations. Weights of Observations are numbers proportional to their degrees of precision, so that one observation of weight p is worth as much as p observations of weight unity. When there are n weighted observations, M, with weight p_1, M_2 with weight p_2, and so on, these being made directly upon the same quantity, then the most probable value of the quantity is the weighted mean z, or

$$z = \frac{p_1 M_1 + p_2 M_2 + \ldots + p_n M_u}{p_1 + p_2 + \ldots + p_n} = \frac{\Sigma p M}{\Sigma p}$$

To find the probable errors, let each observation be subtracted from this mean z, the difference being a residual v, square each residual, multiply each square by its weight, and find the sum $\Sigma p v^2$, then

$$r = 0.6745 \sqrt{\frac{\Sigma p v^2}{n-1}} \quad \text{and} \quad r_0 = \frac{r}{\sqrt{\Sigma p}}$$

the first being the probable error of an observation of the weight unity and the second being the probable error of the weighted mean Z.

Example: Let six observations on the same quantity be made, with weights as in the first line, the sum of these weights being 21. Multiplying each observation M by its weight p, gives the quantities in the third line the sum of which is 3741.36. Then the most probable value of the observed quantity is $z = 3741.36/21 = 178.16$. Subtracting each

M from this gives the residuals in the fourth line. The weighted squares of the residuals are in the last line, their sum being $\Sigma pv^2 = 62.95$. Then the above formulas give the

p	5	4	1	4	3	4
M	178.26	176.30	181.06	177.95	176.20	180.85
pM	891.30	705.20	181.06	711.80	528.60	723.40
v	0.10	1.86	2.90	0.21	1.96	2.69
v^2	0.010	3.460	8.410	0.441	3.842	7.236
pv^2	0.05	13.84	8.41	0.18	11.53	28.94

probable error of an observation of the weight unity as $r = 2.39$ and the probable error of the weighted mean as $r_0 = 0.52$. The final result then is $z = 178.16 \pm 0.52$.

Observation Equations arise when measurements are made to find the values of several quantities. For example, let O be a given bench and Z_1, Z_2, Z_3 three points whose elevations above O are to be determined. Let z_1, z_2, z_3 be the most probable elevations of Z_1, Z_2, Z_3; let five lines of levels be run giving

$$
\begin{array}{llll}
Z_1 \text{ above } O = 10 \text{ feet} & \text{or} & z_1 & = 10 \\
Z_1 \text{ above } Z_3 = 2 \text{ feet} & \text{or} & z_1 - z_3 & = 2 \\
Z_2 \text{ above } O = 18 \text{ feet} & \text{or} & z_2 & = 18 \\
Z_2 \text{ above } Z_3 = 9 \text{ feet} & \text{or} & z_2 - z_3 & = 9 \\
Z_1 \text{ above } Z_3 = 2 \text{ feet} & \text{or} & -z_1 + z_2 & = 7
\end{array}
$$

The problem is to determine the most probable values of z_1, z_2, z_3.

Normal Equations are derived from observation equations, when these are of equal weight, by the following rule: To find a normal equation for z_1, multiply each observation equation by the coefficient of z_1 in that equation and add the results; similarly a normal equation for each of the other unknown quantities is found; thus, for the above level observations the normal equation for z_1 is found by multiplying the first equation by $+1$, the second by $+1$, the third by 0, the fourth by 0, and the fifth by -1; the addition of these gives $3z_1 - z_2 - z_3 = 5$ as the normal equation for z_1. Similarly the normal equation for z_2 is $-z_1 + 3z_2 - z_3 = 34$, and that for z_3 is $-z_1 - z_2 + 2z_3 = -11$. These three normal equations contain three unknown quantities, and their solution gives $z_1 = 10\frac{3}{8}$, $z_2 = 17\frac{5}{8}$, $z_3 = 8\frac{1}{2}$, which are the most probable elevations of the points Z_1, Z_2, Z_3 above the bench mark O.

General Method. Let the n observation equations be:

$$
\begin{array}{ll}
a_1 z_1 + b_1 z_2 + c_1 z_3 + \ldots = M_1 & \text{with weight } p_1 \\
a_2 z_1 + b_2 z_2 + c_2 z_3 + \ldots = M_2 & \text{with weight } p_2 \\
\cdots\cdots\cdots\cdots & \\
a_n z_1 + b_n z_2 + c_n z_3 + \ldots = M_n & \text{with weight } p_n
\end{array}
$$

in which M_1, M_2 . . . M_n are the observed quantities, z_1, z_2, z_3 are quantities required to be found, and a, b, c, are known coefficients. For each unknown quantity form a normal equation by multiplying each observation equation by the coefficient of that unknown quantity in that equation and also by its weight, and adding the results. If the number of unknown quantities is q the number of normal equations will also be q, thus:

$$
\begin{array}{l}
\Sigma pa^2.z_1 + \Sigma pab.z_2 + \Sigma pac.z_3 + \ldots = \Sigma paM \\
\Sigma pab.z_1 + \Sigma pb_2.z_2 + \Sigma pbc.z_3 + \ldots = \Sigma pbM \\
\Sigma pac.z_1 + \Sigma pbc.z_2 + \Sigma pc_2.z_3 + \ldots = \Sigma pcM
\end{array}
$$

in which $\Sigma pa^2 = p_1a_1^2 + p_2a_2^2 + \ldots p_na^2_n$ and similarly $\Sigma pab = p_1a_1b_1 + p_2a_2b_2 + \ldots p_na_nb_n$. The solution of these normal equations gives the most probable values of the unknown quantities z_1, z_2, z_3, . . . z_q.

After these probable values are found, let them be inserted in the observation

equations and the second members be subtracted from the first, thus giving small residuals $v_1, v_2, \ldots v_n$. Then

$$r = 0.6745\sqrt{\frac{\Sigma pv^2}{n-q}} \quad \text{and} \quad r' = \frac{r}{\sqrt{p'}}$$

the first being the probable error of an observation of weight unity, while the second is the probable error of an observation of weight p'.

For an example in which the weights are unequal, see the adjustment of angles at a station in Art. 49 of Sect. 2. When the weights are equal p is to be taken as unity.

Probable Error of a Line. When a line is measured the probable error of an observation increases as the square root of the length of the line. Thus $R = r\sqrt{l}$, where r is the probable error of a line one unit long, and R is the probable error of a line l units long.

The same rule holds good for discrepancies or apparent errors which are found in duplicate measurements of a line. The discrepancy r for a line one unit long may be found as follows: Let a line of length l_1 be measured twice. R_1 being the difference of the observed lengths; let a second length, l_2, be measured twice, R_2 being the difference of the observed lengths. Then $r = (R_1 - R_2)/(\sqrt{l_1} - \sqrt{l_2})$.

The same rule holds for duplicate lines of levels. Thus, if r is the difference in the results for a line one unit long, then the difference R for a line of length l ought to be $R = r\sqrt{l}$. The practical rules stated at foot of page 83 are derived from this principle.

STATICS

15. Forces and Moments

Force. An action of one body upon another which changes or tends to change the state of rest or motion of the body acted upon, is called force. A force has magnitude, direction, and place of application. When the extent of the place of application is negligible, and the force is regarded as applied or concentrated at a point, this is the point of application; and a line thru the point parallel to the direction of the force is the line of action. The word "sense" as applied to forces refers to one of the two directions along the line of action of the force. The unit of force commonly used in America is the "pound," which is a force equal to the earth's attraction on the standard of weight, also called pound; this unit of force varies slightly from place to place on the earth, but the variation is negligible in most engineering calculations. Any number of forces considered collectively is a system of forces; a system is CONCURRENT, or nonconcurrent, according as the lines of action of the forces do or do not intersect in a point, and it is COPLANAR, or noncoplanar, according as they do or do not lie in a plane. The resultant of a system of forces is the single force which is equivalent to that system, but if a system has no single force equivalent, then the simplest equivalent system may be called the resultant; a resultant never includes more than two forces. The process of determining the resultant is called composition (Art. 16).

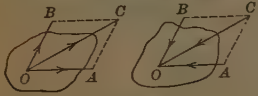


Fig. 56

Parallelogram Law. If magnitudes, lines of action, and senses of two concurrent forces acting on a rigid body be represented by OA and OB (Fig. 56), then the magnitude, line of action, and sense of their resultant is represented by the diagonal OC of the parallelogram $OABC$. The points of application of the forces may be anywhere on the body in the lines OA, OB, and OC, or their extensions. The arrowheads on the lines on AO, OB, and OC all point toward or all away from the poin

of concurrence O. TRIANGLE LAW: If the magnitudes and directions of two concurrent forces are represented by AB and BC, (Fig. 57 or 58) then the magnitude and direction of the resultant is represented by the side AC of the triangle ABC. The arrowheads on the sides AB and BC are confluent (point

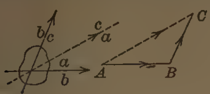


Fig. 57

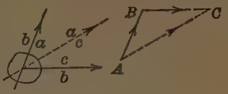


Fig. 58

the same way around), but the arrowhead on AC is not confluent with the others. The two forces and their resultant being concurrent, the line of action of the resultant is ac, parallel to AC.

The resultant of two concurrent forces may be determined algebraically, thus: let P and Q be the forces and α that angle between their lines of action in which the resultant lies, R the resultant and θ the angle between R and P; then $R = (P^2 + Q^2 + 2\,PQ \cos \alpha)^{1/2}$ and $\tan \theta = (Q \sin \alpha)/(P + Q \cos \alpha)$. If the angle between the given forces is 90°, then $R = \sqrt{P^2 + Q^2}$ and $\tan \theta = Q/P$.

Parallelopiped Law. If the magnitudes and lines of action of three noncoplanar concurrent forces be represented by OA, OB, and OC, then the magnitude and the line of action of the resultant is represented by the diagonal OD of the parallelopiped $OABC - D$ (Fig. 59). This is not a practical device to get numerical results; for a better, see Art. 16. When the three forces are mutually at right angles then the parallelopiped is right-angled, and the resultant can be conveniently determined algebraically thus: let P, Q, and S denote the three forces, R the resultant and α, β and γ the angles between R and the three forces respectively; then $R = (P^2 + Q^2 + S^2)^{1/2}$, $\cos \alpha = P/R$, $\cos \beta = Q/R$ and $\cos \gamma = S/R$.

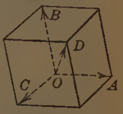


Fig. 59

Resolution of a Force. Two or more forces which together are equivalent to a given force are components of the force. The process of determining components of a force is called resolution; resolution of a force into two components at right angles to each is the common case; such components are "rectangular" and each is a "resolved part" of the given force. (1) Resolution into concurrent components. A force may be resolved into two concurrent components by applying the parallelogram law inversely; thus to resolve the 10 lbs (Fig. 60) into two components: lay off AB by scale to

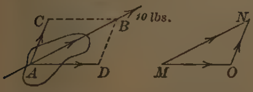


Fig. 60

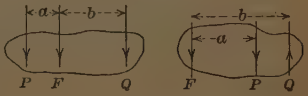
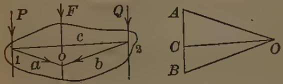


Fig. 61

represent 10 lbs; construct a parallelogram on AB as diagonal; then AC and AD represent components of the given force. Or, by means of the triangle law, thus: lay off MN by scale to represent the magnitude and direction of the given force; construct a triangle on MN as one side; then MO and ON represent magnitudes and directions of two components of the given force, the action lines being concurrent with that of the 10-lb force. Two rectangular components of a force can readily be found algebraically; each equals

the product of the force and the cosine of the acute angle between the force and that component. Three rectangular components of a force can be readily computed if the angles between the force and the desired components are known; each component equals the product of the force and the cosine of the acute angle between the force and that component. (2) Resolution into two components parallel to the force, their lines of action being specified: Let F (Fig. 61) be the force to be resolved into two components P and Q (values and senses unknown), a and b the distances from F to P and Q respectively, and c the distance between P and Q; the principle that the moment of F about a point on either force equals the moment of the other about the same point determines the senses of the components as shown, also their values, $P = Fb/c$ and $Q = Fa/c$. Or, graphically, suppose F (Fig. 62) to be



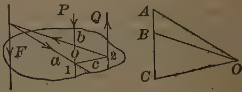


Fig. 62

the force and P and Q the components; resolve AB (representing magnitude and direction of F) into any two concurrent components AO and OB, action lines in ao and ob; at 1 resolve AO along P and the line 12, and at 2 resolve OB along Q and the line 12; the two components along 12 balance, and those along P and Q represented by AC and CB in value and direction are the desired components. (3) To resolve a force into three nonconcurrent nonparallel forces coplanar with the force: Let P (Fig. 63) be the force, the components to act in F_1, F_2 and F_3; join the intersection of P and F_1 with that of F_2 and F_3; draw AB to represent P and then resolve P into two

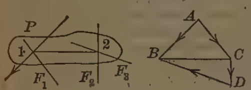


Fig. 63

components along F_1 and the line 12 (represented by AC and CB); then resolve CB into two components along F_2 and F_3 (represented by CD and DB); then AC, CD, and DB are the magnitudes and directions of the desired components.

The Moment of a Force with Respect to a Point is the product of the magnitude of the force and the perpendicular distance between the force and the point; this distance is the "arm" of the force with respect to the point and the point is the origin or center of moments. Moments are commonly given signs, those corresponding to forces which tend to turn the body acted upon in one direction (clockwise) being given the same sign and all others the opposite. The moment of a force with respect to a point may also be computed by taking the algebraic sum of the moments of its two rectangular components with respect to that point; and it will often be convenient to resolve the force so that one of the components will act thru the origin of moments, so that that component will have no moment.

The Moment of a Force with Respect to a Line is the product of its rectangular component perpendicular to the line (the other being parallel) and the distance between the line and the perpendicular component (or the force); the line is the "axis" of moments. If a force is parallel to the axis of moments or if it cuts the axis, then its moment with respect to that axis is zero. Moments of forces about the same axis are commonly given signs; those corresponding to the forces which tend to turn the body acted upon about the axis (regarded as a shaft) in the same direction (clockwise) being given the same sign and others the opposite. The moment of a force with respect to an axis may also be computed by resolving the force into three rectangular components, one being parallel

the axis, the other two perpendicular to it; then the moment of the given force equals the algebraic sum of the moments of the two perpendicular components. If the resolution is made so that one of the perpendicular components cuts the axis, then the moment of the given force equals the moment of the other perpendicular component.

The moment of the resultant of any coplanar forces about a point in their plane (or of any noncoplanar forces about a line) equals the algebraic sum of the moments of those forces about the point (or line). This is called the principle of moments for forces. The word TORQUE is frequently used as synonymous with moment of a force, especially when the force is distributed around a circumference.

Couples. Two equal, parallel, and opposite forces are called a couple; the perpendicular distance between the forces is the "arm" of the couple. The moment or torque of a couple with respect to any point or origin in their plane is the algebraic sum of the moments of the two forces with respect to that point; this sum or moment, the same for all origins in the plane, always equals the product of one of the forces and the arm of the couple. Moments of couples whose planes are parallel are sometimes given signs; those corresponding to couples which tend to turn the body in the same direction are given the same sign, and the others the opposite sign. A couple may be represented sufficiently for statical purposes by means of a single vector; the vector is drawn perpendicular to the plane of the couple, and the arrowhead is so placed that it points toward the place from which the rotation appears, say counter-clockwise. Two couples whose vectors are equal, same in length and direction, have equal moments (sign included) and their planes coincide or are parallel. Such are equivalent couples; that is, either may be substituted for the other without change of effect on the body acted upon if rigid. The resultant of a number of couples is a couple. If the planes of the given couples are parallel or coincident, the resultant couple is one whose plane is parallel to the others and whose moment (with sign) equals the algebraic sum of the moments of the given couples. If the planes of the given couples are not parallel or coincident, then the resultant can be determined from the vectors representing the different couples; thus, add the vectors, that is, find their resultant; this resultant vector represents the resultant couple. A couple can be resolved into component couples thus; resolve the vector of the given couple into component vectors (Art. 16) which are perpendicular to the planes of the desired components; these component vectors represent the several component couples.

16. Composition of Forces

The Force Polygon for a system of forces is the figure formed by drawing consecutively lines representing the magnitudes and directions of the forces of a system; the order in which the lines are drawn is immaterial, but the arrowheads on the lines (to indicate the senses of the forces) must be confluent, that is, pointing the same way around. A force polygon, unlike a geometrical one, need not be a closed figure. For a given system, as

Fig. 64

many different force polygons may be drawn as there are orders of taking the forces; if the number of forces is n, then the number of possible force polygons is $n!$ (see Art. 1). In Fig. 64 there are three different force polygons for the system acting upon the body shown at the left. If a system is not coplanar, then its force polygon is not plane, but is called "gauche."

Concurrent Systems. (1) Graphic Method: Draw a force polygon for the system; the magnitude of the resultant is represented by the length of the line joining the ends of the polygon; the sense of the resultant is represented by an arrowhead on that line not confluent with the other arrowheads, and the resultant is concurrent with the given forces. If the forces are noncoplanar, then this method is not practical but it can be used by drawing the force polygon in "plan and elevation." (2) Algebraic Method: If the forces F are coplanar, resolve each into components F_x and F_y along axes x and y at right angles to each other; get the algebraic sums of the x and y components, ΣF_x and ΣF_y; then the resultant being called R, and its angle with the x axis α, $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$ and $\tan \alpha = (\Sigma F_y)/(\Sigma F_x)$. The approximate direction of the resultant is apparent from the directions of its x and y components, which respectively equal ΣF_x and ΣF_y. If the forces F are noncoplanar, then each force should be resolved into components parallel to three rectangular axes x, y, and z. $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2$, $\cos \alpha = (\Sigma F_x)/R$, $\cos \beta = (\Sigma F_y)/R$, $\cos \gamma = (\Sigma F_z)/R$, α, β, and γ being the angles between R and the x, y, and z axes respectively.

Nonconcurrent Coplanar Systems. (1) Graphic Method: If the forces are not parallel or not nearly parallel, find the resultant R_1 of any two forces (Triangle Law, Art. 15), then the resultant R_2 of R_1 and the third force, etc., until all the forces have been compounded. If the forces are parallel or nearly so, the method just explained fails because the lines of action of the several resultants cannot be determined readily on account of inaccessible intersections. In this case, each force may be replaced by two components in a certain way, and then the resultant of these components can be found. Thus, to find the resultant of the three forces ab, bc, and cd acting on the body in Fig. 65: draw a force polygon, as $ABCD$, for the given forces; resolve AB into AO and OB; BC into BO and OC, etc.; O having been taken anywhere; all components except the first and last occur in pairs and the forces of each pair are equal and opposite, thus OB and BO, OC and CO, etc.; choose the action lines of these components so that those of any one pair shall be colinear; thus, insert the components of AB at pleasure as at oa and ob, the components of BC so that the component BO shall act in ob, and hence the component OC is oc, etc.; thus the pairs of components consisting of equal, colinear, and opposite forces each balance, leaving only the first and last components AO and OD acting in ao and od; and their resultant (which is also the resultant of the given forces) is AD (magnitude and direction) ad (action line). The point O (Fig. 65) is called pole, lines OA, OB, OC, etc., are rays; oa, ob, oc, etc., are strings, and all the strings constitute the string polygon for the forces. This polygon is also called "funicular," "link" and "equilibrium polygon," the last being especially appropriate when the given forces are in equilibrium. The part of the drawing which represents the body and the lines of action of the

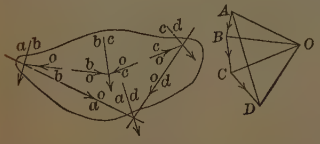


Fig. 65

forces is the space diagram; that representing force magnitudes, the vector diagram. If the force polygon closes, then the resultant is in general a couple, the forces of the couple acting in the first and last strings of the string polygon, the magnitude of the forces being represented by the corresponding rays. If it happens that the first and last strings coincide, then the string polygon is closed and the resultant vanishes.

(2) Algebraic Method. When the forces F are parallel, give to the forces in the one direction the same sign, and to the others the opposite; then the resultant R equals ΣF, the sense of R being indicated by the sign of ΣF. The position or line of action of R may be fixt by means of the arm of R, a, with respect to any origin O in the plane of the forces; thus if ΣM denotes the sum of the moments of the given forces with respect to O, then $a = (\Sigma M)/R$, a being measured in such direction from O that the sign of the moment of R will be the same as that of ΣM. If $\Sigma F = 0$, then the resultant of the system is a couple whose moment equals ΣM. When the parallel forces are two in number, P and Q (Fig. 66), then if P and Q act in the same direction, R cuts any line AB internally, and if P and Q are opposite, then externally on the side of the larger force; and in each case the segments of AB are inversely proportional to P and Q, that is, $AC/BC = Q/P$. When the forces are not parallel, compute the algebraic sums of the x and y components of the forces (ΣF_x and ΣF_y) and the algebraic sum of the moments of the forces with respect to any origin O in their plane (ΣM). Then the resultant $R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$, its angle with the x axis $= \tan^{-1}(\Sigma F_y)/(\Sigma F_x)$ and its arm with respect to O is $a = \Sigma M/R$. The general direction of R is apparent from the directions of its components ΣF_x and ΣF_y; a must be measured in such a direction from O that the sign of the moment of R will be the same as that of ΣM. If $\Sigma F_x = \Sigma F_y = 0$, the resultant is in general a couple whose moment $= \Sigma M$.

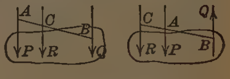


Fig. 66

Nonconcurrent Noncoplanar Systems. The graphic method is generally not advantageous; the algebraic is here given. (1) Forces Parallel. Give to the forces F acting in the same direction one sign and to the others the opposite sign; then the resultant $R = \Sigma F$, the sense of R being indicated by the sign of ΣF. Next compute the sums of the moments of the forces with respect to two axes (x and y, say) perpendicular to the forces; call these sums ΣM_x and ΣM_y and the arms of R with respect to those axes respectively a_x and a_y; then $a_x = (\Sigma M_x)/R$ and $a_y = (\Sigma M_y)/R$. The signs in these ratios may be disregarded; a_x and a_y have such positions that the moments of R with respect to the x and y axis have the same signs as those of ΣM_x and ΣM_y respectively. If $\Sigma F = 0$, the resultant in general is a couple which can be determined by finding the resultant of all the forces but one; the resultant and the omitted force constitute the resultant couple.

(2) Forces not Parallel. In general, the resultant is not a single force, but the system can be reduced to a force R acting thru any point of the body selected and a couple C; and if desired, R and C can in general be compounded into two nonparallel nonconcurrent forces. To determine R and C: select a set of coordinate axes (x, y, and z) in the body, the origin O being at the selected point referred to; determine the sums of the x, y, and z components of the given forces (ΣF_x, ΣF_y, and ΣF_z) and the algebraic sums of the moments of the forces with respect to the x, y, and z axes (ΣM_x, ΣM_y, and ΣM_z); ΣF_x, ΣF_y, and ΣF_z are the x, y, and z components of R, and ΣM_x, ΣM_y, and ΣM_z are the moments of the components of C perpendicular to the x, y, and z axes respectively.

$R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2$ and $C^2 = (\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2$; if α_1, α_2, and α_3 denote the angles between R and the x, y, and z axes, and θ_1, θ_2, and θ_3 the angles between the vector representing C (Art. 15) and the x, y, and z axes respectively, then

$$\cos\alpha_1 = (\Sigma F_x)/R \qquad \cos\alpha_2 = (\Sigma F_y)/R \qquad \cos\alpha_3 = (\Sigma F_z)/R$$
$$\cos\theta_1 = (\Sigma M_x)/C \qquad \cos\theta_2 = (\Sigma M_y)/C \qquad \cos\theta_3 = (\Sigma M_z)/C$$

The resultant force R and the resultant couple C can be compounded into two force as follows: take the plane of the couple so that one of the forces of the couple intersects R find the resultant of this force and R; this resultant and the other force of C are the tw forces sought. In general the final two forces are skewed. If the plane of C is parall to R, then C and R may be compounded into a single force as follows: take forces of th couple so that they are parallel to R; then find the resultant of those forces and R; this a single force.

17. Principles of Equilibrium

Conditions of Equilibrium. A force exerted on a body (definite portio of matter) by another body is an external force with reference to the fir body. A force exerted upon one part of a body by another part of the sam body is an internal force. All the external forces applied to a body at re constitute a system said to be in equilibrium. When a force system is i equilibrium, its resultant is zero; this is the general condition of equilibriur Detailed conditions for the various kinds of force systems follow, the notatio being: F denotes force, F_x, F_y, and F_z, x, y, and z components of F, M denot moment of F, M_a, M_b, and M_c moments of F with respect to points a, b, and M_x, M_y, and M_z moments of F with respect to x, y, and z axes respectivel (1) Colinear System: $\Sigma F = 0$ or $\Sigma M_a = 0$ (a is not to be taken on th forces). (2) Coplanar Concurrent System: $\Sigma F_x = 0$ and $\Sigma F_y = 0$; $\Sigma F_x = 0$ and $\Sigma M_a = 0$ (the x axis must not be perpendicular to the li joining a and the point of concurrence of the forces); or $\Sigma M_a = 0$ and $\Sigma M_b =$ (a, b, and the point of concurrence must not be colinear). For the ca of three forces: $F_1 : F_2 : F_3 :: \sin \alpha_1 : \sin \alpha_2 : \sin \alpha_3$; F_1, F_2, and F_3 deno the forces, α_1, α_2, and α_3 the acute angles between F_2 and F_3, F_3 and F_1, a F_1 and F_2 respectively. (3) Coplanar Parallel System: $\Sigma F = 0$ a $\Sigma M_a = 0$; or $\Sigma M_a = 0$ and $\Sigma M_b = 0$ (the line joining a and b must not parallel to the forces). (4) Coplanar Nonconcurrent Nonparall System: $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$; or $\Sigma F_x = 0$, $\Sigma M_a = 0$, and $\Sigma M_b =$ (the x axis must not be perpendicular to the line joining a and b); or ΣM $= 0$, $\Sigma M_b = 0$, and $\Sigma M_c = 0$ (a, b, and c must not be colinear). (5) No coplanar Concurrent System: $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$. (Noncoplanar Parallel System: $\Sigma F = 0$, $\Sigma M_x = 0$, and $\Sigma M_y = 0$ (x and y axes are not parallel to the forces or to each other). (7) Non planar Nonconcurrent Nonparallel System: $\Sigma F_x = 0$, $\Sigma F_y =$ $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$ and $\Sigma M_z = 0$.

Also, based on graphic methods, these conditions of equilibrium: For concurrent s tems, the force polygon closes; for coplanar nonconcurrent systems, the force and str polygons close. Special Principles, applicable in either algebraic or graphic analy (1) If three forces are in equilibrium, then they are coplanar, and concurrent or para (2) If four coplanar nonconcurrent nonparallel forces are in equilibrium, then the result of any two is concurrent with the other two.

Virtual Work. Any imaginary displacement of a body or system of bo is a virtual displacement. The work done by a force during a virtual placement of its point of application is called the virtual work of that fo Virtual works are computed according to the definitions and rules for c puting real works (see Art. 28). The quantity here called virtual worl also called "virtual moment." (1) If a rigid body is in equilibrium, then any infinitesimal virtual displacement the algebraic sum of the virtual w of the external forces equals zero. The work of a force for a displacen of its application point at right angles to the force is zero; and so in appl the principle of virtual work to determine a particular force of a system equilibrium, it is generally advantageous to take a virtual displacemen that the displacements of the application points of as many forces (particul unknowns but excepting the one in question) as possible shall be at r

angles to the corresponding forces. (2) If any system of particles (constituting a rigid body, a deformable body, or a collection of such bodies) is in equilibrium, then for any infinitesimal virtual displacement of the system the algebraic sum of the virtual works of all external and internal forces acting upon it equals zero. Internal forces occur in pairs, and the forces of any pair are equal, colinear, and opposite. Let S denote the magnitude of either force of a pair (regarded as positiv if they are pulls and negativ if pushes), and let ds denote the change in the distance between the application points of the forces for infinitesimal virtual displacement of the points (regarded as positiv or negativ according as the distance is increased or decreased); then in such a displacement the work of the pair is $-Sds$. For any infinitesimal displacement of a rigid body ds is zero for all pairs of internal forces, and the work of each pair (and of all pairs) is zero. In applying the principle to a collection of rigid bodies which press against one another or are connected as by hinges or strings, the equation of virtual work must in general include, besides the external forces, those internal forces which the bodies exert upon each other. But the virtual works of these forces may be zero; thus the virtual works of the pressures at a frictionless contact is zero for any virtual displacement which preserves the contact, and the virtual work of the binding forces of a string is zero for any virtual displacement which leaves the string taut and unchanged in length.

Stability. When a body (or collection of bodies) is in equilibrium and the state is such that if when displaced slightly in any way the body returns of itself to its original position, then the equilibrium is stable; if when displaced slightly the body moves farther from its original position, then the equilibrium is unstable; and if when displaced slightly it remains in that displaced position, the equilibrium is neutral, or indifferent. The body or collection is also said to be stable, unstable, or neutral (or indifferent) respectively. When a body or collection is stable, its potential energy is a minimum; when unstable, a maximum; and when neutral, constant (or stationary); the converse statements also are true. When the potential energy is gravitational, that is, due to weight, then when a body or collection is stable its center of gravity is in a lowest position; when unstable, in a highest position; and when neutral, at a uniform height, that is, it moves in a horizontal plane if the body or collection is slightly displaced. When a body rests on a number of points, the smallest polygon including all the points is called the supporting base or, simply, base. If the resultant of all forces acting on the body including its own weight but not the supporting forces, cuts the base, the equilibrium is stable, and the moment of the resultant about the side of the base nearest the resultant is a measure of the stability.

Properties of the Equilibrium or String Polygon. (See also Art. 16.) In the following the force-systems are not assumed to be in equilibrium except where so stated. (1) The line of action of the resultant of any number of coplanar forces which are represented consecutively in a force polygon passes thru the intersection of the two strings of the equilibrium polygon which are parallel to the two rays embracing those forces in the force polygon. Thus the resultant of AB, BC, and CD (Fig. 67), acting in ab, bc, and cd, acts thru the intersection of ao and od; the resultant of BC, CD, and DE acts thru the intersection of ob and oe; etc. (2) If a pole is taken

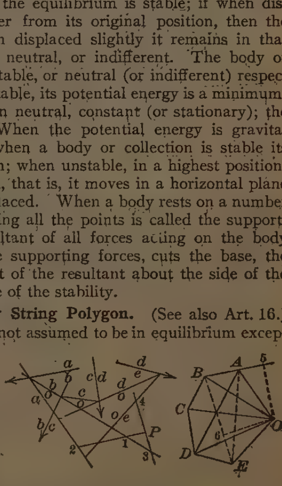


Fig. 67

at the beginning of a force polygon for a given force-system, then each strin of a corresponding equilibrium polygon is the action line of all the forces fron (and including) the first up to that string. Thus, the string od (Fig. 68) i the line of action of the forces AB, BC, and CD, actin in ab, bc, and cd; and ae is the line of action of AB, BC CD, and DE. (3) If two equilibrium polygons be draw for a given force-system from the same force polygon bu with different poles, then the intersections of correspond ing strings will lie on a straight line parallel to that joinin the poles. Thus in Fig. 69 there are represented force AB, BC, and CD, acting in ab, bc, and cd, and two equ librium polygons are shown corresponding to poles P an Q; corresponding strings intersect in points 1, 2, 3, and all being in a line parallel to PQ. (4) If an equilibriu polygon for a system of forces in equilibrium be regarded as a series of link jointed at the intersections of the segments of the polygon, by means of whic the forces react upon each other, the series would remain at rest under th action of the forces; each link would be under te sion or compression, and the ray corresponding any particular link represents the amount of th tension or compression. In Fig. 70 each link under compression.

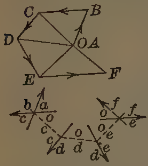


Fig. 68

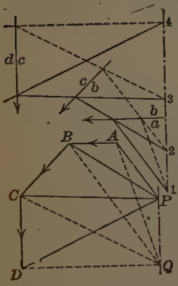


Fig. 69

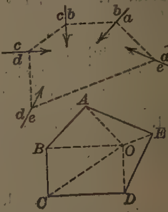


Fig. 70

An equilibrium polygon for a coplanar force-system furnishes a ready means of obtaining the moment of any one of the forces, and of the resultant of any of the forces consecutive in the force polygon. Thus the moment of any force with respect to any origin is the product of its "intersept" and "pole distance"; by intersept of a force is meant the distance (by the space scale) intercepted by the strings corresponding to the force from the line drawn parallel to the fo thru the origin of moments, and the pole distance of a force is the perpendicular dista (by the force scale) from the pole to the line representing the force in the force polyg Thus the moment of AB (Fig. 67) acting in ab, about P, is $\overline{12} \times \overline{O5}$, and the momen the resultant of BC, CD, and DE about P is $\overline{34} \times \overline{O6}$. Both moments are counterclockwise, determined from the lines of action and senses of forces as related to the origin of moments.

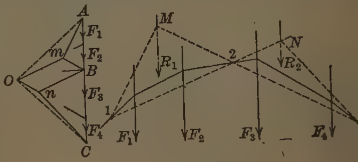


Fig. 71

An equilibrium polygon for a coplanar system of forces can be drawn so as to pass thru any three points of the plane of the forces. Thus, to draw one thru points 1, 2, and 3 for the forces F_1, F_2, F_3, F_4 (Fig. 71): find the resultants R_1 an of the forces whose lines of action are cut by the lines joining 1 and 2, and 2 and 3 first acts in R_1 and the second in R_2 (construction for position not shown), and values are represented by AB and BC respectively in the force polygon for the fo Extend lines R_1 and $\overline{32}$ to their intersection M, and lines R_2 and $\overline{12}$ to their intersectio

From A and B draw lines parallel to $\overline{M1}$ and $\overline{M2}$ to their intersection m, and from B and C draw lines parallel to $\overline{N2}$ and $\overline{N3}$ to their intersection n. Complete the parallelogram $BmnO$; then O is a pole and the corresponding equilibrium polygon, if started thru one of the three specified points, will pass thru the other two. (It is advisable to draw as first string the one thru point 2, parallel to OB.)

18. Typical Problems

In each of the problems following, some of the forces of the system in equilibrium are unknown in sense. In writing an equilibrium equation for the system, senses may be assumed when unknown; if the computed value of a force comes out positiv, the sense was guessed correctly; if negativ, then incorrectly. In the figure, incorrectly assumed senses are indicated by a short line thru the arrowhead.

A Coplanar Concurrent Force-System is in Equilibrium and the forces are all known except two whose action lines only are known; to determine these two completely. This is a common problem in the determination of the stresses of a roof or bridge truss, and the numerical illustration is from a truss, but the method of solution is as general as the statement of the problem.

(1) Algebraic Solution: Three sets of equilibrium equations are available (see Art. 17). No general rule covering all cases can be laid down as to which set is best in a particular case, but if a resolution equation ($\Sigma F_x = 0$ or $\Sigma F_y = 0$) is taken first, it is advantageous to take the resolution axis perpendicular to one of the unknown forces. If a moment equation ($\Sigma M = 0$) is taken first, it is advantageous to take the moment origin on the action line of one of the unknowns. Fig. 72 represents a joint of a truss under the action of a load of 1600 lb, a known pull of a member, 2000 lb, and two unknown forces F_1 and F_2; to find these two: Choosing $\Sigma F_x = 0$ and $\Sigma F_y = 0$, with the x axis horizontal, $\Sigma F_y = -1600 + F_1 \sin 30° = 0$, or $F_1 = +3200$ lb, the positiv sign indicating that F_1 acts as assumed in the figure. Next $\Sigma F_x = -2000 + 3200 \cos 30° + F_2 = 0$, or $F_2 = -771$ lb, the negativ sign indicating that F_2 acts toward the left. Or, beginning with $\Sigma M = 0$, the origin being on F_2, 10 feet to the right of O, say, then $\Sigma M = -1600 \times 10 + F_1 \times 10 \sin 30° = 0$, or $F_1 = 3200$; F_2 may now be determined as before or from another moment equation with origin anywhere except on F_2. When there are only three forces in the system, then a special condition of equilibrium (Art. 17) may be applied thus: suppose that the three forces are 1600 lb, F_1, and F_2 (Fig. 72); $F_1/\sin 90° = F_2/\sin 60° = 1600/\sin 30°$, which equations furnish values of F_1 and F_2.

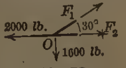


Fig. 72

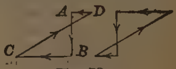


Fig. 73

(2) Graphic Solution: The condition of equilibrium is that the force polygon for the force system must close; constructing the polygon and making it close will determine the unknown forces. The order in which the forces are represented in the polygon is immaterial, but the knowns must be drawn first of course. To construct a force polygon for the forces: draw AB (Fig. 73) to represent the 1600-lb force, BC to represent the 2000-lb force; then from A and C lines parallel to the other two forces; the intersection of these two lines is D, and CD and DA represent the values and directions of the two unknowns. The unlettered polygon in Fig. 73 is another possible force polygon, giving the same results as the one explained.

A Coplanar Parallel Force-System is in Equilibrium and all the forces are known except two whose action lines only are known; to determine completely these two. The determination of the reactions on a beam or truss on horizontal supports and under vertical loads is a problem of this sort. Such a beam is used as an illustration, but the method of solution is as general as the statement of the problem.

(1) Algebraic Solution: Either one of two sets of equilibrium equations is available (Art. 17). It will be well to use the two moment equations with origins on the action lines of the two unknown forces. Then after the unknowns have been determined one might test, as a check, whether ΣF is zero. Fig. 74 represents a beam supported at R_1 and R_2, the beam bearing a concentrated load at the left end, a uniform load as shown, and its own weight, 1800 lb; to determine the reactions. With origin at R_2, the

moment equation is $-10\,000 \times 18 - 40\,000 \times 6 - 1800 \times 9 + R_1 \times 10 = 0$, or $R_1 =$ 43 620 lb; with origin at R_1, it is $-10\,000 \times 8 + 40.000 \times 4 + 1800 \times 1 - R_2 \times 10 = 0$ or $R_2 = 8180$ lb. As algebraic sum of loads and reactions is zero, the computation check

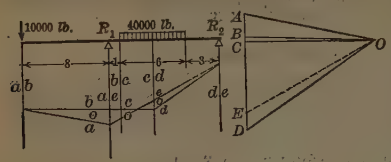


Fig. 74

(2) Graphic Solution. The conditions of equilibrium are that the force and string polygons close; if these be constructed and made to close, the problem will have been solved. Using the preceding illustration again, the 40 000-lb load is regarded as concentrated at its center and also the weight of the beam at the middle of the beam; first $ABCD$ the polygon for the known forces, is laid off; the right reaction will be taken next and called DE and the other EA, but E is as yet unknown; next a pole O chosen, rays are drawn and then the strings or oa, ob, oc, and od; od should be extended to de, and oa to ea. The line oe is the closing string, and a ray parallel to it fixes the point E. Then DE and EA represent the magnitudes of the reactions.

A Coplanar Nonconcurrent Nonparallel Force-System is in Equilibrium, and all the forces are known except two; the action line of one of these and a point in that of the other are known; required to determine the unknowns completely. This problem occurs in the determination of the reactions on a roof truss sustaining wind pressures, the truss being fixt at one end and resting on rollers at the other. This case is used in illustrations below but the solutions are as general as the statement of this problem.

(1) Algebraic Solution: Calling the first described unknown P and the second Q, imagine Q replaced by two rectangular components Q_x and Q_y acting at the given point of Q; then the unknowns of the system are the magnitudes and senses of P, Q_x, and Q_y. Any one of three sets of equilibrium equations may be used (Art. 17); generally it is advantageous to begin with a moment equation, the origin being at the known point of Q, as such an equation will furnish P directly. Q_x and Q_y can be determined from the other two equations of the set selected, and then Q itself from its components. Fig. 75 represents a roof truss whose span is 60 ft, rise 12 ft, resting on rollers at the left end and pinned to the support at the right; there are four loads as shown; required the reactions. The reaction at the roller end can be vertical only; that at the other end may have any direction. The first is P and the second Q, but Q is represented in the figure by its two unknown components Q_x and Q_y.

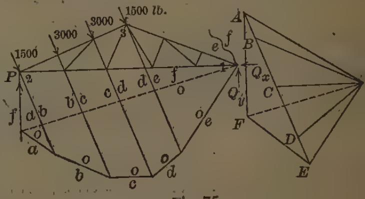


Fig. 75

The moment equation for the system with origin at 1 is $P \times 60 - 1500 \times 55.71 - 3000 \times 44.94 - 3000 \times 34.17 - 1500 \times 23.40 = 0$, or $P = 5933$. Then from $\Sigma M_2 = 0$, or $\Sigma F_y = 0$, Q_y may be found to be 2423, and from $\Sigma F_x = 0$, or $\Sigma M_3 = 0$, Q_x may be found to be 3342.

(2) Graphic Solution: The conditions of equilibrium are that the force and string polygons must close; constructing them and making them close will determine the unknowns. In order to make the construction possible, the first string drawn must be corresponding to the unknown force Q, one point of which is known, and it must be drawn thru that point. The following special graphic solution is simpler in principle. First determine the resultant R of the known forces, and imagine the knowns replaced by that resultant; then the system consists of three forces, namely that resultant and the two unknowns. If R and the unknown P whose action line is known are not parallel, t

the three forces R, P, and Q are concurrent and the action line of Q is determined. The solution of the three-force system can then be made as explained in the first paragraph. If R and P are parallel, then Q is also parallel to P and R; P and Q can be determined most readily algebraically, and graphically by constructing the force and funicular polygons for the three forces. (When R, P, and Q are parallel, this special graphical method is no simpler than the general method first described.) As illustration of the general method the reactions on the truss shown in Fig. 75 are determined thus: The force polygon for the known forces is $ABCDE$; calling the reaction at the fixt end ef and the other fa, the first string drawn is oe; then od, oc, ob, oa, and of the closing string. Next the ray parallel to of is drawn and its intersection with AF determines F; EF and FA represent the magnitudes and directions of the reaction at the fixt and roller ends respectively.

A coplanar noncurrent nonparallel force-system is in equilibrium and all the forces are known except three whose action lines only are known. Required to determine these three completely. (This problem is indeterminate if the three unknowns are concurrent or parallel.)

(1) Algebraic Solution: Any one of three sets of equilibrium equations may be used (Art. 15). In general it is advantageous to use a moment equation first, the origin being at the intersection of two of the unknowns; the choice of the other two equations will depend on the particular problem under consideration. For example, consider the overhanging truss (Fig. 76) which sustains three loads as shown and is supported at 1 so that the reaction there acts along the line marked R_1 and at 2 by two forces R_2 and R_3 which are horizontal and vertical respectively. Required R_1, R_2, and R_3. $\Sigma M_1 = -800 \times 20 - 1500 \times 10 + R_2 \times 10 = 0$, or $R_2 = 3100$ lb; $\Sigma M_3 = 1500 \times 10 + 1000 \times 20 - R_3 \times 20 = 0$, or $R_3 = 1750$ lb; and $\Sigma F_x = R_1 \cos a - 3100 = 0$, or $R_1 = 3466$ lb.

(2) Graphic Solution: The conditions of equilibrium are that the force and equilibrium polygons must close. In order to construct the equilibrium polygon, the first string must be drawn thru the intersection of two of the unknowns. Using the preceding illustration the force polygon for the three known forces is $ABCD$ (Fig. 76). The reaction at 1 is called de and the reactions at 2 are called ef and fa. The first string is oa

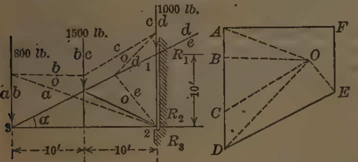


Fig. 76

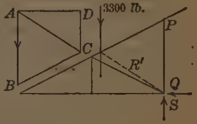


Fig. 77

and it is drawn thru the intersection of fa and ef; then ob, oc, and od are drawn, the latter to its intersection with de. The closing string is oe, and a ray parallel to oe determines E, in the line through D parallel to de. Then completing the force polygon by line through A and E parallel to R_2 and R_3 respectively determines F, and $FA = R_2$ and $EF = R_3$. The senses of R_1, R_2 and R_3, apparent in this example, are given by arrow heads on DE, EF and FA confluent with the senses of AB, BC and CD.

The following special method is simpler than the foregoing: Determine the resultant R of the known forces, and imagine them replaced by their resultant; then the system consists of R and the three unknowns P, Q, and S. Note that R, P, and the resultant R' of Q and S are concurrent, and by a force triangle determine P and R'; finally resolve R' into its components Q and S. For example, in the preceding illustration the resultant of the known forces is 3300 lb as shown in Fig. 77. The resultant R' of Q and S is concurrent with R and P, and acts in the dotted line. The force triangle for R, P, and R' is $ABCA$, AB representing 3300 lbs, BC representing P, and CA representing R'. Then resolving CA into Q and S, it is found that CD represents S and DA represents Q.

19. Shear and Moment in Beams

Beams and Trusses are usually subjected to vertical loads and reactions. In such cases, the VERTICAL SHEAR at any cross-section of the beam or truss is the algebraic sum of all the loads and reactions on either side of the section; if the shear is computed from the forces (loads and reactions) to the left of the section, then upward forces are given the positiv sign, but if from those on the right, then the upward forces are taken as negativ. The BENDING MOMENT at any cross-section of a beam or truss is the algebraic sum of the moments of all the loads and reactions on either side of the section, the origin of moments being taken in the section; if the bending moment is computed from the forces to the left of the section, clockwise moments are regarded as positiv, but if from those to the right, clockwise moments are taken as negativ. V and M are used to denote shear and moment respectively.

Fig. 78*a* represents a cantilever sustaining a concentrated load of 1000 lb at the free end and uniform load of 2000 lb on half its length as shown. Fig. 78*b* is a shear diagram for the cantilever as loaded, showing how the external shear varies from section to section; at a section just to the right of the concentrated load $V = -1000$, at the wall $V = -3000$ lb. Fig. 78*c* is a moment diagram for the cantilever as loaded, showing how the bending moment varies from section to section; at the middle $M = 5000$ and at the wall $M = 15\ 000$ ft lb. Fig. 79*a* represents a beam resting on two supports A and B and bearing a uniform load of 1000 lb per ft between the supports, and a concentrated load of 2500 lb at the right end; the reactions at A and B are respectively 4000 and 8500 lb. Fig. 79*b* is a shear diagram for the beam so loaded; just to the right of A, $V = +4000$ lb, just to the left of B, $V = -6000$ lb, and at any section to the right of B, $V = +2500$ lb. Fig. 79*c* is a moment diagram for the beam as loaded; at B, $M = -10\ 000$ ft-lb and the greatest positiv value of M is 8000 ft-lb, at the section 4 ft from the left end.

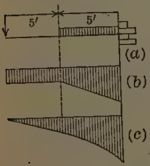


Fig. 78

The equilibrium polygon is a moment diagram for the beam as loaded, the vertical ordinates in it being proportional to the bending moments at the corresponding sections. The bending moment at any particular section is the product of the corresponding ordinate in the equilibrium polygon (according to the scale of the drawing of the beam) and the distance from the pole to the "load line" (according to the scale of the force polygon).

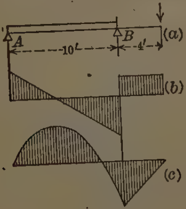


Fig. 79

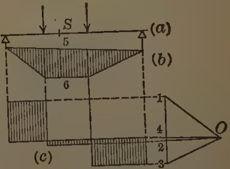


Fig. 80

For example, in Fig. 80, the beam represented is supported at each end and sustai two loads of values 12 and 23; the perimeter of the upper shaded part is an equilibriu polygon and constitutes a moment diagram for the beam as loaded. The bending m ment at any section as S equals the product of the distance represented by the ordina 5-6 and the force represented by the perpendicular from O to the line 1-2-3. The she diagram (Fig. 80*c*) can be constructed from the force polygon by obvious means.

For a beam bearing a distributed load an approximate moment diagram may be constructed thus: draw an equilibrium polygon for the beam under an approximately equivalent series of concentrated loads obtained by imagining the uniform load divided into parts and each part replaced by a concentrated load equal to that part and applied at its center. See Fig. 81, in which the distributed load is divided into three parts. The true bending moment line is curved below the distributed load, and the curve is tangent to the equilibrium polygon at points immediately below the lines of division of the load.

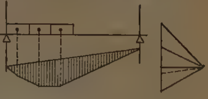


Fig. 81

Properties of Shear and Moment Diagrams. For any part of a beam bearing no load and weight of beam being disregarded, the shear line is horizontal and the moment line is generally inclined; for any part bearing a uniformly distributed load, the shear line is inclined downward to the right, and the moment line is a parabola, convex upward and axis vertical. The shears on either side of a concentrated load differ by an amount equal to the load, and the moment line changes direction there suddenly. At each end of a beam the shear and moment are zero. Where the shear changes sign, there the moment has a maximum or minimum value.

20. Simple Frameworks

A Truss is a framework intended to carry loads, while each member of the truss is subjected only to longitudinal stress, either tensile or compressive. In this article it is assumed, except as otherwise noted, that (a) the truss under consideration is pin-jointed, that is, the members have "eyes" and are pinned together at the joints, (b) each member is continuous between two joints only, and (c) the loads and reactions are applied to the truss at the joints. When these assumptions are fulfilled the forces acting upon any member, consisting of loads, reactions or pin pressures, are applied at its ends only; and the resultants, R_1 and R_2, of the forces at each end act thru both ends, that is, R_1 and R_2 are colinear, and they are equal and opposite. When they are pulls, the member is in tension, and any two parts exert pulls upon each other (Fig. 82); when they are pushes, the member is in compression, and any two parts exert pushes upon each other (Fig. 82). These internal pulls and pushes are each equal to the end pulls or pushes and colinear with them. By force or stress in a member is meant either of the forces which either of the two parts of the member exerts on the other part; the magnitude of a stress is the magnitude of one of the two forces referred to. By analysis of a truss for certain loads is meant the determination of the stresses in its members due to these loads.

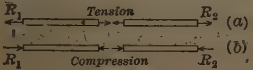


Fig. 82

The assumptions stated above are not realized in all actual trusses; yet if either (a) or (b) is not realized but the joints are properly made, then the methods of analysis here explained, or their equivalent, are used and without practical error; if (c) is not realized, the methods here given require amplification only.

To Determine the Stress in any particular member of a truss due to certain loads: First, determine the reactions on the truss due to the loads; second, imagine the truss separated into two distinct parts (that is pass a section thru the truss) so that the member under consideration is one of the members cut and so that the system of forces, including stresses, acting on either part of the truss is solvable for the desired stress; third, solve the system. (For plane trusses, the system will be coplanar and concurrent, or

nonconcurrent; the first kind can be solved completely if it includes not more than two unknown stresses, and the second if not more than three except when these three are concurrent or parallel.)

To illustrate how to pass the section, suppose the stress in HI (Fig. 83) is required, the truss being supported at its ends and bearing five loads L and one P, and suppose the reactions determined. Trying section 1-1, the force system on the left part of the truss (Fig. 83b) is a nonconcurrent one of seven forces, and includes four unknown stresses, S_1, S_2, S_3, and S_4; it is not solvable for the desired stress S_1. Trying section 2-2, the force system on the lower part (Fig. 83c) is a concurrent one, and includes four unknown

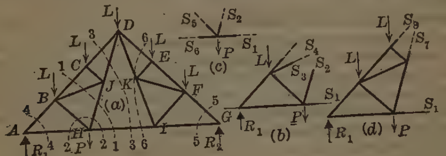


Fig. 83

stresses, S_1, S_2, S_5, and S_6; it is not solvable. Trying section 3-3, the force system on the left part (Fig. 83d) is nonconcurrent with three unknown stresses, S_1, S_7, and S_8; it is solvable. In some instances different sections may be used, each leading to a solution.

An Algebraic Analysis of a Truss is carried out by solving the various force-systems resulting from sections past, as explained in the foregoing, by algebraic methods. The problems to be solved are generally like one of Art. 18, where methods for their solution are given. In the following the unknown stresses will be assumed to be pulls always; then positiv computed stresses will be tension and negativ ones will be compression. In a truss with horizontal chords, the stress in any diagonal equals the vertical shear in the panel in which the member is, multiplied by the secant of the angle which the member makes with the vertical. The stress in either chord member of any panel equals the bending moment at the point where the other chord member and the strest web member of that panel intersect, divided by the height of the truss.

(1) **Roof Truss.** In Fig. 83 angles A and G are 45°, $AH = HI = IG =$ 16 ft; AB, BC, CD, DE, EF, and FG are equal, also HJ, JD, IK, and KD. Loads $L = 800$ lb, load $P = 1200$ lb. The reaction R_1 is found thus: $R_1 \times 48 - 4000 \times 24 - 1200 \times 32 = 0$, or $R_1 = 2800$ lb; also $R_2 = 2400$ lb.

To determine the stresses: Passing section 4-4, the force system on the part within the section (Fig. 84a) is concurrent with two unknowns S_1 and S_2. Taking a vertical y axis, $\Sigma F_y = S_2 \sin 45° + 2800 = 0$, or $S_2 = -3960$ lb, the negativ sign indicating that S_2 is compressive; next with the corrected direction of S_2, $\Sigma F_x = +S_1 - 3960 \cos 45° = 0$, or $S_1 = +2800$ lb, the positiv sign indicating that S_1 is tensile. In a similar manner the stresses in GF and GI might be determined; they are respectively 3394 lb compression and 2400 lb tension. No other section than 4-4 or 5-5 can be past so that the force-system acting on either part of the truss will be concurrent including only two unknown stresses; in fact the only sections leading to solvable force-systems are 3-3 and 6-6. On the left of 3-3 (Fig. 84b) the force-system is nonconcurrent with three unknown: S_3, S_1, and S_5. To determine S_3: $\Sigma M_D = -2800 \times 24 + 1200 \times 8 + 800 \times 8 + 80$ $\times 16 + S_3 \times 24 = 0$, or $S_3 = +1600$ lb tension. One might now solve the system for S_4 and S_5 or pass a section around joint H or I (Fig. 83), each furnishing a solvable con current system. But continuing with Fig. 84b, $\Sigma M_H = -2800 \times 16 + 800 \times 8 - S$ $\times BH = 0$, or $S_5 = -3394$ lb compression; and $\Sigma M_A = 0$ gives $S_4 = +2530$ lb ten sion. Next a section may be past about H or I as stated, or about C; section about H gives a concurrent system (Fig. 84c) with two unknowns, S_6 and S_7. Taking an x axi

at right angles to S_7, $\Sigma F_x = -1200 \cos 45° - 2800 \cos 45° + 1600 \cos 45° + S_6 \cos (71° 34' - 45°) = 0$, or $S_6 = +1897$ lb tension; and taking a vertical y axis, $\Sigma F_y = -1200 + S_7 \cos 45° + 1897 \sin 71° 34' = 0$, or $S_7 = -847$ lb compression. Next passing a section about C, the force-system is concurrent (Fig. 84d) with two unknowns S_8 and S_9. Two resolution equations, the axes being taken along the unknown stresses are simple; they give $S_8 = -556$ lb compression, and $S_9 = -3960$ lb compression. Next passing a section about B gives a concurrent system with one unknown (Fig. 84e) S_{10}. Or, one might pass next a section about J and get a solvable system (Fig. 84f). In similar manner the stresses in the members of the right half of the truss may be determined. When, in the analysis, a force system is reached in which there are fewer unknown stresses than the number of conditions of equilibrium for the system, as in Figs. 82e and f, then a partial check on the preceding computations may be made, thus: determine the unknown stress or stresses and then test whether the force-system satisfies the superfluous or extra equation or equations of equilibrium. Thus for Fig. 84e with the x axis along the two equal stresses,

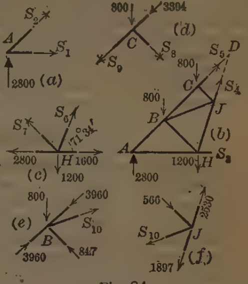


Fig. 84

$\Sigma F_x = S_{10} \times \cos 26° 34' - 800 \cos 45° = 0$, or $S_{10} = +633$ lb tension, and $\Sigma F_y = 847 - 800 \sin 45° - 633 \sin 26° 34' = -2$, or nearly zero, and so the check is satisfactory.

(2) **Howe Bridge Truss.** The truss represented in Fig. 85 is generally constructed all of wood except the "verticals." The "diagonals" are not connected to other members at the various joints but simply butt up against bearing blocks at their ends, and so can be subjected only to compression. Two diagonals in any panel cannot be strest at the same time; when the loading is symmetrical with respect to the middle of the truss, those diagonals represented are the ones strest and they are the main diagonals. The others, not shown, may be strest only when the truss is partially loaded, and they are counter diagonals, or braces, or simply counters.

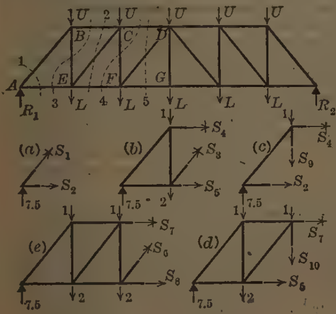


Fig. 85

In the following example the truss is supposed to be loaded symmetrically (see Fig. 85) and the counters are not mentioned. The span is 72 ft, height of truss 15 ft, each upper load U is 1 ton and each lower load L is 2 tons; then each reaction is 7.5 tons. The stress in any diagonal may be found by passing a vertical section thru the truss so as to cut the member under consideration and then solving the force system acting on either part of the truss. Thus, to determine the stress in AB, pass the section 1, and solve the system shown in Fig. 85a for S_1; to determine the stress in EC, pass section 2 and solve the system shown in Fig. 85b for S_3; similarly to determine the stress in FD, etc. It will be noticed that the vertical component of the stress in any diagonal equals the external shear at the section cutting the diagonal; thus (see Fig. 85b) $S_3 \times \cos \alpha = 7.5 - 2 - 1 = 4.5$. The stress in any vertical, except the middle one if there be such, may be found by passing

a section so as to cut only that member and adjaçent chord members and then solving the system of forces acting on either part of the truss for the desired stress. Thus, to determine the stress in BE, pass section 3, and from the system shown in Fig. 85c get $S_9 = 7.5 - 1 = 6.5$ tons; to determine the stress in CF, pass section 4, and from the system shown in Fig. 85d get $S_{10} = 7.5 - 2 - 1 - 1 = 3.5$ tons. The stress in the middle vertical equals the load at its lower end, 2 tons. The stress in any chord member may be found by passing a section cutting that member and the others in the same panel, and then solving the force-system acting on either part of the truss for the desired stress; the solution is easily made from a moment equation, the origin being taken at the intersection of the other two members cut. Thus to determine the stress in CD, pass section 5; the moment equation for the system shown in Fig. 85e with origin at the intersection of S_6 and S_8 is $7.5 \times 24 + 3 \times 12 - S_7 \times 15 = 0$, or $S_7 = 9.60$ tons.

21. Stress Diagrams for Trusses

Graphic Methods for Analyzing Trusses are especially well adapted for solving problems like the preceding. As in the algebraic method, the truss is imagined separated into two parts and then the attention is directed to the forces acting upon either part. Graphic instead of algebraic conditions of equilibrium are then applied to the system of forces to determine the unknowns. In making the imaginary separations of the truss, care should be taken to cut not more than three members, the forces in which are unknown. It is advantageous to make the separation so that not more than two such members are cut. If that be done, a single force polygon will determine the two unknowns, while if three be cut, a force polygon and an equilibrium polygon, or the equivalent, are necessary for determining the three unknowns. In drawing the force polygon, it will be advantageous to represent the forces in the order in which they occur about the joint. A force polygon so drawn will be called a polygon for the joint; and for brevity, if the order taken is clockwise, the polygon will be called a clockwise polygon and if counter-clockwise, it is called a counter-clockwise polygon. If the polygons for all the joints of a truss are drawn separately, then the stress in each member will have been represented twice. It is possible to combine the polygons so that it will not be necessary to represent the stress in any member more than once, thus reducing the number of lines to be drawn. Such a combination of force polygons is called a stress diagram. Each triangular space in the truss diagram is marked by a small letter, also the space between consecutive action lines of the loads and reactions. Then the two letters on opposite sides of any line serve to designate that line, and the same large letters are used to designate the magnitude of the corresponding force.

To construct a stress diagram for a truss under given loads:

(1) Determine the reactions. (2) Letter the truss diagram as directed. (3) Construct a force polygon for all the external forces applied to the truss (loads and reactions), representing them in the order in which their application points occur about the truss, clockwise or counter-clockwise. (4) On the sides of that polygon construct the polygons for all the joints. They must be clockwise or counter-clockwise ones according as the polygon for the loads and reactions was drawn clockwise or counter-clockwise. (The first polygon drawn must be for a joint at which but two members are fastened; the joints at the supports are usually such. Next that joint is considered and its polygon is drawn, at which not more than two stresses are unknown.

(1) **Roof Truss.** Fig. 86 represents a truss sustaining loads of 600, 1000, 1200 and 1800 lb; the right reaction is 2100 lb and the left 2500 lb. $ABCDEFA$ is a polygon for the loads and reactions, these being represented in the order in which their points of application occur about the truss. The polygon for joint 1 is $FABGF$; the force BG acts toward the joint, hence bg is under compression, and GF acts away from the joint, hence

gf is in tension. The polygon for joint 2 is $CDEHC$; the force EH acts away from the joint, hence eh is in tension; and HC acts toward the joint, hence hc is in compression. The polygon for joint 3 is $HEFGH$; the force GH acts away from the joint and hence gh is in tension. If the work has been correctly done, GH is parallel to gh. (In Fig. 86a the polygons are all clockwise, and in Fig. 86b counter-clockwise.)

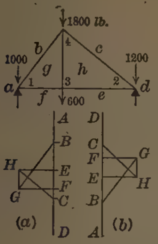


Fig. 86

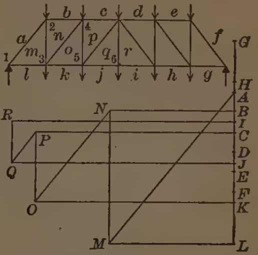


Fig. 87

(2) **Howe Truss.** Fig. 87 represents a Howe truss, 72 ft span and 15 ft high, under five 1-ton loads on the upper chord and five 2-ton loads on the lower. $ABCDEFGHIJKLA$ is a force polygon for all the loads and reactions. Polygon for joint 1 is $LAML$; for joint 2, $MABNM$; for joint 3, $KLMNOK$; for joint 4, $ONBCPO$; for joint 5, $JKOPQJ$; and for joint 6, $IJQRI$.

(3) **Derrick.** Fig. 88a represents a stiff-leg derrick, only one stiff leg, da, shown, and the boom and shown stiff leg in the same vertical plane. For the analysis it is not necessary to determine the reactions first. The polygon for joint 1 may be drawn first; it is $ABCA$, and BC and CA represent compression and tension respectively. The polygon for joint 2 may be drawn next; it is $ACDA$, and CD and DA represent compression and tension respectively. The reaction at 3 equals the resultant of CB and DC, that is DB.

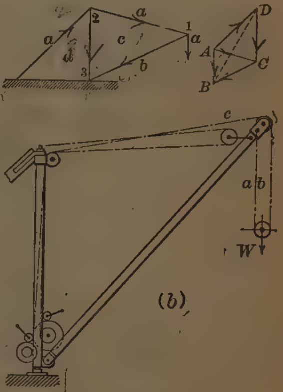


Fig. 88

The foregoing analysis is imperfect because it assumes, in part, a single stay along ac, whereas such derricks usually have a multiple stay and a hoisting cable as shown in Fig. 88b. To determine the reaction P_1 at the top of the mast exerted by the stiff leg and that P_2 at its base: P_1 acts nearly along the axis of the stiff leg, and P_2 in a direction unknown as yet; these forces along with the weights of the mast, boom, and load, constitute a system in equilibrium, and it may be solved for P_1 and P_2 as explained in Art. 18. (If hoisting is accomplished not by a winze mounted on the derrick, as assumed, but by an engine, then there must be included in the system described the pull of the engine.) To determine the reaction Q_1 at the base of the boom and the pull Q_2 of the top stay: Q_2 acts along the stay and Q_1 in a direction unknown as yet; these forces along with the weight of the boom, a pull of $\frac{1}{2}W$ in line a, and a pressure at the top pulley pin identical with the resultant of the pulls $\frac{1}{2}W$ in the lines b and c, constitute a system in equilibrium which may be solved for Q_1 and Q_2. (Art. 18.)

GRAVITY AND INERTIA FUNCTIONS

22. Principles of Center of Gravity

Definitions. The resultant of the weights, or parallel forces of gravity upon all the particles of a body, always passes thru a certain particle or point fixt with reference to the body, no matter how the body is turned about; this particle or point is the CENTER OF GRAVITY of the body. If the body can be supported at its center of gravity, then so supported it would remain at rest in any position. Center of gravity is also called center of mass and center of inertia, especially in discussions relating to the motion of the body. The center of gravity of a line (or length), surface (or area), solid (or volume) is the center of gravity of the line, surface, or solid imagined materialized, that is, conceived as a very slender wire, thin plate, or homogeneous body. Centroid is also used in place of center of gravity as applied to lines, surfaces, and solids. The center of gravity of a line, surface, solid or homogeneous body is a mean point; that is, the distance of the center of gravity from any reference plane is the mean of the distances of all the equal elementary parts of the line, surface, solid, or body from the plane. If the reference plane cuts the line, surface, solid, or body, distances on opposite sides of the plane must be regarded as opposite in signs. SYMMETRY: Two points are symmetrical with respect to a third point if the line joining the two is bisected by the third. Two points are symmetrical with respect to a line or a plane if the line joining them is perpendicular to the given line or plane and is bisected by it. A body, line, surface, or volume is symmetrical with respect to a point, a line, or a plane if all the points of the body, line, surface, or volume can be paired off so that each pair is symmetrical with respect to the point, line, or plane. If a line, surface, solid or homogeneous body is symmetrical with respect to a point, line, or plane, then its center of gravity is at the point in the line, or plane. The STATICAL MOMENT of a body (or weight), line (or length), surface (or area), or solid (or volume) with respect to any plane is the product of the weight, length, area, or volume and the distance of the center of gravity of the body, line, surface, or solid from the plane. The statical moment of a plane line (or length) or plane surface (or area) with respect to a straight line in the plane is the product of the length or area and the distance of the center of gravity of the line or surface from the reference line. A statical moment is regarded as positiv or negativ according as the corresponding center of gravity is on the positiv or negativ side of the reference plane or line. In the foregoing definitions and in the following, the words body, line, surface, and solid are used broadly to include what would ordinarily be described as a collection of bodies, lines, surfaces, or solids.

Methods for Locating Center of Gravity. The statical moment of a body, line, surface, or solid equals the algebraic sum of the statical moments of all the parts into which it is, or is imagined, divided. This principle is the basis of all formulas for locating centers of gravity. If the sum of the statical moments is zero, then the center of gravity is in the reference plane or line as the case may be. The position of a center of gravity is generally conveniently specified by its rectangular coordinates $\bar{x}$, $\bar{y}$ and $\bar{z}$.

(1) The formulas for the coordinates of the center of gravity of a body are

$$W\bar{x} = \int x\,dW \qquad W\bar{y} = \int y\,dW \qquad W\bar{z} = \int z\,dW,$$

in which W denotes the weight of the body, dW the weight of any elementary portion, and x, y, and z the coordinates of the center of gravity of that element; the limits of integration must be assigned so that the integration includes

all elementary parts of the body. The formulas furnish values of $\bar{x}$, $\bar{y}$, and $\bar{z}$ in any case if the body is mathematically regular and such that the integrations can be performed. The following are corresponding formulas for the coordinates of the center of gravity of lines, surfaces, and solids:

$$L\bar{x} = \int x\,dL \qquad L\bar{y} = \int y\,dL \qquad L\bar{z} = \int z\,dL$$

$$A\bar{x} = \int x\,dA \qquad A\bar{y} = \int y\,dA \qquad A\bar{z} = \int z\,dA$$

$$V\bar{x} = \int x\,dV \qquad V\bar{y} = \int y\,dV \qquad V\bar{z} = \int z\,dV$$

in which L, A, and V denote length, area, and volume respectively.

(2) If a body consists of finite parts whose weights and centers of gravity are known, then the coordinates of the center of gravity of the body can be computed, without integration, from

$$W\bar{x} = W_1\bar{x}_1 + W_2\bar{x}_2 + \cdots \qquad W\bar{y} = W_1\bar{y}_1 + W_2\bar{y}_2 + \cdots$$

$$W\bar{z} = W_1\bar{z}_1 + W_2\bar{z}_2 + \cdots$$

in which W denotes the weight of the body, W_1, W_2, etc., the weights of its parts; $\bar{x}_1$, $\bar{y}_1$, $\bar{z}_1$, the coordinates of the center of gravity of W_1; $\bar{x}_2$, $\bar{y}_2$, $\bar{z}_2$ those of the center of gravity of W_2; etc. The following are corresponding formulas for lines, surfaces, and solids:

$$L\bar{x} = L_1\bar{x}_1 + L_2\bar{x}_2 + \cdots \qquad L\bar{y} = L_1\bar{y}_1 + L_2\bar{y}_2 + \cdots \qquad L\bar{z} = L_1\bar{z}_1 + L_2\bar{z}_2 + \cdots$$

$$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2 + \cdots \qquad A\bar{y} = A_1\bar{y}_1 + A_2\bar{y}_2 + \cdots \qquad A\bar{z} = A_1\bar{z}_1 + A_2\bar{z}_2 + \cdots$$

$$V\bar{x} = V_1\bar{x}_1 + V_2\bar{x}_2 + \cdots \qquad V\bar{y} = V_1\bar{y}_1 + V_2\bar{y}_2 + \cdots \qquad V\bar{y} = V_1\bar{z}_1 + V_2\bar{z}_2 + \cdots$$

in which the L's, A's, and V's denote lengths, areas, and volumes.

The center of gravity of two bodies, lines, surfaces, or solids is on the straight line joining the centers of gravity of the two, and the center of gravity of the two divides the joining line into segments inversely proportional to their weights, lengths, areas, or volumes. The center of gravity of three bodies, lines, surfaces, or solids is in the plane of the centers of gravity of the three.

If the center of gravity of the parts of a body, line, surface, or solid lie in a plane, the center of gravity of the whole may be gotten graphically. (1) Let a, b, c, etc., be the centers of gravity of the parts and A, B, C, etc., their weights, lengths, areas, or volumes; then imagine parallel forces of values A, B, C, etc., to act thru a, b, c, etc., and find the line of action of their resultant; repeat the operation for the parallel forces at an angle (as 90°) with their first positions. The intersection of the resultants is the center of gravity sought. (2) Choose any point O (Fig. 89) in the plane of the centers of gravity as origin; measure Oa, Ob, Oc, etc., and form the products $A\overline{Oa}$, $B\overline{Ob}$, $C\overline{Oc}$, etc. Imagine forces whose values equal the products respectively, to act from O in the lines Oa, Ob, Oc, etc., and find their resultant R (Art. 16). The center of gravity sought, r, lies in the line of action of R at a distance from O equal to $R \div (A + B + C + \ldots)$.

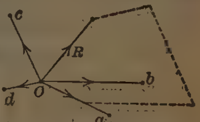


Fig. 89

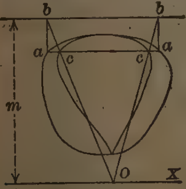


Fig. 90

The center of gravity of an irregular plane figure may be found by cutting out the figure from stiff paper and then determining the center of gravity of the paper experimentally by balancing (see below); this center of gravity gives that of the figure. Or, proceed as follows (Fig. 90): Take a point O and a line bb on opposite sides of the figure at any convenient distance m apart; project any width of the figure parallel to bb as aa on bb, connect the projections bb with O and note the intersections cc; determine other points cc and draw a smooth curve thru them as

shown; measure the area A' within the curve cc; then $A'm$ is the statical moment of the given figure with respect to OX; if A is the area of the given figure and y the distance of its center of gravity from OX, $y=A'm/A$. In a similar way the distance of the center of gravity from a line perpendicular to OX can be determined

Experimental Determination of center of gravity must be resorted to when the body is so irregular that the appropriate formulas foregoing cannot be applied. (1) Method of Suspension: The body is suspended from one point of it, and the direction of the suspending cord is then marked in some way on the body; the operation is repeated for another point of suspension. The center of gravity is at the intersection of the two lines or directions so fixt in the body. (2) Method of Balancing: The body is balanced on a straight-edge, and the vertical plane containing the edge is marked on the body; the operation is re-

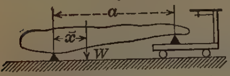


Fig. 91

peated for two more balancing positions of the body. The center of gravity is at the common point of the three planes so fixt in the body. This method is readily applied to a body in the form of a thin plane plate; practically only two balancings are necessary. (3) Method of Weighing: The weigh W of the body is determined, and then it is supported on a knife edg (Fig. 91) and on a point support which rests upon a platform scale; the re action R of the point support is weighed, the horizontal distance a of th point from the knife edge is measured; then the distance from center c gravity to knife edge $\bar{x}$ is Ra/W.

23. Centers of Gravity of Some Lines and Areas

Circular Arc (Fig. 92): the center of gravity is on the axis of symmetry its distance from the center is $\bar{x}=rc/s=r\sin\alpha/\alpha$, the last α being express in radians (1 degree=0.0175 radian). For a semicircle $\bar{x}=2r/\pi=0.6366\,r$; for a quadrant $\bar{x}=2r\sqrt{2}/\pi=0.9003\,r$, and the distance of the center of gravity from the radius drawn to either end of the arc is $2r/\pi=0.6366\,r$. For flat arcs, α small, the distance from its center of gravity to the chord c is closely equal to $2/3h$; the error is less than $1/2\%$ when $\alpha=30°$, and less than 1.1% when $\alpha=45°$.

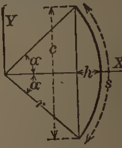


Fig. 92

Triangle: The center of gravity is at the intersections of the medians; its distance (perpendicular) from any side equals one-third the altitude of the triangle measured from that side. When x_1, x_2, x_3 are parallel distances from the vertexes to a plane, then the distance of the center of gravity is $1/3\,(x_1+x_2+x_3)$.

Trapezoid; Let a and b be the parallel bases and h the altitude. For t left-hand diagram in Fig. 92½, where m is the overhang of the right side, t horizontal distance of the center gravity from the left corner is $\bar{x}=2/3b$ $1/3m-1/3a\,(a-m)/(a+b)$. For the rig diagram the same formula applies if is taken as negative. For all cases t distance of the center of gravity abo the base b is $\bar{y}=1/3h\,(b+2a)/(a+$ When a equals b, then $x=1/2\,(b\pm m)$ a $\bar{y}=1/2h$. Geometric determinations

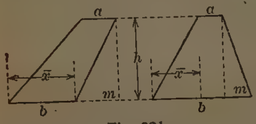
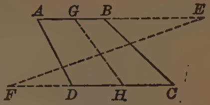


Fig. 92½.

center of gravity: (1) Extend AB (Fig. 93) so that $BE=CD$, and in the o posite direction extend CD so that $DF=AB$; the intersection of FE and median GH is the center of gravity. (2) Divide the trapezoid (Fig. 93) i

triangles by a diagonal as AC; find the centers of gravity G_1 and G_2 of the triangles (construction indicated in the figure); the intersection of G_1G_2 with the median EF is the center of gravity sought.



Quadrilateral: (a) Divide the quadrilateral into triangles by a diagonal AC (Fig. 94) and find their centers of gravity G_1 and G_2; divide it into triangles by the other diagonal and find their centers of gravity G_3 and G_4; the intersections of the lines G_1G_2 and G_3G_4 is the center of gravity sought. (b) Divide the sides into thirds (Fig. 96) and draw lines thru the third points as shown; these lines form a parallelogram whose diagonals intersect at the center of gravity of the quadrilateral.

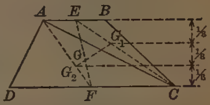


Fig. 93

Circular Sector (Fig. 95a). The center of gravity is on the axis of symmetry at a distance from the center equal to $\bar{x} = \frac{2}{3} rc/s = \frac{2}{3} r \sin\alpha/\alpha$, the last α being exprest in radians (1 degree = 0.0175 radian). For a semicircle $\bar{x} = 4r/3\pi = 0.4244\,r$. For a quadrant $\bar{x} = 4\sqrt{2}\,r/3\pi = 0.6002\,r$, and distance of the center of gravity from each bounding radius is $4r/3\pi = 0.4244\,r$.

Circular Segment (Fig. 95b). The center of gravity is on the axis of symmetry at a distance from the center equal to $\bar{x} = c^3/12A = (2r^3\sin^3\alpha)/3A$, in which A is the area of the segment, or $\frac{1}{2}r^2(2\alpha - \sin 2\alpha)$, the first α being exprest in radians (1 degree = 0.0175 radian).

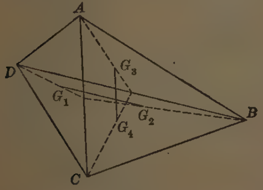


Fig. 94

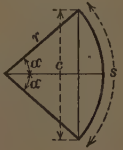


Fig. 95a

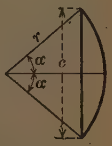


Fig. 95b

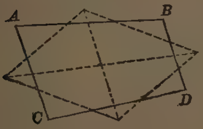


Fig. 96

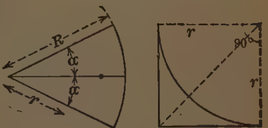


Fig. 97

Sector of a Circular Ring (Fig. 97). The center of gravity is on the axis of symmetry at a distance from the center of the circle which is given by $\bar{x} = \frac{2}{3}(R^3 - r^3)\sin\alpha/(R^2 - r^2)\,\alpha$, the last α being in radians.

The Surface (Fig. 97) bounded by a circular quadrant and the tangents at its extremities. The center of gravity is on the axis of symmetry at a distance from each tangent equal to $0.223\,r$.

Parabolic Segment (Fig. 98): G_1 and G_2 are the centers of gravity of $OXCO$ and $OYCO$ respectively: $x_2 = 3/5\,a$, $y_1 = 3/8\,b$, $x_1 = 3/10\,a$, $y_2 = 3/4\,b$.

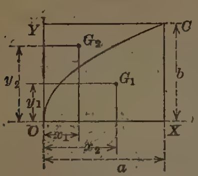


Fig. 98

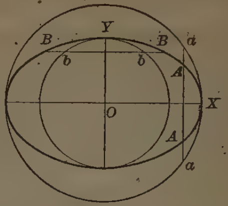


Fig. 99

Symmetric Elliptic Segment (Fig. 99). The center of gravity of $YBBY$ coincides with that of the circular segment $YbbY$, and the center of gravity of $XAAX$ coincides with that of the circular segment $XaaX$.

24. Centers of Gravity of some Volumes

Right Circular Cylinder (Fig. 100). The base XOA is normal to the axis of the cylinder, and the top makes an angle α with the base; the radius of the base is r and the mean height is h; then

$$\bar{x} = (r^2 \tan \alpha)/4\,h;\ \bar{y} = \tfrac{1}{2}\,h + (r^2 \tan^2 \alpha)/8\,h.$$

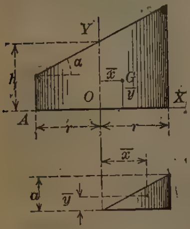


Fig. 100

If the oblique top cuts the base in a diameter, $\bar{x} = 3/16\,\pi r$, and $\bar{y} = 3/32\,\pi a$.

Cone and Pyramid. The center of gravity of the surface (not including base) is on a line joining the apex with the center of gravity of the perimeter of the base at a distance two-thirds the length of that line from the apex. The center of gravity of the solid cone or pyramid is on the line joining the apex with center of gravity of the base three-fourths the length from the apex.

Frustum of a Circular Cone. Let R = radius larger base, r = radius smaller, a = altitude; then distance of center of gravity of the conical surface from larger base is $\tfrac{1}{3}\,a(R + 2\,r)/(R + r)$, from smaller base $\tfrac{1}{3}\,a(2\,R + r)/(R + r)$, from a plane midway between bases $\tfrac{1}{6}\,a\,(R - r)/(R + r)$. The distance from the center of gravity of the solid frustum to the larger base is $\tfrac{1}{4}\,a\,(R^2 + 2\,Rr + 3\,r^2)/(R^2 + Rr + r^2)$.

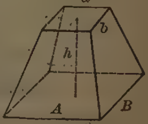


Fig. 101

Frustum of a Pyramid. If the pyramid has regular bases, let R and r be the lengths of sides of the larger and smaller bases, and h the altitude; then the distance from the center of gravity of the surface (not including bases) from the larger base is $\tfrac{1}{3}\,h\,(R + 2\,r)/(R + r)$. If A and a are the areas of the large and small bases of the frustum of any pyramid and h the altitude, the distance from the center of gravity of the solid from the larger base is $\tfrac{1}{4}\,h\,\left(A + 2\sqrt{Aa} + 3\,a\right)/\left(A + \sqrt{Aa} + a\right)$.

Obelisk and Wedge (Fig. 101). The distances from the center of gravity to the base AB is $\tfrac{1}{2}\,h\,(AB + Ab + aB + 3\,ab)/(2\,AB + Ab + aB + 2\,ab)$. If

$b = 0$; the solid is a wedge, and the distance from the center of gravity to the base is $\frac{1}{3} h (A + a)/(2A + a)$.

Sphere Parts. The center of gravity of any zone (surface) (Fig. 102) of a sphere is midway between the bases. Segment (solid): height h (Fig. 103),

Fig. 102

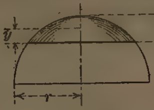


Fig. 103

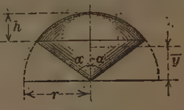


Fig. 104

$\bar{y} = \frac{1}{4} h (4r - h)/(3r - h)$; when $h = r$ (hemisphere), $\bar{y} = \frac{3}{8} r$. Sector (Fig. 104): $\bar{y} = \frac{3}{8} (1 + \cos \alpha)\, r = \frac{3}{8} (2r - h)$.

Ellipsoid. Let the three axes be taken as x, y, and z coordinate axes, and a, b, and c denote the semi-lengths of the corresponding axes of the ellipsoid; the center of gravity of one octant of the solid is given by $\bar{x} = \frac{3}{8} a$, $\bar{y} = \frac{3}{8} b$, and $\bar{z} = \frac{3}{8} c$.

Paraboloid of Revolution formed by revolving a parabola about its axis. Let h = height of the paraboloid, the distance from its apex to the base; then the distance from the center of gravity of the solid to the base is $\frac{1}{3} h$.

25. Principles of Moment of Inertia

Definitions. The MOMENT OF INERTIA of a surface (figure or area) with respect to or about a line is the sum of the products obtained by multiplying the area of each element of the surface by the square of its distance from the line. Thus, if I denotes moment of inertia, A area and r distance of any element dA from the line or axes with respect to which I is taken, then $I = \int r^2\, dA$; the limits of integration are to be so chosen that the integration will include all products like $r^2\, dA$ for the surface. The moment of inertia, obviously, of any surface or figure is the sum of the moments of inertia of its parts. The moment of inertia of a plane figure with respect to a line in the plane is called rectangular, and one with respect to a line perpendicular to the plane is called polar; these are the only moments of inertia of surfaces that are of practical importance A unit moment of inertia is four "dimensions" in length, and is called quadric inch, foot, etc., according as the inch or foot is used as unit length; the corresponding abbreviations are in^4, ft^4, etc. The RADIUS OF GYRATION of a surface (figure or area) with respect to a line is such a length whose square multiplied by the area of the surface equals the moment of inertia of the surface with respect to the same line. Thus if k denotes radius of gyration, A area, and I moment of inertia, $k^2 A = I$, or $k = \sqrt{I/A}$. The square of the radius of gyration of a figure with respect to a line is the mean of the squares of the distances of all the elementary parts of the figure from the line. (See also Art 27.)

The Product of Inertia of a plane surface (figure or area) with respect to a pair of coordinate axes in the plane is the sum of the products obtained by multiplying the area of each element of the surface by its coordinates. Thus if J denotes product of inertia with respect to x and y axes, A area, and x and y the coordinates of any element-area dA, then $J = \int xy\, dA$; the limits of integration are to be so chosen that the integration will include all products like $xy\, dA$ for the surface. A unit product of inertia, like a unit moment of inertia (see foregoing), is four dimensions in length. Unlike moments of inertia, prod-

ucts of inertia may be zero or negativ as well as positiv. If a figure has an axis of symmetry, then its product of inertia with respect to that axis and one perpendicular thereto is zero.

Graphic Determination of Moment of Inertia. When the outline of a surface is so irregular that the integration in the expression for I cannot be performed, then the following may be resorted to: Let $aaaa$ (Fig. 105) be the outline and XX' the axis with respect to which the moment of inertia is desired; at any convenient distance m from XX' draw two parallels (but if XX' does not cut the figure, then only one parallel, the one on the opposite side of the figure from XX'); draw any line as aa parallel to XX' and project the points aa on the nearer parallel; join the projections bb to any point O in XX' and note the intersections cc on aa; project cc on the same parallel; join the projections dd with O and note the intersections ee on aa. In a similar manner determine points like ee for other widths like aa, and connect all points e as shown. Then measure the area of the loops OPO and OQO; denoting this combined area by A'', $I = A''m^2$. (There will be only one loop if only one parallel, bb, is used.)

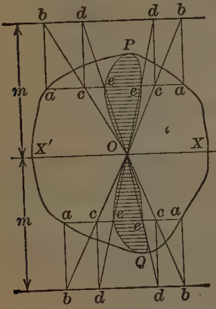


Fig. 105

Transformation Formulas. (1) Let I = moment of inertia of a figure with respect to any line or axis, $\bar{I}$ = that with respect to a parallel axis passing thru the center of gravity of the figure; d = distance between the axes, k and $\bar{k}$ = the radii of gyration with respect to the same axes respectively, and A = area of the figure; then

$$I = \bar{I} + Ad^2 \quad \text{and} \quad k^2 = \bar{k}^2 + d^2$$

These show that with respect to all parallel axes the moment of inertia and the radius of gyration is least for the one passing thru the center of gravity of the figure. (2) Let I_x, I_y, and I_z = the moments of inertia of a plane figure with respect to x, y, and z axes respectively, the axes being at right angles to each other and the x and y axes in the plane; and let k_x, k_y, and k_z = the corresponding radii of gyration; then

$$I_x + I_y = I_z \qquad k_x^2 + k_y^2 = k_z^2$$

(3) Let J = the product of inertia of a plane figure with respect to a pair of coordinate axes in the plane, and $\bar{J}$ = that with respect to a parallel pair whose origin is at the center of gravity; $\bar{x}$, $\bar{y}$ the coordinates of the center of gravity referred to first pair, and A the area of the figure; then $J = \bar{J} + A\bar{x}\bar{y}$. (4) Let XOY and UOV (Fig. 106) be two sets of rectangular coordinate axes with a common origin and in a given plane figure; I_x, I_y, I_u, I_v = moments of inertia of the figure with respect to x, y, u, and v axes respectively; J_{xy} and J_{uv} = its products of inertia with respect to the sets of axes respectively; α = the angle thru which the x axis must be rotated to bring it into the u axis, regarded as positiv or negativ according as the turning is counter-clockwise or clockwise. Then $I_u + I_v = I_x + I_y$, and

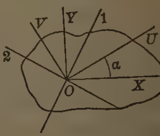


Fig. 106

$$I_u = I_x \cos^2\alpha + I_y \sin^2\alpha - J_{xy} \sin 2\alpha$$
$$J_{uv} = \tfrac{1}{2}(I_x - I_y) \sin 2\alpha + J_{xy} \cos 2\alpha.$$

Principal Axes of Inertia. In general, the moments of inertia of a plane figure with respect to different lines thru any point of the plane are unlike; for one axis the moment is greater and for another less than for any other axis thru the point. The principal axes for a plane figure at a particular point of the plane are the two axes for which the moments of inertia are greater and less than for any other axis thru the point and in the plane; the corresponding moments of inertia are called the PRINCIPAL MOMENTS OF INERTIA of the figure at the point. The principal axes are always at right angles to each other.

With respect to the principal axes, the product of inertia is zero; from this principle the principal axes can readily be found in some cases. Thus, at a corner of a square the principal axes are the diagonal thru that corner and a line perpendicular to it, for with respect to those lines the product of inertia of the square is zero (see under Product of Inertia). In any case, the principal axes of a figure at a point of it can be found from the following formula if the moments of inertia and the product of inertia of the figure with respect to two rectangular axes thru the point and in the plane are known; thus let the two rectangular axes be OX and OY (Fig. 106), I_x, I_y the corresponding moments of inertia, J_{xy} the corresponding product of inertia, and θ the (unknown) angle thru which OX must be turned counter-clockwise to bring it into either principal axis, $O1$ or $O2$; then $\tan 2\theta = 2\,J_{xy}/(I_y - I_x)$, which gives always two values of θ differing by 90° unless J_{xy} and $I_y - I_x$ are both zero. In that case the figure has no principal axis at the point, and the moments of inertia with respect to the different lines thru the point are all equal.

The Principal Moments of Inertia I_1 and I_2 can be computed from

$$I_1 = I_x \cos^2\theta_1 + I_y \sin^2\theta_1 - J_{xy}\sin 2\theta_1,$$

and

$$I_2 = I_x \cos^2\theta_2 + I_y \sin^2\theta_2 - J_{xy}\sin 2\theta_2$$

θ_1 and θ_2 being the two values of θ given by the formula above for principal axes. Or, after I_1 is determined, $I_2 = I_x + I_y - I_1$.

The Inertia-Circle and Ellipse. The inertia-circle is a device for determining the moment of inertia of a plane figure with respect to any line of the plane and the principal axes and principal moments of inertia at any point graphically. To construct the circle, it is necessary to know the moments of inertia and the product of inertia with respect to two rectangular axes thru the point and in the plane figure. Thus if I_u and I_1 and I_2 at O (Fig. 107) are desired, I_x, I_y, and J_{xy} being known: By any scale, lay off $OX = I_x$, $OY = I_y$ and $YA = J_{xy}$, upwards or downwards from XY according as J_{xy} is positiv or negativ; bisect XY and from the middle point C as center describe a circle passing thru A; this is the inertia-circle for the axes xOy. Draw a secant thru A parallel to the u axis, and from its intersection B with the circle draw a perpendicular BU to XY; then $OU = I_u$, by the scale used, (and $BU = J_{uv}$). Lines thru O parallel to AP and AQ are the principal axes at O; and OP and OQ represent the corresponding principal moments of inertia respectively by the scale used.

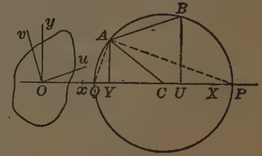


Fig. 107

The inertia-ellipse like the inertia-circle shows the relations between the moments of inertia of a plane figure for different axes thru a point of the plane; for qualitative results the ellipse is preferable, generally, but for quantitative results, the circle. The inertia-ellipse for a plane surface at any particular point has its center at the point and is

so drawn that the distance from any diameter to either parallel tangent equals the radius of gyration of the figure with respect to that diameter. An inertia-ellipse of a figure at its center of gravity is the central inertia-ellipse. In general, the ellipse is determined most readily thus: determine the principal axes at the point in question and the corresponding principal radii of gyration; from the point lay off on each axis in each direction distances equal (by some scale) to the radius of gyration with respect to the other axis; construct the ellipse on those two lengths as axes (Art. 11).

26. Moments of Inertia of Some Plane Figures

Rectangle. Let b = base and h = altitude; about a line thru center parallel to b, $I = \frac{1}{12}bh^3$; about a line thru the center parallel to h, $I = \frac{1}{12}hb^3$; about side b, $I = \frac{1}{3}bh^3$; about side h, $I = \frac{1}{3}hb^3$; about a diagonal $I = \frac{1}{6}b^3h^3/(b^2+h^2)$; about a line thru the center perpendicular to the diagonal $I = \frac{1}{12}(bh^3+hb^3)$.

Square. Make $b = h$ in foregoing. The moment of inertia is $\frac{1}{12}h^4$ for all axes in the plane of the square and passing thru the center.

Hollow Rectangle. Let B and b = outer and inner breadths, and D and d = outer and inner depths; about an axis parallel to B and b and passing thru the center $I = \frac{1}{12}(BD^3 - bd^3)$.

Triangle. Let b = base and h = altitude; about the base $I = \frac{1}{12}bh^3$; about a line thru the center of gravity parallel to the base $I = \frac{1}{36}bh^3$; about a line thru the vertex parallel to the base $I = \frac{1}{4}bh^3$.

Regular Polygon. Let A = area, R = radius of circumscribed circle, r = radius of inscribed circle, and s = length of a side; about any axis thru the center and in the plane of the polygon $I = \frac{1}{24}A(6R^2 - s^2) = \frac{1}{48}A(12r^2 + s^2)$; about a line perpendicular to the plane of the polygon passing thru the center I = double the preceding I.

Trapezoid. Let B = long base, b = short base, h = altitude; about the long base $I = \frac{1}{12}(B + 3b)h^3$; about the short base $I = \frac{1}{12}(3B + b)h^3$; about a line thru center of gravity and parallel to bases $I = \frac{1}{36}(B^2 + 4Bb + b^2)h^3/(B+b)$.

Circle. Let d = diameter and r = radius; about a diameter $I = \frac{1}{64}\pi d^4 = \frac{1}{4}\pi r^4$, and $k^2 = \frac{1}{16}d^2 = \frac{1}{4}r^2$; about a line thru the center and perpendicular to the circle $I = \frac{1}{32}\pi d^4 = \frac{1}{2}\pi r^4$, and $k^2 = \frac{1}{8}d^2 = \frac{1}{2}r^2$.

Semicircle. Let d = diameter and r = radius; about the bounding diameter or about the line of symmetry $I = \frac{1}{128}\pi d^4 = \frac{1}{8}\pi r^4$; about a line thru the center of gravity parallel to the bounding diameter $I = (9\pi^2 - 64)d^4/1152\pi = 0.00686\,d^4 = 0.110\,r^4$.

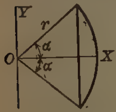


Fig. 108

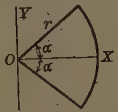


Fig. 109

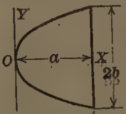


Fig. 110

Hollow Circle. Let D and d = outer and inner diameters, and R and r = outer and inner radii; about a diameter $I = \frac{1}{64}\pi(D^4 - d^4) = \frac{1}{4}\pi(R^4 - r^4)$, and $k^2 = \frac{1}{16}(D^2 + d^2) = \frac{1}{4}(R^2 + r^2)$; about a line thru the center and normal to circle $I = \frac{1}{32}\pi(D^4 - d^4) = \frac{1}{2}\pi(R^4 - r^4)$, and $k^2 = \frac{1}{8}(D^2 + d^2) = \frac{1}{2}(R^2 + r^2)$.

Circular Segment (Fig. 108). Let A = area of the segment;

$$I_x = \tfrac{1}{4}Ar^2[1 - \tfrac{2}{3}(\sin^3\alpha\cos\alpha)/(\alpha - \sin\alpha\cos\alpha)],$$
$$I_y = \tfrac{1}{4}Ar^2[1 + (2\sin^3\alpha\cos\alpha)/(\alpha - \sin\alpha\cos\alpha)].$$

Circular Sector (Fig. 109). Let A = area of the sector; $I_x = \frac{1}{4}Ar^2(1 - \sin\alpha\cos\alpha/\alpha)$, $I_y = \frac{1}{4}Ar^2(1 + \sin\alpha\cos\alpha/\alpha)$; with respect to a line thru O perpendicular to the sector, $I = \frac{1}{2}Ar^2$.

Parabolic Segment (Fig. 110). $I_x = \frac{4}{15} ab^3$, $I_y = \frac{4}{7} ba^3$.

Ellipse. Let $2a$ and $2b$ = lengths of the axes of the ellipse; about the $2a$ axis $I = \frac{1}{4}\pi ab^3$; about the $2b$ axis $I = \frac{1}{4}\pi ba^3$; about a line thru the center and perpendicular to the ellipse $I = \frac{1}{4}\pi ab(a^2 + b^2)$.

27. Moment of Inertia of Bodies

Definitions. The MOMENT OF INERTIA OF A BODY with respect to or about a line is the sum of the products obtained by multiplying the mass of each elementary part by the square of its distance from the line. Thus I denoting moment of inertia, m mass, and p the distance of any element dm from the line of reference, $I = \int p^2\,dm$. The moment of inertia of a body is, obviously, the sum of the moments of inertia of its parts. A unit moment of inertia of a body is one dimension in mass and two in length. The CENTER OF GYRATION of a body with respect to a line is a point at such a distance from the line that if the entire mass of the body were concentrated there, the moment of inertia of the mass-point would be the same as that of the body; the distance of the center of gyration from the line is the radius of gyration with respect to the line. Thus k denoting radius of gyration, $k^2m = I$, or $k = \sqrt{I/m}$. The PRODUCT OF INERTIA of a body with respect to two coordinate planes is the sum of the products obtained by multiplying the mass of each element of the body by the two coordinates of the element with reference to those planes. Thus with respect to YOZ and ZOX (Fig. 111), ZOX and XOY, and XOY and YOZ planes, the products of inertia are respectively $\int xy\,dm$, $\int yz\,dm$, and $\int zx\,dm$. A unit product of inertia is, like a unit moment of inertia, one dimension in mass and two in length. Unlike moments, products of inertia may be zero or negativ as well as positiv.

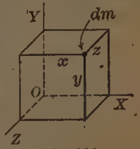


Fig. 111

Transformation Formulas. (1) If I denotes the moment of inertia of a body with respect to any line or axis, $\bar{I}$ that with respect to a parallel line thru the center of gravity, d the distance between the axes, and m the mass of the body, then

$$I = \bar{I} + md^2; \quad \text{also} \quad k^2 = \bar{k}^2 + d^2$$

k and $\bar{k}$ denoting radii of gyration corresponding to the axes named respectively. (2) Let I_x, I_y, and I_z denote the moments of inertia of a body with respect to rectangular axes x, y, and z respectively; J_{xy}, J_{yz}, and J_{zx} its products of inertia with respect to yz and zx planes, zx and xy planes, and xy and yz planes respectively; I the moment of inertia of the body with respect to a line thru the origin of coordinates having direction-angles α, β, and γ; then

$$I = I_x \cos^2\alpha + I_y \cos^2\beta + I_z \cos^2\gamma - 2\,J_{yz}\cos\beta\cos\gamma - 2\,J_{zx}\cos\gamma\cos\alpha - 2\,J_{xy}\cos\alpha\cos\beta$$

Principal Axes and Moments of Inertia. The values of the moments of inertia of a body for all axes thru a given point are in general unequal; for one axis the moment of inertia is greater and for another it is less than for any other axis thru the point. These two axes are at right angles, and they together with one at right angles to their plane and passing thru the point are the principal axes of the body at the point; the corresponding moments of inertia are the principal moments of inertia of the body at the point. If

the point is the center of gravity of the body, then the axes and moments are called central principal axes and central principal moments of inertia.

If $J_{xy} = J_{yz} = 0$, the y axis is a principal axis at the origin,
If $J_{yz} = J_{zx} = 0$, the z axis is a principal axis at the origin,
if $J_{zx} = J_{xy} = 0$, the x axis is a principal axis at the origin.

If a body has a plane of symmetry, then any perpendicular to the plane is a principal axis of the body at the point where the line pierces the plane. If a body has two planes of symmetry at right angles to each other, then their intersection is a principal axis at any point of the intersection, the other two being in the planes of symmetry. If a body has three planes of symmetry their lines of intersection are the central principal axes of the body.

Special Cases. The bodies are supposed to be homogeneous; m = the mass of the body in each instance and δ = its density, that is its mass per unit of volume. In any system, like the C.G.S. or Engineers' (see Art. 29), $m = W/g$ and $\delta = w/g$; wherein W denotes the weight of the body, w its weight per unit volume, and g the acceleration of a freely falling body.

Straight Rod. Let l = its length; about a line making an angle α with the axis of the rod and passing thru its center of gravity $I = \frac{1}{12} ml^2 \sin^2 \alpha$; about a line thru one end of the rod $I = \frac{1}{3} ml^2 \sin^2 \alpha$.

Rod Bent into a Circular Arc (Fig. 112). $I_x = \frac{1}{2} mr^2 [1 - (\sin \alpha \cos \alpha)/\alpha]$; $I_y = \frac{1}{2} mr^2 [1 + (\sin \alpha \cos \alpha)/\alpha]$; about a line perpendicular to the plane of the arc and thru the center of the circle, $I = mr^2$.

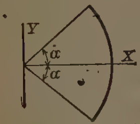


Fig. 112

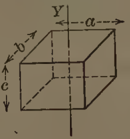


Fig. 113

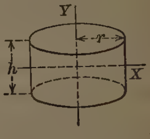


Fig. 114

Right Prism. Let h = its altitude and A = area of base; about any lin perpendicular to the bases, $I = \delta h \times$ moment of inertia of a base about th same line; about a line thru its center of gravity and perpendicular to th axis of the prism $I = \frac{1}{12} \delta A h^3 + \delta h \times$ the moment of inertia of the central cros section about the line.

Right Parallelopiped (Fig. 113). $I_y = \frac{1}{12} m (a^2 + b^2) = \frac{1}{12} abc\delta (a^2 + b^2)$; th y axis passes thru the center of gravity and is parallel to the edge c.

Right Circular Cylinder (Fig. 114). $I_y = \frac{1}{2} mr^2 = \frac{1}{2} \pi r^4 h\delta$; $I_x = \frac{1}{12}$ $(3r^2 + h^2) = \frac{1}{12} \pi r^2 h\delta (3r^2 + h^2)$. The y axis is the axis of the cylinder, and tl x axis is perpendicular to it and passes thru the center of gravity.

Hollow Right Circular Cylinder. Let R and r be the outer and inn radii, and axes taken as in Fig. 114. $I_y = \frac{1}{2} m (R^2 + r^2) = \frac{1}{2} \pi h\delta (R^4 - r$ $I_x = \frac{1}{4} m (R^2 + r^2 + \frac{1}{3} h^2) = \frac{1}{4} \pi (R^2 - r^2) h\delta (R^2 + r^2 + \frac{1}{3} h^2)$.

Right Rectangular Pyramid (Fig. 115). $I_y = \frac{1}{20} m (a^2 + b^2) = \frac{1}{60} abh\delta (a^2$ $b^2)$ and $I_x = \frac{1}{20} m (\frac{3}{4} h^2 + b^2) = \frac{1}{60} abh\delta (\frac{3}{4} h^2 + b^2)$; the y is the axis of the pyı mid, and the x axis passes thru the center of gravity and is parallel to side a

Right Circular Cone (Fig. 116). $I_y = \frac{3}{10} mr^2 = \frac{1}{10} \pi r^4 h\delta$, $I_x = \frac{3}{20} m (r^2$ $\frac{1}{4} h^2) = \frac{1}{20} \pi r^2 h\delta (r^2 + \frac{1}{4} h^2)$, $I_z = \frac{3}{20} m (r^2 + 4 h^2)$; the y is the axis of the co the x is parallel to the base and passes thru the center of gravity, and z is parallel to the base and passes thru the apex.

Frustum of a Cone. Let R and r = radii of larger and smaller bases, h = altitude; about the axis of the frustum $I = \frac{3}{10} m (R^5 - r^5)/(R^3 - r^3) = \frac{1}{10} \pi h \delta (R^5 - r^5)/(R - r)$.

Sphere. Let r = its radius; about any diameter $I = \frac{2}{5} mr^2 = \frac{8}{15} \pi r^5 \delta$.

Hollow Sphere. Let R and r = the outer and inner radii; about any diameter $I = \frac{2}{5} m (R^5 - r^5)/(R^3 - r^3) = \frac{8}{15} \pi \delta (R^5 - r^5)$.

Ellipsoid. Let $2a$, $2b$, and $2c$ = the lengths of the axes; about the axis $2c$, $I = \frac{1}{5} m (a^2 + b^2) = \frac{4}{15} \pi abc\delta (a^2 + b^2)$.

Paraboloid generated by revolving a parabola about its axis. Let h = its height and r = radius of base; about the axis of revolution $I = \frac{1}{3} mr^2 = \frac{1}{6} \pi h r^4 \delta$.

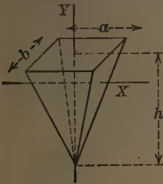


Fig. 115

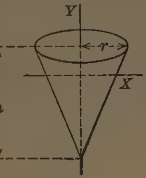


Fig. 116

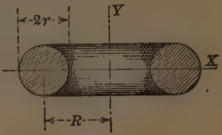


Fig. 117

Ring (Fig. 117). $I_y = m (R^2 + \frac{3}{4} r^2) = \frac{1}{2} \pi^2 R r^2 (4 R^2 + 3 r^2) \delta$, $I_x = m (\frac{1}{2} R^2 + \frac{5}{8} r^2) = \pi^2 R r^2 (R^2 + \frac{5}{4} r^2) \delta$; both axes, x and y, pass thru the center of gravity as shown.

DYNAMICS

28 Dynamical Quantities

Weight and Mass. In common parlance the word weight is used in at least two senses: thus a body is said to be heavy, its weight is 500 pounds, etc., the reference being to the earth-pull or gravity on the body; also a cask is said to contain much sugar, its weight is 500 pounds, etc., the reference here being to quantity of material, matter or stuff of a certain kind (sugar). Doubtless the legal definitions of our standards of weight imply the second sense, since they were framed primarily to standardize weight measures of commodities made in trade. For the sake of clearness, many writers restrict the use of the word weight to one meaning, namely the first, that is earth-pull; and to denote the second, quantity of substance, they employ the term mass. This usage is employed in the present chapter.

Force has been previously defined (Art. 15). Every so-called practical unit of force is a force equal to the earth-pull on a standard of mass, and the name given to such unit force is the same as the name given to the standard or unit of mass. Thus, a force equal to the weight or gravity of a pound mass is a unit of force, and that unit is called a pound force; a force equal to the weight or gravity of a kilogram mass is a unit of force and is called a kilogram force, etc. These units are called gravitational units.

Some writers seeking to make gravitational units of force absolute specify that the pound force, for example, is the weight or gravity of the pound mass at London or at sea level 45° latitude. However, the actual pound forces used in different places are the earth-pulls on pound masses at those places, and hence gravitation units of force as used are not absolute; the magnitudes of the units for two places are as the values of g for those places. To make the unit force absolute and to simplify certain dynamical equations, units of force have been proposed based on the following: In any system of dynamical units the

unit force is one which applied to the unit mass of that system produces the unit acceleration of that system. Thus in the C.G.S. system the unit of force is that force which applied to a gram mass gives it an acceleration of one cm per sec per sec; this unit is called dyne. And, in the F.P.(mass) S. system (never widely used, and now losing favor), the unit is the force which applied to the pound mass gives it an acceleration of one ft per sec per sec; it is called poundal.

Work. When the point of application of a force moves, so that the force has a component along the displacement of its application point, the force is said to do work; also the body exerting the force is said to do work. If the force is constant in magnitude and in direction and the displacement is straight, then the magnitude of the work is the product of the component of the force along the displacement and the displacement; if this component is in the direction of the displacemen the work is regarded as positiv; if opposite, the work is negativ. Thus in the displacements of the body C (Fig. 118) from A to B, the works of the several forces are respectively $+F_1s$, $-F_2s$, $+F_3 \cos \theta \cdot s$, $-F_4 \cos \phi \cdot s$, and o. If the force varies in magnitude or direction, or if the displacement of its application point is curved, the work must in general be computed by an integration; the general expression for work in

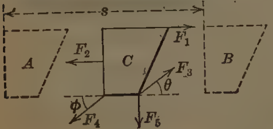


Fig. 118

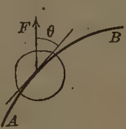


Fig. 119

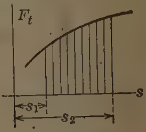


Fig. 120

this case is $\int F \cos \theta \cdot ds$, in which F denotes the force, ds elementary part of the displacement, and θ the angle between F and the direction of the elementary displacement (Fig. 119); the limits of integration must be assigned so as to include the entire displacement A to B. $F \cos \theta$ is the component of F along the tangent to the curve of displacement; denoting this component by F_t, the work is also given by $\int F_t ds$. The work can also be regarded as the product of the force and the component of the displacement along the action line of the force; this component is called the distance thru which the force acts, and so work is also said to equal force times distance thru which it acts. The work done by a force can be represented by a work diagram, which is constructed by plotting a line showing how the component of the force along the displacement F_t varies with the displacement (Fig. 120). The area included between the curve, the displacement axis, and two ordinates represents the work done by the force during the corresponding displacement $s_2 - s_1$. This diagram suggests also that the work done by a force equals the product of the average value of the tangential or working component and the displacement.

The Unit of Work depends on the units used for force and distance; thus, there are the foot-pound, the foot-ton, the dyne-centimeter (for which there is a special name, erg), etc. The joule is a practical electrical unit of work and equals 10 000 000 ergs; also the kilogrammeter, equal to 100 000 ergs. Work is also exprest in horse-power-hours, watt-hours, etc.; the horse-power-hour is the amount of work done in one hour at the rate of one horse-power, and the watt-hour is the amount of work done in one hour at the rate of one watt.

Power. By power of a force or agent doing work is meant the time-rate at which the work is done. Some units of power: foot-pound per second;

dyne-centimeter (or erg) per second; horse-power (abbreviated h.p.) = 550 foot-pounds per second, or 33 000 foot-pounds per minute; watt = one joule (10 000 000 ergs) per second; kilowatt = 1000 watts; the metric or French horse-power = 75 kilogram-meters per second.

1 ft-lb per sec = 0.13820 kg-m per sec	1 kg-m per sec = 7.233 ft-lb per sec
1 Engl. h.p. = 746 watts	1 kilowatt = 1.34 Engl. h.p.
1 Engl. h.p. = 1.0136 Fr. h.p.	1 Fr. h.p. = 0.9863 Engl. h.p.

Energy. When the condition or state of a body, or system, is such that it can do work, it is said to possess energy; and by its amount or store of energy is meant the amount of work which the body can do in passing to some standard state. Thus, a body in motion has energy, and the amount of its energy is the amount of work it can do in coming to rest; also a stretched spring has energy, and the amount of its energy is the amount of work which it can do in assuming its natural unstretched state. Energy is exprest in the same units as work, foot-pound, foot-ton, dyne-centimeter (erg), etc. Energy which a system has in virtue of its velocity is called kinetic energy. The KINETIC ENERGY of a particle of mass m moving with velocity v is $\frac{1}{2} mv^2$, and the kinetic energy of any body is the sum of the kinetic energies of its particles. In translation: kinetic energy = $\frac{1}{2} Mv^2 = \frac{1}{2} (W/g) v^2$, M denoting mass of the body, W its weight, v its velocity, and g acceleration of gravity; the last form gives energy in ft-lbs if W is exprest in lbs, v in ft per sec, and g is taken as 32.2. In rotation: kinetic energy = $\frac{1}{2} I\omega^2 = \frac{1}{2} Mk^2\omega^2 = \frac{1}{2} (W/g) k^2\omega^2 = (W/g) k^2 2\pi^2n^2$, I being the moment of inertia of the body with respect to the axis of rotation, k the corresponding radius of gyration, ω its angular velocity exprest in radians per unit time, W its weight, g the acceleration of gravity, and n the number of revolutions per unit time; the last form gives energy in ft-lbs if W is exprest in lbs, k in ft, n in revolutions per sec, and g is taken as 32.2. POTENTIAL ENERGY is that energy which a system has in virtue of its configuration. The earth and a body elevated above the earth's surface constitute a system having potential energy. Inasmuch as the energy can be withdrawn only from the elevated body it is said to possess the energy. The amount of potential energy possest by an elevated body (rigid or not) $= Wh$, W denoting the weight of the body and h the vertical distance thru which its center of gravity can descend.

Kinetic energy and potential energy are also called mechanical energy. There are other forms of energy which may not be mechanical, as thermal, chemical, and electrical. But the thermal energy of a body is generally regarded as due to the motions of the ultimate particle constituents of the body, thus being kinetic; much of chemical energy is regarded as due to the relative positions of ultimate particles, thus potential; the nature of electric energy is even less understood than that of chemical and thermal energy.

The Impulse of a force which remains constant in magnitude and direction for any time is the product of the magnitude of the force and the time. The unit of impulse depends on the units used to express the force and the time; thus there are the pound-second, dyne-second, etc. If the force F varies in magnitude but is constant in direction, then its impulse for the interval of time $t_2 - t_1$ is $\int_{t_1}^{t_2} F\,dt$. If the force varies in magnitude and in direction, its impulse can be computed from the impulses of its x, y, and z components; thus if F_x, F_y, and F_z denote these components of F, then the component impulses are $\int F_x\,dt$, $\int F_y\,dt$, and $\int F_z\,dt$, and the impulse of F equals the square root of the sum of the squares of the components. The angular impulse of a force about any line for an element of time is the product of the moment of the force about the line and the element of time,

that is, $M\,dt$, if M denotes the moment of the force; and the angular impulse for any interval of time $t_2 - t_1$ is $\int_{t_1}^{t_2} M\,dt$.

The Momentum of a particle is the product of its mass and velocity. The unit of momentum depends on the units of mass and velocity used; the dimensions of unit momentum are the same as those of unit impulse (see preceding). The momentum of a body is the resultant of the momentums of its particles. This resultant is not the scalar but the vector-sum of the momentums, that is, the resultant is to be computed as the resultant of a number of forces is, the separate momentums being regarded as having the directions of the velocities of the several particles and acting at the particles. In a translation the momentum is $Mv = (W/g)\,v$, M denoting the mass of the body, W its weight, v its velocity, and g the acceleration of gravity; the second form gives momentum in lb-sec if W is exprest in lbs, v in ft per sec, and g is taken as 32.2.

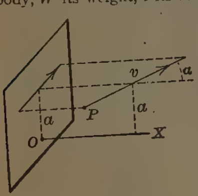


Fig. 121

The angular momentum of a particle about a line is the moment about that line of its momentum; thus, if the particle at P (Fig. 121) has a mass m and a velocity v, its momentum is mv acting in the line v and its angular momentum about OX is $(mv \sin \alpha)\,a$. The angular momentum of a body about a line is the algebraic sum of the angular momentums of its constituent particles. A rotating body has angular momentum about the axis of rotation equal to $I\omega = Mk^2\omega = (W/g)\,k^2\omega = (W/g)\,k^2\,2\pi n$, I denoting the moment of inertia of the body with respect to the axis, k its corresponding radius of gyration, W its weight, ω its angular velocity in radians per unit time and n in revolutions per unit time, and g the acceleration of gravity.

Dimensions of Units. All mechanical and nearly all physical units can be definec in terms of one or more of three units arbitrarily defined, that is, without reference tc other units. These three are fundamental units and the others derived units. Ir theoretical mechanics and physics the fundamental units chosen are those of length mass, and time; in applied mechanics, units of length, force, and time are generally more convenient. A statement of the way in which a derived unit depends on the fundamenta ones is a statement of the dimensions of the derived. Thus a unit of velocity depend on the units of length and time used, but is independent of the third fundamental unit and the magnitude of a unit of velocity varies directly as the unit of length and inversel as the unit of time used. Denoting the first set of fundamental units by L, M and T respectively, and the unit of velocity by V, the dimensional statement is written thus $V = L^1M^0T^{-1}$, and is called the dimensional formula for V.

The dimensional formulas may be used to test the accuracy of equations betwee mechanical quantities. Such an equation, if rationally and correctly deduced, is HOMOGENEOUS; that is, its terms are of the same kind. To ascertain whether terms are th same in kind, substitute for each quantity the dimensional formula for its unit, the treating the symbols L, M or F, and T as quantities, reduce each term; if the reduced term are alike, then the equation is correct dimensionally. Thus in the equation $24\,EIy$ $W(4l^3x - x^4)/l$, E denotes modulus of elasticity, I moment of inertia of an area, W a loa l, x, and y lengths, and the dimensional form is (dropping abstract members) $L^{-2}F^1L^4L^1$ $F(L^3L^1 - L^4)/L^1$, which reduces to $L^3F = L^3F - L^3F$, and so the original equation is corre dimensionally.

29. Dynamical Principles

Laws of Motion. (1) The normal state of the center of mass of a boc is one of rest or uniform rectilinear motion; a departure from this state due to force applied to the body. (2) When a single force acts upon a bod the center of mass sustains an acceleration in the direction of the force ar proportional to the force directly and to the mass of the body inversely;

several forces act on the body, the acceleration is given by the vector sum of the accelerations which would be produced by the forces acting singly. (By vector sum is meant the result reached by adding according to the parallelogram law.) (3) When one body exerts a force upon another the second also exerts one upon the other first, and the two forces are equal, colinear, and opposite. If F = force, m = mass, and a = acceleration, then the second law can be written $a \propto F/m$ or $a = kF/m$, k being a constant whose value depends on units used in F, m, and a. It is possible to make $k = 1$ by proper choice of units. Such choice was made in the so-called absolute systems of dynamical units; first units of length, mass, and time were selected and then as unit of F such a force which produces unit acceleration in unit mass. For example, in the C.G.S. system these units are respectively the centimeter, gram, second, and dyne, the latter about $\frac{1}{981}$ gram weight. The constant k may also be made unity in so-called gravitational systems; units of length, force, and time are first taken (that of force as the weight of something), and then the unit of m as that mass which sustains unit acceleration when acted on by unit force. For example, in the ENGINEERS' SYSTEM, these units are respectively foot, pound force, second, and the unit of m a mass equal to about 32.2 pounds. In any system in which $F = ma$, then also $m = W/g$, m and W being the mass and weight of the body and g the acceleration of gravity, all three quantities being exprest in units of that system.

Laws of Impulse and Momentum. When external forces act upon a body or collection of bodies they produce in general a change in the momentum of the body or collection. In any interval, the change in the component of the linear momentum along any line equals the algebraic sum of the components of the impulses of the forces along that line for the same time; and the change in any interval in the angular momentum about any line equals the algebraic sum of the angular impulses of the forces about that line for the same time. When no external forces are acting, the component linear momentum along any line and the angular momentum about any line remain constant; these are the principles of conservation of linear and angular momentum.

Principles of Work and Energy. In general, the kinetic energy of a body changes value during a change of its position or form, and the external and the internal forces acting on the body do work. The increment in the kinetic energy equals the sum total of work done by all the external and internal forces. In applying this principle, signs must be given to the works of the various forces as explained on p. 1480; then if the total work is positiv, the increment is positiv and there is a real gain in kinetic energy, and if negativ then there is a loss. If the body is a rigid one, the internal forces do no work in any displacement of the body, and the increment in kinetic energy equals the total work done by the external forces acting on the body. If a body or system of bodies is isolated so that it neither receives nor gives out energy, then its total store of energy, all forms included, remains constant; there may be a transfer of energy from one part of the system to another, but the total gain or loss in one part is exactly equivalent to the loss or gain in the remainder. This is the principle of conservation of energy.

30. Rectilinear Motion

Motion of a Point. The velocity of a moving point at any instant is the time-rate at which it is describing distance then; the symbol is v. In uniform motion (equal distances described in all equal intervals of time) the rate is constant and $v = \Delta s/\Delta t$, Δs denoting the distance described in the interval Δt. In non-uniform motion the rate varies, and $\Delta s/\Delta t$ is the average

velocity for the interval Δt; the actual velocity at any instant is the value of ds/dt for that instant. The acceleration of a point at any instant is the rate at which its velocity is changing then; the symbol is a. If v changes uniformly the rate is constant, and $a = \Delta v/\Delta t$, Δv denoting the velocity-change occurring in the interval Δt. If v does not change uniformly $\Delta v/\Delta t$ is the average acceleration for the interval Δt; the actual acceleration at any instant is the value of dv/dt for that instant. The general differential equations of rectilinear motion are $v = ds/dt$, $a = dv/dt = d^2s/dt^2$, $vdv = ads$; their integral forms are

$$s_2 - s_1 = \int_{t_1}^{t_2} v\,dt \qquad v_2 - v_1 = \int_{t_1}^{t_2} a\,dt \qquad v_2^2 - v_1^2 = \int_{s_1}^{s_2} a\,ds$$

in which s_1, v_1, and t_1 are corresponding or simultaneous values of s, v, and t, likewise s_2, v_2, and t_2.

Space-time, velocity-time, acceleration-time, velocity-space, acceleration-space curves respectively are graphs showing the relations between s and t, v and t, a and t, v and s, a and s (Figs. 122–126; they do not correspond to the same motion). (1) In the s-t diagram, the slope of the curve at any point represents the corresponding velocity. If AB

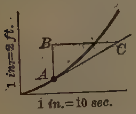


Fig. 122

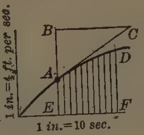


Fig. 123

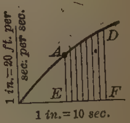


Fig. 124

and BC are measured by the s and t scales of the drawing respectively, then the slope equals the velocity; thus if $AB = 0.2$ in and $BC = 0.4$ in, $v = 0.4 \div 4 = 0.1$ ft per sec. (2) In the v-t diagram, the slopes represent the accelerations; if AB and BC are measured by the v and t scales respectively, then the slope equals the acceleration; thus if $AB = 0.3$ in and $BC = 0.5$ in, $a = 1.2 \div 5 = 0.24$ ft per sec per sec. The area included between any two ordinates (as AE and DF), the curve and the t axis represents the displacement of the moving point in the time EF. If the area is computed by multiplying its average ordinate measured by the velocity scale (thus giving the average velocity) by EF measured by the time scale, then the product equals the displacement; thus if the average ordinate is 0.35 in, and EF is 4 sec; the displacement $= 1.4 \times 4 = 5.6$ f

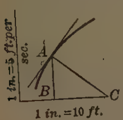


Fig. 125

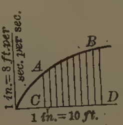


Fig. 126

(3) In the a-t diagram the slope represents the rate at which the acceleration is changing. The area included between any two ordinates (as AE and DF), the curve, and the t axis represents the velocity change in the time EF. Thus if the average ordinate is 0.3 in and EF is 2 sec then the velocity change $= (0.3 \times 20) \times 2 = 12$ ft per sec. (4) In the v-s diagram, the subnormals represent the accelerations. If the length of the subnormal is multiplied by the square of the velocity scale number and the product divided by the space scale number, the result will equal the acceleration; thus suppose that the subnormal $BC = 1/3$ inch, then $a = (1/3 \times 25) \div 10 = 0.83$ ft per sec per sec. (5) In the a-s diagram, the area included between two ordinates (as AC and BD), the curve, and the s axis represents the change in the velocity square. If the area is computed by multiplying the mean ordinate measured by the acceleration scale by CD measured by the space scale, the product equals the change in the velocity square; thus if the average ordinate $= 0.3$ in, and $CD = 0.4$, then the change $= 2.4 \times 4 = 9.$

Uniformly Accelerated Motion, constant acceleration. Let v_1 and v_2 denote the velocities at times t_1 and t_2, s_1 and s_2 the corresponding values of space, an

a = acceleration. Then the velocity change is $v_2 - v_1 = a(t_2 - t_1)$; the space traversed is $s_2 - s_1 = \frac{1}{2}(v_1 + v_2)(t_2 - t_1) = v_1(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$; also $v_2^2 - v_1^2 = 2a(s_2 - s_1)$.

Falling Body. The acceleration due to gravity (denoted by g) is approximately 32.2 ft per sec per sec. It changes slightly with elevation above sea level and with latitude; denoting elevation in feet by e and latitude by ϕ, then in ft per sec per sec

$$g = 32.0894\,(1 + 0.0052375 \sin \phi)(1 - 0.0000000957\, e)$$

At the equator sea level $g = 32.0894$ and at the poles 32.254; the extreme values for the United States are roughly 32.19 (in latitude 49° sea level) and 32.09 (in latitude 25° and 10 000 ft above sea).

A body dropt without initial velocity falls in a vacuum according to the laws

$$v = gt \qquad h = \tfrac{1}{2}gt^2 \qquad v^2 = 2gh$$

wherein v = velocity and h the descent at the time t. A body projected downward in a vacuum with initial velocity v_1 moves according to

$$v = v_1 + gt; \qquad h = v_1t + \tfrac{1}{2}gt^2, \qquad v^2 = v_1^2 + 2gh.$$

These three formulas apply to upward projection if the sign + is changed to −; h then denotes ascent in the time t, after projection. The total ascent (to highest position) and the corresponding time are respectively $v_1^2/2g$ and v_1/g.

Simple Harmonic Motion. If a point moves in a circle with constant speed, then the motion of the projection of the point on any diameter of the circle is simply harmonic; other motions resembling this are also called simply harmonic. Simple harmonic motion may also be defined as any rectilinear motion in which the acceleration is always directed toward a fixt point in the path and is proportional to the distance between that and the moving point. Taking the first definition, let P (Fig. 127) be the point moving in the circle and regard the motion of its projection on the vertical diameter. The period is the time of one revolution of P, or one complete oscillation of Q; the frequency is the number of revolutions of P, or oscillations of Q, per unit time; the amplitude is half the length of the path of Q or the radius of the circle; the displacement at any time is the ordinate of Q from the center of the path; the phase is the angle at any time from OX to OP (thus when Q is at the top of its path the phase is 90°, when at the center going down, 180°, when at the lowest point, 270° or − 90°). If, in the description of a simple harmonic motion, time is reckoned from the instant when the phase angle is not zero, then that motion is said to have a lead or a lag according as the initial phase (angle) is between 0 and 180° or 0 and −180°, and the value of the angle is called lead or lag as the case may be. Let ε = lead or lag, t = time elapsed after starting (motion from P_0 to P), ω = angular velocity of OP; T = period, n = frequency, r = amplitude, y = displacement, v = velocity of Q and a = its acceleration. The following are the general relations between the quantities involved:

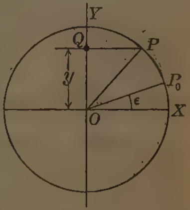


Fig. 127

$$n = 1/T \qquad \omega = 2\pi n = 2\pi/T \qquad y = r \sin(\omega t + \varepsilon)$$

$$v = \omega r \cos(\omega t + \varepsilon) = \omega\sqrt{r^2 - y^2} \qquad a = -\omega^2 r \sin(\omega t + \varepsilon) = -\omega^2 y$$

If the time is reckoned from the instant when Q is in its mid-position, $\epsilon = 0$. The three curves (Fig. 130) OA, $O'B$, OC are the space-time, velocity-time, and acceleration-time curves for one complete period of a simple harmonic motion; $\epsilon = 0$; Ot represents the period; the values of y, v, and a marked are for position Q shown. In Fig. 128 the curve is the

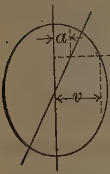


Fig. 128

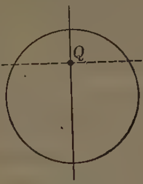


Fig. 129

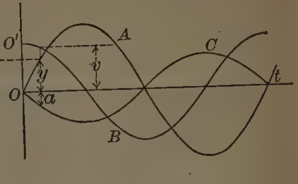


Fig. 130

velocity-space curve, and the inclined line the acceleration-space curve. They show how v and a vary with the displacement of the moving point; thus for the moving point Q v and a have values as marked.

Translation is a motion of a body such that the lines joining all pairs of points of the body remain fixt in direction. In any interval of time all points describe identical paths; at any instant all points have identical velocities and identical accelerations. Translations in which each point of the body describes straight lines are the common ones, and translation is defined sometimes as any rectilinear motion of a body. The resultant of all the forces acting on the body passes thru the center of gravity of the body; that force and the acceleration have at each instant the same direction, and $R = ma = (W/g)\,a$, wherein R = resultant force, a = acceleration, m and W = mass and weight of the body respectively, and g = the acceleration of gravity. Also if a_x = the x component of the acceleration at any instant and ΣF_x = the algebraic sum of the x components of all the external forces, then $\Sigma F_x = ma_x$; and similarly for y and z axes, the three axes being fixt and rectangular.

For example, consider the motion of a parallel rod of a locomotive, weighing 275 lb running at constant velocity of 60 mi per hr on a level track, the driver diameter being 5.5 ft and the crank length 1 ft. The forces acting on the rod are its weight and the pressures of the crank pins at its ends; these latter each are represented (Fig. 131) by their horizontal and vertical components. Since the resultant of all these forces acts thru the center of gravity, $V_1 = V_2$; also $2\,V_1 - 275 = (W/g)\,a_y = 8.55\,a_y$ and $H_1 - H_2 = (W/g)\,a_x = 8.55\,a_x$. To determine a_x and a_y: The velocity of the center of either crank pin relative to the locomotive is $(88 \times 1)/2.75 = 32$ ft per sec (60 miles per hr = 88 ft per sec), and the relative motion of the pin being circular at constant velocity, the relative acceleration is toward the center of the crank-pin circle at all times and equals $32^2/1 = 1024$ ft per sec per sec. This is also the absolute acceleration of the crank pin, since the locomotive is assumed to have no acceleration. But the rod has the same acceleration as the crank pin; hence $a_x = 1024 \sin\theta$ and $a_y = 1024 \cos\theta$. In the lowest position of the rod $\theta = 0$, $a_x = 0$, $a_y = 1024$, $H_1 = H_2$, $V = \frac{1}{2}(8755 + 275) = 4515$. In a mid-position when $\theta = 90°$, $a_x = 1024$, $a_y = 0$, $H_1 - H_2 = 8755$, and $V = \frac{1}{2}\,275 = 137.5$. In the highest position, $\theta = 180°$, $a_x = 0$, $a_y = -1024$, $H_1 = H_2$, and $V = \frac{1}{2}(275 - 8755) = -4240$, the negativ sign meaning that V acts downward on the rod.

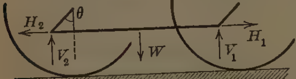


Fig. 131

If the translation is rectilinear and R is constant in value, then

$$Rt = mv_2 - mv_1 \quad \text{and} \quad Rs = \tfrac{1}{2}\,mv_2^2 - \tfrac{1}{2}\,mv_1^2$$

wherein v_1 and v_2 are the values of v at ends of any interval of time t, and s the distance described in that time. These formulas are the principles of impulse and momentum, and of work and energy for this special motion. If the translation is not rectilinear: According to the principle of impulse and momentum $\Sigma \int_{t_1}^{t_2} F_x dt = m v''_x - m v'_x$ and similar equations for y and z directions; the left-hand member is the algebraic sum of the x components of the impulses of the forces acting on the body for the time $t_2 - t_1$, and v'_x and v''_x are the x components of the velocity at the times t_1 and t_2. According to the principle of work and energy $\Sigma \int_{s_1}^{s_2} F_t \, ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$; the left-hand member is the algebraic sum of the works done by all the forces acting on the body whilst its velocity changes from v_1 to v_2, and the displacement is $s_2 - s_1$.

31. Curvilinear Motion

Curvilinear Motion of a Point. If the motion is uniform (equal distances traversed in all equal intervals of time), $v = \Delta s / \Delta t$, and if non-uniform, $v = ds/dt$, just as in rectilinear motion; the velocity of the moving point is directed along the tangent to the path at the point. But acceleration formulas for rectilinear motion do not hold for this case; here it is important to note that the velocity varies in direction continually, and that acceleration is the rate at which the velocity-changes (including magnitude and direction) occur. These complete velocity-changes for a given motion are exhibited by the HODOGRAPH for the motion. A hodograph is constructed by drawing from any point O' (Fig. 132) vectors which represent the velocities of the moving point at a number of its successive positions and then joining the free ends of the vectors by a smooth curve. The velocity-change occurring in any motion as from A to B is represented by the vector $A'B'$. If Δt is the time in which this occurred, then the magnitude of the average acceleration for that time is (chord $A'B'$)$/\Delta t$, $A'B'$ being measured by the scale of the hodograph diagram, and the direction of that average acceleration is that of the vector $A'B'$. The actual acceleration at A, say, is the limit of this average as B is taken closer and closer to A; that is, a at A is the value of ds'/dt at A' (s' denoting distance on the hodograph), and the direction of a is that of the tangent at A'. It will be noticed that the acceleration of the moving point is represented by the velocity of its corresponding point in the hodograph (A' is A's corresponding point).

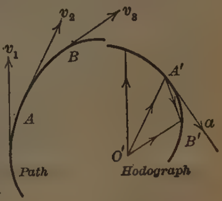


Fig. 132

Velocities and accelerations may be resolved; let α, β, and γ denote the direction angles of v (with respect to x, y, and z axes), v_x, v_y, and v_z the components of v parallel to these axes (axial components), x, y, and z the coordinates of the moving point. Then $v_x = v \cos \alpha = dx/dt$, $v_y = v \cos \beta = dy/dt$, $v_z = v \cos \gamma = dz/dt$. Let a_x, a_y, and a_z denote the axial components of a, and λ, μ, and ν the direction angles of a; then $a_x = a \cos \lambda = dv_x/dt = d^2x/dt^2$, $a_y = a \cos \mu = dv_y/dt = d^2y/dt^2$, $a_z = a \cos \nu = dv_z/dt = d^2z/dt^2$. Let a_t and a_n denote the components of a along the tangent and normal to the path, ϕ the angle between the tangent and a, θ that between normal and a, and r the radius of curvature at the moving point; then $a_t = a \cos \phi = dv/dt = d^2s/dt^2$, $a_n = a \cos \theta = v^2/r$. The tangential acceleration corresponds to change in value of velocity, and the normal acceleration to change in direction of velocity. When a point moves in a circle with constant speed v, then $a_t = 0$ and $a = a_n = v^2/r$, and a is directed along the radius toward the center of the circle.

Motion of a Projectile. In the following formulas air resistance is neglected; v_0 = velocity of projection, θ = angle of projection (Fig. 133), x and

y = coordinates of the projectile at any time t after projection, v = velocity, v_x and v_y = x and y components respectively of v, r = range on the horizontal plane thru O, and h = greatest height attained. The path of the projectile, or the trajectory, is a parabola as represented, and its equation is $y = x \tan \theta - gx^2/2\, v_0^2 \cos^2 \theta$. Also

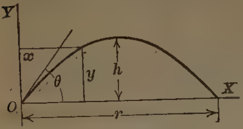


Fig. 133

$$v_x = v_0 \cos \theta \quad v_y = v_0 \sin \theta - gt \quad v = \sqrt{v_0^2 - 2\,gy}$$
$$x = v_0 \cos \theta \cdot t \qquad y = v_0 \sin \theta \cdot t - \tfrac{1}{2}\, gt^2$$
$$h = \sin^2 \theta \cdot v_0^2/2\, g \qquad r = \sin 2\,\theta \cdot v_0^2/g$$

If the direction of projection is horizontal, $\theta = 0$; the equation of the path is $y = -gx^2/2\, v_0^2$, and $x = v_0 t$ and $y = -\tfrac{1}{2}\, gt^2$.

Relative Motion. Description of motion requires the use of coordinate axes or other equivalent means of reference for the specification of positions of the moving point. Motion relative to a body is described by means of coordinate axes fixt in or on the body. Thus, suppose that in a given time a body A (Fig. 134) moves from A' to A'', and a point B from B' to B'': In the first position the coordinates of B are 0.3 and 0.2 in, and in the second 0.5 and 0.4. Then the x and y components of the relative displacement of B with respect to A are $0.5 - 0.3 = 0.2$ and $0.4 - 0.2 = 0.2$, and the total relative displacement is $\sqrt{0.2^2 + 0.2^2} = 0.28$ in. Motion relative to a point is described by means of coordinate axes having the point as origin and their directions remaining constant (relative to the body to which the motion of the point itself is referred). Thus suppose that in a certain time a point A moves from A' to A'' and another B from B' to B'' (Fig. 135); the x and y component displacements of B relative to A are 0.2 and 0.1 in respectively and the displacement of B relative to A is $\sqrt{0.2^2 + 0.1^2}$. The displacements, velocities, and accelerations of two points relative to each other at any instant are equal and opposite. The velocities of three points A, B, and C relative to each other are related as shown in Fig. 136; V_{ab} means velocity of A relative to B and V_{ba} means velocity of B relative to A. Among three points there are six relative velocities, and

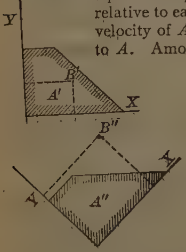


Fig. 134

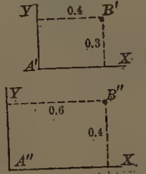


Fig. 135

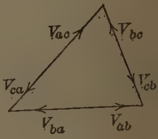


Fig. 136

when any two (except the velocities of two points with respect to each other) are known, the others can be computed by compounding as may be seen from the figure. Among three points there are six relative accelerations; their relations are similar to those of the velocities, and the figure shows the relation, V being now taken to mean acceleration.

Force and Acceleration. The forces acting on a body and the motion of its center of gravity are simply related; the algebraic sum of the components of all the external forces acting on the body along any line equa

the product of the mass of the body and the component of acceleration of its center of gravity along that line. In general, the principle leads to

$$\Sigma F_x = m\bar{a}_x, \quad \Sigma F_y = m\bar{a}_y, \quad \Sigma F_z = m\bar{a}_z$$

wherein m = mass = weight ÷ g; the left-hand members are the sums of the components of the forces along three rectangular axes, and $\bar{a}_x$, $\bar{a}_y$, and $\bar{a}_z$ are the corresponding components of the acceleration of the center of gravity. Or, if the directions of resolution be taken along the tangent to the path described by the center of gravity, the principal normal and a line perpendicular to the principal normal and tangent (the binormal), then

$$\Sigma F_t = m\bar{a}_t \quad \Sigma F_n = m\bar{a}_n \quad \Sigma F_b = 0.$$

The resultant of the normal components, ΣF_n, is called the centripetal force on the body; it is always directed inward toward the center of the curved path just as a_n is. The reaction corresponding to ΣF_n, exerted by the body on those that exert the centripetal force, is called centrifugal force.

For example, consider a car rounding a curve of radius r at a constant speed v. Imagine the rail pressure on each wheel resolved into three components, one parallel to the rails, one parallel to the ties, and one perpendicular to the track; call the resultants of these sets of components respectively R_1, R_2, and R_3 (Fig. 137). Also let P_1 and P_2 denote the pulls at the front and rear ends of the car. The components of the rail pressures parallel to the rails, P_1, and P_2, are practically parallel to the tangent to the path of the center of gravity of the car at that point. The velocity of the car is constant, so $\bar{a}_t = 0$; also $\bar{a}_n = v^2/r$. Hence

$$\Sigma F_t = P_1 - P_2 - R_1 - (W/g)\,\bar{a}_t = 0$$
$$\Sigma F_n = R_2 \cos\delta + R_3 \sin\delta = (W/g)\,\bar{a}_n = (W/g)\,v^2/r$$
$$\Sigma F_b = R_3 \cos\delta - R_2 \sin\delta - W = 0.$$

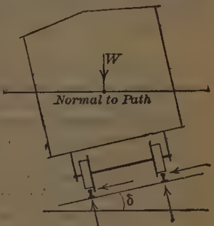


Fig. 137

The first equation shows that $P_1 - P_2 = R_1$, and the second and third solved simultaneously give $R_2 = (W/g)(v^2/r)\cos\delta - W\sin\delta$ and $R_3 = (W/g)(v^2/r)\sin\delta + W\cos\delta$. To make $R_2 = 0$, $(W/g)(v^2/r)\cos\delta = W\sin\delta$ or $\tan\delta = v^2/gr$. This value of tilt of track will not necessarily make each component rail pressure parallel to ties zero, but the resultant of all such components will be zero.

32. Rotation

Rotation of a Body is motion such that one straight line in the body or in its extension remains fixt. This line is the axis of the rotation, and all points of the body not on the line describe circles whose centers are on the line. All lines of the body perpendicular to the axis describe equal angles in equal times; angle described by the body means the angle described by any of the lines just mentioned. The angular velocity of the body at any instant is the rate at which it is describing angle then; the symbol is ω. In uniform rotation (equal angles described in all equal intervals of time) the rate is constant and $\omega = \Delta\theta/\Delta t$, $\Delta\theta$ denoting the angle described in the interval Δt. In non-uniform rotation, the rate varies and $\Delta\theta/\Delta t$ is the average angular velocity for the interval; the actual value of ω at any instant is the value of $d\theta/dt$ for that instant. The angular acceleration of a rotating body is the rate at which its angular velocity is changing; the symbol is α. If the ω changes uniformly the rate is constant and $\alpha = \Delta\omega/\Delta t$, $\Delta\omega$ denoting the velocity-change in the interval Δt. If ω does not change uniformly, then the rate varies, and $\Delta\omega/\Delta t$ is the average angular acceleration for the interval; the actual value of α at any instant is the value of $d\omega/dt$ at that instant.

The general differential equations of rotation are $\omega = d\theta/dt$, $\alpha = d\omega/dt = d^2\theta/dt$
$\omega d\omega = \alpha d\theta$; their integrated forms are

$$\theta_2 - \theta_1 = \int_{t_1}^{t_2} \omega\, dt \qquad \omega_2 - \omega_1 = \int_{t_1}^{t_2} \alpha\, dt \qquad \omega_2^2 - \omega_1^2 = 2\int_{\theta_1}^{\theta_2} \alpha\, d\theta$$

in which θ_1, ω_1, and t_1 are corresponding or simultaneous values of θ, ω, an
t, likewise θ_2, ω_2, and t_2. If the angular velocity is constant, the angul
displacement in the interval $t_2 - t_1$ is $\theta_2 - \theta_1 = \omega(t_2 - t_1)$; if the angul
acceleration is constant the velocity-change $\omega_2 - \omega_1 = \alpha(t_2 - t_1)$, the chan
in the velocity square, $\omega_2^2 - \omega_1^2 = 2\alpha(\theta_2 - \theta_1)$, and $\theta_2 - \theta_1 = ½(\omega_1 +$
$(t_2 - t_1)$ The relations between θ, ω, and t can
represented by curves exactly analogous to the cur
described on page 1486. The corresponding variab
are θ and s, ω and v, and α and a.

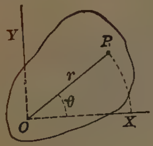


Fig. 138

Let the irregular line in Fig. 138 represent a body rota
about a line thru O perpendicular to the paper, OP a lin
the body and OX and OY fixt axes of reference; also θ = a
XOP, $r = OP$, v and a velocity and acceleration respectivel
P, a_t and a_n the components of a perpendicular and parall
OP, and ω and α the angular velocity and acceleration of
body. If the radian is used in θ, ω, and α, then

$$v = r\omega, \quad a_t = r\alpha, \quad a_n = r\omega^2, \quad a = r(\alpha^2 + \omega^4)^{1/2}$$

any units of time and length may be used. If n = the angular velocity in revolu
per unit time, $\omega = 2\pi n$, and this value may be substituted for ω in the foregoing
the rotation is uniform $\alpha = 0$, and $a = r\omega^2 = v^2/r = 4r\pi^2 n^2$.

Relation between Forces and Motion. The angular acceleration
rotating body depends only on the moments of all the external forces a
on the body about the axis of rotation and on the moment of inertia o
body about the same axis. If ΣM = the algebraic sum of moments o
forces, I the moment of inertia, m the mass of the body, W its weight, a
its radius of gyration with respect to the axis of rotation, then

$$\Sigma M = I\alpha = mk^2\alpha = (W/g)\, k^2\alpha$$

Let ω_1 and ω_2 = the angular velocities of a rotating body at time t_1 a
later time t_2; then according to the principle of angular impulse and mome

$\Sigma \int_{t_1}^{t_2} M\, dt = I\omega_2 - I\omega_1$. The left-hand member is the algebraic sum o
angular impulses, about the axis of rotation, of all the external forces f
time $t_2 - t_1$. If the moments of all the forces are constant during the
the left-hand member becomes $\Sigma M(t_2 - t_1)$. According to the pri

of work and energy $\Sigma \int_{s_1}^{s_2} F_t ds = ½ I\omega_2^2 - ½ I\omega_1^2$. The left-hand mem
the algebraic sum of the works done on the body by all the external
for the interval $t_2 - t_1$.

Centrifugal Force. Suppose that P is a particle of a body rotatin
angular velocity ω, and let dm = mass of P, r = the radius of the circle de
by P. The resultant of all the forces acting on P may be resolved in
components, one along the radius r and one along the tangent to the
P; the radial or normal component = $dmr\omega^2$ and acts from P toward
of rotation; the tangential component = $dmr\alpha$. The normal compo
called the centripetal force on P; it (and the other component also) is
on P by other things (neighboring particles, usually). P reacts on th
things with a force whose radial and tangential components equal
above but in the opposite directions. The normal component, dm

ward, is called the centrifugal force of P; the resultant of the centrifugal forces of all the particles is the centrifugal force of the body; in general the resultant of the centrifugal forces of all the particles is not a single force, but in certain common cases there is a single force resultant. Thus if m = mass of the body($= W/g$), $\bar{r}$ the radius of the circle described by the center of gravity G (Fig. 139), n = revolutions per unit time ($\omega = 2\pi n$), and the body is homogeneous, then (*a*) when the body is symmetrical about a plane perpendicular to the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$ and acts in the line OG; (*b*) when the body is symmetrical about a line parallel to the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$ and it acts in the line OG; (*c*) when the body is symmetrical about a plane containing the axis of rotation, the centrifugal force $= m\bar{r}\omega^2$, acts in the plane of symmetry, is parallel to OG at a distance from any XY plane equal to $\left(\int dmxz\right) \div m\bar{r}$. If ω is in radians per sec, n in rev per min, $\bar{r}$ in ft, W (weight of body) in lb, $\bar{v}$ (velocity of center of gravity) in ft per sec, then centrifugal force in lbs is

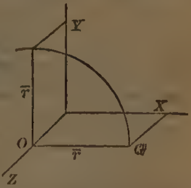


Fig. 139

$$\frac{W}{g}\bar{r}\omega^2 = \frac{W}{g}\frac{\bar{v}^2}{\bar{r}} = \frac{W\bar{r}}{g}\frac{4\pi^2 n^2}{3600} = \frac{W\bar{r}n^2}{2937} = 0.000341\, W\bar{r}n^2$$

When a body is rotating with constant angular velocity, then the resultant of all the actual external forces acting on the body (weight, reactions of axle bearings, belt pulls, etc.) is equal and opposite to the resultant centrifugal force of the body. For example, in Fig. 140 PC is a rod supported by a pivot at P and by a cord AB so that the rod can be rotated about AP. During rotation it is under the action of the pivot pressure represented by components P', P'', and P''' (not shown), the pull of the cord T, the weight of the rod W, air resistance, and the driving force F (not shown). When the speed is constant, then the resultant of all these is as described under (*c*) above. If air resistance and pivot friction are negligible, no driving force is required to maintain speed, and $P''' = 0$; T, P', and P'' can be determined as follows (supposing $PC = 6$ ft, $PB = 5$ ft, $PG = 3$ ft, $ABP = 90°$, $APB = 30°$, $W = 100$ lbs, and $n = 60$ rev per min): Since the resultant of T, P', P'', and W is horizontal, the sum of their vertical components $= 0$, or $T \sin 30 + P'' - 100 = 0$; the sum of their horizontal components $= m\bar{r}\omega^2$, or $T \cos 30° + P' = (100/32.2)\, 1.5\, (2\pi 60/60)^2 = 184$; the resultant acts at a point Q whose distance above PX equals $\int dmxz \div m\bar{r} = 3.46$ ft, and $PQ = 4$ ft; the moment sum of all the forces about $Q = 0$, or $-T \times 1 - 100 \times 1/2 + P' \times 3.46 + P'' \times 2 = 0$. The three equations solved simultaneously give $T = 157.5$, $P' = 47.5$, and $P'' = 22.5$ lb.

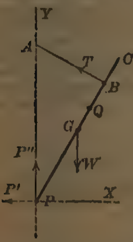


Fig. 140

33. Balancing

The reacting forces which support a rotating body depend in general not merely on the weight of the body and applied forces but also on the speed, the mass, and shape of the body; moreover, the components of the supporting forces which depend on the speed, mass, and shape change direction continually. Thus in the preceding illustration, when the rod is not rotating $T = 30$ lb, $P' = 26$ lb, and $P'' = 85$ lb, values quite different from those for motion. Or, consider a body weighing 100 lbs mounted on a shaft supported in bearings equally distant from the body and the center of gravity 1 in from the axis of the shaft. When the system is at rest, the bearing

pressures (due to the body) = 50 lbs each and act vertically, but when the system rotates at constant speed the bearing pressures have additional components each equal to ½ $(100/32.2)(1/6)\,4\pi^2n^2$, n being the speed in rev per sec. These components are said to be due to centrifugal action or centrifugal force, and will be called centrifugal components. If the center of gravity of the rotating system is in the axis of rotation, the system is said to have standing balance; such a system with shaft horizontal would not turn of itself from any position into which it might be placed (see Fig. 141). If the centrifugal forc of the rotating system is zero, then th system is said to have or be in runnin

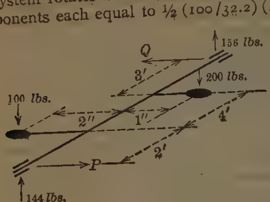


Fig. 141

balance. Standing balance is a necessary condition for running balance bu not a sufficient one; thus in Fig. 141 there is standing but obviously not run ning balance. In fact the centrifugal forces of the two bodies are equa $(100/32.2)(1/6)\,4\pi^2n^2$ and $(200/32.2)(1/12)\,4\pi^2n^2$ or $20.4\,n^2$, and opposite bu not colinear and so they constitute a couple. The centrifugal components o the reactions of the bearings are therefore each $20.4\,n^2 \times 4 \div 9 = 9.07\,n^2$, an directed as shown, P and Q. If these bearings were not firmly supported, the the rotating system would wobble.

Balancing a system out of running balance requires the rearrangement of the rotati material or the addition or removal of material. Car wheel-pairs are balanced expe mentally by running them in spring-supported bearings and adding material here a there to the wheels, by trial, until steady running is secured. An unbalanced system c always be balanced by the addition of two bodies rotating about any two arbitrar selected points on the axis of rotation; moreover the weights of these bodies and th exact positions relative to the rotating system can be computed provided that the centr ugal force of the unbalanced system is known or if the unbalanced system can be divic up into parts the centrifugal forces of which are known. Thus suppose that it is desi to balance the body of weight W (Fig. 142 or 143), out of center a distance r (center

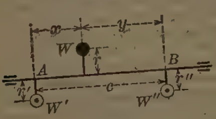


Fig. 142

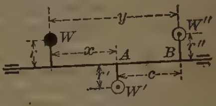


Fig. 143

gravity to axis of shaft $= r$) by means of two bodies rotating about A and B. In o that there may be running balance: (1) the centers of gravity of the three bodies and axis of the shaft must be in a plane, and (2) the sum of moments of the centrifugal fo of the three bodies must be zero about any point in the plane. Condition (2) requ that the middle one of the three bodies, counting along the shaft, shall be alone on side of the shaft, and further, that

$$W'r' = Wry/c \quad \text{and} \quad W''r'' = Wrx/c$$

where W' and W'' are the weights of the balancing bodies rotating about A and B re tively and r' and r'' are their distances out of center. Any unit of weight and of dis may be used in these formulas.

For example, consider the balancing of the cranks and crank pins on a pair of locom drive wheels. In Fig. 144 the cranks are set 90° apart, the planes of rotation of centers of gravity are 62 in apart, and the planes of rotation of the centers of grav the pins are 72 in apart; for each crank pin $W = 25$ lbs and $r = 12$ in; for the unbal part of each crank $W = 30$ lbs and $r = 5$ in.

The counter (or balancing) weights are to rotate about A and B with radii $=r$ inches say. To balance P_1 requires a_1 and b_1 (see figure), such that for a_1, $Wr = (25 \times 12)66/60 = 330$, and for b_1, $Wr = (25 \times 12)6/60 = 30$ in-lbs; similarly to balance P_4 requires a_4 and b_4, and for a_4, $Wr = 30$ and for b_4, $Wr = 330$ in-lbs. To balance P_2 requires a_2 and b_2 and so that for a_2, $Wr = (30 \times 5)61/60 = 152.5$ in-lbs, and for b_2, $Wr = (30 \times 5)1/60 = 2.5$ in-lbs; similarly to balance P_3 requires a_3 and b_3, and for a_3, $Wr = 2.5$ and for b_3, $W_r = 152.5$ in-lbs. These eight bodies a_1, b_1, a_2, b_2, etc., would balance the cranks and pins; it remains to find two substitutes for the eight. Now a_1 and a_2 can be combined, giving $Wr = 330 + 152.5 = 482.5$; a_3 and a_4, giving $2.5 + 30 = 32.5$; b_1 and b_2, giving 32.5; and b_3 and b_4, giving 482.5 in-lbs. The centrifugal forces of a' and a'' (whose $Wr = 482.5$ and 32.5) have a simple resultant, and a single body whose centrifugal force is identical with that resultant can be found thus: treat 482.5 and 32.5 as tho they were forces acting from A to a' and A to a'' and find their resultant; it is 484 in-lbs directed along Aa, the angle $a'Aa$ being nearly 4°; then any body the radius to whose center of gravity lies along Aa and whose $Wr = 484$ in-lbs will serve as one counterweight. In a similar manner the other counterweight may be determined; its $Wr = 484$ and its radius lies along Bb. (It should be noted that the effects of coupling and connecting rod are not considered.)

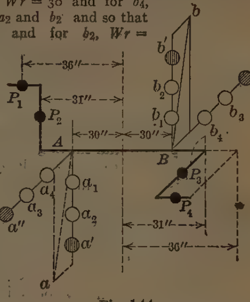


Fig. 144

34. Pendulums.

The Compound or Physical Pendulum is a body suspended on a horizontal axis so it can oscillate freely under the influence of gravity. Let G (Fig. 145) be the center of gravity of the pendulum, O the center of suspension, $a = OG$, and $k =$ the radius of gyration of the pendulum with respect to its axis of suspension, and $T =$ the time of one oscillation (from one extreme position of the pendulum to the opposite extreme position); then if the oscillation is small, $T = \pi\sqrt{k^2/ag}$. If $\alpha =$ the maximum angle which OG makes with the vertical, that is, if 2α is the complete angle described by the pendulum in one oscillation, then more correctly,

Fig. 145

$$T = \pi\sqrt{\frac{k^2}{ag}}\left[1 + (½)^2 \sin^2\frac{\alpha}{2} + (¼ \cdot ¾)^2 \sin^4\frac{\alpha}{2} + \cdots\right]$$

If $\alpha = 8°$, the bracket is only 1.00122 and nearer unity for smaller values of α. T for a given pendulum is hence practically independent of a, if a be small; that is, for small angles a pendulum is practically isochronous. The point Q in the line OG extended and $OQ = k^2/a$ is called the center of oscillation corresponding to the center of suspension O; it is also called center of percussion (Art. 37). The centers of suspension and oscillation are interchangeable, that is, the times of oscillation about axes thru O and Q are equal. This property is made use of in pendulum determinations of g as follows: the pendulum is made so that it can be suspended from two points O at a known distance d apart; then the times of oscillation for the two points O are compared and by trial are made equal by shifting a weight along the stem of the pendulum; either of these points then is the center of oscillation for the other as center of suspension, and $d = k^2/a$; then $g = \pi^2 d/T^2$.

A Simple Pendulum is a very small sphere supported by a cord; its length is the distance from the center of the sphere to the point of suspension. A physical and a simple pendulum whose times of oscillation are equal are said to be equivalent. A seconds pendulum is one whose time of oscillation is one second. The length of the equivalent seconds pendulum at New York is 39.101 inches.

The Conical Pendulum consists of a body suspended from a fixt point and so that it can rotate about a vertical axis thru the point. The height of the pendulum h (Fig. 146) depends only on the angular velocity; thus for n revolutions per unit time $h = g/4\,\pi^2 n^2$ If g is taken as 32.2 ft per sec per sec, then n must be taken in rev per sec; h will be in feet. If n is less than $\sqrt{g/l} \div 2\,\pi$, the ball will not fly out or remain in any deflected position. If P = tension in the cord, l = length of pendulum, W = weight, then $P = Wl\;4\,\pi^2 n^2/g$.

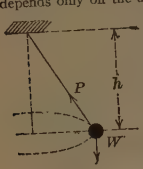


Fig. 146

35. Uniplanar Motion

Uniplanar Motion is a motion such that each point of the moving body remains at a constant distance from a fixt plane. Examples: motion of the connecting rod of a steam engine, rolling of a wheel, rotation of a fly wheel, etc. All straight lines of the body which are parallel to the plane of the motion describe equal angles during any displacement of the body, and by angle described by the body is meant the angle described by any of the lines mentioned. The angular velocity of the body at any instant is the rate at which it is describing angle then. The angular acceleration at any instant is the rate at which its angular velocity is changing then. Any displacement may

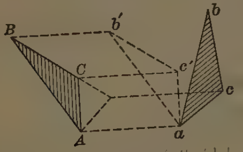


Fig. 147

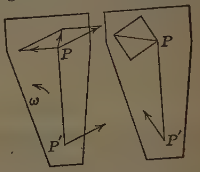


Fig. 148

be accomplished by a translation of the body which will bring one line of it which is perpendicular to the plane of the motion into final position followed by a rotation of the body about that line into final position. T the displacement of a body from ABC to abc (Fig 147) may be accomplish by a translation from ABC to $ab'c'$ followed by rotation about a into posi abc. The amount of translation depends on the line of the body selected axis of the rotation; the amount of the rotation does not The actual state of (uniplanar) motion of a body at any instant may be regarded as consisting of two components, a translational and a rotational. And the velocity v of any point P of the body (Fig. 148) may be regarded as the resultant of two velocities, one the actual velocity v' of a second point P', and the other the velocity of P about (relative to) P', $r\omega$, where ω = angular velocity of the body and $r = PP'$. Likewise the acceleration a of P may be regarded as the resultant of the actual acceleration a' of P' and the acceleration of P about (relative to) P'.

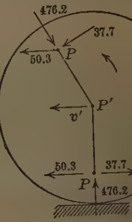


Fig. 149

For example, when a wheel 8 ft in diameter rolls uniformly at 2 rev per sec, or $\omega = 12.56$ radians per sec, then taking P' at the center (Fig. 149), $v' = \pi 8 \times 2 = 50.3$ ft per sec, and $a' = 0$; the velocity of a point P 3 ft from the center relative to P' is 37.7 ft per sec direc shown, and the actual velocity of P is the resultant of 50.3 and 37.7; the accelerat P relative to P' is $3\,\omega^2 = 476.27$ ft per sec per sec (see page 1490) directed as and since $a' = 0$, the acceleration of P is 476.2.

Uniplanar Displacement can also be accomplished by a single rotation about some line. Thus the displacement from ABC to abc (Fig. 147) can be made by a rotation about the intersection of the perpendiculars to Aa and Bb at their centers. Any uniplanar motion may be regarded as a continual rotation about a line which in general is continually shifting its position in space and in the body or its extension This line is called INSTANTANEOUS AXIS (of rotation), and any point of it the instantaneous center. In uniplanar motion, the velocity of any point of the body at any instant equals the product of the angular velocity of the body then and the distance of the point from the instantaneous center, and the direction of the velocity is perpendicular to the line joining the point and the instantaneous center; the velocities of different points of the body are as their distances from the instantaneous axis. The instantaneous center may be determined when the directions of the velocities of any two points of the body (the line joining which is parallel to the plane of the motion) are known, provided these directions are not parallel, as follows: at each of the two points draw a line parallel to the plane of motion and perpendicular to the direction of the velocity of that point, and determine their intersection; this is the instantaneous center.

For example, consider the motion of the connecting rod of an engine. The velocity of the crank end of the rod is perpendicular to the crank and the velocity of the other end is parallel to the guides; hence for any particular position of the rod the instantaneous center is at the intersection of the crank (radius) extended and a line perpendicular to the guides at the guide end of the rod.

The relation between the external forces and the motion produced is exprest thus: take x and y axes of reference parallel to the plane of motion; then

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma M = I\alpha = mk^2\alpha = (W/g)\,k^2\alpha$$

wherein ΣF_x and ΣF_y denote the sums of the x and y components of all the external forces, $\bar{a}_x$ and $\bar{a}_y$ the x and y components of the acceleration of the center of gravity, α the angular acceleration of the body, m its mass, W its weight, ΣM the algebraic sum of the moments of the external forces about an axis thru the center of gravity and perpendicular to the plane of motion, I the moment of inertia, and k the radius of gyration of the body with respect to the same axis. For example, consider a cylinder of radius r and weight W rolling down an inclined plane. The forces acting on the cylinder are its weight W and the reaction of the plane regarded as replaced by its two components N and F (Fig. 150); taking the x axis along the plane and the y normal to it, the three formulas become $(W \sin\phi - F) = (W/g)\bar{a}$, $N - W\cos\phi = 0$, and $Fr = \frac{1}{2}(W/g)\,r^2\alpha$, $\frac{1}{2}(W/g)\,r^2$ being the moment of inertia of the cylinder with respect to its axis. (It is assumed that there is no indentation of the roadway, that is, no rolling resistance.) If there is no slipping, then $\bar{a} = r\alpha$, which equation combined with first and third above gives $\bar{a} = \frac{2}{3}g\sin\phi$, $\alpha = \frac{2}{3}(g/r)\sin\phi$, and $F = \frac{1}{3}W\sin\phi$; also $N = W\cos\phi$.

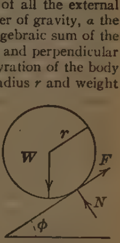


Fig. 150

The Kinetic Energy of translation and rotation combined is $\frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, where m denotes the mass of the body, $\bar{v}$ the velocity of its center of gravity, $\bar{I}$ its moment of inertia with respect to a line thru the center of gravity and perpendicular to plane of the motion, and ω its angular velocity. The first term is sometimes called the translational and the second the rotational part of the kinetic energy. It is also given by $\frac{1}{2}I\omega^2$, where I denotes the moment of inertia of the body with respect to its instantaneous axis (at the instant for which the energy is to be figured) and ω is the angular velocity.

For example, take a cylinder of radius r and weight W rolling at n revolutions per unit time: $m = W/g$, $v = 2\pi rn$, $\bar{I} = \frac{1}{2}(W/g)\,r^2$ (see page 1478), and $\omega = 2\pi n$; hence the kinetic energy $= 3\,(W/g)\,\pi^2r^2n^2$, one-third of which is rotational. Also $I = \bar{I} + (W/g)r^2 = \frac{3}{2}(W/g)r^2$; hence according to the second formula the kinetic energy is $\frac{1}{2}(3\,W/2\,g)\,r^2\,4\,\pi^2n^2 = 3\,(W/g)\,\pi^2r^2n^2$ as before.

36. Three-dimensional Motion

Any motion of a body may be regarded as consisting of two compone one a translation equal to that of the center of gravity and the oth rotation about some axis thru the center of gravity. These motions ma said to be produced independently by the forces acting on the body; (a) the acceleration of the center of gravity is the same as if the whole were concentrated at the center of gravity and acted upon by forces e in magnitude to and same in direction as the actual external forces; and (b angular acceleration is the same as if the center of gravity were fixt anc actual external forces applied. The reasonableness of this will probabl seen from this: imagine each force acting on the body replaced by a acting at the center of gravity G and a couple; the resultant of all the f acting at G is a single force R, and the resultant of all the couples is a s couple C; R cannot turn the body but gives it a motion of translation and C cannot move G but merely turns the body about some line thr In general C does not cause turning about a line perpendicular to the of C, only so if the plane of C is perpendicular to one of the principal tral axes of the body (see page 1491). To determine the acceleration center of gravity, take fixt x, y, and z axes outside the body and resol external forces F_1, F_2, etc., into x, y, and z components; then

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma F_z = m\bar{a}_z$$

m denoting the mass of the body. To determine the angular accele of the body, take moments of all the forces F_1, F_2, etc., about the three c principal axes; calling the sums of the moments about these axes ΣM_1, and ΣM_3; the components of the angular acceleration α_1, α_2, and α components of the angular velocity ω_1, ω_2, and ω_3; then

$$\Sigma M_1 = I_1\,\alpha_1 + (I_3 - I_2)\,\omega_2\,\omega_3, \quad \Sigma M_2 = I_2\,\alpha_2 + (I_1 - I_3)\,\omega_3\,\omega_1,$$
$$\Sigma M_3 = I_3\,\alpha_3 + (I_2 - I_1)\,\omega_1\,\omega_2$$

wherein I_1, I_2 and I_3 denote the three central principal moments of of the body. In any motion of a body: the kinetic energy may be co in two parts, (1) the kinetic energy of the whole body moving with a equal to that of the center of gravity and (2) the sum of the kinetic e of the constituent particles of the bodies their velocities relative to the center of gr

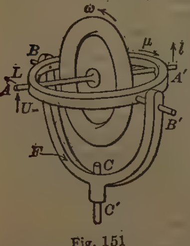


Fig. 151

Gyrostat (Fig. 151). The wheel ω a can rotate about AA', the supporting ring can rotate about BB', and the supportin F can rotate about CC'. If the disk or spinning rapidly in the direction indicat and an upward force U be applied at A, AA' will not rotate about BB' but abou the direction indicated by the arrowhe downward force causes rotation in the direction. A horizontal force L to the le cause the axle AA' to rotate not about about BB' in the direction indicated arrow l; a force to the right would cause in the opposite direction. These rotations of the axle of spin are ca cessional motions, the wheel being said to precess. The precession o of spin, AA', tends to convert the existing spin of the wheel into a s the axis of the applied moment and in the direction required by that

A Mono-rail Car, as its name implies, runs on a single rail; and the Brennan car is wholly above the rail. Stability of this car is furnished by two gyrostats mounted in the car as shown in plan and elevation (Fig. 152). W is one of the gyrostat wheels (W' the other); it is mounted rigidly on the axle A, the bearings of which are on the frame or case C. (These cases wholly enclose the gyrostat wheels, and a vacuum is maintained in the cases to reduce the windage of the wheels; the wheels are driven electrically—indeed the wheels are motor armatures—in opposite directions.) The case C has trunnions T and T supported in bearings on the double frame F, which also supports the other gyrostat case C'. The double frame F is supported on an axle X parallel to the rail, on which the whole gyrostat system can turn. To the two upper trunnions T and T' are keyed two spur-toothed segments which mesh at G; thus the cases C and C' can rotate on their trunnions only together, in opposite directions, and equal amounts. R and R' are wheels or rollers which turn on bearings fixt to the cases C and C'. Rigidly fastened to the car are four shelves M, M', N, and N'; as shown in the plan these do not extend beyond the line OO'. When for any reason the axles A and A' are at OP and $O'P'$ say, or OQ and $O'Q'$, and the car is tipping either way relative to the gyrostat system, then one shelf, and only one, will come up against an axle end or roller; for instance when the axles are at OP and $O'P'$ and the car tips clockwise, then the shelf M comes up against the axle A.

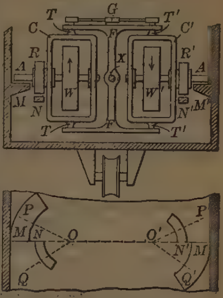


Fig. 152. (After Perry)

When the car is running straight, erect, and balanced, the axles A and A' are in line, across the car, and this is the normal position of the gyrostats. Now suppose a disturbance, as a wind pressure on the left side, tips the car clockwise; the shelf M comes up against the axle A and causes precession of A away from the reader, which continues as long as there is contact at A. The rapidly rotating axle slips on the shelf M and is itself subjected to a friction which tends to hurry the precession, the effect of which is to increase the pressure on the shelf. This pressure down arrests the tipping and pushes the car back against the wind to the neutral position, in which the wind and gravity moments are balanced. This neutral position is reached with a certain momentum, and the car is carried past it, the shelf M recedes from A with the axles of the gyrostats in positions as OP and $O'P'$, and eventually the shelf N' comes up under the roller R'. The upward pressure on the roller causes precession of A' toward the reader, which continues as long as there is contact at the shelf N'. The friction of the roller on its axle tends to retard this precession, and the correcting pressure of the roller on the shelf N' is not so great as that of the axle A on M (in similar circumstances), and the gyrostats are returned to the normal position with the car still tilted to the left of the neutral position. But the gyrostats reach the normal position with velocity and swing beyond it; the roller R' runs off the end of the shelf N' and the axle A' comes into contact with the shelf M'. The pressure of M' on A' causes precession of A' toward the reader, the friction on A' tends to hurry the precession, and A' reacts strongly on the shelf M', bringing it to its neutral position and beyond. Eventually M' recedes from A', the shelf N coming up against the roller R, with the gyrostats in the positions OQ and $O'Q'$. The shelf N, like N', brings the gyrostats into normal position and then shelf M into play; and so the operations described are repeated. But the oscillations of the car and the swings of the gyrostats rapidly decay, and the car settles down into its neutral position quickly. Geared together and spinning in opposite directions, the two gyrostats work together in the above described actions. A single gyrostat might furnish the stability on a straight track, but two gyrostats are essential for running on curves. In Oct., 1909, Mr. Brennan operated a mono-rail car 40 ft long, weighing 22 tons, and designed to carry a load of 10 to 15 tons; it is furnished with an 80 and a 20 H.P. petrol-electric set for running the car and the gyrostats; each gyrostat weighs about 3/4 ton, is 3 ft 6 in in diameter, and is run at 3000 R.P.M.; the vacuum of the gyrostat cases is equivalent to about 1/2 to 5/8 in of mercury. The car was run on a curve of 105 ft radius at 7 miles per hour, and over reverse curves of 35 ft radius without "appreciable disturbance" of the level of the car floor.

37. Impact

Direct Central Impact is such that before collision the centers of gravi of the bodies move along the same straight line. Central impact is collisio during which the forces which the bodies exert on each other are direct along the line joining the centers of gravity. In any collision, the forc which the two bodies exert on each other are equal and opposite at ea instant; hence the total impulses of these forces during the collision are equ and opposite, and according to the principle of impulse and momentum t changes in the momentum of the bodies produced by the collision must equal and opposite; or, otherwise stated, the total momentum of the t bodies is unchanged by the collision. Or, for direct central impact:

$$m_1v_1 + m_2v_2 = m_1V_1 + m_2V_2$$

wherein m_1 and m_2 = the masses of the bodies, v_1 and v_2 their velocities bef and V_1 and V_2 their velocities after the collision; but in numerical substi tion velocities in one direction are given the same sign and those in the ot direction the opposite sign.

Experiments on direct central impact of spherical bodies have shown that the rela velocities of spheres after impact are always less than before the impact and that t relative velocities are opposite in direction. The ratio of the relative velocities impact to that before impact is called coefficient of restitution; it seems to depend on the material of the impinging spheres. For glass the coefficient is 15/16, for steel cork 5/9, ivory 8/9, wood about 1/2, clay and putty 0. If e = the coefficient, then

$$(v_1 - v_2) = -e\,(V_1 - V_2)$$

This equation and the preceding one solved simultaneously show that

$$V_1 = v_1 - \frac{(1+e)\,m_2}{m_1+m_2}(v_1 - v_2) \qquad V_2 = v_2 - \frac{(1+e)\,m_1}{m_1+m_2}(v_2 - v_1)$$

During impact there is, in general, loss of kinetic energy; the loss is $\frac{1}{2}(v_1 - v_2)^2(1 - e^2)\,m$ (m_1+m_2). Bodies for which $e = 0$ are said to be inelastic; and those for wh is nearly 1 are said to be nearly perfectly elastic. When a sphere is dropt on a zontal surface of a large body from a height h, and if H = the height of rebound, $H = e^2h$. This equation furnishes a means of computing e.

Force of a Blow. By this is meant the actual pressure between bodies in collision. This pressure or force varies during the collision zero at the beginning up to a maximum value and then down to ze the end. Curves showing how the force varies with respect to the tim with respect to the displacement of the point of contact during the coll are quite dissimilar. The average values of the force as shown by thes curves are unequal. The first is called the time-average and the secon space-average force of the blow. Let F_t and F_s denote these averages re tively and t and s the total time and displacement; then F_tt = the im exerted on either body, and if the impact is direct and central, F_ss = the done on either body by the force of the blow. When any object is stru a hand-hammer for example, the momentum of the hammer is changed; c the change M, then $F_tt = M$, or $F_t = M/t$. That is, for a given char momentum the time-average force of a blow varies inversely as the ti the impact. The energy of the hammer is also changed, and callin change E, then if the energy dissipated (in vibration of the hammer) is gible, $F_ss = E$, or $F_s = E/s$. That is, for a given change in energy the average force of a blow varies inversely as the space thru which the forc (It is assumed in the preceding that no other force than that of the affects the momentum and the energy changes.)

A pile-driver hammer of weight W falling a height h to a pile which it drives a s is given an amount of energy equal to $W(h+s)$ by gravity before the hammer is a

This energy is delivered to the pile and is wasted against resistance between earth and the ile, in vibration of the pile and adjacent earth. Calling the space-average resistance R nd neglecting the energy lost in vibration, then $Rs = W(h+s)$, or $R = W(1+h/s)$. This ormula is in use for computing the bearing capacity of a pile; h being the drop of the ammer and s the penetration of the pile in the last blow (or average penetration for ast few blows), W weight of hammer, and R ultimate bearing capacity, safe capacity eing generally taken as ⅕ or ⅙ of R. As 1 is small compared to h/s, the formula may be ritten, safe load = ⅕ or ⅙ Wh/s.

Ballistic Pendulum. This is a device formerly used to measure the velocity f a projectile; it is essentially a pendulum with a massive bob into which the rojectile may be fired and arrested (Fig. 153). Let v = velocity of the proectile, m its mass, M the mass of the pendulum ($=W/g$, W denoting its eight and g acceleration of gravity), k its radius of gyration with respect o the axis of suspension (for determination of which see below), a the istance from the axis to the center of gravity and r that to the path of the rojectile, and θ deflection of pendulum caused by projectile; then

$$v = \sqrt{2ga(1-\cos\theta)}\,(m + Mk^2/r^2) \div (mk/r)$$

f the weight of the bob is large compared to that of the suspending rod, then $k/r = 1$ nearly. If a pendulum in its vertical position is struck a horizontal low thru the center of gravity and perpendicular to the axis of suspension, he whole pendulum tends to move in the direction of the blow. But such ranslation is prevented by the support which exerts a sudden reaction oppoite to the blow. If the pendulum is struck below the center of gravity the low tends to produce a translation in its direction and a rotation about the enter of gravity. There would still be a sudden reaction of the support on he pendulum to overcome the tendency to the motion there unless the ranslation and the rotational velocities of the center of suspension, which he blow tends to produce, are equal. That point f impact for which the blow produces no sudden eaction at the center of suspension is called CENTER F PERCUSSION; it coincides with the center of oscilation (see page 1493). Its distance from the center f suspension $= k^2/a$.

Fig. 153

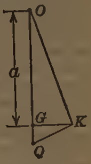


Fig. 154

The distance k^2/a can be determined experimentally thus: et the pendulum oscillate and ascertain the time T of one wing, extreme to extreme position; then $k^2/a = gT^2/\pi^2$; if is taken in ft per sec per sec and T in sec, then k^2/a is in ft. et k_g be the radius of gyration of the pendulum with respect o a line thru its center of gravity and parallel to the axis of uspension. When k_g is known the center of percussion (Fig. 154) can be located graphically thus: at G draw GK perpendicular to OG and make $OG = k_g$; join O and K and draw a perpendicular to OK at K; Q is the intersection f this perpendicular and OG extended.

The Moment of Inertia I of an irregular or nonhomogeneous body with espect to a specified line can be determined best experimentally; thus proeed as described above to determine k^2/a, then $I = (W/g)k^2 = WT^2/a\pi^2$. f the specified line passes through the center of gravity of the body then, ince the body would not oscillate about such line, proceed thus: determine I about any parallel line distant a from the center of gravity, then the desired moment of inertia $= I - (W/g)a^2$. If it is impossible or inconvenient to suspend the body at the specified line, then suspend and oscillate it about any ine parallel to the specified line, and determine I as explained about such parallel line; calling the distances of the parallel and specified lines from the center of gravity a and b respectively, then the desired moment of inertia $= I - (W/g)a^2 + (W/g)b^2 = I - (a^2 - b^2)W/g$.

FRICTION

38. Static Friction

Static Friction, or FRICTION OF REST is the friction between two bodies when there is a tendency to but not an actual slipping of one relative to the other. Let P (Fig. 155) = a pull applied to a body A which is supported by a body B, R = the reaction of B on A; then the component F of R along the surface of contact is the friction, and the component N perpendicular to the surface is the normal pressure. So long as there is no slipping $F = P$, and the greatest value of F obtains when slipping impends; this value of F is called limiting friction. The coefficient of static friction for two bodies is the ratio of their limiting friction to the accompanying normal pressure; it is denoted by f. The angle between R and N changes as F changes, and its greatest value obtains when motion impends; this value is called the angle of friction for the two bodies, and it will be denoted by ϕ. When a body A is supported on an inclined plane B, and no forces act on A except its own weight and the supporting force, then that inclination of the plane to the horizontal which will just cause slipping of A is called the angle of repose for A and B; it also will be denoted by ϕ because the angles of repose and friction for two bodies are equal. Also the tangents of these angles equal the coefficient of friction,

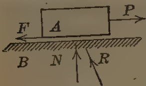


Fig. 155

$$F \leqq fN \qquad\qquad f = \tan\phi$$

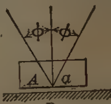


Fig. 156

Suppose the body A (Fig. 156) is supported by a body B, let Q denote the resultant of all the forces acting on A excepting the supporting force, and suppose that the line of action of Q cuts the surface of contact at a; then the cone with vertex at a, axis normal to the contact surface and apex angle equal to double the angle of friction, is called a cone of friction for the bodies A and B. If Q fall within the cone, motion will not ensue for any value of Q, but if without, then Q will move A.

The coefficient of static friction for two bodies A and B may be determined thus: (1) Place A on B as in Fig. 155 and determine the pull P which will just start A; then $f = P$ divided by the weight of A. Or (2) tilt B and determine the inclination at which gravity will start A down; then f = the tangent angle of that of inclination. In either method, several determinations must be made to obtain a fair average value of the coefficient.

Coefficients of Static Friction depend on the nature of the materials, character of rubbing surfaces, and kind of lubricant, if any be used. Early experimenters reported (Coulomb 1871, Rennie 1828, Morin 1834, and others) that the coefficient is independent of the intensity of normal pressure and altho this announcement was clearly subject to the limitation of the range of the experiments performed, yet it was generalized and long accepted as a universal law of friction. But the universality of the law has been questioned. Morin himself pointed out that length of the time of contact of the two bodies influences the coefficient and obviously the coefficient changes when the intensities of pressure get so high as to affect the character of the surfaces in contact. Messiter and Hanson report practical constancy of coefficient for yellow pine and spruce (Eng. News, 1895, vol. 33, p. 322), the first for a range from 100 to 1500 lbs per sq in and the second for a range from 100 to 800. They report for planed or sandpapered yellow pine: f = 0.25–0.33, average 0.29, for 100 to 1000 lb per sq in; for spruce: f = 0.18–0.53; average 0.42, for 100 to 800 lb per sq in. The variation depends on relation of grain of wood to direction of slide.

Coefficients of Static Friction

Compiled by Rankine from Experiments by Morin and others

Dry masonry and brickwork	0.6 to 0.7	Masonry on dry clay	0.51
Masonry and brickwork, with damp mortar	0.74	Masonry on moist clay	0.33
Timber on stone	about 0.4	Earth on earth	0.25 to 1.0
Iron on stone	0.7 to 0.3	Earth on earth, dry sand, clay, and mixt earth	0.38 to 0.75
Timber on timber	0.5 to 0.2	Earth on earth, damp clay	1.0
Timber on metals	0.6 to 0.2	Earth on earth, wet clay	0.31
Metals on metals	0.25 to 0.15	Earth on earth, shingle and gravel	0.81 to 1.11

Coil Friction. When a rope wholly or partially encircles a pulley or drum (Fig. 157) the tensions P_1 and P_2 may be quite different on account of the friction on the rope. The difference is greatest when slipping impends; if α = arc of contact in radians, ε = Naperian base = 2.718, the maximum ratio $P_2/P_1 = \varepsilon^{f\alpha}$.

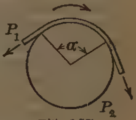


Fig. 157

Maximum Ratios P_2/P_1 (Slipping Impending)

$\alpha/2\pi$	$f=\frac{1}{4}$	$f=\frac{1}{3}$	$f=\frac{1}{2}$	$\alpha/2\pi$	$f=\frac{1}{4}$	$f=\frac{1}{3}$	$f=\frac{1}{2}$
0.1	1.17	1.23	1.37	0.7	3.00	4.33	9.00
0.2	1.37	1.51	1.87	0.8	3.51	5.34	12.34
0.3	1.60	1.87	2.57	0.9	4.11	6.58	16.90
0.4	1.87	2.31	3.51	1.0	4.81	8.12	23.14
0.5	2.19	2.85	4.81	2.0	23.1	66.0	535.5
0.6	2.57	3.51	6.59	3.0	111.	535	1239.0

Belting. The foregoing formula, $P_2/P_1 = \epsilon^{f\alpha}$, applies also. Single (thickness) leather belt varies from 3/16 to 7/32 inch thick; double leather (two thicknesses glued) belt varies from 3/8 to 1/2 inch thick. Cement splices are nearly as strong as the belt; lace joints average about 70% as strong as the belt, and metal fastenings generally less. Working strengths of 400 lbs per sq in for endless (cement spliced) belts and 200 to 300 for laced belts are safe, but smaller values are advised by F. M. Taylor (Trans. Am. Soc. M. E., 1894, vol. 15, p. 204). These figures correspond to about 100 lbs per inch of width for cemented single belt, 50 to 75 for laced single belt, and twice these values for double belt.

The horse-power which a belt transmits is given by $(T-Z)\,Cv/550$, wherein T = total tension in tight side of belt in pounds, v = velocity of belt travel in ft per sec, Z = a quantity depending on centrifugal action (table below), C = a quantity depending on coefficient of friction and angle of wrap on the pulley (table below). The formula is Nagle's (Trans. Am. Soc. M. E., 1881, vol. 2, p. 91) but modified by Kimball and Barr.

For a leather belt which weighs 0.035 lb per cu in when:

v (ft per sec) =	30	60	70	80	90	100	110	120	130	140	150
Z (lbs) =	32.5	47.0	64.2	83.4	105	130	158	188	220	256	293

Values of C, in formula h.p. $=(T-Z)\,Cv/550$

f \ α	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
0.15	0.210	0.230	0.250	0.270	0.288	0.307	0.325	0.342	0.359	0.376
0.25	.325	.354	.381	.407	.432	.457	.480	.503	.524	.544
0.35	.423	.457	.489	.520	.548	.575	.600	.624	.646	.667
0.45	.507	.544	.579	.610	.640	.667	.692	.715	.737	.757
0.55	.578	.617	.652	.684	.713	.739	.763	.785	.805	.822
1.00	.792	.825	.853	.877	.897	.913	.927	.937	.947	.956

39. Sliding Friction

Kinetic Friction, Friction of Motion, or sliding friction, is the friction between two bodies when sliding actually occurs. The coefficient of kinetic friction for two bodies is the ratio of the kinetic friction to the corresponding normal pressure between them. One of the so-called laws of friction states that the kinetic coefficient is less than the static coefficient and implies that there is a sudden or abrupt change in the values of the coefficients. Experiments by Jenkin and Ewing (Phil. Trans. Roy. Soc., 1877, vol. 167, Part 2) on the kinetic coefficients at speeds as low as 0.0002 ft per sec, about ¾ ft per hour, lead them to conclude that "it is highly probable that the kinetic coefficient gradually increases when the velocity becomes extremely small, so as to pass without discontinuity into the static coefficient." Experiments by Kimball (Am. Jour. Sci., 1877, vol. 13, p. 353) also indicate that there is no abrupt change from static to kinetic coefficient. Moreover they show that the kinetic coefficient may be greater than the static. For dry surfaces, the kinetic coefficient probably decreases progressively from the value of the static coefficient, as the velocity increases as indicated by the following table from Galton and Westinghouse's experiments (Proc. Inst. Mech. Engrs., 1879).

Coefficients of Friction at Various Speeds for Cast-iron Brake Shoes and Steel-tired Wheels

Velocity		Coefficients			Number of tests
Mi per hr	Ft per sec	Max.	Min.	Mean	
0+	0+	....	....	0.330	....
5−	7−	0.340	0.156	.273	20
7.5	11	.325	.123	.244	28
10	14.5	.281	.161	.242	54
15	22	.280	.131	.223	78
20	29	.240	.133	.192	69
25	36.5	.205	.108	.166	70
30	44	.196	.098	.164	94
35	51	.197	.087	.142	80
40	59	.194	.088	.140	70
45	66	.179	.083	.127	77
50	73	.153	.050	.116	55
55	81	.136	.060	.111	67
60	88	.123	.058	.074	12

The foregoing given coefficients were obtained from measurements taken very soon after application of brakes.

The rubbing of dry surfaces abraids them and decreases the kinetic coefficient. The following table shows how this coefficient changed with lapse of time after rubbing began.

Coefficient of Friction as Affected by Time of Rubbing

Cast-iron brake shoes on steel-tired wheels

Miles per hour	Time after applying brakes				
	0+	5 sec	10 sec	15 sec	20 sec
20	0.182	0.152	0.133	0.116	0.099
27	.171	.130	.119	.081	.072
37	.152	.096	.083	.069	....
47	.132	.080	070	....	....
60	.072	.063	058	....	....

The discrepancies between the two tables are due in part to the fact that the values at time 0+ in the second table are averages based on comparatively few experiments. In the second table preceding there is a wide variation from the mean value of the coefficient for each velocity. The variation was due in part at least to differences in intensities of pressure in the various runs of each velocity. In general, the coefficients decrease with increase of intensity.

Coefficients of Friction for Wheels Skidded on Rails

Galton-Westinghouse Experiments

Wheels skidded on rails gave much smaller coefficients than brake shoes, as shown by the adjacent table.

Approximate velocity		Steel tire on	
Ft per sec	Mi per hr	Steel rail	Iron rail
0+	0+	0.242	0.247
10	6.8	.088	.095
20	13.6	.072	.073
40	27.3	.070	....
50	34.1	.065	.070
60	40.9	.057	....
70	47.7	.040	.060
80	54.5	.038	....
88	60	.027	....

Kinetic Coefficients for Brake Shoes

From Experiments by Ernest Wilson (Eng. News, 1909, vol. 62, p. 736)

The adjacent table furnishes additional information on the variation of the kinetic coefficient with the intensity of pressure and on the velocity; also on the influence of water lubrication.

Materials	Pressure, lb per sq in	Vel., mi per hr		Lubrication
		7	15	
Cast iron......	10	0.43	0.37	none
Cast iron......	40	.36	.30	none
Oak..........	10	.60	.55	none
Oak..........	40	.43	.40	none
Poplar........	10	....	.72	none
Poplar........	40	....	.53	none
Cast iron......	20	.32	.28	water
Cast iron......	80	.30	.26	water
Oak..........	40	.037	.032	water
Oak..........	120	.073	.055	water
Poplar........	40	.041	.038	water
Poplar........	120	.070	.053	water

Coefficients of Kinetic Friction (rough averages)

Compiled by Rankine from Experiments by Morin and others

Wood on wood,	dry	0.25 to 0.50	Leather on oak	....	0.27 to 0.38
	soapy....	.2	Leather on metals,	dry.....	.56
Metals on oak,	dry......	.5 to .6		wet	.36
	wet......	.24 to .26		greasy..	.23
	soapy....	.2		oily	.15
Metals on elm,	dry......	.2 to .25	Metals on metals,	dry	.15 to .2
Hemp on oak,	dry......	.53		wet	.3
	wet......	.33			

Pivots. Let W = load (lbs), μ = coefficient of kinetic friction, n = speed (r.p.m.), m = moment of the frictional resistance about the axis of the shaft (in-lbs), w = work done against friction per revolution (in-lbs), P = power lost

thru friction (h.p.), dimensions as marked in figures (ins). Flat Pivot (Fig. 158): $m = \frac{2}{3}\mu Wr$; $w = \frac{4}{3}\pi\mu Wr = 4.187\,\mu Wr$; $P = \frac{4}{3}\pi\mu Wrn/396\,000 = \mu Wrn/94\,700$. Collar Bearing (Fig. 159): $m = \frac{2}{3}\mu W(R^3 - r^3)/(R^2 - r^2)$;

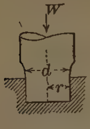


Fig. 158

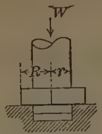


Fig. 159

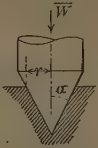


Fig. 160

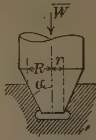


Fig. 161

$w = 2\pi m$; $P = 2\pi mn/396\,000 = mn/63\,000$. Conical Pivot (Fig. 160): $m = \frac{2}{3}\mu Wr/\sin\alpha$; $w = \frac{4}{3}\pi\mu Wr/\sin\alpha$; $P = \frac{4}{3}\pi\mu Wrn/396\,000 \sin\alpha = \mu Wrn/94\,700 \sin\alpha$. Frustrated Conical Pivot (Fig. 161): $m = \frac{2}{3}\mu W(R^3 - r^3)/(R^2 - r^2)\sin\alpha$; $w = 2\pi m$; $P = 2\pi mn/396\,000 = mn/63\,000$.

Journal Friction. The coefficient of journal friction is the ratio of the frictional resistance at the journal to the pressure between journal and bearing. The pressure is not uniformly distributed over the surface of contact between journal and bearing. By nominal (intensity) of pressure is meant the whole pressure divided by product of length and diameter of bearing.

For example, suppose a fly wheel and shaft weighing 6000 lbs are supported midway between two bearings, the diameter of the wheel is 8 ft (or 96 in), that of the journal is 6 in, and that a force of 10 lbs applied to the rim is necessary to overcome friction to maintain a constant speed. Then the frictional resistance is $(10 \times 48) \div 3 = 160$ lbs, 80 at each journal; the pressure at each journal is 3000 lbs, and the coefficient of journal friction is $160 \div 3000 = 0.053$. If the length of bearing is 10 in, the nominal (intensity) of pressure is $3000 \div (6 \times 10) = 50$ lbs per sq in.

Energy lost at journals, work done against friction. Let W = total load on bearing (lbs), d = diameter of journal (ins), f = coefficient of journal friction, n = number of revolutions per minute; then

work done per revolution (ft-lbs) $= fW\pi d/12$
work done per minute ft-lbs $= fW\pi dn/12$
power lost (h.p.) $= fW\pi dn/396\,000 = fWdn/126\,000$

Coefficients of Journal Friction depend on (1) the method of lubrication, (2) the lubricant, (3) its temperature, (4) the velocity of rubbing, and (5) the intensity of pressure (lbs per sq in) on the bearing.

(1) The relative intensities of friction for different methods of lubrication are as follows according to Tower (Proc. Inst. Mech. Engs., 1883) and Goodman (Mechanics Applied to Engineering, 1896).

Method	Tower	Goodman
Bath	1.00	1.00
Saturated pad	....	1.32
Ordinary pad	6.48	2.21
Siphon	7.06	4.20

Tower found that in bath lubrication the load is completely oil borne, in other words that a continuous film separates journal and bearing. For any given pressure there seems to be a minimum velocity below which the oil film will not form. H. F. Moore also found (Am. Mach., 1903) that this pressure-velocity relation is approximately $p = 7.47\sqrt{v}$, p being in lbs per sq in, and v in ft per min. Moore used mineral oil at 90° temperature. The friction then is wholly fluid friction and not solid. (2) In the following table the figures are based on mean actual frictional resistances at the surface of the journal at speed of 300 r.p.m. with all nominal loads from 100 to 310 lbs per sq in; the temperature was 90°; the journal was steel, 4 in diameter and 6 in long; the bearing

a gun-metal brass embracing somewhat less than half the circumference of the journal. The relative frictional resistances for various lubricants, according to Tower, are 1.00 for sperm oil, 1.06 for rape oil, 1.29 for mineral oil, 1.35 for lard oil and olive oil, 2.17 for mineral grease. Tower states also: "the numbers represent the relative thickness or body of the various oils, and also in their order, tho perhaps not exactly in their numerical proportions, their relative weight carrying power. Thus sperm oil, which has the highest lubricating power, has the least weight-carrying power; and tho the best oil for light loads, would be inferior to the thicker oils if heavy pressures or high temperatures were to be encountered." (3) The coefficient decreases with increased temperature; see Figs. 162 and 164. But if the temperature gets so high as greatly to lower the viscosity, the lubricant gets squeezed out and then the coefficient increases. (4) In general the coefficient increases with increase of speed; see Figs. 163 and 165 and accompanying explanations. But at the lower speeds the coefficient may decrease with increase of speed (see Fig. 163). (5) The coefficient decreases with increase of intensity of pressure (see Figs. 163 and 166). But the intensity may become so great that the lubricant gets squeezed out and then the coefficient increases and seizing occurs.

Coefficient of friction

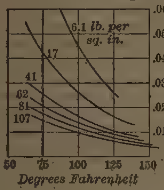


Fig. 162

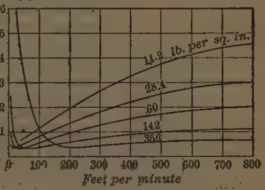
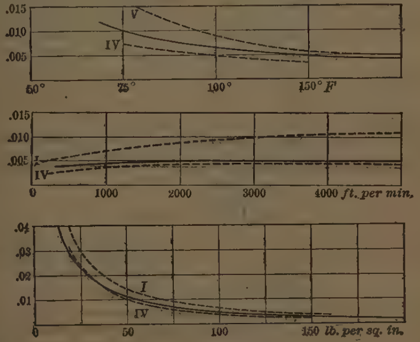


Fig. 163



Figs. 164, 165, 166

Figs. 162 and 163 show the relation between coefficient and temperature, and coefficient and velocity. Sellers bearing, length 13 in and diameter 2.75 in; ring oiler; "gas motor oil"; oil temperature 77°; velocity 846 ft per min. (Stribeck, Zeit. Ver. Deutsch. Ing., 1902, vol. 46. p. 1341.) Figs. 164, 165, and 166 show respectively relations between coefficient and temperature, velocity, and pressure. In 164, $p = 92.5$ lbs per sq in and $v = 197$ ft per min; in 165, $p = 92.5$ lbs per sq in and $t = 112°$; in 166, $t = 112°$; and $v = 197$ ft per min. The heavy line represents an average law for forced lubrication on five differ-

ent journals as described below, and the other curves relate to the two journals which departed most widely from the average. (Lasche, Zeit. Ver. Deutsch. Ing., 1902, vol. 46.)

Number	Journal	Bearing
I	steel	white metal
II	nickel steel	white metal
III	nickel steel	mercury alloy
IV	nickel steel	bronze
V	ingot iron	white metal

Coefficients of Journal Friction (Tower's Experiments)

Lubrication	Velocity, ft per min	Nominal pressure, pounds per square inch					
		100	153	205	310	415	520
Olive oil by bath	105	0.0036	0.0023	0.0018	......	......	......
	157	.0045	.003	.0021	0.0015	0.0012	0.0008
	471	.0089	.0057	.004	.0027	.0024	.0017
Lard oil by bath	105	.0035	.0022	.0017	......	......	......
	157	.0042	.0027	.0020	.0014	.0012	.0009
	471	.009	.0052	.0042	.0029	.0021	.0017
Mineral grease by bath	105	.0054	.0028	.0026	.002	......	......
	157	.0076	.0038	.0034	.0022	.0016	.0014
	471	.0151	.0083	.0066	.004	.0027	.0022
Sperm oil by bath	105	.0025	.0016	.0013	......	......	Seized
	157	.003	.0019	.0016	.0011	.0015	......
	471	.0064	.0037	.0027	.0019	.0021	......
Rape oil by bath	105	.0028	.0016	.0011	......	......	......
	157	.0036	.0020	.0014	.0008	.0009	.0010
	471	.0071	.0040	.0024	.0016	.0016	.0015
Mineral oil by bath	105	.0033	......	.0018	......	......	......
	157	.0041	......	.0020	.0014	.0012	.0012
	471	.0073	......	.0035	.0024	.002	.0018
Rape oil by siphon	105	.0144	......	.0132	......	......	......
	157	.0125	......	.0098	.0056*	......	......
	314	.0163	......	.0082	.0068*	......	......
Rape oil by pad	105	.0105	......	......	......	......	......
	157	.0099	.009	.0105	.0099	......	......
	314	.0133	.0105	.0078	.0099	......	......

* For 258 pounds per square inch.

40. Rolling Friction

Rolling Resistance (OR FRICTION). When a roller or wheel is made to roll, the reaction of the roadway has a component opposite to the direction of rolling, and this component is the rolling resistance. Thus R (Fig. 167) represents the reaction of the roadway, and its horizontal component is the rolling resistance. The distance c is called coefficient of rolling resistance. Let W = weight of roller or the load on the roadway and P = driving force; when applied as in Fig. 167, then $Pr = Wc$; when at the top of the roller $P\,2r = Wc$. When a roller is interposed between a load and roadway so that there is rolling resistance at two places, then $P\,2r = W\,2\,c$. Few determinations of the coefficient c have been made; it has been reported as independent of W and r, and also as independent of W but varying with $\sqrt{r}$. "Laws" of rolling

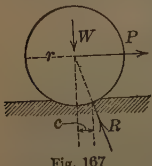


Fig. 167

resistances certainly remain to be established. The following are some reported values of the coefficient of rolling resistance.

Lignum vitæ roller on oak track, 0.019 inch.
Elm roller on oak track, .032.
Cast-iron wheel (20 in diam) on cast-iron rail, 0.018–0.019.
Railroad wheels (39.4 in diam) 0.020–0.022.
Iron or steel wheels on wood track, 0.06–0.10.

Roller Bearings offer less resistance than ordinary bearings. C. H. Benjamin has made a comparison (Machinery, N. Y., Oct., 1905) between the friction in a flexible roller, a solid roller, and ordinary bearing. The speed was 560 revolutions per minute and the loads varied from 113 to 456 lb. Under 470 lb load the flexible bearing developed end thrust of 13.5 lb and the solid roller bearing one of 11 lb.

Coefficients of Journal Friction

Diameter of journal	Flexible rollers			Solid rollers			Babbitt bearing		
	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	Mean
$1\frac{1}{16}$	0.032	0.012	0.018	0.033	0.017	0.022	0.074	0.029	0.043
$2\frac{3}{16}$	.019	.011	.014	......	......	......	.088	.078	.082
$2\frac{7}{16}$	.042	.025	.032	.028	.015	.021	.114	.083	.096
$2\frac{15}{16}$	.029	.022	.025	.039	.019	.027	.125	.089	.107

Stribeck (Zeit. Ver. Deutsch. Ing., 1902, vol. 46, p. 1463) from experiments of four different makes of roller bearings, proposes the following: Let W = load on bearing (lb), n = number of rollers, l = length of each (in), r = radius of convex bearing surface for rollers (in), d = diameter of rollers (in), D = diameter of circle described by centers of rollers (in), $2r+d=D$, c = coefficient of rolling resistance, f = coefficient of friction such that fW = total resistance to turning as tho applied tangentially at surface of bearing and fWr = moment of resistance to turning; then $f=1.2\,cD/rd$, and c has following values:

for $5W/lnd$ =	50	75	100	150	200 lb per sq in
c =	0.0041	.0032	.0028	.0022	.0019 inch

He also states that the resistance of roller bearings is nearly independent of velocity; that the value of the highest roller pressure is 5 times the average, that is, $5\,W/n$; and he proposes as formula for safe load in lbs on the bearing $kldn/5$, where k may be taken from 85 to 155 lbs per sq in for unhardened rollers and bearing surfaces. F. R. Jones proposes (Machine Design, Part II) as formula for safe load for a well-made solid roller journal bearing of six or more rollers $100\,000\,d\text{-}ln/3\,v$, where v is the velocity of the convex bearing surface in ft per min; but values of v less than 50 should not be used in the formula; for such velocity the safe load is practically the same as for $v=50$.

Ball Bearings. Stribeck reports extensive tests (Zeits. Ver. Deutsch. Ing., 1901, vol. 45, p. 121) from which the following is taken. The ball cases were grooves, cross-sections of which were arcs of circles of radius $\frac{2}{3}\,d$, thus affording two points of contact for each ball, $d=\frac{7}{8}$ in, $D=4$ in.

Coefficients of Friction for a Ball Bearing

Load in pounds	Revolutions per minute		
	65	385	780
840	0.0033	0.0035	0.0037
1 870	.0020	.0021	.0022
2 420	.0017	.0018	.0019
3 480	.0016	.0016	.00165
4 500	.0015	.0015	.0015
6 600	.0015	.0013	.0013
10 800	....	....	.0011

Stribeck also states that when the number of balls, n, in a race is between 10 and 20, the greatest pressure on any ball is about $5\,W/n$, where W is the load per race; then if P is the safe load for a single ball, the safe load per race is $Pn/5$. Of course, in a good thrust bearing the load is uniformly distributed among the balls. He also recommends as safe loads per ball $2100\,d^2$ for two-point bearing balls, and from $500\,d^2$ to $750\,d^2$ for three- or four-point bearings.

41. Efficiency of Machines

Efficiency. A machine in operation receives and transmits or deliv energy. The energy received is called input, that transmitted or delive is called output; the latter is also called the useful work of the machi The output is always less than the input, some of the energy miscarry as it were. The difference between output and input is the lost work, simply loss. The efficiency of a machine is the ratio of output to inp The efficiency of a combination or succession of machines, the first receiv energy, transmitting to the second, the second to the third, etc., is the c tinued product of their separate efficiencies.

Efficiency of Some Machine Elements

(Kimball and Barr, Elements of Machine Design.)

Common bearing, singly	96–98
Common bearing, long lines of shafting	95
Roller bearing	98
Ball bearings	99
Spur gear cast teeth, including bearings	93
Spur gear cut teeth, including bearings	96
Bevel gear cast teeth, including bearings	92
Bevel gear cut teeth, including bearings	95
Belting	96–98
Pin-connected chains, as used on bicycles	95–97
High-grade transmission chains	97–99

In some simple machines, the energy is supplied by means of a single force calle effort; for example in a hoisting tackle, the pull applied is the effort. The force ag which the useful work is done is called load; in the illustration the weight of the body is the load. The mechanical advantage of a machine is the ratio of the load effort. While the effort works or acts thru any particular distance, the load acts t definite distance also; the ratio of the former to the latter distance is called the ve ratio of the machine. Mechanical advantage depends on efficiency of the mac velocity ratio does not. In any case mechanical advantage = ve ratio × efficiency.

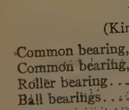


Fig. 168.

Inclined Plane and Wedge. Let W = weight of f = coefficient of static friction, ϕ = angle of repose = of friction and $f = \tan\phi$, α = inclination of plane and clination of P (Fig. 168), θ being positiv as shown and tiv when the inclination of P to the horizontal is less th (1) To start the body up, $P = W \sin(\phi+\alpha)/\cos(\theta$ P is least when $\theta = \phi$, its value then being $W \sin(\phi+\alpha)$. (2) α is greater than ϕ, gravity would overcome friction and the body, prevented by some pull as P, would slip down. To prevent slipping

$$P = W \sin(\alpha-\phi)/\cos(\theta+\phi)$$

P is least when $\theta = -\phi$, its value then being $W \sin(\alpha-\phi)$. When α than ϕ, there is no danger of slipping. To start the body down

$$P = W \sin(\phi-\alpha)/\cos(\theta-\phi)$$

this is least when $\theta = -\phi$, its value then being $W \sin(\phi-\alpha)$. When th is being dragged up, let ϕ = angle of kinetic friction, that is, the angle tangent equals the kinetic coefficient, and e = efficiency. Then

$$e = \sin\alpha \cos(\theta-\phi)/\cos\theta \cdot \sin(\phi+\alpha)$$

e is maximum when P is minimum, that is, when $\theta = \phi$.

To start the wedge (Fig. 169) against the resistances R, $P = 2\,R \tan(\alpha + \phi)$, ϕ being the angle of static friction. If the forces R continue to act and P ceases, the wedge will be pushed up if α is greater than ϕ. If α is less than ϕ, the pull required to start the wedge out is $P = 2\,R \tan(\phi - \alpha)$. The efficiency of the wedge is $\tan \alpha / \tan(\alpha + \phi)$. In Fig. 170 there are three rubbing surfaces here assumed equally rough. To start the wedge in, $P = R \tan(\alpha + 2\,\phi)$ and the efficiency is $(\tan \alpha) / \tan(\alpha + 2\,\phi)$. To pull the wedge out, when α is less than $2\,\phi$, $P = W \tan(2\,\phi - \alpha)$.

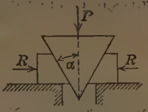


Fig. 169

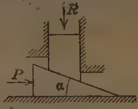


Fig. 170

Screw. Let p = pitch, r = mean radius of screw thread, d = diameter, α = angle of pitch ($\tan \alpha = p / \pi d$), f = static, and μ = kinetic coefficient, and ϕ_s and ϕ_k = the corresponding angles of friction ($f = \tan \phi_s$ and $\mu = \tan \phi_k$). The turning moment required to raise or lower a load W is

$$Wr\,(f \cos \alpha \pm \sin \alpha) / (\cos \alpha \mp f \sin \alpha)$$

the upper sign for raising and the lower for lowering the load. The efficiencies for raising and lowering are $(\tan \alpha) / \tan(\alpha + \phi_k)$ and $[\tan(\alpha - \phi_k)] / \tan \alpha$.

The following table gives highest and lowest efficiencies of some screws tested by Albert Kingsbury (Trans. Am. Soc. M. E., 1896, vol. 17, p. 96). Mean diameter of thread = 1.352 in, pitch = 1/8 in, depth of nut 1 1/16 in, area of rubbing surface of thread about 1 sq in; the threads were cut carefully and worn to good condition before tested; speed about one-half r.p.m. There were 5 screws and 4 nuts as follows: S1 mild steel; S2 wrought iron; S3 cast iron; S4 cast bronze; S5 mild steel, case-hardened; N1 mild steel; N2 wrought iron; N3 cast iron; N4 cast bronze.

Efficiencies of Square Threaded Screws (Kingsbury)

Pressure on thread, lbs per sq in	Lubrication	Highest	Lowest
10 000	machinery oil	0.20, S5 N4	0.11, S3 N3
10 000	lard oil	0.25, S4 N4	0.09, S3 N3
10 000	machinery oil and graphite	0.15, S5 N1	0.03, S5 N4
3 000	machinery oil	0.19, S5 N2	0.11, S2 N4

Tackle. Fig. 171 represents a fixt pulley, Fig. 172 a movable pulley lifting and Fig. 173 a movable pulley lowering the load. Let P = pull in lead line or off side, Q = that in following or on side; then $k = P / Q$, where k is a coefficient always greater than unity. Its value depends on the stiffness or rigidity of the rope and the pin friction; this has been proposed:

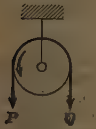


Fig. 171

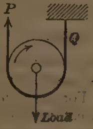


Fig. 172

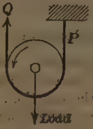


Fig. 173

$$k = 1 + Cd^2/a + 2\,fr/a$$

where d = diameter of rope, a = distance center of pin to center of rope, r = pin radius, f = coefficient of axle or pin friction, C an experimental coefficient. $C = 0.46$ has been recommended for hemp rope; with that value, and $a = 4\,d$, $r = d/2$, and $f = 0.08$,

for d =	1/2	3/4	1	1 1/2 inches
k =	1.08	1.11	1.13	1.19

Some experiments by the American Bridge Co. (Trans. Am. Soc. C. E. 1903, vol. 51, p. 161) indicate that C itself depends on rope diameter. Th following table gives values of C adopted and k computed from the formul preceding with $f=0.08$; values of d, a, and r are in inches.

	Hemp rope				Wire rope
d	1¼	1½	1¾	2	¾
a	3⅞	4⁹⁄₁₆	5⅜	6½	7⅜
r	⅞	1	1⅛	1⅜	2½
C	0.46	0.40	0.38	0.34	1.8
k	1.20	1.21	1.23	1.23	1.15

It may be noted that the pin friction contributes little to the value of k; in the cases abov 0.02 for the hemp and about 0.01 for the wire rope. The efficiencies for a single pull are: if fixt $1/k$, movable lifting $(1+k)/2k$, movable lowering $2/(1+k)$. Following tab gives the ratios of load to pull for tackles of manila rope, as determined by experimen of American Bridge Co., see Engr. Record, 1903, vol. 48, p. 307.

Ratios of Load to Lead-line Pull for Manila Rope

No. of Parts	Diameter of rope in inches							
	¾	⅞	1	1¼	1½	1¾	2	2¼
2	1.93	1.92	1.93	1.92	1.91	1.91	1.91	1.90
3	2.73	2.68	2.74	2.68	2.67	2.64	2.65	2.63
4	3.48	3.37	3.50	3.37	3.36	3.30	3.32	3.28
5	4.12	3.95	4.16	3.95	3.93	3.84	3.87	3.80
6	4.71	4.48	4.77	4.48	4.45	4.33	4.37	4.28
7	5.23	4.92	5.30	4.92	4.89	4.72	4.78	4.65
8	5.71	5.32	5.80	5.32	5.28	5.08	5.14	5.00
9	6.12	5.66	6.23	5.65	5.61	5.37	5.45	5.27
10	6.50	5.96	6.63	5.96	5.91	5.64	5.72	5.52
11	6.83	6.22	6.98	6.21	6.15	5.85	5.94	5.72
12	7.14	6.45	7.30	6.44	6.38	6.04	6.15	5.90
13	7.40	6.64	7.58	6.63	6.56	6.20	6.31	6.04
14	7.64	6.82	7.85	6.81	6.73	6.34	6.46	6.17

Number of parts means number of runs of rope to movable block. Efficiency in cent for any case = ratio divided by number of parts. Example: The fixt and mova blocks of a tackle are each double (two sheaves in each), and the rope is fastened to fixt block; rope diameter is one inch. Then the number of parts is 4 and the load is 3 times the pull; the efficiency is $3.50/4=87.5$ per cent.

42. Muscular Exertion or Labor

Power and Work of Men and Animals. The power or rate of work of any agent (man or beast) depends on several factors, the principal ones which are the duration and form of the exertion. For example, in raising own weight (climbing a stairs) man has worked (against gravity) at the aver rate of 1½ horse-power for 6 seconds; but only at the average rate of ab ⅛ horsepower for a day of 10 hours, and in this case the (necessary) desce were made without effort on his part. In one day a man can do about t times as much work pulling horizontally as in lifting weights.

Daily total work or performance depends on resistance overcome, velo at which it is overcome, and length of the working day. The possible d performance of any agent in any given form of exertion depends on pr choice or adjustment of these three factors. This is well illustrated by a c

brated example developed by Dr. J. F. Taylor: At a certain steel works the regular performance of men loading pig iron onto cars from piles on the ground was 12½ tons per day; the men best adapted to such labor were induced to work according to a suitable program of working and resting, whereby their output reached 47½ tons per day. It may be noted that this performance was not all work in the mechanical sense, but partly transport, see following paragraphs.

In trials, horses have overcome resistance equal to one-half their own weight thru 100 ft of distance. A draft or resistance of one-fourth the weight of the horse is regarded as heavy. For steady work during a day of 10 hours, a draft equal to $^1/_{10}$ to $^1/_8$ of the weight of the horse exerted at 2.5 miles per hour, is regarded as full demand on the animal. Under these circumstances a 1000-lb horse works at 0.67 to 0.83 horsepower.

The following two tables in the main are from Rankine on the authority principally of Coulomb, Navier and Poncelet.

Work by Man and Horse

Kind of Exertion	R	v	Rv	T	Rvt
Man					
1. Raising own weight up stair or ladder	143	0.5	72.5	8	2 088 000
2. Hoisting weight with rope and pulley	40	0.75	30	6	648 000
3. Lifting weights by hand	44	0.55	24.2	6	522 720
4. Carrying weights upstairs, returning unloaded	143	0.13	18.5	6	399 600
5. Shovelling up earth to height 5 ft 3 in	6	0.13	7.8	10	280 800
6. Wheeling earth in barrow up slope 1 : 12, returning unloaded	132	0.075	9.9	10	356 400
7. Pushing or pulling horizontally (capstan or oar)	26.5	2.0	53	8	1 526 400
8. Turning a crank or winch	12	5.	62.5	?	
	18	2.5	45	8	1 296 000
9. Working a pump	20	14.4	288	2 min.	
	13.2	2.5	33	10	1 188 000
10. Hammering	15	?	?	?	480 000
Horse					
11. Cantering and trotting, drawing light railway carriage	22½	14⅔	447⅔	4	6 444 000
	30½				
12. Walking, drawing cart or boat	50				
	120	3.6	432	8	12 441 000
13. Walking, drawing a gin or mill	100	3.0	300	8	8 640 000
		6.5	429	4½	
14. Trotting, drawing a gin or mill	66	......	......	......	6 950 000

Explanation of Table. R = resistance overcome in pounds; v = effective velocity in feet per second, or distance through which R is overcome divided by total time occupied, including time of moving unloaded if any; Rv = effective power, foot-pounds per second; T = hours per working day; Rvt = daily work, in foot-pounds. In item 6, 132 is the weight of earth in the barrow, and .075 is the vertical velocity of the laborer when pushing the loaded barrow. Therefore Rv and Rvt (for this item) do not include all the mechanical work done by the laborer.

Transport by Men and Draft Animals. When a man stands and support a load on his back he is subject to more or less fatigue and ordinarily he regard himself as working. But he is doing no work in the technical or mechanica sense, "overcoming resistance thru distance." When he is walking under thi load, he does some mechanical work, for he lifts the load slightly at each step but even this is a small part of his exertion. In order to express amount o exertion involved in transporting loads horizontally, where the main effort i to support the load, Rankine uses product of load and distance transported, an he calls the product transport. Thus like work transport is product of forc and distance but in the latter case the force and distance are at right angles t each other, whereas in the former they are directed along the same line. Trans port then may be exprest in foot-pounds. Railroad traffic engineers use similar concept, freight (hauled) or traffic, and measure it in ton-miles.

Transport by Man and Horse

Kind of Exertion	L	v	Lv	T	Lvt
Man					
1. Walking unloaded, transferring own weight	140	5	700	10	25 200 000
2. Carrying burden, returning unloaded	140	$1\frac{2}{3}$	233	6	5 032 800
3. Traveling with burden	90	$2\frac{1}{2}$	225	7	5 670 000
4. Carrying burden, 30 seconds only	252	0	0		
	126	11.7	1574.2		
	0	23.1	0		
5. Wheeling load L in 2-wheeled barrow, returning unloaded	224	$1\frac{2}{3}$	372	10	13 428 000
6. Wheeling load L in 1-wheel barrow, returning unloaded	132	$1\frac{2}{3}$	220	10	7 920 000
Horse					
7. Carrying burden, walking	270	3.6	972	10	34 992 000
8. Carrying burden, trotting	180	7.2	1 296	7	32 659 200

Explanation of Table. L is the weight or load transported in pounds; v is "effecti velocity" as in preceding table; Lv = rate of transport in foot-pounds per second; T hours per working day; and Lvt = transport in foot-pounds per day.

SECTION 13

PHYSICS, METEOROLOGY, WEIGHTS AND MEASURES

BY

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CHEMISTRY

1. The Chemical Elements

The following atomic weights were published by the International Committee on Atomic Weights in 1917. No table appeared in 1918.

Names of the more important elements, including those of metallurgical interest, are in SMALL CAPITALS.

Name	Symbol	Atomic weight	Nature
ALUMINUM	Al	27.1	Metal
ANTIMONY (stibium)	Sb	120.2	Metal
Argon	A	39.88	Inert gas
ARSENIC	As	74.96	Metalloid
BARIUM	Ba	137.37	Metal
Bismuth	Bi	208.0	Metal
Boron	B	11.0	Metalloid
Bromine	Br	79.92	Liquid
Cadmium	Cd	112.40	Metal
CALCIUM	Ca	40.07	Metal
CARBON	C	12.005	Metalloid
Cerium	Ce	140.25	Metal
Cesium	Cs	132.81	Metal
CHLORINE	Cl	35.46	Gas
CHROMIUM	Cr	52.0	Metal
Cobalt	Co	58.97	Metal
Columbium (Niobium)	Cb	93.1	Metal
COPPER (cuprum)	Cu	63.57	Metal
Dysprosium	Dy	162.5	Metal
Erbium	Er	167.7	Metal
Europium	Eu	152.0	Metal
FLUORINE	F	19.0	Most active gas
Gadolinium	Gd	157.3	Metal
Gallium	Ga	69.9	Metal
Germanium	Ge	72.5	Metal
Glucinum (Beryllium)	Gl	9.1	Metal
GOLD (aurum)	Au	197.2	Metal
Helium	He	4.00	Inert gas
Holmium	Ho	163.5	Metal
HYDROGEN	H	1.008	Lightest gas
Indium	In	114.8	Metal
Iodine	I	126.92	Metalloid
Iridium	Ir	193.1	Metal
IRON (ferrum)	Fe	55.84	Metal
Krypton	Kr	82.92	Inert gas
Lanthanum	La	139.0	Metal
LEAD (plumbum)	Pb	207.20	Metal
Lithium	Li	6.94	Lightest metal
Lutecium	Lu	175.0	Metal
MAGNESIUM	Mg	24.32	Metal

Name	Symbol	Atomic weight	Nature
MANGANESE	Mn	54.93	Metal
MERCURY (hydrargyrum)	Hg	200.6	Metal
MOLYBDENUM	Mo	96.0	Metal
Neodymium	Nd	144.3	Metal
Neon	Ne	20.2	Inert gas
NICKEL	Ni	58.68	Metal
NITROGEN	N	14.01	Gas
Osmium	Os	190.9	Metal
OXYGEN	O	16.00	Gas
Palladium	Pd	106.7	Metal
PHOSPHORUS	P	31.04	Metalloid
Platinum	Pt	195.2	Metal
POTASSIUM (kalium)	K	39.10	Metal
Praseodymium	Pr	140.9	Metal
Radium	Ra	226.0	Metal
Rhodium	Rh	102.9	Metal
Rubidium	Rb	85.45	Metal
Ruthenium	Ru	101.7	Metal
Samarium	Sa	150.4	Metal
Scandium	Sc	44.1	Metal
Selenium	Se	79.2	Metalloid
SILICON	Si	28.3	Metalloid
SILVER (argentum)	Ag	107.88	Metal
SODIUM (natrium)	Na	23.00	Metal
Strontium	Sr	87.63	Metal
SULFUR	S	32.06	Metalloid
TANTALUM	Ta	181.5	Metal
Tellurium	Te	127.5	Metalloid
Terbium	Tb	159.2	Metal
Thallium	Tl	204.0	Metal
Thorium	Th	232.4	Metal
Thulium	Tm	168.5	Metal
TIN (stannum)	Sn	118.7	Metal
TITANIUM	Ti	48.1	Metal
TUNGSTEN (Wolfram)	W	184.0	Metal
Uranium	U	238.2	Metal
VANADIUM	V	51.0	Metal
Xenon	Xe	130.2	Inert gas
Ytterbium (Neoytterbium)	Yb	173.5	Metal
Yttrium	Yt	88.7	Metal
ZINC	Zn	65.37	Metal
Zirconium	Zr	90.6	Metal

A Chemical Element is a substance that cannot by any means at our disposal b separated into two or more different substances. CHEMICAL COMPOUNDS are pure sub stances which can, by chemical means, be separated or decomposed into different element

or which can be made by combining different elements. The MOLECULES of a compound are regarded as aggregates of smaller particles of the elements, known as ATOMS, into which the compound can be broken up, the atoms of any given element being regarded as all precisely alike in every respect. The atoms are considered the smallest mass-elements which occur separately in the structure of molecules of either compounds or elementary substances, so far as can be determined by ordinary analysis. The molecule of an element consists of a definite (usually small) number of its atoms. The molecule of a compound consists of one or more atoms of each of its several elements, the numbers of the various kinds of atoms and their arrangement being definite and fixt, and determining the character of the compound.

2. Chemical Reactions

A Chemical Compound is represented by means of the symbols of the elements concerned. Thus, HCl, hydrochloric or muriatic acid gas, indicates a compound made up of one atom each of hydrogen and chlorine; NaOH, sodium hydroxide or caustic soda, contains sodium, oxygen, and hydrogen. The elements do not always combine atom for atom, as in these examples, but several atoms of each of different kinds may combine; as, H_2O, water; HNO_3, nitric acid; Na_2SO_4, sodium sulfate, etc. Each of these formulas represents a molecule or the smallest portion of the compound that can exist. Chemical reactions are represented by means of equations thus: $2\ H + O = H_2O$ represents the formation of water by the union of two atoms of hydrogen with one of oxygen. $CaCO_3 + 2\ HCl = CaCl_2 + H_2O + CO_2$ indicates that when one molecule of calcium carbonate ($CaCO_3$) is acted on by two molecules of hydrochloric acid, the products formed are calcium chloride, water, and carbon dioxide gas. The masses of the atoms of different elements are found to bear constant, definite ratios to one another. Oxygen, which is taken as the standard, has an atomic mass of 16. The atomic weight of carbon is 12, which means that its atom has $12/16 = 3/4$ the mass of an atom of oxygen. So, too, the atomic mass of bismuth is $208 = 208/16 = 13$ times that of oxygen.

When two or more elements unite, the molecule of the compound formed must have a mass equal to the sum of the masses of the constituent atoms. The molecular mass of phosphorus pentoxide, P_2O_5, for instance, can be calculated as follows: $2\ P = 2 \times 31.04 = 62.08$, and $5\ O = 5 \times 16 = 80$; therefore $P_2O_5 = 62.08 + 80 = 142.08$. A chemical equation can be interpreted not only qualitatively but quantitatively, and from the relative masses of the atoms of the reacting substances the amount of each product formed may be calculated. The equation $C + 2\ O = CO_2$ means, then, that one atom of carbon (atomic weight = 12.005) combines with two atoms of oxygen to form one molecule of carbon dioxide (carbonic acid gas). This molecule must have the molecular weight $12.005 + (2 \times 16) = 44.005$. In other words 12.005 parts by weight of carbon yield 44.005 parts by weight of carbon dioxide. When calcium carbonate (limestone) is heated we get quicklime and carbon dioxide, as represented thus: $CaCO_3 = CaO + CO_2$. Consulting the table of atomic weights it is seen that $40.07 + 12.005 + (3 \times 16) = 100.075$ parts of calcium carbonate will yield $40.07 + 16 = 56.07$ parts of quicklime and $12.005 + (2 \times 16) = 44.005$ parts of carbon dioxide. Or, to get 56.07 parts of lime 100.075 parts of calcium carbonate are required. In other words, 1 ton of pure limestone will yield the amount of lime determined by the proportion $100.075 : 56.07 :: 1$ ton : $x = 0.56$ ton. Or the amount of material required to produce 1 ton of lime may be calculated as follows: $100.075 : 56.07 :: x : 1$ ton. Here $x = 1.78$ tons of calcium carbonate. These calculations assume that the materials are free from moisture and other impurities. A limestone containing only 90 percent of calcium carbonate would yield only 90 percent as much actual quicklime tho the weight obtained would be equal to that of the quicklime plus the mineral impurities not driven off in burning.

Classification of the Elements. The chemical elements are usually divided roughly into base-forming and acid-forming. These correspond, in general, with the metals and non-metals or metalloids. This division does not hold strictly, for there are many metals that yield compounds with

acid properties and some of the metalloids form bases. Acids are compounds of the elements with hydrogen and often with oxygen also. The hydrogen can be replaced by metals, forming salts. Bases are compounds of metals with oxygen and hydrogen. When a base and an acid react, the hydrogen and oxygen of the base combine with the hydrogen of the acid to form water, while the metal and the remainder of the acid combine to form a salt. Thus, sodium hydroxide, $NaOH$, combines with HCl, hydrochloric acid, to form H_2O, water, and $NaCl$, sodium chloride (common salt): $NaOH + HCl = H_2O + NaCl$. Another example is the action of sulfuric acid, H_2SO_4, upon calcium hydroxide (slaked lime), CaO_2H_2, the products being calcium sulfate, $CaSO_4$, and water: $CaO_2H_2 + H_2SO_4 = CaSO_4 + 2H_2O$.

It will be noticed that the sodium atom of sodium hydroxide replaces the hydrogen atom of hydrochloric acid, while in calcium sulfate a calcium atom has replaced two hydrogen atoms of sulfuric acid. Other metal atoms have the power of replacing still more hydrogen atoms of acids, as shown by the compounds aluminum chloride, $AlCl_3$, and platinic chloride, $PtCl_4$. Any atom which can be combined with or substituted for a single hydrogen atom is said to be univalent, hydrogen being the standard univalent atom. Any atom, e.g. oxygen or calcium which can replace or combine directly with two univalent atoms, is said to be bivalent, etc. The number of univalent atoms which any given atom can replace or directly combine with is known as valence of the given atom. We say that chlorine is uni- (mono-) valent because it forms the compound HCl; oxygen in water, H_2O, is bi- (di-) valent; nitrogen in ammonia, NH_3, is tri- (ter-) valent; carbon in methane, CH_4, is quadri- (tetra-) valent. The metal calcium is said to be bivalent because it combines with two atoms of chlorine to form $CaCl_2$ or one atom of oxygen to form CaO. A more complex instance is aluminium oxide, Al_2O_3. Here three oxygen atoms having the combined valence 6 unite with two aluminium atoms, the valence of each of the latter being ½ of $6 = 3$.

A given element may have different valences in different compounds. Copper, for instance, forms two chlorides, $CuCl$ and $CuCl_2$, in which its valence is 1 and 2 respectively. Sulfur yields the oxides SO_2 in which its valence is 4, and SO_3 in which it is 6.

3. Chemistry of Lime and Plaster

Quicklime, or calcium oxide (CaO), is made by heating limestone, marble, or shells (oysters, etc.) to a high temperature in kilns. The calcium carbonate ($CaCO_3$), of which the raw material is mainly composed, loses carbon dioxide gas (CO_2), leaving behind calcium oxide mixt with mineral impurities either in their original state or as new compounds formed at the high temperature of the kiln. SLAKED LIME, or calcium hydroxide (CaO_2H_2), is formed by the direct combination of quicklime with water. When only a little more than the exact amount of water necessary for the reaction is used, the slaked lime may be obtained as a fine, dry powder. Great heat is evolved at the same time, and fires have been caused by the accidental wetting of stored quicklime. Lime exposed to the air readily absorbs moisture, forming calcium hydroxide, and this then takes up carbon dioxide, which regenerate calcium carbonate. Hence, air-slaked lime is a mixture of calcium hydroxide and calcium carbonate, or if sufficient time has elapsed, calcium carbonat alone. As air-slaked lime is of no value for mortar, all quicklime should b tested before use by placing several lumps in a little water to see whether i slakes readily.

In making common mortar, a considerable excess of water is used, so that the sand ma be more thoroly incorporated with it and to give a better bond with the bricks. The fir setting of mortar is due to loss of the excess water, partly by evaporation, but mainly b its being absorbed by the pores of the bricks. The carbon dioxide of the air also begi to change the outer layer into calcium carbonate. This action proceeds more rapid as the mortar becomes drier and more porous, until finally only calcium carbonate and t sand are left. The mortar in very old buildings contains considerable calcium silica formed by the slow action between the calcium carbonate and the silica of the sand.

The dampness of recently plastered walls is largely due to the water formed by the action of carbon dioxide on slaked lime ($CaO_2H_2 + CO_2 = CaCO_3 + H_2O$). The burning of charcoal in such rooms hastens the drying, not so much by raising the temperature as by furnishing carbon dioxide in much larger amount than is normally present in the air.

Plaster of Paris is made by heating natural gypsum, a form of calcium sulfate containing "water of crystallization" ($CaSO_4$), to about 125° C. until three-fourths of the water has been driven off. The resultant plaster is chiefly $CaSO_4 . \frac{1}{2} H_2O$, with some $CaSO_4$. It sets very rapidly when mixt with the proper amount of water. When gypsum is burnt at higher temperatures than 125° C., it loses more and more of its water and sets with increasing slowness. At 200° C., it loses all its water and becomes "dead-burnt," having lost its power of setting.

The setting process seems to be due to the solution of part of the $CaSO_4 . \frac{1}{2} H_2O$, which then combines with water to re-form $CaSO_4 . 2 H_2O$, which is less soluble and crystallizes out in the form of fine needles. More of the $CaSO_4 . \frac{1}{2} H_2O$ then dissolves, and so on, until the reaction is complete, each successive lot of crystals interlacing with those previously formed. The setting is accompanied by about one percent increase of volume, which accounts for the sharp outlines obtained in making plaster casts. Theoretically, plaster should be mixt with a little more than 18.3 percent of its weight of water, but twice as much as this is at times used to increase its plasticity and to retard its setting. The hardness of plaster may be increased by mixing it with solutions of alum, borax, and so forth.

Stucco is simply plaster mixt with a dilute solution of glue; it sets comparatively slowly. Floor plaster is made by heating gypsum to much higher temperatures than that at which plaster of Paris is made, it being changed first into "dead-burnt" plaster and then into another modification known as "floor" or "estrich" plaster. The product formed is probably a basic sulfate of calcium or else a mixture of calcium sulfate and oxide. It requires several days to set, which it does with no change of volume, forming a mass considerably harder and stronger than ordinary plaster.

4. Combustion and Fuels

Carbon is the principal constituent of solid, liquid, and gaseous fuels, it being either free, as in charcoal and coke, or combined with hydrogen, oxygen, or with both. In burning, the carbon and hydrogen combine with oxygen from the air, yielding, when combustion is complete, carbon dioxide and water vapor. When insufficient air is admitted over the bed of fuel, or into the fire-box when liquid or gaseous fuels are used, much of the fuel may be lost, either as free carbon (smoke, soot) or in the form of unburned gases, especially carbon monoxide. It is almost impossible to attain complete combustion of solid fuels by forcing air under the grate and thru the bed of coals, because part of the carbon dioxide formed near the grate is reduced to carbon monoxide on passing thru the overlying layers of hot fuel. This monoxide, which has a high heating value, can be burned completely only when a plentiful supply of air is admitted over the bed of fuel. The blue flames seen when anthracite, coke, or charcoal is burned are due to carbon monoxide.* It is not economical to supply sufficient air by forced draft thru the fuel, because the excessive temperatures rapidly burn out the grate bars, and also because of the increased mechanical wear on the boiler tubes due to the greater number of fine particles of coal and ash driven thru them.

Wood consists mainly of lignin and cellulose, compounds of carbon with hydrogen and oxygen, together with varying amounts of water and mineral matter. The latter largely remains behind in the ash.

Charcoal is made by piling wood into heaps which are covered with earth, leaving a few small openings to admit a limited amount of air and allow the products of combustion to escape when the wood is ignited. When sufficient

wood has burned to insure thoro charring ("destructive distillation") of the remainder, the openings are closed and the pile allowed to cool completel By this method of making charcoal only a little tar is obtained and all th volatile constituents are allowed to escape. When wood is heated in close retorts, large amounts of tar, creosote, wood or methyl alcohol, acetone, an pyroligneous (acetic) acid, etc., are obtained. The yield of charcoal is al nearly doubled. Charcoal consists of carbon and the mineral matter of th wood. Its value in metallurgy is due to its low content of phosphorus an sulfur. The calorific value of charcoal is about 75 percent that of anthracit

Peat is formed by the partial decay of mosses and other bog plants under wat Even when comprest and dried it contains much water and its mineral content may high. Its calorific value is 3000 to 4000 cal per kg.

Lignite, or brown coal, is a stage beyond peat in the formation of coal. It contai much moisture and ash and is often high in sulfur content. Its calorific value is 4000 5500 cal per kg. Owing to its large amount of volatile matter, lignite burns with a lo smoky flame.

Bituminous Coal is formed by the further transformation of lignite by he and pressure. It comprises many varieties, including gas, coking, steam, a cannel coals. They differ principally in their content of volatile matter, t "fat" coals having at times as high as 50% of compounds of carbon and hydr gen, which are readily driven off by heating. The length of flame of burni bituminous coal depends on the percentage of volatile matter.

Anthracite Coal. Produced by the further action of heat and pressu upon bituminous coal, whereby nearly all the volatile constituents are driv off, leaving mainly carbon and mineral matter. These coals burn with lit flame and smoke, and do not cake. Their calorific value may be 9000 to 9 cal per kg.

Coke. As charcoal is the residue left by heating wood in retorts or partia burning it with a limited air supply, so coke is made by heating bitumin coals. In some types of coke ovens the gaseous and liquid products form by the destructive distillation are allowed to escape into the air; with ot types of ovens this loss is not permitted and valuable by-products are obtain such as ammonia, fuel and illuminating gas, and coal tar. Coke is mai carbon, but contains also the mineral constituents of the coal. It is low volatile matter and sulfur. Upon this and its infusibility and resistance crushing depends its value as a fuel in blast furnaces. Its calorific va is about 90 per cent that of the much more expensive anthracite.

Chemical Examination of Coal and Coke. The heating value of any can be determined conveniently by means of one of the numerous form bomb calorimeters. But this leaves unanswered many questions which h a very practical bearing, for example the percentages of volatile mat sulfur, and ash, and the amount of coke the coal will yield. In gene the lower the percentage of ash the better the quality of the fuel. The mineral constituent that has any heating value is the sulfur of pyr But as sulfur is injurious in practically all metallurgical operations and oxides of sulfur have a corroding effect upon boiler tubes, etc., a coal in either pyritic or organic sulfur is undesirable. Gas coals and coal use in certain metallurgical operations requiring long reducing flames sh be high in volatile matter. Coals that are to be worked economically coke may be low in volatile matter, but must possess the property of part fusing or caking together when heated in the ovens.

In taking samples, for calorimetric determinations or for chemical analysis, the acce method is to select a large number of pieces, representing as nearly as possible in etc., the whole load. These must be chosen from all parts of the coal to be tested and t

from the top and down thru the whole mass. These lumps should then be broken into nearly uniform size, thoroly mixt together, and a smaller sample obtained by "quartering." If this sample is still too large it should be further broken up and again quartered until a sample that can readily be enclosed in air-tight jars is obtained. In sampling at the mine, the coal should be cut from a freshly exposed face, and the sample, after quartering to suitable size, should fairly represent not only the actual coal but also the interpenetrating veins of shale, etc., if these are regularly mined with the coal. It is very important to prepare the sample, not only with great care but also as rapidly as possible, to minimize the inevitable loss of moisture in breaking up the lumps. This explains the necessity of placing the fuel sample in air-tight receptacles, such as fruit jars with rubber rings. The volatile matter in coal is highly explosive when mixt with air in the proper proportion, so that care should be taken to ensure thoro ventilation of all places where coal, especially bituminous, is stored.

5. Liquid and Gaseous Fuels

Crude Petroleum is the most important of the liquid fuels. It owes its importance not only to its comparative cheapness but also to the ease with which it is handled and its high efficiency, which is two or more times that of anthracite. It is usually burned in the form of a spray obtained by means of a blast of air or superheated steam. Petroleum residues and coal-tar residues are also burned to some extent. Their calorific value is not as great as that of crude petroleum, but may run as high as 16 000 cal.

Gasoline is one of the lower-boiling distillates obtained from crude petroleum. It consists of a mixture of several hydrocarbons containing varying percentages of carbon and hydrogen. Its value as a fuel depends on its great volatility. When mixt with the proper amount of air the vapors form a mixture which is readily ignited and burns with explosive violence. If the vapor is largely in excess of the proportion needed for complete combustion the force of the explosion is weakened, so that, apart from the actual loss of unburned gases, the fuel is not used economically. There is a similar loss in power when too much air is present. For the complete combustion of one cubic foot of the vapor of the hydrocarbon hexane, C_6H_{14}, 45.2 cubic feet of air are required, while the same volume of heptane (C_7H_{16}) vapor requires 52.4 cubic feet of air, or 16 percent more.

Denatured Alcohol as a fuel for explosion engines is used to only a very limited extent at present. Whether or not it can ever compete successfully with gasoline will depend on its cost, relative efficiency, etc. Denatured alcohol is grain alcohol which has been rendered unfit for drinking by the addition of bone oil, wood alcohol, or other ill-smelling substances prescribed by law.

Natural Gas is the most efficient as well as the cheapest of all fuels, tho its use is of course limited by the distance to which it can be economically piped. It consists mainly of methane, CH_4, with 10 percent or less of hydrogen and other gases. Methane is also known as marsh gas, from its abundant formation when vegetable matter decays under water. The name fire damp refers to its occurrence in coal mines, where it is one of the causes of explosions.

Coal Gas, which is made by distilling bituminous coal in retorts, contains 80 to 85 percent of a mixture of nearly equal parts of hydrogen and methane, with much smaller amounts of oxygen, nitrogen, carbon monoxide and dioxide, etc. It is used to some extent in gas engines, and as a fuel.

Water Gas is formed by the action of superheated steam upon white-hot coal or coke. The steam gives up its oxygen to the carbon of the fuel, forming carbon monoxide, CO, and leaving hydrogen, thus, $C + H_2O = CO + 2\ H$. The reaction is endothermic, that is, it requires the addition of heat, so that it is necessary to cut off the steam every few minutes and reheat the fuel by an air blast. Water gas consists of about 45 percent each of hydrogen and carbon monoxide, with small percentages of oxygen, nitrogen, carbon

dioxide, etc. The first two gases burn with very hot, non-luminous flames. For use a
an illuminant it must be "enriched" with naphtha or other similar oil.

Producer Gas is made in much the same way as water gas, except that only air and n
steam is past thru the incandescent coal or coke. The carbon is burned to carbo
monoxide, which makes about 25 percent of the gas. Small amounts of hydrogen
methane, and carbon dioxide are present. There is also nearly 65 percent of nitroge
from the air which is used. This is unavoidable, tho the presence of such a large amoun
of inert gas reduces the thermal efficiency. The reaction whereby carbon is burned t
carbon monoxide is accompanied by the evolution of about one-third the total heatin
value of the fuel. It is evident that if the gas can be burned without allowing it to coo
a great saving of heat can be effected. This is not always feasible and it is the practis
with some forms of producers, to pass some steam with the air, thus making a mi
water-producer gas. The heat which would otherwise be lost is used up in forming wat
gas, and the resultant fuel gas has an increased fuel value. It is much more economic
to convert the fuel into producer gas and use it in explosion engines than to burn it und
steam boilers.

6. Explosives

The Fundamental Property of an explosive is that when ignited or su
jected to a sudden shock it shall decompose, or its components rea
suddenly yielding a relatively large volume of highly heated gas. Th
definition includes not only gunpowder, nitroglycerin, and similar substanc
but mixtures of inflammable gases and vapors with air; or even coal dust, fi
sawdust, or flour suspended in the air. The last three have all been the cau
of disasters, the reason being that when some of the particles are ignit
the flame is rapidly communicated to adjacent ones, yielding large volum
of highly heated gaseous products of combustion, in addition to which t
surrounding air is also heated. Thus, one gram of anthracite, of speci
gravity 1.5, occupies a volume equal to only ⅔ cubic centimeter. If it co
tains 95 percent of carbon, it will yield when burnt about 1761 cc, or 2642 tim
its own volume, of carbon dioxide measured at 0° C. and 760 mm pressu
If suspended as dust in a large volume of air and burned in a fraction o
second, it is evident that the large amount of hot gases must expand w
explosive violence.

Gunpowder is a mixture of 75 parts by weight of saltpeter, or potassiu
nitrate, 15 parts of charcoal, and 10 parts of sulfur, made by grinding
ingredients together with enough water to moisten the mass. It is th
comprest into a cake and broken into grains, which are glazed by revolv
with graphite and sorted into sizes by sieves. The larger grains are used
blasting, and the smaller ones for small arms. BLASTING POWDER is frequen
made with the cheaper Chile saltpeter, or sodium nitrate, which produce
cheaper and less powerful powder than that made from ordinary saltpe
Chile saltpeter, however, has the disadvantage of absorbing moisture fr
the air, and powder made from it cannot be kept too long or stored in a da
place. The proportions used are 73 parts of Chile saltpeter, 16 parts
charcoal, and 11 parts of sulfur.

Guncotton, Nitrocellulose, typical of the second class of explosives, is m
by the action of a mixture of nitric and sulfuric acids upon cotton. W
only moderately strong acids are allowed to act on the cotton for a sl
time, the product is pyroxylin, or soluble nitrocellulose, used for mak
collodion and celluloid. By longer action with more concentrated ac
guncotton is formed. It is then washt in a machine of the kind used
making paper pulp to remove all traces of acids that might cause spontane
explosions. While still moist, it is comprest into blocks or sticks. G
cotton is usually stored and transported in a moist condition, and can be

ploded without drying. It is comparatively safe to handle, as ordinary shocks do not explode it readily. In the open, it burns with extreme rapidity.

Nitroglycerin is made by the cautious addition of glycerin to a well-stirred and cooled mixture of the strongest nitric and sulfuric acids. The oily product is washt to remove all traces of acids that might cause spontaneous explosion. Under the most favorable conditions, nitroglycerin is not safe to handle. The fact that it is a liquid with consequent liability to leakage from containers greatly increases the danger of transportation and storage. For this reason, it is commonly mixt with some absorbent or transformed into a gelatinous mass.

Dynamite is a mixture of nitroglycerin with infusorial earth, powdered "rottenstone," or similar porous material, known as "dope." Instead of these inactive dopes that take no part in the explosion, explosive mixtures are often used to absorb the nitroglycerin. Gunpowder is one of these. Dynamite consisting of 40 percent nitroglycerin, 44 percent sodium nitrate, 15 percent wood pulp, and 1 percent calcium carbonate, is an example of dynamite with an active dope.

Explosive gelatin is a jelly-like mass made from a solution of soluble nitrocellulose in nitroglycerin. Too powerful for common work; it is used with success for very hard rock in tunnels. Gelatin dynamite is a mixture of explosive gelatin with a dope such as sodium nitrate and wood pulp; it is not so powerful as the straight gelatin. Smokeless powder is a general term covering many modifications of explosive gelatin, and mixtures of nitrocellulose with nitrobenzene, etc.; they are usually given fanciful names, as ballistite, cordite, indurite, and so forth. Nitroglycerin and mixtures containing it are all liable to freeze at moderately low temperatures. They cannot be used satisfactorily in that condition, and should not be thawed by placing them near a fire or on steam pipes but by leaving them in a warm chamber kept at a temperature not over 90° F.

Picric Acid, or trinitrophenol, is made by the action of nitric and sulfuric acids upon phenol (carbolic acid). It is a yellow, crystalline substance, formerly used only as a dye for silks and so forth. For years it was not known as an explosive, but it is now known that it will explode with great violence when detonated. If ignited, it usually burns without exploding and is not very susceptible to shock. Lyddite, melinite, and shimose are composed of picric acid. Some of the salts, or picrates, are exploded by slight blows.

Nitrocellulose, Nitroglycerin, and Nitrostarch, are true nitrates, as they all contain the atomic group NO_3. They are chemically quite different from the true nitro-compounds, such as picric acid, which contain the atomic group NO_2. Benzene, toluene, naphthalene, and other substances obtained from coal tar yield nitro-compounds when treated with nitric acid. The best known of these is trinitrotoluene, or "TNT," which was used in such enormous quantities in the great war. These are all more or less unstable and are used as components of explosives, mixt either with ammonium nitrate or other nitrates, or with chlorates, which are good oxidizing agents, or they may be used in dynamite because they lower the freezing point of the nitroglycerin. Rack-a-rock, roburite, bellite, and securite are typical of the explosives made from these nitro-compounds and oxidizing agents.

A Detonator contains a high explosive, too powerful and unstable to be employed alone, which by its sudden disruptive force brings about the instantaneous explosion of a large amount of a more stable explosive. The ones commonly in use consist of copper capsules containing a definite amount of a mixture of chlorate of potash and mercury fulminate, which is exploded either by a fuse or a wire heated electrically. The fulminate is made by mixing a solution of mercury in strong nitric acid with alcohol. The gray, crystalline powder which is precipitated must be well washed to remove all acid. It is sensitive to shock and may explode even when wet.

Explosives must be selected with reference to the character of the work. For quarryi[ng] building stone, those that act slowly, with little shattering effect, must be chosen. Wh[en] the stone is to be crushed after quarrying, or for breaking up rock so that it can be handl[ed] by a steam shovel, a quick shattering effect is desired. In all open work, the charact[er] of the gases arising from the explosion may be disregarded, but in tunnels or min[es] especially if not well ventilated, this factor is of great importance. No explosive is abs[o]lutely safe in this respect. In coal mines, where the presence of fire damp (methan[e]) is a menace, no explosive giving a long flame or a high heat of detonation should [be] used. Even in the absence of gas, there is danger of igniting the coal dust.

Explosives should be stored in a dry place so that the sodium or ammonium nitra[te] will not take up enough moisture to lessen their power. But if in too dry a place, th[ey] may lose the moisture they naturally contain, which will change their speed of explosi[on] and thus modify the character of the results obtained. Explosives should not be sto[red] for a longer time than absolutely necessary, on account of the possibility of chemi[cal] changes taking place in the nitro-compounds most of them contain.

PHYSICS

7. Physical Properties of Solids

The properties of substances nominally the same differ so widely tha[t it] would be misleading to give more than two or three significant figures [in] most cases without such detailed specification of conditions as would ma[ke] the tables too voluminous. Lower and upper limits of values found [for] different specimens are given when warranted by the data available.

Physical Properties of Rocky Materials

Substance	Specific gravity or density	* Coefficient of cubical expansion (Mean 0–100° C.)	Specific heat (Mean 0–100° C.)
Asphaltum	0.9 1.7	....	....
Basalt	2.7 3.2	....	0.20 0.24
Brick	1.4 2.3	0.18	0.22
Cement,† loose	1.3 2.0	....	....
Cement,† set	2.7 3.2	0.30 0.42	0.2+
Coal, anthracite	1.4 1.8	.... 0.6	....
Coal, bituminous	1.2 1.5	....	....
Concrete	1.8 2.5	0.30 0.43	....
Glass	2.4 4.5	0.17 0.29	0.19±
Glass (Quartz)	.....	0.015	0.18
Glass (Jena 16III)	.....	0.253	....
Gneiss	2.4 2.7	....	0.20 ....

Substance	Specific gravity or density	* Coefficient of cubical expansion (Mean 0–100° C.)	Spec[ific] he[at] (Me[an] 0–100° [C.])
Granite	2.5 3.1	.... 0.26	0.[illegible]
Graphite	1.9 2.3	0.23	0.[illegible]
Greenstone	2.9 3.0	....	...
Limestone	2.4 2.9	0.075 0.24	... 0.[illegible]
Marble	2.5 2.8	0.13 0.25	... 0.[illegible]
Porcelain	2.3 2.5	0.09 0.11	...
Sandstone	2.2 2.5	0.18 0.36	0.[illegible]
Serpentine	2.4 2.7	....	0.[illegible]
Slate	2.6 2.9	.... 0.31	0.[illegible] 0.[illegible]
Soapstone	2.6 2.8		
Terra cotta	1.9	....	..
Trap	2.7	....	..

* Divide each number in this column by 10 000. † Portland.

Mohs's Scale of Hardness: 1. Talc, 2. Gypsum, 3. Calc spar, 4. Fluorspar, 5. Apatite, 6. Feldspar, 7. Quartz, 8. Topaz, 9. Sapphire, 10. Diamond.

Density. The terms True Density and Apparent Density are used in describing certain porous bodies to distinguish between the density of the substance and the average density of the substance plus the pores. The terms Density and Specific Gravity are synonymous in engineering work.

Thermal Conductivity. The value given in the tables is the number of gram-calories that will pass per second thru every square centimeter of a plane section within the substance when the temperature is uniform over the section and falls along the normal to it at the rate of 1° C. per cm.

1 gm-cal per sec per sq cm for a temperature gradient of 1° C. per cm = 360 kg-cal per hr per sq m for a temperature gradient of 1° C. per m = 2.91×10^3 Btu per hr per sq ft for a temperature gradient of 1° F. per in.

Electrical Resistivity. Each value given in the table of metals and alloys is the resistance in microhms between the opposite faces of a cube 1 cm on each edge when at 18° C. (64.4° F.). This increases bR for every degree C., or $5/9\ bR$ for every degree F., that the temperature of the substance exceeds 18° C. 1 microhm to the sq cm of cross-section per cm of length = 6.015 ohms to the circular mil of cross-section per ft of length.

Physical Properties of Woods

Kind	Specific gravity or density		* Coefficient of linear expansion 2°–34° C.	
	Dry	Green	Parallel to fibers	Perpendicular to fibers
Acacia	0.58	0.75		
	0.85	1.00		
Alder	0.42	0.63		
	0.68	1.01		
Ash	0.57	0.70		
	0.94	1.14	0.095	
Beech	0.62	0.85	0.026	0.61
	0.90	1.25	0.06	
Birch	0.51			
	0.77			
Blue gum	0.84			
Box	0.91	1.20		
	1.16	1.26	0.026	0.61
Butternut	0.38			
Cedar	0.49			
	0.57			
Cherry	0.70	1.05		
	0.90	1.18		
Chestnut	0.58		0.065	0.33
Cork	0.22			
	0.26			
Ebony	1.11			
	1.33		0.097	
Elm	0.54	0.78		
	0.82	1.18	0.057	0.44
Fir	0.31	0.38		
	0.85	1.08	0.037	0.58
Hickory	0.60			
	0.93			
Lancewood	0.68			
	1.00			
Larch	0.47			
	0.56	0.81		
Lignum Vitæ	1.17			
	1.33			
Linden or lime	0.32	0.58		
	0.59	0.87		
Locust	0.67			
	0.71			
Mahogany	0.56			
	1.06		0.036	0.40
Maple	0.53	0.83		
	0.81	1.05	0.064	0.48
Oak	0.60	0.93		
	1.07	1.28	0.049	0.54
Pear	0.61	0.96		
	0.73	1.23		
Pine	0.35	0.40		
	0.85	1.07	0.054	0.34
Poplar	0.35	0.61		
	0.59	1.07	0.039	0.37
Satinwood	0.95			
Spruce	0.48			
	0.70			
Sycamore	0.40			
	0.60			
Teak	0.66			
	0.98			
Walnut	0.60	0.91		
	0.81	0.92	0.066	0.48
Willow	0.40			
	0.60	0.79		

* Numbers in these columns to be divided by 10 000.

Physical Properties of Metals and Alloys

Substance	Specific gravity or density	Hardness (Mohs)	* Freezing point (Centigrade)	†Coefficient of cubical expansion (Mean 0—100° C.)	Specific heat (Mean 0—100° C.)	Thermal conductivity (Mean 0—100° C.)	Electrical resistivity at 18° C. Microhm per cm cu	Electrical resistivity at 18° C. Temperature coefficient %
Aluminum. . Al	2.7	3−	**658**	0.70	0.22	0.48	2.8 3.2	0.
Antimony. . . Sb	6.7	3+	**630**	0.27 0.50	0.050	0.042	45.	0.
Bismuth. . . . Bi	9.8	2+	270	0.37 0.49	0.030	0.018	120.	0.
Brass.	8.1 8.7	3+	900±	0.55 0.64	0.092	0.15 0.30	6. 9.	. .
Bronze.	7.4 8.9	3+	900±	0.51 0.57	0.086	. . . .	. . . .	. . .
Cobalt. . . . . . Co	8.6	6	**1490**	0.37	0.107	. . . .	. . . .	. . .
Constantan. . . . ("Advance")	8.8	. . . .	. . . .	0.45	0.10	0.6	47. 50.	−0 +0
Copper. . . . . Cu	8.5 9.0	3	**1083**	0.51	0.093	0.89	1.7 1.8	0
German silver.	8.3 8.7	3+	1000±	0.55	0.095	0.07 0.09	16. 49.	0 0
Gold. . . . . . . Au	19.3	3−	**1063**	0.44	0.032	0.70	2.3	0
Iron. . . . . Fe	7.9	4	1520	0.36 0.43	0.113	0.11 0.18	9. 15.	up c
Iron (cast).	7.6 7.7	6 8	1100 1300	0.32 0.35	0.11 0.13	. . . .	56. 114.	. . . . . .
Iridium. . . . . Ir	22.4	. . . .	2300±	0.20	0.032	0.34	5.3	c
Lead. . . . . . . Pb	11.3	2−	**327**	0.88	0.031	0.082	21.	
Manganin.	8.5	. . . .	. . . .	0.54	. . . .	0.6	39. 46.	<
Mercury. . . . Hg	13.6	. . . .	−39	1.82	0.033	0.018	95.8	
Molybdenum . . . . Mo	8.4 9.9	. . . . .	2500±	. . . .	0.072	. . . .	6.	. .
Nichrome.	8.2	. . . .	1500±	. . . .	. . . .	. . . .	96.	
Nickel. . . . . . Ni	8.3 9.2	4+	**1450**	0.39	0.109	0.14	11. 15.	
Osmium. . . . Os	22.5	. . . .	>2500	0.20	0.031	. . . .	10.	
Palladium. . Pd	12.0	. . . .	1550	0.36	0.058	0.17	10.7	
Platinum. . . Pt	21.2 21.7	4+	**1755**	0.27	0.032	0.17	8. 16.	
Rhodium. . . Rh	12.4	. . . .	1900±	0.26	0.058	0.30	6.0	
Silver. . . . . . Ag	10.5	2+	**961**	0.58	0.056	1.00	1.6	
Steel.	7.9	4 9	1300 1450	0.30 0.41	0.114 0.117	0.06 0.14	10. 50.	
Tantalum. . . Ta	14.1 16.6	. . . . .	2900	0.24	0.036	. . . .	15.	
Tin. . . . . Sn	7.3	2	**232**	0.68	0.056	0.15	. . . .	
Tungsten. . . W	18.8	. . . .	3000	. . . .	0.033	. . . .	6.	
Zinc. . . . . . . Zn	7.1	3+	**419**	0.87	0.094	0.26	6.1	

* Heavy type shows values suitable for standardizing pyrometers.
† Numbers in this column to be divided by 10 000.

Materials for Thermal Insulation and Furnace Linings

Substance	Specific gravity or density		Thermal conductivity (Mean)	
	True	Apparent	Coefficient	Range
Asbestos wool	....	0.2	0.0003	....
Ashes (wood)	....	0.5	0.0002	....
Bauxite brick	3.2	1.9	0.0033	150–1150° C.
Building brick	2.5	1.9	0.002	0–100°
			0.003	100–1100°
Carborundum brick	2.9	2.0	0.003	150–1200°
			0.015	
Checker brick	2.7	1.9	0.004	100–800°
Chromite brick	4.0	3.1	0.006	100–1200°
Cork slab	....	0.2	0.0001	....
			0.0004	
Felt (hair)	....	0.2	0.0002	....
Firebrick	2.6	1.8	0.0014	0–500
	....	....	0.003	0–1300
	....	....	0.004	200–1200
Gas retort brick	2.6	1.9	0.004	100–1100
Graphite brick	2.4	1.8	0.025	300–700
Infusorial earth brick	2.5	1.0	0.0018	100–900
Magnesia brick	3.4	2.0	0.0062	0–1300
	....	....	0.003	50–1100
	....	....	0.007	19–100
Sand (quartz)	....	....	0.00060	18–98
Sawdust (mahogany)	....	0.3	0.0002	0–100
Silica brick	2.7	1.6	0.002	150–1000
Stoneware brick	2.4	2.0	0.004	100–1000
Terra cotta	....	....	0.003	100–1100

8. Properties of Water and Gases

1 Standard Atmosphere is the pressure that will support a column of mercury 76 m = 29.921 in high at 0° C. at a place where $g = g_0 = 980.665$ cm per sec per sec.

Specific Heat of Water. (From Kohlrausch, 1910)

0° C.	1.008	40°	0.9990	140°	1.025	7°	1.0033	15°	1.0000
5	1.0044	50	1.0000	160	1.036	8	1.0028	16	0.9998
10	1.0018	60	1.0017	180	1.048	9	1.0023	17	0.9995
15	1.0000	70	1.0034	200	1.062	10	1.0018	18	0.9993
20	0.9989	80	1.005	220	1.077	11	1.0014	19	0.9991
25	0.9984	90	1.007	240	1.094	12	1.0010	20	0.9989
30	0.9983	100	1.010	260	1.113	13	1.0007	21	0.9987
35	0.9985	120	1.017	280	1.133	14	1.0003	22	0.9986
40	0.9990	140	1.025	300	1.155	15	1.0000	23	0.9985

Specific Volume of Water. (From Kohlrausch, 1910)

°C.	cu cm per gm	°C.	cu cm per gm	°C.	cu cm per gm	°C.	cu cm per gm	°C.	cu cm per gm
0	1.000 13	45	1.009 85	90	1.035 90	140	1.079	230	1.215
4	1.000 00	50	1.012 07	95	1.039 59	150	1.090	240	1.236
10	1.000 27	55	1.014 48	99	1.042 65	160	1.102	250	1.26
15	1.000 87	60	1.017 05	100	1.043 43	170	1.114	260	1.28
20	1.001 77	65	1.019 79	101	1.044 22	180	1.128	270	1.30
25	1.002 94	70	1.022 70	102	1.045 01	190	1.143	280	1.34
30	1.004 35	75	1.025 76	110	1.051	200	1.159	290	1.38
35	1.005 98	80	1.028 99	120	1.060	210	1.177	300	1.42
40	1.007 82	85	1.032 37	130	1.069	220	1.195	310	1.46

Boiling Point of Water in Centigrade Degrees. (From Wiebe, 1910)

	Height of mercurial barometer in mm											
	680	690	700	710	720	730	740	750	760	770	780	790
0	96.92	7.32	7.71	8.11	8.49	8.88	9.26	9.63	100.00	0.37	0.73	1.09
1	6.96	7.36	7.75	8.14	8.53	8.91	9.29	9.67	0.04	0.40	0.76	1.12
2	7.00	7.40	7.79	8.18	8.57	8.95	9.33	9.70	0.07	0.44	0.80	1.16
3	7.04	7.44	7.83	8.22	8.61	8.99	9.37	9.74	0.11	0.48	0.84	1.19
4	7.08	7.48	7.87	8.26	8.65	9.03	9.41	9.78	0.15	0.51	0.87	1.23
5	7.12	7.52	7.91	8.30	8.69	9.07	9.44	9.82	0.18	0.55	0.91	1.26
6	7.16	7.56	7.95	8.34	8.72	9.10	9.48	9.85	0.22	0.58	0.94	1.3
7	7.20	7.60	7.99	8.38	8.76	9.14	9.52	9.89	0.26	0.62	0.98	1.3
8	7.24	7.63	8.03	8.42	8.80	9.18	9.56	9.93	0.29	0.66	1.02	1.3
9	7.28	7.67	8.07	8.45	8.84	9.22	9.59	9.96	0.33	0.69	1.05	1.4

Common Gases. (From Kohlrausch, 1910)

Gas	Specific gravity or density*	Molecular mass	Specific heat (0°–200°C.) constant pressure	$\frac{c_p}{c_v}$ ‡	Melting point C°.	Boiling point C°.	Water dissolves cu cm †		Sy bo
							At 0° C.	At 20° C.	
Air (free of CO_2)	1.2928	28.98	0.238	1.40	....	−193	29	19	Ai
Acetylene.....	1.1759	24.02	......	1.26	− 81.5	− 83.6	(12×10^5)	(7×10^5)	C_2
Ammonia......	0.7708	17.03	0.52	1.32	− 78	− 33.5	(1800)	(900)	N
Carbon dioxid..	1.9768	44.00	0.218	1.30	− 57	− 78.2	35.4	23.2	C
Carbon monoxid	1.2503	28.00	0.243	1.41	−207	−190.0	(4600)	(2300)	C
Chlorine.......	3.2197	70.92	0.121	1.32	−102	− 33.4	21.1	18.1	Cl
Hydrogen	0.08985	2.016	3.41	1.41	−259	−252.6	23.5	15.4	H
Nitrogen......	1.2507	28.02	0.244	1.41	−210.5	−195.7	1300	650	N
Nitrous oxid...	1.9777	44.02	0.225	1.28	−103	− 90	48.9	31.0	N
Oxygen.......	1.4292	32.00	0.220	1.40	−227	−182.8			O

* Numbers in this column to be divided by 1000. † These columns contain the n ber of cubic centimeters of gas that will be dissolved at a barometric pressure of 76 c meters in one liter of water. ‡ This column gives the ratio of the specific heat at cons pressure to that at constant volume.

Freezing Mixtures. (From Hütte)

Mixture	Parts by mass	Temperature falls ° C. From	Temperature falls ° C. To	Parts by mass	Temperature falls ° C. From	Temperature falls ° C. To
Common salt (NaCl)........ Snow.....................	1 3	0	−17.7	1 1	0	−18
Calcium chlorid ($CaCl_2$)...... Snow.....................	3 2	0	−33	2 1	0	−42
Sal ammoniac (NH_4Cl) ...... Saltpeter (KNO_3)........... Water....................	5 5 16	+10	−12	1 1 1	+8	−24
Ammonium nitrate (NH_4NO_3) Water....................	1 1	+10	−16	.....	......	......
Potassium hydroxide (KOH) . Snow.....................	4 3	0	−37	.....	......	......

9. Light and Illumination

Speed of Light in a vacuum is approximately 300 000 kilometers per second. All kinds of light move with the same speed in a vacuum.

The waves of the easily visible portion of the spectrum vary in length from 0.0000 33 to 0.0000 77 centimeters. According to Rowland the length of a wave corresponding to Frauenhofer's band *A* is between 0.000 7594 and 0.0000 7671 cm.

When a ray of light strikes the plane surface of a transparent body, it is refracted toward the normal to that plane. The ratio of the sine of the angle between the original ray and the normal to the sine of the angle between the refracted ray and the normal is called the index of refraction, the value of which is 1.0003 for air, 1.33 for water, 1.51 for soft crown glass and 1.55 for rock salt for sodium light entering from a vacuum.

The ratio of the speed of light in a vacuum to that in any substance is equal to the index of refraction of the substance.

The ratio of the electromagnetic to the electrostatic units of electrical quantity and current is equal to the speed of light in a vacuum exprest in centimeters per second, that is, approximately 3×10^{10}.

The International Candle is the unit of light employed since July 1, 1909, for all photometric standardization by the national standardizing laboratories of America, France, and Great Britain. By May 1, 1910, it was in use by all the large manufacturers of electric lamps in the United States as the unit for rating lamps. This is 1.6% smaller than the unit previously employed in the United States, but the same as the former pentane candle of England and the bougie décimale of France. The Hefner unit of Germany is 9/10 of the international candle.

1 International Candle = 1 Pentane Candle = 1 Bougie Décimale = 1 American Candle = 1.11 Hefner Unit = 0.104 Carcel Unit.

The Energy consumed by incandescent electric lamps producing a given illumination has been materially reduced by the introduction of graphitized and especially of metallic filaments capable of standing higher temperatures than the old-type carbon filament. The watts per mean horizontal candle power required on the average by the best makes of lamps are as follows for four kinds of filaments: 3.1 for standard carbon, 2.5 for metallized (graphitized) carbon, 2.0 for tantalum, and 1.25 for tungsten.

Lamps with Frosted Globes fall to 80% of their initial candle power (which marks the end of their useful life) in about 60% to 70% of the time required for plain lamps to deteriorate to the same extent. Hyde has shown that most, if not all, of this effect can be accounted for by the diffusing action of the frosting causing a much greater proportion of the light to pass several times thru the strongly absorbing film of carbon that is deposited on the inside surface of the bulb. (Bull. Bureau of Standards, 3, 341, 1907.)

Standard Specifications for the purchase of incandescent electric lamps are published by the Bureau of Standards, Washington, D.C.

10. Microscopes and Telescopes

Magnification, increasing visibility of detail, is secured by bringing the image of an object nearer to the eye, or by any other means of increasing the visual angle which the object subtends. Vision with the unaided normal eye is, however, most distinct at a distance of 25 to 30 cm, because the accommodating mechanism of the eye is unable to focus sharply on the retina points nearer than this. A magnifying optical system produces in effect the required increase in the visual angle while forming an image (real or virtual) farther from the eye than the least distance of distinct vision. Often the arrange-

ment is such that the eye views a virtual image at an infinite distance, so the muscles of accommodation may be completely relaxed.

The Simple Microscope (Fig. 1). A single converging lens if pl closer to an object than the principal focal length produces an enla virtual image, which is seen on looking thru the lens. The magnifica produced is $1 + d/f$ for an eye whose least distance of distinct vision A simple plano-convex lens, with the plane side toward the eye, gives images for magnification less than eight diameters, that is, with focal len greater than a 3 cm. The image be much impr especially where magnification is siderable, by the special combina of lenses design reduce spherical chromatic aber so as to give a fairly large field of view approximately free from dist and color.

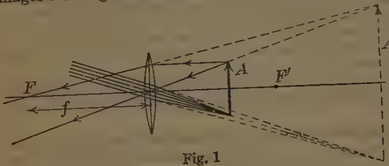

Fig. 1

The Ramsden, or Positiv, Eyepiece (Fig. 2) consists of two conv lenses, usually plano-convex, with their convex surfaces facing each of equal focal length, and separated by ⅔ the focal length of either. A virtual image of the object or real image A is formed by the field-lens L_1 at A'. The eye-lens L_2 forms an image of this at infinity. This eyepiece is fairly, but not quite, achr

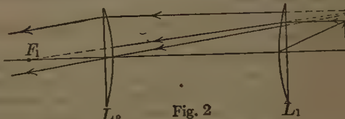

Fig. 2

The Huyghens, or Negativ, Eyepiece (Fig. 3). Two converging usually plano-convex, with the plane surfaces toward the eye, are so ar as to divide equa tween them the de produced on i light parallel to a to the axis. Th lens L_1 has thre the focal length eye-lens L_2, and are separated difference in the lengths. Light which if unhindered would converge at A is deviated form an image at A', of which L_2 forms an image at infinity. This is highly achromatic and free from disturbing spherical aberration.

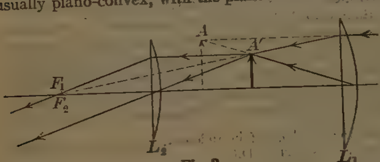

Fig. 3

For Measuring Microscopes and Telescopes, in which the eyepiece is fi cross-hairs, the positiv eyepiece is far more suitable than the negativ because of the hairs being formed by both lenses is corrected for both chromatic and aberration, and because the cross-hairs can be easily adjusted to suit differen altering their distance from the eyepiece.

The Compound Microscope (Fig. 4) in its simplest form consists of two c lenses. The objective L_1 forms within the tube a real, inverted, magnified of the object A. This image is viewed thru the eyepiece L_2 and further mag **microscope** is usually fitted with either a Huyghens or a Ramsden eyepiece, ac

the purpose for which it is to be used. The objective is also generally a combination of several lenses to overcome spherical and chromatic aberration while admitting as much light as possible. In microscopes of the highest power a drop of oil of cedar is placed between the slide and the objective; this is known as "immersion." The smallest interval that can be optically resolved is about 0.00005 mm, and the limit of resolution of the microscope is attained when the total magnification is about 1200.

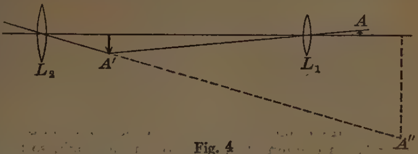

Fig. 4

The Astronomical Refracting Telescope differs from the compound microscope in that the objective forms a reduced image of a distant object. The objective is generally a compound lens consisting of a convex lens of crown and a concave lens of flint glass. A Huyghens or a Ramsden eyepiece is ordinarily used; but the best instruments employ eyepieces embodying later improvements.

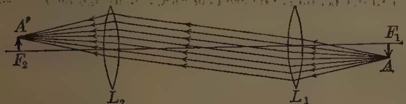

Fig. 5

The Terrestrial Telescope (Fig. 5) produces an erect image by an inverting system between the eyepiece and the inverted image formed by the objective. One form of inverting system consists of two converging lenses of equal focal length so placed that the inverted image A formed by the objective is at the principal focus of the first lens. An erect image A' is then formed at the principal focus of the second lens, and is magnified by an eyepiece.

Galileo's Telescope (Fig. 6) consists of a convex lens L_1 for objective and a concave lens L_2 for eyepiece. The light from L_1 converging so that if unhindered it would form at A' a real, inverted image of the distant object A, is intercepted by L_2 and rendered parallel or slightly divergent as if it came from A'', which is a virtual, erect image of A. The use of the diverging eye-lens limits considerably the angular field of view. Ordinary field-glasses and opera-glasses are Galilean telescopes.

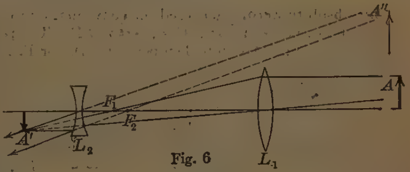

Fig. 6

The Prism Binocular (Fig. 7) secures the wide field of view that accompanies the use of a converging eyepiece, while at the same time it avoids the inconvenient length of the ordinary terrestrial telescope. This is accomplished by employing four total reflections within two right-angled prisms to invert the image formed by the objective. Otherwise the construction is the same as that of the astronomical telescope. This prism construction permits a considerable shortening of the telescope by separating the prisms, since the light traverses the distance between them three times. In addition the stereoscopic effect due to binocular vision may be greatly increased by placing the centers of the objectives much farther apart than the pupils of the eye. The increased field of view is obtained at a sacrifice of illumination.


Fig. 7

In **Reflecting Telescopes** the object lens is replaced by a concave mirror. Being strictly achromatic, reflecting telescopes are valuable for certain classes of astronomical work. Mirrors as large as six feet in aperture have been made, but the best work has

been done by those about two feet in diameter. The mirror is of glass, upon wh
thin film of silver has been deposited.

11. Determination of Temperature

The International Hydrogen Scale adopted in 1887 by the Internati Committee on Weights and Measures is the standard thruout the worl designating temperature. The temperature of melting ice is designate 0° C. and that of the saturated vapor of distilled water boiling at stan atmospheric pressure is 100° C. Other temperatures are defined as pro tional to the increments of pressure exerted by any fixt mass of hydr confined at constant volume, the pressure when at 0° C. to be that due t weight of a column of mercury one meter high; so that when the gas e a pressure p, its temperature will be designated by as many degrees c grade as 1/100 of the increase of pressure on warming from 0° to 10 contained in the amount p exceeds algebraically the pressure when at 0°

Between 100° and 1500° C. nitrogen is used instead of hydrogen, as the latter c be employed at high temperatures because of its great chemical activity and the rea with which it permeates the walls of containing vessels. Temperatures beyon range of the nitrogen thermometer are specified by radiation methods. Within the for which it is adapted, each of these scales agrees with Kelvin's thermodynamic as closely as it is possible to determine temperatures at the present time (1919), an be regarded as representing thermodynamic temperature within its own interval.

Temperatures used in specifying scientific data are almost unive exprest in degrees of the International Centigrade Scale, except by English and American engineers, who use the designation of Fahre If the same temperature is designated by C.° Centigrade (or Celsius Fahrenheit, and R.° Reamur, then

$$C/100 = (F - 32)/180 = R/80$$

The Absolute Zero, often used for convenience in making thermody computations, is about −273° C. or −459°.6 F. Temperatures specified this zero are called Absolute Temperatures (C. or F.), and on the cent scale are sometimes indicated by the letter K following the number of de

A Thermometer, or instrument to indicate the temperature of a b constructed by using any physical property whatever that varies wi temperature, provided that this property always has the same value same temperature, and that there is not more than one temperature the range thru which the thermometer is to be employed at which this erty has any one particular value. Convenience, sensitiveness, reliabi indications, and other considerations dictate the type of thermomete suited for a particular purpose. The graduations of a thermomete indicate degrees directly, or they may require translation by means of a Instruments, even when apparently alike, often differ considerably in th dications when at the same temperature. For all work where it is im to know temperatures accurately, an instrument should be employed th been calibrated (better both before and after use) by the Bureau of Sta at Washington, or by one of the other national physical laboratories, a vided with a statement of the conditions to be observed in its use as with a certified table of corrections.

Standard Temperatures of Reference for checking the accuracy o mometers are generally the freezing or the boiling points of certain sub easily obtainable in the pure state, which have been accurately dete in degrees of the standard scale. Standard boiling points under pres 76.0 cm mercury are as follows, the quantity following the + sign be increase for each cm of pressure:

Water .... 100.0° C. + 0.4°	Benzophenone 305.9° C. + 0.75°
Aniline ...... 184.1° C. + 0.5°	Sulfur......... 444.9° C. + 0.90°
Naphthalene .. 218.0° C. + 0.6°	Zinc........ 920° C. + 1.5°

A Pyrometer is a thermometer intended for use near and above the temperature at which heated bodies begin to glow. Various devices for determining temperature are on the market or used in scientific investigations; but most of those not enumerated below are either too crude and unreliable, or too complicated in construction or manipulation, to be comparable with them for engineering and industrial purposes.

An Ordinary Mercurial Thermometer is usable from about −39° C. to about 300° C. The upper limit can be extended to 550° C. by filling the capillary above the mercury with nitrogen or carbon dioxid under high pressure. For low temperatures alcohol (−70° C. to +50° C.), toluene (−90° to +20°), pentane or petroleum ether (−200° to +20°), etc., are used in place of mercury. Between 0° and 100° the indications of the best mercurial thermometers should differ from those of the standard hydrogen thermometer by less than 0.1°, but the ordinary commercial instruments may be out several degrees.

The Joly Meldometer determines the fusion points of solids very rapidly and very accurately. A minute specimen is placed on a slightly stretched strip of platinum gradually heated by a regulated electric current. The length of the strip at the instant of melting indicates the temperature. The meldometer is also useful for identifying substances of known melting points. Temperatures in the neighborhood of the fusion point of gold (1063° C.) are easily determined within a few degrees by means of this instrument; but great care is required to prevent injury to the platinum strip either thru melting, overstretching, or contamination with fused metals. The necessity of recalibration every time a strip is injured is avoided by the method of Burgess (Bull. Bureau of Standards, vol. 3, p. 346, 1906), in which observing with an optical pyrometer replaces measuring the length of the strip.

In several commercial thermometers the difference in the expansions of two solids, or the expansion of air in a bulb, operates a pointer moving over a dial graduated in degrees. The upper limit of these instruments is about 800° C. (1500° F.), and they deteriorate rapidly when used at the higher temperatures.

The Electrical Resistance of a coil of wire may be made a very sensitive and very reliable indicator of temperature. Platinum is usable from the lowest obtainable temperatures to 1000° C. Other metals, such as nickel, are suitable for lower temperatures.

A Thermoelement indicates temperature by the e.m.f. set up in a closed electrical circuit of two wires of different materials in series when the two junctions are at different temperatures. One junction is exposed to the temperature to be determined, while the other is kept at constant temperature. The particular combination of metals best adapted to any given case will depend on the range of temperature to be indicated. An element composed of one wire of pure platinum and the other of platinum alloyed with 10 % of rhodium is best for temperatures above 300° C., and can be used up to 1600° C. Constantan with copper or iron is more suitable under 300° C. Other combinations are found in commercial instruments.

Radiation Pyrometers are the only ones usable at temperatures high enough to melt or to change permanently any thermometric substance subjected to them, and in other cases where the hot body cannot be touched with the instrument. They are merely pointed at the body from a distance. They are useful from dull red to the highest temperatures obtainable, and are capable of considerable precision (about 1 %). Optical Pyrometers depend on the fact that the intensity of the light of any given color emitted by a hot body is a definite function of the temperature. They are essentially pho-

tometers graduated to indicate temperatures instead of brightness of illumination. Le Chatelier, Féry, Wanner, Holborn and Kurlbaum, and Mo have devised very satisfactory instruments of this type. The Féry THERMOELECTRIC TELESCOPE represents a second class, and is based on the f that the total energy of the radiation emitted per unit area of a body is a defin function of the temperature. In this instrument the radiation from a limi area of the hot body is focused by means of a concave mirror (or by a fluo or glass lens) upon a small thermoelement in series with a portable gal nometer, whose scale is graduated to indicate temperatures directly. F has also devised a simpler instrument in which the thermoelement is repla by a thin coiled strip composed of two differently expanding metals t causes a pointer to move over a graduated dial.

The Color of Glowing Bodies affords a very crude means of estimating temperat Approximate values are

First visible red	525° C.	Dull orange	1100° C
Dull red	700	Bright orange	1200
Turning to cherry	800	White	1300
Cherry proper	900	Brilliant white	1400
Bright cherry	1000	Dazzling white	1500

Workmen can sometimes guess to better than 25° C. up to 800° C. At 1200° erro over 100° will be made.

Seger's Pyrometric Cones are often used for indicating the maximum temperat reached in pottery kilns. They are merely little pyramids of clay whose pointed soften and curl over at fairly definite temperatures determined by their composi They are on the market with softening temperatures ranging by steps of about 2 from 590° to 1850° C.

The Calorimetric Method of observing temperature determines the given up to a mass of water when a piece of metal is suddenly transferre it after heating in the furnace or other region whose temperature is requ If a mass m_2 of water is warmed from $t_2°$ to $t_3°$ by dropping into it a mas of metal with an initial temperature $t_1°$ and a mean specific heat c_{13} betw $t_1°$ and $t_3°$, then t_1 may be found from the relation $m_1c_{13}(t_1 - t_3) = m_2(t_3$ Platinum and nickel are about the only metals suitable for high temperat iron, tho frequently used, is too readily changed by oxidation. The me is too crude and too time-consuming to be recommended except for occas rough determinations.

Thermometers for Recording and Use at a Distance. All the thermon described above except the optical and the fusion-point ones may be made to recor indications automatically, either autographically or photographically; but those l themselves most readily to this use are the electrical thermometers and the Féry th electric telescope. These have the further advantage that any number of thermo can be connected by a switchboard with one recording or indicating instrument, may be placed at any convenient distance from the body whose temperature is indicated.

12. Quantities of Heat

The 15° Gram Calorie (gm-cal) is the unit for measuring quantity o most frequently used in scientific work, and is the heat necessary to rai temperature of 1 gm of water from 14.5° C. to 15.5° C. Q gm-cal is the qu of heat necessary to raise Q gm of water thru the same interval, not the tity necessary to raise 1 gm of water thru Q degrees. The KILOGRAM CA (kg-cal), or large calorie, is 1000 gm-cal, and is the common unit for eng ing purposes in countries where the metric system is in use. In the c water, it happens that the quantity of heat necessary to raise a given m is very nearly the same at all temperatures between 0° and 100° C.

experiments of Barnes in 1902 indicate that this is a minimum in the neighborhood of 37.5° C. and a maximum at 0°; but the extreme variation found amounted to scarcely more than 1 %. The BRITISH THERMAL UNIT, often abbreviated Btu, is the heat required to raise 1 lb of water 1° F. The temperature does not need to be specified for the accuracy necessary in most engineering computations. 1 Btu = 0.252 kg-cal.

Mechanical Equivalent of Heat. When heat has to be compared quantitatively with energy in some other form, it is usually most convenient to measure all the quantities in terms of some unit of work. A 15° gm-cal is very nearly equivalent to 4.19 joules or 0.427 m-kg, and a Btu is nearly equivalent to 778.4 ft-lb. This mechanical equivalent of heat is usually represented by the symbol J (in honor of Joule), so that if W units of work is equivalent to Q units of heat, then $W = JQ$.

Thermal Capacity and Specific Heat. If ΔQ represents the heat added to a body and $\Delta T = T' - T$ the change in the body's temperature, then $\Delta Q / \Delta T$ is called the mean thermal capacity of the body between the temperatures T and T'; while the limit that this approaches as ΔT approaches zero is called the true thermal capacity of the body at the temperature T.

The Specific Heat of a homogeneous substance is its thermal capacity per unit mass. This may have any value whatever from $-\infty$ to $+\infty$ and is not determinate till it is specified how the volume of and pressure on the substance shall vary during the process of heating or cooling; because the heat transferred to a body during any given temperature change will depend not only on the temperature interval, but also on how much work is done by or on the body at the same time. This specification is especially necessary in the case of a gas or a vapor, where it usually suffices to distinguish between the specific heats c_p, at constant pressure, and c_v, at constant volume, tho it is sometimes desirable to know the specific heat of a vapor maintained in the condition of saturation during change of temperature.

Because of the way in which the unit of heat has been defined, it follows that the thermal capacity of a body is often exprest by almost the same number as what is known as the WATER EQUIVALENT of the body, that is, the mass of water whose temperature would be raised the same number of degrees by the addition of the same quantity of heat. Also, the mean specific heat of the body is often nearly the same as the ratio of the quantity of heat necessary to effect a given change in the body's temperature to the quantity of heat necessary to effect the same change in the temperature of an equal mass of water; but it may be far from this.

The Specific Heat of a solid or of a liquid is in nearly all cases practically independent of the manner in which heating takes place; because the external work involved in its thermal change of volume is negligible. While the specific heat of water is the same within about 1% at all temperatures between 0° and 100° C., that of most solids and liquids increases slowly with the temperature, tho in some instances, such as iron and diamond, the change is rapid. A marked change in specific heat also accompanies change in state of aggregation.

An Abnormal Specific Heat occurs during the more or less rapid softening that usually precedes the complete liquefaction of a solid as it is heated; and it becomes infinite when the change in state of aggregation occurs abruptly without change of temperature until the process is completed, as during fusion of some chemically simple crystalline substances, such as ice, tin, zinc, copper, silver, and gold, or during boiling and sublimation. When the transition is gradual, it is impossible to state what part of the heat absorbed is to be attributed to the change in state of aggregation and what merely to the accompanying change in temperature. The term LATENT HEAT is in use to designate the number of heat units absorbed or liberated per unit mass of the substance during any of those special changes of state that occur

without change in temperature, since the specific heat becomes unmanageab when it assumes an infinite value. Altho the term is usually restricted such very marked transitions as changes of state of aggregation, it is equal applicable to any isothermal change: for example, to the isothermal expansi of a gas against outside pressure. As special instances it is convenient distinguish heat of fusion, heat of vaporization, heat of sublimation, heat solution, heat of chemical reaction (with heat of combustion as a special cas heat of strain, and heat of recalescence.

Cooling and Heating Curves are obtained by plotting as coordinates t time a body had been cooling or heating under uniform external conditio and the instantaneous value of its temperature. They afford a convenie means of determining TRANSITION REGIONS, or temperature intervals with which internal physical or chemical changes take place, since the rapid absor tion of heat during rise, or liberation during fall, of temperature is indicat by a more or less sudden decrease in the angle the curve makes with the ti axis, followed by a return to a greater slope when the change is complete Such transformations are very common in solids far below their melti points, occurring commonly in alloys, and particularly in steels. They a accompanied and made evident by changes in physical properties such hardness, density, resistivity, or permeability. The recalescence point of ir is a familiar example. As such changes are generally slow, they may be o scured by being spread over too great a temperature interval if the cooli or heating is too rapid.

13. Transference of Heat

Radiation. All bodies, whatever their temperatures may be, are cont ually radiating energy which is propagated thru space with the speed light. This energy appears to be transferred by some sort of periodic d turbances, or wave-trains, of various frequencies. The greater the temp ature of a body the greater both the total quantity of this RADIANT ENER and the relative intensity of the waves of short period. Unless the body is least almost as hot as the sun, the most energetic of the waves that radi from it are of somewhat longer period than those that affect the eye with sensation of light. During the time that the body loses energy by transforn tion of some of its heat into the radiation it emits, its temperature must crease unless the supply of heat is replenished either by energy transformati going on within or by energy received from without. When radiant ene falls upon a body, that part which is neither reflected nor transmitted is sorbed by the body, and some or all of it is transformed into heat. Thus effect, the process of radiation transfers heat almost instantaneously from body to another at a lower temperature not visibly connected with it, and e at a considerable distance: the warmer radiates more energy than it recei from the colder, and the colder receives more than it radiates to the hot When, in this way, radiant energy causes the transference of heat, it is u ally misnamed RADIANT HEAT; but it should be remembered that the ene while traveling from one body to the other is not in the form of heat at and does not heat the bodies into which it passes except in so far as t absorb and transform some of it into heat. The behavior of all these wa is described by exactly the same general laws as those that describe the havior of that limited portion of them that has such periods as to be capa not only of heating bodies but also of producing the sensation of light, o exciting chemical activity.

The fraction of the radiant energy absorbed, the fraction transmitted, and the frac reflected by a given body will depend upon the periods of the incident waves, as

s upon the nature, surface, thickness, temperature, etc., of the body, and the angle at which the radiation is incident. Bodies that either reflect or transmit readily are poor radiators; and vice versa, opaque bodies if good absorbers are good radiators. Emission, absorption, and transmission of radiation are always most intense in a direction perpendicular to the radiating surface, while reflection increases with the obliquity of incidence; but the influence of direction is less the greater the absorptivity of the body; and it has no influence whatever in the case of a perfect absorber, or theoretically black body, that is, one that completely absorbs all the waves incident upon it regardless of their periods. Such a body is almost perfectly realized experimentally by a small hole leading into a relatively large cavity bounded by opaque walls of uniform temperature, no matter what their nature may be, as practically none of the radiation entering the hole would ever succeed in getting out again.

The transparency and also the reflectivity of a substance for those vibrations ordinarily called light afford no criteria whatever for predicting the behavior of the substance toward waves of longer or of shorter period. Thus, hard rubber, while completely opaque to visible radiations even in extremely thin layers, is fairly transparent to the waves of longer period; colorless glass and water, on the other hand, absorb strongly most of the invisible heating radiations; while rock-salt is very transparent to both visible and invisible. Polished metals are excellent reflectors of nearly all waves of longer period than those of the visible spectrum, and many of them, notably silver, have a high reflecting power in the visible as well.

Convection usually plays the most important part in the transference of heat from one portion of a fluid to another and in equalizing the temperature of the whole. After the particles adjacent to the sources of heat become warmed, they are carried away bodily by convection currents and distributed more or less uniformly thruout the entire extent of the fluid. These currents may be caused by changes in buoyancy following expansion or contraction, by the slow process of diffusion, or by mechanical stirring.

Conduction. All bodies transmit some heat from their warmer to their colder portions by what is called conduction. The number of heat units that will pass per unit time thru a plane section of area A within a substance when the temperature T is uniform over the section and falls along the normal N to it at the rate $-dT/dN$ is $dQ/dt = -kA\,dT/dN$, where k is called the THERMAL CONDUCTIVITY of the substance. The rate $-dT/dN$ is called the TEMPERATURE GRADIENT normal to the section. The conductivity k is a function of the temperature, but varies so little that in most problems it may be treated as constant unless very large temperature intervals are involved.

The thermal conductivities of different substances differ widely, being greatest in metals, especially silver. Non-metallic liquids, gases, and many solids are very poor conductors; but no substance is a complete insulator. The thermal conductivities of the different pure metals are to one another roughly as their electrical conductivities. (Law of Wiedemann and Franz.) Alloys, however, show large deviations from this relation. Bodies having finely porous structures such as powders, wood, wool, cloth, and paper are as a rule poor conductors. (See tables of Physical Properties of Substances.)

The Problem of Thermal Insulation is essentially the same whether it be desired to keep a hot body from losing heat or a cold body from gaining it. In the former case, however, the choice of insulating materials may be seriously limited by the temperatures they are capable of withstanding without troublesome deterioration. Whenever possible, insulating jackets should be enclosed between metallic walls highly polished on their outside surfaces to prevent radiation and absorption; the space between should be filled with a sufficiently thick layer of the poorest conducting substance that will stand the highest temperature to which it may be subjected; and this substance should be of a finely porous nature. The jacketing for a very highly heated body such as a furnace is best made in layers, some excellent insulator such as infusorial earth (diatomaceous earth, Kieselguhr) or light calcined magnesia being used for the outside and extending as far in as it can stand the temperature without shrinkage or fusion. Such heat-resisting materials as carborundum fire-sand or powdered charcoal may have to be used for the most highly heated layers; but they should in general be avoided when the temperature

is low enough to permit the employment of better insulators. The corrosive action o furnace gases is often the most important factor in determining the selection of the linin exposed to them. The best of all thermal insulation for low and moderate temperature is a very high vacuum between thinly silvered walls, as illustrated by the well-know Dewar vessels for containing liquid air, etc. In a good Dewar flask half a gallon o liquid air will take a week to evaporate, altho separated from the atmosphere of the roor by only ½ to ¾ in. With rising temperature, however, radiation plays an increasingl important part, until at furnace heats and above it becomes the principal means of effec ing heat-transfer; and radiation takes place most readily of all thru a vacuum. A shee of bright tin plate with free air circulation behind it is frequently a better protection to wooden wall than asbestos applied directly to the wood unless the asbestos is very thic In engineering practice much heat is continually wasted because of insufficient attentio to thermal insulation.

14. Changes Due to Heating

Heat and Expansion. Altho most substances expand as they are warmed or if hindered from expanding freely will exert pressure upon their container water between 0° and 4° C., quartz-glass below −84° C., and some other sub stances are exceptions to the general rule. When heating causes a body t change its state of aggregation (melt, vaporize, etc.) there is also change i volume. This may be either an expansion or a contraction, and may b accompanied by little or no change of temperature until after the process completed.

Thermal Hysteresis. When a solid has its temperature changed, e pecially by rapid cooling, slow changes in its dimensions continue long aft it has attained the same temperature thruout its entire extent. These chang are often so minute as to require extremely delicate means for detection; b in such cases as glass they are very marked. The gradual contraction thermometer bulbs has been observed to continue for over a quarter of century, causing the zero reading to rise. The zero point of even a goo thermometer may be shifted many degrees in a few minutes by heating several hundred degrees. (Bul. Bureau Standards, vol. 2, 1906, p. 18 Most of this after-effect disappears in a few hours or days, and the process considerably accelerated by prolonged heating at a high temperature follow by slow annealing.

Laws of Change. As long as a physically homogeneous body is n subjected to treatment that causes more or less permanent alterations in structure, its volume appears to be a definite function of its temperature a pressure, when these are uniform thruout its entire extent. The amou of expansion under constant pressure caused by a given change of temperatu depends upon the material of the body. Solids and liquids differ conside ably among themselves and follow no regular law. Gases, on the other han show remarkable uniformity, all expanding about $100/273 = 0.367$ of th volumes at 0° C. when warmed from this temperature to 100° C. (Law Gay-Lussac.) Besides, all gases expand very nearly alike thruout gre ranges of temperature; and the particular pressure under which a gas warmed makes very little difference.

Coefficients of Expansion. If the volume at $t°$ is V and at 0° is then the rate at which unit volume changes under constant pressure w change of temperature is $(dV/dt)/V$, which is called the true coeffici of cubical expansion at $t°$; while $(V-V_0)/V_0t = \alpha$ is called the mean-ze coefficient from 0° to $t°$. Similarly, if L represents the length of a solid, th $(L-L_0)/L_0t = \beta$ is the mean-zero coefficient of linear expansion from 0° to These definitions give $V = V_0(1+\alpha t)$ and $L = L_0(1+\beta t)$. Usually α and are so small that one may assume $\alpha = 3\beta$.

Values of the coefficient of cubical expansion for solids are given above in Art. 7. The coefficient of linear expansion is one-third of the cubical. Art. 7 gives values for the Centigrade degree. To find coefficients of linear expansion for the Fahrenheit degree, multiply tabular values by 0.185. Thus, for steel, the table gives coefficients of cubical expansion ranging from 0.000030 to 0.000041, hence coefficient of linear expansion for the Fahrenheit degree ranges from 0.0000055 to 0.000076, while mean value given for steel on p. 404 is 0.0000065.

Dalton's Law. In a mixture of several gases or vapors that do not react upon each other chemically the pressure exerted is approximately the same as if each constituent exerted the full pressure it would exert if it alone filled the entire volume; that is to say, if volumes V_1, V_2, V_3, . . . , of different gases and vapors, all under the same pressure p and at the same temperature, when mixt fill a volume V, then $p=p_1+p_2+p_3+\ldots$, approximately, where $p_1=pV_1/V$; $p_2=pV_2/V$; etc. This relation is more accurately fulfilled the farther all the vapors are from the conditions under which they liquefy and the less their liquids dissolve one another. p_1, p_2, etc., are spoken of as the partial pressures exerted by the different components.

15. Sound

Velocity of Sound in air at 0° F is 1050 ft per sec, and it increases 1.1 ft per sec for each 1° F rise in temperature. A few values are

Temperature F,	−10°	+10°	+30°	+50°	+70°	+90°
Velocity, ft per sec,	1039	1061	1083	1105	1127	1149

At 0° C the velocity of sound in dry air is 331 meters per sec and it increases 0.609 meters per sec for a rise of each degree of the Centigrade scale. At + 30° C the velocity is about 349 meters per sec.

The velocity of sound in air is independent of the barometric pressure, and hence is the same on high mountains as at sea level, the temperature being the same in both cases. It is said to be little influenced by fog or rain but materially altered by aqueous vapor and also by wind.

A formula for velocity of sound in any solid or liquid is $v=\sqrt{Eg/w}$, where $E=$ modulus of elasticity, $w=$weight per cubic unit of the substance, and $g=$acceleration of gravity. Mean values of v are as follows in ft per sec:

Fresh water	4 700	Copper	11 700
Salt water	4 765	Cast iron	12 400
Timber	13 260	Wrought iron	15 500
Silver	8 550	Steel	17 200

Stress is transmitted thru a solid with the same velocity as sound.

The velocities of sound in different gases at the same pressure and temperature vary inversely as the square roots of their densities. Thus, using densities of air and hydrogen as given on p. 1526, the velocity in hydrogen at 0° F is 3980 ft per sec.

In open air the intensity of sound varies inversely as the square of the distance of the source from the ear. For air in a pipe this law does not hold, but the intensity remains the same for considerable distances, it being diminished only by internal frictional resistances. The intensity also depends on the medium; the ticking of a watch is heard 2½ times as far under water as in air. It also depends on the volume of sound produced by the sonorous body; the explosion of a volcano has been heard at a distance of 300 miles.

When a sonorous body approaches the ear the tone perceived is higher than the true, and when it recedes the tone is lower. This is because high tones are produced by short waves, and in the case of approach to the ear, the sound waves are crowded together or shortened.

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16. Signals of United States Weather Bureau

Flag Signals for ordinary weather predictions are shown in Fig. 8.

No. 1. Fair weather. No. 2. Rain or snow. No. 3. Local rain or snow. No. 4. Temperature. No. 5. Cold wave. Col key

Fig. 8. Flags for Weather Predictions

No. 1, alone, indicates fair weather, stationary temperature.
No. 2, alone, indicates rain or snow, stationary temperature.
No. 3, alone, indicates local rain or snow, stationary temperature.
No. 1, with No. 4 above it, indicates fair weather, warmer.
No. 1, with No. 4 below it, indicates fair weather, colder.
No. 2, with No. 4 above it, indicates rain or snow, warmer.
No. 2, with No. 4 below it, indicates rain or snow, colder.
No. 3, with No. 4 above it indicates local rain or snow, warmer.
No. 3, with No. 4 below it, indicates local rain or snow, colder.

Whistle Signals. A warning blast of from fifteen to twenty seconds c tion is sounded to attract attention. After this warning the longer b (of from four to six seconds duration) refer to weather, and shorter blast from one to three seconds duration) refer to temperature; those for we are sounded first.

Blasts	Indicate	Blasts	Indicate
One long	Fair weather	One short	Lower temperat
Two long	Rain or snow	Two short	Higher tempera
Three long	Local rain or snow	Three short	Cold wave.

Storm and Hurricane Warnings are given by the flags in Fig. 9.

N. E. Winds. S. E. Winds. N. W. Winds. S. W. Winds. Hurricane Color K

Fig. 7. Storm and Hurricane Flags

A red flag with a black center indicates that a storm of marked violence is exp The pennants displayed with the flags indicate the direction of the wind; red, ea (from northeast to south); white, westerly (from southwest to north). The pennant the flag indicates that the wind is expected to blow from the northerly quadrants; from the southerly quadrants. By night a red light indicates easterly winds, and a light below a red light, westerly winds.

Two red flags with black centers, displayed one above the other, indicate the ex approach of a tropical hurricane, or one of those extremely severe and dangerous which occasionally move across the Lakes and northern Atlantic coast. No nigh ricane warnings are displayed.

Wind Pressure on Flags. Tests at the U. S. Navy Yard, Washington, D. C. the largest flags that could be handled in the wind tunnel, form the basis for an em formula for determining the pressure of wind on flags, for use in designing flagpoles. sizes of flags were used—one $3 \times 5\frac{1}{2}$ ft, and the other $2\frac{1}{2} \times 4\frac{1}{4}$ ft—and velocities v from 20 to 60 miles per hour were applied. The following formula was found o rep the results, the constant varying but slightly with size of flags, being less for the large

$$R = 0.0003\, A V^{1.9}$$

which R is the resistance in pounds, A is the area of flag in square feet, and V is the elocity of wind in miles per hour. This formula is for steady wind pressure. It was ot found practicable to measure the forces produced by wind gusts.

17. Local Weather Predictions

The wind and barometer indications for the United States are generally ımmarized in the following table (E. B. Garriott, U. S. Dept. Agriculture).

Wind direction	Barometer reduced to sea level	Character of weather indicated
SW. to NW.	30.10 to 30.20 and steady	Fair, with slight temperature changes, for 1 to 2 days.
SW. to NW.	30.10 to 30.20 and rising rapidly	Fair, followed within 2 days by rain.
SW. to NW.	30.10 to 30.20 and falling slowly	Warmer, with rain within 24 to 36 hours.
SW. to NW.	30.10 to 30.20 and falling rapidly	Warmer, with rain within 18 to 24 hours.
SW. to NW.	30.20 and above and stationary	Continued fair, with no decided temperature change.
SW. to NW.	30.20 and above and falling slowly	Slowly rising temperature and fair for 2 days.
. to SE.	30.10 to 30.20 and falling slowly	Rain within 24 hours.
. to SE.	30.10 to 30.20 and falling rapidly	Wind increasing in force, with rain within 12 to 24 hours.
SE. to NE.	30.10 to 30.20 and falling slowly	Rain in 12 to 18 hours.
SE. to NE.	30.10 to 30.20 and falling rapidly	Increasing wind, and rain within 12 hours.
E. to NE.	30.10 and above and falling slowly.	In summer, with light winds, rain may not fall for several days. In winter, rain within 24 hours.
E. to NE.	30.10 and above and falling rapidly.	In summer, rain probable within 12 to 24 hours. In winter, rain or snow, with increasing winds, will often set in when the barometer begins to fall and the wind sets in from the NE.
SE. to NE.	30.00 or below and falling slowly	Rain will continue 1 to 2 days.
SE. to NE.	30.00 or below and falling rapidly	Rain, with high winds, followed, within 36 hours, by clearing, and in winter by colder.
S. to SW.	30.00 or below and rising slowly.	Clearing within a few hours, and fair for several days.
S. to E.	29.80 or below and falling rapidly	Severe storm imminent, followed, within 24 hours, by clearing, and in winter by colder.
E. to N.	29.80 or below and falling rapidly	Severe northeast gale and heavy precipitation; in winter, heavy snow, followed by a cold wave.
Going to W.	29.80 or below and rising rapidly	Clearing and colder.

As a rule winds from the east quadrants and falling barometer indicate oul weather; and winds shifting to the west quadrants indicate clearing and air weather. The rapidity of the storm's approach and its intensity are ndicated by the rate and the amount in the fall of the barometer.

The indications afforded by the wind and the barometer are the best guides or determining future weather conditions. As low barometer readings sually attend stormy weather, and high barometer readings are generally ssociated with clearing or fair weather, it follows that falling barometer ndicates precipitation and wind, and rising barometer, fair weather or the pproach of fair weather. As atmospheric waves or crests (areas of high arometer) and troughs or depressions (areas of low barometer) are, by

natural laws, caused to assume circular or oval forms, the wind direc with reference to areas of low barometer are spirally and contraclocl inward toward the region of lowest atmospheric pressure, as indicate readings of the barometer. Areas of low barometric pressure are, in whirlwinds of greater or less magnitude and intensity, depending upo steepness of the barometric gradient. The atmospheric crests, or are high barometer, on the contrary, show winds flowing spirally clockwise ward from the region of highest barometric pressure.

The wind directions thus produced give rise to, and are responsible for, all local w signs. The south winds bring warmth, the north winds cold, the east winds, in the latitudes, indicate the approach from the westward of a low barometer, or storm and the west winds show that the storm area has past to the eastward. The indi of the barometer generally forerun the shifts of the wind. This much is shown b observations.

During the colder months, when the land temperatures are below the water t atures of the ocean, precipitation will begin along the seaboards when the winc and blows steadily from the water over the land without regard to the height of the eter. In such cases the moisture in the warm ocean winds is condensed by the the continental area. During the summer months, on the contrary, the onshore are not necessarily rain winds. for the reason that they are cooler than the land s and their capacity for moisture is increased by the warmth that is communicated t by the land surface. In such cases thunderstorms commonly occur when the ocea are intercepted by mountain ranges or peaks. If, however, the easterly winds of s increase in force, with falling barometer, the approach of an area of low bar pressure from the west is indicated and rain will follow within a day or two.

From the Mississippi and Missouri valleys to the Atlantic coast, and on the coast, rain generally begins on a falling barometer, while in the Rocky Mount Plateau districts, and on the eastern Rocky Mountain slope, precipitation seldom until the barometer begins to rise, after a fall. This is true as regards the eastern the country, however, only during the colder months, and in the presence of storms that may occur at other seasons. In the warmer months summer show thunderstorms usually come about the time the barometer turns from falling to The fact that during practically the entire year precipitation on the great wester and in the mountain regions that lie between the plains and the Pacific coast does not begin until the center of the low barometer area has past to the eastward o ward, and the wind has shifted to the north quadrants, with rising barometer, is portant one to note.

18. Weather Observations

Mean Temperature. When maximum and minimum readings of t ature are taken, the mean of the two is the mean temperature of th The mean of all the daily means in a month is the mean temperature month. When a recording thermometer is used the area between th and an axis of abscissas is to be divided by the length of that axis in to obtain the mean temperature for the elapsed time.

Rainfall. The rain gage used by voluntary observers consists of a drical receiver 8 inches in diameter which has a funnel-shaped botto discharges into a tube 2.53 inches in diameter. The cross section is one-tenth that of the receiver, and hence height of water in tube is te as great as actual rainfall. The depth in the tube is measured by which is so graduated as to read the true rainfall in inches. The 20 inches long so that a precipitation of 2 inches or less can be measure out emptying it.

Self-registering rain gages catch the water and weigh it, these being used at main of U. S. Weather Bureau. Snowfall is caught, melted, and then measured; rou inches of snow make one inch of water. A rain-gage at the top of a building giv rainfall than one on the ground.

Voluntary observers of the Weather Bureau record maximum and minimum

Temperature in the United States to Jan. 1, 1919

Prepared by the Weather Bureau, U. S. Department of Agriculture

States and territories	Stations	Temperature F.			
		Mean		Extremes	
		Jan.	July	Highest	Lowest
Ala.	Birmingham	45	80	104	−10
	Mobile	50	80	102	−1
Ariz.	Flagstaff	27	65	93	−22
	Phoenix	50	90	119	12
	Yuma	55	91	120	22
Ark.	Fort Smith	38	81	118	−15
	Little Rock	41	81	106	−12
Cal.	Fresno	45	82	115	17
	Los Angeles	53	67	109	28
	Sacramento	46	72	110	19
	San Diego	54	67	110	25
	San Francisco	50	57	101	29
Col.	Denver	29	72	105	−29
	Grand Junc.	25	79	104	−19
	Pueblo	29	74	104	−27
Conn.	New Haven	27	72	100	−14
D.C.	Washington	33	77	106	−15
Fla.	Jacksonville	54	81	104	10
	Key West	69	84	100	41
	Pensacola	52	81	103	7
	Tampa	57	80	98	19
Ga.	Atlanta	42	78	100	−8
	Augusta	46	80	105	3
	Savannah	50	80	105	8
Idaho	Boise	29	73	111	−28
Ill.	Cairo	35	79	106	−16
	Chicago	24	72	103	−23
	Springfield	26	76	107	−24
Ind.	Indianapolis	28	76	106	−25
Iowa	Des Moines	20	76	110	−30
	Dubuque	18	75	106	−32
	Keokuk	24	77	108	−27
Kan.	Dodge	27	78	108	−26
	Wichita	30	79	107	−22
Ky.	Louisville	34	79	107	−20
La.	New Orleans	53	81	102	7
	Shreveport	46	82	110	−5
Me.	Portland	22	68	103	−21
Md.	Baltimore	33	77	105	−7
Mass.	Boston	27	71	104	−14
Mich.	Detroit	24	72	104	−24
	Marquette	16	65	108	−27
	Port Huron	22	69	104	−25
Minn.	Duluth	10	66	99	−41
	St Paul	12	72	104	−41
Miss.	Vicksburg	47	80	101	−1
Mo.	Kansas City	26	78	108	−22
	St. Louis	31	79	107	−22
	Springfield	31	76	106	−29
Mont.	Helena	20	67	103	−42
Mont.	Kalispell	20	64	97	−34
	Miles City	14	73	111	−49
Neb.	N. Platte	21	74	107	−35
	Omaha	20	76	110	−32
Nev.	Winnemucca	29	72	104	−28
N. C.	Charlotte	40	79	102	−5
	Hatteras	46	79	93	8
	Wilmington	46	79	103	−5
N.D.	Bismarck	7	70	107	−45
N.H.	Concord	21	69	102	−23
N.J.	Atlantic City	32	72	104	−7
	Cape May	34	73	98	−7
N.Mex.	Sante Fe	28	69	97	−13
N.Y.	Albany	22	72	104	−24
	Binghamton	23	70	99	−26
	Buffalo	25	70	95	−14
	N. Y. City	30	74	102	−13
	Oswego	24	70	100	−23
Ohio	Cincinnati	32	78	105	−17
	Columbus	29	75	104	−20
	Toledo	26	74	103	−16
Okla.	Oklahoma	35	80	108	−17
Oreg.	Portland	39	66	102	−2
Pa.	Erie	26	72	96	−16
	Phila.	32	76	106	−6
	Pittsburg	31	73	103	−20
R.I.	Block Island	31	68	92	−6
S.C.	Charleston	49	81	104	7
S.D.	Huron	10	72	108	−43
	Yankton	16	75	107	−36
Tenn.	Chattanooga	41	78	101	−10
	Memphis	40	81	104	−9
	Nashville	38	79	104	−13
Tex.	Abilene	43	82	110	−6
	Amarillo	34	76	106	−16
	El Paso	44	80	113	−5
	Galveston	53	83	99	8
	San Antonio	51	82	108	4
Utah	Salt Lake City	29	76	102	−20
Vt.	Burlington	16	68	100	−27
Va.	Lynchburg	36	77	105	−7
	Norfolk	40	78	105	2
Wash.	Seattle	39	64	96	11
	Spokane	27	69	104	−30
	Walla Walla	33	74	113	−17
W.Va.	Elkins	29	70	99	−28
	Parkersburg	31	76	106	−27
Wis.	La Crosse	15	73	104	−43
	Milwaukee	20	70	102	−25
Wyo.	Cheyenne	26	67	100	−38

ature, precipitation, wind direction, general character of day, and miscellaneous phenomena such as halos, dates of frost, hail, sleet, auroras, and tornadoes. The general character of the day is recorded "clear" when the sky is $^3/_{10}$ or less obscured, "partly cloudy" when from $^4/_{10}$ to $^7/_{10}$ is obscured, and "cloudy" when more than $^7/_{10}$ is obscured.

19. Rainfall and Evaporation

Mean Annual and Monthly rainfall at many stations are given in Sect. 9, Art. 40. The term Rainfall includes both rain and melted snow.

Measurements of Evaporation are made by placing water-tight pans at the level of the ground and noting daily the variations in depth, together with the rainfall. On a water surface similar measurements may be made by floating boxes. It is found that the evaporation from water surfaces is greater than that from land, that it is greater in dry and desert regions than in cultivated ones, that it is less in low lands than on mountains, that it decreases as the humidity of the air increases, and that it increases with the temperature of the air and the velocity of the wind. In the North Atlantic states the annual evaporation from land surfaces is, on the average, about 40 percent, while that from water surfaces is about 60 percent of the annual rainfall. In low and level localities these percentages are decreased, while for high regions and steep slopes they are increased. In some arid localities west of the Rocky Mountains nearly all the rainfall evaporates from land surfaces, while the evaporation from water surfaces may be several times as great as the rainfall.

Experiments made in 1909–10 by the U. S. Department of Agriculture gave the following figures for the annual evaporation at twenty places in the United States, the evaporating pan being at or very near the surface of the ground or water.

Place	Position of pan	Diameter of pan, feet	Annual evaporation, inches
Salton Sea, Cal.	1500 ft inland	2	164.50
Salton Sea, Cal.	500 ft at sea	4	108.65
Salton Sea, Cal.	7500 ft at sea	4	106.45
Indio, Cal.	15 miles from Salton Sea	6	119.33
Mecca, Cal	½ mile from Salton Sea	6	107.81
Brawley, Cal.	20 miles from Salton Sea	6	103.55
Mammoth, Cal.	40 miles from Salton Sea	6	125.53
N. Yakima, Wash.	½ mile west of city	4	67.96
Hermiston, Ore.	On raft in reservoir	4	68.05
	On ground	3	97.29
Granite Reef, Ariz.	Floating in Salt River	4	97.74
	On ground	4	115.18
California, O.	Floating in reservoir	4	45.99
Birmingham, Ala.	Floating in reservoir	4	51.74
Dutch Flats, Neb.	A few miles from Mitchell	4	65.67
Deer Flat, Id.	On raft near water edge	4	77.43
	On ground of embankment	3	79.00
Ady, Ore.	Floating in borrow pit	4	53.45
Fallon, Nev.	Floating in canal	4	53.65
Lake Tahoe, Cal.	2 ft above water	4	42.21
Elephant Butte, N. Mex.	Near Rio Grande River	4	86.95
Carlsbad, N. Mex.	In the city	4	107.25
	In an alfalfa field	4	94.35
Lake Avalon, N. Mex	A few miles from Carlsbad	4	94.51

The evaporation from a pan 2 feet in diameter is about 75 percent, that from a pan 4 feet in diameter is about 50 per cent, and that from a pan 6 feet in diameter is about 30 percent greater than the evaporation from a large pond or lake. The above figures may be roughly corrected by using these percentages; thus, at Birmingham, Ala., the true annual evaporation is 34.50 inches.

The U. S. Weather Bureau maintains at selected locations hook gauge measurements of evaporation losses from cylindrical pans 10 in deep and 4 ft diameter, exposed on wood frames, bottom of pans 1 in above the ground, generally on level ground, and in full sunshine. Detailed description published in Monthly Weather Review, December, 1916, or in Circular L, Instrument Division, Weather Bureau No. 559. From comparisons available, it appears that losses from bodies of water of considerable area are 50 to 60 percent as great as from pans exposed as above. The records are published in detail in the reports issued by the state section directors of the U. S. Weather Bureau.

Maximum Intensity of Rainfall

Station	Inches per hour for			Station	Inches per hour for		
	5 minutes, inches	10 minutes, inches	60 minutes, inches		5 minutes, inches	10 minutes, inches	60 minutes, inches
Bismarck, N. D..	9.00	6.00	2.00	Chicago, Ill.....	6.60	5.92	1.60
St. Paul, Minn...	8.40	6.00	1.30	Galveston, Tex..	6.48	5.58	2.55
New Orleans, La.	8.16	4.86	2.18	Omaha, Neb....	6.00	4.80	1.55
Milwaukee, Wis..	7.80	4.20	1.25	Dodge City, Iowa	6.00	4.20	1.34
Kansas City, Mo.	7.80	6.60	2.40	Norfolk, Va.....	5.76	5.46	1.55
Washington, D.C.	7.50	5.10	1.78	Cleveland, O....	5.64	3.66	1.12
Jacksonville, Fla.	7.44	7.08	2.20	Atlanta, Ga.....	5.46	5.46	1.50
Detroit, Mich....	7.20	6.00	2.15	Key West, Fla...	5.40	4.80	2.25
New York City..	7.20	4.92	1.60	Philadelphia, Pa.	5.40	4.02	1.50
Boston, Mass....	6.72	4.98	1.68	St. Louis, Mo...	4.80	3.84	2.25
Savannah, Ga....	6.60	6.00	2.21	Cincinnati, O....	4.56	4.20	1.70
Indianapolis, Ind.	6.60	3.90	1.60	Denver, Colo....	3.60	3.30	1.18
Memphis, Tenn..	6.60	4.80	1.86	Duluth, Minn...	3.60	2.40	1.35

This table has been compiled from all the available records at stations of the U. S. Weather Bureau which are equipped with self-registering rain gages.

20. Speed of Winds in the United States

U. S. Weather Bureau Records of the average speed of wind in miles per hr at selected stations, and the highest speeds ever reported for a period of five minutes.

Station	Average	Highest	Station	Average	Highest
Abilene, Texas....	11	66	Denver, Colo.....	7	75
Albany, N. Y.....	6	70	Detroit, Mich.....	9	86
Alpena, Mich.....	9	72	Dodge City, Kan..	11	75
Atlanta, Ga......	9	66	Dubuque, Iowa...	5	60
Bismarck, N. D...	8	74	Duluth, Minn.....	7	78
Boise, Idaho......	4	55	Eastport, Me......	9	78
Boston, Mass.....	11	72	El Paso, Tex......	5	78
Buffalo, N. Y.....	11	92	Fort Smith, Ark...	5	74
Charlotte, N. C...	5	72	Galveston, Texas..	10	93
Chattanooga, Tenn.	6	66	Havre, Mont.......	11	74
Chicago, Ill.......	9	84	Helena, Mont.....	6	70
Cincinnati, O.....	7	59	Huron, S. D......	10	72
Cleveland, O......	9	73	Jacksonville, Fla...	[illegible]	75

Station	Average	Highest	Station	Average	Highest
Kansas City, Mo ..	9	74	Pittsburgh, Pa......	6	69
Keokuk, Iowa......	8	60	Portland, Me........	5	61
Knoxville, Tenn...	5	84	Red Bluff, Cal.....	7	60
Louisville, Ky......	7	74	Rochester, N. Y...	11	78
Lynchburg, Va.....	4	63	St. Louis, Mo.......	11	80
Memphis, Tenn....	6	75	St. Paul, Minn.....	7	102
Miles City, Mont..	6	62	Salt Lake Cy. Utah	5	66
Montgomery, Ala..	5	54	San Diego, Cal.....	6	54
Moorehead, Minn..	11	75	San Francisco, Cal.	9	[illegible]4
Nashville, Tenn....	6	75	Sante Fe, N. M....	6	53
New Orleans, La...	7	86	Savannah, Ga.....	7	88
New York, N. Y...	9	96	Spokane, Wash....	4	52
Nor. Platte, Neb...	9	96	Toledo, Ohio......	9	84
Omaha, Neb.......	8	66	Vicksburg, Miss....	6	62
Palestine. Tex.....	8	60	Washington, D. C..	5	68
Philadelphia, Pa...	1[illegible]	7[illegible]	Wilmington, N. C..	7	72

The Beaufort Scale is used by seamen. In following table the corresponding velocity per hour in statute miles and in nautical miles is added.

Intensity or force of wind. Beaufort's scale	Velocity	
	Statute miles per hour	Nautical miles per hour
0. Calm. Full-rigged ship, all sail set, no headway	0 to 3	0 to 2.6
1. Light Air. Just sufficient to give steerageway..	8	6.9
2. Light Breeze. Speed of 1 or 2 knots, "full and by"	13	11.3
3. Gentle Breeze. Speed of 3 or 4 knots, "full and by"	18	15.6
4. Moderate Breeze. Speed of 5 or 6 knots, "full and by"	23	20.0
5. Fresh Breeze. All plain sail, "full and by"..	28	24.3
6. Strong Breeze. Topgallant sails over single-reefed topsails	34	29.5
7. Moderate Gale. Double-reefed topsails......	40	34.7
8. Fresh Gale. Treble-reefed topsails (or reefed upper topsails and courses)................	48	41.6
9. Strong Gale. Close-reefed topsails and courses (or lower topsails and courses)............	56	48.6
10. Whole Gale. Close-reefed main topsail and reefed foresail (or lower main topsail and reefed foresail)................................	65	56.4
11. Storm. Storm staysails......................	75	65.1
12. Hurricane. Under bare poles	90 and over	78.1 and over

The words "intensity" and "force," used in connection with this scale, have no direc relation to pressure, but refer to speed or velocity.

The Pressure exerted by the wind against a plane area normal to its direction varies closely with the square of the velocity; thus, $p = av^2$. When v i in meters per second and p is in kilograms per square meter, $a = 0.075$ when v is in statute miles per hour and p is in pounds per square foot, $a = 0.00$

21. Barometric Observations

Whenever pressure is specified in terms of the height of a column of mercury, it is always tacitly understood that it is the height the column balancing the pressure would have under "standard conditions," that is, at 0° C. and where g has the standard value $g_0 = 980.665$ cm per sec per sec adopted by the International Committee of Weights and Measures. In all cases where the mercury column (if of average barometric height and at ordinary atmospheric temperatures) is to be read closer than 2 or 3 mm (0.1 in) one or more of the corrections described below must be made or the accuracy of the reading will be imaginary.

Corrections. Let l be the height of the mercury column as read at t° C. with a scale correct at t_0° C. whose coefficient of linear expansion is β. Let ϕ be the latitude and H the elevation in meters above sea level.

(1) Temperature of the Mercury. Subtract $0.000182\, lt$.

(2) Temperature of the Measuring Scale. Add $\beta\,(t-t_0)\,l$. For brass $\beta=0.000019$; for glass $\beta=0.000008$. If, as is usual, the scale is correct at 0° C., the complete correction, (1) and (2), for the expansion of both the mercury and the scale may be made by subtracting from the observed reading $(0.000182-\beta)\,lt$, which gives

$0.000163\, lt$ for a brass scale, and
$0.000174\, lt$ for a glass scale.

Under ordinary conditions the correction may amount to as much as 4 mm.

(3) Capillary Depression in a Cistern Barometer. Add to the reading of the top of the meniscus the amount given in the table below corresponding to the internal diameter of the tube and the height of the meniscus. This somewhat uncertain correction can be avoided by using a tube at least 25 mm in diameter.

Capillary Depression of Mercury

(After Mendeléeff and Gutkowsky. Kohlrausch, 1910)

Diameter	Height of the meniscus in mm							
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
mm	mm	mm	mm	mm	mm	mm	mm	mm
4	0.83	1.22	1.54	1.98	2.37	......	......	......
5	0.47	0.65	0.86	1.19	1.45	1.80	......	......
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	......
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	......	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	......	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	......	......	0.15	0.20	0.25	0.29	0.33	0.37
11	......	......	0.10	0.14	0.18	0.21	0.24	0.27
12	......	......	0.07	0.10	0.13	0.15	0.18	0.19
13	......	......	0.04	0.07	0.10	0.12	0.13	0.14

(4) Pressure of the Mercury Vapor. This causes a slight depression at high temperatures, but is less than 0.01 mm under 40° C.

(5) Variation of Weight with Latitude and Elevation. Multiply by $g/g_0 = (1-0.0026 \cos 2\phi - 0.000\,000\,2\,H)$, the local height obtained by applying the above corrections to the reading. The correction for elevation is only 0.1 mm at 700 m, but the correction for latitude may amount to as much as 2 mm.

Mean Barometer-Height b at an Elevation of H Meters above Sea Level
(Kohlrausch)
Air at 10° C. (50° F.) $b_0=760$ mm.

$H=$	0	100	200	300	400	500	600	700	800	900	1000 m
$b=$	760	751	742	733	724	716	707	699	690	682	674 mm
$H=$	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000 m
$b=$	674	666	658	650	642	635	627	620	612	605	598 mm

The international meteorological formula for reducing height b of mercurial barometer at t° C. and latitude ϕ to height b_0 at 0° C. and latitude 45° is

$$\log_e b_0=\log_e b+\frac{H(1-3/8\ \epsilon/b)}{(18\,429+67.5\,t+0.003\,H)(1+0.0026\cos 2\,\phi)}$$

22. Laws of Flight of an Aeroplane

Abstract from The Present Status of Military Aeronautics,
by General Geo. O. Squier, Signal Corps, U. S. A.

Support. In this class of flying machines, since the buoyancy is practically insignificant, support must be obtained from the dynamic reaction of the atmosphere itself. In its simplest form, an aeroplane may be considered as a single plane surface moving thru the air. The law of pressure on such a surface may be exprest by $L=2\,k\delta A V^2 \sin\alpha$ (1)

in which L is the lift (vertical force) acting upon the plane, k is the lift coefficient involving also the effect of aspect ratio (i.e., the ratio of length of plane to breadth), δ the density of the air, A the area of the plane, V the relative velocity of translation of the plane with respect to the air, and α the angle of attack; that is, the angle made by the plane with the direction of flight or translation.

This is the form taken by Duechemin's formula for small angles of attack such as are usually employed in practise. The equation shows that the upward pressure on the plane varies directly with the area of the plane, with the sine of the angle of attack, with the density of the air, and also with the square of the relative velocity of translation.

It is evident that the total upward force developed must be at least equal to the weight of the plane and its load, in order to support the system. If L is greater than the weight the machine will ascend; if less, it will descend.

The constant k depends upon the shape and aspect of the plane and should be determined by experiment. It approximates a constant value for small values of α; for example, with a plane 1 ft square $k\delta=0.00167$ as determined by Langley, when L is exprest in pounds per square foot and V in feet per second.

Equation (1) may be written: $AV^2=L/2k\delta\sin\alpha$.

If L and α are kept constant, then the equation has the form

$$AV^2=\text{constant}.$$

An interpretation of (2) shows that the supporting area varies inversely as the square of the velocity. In the Wright aeroplane, the supporting area at 40 miles per hour is 500 sq ft, while if the speed is increased to 60 miles per hour this area need be only $500/2.25=222$ sq ft, or less than one-half of its present size. At 80 miles per hour the area would be reduced to 125 sq ft, and at 100 miles per hour only 80 sq ft of supporting area is required. It thus appears that if the angle of attack be kept constant while the speed is increased to one hundred miles per hour, the total supporting area may be reduced to 80 sq ft, or two surfaces each measuring about 2.5 by 16 ft, or 4 by 10 ft if preferred.

In the case of a bird's flight, its wing surface is "reefed" as its velocity is increased, which action serves to reduce its head resistance and skin-frictional area, and the consequent power required for a particular speed.

Determination of k for Arched Surfaces. Since arched surfaces are commonly used in aeroplane construction, and as the above equation (1) applies to plane surfaces only, it is important to determine experimentally the value of the coefficient of figure k, for each type of arched surface employed, especially

as k is shown in some cases to vary with the angle of attack, that is, with the inclination of the chord of the surface to the line of translation.

Assuming α constant, the lift of any particular arched surface with a plane surface of the same projected plan and angle of attack may be compared. To illustrate, in the case of the Wright aeroplane, assume $L = 1000$ lb = total weight = W, $A = 500$ sq ft, $V = 40$ miles per hour = 60 ft per second, and $\alpha = 7°$ approximately. Whence

$$k = \frac{L}{2\,A V^2 \sin\alpha} = \frac{1000}{2 \times 500 \times 60^2 \times \frac{1}{8}} = 0.0022\ (V = \text{ft per sec}) = 0.005\ (V = \text{mi per hr}).$$

Comparing this value of k with Langley's value 0.004 for a plane surface, V being in miles per hour, the lift for the arched surface is 25 per cent greater than for a plane surface of the same projected plan. That is, the arched surface is dynamically equivalent to a plane surface of 25 per cent greater than the projected plan. Such a plane surface may be defined as the "equivalent plane."

Resistance and Propulsion. The resistance of the air to the motion of an aeroplane is composed of two parts: (*a*) the resistance due to the framing and load; (*b*) the necessary resistance of the sustaining surfaces, that is, the drag, or horizontal component of pressure, and the unavoidable skin friction. Disregarding the frame, and considering the aeroplane as a simple plane surface, we may express the resistance by the equation

$$R = W \tan\alpha + 2\,fA \qquad (3)$$

in which R is the total resistance, W the gross weight sustained, α the angle of attack, f the friction per square unit of area of the plane, A the area of the plane. The first term of the second member gives the drag, the second term the skin friction. The power required to propel the aeroplane is $H = RV$, in which H is the power, V the velocity. Now W varies as the second power of the velocity, as shown by equation (1), and f varies as the power 1.85, as will be shown later. Hence the total resistance R of the air to the aeroplane varies approximately as the square of its speed, and the propulsive power practically as the cube of speed.

Most Advantageous Speed and Angle of Attack. Again regarding W and A as constant, α may, by equation (1), be computed for various values of V, and f be found for those velocities from the skin-friction table given on page 1548. Thus δ, R and H may be found for various velocities of flight, and their magnitudes compared. In this way, the values in the following were computed for a soaring plane 1 foot square weighing 1 lb, assuming $k\delta = 0.004$, which is approximately Langley's value when V is in miles per hour.

Computed Power Required to Tow a Plane One Foot Square, Weighing One Pound, Horizontally thru the Air

Velocity miles per hour	Angle of attack degrees	Computed resistance in pounds			Tow-line power ft-lb per sec	Lift per tow-line horse-power, lb
		Drag	Friction	Total		
30	8.25	0.145	0.0170	0.162	7.13	77.1
35	5.94	0.104	0.0226	0.1266	6.51	84.3
40	4.52	0.0790	0.0289	0.1079	6.32	86.7
45	3.55	0.0621	0.0360	0.0981	6.39	86.1
50	2.88	0.0500	0.0439	0.0939	6.89	80.2
60	2.03	0.0354	0.0614	0.0968	8.50	64.7
70	1.47	0.0257	0.0814	0.1071	11.00	50.0
80	1.12	0.0195	0.1045	0.1240	14.56	35.8
90	0.88	0.0154	0.1300	0.1454	19.17	28.7
100	0.71	0.0124	0.1584	0.1708	25.00	22.0

Column two, giving values of α for various speeds, is computed from equation (1). Thus, at 30 miles per hour,

$$\sin\alpha = \frac{W}{2\,k\delta A V^2} = \frac{1}{2 \times 0.004 \times 1 \times 30^2}$$

whence $\alpha = 8°.25$. Column three is computed from the term $W \tan \alpha$ in equation (3), thus, Drag $= W \tan \alpha = 1 \times \tan 8°.25 = 0.145$. Column four is computed from the term $2fA$ in equation (3), f being taken from the skin-friction table given below.

The table shows that if a thin plane 1 ft square, weighing 1 lb, be towed thru the air so as just to float horizontally at various velocities and angles of attack, the total resistance becomes a minimum at an angle of slightly less than 3°, and at a velocity of about 50 miles per hour; also that the skin friction approximately equals the drag at this angle. The table also shows that the propulsive power for the given plane is a minimum at a speed of between 40 and 45 miles per hour, the angle of attack then being approximately 4.5°.

The last column of the table shows that the maximum weight carried per horse-power is less than 90 lb. This horse-power may be increased by changing the foot-square plane to a rectangular plane and towing it long-side foremost; also by lightening the load, and letting the plane glide at a lower speed; but, best of all, perhaps, by arching it like a vulture's wing and also towing it long-side foremost, as in the prevailing practise with aeroplanes.

Skin Friction in Air. Even as late as Langley's experiments, skin friction in air was regarded as a negligible quantity, but the extensive and reliable experiments of Zahm in air showed that it was appreciable. As a result of his research on atmospheric friction the following equations are deduced:

$$f = 0.00000778\, l^{-0.07} v^{1.85} \; (v = \text{ft per sec})$$

$$f = 0.0000158\, l^{-0.07} v^{1.85} \; (v = \text{mi per hr})$$

in which f is the average skin friction per square foot and l the length of surface. From this equation the accompanying table of frictions was computed.

Friction per Square Foot for Various Speeds and Lengths of Surface

Wind speed, miles per hour	Average friction in pounds per square foot					
	1-ft plane	2-ft plane	4-ft plane	8-ft plane	16-ft plane	32-ft plane
5	0.000303	0.000289	0.000275	0.000262	0.000250	0.000238
10	0.00112	0.00105	0.00101	0.000967	0.000922	0.000878
15	0.00237	0.00226	0.00215	0.00205	0.00195	0.00186
20	0.00402	0.00384	0.00365	0.00349	0.00332	0.00317
25	0.00606	0.00579	0.00551	0.00527	0.00501	0.00478
30	0.00850	0.00810	0.00772	0.00736	0.00701	0.00668
35	0.01130	0.0108	0.0103	0.0098	0.00932	0.00888
40	0.0145	0.0138	0.0132	0.0125	0.0125	0.0114
50	0.0219	0.0209	0.0199	0.0190	0.0181	0.0172
60	0.0307	0.0293	0.0279	0.0265	0.0253	0.0242
70	0.0407	0.0390	0.0370	0.0353	0.0337	0.0321
80	0.0522	0.0500	0.0474	0.0452	0.0431	0.0411
90	0.0650	0.0621	0.0590	0.0563	0.0536	0.0511
100	0.0792	0.0755	0.0719	0.0685	0.0652	0.0622

The numbers within the rules represent data coming within the range of observation. These observations show that "the frictional resistance is at least as great for air as water, in proportion to their densities. In aeronautics it is one of the chief elements of resistance both to hull-shaped bodies and to aero-surfaces gliding at small angles of flight."

WEIGHTS AND MEASURES

23. The Metric System

The Unit of Length in the metric system is the meter, a length very nearly equal to the one ten-millionth of the distance between the equator and the pole. The original meter was a platinum bar constructed and deposited in the Archives of the French Republic in 1799. At the same time a platinum

weight was constructed which was made as nearly as possible equal to the mass of a cube of pure water at 4° C., the sides of the cube being one tenth the length of the meter. This weight, which is equal to one thousand units in the metric system, is called the kilogram.

The use of the meter as the basis of geodetic surveys had become so general thruout Europe that a conference was called in Paris, France, in 1870, for the purpose of establishing a central bureau where the standards of the different countries could be intercompared. As a result of this conference an International Bureau of Weights and Measures was established near Paris, France, in 1875, by the concurrent action of the principal nations of the world. One of the first tasks undertaken by the Bureau was the construction of exact copies of the meter and kilogram deposited in the Archives. Thirty-one standard meters of iridio-platinum and forty kilograms of the same alloy were constructed and carefully compared with the standards of the Archives and with one another. This great work was completed in 1889, and the meter and kilogram which agreed most nearly with the original standards were called international prototypes, and were deposited at the International Bureau, where they are maintained to-day subject to the authority of the International Committee on Weights and Measures. The remaining meters and kilograms were distributed by lot to the different nations which contributed to the support of the Bureau. The United States secured two copies of the meter and two copies of the kilogram, which are in the custody of the Bureau of Standards at Washington. One of the meters, known as No. 27, and one kilogram, No. 20, were selected as the United States standards, while the other meter and kilogram are used as working standards. It was the declared intention of the International Committee that the various national prototypes should be returned to the International Bureau at regular intervals for the purpose of recomparing them with the international standards and with one another. In this way all measurements based upon metric standards thruout the world are ultimately referred to the international meter and kilogram.

The Unit of Capacity in the metric system is the liter, which is defined as the volume of one kilogram of pure water at the temperature of maximum density (4° C.) under a pressure of 76 cm of mercury. For all practical purposes the cubic decimeter and the kilogram of water may be regarded as identical, the difference between them being less than three parts in one hundred thousand, the kilogram of water having the larger volume.

24. The English System

Units of Length and Mass. While it is the common impression that the customary system of weights and measures in use in the United States is identical with the system in everyday use in Great Britain, there are important differences in some of the units. For all practical purposes the units of length and mass are the same, altho the fundamental standards are quite different in the two countries. In Great Britain the standard yard is a bronze bar in the custody of the Board of Trade and preserved in the Standards Office, Westminster. In terms of the International meter, in England, one yard equals 36/39.370113 meters. In the United States the yard is defined as 3600/3937 meters. The difference amounts to only a little more than one part in 400 000 which is about the difference found in comparing the Imperial yard with its authentic copies at different periods.

It is therefore doubtful whether there is a real difference between the English and United States yards. The difference is merely one of standards, the standard of Great Britain being the bronze bar previously referrred to, while in the United States the fundamental standard is the International meter, of which the U. S. yard is a certain definite fraction, easily measured with the proper apparatus. A similar condition exists with respect to the avoirdupois pound; in England a certain platinum cylinder deposited in the Standards Office is regarded as the standard pound, while in the United States the pound is defined as 453.5924277 grams of which the International kilogram contains 1000. This is the same as the legal value in Great Britain, so that the only difference between the British system in the two countries is one of fundamental standards.

The Capacity Measures of the two countries are, however, quite different. The United States gallon of 231 cubic inches and the Winchester bushel of

2150.42 cubic inches have not been legal measures in Great Britain since 1825, having been superseded by the present Imperial gallon, which is defined as the volume of 10 lbs of pure water at 62° Fahrenheit, weighed against brass weights in air at the same temperature as the water, and with the barometer at 30 in. The Imperial bushel is defined as the volume of 80 lbs of water weighed under the same conditions as the gallon. The bushel of Great Britain is, therefore, exactly equal to eight gallons. The volume of the Imperial gallon, according to the most reliable data available at this time, is 277,420 cubic inches and the bushel 2219.36 cubic inches, the first being larger than the United States gallon by approximately 20% and the latter larger than the United States bushel by 3.2%.

25. Chinese Measures

The following gives the new system of weights and measures adopted by China in pursuance of the imperial rescript of August 28, 1908.

Measures of Length

Chinese		Metric	
1 Hao	equals	0.0032	Centimeter
1 Li	"	0.032	"
1 Fen	"	0.32	"
1 Ts'un	"	3.2	Centimeters
1 Ch'ih	"	0.32	Meter
1 Pu	"	1.6	Meters
1 Chang	"	3.2	"
1 Li	"	576.	"

Measures of Surface

Chinese		Metric	
1 Hao	equals	0.006144	Are
1 Li	"	0.06144	"
1 Fen	"	0.6144	"
1 Fang pu	"	0.0256	"
1 Mu	"	6.144	Ares
1 Ch'ing	"	614.4	"

Measures of Capacity

Chinese		Metric	
1 Shao	equals	0.0104	Liter
1 Ko	"	0.1035	"
1 Sheng	"	1.0355	Liters
1 Tou	"	10.355	"
1 Hu	"	51.7734	"
1 Tan or Picul	"	103.5469	"

Measures of Weight

Chinese		Metric	
1 Hao	equals	0.0037301	Gram
1 Li	"	0.037301	"
1 Fen	"	0.37301	"
1 Ch'ien	"	3.7301	Grams
1 Liang	"	37.301	"
1 Chin or catty	"	596.816	"

26. Japanese Measures

The Kwan, equal to 3.75 kg or 8.267 pounds avoirdupois, is the unit of mass. It is divided into 1000 mommes, the momme into 10 funs and the fun into 10 rins, the rin into 10 mos, and the mo into 10 shis.

The Shaku is the unit of length, equal to $^{10}/_{33}$ (0.30303) meter, or 0.994 U. S. ft. The shaku is decimally divided into sun, bu, rin, mo, shi. Multiples of the shaku are the ken, equal to 6 shaku; the cho equal to 60 ken, and the ri, equal to 36 cho, or 12 960 shaku.

The shaku (land measure) is equal to 0.00033 are, or 0.9884 sq ft, ten shaku equal one go and ten gos equal one bu or tsubo, thirty bu equal one sé, ten sé one tan, and ten tan one cho.

The shaku (capacity measure) equals 0.01804 liter, or 0.0187 U. S. liquid quart, ten shakus equal one go, ten gos one sho, ten shos one to, and ten tos one koku. The metric weights and measures are recognized as legal in accordance with the above equivalents.

27. Russian Measures

The Funt, equal to 0.4095 kilogram, or 0.9028 lb avoirdupois, is the basis of Russian weights. The funt is divided into 96 zolotniks or 32 loths, and the zolotnik is divided into 96 dolias; 40 funts equal one pood.

The Archine, equal to 28 inches or 0.71120 meter, is the basis of the length measure. The archine contains 16 verskops; three archines equal one sagene and 500 sagenes equal one verst.

The dessiatine, equal to 2400 square saganes or 2.6997 acres, is the unit for land measure.

The tchetvert, equal to 2.099 hectoliters, or 5.9567 U. S. bushels, is the unit for dry measure. The tchetvert is divided into 8 tchetveriks, and the tchetverik into 8 garnetz. Twelve tchetverts equal one last.

The vedro, equal to 12.2993 liters, or 3.2491 U. S. gallons, is used for liquids.

28. Special Commercial Units

For Logs the Doyle rule, known in some sections as the Connecticut River Rule, the St. Croix Rule, the Thurber Rule, the Moore and Beeman Rule, and the Scribner Rule, is more generally employed than any other: Deduct 4 inches from the diameter of the log, as an allowance for slab; square one-quarter of the remainder and multiply the result by the length of the log in feet. It is the usual custom to measure the diameter inside the bark at the small end.

The Miner's Inch is defined as the quantity of water that will pass thru an orifice one square inch in cross-section under a given head. The head has been fixt in various localities as from four to six and one-half inches to the center of the orifice.

Bushel (dry measure) = 4 pecks = 8 gallons = 32 quarts, 1 struck bushel = 1.24445 cu ft = 2150.4 cu in.

Barrel. The crude oil barrel is 42 gallons, while the refined product is bought and sold on the basis of 50 gallons to the barrel. The large lime barrel contains 280 lb; the small one 180 lb. Legal net weight of a barrel of flour = 196 lbs. For natural cement barrels contain 282 lb, for portland cement 376 lb.

A legal standard barrel for fruits, vegetables and other dry commodities, except cranberries, contains 7056 cu in. Its outside dimensions are: Length of stave, 28½ in; distance between heads, 26 in; diameter of head, 17⅛ in; circumference of bulge, 64 in; outside measurement and thickness of staves not greater than 0.4 in. For cranberries the corresponding dimensions of the barrel are 28½ in, 25¼ in, 16¼ in, 58½ in.

Firewood is mostly sold by the cord of 128 cu ft.

Old Foreign Units, still sometimes used, with their U. S. equivalents, are as follows:

Central America: libra = 1.014 lb, arroba = 25.36 lb, quintal = 101.41 lb, vara = 32.87, 32.99, and 33 in.

Cuba: libra = 1.016 lb, arroba = 25.37 lb, vara = 33.38 in.

Danish West Indies: pund = 1.023 lb, fod = 1.029 ft, tõnde = 3.948 bu, pot = 0.2552 gal, mil = 4.68 miles.

Mexico: libra = 1.015 lb, arroba = 25.366 lb, vara = 32.992 in, frasco = 2.5 qt.

South Africa: leaguer = 153.64 gal, muid = 3.095 bu, morgen = 2.116 acres.

South America: libra = 1.014 lb, arroba = 25.36 and 25.40 lb, quintal = 100 and 101.42 lb, vara = 32.91 and 33.38 in.

Spain: libra = 1.0143 lb, arroba = 25.36 and 25.40 lb, vara = 32.913 in, butt = 140 gal.

Texas and States on Mexican border: vara = 33⅓ in = 2.778 ft, 5645.376 sq varas = 1 acre, labor = 1 000 000 sq varas = 177.136 acres, league = 25 000 000 sq varas = 4428.4 acres.

Turkey: oke = 2.819 lb, kilé = 1.1 bu, pic = about 27 in.

29. Equivalents of Units

In the following tables the quantities in the same line are equivalents. For example, 1 kilogram equals 2.20462 pounds or 0.980665 megadynes.

Numbers in **boldface** type are exact values by definition. The notation 0.(³)1259 indicates that the (³) is to be replaced by three ciphers.

Density

Gm per cu cm	Lb per cu ft	Short ton per cu yd	Lb per U. S. gal
1	62.4281 1.79538*	0.84278 $\bar{1}$.92572*	8.3454 0.92145*
0.01602 $\bar{2}$.20462*	1	0.0135 $\bar{2}$.13033*	0.13368 $\bar{1}$.12607*
1.18655 0.07428*	74.074 1.86967*	1	9.9023 0.99574*
0.11983 $\bar{1}$.07855*	7.4805 0.87393*	0.10099 $\bar{1}$.00428*	1

* Logarithm of the number immediately above.

Force

Megadynes	Kilograms	Pounds
1	1.01972 0.008481*	2.248089 0.351813*
0.980665 $\bar{1}$.991490*	1	2.20462 0.343334*
0.444822 $\bar{1}$.648186*	0.453592 $\bar{1}$.656666*	1

1 Megadyne = 10^6 dynes.

* Logarithm of the number immediately above.

Length

1 meter = 10 decimeters = 100 centimeters = 1000 millimeters = 10^6 microns = 0.1 dekameter = 0.01 hectometer = 0.001 kilometer = 0.0001 myriameter.

1 U. S. yard = 3600/3937 meters (by definition); log = $\bar{1}.9611371$.

Meters	Inches	Feet	Yards	Links	Rods, poles, or perches	Chains, Gunter's	Statute miles U. S.	Nautical miles U. S.
1	39.37 1.59517*	3.2808 0.51598*	1.0936 0.03886*	4.971 0.69644*	0.1988 $\bar{1}.29850$*	0.04971 $\bar{2}.69644$*	0.(3)6214 $\bar{4}.79335$*	0.(3)5396 $\bar{4}.73207$*
0.0254 $\bar{2}.40483$*	1	0.08333 $\bar{2}.92082$*	0.02778 $\bar{2}.44370$*	0.1263 $\bar{1}.10127$*	0.005051 $\bar{3}.70333$*	0.001263 $\bar{3}.10127$*	0.(4)1578 $\bar{5}.19818$*	0.(4)1371 $\bar{5}.13690$
0.3048 $\bar{1}.48402$*	12 1.07918*	1	0.3333 $\bar{1}.52288$*	1.515 0.18046*	0.06061 $\bar{2}.78252$*	0.01515 $\bar{2}.18046$*	0.(3)1894 $\bar{4}.27737$*	0.(3)1645 $\bar{4}.21608$*
0.9144 $\bar{1}.96114$*	36 1.55630*	3 0.47712*	1	4.545 0.65758*	0.1818 $\bar{1}.25964$*	0.04545 $\bar{2}.65758$*	0.(3)5682 $\bar{4}.75449$*	0.(3)4934 $\bar{4}.69320$*
0.2012 $\bar{1}.30356$*	7.92 0.89873*	0.66 $\bar{1}.81954$*	0.22 $\bar{1}.34242$*	1	0.04 $\bar{2}.60206$*	0.01 $\bar{2}.00000$*	0.(3)1250 $\bar{4}.09691$*	0.(3)1086 $\bar{4}.03564$
5.029 0.70150*	198 2.29667*	16.5 1.21748*	5.5 0.74036*	25 1.39794*	1	0.25 $\bar{1}.39794$*	0.(2)3125 $\bar{3}.49485$*	0.(2)2714 $\bar{3}.43357$*
20.12 1.30356*	792 2.89873*	66 1.81954*	22 1.34242*	100 2.00000*	4 0.60206*	1	0.0125 $\bar{2}.09691$*	0.01086 $\bar{2}.03564$*
1609.3 3.20665*	63360 4.80182*	5280 3.72263*	1760 3.24551*	8000 3.90309*	320 2.50515*	80 1.90309*	1	0.8684 $\bar{1}.93872$
1853.25 3.26793*	72962 4.86310*	6080.2 3.78392*	2026.73 3.30680*	9212 3.96437*	368.5 2.56643*	92.12 1.96437*	1.1516 0.06128*	1

1 nautical mile of the British admiralty = 6080 ft. 1 furlong = 1/8 mile = 660 feet. 1 league = 3 miles = 24 furlongs. 1 fathom = 2 yards = 6 feet.

* Logarithm of the number immediately above.

Area

1 hectare = 100 ares = 10 000 centares or square meters.

Square meters	Square inches	Square feet	Square yards	Square rods	Square chains	Acres	Square miles or sections
1	1550 3.19033*	10.764 1.03197*	1.1960 0.07773*	0.03954 $\bar{2}.59700$*	0.(2)2471 $\bar{3}.39288$*	0.(3)2471 $\bar{4}.39288$*	0.(6)3861 $\bar{7}.38670$*
0.(3)6452 $\bar{4}.80967$*	1	0.006944 $\bar{3}.84164$*	0.(3)7716 $\bar{4}.88740$*	0.(4)2551 $\bar{5}.40667$*	0.(5)1594 $\bar{6}.20255$*	0.(6)1594 $\bar{7}.20255$*	0.(9)2491 $\overline{10}.39637$*
0.09290 $\bar{2}.96803$*	144 2.15836*	1	0.1111 $\bar{1}.04576$*	0.(2)3673 $\bar{3}.56503$*	0.(3)2296 $\bar{4}.36091$*	0.(4)2296 $\bar{5}.36091$*	0.(7)3587 $\bar{8}.55473$*
0.8361 $\bar{1}.92227$*	1296 3.11260*	9 0.95424*	1	0.03306 $\bar{2}.51927$*	0.(2)2066 $\bar{3}.31515$*	0.(3)2066 $\bar{4}.31515$*	0.(6)3228 $\bar{7}.50898$*
25.29 1.40300*	39204 4.5933*	272.25 2.43497*	30.25 1.48072*	1	0.0625 $\bar{2}.79588$*	0.00625 $\bar{3}.79588$*	0.(5)9766 $\bar{6}.98970$*
404.69 2.60712*	627264 5.79745*	4356 3.63909*	484 2.68484*	16 1.20412*	1	0.1 $\bar{1}.00000$*	0.(3)1562 $\bar{4}.19382$*
4046.9 3.60712*	6272640 6.79745*	43560 4.63909*	4840 3.68484*	160 2.20412*	10 1.00000*	1	0.001562 $\bar{3}.19382$*
2589998 6.41330*		27878400 7.44527*	3097600 6.49102*	102400 5.01030*	6400 3.80618*	640 2.80618*	1

* Logarithm of the number immediately above.

Mass (Physicists' system) or **Force** (Engineers' system)

1 kilogram = 1000 grams = 0.001 metric ton. 1 gm = 10 decigrams = 100 centigrams = 1000 milligrams = 0.1 dekagram = 0.01 hectogram = 0.001 kilogram = 0.0001 myriagram.
1 U. S. Avoirdupois pound = 0 4535924277 kg = (by definition) 7000/5760 troy pounds.

Kilograms	Grains	Ounces Avoir.	Ounces Troy and apoth.	Pounds Troy and apoth.	Pounds Avoir.	Tons Short, 2000 lb	Tons Long, 2240 lb	Tons Metric, 1000 kg
1	15432. 4.18843*	35.274 1.54745*	32.151 1.50719*	2.6792 0.42801*	2.20461 0.34333*	0.001102 $\bar{3}$.04230*	0.(3)9842 $\bar{4}$.99309*	0.001 $\bar{3}$.00000*
0.(4)6480 $\bar{5}$.81157*	1	0.(2)2286 $\bar{3}$.35902*	0.002083 $\bar{3}$.31876*	0.(3)1736 $\bar{4}$.23958*	0.(3)142 $\bar{4}$.15490*			
0.028349 $\bar{2}$.45255*	437.5 2.64098*	■	0.91146 $\bar{1}$.95974*	0.075955 $\bar{2}$.88056*	0.0625 $\bar{2}$.79588*	0.(4)3125 $\bar{5}$.49485*	0.(4)2790 $\bar{5}$.44563*	0.(4)2835 $\bar{5}$.45255*
0.031103 $\bar{2}$.49281*	480 2.68124*	1.0971 0.04026*	■	0.083333 $\bar{2}$.92082*	0.068571 $\bar{2}$.83614*	0.(4)3429 $\bar{5}$.53511*	0.(3)3061 $\bar{4}$.56508*	0.(4)3110 $\bar{5}$.49281*
0.37324 $\bar{1}$.57199*	5760 3.76042*	13.166 1.11944*	■ 1.07918*	1		0.(3)4114 $\bar{4}$.61429*	0.(3)3673 $\bar{4}$.56508*	0.(3)3732 $\bar{4}$.57199*
0.45359 $\bar{1}$.65667*	7000 3.84510*	16 1.20412*	14.583 1.16386*	1.2153 0.08468*	1	0.0005 $\bar{4}$.69897*	0.(3)4464 $\bar{4}$.64975*	0.(3)4536 $\bar{4}$.65667*
907.18 2.95770*		32000 4.50515*	29167. 4.46489*	2430.6 3.38570*	2000 3.30103*	■	0.89286 $\bar{1}$.95078*	0.90718 $\bar{1}$.95770*
1016.1 3.00691*		35840 4.55437*	32667 4.51410*	2722.2 3.43492*	2240 3.35025*	1.12 0.04922*	■	1.0160 0.00691*
1000 3.00000*		35274 4.54745*	32151 4.50719*	2679.2 3.42801*	2204.6 3.34333*	1.1023 0.04230*	0.98421 $\bar{1}$.99309*	■

1 quarter = 28 lb avoir. 1 pennyweight = 24 gr = 0.05 oz troy. 1 oz avoir. = 16 drams avoir. = 437.5 gr. 1 stone = 14 pounds. 1 cental = 100 pounds. 1 hundredweight = 112 pounds. 1 apothecaries' ounce = 8 apoth. drams = 24 scruples = 480 grains.
* Logarithm of the number immediately above.

Power

1 kilowatt = 1000 watts = 1000 joules per second.
1 horse-power = 550 foot-pounds per second.
1 cheval-vapeur = 75 kilogram-meters per second.

Kilowatts	Cheval-vapeur	Poncelet	Horse-power	M-kg per sec	Ft-lb per sec	Kg cal per sec	Btu per sec
1	1.3600 0.13341*	1.0197 0.00848*	1.341 0.12743*	101.97 2.00848*	737.5 2.86780*	0.2388 $\bar{1}$.37803*	0.9475 $\bar{1}$.97660*
0.7355 $\bar{1}$.86659*	■	0.75 $\bar{1}$.87506*	0.9863 $\bar{1}$.99402*	75 1.87506*	542.5 2.73438*	0.1756 $\bar{1}$.24456*	0.6969 $\bar{1}$.84318
0.980665 $\bar{1}$.99152*	1.333 0.12493*	■	1.3151 0.11896*	100 2.00000*	723.3 2.85932*	0.2342 $\bar{1}$.36951*	0.9292 $\bar{1}$.96812*
0.7457 $\bar{1}$.87257*	1.0139 0.00598*	0.7604 $\bar{1}$.88104*	■	76.04 1.88104*	550 2.74036*	0.1780 $\bar{1}$.25055*	0.7066 $\bar{1}$.84916*
0.009807 $\bar{3}$.99152*	0.01333 $\bar{2}$.12493*	0.01 $\bar{2}$.00000*	0.01315 $\bar{2}$.11896*	1	7.233 0.85932*	0.002342 $\bar{3}$.36951*	0.009292 $\bar{3}$.96812*
0.001356 $\bar{3}$.13220*	0.001843 $\bar{3}$.26562*	0.00138 $\bar{3}$.14068*	0.001818 $\bar{3}$.25964*	0.1383 $\bar{1}$.14068*	1	0.0003237 $\bar{4}$.51016*	0.001285 $\bar{3}$.10880*
4.188 0.62201*	5.694 0.75542*	4.271 0.63049*	5.616 0.74945*	427.1 2.63049*	3089 3.48984*	1	3.968 0.59861*
1.055 0.02340*	1.435 0.15682*	1.076 0.03188*	1.415 0.15084*	107.62 2.03188*	778.4 2.89120*	0.2520 $\bar{1}$.40139*	1

* Logarithm of the number immediately above.

Speed and Velocity

Cm per sec	Km per hour	Ft per sec	Ft per min	Miles per hour	Knots
1	0.036 $\bar{2}$.55630*	0.03281 $\bar{2}$.51598*	1.9685 0.29413*	0.02237 $\bar{2}$.34965*	0.01942 $\bar{2}$.28825*
27.7778 1.44370*	1	0.91134 $\bar{1}$.95968*	54.6806 1.73783*	0.62137 $\bar{1}$.79335*	0.53960 $\bar{1}$.73207*
30.4801 1.48402*	1.0973 0.04032*	1	60 1.77815*	0.68182 $\bar{1}$.83367*	0.59209 $\bar{1}$.77238*
0.5080 $\bar{1}$.70586*	0.01829 $\bar{2}$.26217*	0.01667 $\bar{2}$.22185*	1	0.01136 $\bar{2}$.05553*	0.009868 $\bar{3}$.99423*
44.7041 1.65035*	1.6093 0.20670*	1.46667 0.16633*	88 1.94448*	1	0.86839 $\bar{1}$.93872*
51.4971 1.71178*	1.8532 0.26793*	1.68894 0.22761*	101.337 2.00577*	1.15155 0.06128*	1

1 knot = 1 nautical mile per hour.
* Logarithm of the number immediately above.

Volume and Capacity

1 liter = 1 cubic decimeter = 1000 cubic centimeters = 10 deciliters = 100 centiliters = 1000 milliliters = 0.1 dekaliter = 0.01 hectoliter = 0.01 kiloliter = 0.001 cubic meters or steres.

Cubic inches	Cubic feet	Cubic yards	U. S. quarts		Gallons		Bushels U. S.	Liters
			Liquid	Dry	U. S. liquid	U. S. dry		
1	0.(3)57870 $\bar{4}$.76246*	0.(4)2143 $\bar{5}$.33109*	0.017316 $\bar{2}$.23845*	0.014881 $\bar{2}$.17263*	0.004329 $\bar{3}$.63639*	0.003720 $\bar{3}$.57057*	0.(3)4650 $\bar{4}$.66748*	0.01638 $\bar{2}$.21450
1728 3.23754*	1	0.037037 $\bar{2}$.56864*	29.922 1.47599*	25.714 1.41017*	7.4805 0.87393*	6.4285 0.80811*	0.80356 $\bar{1}$.90502*	28.317 1.45205
46656 4.66891*	27 1.43136*	1	807.90 2.90736*	694.28 2.84153*	201.97 2.30530*	173.57 2.23948*	21.696 1.33638*	764.56 2.88341
57.75 1.76155*	0.033420 $\bar{2}$.52401*	0.001238 $\bar{3}$.09026*	1	0.85937 $\bar{1}$.93418*	0.25 $\bar{1}$.39794*	0.21484 $\bar{1}$.33212*	0.026855 $\bar{2}$.42903*	0.94636 $\bar{1}$.97606
67.201 1.82737*	0.038889 $\bar{2}$.58983*	0.001440 $\bar{3}$.15847*	1.1637 0.06582*	1	0.29091 $\bar{1}$.46376*	0.25 $\bar{1}$.39794*	0.03125 $\bar{2}$.49485*	1.1012 0.04188
231 2.36361*	0.13368 $\bar{1}$.12607*	0.004951 $\bar{3}$.69471*	4 0.60206*	3.4375 0.53624*	1	0.85937 $\bar{1}$.93418*	0.10742 $\bar{1}$.03109*	3.7854 0.57812
268.80 2.42943*	0.15556 $\bar{1}$.19189*	0.005761 $\bar{3}$.76053*	4.6546 0.66788*	4 0.60201*	1.1637 0.06582*	1	0.125 $\bar{1}$.09691*	4.4049 0.64394
2150.4 3.33253*	1.2445 0.09498*	0.046091 $\bar{2}$.66362*	37.237 1.57097*	32 1.50515*	9.3092 0.96891*	8 0.90309*	1	35.239 1.54703
61.023 1.78550*	0.035315 $\bar{2}$.54795*	0.001308 $\bar{3}$.11659*	1.0567 0.02394*	0.90808 $\bar{1}$.95812*	0.26417 $\bar{1}$.42188*	0.22702 $\bar{1}$.35606*	0.028377 $\bar{2}$.45297*	1

1 U. S. liquid quart = 2 pints = 8 gills = 32 fluid ounces = 256 fluid drams = 768 fluid scruples. 1 bushel = 4 pecks.

1 Imperial gallon = 1.201 U. S. gallons = 0.1605 cu ft = 4.5460 liters.

1 U. S. gallon = 0.8327 Imperial gallons. 1 cubic foot = 6.229 Imperial gallons.

1 British bushel = 1.2837 cubic feet.

Shipping Measure: 1 register ton = 100 cu ft. 1 U. S. shipping ton = 40 cu ft.

1 British shipping ton = 42 cu ft.

* Logarithm of the number immediately above.

Pressure

Kilograms per sq cm	Pounds Per sq in	Pounds Per sq ft	Short tons, per sq ft	Atmospheres	Columns of mercury ‡ Meters	Columns of mercury ‡ Inches	Columns of water† Meters	Columns of water† Feet
1	14.223 1.15300*	2048.2 3.31137*	1.0241 0.01034*	0.96781 $\bar{1}$.98579*	0.73553 $\bar{1}$.86660*	28.958 1.46177*	10.009 1.00038*	32.837 1.51636*
0.070307 $\bar{2}$.84700*	1	144 2.15836*	0.072 $\bar{2}$.85733*	0.06804 $\bar{2}$.83279*	0.051713 $\bar{2}$.71360*	2.0359 0.30876*	0.70368 $\bar{1}$.84738*	2.3087 0.36336*
0.(3)4882 $\bar{4}$.68863*	0.006944 $\bar{3}$.84164*	1	0.0005 $\bar{4}$.69897*	0.(3)4725 $\bar{4}$.67442*	0.(3)3591 $\bar{4}$.55524*	0.014138 $\bar{2}$.15040*	0.004887 $\bar{3}$.68901*	0.016032 $\bar{2}$.20500*
0.97648 $\bar{1}$.98966*	13.889 1.14267*	2000 3.30103*	1	0.94504 $\bar{1}$.97545*	0.71823 $\bar{1}$.85627*	28.277 1.45143*	9.7734 0.99004*	32.065 1.50603*
1.0333 0.01421*	14.697 1.16722*	2116.3 3.32558*	1.0582 0.02955*	1	0.76 $\bar{1}$.88081*	29.921 1.47598*	10.342 1.01459*	33.929 1.53058*
1.3596 0.13340*	19.338 1.28640*	2784.6 3.44476*	1.3923 0.14373*	1.3158 0.11919*	1	39.37 1.59517*	13.607 1.13378*	44.644 1.64976*
0.034533 $\bar{2}$.53823*	0.49118 $\bar{1}$.69124*	70.729 1.84960*	0.035365 $\bar{2}$.54857*	0.033421 $\bar{2}$.52402*	0.025400 $\bar{2}$.40484*	1	0.34563 $\bar{1}$.53861*	1.1340 0.05460*
0.099913 $\bar{2}$.99962*	1.4211 0.15262*	204.64 2.31099*	0.10232 $\bar{1}$.00996*	0.096697 $\bar{2}$.98541*	0.073489 $\bar{2}$.86622*	2.8933 0.46139*	1	3.2808 0.51598*
0.030453 $\bar{2}$.48364*	0.43315 $\bar{1}$.63664*	62.374 1.79500*	0.031187 $\bar{2}$.49397*	0.029473 $\bar{2}$.46942*	0.022399 $\bar{2}$.35024*	0.88187 $\bar{1}$.94540*	0.30480 $\bar{1}$.48402*	1

* Logarithm of the number immediately above. † At 15° C. and $g = g_0$. ‡ At 0° C.

Energy

Joules $= 10^7$ erg	Meter-kilograms	Foot-pounds	Kilowatt-hours	Cheval-vapeur-hours	Horse-power-hours	British thermal units
1	0.10197 $\bar{1}$.00848*	0.73756 $\bar{1}$.86780*	0.(6)27778 $\bar{7}$.44370*	0.(6)37767 $\bar{7}$.57711*	0.(6)37251 $\bar{7}$.57113*	0.(3)9475 $\bar{4}$.97660*
9.80665 0.9915207*	1	7.2330 0.85932*	0.(5)27241 $\bar{6}$.43522*	0.(5)37937 $\bar{6}$.56863*	0.(5)36530 $\bar{6}$.56265*	0.009292 $\bar{3}$.96812*
1.3558 0.13220*	0.13826 $\bar{1}$.14068*	1	0.(6)37662 $\bar{7}$.57590*	0.(6)51206 $\bar{7}$.70932*	0.(6)50505 $\bar{7}$.70333*	0.001285 $\bar{3}$.10880*
3.6×10^6 6.55630*	3.6710×10^5 5.56478*	2.6552×10^6 6.42410*	1	1.3596 0.13342*	1.3410 0.12743*	3411. 3.53290*
2.6478×10^6 6.42288*	270000 5.43136*	1.9529×10^6 6.29068*	0.73550 $\bar{1}$.86658*	1	0.98631 $\bar{1}$.99401*	2509. 3.39948*
2.6845×10^6 6.42887*	2.7375×10^5 5.43735*	1.98×10^6 6.29667*	0.74571 $\bar{1}$.87257*	1.0139 0.00598*	1	2544. 3.40547*
1055. 3.02340*	107.6 2.03188*	778.4 2.89120*	0.0(3)2932 $\bar{4}$.46710*	0.0(3)3986 $\bar{4}$.60051*	0.0(3)3931 $\bar{4}$.59453*	1

* Logarithm of the number immediately above.

30. Conversion Tables

The following tables give multiples for transferring English to metric measures or metric to English measures. For example, to reduce 39.37 inches to centimeters, take two numbers from the table at foot of page 1560 and add them together, thus 39 in = 99.06 cm, and 0.37 in = 0.94 cm; then 39.37 inches = 99.06 + 0.94 = 100.00 centimeters.

INDEX TO CONVERSION TABLES

LENGTH

AREA

VOLUME AND CAPACITY

WEIGHT AND MASS

PRESSURE

VELOCITY AND SPEED

ENERGY AND WORK

POWER

Cubic Feet to U. S. Liquid Gallons

Cu. Ft	0	1	2	3	4	5	6	7	8	9
		7.4805	14.961	22.442	29.922	37.403	44.883	52.364	59.844	67.325
10	74.805	82.286	89.766	97.247	104.73	112.21	119.69	127.17	134.65	142.13
20	149.61	157.09	164.57	172.05	179.53	187.01	194.49	201.97	209.45	216.94
30	224.42	231.90	239.38	246.86	254.34	261.82	269.30	276.78	284.26	291.74
40	299.22	306.70	314.18	321.66	329.14	336.62	344.10	351.58	359.06	366.55
50	374.03	381.51	388.99	396.47	403.95	411.43	418.91	426.39	433.87	441.35
60	448.83	456.31	463.79	471.27	478.75	486.23	493.71	501.19	508.68	516.16
70	523.64	531.12	538.60	546.08	553.56	561.04	568.52	576.00	583.48	590.96
80	598.44	605.92	613.40	620.88	628.36	635.84	643.32	650.81	658.29	665.77
90	673.25	680.73	688.21	695.69	703.17	710.65	718.13	725.61	733.09	740.57

U. S. Liquid Gallons to Cubic Feet

U.S. Gal.	0	1	2	3	4	5	6	7	8	9
		0.1337	0.2674	0.4010	0.5347	0.6684	0.8021	0.9358	1.0694	1.2031
10	1.3368	1.4705	1.6042	1.7378	1.8715	2.0052	2.1389	2.2726	2.4063	2.5399
20	2.6736	2.8073	2.9410	3.0747	3.2083	3.3420	3.4757	3.6094	3.7431	3.8767
30	4.0104	4.1441	4.2778	4.4115	4.5451	4.6788	4.8125	4.9462	5.0799	5.2135
40	5.3472	5.4809	5.6146	5.7483	5.8819	6.0156	6.1493	6.2830	6.4167	6.5503
50	6.6840	6.8177	6.9514	7.0851	7.2188	7.3524	7.4861	7.6198	7.7535	7.8872
60	8.0208	8.1545	8.2882	8.4219	8.5556	8.6892	8.8229	8.9566	9.0903	9.2240
70	9.3576	9.4913	9.6250	9.7587	9.8924	10.026	10.160	10.293	10.427	10.561
80	10.694	10.828	10.962	11.095	11.229	11.363	11.497	11.630	11.764	11.898
90	12.031	12.165	12.299	12.532	12.566	12.700	12.833	12.967	13.101	13.234

Imperial Gallons to U. S. Liquid Gallons

Imp. Gal.	0	1	2	3	4	5	6	7	8	9
		1.2010	2.4019	3.6029	4.8038	6.0048	7.2057	8.4067	9.6076	10.809
10	12.010	13.210	14.411	15.612	16.813	18.014	19.215	20.416	21.617	22.818
20	24.019	25.220	26.421	27.622	28.823	30.024	31.225	32.426	33.627	34.828
30	35.029	37.230	38.430	39.631	40.832	42.033	43.234	44.435	45.636	46.837
40	48.038	49.239	50.440	51.641	52.842	54.043	55.244	56.445	57.646	58.847
50	60.048	61.249	62.450	63.650	64.851	66.052	67.253	68.454	69.655	70.856
60	72.057	73.258	74.459	75.660	76.861	78.062	79.263	80.464	81.665	82.866
70	84.067	85.268	86.469	87.669	88.870	90.071	91.272	92.473	93.674	94.875
80	96.076	97.277	98.478	99.679	100.88	102.08	103.28	104.48	105.68	106.88
90	108.09	109.29	110.49	111.69	111.89	114.09	115.29	116.49	117.69	118.89

U. S. Liquid Gallons to Imperial Gallons

U.S. Gal.	0	1	2	3	4	5	6	7	8	9
		0.8327	1.6653	2.4980	3.3307	4.1634	4.9960	5.8287	6.6614	7.4941
10	8.3267	9.1594	9.9921	10.825	11.657	12.490	13.323	14.155	14.988	15.821
20	16.653	17.486	18.319	19.151	19.984	20.817	21.649	22.482	23.315	24.148
30	24.980	25.813	26.646	27.478	28.311	29.144	29.976	30.809	31.642	32.474
40	33.307	34.140	34.972	35.805	36.638	37.477	38.303	39.136	39.968	40.801
50	41.634	42.466	43.299	44.132	44.964	45.797	46.630	47.462	48.295	49.128
60	49.960	50.793	51.626	52.458	53.291	54.124	54.956	55.789	56.622	57.454
70	58.287	59.120	59.952	60.785	61.618	62.450	63.283	64.116	64.948	65.781
80	66.614	67.446	68.279	69.112	69.944	70.777	71.610	72.443	73.275	74.108
90	74.941	75.773	76.606	77.439	78.271	79.104	79.937	80.769	81.602	82.435

Kilograms to Pounds Avoirdupois

Kg	0	1	2	3	4	5	6	7	8	9
		2.20	4.41	6.61	8.82	11.02	13.23	15.43	17.64	19.
10	22.05	24.25	26.46	28.66	30.86	33.07	35.27	37.48	39.68	41.
20	44.09	46.30	48.50	50.71	52.91	55.12	57.32	59.52	61.73	63.
30	66.14	68.34	70.55	72.75	74.96	77.16	79.37	81.57	83.78	85
40	88.18	90.39	92.59	94.80	97.00	99.21	101.41	103.62	105.82	108
50	110.23	112.44	114.64	116.84	119.05	121.25	123.46	125.66	127.87	130
60	132.28	134.48	136.69	138.89	141.10	143.30	145.51	147.71	149.91	152
70	154.32	156.53	158.73	160.94	163.14	165.35	167.55	169.76	171.96	174
80	176.37	178.57	180.78	182.98	185.19	187.39	189.60	191.80	194.01	196
90	198.42	200.62	202.83	205.03	207.23	209.44	211.64	213.85	216.05	218

Pounds Avoirdupois to Kilograms

Lbs	0	1	2	3	4	5	6	7	8	9
		0.4536	0.9072	1.361	1.814	2.268	2.722	3.175	3.629	4.
10	4.536	4.990	5.443	5.897	6.350	6.804	7.257	7.711	8.165	8.
20	9.072	9.525	9.979	10.433	10.886	11.340	11.793	12.247	12.701	13.
30	13.608	14.061	14.515	14.969	15.422	15.876	16.329	16.783	17.237	17.
40	18.144	18.597	19.051	19.504	19.958	20.412	20.865	21.319	21.772	22.
50	22.680	23.133	23.587	24.040	24.494	24.948	25.401	25.855	26.308	26.
60	27.216	27.669	28.123	28.576	29.030	29.484	29.937	30.391	30.844	31
70	31.751	32.205	32.659	33.112	33.566	34.019	34.473	34.927	35.380	35
80	36.287	36.741	37.195	37.648	38.102	38.555	39.009	39.463	39.916	40
90	40.823	41.277	41.731	42.184	42.638	43.091	43.545	43.998	44.452	44

Metric Tons to Short Tons (2000 Pounds)

Metric tons	0	1	2	3	4	5	6	7	8	
		1.102	2.205	3.307	4.409	5.512	6.614	7.716	8.818	9
10	11.023	12.125	13.228	14.330	15.432	16.535	17.637	18.739	19.842	20
20	22.046	23.149	24.251	25.353	26.455	27.558	28.660	29.762	30.865	31
30	33.069	34.172	35.274	36.376	37.479	38.581	39.683	40.786	41.888	42
40	44.092	45.195	46.297	47.399	48.502	49.604	50.706	51.809	52.911	54
50	55.116	56.218	57.320	58.422	59.525	60.627	61.729	62.832	63.934	6
60	66.139	57.241	68.343	69.446	70.548	71.650	72.753	73.855	74.957	7
70	77.162	78.264	79.366	80.469	81.571	82.673	83.776	84.878	85.980	8
80	88.185	89.287	90.390	91.492	[illegible]2.594	93.696	94.799	95.901	97.003	9
90	99.208	100.31	101.41	102.51	103.62	104.72	105.82	106.92	108.03	1

Short Tons (2000 Pounds) to Metric Tons

Short tons	0	1	2	3	4	5	6	7	8	
		0.907	1.814	2.722	3.629	4.536	5.443	6.350	7.257	
10	9.072	9.979	10.886	11.793	12.701	13.608	14.515	15.422	16.329	1
20	18.144	19.051	19.958	20.865	21.772	22.680	23.587	24.494	25.401	2
30	27.216	28.123	29.030	29.937	30.844	31.751	32.659	33.566	34.473	3
40	36.287	37.195	38.102	39.009	39.916	40.823	41.731	42.638	43.545	4
50	45.359	46.266	47.174	48.081	48.988	49.895	50.802	51.710	52.617	5
60	54.431	55.338	56.245	57.153	58.060	58.967	59.874	60.781	61.689	6
70	63.503	64.410	65.317	66.224	67.132	68.039	68.946	69.853	70.760	7
80	72.575	73.482	74.389	75.296	76.204	77.111	78.018	78.925	79.832	8
90	81.647	82.554	83.461	84.368	85.275	86.183	87.090	87.997	88.904	8

Metric Tons to Long Tons (2240 Pounds)

Metric tons	0	1	2	3	4	5	6	7	8	9
		0.984	1.968	2.953	3.937	4.921	5.905	6.889	7.874	8.858
10	9.842	10.826	11.810	12.795	13.779	14.763	15.747	16.732	17.716	18.700
20	19.684	20.668	21.653	22.637	23.621	24.605	25.589	26.574	27.558	28.542
30	29.526	30.510	31.495	32.479	33.463	34.447	35.431	36.416	37.400	38.384
40	39.368	40.352	41.337	42.321	43.305	44.289	45.273	46.258	47.242	48.226
50	49.210	50.195	51.179	52.163	53.147	54.131	55.116	56.100	57.084	58.068
60	59.052	60.037	61.021	62.005	62.989	63.973	64.958	65.942	66.926	67.910
70	68.894	69.879	70.863	71.847	72.831	73.815	74.800	75.784	76.768	77.752
80	78.737	79.721	80.705	81.689	82.673	83.658	84.642	85.626	86.610	87.594
90	88.579	89.563	90.547	91.531	92.515	93.500	94.484	95.468	96.452	97.436

Long Tons (2240 Pounds) to Metric Tons

Long tons	0	1	2	3	4	5	6	7	8	9
		1.016	2.022	3.048	4.064	5.080	6.096	7.112	8.128	9.144
10	10.160	11.177	12.193	13.209	14.225	15.241	16.257	17.273	18.289	19.305
20	20.321	21.337	22.353	23.369	24.385	25.401	26.417	27.433	28.449	29.465
30	30.481	31.497	32.514	33.530	34.546	35.562	36.578	37.594	38.610	39.626
40	40.642	41.658	42.674	43.690	44.706	45.722	46.738	47.754	48.770	49.786
50	50.802	51.818	52.834	53.850	54.867	55.883	56.899	57.915	58.931	59.947
60	60.963	61.979	62.995	64.011	65.027	66.043	67.059	68.075	69.091	70.107
70	71.123	72.139	73.155	74.171	75.187	76.204	77.220	78.236	79.252	80.268
80	81.284	82.300	83.316	84.332	85.348	86.364	87.380	88.396	89.412	90.428
90	91.444	92.460	93.476	94.492	95.508	96.524	97.541	98.557	99.573	100.59

Fractions of an Inch to Millimeters

16ths	32nds	64ths	Millimeters	16ths	32nds	64ths	Millimeters	16ths	32nds	64ths	Millimeters
		1	0.397			23	9.128			45	17.859
	1	2	0.794	6	12	24	9.525		23	46	18.256
		3	1.191			25	9.922			47	18.653
1	2	4	1.588		13	26	10.319	12	24	48	19.050
		5	1.984			27	10.716			49	19.447
	3	6	2.381	7	14	28	11.113		25	50	19.844
		7	2.778			29	11.509			51	20.241
2	4	8	3.175		15	30	11.906	13	26	52	20.638
		9	3.572			31	12.303			53	21.034
	5	10	3.969	8	16	32	12.700		27	54	21.431
		11	4.366			33	13.097			55	21.828
3	6	12	4.763		17	34	13.494	14	28	56	22.225
		13	5.159			35	13.891			57	22.622
	7	14	5.556	9	18	36	14.288		29	58	23.019
		15	5.953			37	14.684			59	23.416
4	8	16	6.350		19	38	15.081	15	30	60	23.813
		17	6.747			39	15.478			61	24.209
	9	18	7.144	10	20	40	15.875		31	62	24.606
		19	7.541			41	16.272			63	25.003
5	10	20	7.938		21	42	16.669	16	32	64	25.400
		21	8.334			43	17.066				
	11	22	8.731	11	22	44	17.463				

Millimeters to Decimals of an Inch

Millimeters	0	1	2	3	4	5	6	7	8	9
		.0394	.0787	.1181	.1575	.1968	.2362	.2756	.3150	.3543
10	0.3937	.4331	.4724	.5118	.5512	.5905	.6299	.6693	.7087	.7480
20	0.7874	.8268	.8661	.9055	.9449	.9842	1.0236	1.0630	1.1024	1.1417
30	1.1811	1.2205	1.2598	1.2992	1.3386	1.3780	1.4173	1.4567	1.4961	1.5354
40	1.5748	1.6142	1.6534	1.6929	1.7323	1.7716	1.8110	1.8504	1.8898	1.9291
50	1.9685	2.0079	2.0472	2.0866	2.1260	2.1654	2.2047	2.2441	2.2835	2.3228
60	2.3622	2.4016	2.4409	2.4803	2.5197	2.5590	2.5984	2.6378	2.6772	2.7165
70	2.7559	2.7953	2.8346	2.8740	2.9134	2.9528	2.9921	3.0315	3.0709	3.1102
80	3.1496	3.1890	3.2283	3.2677	3.3071	3.3464	3.3858	3.4252	3.4646	3.5039
90	3.5433	3.5827	3.6220	3.6614	3.7008	3.7402	3.7795	3.8189	3.8583	3.8976

Hundredths of an Inch to Millimeters

100ths of an inch	0	1	2	3	4	5	6	7	8	9
		0.254	0.508	0.762	1.016	1.270	1.524	1.778	2.032	2.286
10	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
20	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
30	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
40	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
50	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
60	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
70	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
80	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
90	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Centimeters to Inches

Centimeters	0	1	2	3	4	5	6	7	8	9
		0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.54
10	3.937	4.331	4.724	5.118	5.512	5.906	6.299	6.693	7.087	7.48
20	7.874	8.268	8.661	9.055	9.449	9.843	10.236	10.630	11.024	11.41
30	11.811	12.205	12.598	12.992	13.386	13.780	14.173	14.567	14.961	15.35
40	15.748	16.142	16.535	16.929	17.323	17.717	18.110	18.504	18.898	19.29
50	19.685	20.079	20.472	20.866	21.260	21.654	22.047	22.441	22.835	23.22
60	23.622	24.016	24.409	24.803	25.197	25.591	25.984	26.378	26.772	27.16
70	27.559	27.953	28.346	28.740	29.134	29.528	29.921	30.315	30.709	31.10
80	31.496	31.890	32.283	32.677	33.071	33.465	33.858	34.252	34.646	35.03
90	35.433	35.827	36.220	36.614	37.008	37.402	37.795	38.189	38.583	38.97

Inches to Centimeters

Inches	0	1	2	3	4	5	6	7	8	9
		2.54	5.08	7.62	10.16	12.70	15.24	17.78	20.32	22.
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172.72	175.
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.

Meters to Feet

Meters	0	1	2	3	4	5	6	7	8	9
		3.28	6.56	9.84	13.12	16.40	19.68	22.97	26.25	29.53
10	32.81	36.09	39.37	42.65	45.93	49.21	52.49	55.77	59.06	62.34
20	65.62	68.90	72.18	75.46	78.74	82.02	85.30	88.58	91.86	95.14
30	98.42	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.95
40	131.23	134.51	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.32	170.60	173.88	177.16	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.41	206.69	209.97	213.25	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.50	242.78	246.06	249.34	252.62	255.90	259.19
80	262.47	265.75	269.03	272.31	275.59	278.87	282.15	285.43	288.71	291.99
90	295.28	298.56	301.84	305.12	308.40	311.68	314.96	318.24	321.52	324.80

Feet to Meters

Feet	0	1	2	3	4	5	6	7	8	9
		0.305	0.610	0.914	1.219	1.524	1.829	2.134	2.438	2.743
10	3.048	3.353	3.658	3.962	4.267	4.572	4.877	5.182	5.486	5.791
20	6.096	6.401	6.706	7.010	7.315	7.620	7.925	8.230	8.534	8.839
30	9.144	9.449	9.754	10.058	10.363	10.668	10.973	11.278	11.582	11.887
40	12.192	12.497	12.802	13.106	13.411	13.716	14.021	14.326	14.630	14.935
50	15.240	15.545	15.850	16.154	16.459	16.764	17.069	17.374	17.678	17.983
60	18.288	18.593	18.898	19.202	19.507	19.812	20.117	20.422	20.726	21.031
70	21.336	21.641	21.946	22.250	22.555	22.860	23.165	23.470	23.774	24.079
80	24.384	24.689	24.994	25.298	25.603	25.908	26.213	26.518	26.822	27.127
90	27.432	27.737	28.042	28.346	28.651	28.956	29.261	29.566	29.870	30.175

Kilometers to U. S. Statute Miles

Km	0	1	2	3	4	5	6	7	8	9
		0.621	1.243	1.864	2.485	3.107	3.728	4.350	4.971	5.592
10	6.214	6.835	7.456	8.078	8.699	9.321	9.942	10.563	11.185	11.806
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.777	17.398	18.020
30	18.641	19.262	19.884	20.505	21.127	21.748	22.369	22.991	23.612	24.233
40	24.855	25.476	26.098	26.719	27.340	27.962	28.583	29.204	29.826	30.447
50	31.068	31.690	32.311	32.933	33.554	34.175	34.797	35.418	36.039	36.661
60	37.282	37.904	38.525	39.146	39.768	40.389	41.010	41.632	42.253	42.875
70	43.496	44.117	44.739	45.360	45.981	46.603	47.224	47.845	48.467	49.088
80	49.710	50.331	50.952	51.574	52.195	52.816	53.438	54.059	54.681	55.302
90	55.923	56.545	57.166	57.787	58.409	59.030	59.652	60.273	60.894	61.516

U. S. Statute Miles to Kilometers

Miles	0	1	2	3	4	5	6	7	8	9
		1.61	3.22	4.83	6.44	8.05	9.66	11.27	12.87	14.48
10	16.09	17.70	19.31	20.92	22.53	24.14	25.75	27.36	28.97	30.58
20	32.19	33.80	35.41	37.01	38.62	40.23	41.84	43.45	45.06	46.67
30	48.28	49.89	51.50	53.11	54.72	56.33	57.94	59.55	61.16	62.76
40	64.37	65.98	67.59	69.20	70.81	72.42	74.03	75.64	77.25	78.86
50	80.47	82.08	83.69	85.30	86.90	88.51	90.12	91.73	93.34	94.95
60	96.56	98.17	99.78	101.39	103.00	104.61	106.22	107.83	109.44	111.04
70	112.65	114.26	115.87	117.48	119.09	120.70	122.31	123.92	125.53	127.14
80	128.75	130.36	131.97	133.58	135.19	136.79	138.40	140.01	141.62	143.23
90	144.84	146.45	148.06	149.67	151.28	152.89	154.50	156.11	157.72	159.33

Square Centimeters to Square Inches

Sq cm	0	1	2	3	4	5	6	7	8	9
		0.155	0.310	0.465	0.620	0.775	0.930	1.085	1.240	1.395
10	1.550	1.705	1.860	2.015	2.170	2.325	2.480	2.635	2.790	2.945
20	3.100	3.255	3.410	3.565	3.720	3.875	4.030	4.185	4.340	4.495
30	4.650	4.805	4.960	5.115	5.270	5.425	5.580	5.735	5.890	6.045
40	6.200	6.355	6.510	6.665	6.820	6.975	7.130	7.285	7.440	7.595
50	7.750	7.905	8.060	8.215	8.370	8.525	8.680	8.835	8.990	9.145
60	9.300	9.455	9.610	9.765	9.920	10.075	10.230	10.385	10.540	10.695
70	10.850	11.005	11.160	11.315	11.470	11.625	11.780	11.935	12.090	12.245
80	12.400	12.555	12.710	12.865	13.020	13.175	13.330	13.485	13.640	13.795
90	13.950	14.105	14.260	14.415	14.570	14.725	14.880	15.035	15.190	15.345

Square Inches to Square Centimeters

Sq in	0	1	2	3	4	5	6	7	8	9
		6.45	12.90	19.35	25.81	32.26	38.71	45.16	51.61	58.06
10	64.52	70.97	77.42	83.87	90.32	96.77	103.23	109.68	116.13	122.58
20	129.03	135.48	141.94	148.39	154.84	161.29	167.74	174.19	180.65	187.10
30	193.55	200.00	206.45	212.90	219.36	225.81	232.26	238.71	245.16	251.61
40	258.07	264.52	270.97	277.42	283.87	290.32	296.77	303.23	309.68	316.13
50	322.58	329.03	335.48	341.94	348.39	354.84	361.29	367.74	374.19	380.65
60	387.10	393.55	400.00	406.45	412.90	419.36	425.81	432.26	438.71	445.16
70	451.61	458.07	464.52	470.97	477.42	483.87	490.32	496.78	503.23	509.68
80	516.13	522.58	529.03	535.48	541.94	548.39	554.84	561.29	567.74	574.19
90	580.65	587.10	593.55	600.00	606.45	612.90	619.36	625.81	632.26	638.7

Square Meters to Square Feet

Sq m	0	1	2	3	4	5	6	7	8	9
		10.76	21.53	32.29	43.06	53.82	64.58	75.35	86.11	96.8
10	107.64	118.40	129.17	139.93	150.69	161.46	172.22	182.99	193.75	204.5
20	215.28	226.04	236.81	247.57	258.33	269.10	279.86	290.62	301.39	312.1
30	322.92	333.68	344.44	355.21	365.97	376.74	387.50	398.26	409.03	419.7
40	430.55	441.32	452.08	462.85	473.61	484.37	495.14	505.90	516.67	527.4
50	538.19	548.96	559.72	570.48	581.25	592.01	602.78	613.54	624.30	635.0
60	645.83	656.60	667.36	678.12	688.89	699.65	710.42	721.18	731.94	742.7
70	753.47	764.23	775.00	785.76	796.53	807.29	818.05	828.82	839.58	850.3
80	861.11	871.87	882.64	893.40	904.16	914.93	925.69	936.46	947.22	957.9
90	968.75	979.51	990.28	1001.0	1011.8	1022.6	1033.3	1044.1	1054.9	1065.

Square Feet to Square Meters

Sq ft	0	1	2	3	4	5	6	7	8	9
		0.0929	0.1858	0.2787	0.3716	0.4645	0.5574	0.6503	0.7432	0.836
10	0.9290	1.0219	1.1148	1.2077	1.3006	1.3936	1.4865	1.5794	1.6723	1.765
20	1.8581	1.9510	2.0439	2.1368	2.2297	2.3226	2.4155	2.5084	2.6013	2.694
30	2.7871	2.8800	2.9729	3.0658	3.1587	3.2516	3.3445	3.4374	3.5303	3.623
40	3.7161	3.8090	3.9019	3.9948	4.0878	4.1807	4.2736	4.3665	4.4594	4.552
50	4.6452	4.7381	4.8310	4.9239	5.0168	5.1097	5.2026	5.2955	5.3884	5.481
60	5.5742	5.6671	5.7600	5.8529	5.9458	6.0387	6.1316	6.2245	6.3174	6.410
70	6.5032	6.5961	6.6890	6.7819	6.8749	6.9678	7.0607	7.1536	7.2465	7.339
80	7.4323	7.5252	7.6181	7.7110	7.8039	7.8968	7.9897	8.0826	8.1755	8.268
90	8.3613	8.4542	8.5471	8.6400	8.7329	8.8258	8.9187	9.0116	9.1045	9.197

Square Meters to Square Yards

Sq m	0	1	2	3	4	5	6	7	8	9
		1.196	2.392	3.588	4.784	5.980	7.176	8.372	9.568	10.764
10	11.960	13.156	14.352	15.548	16.744	17.940	19.136	20.332	21.528	22.724
20	23.920	25.116	26.312	27.508	28.704	29.900	31.096	32.292	33.488	34.684
30	35.880	37.076	38.272	39.468	40.663	41.859	43.055	44.251	45.447	46.643
40	47.839	49.035	50.231	51.427	52.623	53.819	55.015	56.211	57.407	58.603
50	59.799	60.995	62.191	63.387	64.583	65.779	66.975	68.171	69.367	70.563
60	71.759	72.955	74.151	75.347	76.543	77.739	78.935	80.131	81.327	82.523
70	83.719	84.915	86.111	87.307	88.503	89.699	90.895	92.091	93.287	94.483
80	95.679	96.875	98.071	99.267	100.46	101.66	102.85	104.05	105.25	106.44
90	107.64	108.83	110.03	111.23	112.42	113.62	114.81	116.01	117.21	118.40

Square Yards to Square Meters

Sq yd	0	1	2	3	4	5	6	7	8	9
		0.836	1.672	2.508	3.345	4.181	5.017	5.853	6.689	7.525
10	8.361	9.197	10.034	10.870	11.706	12.542	13.378	14.214	15.050	15.886
20	16.723	17.559	18.395	19.231	20.067	20.903	21.739	22.576	23.412	24.248
30	25.084	25.920	26.756	27.592	28.428	29.265	30.101	30.937	31.773	32.609
40	33.445	34.281	35.117	35.954	36.790	37.626	38.462	39.298	40.134	40.970
50	41.807	42.643	43.479	44.315	45.151	45.987	46.823	47.659	48.496	49.332
60	50.168	51.004	51.840	52.676	53.512	54.348	55.185	56.021	56.857	57.693
70	58.529	59.365	60.201	61.038	61.874	62.710	63.546	64.382	65.218	66.054
80	66.890	67.727	68.563	69.399	70.235	71.071	71.907	72.743	73.580	74.416
90	75.252	76.088	76.924	77.760	78.596	79.432	80.269	81.105	81.941	82.777

Square Kilometers to Square Miles

Sq km	0	1	2	3	4	5	6	7	8	9
		0.386	0.772	1.158	1.544	1.931	2.317	2.703	3.089	3.475
10	3.861	4.247	4.633	5.019	5.405	5.792	6.178	6.564	6.950	7.336
20	7.722	8.108	8.494	8.880	9.266	9.653	10.039	10.425	10.811	11.197
30	11.583	11.969	12.355	12.741	13.127	13.514	13.900	14.286	14.672	15.058
40	15.444	15.830	16.216	16.602	16.988	17.375	17.761	18.147	18.533	18.919
50	19.305	19.691	20.077	20.463	20.849	21.236	21.622	22.008	22.394	22.780
60	23.166	23.552	23.938	24.324	24.710	25.097	25.483	25.869	26.255	26.641
70	27.027	27.413	27.799	28.185	28.571	28.958	29.344	29.730	30.116	30.502
80	30.888	31.274	31.660	32.046	32.432	32.819	33.205	33.591	33.977	34.363
90	34.749	35.135	35.521	35.907	36.293	36.680	37.066	37.452	37.838	38.224

Square Miles to Square Kilometers

Sq miles	0	1	2	3	4	5	6	7	8	9
		2.59	5.18	7.77	10.36	12.95	15.54	18.13	20.72	23.31
10	25.90	28.49	31.08	33.67	36.26	38.85	41.44	44.03	46.62	49.21
20	51.80	54.39	56.98	59.57	62.16	64.75	67.34	69.93	72.52	75.11
30	77.70	80.29	82.88	85.47	88.06	90.65	93.24	95.83	98.42	101.01
40	103.60	106.19	108.78	111.37	113.96	116.55	119.14	121.73	124.32	126.91
50	129.50	132.09	134.68	137.27	139.86	142.45	145.04	147.63	150.22	152.81
60	155.40	157.99	160.58	163.17	165.76	168.35	170.94	173.53	176.12	178.71
70	181.30	183.89	186.48	189.07	191.66	194.25	196.84	199.43	202.02	204.61
80	207.20	209.79	212.38	214.97	217.56	220.15	222.74	225.33	227.92	230.51
90	233.10	235.69	238.28	240.87	243.46	246.05	248.64	251.23	253.82	256.41

Hectares to Acres

Ha	0	1	2	3	4	5	6	7	8	9
		2.47	4.94	7.41	9.88	12.36	14.83	17.30	19.77	22.
10	24.71	27.18	29.65	32.12	34.59	37.07	39.54	42.01	44.48	46.
20	49.42	51.89	54.36	56.83	59.31	61.78	64.25	66.72	69.19	71.
30	74.13	76.60	79.07	81.54	84.02	86.49	88.96	91.43	93.90	96.
40	98.84	101.31	103.78	106.26	108.73	111.20	113.67	116.14	118.61	121.
50	123.55	126.02	128.49	130.97	133.44	135.91	138.38	140.85	143.32	145.
60	148.26	150.73	153.21	155.68	158.15	160.62	163.09	165.56	168.03	170.
70	172.97	175.44	177.92	180.39	182.86	185.33	187.80	190.27	192.74	195.
80	197.68	200.15	202.63	205.10	207.57	210.04	212.51	214.98	217.45	219.
90	222.39	224.86	227.34	229.81	232.28	234.75	237.22	239.69	242.16	244.

Acres to Hectares

Acres	0	1	2	3	4	5	6	7	8	9
		0.405	0.809	1.214	1.619	2.023	2.428	2.833	3.237	3.
10	4.047	4.452	4.856	5.261	5.666	6.070	6.474	6.880	7.284	7.
20	8.094	8.498	8.903	9.308	9.712	10.117	10.522	10.927	11.331	11.
30	12.141	12.545	12.950	13.355	13.759	14.164	14.569	14.973	15.378	15.
40	16.187	16.592	16.997	17.402	17.806	18.211	18.616	19.020	19.425	19.
50	20.234	20.639	21.044	21.448	21.853	22.258	22.662	23.067	23.472	23.
60	24.281	24.686	25.091	25.495	25.900	26.305	26.709	27.114	27.519	27.
70	28.328	28.733	29.137	29.542	29.947	30.352	30.756	31.161	31.566	31.
80	32.375	32.780	33.184	33.589	33.994	34.398	34.803	35.208	35.612	36.
90	36.422	36.827	37.231	37.636	38.041	38.445	38.850	39.255	39.659	40.

Cubic Meters to Cubic Feet

Cu m	0	1	2	3	4	5	6	7	8	9
		35.3	70.6	105.9	141.3	176.6	211.9	247.2	282.5	31
10	353.1	388.5	423.8	459.1	494.4	529.7	565.0	600.3	635.7	67
20	706.3	741.6	776.9	812.2	847.5	882.9	918.2	953.5	988.8	102
30	1059.4	1094.7	1130.1	1165.4	1200.7	1236.0	1271.3	1306.6	1341.9	137
40	1412.6	1447.9	1483.2	1518.5	1553.8	1589.2	1624.5	1659.8	1695.1	173
50	1765.7	1801.0	1836.4	1871.7	1907.0	1942.3	1977.6	2012.9	2048.2	208
60	2118.9	2154.2	2189.5	2224.8	2260.1	2295.4	2330.8	2366.1	2401.4	24
70	2472.0	2507.3	2542.6	2578.0	2613.3	2648.6	2683.9	2719.2	2754.5	27
80	2825.2	2860.5	2895.8	2931.1	2966.4	3001.7	3037.0	3072.4	3107.7	31
90	3178.3	3213.6	3248.9	3284.2	3319.6	3354.9	3390.2	3425.5	3460.8	34

Cubic Feet to Cubic Meters

Cu ft	0	1	2	3	4	5	6	7	8	9
		0.0283	0.0566	0.0850	0.1133	0.1416	0.1699	0.1982	0.2265	0.
10	0.2832	0.3115	0.3398	0.3681	0.3964	0.4248	0.4531	0.4814	0.5097	0.
20	0.5663	0.5947	0.6230	0.6513	0.6796	0.7079	0.7362	0.7646	0.7929	0.
30	0.8495	0.8778	0.9061	0.9345	0.9628	0.9911	1.0194	1.0477	1.0760	1.
40	1.1327	1.1610	1.1893	1.2176	1.2459	1.2743	1.3026	1.3309	1.3592	1.
50	1.4159	1.4442	1.4725	1.5008	1.6291	1.5574	1.5858	1.6141	1.6424	1.
60	1.6990	1.7273	1.7557	1.7840	1.8123	1.8406	1.8689	1.8972	1.9256	1.
70	1.9822	2.0105	2.0388	2.0671	2.0955	2.1238	2.1521	2.1804	2.2087	2.
80	2.2654	2.2937	2.3220	2.3503	2.3786	2.4069	2.4353	2.4636	2.4919	2.
90	2.5485	2.5768	2.6052	2.6335	2.6618	2.6901	2.7184	2.7468	2.7751	2.

Liters to U. S. Liquid Gallons

Liters	0	1	2	3	4	5	6	7	8	9
	·	0.2642	0.5283	0.7925	1.0567	1.3209	1.5850	1.8492	2.1134	2.3775
10	2.6417	2.9059	3.1700	3.4342	3.6984	3.9626	4.2267	4.4909	4.7551	5.0192
20	5.2834	5.5476	5.8118	6.0759	6.3401	6.6043	6.8684	7.1326	7.3968	7.6609
30	7.9251	8.1893	8.4535	8.7176	8.9818	9.2460	9.5101	9.7743	10.038	10.303
40	10.567	10.831	11.095	11.359	11.624	11.888	12.152	12.416	12.680	12.944
50	13.209	13.473	13.737	14.001	14.265	14.528	14.794	15.058	15.322	15.586
60	15.850	16.114	16.379	16.643	16.907	17.171	17.435	17.699	17.964	18.228
70	18.492	18.756	19.020	19.284	19.549	19.813	20.077	20.341	20.605	20.869
80	21.134	21.398	21.662	21.926	22.190	22.454	22.719	22.983	23.247	23.511
90	23.775	24.040	24.304	24.568	24.832	25.096	25.360	25.625	25.889	26.153

U. S. Liquid Gallons to Liters

U.S. Gal.	0	1	2	3	4	5	6	7	8	9
		3.7854	7.5709	11.356	15.142	18.927	22.713	26.498	30.283	34.769
10	37.854	41.640	45.425	49.211	52.996	56.781	60.567	64.352	68.138	71.923
20	75.709	79.494	83.279	87.065	90.850	94.636	98.421	102.21	105.99	109.78
30	113.56	117.35	121.13	124.92	128.70	132.49	136.28	140.06	143.85	147.63
40	151.42	155.20	158.99	162.77	166.56	170.34	174.13	177.92	181.70	185.49
50	189.27	193.06	196.84	200.63	104.41	208.20	211.98	215.77	219.55	223.34
60	227.13	230.91	234.70	238.48	242.27	246.05	249.84	253.62	257.41	261.19
70	264.98	268.77	272.55	276.34	280.12	283.91	287.69	291.48	295.26	299.05
80	302.83	306.62	310.41	314.19	317.98	321.76	325.55	329.33	333.12	336.90
90	340.69	344.47	348.26	352.04	355.83	359.62	363.40	367.19	370.97	374.76

Hectoliters to U. S. Bushels

Hl	0	1	2	3	4	5	6	7	8	9
		2.84	5.68	8.51	11.35	14.19	17.03	19.86	22.70	25.54
10	28.38	31.22	34.05	36.89	39.73	42.57	45.40	48.24	51.08	53.92
20	56.75	59.59	62.43	65.27	68.11	70.94	73.78	76.62	79.46	82.29
30	85.13	87.97	90.81	93.65	96.48	99.32	102.16	105.00	107.83	110.67
40	113.51	116.35	119.19	122.02	124.86	127.70	130.54	133.37	136.21	139.05
50	141.89	144.72	147.56	150.40	153.24	156.08	158.91	161.75	164.59	167.43
60	170.26	173.10	175.94	178.78	181.62	184.45	187.29	190.13	192.97	195.80
70	198.64	201.48	204.32	207.16	209.99	212.83	215.67	218.51	221.34	224.18
80	227.02	229.86	232.69	235.53	238.37	241.21	244.05	246.88	249.72	252.56
90	255.40	258.23	261.07	263.91	267.75	269.59	272.42	275.26	278.10	280.94

Bushels to Hectoliters

Bu	0	1	2	3	4	5	6	7	8	9
		0.352	0.705	1.057	1.410	1.762	2.114	2.467	2.819	3.172
10	3.524	3.876	4.229	4.581	4.933	5.286	5.638	5.991	6.343	6.695
20	7.048	7.400	7.753	8.105	8.457	8.810	9.162	9.515	9.867	10.219
30	10.572	10.924	11.277	11.629	11.981	12.334	12.686	13.039	13.391	13.743
40	14.096	14.448	14.800	15.153	15.505	15.858	16.210	16.562	16.915	17.267
50	17.620	17.972	18.324	18.677	19.029	19.382	19.734	20.086	20.439	20.791
60	21.144	21.496	21.848	22.201	22.553	22.906	23.258	23.610	23.963	24.315
70	24.667	25.020	25.372	25.725	26.077	26.429	26.782	27.134	27.487	27.839
80	28.191	28.544	28.896	29.249	29.601	29.953	30.306	30.658	31.011	31.363
90	31.715	32.068	32.420	32.773	33.125	33.477	33.830	34.182	34.534	34.887

Centimeters per Second to Feet per Minute

Cm per sec	0	1	2	3	4	5	6	7	8	9
		1.97	3.94	5.91	7.87	9.84	11.81	13.78	15.75	17.72
10	19.69	21.65	23.62	25.59	27.56	29.52	31.50	33.46	35.43	37.40
20	39.37	41.34	43.31	45.28	47.24	49.21	51.18	53.15	55.12	57.09
30	59.06	61.02	62.99	64.96	66.93	68.90	70.87	72.83	74.80	76.77
40	78.74	80.71	82.68	84.65	86.61	88.58	90.55	92.52	94.49	96.46
50	98.43	100.39	102.36	104.33	106.30	108.27	110.24	112.20	114.17	116.14
60	118.11	120.08	122.05	124.02	125.98	127.95	129.92	131.89	133.86	135.83
70	137.80	139.76	141.73	143.70	145.67	147.64	149.61	151.57	153.54	155.51
80	157.48	159.45	161.42	163.39	165.35	167.32	169.29	171.26	173.23	175.20
90	177.17	179.13	181.10	183.07	185.04	187.01	188.98	190.94	192.91	194.88

Feet per Minute to Centimeters per Second

Ft per min	0	1	2	3	4	5	6	7	8	9
		0.508	1.016	1.524	2.032	2.540	3.048	3.556	4.064	4.572
10	5.080	5.588	6.096	6.604	7.112	7.620	8.128	8.636	9.144	9.652
20	10.160	10.668	11.176	11.684	12.192	12.700	13.208	13.716	14.224	14.732
30	15.240	15.748	16.256	16.764	17.272	17.780	18.288	18.796	19.304	19.812
40	20.320	20.828	21.336	21.844	22.352	22.860	23.368	23.876	24.384	24.892
50	25.400	25.908	26.416	26.924	27.432	27.940	28.448	28.956	29.464	29.972
60	30.480	30.988	31.496	32.004	32.512	33.020	33.528	34.036	34.544	35.052
70	35.560	36.068	36.576	37.084	37.592	38.100	38.608	39.116	39.624	40.132
80	40.640	41.148	41.656	42.164	42.672	43.180	43.688	44.196	44.704	45.212
90	45.720	46.228	46.736	47.244	47.752	48.260	48.768	49.276	49.784	50.292

Feet per Second to Miles per Hour

Feet per sec	0	1	2	3	4	5	6	7	8	9
		0.682	1.364	2.045	2.727	3.409	4.091	4.773	5.455	6.136
10	6.818	7.500	8.182	8.864	9.545	10.227	10.909	11.591	12.273	12.946
20	13.636	14.318	15.000	15.682	16.364	17.045	17.727	18.409	19.091	19.773
30	20.455	21.136	21.818	32.500	23.182	24.864	24.545	25.227	25.909	26.591
40	27.273	27.955	26.636	29.318	30.000	30.682	31.364	32.045	32.727	33.409
50	34.091	34.773	35.455	36.136	36.818	37.500	38.182	38.864	39.545	40.227
60	40.909	41.591	42.273	42.955	43.636	44.318	45.000	45.682	46.364	47.045
70	47.727	48.409	49.091	49.773	50.455	51.136	51.818	52.500	53.182	53.864
80	54.545	55.227	55.909	56.591	57.273	57.955	58.636	59.318	60.000	60.682
90	61.364	62.045	62.727	63.409	64.091	64.773	65.455	66.136	66.818	67.500

Miles per Hour to Feet per Second

Miles per hr	0	1	2	3	4	5	6	7	8	9
		1.47	2.93	4.40	5.87	7.33	8.80	10.27	11.73	13.20
10	14.67	16.13	17.60	19.07	20.53	22.00	23.47	24.93	26.40	27.87
20	29.33	30.80	32.27	33.73	35.20	36.67	38.13	39.60	41.07	42.53
30	44.00	45.47	46.93	48.40	49.87	51.33	52.80	54.27	55.73	57.20
40	58.67	60.13	61.60	63.07	64.53	66.00	67.47	68.93	70.40	71.87
50	73.33	74.80	76.27	77.73	79.20	80.67	82.13	83.60	85.07	86.53
60	88.00	89.47	90.93	92.40	93.87	95.33	96.80	98.27	99.73	101.20
70	102.67	104.13	105.60	107.07	108.53	110.00	111.47	112.93	114.40	115.87
80	117.33	118.80	120.27	121.73	123.20	124.67	126.13	127.60	129.07	130.53
90	132.00	133.47	134.93	136.40	137.87	139.33	140.80	142.27	143.73	145.20

Radians per Second to Revolutions per Minute

Radians per sec	0	1	2	3	4	5	6	7	8	9
		9.55	19.10	28.65	38.20	47.75	57.30	66.85	76.39	85.94
10	95.49	105.04	114.59	124.14	133.69	143.24	152.79	162.34	171.89	181.44
20	190.99	200.54	210.08	219.63	229.18	238.73	248.28	257.83	267.38	276.93
30	286.48	296.03	305.58	315.13	324.68	334.23	343.77	353.32	362.87	372.42
40	381.97	391.52	401.07	410.62	420.17	429.72	439.27	448.82	458.37	467.92
50	477.46	487.01	496.56	506.11	515.66	525.21	534.76	544.31	553.86	563.41
60	572.96	582.51	592.06	601.61	611.15	620.70	630.25	639.80	649.35	658.90
70	668.45	678.00	687.55	697.10	706.65	716.20	725.75	735.30	744.85	754.39
80	763.94	773.49	783.04	792.59	802.14	811.69	821.24	830.79	840.34	849.89
90	859.44	868.99	878.54	888.08	897.63	907.18	916.73	926.28	935.83	945.38

Revolutions per Minute to Radians per Second

Rev per min	0	1	2	3	4	5	6	7	8	9
		0.1047	0.2094	0.3142	0.4189	0.5236	0.6283	0.7330	0.8378	0.9425
10	1.0472	1.1519	1.2566	1.3614	1.4661	1.5708	1.6755	1.7802	1.8850	1.9897
20	2.0944	2.1991	2.3038	2.4086	2.5133	2.6180	2.7227	2.8274	2.9322	3.0369
30	3.1416	3.2463	3.3510	3.4558	3.5605	3.6652	3.7699	3.8746	3.9794	4.0841
40	4.1888	4.2935	4.3982	4.5029	4.6077	4.7124	4.8171	4.9218	5.0265	5.1313
50	5.2360	5.3407	5.4454	5.5501	5.6549	5.7596	5.8643	5.9690	6.0737	6.1785
60	6.2832	6.3879	6.4926	6.5973	6.7021	6.8068	6.9115	7.0162	7.1209	7.2257
70	7.3304	7.4351	7.5398	7.6445	7.7493	7.8540	7.9587	8.0634	8.1681	8.2729
80	8.3776	8.4823	8.5870	8.6917	8.7965	8.9012	9.0059	9.1106	9.2153	9.3201
90	9.4248	9.5295	9.6342	9.7389	9.8437	9.9484	10.053	10.158	10.263	10.367

Kilograms per Square Centimeter to Pounds per Square Inch

Kg per sq cm	0	1	2	3	4	5	6	7	8	9
		14.2	28.4	42.7	56.9	71.1	85.3	99.6	113.8	128.0
10	142.2	156.5	170.7	184.9	199.1	213.4	227.6	241.8	256.0	270.2
20	284.5	298.7	312.9	327.1	341.4	355.6	369.8	384.0	398.3	412.5
30	426.7	440.9	455.1	469.4	483.6	497.8	512.0	526.3	540.5	554.7
40	568.9	583.2	597.4	611.6	625.8	640.1	654.3	668.5	682.7	696.9
50	711.2	725.4	739.6	753.8	768.1	782.3	796.5	810.7	825.0	839.2
60	853.4	867.6	881.9	896.1	910.3	924.5	938.7	953.0	967.2	981.4
70	995.6	1009.9	1024.1	1038.3	1052.5	1066.8	1081.0	1095.2	1109.4	1123.6
80	1137.9	1152.1	1166.3	1180.5	1194.8	1209.0	1223.2	1237.4	1251.7	1265.9
90	1280.1	1294.3	1308.6	1322.8	1337.0	1351.2	1365.4	1379.7	1393.9	1408.1

Pounds per Square Inch to Kilograms per Square Centimeter

Lb per sq in	0	1	2	3	4	5	6	7	8	9
		0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
10	0.7031	0.7734	0.8437	0.9140	0.9843	1.0546	1.1249	1.1952	1.2655	1.3358
20	1.4061	1.4764	1.5467	1.6171	1.6874	1.7577	1.8280	1.8983	1.9686	2.0389
30	2.1092	2.1795	2.2498	2.3201	2.3904	2.4607	2.5310	2.6014	2.6717	2.7420
40	2.8123	2.8826	2.9529	3.0232	3.0935	3.1638	3.2341	3.3044	3.3747	3.4450
50	3.5153	3.5856	3.6559	3.7263	3.7966	3.8669	3.9372	4.0075	4.0778	4.1481
60	4.2184	4.2887	4.3590	4.4293	4.4996	4.5699	4.6402	4.7105	4.7809	4.8512
70	4.9215	4.9918	5.0621	5.1324	5.2027	5.2730	5.3433	5.4136	5.4839	5.5542
80	5.6245	5.6948	5.7651	5.8355	5.9058	5.9761	6.0464	6.1167	6.1870	6.2573
90	6.3276	6.3979	6.4682	6.5385	6.6088	6.6791	6.7494	6.8197	6.8901	6.9604

Kilograms per Square Meter to Pounds per Square Foot

Kg per sq m	0	1	2	3	4	5	6	7	8	9
		0.205	0.410	0.614	0.819	1.024	1.229	1.434	1.639	1.843
10	2.048	2.253	2.458	2.663	2.867	3.072	3.277	3.482	3.687	3.892
20	4.096	4.301	4.506	4.711	4.916	5.120	5.325	5.530	5.735	5.940
30	6.145	6.349	6.554	6.759	6.964	7.169	7.373	7.578	7.783	7.988
40	8.193	8.397	8.602	8.807	9.012	9.217	9.422	9.626	9.831	10.036
50	10.241	10.446	10.650	10.855	11.060	11.265	11.470	11.675	11.879	12.084
60	12.289	12.494	12.699	12.903	13.108	13.313	13.518	13.723	13.928	14.132
70	14.337	14.542	14.747	14.952	15.156	15.361	15.566	15.771	15.976	16.181
80	16.385	16.590	16.795	17.000	17.205	17.409	17.614	17.819	18.024	18.229
90	18.434	18.638	18.843	19.048	19.253	19.458	19.662	19.867	20.072	20.277

Pounds per Square Foot to Kilograms per Square Meter

Lb per sq ft	0	1	2	3	4	5	6	7	8	9
		4.88	9.76	14.65	19.53	24.41	29.29	34.18	39.06	43.94
10	48.82	53.71	58.59	63.47	68.35	73.24	78.12	83.00	87.88	92.77
20	97.65	102.53	107.41	112.30	117.18	122.06	126.94	131.83	136.71	141.59
30	146.47	151.35	156.24	161.12	166.00	170.88	175.77	180.65	185.53	190.41
40	195.30	200.18	205.06	209.94	214.83	219.71	224.59	229.47	234.36	239.24
50	244.12	249.00	253.89	258.77	263.65	268.53	273.41	278.30	283.18	288.06
60	292.94	297.83	302.71	307.59	312.47	317.36	322.24	327.12	332.00	336.89
70	341.77	346.65	351.53	356.42	361.30	366.18	371.06	375.95	380.83	385.71
80	390.59	395.48	400.36	405.24	410.12	415.00	419.89	424.77	429.65	434.53
90	439.42	444.30	449.18	454.06	458.95	463.83	468.71	473.59	478.48	483.36

Kilograms per Square Centimeter to Short Tons per Square Inch

Kg per sq cm	0	1	2	3	4	5	6	7	8	9
		.00711	.01422	.02134	.02845	.03556	.04267	.04978	.05689	.06401
10	.07112	.07823	.08534	.09245	.09956	.10668	.11379	.12090	.12801	.13512
20	.14223	.14935	.15646	.16357	.17068	.17779	.18490	.19202	.19913	.20624
30	.21335	.22046	.22757	.23469	.24180	.24891	.25602	.26313	.27024	.27736
40	.28447	.29158	.29869	.30580	.31291	.32003	.32714	.33425	.34136	.34847
50	.35558	.36270	.36981	.37692	.38403	.39114	.39826	.40537	.41248	.41959
60	.42670	.43381	.44093	.44804	.45515	.46226	.46937	.47648	.48360	.49071
70	.49782	.50493	.51204	.51915	.52627	.53338	.54049	.54760	.55471	.56182
80	.56894	.57605	.58316	.59027	.59738	.60449	.61161	.61872	.62583	.63294
90	.64005	.64716	.65428	.66139	.66850	.67561	.68272	.68983	.69695	.70406

Short Tons per Square Inch to Metric Tons per Square Centimeter

Short tons per sq in	0	1	2	3	4	5	6	7	8	9
		0.1406	0.2812	0.4218	0.5625	0.7031	0.8437	0.9843	1.1249	1.2655
10	1.4061	1.5467	1.6874	1.8280	1.9686	2.1092	2.2498	2.3904	2.5310	2.6717
20	2.8123	2.9529	3.0935	3.2341	3.3747	3.5153	3.6559	3.7966	3.9372	4.0778
30	4.2184	4.3590	4.4996	4.6402	4.7809	4.9215	5.0621	5.2027	5.3433	5.4839
40	5.6245	5.7651	5.9058	6.0464	6.1870	6.3276	6.4682	6.6088	6.7494	6.8901
50	7.0307	7.1713	7.3119	7.4525	7.5931	7.7337	7.8743	8.0150	8.1556	8.2962
60	8.4368	8.5774	8.7180	8.8586	8.9993	9.1399	9.2805	9.4211	9.5617	9.7023
70	9.8429	9.9835	10.124	10.265	10.405	10.546	10.687	10.827	10.968	11.108
80	11.249	11.390	11.530	11.671	11.812	11.952	12.093	12.233	12.374	12.515
90	21.655	12.796	12.936	13.077	13.218	13.358	13.499	13.639	13.780	13.921

Meter-kilograms to Foot-pounds

M-kg	0	1	2	3	4	5	6	7	8	9
		7.23	14.47	21.70	28.93	36.16	43.40	50.63	57.86	65.10
10	72.33	79.56	86.80	94.03	101.26	108.49	115.73	122.96	130.19	137.43
20	144.66	151.89	159.13	166.36	173.59	180.82	188.06	195.29	202.52	209.76
30	216.99	224.22	231.46	238.69	245.92	253.15	260.39	267.62	274.85	282.09
40	289.32	296.55	303.79	311.02	318.25	325.48	332.72	339.95	347.18	354.42
50	361.65	368.88	376.12	383.35	390.58	397.81	405.05	412.28	419.51	426.75
60	433.98	441.21	448.45	455.68	462.91	470.14	477.38	484.61	491.84	499.08
70	506.31	513.54	520.78	528.01	535.24	542.47	549.71	556.94	564.17	571.41
80	578.64	585.87	593.11	600.34	607.57	614.80	622.04	629.27	636.50	643.74
90	650.97	658.20	665.44	672.67	679.90	687.13	694.37	701.60	708.83	716.07

Foot-pounds to Meter-kilograms

Ft-lb	0	1	2	3	4	5	6	7	8	9
		0.138	0.277	0.415	0.553	0.691	0.830	0.968	1.106	1.244
10	1.383	1.521	1.659	1.797	1.936	2.074	2.212	2.350	2.489	2.627
20	2.765	2.903	3.042	3.180	3.318	3.456	3.595	3.733	3.871	4.009
30	4.148	4.286	4.424	4.562	4.701	4.839	4.977	5.115	5.254	5.392
40	5.530	5.668	5.807	5.945	6.083	6.221	6.360	6.498	6.636	6.775
50	6.913	7.051	7.189	7.328	7.466	7.604	7.742	7.881	8.019	8.157
60	8.295	8.434	8.572	8.710	8.848	8.987	9.125	9.263	9.401	9.540
70	9.678	9.816	9.954	10.093	10.231	10.369	10.507	10.646	10.784	10.922
80	11.060	11.199	11.337	11.475	11.613	11.752	11.890	12.028	12.166	12.305
90	12.443	12.581	12.719	12.858	12.996	13.134	13.273	13.411	13.549	13.687

Gram-calories to Joules

Gm-cal	0	1	2	3	4	5	6	7	8	9
		4.19	8.38	12.56	16.75	20.94	25.13	29.32	33.50	37.69
10	41.88	46.07	50.26	54.44	58.63	62.82	67.01	71.20	75.38	79.57
20	83.76	87.95	92.14	96.32	100.51	104.70	108.89	113.08	117.26	121.45
30	125.64	129.83	134.02	138.20	142.39	146.58	150.77	154.96	159.14	163.33
40	167.52	171.71	175.90	180.08	184.27	188.46	192.65	196.84	201.02	205.21
50	209.4	213.6	217.8	222.0	226.2	230.3	234.5	238.7	242.9	247.1
60	251.3	255.5	259.7	263.8	268.0	272.2	276.4	280.6	284.8	289.0
70	293.2	297.3	301.5	305.7	309.9	314.1	318.3	322.5	326.7	330.9
80	335.0	339.2	343.4	347.6	351.8	356.0	360.2	364.4	368.5	372.7
90	376.9	381.1	385.3	389.5	393.7	397.9	402.0	406.2	410.4	414.6

Joules to Gram-calories

Joules	0	1	2	3	4	5	6	7	8	9
		0.239	0.478	0.716	0.955	1.194	1.433	1.671	1.910	2.149
10	2.388	2.627	2.865	3.104	3.343	3.582	3.820	4.059	4.298	4.537
20	4.776	5.014	5.253	5.492	5.731	5.969	6.208	6.447	6.686	6.925
30	7.163	7.402	7.641	7.880	8.118	8.357	8.596	8.835	9.074	9.312
40	9.551	9.790	10.029	10.267	10.506	10.745	10.984	11.223	11.461	11.700
50	11.939	12.178	12.416	12.655	12.894	13.133	13.372	13.610	13.849	14.088
60	14.327	14.565	14.804	15.043	15.282	15.521	15.759	15.998	16.237	16.476
70	16.714	16.953	17.192	17.431	17.670	17.908	18.147	18.386	18.625	18.863
80	19.102	19.341	19.580	19.819	20.057	20.296	20.535	20.774	21.012	21.251
90	21.490	21.729	21.968	22.206	22.445	22.684	22.923	23.161	23.400	23.639

Kilogram-calories to Meter-kilograms

Kg-cal	0	1	2	3	4	5	6	7	8	9
		427	854	1 281	1 708	2 135	2 562	2 989	3 416	3 844
10	4 271	4 698	5 125	5 552	5 979	6 406	6 833	7 260	7 687	8 114
20	8 541	8 968	9 395	9 822	10 249	10 676	11 103	11 531	11 958	12 385
30	12 812	13 239	13 666	14 093	14 520	14 947	15 374	15 801	16 228	16 655
40	17 082	17 509	17 936	18 363	18 791	19 218	19 645	20 072	20 499	20 926
50	21 353	21 780	22 207	22 634	23 061	23 488	23 915	24 342	24 769	25 196
60	25 623	26 050	26 478	26 905	27 332	27 759	28 186	28 613	29 040	29 467
70	29 894	30 321	30 748	31 175	31 602	32 029	32 456	32 883	33 310	33 738
80	34 165	34 592	35 019	35 446	35 873	36 300	36 727	37 154	37 581	38 008
90	38 435	38 862	39 289	39 716	40 143	40 570	40 997	41 425	41 852	42 279

Meter-kilograms to Kilogram-calories

M-kg	0	1	2	3	4	5	6	7	8	9
		.00234	.00468	.00702	.00937	.01171	.01405	.01639	.01873	.02107
10	0.02342	.02576	.02810	.03044	.03278	.03512	.03747	.03981	.04215	.04449
20	0.04683	.04917	.05152	.05386	.05620	.05854	.06080	.06322	.06556	.06791
30	0.07025	.07259	.07493	.07727	.07961	.08196	.08430	.08664	.08898	.09132
40	0.09366	.09601	.09835	.10069	.10303	.10537	.10771	.11006	.11240	.11474
50	0.11708	.11942	.12176	.12411	.12645	.12879	.13113	.13347	.13581	.13815
60	0.14050	.14284	.14518	.14752	.14986	.15220	.15455	.15689	.15923	.16157
70	0.16391	.16625	.16860	.17094	.17328	.17562	.17796	.18030	.18265	.18499
80	0.18733	.18967	.19201	.19435	.19669	.19904	.20138	.20372	.20606	.20840
90	0.21074	.21309	.21543	.21777	.22011	.22245	.22479	.22714	.22948	.23182

British Thermal Units to Foot-pounds

Btu	0	1	2	3	4	5	6	7	8	9
		778	1 557	2 335	3 114	3 892	4 670	5 449	6 227	7 006
10	7 784	8 562	9 341	10 119	10 897	11 676	12 454	13 233	14 011	14 789
20	15 568	16 346	17 125	17 903	18 681	19 460	20 238	21 017	21 795	22 573
30	23 352	24 130	24 909	25 687	26 465	27 244	28 022	28 800	29 579	30 357
40	31 136	31 914	32 692	33 471	34 249	35 028	35 806	36 584	37 363	38 141
50	38 920	39 698	40 476	41 255	42 033	42 811	43 590	44 368	45 147	45 925
60	46 703	47 482	48 260	49 039	49 817	50 595	51 374	52 152	52 931	53 709
70	54 487	55 266	56 044	56 823	57 601	58 379	59 158	59 936	60 714	61 493
80	62 271	63 050	63 828	64 606	65 385	66 163	66 942	67 720	68 498	69 277
90	70 055	70 834	71 612	72 390	73 169	73 947	74 726	75 504	76 282	77 061

Foot-pounds to British Thermal Units

Ft-lb	0	1	2	3	4	5	6	7	8	9
		.00128	.00257	.00385	.00514	.00642	.00771	.00899	.01028	.01156
10	0.01285	.01413	.01542	.01670	.01799	.01927	.02056	.02184	.02312	.02441
20	0.02569	.02698	.02826	.02955	.03083	.03212	.03340	.03469	.03597	.03726
30	0.03854	.03983	.04111	.04240	.04368	.04496	.04625	.04753	.04882	.05010
40	0.05139	.05267	.05396	.05524	.05653	.05781	.05910	.06038	.06167	.06295
50	0.06424	.06552	.06680	.06809	.06937	.07066	.07194	.07323	.07451	.07580
60	0.07708	.07837	.07965	.08094	.08222	.08351	.08479	.08608	.08736	.0886[illegible]
70	0.08993	.09121	.09250	.09378	.09507	.09635	.09764	.09892	.10021	.10149
80	0.10278	.10406	.10535	.10663	.10791	.10920	.11048	.11177	.11305	.11434
90	0.11562	.11691	.11819	.11948	.12076	.12205	.12333	.12462	.12590	.12719

Kilowatts to Horse-power

Kw.	0	1	2	3	4	5	6	7	8	9
		1.341	2.682	4.023	5.364	6.705	8.046	9.387	10.728	12.069
10	13.410	14.751	16.092	17.433	18.774	20.115	21.456	22.797	24.138	25.479
20	26.820	28.161	29.502	30.843	32.184	33.525	34.866	36.208	37.549	38.890
30	40.231	41.572	42.913	44.254	45.595	46.936	48.277	49.618	50.959	52.300
40	53.641	54.982	56.323	57.664	59.005	60.346	61.687	63.028	64.369	65.710
50	67.051	68.392	69.733	71.074	72.415	73.756	75.097	76.438	77.779	79.120
60	80.461	81.802	83.143	84.484	85.825	87.166	88.507	89.848	91.189	92.530
70	93.871	95.212	96.553	97.894	99.235	100.58	101.92	103.26	104.60	105.94
80	107.28	108.62	109.96	111.30	112.65	113.99	115.33	116.67	118.01	119.35
90	120.69	122.03	123.37	124.71	126.06	127.40	128.74	130.08	131.42	132.76

Horse-power to Kilowatts

Hp.	0	1	2	3	4	5	6	7	8	9
		0.746	1.491	2.237	2.983	3.729	4.474	5.220	5.966	6.711
10	7.457	8.203	8.948	9.694	10.440	11.186	11.931	12.677	13.423	14.168
20	14.914	15.660	16.405	17.151	17.897	18.643	19.388	20.134	20.880	21.625
30	22.371	23.117	23.862	24.608	25.354	26.100	26.845	27.591	28.337	29.082
40	29.828	30.574	31.319	32.065	32.811	33.557	34.302	35.048	35.794	36.539
50	37.285	38.031	38.776	39.522	40.268	41.014	41.759	42.505	43.251	43.996
60	44.742	45.488	46.233	46.979	47.725	48.471	49.216	49.962	50.708	51.453
70	52.199	52.945	53.691	54.436	55.182	55.928	56.673	57.419	58.165	58.910
80	59.656	60.402	61.148	61.893	62.639	63.385	64.130	64.876	65.622	66.367
90	67.113	67.859	68.605	69.350	70.096	70.842	71.587	72.333	73.079	73.824

Cheval-vapeur to Horse-power

Cheval-vapeur	0	1	2	3	4	5	6	7	8	9
		0.986	1.973	2.959	3.945	4.932	5.918	6.904	7.891	8.877
10	9.863	10.849	11.836	12.822	13.808	14.795	15.781	16.767	17.754	18.740
20	19.726	20.713	21.699	22.685	23.672	24.658	25.644	26.631	27.617	28.603
30	29.590	30.576	31.562	32.548	33.535	34.521	35.507	36.494	37.480	38.466
40	39.453	40.439	41.425	42.412	43.398	44.384	45.371	46.357	47.343	48.330
50	49.316	50.302	51.289	52.275	53.261	54.247	55.234	56.220	57.206	58.193
60	59.179	60.165	61.152	62.138	63.124	64.111	65.097	66.083	67.070	68.056
70	69.042	70.029	71.015	72.001	72.988	73.974	74.960	75.946	76.933	77.919
80	78.905	79.892	80.878	81.864	82.851	83.837	84.823	85.810	86.796	87.782
90	88.769	89.755	90.741	91.728	92.714	93.700	94.687	95.673	96.659	97.645

Horse-power to Cheval-vapeur

Horse-power	0	1	2	3	4	5	6	7	8	9
		1.014	2.028	3.042	4.055	5.069	6.083	7.097	8.111	9.125
10	10.139	11.153	12.166	13.180	14.194	15.208	16.222	17.236	18.250	19.264
20	20.277	21.291	22.305	23.319	24.333	25.347	26.361	27.375	28.388	29.402
30	30.416	31.430	32.444	33.458	34.472	35.486	36.499	37.513	38.527	39.541
40	40.555	41.569	42.583	43.596	44.610	45.624	46.638	47.652	48.666	49.680
50	50.694	51.707	52.721	53.735	54.749	55.763	56.777	57.791	58.805	59.818
60	60.832	61.846	62.860	63.874	64.888	65.902	66.916	67.929	68.943	69.957
70	70.971	71.985	72.999	74.013	75.027	76.040	77.054	78.068	79.082	80.096
80	81.110	82.124	83.137	84.151	85.165	86.179	87.193	88.207	89.221	90.235
90	91.248	92.262	93.276	94.290	95.304	96.318	97.332	98.346	99.359	100.373

31. Values of Foreign Coins

The following equivalents in terms of the U. S. gold dollar were established by the Secretary of the Treasury on April 1, 1919, for use in estimating the value of all foreign merchandise exported to the United States and expressed in such metallic currencies.

Country	Monetary unit	Value in terms of U. S. money
Argentine Republic	Peso	$0.9648
Austria-Hungary	Krone	.2026
Belgium	Franc	.1930
Bolivia	Boliviano	.3893
Brazil	Milreis	.5462
British Colonies in Australasia and Africa	Pound sterling	4.8665
Canada	Dollar	1.0000
Central Amer. States		
Costa Rica	Colon	.4653
British Honduras	Dollar	1.0000
Nicaragua	Cordoba	1.0000
Guatemala, Honduras, Salvador	Peso*	.7234
Chile	Peso	.3650
China	Tael* — Amoy	1.1859
	Tael* — Canton	1.1823
	Tael* — Cheefoo	1.1342
	Tael* — Chin Kiang	1.1585
	Tael* — Fuchau	1.0970
	Tael* — Haikwan (customs)	1.2066
	Tael* — Hankow	1.1096
	Tael* — Kiaochow	1.1492
	Tael* — Nankin	1.1735
	Tael* — Niuchwang	1.1121
	Tael* — Ningpo	1.1402
	Tael* — Peking	1.1561
	Tael* — Shanghai	1.0832
	Tael* — Swatow	1.0955
	Tael* — Takau	1.1934
	Tael* — Tientsin	1.1492
	Yuan	.7771
	Dollar* — Hongkong	.7800
	Dollar* — British	.7800
	Dollar* — Mexican	.7857
Colombia	Dollar	.9733
Cuba	Peso	1.0000
Denmark	Krone	.2680
Ecuador	Sucre	$.486
Egypt	Pound (100 piasters)	4 943
Finland	Markka	.193
France	Franc	.193
German Empire	Mark	.238
Great Britain	Pound sterling	4 866
Greece	Drachma	.193
Haiti	Gourde	.250
India (British)	Rupee	.324
Indo-China	Piaster*	.781
Italy	Lira	.193
Japan	Yen	.498
Liberia	Dollar	1 000
Mexico*	Peso	.498
Netherlands	Guilder (Florin)	.40
Newfoundland	Dollar	1.00
Norway	Krone	.268
Panama	Balboa	1.00
Paraguay	Peso (Argentine)	.96
Persia	Achrefi	.09
	Kran*	.13
Peru	Libra	4 86
Philippine Isl'ds.	Peso	.50
Portugal	Escudo	1 08
Roumania	Leu	.19
Russia	Ruble	.51
Santo Domingo	Dollar	1.00
Servia	Dinar	.19
Siam	Tical	.37
Spain	Peseta	.19
Straits Settlem't	Dollar	.56
Sweden	Krona	.26
Switzerland	Franc	.19
Turkey	Piaster	.04
Uruguay	Peso	1.03
Venezuela	Bolivar	1.93

* Silver standard; other countries have the gold standard.

SECTION 14

STEAM AND ELECTRIC ENGINEERING

The following outlines of fundamental principles and facts have been prepared, not for mechanical and electrical engineers, but for civil engineers whose knowledge of the subjects is limited.—EDITOR-IN-CHIEF.

THERMODYNAMICS

BY

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THERMODYNAMICS

ELECTRICITY

THERMODYNAMICS

1. Preliminary Statements and Definitions

Heat Energy. The mechanical theory of heat asserts that heat is a form of energy due to the motion or configuration of the molecules of a body. Like mechanical energy, heat energy may be of the kinetic or of the potential form. The summation $\Sigma \frac{1}{2} mv^2$ extended to the moving molecules of a system gives the thermal kinetic energy of the system. Considerations derived from the kinetic theory of gases show that the temperature of a body is a measure of its thermal kinetic energy. When the temperature rises, it is inferred that the thermal kinetic energy is increased, and vice versa. Thermal potential energy is due to the position or configuration of the molecules of a body. Thus when the volume of the body is increased, work is required to separate the molecules against their mutual attractions, and this work is stored as potential energy. Again, when the state of aggregation is changed, as in fusion or vaporization, work is required to break down the molecular structure, and this work is stored in the system as potential energy. In the case of gases, like air and nitrogen, the attractive forces between the molecules are so small that the thermal potential energy is practically zero. The internal energy of a gas is therefore assumed to be wholly of the kinetic form.

Units of Energy. The conventional units used in this chapter are (1) for mechanical energy, the foot-pound and the horsepower-hour (h p-hr), which is equal to 1 980 000 ft-lb; (2) for heat energy, the British thermal unit (B t u), which is defined as the heat required to raise the temperature of one pound of water from 63° to 64° F.

Mechanical Equivalent of Heat. The numerical relation between the unit of heat and the unit of work has been determined very accurately from experiments. The accepted relations are

$$1 \text{ B t u} = 777.64 \text{ ft-lb} \qquad 1 \text{ h p-hr} = 2546.2 \text{ B t u}$$

In ordinary calculations the integral values 778 and 2546 are sufficiently accurate. The mechanical equivalent is denoted by the symbol J, and the reciprocal of it by A; thus, $J = 778$, and $A = 1/J = 1/778$.

State of a System. A thermodynamic system is defined as a body or group of bodies capable of receiving and giving out heat or other forms of energy. Examples of such systems are the media used in heat engines, as water vapor, air, ammonia, etc. In order that a system may receive or give up energy, its state must change; hence the magnitudes that determine the state must change. It is assumed ordinarily that the system is homogeneous and of uniform density and temperature thruout; also that it is subjected to uniform pressure. Then the magnitudes that describe the state of a unit mass are: the pressure p, the temperature t, and the volume v. In the case of a homogeneous mixture of vapor and liquid, as wet steam, a fourth variable is required; this is the ratio x of the weight of vapor to the weight of the mixture. These magnitudes, p, v, t, and x, are called the coordinates of the system.

Absolute Temperature. Many of the equations of thermodynamics are simplified by taking the temperatures from absolute zero instead of from the zero of the F. or C. scale. The position of the absolute zero, as determined by experiments on actual gases, is about 273.1° below 0 C., or 459.6° below 0 F. Hence, denoting ordinary temperatures by t and absolute temperatures by T,

$$T = t + 273.1 \text{ for the C. scale,}$$
$$T = t + 459.6 \text{ for the F. scale.}$$

Characteristic Equations. Of the three coordinates, p, v, T, any two may be, in general, taken as independent and the third is then a function of these two.

Thus in the case of a confined gas, the pressure p may be kept at any desired value and by the addition of heat the temperature T may be raised to any predetermined point. The volume v must, however, depend upon the values given to p and T; that is, v is a function of p and T, as $v = f(p, T)$. Likewise, taking v and T as independent, a functional relation $p = \phi(v, T)$ must exist. For any substance there is such a functional relation between the coordinates, and this relation is called the characteristic equation of the substance. The simplest characteristic equation is that which applies to an ideal perfect gas, namely

$$pv = BT.$$

For certain imperfect gases van der Waal's equation

$$p = \frac{BT}{v-b} - \frac{a}{v^2}$$

represents the relation between p, v, and T. The behavior of superheated steam is, within certain limits, represented accurately by the empirical equation

$$p(v-c) = BT - p(1 + ap^{\frac{1}{2}})\frac{m}{T^n}.$$

The characteristic equation $\phi(p, v, T) = 0$ having three variables may be represented geometrically by a surface. Any possible state of the substance is represented by a point on the surface and a change of state is represented by a path lying in the surface. In the case of a mixture of liquid and vapor (as wet steam), the pressure is a function of the temperature only, and the volume depends upon the temperature and the ratio x. The characteristic surface of such a mixture is a cylindrical surface cutting the pt-plane in the curve $p = f(t)$.

Changes of State. Certain changes of state of a thermodynamic system are of special importance.

1. Constant volume. The volume remaining constant, the pressure varies with the temperature and, in general, heat is absorbed or rejected by the system.
2. Constant pressure. A change of volume involves a change of temperature. Heat is absorbed or rejected.
3. Isothermal. If the temperature remains constant during a change of state, the change is said to be isothermal. In general, heat is absorbed or rejected.
4. Adiabatic. An adiabatic change is one in which the system neither receives nor gives out heat.
5. Isodynamic. In an isodynamic change of state the internal energy of the system remains constant.

Specific Heat. The heat required to raise the temperature of a unit weight of a body one degree under given external conditions is called the thermal capacity of the body. The ratio of the thermal capacity of a substance at the temperature t to the thermal capacity of water at a chosen standard temperature (17.5° C. or 63.5° F.) is the specific heat of the substance. Since at 63.5° F. the thermal capacity of water is 1 B t u, it follows that the specific heat of a substance at temperature t is numerically equal to the thermal capacity at the same temperature. In the case of gaseous substances two specific heats are of special importance: the specific heat at constant volume denoted by c_v, and that at constant pressure denoted by c_p. For solids and liquids, these specific heats are practically identical, but for gases they are considerably different; thus in the case of air, $c_p = 1.4\, c_v$.

Latent Heat. During a change of state of aggregation the heat added to a substance is expended in performing work of disgregation, and none of it is used in raising temperature; that is, the heat absorbed goes to increase the potential energy of the system. Heat thus absorbed during fusion or vaporization is

called latent heat. The heat required to vaporize a unit weight of a liquid depends upon the pressure under which the vapor is formed. The following are latent heats of fusion for various substances in British thermal units per pound:

Ice	144	Cast iron, grey	41.4
Lead	9.66	Cast iron, white	59.4
Tin	25.65	Zinc	50.6
Silver	37.93	Sulfur	16.86

2. Fundamental Laws of Thermodynamics

Transformations of Energy. All transformations of energy are subject to two far-reaching general laws: (1) The law of CONSERVATION OF ENERGY, of which the following is a statement: The total energy of an isolated system remains constant and cannot be increased or diminished by any physical process whatever. (2) The law of DEGRADATION OF ENERGY. According to this law, the result of any transformation of energy is to reduce the quantity of energy that may be usefully transformed into mechanical work.

Examples of the law of degradation are abundant. Work is transformed into heat thru friction and only a small part can be recovered; electrical energy is rendered unavailable when transformed into heat in the conducting system; heat flows from a body of higher temperature to one of lower temperature and as a result a smaller fraction of it becomes available for transformation into work; in the transformation of work into electrical energy or of electrical energy into work, some of the high-grade energy is transformed into heat and is rendered unavailable. The term thermodynamic degeneration is sometimes applied to the inevitable loss of available energy in any transformation of energy; and the law of degradation may be stated as follows: Every natural process is accompanied by thermodynamic degeneration.

The First Law of Thermodynamics is merely the law of conservation applied to the transformation of heat into work. It may be stated as follows:

> When work is expended in producing heat the quantity of heat generated is equivalent to the work done; and conversely, when heat is employed to do work, a quantity of heat precisely equivalent to the work done disappears.

Denoting by Q the heat converted into work, and by W the work thus obtained, the first law is exprest symbolically by the equations $W = JQ$ and $Q = AW$.

The Energy Equation. Let heat be absorbed by a body, as a given mass of air or a saturated vapor. If the volume of the body remains constant, the energy absorbed must be added to the intrinsic energy of the system, and as a result the temperature will rise, or, in the case of a liquid, vaporization will ensue. If, however, the volume of the system changes, external work will be done, and the heat absorbed will in part be used in doing this work, while the remainder will increase the intrinsic energy. In general, therefore,

$$\text{heat absorbed} = \text{increase of energy} + \text{external work.}$$

The change of energy depends upon the initial and final states of the system only. The external work depends, however, on the relation between p and v during the change of state, that is, upon the path; hence the heat absorbed also depends upon the path.

If a system passes thru a closed cycle of processes and returns to its initial state, the change of energy for the cycle is zero; hence for a closed cycle the heat absorbed by the system is the equivalent of the external work. If the change of state is adiabatic, the heat absorbed is zero and the external work is gained at the expense of the intrinsic energy of the system.

The Second Law of Thermodynamics is essentially the law of degradation of energy. While the first law gives a relation that must be satisfied in any

transformation of energy, it is the second law that gives information regarding the possibility of transformation and the availability of a given form of energy for transformation into work. A general statement of the second law is

> No change in a system of bodies that takes place of itself can increase the available energy of the system.

A more concrete statement is that of Kelvin, namely: It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of surrounding objects. In effect, Kelvin's statement denies the possibility of deriving work directly from the heat contained in the atmosphere.

Carnot's Cycle. The availability of a given quantity of heat energy for transformation into work is given by the efficiency of the ideal Carnot engine. In the ideal cycle described by Carnot, the medium is subjected to four processes: (1) It is placed in contact with a source of heat at a higher temperature T_1 and expands isothermally while absorbing heat Q_1 from the source. (2) It is removed from the source and expands adiabatically until the temperature reaches the lower value T_2. (3) It is placed in contact with a refrigerator at temperature T_2, is compresst isothermally and rejects to the refrigerator the heat Q_2. (4) It is compresst adiabatically and arrives at the initial state. The heat $Q_1 - Q_2$ is transformed into work, and the efficiency of the cycle is therefore the fraction $e = (Q_1 - Q_2)/Q_1$. According to Carnot's principle, which may be proved by the second law, all ideal reversible engines working between the same temperature limits T_1 and T_2 have the same efficiency; that is, the efficiency is independent of the working medium and depends upon T_1 and T_2 only. Hence $e = f(T_1, T_2)$. Kelvin proposed as a definition of temperature the relation $Q_2/Q_1 = T_2/T_1$. From this definition, $e = (T_1 - T_2)/T_1 = 1 - T_2/T_1$. Given a quantity of heat Q in a body having the temperature T, and let T_0 denote the lowest available temperature (e.g., the temperature of the atmosphere). No device can transform a greater part of Q into work than the ideal Carnot engine; hence the available part of Q is $Q(1 - T_0/T)$ and the remainder QT_0/T must inevitably be wasted. For example, if the absolute temperature of the source is 900° and that of the atmosphere is 540°, the available energy is $1 - 540/900 = 0.4$ of the total energy; therefore of 1000 B t u taken from the source not more than 400 B t u can by any possible means be transformed into work and at least 600 B t u is unavailable.

Entropy. Experience shows that any actual physical process, as the change of state of a system, is irreversible and is accompanied by frictional effects. A strictly reversible frictionless process is an ideal that may be approched but never attained. In the case of the ideal reversible process, there is no change in the quantity of available energy; but an actual irreversible process is always accompanied by a decrease of the amount of energy available for transformation, or, what is the same thing, an increase of unavailable energy. An investigation of special cases shows that the increase of unavailable energy is the product of two factors: one is T_0, the lowest absolute temperature available in a refrigerator, the other is a term of the form Q/T or $\int T^{-1}\,dQ$. This second factor is called the increase of entropy of the system under consideration.

When the conception of increase of entropy is applied to the system composed of all the bodies involved in a change, that is, to an isolated system, it appears that the increase of entropy is a measure of the thermodynamic degeneration produced by the change. According to the law of degradation every natural change is accompanied by thermodynamic degeneration, therefore it is accompanied by an increase of entropy. The following important principle is evident: The direction of a process, physical or chemical,

that occurs of itself is such as will bring about an increase of entropy of the system. This principle is the foundation of the application of thermodynamics to chemistry.

The Change of Entropy of a body which is in thermal communication with other bodies (as a pound of air, or the steam in an engine cylinder) is illustrated as follows: Let the ordinate A_2A (Fig. 1) represent the absolute temperature T_1 of the body in its initial state. Suppose the body to receive heat from external sources while its state changes reversibly; and let the ordinate A_1A increase in length so as to represent the successive values of T during the heating and at the same time move horizontally in such a way that the area swept over represents the heat absorbed. The end of the ordinate must move in a definite path, as AB. The horizontal distance A_1B_1 thru which the ordinate moves represents the increase of entropy. If OA_1 is chosen arbitrarily to represent the entropy S_1 of the body in the initial state, then OB_1 represents the entropy S_2 in the final state. The area A_1ABB_1 represents the heat Q absorbed (provided the process is reversible), and the following relations between Q and the change of entropy exist:

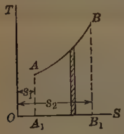

Fig. 1

$$Q = \int_{S_1}^{S_2} T\,dS, \quad S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T}.$$

The entropy of a system, as thus defined, depends upon the state only; hence S may be used with p, v, and T as a coordinate. Graphical representations on the TS-plane are specially advantageous, as the areas involved represent heat entering or leaving the system. If the process in question is not reversible (for example, as in the case of the flow of steam in a nozzle), this graphical representation fails. The area in this case does not represent the heat entering the system.

An isothermal process is represented on the TS-plane by a straight line parallel to the S-axis, as AB and CD, Fig. 2; reversible adiabatic processes by lines parallel to the T-axis, as BC and DA. The closed cycle $ABCD$ is the ideal Carnot cycle. Area A_1ABB_1 represents the heat Q_1 absorbed from the source during the isothermal expansion AB, area B_1CDA_1 represents the heat Q_2 rejected to the refrigerator during the isothermal compression CD, and the cycle area $ABCD$ represents the heat $Q_1 - Q_2$ transformed into work. The expression for efficiency $e = (Q_1 - Q_2)/Q_1 = (T_1 - T_2)/T_1$ follows from the geometry of the figure. When the order of events in the cycle of Fig. 2 is reversed, the cycle represents the operation of a refrigerating machine under ideal conditions. During the isothermal expansion DC heat Q_2 is absorbed from the cold body, and during the isothermal compression heat Q_1 is rejected to a body at temperature T_1. The cycle area represents the equivalent of the work W that must be furnished from external sources; and $Q_1 = Q_2 + AW$.

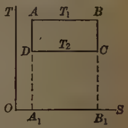

Fig. 2

Heat Content. The functions $u + pv$ plays an important part in many technical applications of thermodynamics. The energy u is measured in mechanical units (foot-pounds) and the product pv has necessarily the same unit. The heat equivalent $A(u + pv)$ is denoted by i and is called the heat content of the system (per unit weight). Evidently i, like u and s, is a function of the state only. The heat absorbed by a unit weight of a substance at constant pressure is the change in heat content $i_2 - i_1$. It is frequently convenient to represent changes of state on a plane having i and s as coordinates. In this scheme of representation, quantities of heat are represented by segments of lines instead of by areas.

3. Properties of Gases

Experimental Laws. The so-called permanent gases obey quite closely the laws of Boyle, Charles, and Joule, namely:

BOYLE'S LAW. At constant temperature, the volume of a given mass of gas varies inversely as the pressure. That is, the product pv is constant when T is constant.

CHARLES'S LAW. At constant pressure, the change of volume of a gas is proportional to the change of temperature. That is, $(\Delta v/\Delta t)p$ = constant.

JOULE'S LAW. The intrinsic energy of a gas is independent of the volume of the gas and depends upon the temperature only. Denoting the energy of unit weight of gas by u, Joule's law is exprest symbolically by the relations: $u = f(t)$, $\partial u/\partial v = 0$.

Characteristic Equation. By a combination of the laws of Boyle and Charles the relation $pv = BT$ is obtained as the characteristic equation of a gas that strictly obeys those laws. Here v denotes the volume of unit weight. If V is used to denote the volume of M lb of gas, the equation takes the useful form $pV = MBT$. The homogeneous form $p_1V_1/T_1 = p_2V_2/T_2$ is advantageous in the solution of problems that involve two states of the gas. The numerical value of the constant B is determined from the equation by inserting known values of p, v, and T corresponding to some assumed standard state. The following are the values of B for certain gases (English units):

Air	53.34	Hydrogen	765.86
Oxygen	48.25	Carbon dioxide	35.09
Nitrogen	54.99	Carbon monoxide	55.14

The gas equation may be written in the form $p = B\gamma T$, where $\gamma = 1/v$ denotes the weight of unit volume. For a chosen pressure and temperature, the product $B\gamma$ is the same for all gases to which the equation applies; and since γ is directly proportional to the molecular weight m, the product Bm is likewise the same for such gases. This product is denoted by R and it is called the universal gas constant. For English units the numerical value of $R = 1544$; hence the gas equation may be written in the form $pv = (1544/m)T$.

It is frequently desirable to refer the specific volume or the specific weight of a gas to a standard state, namely, atmospheric pressure and a temperature of 32° C. In this state v and γ are exprest in terms of the molecular weight m of the gas by the relations $v_0 = 358.65/m$ and $\gamma_0 = 0.002788\ m$.

Specific Heat of Gases. The specific heat of a gas that obeys the law $pv = BT$ must be independent of the volume and also of the pressure, but it may vary with the temperature. For moderate ranges of temperature the specific heats c_p and c_v may be assumed constant without serious error. The following are mean values for the range 0° to 200° C. (32° to 392° F.):

	c_p	c_v	$k = c_p/c_v$
Air	0.240	0.171	1.4
Hydrogen	3.424	2.446	1.4
Nitrogen	0.2438	0.174	1.4
Oxygen	0.2175	0.155	1.4
Carbon monoxide	0.2426	0.162	1.3
Ammonia	0.5106	0.387	1.32

When the temperature range is large, as exemplified in the internal combustion engine, the assumption of constant specific heat is no longer justified. The experiments of Mallard and Le Chatelier, Langen, and others, show that both c_p and c_v increase with the temperature according to a law that is exprest sufficiently well by the linear relation $c = a + bt$. According to Langen's experiments the variation of specific heat with temperature is represented by the following equations, temperatures being Fahrenheit and m being the molecular weight of the gas:

1. For simple gases, as air, nitrogen, hydrogen, etc.:

$$c_v = \frac{1}{m}(4.77 + 0.000667\,t) = \frac{1}{m}(4.48 + 0.000667\,T),$$

$$c_p = \frac{1}{m}(6.75 + 0.000667\,t) = \frac{1}{m}(6.46 + 0.000667\,T).$$

2. For carbon dioxide:

$$c_v = 0.15 + 0.000066\,t = 0.12 + 0.000066\,T,$$
$$c_p = 0.195 + 0.000066\,t = 0.165 + 0.000066\,T.$$

3. For superheated water vapor:

$$c_v = 0.324 + 0.000133\,t = 0.263 + 0.000133\,T,$$
$$c_p = 0.435 + 0.000133\,t = 0.374 + 0.000133\,T.$$

Changes of State. For any change of state of a permanent gas the following relations hold. Denoting by M the weight of gas under consideration, the change of energy is

$$U_2 - U_1 = JMc_v(T_2 - T_1) = (p_2V_2 - p_1V_1)/(k - 1),$$

the change of heat content is

$$I_2 - I_1 = Mc_p(T_2 - T_1) = A\,(p_2V_2 - p_1V_1)k/(k - 1),$$

and the change of entropy per unit weight is

$$s_2 - s_1 = c_p \log_\epsilon \frac{v_2}{v_1} + c_v \log_\epsilon \frac{p_2}{p_1}.$$

For certain important special changes of state the following relations exist: ($W =$ external work, $Q =$ heat absorbed.)

(a) Constant volume:

$p_2/p_1 = T_2/T_1$. $W = 0$. $Q = Mc_v(T_2 - T_1) = (U_2 - U_1)A$.

$$S_2 - S_1 = Mc_v \log_\epsilon \frac{T_2}{T_1}.$$

(b) Constant pressure:

$V_2/V_1 = T_2/T_1$. $W = p(V_2 - V_1) = MB(T_2 - T_1)$.

$$U_2 - U_1 = \frac{p(V_2 - V_1)}{k - 1} = \frac{W}{k - 1}. \quad Q = Mc_p(T_2 - T_1) = I_2 - I_1.$$

$$S_2 - S_1 = Mc_p \log_\epsilon \frac{T_2}{T_1}.$$

(c) Isothermal change of state:

$$p_1V_1 = p_2V_2. \quad U_2 - U_1 = 0. \quad W = MBT \log_\epsilon \frac{V_2}{V_1} = p_1V_1 \log_\epsilon \frac{V_2}{V_1}.$$

$$Q = AW. \quad S_2 - S_1 = \frac{Q}{T} = ABM \log_\epsilon \frac{V_2}{V_1}.$$

(d) Adiabatic change of state: In the adiabatic change, the equation of the expansion curve is $pv^k =$ const. Combining this with the equation $pv = BT$,

$$Tv^{k-1} = \text{const., or } \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{T_1}{T_2}; \quad \frac{p^{\frac{k-1}{k}}}{T} = \text{const., or } \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \frac{T_2}{T_1}.$$

$$Q = 0. \quad W = U_1 - U_2 = JMc_v(T_1 - T_2) = \frac{p_1V_1 - p_2V_2}{k - 1}. \quad S = \text{const.}$$

(e) Polytropic change of state: This change is defined by the relation $pV^n =$ const., in which n is a constant.

$$(V_2/V_1)^{n-1} = T_1/T_2. \quad (p_2/p_1)^{\frac{n-1}{n}} = T_2/T_1$$

$$W = \int_{v_1}^{v_2} p\,dV = p_1V_1^n \int_{v_1}^{v_2} V^{-n}dV = \frac{p_2V_2 - p_1V_1}{1-n}. \quad U_2 - U_1 = \frac{p_2V_2 - p_1V_1}{k-1}.$$

$$JQ = U_2 - U_1 + W = \frac{k-n}{(k-1)(1-n)}(p_2V_2 - p_1V_1).$$

$$W : U_2 - U_1 : JQ = k - 1 : 1 - n : k - n.$$

For example, let air be compress according to the law $pV^{1.3} =$ const. Taking $k = 1.4$, $W : U_2 - U_1 : JQ = 0.4 : -0.3 : 0.1$. That is, three-fourths of the work of compression is stored in the air and is manifested by a rise of temperature and one-fourth of it is taken away by the water jacket.

The specific heat corresponding to the polytropic change is constant and is given by the relation $c_n = c_v(k-n)/(1-n)$. For values of n lying between 1 and k, c_n is negative. The work W and heat absorbed may be exprest in terms of c_n; thus

$$W = JM(c_n - c_v)(T_2 - T_1) = \frac{MB}{n-1}(T_2 - T_1). \quad Q = Mc_n(T_2 - T_1).$$

Also $$S_2 - S_1 = Mc_n \log_e \frac{T_2}{T_1}.$$

To determine the value of n for an experimental curve, which is assumed to follow the law $pV^n =$ const., measure to any convenient scale p_1, V_1 and p_2, V_2 at two assumed points A and B. Then compute $n = (\log p_2 - \log p_1)/(\log V_1 - \log V_2)$.

Air Compression. The ideal indicator diagram of an air compressor without clearance is shown in Fig. 3. The curve AB represents the change of state of the air during compression. Without a water jacket, curve AB may be taken as an adiabatic. If a water jacket is used, AB is represented by an equation $pV^n =$ const., and the value of n lies between 1.4 and 1.0. In practice, n is usually about 1.3.

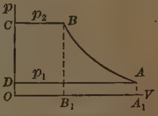

Fig. 3

The work represented by area $ABCD$ is given by the expression $W = n(p_1V_1 - p_2V_2)/(n-1)$, in which V_1 and V_2 denote the volumes at points A_1 and B_1 respectively. The heat absorbed by the water in the jacket during the compression AB is given by $Q = AW(k-n)/(k-1)$. The work of compression is reduced: (a) by water-jacketing; (b) by compression in two or more stages with cooling between the stages.

4. Saturated and Superheated Vapors

Characteristics of Vapors. When sufficient heat is applied to a liquid the state of aggregation is changed from the liquid to the gaseous. The process is called vaporization, and the resulting gaseous product is called a vapor. As long as the vapor is in contact with the liquid from which it is formed it is said to be saturated. The leading characteristic of a saturated vapor is that its temperature depends upon the pressure only, that is, $t = f(p)$. If the vapor is removed from the liquid and is further heated at constant pressure, the temperature rises above the saturation temperature given by $t = f(p)$, and the specific volume increases. The vapor is then said to be superheated, and the difference between the temperature of the vapor and the saturation temperature is the degree of superheat. In the case of saturated vapor, the volume of one pound is a function of the pressure only ($v = \phi(p)$), while in the case of a superheated

vapor the volume is a function of pressure and temperature ($v = F(p, t)$). The so-called permanent gases, air, nitrogen, etc., are in reality superheated vapors far removed from the saturation state.

Vapor and Liquid Mixtures. The process of vaporization and superheating is shown graphically on the TS-plane in Fig. 4. Liquid at 32° F. represented by point A (OA = 491.6) is heated under constant pressure. The process is represented by AB. At B the saturation temperature is reached, vaporization begins and proceeds till all the liquid is changed to vapor, as indicated by point C. Further application of heat superheats the vapor, as indicated by CD. Point B represents liquid at the boiling temperature, point C, saturated vapor, and point D, superheated vapor. For different pressures, points B and C will move along curves s' and s'', called the liquid and saturation curves, respectively. In raising the temperature from 32° to the boiling point, heat represented by area $OABB_1$ is absorbed; this is the heat of the liquid (q'). To vaporize the liquid, heat represented by area B_1BCC_1 is required; this is the heat of vaporization (r). The sum of these is the total heat of the vapor (q''). The area C_1CDD_1 represents the heat absorbed during the superheating. OB_1 represents the increase of entropy (s') during the heating of the liquid, and B_1C_1 the further increase of entropy during vaporization. A point M between B and C represents a mixture of vapor and liquid, the ratio BM/BC being the ratio of the weight of vapor to the weight of the mixture. This ratio is denoted by x and is called the quality of the mixture.

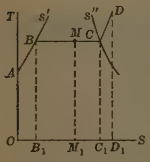

Fig. 4

The latent heat of vaporization r is separable into two parts: (1) the external latent heat, that is, the heat equivalent of the work done during the vaporization process; (2) the internal latent heat ρ, which is the heat required to break up the molecular structure and is stored in the steam in the form of potential energy. The total heat of steam q'' is practically equal to the heat content i'', and in recent steam tables i'' rather than q'' is given.

For a mixture having the quality x (as indicated by point M, Fig. 4) the total heat is $q' + xr$ B t u per lb, and the energy above that of water at 32° is $q' + x\rho$ B t u per lb. The entropy of the mixture per pound is $s' + xr/T$.

Thermal Properties of Saturated and Superheated Steam. The magnitudes i', i'', r, ρ, v'', s', and s'', defined in the preceding articles, are dependent on the temperature of vaporization. For several vapors of technical importance these properties have been determined experimentally with more or less accuracy, and empirical formulæ have been deduced by means of which the properties are exprest as functions of the temperature. In the case of water vapor, the older tables based on Regnault's experiments are now known to be inaccurate.

The relation between pressure and temperature of saturated steam is given very accurately by the formula

$$\log p = 10.5688080 - \frac{4876.643}{T} - 0.0155 \log T - 0.00406258\,T$$
$$+ 0.00000140055\,T^2 - 0.00002\left[10 - 10\left(\frac{t-370}{100}\right)^2 + \left(\frac{t-370}{100}\right)^4\right]$$

The relation between the volume, pressure and temperature of superheated steam is given by the equation

$$v - 0.017 = 0.59495\frac{T}{p} - (1 + 0.05129\,p^{\frac{1}{2}})\frac{66834 \times 10^6}{T^4}$$

in which p is in lb per sq in.

The following formulas give the heat content and entropy of superheated steam.

$$i = 0.320\,T + 0.000063\,T^2 - \frac{23583}{T} - p(1 + 0.0342\,p^{\frac{1}{2}})\frac{6188 \times 10^7}{T^4} + 948.5,$$

$$s = 0.73683 \log T + 0.000126\,T - \frac{11792}{T^2}$$

$$-0.254 \log p - (1 + 0.03420\,p^{\frac{1}{2}})\frac{49504 \times 10^6}{T^5} - 0.0811.$$

With p in lb per sq in the energy u is obtained from the relation

$$u = i - 0.1852\,pv.$$

If in the preceding formulas corresponding values of p and T at the saturation state are inserted, the volume, heat content, and entropy (v'', i'', s'') of saturated steam are obtained.

Properties of Saturated Steam

(Condensed from Goodenough's "Properties of Steam and Ammonia")

Absolute Pressure, Inches of Mercury	Temp. Fahr.	Heat Content Btu		Latent Heat Btu.		Entropy			Volume of 1 lb cu ft
		of Liquid	of Vapor	Total	Internal	of Liquid	of Vaporization	of Vapor	
p	t	i'	i''	r	ρ	s'	r/T	s''	v''
1.0	79.06	47.1	1095.0	1047.9	988.7	0.0915	1.9455	2.0370	652
2.0	101.2	69.2	1105.1	1036.0	974.3	0.1316	1.8474	1.9790	338.9
3.0	115.1	83.0	1111.4	1028.3	965.2	0.1561	1.7893	1.9454	231.4
4.0	125.4	93.4	1115.9	1022.5	958.3	0.1739	1.7478	1.9217	176.5
5.0	133.8	101.7	1119.6	1017.9	952.8	0.1880	1.7154	1.9034	143.2
10	161.5	129.4	1131.4	1002.1	934.1	0.2336	1.6134	1.8470	74.8
15	179.1	147.0	1138.8	991.7	922.0	0.2617	1.5526	1.8143	51.1
20	192.4	160.3	1144.1	983.8	912.7	0.2822	1.5089	1.7912	39.1
25	203.1	170.1	1148.3	977.3	905.2	0.2986	1.4747	1.7733	31.7
Lb per sq in									
14.7	212.0	180.0	1151.7	971.7	898.8	0.3120	1.4469	1.7589	26.81
15	213.0	181.0	1152.2	971.2	898.1	0.3135	1.4438	1.7573	26.30
16	216.3	184.3	1153.4	969.1	895.8	0.3184	1.4337	1.7521	24.76
17	219.4	187.5	1154.6	967.1	893.5	0.3230	1.4242	1.7473	23.40
18	222.4	190.5	1155.7	965.2	891.4	0.3274	1.4153	1.7427	22.18
19	225.2	193.3	1156.7	963.4	889.3	0.3316	1.4068	1.7384	21.09
20	228.0	196.0	1157.7	961.7	887.3	0.3356	1.3987	1.7343	20.10
22	233.1	201.2	1159.6	958.4	883.6	0.3430	1.3837	1.7267	18.38
24	237.8	206.0	1161.3	955.3	880.1	0.3499	1.3698	1.7197	16.95
26	242.2	210.1	1162.8	952.4	876.8	0.3563	1.3570	1.7133	15.73
28	246.4	214.6	1164.3	949.7	873.7	0.3622	1.3452	1.7074	14.67
30	250.3	218.6	1165.7	947.1	870.7	0.3679	1.3340	1.7019	13.76
32	254.0	222.4	1166.9	944.6	867.9	0.3731	1.3236	1.6967	12.95
34	257.6	225.9	1168.1	942.2	865.2	0.3781	1.3137	1.6918	12.24
36	260.9	229.4	1169.2	939.9	862.7	0.3829	1.3044	1.6873	11.60
38	264.2	232.6	1170.3	937.7	860.2	0.3874	1.2956	1.6830	11.03

Properties of Saturated Steam—*Concluded*

Absolute Pressure, Pounds per sq in	Temp. Fahr.	Heat Content Btu		Latent Heat Btu		Entropy			Volume of 1 lb cu ft
		of Liquid	of Vapor	Total	Internal	of Liquid	of Vaporization	of Vapor	
p	t	i'	i''	r	ρ	s'	r/T	s''	v''
40	267.2	235.8	1171.3	935.5	857.8	0.3917	1.2871	1.6788	10.51
42	270.2	238.8	1172.2	933.5	855.5	0.3958	1.2791	1.6749	10.04
44	273.0	241.7	1173.2	931.5	853.3	0.3998	1.2714	1.6712	9.61
46	275.8	244.5	1174.0	929.6	851.2	0.4036	1.2640	1.6676	9.22
48	278.4	247.2	1174.8	927.7	849.1	0.4072	1.2570	1.6642	8.86
50	281.0	249.8	1175.6	925.9	847.1	0.4108	1.2501	1.6609	8.53
55	287.1	255.9	1177.5	921.5	842.3	0.4190	1.2342	1.6532	7.80
60	292.7	261.7	1179.1	917.4	837.8	0.4267	1.2195	1.6462	7.18
65	298.0	267.1	1180.6	913.5	833.5	0.4338	1.2058	1.6397	6.66
70	302.9	272.2	1182.0	909.8	829.5	0.4405	1.1931	1.6336	6.22
80	312.0	281.6	1184.4	902.8	821.9	0.4527	1.1700	1.6227	5.48
85	316.3	286.0	1185.5	899.6	818.4	0.4583	1.1595	1.6178	5.18
90	320.3	290.1	1186.5	896.4	815.0	0.4636	1.1495	1.6131	4.905
95	324.1	294.1	1187.5	893.4	811.7	0.4687	1.1400	1.6087	4.663
100	327.8	297.9	1188.4	890.5	808.6	0.4736	1.1309	1.6045	4.442
105	331.4	301.6	1189.2	887.6	805.5	0.4782	1.1222	1.6004	4.240
110	334.8	305.1	1190.0	884.8	802.6	0.4827	1.1138	1.5695	4.057
115	338.1	308.6	1190.7	882.1	799.7	0.4870	1.1058	1.5928	3.889
120	341.3	311.9	1191.4	879.5	796.9	0.4911	1.0982	1.5893	3.735
125	344.4	315.1	1192.0	876.9	794.2	0.4950	1.0908	1.5858	3.593
130	347.4	318.2	1192.6	874.4	791.6	0.4989	1.0836	1.5825	3.461
140	353.1	324.2	1193.7	869.6	786.4	0.5062	1.0700	1.5762	3.226
150	358.5	329.8	1194.7	864.9	781.6	0.5131	1.0573	1.5704	3.020
160	363.6	335.2	1195.7	860.5	776.9	0.5196	1.0453	1.5649	2.839
170	368.5	340.3	1196.5	856.2	772.4	0.5258	1.0339	1.5597	2.679
180	373.1	345.2	1197.2	852.0	768.0	0.5316	1.0231	1.5547	2.536
190	377.6	350.0	1197.9	847.9	763.9	0.5372	1.0128	1.5500	2.408
200	381.9	354.5	1198.5	844.0	759.8	0.5426	1.0030	1.5456	2.292
210	386.0	358.8	1199.0	840.2	755.9	0.5477	0.9936	1.5413	2.186
220	390.0	363.0	1199.5	836.5	752.1	0.5526	0.9846	1.5372	2.090
230	393.8	367.1	1199.9	832.8	748.3	0.5573	0.9760	1.5333	2.002
240	397.5	371.0	1200.3	829.3	744.7	0.5619	0.9676	1.5295	1.921
250	401.1	374.9	1200.6	825.8	741.2	0.5663	0.9595	1.5258	1.846
260	404.5	378.6	1201.0	822.4	737.7	0.5706	0.9517	1.5223	1.777
270	407.9	382.2	1201.2	819.1	734.4	0.5747	0.9442	1.5189	1.713
280	411.2	385.7	1201.5	815.8	731.1	0.5787	0.9369	1.5156	1.654
290	414.4	389.1	1201.7	812.6	727.9	0.5826	0.9298	1.5123	1.598
300	417.5	392.4	1201.9	899.4	724.7	0.5863	0.9229	1.5092	1.545
350	431.9	408	1202.5	794.5	709.7	0.6036	0.8912	1.4949	1.327
400	444.8	422	1202.5	780.6	695.9	0.6190	0.8631	1.4821	1.162

Properties of Superheated Steam

[Condensed from Goodenough's "Properties of Steam and Ammonia"]

i = heat content, B t u per pound; s = entropy; v = volume, cubic feet per pound

Absolute Pressure Lb per sq in		Temperature Fahr: Sat	300	350	400	450	500	550	600	650	700
20 [228.0]*	i	1157.7	1192.7	1216.5	1240.2	1263.8	1287.4	1311.1	1335.0	1359.1	1383.3
	s	1.7343	1.7827	1.8131	1.8414	1.8681	1.8934	1.9175	1.9406	1.9628	1.9841
	v	20.10	22.37	23.91	25.44	26.96	28.47	29.97	31.47	32.97	34.47
40 [267.2]	i	1171.3	1188.0	1212.9	1237.3	1261.5	1285.6	1309.7	1333.8	1358.1	1382.4
	s	1.6788	1.7013	1.7331	1.7624	1.7897	1.8155	1.8399	1.8633	1.8856	1.9071
	v	10.51	11.05	11.85	12.64	13.42	14.19	14.95	15.71	16.46	17.22
60 [292.7]	i	1179.1	1183.0	1209.0	1234.3	1259.1	1283.7	1308.1	1332.5	1357.0	1381.6
	s	1.6462	1.6513	1.6845	1.7148	1.7429	1.7692	1.7940	1.8176	1.8402	1.8618
	v	7.18	7.27	7.83	8.37	8.90	9.42	9.94	10.45	10.96	11.47
80 [312.0]	i	1184.4	......	1205.0	1231.1	1256.6	1281.7	1306.5	1331.2	1355.9	1380.7
	s	1.6227	......	1.6487	1.6801	1.7089	1.7357	1.7609	1.7848	1.8076	1.8294
	v	5.48	......	5.81	6.23	6.64	7.04	7.43	7.82	8.21	8.59
100 [327.8]	i	1188.4	......	1200.8	1227.8	1254.0	1279.6	1304.8	1329.8	1354.8	1379.7
	s	1.6045	......	1.6199	1.6523	1.6820	1.7093	1.7349	1.7592	1.7822	1.8042
	v	4.44	......	4.60	4.95	5.28	5.61	5.93	6.24	6.55	6.86
120 [341.3]	i	1191.4	......	1196.4	1224.4	1251.3	1277.4	1303.0	1328.4	1353.6	1378.7
	s	1.5893	......	1.5955	1.6291	1.6594	1.6874	1.7134	1.7379	1.7612	1.7833
	v	3.74	......	3.79	4.09	4.38	4.65	4.92	5.19	5.45	5.71
140 [353.1]	i	1193.7	......	......	1220.9	1248.5	1275.2	1301.2	1326.9	1352.4	1377.7
	s	1.5762	......	......	1.6087	1.6400	1.6685	1.6949	1.7198	1.7433	1.7656
	v	3.23	......	......	3.48	3.73	3.97	4.21	4.44	4.66	4.89
160 [363.6]	i	1195.7	......	......	1217.3	1245.6	1272.9	1299.3	1325.4	1351.1	1376.7
	s	1.5649	......	......	1.5906	1.6226	1.6518	1.6787	1.7039	1.7276	1.7501
	v	2.84	......	......	3.01	3.24	3.46	3.67	3.87	4.07	4.27
180 [373.1]	i	1197.2	......	......	1213.6	1242.7	1270.5	1297.4	1323.8	1349.8	1375.6
	s	1.5547	......	......	1.5741	1.6070	1.6368	1.6641	1.6896	1.7136	1.7364
	v	2.54	......	......	2.65	2.86	3.06	3.25	3.43	3.61	3.79
200 [381.9]	i	1198.5	......	......	1209.8	1239.7	1268.1	1295.5	1322.2	1348.5	1374.5
	s	1.5456	......	......	1.5589	1.5927	1.6231	1.6509	1.6768	1.7010	1.7239
	v	2.29	......	......	2.36	2.56	2.74	2.91	3.08	3.24	3.40
220 [390.0]	i	1199.5	......	......	1205.9	1236.6	1265.6	1293.5	1320.6	1347.1	1373.4
	s	1.5372	......	......	1.5447	1.5794	1.6105	1.6388	1.6650	1.6895	1.7126
	v	2.09	......	......	2.13	2.31	2.47	2.64	2.79	2.94	3.09
240 [397.5]	i	1200.3	......	......	1202.0	1233.5	1263.1	1291.4	1318.9	1345.8	1372.3
	s	1.5295	......	......	1.5314	1.5670	1.5987	1.6275	1.6541	1.6789	1.7022
	v	1.92	......	......	1.93	2.10	2.26	2.41	2.55	2.69	2.83
260 [404.5]	i	1201.0	......	......	......	1230.3	1260.5	1289.4	1317.2	1344.4	1371.1
	s	1.5223	......	......	......	1.5553	1.5877	1.6170	1.6439	1.6690	1.6925
	v	1.777	......	......	......	1.922	2.071	2.212	2.347	2.477	2.605
280 [411.2]	i	1201.5	......	......	......	1227.0	1257.9	1287.2	1315.5	1342.9	1369.9
	s	1.5156	......	......	......	1.5442	1.5773	1.6071	1.6344	1.6597	1.6835
	v	1.654	......	......	......	1.770	1.911	2.045	2.172	2.295	2.414
300 [417.5]	i	1201.9	......	......	......	1223.7	1255.2	1285.1	1313.7	1341.5	1368.7
	s	1.5092	......	......	......	1.5336	1.5674	1.5977	1.6254	1.6510	1.6750
	v	1.545	......	......	......	1.638	1.773	1.900	2.020	2.136	2.249

* The number in brackets is the saturation temperature corresponding to the pressure.

Example 1. Let a mixture of steam and water having a quality $x = 0.96$ expand adiabatically from a pressure of 120 lb per sq in absolute to atmospheric pressure. Required the properties of the mixture in the initial and final states and the work of expansion.

In the initial state the heat content is $i_1' + x_1 r = 311.9 + 0.96 \times 879.5 = 1156.2$ B t u, and the energy of the mixture is $i_1' + x_1 \rho_1 = 311.9 + 0.96 \times 796.9 = 1076.8$ B t u. The entropy is $s_1' + x_1 r_1/T_1 = 0.4911 + 0.96 \times 1.0982 = 1.5454$. The volume of 1 lb of dry steam at 120 lb pressure is 3.735 cu ft, and the volume of a pound of the mixture is $3.735 \times 0.96 = 3.586$ cu ft.

In the adiabatic expansion the entropy remains constant, and in the final state, therefore, $s_2' + x_2 r_2/T_2 = 1.5454$, whence $x_2 = 0.852$; that is, at the end of the expansion the mixture contains nearly 15% water. The heat content in the second state is $180 + 0.852 \times 971.7 = 1007.9$ B t u, the energy is $180 + 0.852 \times 898.9 = 945.9$ B t u, and the volume of 1 lb is $26.81 \times 0.852 = 22.84$ cu ft. The external work is the equivalent of the decrease in energy, or $778\ (1076.8 - 945.9) = 101\ 840$ ft-lb per pound of mixture. The difference $i_1 - i_2 = 1156.2 - 1007.9 = 148.3$ B t u, is the available heat, that is, the heat that may be transformed into work in an ideal engine working between the pressure limits under consideration. It is also the equivalent of the kinetic energy $w^2/2g$ of a jet flowing from a region of 120 lb pressure into a region of atmospheric pressure.

Example 2. Let steam at 200 lb per sq in absolute pressure superheated to 560° expand adiabatically to a pressure of 3 in of mercury. Required the final quality, the change in heat content, and the change of energy.

By interpolation the following values are obtained for the steam in the initial state: $i = 1300.9$, $s = 1.6562$, $v = 2.95$. Consequently the energy per pound is $1300.9 - 0.1852 \times 200 \times 2.95 = 1191.6$ B t u $\quad s_2 = 1.6562 = s_2' + x_2 r_2/T_2 = 0.1561 + 1.7893 x_2$, whence $x_2 = 0.838$, the final quality. $i_2 = 83 + 0.838 \times 1028.3 = 945.1$, $u_2 = 83 + 0.838 \times 965.2 = 892.2$ B t u. The change in energy is therefore $1191.6 - 892.2 = 299.4$ B t u, and the change in heat content is $1300.9 - 945.1 = 355.8$ B t u.

Adiabatic Changes. An adiabatic expansion of a saturated or superheated vapor may be represented approximately by the equation $pv^m = \text{const.}$ For saturated steam m depends upon the initial quality and pressure and is given by the equation $m = 1.059 + 0.000315\,p + o(0.0706 + 0.000376\,p)x$. For superheated steam m may be taken as 1.31, for superheated ammonia at 1.333, and for superheated sulfur dioxide as 1.282. The final volume V_2 is given by the relation $V_2 = V_1(p_1/p_2)^{\frac{1}{m}}$ and the external work is then approximately $W = (p_1 V_1 - p_2 V_2)/(m - 1)$.

5. Flow of Elastic Fluids

General Equation of Flow. Let an elastic fluid, as air or steam, flow along a horizontal tube, Fig. 5. At an assumed cross-section F_1 the pressure of the fluid is p_1 and the mean velocity is w_1; at a second section F_2 the pressure is p_2 and the velocity is w_2. The principal equation of flow is $(w_2^2 - w_1^2)/2g = J(i_1 - i_2)$, in which i_1 and i_2 denote, respectively, the heat content per pound of fluid at sections F_1 and F_2. If section F_1 be taken in the boiler or reservoir from which the fluid is flowing, then $w_1 = 0$, and the equation becomes

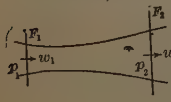

Fig. 5

$$w^2/2g = J(i_1 - i_2); \quad \text{or} \quad w_2 = 223.7\sqrt{i_1 - i_2}.$$

In the flow of elastic fluids, the heat content i plays the same part as the head h in the flow of liquids.

Example. Steam at 120 lb per sq in, quality 0.96. flows into the atmosphere thru a properly proportioned nozzle. Required the velocity of flow. The heat content i_1 is 1156.2 B t u per lb, and if the flow is adiabatic and frictionless the final heat content i_2 is 1007.9 B t u. Hence $w = 223.7\sqrt{1156.2 - 1007.9} = 2724$ ft per sec.

Formulas of Discharge. The weight M of fluid flowing past a cross-section per second is given by the equation of continuity $Fw = Mv$, in which

F denotes the area of the section and v the specific volume of the fluid at this section. If the flow is thru an orifice or short tube, it is found that the pressure in the plane of the orifice never falls below a certain value p_m called the critical pressure. The ratio p_m/p_1 (p_1 being the pressure in the boiler or reservoir) is called the critical ratio. For air its value is about 0.53; for saturated steam, about 0.57. If the pressure p_2 of the region into which the fluid is flowing is greater than p_m, the pressure at the orifice is p_2; but if p_2 is less than p_m, then the pressure at the orifice is p_m and the discharge M is the same whatever the value of p_2. The following formulas for discharge are in current use:

(a) Fliegner's equations for air.

When $\quad p_2 < 0.53\,p_1, \qquad M = 0.53\,F\dfrac{p_1}{\sqrt{T_1}}.$

When $\quad p_2 > 0.53\,p_1, \qquad M = 1.06\,F\sqrt{\dfrac{p_2(p_1 - p_2)}{T_1}}.$

(b) Grashof's equation for steam. Taking the area of the orifice in square inches and the pressure p_1 in lb per sq in,

$$M = 0.0165\,Fp_1^{0.97}. \qquad p_2 < 0.57\,p_1.$$

(c) Rateau's equation. This equation is empirical and is based on experimental data.

$$M = \frac{Fp_1}{1000}(16.367 - 0.96\log p_1). \qquad p_2 < 0.57\,p_1.$$

(d) Napier's equations. These are also empirical and rather inaccurate.

When $\quad p_1 > \frac{5}{3}p_2, \quad M = \dfrac{Fp_1}{70}.$

When $\quad p_1 < \frac{5}{3}p_2, \quad M = \dfrac{Fp_2}{42}\sqrt{\dfrac{3(p_1 - p_2)}{2\,p_2}}.$

Example. Required the discharge in pounds per minute of saturated steam at 100 lb pressure (absolute) thru an orifice having an area of 0.4 sq in. The back pressure is less than the critical pressure, 57 lb per sq in.

By Grashof's formula:

$$M = 60 \times 0.0165 \times 0.4 \times 100^{0.97} = 34.493 \text{ lb}.$$

By Rateau's formula:

$$M = \frac{60 \times 0.4 \times 100}{1000}(16.367 - 0.96 \times 2) = 34.673 \text{ lb}.$$

By Napier's formula:

$$M = \frac{0.4 \times 100}{70} \times 60 = 34.286 \text{ lb}.$$

The discharge may be found from the two fundamental formulas:

$$w = 223.7\sqrt{i_1 - i_2}, \quad \text{and} \quad M = Fw/v.$$

The critical pressure p_m is 57 lb per sq in. From the steam table, i_1 (for 100 lb) = 1188.4 B t u; i_m (for 57 lb) = 1144.1 B t u; $x_m = 0.963$; $v_m = 7.26$ cu ft.

Then $w = 223.7\sqrt{1188.4 - 1144.1} = 1490$ ft per sec. and $M = 60 \times \dfrac{0.4}{144} \times \dfrac{1490}{7.26} =$ 34.18 lb.

Flow in Nozzles. When the pressure p_2 of the region into which a jet is flowing is less than the critical pressure p_m, which is the pressure existing in the emerging fluid, the difference $p_m - p_2$ gives rise to a lateral spreading of the jet. This spreading may be prevented by the addition of a properly proportioned tube (Fig. 6). The tube must diverge so as to permit the expansion of the fluid required by the drop in pressure from p_m at the smallest section a to the

external pressure p_2 at the end section b. The effect of the diverging nozzle is to cause a solid jet to emerge with a velocity w_2 given by the fundamental relation $w_2 = \sqrt{2\,gJ(i_1 - i_2)} = 223.7\sqrt{i_1 - i_2}$. If the external pressure p_2 is greater than p_m, the diverging tube is unnecessary and a simple orifice is used. In the Rateau turbine, for example, the drop in pressure from cell to cell is so arranged that $p_2/p_1 > 0.57$, and the steam flows thru orifices rather than nozzles.

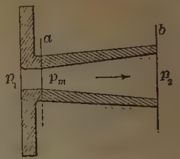

Fig. 6

On account of the length of a nozzle there is necessarily some loss of energy due to frictional resistances. The work of friction is converted into heat, which enters the flowing fluid and increases the final heat content i_2 to a larger value i_2'. It is customary to take as a friction coefficient the ratio of the loss of energy to the kinetic energy of the jet without friction. Denoting the coefficient by y, its formula is

$$y = (i_2' - i_2)/(i_1 - i_2)$$

and the actual velocity of exit w_2' is given by the relation

$$w_2' = \sqrt{2\,gJ(1 - y)\,(i_1 - i_2)} = 223.7\sqrt{(1 - y)\,(i_1 - i_2)}.$$

The quality of the steam at exit is increased by the heat generated thru friction and is given by the equation

$$x_2' = x_2 + (i_1 - i_2)y/r_2,$$

in which x_2 denotes the quality on the assumption of frictionless adiabatic expansion and is given by $s_1' + x_1r_1/T_1 = s_2' + x_2r_2/T_2$. Experiments indicate that y may vary from 0.08 to 0.20, depending on the size of nozzle.

Throttling. The throttling or wiredrawing of a fluid is merely a special case of flow, in which the loss by friction is excessive. The fluid in the region of higher pressure p_1 passes thru a valve or constricted passage into a region of lower pressure p_2. The velocity is increased from w_1 to w_2, but the increased kinetic energy is dissipated as the fluid passing the orifice enters and mixes with the fluid in the second region. Ultimately the velocity w_2 is sensibly equal to the original velocity w_1, and therefore $i_2 = i_1$. For a mixture of saturated vapor and liquid, the equation of throttling is $i_1' + x_1r_1 = i_2' + x_2r_2$. From this equation the value of x_2 is found, and having x_2, the increase of entropy is determined; and the increase of entropy multiplied by T_0 gives the loss of available energy resulting from the throttling.

If steam at the higher pressure p_1 is nearly dry, it becomes superheated when throttled to a much lower pressure. The equation, in this case, becomes $i_1' + x_1r_1 = i_2'' + c_p(t_2' - t_2)$, in which t_2 denotes the saturation temperature for the pressure p_2, t_2' the temperature of the superheated steam, and c_p the mean specific heat for the range $t_2' - t_2$. By means of this equation the initial quality x_1 may be determined from observed values of p_2 and t_2'.

6. Steam Boilers

Types of Boilers. Steam boilers may be divided into two general classes: (1) fire-tube boilers, in which the hot gases from the furnace pass thru the tubes and the water surrounds the tubes; (2) water-tube boilers, in which the water flows thru the tubes. A second classification takes account of the location of the furnace. In an externally fired boiler, the furnace is placed in a brick chamber external to the boiler. The internally fired boiler has its furnace enclosed in the steel shell of the boiler itself. Marine and locomotive boilers are examples of the latter type.

Horse-power Rating. The capacity of a boiler for the generation of steam is measured in terms of an arbitrary unit called horse-power. As defined b

the American Society of Mechanical Engineers, a "boiler horse-power" is equivalent to the evaporation of 34.5 lb of water per hour from and at 212° F. Taking 970.4 B t u as the latent heat of steam at 212°, a horse-power is thus equivalent to the development of 33 480 B t u per hour.

In the original report of the committee, the unit was given as the evaporation of 30 lb of water per hour from feed water at 100° F. into dry steam at a pressure of 70 lb per sq in gage. With the older values of the latent heat of steam, this unit was equivalent to 33 305 B t u per hour, and practically equivalent to the evaporation of 34.5 lb of water from and at 212° F.

Heating and Grate Area. The heating surface of a boiler includes all parts of the boiler that have water on one side and hot gases on the other. In the fire-tube boiler, this includes about two-thirds of the shell and tube sheets, and the external surface of all the tubes. It is customary to allow 10 to 12 sq ft of heating surface per boiler horse-power; hence with 34.5 lb of water evaporated per horse-power, 1 sq ft of heating surface gives an evaporation of about 3 lb of water from and at 212°. It is possible by increasing the speed of the hot gases along the tubes to increase considerably the rate of absorption of heat and thereby increase the capacity of the boiler at some sacrifice of economy.

The grate area required depends upon the character of the fuel used and the draft. With ordinary chimney draft, the grate area should be ⅓ sq ft per horse-power, or more. With forced draft, the area may be considerably smaller. The rate of combustion is ordinarily 10 to 15 lb of coal per hour per sq ft of grate for anthracite coal, and 15 to 20 lb for bituminous coal. With forced draft, these rates may be largely exceeded.

Boiler Economy. The measure of the economic performance of a steam boiler is the weight of water evaporated per pound of coal. In order that boilers working under different conditions may be compared, it is customary to reduce the actual evaporation to equivalent evaporation from and at 212°. If under actual conditions the feed water enters the boiler at temperature t_2 and steam of quality x is generated at pressure p_1, the heat required per pound is $q_1' + xr_1 - q_2'$, where q_2' is the heat of the liquid corresponding to the temperature t_2. To evaporate 1 lb from and at 212° requires 971.7 B t u; hence if M is the actual evaporation and M_e is the equivalent evaporation from and at 212°, $971.7\ M_e = M(q_1' + xr_1 - q_2')$, or $M_e = M(q_1' + xr_1 - q_2')/971.7$. The weight of water evaporated per pound of coal depends (1) upon the efficiency of the boiler and (2) upon the quality of coal used. With high grade bituminous coal having a heating value of 15 000 B t u per lb, the maximum evaporation from and at 212° is 15 000 ÷ 971.7 = 15.44 lb per lb of coal, and the actual evaporation will vary from 9 to 12 lb. With a low-grade coal having a heating value of 10 000 to 11 000 B t u per lb, the actual evaporation from and at 212° may fall as low as 5 or 6 lb.

Losses in Boilers. Coal always contains a certain amount of moisture and ash; and the term combustible is used to denote coal without these incombustible constituents. In tests of boilers the performance is based upon the combustible rather than upon the coal as fired. The total weight of combustible fired multiplied by the heating value of 1 lb of combustible gives the maximum amount of heat that can be produced in the furnace. The heat actually produced is less than this maximum because of incomplete combustion: (1) Part of the carbon may burn to CO instead of to CO_2. (2) With bituminous coal, some of the volatile hydrocarbons may escape unburned. These furnace losses may be reduced to a minimum by proper design of the furnace and by the use of suitable mechanical stokers.

Of the heat actually generated in the furnace, only a part is absorbed by the water in the boiler. The following are the losses of heat: (1) Loss due to moisture in coal

(2) Loss due to moisture formed by burning hydrogen. (3) Loss due to heat carried away by chimney gases. (4) Loss due to radiation. The data required for the determination of these losses are an analysis of the coal with the determination of its heating value, an analysis of the chimney gases, and the temperatures of the outside air and of the chimney gases, respectively.

The composition of the chimney gases furnishes an indication of the conditions as regards economy under which the boiler operates. The CO_2 content of the gas is of special significance. Under proper conditions of operation, the CO_2 may lie between 9 and 12%. A lower value of the CO_2 content indicates either excessive air supply to the furnace or leakage of air thru the brick setting. Many large boiler plants are provided with autographic CO_2 recorders for the continuous recording of the percentage of CO_2 in the gases.

Efficiencies. The efficiency of the furnace is the ratio of the heat actually generated in the furnace to heating value of the combustible fired. The efficiency of the boiler is the ratio of the heat absorbed by the boiler and carried away in steam to the heat actually generated in the furnace. The product of these two efficiencies is the over-all efficiency, that is, the efficiency of the boiler and furnace combined; it may be defined as the ratio of the heat absorbed by the water in the boiler to the heating value of the combustible. The over-all efficiency in ordinary practices is 0.65 to 0.72, tho values as high as 0.80 have been reached.

Boiler Tests. The object of a boiler test is to determine the efficiency and capacity of the boiler. The test consists essentially in measuring accurately the coal consumed and the water evaporated, and in observing the conditions under which the steam is generated. A standard code of rules for conducting boiler tests has been published by the Amer. Soc. of M. E. The following is a brief outline:

I. Determine at the outset the specific object of the trial.

II, III. Examine the boiler, ascertain important dimensions, notice the general condition of boiler and equipment.

IV. Determine the character of the coal to be used.

V. Establish the correctness of all apparatus used in the test; as scales, tanks, water-meters, thermometers, pyrometers, gages.

VI. See that the boiler is thoroly heated before the trial.

VII. The boiler and connections should be proved to be free from leaks before beginning the test, and all water connections except the pipe thru which the water is fed shoulc be disconnected.

VIII. The duration of the test should be ten hours of continuous running.

IX–XIII. These rules refer to methods of starting and stopping the test, uniformit of conditions, and the keeping of records.

XIV. The percentage of moisture in the steam should be determined by the use of a calorimeter.

XV–XVII. A fair sample of the coal should be obtained. The percent of moistur should be determined by air drying and weighing. The quality of the coal should b determined either by heat test, by analysis, or both. The ashes and refuse are to b weighed in the dry state. For elaborate trials a complete analysis of the refuse and as should be made.

XVIII. The analysis of the flue gases is especially desirable. Care should be taken t procure average samples.

XXII. An approximate "heat balance" may be included in the report of the test i the following form:

Total Heat Value of 1 lb of Combustible

	Btu	Per cent
1. Heat absorbed by boiler		
2. Loss due to moisture in coal		
3. Loss due to moisture formed by burning of hydrogen		
4. Loss due to heat carried away by dry chimney gases		
5. Loss due to incomplete combustion of carbon		
6. Loss due to unconsumed hydrogen and hydrocarbons, to heating the moisture in the air, to radiation, and unaccounted for		

7. Steam Engines

Types of Steam Engines. Reciprocating steam engines may be grouped in classes in several ways. The type of valve gear furnishes one basis of division; thus engines may be classed as slide-valve, Corliss, four-valve, etc. Slide-valve engines may be either throttling or automatic, according as the governor regulates the work in the cylinder by throttling the steam or by changing the point of cut-off. Engines in which the complete expansion of steam takes place in one cylinder are simple engines; if the expansion requires two, three, or four cylinders, the engines are termed compound, triple, and quadruple, respectively. Engines are condensing when the exhaust steam passes into a condenser, non-condensing when it passes directly into the atmosphere. Most steam engines are double-acting, that is, work is done during both strokes of the piston.

Ideal Cycle. The ideal cycle of a steam engine without clearance may be represented most advantageously on the temperature-entropy plane. The point A (Fig. 7) represents the state of one pound of water entering the boiler; curve AB represents the heating of the water to the boiling temperature T_1, BC represents vaporization, CD adiabatic expansion in the engine cylinder, and DA condensation of the exhaust steam. The cycle area $ABCD$ represents the available heat, that is, the part of the total heat supplied that may be transformed into work. Denoting by i_1 the heat content of the steam in state C and by i_2 the heat content after adiabatic expansion to D, the available heat is the difference $i_1 - i_2$. The weight of steam consumed per h p-hr is $2546/(i_1 - i_2)$.

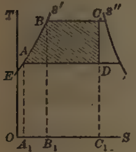

Fig. 7

Example. Referring to the examples on p. 1586, it was shown that in the case of an engine using steam at 120-lb pressure, quality 0.96, and exhausting at atmospheric pressure, the available heat was 148.3 B t u per lb. For an engine using steam at 200-lb pressure superheated to 560° and with a condenser pressure of 3 in of mercury, the available heat was 355.8 B t u per lb. In the first case the ideal steam consumption is $2546/148.3 = 17.2$ lb and in the second case it is $2546/355.8 = 7.16$ lb per hp-hr.

This ideal cycle is called the Rankine cycle and is used as a standard by which to measure engine performance. With given limiting pressures p_1 and p_2 and initial quality x_c, the available heat is the maximum amount of heat that can be transformed into work under these conditions, and the ratio of the heat actually transformed to the available heat indicates the real efficiency of the engine from the point of view of thermodynamics.

Indicator Diagram. The actual cycle of the steam engine differs from the ideal Rankine cycle in several particulars: (1) The metal of the cylinder walls and piston conduct heat, and there is thus an active interchange of heat between the metal and steam, thus making adiabatic expansion impossible. (2) The valves do not act instantly and there is wiredrawing of steam in passing thru the ports. (3) The cylinder must have clearance. The result of these modifications is a cycle having a smaller area than the ideal Rankine cycle.

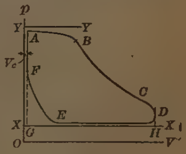

Fig. 8

On the pv-plane the actual cycle of the engine is the diagram drawn by the indicator (Fig. 8). In this diagram, OV represents the line of zero pressure, XX the atmospheric line, OX representing atmospheric pressure, and YY the line of boiler pressure. The length GH represents the volume swept thru by the piston, and XG the clearance volume V_c. AB is the steam line, B the point of cut-off, BC the expansion curve, C the point of release, CD the exhaust line, DE the back-pressure line, E the point of exhaust closure, EF the compression curve, and FA the admission line.

A single indicator diagram shows graphically the pressures exerted by the steam on one side of the piston during two strokes. To determine the net steam pressure transmitted to the piston rod, diagrams taken simultaneously from opposite ends of the cylinder must be superimposed.

The expansion curve BC of the actual diagram may be represented by an equation of the form $pV^n =$ const. The usual statement that the curve is an equilateral hyperbola, in other words, that $n = 1$, has no foundation. Experiments show that the value of n depends upon the quality of the mixture in the cylinder at cut-off; if there is a large per cent of water in the mixture, n falls as low as 0.85 or 0.9, while if there is little or no water present, due to use of superheated steam, n may rise to 1.2.

The indicator diagram has two principal uses: (1) It shows the distribution of the steam to the cylinder, and reveals errors in the setting of the valves. (2) It gives the horse-power developed in the cylinder, the so-called indicated horse-power (i h p).

Indicated Horse-power. The area of the indicator diagram is the integral $\int p\,dV$ taken around the closed cycle; hence it represents the work of the cycle. The mean ordinate of the diagram (see Fig. 8) is found by any convenient method. Ordinarily the area is determined by a planimeter, and this area divided by the length GH gives the mean ordinate. Multiplying by the scale of the indicator spring (30, 40, or 60 lb per sq in), the resulting product is the mean effective pressure (m e p) in lb per sq in.

Let p denote the mean effective pressure, a the area of piston in sq in, l the length of stroke in feet, and n the revolutions per minute. Then the indicated horse-power is

$$\text{i h p} = 2\,plan/33\,000.$$

For a single-acting engine, the factor 2 is dropped. For accurate calculations, the area a should be reduced by one-half of the cross-section area of the piston rod if the rod passes thru one head and by the whole area if the rod passes thru two heads. The piston speed is given by $2\,ln$; hence i h p $= paS/33\,000$.

Brake Horse-power is the horse-power of the engine delivered at the flywheel. The difference between the i h p and b h p is the friction horse-power, and the ratio b h p/i h p is the mechanical efficiency of the engine. The brake horse-power is determined experimentally by the use of a band brake or an absorption dynamometer.

Losses in the Steam Engine. The following are the principal sources of loss of available heat in the operation of the engine: (1) Wiredrawing in ports and valves, and friction in pipes. (2) Leakage past the piston and valves. (3) Loss due to clearance. (4) Radiation and conduction from the cylinder. (5) Initial condensation.

The losses indicated in the first four items may be reduced to a minimum by careful design and construction. The most serious loss is due to the interchange of heat between steam and metal, resulting in initial condensation and subsequent evaporation. The magnitude of this loss depends upon the conditions of operation. It is greater the slower the rotative speed of the engine, the earlier the cut-off, the greater the temperature range in the cylinder, and the greater the ratio of cylinder-wall surface to cylinder volume. The following are the means used to reduce the loss due to initial condensation.

(a) Steam Jackets. The cylinder is surrounded by an annular space containing steam at boiler pressure. The condensation takes place in this jacket instead of in the cylinder. The jacket is most effective in slow-speed engines having large ratios of expansion.

(b) Compounding. The large temperature range that accompanies a large ratio of expansion may be divided between 2, 3, or 4 cylinders, thus giving a relatively small range in each cylinder. The total loss from condensation in all the cylinders is less than the loss would be if the steam were expanded in a single cylinder. The use of two or more cylinders is also advantageous mechanically; the pressures on cranks, shafts, etc., are reduced, and by setting the cranks at proper angles a more uniform turning moment on the shaft is obtained.

(c) Superheating. The most effective means of reducing initial condensation is the use of steam initially superheated. By a sufficient degree of superheat dry steam at cut-off may be insured, and as the rate of absorption of heat by a gas is small, the activity of heat interchange between steam and metal is greatly reduced.

Efficiency Standards. The term efficiency when applied to the steam engine may mean any one of several ratios. Let q = heat supplied to an engine per lb of steam, q_a = available heat, the maximum quantity of heat that can be transformed into work in the ideal Rankine cycle, q_e = heat transformed into work in actual engine, $W_e = Jq_e$ the indicated work per lb of steam, W_b = the work obtained at the brake per lb of steam. Then $e_a = q_a/q$ = thermal efficiency of Rankine engine, $e_e = q_e/q$ = thermal efficiency of actual engine, $e_i = e_e/e_a = q_e/q_a$ = efficiency ratio (sometimes called potential efficiency), $e_b = W_b/Jq_e$ = brake efficiency ratio (based on work at brake), $e_m = W_b/W_a$ = mechanical efficiency. Two other standards are coming into use. These are: B t u consumed per i h p-hour, and B t u per b h p-hour. Since 1 h p-hr = 2546 B t u, the first ratio is equal to 2546 divided by the indicated thermal efficiency; thus if the thermal efficiency is 0.20, the engine consumes 2546/0.20 = 12 730 B t u per hour per i h p.

The really useful criterion of engine performance is the ratio e_b which is equal to the product $e_i \times e_m$. The ratio e_i is a measure of the extent to which the engine transforms into work the heat q_a that is available for transformation, and the ratio e_m measures the mechanical perfection of the engine. The product $e_i \times e_m$, therefore, is an indication of the quality of the engine both thermodynamically and mechanically.

The Rankine efficiency e_a depends upon the pressure limits used and may vary from 0.08 to 0.30. The thermal efficiency of the actual engine varies from 0.03 to as high as 0.25. The efficiency ratio e_e usually lies between 0.60 and 0.75, but in exceptional cases the extremely high value 0 88 has been attained. At full load the mechanical efficiency e_m should lie between 0.80 and 0 90. Since the engine friction remains nearly constant at all loads, e_m may fall as low as 0.50 at light loads.

Performance of Steam Engines. The following gives approximate values of steam consumption of various classes of steam engines (Allen and Bursley's Heat Engines, p. 160):

Steam Consumption.	Pounds per i h p-hr.
Simple throttling engine, non-condensing	44–45
Simple automatic engine, non-condensing	30–35
Simple Corliss engine, non-condensing	26–28
Simple automatic engine, condensing	22–26
Simple Corliss engine, condensing	22–24
Compound automatic engine, non-condensing	25–30
Compound automatic engine, condensing	18–20
Compound Corliss engine, condensing	14–16
Triple Corliss engine, condensing	12¼–13

The Economy of Pumping Engines is usually exprest in terms of duty, which is defined as follows: The duty is the number of foot-pounds of work obtained from the pump cylinders per million B t u furnished to the engine by the boilers. The duty that may be expected of various forms of pumping engines is as follows (Allen and Bursley's Heat Engines, p. 163):

Small duplex non-condensing pumps	10 000 000
Large duplex non-condensing pumps	25 000 000
Small simple flywheel pumps, condensing	50 000 000
Large simple flywheel pumps, condensing	65 000 000
Small compound flywheel pumps, condensing	85 000 000
Large compound flywheel pumps, condensing	120 000 000
Large triple-expansion flywheel pumps, condensing	150 000 000
Large triple-expansion flywheel pumps, condensing, of exceptional economy	165 000 000

At the Deer Island sewage pumping station, Boston, Mass., a 45 000 000-gallon centri ugal pump showed upon test a duty of nearly 96 000 000.

A table by C. V. Kerr (Trans. A. S. M. E., vol. 25, 1904) gives the results of a numbe of tests on various types of steam engines. It appears from these results that the efficienc ratio of a good steam engine should lie between 0.62 and 0.75. Under exceptional con ditions it may rise to 0.80. In general, the steam consumption per h p-hr and the efficienc ratio vary with the load. At light loads the efficiency decreases rapidly as the load decreased; for loads in excess of the engine's rating, the efficiency also decreases but a slower rate.

Steam-engine Testing. The object of the test is to determine the stea consumption of the engine and the efficiency ratio. If a surface condenser used the steam consumption is determined by weighing the steam condense otherwise, the water fed to the boilers must be measured, care being taken th all the steam produced from the feed water goes to the engine. The length the engine test should be at least 5 hours, and if the test includes the boilers, should continue 24 hours. The indicated horse-power is determined from ind cator diagrams taken at intervals of 10 or 15 minutes; and the brake horse-pow is obtained by the use of a brake or dynamometer. The weight of water reduc to pounds per hour is divided by the average i h p and the quotient is the stea consumption in lb per i h p-hr. The steam consumption for the ideal Rankin engine is readily found for the same conditions and the ratio of these gives t efficiency ratio.

A code of rules for conducting steam-engine tests is contained in the Transactions of t A. S. M. E., vol. xxiv. The following are extracts from the report of the committee.

From Introduction to Report: The heat consumption of a steam-engine pla is ascertained by measuring the quantity of steam consumed by the plant, calculati the total heat of the entire quantity, and crediting this total with that portion of the he rejected by the plant which is utilized and returned to the boiler. The term engine pla as here used should include the entire equipment of the steam plant which is concern in the production of the power, embracing the main cylinder or cylinders; the jackets a reheaters; the air, circulating, and boiler-feed pumps, if steam driven; and any other stea driven mechanism or auxiliaries necessary to the working of the engine.

X. Measurement of Feed Water. The method of determining the steam co sumption applicable to all plants is to measure all the feed water supplied to the boile and deduct therefrom the water discharged by separators and drips, as also the water a steam which escape on account of leakage of the steam main and branches connecti the boiler and engine. In plants where the engine exhausts into a surface condenser steam consumption can be measured by determining the quantity of water discharged the air pump, corrected for any leakage of the condenser, and adding thereto the ste used by jackets, reheaters, and auxiliaries as determined independently.

XIII. Indicated Horse-power. The indicated horse-power should be determi from the average m e p of diagrams taken at intervals of 20 minutes, and at more frequ intervals if the nature of the test makes this necessary, for each end of the cylinder.

XXI. Standards of Economy and Efficiency. The hourly consumption of h . . . divided by the indicated and brake horse-power, that is, the number of heat u consumed per i h p and per b h p per hour, are the standards of engine efficiency rec mended by the committee.

XXIV. Ratio of Economy of an Engine to That of an Ideal Engine. The i engine recommended for obtaining this ratio is . . . one which follows the Ran cycle, where steam at constant pressure is admitted into the cylinder with no cleara

d after the point of cut-off is expanded adiabatically to the back pressure. In obtain-g the economy of this engine the feed water is assumed to be returned to the boiler exhaust temperature.

8. Steam Turbines

Types of Steam Turbines. Steam turbines may be divided into two general asses: (1) The impulse or velocity turbine, which is analogous to the Pelton ater wheel. Steam expands in a nozzle until the pressure drops to the essure in the region in which the turbine wheel rotates, and the jet issuing ith relatively high velocity is directed against the blades of the wheel. (2) The action or pressure turbine, which is analogous to the water turbine. In this pe of turbine the steam flows thru alternate guide blades and moving blades d the pressure gradually falls thru both sets of blades. The essential differ-ces between the two types are shown by the following comparison:

Impulse or Velocity	Reaction or Pressure
1. Drop of pressure in nozzles only.	1. Drop of pressure continuous in guides and moving blades.
2. Jet fills only part of wheel circumference.	2. Turbine runs full.
3. Blades usually symmetrical.	3. Blades necessarily unsymmetrical.
4. Jet velocity high, 1000 to 3500 ft per sec.	4. Steam velocities low, 300 to 900 ft per sec.
5. Speed of blade nearly one-half of jet velocity for highest efficiency.	5. Speed of blade nearly equal to velocity of steam.

Compounding. The high velocity of the steam jet resulting from a consider-le drop of pressure renders desirable some method of compounding in order at the peripheral speed of the turbine wheels may be kept within reasonable nits. In most turbines pressure compounding is used. The total drop of essure $p_1 - p_2$ is divided among several wheels, thus reducing the velocity the jet at each wheel. The arrangement is shown in Fig. 9. Steam passes ru orifices m_1, m_2, etc., in the partitions which divide the interior of the turbine to wheel chambers. The pressure drops from p_1 to p_2 in passing thru the st orifices, then from p_2 to p_3, etc. The curves p and w show roughly the essure and velocity changes. The principle of velocity compounding is shown Fig. 10. The steam is expanded in a nozzle to the back pressure p_2, thus ving a jet of relatively high velocity. The jet passes into the first moving heel, then thru a fixt guide, where its direction is reversed, then into a second oving wheel. A second guide and a third wheel are sometimes added. The rves p and w show the changes of pressure and velocity.

Leading Commercial Turbines. The De Laval turbine is a velocity rbine having a single stage. The steam expands in a diverging nozzle to the al pressure and the jet passes thru a single wheel.

The Rateau turbine is a velocity turbine with pressure compounding, g. 9. The ratio of pressures between successive stages is kept above the itical ratio, so that diverging nozzles are not necessary. The number of essure stages is relatively large.

The Zoelly turbine differs from the Rateau turbine only in constructive tails and in having a smaller number stages.

The Parsons turbine is a reaction or pressure turbine. The moving blades e mounted on a drum or rotor and the rows of moving blades alternate with ws of stationary blades attached to the casing. The entire annular space tween rotor and casing is filled with steam and the pressure drops continuously the steam passes thru the stages. The radial length of the blades is increased om stage to stage to provide for the increasing specific volume of the steam.

The Parsons turbine is made in the United States by the Westinghouse Company and by the Allis-Chalmers Company.

The WESTINGHOUSE DOUBLE-FLOW TURBINE is a combination of a velocity and pressure turbine. The steam at boiler pressure is expanded in a nozzle to a much lower pressure and the jet at high velocity is past thru two sets of moving blades with intermediate guide blades (see Fig. 10). The steam is then led to two rotors on opposite ends of the shaft, and the remainder of the expansion is the same as in the Parsons turbine.

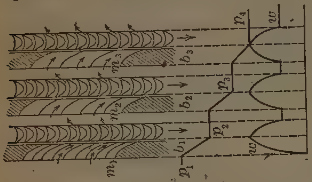

Fig. 9

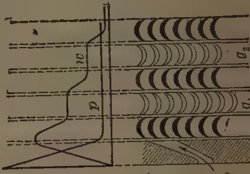

Fig. 10

The CURTIS TURBINE has from 4 to 6 pressure stages, with 2 velocity stage in each pressure stage. That is, there are 4 to 6 wheels, each in a chamber, an each wheel carries 2 rows of blades, with an intermediate row of stationar blades (Fig. 10) attached to the casing. The larger Curtis turbines have ver tical shafts. The shaft is supported by a step bearing which is supplied wit oil under a pressure sufficient to support the weight of the shaft and rotatin parts.

Performance of Steam Turbines. The economy of a steam turbine mea ured in steam consumption per h p-hr is not much different from the econom of a reciprocating engine under the same conditions. With a good vacuum (2 to 29 in of mercury) and with superheated steam the consumption per b h p-h should lie between 10.5 and 14 lb. The efficiency ratios shown by numerou tests lie between 0.60 and 0.72.

Low-pressure Steam Turbines. Because of the limitation of cylind volume, the reciprocating engine is unable to make effective use of an extreme high vacuum. No such restriction applies to the steam turbine, as the blades the final stages may be made long enough to pass the required volume of stea at the lowest pressures obtainable. In general, the reciprocating engine is mo economical than the turbine at high pressures, but the reverse is true at lo pressures; hence a combination of reciprocating engine, to work between boil pressure and atmospheric pressure, and a turbine to take steam from the engi and expand it to condenser pressure, is more efficient than the engine alone turbine alone. Such combinations have found favor in marine engineering. many cases low-pressure turbines have been installed with excellent results existing reciprocating engine plants (see paper by Stott and Pigott, Tra A. S. M. E., 1910). Rateau has applied the low-pressure turbine in cases whe the steam supply is intermittent, as in rolling mills, mine hoists, etc. The engi exhausts at about atmospheric pressure into a heat accumulator, which simply a tank partly filled with water, and the turbine is supplied from th accumulator (see paper by Rateau, Trans. A. S. M. E., vol. 25.).

Relative Fields of Engines and Turbines. The reciprocating engine is most eff tive in any service of an irregular character, where varying speed is required, or where t direction of rotation must be reversed. The moderate rotative speed of the engine is a a decided advantage in many classes of service. The field in which the turbine is preë nent is in central power-station service. The high rotative speed of the turbine facilita

direct connection to electric generators. The turbine can be closely regulated, its efficiency is nearly constant under widely varying loads, it requires small floor space and comparatively inexpensive foundations. For these reasons the turbine has practically displaced the reciproacting engine in central-station service. The turbine has also been applied successfully to marine service both alone and in combination with reciprocating engines. Rateau has been successful in applying the turbine to centrifugal pumps and air compressors; and an extension of this field may be expected.

9. The Internal Combustion Engine

Heating by Internal Combustion. An air engine is one that uses air as the working fluid. The term hot-air engine is usually applied, however, to an engine in which the air is separated from the furnace by a metal wall. Such engines have been failures for the reason that the air absorbs heat slowly and it is impossible to maintain a high temperature in the working fluid. By the method of heating by internal combustion the rapid chemical action supported by the medium itself permits the rapid heating of large quantities of air to a very high temperature. The medium and the furnace being within the cylinder, the metal walls can be kept at a sufficiently low temperature by a water jacket, and the inner surface may be exposed to high temperature without danger of destruction. Engines that make use of the principle of internal combustion are in reality air engines, since the working medium is principally air, but they are generally known as gas engines and oil engines.

Types of Gas Engines. Internal combustion engines fall under two chief classes: (1) The explosion type, in which the mixture of fuel and air is drawn into the cylinder and ignited, thus producing an explosion and a sudden rise of pressure at nearly constant volume. (2) The slow burning type, in which the fuel is introduced gradually and burned quietly without increase of pressure. The Otto engine is a representative of the first type, the Diesel engine of the second type.

Another classification is based on the number of strokes of the piston required for the completion of the cycle. In the four-cycle engine four strokes are required, in the two-cycle engine, two strokes. Engines of small power are usually single-acting; large engines, are, however, usually double-acting. The Nürnberg, Westinghouse, and Snow engines are examples of double-acting four-cycle engines; the Koerting and Oechelhaeuser engines are examples of double-acting two-cycle engines. (For descriptions of various types of gas engines, see Levin's Modern Gas Engine, chap. xiv.)

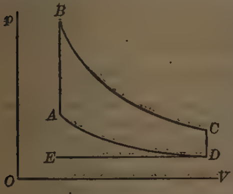

Fig. 11

The Otto Cycle. In Fig. 11 is shown the ideal indicator diagram of the four-cycle Otto engine. The operations are as follows: The explosive mixture is drawn into the cylinder, as represented by ED, and comprest adiabatically (DA). The mixture is ignited, causing a rise of pressure, as shown by AB. The gases in the cylinder, which are products of combustion with an excess of air, expand adiabatically, as shown by BC. Finally, the burned gases are in part expelled from the cylinder, as indicated by DE. As a first approximation it is assumed that the medium thruout the cycle has the properties of air, that the specific heat is constant, and that the medium during the process AB receives a quantity of heat equal to that developed by the combustion of the fuel in the actual cycle. With this assumption, the ideal efficiency of the engine is

$$e = 1 - \frac{T_d}{T_a} = 1 - \left(\frac{V_a}{V_d}\right)^{k-1} = 1 - \left(\frac{p_d}{p_a}\right)^{\frac{k-1}{k}}.$$

It is seen that the higher the compression pressure p_a, the greater the ideal efficiency.

For the ideal case the temperature and pressure at the point B are determined as follows: If q_1 is the heat absorbed per pound of air during the process AB,

$$q_1 = c_v(T_b - T_a), \quad \text{whence} \quad \frac{T_b}{T_a} = \frac{q_1}{C_v T_a} + 1; \quad \text{and since } V_a = V_b,$$

$$\frac{p_b}{p_a} = \frac{T_b}{T_a}, \quad \text{or} \quad p_b = p_a\left(\frac{q_1}{C_v T_a} + 1\right).$$

The temperature and pressure as thus calculated are never realized in practice. In the first place, part of the heat q_1 is absorbed by the water jacket; secondly, the properties of the actual fluid in the cylinder are not the same as the properties of air, and the specific heat of the fluid is not constant but increases with the temperature. The efficiency of the cycle when the calculation is made with the actual contents of the cylinder taken into consideration is not more than 0.8 of the efficiency deduced for the cycle with air as the medium; and the efficiency of the actual engine is still smaller.

Diesel Cycle. In the Diesel oil engine, air without fuel is comprest to a pressure of about 500 lb per sq in. The fuel is introduced into this air by the aid of a separate small air compressor, and burns without explosion. No igniting device is required, as the temperature of the air is raised by compression above the temperature of ignition. The fuel may be cut off early or late, depending on the load. The De La Vergne oil engine resembles the Diesel engine in its general features, but a lower compression pressure is employed.

Horse-power of Gas Engines. The indicated horse-power of a gas engine is given by the formula i h p = $plan/33\,000$, in which n denotes the number of explosions per minute. The mean effective pressure that may be obtained depends upon the kind of fuel. It may vary from 60 to 70 lb per sq in for producer and blast-furnace gas, 85 to 95 lb for natural gas and gasoline, and 95 to 110 lb for alcohol. The brake horse-power of automobile engines is given approximately by the rule of the Association of Automobile Manufacturers, namely: b h p = $d^2N/2.5$, in which d denotes the cylinder diameter in inches and N the number of cylinders.

Performance of Gas Engines. The internal combustion engine has intrinsically a higher thermal efficiency than the steam engine. Based on coal used, the consumption of a gas engine with producer gas is about 30% less than the consumption of a steam engine. This result is obtained under most favorable conditions for both engines.

A large number of tests of a gas engine and gas producer were made by the U. S. Geological Survey at the St. Louis exhibition. The producer was rated at 250 horse-power capacity and the engine at 235 h p at 200 r p m. Tests were made with a large number of coals. With high-grade bituminous coal the weight of coal consumed by the producer per brake horse-power delivered by the engine ranged from 0.95 lb to 1.32 lb. A steam engine of the same power would consume about double the weight of coal.

The following are the results of tests of gas and oil engines of the larger sizes:

Engine	Fuel	Brake h p	Thermal efficiency Per i h p	Thermal efficiency Per b h p
Westinghouse	Natural gas	606	28.6	25.5
Snow	Natural gas	595	29.4	23.7
Borsig-Oechelhaeuser	Coke-oven gas	628	33.0	27.5
Premier	Producer gas	368	33.7	25.6
Koerting	Producer gas	341	34.0	24.1
Westinghouse	Producer gas	500	30.1	25.2
Cockerill	Blast-furnace gas	725	31.5	26.0
Nürnberg	Blast-furnace gas	1186	33.9	28.2
Diesel	Oil	297	45.8	32.2

ELECTRICITY

10. Definitions, Units, Fundamental Laws

Systems of Units. There are two systems of electrical units, both of which are derived from the fundamental or centimeter-gram-second (C G S) system. The ELECTROSTATIC SYSTEM is based on the force exerted between two charges of electricity; the ELECTROMAGNETIC SYSTEM is based on the force exerted between a magnetic field and a conductor carrying a current placed in that field. Many of these units are inconvenient to use in practice, and certain modifications, known as practical units, have been adopted and legalized in a number of countries by international agreement. These practical units are used in all engineering work; C G S units are frequently used in physical and research work.

Electromotive force (symbols E, e, and e m f) is that which causes electricity to flow or tend to flow, and is analogous to hydrostatic head. The practical unit is the VOLT, that electromotive force which, when applied to a circuit of one ohm resistance, will cause a current of one ampere to flow. The standard volt is represented by the Clark and Weston standard cells. The legal value of the Clark cell, when made according to certain prescribed directions, is 1.434 volts at 15° C. The value of the Weston cell, adopted by the U. S. Bureau of Standards, Jan. 1, 1911, is 1.0183 volts at 20° C.

The Volt is equal to 10^8 C G S units. When very small electromotive forces or potentials are to be measured, the unit millivolt, one thousandth of a volt, is frequently used. High potentials are often exprest in kilovolts or thousands of volts. DIFFERENCE OF POTENTIAL is the difference in electrical pressure between two points in an electric circuit and is measured in volts. This term is often incorrectly used instead of electromotive force.

Current (I, i) is the rate of flow of electricity. The practical unit is the AMPERE, which is the current flowing in a circuit of one ohm resistance when an electromotive force of one volt is impressed on it. The legal standard ampere is that steady current which will deposit 0.001118 grams of silver per second when past thru a silver nitrate solution under certain prescribed conditions.

The Ampere is equal to 10^{-1} C G S units. A milliampere is one thousandth part of an ampere.

Resistance (R, r) is that property of a material that opposes the flow of electricity thru it. It varies directly with the length and inversely with the cross-sectional area. The practical unit is the OHM, which is the resistance of a circuit in which a current of one ampere flows when subjected to an electromotive force of one volt. The legal standard ohm is the resistance of a column of mercury 106.3 cm long and 14.4521 grams mass at 0° C. Working standard resistances are made of resistance wire or ribbon carefully adjusted and standardized by comparison with the legal standard. Manganin, a copper alloy, is most extensively used because of its permanency. The usual sizes range from 100 000 ohms to 0.00001 ohm, and always in powers of 10.

The Ohm is equal to 10^9 C G S units. Insulation resistances are usually exprest in terms of the megohm, or one million ohms, and very small resistances in microhms, or millionths of an ohm. RESISTIVITY (ρ) or specific resistance of a material is the resistance between opposite faces of a centimeter cube or an inch cube. CONDUCTANCE is the reciprocal of resistance, and CONDUCTIVITY (γ) is the reciprocal of resistivity. RELATIVE CONDUCTIVITY (usually incorrectly abbreviated to "conductivity") is the term most used in engineering. It is the percentage ratio of the true conductivity (i.e., reciprocal of resistivity) of the material to that of copper of a certain specific gravity and specific resistance known as the International Annealed Copper Standard. Materials are classed as conductors or insulators according as they are of low or high resistivity, respectively INSULATION RESISTANCE refers to the resistance of insulating material measured in

megohms. The DIELECTRIC STRENGTH of an insulating material is the high-potential voltage which ruptures that material and is usually express in volts per mil or per millimeter.

Quantity of Electricity (Q, q) is the product of current and time. The practical unit is the COULOMB, which is one ampere flowing in a circuit for one second.

The Coulomb is equal to 10^{-1} C G S units. The unit in more common use commercially is the ampere-hour, equal to 3600 ampere-seconds or coulombs.

Electrical Energy (W) is the work done in a circuit or an apparatus by current flowing thru it. The practical unit is the JOULE or WATT-SECOND, which is the work done when one ampere flows thru a resistance of one ohm for one second.

The commercial unit is the watt-hour, equal to 3600 joules; also the kilowatt-hour, equal to 1000 watt-hours.

Electrical Power (P) is the rate of expending electrical energy or the rate of doing work per unit of time. The practical unit is the WATT, which is the work done in one second when one ampere flows in a circuit under a pressure of one volt, or $P = I \times E = I^2R$.

The commercial unit is the kilowatt (one thousand watts). One horse-power is equal to 746 watts.

Capacitance or Electrostatic Capacity (C, c) of a circuit or apparatus is the property by virtue of which it can hold a charge of electricity. The practical unit is the FARAD, which is the capacity of a circuit or apparatus that will be charged to a potential of one volt by one coulomb of electricity.

The farad is so large a quantity that commercially the microfarad is generally used The farad is equal to 10^{-9} C G S units; the microfarad to 10^{-15} C G S units.

Self-induction of a circuit is that property which opposes any change in the value of the current flowing thru it. It is analogous to inertia and is due to the magnetic field which surrounds a conductor carrying current. When the current is established or changes, the field increases or decreases and the lines of force cut the conductor, inducing a counter e m f which opposes the change in the current.

Inductance (L, l.), or coefficient of self-induction, of a circuit is the constant by which the time rate of change of the current in the circuit must be multiplied to give the e m f induced in the circuit by such change. The practical unit is the HENRY, which is the inductance of a circuit where a change of one ampere per second will induce one volt e m f.

A henry is equal to 10^9 C G S units. The unit ordinarily used is the millihenry (one-thousandth of a henry).

Impedance (Z) of a circuit is the resistance offered to the passage of alternating current. It depends on the frequency of the current and the resistance, inductance, and capacity of the circuit. It is measured in ohms and is equal to $\sqrt{R^2 + X^2}$.

Reactance (X) of a circuit is that part of the resistance offered to the passage of alternating current which is due to the inductance and capacity of the circuit.

It is measured in ohms and is equal to $2\pi nL - \frac{1}{2\pi nC}$.

Frequency (n) of an alternating-current circuit is the number of complete reversals, or cycles, of the current per second. It is equal to the product of half

the number of poles on the generator and the revolutions per second. An alternation is half a cycle.

The standard frequencies in this country are 25 and 60 cycles, altho 40-cycle systems are in operation. A very few of the early 133-cycle systems are also to be found, and 15 cycles has been advocated for alternating current railways.

Phase refers to the time relation between the current and potential in an alternating-current circuit, or to the time relation between the potentials in two or more circuits.

Thus, if the current and potential in a circuit reverse at the same instant, they are in phase. When the current reverses after the potential, it is said to be out of phase. and a lagging current; when it reverses before the potential, it is out of phase, and a leading current. A two-phase generator has two circuits in its armature and the e m f's generated are ¼ cycle or 90 electrical degrees apart. Similarly, a three-phase generator has three circuits and the e m f's are ⅓ of a cycle or 120° apart.

Power Factor of a circuit is the ratio of the true power passing thru it to the product of the volts and amperes. It is equal to the cosine of the time angle at any instant between the potential and current.

When the time angle is zero, that is, when the potential and current are in phase, the power factor is 1.00 (cosine 0°), often exprest 100%. If they are out of phase by 90°, the power factor will be zero (cosine 90°).

Load Factor. The daily or yearly (or other period) load factor of a machine, plant or system is the ratio of the average power to the maximum power during the period indicated.

The **demand factor** of an installation is the ratio of the maximum load actually taken by the installation to the total connected load (that is the sum of the ratings of all apparatus in the installation).

The **diversity factor** of an installation is the ratio of the sum of the maximum demands of the various loads in the installation to the maximum demand of the whole installation.

Hysteresis is that property of magnetic material which causes the induction corresponding to a given magnetizing force to be greater when the latter is decreasing than when it is increasing.

Hysteresis loss is the energy expended in the material because of hysteresis when the magnetizing force is changed from one direction to the other and back again. It appears in the form of heat.

Eddy Currents are stray local currents induced in those metal parts of electrical apparatus which are in rapidly changing magnetic fields.

Eddy-current loss, which appears in the form of heat, is the power expended by eddy currents flowing in these metal parts, and is equal to the product of the square of the current and the resistance. It is one of the largest losses in alternating current apparatus, and therefore parts in which this loss occurs are laminated in order to reduce the length of the current paths, thereby increasing the resistance and decreasing the currents.

A Series Circuit is one in which the total current passes thru each part of the circuit. A PARALLEL OR MULTIPLE CIRCUIT is one in which the total current is divided among the various parts of the circuit.

Fundamental Laws. The following are the more important laws and principles of electricity and magnetism underlying the practical applications of electricity.

Ohm's Law. The current in a circuit containing no source of e m f is equal to the potential applied at the terminals of the circuit divided by the resistance, or $I = E/R$.

This law applies to alternating-current circuits only when they are non-inductive, that is, contain resistance only. Otherwise the current is equal to the potential divided by the impedance, or $I = E/Z$.

Laws of Series and Parallel Circuits Carrying Direct Current. In a series circuit the total resistance is equal to the sum of the resistances of its component parts. In a parallel circuit the current in each circuit is inversely proportional to the resistance of each branch of the circuit, and the reciprocal of the total resistance of the circuit is equal to the sum of the reciprocals of the branch resistances.

Capacitances. When two or more capacitances (condensers) are connected in series, the reciprocal of the total capacitance is equal to the sum of the reciprocals of the capacitances. If they are connected in parallel, the total capacitance is equal to the sum of the capacitances. It is to be noted that the law is just the reverse of that for resistances.

Conductor in a Magnetic Field. When a free conductor is placed in a magnetic field and current is passed thru it, the conductor will move. The force exerted will be proportional to the field strength and the current. This is the fundamental principle of MOTORS.

Electromagnetic Induction. When a conductor is moved thru a magnetic field or a magnetic field is moved thru a conductor, an e m f will be induced in it which will be proportional to the field strength, the rate of motion, and the length of conductor cutting the field. This is the fundamental principle of GENERATORS.

Fundamental Equation for the Generation of an EMF. The following is the equation for the e m f generated in (*a*) a coil rotating in a magnetic field at a uniform speed or (*b*) a coil which incloses an alternating magnetic flux varying in such a manner that a sine wave e m f is generated (see Art. 11).

$$E = 4.44\,\frac{n\phi f}{10^8}$$

where E = e m f, mean effective value (see Art. 11) in volts;

n = number of complete turns in coil;

ϕ = total magnetic flux (maximum instantaneous value if produced by alternating current) inclosed by the circuit. If the magnetic flux density is uniform (the usual case) throughout the space occupied by the coil, $\phi = BA$ where B = flux density in lines per square centimeter and A = area inclosed by coil in square centimeters.

f = revolutions of coil per second or frequency in cycles per second of magnetizing current.

Law of Magnetic Circuits. In a magnetic circuit the total magnetic flux is equal to the magnetomotive force divided by the reluctance of the circuit, the relation being similar to Ohm's law. The total magnetic flux (lines of force) corresponds to current (amperes), the magnetomotive force (gilberts) to electromotive force (volts), and magnetic reluctance (oersteds) to resistance (ohms). In practice it is customary to deal with the magnetizing force per unit length of magnetic circuit, which, when the magnetic circuit is of non-magnetic material, is equal to $H = 4\pi NI/10\,l$, where H = field strength (gausses), NI = ampere turns and l = length. When the material is magnetic, the magnetic induction (gausses) is $B = kH$, where k = permeability, a quantity which varies with the material and with B.

Galvanic Electricity. If two unlike metals are immersed in an acid or salt solution an electromotive force will be generated between them. The magnitude of the e m f will depend upon the metals and the solution used. This phenomenon is the basis of all electric batteries.

Electrolysis. When electric current (direct) is passed thru a solution (called electrolyte), it is decomposed. According to Faraday's law, the amount of decomposition is directly proportional to the quantity of electricity (product of current and time) passed thru the solution.

Thermoelectric Effect. When a circuit is made up of two dissimilar metals and the two junctions are maintained at different temperatures, an e m f will be established which will be very nearly proportional to the difference in temperature between the two junctions. Conversely if current is passed thru a circuit consisting of two dissimilar metals, heat will be developed at one junction and absorbed at the other junction. This is known as Peltier effect.

Heating Effect. When an electric current is passed thru a solid conductor, the electric energy expended in the conductor is entirely converted to heat energy. The amount of heat developed is proportional to the resistance, the square of the resistance and the time. (Joule's law). That is,

$$H = 0.2928\, I^2RT,$$

where H = heat in British thermal units;
I = current in amperes;
R = resistance in ohms;
T = time in hours.

11. Commercial Electrical Measurements

Commercial and Ordinary Engineering Measurements are made with various indicating instruments which are calibrated by various more or less direct methods against the primary standard or standards representing the quantity indicated by the instrument.

Direct current and potential are measured with ammeters and voltmeters respectively. They are almost universally of the D'Arsonval type, in which a small fine wire coil is free to move in the field of a permanent magnet. In ammeters this coil is connected to a specially made low resistance called a "shunt." Its deflections are proportional to the fall of potential across this shunt and therefore to the current flowing thru it. Voltmeters differ from ammeters only in that the moving coil is connected in series with a high resistance and across the circuit to be measured instead of in parallel to a low resistance which is in series with the circuit.

For moderate current (25 amperes and less) the shunt is contained in the instrument case, but for larger currents it is external to the instrument and connected to the latter by small flexible wires. Thus, shunts as high as 25 000 amperes capacity can be placed in the circuit at the most convenient point. Instruments permanently located on switchboards are called switchboard or station instruments, as distinguished from portable instruments used for testing purposes.

Alternating current is measured with ammeters of several types, the more usual being the iron-disk, inclined-coil dynamometer, and hot-wire types. The principle of iron-disk instruments is the repulsion between the eddy currents induced in a very small iron disk, and the field which produces them. The principle of the inclined-coil and dynamometer instruments is the attraction and repulsion between two coils carrying current. In hot-wire instruments the expansion of a taut wire, when heated by the passage of a current, is utilized to measure the current. Some of the types are made self-contained up to 300 amperes, but current transformers are usually used with larger currents. In all high-tension circuits (over 440 volts) current transformers are used irrespective of the size of the current in order to insulate the instrument from the circuit.

The several VALUES OF AN ALTERNATING CURRENT (and e m f) are (*a*) *instantaneous value* or value at any instant during a cycle, (*b*) *mean effective value* which is the square root of the sum of the squares of the instantaneous values during one cycle (also called effective value and root-mean-square value), (*c*) *average value* which is the arithmetical average of the instantaneous values during one alternation or half cycle, (*d*) *maximum value* which is the maximum of the instantaneous values during one cycle. When the wave form is a sine curve (that is, when the variation of the instantaneous values during one cycle with respect to time follows the sine law) the relations between these various values are as follows:

$$\text{Maximum value} = \sqrt{2} \times \text{effective value};$$

$$\text{Maximum value} = \frac{\pi}{2} \times \text{average value};$$

$$\text{Average value} = \frac{2\sqrt{2}}{\pi} \times \text{effective value};$$

$$\frac{\text{Effective value}}{\text{Average value}} = \frac{\pi}{2\sqrt{2}} \doteq 1.11 \ (= \text{form factor});$$

$$\frac{\text{Maximum value}}{\text{Effective value}} = \sqrt{2} = 1.414 \ (= \text{crest, peak or amplitude factor}).$$

All of these values are employed in electrical engineering but the *effective value* is the one ordinarily measured and used. This particular value is used because it is the value which will produce the same heating effect as a direct current of the same magnitude, and it is the value indicated by all alternating-current instruments used in all ordinary measurements.

Alternating Potentials are measured with voltmeters of the same types as the ammeters, being wound with many turns of small wire instead of a few turns of relatively large wire. Voltage transformers which step-down the voltage to the standard value of 110 volts are used when measuring high voltages.

The secondaries of current transformers are usually 5 amperes capacity, but the instrument scales are marked to read the primary current. Similarly, the scale of the voltmeter may be marked to indicate the line potential altho the voltage on the instrument is usually about 110 volts.

Power in direct current circuits is usually measured with an ammeter and a voltmeter, the power in watts being the product of the amperes and the volts. In alternating current circuits however, power is usually measured with a *wattmeter*. It is a two-circuit instrument, one circuit being a fixed coil of a relatively few turns of relatively large wire, connected in series with the circuit to be measured. The other circuit consists of a coil of many turns of fine wire so mounted as to be free to move within the magnetic field produced by the fixed coil. It is connected in series with a certain amount of resistance and across the circuit to be measured.

In a single-phase alternating-current circuit, having a power factor of 1.00 (which is practically the case where the load is incandescent lamps), the power may be measured with an ammeter and a voltmeter because in that case the power is equal to the product of the amperes and the volts. If, however, the power factor is less than 1.00 (which is always the case where the load is all or partially motors, arc lamps, etc.), the power is equal to the product of amperes, volts and power factor, and a wattmeter is used. (The power may be measured by the less convenient "three-ammeter" or "three-voltmeter" method indicated below.)

In polyphase alternating current circuits, wattmeters are always used because the condition where the power could be measured with a voltmeter and an ammeter, that is unity power factor, is practically never found.

Wattmeter Connections. The following paragraphs show the methods of connecting wattmeters in various kinds of alternating current circuits. Current and potential transformers are used with wattmeters where necessary, the same set of transformers being often used for all three instruments (wattmeter, ammeter and voltmeter) at the same time.

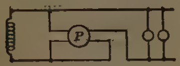

Fig. 12

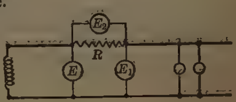

Fig. 13

Single-phase Circuit. One wattmeter connected as shown in Fig. 12 will read true watts. The power may also be measured with three voltmeters or three ammeters.

In the three-voltmeter method, a known non-inductive resistance, R, is connected in series with the load as shown in Fig. 13, where E, E_1, and E_2 are points where voltmeter readings are to be taken. The power in watts is

$$W = \frac{E^2 - E_1^2 - E_2^2}{2R} \qquad \text{(watts)}$$

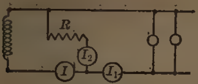

Fig. 14

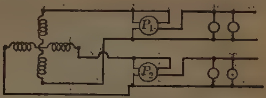

Fig. 15

Similarly, in the three-ammeter method, Fig. 14, the power in watts is

$$W = R\left(\frac{I^2 - I_1^2 - I_2^2}{2}\right) \qquad \text{(watts)}$$

Two-phase, Four-wire Circuit (not interconnected). Two wattmeters, connected as shown in Fig. 15 are sufficient, these conditions being equivalent to two single-phase circuits. The total power is obviously the arithmetical sum of the readings of the two instruments.

Two-phase, Three-wire Circuit. Two wattmeters should be connected as shown in Fig. 16, the total power being the algebraic sum of the two readings. This connection is

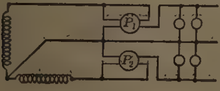

Fig. 16

Fig. 17

correct for all conditions of load, balance and power-factor. One wattmeter may be used as in Fig. 17, provided there is no load across the outer conductors and the phases are balanced as to load and power factor.

Two-phase, Four-wire, Interconnected Circuit. Three wattmeters should be used, connected as in Fig. 18, the total power being the algebraic sum of the three readings. This connection is correct for all conditions of load, balance and power-factor. Two wattmeters, one in each phase, will give the true power only when the load is balanced.

Three-phase, Three-wire Circuits. Two wattmeters should be used, connected as indicated in Fig. 19, the total power being the algebraic sum of the two readings. With

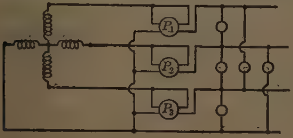

Fig. 18

Fig. 19

a balanced load, each instrument will indicate half the total power at unity power-factor, and at 50 per cent power-factor one instrument will indicate the total power, the other instrument indicating zero. At less than 50 per cent power-factor, one instrument will read negative.

Three-phase, Three-wire Circuits, Balanced Load. When the load is balanced, the power may be measured with one wattmeter by the following methods:

(*a*) With "star" box or artificial neutral as shown in Fig. 20. The total power is three times the reading of the wattmeter. The resistance in each leg of the star box should be non-inductive and small compared with that of the potential circuit of the wattmeter, so that the current taken by the latter will not disturb the potential at the neutral point.

(*b*) With "Y" box as shown in Fig. 21. The total power is three times the wattmeter reading. This arrangement is similar to (*a*), one leg of the star box being re-

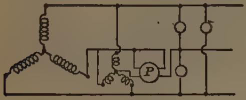

Fig. 20

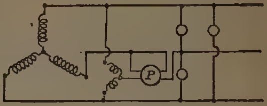

Fig. 21

placed with the potential circuit of the wattmeter itself. The other two legs have the same resistance as the potential circuit of the wattmeter.

(*c*) With a "T" reactance coil as shown in Fig. 22. The total power is twice the wattmeter reading. The impedance of the reactance coil must be small compared with that of the potential circuit of the wattmeter, so that the current taken by the potential circuit will not disturb the potential at *O*.

Three-phase, Four-wire Circuits. Three wattmeters are used as shown in Fig. 23. The total power is the algebraic sum of the three readings. This method is correct

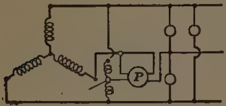

Fig. 22

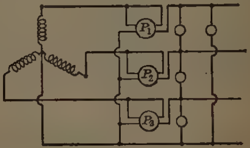

Fig. 23

for all conditions of load, balance and power-factor. A three-phase "star" system with a grounded neutral is virtually a four-wire system and the power should be measured with three wattmeters. Obviously, if the load is balanced, one wattmeter can be used, the total power being the indication of the wattmeter multiplied by three. In that case, the current coil should be connected in series with one conductor or phase wire and the potential coil between that conductor and the neutral.

The Power Factor of a single-phase circuit and of each phase of a two-phase circuit is the ratio of the watts, indicated by a wattmeter, to the product of the volts and amperes. In a three-phase circuit it is the ratio of the total watts to the product of the average volts across the three phases, the average current in the three conductors and 1.73 $(\sqrt{3})$.

Energy is measured with watt-hour meters. They are essentially small motors in which the speed is proportional to the power and the total revolutions is proportional to the energy being consumed. The moving element is geared to a suitable registering mechanism which indicates the kilowatt-hours of energy past thru the meter. On direct-current circuits the whole current must pass thru the instrument. Alternating current meters can be used with potential and current transformers, and in such cases the meter readings are multiplied by the product of the ratios of these transformers. Single-phase energy is measured with one meter, two-phase with one meter in each phase, or with a poly-

phase meter, and three-phase energy is measured either with two meters connected in the same manner as two indicating wattmeters or with a polyphase watt-hour meter which consists essentially of two single-phase meters with a common shaft.

Watt-hour meters are connected into the circuit just the same as watt meters as indicated in Figs. 12 to 23 inclusive. A polyphase meter has separate terminals for each element and is connected just as two wattmeters would be connected.

12. Conductors

Electrical Conductors are materials of comparatively low resistance, thru which electricity will flow in appreciable quantities, as distinguished from high-resistance materials called INSULATORS, thru which electricity will not pass except in minute quantities. There is no sharp distinction between the two, there being good and poor conductors, and good and poor insulators. Most metals are conductors, but only a few are used commercially. Silver is the best conductor, but copper is very nearly as good, and being comparatively plentiful, cheap, and otherwise very suitable, it is by far the most generally used metal for conductors. Aluminum, which has about 62% of the conductivity of copper, is being used to some extent in high-voltage transmission and on heavy current switchboards where minimum weight is desirable. Iron and steel have 10 to 20% of the conductivity of copper and are used in telephone and telegraph lines and in special cases where the current is small and low cost or greater strength is important. There has recently come into commercial use a bimetallic conductor consisting of steel wire with a shell of copper welded on the outside. This material has a conductivity of 30 to 40% of that of copper alone, and a much

Specific Resistance, Conductivity and Resistance-Temperature Coefficient of Conductors

Material	Spec. res., microhms per cm cube at 0° C.	Ohms resistance per mil-foot at 0° C.	Relative conductivity, per cent*	Temp. coef., increase per degree C. from 0° C.
Silver	1.47	8.84	108.2	0.40
Copper, soft †	1.59	9.56	100.0	0.43
Copper, hard drawn	1.63	9.80	97.6	....
Copper, cast ‡	1.8 to 16	10.8 to 96	88.5 to 100	....
Gold	2.22	13.35	71.6	0.37
Aluminum	2.62	15.76	60.6	0.42
Zinc	5.75	34.6	27.6	0.40
Platinum	10.96	65.92	14.5	0.37
Iron	8.85	53.2	18.0	0.63
Iron, soft cast	75.0	451.0	2.1	....
Iron, hard cast	98.0	589.0	1.6	....
Steel, soft	15.9	95.6	10.0	0.42
Steel, glass hard	46	2.76	3.5	0.16
Nickel	6.93	41.7	22.9	0.62
Tin	13.0	78.2	12.2	0.46
Lead	20.4	122.7	7.8	0.43
Mercury	94.1	566	1.7	0.09

* In terms of the International Annealed Copper Standard, (see "Resistance," article 10).

† International Annealed Copper Standard.

‡ In order to make sound copper castings, it is necessary to add small amounts of other materials and these invariably reduce the conductivity.

Bare Copper Wire

(Principally from Circular No. 31, Bureau of Standards)

Size, B & S gage*	Diameter, mils	Area, cir. mils.	Weight, pounds		Length, feet per pound	Resistance, ohms at 25° C. (77° F.)	
			Per 1000′	Per mile		Per 1000′	Per mile
.....	1152	1 000 000	3090	16 320	0.323	0.0108	0.0570
.....	1031	800 000	2470	13 040	0.405	0.0135	0.0713
.....	964	700 000	2160	11 400	0.463	0.0154	0.0813
.....	893	600 000	1850	9 770	0.540	0.0180	0.0950
.....	814	500 000	1540	8 130	0.650	0.0216	0.114
.....	728	400 000	1240	6 550	0.806	0.0270	0.1425
.....	575	250 000	772	4 075	1.295	0.0431	0.2275
0000	528	212 000	653	3 448	1.532	0.0509	0.269
000	470	168 000	518	2 734	1.93	0.0642	0.339
00	418	133 000	411	2 170	2.435	0.0811	0.428
0	373	106 000	326	1 721	3.07	0.102	0.539
1	332	83 700	258	1 362	3.88	0.129	0.681
2	292	66 400	205	1 082	4.88	0.162	0.855
3	260	52 600	163	861	6.14	0.205	1.082
4	232	41 700	129	686	7.75	0.259	1.365
5	206	33 100	102	538.5	9.80	0.326	1.720
6	184	26 300	81	427.5	12.4	0.410	2.165
7	164	20 800	64.3	339.5	15.6	0.519	2.74
8	128.5	16 500	50.0	264	20.0	0.641	3.38
9	114.4	13 100	39.6	209	25.2	0.808	4.27
10	101.9	10 380	31.4	165.5	31.8	1.02	5.39
11	90.7	8 234	24.9	131.5	40.1	1.28	6.76
12	80.8	6 530	19.8	104.5	50.6	1.62	8.55
13	72.0	5 178	15.7	82.9	63.8	2.04	10.75
14	64.1	4 107	12.4	65.5	80.4	2.58	13.62

* Down to and including No. 7 B & S is stranded; No. 8 B & S and smaller is solid. The mass and resistance of stranded wire is assumed to be 2 per cent greater than that of the equivalent solid wire.

These data apply to bare copper wire of 100% relative conductivity (International Annealed Copper Standard). The resistance of hard drawn wire may be considered as about 2.5% higher than these values. The corresponding data for aluminum wire of 61% conductivity may be obtained by multiplying weights by 0.304, feet per pound by 3.29 and resistance by 1.64.

The data for sizes larger than No. 0000 B & S refer to stranded cable and are approximate. They may be in error from 1% to 3%.

greater tensile strength. Poor conductors, such as nickel, nickel alloys of various kinds, and platinum, are used to a great extent where the comparatively high resistance is desirable, as in rheostats and heating devices. Wire made of these materials is known as resistance wire.

The cross-section of electrical conductors is usually measured in circular mils. A circular mil is the area of a circle 0.001 inch diameter. The specific resistance is often exprest in terms of a mil-foot, which is the resistance of a wire one circular mil in cross-section and one foot long. Telephone and telegraph engineers use the term pounds per mile-ohm, which is the weight in pounds of a conductor one mile long and having a resistance of one ohm. The resistance of conductors varies with the temperature, usually increasing in direct proportion to the temperature up to at least 100° C. The increase in resistance per ohm per degree increase in temperature above a given standard

(0° or 20° C.) is the resistance temperature coefficient. The preceding tables give data for the more common conductor materials and copper wire of standard sizes.

Resistance Wires. The following materials are representative of this class of conductors:

German Silver is one of the first materials used for resistance purposes. Alloy of copper, nickel, and zinc (the percentage in the table refers to percentage of nickel). Tends to become brittle if repeatedly heated and cooled and has comparatively high temperature coefficient (0.0002 to 0.0004 per degree C.). Resistance per mil-foot, 195 and 290 ohms, respectively.

Manganin. Alloy of copper, nickel, and manganese. Very low temperature coefficient (0.00001 per degree C.). Permanent if not heated excessively. Used extensively for standard resistances. Resistance per mil-foot, 250 ohms.

"Nichrome." Trade name for a nickel-chromium alloy. Practically non-corrosive and has extremely high melting point (2800 deg. F.). Used extensively in heating appliances and small electric furnaces. Resistance per mil-foot, 600 ohms.

"Advance." Trade name for a copper-nickel alloy. Temperature coefficient practically nil. Durable at moderate temperatures. Used extensively in electrical instruments. Resistance per mil-foot, 295 ohms.

"Therlo." Trade name for a copper-manganese-aluminum alloy. Temperature coefficient practically nil and has a very low thermal e m f against copper. Used in electrical instruments, particularly shunts of ammeters. Resistance per mil-foot, 280 ohms.

Resistance Wire Table

Size, B & S	Diam., mils	Ohms per 1000 feet					
		18% German silver	30% German silver	Manganin	"Nichrome"	"Advance"	"Therlo"
14	64.1	47	71	60	146	71.7	68.3
16	50.8	75	112	94	230	113	107
18	40.3	119	179	153	375	184	175
20	32.0	190	285	244	586	287	273
22	25.3	302	453	382	937	460	437
24	20.1	480	720	605	1 485	725	693
26	15.9	764	1 140	970	2 375	1 160	1 107
28	12.6	1 210	1 820	1 540	3 780	1 850	1 765
30	10.0	1 930	2 890	2 450	6 000	2 940	2 800
32	8.0	3 070	4 610	3 825	9 375	4 600	4 375
34	6.3	4 880	7 330	6 173	15 110	7 400	7 050
36	5.0	7 770	11 600	9 790	24 000	11 760	11 200
38	4.0	12 300	18 500	15 690	37 500	18 400	17 500
40	3.1	19 600	29 400	25 000	66 700	32 700	31 100

13. Direct-current Generators and Motors

Definitions. A GENERATOR is a machine for converting mechanical power into electrical power, and a MOTOR is a machine which converts electrical power into mechanical power. A direct-current generator can also be operated as a motor, but commercially it is not customary to make motors and generators interchangeable because there are minor differences in design.

When a loop of wire is revolved in a magnetic field, an electromotive force is induced, the value of which, at any instant, depends upon the speed of rotation, the strength of the magnetic field, and the size of the loop. The direction of the e m f in such a loop will reverse twice each revolution; hence to obtain direct current a COMMUTATOR is used to

reverse the connection between the external circuit and this coil at the proper moment, thus keeping the polarity of the potential at the terminals the same. A number of such loops wound on an iron or steel CORE together with a suitable commutator constitute the ARMATURE of a generator. The magnetic field is produced by two, or any multiple of two, electromagnets, called POLES, which surround the armature. These poles and the supporting frame which completes the magnetic circuit constitute the FIELD of the machine.

Types of Generators. The various types of direct-current generators may be classified according to the method of exciting the field. SERIES GENERATORS have the field windings connected in series with the armature, and hence the voltage depends upon the load. This type can be designed to give potentials of 2000 to 6000 volts, and a special form is used extensively for series arc-lighting systems. SHUNT GENERATORS have their field windings connected in parallel with the armature. The field contains many turns of relatively small wire and requires only a small current. The voltage is regulated independently of the load by increasing or decreasing the field current by means of a variable resistance (rheostat) which is in series with the field. This type is little used, the COMPOUND GENERATOR being the standard type. In this, both a shunt field and a series field are provided, one winding being placed over the other.

In the plain shunt generator the voltage decreases somewhat as the load increases unless the field current is increased. By winding a few turns of heavy conductor on the poles and passing the load current thru them, the field is automatically increased so that the voltage can not only be kept automatically constant, but even raised to compensate for the drop in voltage in the feeders and mains. This form of compound generator is said to be over-compounded. The compound generator is universally used for lighting and power at 125 and 250 volts, and for railway purposes at 550 to 1200 volts. Any number of compound generators can be operated in parallel, but a special connection between the series fields of the several machines, called an equalizer, is necessary to obtain satisfactory operation.

A Separately Excited Generator is a generator in which the field current is obtained from a source other than the generator itself. It is used only in special cases, such as low-voltage generators for electroplating and for storage-battery charging. BOOSTERS are separately excited low-voltage generators in which the armature is connected in series with a circuit in which it is desired to raise the voltage. They are frequently used on circuits where the voltage is below normal on account of the excessive drop due to heavy loads or long length. MAGNETO GENERATORS have permanent magnets and are only built in small sizes for telephone signals, gas-engine ignition, and similar purposes.

Types of Motors. There are three general classes of motors,—SERIES, SHUNT, and COMPOUND; like generators, they are distinguished by the method of exciting the field. The field of the SERIES MOTOR is in series with the armature and the characteristic feature is a large torque at low speed, which makes it particularly adapted for railways, hoists, and cranes where a high torque is required at starting. SHUNT MOTORS have the fields in parallel with the armature and are essentially shunt generators operated as motors. This is the type in most common use for general power purposes, as its speed decreases only a few per cent from no load to full load. Fig. 24 shows the characteristic relation between speed and torque, also between current and torque for both series and shunt motors. COMPOUND MOTORS have both shunt and series fields, and are either accumulative or differential, according as the series field strengthens or weakens the total field when the load increases. The accumulative form is ordinarily used, for it combines the high-starting-torque feature of the series motor with the constant-speed feature of the shunt motor. Where very uniform speed or an increase in speed is desired, the differential form is used. To prevent excessive sparking in motors subject to rapidly changing loads over wide

ranges, interpoles are provided. These are small poles placed between the regular poles and provided with a winding which is in series with the armature.

Speed Regulation can be effected by changing the voltage applied to the armature or the amount of magnetic flux passing thru it. The former is done either with a regulating rheostat in series with the armature or by having several different voltages available to which the armature can be connected. The usual method of changing the magnetic flux is to change the field current by means of a rheostat. There are also in commercial use, in small motors, methods in which the distance between the pole pieces and the armature can be readily changed with a hand wheel and thus alter the field strength.

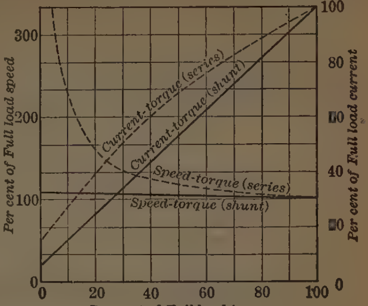

Fig. 24

Starting. Motors are not connected directly to the line when starting because the resistance is so low that an excessive current would flow which would injure the motor. When the motor is running, a generator or counter e m f is produced which opposes the line voltage, and limits the current to the amount required by the load. A motor is started by reducing the line voltage to a low value by inserting resistance (starting rheostat) in series with the armature, the resistance being gradually cut out as the motor comes up to speed. These rheostats are usually provided with attachments which automatically cause the lever to return to the starting position if the voltage is cut off the line.

The Capacity of generators and motors is determined by the temperature that their insulation will withstand continuously without injury. The rating of such machines is therefore more or less indefinite, but the Amer. Inst. Elec. Eng. prescribes in its Standardization Rules that, at rated load and with a room temperature not exceeding 40° C., the maximum rise in temperature above the room of any part of the insulation shall not exceed certain stipulated values for the various types of insulation in general use. These range from 55° to 85° C.

Specifications for generators should state the class of service on which the machine is to operate; type of machine; kind of prime mover; and whether direct-connected or belted; kilowatt capacity; voltage; speed; allowable temperature rise of commutator, field, and armature under continuous operation at full load and two hours at 25% overload; that no sparking should occur at any load from no load to full load with brushes stationary; that the machine should carry 50% overload one hour without injury or serious sparking; efficiency at 25, 50, 75, 100, and 125% loads; and change in voltage from no load to full load. Specifications for motors include similar clauses, except that speed regulation replaces voltage regulation.

Cost and Efficiency of generators and motors will vary with the conditions of use. In the table on p. 1612 are given figures for average, conservative conditions.

The cost of motors and generators varies greatly with the speed. Standard slow-speed machines are 30 or 40% more expensive than standard high-speed machines of the same capacity. The efficiency falls off, with decrease in load, about 1% at 75% load, 2 to 3% at 50% load, and 7 to 15% at 25% load.

14. Alternating-current Generators and Motors

Alternating-current Generators, frequently called alternators, are essentially direct-current machines without commutators, the armature circuit being con-

Average Efficiency and Cost of Direct-Current Generators and Motors

Generators			Motors		
Size, k w	Efficiency at full load, per cent	Cost, dollars per kw	Size, h p	Efficiency at full load, per cent	Cost, dollars per h p
5	87	70	1	75	70
10	88.5	50	2	80	65
25	90	40	3	83	60
50	91	35	5	85	50
100	92	30	7.5	87	42.5
200	93	25	10	88	35
500	94	22.5	15	89	30
1000	94.5	20	20	89	25
1500	95	19	50	90	20
2000	95	18	100	91	17.5

For a standard starting rheostat, add for example about 10% for a 1-h p motor and about 5% for a 100-h p motor.

nected directly to the outside circuit. The connection between the armature and the outside circuit is not reversed, every time the winding passes from a north pole to a south pole, and vice versa, as in a direct-current generator. Thus an e m f is produced at the machine terminals which rises from zero to a maximum value in one direction, decreases to zero, reverses, increases to a maximum in the other direction, and again decreases to zero in the time that the winding passes under one north pole and one south pole. Such a cyclic period is called a CYCLE and the FREQUENCY of the current is the number of cycles per second.

Types of Generators. The most common forms of alternating-current generators may be divided into two types. In the REVOLVING-ARMATURE type, the armature is the moving member and the field is stationary, current being delivered to the circuit by means of BRUSHES resting on the revolving collector rings. In the REVOLVING-FIELD type, the field is the moving member, while the armature is stationary. It is wound in the frame which surrounds the field.

The revolving-armature type is confined to generators of less than 200 or 300 kw capacity. The stationary-armature type permits the direct generation of potentials up to 15 000 volts and the economical construction and operation of large capacity generators for a wide range of speeds, from that of the low-head water wheel to that of the steam turbine.

Windings. There are three kinds of alternators, based on the number of circuits in the armature. A SINGLE-PHASE generator has only one circuit in its armature and two terminals. In a TWO-PHASE generator there are two circuits which generate e m f's 90 electrical degrees apart. When each circuit or phase is brought out separately there are four terminals, for connection to a two-phase four-wire circuit. Frequently, the two phases are connected to each other and to the middle one of three terminals, for connection to a two-phase three-wire circuit. In a THREE-PHASE generator there are three circuits or phases in which three e m f's 120 electrical degrees apart are generated. These circuits are interconnected with either a delta or a star connection requiring three or four terminals respectively.

The following diagrams indicate the relation between the currents and voltages in these various forms of generators and the circuits to which they are ordinarily connected. I and i are the amperes per phase, and E and e the volts per phase in armature and circuit respectively.

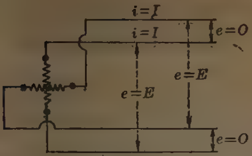

FIG. 25. Two-phase Four-wire Circuit

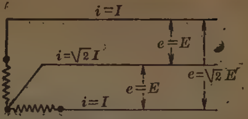

FIG. 26. Two-phase Three-wire Circuit

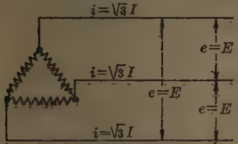

FIG. 27. Three-phase, Three-wire Circuit, Delta connected

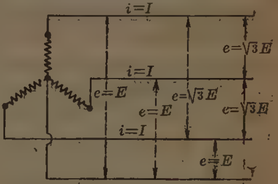

FIG. 28. Three-phase Four-wire Circuit, Star connected

Field Excitation. Alternators are usually separately excited. Small ma-:hines are sometimes provided with a commutator device by means of which ;ufficient current is rectified to excite the field. A common practice is to pro-vide a separate generator or exciter for each alternator. This machine is driven rom the alternator shaft by a belt or by direct mechanical connection. In large tations, a source of excitation common to all the generators is usually provided.

Efficiency and Cost. The following table gives average conservative figures for the fficiency and cost of polyphase alternators. The cost figures, in particular, are neces-arily approximate. Slow-speed machines cost more than high-speed machines, and high-oltage more than low-voltage machines. Hence there may be a difference of at least 5% in the cost of generators of the same capacity.

Efficiency and Cost of Polyphase Alternators

Size, kw	Efficiency			Cost, dollars per kw
	50% load	75% load	100% load	
50	81.0	83.0	86.0	18.00
75	82.5	84.5	87.5	15.00
100	84.5	86.5	89.0	13.00
200	85.5	87.5	90.0	11.00
300	87.0	89.0	91.0	10.00
500	88.0	90.5	92.0	9.50
750	89.0	91.5	93.0	9.25
1000	90.5	93.0	94.0	9.00
1500	91.0	93.5	94.5	8.00
2000	91.5	94.0	95.0	7.50
3000	92.0	94.5	95.5	6.50
5000	92.5	95.0	96.0	5.00

Motors. There are two general types of alternating-current motors,—the ynchronous and the induction. The SYNCHRONOUS MOTOR is essentially an

alternator operated as a motor. It has to be brought up to synchronous speed (the speed at which it would run if operated as a generator at the line frequency) by external means before it can be connected to the line. It will operate only at synchronous speed.

Synchronous motors are used as a rule only in large sizes (200 h p and over) and where it is not necessary to stop the machine frequently. Such cases are in substations where power is converted from one frequency to another or from alternating current to direct current, and in large factories.

Induction Motors are, theoretically, transformers in which the core and the secondary winding are free to move, and the force which the windings of a static transformer exert on each other is utilized to produce mechanical power. When the rotating member (rotor) runs at synchronous speed, no magnetic flux from the stationary member (stator), which is connected to the line, cuts the conductors, and hence no torque is exerted. When a load is applied to the motor, the speed falls below synchronism (called slip), and a current is induced in the rotor conductors which will produce the necessary torque to carry the load. Fig. 29 shows the characteristic relation between speed and torque, also between current and torque, of a polyphase induction motor.

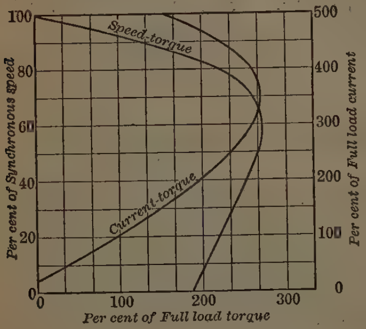

Fig. 29

Polyphase Induction Motors (both two and three phases) are the most commonly used type. Their characteristics are similar to those of the shunt motor, in that they have a comparatively low starting torque and operate at approximately constant speed. Polyphase motors are built in all sizes from fan motors to 1000 h p and over, and are used for all power purposes. There are two forms,—the SQUIRREL CAGE, in which the winding of the rotor consists of heavy copper bars short circuited on each other, and with no external connections; in the SLIP-RING form, the rotor has a regular polyphase distributed winding, into the phases of which an external variable resistance is connected thru collector rings on the shaft. The slip-ring form has a larger torque at starting than the squirrel-cage form, but its efficiency is somewhat lower. When induction motors are overloaded a certain definite amount they will stop. This load is called the maximum torque, or "stalling" load and is two or three times normal load.

Single-phase Motors are infrequently used because special devices are necessary to make them self-starting, and they are more expensive and less efficient than polyphase motors.

The Series Alternating-current Motor is essentially a direct-current series motor with certain modifications which are necessary for operation on alternating current. It is a single-phase motor and is used in traction work where variable speed and variable torque are required. The repulsion motor is one of several forms of the series alternating-current motor.

Operating Data and Cost. In the following table are given approximate average values for the slip, power factor, efficiency, and cost of standard 60-

cycle polyphase motors. The efficiency and cost of 25-cycle motors are slightly higher. As with all rotating electrical apparatus, the cost varies greatly with the speed—as much as 50% in some cases.

Polyphase Induction Motor Data

Standard sizes, h p	Slip, per cent	Power factor, per cent	Efficiency, per cent	Cost, dollars per h p
¼	20	70	65	90
½	15	75	70	80
1	10	75	75	70
2	8	78	78	60
3	7	80	80	55
5	6	82	82	45
7½	6	84	84	40
10	6	85	85	35
15	6	86	85.5	30
20	5.5	87	86	25
30	5	88	87	20
50	4.5	89	88	17.5
75	4.0	90	88.5	15
100	3.5	91	89	13
150	3.0	91	90	12.5
200	2.5	92	90	12
300	2.0	92	90.5	10.5
500	2.0	92	91	9

Starting. If an induction motor is connected directly to the line when starting it will take a current two to four times the normal full-load running current. To avoid the disturbance which this produces in the system, it is customary to start at a reduced voltage. This is obtained by means of a COMPENSATOR, a single-winding transformer with several taps on it to which the motor is successively connected. The slip-ring type is started with full-line voltage on the motor but with resistance in the rotor circuits. This resistance is gradually cut out as the motor speed increases.

Speed Variation. The inherent characteristics of induction motors are such that the speed is practically constant at a value depending on the frequency and the number of poles in the stator winding. Certain fixt changes in speed are sometimes obtained by providing a special winding by means of which the number of poles can be changed. Change in speed may also be obtained with a variable resistance in the rotor circuits of a slip-ring motor, but the efficiency is materially decreased.

Rating. Alternating-current apparatus, like direct-current apparatus, is rated on the basis of the load which it will carry without overheating. In alternators, this rating is the output in kilovolt-amperes, and in motors it is horse-power. The recommendations of Amer. Inst. Elec. Eng for the limiting temperature rise above a room temperature not exceeding 40° C., vary from 55° C. to 85° C, depending upon the class of insulation used.

Specifications for ALTERNATORS should include, in addition to a description of the type of machine desired and the class of service in which it is to be used, certain performance requirements as follows: Efficiency at various loads from 50% to 125% normal load; voltage regulation (per cent rise in voltage when the load is reduced from normal value to zero) at 100% and 80% power factor; maximum voltage required to excite the field with 125% normal load at 80% power factor; temperature rise of the various parts after continuous operation at full load and 25% overload; ability to withstand 50% overload for one hour without injury; and ability to withstand running at 100% overspeed without injury. Specifications for INDUCTION MOTORS should include: description of type of motor desired; purpose for which they are to be used; efficiency; power factor and slip at various

loads up to 25% overload; maximum torque at starting when connected directly to the line; maximum current required when starting; maximum torque and maximum load which motor will carry without stopping; temperature rise after continuous operation at full load and after two hours at 25% overload; and ability to carry 50% overload one hour without injury.

15. Power Plants

The term Central Station usually refers to a plant operating under a franchise, for the manufacture and sale of electricity to the general public. It is therefore a semipublic enterprise as distinguished from a private plant, in which electricity is generated for private purposes. Central stations vary in size from 50-kw plants in small villages to the large stations in New York and Chicago of over 100 000-kw capacity. Private plants range from single-unit plants for lighting buildings to the large traction plants of over 50 000 kw capacity.

Location. The location of HYDRAULIC PLANTS is obviously at the source of water power. Many conditions govern the location of a STEAM-DRIVEN central station. The more important are: kind of service to be rendered, whether alternating or direct current; concentrated or scattered area of distribution; size of plant; cost of real estate; and transportation facilities. Large steam plants are located on waterways wherever possible on account of cheap transportation and ample water supply for condensers and boilers. Where a waterway is not available, cheap land, railroad facilities, and water free from scale-forming impurities are the more important considerations. On the other hand, it might be more economical for a smaller plant to be located as near the center of distribution as possible, regardless of other disadvantages. GAS-DRIVEN plants are usually private plants, and hence are located on the same premises as the load. Private plants in buildings primarily used for other purposes should be located with due regard to the following points: effect of vibration and noise, ventilating facilities, facilities for handling coal and ashes; value of the space for other purposes, and coal storage facilities.

Prime Movers. The prime movers in common use for electric power generation are reciprocating steam engines, high- and low-pressure steam turbines, internal combustion engines, and hydraulic turbines.

The Reciprocating Steam Engine is the most commonly used steam-driven prime mover. The principal types used for electric power generation are the simple and compound high-speed engines in small sizes, the various forms of slow-speed horizontal Corliss engines in medium sizes, and the special vertical types which have been developed in 5000 to 10 000 h p sizes for large central stations. Small single-cylinder engines receive steam at 75 to 100 lb pressure per sq in, and exhaust it at about atmospheric pressure. Large engines receive steam at 100 to 200 lb pressure and discharge it into a vacuum at 2 to 4 lb absolute pressure.

In Steam Turbines the steam issues from nozzles at a high velocity and impinge on blades set in the periphery of a disk free to move. Turbines are rapidly supersedin reciprocating engines for power-station purposes because they are simpler, occupy muc less space, are more efficient, can be used with steam superheated to a much highe degree than is permissible with reciprocating engines, and can be built in much large units,—machines up to 70 000 kw capacity are now in daily operation. Furthermore high-speed generators, which are cheaper than low-speed generators, can be used.

Low-pressure turbines receive steam at about atmospheric pressure and discharge it int a vacuum. A much greater percentage of the energy in steam below atmospheric pressur can be utilized in a turbine than in the low-pressure cylinder of a reciprocating engine The capacity of existing plants using condensing engines can be increased 10 to 30% by exhausting from the engines at atmospheric pressure into low-pressure turbine This additional capacity is obtained without additional fuel consumption. At th

same time the range of loads at which maximum economy is obtained is greatly increased.

Internal Combustion Engines are operated with natural, illuminating, producer, and blast-furnace gases, with kerosene oil and with gasoline. Where liquid fuels are used, an auxiliary device vaporizes the liquid before it is admitted to the cylinders. This very efficient class of prime movers has been handicapped for electric power plant work because of non-uniform angular velocity, unreliability of operation, limited overload capacity, and high maintenance cost. These faults are rapidly being overcome and gas engines are being used quite extensively, particularly where very cheap fuel is available, such as the waste gases from blast furnaces. Engines of 2000-h p capacity are in successful operation on this fuel. The thermal efficiency of the gas engine is from 20 to 30% and is the highest of all heat engines. The cost of power is therefore very low even in small sizes, especially in connection with producer-gas plants, where cheap grades of coal can be utilized.

Hydraulic Turbines in power plants where the head is less than 150 ft are usually of the pressure type. The turbines in the older plants are of the vertical type, but the lower cost and more reliable operation of horizontal generators has led to the general use of horizontal turbines. Where the head is high the impulse type of turbine is used.

Approximate Steam and Fuel Consumption of Prime Movers

	Pounds of steam per kw-hr	Pounds of coal per kw-hr
Steam Reciprocating Engines:		
Simple, non-condensing, 25 to 200 h p	200 to 75	30 to 10
High-speed, automatic, compound, condensing, 200 to 1000 h p	75 to 50	10 to 6
Simple, Corliss, condensing, 150 to 500 h p	60 to 40	7 to 5
Compound, Corliss, condensing, 500 to 5000 h p	40 to 20	5 to 2.25
Compound and triple, four-cylinder, condensing, 2000 to 10 000 h p	22 to 17	2.2 to 1.7
Steam Turbines, high pressure:		
Non-condensing, 200 to 500 kw	60 to 40	10 to 5
Condensing, 500 to 3000 kw	25 to 18	3 to 1.8
Condensing, 5000 to 20 000 kw	17 to 13	1.7 to 1.3
Steam Turbines, low pressure, combined with reciprocating engines:		
One 1000-kw engine with one turbine	20	2.5
8000-kw engine plant with one turbine	14	1.5
One 7500-kw engine with one turbine	12	1.2
Gas Engines:		
Natural gas, 50 to 200 h p	....	25 to 15 cu ft
Producer gas, 50 to 200 h p*	....	3.0 to 2.0
Illuminating gas, 10 to 75 h p	....	35 to 25 cu ft
Gasoline, 10 to 75 h p	....	2.0 to 1.2 pints
Oil Engines:		
100 to 500 h p	....	1.5 to 1.0 lb oil

* Complete plant.

Boilers for the production of steam are built in several different types, the most commonly used for power-plant purposes being the water-tube and fire-tube. The WATER-TUBE BOILER is the safest, most efficient, and can be operated at the highest pressure. The small FIRE-TUBE BOILER is cheaper and easier to operate than the small water-tube boiler, and hence is preferable for small plants. Boilers are arranged to reduce the piping to a minimum and make firing as convenient as possible. They are usually placed side by side in one row or in two rows facing each other. All are piped to a common pipe or header, but in such a way that each boiler and group of boilers can be cut off by conveniently located valves.

Rating. Boilers are rated in horse-power, an arbitrary unit equal to the evaporation of 34.5 lb of water per hour from water at 212° F. to steam at 212°. Boilers properly rated should develop their rated horse-power when fired by an ordinary fireman and with ordinary steam coal. They should be capable of giving 30 to 50% over their rating when forced.

Steam Piping should be well lagged and free from pockets where condensed steam can collect. All valves should be easily accessible. The following table gives the approximate size of piping required to deliver any quantity of steam at any distance from the boiler. This table is based on Babcock's formula which is as follows:

$$p = 0.0001321\left(1 + \frac{3.6}{d}\right)\frac{M^2L}{Dd^5}$$

where p = difference in pressure between the two ends of the pipe, lb per sq in;
d = internal diameter of pipe, inches;
M = amount of steam, lb per min;
L = length of pipe, feet;
D = mean density of steam, lb per cu ft.

It is to be noted that authorities do not agree on what is the correct formula for steam flow so that for exact calculations reference should be made to the more extensive discussions of the subject in Lucke's "Engineering Thermodynamics," Marks' "Mechanical Engineer's Handbook" (p. 360), etc.

Flow of Steam through Pipes (Babcock)

Pounds per minute with 1 lb loss of pressure

Initial gage pressure, lb per sq in	Diameter of pipe, inches. Length, 240 diameters								
	1	1½	2	3	4	5	6	8	10
50	4.04	11.2	20.01	49.48	91.34	150.8	226.0	412.2	665.0
60	4.32	11.9	21.38	52.87	97.60	161.1	241.5	440.5	710.6
70	4.58	12.6	22.65	56.00	103.37	170.7	255.8	466.5	752.7
80	4.82	13.3	23.82	58.91	108.74	179.5	269.0	490.7	791.7
90	5.04	13.9	24.92	61.62	113.74	187.8	281.4	513.3	828.1
100	5.25	14.5	25.96	64.18	118.47	195.6	293.1	534.6	862.6
120	5.63	15.5	27.85	68.87	127.12	209.9	314.5	573.7	925.6
150	6.14	17.0	30.37	75.09	138.61	228.8	343.0	625.5	1009.2

For any other loss of pressure, multiply by the square root of the proposed loss. For any other length of pipe, divide 240 by the given length expresst in diameters, and multiply the table figures by the square root of this quotient to get the flow for one pound loss of pressure.

The resistance due to steam entering pipe = 60 diameters additional length; to a globe valve = 60; to an elbow = 40, or two-thirds of a globe valve.

Furnaces should be designed for the particular class of fuel to be used. The object of a furnace is to transfer the heat in the fuel to the water in the boiler with the least loss; hence there is a certain kind of grate and arrangement of fire-walls, baffles, plates, etc., best suited for each fuel. The average grate surface provided per boiler horse-power is about ¼ to ½ sq ft. Furnaces for oil or gas fuel are the same as coal-burning furnaces, except that special burners are substituted for grates.

Combustion. The sources of inefficiency in a furnace are, coal passing thru the grates unburnt and incomplete combustion. The unburnt coal in the ashes should not exceed 2 or 3%. Complete combustion means that the carbon — the source of heat in the fuel — is completely oxidized to carbon dioxide. If oxidized only to the monoxide, about two-thirds of the available heat is wasted. Incomplete combustion is usually caused by insufficient air supply. Too much air, on the other hand, not only lowers the temperature of the furnace but absorbs heat which is carried up the chimney. Frequent

analysis of the flue gases and ash is necessary to insure that the furnace is being properly operated.

Draft. Air is supplied to the furnaces by natural draft thru chimneys or by forced draft. NATURAL DRAFT is more commonly used and various formulæ have been developed to express the relation between the internal area and height of chimney required per horsepower of boilers. Kent's formula, based on the rather liberal allowance of 5 lb of coal per boiler h p, is $HP = 3.33\,(A - 0.6\sqrt{A})\,H$, where HP = horse-power of boiler plant, A = inside area in square feet and H = height in feet. Self-supporting steel chimneys cost about one-third as much as brick chimneys, and are therefore replacing brick for plants of 100-h p capacity and less. FORCED DRAFT, either by pressure in the ash pit made air-tight or by suction in the flues, is used to a considerable extent in large power plants. The maximum capacity of the plant can be greatly increased with forced draft, and greater economy, under fluctuating load conditions, is obtained.

Mechanical Stokers are devices for automatically carrying the coal under the boilers, properly burning it, and depositing the ashes in the ash pit. The principal forms are the rocking grate, endless chain grate, and the screw feed. They have many advantages when used in large plants, principal among which are economy in labor, economy in fuel because cheaper fuels may be used, and more uniform fires. Their use is not advisable in comparatively small plants where the total load varies thru wide ranges, hand firing under such conditions being more flexible and economical.

Approximate Cost and Heating Value of Fuels (1912)

Fuel	Cost, dollars	B t u	B t u per dollar
Anthracite	6.50 per ton	13 000 per lb	4 000 000
Bituminous	3.00 per ton	14 000 per lb	9 300 000
Coke	5.00 per ton	13 100 per lb	5 250 000
Wood, hard, dry	4.00 per cord	6 250 per lb	7 000 000
Crude petroleum	0.035 per gal	19 000 per lb	4 000 000
Kerosene	0.12 per gal	17 000 per lb	1 100 000
Illuminating gas	1.00 per M cu ft	750 per cu ft	750 000
Natural gas	0.25 per M cu ft	1 000 per cu ft	4 000 000
Gasoline	0.15 per gal	18 000 per lb	620 000

Generators. The type of generator used in a power plant varies with the conditions. Where the load is concentrated, near at hand, and mixt lighting and power, direct-current generators are used. Since, however, the cost of power production decreases with the size of the units and the size of the plant, the trend of modern engineering is toward the concentration of power production in large stations with large alternating-current generator units (single units of 70 000 kw capacity of the compound type are now in service), the power being distributed at high tension to distributing centers called substations. The selection of the size and number of units for a plant requires careful consideration. Both prime movers and generators decrease rapidly in efficiency below 50 or 60% load, hence the sizes should be such that those in use will always be well loaded. There should be enough units to carry the maximum load, with one as a reserve. For example, a 200-kw plant ought to have three 100-kw units, a 600-kw plant four 200-kw units, and a 1000-kw plant three 500-kw units.

The Foundations for all apparatus should be separate from that of the building. They should be of ample dimensions and never less than those recommended by the manufacturer. Direct-connected generators, motor generators, and rotary converters, altho self-contained, should have substantial foundations in order to reduce the vibration and consequent wear on the bearings and insulation. Concrete foundations are the best and cheapest: they will safely carry loads of 7500 lb per sq ft. The safe bearing loads of various soils

are as follows: clay 4000 lb, coarse gravel and sand 2500 to 3500 lb, rock 10 000 to 30 000 lb.

Switchboards are designed to provide not only independent, easy and quick control of each generator, exciter, and feeder, but to show by means of instruments mounted on them the amount and character of the load. They are made up in panels of slate, marble, and artificial stone placed side by side, one panel for each machine and feeder. The board is usually installed along one side of the generator room with ample space in the front and rear. In low-tension alternating-current and direct-current plants, all circuits are brought to the switches and instruments on the board. Buses or heavy copper bars are mounted on the back of the board (often in duplicate) and connected to the various panels so that any number of generators and feeders may be connected in parallel. The circuits of high-tension plants are not brought to the switch-board but are controlled by switches under oil which are operated mechanically or electrically from the switchboard, the instruments being connected in the secondary circuits of instrument transformers.

The following distribution of items which make up the cost of power in various kinds of plants has been made by H. G. Stott (Power-Plant Economies, vol. xxv, Trans. Amer. Inst. Elec. Eng.).

Relative Cost of Power

Items	Reciprocating engines	Steam turbines	Reciprocating engines and steam turbines combined	Gas-engine plants	*Gas engines and steam turbines combined
Maintenance:					
1. Engine, room, mechanical	2.57	0.51	1.54	2.57	1.54
2. Boiler room or producer room	4.61	4.30	3.52	1.15	1.95
3. Coal- and ash-handling apparatus	0.58	0.54	0.44	0.29	0.29
4. Electrical apparatus	1.12	1.12	1.12	1.12	1.12
Operation:					
5. Coal- and ash-handling labor	2.26	2.11	1.74	1.13	1.13
6. Removal of ashes	1.06	0.94	0.80	0.53	0.53
7. Dock rental	0.74	0.74	0.74	0.74	0.74
8. Boiler-room labor	7.15	6.68	5.46	1.79	3.03
9. Boiler-room oil, waste, etc.	0.17	0.17	0.17	0.17	0.17
10. Coal	61.30	57.30	46.87	26.31	25.77
11. Water	7.14	0.71	5.46	3.57	2.14
12. Engine-room, mechanical labor	6.71	1.35	4.03	6.71	4.03
13. Lubrication	1.77	0.35	1.01	1.77	1.06
14. Waste, etc.	0.30	0.30	0.30	0.30	0.30
15. Electrical labor	2.52	2.52	2.52	2.52	2.52
Relative cost of maintenance and operation	100.00	79.64	75.72	50.67	46.32
Relative investment	100.00	82.50	77.00	100.00	91.20

* A proposed combination where the exhaust gases from the gas engine would be used to generate low-pressure steam for low-pressure turbines. Both of these machines have a high efficiency and each would be used under the most favorable conditions.

The very large modern stations generate high-tension alternating current and all generators and feeders are controlled from a miniature remote-control switchboard. The main switches are operated by motors controlled by small switches on this control board. One operator can thus handle all of the outgoing power of a very large plant.

Direct-current generator and feeder panels are usually provided with knife switches, an automatic overload circuit breaker, a voltmeter and ammeter. Low-tension alternating-current panels are similarly equipped, with the addition of a wattmeter and a power-factor meter.

Cost of Power Plants. The cost of power plants depends upon location, type of plant, class of service, size, etc. HYDRAULIC PLANTS are on the whole less expensive than steam plants, tho in many cases the cost per kilowatt has far exceeded that of the best steam plants because of the great cost of impounding the water. The average investment per kilowatt will range from $50 to $150 per kilowatt, but some plants cost $400 per kilowatt. This is divided, in a typical case, about as follows: hydraulic works 50%, wheels and fittings 15%, building 5%, generators, exciters, and switchboard 20%, and step-up transformers 10%. STEAM PLANTS cost from $100 to $200 per kilowatt, and an average approximate division of the cost is: boilers and piping 15%, engines and condensers 25%, pumps and other auxiliaries 5%, generators and switchboard 20%, building, foundations, and smokestack 25%, coal-handling plant 5%,and engineering 5%.

The Cost of Producing Energy depends upon a great many factors, principal among which are the kind and size of plant, the cost of fuel, labor, and supplies, and the load factor. In hydraulic plants it will range from 0.4 to 1.0 cent per kilowatt-hour delivered to the transmission line. An average division of cost will be about as follows: interest and depreciation 65%, labor 15%, maintenance and supplies 20%. The average cost per kilowatt-hour at the bus-bars in steam plants may be taken at about 3 cents for 250-kw plants, 1.75 cents for 500-kw plants, 1.5 cents for 1000-kw plants, and 1.25 cents for 2000-kw plants. Very large plants of 50 000 kw and over probably produce power for less than 0.5 cent per kilowatt-hour. The cost in gas-engine plants will be less than in steam plants of the same size. This is particularly true of the smaller sizes. Producer-gas plants of 400 or 500 kw can produce energy at a cost of 1 cent per kilowatt-hour.

16. Transmission of Power

Losses of Electrical Power occur in transmission, the principal one of which is that due to the resistance of the conductors. The amount of loss which may be economically allowed will depend upon several factors such as cost of producing the power, the line investment, size of the load, load factor and the value of the power at the point of delivery. Much greater losses may be allowable and much larger distances covered under certain conditions than would be financially permissible under others.

When current flows thru a conductor, a loss occurs which is equal to the product of the current and the fall in potential between the ends of the conductor. From Ohm's law it follows that with a given percentage loss the distance that power may be transmitted will be proportional to the square of the voltage. High-voltage direct current is not commercially practicable, for altho a moderately high voltage may be produced by connecting generators in series, it cannot be readily reduced to low voltage for general distribution. On the other hand, alternating current can be transformed from low to very high voltage and vice versa with very simple and efficient apparatus called transformers.

It is now standard practice in large concentrated systems to generate all of the power in one station and transmit it at 2200 to 15 000 volts to various substations from which it is distributed at low potentials. The distance in such cases is, however, comparatively short. Where the electricity is generated by water power, it is usually necessary to transmit long distances to reach a sufficiently large market. In such systems it is customary to generate at 2200 to 15 000 volts and transform to 6600 to 150 000 volts, depending upon the distance. About 150 000 volts is at present the highest voltage in commercial use

and about 500 miles the greatest distance of transmission. Transmission at 200 000 volts is being considered but at this voltage the energy dissipated into the air (corona loss) begins to increase rapidly, particularly at high altitudes and specially designed conductors may have to be employed.

Transformers are essentially large induction coils without interrupters. Two coils are wound on a common core built up of laminated steel. They are thoroly insulated from the core and from each other with varnished cambric and similar materials. When alternating voltage is applied to one coil, the primary winding, an alternating flux is produced in the core. This flux induces an e m f in the other coil, the secondary winding. The ratio of the voltages will be practically the same as the ratio of the number of turns of wire in the two coils.

Classification. Transformers may be classified according to their application, whether for power purposes or for use with instruments. Power transformers may be either constant potential or constant current. The CONSTANT-POTENTIAL transformer is the most usual form and is used for all general power transmission at constant potential. Power from two-phase generators is transmitted with two transformers, one in each phase. Power from three-phase generators is transmitted with three transformers connected delta or star and with three-phase transformers in which three transformers are combined by placing three sets of windings on three parts of a common magnetic circuit. The three-phase transformer is cheaper than three single transformers, but the latter method is more flexible and permits operation with only two transformers if one burns out. CONSTANT CURRENT transformers convert power from low-voltage constant potential to constant current at variable high voltage. They are used for series lighting systems. INSTRUMENT TRANSFORMERS are used only to insulate instruments from the line and have only 10 to 200 watts power capacity. VOLTAGE transformers are connected to voltmeters and wattmeters. They transform the high voltage of the line to about 110 volts. CURRENT transformers are connected to ammeters and wattmeters. They insulate the instrument from the line, and also transform the current to a small value, usually 5 amperes.

Rating. The capacity of transformers is determined by the maximum temperature reached in operation. This is fixt at values which will not cause deterioration of the insulating materials. The Amer. Inst. Elec. Eng. recommends a temperature limit corresponding to 55° to 85° C. rise above a standard room temperature of not exceeding 40° C. after continuous operation, the limit of permissible rise depending upon the class of insulation employed. Various methods are used to dissipate the heat and thus increase the capacity. In AIR-COOLED transformers (a type which is, however, falling into disuse because of the greater space required per kw, lower efficiency, higher cost, and greater fire risk) the case is open at the top and bottom and air is forced thru the core and windings by blowers. In SELF-COOLED transformers the case is filled with oil. The oil, in addition to increasing the insulation, carries the heat by natural circulation from the core and coils to the sides of the tanks, from which it is dissipated by radiation and convection. The surface of the tank is made as large as possible by corrugating and by the addition of external pipes or reservoirs connected to the tank at the top and bottom. The WATER-COOLED type is the same as the oil-cooled type, with the addition of coils of pipe at the top thru which water flows and carries off the heat. Transformers up to 500 or 600 kw capacity are usually self-cooled, larger transformers are usually water cooled.

Losses and Efficiency. The losses in transformers are of two kinds: COPPER LOSS is due to the resistance of the windings and varies with the square of the load. The CORE LOSS is due to the rapid reversal of the magnetic flux in the steel core and is constant at all loads. In well-designed transformers these losses are about equal, except in distribution transformers (50 kw and under) where the iron loss is usually not over half the copper loss. The iron loss in a transformer is constant irrespective of the load and consequently is made a smaller proportion in distribution transformers for light and power (see article 17) where the load is on only a few hours a day but the iron loss is constant for 24 hours. In other words, the "all-day" efficiency becomes more important. The efficiency of transformers is comparatively high, as indicated by the following table:

Efficiency of Power Transformers

Size, kw	Per cent of normal load				
	125	100	75	50	25
5	96.75	97.0	97.0	97.0	96.0
50	97.75	98.0	98.0	98.0	97.5
100	97.0	97.0	97.0	96.25	93.5
500	98.0	98.0	98.0	97.5	96.0
1000	98.0	98.0	98.0	97.5	96.0
1500	98.25	98.25	98.25	98.0	97.0
3000	98.5	98.5	98.5	98.25	97.5
5000	98.75	98.75	98.75	98.5	97.75

These are approximate average values. The efficiency is slightly lower for 25 cycles than for 60 cycles. Also the efficiency is lower for high voltages than for low voltages. In the above table, the 5-kw and 50-kw values are for distribution transformers for 2200-volts; the 100-kw and 500-kw values are for 22 000-volt transmission transformers and the others are for 66 000-volt transmission transformers.

Regulation. With a constant primary voltage, the secondary voltage of a transformer will increase as the load decreases due to the resistance and inductance of the windings. The ratio of this increase to the voltage at full load is the REGULATION of the transformer. In designing, the aim is to keep the regulation as small and the efficiency as high as possible. The regulation at 100% power factor of well-designed transformers will range from 3% for a 5-kw transformer to 1.5% for a 1000-kw. It will increase very rapidly as the power factor decreases.

Specifications usually include the following: purpose for which the transformer is to be used, capacity, voltage, kind of cooling, efficiency at various 100% power-factor loads, regulation at 100% and 80% power factor, temperature after continuous operation at full load and after 2 hours at 25% overload, high-potential test of insulation at double normal voltage and an over-potential test by operating at double voltage for 5 minutes.

Cost. The cost of transformers of a given rating will depend upon the high-tension voltage, the frequency and the method of cooling. For example the approximate cost of 5 kw, 2200-volt distribution transformers is of the order of $25 per kw but for 22 000 volts, the cost is $80 or $90. The cost of a 3000-kw water-cooled transformer will vary from $1.75 to $2.00 per kw at 22 000 volts to $3.50 or $4.00 at 66 000 volts. Self-cooled transformers cost on the average 50% more than water-cooled. Sixty-cycle transformers are slightly cheaper than 25-cycle.

The Transmission Line is the weakest part of a transmission system, and therefore, where continuity of service is at all important, requires great care in designing. It is essential to have ample insulation under all weather conditions; ample factor of safety in poles, conductors and insulators under all temperature, wind, and sleet conditions; effective protection against lightning.

The earlier lines are run on wood poles with wood cross-arms. They are set 100 to 200 ft apart and the conductors are carried on rigid porcelain insulators. Modern lines are carried on towers built up of steel shapes thruout and set 400 to 800 ft apart. The conductors are suspended from the cross-arms by insulators made up of 3 to 8 porcelain disks fastened together with flexible joints. On large systems these towers are either rectangular, square, or triangular in shape and are very substantially built. The base will occupy a space 10 to 15 ft square, the height will vary from 40 to 60 ft, and they will withstand loads of 10 000 to 15 000 lb applied at the top of the tower. A two-legged flexible steel tower is being extensively used on transmission lines where a low investment charge is essential. Obviously such towers will not carry any stresses in the direction of the line so that substantial "anchor" towers or similar structures must be provided at intervals to give the necessary stiffness.

A very high voltage transmission line is usually run over a private right-of-way 50 to 100 ft wide with all tall and dead trees cut away on each side. To guard against complete shut down due to an accident to the line, large systems are frequently provided with a duplicate line. This line may be run beside the other, but is preferably run thru a different part of the country so that both lines would not ordinarily be subjected to a lightning storm at the same time.

The kind of material to be used for the conductors depends upon various factors such as topography (rough country may require some long spans where the strength of the conductors would be unusually important), climate (excessive wind and sleet storms would require high-strength conductors), importance of continuity of service, length of line and the economics of the proposition. Copper, aluminum, stranded steel, iron and copper-clad steel are used, but hard-drawn copper is by far the most generally employed.

Size of Conductor. The size of the conductors will depend upon many conditions including most economical energy loss, load factor, character of the installation (that is, whether temporary or permanent), strength required and voltage regulation necessary. With very high voltage especially at high altitutes, loss due to corona (which is greater with smaller conductors) must also be considered. So far as the energy loss is concerned, Lord Kelvin propounded the law that "the most economical area of conductor will be that for which the annual interest on capital outlay equals the annual cost of energy wasted."

Line Drop, Efficiency and Regulation. The line drop is the difference between the voltage at the sending end of the line and that at the receiver end under prescribed load conditions. It will depend upon the resistance and reactance of the line. The line efficiency is the ratio of the power delivered by the line (that is, receiver load) to the power input at the sending end under prescribed conditions at the receiver end. The regulation is the change in voltage at the receiver end where the prescribed load changes from no load to full load, voltage being maintained constant at the generator or sending end. It will depend upon, in addition to the resistance, the inductive reactance and capacitance reactance, which will vary with the frequency, and with the size and spacing of the conductors.

For calculations of these quantities for long lines and very high voltages, reference should be made to the numerous reference books on the subject. The following formulas will, however, be approximately correct for short lines of moderate voltage where capacitance reactance and capacitance current is negligible. E_s = voltage, sending end; E_r = voltage, receiver end; θ = angle of lag between current and voltage, receiver end; I = current in one conductor in amperes; R = total resistance of one conductor in ohms, x = total reactance of one conductor in ohms.

$$E_s = \sqrt{(E_r \cos\theta + IR)^2 + (E_r \sin\theta + IX)^2} \text{ (volts).}$$

Line drop $= E_s - E_r$ (volts);

Regulation $= \dfrac{E_s - E_r}{E_r} \times 100$ (per cent);

Power, receiver end $= E_r I \cos\theta$ (kw);

Power, sending end $= E_r I \cos\theta + I^2R$ (kw);

Power factor, sending end $= \dfrac{E_r \cos\theta + IR}{E_s} \times 100$ (per cent):

Efficiency $= \dfrac{E_r I \cos\theta}{E_r I \cos\theta + I^2R} \times 100$ (per cent).

In application, it is more convenient to use conductor-to-neutral rather than conductor-to-conductor voltage and the power in one wire only, converting back again in the final results. In a single-phase circuit this means ½ line voltage and ½ total power respectively, and in three-phase circuits, $\frac{1}{\sqrt{3}}$ line voltage and ⅓ power respectively. Resistance and reactance tables will be found in various handbooks.

Conductor Spacings are fixed by experience, but for moderate voltages the distance between conductors in inches may be made 20 + voltage in kilovolts between conductors.

Conductor Stresses. The general formula usually applied to the spans assumes that the conductor takes the shape of a parabolic curve. It is ordinarily approximately true. This formula is

$$T = \frac{L^2 w}{8 d}$$

where T = total tension at center of span in lb;
L = distance between supports (assumed same height) in ft;
w = weight of conductor in lb per ft;
d = sag at center of span in ft.

The tension at the point of support is:

$$T' = T + wd.$$

The length of the conductor between supports is approximately

$$L' = L + \frac{8 D^2}{3 L}.$$

The important factor of added load due to wind, sleet, snow and temperature changes depends of course upon the climate and for data, reference should be made to electrical engineering hand-books. This added load is taken as high as 0.75 inch of ice plus 11 lb wind pressure per sq ft projected area (including ice).

Lightning Storms are the cause of most interruptions to the service on high-tension transmission lines. The poles, conductors, and insulators are often struck by direct discharges. The potential of the conductors may be raised an excessive amount above that of the earth by electrostatic induction from the thunder cloud. This may break down insulators, cause arcing over and grounding of the circuit, or set up waves of high potential which would travel along the conductors to the power station or substation and seriously damage apparatus. LIGHTNING ARRESTERS are devices for allowing excessive voltages to be quietly dissipated without interfering with the service. The usual form of protection for a transmission line is an iron wire along the topmost points of the poles or towers and connected to the ground at frequent intervals. This is supplemented with special arresters wherever apparatus is connected to the line.

The voltage at the receiver end of the line will depend upon the inherent regulation of the line as mentioned above and the character of the load. With a low-power-factor load, it is customary to connect either a static or a synchronous condenser (a form of synchronous motor operated without load and with a field current above the normal value) to the receiver end of the line, thus neutralizing the lagging current and improving the voltage regulation by raising the power factor.

Substations are stations at the various centers of distribution where the high-tension power is transformed by "step-down" transformers to low-tension power. It is then distributed either as alternating or direct current, by any of the methods described in article 17.

With alternating-current distribution, the substation contains only the transformers which lower the voltage to the required value, and switchboards controlling the incoming and outgoing lines. For direct-current distribution, the alternating current is transformed to direct current either by rotary converters or motor generators. A ROTARY CONVERTER is essentially a double-current generator, a machine which, when driven as a generator, will give direct current at one end and alternating current at the other end. If the alternating-current end is connected to an alternating-current supply and operated as a motor, the machine becomes a rotary converter and direct current will be produced at the other end. The relation between the imprest alternating-current voltage and the direct-current voltage is fixt. Variation of the direct-current voltage must, therefore, be made with auxiliary apparatus which will alter the alternating-current voltage. The principal devices of this kind are the SYNCHRONOUS BOOSTER, a generator in series with the alternating-current supply; the INDUCTION REGULATOR, a single-winding transformer with a movable secondary coil by means of which the supply of voltage can be raised or lowered in small steps; and the SPLIT-POLE FIELD by means of which the wave

form of the machine and therefore the alternating-current voltage can be altered. A MOTOR GENERATOR is a synchronous or induction motor mechanically connected directly to a direct-current generator. This method gives complete electrical separation of the direct-current from the alternating-current systems, and greater control of the direct-current supply. The rotary converter, on the other hand, has a higher efficiency, especially at loads less than full load, occupies less floor space, and is somewhat lower in cost. The switchboards in substations, like those in central stations, are designed to give control over each machine and outgoing feeder. Control of the incoming high-tension current is provided by means of remote-control oil switches.

Rectifiers. Alternating currents of relatively small magnitudes are transformed to unidirectional current by several other methods. In the MERCURY-ARC RECTIFIER, the operation depends upon the fact that if a glass (or steel) tube is exhausted to a low pressure, filled with mercury vapor and has a pool of mercury at one end with a metallic electrode at the other, the resistance to the flow of current is low in one direction and high in the other. Thus a valve action is obtained which in the simple case and with alternating current, would permit current to flow only every other half cycle. It is necessary to start the action by first producing a momentary arc between the mercury and the other electrode which is usually accomplished by tipping the tube for an instant. In commercial rectifiers there are two positive electrodes (the mercury being the negative electrode) which are so connected to a transformer, suitable reactances and the direct current load, that both halves of the wave are utilized and a continuous but pulsating unidirectional current is obtained. They are built for use on two- and three-phase circuits as well as single-phase.

Mercury-arc rectifiers are available with current capacities up to 40 or 50 amperes with glass tubes and 300 or 400 amperes with steel tubes. Their most common application is for charging storage batteries in electric vehicles, operating moving picture arc lamps, etc., but the larger rectifiers have been used on electric locomotives, etc. High-voltage rectifiers are employed in conjunction with constant-current transformers to operate series, direct-current arc-lighting circuits (see article 18).

The "TUNGAR RECTIFIER" employs the principle of the thermionic valve which is as follows: If one of two electrodes in a vacuum tube is heated to a red heat, the resistance to the passage of current from one electrode to the other is very low in one direction and very high in the other. These rectifiers are limited to about 5 or 6 amperes capacity at voltages from 7.5 to 75.

MECHANICAL RECTIFIERS of various types have been developed in which the connection between the alternating-current circuit and the direct-current load is reversed each half cycle by means of a commutator driven by a synchronous motor having the same number of poles as the commutator has segments, or by means of alternating current electromagnets and a pivoted, contact-making, polarized magnet which changes the connection every half cycle. The former type is used with X-ray apparatus and the latter for charging automobile ignition batteries which require small currents at low voltages.

Auxiliary Steam Plants. Many water-power transmission systems have auxiliary steam plants connected to the system. These plants, in addition to assisting the water-power plant at times of low water, are kept under a low head of steam, so that they will be available on short notice in case of failure of the transmission line.

17. Power Distribution

Distribution Systems. Electrical power is distributed to consumers in two forms, direct current and alternating current. DIRECT-CURRENT systems are the most suitable for general service purposes and are generally used in closely settled districts where the distances are short. Direct-current motors, particularly in small sizes, are somewhat more efficient and convenient to operate than alternating-current motors. Direct current may also be used for many purposes where alternating current would be unsuitable, such as battery charging and electroplating. ALTERNATING-CURRENT systems are used where scattered districts are served because the distribution can be made at a higher voltage and thus greatly reduce the investment in copper. The power is usually distrib-

uted at 2200 volts and reduced to a lower voltage at the consumers' premises by small transformers called distribution transformers at each building or group of buildings. Direct current is distributed by series two-wire and three-wire systems. Alternating current is distributed by the series single-phase; two- and three-wire single-phase; three- and four-wire two-phase; and three- and four-wire three-phase systems.

Series Systems. In both direct- and alternating-current systems all apparatus is connected in series and the same current passes thru all. This system is used only for street lighting, where the smaller investment in copper required is a great advantage, The current is kept constant by automatic apparatus in the station and the voltage varies. with the number of lamps in the circuit. Open-arc lamps require 45 to 50 volts each, inclosed-arc lamps 75 to 80 volts each, and series incandescent lamps 15 to 30 volts each. Forty to one hundred lamps may be burned on one circuit, requiring a total of 2000 to 6000 volts. Provision is made in each lamp for automatically keeping the circuit closed when the electrodes of an arc lamp become too short or the filament of an incandescent lamp breaks. Incandescent lamps of different current ratings (and therefore different candle-power ratings) are used on the same alternating-current circuit by employing specially designed individual current transformers.

Two-and Three-wire Systems. The series system is not feasible for general distribution because lamps cannot be turned on and off individually without affecting the others on the circuit, series motors are not suitable for general use, and the high voltage of the series system would be dangerous to life and property if carried into buildings. Distribution to and within buildings is therefore made with constant-potential multiple circuits, either two-wire or three-wire.

In a Two-wire Direct-current System, pairs of cables (feeders) radiate from the central station, each pair dividing and subdividing into other pairs (mains) running to the premises of the consumers. The voltage varies from 100 to 125 volts and each lamp and motor is connected directly across these lines, each being independent of the others. Single-phase alternating current is similarly distributed at 2200 volts to transformers just outside of the consumers' premises and from which secondary distribution is furnished at 100 to 125 volts. The two-wire system requires a large copper investment to keep the potential drop within reasonable limits. Excessive drop in potential due to too high resistance in the circuit means not only lost power but flickering of incandescent lamps, since the candle-power of the latter changes 4 to 6% with 1% change in voltage.

The Three-wire System was developed to decrease the amount of copper required. Two direct-current generators are connected in series and feeders are run from each outer terminal of the two machines and the common connection between them. The distribution to the consumers' premises is made with three instead of two feeders and mains. The lighting load is divided, half being connected between each outer wire and the middle or neutral wire. Motors are connected to the outer wires in order to decrease the flickering of lamps which fluctuating motor loads produce. Furthermore, 250-volt motors are somewhat cheaper than 125-volt motors. In the three-wire system the advantages of the two-wire system are retained and at the same time only 38% as much copper is required for the same percentage loss. By keeping the load fairly evenly divided, the neutral feeders and mains may be made smaller and an additional saving affected.

Secondary distribution on single-phase alternating-current systems is made by the same method by winding the secondary of transformers for 200 to 250 volts, and connecting the third wire to the center of the winding.

Two-phase Systems. The two-phase system of distributing alternating current is very generally used where the load is both lamps and motors, the lamps being connected across each phase separately and motors across both phases together. Single-phase motors are not practicable except in small sizes, hence the two-phase system combines the advantages of a polyphase system for motors with those of the two- and three-wire systems for lamps. The current is generated with two-phase alternators and is distributed over four or three wires. In the four-wire system, each phase is distributed independently thru two wires. In the three-wire system, one wire of each phase is common.

Three-phase Systems of distribution are only used where the load is principally motors and where the power has to be transmitted considerable distances. About 25% less copper is required than for two-phase, four-wire, systems under the same conditions.

The current is generated in three phase alternators, usually at 25 or 60 cycles, and may be distributed directly to consumers up to 440 volts or transformed to a higher voltage for transmission to distributing centers, or directly to the consumers premises, where it is transformed to a low voltage. The primary distribution is always three-wire, but the secondary distribution may be made with four wires where there is a considerable lighting load. In such cases, lamps are connected between each of the three main wires and the neutral, while motors are connected to the three main wires.

Distribution Methods. The power is delivered to the consumers' premises from the power plant or substation thru feeders and mains carried on poles or placed underground. The former method is very much cheaper and is the general practice outside of large cities. In congested districts, the crowded condition of the streets, the unsightly appearance of pole lines, and the handicapping of firemen have caused the development of the underground method.

Overhead Distribution is made on wood, built-up steel, cast-iron, or reinforced concrete poles 30 to 60 ft high, and spaced 50 to 200 ft apart, depending upon the local conditions. The wires are carried on insulators attached to wood cross-arms. Glass insulators are used for low-voltage circuits and small porcelain insulators for 2200-volt circuits. Wire with weather proof insulation is commonly used. The cost of overhead distribution depends upon the kind, quality, height, and spacing of the poles, upon the number of circuits, condition of the streets, and so forth. It varies from $750 to $2000 per mile exclusive of the cost of the wire.

Underground Distribution can be divided into two general classes. In one class the conductors with their protecting covering are placed directly in the ground, and in the other class, called conduit systems, tubes are placed in the ground through which the conductors are afterwards drawn. Examples of the first class are the Edison tube system, now little used, and the wood-trough construction. In the Edison tube system, wires properly spaced in iron pipes are surrounded with a solid insulating compound poured in while hot. The lengths are joined together thru special junction boxes. The wood-trough construction consists of a wooden trough in which the conductors are laid on insulating spacers. The trough is filled with a hot compound which becomes solid when cold. This is the cheapest type of construction for underground distribution. Conduits are made of wood impregnated with creosote, sheet-iron pipe lined with cement, fiber, and vitrified clay. Wood conduit and sometimes fiber conduit is laid directly in the soil, while the other conduits mentioned are usually encased in concrete or cement mortar, a number of conduits being laid side by side. The conduit material most in use is vitrified clay. It is made in single-duct pieces about 18 in long, and in multiduct pieces, with any number of round or square holes (from two to sixteen), about 30 in long. On account of the greater strength and smaller number of joints, the multiduct conduit is frequently laid directly in the ground without cement encasement, the joints being wrapped in burlap, and well asphalted. Manholes built of brick or concrete are provided every 100 to 500 ft where service connections may be made, cables pulled in and out for repairs, and splices made. If distribution is made at 2200 volts, the step-down distribution transformers are installed in these manholes.

Interior Wiring. The system used for the distribution is usually continued thru the building. The materials and methods used in wiring buildings on which insurance is carried must conform to the rules of the National Board of Fire Underwriters. The building departments of all large cities also have rules which must be observed. The standard methods are open or cleat work, molding, rigid steel conduit, flexible steel conduit, non-metallic tubing and armored cable.

In Cleat Work, the wires are exposed and supported on walls and ceilings by porcelain cleats or knobs. Porcelain tubes are used where the wires pass thru walls or ceilings. This class of wiring is the cheapest but is entirely satisfactory when the work is properly done and appearance is of secondary importance.

In Molding Work the wires are imbedded in grooved strips of wood or metal, and covered with a thin strip of the same material. Special porcelain fittings are used where apparatus is connected. (Wood molding is no longer allowed in some of the larger cities.)

Rigid Steel Conduit Work consists of special soft-steel pipe run thruout the building and thru which the wires are pulled. Steel junction boxes are provided thru which the wires are pulled into the pipe and defective ones pulled out. This is the most expensive form of wiring, in first cost, but it completely protects the wires from mechanical injury, reduces the fire risk on the building to a minimum, and the maintenance cost is small, for old and defective wires can be readily replaced without injury to the cónduit.

Flexible Steel Conduit is a tubing of steel ribbon wound spirally in such a manner as to produce a very flexible steel tubing which is reasonably tight but not water-tight. Steel outlet boxes and junction boxes are used as with the rigid conduit. Altho this is a cheaper construction than the rigid pipe system it eliminates the unsafe features of molding work. It is particularly applicable in wiring old buildings.

Non-metallic Tubing is a stiff though sufficiently flexible hose of fiber composition which is a cheap substitute for the metal conduit. It is obviously not waterproof.

Armored Cable is similar to flexible steel conduit with the conductors included. The conductors are covered with suitable insulation and the steel ribbon wound directly over this insulation. This form of wiring is rapidly replacing the flexible conduit method for old buildings because conduit and wire are installed together.

Wires and Cables. There are three general classes of wires and cables used in electrical work: weatherproof, rubber insulated, and lead covered.

Weatherproof Wire is insulated with two or three layers of cotton braid impregnated with insulating compounds. It is used only for exterior wiring, principally for overhead distribution. It is the cheapest form of insulation, but also the least efficient. While sufficient protection is provided for pole-line purposes where the wires are supported on insulators, it is entirely inadequate for interior wiring.

Rubber-covered Wire forms the greatest percentage of wire used for electrical purposes. It is used exclusively for wiring buildings and extensively for high-tension cables. There are three standard grades of rubber insulation: National Electric Code Standard, thirty per cent Para, and Navy Standard. All grades consist of rubber, ground and mixt with various kinds of powdered mineral matter together with a certain amount of sulfur. After being applied to the wire by running thru dies, it is vulcanized by subjection to a high temperature. The difference in the grades is in the quality and amount of rubber used. National Electric Code insulation is made with various kinds of old and reclaimed rubber and rubber substitutes but must contain about 20% Para rubber. Before it can be used, it must be tested, past, and sealed by the inspector of the National Board of Fire Underwriters. Thirty per cent Para insulation contains at least 30% of new Para rubber gum. It is more durable than the National Electric Code wire and is used in all first-class interior-wiring work. Navy Standard insulation contains not less than 40% of new Para gum. It is used by the government on all battleships and is the highest grade of wire regularly manufactured.

Lead-covered Wires and Cables include various forms of insulation and are made up in a great variety of ways to suit any requirement for insulation and mechanical protection. Lead-covered cables are used where protection from mechanical or chemical injury is essential, as in underground distribution and under water. One, two, or three conductors may be in one cable. The conductors in two- or three-conductor cables may be side by side or one inside the other (concentric). The standard insulations are 30% rubber, paper, and varnished cambric. Paper insulation is applied in a narrow strip wound spirally on the copper to any desired thickness and saturated with suitable oil to increase the insulation and render the cable flexible. Varnished-cambric insulation is applied in the same manner. The capacity of wires and cables is fixt by the temperature at which the life of the insulation will be affected. The table on p. 1630 shows the carrying capacity of various kinds of wires and cables based on safe temperature limits.

The second column of this table gives capacity for wire in still air, at 125° F. The third and fifth columns give the capacities allowed by the Fire Insurance Underwriters (National Electric Code). The limits given for rubber are those beyond which deterioration (when exposed to air) may begin. Seventh column gives capacity at 125°F. and eighth column gives capacity at 175° F for still air at about 70° F. These data apply substantially to the conditions existing in a standard, single-duct, underground conduit. If the conduit is multi-duct with a loaded cable in each duct, the capacity

Current Capacity of Wires and Cables

Size, B & S gage*	Bare copper wire	Interior wiring				Low-tension underground cable, lead covered	
		"Code" rubber insulation	Corresponding volts, drop per 1000 ft	All other insulations	Corresponding volts drop per 1000 ft	Rubber insulaton	Paper and cambric insulation
14	12	12	30.0	16	39.7	....	....
12	17	17	26.5	23	35.7	....	....
10	24	24	23.5	33	31.4	20	24
8	42	33	20.6	46	28.6	30	36
6	59	46	17.6	65	25.0	50	60
4	83	65	15.8	92	22.5	75	95
2	118	90	13.7	131	20.0	110	130
0	170	127	12.7	185	17.7	160	200
00	202	150	11.4	220	16.7	190	240
000	240	177	10.8	262	16.0	235	285
0000	286	210	10.1	312	15.0	280	340
300 000	373	270	9.5	406	14.0	370	450
400 000	463	330	8.7	500	13.8	460	560
500 000	549	390	8.2	590	12.4	550	660
600 000	631	450	7.9	682	11.7	625	760
700 000	708	500	7.5	760	11.4	700	860
800 000	781	550	7.2	840	11.0	770	960
1 000 000	922	650	6.8	1000	10.5	900	1150

* Sizes larger than No. 0000 B & S are stranded and are given in circular mils.

of each cable will be reduced from 5 to 25 or 30%, depending on the number and arrangement of ducts and cables.

The temperature of paper cables will be about 10% higher than that of rubber cables for the same current and thickness of coverings. Paper cables should not be operated over 190° F and rubber cables over 160° F. Cables immersed in water will carry, for the same temperature rise, 40 to 50% more current than indicated in the table, and if buried in moist earth, 10 to 25% more current.

The capacity of each conductor in a two-conductor, non-concentric cable is about 85% of that of a single-conductor cable; in a two-conductor, concentric cable, 80%; in a three-conductor, non-concentric cable, 75%; and in a three-conductor, concentric cable, 60%.

High tension cables should not be operated over 125° F. on account of the rapid decrease in dielectric strength with increase in temperature.

Wiring Table. The table on p. 1631 shows how far various currents may be transmitted with different sizes of wire and with given percentages of loss. Assuming the usual allowable loss of 2% and given the distance from the entrance of the service to the building, the size of a wire for a given load is at once found in the table. If the size found is less than that allowed by the Underwriters' rules for that current, the latter size should be used.

The current required at 110 volts by motors and lamps is as follows: Motors: 8 to 9 amperes per h p. Lamps: 16-c p carbon, 0.5 amp; 32-c p carbon, 1.0 amp: 40-watt (14 c p) metallized, 0.4 amp; 25-watt tantalum (12 c p), 0.25 amp; 40-watt tantalum (21-c p), 0.4 amp; 25-watt tungsten (18-c p), 0.25 amp; 40-watt tungsten (30-c p), 0.4 amp.

The Sale of Power. Systems of selling power are based on the direct cost of manufacturing; on the investment in power-station and distribution equipment; and on the reserve capacity of the plant required to meet unusual demands of the consumer. Among the numerous systems in use, the principal ones are FLAT RATE, STRAIGHT METER BASIS,

Wiring Table

Distances in feet which power at 100 volts may be transmitted with various sizes of wire at various percentages of loss.

Size, B & S gage, per cent loss			Current in amperes							
2	5	10	2	4	8	12	20	40	60	100
.....	.....	0000	48 900	24 450	12 225	8150	4890	2445	1630	978
.....	.....	000	38 750	19 400	9 700	6460	3875	1940	1290	775
.....	.....	00	30 800	15 400	7 700	5130	3080	1540	1025	616
.....	0000	0	24 450	12 220	6 110	4075	2445	1220	815	489
.....	000	1	19 450	9 720	4 860	3240	1945	972	648	389
.....	00	2	15 450	7 720	3 860	2575	1545	772	515	309
.....	0	3	12 250	6 120	3 060	2040	1225	612	408	245
0000	1	4	9 700	4 850	2 425	1615	970	485	323	194
000	2	5	7 700	3 850	1 925	1280	770	385	257	154
00	3	6	6 100	3 040	1 520	1015	610	304	203	122
0	4	7	4 850	2 420	1 210	810	485	242	161	97
1	5	8	3 845	1 920	960	640	385	192	128	76
2	6	9	3 050	1 520	760	507	305	152	102	61
3	7	10	2 420	1 210	605	403	242	121	81	48
4	8	11	1 920	960	480	320	192	96	64	38
5	9	12	1 520	760	380	253	152	76	51	30
6	10	13	1210	605	302	200	121	60	40	24
8	12	15	760	380	190	127	76	38	25	15
10	14	.....	480	240	120	80	48	24	16	10
12	16	.....	302	151	75	50	30	15	10	6

and MAXIMUM DEMAND. In the FLAT-RATE method a fixt price, depending upon the size of the installation, is charged per month, irrespective of the hours of use. This is used only where power is cheap or the loads small. The METER SYSTEM is the usual system employed by central stations. Various discount methods, based on the total kw-hr consumption per month and on the size of the installation, are used. The MAXIMUM-DEMAND system makes the price paid depend upon the maximum power taken during any given period, as for instance during any 5-minute period in 24 hours. This system is extensively used where large quantities of energy are being consumed and is designed to take care of the investment in equipment not used except for a short time during each day.

Meters. The energy consumed is measured with watt-hour meters or ampere-hour meters. WATT-HOUR METERS, both direct and alternating current, are very small motors geared to a recording mechanism similar to that on gas and water meters. The speed is proportional to power and each revolution represents a certain amount of energy. Direct-current meters have their armatures connected across the circuit and their fields connected in series with the circuit. Alternating-current meters are induction motors with an armature consisting of a metal disk and a field produced by one coil across the circuit and another in series with the circuit. AMPERE-HOUR METERS measure the product of amperes and hours. They are of two types, the ELECTROLYTIC METER and MOTOR METER. The principle of the electrolytic meter is the decomposition of liquids by the passage of current. This decomposition is strictly proportional to the current, and by noting the decrease in volume a measure of the ampere-hours is obtained. In the motor ampere-hour meter the field is produced by permanent magnets and the armature consists of a metallic disk floating in mercury thru which the current flows to and from the disk. Ampere-hour meters may be made to read in kilowatt-hours by assuming a constant voltage and introducing the proper constant in the calibration.

18. Electric Lighting and Illumination

Lighting by electricity is accomplished by three methods: by the arc between two carbon pencils automatically kept a short distance apart, by the passage of current thru rarified gases, and by the heating of a refractory material to incandescence.

The intensity of a source of light is measured in terms of a unit source called a *candle-power*. The four principal standard sources of light are (a) the British standard candle, a spermaceti candle, (b) the English Harcourt lamp, which burns pentane vapor mixt with air in an Argand burner (c), the French Carcel lamp, which burns colza oil with a wick, and (d) the German amylacetate lamp, which burns amylacetate with a wick. The British candle is nominally the standard in this country but the amylacetate lamp is probably the most generally used standard lamp. All of these standards are of course arbitrary, and are made and used in strict accordance with very detailed specifications in order to insure reliable and reproducible results.

Illuminants are measured with a *photometer*, an instrument which gives the value of the unknown source in terms of a standard lamp. Because of the much greater convenience in commercial work, this standard is usually an electric incandescent lamp that has been carefully standardized against one or more of the above primary standards.

Rating of Illuminants. The theoretical standard of intensity of light, the candle-power, is a point of light giving a candle-power in each direction. *Mean horizontal candle-power* is the average candle-power in all directions in a horizontal plane thru the center of the source. *Mean spherical candle-power* is the average candle-power in all directions from the center of the source. *Mean hemispherical candle-power* is the average candle-power in all directions in a hemisphere with the source as the center.

Illuminants are sometimes rated in terms of the *lumen* which is the unit of light radiation. A lumen is equal to the light flux emitted by a source of one candle-power in unit solid angle. A source of one candle power emits a total light flux of 4π lumens.

All commercial illuminants have a more or less irregular distribution curve, that is, the candle-power is greater in some directions than in others, and in making comparisons between different illuminants it is therefore necessary to be specific. Formerly, it was customary to rate electric lamps on a basis of the maximum candle-power obtainable, Later incandescent lamps were rated on a mean horizontal candle-power basis and other illuminants on a mean spherical or mean hemispherical candle-power basis. The present practice, however, is to rate all electric illuminants on the basis of the nominal watts consumed. This applies particularly to incandescent lamps which are all labeled and described on this basis, the candle-power being omitted. This practice is the result of the wide use of reflectors, each form of which gives a different apparent candle-power.

Arc Lamps are extensively used for lighting streets, large interiors, and similar places where the illumination must be general but not necessarily high.

In the OPEN ARC, the earliest form of electric light, an arc is maintained between solid carbon rods in the open air. They are usually operated 40 to 100 in series, on both direct and alternating constant-current circuits. The standard sizes are 6.6 and 9.6 ampere lamps, requiring 45 to 50 volts. Altho this type is relatively very efficient as a source of light, it has been largely superseded by the ENCLOSED-ARC lamp, a lamp which is less efficient but also much less expensive to operate and giving a much better distribution of light. The arc is surrounded by a small, close-fitting glass globe which excludes the air and retards the consumption of the carbons, thereby increasing their life eight to ten times. These lamps are operated on both direct and alternating current, but more often on multiple circuits at 110 volts than on series circuits. The voltage per lamp is 75 to 80 and the current varies from 3.5 to 7.5 amperes. The MAGNETITE-ARC lamp is a direct-current open arc in which the electrodes are metallic copper and an oxide of iron. Its special features are long life of electrodes (about 150 hours), high efficiency, and light distribution particularly suited for street lighting. The FLAMING-ARC lamp is an alternating and direct-current open arc with small-diameter carbons impregnated with sodium or other similar salts which enormously increase the light emitted. They are the highest candle-power units in general use, 2500 to 4000 c p being given in the maximum direction, and are extensively used for spectacular and show purposes. MINIATURE ARC lamps are enclosed carbon-electrode lamps which have been developed to meet the demand for a relatively small unit of high efficiency for interior illumination.

Lamps with Luminous Rarified Gases are not used for general illumination but only where the special characteristics of the light are advantageous.

The Cooper Hewitt lamp consists of an arc in a tube 2 to 4 ft long, in which the cathode is metallic mercury and the arc passes thru rarified mercury vapor. The light is not due to the temperature of the arc, as in the ordinary arc lamps, but to the luminescence of the vapor. The light is therefore green in color, red and yellow being almost entirely absent from the spectrum. The actinic value of the light is high, hence it is used for photographing and blue-printing. It is also said to be less fatiguing to the eyes than white light, and is therefore desirable in drafting rooms. On account of the unnatural greenish tint of the light it is not suitable for general use. When a quartz tube is used instead of glass, the lamp can be operated at a much higher current density, and a higher efficiency as well as a whiter light is obtained. The Cooper Hewitt lamp operates on either direct or alternating current.

The Moore light is practically a Geissler tube about 1½ in in diameter and 100 or more feet long. It is placed around the room to be lighted in one continuous length and operated from a small high-tension transformer. The color depends upon the gas used, pure nitrogen giving a light which has practically the same composition as white light and is therefore particularly suitable for matching colors.

Incandescent Lamps are almost exclusively used for interior illumination where relatively high and uniformly distributed illumination is required.

An Incandescent Lamp is an illuminant where the incandescent material is in the form of a long thin wire of refractory material placed in a glass bulb. The types in most general use are distinguished by the material of which the filament is made, the carbon filament being the oldest. The others in the order of development are metallized (specially treated carbon), and tungsten. The essential difference between these is in the temperature at which the materials may be operated, the temperature of carbon being the lowest and that of tungsten the highest. For the same amount of light, thinner filaments may be used as the permissible temperature increases, and therefore less power is required. Where the power required per candle-power in carbon lamps is about 3.1 watts, that for tungsten lamps (gas-filled) is about 1 watt. All of these types operate equally well on alternating and direct current. The thin filament types of lamps require more careful handling than the carbon, altho they are now sufficiently rugged to be used on trolley cars and other locations subject to vibration. Rapid improvements in the manufacture of tungsten lamps are, however, making them more and more rugged.

The tantalum lamp was developed next after the metallized carbon lamp and came into quite extensive use but it has been completely superseded by the tungsten lamp. Osmium, osram, and zirconium lamps are similar high-efficiency lamps developed abroad in which the filaments are of those metals. They have also become practically obsolete. "Gem" and "Mazda" are trade names for metallized carbon and tungsten lamps respectively made by certain groups of manufacturers.

There are two types of tungsten lamps. In one type (known commercially as type "B") the filament operates in a vacuum while in the other type the bulb is filled with nitrogen gas at about atmospheric pressure ("type C"). The latter is operated at a slightly higher temperature and therefore a slightly higher efficiency than the vacuum type. The gas-filled type is made principally in the larger sizes where the increase in efficiency is most pronounced.

The table on p. 1634 shows the approximate candle-power, power consumption in watts per candle-power, and cost of carbon, metallized, and tungsten lamps. The sizes given are those most commonly used. The prices are approximate for a basis of not less than 5000 lamps purchased per year.

The Nernst Lamp consists of a short stout filament (glower) composed of highly refractory materials (thorium, osmium, etc.) and is operated in the open air. The resistance of the glower is high at normal temperatures, but decreases rapidly at higher temperatures. An auxiliary heating device is therefore provided to heat the glower to the temperature where its resistance will be sufficiently low to permit the passage of line current. The lamp is manufactured with 1, 2, 4, and 6 glowers. They are burned on multiple circuits, either direct or alternating current. This lamp has been practically superseded by the tungsten lamp which is more economical both in operation and in maintenance.

Operation of Incandescent Lamps. Incandescent lamps are used on series as well as multiple circuits, but series lamps are a small per cent of the total. Series lamps have

Candle-Power, Efficiency, and Cost of Incandescent Lamps

Nominal watts	Carbon			Metallized ("Gem")		
	Nominal mhcp	Watts per mhcp	Approx. cost	Nominal mhcp	Watts per mhcp	Approx. cost
10	2.0	5.0	$.22	....	....	....
20	....	....	....	5.0	4.0	$.26
30	9.4	3.2	.22	10.0	3.0	.26
50	16.7	3.0	.22	20.0	2.5	.26

Tungsten

Nominal watts	Vacuum ("type B")			Gas Filled ("type C")		
	Nominal mhcp	Watts per mhcp	Approx. cost	Nominal mscp	Watts per mscp	Approx. cost
10	7.2	1.4	$.35	....	....	....
20	18.2	1.1	.35	....	....	....
30	....	....	....	....	....	....
50	48.0	1.0	.35	....	....	....
100	100.0	1.0	.85	100	1.00	$1.10

NOTE: M h c p = mean horizontal candle-power, m s c p = mean spherical candle-power.

short thick filaments, requiring 15 to 30 volts, and are often connected in the same series circuit with arc lamps and placed in side streets, alleys, and similar places where high illumination is not required. Lamps of different sizes, that is different current ratings, can be operated from the same series circuit by employing a special type of current transformer. The continuity of the series circuit is maintained when a filament burns out by means of an automatic short-circuiting device in the lamp socket.

The life and candle-hours of incandescent lamps are seriously affected by changes in voltage for altho raising the voltage on a lamp increases its efficiency, the life is reduced at a much greater rate. The most economical operating voltage is therefore that where a balance is obtained between the decreased cost of energy consumption and the increased cost of lamp replacements. Fluctuating voltage also produces disagreeable flickering, consequently it is essential that the system be well regulated if satisfactory service is to be obtained. The following table shows the effect of change in voltage on the candle-power and life of carbon, metallized, and tungsten lamps (vacuum type).

Effect of Voltage Variation on Incandescent Lamps

Per cent normal voltage	Per cent normal candle-power			Per cent normal life		
	Carbon	Metallized	Tungsten	Carbon	Metallized	Tungsten
90	53	60	68	830	650	350
95	75	78	83	275	250	195
100	100	100	100	100	100	100
105	128	127	120	37	42	48
110	170	157	144	15	22	26

Approximate Candle-power, Watts Consumed, and Life of Electric Illuminants

Kind and type	Candle-power, usual sizes		Watts per candle-power (m.s.)	Approximate life, hours
Incandescent, carbon	5 to 20 (m h)		5.0–3.6	500
Incandescent, metallized	5 to 40 (m h)		5.0–3.0	500
Incandescent, tungsten (vacuum)	7 to 100 (m h)		1.8–1.3	1000
Incandescent, tungsten (gas-filled)	70 to 1500 (m s)		1.1–0.6	1000
Nernst, 1 glower	22	(m s)	4.1	800
Nernst, 3 glower	65	(m s)	4.0	800
Nernst, 6 glower	165	(m s)	3.3	800
Cooper Hewitt	500	(rated max)	0.8	...*
Moore tube	8	(m s per foot)	2.5	...*
Open arc, carbon, 6.6 amp	260	(m s)	1.3	15
Open arc, carbon, 9.6 amp	410	(m s)	1.0	15
Open arc, magnetite, 4.0 amp	250	(m s)	1.3	150
Open arc, magnetite, 6 6 amp	720	(m s)	0.7	100
Open arc, flaming	1000–700	(m s)	0.5–0.7	15–100
Enclosed arc, series a.c., 6.6 amp	130	(m s)	3.3	100–125
Enclosed arc, series a.c., 7.5 amp	155	(m s)	3.1	100–125
Enclosed arc, series d.c., 6.6 amp	260	(m s)	1.9	100–125
Enclosed arc, d.c. multiple	150–250	(m s)	3.0	100–125

* Manufacturers claim several thousand hours.

Notes: m h = mean horizontal candle-power; m s = mean spherical candle-power. 'he watts per candle-power of incandescent lamp decreases as the size of the lamp icreases.

The Relative Cost of Lighting depends not only on the amount of light produced er watt of power consumed, but also upon the cost of power, hours of burning, initial ıvestment, repairs, life, and cost of maintenance (cleaning glassware, replacing incanescent lamps, renewing carbons, glowers, and so forth). Hence the relative standing f the various illuminants will vary with conditions applying to each case, and no general :atement can be made that will be universally applicable. For instance where the cost f energy is low or the lamps are burned only a small fraction of the time, it would cost ess to use carbon lamps instead of any of the more efficient but also more expensive ımps.

Illumination. Illumination is exprest in terms of the FOOT-CANDLE, which is he illumination produced on a surface normal to, and one foot from, a source f one candle-power. The intensity of illumination varies inversely as the quare of the distance from the source.

Illumination of Interiors is usually effected by direct lighting, indirect, and semiıdirect lighting. In direct lighting the illumination is received directly from the light ources but in the indirect method, the light sources are concealed from direct view nd the light is reflected into the room from a highly reflecting surface, usually the ceiling. n the semi-indirect method, a part of the illumination is obtained by reflection and part y direct lighting usually by means of an inverted, translucent reflector or bowl. These wo latter methods have the physiological advantage of removing the light sources from he field of vision of the eye, but it requires considerably more energy for the same amount f illumination. In direct lighting, the intrinsic brilliancy of the sources is (or should be) educed by frosting the bulbs and by the use of enclosing, frosted or opalescent globes. 'his is accompanied, however, by a reduction of 8 to 15% in the amount of light.

The illumination generally accepted as sufficient for some classes of interior lighting indicated in the following table. The Illuminating Engineering Society has prepared ather complete and detailed codes of lighting for factories and school buildings. The

candle-power required in the lamps can be readily computed for a given case if the distance to the lamps and the curve of distribution of light around the lamps are known.

Class of service	Illumination, foot-candles	Class of service	Illumination foot-candles,
Auditoriums	1 to 3.5	Bookkeeping	3 to 5
Theaters	1 to 3	Stores, general illumination	2 to 5
Churches	1 to 3	Stores, clothing	4 to 7
Reading rooms	3 to 4	Drafting	5 to 10
Residences	1 to 3	Engraving	5 to 10
Desk illumination	2 to 5	Factories, gen. illumination	4 to 5

Street Lighting. The best method of illuminating streets is that which will produce the most uniform illumination for the same average illumination. This lies between the two extremes, large units placed far apart and smaller units placed close together. Cost considerations have dictated the very general use of the former method, the lamps being of such a size and so spaced that the minimum illumination midway between lamps will be at least that of average moonlight or about 0.02 foot candle. Obviously, the amount of illumination may be as much greater than this minimum as cost considerations will permit. The prices obtained for street lighting by electric companies vary from \$50 to \$125 per arc lamp per year, and the cost per mile of street will range from \$500 per year in villages to \$5000 for the important avenues in cities. The cost varies greatly even for the same illumination because of the varying conditions such as hours of burning, whether all night or only a part, and whether every night or on a "moonlight schedule"; type of construction, whether ornamental iron poles or wood poles, and overhead or underground distribution; cost of producing energy and so forth.

Reflectors. Efficient artificial illumination requires that as much as possible of the light emitted from the source fall upon the surface or objects to be illuminated. Care should be exercised to select the proper type and size of lamps for the particular case and to locate them where they will be most effective. Reflectors placed back of the lamps can be advantageously used to reflect the light from directions where it is useless to the object to be illuminated. Reflectors are now scientifically designed according to the laws of reflection and refraction. They may be obtained with various forms of distribution curves, either extreme concentration, moderate concentration. or wide diffusion.

19. Batteries

Primary Batteries. If two metals are placed in a liquid, an e m f is generated at the plates, the value of which will depend upon the metals and the liquid. Such a combination constitutes a VOLTAIC COUPLE. Those couples which produce the greatest potentials are used to a large extent as sources of electricity for telegraphing, signals, telephones, call-bell systems, and other purposes where very small amounts of power are required. They are called PRIMARY CELLS, and a number connected together form a primary battery. The various kinds may be divided into two classes: closed circuit and open circuit. Batteries in the first class always use a fluid electrolyte. Those in the second class use either fluid or nonfluid electrolyte.

Closed-circuit Batteries are those which can be discharged continuously. The most common battery of this class is the gravity battery, with electrodes of copper and zinc and a copper-sulfate electrolyte. Its construction is such that it operates best when kept connected to a closed circuit. The voltage of the gravity battery and the similar DANIELL battery is about 1 volt per cell, but the current capacity is only a few hundredths of an ampere. The gravity battery is used extensively for telegraph and railway signal work. POTASSIUM-BICHROMATE batteries use carbon and zinc electrodes with a solution of bichromate of potash. These cells have a potential of about 2 volts each and a comparatively large current capacity. Frequent renewal of the elements and electrolyte is necessary. The EDISON-LELANDE battery consists of zinc and copper electrodes and caustic-soda electrolyte. Its potential is about 1 volt per cell and the largest size will deliver about 7 amperes for nearly 100 hours on one charge of material.

Open-circuit Batteries are intended to be used only on intermittent service, such as call bells and short telephone lines. Very large quantities are used for pocket flash lamps. Practically all batteries in this class are carbon and zinc in a sal-ammoniac solution. The e m f per cell is about 1.5 volts. DRY BATTERIES are carbon-zinc-sal-ammoniac batteries made nonspillable by filling the space between the electrodes with sawdust or similar material which has been saturated with sal-ammoniac solution. They cannot be renewed when exhausted.

Standard Cells are cells used as standards of e m f. The two types in most general use are the Clark cell and the Weston cell. The Clark cell is made with zinc and mercury electrodes and a solution of zinc and mercurous sulfate. Its potential when made up in accordance with certain conditions is 1.434 volts at 15° C. The Weston cell is made with cadmium instead of zinc and the potential is 1.0183 volts at 20° C. Both will remain constant for years if no appreciable current is drawn. The Weston cell has a much lower temperature coefficient and a longer life than the Clark cell.

Storage Batteries are those in which the chemical process is reversible, and which, after being discharged, can be restored to the original chemical condition by sending current thru them in the opposite direction. In the best-known class, the elements or electrodes are metallic sponge lead (negativ plate) and lead peroxide (positiv plate) in a sulphuric-acid electrolyte. While discharging, both elements partially change to lead sulfate and when current is past thru the cell in the opposite direction (charging) they are converted back again to the original state. The two important methods of making the plates are the Planté method and the pasted or Faure method. In the Planté process the lead peroxide and the sponge lead are formed by chemical or electrochemical means directly on lead plates from the plates themselves. In the Faure process various oxides of lead are mechanically applied to the plates and then reduced electrochemically to sponge lead and lead peroxide, respectively. The Planté type batteries are usually used for stationary work where weight and volume are relatively unimportant. For motor vehicles, train lighting, and similar purposes where the maximum capacity per unit of weight and volume is essential, the pasted type is used.

The Capacity of a battery is measured in ampere-hours. A capacity of one ampere-hour means the ability to deliver one ampere continuously for one hour. The capacity of a battery depends upon the amount of active material; the area of the plates; the amount, temperature, and specific gravity of the electrolyte; and the current at which discharge is taking place. In order to get as great capacity as possible per unit volume, the effective surface of the plates is increased by cutting or casting grooves and ridges in the plates and by laminating. Batteries of any capacity are obtained by putting additional plates in the same cells and connecting all positive plates together and all negative plates together. The volts per cell, however, are the same irrespective of the size of the plates or the number connected in parallel, the total voltage required being obtained by connecting cells in series.

The capacity of a battery will vary with the temperature, increasing as the temperature of the electrolyte rises. The change per degree varies with the type of battery. The capacity is greater with currents below the normal rate and smaller with currents above the normal rate. Lyndon gives the following approximate figures for the capacity at high rates: 200% normal rate, 75% normal capacity; 300% normal rate, 58% normal capacity; 400% normal rate, 50% normal capacity.

Rating. The standard method of rating a storage battery is the current which it will give continuously during 8 hours for stationary batteries, and 5 hours for vehicle batteries. Automobile lighting and starting batteries are rated on two bases. One, the lighting rating, is the ampere-hour capacity obtained when discharging at 5 amperes and the other, the starting rating, is the current the battery will deliver continuously for 20 minutes.

Voltage. During discharge at normal rate the voltage falls rapidly at first from about 2.10 or 2.15 to 2.0 volts per cell, and then drops slowly to 1.90 or 1.85 volts. Beyond this point the voltage drops rapidly. In order to maintain constant voltage on the load

END CELL SWITCHES are often used by means of which an additional cell is cut in when the total voltage drops 2 volts.

Efficiency. There are two efficiencies of storage batteries, the WATT-HOUR and the AMPERE-HOUR. Watt-hour efficiency is the ratio of the total energy delivered during discharge to the total energy received during charge. Ampere-hour efficiency is the ratio of the product of amperes and hours during discharge to that of amperes and hours during charge. The watt-hour efficiency of lead batteries averages 75 to 80% and the ampere-hour efficiency about 85 to 90%.

Application. Storage batteries are used extensively in connection with power plants of all sizes. In small, isolated, or private plants, a battery is often used to carry the load during parts of the day when the load is too small to warrant running a generator. In the largest direct-current central stations and substations, large batteries are often installed for the sole purpose of carrying the load in case of an accident necessitating the shutting down of the regular source of power. Such a battery is designed to deliver very large currents for short periods of time. It is often connected across the buses all the time, so that it will deliver current automatically when the bus voltage falls below the normal value.

Care. Storage batteries require intelligent care and use if reasonable life is to be obtained. High rates of charge and discharge, repeated discharging to voltage below 1.70 volts per cell, and charging longer than necessary will rapidly disintegrate the active material and decrease the life. The specific gravity must be kept within the proper limits, the battery must be charged immediately after discharge, and should be charged at occasional intervals when not in regular use.

The most reliable indication of the state of charge or discharge is the specific gravity, hence in large plants readings of the specific gravity are taken at frequent intervals on certain representative cells called pilot cells. Autographic hydrometers have been devised which indicate and record the specific gravity at a point some distance from the cells, as for instance the switchboard.

Cost and Depreciation. The cost of stationary type storage batteries, installed, will range from about $1.50 per ampere per cell for small batteries of 5 amperes capacity to $1.00 for 500 ampere batteries. The cost will decrease slightly with larger capacities. Small portable batteries cost from $4 or $5 to $1.00 per ampere per cell. The depreciation of batteries varies enormously with the care and amount of use which they receive. The rate of depreciation for stationary batteries varies from 8 to 15% and for vehicle batteries from 20 to 50% per year.

The Edison Battery is an alkaline battery quite extensively used for vehicle and other purposes where weight is specially important. The elements are nickel and iron, and the electrolyte is sodium hydrate. It is considerably lighter than a lead battery and, according to the manufacturers, requires less care and attention. On the other hand it is considerably more expensive. The output per pound of cell is about 2.5 watts and 16 watt-hours, compared with about 1.25 watts and 12 watt-hours respectively for the vehicle-type lead battery. The efficiency of the Edison battery is, however, lower than that of the lead battery by about 15 to 20%.

SECTION 15

HIGHWAY ENGINEERING

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GENERAL DATA

1. Preliminary Investigations

A Reconnaissance Survey is of relatively as much importance in highway location as it is in railway location. Altho the elimination of distance is desirable from several standpoints, the distance and grade are so intimately related that in some cases a shortening of the distance will necessitate either the adoption of a steeper grade or an increased cost in the excavation to obtain a grade of a lower rate. The shortest route between two points may not be desirable for esthetic reasons, as is evidenced by a study of the roads of our park systems. Within the city limits and on suburban or country highways thru built-up districts the location already occupied can rarely be changed. A change of location that would improve the grade will sometimes be apparent in making a reconnaisance.

Grades. A maximum grade should be determined which should not be exceeded except in extreme cases. This limiting grade will depend to a large extent upon the locality, the nature of the traffic, and the loads to be hauled. Grades as high as 20% and more are found in mountainous districts, and as high as 15% within some cities. Many of the State highway departments never establish grades over 7%. The maximum grade established determines in many cases the type of road or pavement which may be used: for example, the maximum grades for stone block are 10 to 15%; brick pavement, 6 to 15%; earth, gravel and broken stone roads, 7 to 12%; bituminous concrete, bituminous macadam, bituminous surface, and cement-concrete, 5 to 8%; sheet asphalt, 4 to 5%; wood block, 3 to 4%. Minimum grades are governed by longitudinal surface drainage requirements.

Curves and Widths. All curves on country highways should be carefully investigated with a view to increasing the radius and eliminating obstruction. The First International Road Congress recommended a minimum radius of 164 ft for curves wherever possible. The width should be considered from the standpoint of future requirements in designing new highways. The New York City ordinance prescribes the following widths where no car tracks exist: Street, 60 to 66 ft 8 in, roadway, 50% of street width; street, over 66 ft 8 in, roadway, 80% of street width less 20 ft. The 1912 Standards of Newton, Mass., call for the following relative widths of streets and sidewalks:

30-ft street with curbing, sidewalks 4 ft 6 in; without curbing, 3 ft 6 in.
35-ft street with curbing, sidewalks 5 ft 4 in; without curbing, 4 ft 4 in.
40-ft street with curbing, sidewalks 6 ft; without curbing, 5 ft 6 in.
50-ft street with curbing, sidewalks 7 ft 8 in; without curbing, 7 ft.
60-ft street with curbing, sidewalks 9 ft 6 in; without curbing, 8 ft 6 in.

In English cities and towns a multiple of 9 ft has been adopted for width of traveled way, number of units depending upon number of lines of vehicles. Widths vary from 18 to 36 ft. In the United States interurban and country highways are built with an improved surface from 12 to 20 ft wide; in England, from 18 to 22 ft wide; and in France, National Roads, 23 ft wide.

Materials and Foundations. In connection with the reconnaissance, an investigation should be made relative to the materials suitable for road building that occur thruout the area. A knowledge that a certain material may be found in the vicinity may considerably reduce the cost of construction. The character of the natural foundations and the soil formation encountered along the different routes should also be ascertained. When rebuilding old roads the condition of the present surface should be carefully studied and the same be used, if possible, as a foundation.

Survey and Design. A survey should be made of the different routes determined

upon by the reconnaissance. Full information should be recorded relative to center profile and cross-levels and the location and elevation of curbs, gutters, property lines, manholes, abutting property, intersecting roads, culverts, catch-basins, car-tracks, hydrants, and bridges. Plans and cross-sections should be prepared from the survey notes. Grades can then be assumed and preliminary estimates of the cost of the work can be made. With the help of the plan, a further study should be made of existing conditions in the field in order to determine the best and most economical improvement, after which final estimates can be made. In connection with the proper selection of type of road or pavement, preliminary investigations should be made covering the following points: shade, methods in vogue for cleansing and watering, methods of maintenance, climate and range of temperature, the location of the street as to whether it is in a residential or business district or outside the built-up area, the character and amount of traffic, cost of materials, and character and cost of available labor.

2. Traffic Census

General Information. In connection with the preliminary investigations incident to the construction of a highway, a traffic census should be taken which will cover all classes of traffic to which the road is subjected at different seasons of the year. Other facts relative to traffic should be obtained, as, for example, the amount of loads, the direction of travel, the portion of the road occupied by various classes of vehicles, the relation existing between reduced grades, which incur increased distance between two points, and probable traffic, the kind of shoes worn by horses at various seasons of the year, the use of non-skidding devices employed by motorists, and the enforced traffic regulations governing the use of the road, especially with reference to limitations upon speed and loads to be carried.

The Classification recommended by the Special Committee of the Am. Soc. C. E. on Materials for Road Construction, in its final report in 1918, is as follows: Horse-drawn vehicle traffic: 1-horse vehicles, 2- or 3-horse vehicles, 4-or more-horse vehicles. Motor vehicle traffic: motorcycles, motor runabouts, motor touring cars (open or closed), motor-busses, motor trucks. These classes of traffic are further subdivided as empty vehicles, loaded vehicles and passenger vehicles. An estimate in pounds of the maximum load per inch of tire is also requested. Motor truck traffic should be also subdivided based upon total weight of truck and load, and kinds of tires.

Method of Taking Census. After the classification of the traffic has been adopted, the methods of securing traffic data must be considered. As a practical, economical and efficient plan, the following method is proposed for adoption under average conditions for the season from April to October inclusive for country highways in the northern States. The traffic should be taken during four periods of three days each, one period being in April, May or June, one in July, one in August, and one in September or October. As local conditions may dictate, either Friday, Saturday and Sunday, or Saturday, Sunday and Monday could be taken, thus insuring information relative to the usual abnormal Sunday motor-car traffic and, in some cases, the traffic above the week-day average on Saturdays, while the Friday or Monday traffic would give a fair indication of the normal week-day traffic. From a study of the meteoric records, it will be found that the climatic conditions are favorable to the adoption of the plan proposed. In the months from November to March, inclusive, two 3-day periods would be taken in certain cases; one in November or December, the other in February or March. This distribution of the periods would furnish statistics of the normal traffic in this season and would also afford opportunity for a study of traffic detail and condition of the road during the winter season. In many cases only a month will be available in which to make all the preliminary investigations. Under these conditions the traffic should be observed for 3-day periods using three sets, one of Friday, Saturday and Sunday, and two of Saturday, Sunday and Monday, one period in the first week, one in the

third, and one in the last week. The number of consecutive hours which shoul be taken during the day will depend upon local conditions and the period of th year when observations are made. In many cases 24 hr will be absolutel necessary, while in certain cases 8, 12 or 15 hr will be satisfactory. While it feasible to lay down general recommendations relative to traffic-census perioc for country highways, in the case of city streets the plan adopted will va in detail dependent upon local traffic characteristics and other condition Tillson points out that "traffic has been measured in this country by countir the number of vehicles passing over a street in a given time, so arriving an approximate tonnage without regard to width. In England efforts have bee made to arrive at more definite results, and the tonnage per yard of widt of roadway per day or year has been taken as the unit. This reduces it all to common standard, so that the traffic in one city can be easily compared wi that of another."

Since traffic is only one of several factors which affect the durability of a paveme other facts should be given due weight when examining a surface which has failed. Co siderable data on the relation between the traffic and the wear of the surface must collected and studied before definite conclusions can be reached as to the most economi and efficacious wearing surface to take a known traffic.

The following recommendations contained in the 1918 Report of the Special Co mittee on Materials for Road Construction of the Am. Soc. C. E. should be genera adopted in practise: "Your Committee desires to emphasize the fact that experie has demonstrated the value of traffic censuses taken both preliminary and subsequent the construction of a highway. The traffic census should be considered one of the m important variable factors in the solution of that important problem, the selection that type of road or pavement best suited to local conditions considered from both standpoints of economy and efficiency. In connection with the census returns on a road should be considered the traffic on cross and parallel highways, and the effect improvement of these highways on the traffic of the highways under considerati The bald return of a traffic census, however, should not be the sole basis of the select of the type of construction, but should be considered a guide. In considering the eff of traffic and its relation to the design and cost of maintenance, it is necessary to t into account the speed as well as the weight of the vehicles."

3. Comparison of Roads and Pavements

An Ideal Pavement should be durable, noiseless, sanitary, efficacious road users, easily cleaned and made dustless, provide good foothold for hor be non-slippery for all classes of vehicles under varying climatic conditic yield neither dust nor mud, have a low tractive resistance, low first cost, annual cost, low maintenance charge, and an esthetic and impervious surf The following table gives assigned values for some of these characteristics of different types of roads and pavements on the basis of 10 for the value of characteristic in an ideal pavement:

Ideal Relative Values of Pavements

Characteristic	Ideal Pavement	Sheet Asphalt	Brick on Concrete	Stone Block on Concrete	Wood Block on Concrete	Concrete	Bituminous Concrete	Bituminous Macadam	Bituminous Surface	Macadam with Dust Palliative	Macadam	Gravel
First cost	10	3	5	1	1	6	6	7	8	9	9	9
Ease of traction	10	10	8	3	9	9	9	8	8	6	6	5
Non-slipperiness	10	4	8	7	4	6	7	7	7	8	10	10
Ease of cleaning	10	10	9	7	9	8	9	9	9	3	3	1
Noiselessness	10	7	6	3	9	6	9	9	9	10	10	10
Dustlessness	10	10	9	8	7	7	9	8	8	6	4	3

Earth Roads. In some sections of the country where other material is not ailable, earth roads are the most economical proposition. If proper care is ‹en in their maintenance, they may be kept in fair condition thruout the year. ey are inexpensive and easy to build and maintain. Unless treated, however, th a dust palliative, they will be objectionable from the standpoint of dust. wet weather, if built of certain materials, they will be muddy. Sand-clay ıds, when built with the proper grades of material, give a much more durable ·face than ordinary earth roads, and hence eliminate some of the worst fea- es of the last-named type.

Gravel Roads are not as durable as broken stone roads. They are dusty in y weather unless treated with some dust palliative or built with a bituminous rface. They are practically noiseless. Water readily finds its way thru a avel road. The surface should be well crowned. This type of road furnishes excellent foothold for horses. A gravel road during continued wet weather periods when frost is coming out of the ground will usually have a muddy rface.

Macadam Roads, if properly built, withstand a horse-drawn traffic remark- ly well, but are not able to withstand a high speed heavy motor-car traffic. ‹e gravel roads, unless treated with a dust palliative or constructed with a bitu- nous surface, macadam roads are dusty. They are not noisy. Macadam ıds do not absorb moisture as readily as gravel. Horses can obtain a good othold on a macadam surface. Unless the dust, which results in part from abrasion of the stones by the traffic, is cleaned off, the surface will be muddy wet weather.

Palliatives. If a road is properly treated with a dust palliative, the dust ll be effectively laid. In wet weather, however, certain oils will emulsify with water and cause a muddy condition which is very objectionable. There are o some materials used as dust palliatives which have an offensive odor.

Bituminous Surfaces. A superficial coat of bituminous material, if applied a gravel or macadam road surface, will render the road more durable and better le to withstand the automobile traffic. A superficial coat of tar will not be sty if the road is in an exposed location. In general a surface treated with a avy asphaltic material will not be as dusty as a tarred surface. Roads having a perficial coat of tar are a trifle more noisy than ordinary macadam. The othold furnished and tractive effort required depends principally upon the ture and amount of the bituminous material used. For instance, a certain ıde of tar may furnish a poor foothold, under certain conditions, but decrease tractive effort, while the results would be just the reverse in the case of ·tain asphaltic materials. Both types of surfaces are easily cleaned and are actically impervious.

Bituminous Macadam Pavements are more durable than superficially ated macadam roads. Otherwise their characteristics are practically the ne as those of bituminous surfaces.

Bituminous Concrete Pavements are more durable than bituminous cadam pavements. Their surfaces are impervious, easily cleaned and intained, usually are noiseless, and have a low tractive resistance.

Cement Concrete Pavements are more durable than bituminous macadam vements. They are easily cleaned, practically impervious, non-slippery en dry and when wet if clean, and have a low tractive resistance.

Wood Block Pavements are not as durable as brick. Unless the blocks treated with some preservative, they are liable to decay rapidly. Altho the cks themselves are permeable, this does not seem to be a serious objection.

This type of pavement is practically noiseless, neither the noise from the horses' hoofs nor from the wheels being objectionable. Wood block pavements are very slippery under certain conditions. They afford a smooth surface, with little resistance to traction, and are easily cleaned.

Stone Block Pavements, if laid on a concrete foundation, are suitable to take the heaviest kind of traffic. Thoro sweeping and flushing are necessary to keep them from being dusty. All types of stone block pavements are noisy. These pavements are slippery in wet weather and when the pavements are sufficiently worn so that the surfaces of the blocks become smooth and rounded. Stone block pavements are not as easy to clean as some of the pavements with smoother surfaces. If constructed with a joint filler other than sand or gravel, a stone block pavement is impervious. More tractive effort is required on a good stone block pavement than on a good macadam surface. Durax pavement composed of machine-made cubes, is used in England on many of the inter-urban highways. It is durable, non-slippery, and not productive of much noise.

Brick Pavements on a concrete foundation are not as durable as stone block pavements, but, if correctly built, they are smoother riding. Having a smoother surface than stone block pavements, they are easier to keep clean, and are hence more sanitary. They offer a very good foothold to horse-drawn traffic, and very little resistance to traction provided they have an even surface. When constructed with a bituminous or cement grout filler, these pavements are impervious. If properly constructed and maintained, a brick pavement with a bituminous filler is generally not as noisy as pavements with a cement grout filler.

Sheet Asphalt Pavement is one of the most durable types of pavement next to stone block and brick. It has a smoother surface than any other type. On this account it has some advantages over many of the other pavements in that it is easier to clean and that it offers less resistance to traction. This pavement is impervious, and outside of the noise resulting from the horses' hoofs, it is noiseless. When somewhat dirty and enough rain falls to just moisten the dirt, the pavement becomes particularly slippery. Asphalt block pavements present a surface very similar to that of a sheet asphalt pavement. The surface has about the same characteristics as an asphalt surface, except that it is not so slippery.

4. Capitalized Cost

The Relative Economy of different types of roads and pavements can only be ascertained by comparing the annual costs. The annual cost is a combination of the following variables: interest on the initial cost of the road, the annual maintenance charge and an annuity which will in N years, the so-called life of the road, provide a fund equal to the cost of reconstruction. If we let C = annual cost, A = first cost, r = rate of interest, I = annual maintenance charge, and x = annuity (Sect. 12, Art. 7), the annual cost may be exprest by the formula:

$$C = Ar + I + x$$

In the case of types of roads permitting partial reconstruction every M years, a second annuity y should be included in the above formula to take care of this periodical reconstruction thru N years or the total life of the road. In order to make a fair comparison between the different methods, the same standard of maintenance in each case should be insisted upon. The ideal maintenance which should be striven for in every case is a method by which the surface of the highway is kept in as good condition as when accepted on the completion of construction.

Selection of Pavement. Altho theoretically the pavement giving the lowest annual cost would be the most economical one to build, there are other con

siderations which sometimes make it necessary to select some other type of pavement. For instance, the amount of money at hand for the improvement may not be sufficient to pay for the first cost of construction of a pavement which would give a low annual cost. In large cities the cost of cleaning the pavements is an item that should be taken into account, since the saving in this respect might be more than offset by the difference in annual costs between two pavements. Again, a road or pavement that might be the most economical would require such frequent repairs as to interfere with the traffic and business conducted on it, and would have to be seriously considered in a business district. Esthetics also influence the selection of the type of road or pavement.

Life of Pavements. Fixmer estimates the life in years as follows:

Granite block	30
Creosoted wood block	15–20
Brick	20
Sheet asphalt	16
Asphaltic concrete	12–20
Cement-concrete	6–10
Bituminous macadam	8
Broken stone	4

Application of Formula. In order to show the application of the formula, the annual cost of a granite block pavement laid on a 6-in concrete base will be computed. It will be assumed that the first cost is $3.50 per sq yd; interest at 4%; annual maintenance cost is 2.4 cents; that life is 25 years; that at the end of this time new blocks are laid on the old concrete foundation at a cost of $2.50 per sq yd, and that in this manner the life of the pavement is renewed for another 25 years. The annual cost then for 50 years, considering the whole pavement to be renewed at the end of that time, will be found as follows:

$C = \$3.50 \times \$0.04 + \$0.024 + \$0.023 + \$0.060 = \$0.247$ for the first 25 years,
$C = \$0.187$ for the second 25 years, and the mean of these gives $C = \$0.217$ average for 50 years.

5. Soils

Classification. Soils are formed by the decomposition of mineral, animal, and vegetable matter. They may be designated as sedentary or transported. Sedentary soils are those which remain near their source of formation, while transported soils have been carried by some geological agency from the place where they were first formed to some other. Soil as far as its composition and properties are concerned is extremely variable. The principal constituents of any soil, however, whatever its source, are nearly always silica, with varying amounts of alumina, oxides of iron, lime, magnesia, and the alkalies. A small amount of organic matter is also usually present. Soils are generally classified as gravel, sand, clay, loam, marl, peat, and muck.

Gravel consists of small pieces of rock, worn smooth by abrasive action, mixt with sands and clays in varying proportions. Gravels occur thruout the United States and Canada in those districts which were at one time covered with the glacier. Gravel should contain enough binding material, mixt with the stone particles, to bind the whole into a solid mass. Clay is the most common form of binder found in gravel in its natural state. If present in too large quantities, however, it is detrimental. If gravel does not have sufficient binding material, this can be remedied by adding some cementing material such as clay, shale, marl, loam, or stone screenings. Some gravels contain so much earthy material that it is necessary to screen them before they are suitable for road-making purposes. An indication of the binding qualities for pit gravel may be obtained by noticing the gravel in the pit. If the bank faces are vertical and the gravel breaks off into chunks, good binding qualities may be expected. River

gravel generally contains less clay and more silica than the pit gravel of the same locality. Soil classified as hardpan may mean either a very compact clay, or a gravel which is cemented with clay or an iron oxide.

Sand is largely the result of the breaking-down of sandstone rocks and a sandy soil so-called probably contains over 80% of pure sand. A sandy soil is hard to compact, and possesses little binding power unless wet or mixt with some cementing material such as clay. Quicksand is sand saturated with water, and possesses no stability.

Clay results from the decomposition of feldspar, oligoclase, and micaceous rocks. A clayey soil might be termed such when containing at least 60% of clay. Clay when wet will swell and puddle with water, becoming very plastic. When mixt in the proper proportions with sand or gravel, it makes a very satisfactory road material in localities where stone is unobtainable. Clays used for fire clays in the manufacture of brick, besides being plastic, must possess certain refractory qualities which enable them to withstand long periods of heat at high temperatures without fusing. Shales are chemically the same composition as clays, but they have a laminated structure, and are similar in appearance to slates. Shales however, will rapidly disintegrate on exposure to the atmosphere. In the southwestern States a clay of which mud bricks might be made is called "adobe."

Loams may be any soil between sand and clay. They contain more or less of each of these two materials. They may be classified as heavy clay loams clay loams, sandy loams, and light sandy loams, depending upon the quantity of the sand or clay content. In the Middle West a black loam which contain so much clay as to be sticky when wet is known as "gumbo."

Marl is a term which applies to all calcareous clays containing as a minimum 15% of carbonate of lime, and as a maximum 75% of clay.

Peat and Muck are generally distinguished from other soils by the presence of humus or vegetable matter. They are formed by the decomposition of vegetable matter under water, and are undesirable materials for road building.

6. Stone, Brick, Wood

Rocks used for road-building purposes are trap, granite, limestone, sandstone chert, slate, and field stones. The Special Committee of the Am. Soc. C. E. on Bituminous Materials for Road Construction recommends in its 1912 report that "whatever method of construction may be used, it is essential, as in water bound macadam construction, that a suitable quality of road metal be used."

Diabases and Basalts, which are dark-colored igneous rocks, are commonly know as trap. Trap is extremely hard and tough, and its excellent wearing qualities have cause its widespread use thruout those sections of the country where it is found. When use in the construction of broken-stone roads subjected to a light traffic, the wear on th stones will not be sufficient to make enough binder to hold the stones together. To pre vent the surface from raveling, more binder or a bituminous material must be applied.

Granite is made up chiefly of quartz and feldspar. In trade, gneiss, syenite and por phyry are commonly known as granite, altho not so in a geological sense. If the structur is close, even and granular, these stones make excellent road material for broken-ston roads. If of a coarse structure, they are not so desirable. Granite and syenite are mo largely employed for paving stones than other materials, the latter being one of the bes materials for this purpose. Stone blocks made of porphyry have been found to wea slippery, and their use in Europe is being discontinued.

Limestone possesses excellent binding qualities, but is neither hard nor tough and therefore only suitable for roads taking a light traffic. It is rarely used as a materi for the manufacture of paving-blocks.

Sandstone, due to the fact that it easily breaks up under the action of traffic, and lacking in binding qualities, is generally only considered as a fair material for broke

tone roads. Stone blocks, however, made of Medina, Potsdam, and Colorado sandtone have been used to a considerable extent with excellent results.

Quartzites, which are metamorphosed sandstones, give better results when used for roken-stone roads than sandstones, as they are harder.

Chert is a variety of quartz of a flinty structure. It occurs in certain parts of the ountry either as a solid mass or in a broken-up state, mixed with clay similar to gravel.

Field Stones are boulders which have been carried along by the glacier and are found ainly in those districts which were covered by the glacier. They are composed of a ariety of different kinds of rocks, some of which make good road-building materials. hose stones which show signs of weathering and decomposition should not be used. obblestones used for paving gutters and streets are small boulders which have been elected from field stones.

Slate, an indurated or hardened clay, is of not much value as a road-building stone n account of its fracture and low cementing value.

Chats is a term used in the West to denote the tailings of lead mines. It is a dolomitic mestone, and considered good for use in road construction.

Tests for Broken Stone. By making petrographic and chemical analyses, a ock can be identified and its constituents ascertained. For the purpose of etermining the value of a rock as a road-building material, several tests have een devised which are used both in this country and in Europe. The usual ests made of broken stone are for abrasion, toughness, absorption of water, pecific gravity, hardness, and cementing power. Results of tests on different inds of rocks are given in the Table on page 1648 prepared by the U. S. ureau of Public Roads.

The Abrasion Test, as adopted by the Am. Soc. for Testing Materials, is made y means of the Deval machine, and consists of placing 5 kg of broken stone of certain size in one of the cylinders, the cylinder being rotated for 10 000 revoıtions, at the rate of between 30 and 33 to the minute. The weight of the etritus that will pass a sieve of 1/16-in mesh is determined, and the percent of ear computed. The coefficient of wear, also known as the French coefficient, quals 400/W, in which W is the weight in grams of the detritus passing the bove sieve, obtained per kilogram of rock used. The number 20 was adopted s a standard of excellence. In interpreting results of this test a coefficient of ear below 8 is called low; from 8 to 12, medium; over 12, high. (See Form 168, J. S. Bureau of Public Roads.)

The Test for Cementing Capacity is made by preparing a small cylindrical riquette, 25 mm in height and 25 mm in diameter, of rock powder, which will ass thru a 0.25 mm mesh sieve, mixt with a small amount of water. The ylinders are made in a die and are subjected to a uniform compression of 100 kg er sq cm. The briquettes are dried in the air and steam bath, cooled, and then ested in a specially designed machine. The test consists in noting the number f blows of a 1-kg hammer falling 1 cm that it will take to destroy the briquette. he machine is provided with a self-recording apparatus which measures the umber of blows. The test is interpreted so that cementing values below 25 re called low; from 25 to 75, average; over 75, high. (See Form 168, U. S. ureau of Public Roads.)

The Test for Toughness, as adopted by the Am. Soc. for Testing Materials, made on rock cylinders, 25 mm in diameter by 25 mm in height, which have een bored out of the rock by a diamond core drill. These cylinders are placed n an impact machine underneath a plunger of 1 kg in weight, and the latter is ubjected to the blow of a 2-kg hammer. The hammer falls 1 cm for the first low, and the fall is increased 1 cm for each succeeding blow. The number of lows required to destroy the test piece is used to represent the toughness. Re-

sults of this test are interpreted so that rocks which run below 8 are called low; from 8 to 12, medium; and above 12, high.

The Test for Hardness is made on cores 1 in in diameter, cut from the solid rock, faced off and subjected to the grinding action of sand, fed upon a revolving steel disk against which the test piece is held with a standard pressure. When the disk has made 1000 revolutions, the loss in weight of the sample is determined. One-third of the resulting loss in weight in grams is subtracted from 20; thus a rock losing 6 grams has a hardness of $20 - 6/3$ or 18. Below 15, rocks are called soft; from 15 to 18, medium; above 18, hard.

Crushing Strength of Stone. (Recommended by Special Committee, "Materials for Road Construction," Am. Soc. C. E., 1918 Report.) "Cylinders shall be cut from a suitable block of the material to be tested each of which cylinders shall, as nearly as practicable, be 2 in in diameter and 4 in in length. After cutting, the dimensions of each cylinder shall be accurately measured and recorded. Each cylinder shall then be subjected to compression and the ultimate stress at which its failure occurs shall be noted. This stress divided by the average area in cross-section of the cylinder in sq in shall be reported. It is desirable that the test of the material shall be made on at least three such cylinders separately and the average of the three or more specimens shall be taken as the average resistance to crushing of the material. In making the

Maximum and Minimum Results on Rock Samples, Corrected to January 1, 1915

No. of Samples	Name	Weight Pounds per Cubic Foot		Water Absorbed Pounds per Cubic Foot		Per cent of Wear		French Coefficient of Wear		Hardness		Toughness		Cementing Value	
		Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
24	Amphibolite..	196	172	1.65	0.04	10.3	1.5	41.7	3.9	19.0	13.5	40	7	235	3
69	Andesite.....	184	115	12.50	0.05	17.4	1.4	28.6	2.3	19.4	5.0	44	5	500	9
210	Basalt.......	199	143	6.40	0.02	16.6	1.3	30.4	2.4	19.3	5.7	47	5	500	2
67	Chert.......	187	125	8.27	0.25	29.2	2.7	33.3	1.4	19.7	12.7	26	3	500	2
13	Conglomerate	172	156	3.31	0.36	26.8	3.5	11.6	1.5	18.4	9.3	10	10	500	4
289	Diabase.....	200	165	2.73	0.04	6.3	1.1	36.4	6.4	19.4	10.7	54	4	500	2
85	Diorite......	209	168	1.00	0.05	12.0	1.7	23.8	3.3	19.4	16.6	38	4	164	5
414	Dolomite....	187	143	9.40	0.07	22.5	1.2	33.3	1.8	18.8	0.5	27	2	317	8
9	Eclogite.....	231	184	0.28	0.05	2.9	1.8	22.7	13.8	18.8	17.4	31	14	130	[illegible]
13	Epidosite....	206	168	1.65	0.22	7.4	2.0	19.6	5.4	19.5	10.7	29	18	83	3
12	Felsite.......	178	156	3.13	0.02	3.4	1.9	21.3	11.8	18.7	18.7	16	16	101	2
91	Fieldstone...	....	....	......	....	10.3	2.1	19.0	3.8	....	....	....	....	....	....
50	Gabbro......	228	172	2.62	0.04	5.9	1.3	30.8	6.8	18.8	13.3	23	6	325	5
221	Gneiss.......	200	162	1.28	0.02	16.4	1.7	29.0	2.4	19.5	9.0	26	2	209	1
312	Granite......	187	125	3.00	0.04	24.6	1.1	37.0	1.6	19.7	13.6	33	2	255	2
372	Gravel......	....	....	......	....	....	....	....	....	....	....	....	....	500	1
1032	Limestone...	178	125	13.22	0.02	34.2	1.8	21.7	1.2	19.2	0.0	25	2	500	8
74	Marble......	181	165	2.19	0.06	27.0	2.3	17.5	1.5	17.3	4.5	23	2	85	9
16	Marl........	....	....	......	....	....	....	....	....	....	....	....	....	500	6
19	Mixed Stone.	....	....	......	....	10.3	2.1	19.1	3.9	....	....	....	....	....	....
5	Peridotite....	221	165	1.02	0.27	5.3	3.0	13.2	7.6	15.0	13.3	12	[illegible]	91	25
135	Quartzite....	196	147	2.95	0.04	7.6	1.6	24.5	5.3	19.7	15.3	58	4	200	0
48	Rhyolite.....	181	128	7.15	0.03	9.7	1.7	24.1	4.1	19.7	15.3	42	6	500	1
465	Sandstone....	203	119	14.00	0.07	43.9	1.0	40.8	1.0	19.5	0.0	60	2	500	1
193	Schist.......	200	156	1.87	0.06	23.3	1.3	31.7	1.7	19.2	6.9	44	3	232	5
17	Shale........	172	156	4.80	0.50	35.2	3.2	12.6	1.1	17.7	13.9	12	3	368	28
70	Slag.........	243	125	4.90	0.04	19.1	2.3	17.7	2.1	18.8	9.5	21	2	500	1
85	Slate........	209	150	3.41	0.05	17.1	1.6	24.4	2.3	19.7	1.1	56	1	255	1
32	Syenite......	190	134	3.06	0.05	14.4	1.6	25.6	2.8	19.2	16.4	22	7	375	2

est the cylinder shall be so fixt in the testing machine as to be unsupported on its sides and to rest squarely on its ends and the compressive stress shall be applied cumulatively. The ends of the cylinder shall be at right angles to the long axis of the cylinder and the blocks or pieces of the machine in contact with ends of the cylinder and thru which the pressure is transmitted shall have such position and freedom of movement in the machine as will insure the application of the stress directly along or parallel to the long axis of the cylinder."

Common Commercial Sizes of broken stone are screenings, ⅜-in chips, ½, ¾, 1, 1¼, 1½, 2, 2¼, 2½, and 3 in. The size of the crusht stone depends upon the kind of stone, the crusher, and the screen, and the details of operation. The inclination of the screen and, if a rotary screen, the speed at which it is run, will make a variation in the separation of the sizes of the screened product. The form of specification recommended in 1918 by the Special Committee, "Materials for Road Construction," Am. Soc. C. E., is as follows:

The broken stone shall consist of one product of the operation of a stone-crushing and screening slant, without recombining or mixing, and shall conform to the following mechanical analysis, using laboratory screens:

Passing ...-in screen (having smallest holes selected) from	... to ...%
Passing ...-in screen (having next to largest holes selected) from	... to ...%
Passing ...-in screen (having largest holes selected) from	... to ...%

Example of mechanical analysis to be used in a specification:

Passing ¼-in screen	3 to 10%
Passing 1-in screen and retained on ¼-in screen	80 to 95%
Passing 1¼-in screen and retained on 1-in screen	2 to 10%

Specifications for Stone Blocks should call for a close, fine-grained, homogeneous material, durable, sound and uniform, with no outcrop, soft, brittle or laminated stone. The stone is tested for crushing, abrasion, and toughness. Cobble should be sound, durable and uniform stone, 4 to 8 in in diameter.

Cost of Broken Stone and Stone Blocks varies greatly, being governed to a great extent by local conditions, occurrence, and freight rates. From 1914 and 1915 quotations, the price delivered at New York was 85 cents to $1 per cu yd, and at Boston, 60 to 85 cents f o b at the quarry, varying slightly for different gages, at Pacoima, Cal., 53.5 cents per short ton at the plant. Price quotations of stone block 1910 and 1911 at various points: Quincy granite, $35 to $50 per thousand; granite, $2.15 per sq yd f o b Chicago; Rochester, Medina sandstone, $1.15 per sq yd f o b at the quarry; Cape Ann granite, $1.60 to $1.75 per sq yd delivered in New York; same, drest, $2 to $2.15 per sq yd; Baltimore, granite (1908), $68 per thousand; Toronto (1909), granite, $67 per thousand.

Clays for Making Vitrified Bricks are not often found in a natural state. A clay for this purpose should be fusible, plastic, and be able to be heated to a high temperature without losing its shape. Vitrification is obtained by subjecting the clay to heat, which changes the chemical properties of its constituents, making them coalesce with each other into a new and homogeneous solid. All clays are composed mainly of silica and alumina and certain impurities such as quartz, lime, magnesia, potash, and soda. The impurities, with the exception of quartz, act mainly as fluxes. An excess of silica will cause a weak and brittle specimen, while an excess of alumina will cause shrinkage, cracking, and warping. An excess of lime and magnesia hastens disintegration upon exposure. Shales are also used for the manufacture of paving-bricks. They produce a harder and more brittle brick than one of fire-clay. Paving-bricks are manufactured by crushing and screening the properly mixt clays or shales. This material is then mixt with water in a pug-mill to the right state of consistency, and is then pushed thru a mold, the clay being fed to the mold by means of an auger. The bar of clay as it comes from the mold is cut by machines into the size of brick desired. Paving-bricks are made with plane faces and also with some pro-

jections on the faces so that, when laid, there will always be a space between the faces which will later be filled with the joint filler. Some machines are designed so as to cut the brick with lugs on one side and grooves on the other. Other bricks are represt after being cut, and lugs or grooves, together with the curved edges, are formed by the die used in repressing. The raised letters on the brick serve this same purpose. After being cut or represt, the bricks are dried and then burned for from seven to ten days at temperatures varying from 1500 degrees F. to 2300 degrees F. The bricks are slowly cooled in the kiln after the fire is withdrawn, which serves to anneal and toughen them.

Tests for Paving Brick. Numerous tests have been devised, both abroad and in this country, for testing paving brick, the methods adopted in this country being almost without exception those proposed by the National Paving Brick Manufacturers Association as follows: the rattler test, which gives the amount of wear or abrasion on a number of identical sample test bricks, when subjected to a certain number of revolutions for a given time in a cylindrical hopper; the compression; the cross-breaking; and the absorption tests. Of these the first-named is essential, and according to the 1918 Standard Specifications of the Am. Soc. of Munic. Imps., any sample should not lose by weight more than 22% and the brick should not vary more than 8 points.

Cost of paving brick f o b at plants, 1915, varies from $12 to $21.50 per thousand, the average price being about $15.

Wood Blocks that have not been treated by some preservative process have given unsatisfactory results. Rectangular-faced blocks of Southern long-leafed yellow pine, Norway pine, and tamarack are usually specified, only one kind of wood, however, to be used on any one contract. Black gum and short leaf pine may be satisfactory under certain conditions, but require a different treatment than the woods first mentioned. The Australian hard woods, Jarr and Karrah, are slippery, and are considered too costly for use in this country. The common method of preserving wood blocks is to treat them with some pre-servative fluid, the most common being a pure creosote oil or a water-gas or a coal-tar product. In the United States the blocks are impregnated with th oil under a pressure of from 70 to 200 lb per sq in. The amount of oil absorbec by the wood varies from 10 to 22 lb per cu ft. In Paris one method of incor-porating the preservative fluid, which consists largely of creosote oil, is t immerse the blocks in a bath of this material for a short period. The resultin penetration of the fluid is of small amount.

Tests for Wood Block. The French practise is to conduct careful tests fo resistance to wear when saturated with water, absorption, compression, an impact, but in this country the most common tests of the treated blocks ar the determination of the amount of clear water absorbed after 24 hr and th analysis of the oil content to determine if it conforms to specifications fc creosote oil. Wood paving-blocks should conform to the following specification blocks shall be of sound timber with no sapwood and free from bark, loose c rotten knots; they shall be close-grained, sound and well seasoned; all block for a given contract shall be of same material, and dimensions must not var more than 1/8 in in different samples.

Cost of wood blocks from 1915 quotations is about $2 per sq yd surfac measure f o b New York, blocks to contain 20 lb per cu ft of creosote oil.

7. Bituminous Materials

The Bituminous Materials used in the United States may be classified follows: asphalts, asphaltic and semi-asphaltic oils, coke oven tars, coal-gas tar water-gas tars, combinations of coal-gas and water-gas tars, combinations asphaltic materials and tars, rock asphalts.

Nomenclature of Bituminous Materials and Their Uses. Definitions adopted by the Special Committee on "Materials for Road Construction" of Am. Soc. C. E. are noted thus, †; others adopted by the Am. Soc. Testing Materials are designated thus,* and others proposed by the Committee on "Standard Tests for Road Materials" (Committee D-4) of the Am. Soc. Testing Materials have been indicated thus, ‡.

Asphalt.†* Solid or semisolid native bitumens, solid or semisolid bitumens obtained by refining petroleums, or solid or semisolid bitumens which are combinations of the bitumens mentioned with petroleums or derivatives thereof, which melt on the application of heat, and which consist of a mixture of hydrocarbons and their derivatives of complex structure, largely cyclic and bridge compounds.

Asphalt Block Pavement.† One having a wearing course of previously prepared blocks of asphaltic concrete.

Asphalt Cement.† A fluxed or unfluxed asphaltic material, especially prepared as to quality and consistency, suitable for direct use in the manufacture of asphaltic pavements, and having a penetration of between 5 and 250.

Asphaltenes.†* The components of the bitumen in petroleum, petroleum products, malthas, asphalt cements, and solid native bitumens, which are soluble in carbon disulphide, but insoluble in paraffin naphthas.

Bitumen.*† A mixture of native or pyrogenous hydrocarbons and their non-metallic derivatives, which may be gases, liquids, viscous liquids, or solids, and which are soluble in carbon disulphide.

Bituminous Concrete Pavement.† One composed of broken stone, broken slag, gravel, or shell, with or without sand, portland cement, fine inert material, or combinations thereof, and a bituminous cement incorporated together by a mixing method.

Bituminous Macadam Pavement.† One having a wearing course of macadam with the interstices filled by a penetration method with a bituminous binder.

Bituminous Material.† Material containing bitumen as an essential constituent.

Liquid Bituminous Material.† Bituminous material showing a penetration at normal temperature under a load of 50 grams applied for 1 second of more than 350.

Semisolid Bituminous Material.† Bituminous material showing a penetration at normal temperature under a load of 100 grams applied for 5 seconds of more than 10, and under a load of 50 grams applied for 1 second of not more than 350.

Solid Bituminous Material.† Bituminous material showing a penetration at normal temperature under a load of 100 grams applied for 5 seconds of not more than 10.

Bituminous Pavement.† One composed of broken stone, broken slag, gravel, shell, sand or fine inert material, or combinations thereof, and bituminous cement incorporated together.

Bituminous Surface.† A superficial coat of bituminous material with or without the addition of stone or slag chips, gravel, sand, or material of similar character.

Blown Petroleums.* Semisolid or solid products produced primarily by the action of air upon liquid native bitumens which are heated during the blowing process.

Carbenes.†* The components of the bitumen in petroleums, petroleum products, malthas, asphalt cements, and solid native bitumens, which are soluble in carbon disulphide, but insoluble in carbon tetrachloride.

Coal-tar.†* The mixture of hydrocarbon distillates, mostly unsaturated ring compounds, produced in the destructive distillation of coal.

Coke Oven Tar.†* Coal-tar produced in by-product coke ovens in the manufacture of coke from bituminous coal.

Cut-Back Products.* Petroleum, or tar residuums, which have been fluxed with distillates.

Dead Oils.†* Oils, with a density greater than water, which are distilled from tars.

Dehydrated Tars.†* Tars from which all water has been removed.

Emulsion.† A combination of water and oily material made miscible with water thru the action of a saponifying or other agent.

Fixed Carbon.†* The organic matter of the residual coke obtained upon burning hydrocarbon products in a covered vessel in the absence of free oxygen.

Flux.†* Bitumens, generally liquid, used in combination with harder bitumens for the purpose of softening the latter.

Free Carbon.†* In tars, organic matter which is insoluble in carbon disulphide.

Gas-House Coal-Tar.†* Coal-tar produced in gas-house retorts in the manufactur of illuminating gas from bituminous coal.

Native Asphalt.†* Asphalt occurring as such in nature.

Normal Temperature.†‡ As applied to laboratory observations of the physical characteristics of bituminous materials, is 25° C. (77° F.).

Oil-Gas Tars.†* Tars produced by cracking oil vapors at high temperatures in the manufacture of oil-gas.

Petroleum.‡ Liquid bitumen occurring as such in nature.

Pitch.†* Solid residue produced in the evaporation or distillation of bitumens, the term being usually applied to residue obtained from tar.

Hard Pitch.† Pitch showing a penetration of not more than ten.

Soft Pitch.† Pitch showing a penenetration of more than ten.

Refined Tar.†* A tar freed from water by evaporation or distillation which is con tinued until the residue is of desired consistency, or a product produced by fluxing ta residuum with tar distillate.

Rock Asphalt.†‡ Sandstone or limestone naturally impregnated with asphalt.

Rock Asphalt Pavement.† A wearing course composed of broken or pulverized roc asphalt with or without the addition of other bituminous materials.

Sheet Asphalt Pavement.† One having a wearing course composed of asphalt cemen and sand of predetermined grading, with or without the addition of fine material, in corporated together by a mixing method.

Straight-Run Pitch.* A pitch run to the consistency desired, in the initial proce of distillation, without subsequent fluxing.

Tar.†* Bitumen which yields pitch upon fractional distillation and which is pr duced as a distillate by the destructive distillation of bitumens, pyro-bitumens, or organ material.

Water-Gas Tars.†* Tars produced by cracking oil vapors at high temperatures in th manufacture of carburetted water-gas.

Tests and Specifications for Physical and Chemical Properties. Va ous tests have been devised in order to determine the physical and chemic properties of bituminous materials. Tests are made for control of the man facture of bituminous materials, to obtain a record of the properties of materia used, and are employed in specifications to secure the materials desired f use in the construction and maintenance of roads and pavements.

The Special Committee on "Materials for Road Construction" of the Ame can Society of Civil Engineers, in its 1918 Report, recommended the adopti of the following lists of tests, as including all those probably of value in dete mining and recording the characteristics of the bituminous materials.

Asphalt Cements. Specific gravity at 25° C. (77° F.); flash point; solubil in CS_2 (carbon disulphide); solubility of bitumen in CCl_4 (carbon tetrachloride); so bility of bitumen in petroleum naphtha; penetration 4° C. (39° F.), 200 grams, 1 m penetration 25° C. (77° F.), 100 grams, 5 sec; penetration 46° C. (115° F.), 50 gra 5 sec; float test; melting point by ring and ball method; ductility at 4° C. (39° ductility at 25° C. (77° F.); fixed carbon content; paraffin content; loss on ev oration at 163° C. (325° F.), 5 hours; penetration of residue (same as for asphalt ceme melting point of residue, by ring and ball method, float test on residue; ductility residue at 4° C. (39° F.); ductility of residue at 25° C. (77° F.).

Tar Cements. Water; specific gravity at 25° C. (77° F.); flash point; solubi in CS_2 (carbon disulphide); specific viscosity, Engler; melting point, by cube metl

loat test; distillation by weight and by volume; up to 110° C.; 110° to 170° C.; 70° to 235° C.; 235° to 270° C.; 270° to 300° C.; specific gravity of total distillate t 25° C. (77° F.); melting point of residue, by cube method; float test on residue.

The tests used in a given specification depend upon the kind of bituminous material mployed and the method used. For example, a sepcification for a refined tar to be sed as a bituminous cement in a bituminous concrete pavement in which the aggregate onsists of broken stone composing one product of a stone-crushing plant, would include eference to tests for specific gravity, solubility in carbon disulphide, consistency with he New York Testing Laboratory float apparatus or with a penetrometer, melting point, istillation, specific gravity of total distillate and melting point of pitch residue remaining after distillation. In the case of an asphalt cement to be used in the above type of onstruction, the tests referred to in the specifications would include specific gravity, ash point, penetration at 4° C., 25° C., and 46° C., melting point or consistency with the New York Testing Laboratory float apparatus, loss on evaporation at 163° C. and peneration of the residue from evaporation, solubility in carbon disulphide, solubility of itumen in carbon tetrachloride, solubility of bitumen in paraffin naphtha, and fixed arbon.

For the purposes of this section it is not necessary to describe tests, the names of hich give an indication of the method of performing the tests. Such tests include specific ravity, solubility in carbon disulphide, carbon tetrachloride, petroleum naphtha, evapration, and distillation. For detailed descriptions of methods of conducting all of the ests for tars and asphaltic materials, see 1918 Transactions, Am. Soc. C. E., pages 1448 o 1462.

Flash Point. The material is placed in a cup fitted with a glass cover having a small pening. The temperature of the material is raised and a testing flame is inserted in he opening of the cover from time to time. The appearance, for a few seconds, of a aint bluish flame over the entire surface of the bituminous material will show that the ash point has been reached, and the temperature at this point is recorded.

Melting Point. The material is melted and molded into a ½-in cube. The cube s placed on a wire and suspended one inch above the bottom of a beaker. The temerature of the cube is then raised until the material softens and touches the bottom of he beaker. The temperature at this point of the operation is considered the melting oint of the material.

Consistency. The consistency of bituminous materials is determined by the Engler iscosimeter, the New York Testing Laboratory float apparatus, or the penetrometer.

With the **Engler Viscosimeter** the viscosity of liquid bituminous materials is deternined by noting the time which is required for a given amount of the material, having given temperature, to flow through a very small orifice. The result of the test should e exprest as specific viscosity, which equals the ratio of the number of seconds required or the passage of a given volume of the bituminous material at the temperature used ivided by the number of seconds required for the passage of the same volume of water t 25° C. (77° F.).

The New York Testing Laboratory Float Apparatus consists of an aluminum float nd a brass collar. The collar is filled with bituminous material and screwed into the ottom of the aluminum float and the apparatus placed on the surface of a water bath. s the plug of bituminous material in the collar becomes warm and fluid, due to the eat from the water bath which is maintained at any temperature desired for the test, t is gradually forced upward and out of the collar until water gains entrance to the aucer and causes it to sink. The time in seconds between placing the apparatus on the vater and when the float sinks is taken as the measure of consistency.

The Penetration Test is made by measuring the distance a weighted standard needle vill penetrate into the material at a given temperature in a given period of time. The emperatures, weights, and periods of time which are employed to a considerable extent re as follows: Penetration at 4° C. with a weight of 200 grams for 1 minute; penetration t 25° C. with a weight of 100 grams for 5 seconds; penetration at 46° C. with a weight f 50 grams for 5 seconds. When the penetration of a material is mentioned without eference to temperature, weight of the load, or time, it is understood that reference is nade to the penetration at normal temperature of 25° C. (77° F.) with a weight of 100 rams for 5 seconds. The unit of penetration is 0.1 mm. In literature and specifications he penetration is referred to in terms of the above unit either as a penetration of 6.4 nm or 64.

Ductility. In the ductility test a briquette of the material is formed in a standard briquette mold. The briquette with clips attached is placed in a ductility testing machine filled with water at a temperature of 4° C. or 25° C. The briquette is then pulled apart at a uniform rate and the distance in centimeters registered at the time of rupture of the thread of bituminous material is taken as the measure of ductility.

Fixt Carbon. Fixt carbon is the organic matter of the residual coke obtained upon burning hydrocarbon products in a covered vessel in the absence of free oxygen.

Paraffin. One hundred grams of the material is distilled rapidly in a retort to a dry coke. Five grams of the distillate is then thoroly mixt in a 60 cu cm flask with 25 cu cm of Squibbs' absolute ether. Twenty-five cu cm of Squibbs' absolute alcohol is then added, and the flask packed closely in a freezing mixture of finely crusht ice and salt for at least 30 minutes. The precipitate is filtered out quickly with a suction pump, using a No. 575 C. S. and S. 9 cm hardened filter paper. The flask and precipitate is then rinsed and washed with a mixture of equal parts of Squibbs' alcohol and ether cooled to − 17° C. (1° F.) until free from oil. When sucked dry, the filter paper is removed and the waxy precipitate transferred to a small glass disk and evaporated on a steam bath. The residue (paraffin) remaining on the disk is weighed, and from this weight the percentage on the original 5-gram sample is calculated.

Extraction of Bitumen from Bituminous Aggregates. The aggregate is prepared for analysis by heating it in an enamel-ware pan on a hot plate until it is sufficiently soft to be thoroly disintegrated by means of a large spoon. The disintegrated aggregate is then allowed to cool, after which a sufficient amount is taken to yield on extraction from 50 to 60 grams of bitumen. It is then placed in a mechanical extractor and carbon disulphide poured into the receptacle containing the aggregate. After allowing the material to digest for a few minutes, the machine is started, slowly at first in order to permit the aggregate to distribute uniformly. The speed is then increased sufficiently to cause the dissolved bitumen to flow from the receptacle. When the first charge has drained, the machine is stopt and a fresh portion of disulphide is added. This operation is repeated from four to six times until the liquid flowing from the receptacle is clear. After the aggregate is thoroly dried, it is weighed. The difference between this weight and the original weight taken shows the amount of bitumen extracted.

FOUNDATIONS AND DRAINAGE

8. Subdrainage

Object. It is of the utmost importance that the natural foundation of a road should be kept dry in order to provide a firm and unyielding support. This can only be accomplished by subdrainage in certain instances. If the subdrainage system is correctly designed it may serve to lower the level of the ground water and thus allow the ground to dry out; to remove water which is prevented from flowing off by an impervious stratum which underlies the road surface; to remove water which is always present in a road when the latter is thawing; to reduce the injurious action of frost by removing the moisture from the road; and to intercept water before it reaches the roadbed.

Porous Tile and Vitrified Pipe are used for this purpose. The pipes are cylindrical in shape and are manufactured in 1-ft or 2-ft lengths with varying diameters. The porous tile pipe is made with plain ends, while the vitrified pipe is provided with a bell end. It is good practise not to use a size smaller than 4 or 5 inches in diameter. Where a large quantity of water is expected, if its amount is known, the size of pipe may be determined by one of the well-known formulas for flow of water thru pipes.

The pipes should be laid in a properly constructed trench. The bottom of the trench should be covered with a layer of small-sized gravel or broken stone passing a ½-in screen. The pipe is laid in the trench with open joints, or with the

joints protected with small strips of burlap, and is then covered for a depth of about 1 ft with a material of a size similar to that used in the bottom. The rest of the trench is then filled with large size broken stone. The filling should be carefully tamped around the pipe. Where a pipe with a bell end is used, the bell is placed toward the high end of the trench. On macadam roads it is customary to lay the side drains at a distance of from 1 to 2 ft beyond the edge of the stone. The pipes are generally placed at a depth of from 2½ to 4 ft, and are usually laid to the grade of the road, but with a minimum of 2 in in 100 ft. The outlet end of the pipe should be protected by a small headwall of boulders to prevent the washing out of the pipe at this end. Whether a line of pipe is needed on one or both sides of the road is a matter of judgment. On side-hill work one line of pipe on the uphill side will generally serve. In constructing a road thru broad, flat, wet places, one line of pipe at the side of the road may not be sufficient, and a line on each side of the road will be necessary. Thru cuts two lines are sometimes specified. In streets tile drains should be laid under grass parkings, or, if such do not exist, under the gutters. For sizes of tile pipe under street pavements see Folwell's Sewerage, pp. 44–73. The 1915 cost, f o b factory, per foot for carload lots of small size pipe, either vitrified or porous tile, was approximately as follows: 4-in, 3 cents; 5-in, 4.6 cents; 6-in, 5.8 cents; 8-in, 9.5 cents.

Capacity of Tile Drains in Cubic Feet per Minute

From Spalding's Roads and Pavements, 1911, p. 36

Slope per 100 Ft		Diameter of Pipe in Inches				
Inches	Feet	4	6	8	10	12
2	0.17	4.0	12.0	27.0	49.5	81
4	0.33	5.5	16.5	38.0	70.0	114
6	0.50	6.5	21.0	46.5	86.5	143
9	0.75	8.0	25.5	57.5	106.5	176
12	1.00	9.5	29.5	66.0	122.5	204
24	2.00	13.5	41.5	92.5	173.0	288
36	3.00	16.5	51.0	114.0	212.0	353
48	4.00	19.0	59.0	132.0	245.0	408
60	5.00	21.0	66.0	148.0	275.0	456

Box Drains may be used in place of pipe in localities where stone is available. They should never be built of wood.

Blind Drains. Subdrainage is sometimes accomplished by digging trenches either across or alongside the road, afterward filling them with stone. The depth and distance apart of the trenches will depend upon the conditions encountered.

The V-Drain Foundation of the Massachusetts Highway Commission has given very good satisfaction on poor subsoils. It is built by excavating the full width of the surfaced roadway from 6 to 8 in deeper at the sides and from 12 to 18 in deeper at the center than usual, thus producing a flattened V-shaped trough. This trench is filled with stone varying in size from ½ in to 12 in in longest dimensions. The large stones are placed at the bottom of the trench. The grade of the trench is parallel to the finished grade of the road. The water follows the trench to the low points, where it is led to the sides of the road by a culvert across the road.

Soil Treatment. In places where the soil is very poor and is a hindrance to drainage, an improvement can be made by excavating the soil for a certain

depth, depending upon conditions, and refilling with field stone, broken stone, or a good gravel.

9. Surface Drainage

Surface drainage is accomplished by giving the road or pavement surface a crown or transverse slope, which sheds the water to the side ditches or gutters. The ditches or gutters have a longitudinal grade which generally corresponds to the grade of the center of the road and the water is carried by them to the point where it is discharged. In this manner the flow of water is confined to a small area.

The Crown of the road or pavement is formed by the intersection of two planes or as a parabolic curve. At street intersections, the crown will have to be modified to fit the grades of the intersecting streets. On curves of main trunk highways the crown should consist of one plane sloping up from the inside edge of the curve. On streets that are bordered with curbs the elevations of which cannot be changed, a uniform slope both ways from the center of the road can be obtained in some cases by making one gutter deeper than the other. Frequently, however, it is necessary to use a different slope each side of the center.

Amount of Crown. The Special Committee on "Materials for Road Construction," Am. Soc. C. E., in 1918, recommended the following crowns:

Kind of Roadway	Inch to the Foot		Kind of Roadway	Inch to the Foot	
	Maximum	Minimum		Maximum	Minimum
Asphalt block	¼	⅛	Cement-concrete	⅜	¼
Bituminous surfaces	½	¼	Gravel	1	½
Bituminous concrete	½	¼	Sheet-asphalt	¼	⅛
Bituminous macadam	½	¼	Stone block	½	¼
Brick	⅜	⅛	Wood block	¼	⅛
Broken stone	¾	½			

Crown Formulas. The following formulas, originated by Mr. Andrew Rosewater, M. Am. Soc. C. E., give the crown for brick, stone-block and sheet asphalt pavements. Let C = crown of pavement in feet, W = distance between curbs in feet, and f = grade of street in feet per 100. For brick, stone-block, wood-block, and comprest European rock-asphalt, $C = W(100-4f)/6000$. For American sheet asphalt composed of sand and asphalt, or of comprest natural sand rock, $C = W(100-4f)/5000$. The formulas are based on the laws of the parabola, and the crown C is the total rise at the center above the pavement at the curb. The crown can be found for any other point by deducting from C an amount equal to C times the square of the distance, exprest as a fractional part of the half width, from the center to the point in question.

Ditches for country roads are made by either cutting out a trapezoidal-shaped section at the edge of the shoulder, or by a more gradual rounding off of the road at this point, giving a flatter and shallower ditch. The latter is preferable particularly if a road-scraper is used to any extent in the maintenance of the road. Ditches to be more effective should be given a good bottom slope and should be kept clean. The slope of the ditch is made the same as the grade of the road surface, except in some cases where the grade of the road surface is very slight.

Gutters. Cobblestone, brick and stone-block gutters are laid on steep grades where water may wash out the sides of the road. They are also commonly used on many city streets that are surfaced with macadam, bituminous macadam and bituminous concrete. Gutters are made from 1½ to 6 ft in width, and have the same slope as the road surface. It is better, however, to drop the center so as to form more of a trough section. Concrete curbs and gutters which are built on the spot are coming into more popular use. On streets surfaced with brick, stone-block, wood-block and sheet-asphalt pavements, the pavement is carried to the curb in most cases.

The Minimum Longitudinal Grade recommended by some engineers is 0.5%. If the road surface is kept in good repair and the water has a chance to run off at the sides, a flatter grade than 0.5% can be used.

Methods of Construction. During construction the shape of the surface may be obtained in four ways: by measuring from a string at grade which is stretched across and along the road at intervals; by blocks placed on the subgrade and each course; by using a board template corresponding to the curvature of the cross-section; and by a series of stakes set at intervals across the roadway. Where the cross-section is composed of two planes the string method is rapid and accurate. Where the cross-section is a parabolic arc, the board template or stakes are preferable.

10. Pipe Culverts

Capacity. The size of opening may be determined by formulas, or an estimate of the run-off of water may be made and a size of pipe designed to take care of this amount. If the required area calls for a size of pipe that either is too large to be used, or cannot be obtained, two or three lines of smaller pipes may be substituted for it. It is, however, inadvisable to use a pipe less than 12 inches diameter because a smaller pipe than this is liable to become choked up.

Loads. Culverts are required to support the weight of the material which covers them and the weight of any superimposed loads. They may also be subjected to severe expansive forces caused by water freezing within. The latter, however, would be a very rare occurrence in a well-designed culvert. The amount of load carried to the culvert is indeterminate on account of the unknown action of earth pressure and the distribution of forces thru the same. The load reaching the culvert will have to be assumed. In using standard pipes of cast iron, corrugated metal, or vitrified clay, it is ordinarily not necessary to investigate their strength as far as load-carrying ability is concerned, since they have been used under sufficiently varying conditions to prove that they will resist successfully any load that they are likely to receive, provided the pipes are properly put in.

Construction. Pipe culverts of all types should be laid on a firm bedding. If the soil furnishes a very poor support, the pipes should be bedded in a layer of concrete or broken stone. This is more essential for culverts of vitrified pipe than for those made of the other materials. The trench should be excavated to the grade of the pipe. After the pipe is laid in the trench, good earth or small stone should be filled in around it and carefully tamped so that the pipe will be supported thruout its length. Headwalls should be constructed in every case. Concrete makes the best headwalls, since it is cheap, durable, and can be molded in any form desired. The headwall for small culverts are generally built parallel to the center line of the road, with a thickness of at least 12 in. The bottom of the headwall should be 18 in or more below the bottom of the pipe to prevent the water from flowing around the outside of the pipe and thus washing it out. The headwalls must be made long enough to keep the earth away from the pipe

With the larger-sized culverts it may be advisable under certain conditions to construct the headwalls with wing walls as would be done for an arch culvert, in which case the walls should be designed as retaining-wall sections.

Vitrified Pipes used for culverts should be the best quality salt-glazed sewer pipe of the double strength type, with socket joints. This pipe is made in 2-ft lengths with diameters from 12 to 36 in. The pipes are laid in the trench with the socket end toward the inlet, so as to have at least 15 in of material over the top of the pipe. The joints are sometimes filled with cement. Under conditions ordinarily encountered in highway work vitrified pipe makes a very satisfactory as well as a cheap culvert. During 1911 the approximate costs per foot in less than carload lots were: 12-in, 35 cents; 15-in, 47 cents; 18-in, 66 cents; 20-in, 79 cents; 22-in, $1.05; 24-in, $1.15; 27-in, $1.58; 30-in, $1.93; 36-in, $2.45. The cost per linear foot of vitrified pipe culverts in place, as given by the Massachusetts Highway Commission, is approximately as follows: 12-in, 75 cents; 18-in, $1.50; 24-in, $2.50; 30-in, $3.75.

Cast-Iron Water Pipe with bell and spigot joints has been used in culvert construction for a long time. It is manufactured in 6-ft and 12-ft lengths, and hence is not so easily adaptable for use. It is very strong and will last for many years. This kind of pipe can be placed within 6 in of the road surface without danger of breaking. The principal objection to cast-iron pipe, outside of its cost, is its weight, which makes it expensive to handle. The cost will vary between 1.5 cents and 2 cents per lb. The weight per foot of heavy-weight pipe for some of the sizes from 12 to 72 inches diameter is as follows: 12-in, 85 lb; 18-in, 200 lb; 24-in, 300 lb; 36-in, 500 lb; 48-in, 850 lb; 60-in, 1250 lb; 72-in, 1750 lb. The cost per linear foot of cast-iron pipe culverts in place, as given by the Massachusetts Highway Commission, is approximately as follows: 12-in, $2.25; 18-in, $3.50.

Corrugated Metal Pipe is made in any length desired, ranging by multiples of 2 ft up to 36 ft or is made in nest sections that are later bolted together in the field. The pipe may be laid to within 6 in of the road surface. Since it weighs about one-twentieth as much as cast iron it is much more easily transported. The nest sections are of a particular advantage in this respect. Care should be taken to select pipes made of the proper kind of metal. Wrought iron is superior to steel as far as its non-corrosive properties are concerned, and hence pipes made of iron generally have a longer life. The non-uniformity of results obtained with metal culverts of different makes has been due almost entirely to the different kinds of material used in the manufacture. The approximate cost per linear foot of a few sizes of one type is as follows: 8-in, 54 cents; 12-in, 72 cents; 18-in, $1.04; 24-in, $1.44; 36-in, $2.84; 48-in, $4.14; 60-in, $5.18; 72-in, $6.29. The cost per linear foot of corrugated metal pipe culverts in place, as given in the 1910 Report of the State Highway Commissioner of Maine, is approximately as follows: 12-in, $1; 18-in, $1.40.

Concrete Pipes may be cast and laid as any other form of pipe. The joints are made tapering, or with some form of socket. The pipes are built in lengths of from 4 to 8 ft, with thicknesses varying from 2 to 6 in, depending upon the diameter. Concrete pipes weigh more than cast-iron pipes, but may be constructed so as to cost about one-fourth as much. In the case of concrete pipes constructed in place, the use of reinforcement will not be economical until they exceed 4 ft in diameter.

Special Forms. There are several types of special forms of culverts manufactured of cast iron, all of which are cheaper and more easily handled than the ordinary cast-iron water pipe.

11. Foundations

The loads due to traffic are transmitted by the wearing surface to the foundation. If the foundation fails, the pavement or surface above it will fail. The kind of foundation and its thickness depend upon the amount of traffic, and the nature of the underlying subsoil.

The Different Materials Used as a foundation are the subsoil encountered, gravel, broken stone, slag, broken brick, hydraulic cement concrete, bituminous macadam and bituminous concrete, and old pavements. When a poor subsoil is encountered, the poor material should be excavated for a depth of several inches and refilled with a good gravel. Sandy subsoils may be improved by the addition of clay, and clayey soils by the addition of sand.

Broken Stone. The lower course of a macadam road, when the latter is built in two courses, is the foundation for the upper or wearing course. Ordinarily this course is about 6 in in depth after compaction. Where the subsoil is poor or the road is subjected to heavy traffic, or it is desired to aid the subdrainage of the road, the lower course should be increased in thickness, or an additional layer of broken stone should be used. This extra layer varies from 3 to 10 inches in thickness, and is composed of the larger-sized products of the crusher. Instead of this layer of broken stone, a telford base is sometimes constructed, which consists of placing by hand stones broken into sizes 6 to 8 inches deep, 3 to 8 wide, and 6 to 15 long. The stones are placed on edge with their longest dimensions at right angles to the axis of the road. The spaces between the stones are filled with spawls, after which the whole surface is thoroly rolled. Large stones laid flat instead of on edge are sometimes substituted for telford. The cost of either a broken stone or a telford base depends upon the amount of stone used. A telford base as described above has cost about 35 cents per sq yd. Slag and brickbats, where available, can be used in place of broken stone. Gravel foundations are usually laid 6 to 8 in in thickness.

Cement-Concrete foundations should ordinarily be used under all types of stone block, wood block, brick, sheet asphalt, and bituminous concrete pavements. The thickness of the concrete foundation varies from 4 to 8 in, 6 in usually being employed. An 8-in foundation is necessary when the traffic is exceptionally heavy. The usual proportions of the cement, fine aggregate and coarse aggregate varies from 1 : 2 : 5 to 1 : 3½ : 7. Whenever a concrete foundation is constructed, traffic should be kept from it for 7 to 10 days in order to allow it to set up thoroly. Concrete is manufactured by three methods: mixing, *in situ*, and grouting.

Mixing methods. The proportions having been adopted, the various ingredients are measured out by volume and mixed together with water until the desired consistency is obtained. The concrete thus mixt is placed upon the prepared roadbed to the required thickness. The concrete is then tamped and smoothed with the backs of shovels until the free mortar rises to the surface. In pavements where a layer of some kind of material is interposed between the surface of the foundation and the wearing coarse material, any very slight irregularities in the surface of the foundation will not cause trouble. When a smooth surface is required the concrete should be struck with a template. Altho hand mixing is used, usually mixing machines of the batch or continuous type are employed.

The *in situ* method consists of spreading and rolling a layer of broken stone of the required thickness in a manner similar to the construction of the bottom course of an ordinary broken stone road. A 1 : 3 mixture of cement and sand in a dry state is spread over the surface and swept into the voids. The surface

is flushed with water, rolled, and more dry mortar spread during flushing and rolling until all voids are filled.

In the grouting method, a layer of broken stone, of sufficient depth to make the requisite thickness of concrete, is deposited on the subgrade. The layer is thoroly rolled and is then poured with a 1 : 4 grout. The grouting and rolling are continued until the voids in the stone layer are filled.

Bituminous Macadam and Bituminous Concrete foundations are frequently used under sheet asphalt and certain types of bituminous concrete pavements. They should be used with caution, since they are not as stable as hydraulic cement foundations, and since the economical use of the wearing surfaces of sheet asphalt and certain types of bituminous concrete usually implies a traffic which requires first-class foundations. With a firm natural foundation allowing thoro compaction, good results have been secured.

Old Pavements of brick, cobble and stone block have been used as foundations for asphalt pavements. Where the old surface is firm and unyielding, this method has given fair satisfaction, but where the original surface was very uneven and required much relaying of the brick or stone, or filling of depressions, the resulting surface has not been satisfactory. Old macadam surfaces have also been made to serve as a foundation for asphalt and other bituminous pavements.

Rough Stone foundation in Connecticut, according to C. J. Bennett, consists of a depth of from 12 to 18 in of rough stone laid on the subgrade of the road with frequent outlets to water courses and without great regard to the uniformity of the sizes of the stones themselves. This method should be used in the building of roads thru clayey material, which is liable to become saturated and flow, and should be extended beyond the edge of the roadway surface proper.

ROADS

12. Earth and Gravel Roads

A large percentage of roads in this country are constructed of earth, and these roads will have to be maintained as earth roads for many years to come. An earth road will be defined as one built of native soil, other than gravel. A gravel road will be defined as one built of gravel. Gravel roads are used to a large extent in the construction of park roads and on many of the State, county and town highway systems, where good material is available and the traffic is not very heavy.

Characteristics. Since earth roads may be constructed out of any kind of soil, it is difficult to compare their characteristics. Soils act differently under different conditions. Sand makes the best road in wet weather, and clay a very poor one. All soils contain more or less of each of these materials, so that it is quite customary to find earth roads very dusty in dry weather, and muddy in wet weather. They are, however, the most inexpensive type to build, and under proper maintenance, can be kept in fair and passable condition at a very small cost, provided the traffic is not excessive for this type of road. A gravel road is cheap, fairly durable, noiseless, and offers good foothold for traffic. On the other hand, a gravel road is dusty in dry weather, and muddy during continued wet weather, or when frost is coming out of the ground.

Construction of Earth Roads. Good drainage is an absolute necessity in the construction of earth roads. Proper subdrainage and surface drainage must be provided. In providing for surface drainage the slopes from the center to

the sides should not be made too steep; a slope of from ¾ in to 1 in per ft is used, the latter being a maximum. The surface of the road may be made to conform to the arc of a circle or may be made up of two planes meeting at the center and sloping to the ditches. Care should be taken in constructing the ditches to see that they have sufficient fall to carry the water away. In places where the country is flat and the work involved is simply that of giving a crown to the road, earth roads may be constructed most economically by means of road scrapers or graders. If the soil is not too compact, the use of the plow could be done away with in using the road grader. The latter is usually worked by beginning at the sides, going down one side and back the other, gradually approaching the center, the blade being so adjusted as to plow to the proper depth, and move the earth up from the sides toward the center. The material deposited by the grader is sometimes harrowed and then rolled; when rolling is not resorted to, the material is left to be packed down by traffic. Care must be taken to spread the material evenly and in layers usually recommended not to be over 6 in deep. Where much grading is involved, the bulk of the work will have to be done by the common methods of earth excavation, the road grader being used to finish the road to the desired surface. Among the tools used in the construction of earth roads are drag scrapers, wheel scrapers, buck scrapers, road scrapers, elevating graders, plows and wagons.

Some specifications require that in the construction of any embankment less than 2 ft deep, the old surface shall be broken up and all sod and vegetable matter removed from the area to be occupied by the road, and that no sod will be allowed to be placed in the embankment nearer than 4 ft from the edge of the pavement.

Sand-Clay Roads are constructed somewhat differently, depending upon whether the subsoil is of sand or of clay. The amount of clay necessary is that amount which will just fill the voids in the sand. It will also depend upon the character of both the sand and the clay. It may be approximately determined by finding the quantity of water which is contained in a known volume of sand, the amount of water representing the percentage of voids. Proper drainage must always be provided. The construction of a sand-clay road is a slow process, and the best results can only be obtained by giving the road constant attention for some time after it is first finished.

If a sand-clay road is to be constructed of a sandy subsoil, the road bed is shaped up to the desired crown. The clay is brought onto the road and spread in a layer of 6 to 8 in at the center, tapering off to a thin layer at the sides. If construction is begun at the end of the road near the source of supply of the clay, the road will be somewhat compacted during construction by the teams on the work. It is necessary that the clay be thoroly mixt with the sand. All lumps should be broken up. Altho in many cases this clay layer can be covered with sand and left for the traffic to mix and compact, quicker and better results can be obtained by plowing, harrowing and rolling. Water is necessary to puddle the clay, hence it may be necessary to harrow and further mix the materials soon after or during rainy weather. More sand or more clay may be required in places where the road tends either to be sticky or to pulverize. The surface should be kept in shape by means of road drags or road scrapers.

Maine 1918 State Highway Commission Specifications for construction of sand-clay roads are, in part, as follows:

"Sand and clay shall be furnished by the contractor from sources approved by the engineer. Sand shall be composed of hard, sharp, angular particles, at least 50% of the volume of which, by dry weight, shall be retained on a 50-mesh sieve. Rounded grains of sand will not be accepted. If plastic clay is used it shall be placed upon the road in pieces not larger than 3 in in size. Slaking clay shall be placed in pieces not larger than 6 in in size.

"Sand Subsoil. Wherever the subsoil is sand, 6 in of clay shall be uniformly spread over the surface to a width of 16 ft, unless otherwise directed by the engineer. Each

load of clay should be spread uniformly as soon as deposited and before being driven over. Immediately after the clay is spread it should be covered with a layer of clean sand 6 in in depth unless otherwise directed by the engineer, and shall then be deeply plowed with a heavy plow until all lumps are thoroly broken up and the clay and sand are thoroly mixt to a depth of 14 to 16 in as directed by the engineer.

"Clay Subsoil. Wherever the subsoil is clay, 8 in of sand shall be uniformly spread over the surface to a width of 16 ft unless otherwise directed by the engineer, and on this layer of sand 4 in of clay shall be uniformly spread and then deeply plowed with a heavy plow as above specified to a depth of 14 to 16 in as directed by the engineer."

The Maintenance of Earth Roads consists principally in keeping the surface shaped up and the ditches cleaned out so that the water will not have any chance to stand either in the road or at the sides. Water, if it has a chance to soak into the road, soon softens it to such an extent that it is easily cut up by the action of traffic and is soon destroyed. Two of the most useful tools for the maintenance of earth roads are the road scraper and road drag. There are many different types of these machines. The road drag, which is the simplest, consists of two blades about 7 to 9 ft long set parallel to each other about 30 inches apart. The blades may be made of steel, of plank, or of split logs. This device is so hitched to the team that it may be dragged along the road at an angle at about 45 degrees with the axis of the road. Work with the drag scraper is done in a similar manner as with the road scraper; namely, starting in at the side of the road and working up toward the center. Dragging may be done at all seasons, but should be carried on only after a rain when the road is in a moist condition. If the road is properly drained it will be found that the surface can be kept in excellent condition by the use of the road drag or road scraper at frequent intervals.

The Cost of Earth Roads is an extremely variable quantity, since it depends upon the width and the number of cubic yards involved in the grading. The cost of 10 sand-clay roads built in various sections of the country during the years from 1914 to 1916 under the direction of the U. S. Bureau of Public Roads, ranged from 4 cents to 23 cents per sq yd, with an average of 13 cents. This cost does not include the bulk of the grading, but does cover the construction of the sand-clay surface.

Burnt Clay Roads have been constructed to a limited extent in some of the Southern States where the soil encountered is mainly a plastic, sticky clay known as "gumbo," and the fuel is abundant and cheap. The road is first plowed and graded. Furrows are then dug across the road 4 ft apart for the full width of the roadway. Cord wood is laid across the ridges of these furrows, forming a floor on which is placed more cord wood, built up in a crib formation. The clay is packed on this wood. Successive layers of clay and cord wood are built up in a similar manner until three layers are obtained. The top layer of clay should not be less than 6 to 8 inches thick, and should be tamped and rounded off so as to keep the heat within the flues. The process of burning changes the gumbo to a light clinker which combines well with the plastic clay of the remainder of the road.

Gravel Roads may be constructed in several ways. The road bed should always be well drained and shaped to the desired crown. In one method, the gravel as it comes from the pit or bank is placed at the side of the road or on dumping boards on the road, and spread to the desired depth on this prepared surface. Care should be taken to make the distribution of fine and coarse particles as even as possible the coarser particles being placed in the bottom. The thickness of the gravel at the center is generally more than it is at the sides. The gravel, after it is spread to the desired surface, is thoroly rolled and compacted. A better form of construction is to build a gravel surface in a shallow trench in a manner similar to that used in building broken stone roads. The

subgrade is formed with shoulders to the desired cross-section, which is the same as that of the finished surface. The gravel is spread on this surface in two or three layers, which are each individually rolled. The thickness of the separate courses varies, but the total thickness is about 8 in at the center, and is decreased to about 6 at the sides. In constructing any road with a gravel surface water should be used to help the consolidation of the surface, but care should be taken not to use an excess of water, since it may wash out the binding material or soften the subgrade. Whether or not the gravel should be screened before being placed in the road depends upon the character of the gravel in the bank or the pit. Some gravels have a very uniform composition, while others vary so much that they must be screened in order to produce the best results in the road surface.

Specifications require that no sod, vegetable soil or strippings be mixt with the gravel. Also in many cases that not more than 25% shall be binding material. For road work in Maine, gravel is specified to contain not less than 75% of pebbles that will be retained on a sieve of ⅛-in mesh and pass thru a sieve of either 1½ or 2 inches mesh, depending upon the size required. Some specifications require that the gravel be graded into three or four sizes similar to those generally specified for broken stone for road construction. In this latter case the gravel surface should be constructed exactly the same as one of broken stone.

American Society for Municipal Improvements 1918 Specifications covering quality and sizes of gravel are as follows: "All gravel shall be hard and tough. Gravel which contains over 10% of disintegrated stone shall not be used. No. 1 product (for 2-in top course) shall consist of a mixture of gravel, sand and clay, with the proportions of the various sizes as follows: All to pass a 1½-in screen and to have at least 60 and not more than 75% retained on a ¼-in screen; at least 25 and not more than 75% of the total coarse aggregate, material over ¼ in in size, to be retained on a ¾-in screen; at least 65 and not more than 85% of the total fine aggregate, material under ¼ in in size, to be retained on a 200-mesh sieve. No. 2 product (for middle and bottom courses 3 in each) shall consist of a mixture of gravel, sand and clay, with the proportions of the various sizes as follows: All to pass a 2½-in screen and to have at least 60 and not more than 75% retained on a ¼-in screen; at least 25 and not more than 75% of the total coarse aggregate to be retained on a 1-in screen; at least 65 and not more than 85% of the total fine aggregate to be retained on a 200-mesh sieve."

Maintenance of Gravel Roads. All ruts and pot holes should be filled up as soon as formed with gravel as near the same size as possible as that used in the surface. Water, if allowed to soak into a gravel surface, will soften it and cause its rapid deterioration. The intelligent use of the road drag and road scraper will do much toward keeping a gravel surface in good condition. In resurfacing a gravel road with gravel better compaction will be obtained if thin layers of gravel are put on frequently rather than thick layers not so often.

The Cost of a Gravel Surface varies from about $1 to $1.50 per cu yd in place. The highest price includes screening. The above prices do not include cost of grading old surface.

13. Broken Stone Roads

Characteristics. An ordinary macadam road, if properly built of the right kind of stone, is a very economical and satisfactory surface for medium horse-drawn vehicle traffic. It affords an excellent foothold, is noiseless, does not offer much resistance to traffic, and is comfortable to use. In dry weather, however, a macadam surface is extremely dusty unless the surface is treated with a palliative or coated with bituminous material.

Foundation and Subgrade. The lower or foundation course of a macadam road may be strengthened by a telford base, a V-drain foundation, or by increasing its thickness. The construction of telford and V-drain foundations has

been described in Art. 11. When traffic and soil conditions are favorable, however, it is customary to build the foundation course directly upon the subgrade. The subgrade is a shallow trench composed of two or more planes or a curved surface sloping from the center to the sides of the road. The trench is the same width as the broken stone surface. The sides of the trench are formed by the earth shoulders, generally 3 to 5 ft wide. The subgrade should be brought to true line and grade and be thoroly compacted with a steam roller. Any low spots which appear during compaction should be brought up to grade with good material and rerolled.

Size of Stone. Broken stone roads are ordinarily built in three courses. The sizes of stone and the materials used vary in different specifications. For the first or foundation course the size of the stone is often from 1 to 3 inches in longest dimensions, while sometimes it is specified as from 1¼ to 2½ in. Gravel and slag are sometimes substituted for broken stone in the foundation course. The second course is composed of stone ranging from 1 to 2 inches, or from ½ to 1¼ inches in longest dimensions. The top course consists of screenings varying from ½ in down to dust. Since nearly all of the broken stone used for road construction is screened through a rotary screen, it should be noted that the speed at which the screen is revolved, the pitch, the length and the size of holes in the screen all influence the grading of the stone into different sizes. The sizes of stone as specified in some of the different States are as follows:

State	Foundation Course	Upper Course
Massachusetts	1¼ to 2½ in	½ to 1¼ in.
New Jersey	2½-in stone or stone that will pass a 3-in ring, minimum length 2 in	1½-in stone or stone that will pass a 2-in ring, maximum length 2 in, minimum length 1 in.
New York	2¼ to 3¼ in	1¼ to 2¼ in.
Maryland	2½ to 1 in, maximum length 2½ in.	1 to 2 in, maximum length 2 in.

Laying the Stone. Some engineers advocate laying the larger stone on top of the smaller in cases where the stone is low in hardness and toughness and is liable to be crusht during rolling. Large stones in the surface wear longer than smaller ones, but more rolling is required to secure a smooth surface. In foreign practise larger sizes are used than is the custom in this country. To gage the thickness of a layer, wood cubes of a depth equal to the thickness of the layer are sometimes placed at intervals in the roadway and the stone is filled in to the tops of them. Another method of obtaining the same result is to set longitudinal strings at the proper elevation at the sides and center of the roadway. Stone for the foundation course, if brought in patent-bottom dump-wagons, may be dumped directly upon the subgrade and spread with stone forks or with a stone spreading machine. Stone brought to the work for the second course should be dumped on boards and shoveled from there to the road to prevent the segregation of sizes which might occur if the stone was dumped directly upon the road from the wagons.

Rolling. The first and second courses are each laid to the required thickness and separately rolled, being sprinkled with water to aid compaction if necessary before the next course is placed. The roller used in compacting this material should be at least 10 short tons in weight. The process of rolling should be to begin at one edge of the roadway, roll longitudinally and work toward the center. After reaching the middle of the road, the roller should pass to the other side, again roll longitudinally, and work toward the center. This manner of rolling keeps the road in shape and prevents either pushing the crown out of line or flattening it. Careful rolling is absolutely necessary in order to obtain a good shape to the road surface. Steam or gasoline rollers always should be

used as horse-driven rollers are not sufficiently heavy to properly compact broken stone. Usually 12 to 18-ton rollers should be used for hard, tough rock, and 10 to 12-ton rollers for soft stone.

It is stated in some specifications that the voids in the foundation course be filled with stone screenings, sand or gravel, the binding material to be thoroly swept in and rolled. No surplus material, however, is allowed to remain on the surface of the foundation course. This method of construction provides a firmer foundation than where the voids between the stones are not packed. This same treatment is also frequently used to facilitate the rolling of the foundation course, where the stone is of such a nature that it will not readily compact under the action of the roller without the addition of some mineral binder. If in the first passage of the roller over the surface any low spots are detected, they should be immediately brought to the proper level by the addition of more stone of the same sizes as are used in that course, before further compaction takes place.

Applying Screenings. Having finished and rolled the second course as above described, the surface is covered with a layer of stone screenings and thoroly sprinkled with water in order to fill the voids in the stone with the screenings. More screenings are added as desired and rolling is continued, the surface being sprinkled in front of the roller. When the proper amount of water and screenings have been used, a wave of grout will be pushed along in front of the roller. A coating of screenings should be left over the entire surface, no more being used than is necessary to cover the stone. The stone screenings resulting from the crushing of the rock with which the first two courses are built are generally used for the binder. When this material is unsuitable for this purpose, clay, loam, sand, or screenings of a different rock are substituted for it.

The Thickness of the Courses varies in different specifications, and is governed by the amount of traffic which the road is to receive, and the condition of the subgrade. Common values are 4, 6, and 8 in in total thickness after rolling, where the subgrade furnishes a good support. The upper course is from 2 to 3 in in thickness after rolling. In some States the stone surfacing is the same thickness thruout its width, while in others the thickness is reduced at the sides from 1 to 2 in.

Maintenance. Water, if allowed to stand on a macadam surface, will soften the latter and cause it to wear out rapidly. When the frost is coming out of the ground the surface will also be in a soft condition and require attention. Thoro surface and under drainage is just as essential as it is in the case of an earth or gravel surface. The effect of horses' feet is most frequently observed in the formation of the "horse-path" so-called at the center of the road. If the surface is given a flat crown this will be prevented to some extent, since it will tend to make the traffic use the entire width of surface. The grinding action of the wheels wears the stone and forms dust which, when in a dry state, is swept away by the wind, thus leaving the stone in the top course exposed. The action of traffic will then displace the stone in the surface, and "raveling" results. A heavy traffic of motor-cars traveling at high speed will cause a macadam surface to ravel very quickly when the mosaic of the upper course is exposed. Tracking of traffic will concentrate the wear on the surface and form ruts. This action is particularly disastrous when the road is in a soft condition. To maintain the surface under the above conditions, the side ditches should be kept clear, ruts and pot-holes should be filled up with stone, and the whole surface kept in as perfect shape as possible to facilitate the shedding of water. Sand, stone screenings, or other binding material should be spread on the surface, where lacking, in order to prevent raveling. Two systems of maintenance are in vogue, namely, the continuous and periodic. The continuous method is to repair the surface constantly so that it is always kept in good condition. In maintaining a road by the periodic method, the top surface is allowed to practically wear out and is then resurfaced. A scarifier is especially advantageous

in breaking up the old surface of a macadam road to form a bond with any new stone that may be added. The work will be done better, quicker and at a smaller expense than in any other way.

Material Data. The weight of a cubic yard of broken stone depends upon the specific gravity of the stone and the percentage of voids in the mass. The weight of stone will vary approximately from 2200 lb to 3000 lb per cu yd, measured loose. It will take 1.5 cu yd of broken stone of the ordinary sizes measured loose to make 1 cu yd compacted in place.

The Cost of a Macadam Road per square yard varies with the thickness. The cost per cubic yard of stone compacted in place varies between \$2.50 and \$6. An average cost is \$4.50. The average cost per sq yd, thickness 8 in, is \$1.00.

Slag Roads. Blast furnace slags are produced in the manufacture of iron and steel, and, in some cases, are very similar in appearance to close-grained igneous rocks. In some cases, blast furnace slag may be excavated from slag banks by means of a steam-shovel, which serves to sufficiently break up the material so that it may be screened. The slag from the open-hearth process is generally run into molds. Usually it is broken up in a rock crusher into sizes suitable for road work. The methods used in building slag roads are similar to those described for the construction of broken stone roads.

Shell Roads. The State of Maryland has built many miles of oyster shell roads along the eastern shore of Chesapeake Bay.

Maryland State Roads Commission Specifications stipulate that the subgrade shall be firm and well rolled. The depth of the first course of shells is either 5 in or 5 in at the center and 3 in at the sides. The depth of the second course is either 3 in or 5 in at the center and 3 in at the sides. The shells are spread upon the roadbed with shovels from piles along the road or from a dumping board. They are rolled with an 8-ton roller and are sprinkled with water or bound with sand during the process of rolling until the surface is firmly compacted. The third course is composed of clean, sharp sand, spread just thick enough to cover the second course after the latter has been thoroly compacted. Shell roads cost from 40 to 50 cents per sq yd.

14. Dust Prevention by Palliatives

Classification. Dust preventives, which may be classified as palliatives requiring application with more or less frequency, include water, calcium chloride, salt solutions, tar and oil emulsions, and certain light oils and tars. Palliatives are more especially adapted for urban and suburban districts, for parkways, and for use on highways before races, festivals and processions.

Water is the oldest and most common palliative, its binding power consisting solely in the mechanical bond produced, which varies to great extent with different rocks. The best practice stipulates careful regulation of flow, varying with type of surface, and application one to four times per day, depending on climate, temperature and local conditions.

Horse-drawn carts are equipped with two types of sprinkling devices, one discharging vertically, the other horizontally. For use on pavements, the vertically discharging valves are preferable, as they are easily manipulated to avoid damage to pedestrians, and the action of the water flushes the dust to the gutters without damage to pavements. The vertical discharge has a destructive effect on waterbound broken stone or gravel surfaces, hence horizontal discharge valves should be used.

Sea Water has been tried in a number of instances, being applied with the ordinary water-cart. In one instance it was found that in dry weather it formed a hard, salty scale, while in wet weather the mud contained so much salt that it injured the iron and varnish of vehicles.

Calcium Chloride in granulated form is shipped in airtight steel drums. When applied dry to the road surface, a special distributing apparatus should be employed. (Cost of 1-horse machine with spread of 5 ft, $55, 2-horse machine with spread of 10 ft, $70.) About 1½ lb should be used for an application per square yard. Usually two applications per season in the North will give good results. When applied wet, it is recommended that the calcium chloride should be dissolved at the rate of 1 lb to 1 gal of water, using about ⅓ gal solution per sq yd. For the application of solution ordinary watering-carts are generally used. The usual method is to distribute in two applications along the center of the street and one at the sides. To secure freedom from dust, about ten applications should be used per season in Northern States.

Judges' 1909 and 1910 Reports, The Roads Improvement Association, state that: "We are of opinion that the results of the tests of calcium chloride applied in granular form by the 'dry' method have shown that it is a very effective dust layer; that the treatment has the ill effects of causing, during the winter months, an abnormal quantity of sticky mud, a decided tendency to licking up, and a disintegrating action upon the macadam surface."

Oil Emulsions are made by the addition of some saponifying agent to water which, forming a chemical solution, renders it readily miscible with the oil. They sometimes contain a deliquescent material as an aid in retaining moisture. Their use is common where a light palliative is sought. Alkaline such as potash, soda, ammonia, crude carbolic acid, and various soap solutions are the mediums most commonly used with asphaltic or paraffin oils. Among the numerous processes are: casein added to tar oil; water lyes from wood-pulp factories; fat or grease from wool scourings, emulsified with either deliquescent salt solutions or creosote; an oil emulsion containing a deliquescent salt; waste sulphite cellulose liquor; waste-molasses solutions; and mixtures of saccharine and lime. Tar emulsions are used to a small extent in this country. Distribution is usually made with an ordinary watering-cart on the unprepared surface, altho better results may be obtained by using some type of pressure distributor. In one place a light sand coat was added, but as a rule the surface is left uncovered. A typical emulsion (Wickes) consists of 5% resin soap added to 30% water at 50 degrees Cent. The mixture is thoroly agitated and 65% oil added. A 20% solution mixed at 32 degrees Cent was first used, and for retreatments 5 to 10% solution (see Trans. Am. Soc. C. E., vol. 73). In Boston a 16% solution was first used with 5 to 10% solution for retreatments. Other reports give 5 to 25% solutions, the latter figures being for first application, and the former for retreatments. Applications vary from one to four weeks, according to requirements, generally using about 0.5 gal of material per sq yd.

Light Oils and Tars are generally palliatives. Belonging to this class are vegetable oils, paraffin and asphaltic petroleums, certain tar oils, and the heavy dead or creosote oils. There is sometimes enough binding base to cement the particles, and such materials are preferable, as a number of applications result in an accumulation of binding material at the end of a season. They are fairly efficient, but should not be applied in too great an amount, as a soft greasy surface will result. Heavy rains may either cause washing or an oily, disagreeable mud and pools of oily water. Light coal-gas and water-gas tars have proved satisfactory for use as palliatives. Prior to application of a tar all dirt and surplus dust should be removed by light brushing over the roadway surface with hand brushes or machine sweepers. The materials are usually applied cold. Pressure distributors should be used in order to secure uniform application of a small amount of the material. Two to four applications per season are usual, altho in some instances one coat has lasted for the entire season.

Costs. Watering in Boston (1907) three coats per day for 148 days, was 2.37 cents per sq yd; in 1909, 2.1 cents; East Orange, 2.8 cents per sq yd. Whinery states that 20 to 70 gal of water will cover one season's treatment, and ought not to exceed 3 cents per sq yd for the season. Calcium chloride: Metropolitan Park Commission, Mass., season 1908, 2.5 cents per sq yd; 1909, 1.8 cents; 1910, 3 cents; Pennsylvania, 3 to 4 cents; City of Boston, 1.6 to 4.2 cents; District of Columbia, 2.1 cents. Oil emulsions in numerous American cities (1908 to 1910) ranged from 1.25 to 3.8 cents per sq yd (average, 2 cents for the season). Light oiling is about 3 to 4 cents per sq yd for the season, using from 0.3 to 0.4 gal of material per sq. yd. Treatment with a light tar was from 2.4 to 5 cents per sq yd for the season.

15. Bituminous Surfaces

Bituminous surfaces are used principally on macadam and gravel roads, on bituminous and concrete pavements, and occasionally on brick and wood-block pavements. The different kinds of bituminous materials used are asphaltic oils, cut-back asphalts, refined water-gas tars, refined coal-gas tars, combinations of refined tars, and combinations of refined tars and asphalts.

Characteristics. A bituminous surface renders a road more durable. The ease of traction and foothold are closely connected and depend upon the character and amount of the bituminous material used. Certain materials may increase the traction and offer a good foothold, while the use of others may result in a surface that has about the same characteristics in regard to these two points as asphalt pavement. The surface can usually be easily cleaned. About the same amount of noise will result as is characteristic of a wood-block pavement when asphaltic materials are used. The surface is practically impervious, yields no dust, and is comfortable to use.

Preparation of Road Surface. Before constructing a bituminous surface on a macadam road all depressions, pot-holes, ruts or other irregularities should be carefully repaired by filling the same with bituminous coated stone so that the whole road surface is even. All surplus dust should be removed from the surface by the use of horse sweepers or fine bass brooms, or both, so that the surface of the stones in the upper course is exposed. When the application is made, the surface should be bone-dry. When certain asphaltic materials are used, however, if the surface is slightly damp when the material is applied better results may ensue in obtaining a more even distribution of the material. The road should be closed to traffic during the progress of the work, or if this is impossible, half of the width should be done at one time. Work should be done only on pleasant days.

The Application of the Bituminous Material may be accomplished in a variety of ways, as follows: by hand-pouring cans; by hand-drawn distributors; by pressure tanks to which is attached flexible hose, provided with one or more nozzles; by gravity distributors; by pressure distributors. It is difficult to secure uniform distribution with hand-pouring cans unless the material is brushed after application. The same objection is true of distribution with pressure hose. The selection of either of the last two types of machines will depend upon the nature and amount of material used and local conditions. The uniform distribution of material is accomplished in some cases by hand-brooming or squeegeeing after application by gravity distributors, or by brushes attached to the distributors. Bituminous material suitable for this class of work should set up under average climatic conditions so that tracking is not noticeable in from 36 to 72 hr. Many types of refined tars, combinations of refined tars and asphalts, and certain types of heavy asphaltic materials possess this property.

The Amount of Bituminous Material used per square yard will depend somewhat upon the condition of the surface, but as a general rule from 0.25

to 0.5 gal is used in one treatment. If it is necessary to allow traffic on the road at once, the coat of bituminous material should be covered with a coarse sand, gravel, or screened stone chips varying from 1/8 in to 1/2 in in longest dimensions. The amount of sand, screened stone chips, or gravel used per square yard will depend upon the quantity and nature of the bituminous material. From 5 to 22 lb per sq yd have been used. The coat is improved by rolling. When heavy asphaltic oils are used the covering of mineral matter is an absolute necessity. With some grades of tar, however, satisfactory results have been obtained by omitting the covering of mineral matter. This practice is common in some sections of England where the traffic can be kept from the road until the tar has set up. Gravel roads may be treated in a similar manner except that it is not possible to clean the surface as thoroly as it is in the case of the broken stone roads.

Conclusions relative to Surface Treatments as included in the 1918 Report of the Committee on "Materials for Road Construction," Am. Soc. C. E., 1918, Transactions, page 1404.

"If the surface to be treated is a gravel, broken stone, slag, or other porous and non-bituminous crust, practise has proven that bituminous material of such consistency that it can be applied at a temperature below 52° C. (125° F.) is preferable to heavier material, and that on any crust the application of a quantity in excess of 1/2 gal per sq yd is inadvisable. It is advisable to apply the material in quantities not exceeding 1/4 gal per sq yd at a time. Heavier material in less quantity, however, should be used on bituminous roadways; otherwise there will be tendency toward an objectionable softening of the material previously used in the construction of the roadway.

"The surface to which a bituminous treatment is to be applied should be dry, compact, and free from depressions and dust. The bituminous material, in all cases, should be applied by a pressure distributor designed so that the material will be spread uniformly and with a pressure of not less than 20 or more than 75 lb per sq in. The application, in all cases, should be carried over the outside edges of the rolled metal.

"After the bituminous material is applied it should be covered with the toughest grit obtainable, preferably of a size that will pass thru a screen having openings of not less than 3/8 in nor greater than 5/8 in, just enough of such material being used to cover the bituminous material. It is advantageous, but not entirely necessary, to roll with a steam-roller after the application of the grit."

Details of Methods, as used under various conditions in the United States and Europe, follow:

Extracts from "General Directions for Surface Tarring" recommended in 1911 by The Road Board of England follow: "Surface tarring may be advantageously applied either to an old road surface in good condition or to a new surface after it has been consolidated and dried, but the tarring should never be carried out unless the road is thoroly dry. If there are any depressions, pot-holes, waves, grooves, or other irregularities, these should as far as practicable be made good before tarring is commenced so as to provide an even surface. If it is intended to tar an old surface, it is advisable to take advantage of the early months of the year to scrape or brush the road during wet weather as a preparation for subsequent tarring, and especially to keep the road free from caked mud. The road whilst being tarred should be closed to traffic over half its width, or, where practicable, over its whole width."

"The road should be thoroly brushed and cleaned before application of the tar. Wet brushing should be used some time previous to dry brushing, if there is any caked mud. Any method of brushing may be used which will scour and clean the road thoroly, the best being horse brushing, followed by hand brushing. In order that the tar should be applied to the road as hot as possible, it is advisable, if the method of application is by hand, to use flexible pipes to convey the tar from the boiler to the point of application. If these are not available, it will be found convenient in case of hand pouring, to use 3-gal cans specially constructed for the purpose, fitted with spouts leading direct from the bottom of the cans, and being not less than 1.5 in in diameter at the orifice. Immediately on application the liquid tar should be brushed so far as necessary to ensure regularity

in thickness of the coating. The quantity of tar required will vary according to the physical conditions of the road, but generally, in the case of a road to be treated with tar for the first time, the quantity should be 1 gal to coat from 5 to 7 sq yd."

"If the road must be opened to traffic before the tar has set hard, grit should be spread on the surface to prevent the tar from adhering to the wheels of vehicles, but gritting should be delayed as long as possible, and the quantity of gritting material to be spread should be no more than sufficient to prevent the tar from adhering to wheels. Stone chippings, crusht gravel, coarse sand, or other approved material (free from dust) not larger than will pass thru a ¼-in square mesh should be used for gritting, in quantity not exceeding 1 ton for 300 to 350 sq yd if grit is used, and 1 ton for 200 to 250 sq yd if coarse sand is used."

"On heavily trafficked roads it is advisable to apply a second coat to either the whole width or from 9 to 12 ft of the center of the road in quantity of 1 gal to coat from 8 to 10 sq yd about two or three months after the first application. Surface tarring should be renewed annually on all important roads, and as required on roads with light traffic. On such re-coatings the quantity of tar to be applied will vary with the extent to which the previous coating of tar has been removed by weather or by traffic."

"In all cases careful record should be kept of the condition of the road surfaces in winter and summer, both before and after tarring, the quantity and quality of tar used, the superficial area covered, the state of the weather when the work is being done, the time occupied in actual work, and in waiting whilst work is stopt owing to wet weather, the number of men employed, and full details of the cost of labor and material. Surveyors are recommended to have samples of the tar supplied to them under contracts properly tested by a qualified analytical chemist."

Maryland State Roads Commission 1917 Specifications for Tar and Oil for Cold Application are as follows:

"Sweeping. The road surface shall be swept with a rotary rattan broom drawn by horse or motor power in such a manner as to remove from the surface all dirt or other foreign materials, and should the rotary sweeper not remove all the dirt or foreign material, giving a clean surface, then the contractor will be required to scrape the foreign material from the surface with hoes or other tools, and cleanly sweep it with rattan brooms by hand.

"Applying the Material. No material shall be distributed except by a motor-drawn pressure distributor of a type satisfactory to the engineer, and no material shall be applied when the surface is wet, frosty, or when, in the opinion of the inspector, it is unfit to receive the application. All material will be applied at the rate specified by the inspector, but in no case is more than ⅓ gal of material to the square yard to be applied to the surface in one application and where ½ gal per sq yd is applied, it shall be put on in two applications. When two applications are specified, all the stone chips shall be evenly and uniformly distributed over the surface as hereinunder specified, after the first application of oil or tar has been made, and then the second application shall be made, but in no case shall more than 24 hours elapse between the first and second application of the tar or oil. In order that the traveling public may be accommodated as much as possible, the contractor will apply the oil to one-half of the road surface at a time, and as soon as one truck load is applied to one side of the road it will be covered with chips by the contractor or the State, as the contract provides.

"Spreading Stone Chips. The stone chips or gravel required to be spread on the road after the application of the oil by the contractor will be deposited along the road on one side or the other in approximately even piles, at regular intervals of 25 ft. Approximately 80 tons will be used to a mile of 14-ft surfacing to be treated. The contractor will be required to spread these chips or gravel evenly and uniformly over the surface of the road immediately after the oil is applied and fill in all holes or inequalities, and in no case must more than 2 hours elapse between the application of the oil and the complete covering of the surface with the chips or gravel, and under no circumstances will work be permitted to remain uncovered during the night. The term 'evenly and uniformly' shall be construed to mean that each and every square yard of surfacing shall have the same amount as each other square yard within a variation of not more than 10%."

Cement-Concrete Pavements. A bituminous surface constructed on the surface of a concrete pavement protects the surface of the concrete from abrasive action of traffic, offers a better foothold with certain kinds of bituminous mate-

rials, eliminates the dust which is otherwise liable to form on a concrete surface, and does away with the objectionable glare which results when a strong sunlight shines on the concrete. The bituminous material used is either a refined tar, a tar-asphalt, or an asphalt cement. It is applied to the surface in the amount of ½ gallon per square yard, and spread by means of hand methods or by distributing machines. It is considered that the best bond and most even surface is secured by applying the bituminous material by a pressure machine in two applications of ¼ gallon each. The bituminous material is sometimes swept in with either a rotary sweeper or with hand brooms. It is then covered with sand or fine stone chips to a depth of ⅜ to ½ an inch.

Bituminous Concrete Pavements are in many cases finished with a seal coat either consisting of the same kind of bituminous material as is used in the upper course of the pavement, or a material more suitable for a bituminous surface. Certain types of asphalts, penetration 25° C., 55–75, and combinations of refined tars and asphalts have given excellent results. The amount of bituminous material used per sq yd varies from ⅛ gallon for Topeka bituminous concrete pavement to from ½ to 1 gallon for a bituminous concrete pavement having a mineral aggregate composed of one product of a stone-crushing plant. The best method of applying such a flush coat is by the use of a hand-drawn gravity distributor and a rubber squeegee.

Maintenance. The life of a bituminous surface depends principally upon traffic conditions and the nature of the bituminous material used. With the heavier grades of bituminous materials adaptable for this work, if a broken stone road carries normal traffic, retreatment is necessary every one or two years. Under extreme traffic conditions with the same grade of material, it may be necessary to retreat the road twice each year, as is done in the case of the Avenue du Bois de Bologne, Paris. Retreatments, however, can generally be accomplished by using a smaller amount of bituminous material, about half the amount used in the first treatment. The same care should be taken in preparing the road surface as is done in the original treatment. An average price of maintaining a bituminous surface is 3 to 4 cents per year.

Cost. An average cost of constructing a bituminous surface using 0.5 gal per sq yd of bituminous material, either an asphaltic or tar product, or a combination of the two, is 7 cents per sq yd.

Philadelphia Cost Data. Table I gives the cost of labor and materials from 1913 to 1916, and Table II gives detailed costs of construction in 1916.

Table I

Labor and Material	Unit Costs			
	1916	1915	1914	1913
Torpedo gravel, per 2000 lb, delivered on the road	$2.10	$2.03	$2.14	$2.50
Trap rock chips, per 2000 lb, delivered on the road	2.02	1.94	2.30	2.30
Tarvia A, per gallon applied	0.085	0.085	0.085	0.085
Tarvia B, per gallon applied	0.07	0.0692	0.07	0.07
Ugite hot, per gallon applied	0.08	0.08	0.09	0.09
Ugite cold, per gallon applied	0.064	0.06	0.08	0.08
Asphalt cut-back, per gallon applied	0.1403	0.0791	0.117	0.12
Asphalt, 150 penetration, per gallon applied	0.[illegible]79[illegible]	0.0791	.....	....
Asphaltic road oil, per gallon applied	0.0588	0.0484	0.0523	....
Laborers, per 8-hr day	2.00	2.00	2.00	2.00
Foremen, per 8-hr day	4.00	4.00	4.00	4.00
Asst. foreman, per 8-hr day	3.00	3.00	3.00	3.00
Teams, per 8-hr day	4.80	4.48	4.75	5.20

Table II

Character of Treatment Materials Used	Treatment	Average Materials per Sq Yd: Gallons Bituminous Material	Average Materials per Sq Yd: Pounds of Covering Material	Average Labor Costs per Sq Yd: Sweeping: Hand	Average Labor Costs per Sq Yd: Sweeping: Machine	Average Labor Costs per Sq Yd: Covering	Average Labor Costs per Sq Yd: Cleaning Up	Average Labor Costs per Sq Yd: Total	Average Material Costs per Sq Yd: Bituminous Material	Average Material Costs per Sq Yd: Covering Material	Average Material Costs per Sq Yd: Total	Average Total Cost per Sq Yd
Refined coal tar*	1	.348	18.4	.0066	.0024	.0064	.0020	.0174	.0295	.0187	.0482	.0656
Refined coal tar*	2	.291	16.4	.0058	.0013	.0075	.0022	.0168	.0246	.0168	.0414	.0582
Refined coal tar†	1	.428	17.4	.0063	.0016	.0063	.0018	.0160	.0180	.0180	.0479	.0639
Refined coal tar†	2	.301	16.1	.0032	.0010	.0052	.0006	.0100	.0209	.0168	.0377	.0477
Refined coal tar†	3	.267	15.8	.0032	.0009	.0052	.0007	.0100	.0188	.0163	.0351	.0451
Refined coal tar†	4	.240	15.8	.0030	.0008	.0051	.0006	.0095	.0168	.0164	.0332	.0427
Refined water-gas tar*	1	.34[illegible]	18.4	.0066	.0024	.0064	.0020	.0174	.0278	.0187	.0465	.0639
Refined water-gas tar*	2	.319	15.9	.0026	.0007	.0050	.0004	.0087	.0256	.0164	.0420	.0507
Refined water-gas tar†	1	.428	17.4	.0063	.0016	.0063	.0018	.0160	.0274	.0180	.0454	.0614
Refined water-gas tar†	2	.299	15.4	.0024	.0004	.0053	.0004	.0085	.0193	.0160	.0353	.0438
Refined water-gas tar†	4	.315	16.2	.0011	.0002	.0022	.0006	.0041	.0201	.0168	.0369	.0410
Refined water-gas tar†	5	.200	16.[illegible]	.....	.0002	.0031	.....	.0033	.0128	.0165	.0293	.0326
Asphalt cut-back	1	.267	17.9	.0027	.0004	.0068	.0004	.0103	.0380	.0178	.0558	.0661
Asphalt cut-back	2	.418	17.3	.0038	.0008	.0077	.0008	.0131	.0585	.0162	.0748	.0879
Asphalt cut-back	3	.362	18.0	.0026	.0008	.0045	.0013	.0092	.0471	.0181	.0652	.0744
Asphalt cut-back	4	.320	17.9	.0033	.0008	.0046	.0003	.0090	.0451	.0181	.0632	.9722

*Hot application. †Cold Application.

16. Guard Rails and Road Signs

Guard rails should be placed at the tops of all embankments and at culvert ends where there is the slightest element of danger. Wood, iron and concrete are the materials used in the construction of guard rails. Guard rails on fills are placed 12 inches from the edge of the embankment toward the center of the road; on masonry headwalls they are generally in the center of the masonry.

Wooden Guard Rails are usually built so that the top rail is 3 ft 6 in above the ground. The posts are 6 inches diameter and 7 ft long. They are spaced 8 ft center to center. The top rail is 4 in square, set cornerwise in V-shaped notches sawed in the tops of the posts. The tops of the posts are sometimes sawed off slanting toward the center of the road and the top rail is then 2 in by 6 in, laid flat. The top of the lower rail is placed about 1 ft 3 in below the bottom of the top rail. The lower rail is generally 2 in by 6 in, nailed to the road sides of the posts, which have been previously notched to fit same. The posts are of chestnut or of cedar with the bark shaved off. The rails are planed timber, usually pine or spruce. All joints and exposed surfaces are well painted with a light-colored paint, white lead often being specified for this purpose. The first cost of wooden guard rails in place is ordinarily between 20 cents and 30 cents per

n ft, and the cost of maintenance is from 5 cents to 6 cents per lin ft per ear.

Iron Rods are used sometimes in place of wooden posts where a guard rail needed for a culvert headwall or for a ledge. The top of the rod is forked receive the 4-in square wooden rail. The lower rail is frequently omitted.

Gas Pipe Railing is also used on headwalls of culverts, or on ledge. The pipe sed is 1.5 or 2 inches in diameter. The railing is 3 ft 6 in high, and is built ith two or three lines of pipe. If two lines are used, they are spaced about 21 in part; if three lines are used, they are placed about 13 in apart. The iron posts e spaced generally 8 ft center to center. The pipes are fixt at the bottom by nchoring the bases in the ledge or concrete for a depth of 9 in, or else the ottoms are set in a cast-iron flange which is bolted to the ledge or masonry. he cost of gas-pipe railing, erected, is about 75 cents per lin ft.

Concrete Guard Rails, as described in the 1910 New York State Highway ommission Report, consist of concrete rails set on top of concrete posts. The ils are built with an inverted rectangular trough section, and are 8 ft long y 7 inches deep by 9 wide. Each rail is fitted with three cross diaphragms conecting the sides and top, one being placed at the center and one 4 in back from ach end. The rails are reinforced with four ⅜-in square rods, one being placed t each corner, and the diaphragms are reinforced with a loop of the same size od. The rails are set 3 ft 2 in above the ground on concrete posts, the sides nd end diaphragms of the rail forming a socket into which the top of the post ts. The posts are 6 ft 6 in long and 5 in by 7 in square, and are reinforced ith a ⅜-in square rod in each corner. The posts are set 3 ft 6 in in the ground. he cost of concrete guard rails is estimated to be about 50 cents per lin ft, nd the maintenance cost is practically nothing.

Road Signs. The New York State Highway Commission Standards contain tandard designs for a mile post, danger signal, and guide sign. The mile post built of concrete, 6 ft long and of triangular cross-section. The post is set ft 6 in in the ground. The danger sign is a pine board 20 by 30 inches, attached a galvanized iron pipe which is set into a concrete base. The sign is inscribed ith the word "Danger" in black letters on a large red arrow head. Small igns designating the highway number are also attached to the same standard. uide signs are concrete posts 14.5 ft long, tapering from a 9-in section at the ase to a 6-in section at the top. The posts are set 4 ft into the ground. A 2-in rought iron pipe extends thru the center of the post for practically its entire ength. At a distance of 7 ft above the ground surface the sign boards are ttached, two being placed on one face of the post and two on a face at right ngles to the other. The four sign boards are of white pine, 6 in wide and 3 ft in long, spaced 2 in apart vertically, and faced with a galvanized sign.

In Europe there has been considerable discussion as to the relative advantages of one anger sign meaning reduce speed and a number of symbolic signs describing the specific anger. The signs generally adopted are four in number, each bearing a different symbol esignating respectively, curves in the road; obstacles along the road, such as ditches, umps, and bridges; barriers, such as road crossings or railroad crossings when protected y barriers, except where such crossings should be classed as dangerous crossings; and angerous crossings, such as road crossings or railroad crossings when not protected by arriers. The signs are placed from 600 to 800 ft from the point of danger. The signs n France are 27 inches in diameter, the symbol being painted white on a dark blue ackground.

PAVEMENTS

17. Stone Block Pavements

In the United States, stone block for pavements usually are made from granites and sandstones. Limestone and trap blocks are used to a limited extent. Durax, a special type of stone pavement developed in Europe, was first used in America in 1913. Square blocks and rectangular blocks of a smaller size than is common in this country are also used in some parts of Europe.

Characteristics. The chief advantage of a stone block pavement is its durability, and its chief defect is its rough and noisy surface. It is adaptable to use on grades up to 15% if the joints are filled with bituminous materials, and on grades up to 10% if cement grouted joints are used. Some blocks become slippery from wear. Unless properly constructed and maintained, a block pavement is unsanitary.

Cobblestone pavement is now rarely laid except on unimportant streets or alleys, or as a substitute until a better pavement can be obtained. The stone should be hard and durable, from 4 to 6 inches deep, and from 2 to 4 across the head. The stones are laid on end in a bed of loamy sand about 6 in thick. The stones are set compactly together so as to break joints as much as possible. The surface is then covered with sand which is swept into the joints, after which the stones are tamped with a rammer weighing about 50 lb. When there is no further settlement of the stone the surface is covered with a layer of sand about ½ inch deep. A cobblestone pavement, if subjected to a heavy traffic, soon gets out of shape and the principal maintenance work consists in relaying such portions of the surface. The cost of a cobblestone pavement is about 70 cents to 90 cents per sq yd. One cu yd of sand will be sufficient for about 5 sq yd of pavement.

Belgian Block pavements were constructed with blocks approximately 6 in square on the upper face, with a bottom face smaller than the top, and of variable depths. Confusion as to the meaning of the name Belgian block is caused by the fact that many rectangular block pavements constructed with trap rock are now so named. The method of construction was similar to that described for laying cobblestones. A square yard of pavement required about 27 blocks. The cost per square yard of pavements was $1.25 to $1.45.

Size of Blocks. Pavements are built usually of blocks having rectangular faces. From 1910 to 1918, the depth was reduced from 8 to 5 in in many localities, thus saving in the cost of the blocks. The blocks should be drest so as to be rectangular on the faces, having parallel sides and ends, with right angle corners. Some specifications do not allow bunches or depressions on a face exceeding ¼ in. If the faces are not free from bulges and hollows, it is impossible to get close and even joints in laying the blocks. Blocks are from 5 to 8 inches deep, 3 to 4½ wide, and from 8 to 12 long. The 1918 Report of the Am. Soc. of Mun. Imps, states that the blocks shall be so drest that the joints shall not exceed ½ in in width at the top and for 1 in downward and not more than 1 in in width at any other part of the joint.

Laying the Blocks. The subgrade should be rolled hard with a 10-ton roller, and should be shaped with the same crown as the finished surface. The concrete foundation is then laid as described in Art. 11. The thickness of the foundation is variable, depending upon the traffic conditions, 6 inch being an average value. After the concrete has set, it is covered with a layer of sand 2 to 2½ inches deep, which forms a cushion for the blocks to rest on and allows for any variation in the depths of the blocks. The blocks are placed by hand as closely together as possible, usually in straight parallel courses with the long dimensions per-

pendicular to the curbs. The blocks in one row should break joints with the blocks in another row, by at least 3 in. At street intersections, in order to prevent the traffic of the cross streets from traveling parallel to the long joints and thus forming grooves, the blocks are laid with their long dimensions parallel to one or both of the diagonals of the square formed by the intersecting streets. The work of laying the blocks is started from the curb lines at either side and progresses toward the center of the street. The crown of a stone block pavement is usually parabolic in shape, and the amount of crown can be obtained from formula (see Art. 9). The blocks may be laid to the desired crown by the use of strings, cross-section stakes or a board template. After placing the blocks to the proper lines, they are thoroly tamped with a tamper weighing from 60 to 70 lb until no further settlement occurs. The blocks should all lie perpendicular to the sand bed. If any blocks settle more than others, they should be taken out and relaid. The joints should be filled with a suitable material.

Sand Filler. A filler of sand works out of the top of the joint and leaves the edge of the block exposed so that it soon wears off, thus making a rough pavement. When a sand filler is used, the sand is sometimes spread on the pavement and swept into the joints before ramming is commenced. After ramming, more sand is added to completely fill the joints.

Tar-Gravel Filler. The theory of a tar and gravel joint is that the gravel will keep the blocks in place and the tar compound will fill the voids in the gravel, thus rendering the joint waterproof. It is not possible, however, in practise to uniformly attain this ideal condition, since the gravel will pack in some joints or the tar compound will cool in pouring and thus not enter the joints. The gravel should be of such a size that it will all pass a sieve of ½-in mesh and be retained on a ¼-in mesh. A ¾-in joint is necessary for this type of filler. The gravel should be heated and swept into the joints until they are filled to within 3 in of the top. By using a small iron rod or stick the gravel can be poked down well into the joints. Enough bituminous material is poured to fill the joints to the top of the gravel. The joints are then filled to the top with gravel and poured again with the bituminous material until entirely filled.

Bituminous Fillers are used as joint fillers without previously filling the joints with gravel, but it is necessary in this case, if successful results are to be expected, that the blocks be so drest that very narrow joints can be obtained. Mixtures of sand and asphalt or tar also are used for fillers.

Cement Grout composed of one part of Portland cement and one part of clean sand is also used as a filler. The blocks are laid with thin joints and poured with grout. Traffic should be kept off the pavement for a period of seven days, or until the cement has set up, otherwise good results will not ensue. This type of filler makes a smooth and durable pavement. The surface, however, will be more slippery than where the other types of fillers are used. Repair work will also be more costly, since in removing the blocks more will be broken due to the bond furnished by the cement.

The Maintenance of a stone block pavement, if properly constructed, is practically nothing for the first few years. The life of a granite block pavement constructed on a concrete foundation and with joints well maintained varies from 20 to 30 years.

The Number of Blocks used per square yard will depend upon the size of the block and the width of the joints. Using the ordinary-sized blocks laid with ½-in joints, the number per square yard will vary from 25 to 32.

The Cost of a granite block pavement laid on a concrete base with a bituminous or cement grout filler varies from $2.25 to $3.75 per sq yd, the average

price being about $3.50 per sq yd. When a sand filler is used the cost varies from about $1.50 to $3.00 per sq. yd.

Character and Cost of Stone Block Pavements in 1916 in Several Cities

From Engineering and Contracting, April 4, 1917

City	Square Yards	Price* per Sq Yd	Guarantee, Years	Kind of Filler	Concrete Foundation	
					Thickness, In	Proportions
Malden, Mass.	9 800	$3.17	5	Cement Grout	6	1 : 3 : 6
Springfield, Mass.	9 041	3.70	....	Pitch	5	1 : 3 : 6
Providence, R. I.	16 067	3.83†	....	Cement Grout	6	1 : 3 : 6
Albany, N. Y.	32 391	3.21†	5	" "	6	1 : 3 : 6
Little Falls, N. Y.	669	3.75†	5	Pitch	5	1 : 2½ : 5
Bayonne, N. J.	4 340	3.90	1	Cement Grout	6	1 : 3 : 6
Baltimore, Md.	28 418	3.60	5	" "	6	1 : 3 : 6
Dayton, O.	10 975	3.50	5	" "	6	1 : 3 : 6
Appleton, Wis.	1 880	3.25	....	" "	5	1 : 3 : 6
Duluth, Minn.	2 666	2.30†	5	" "	6	1 : 3 : 6
Louisville, Ky.	24 754	3.48†	5	" "	6	1 : 3 : 6

* Price covers pavement, foundation, and grading. † Does not include grading.

Recut Granite Block Pavements on 5-in concrete foundations cost, according to Howell, in one case in Newark, $1.82 per sq yd, and in the Bronx, N. Y., from 1909 to 1914 an average of $1.72 per sq yd.

Kleinpflaster is used very extensively in Germany both for city streets and highways subjected to a heavy motor-car traffic. The blocks are so cut that when laid they form circular or parabolic arcs. The blocks are roughly 2¼ by 2¼ by 2¾ in. They are laid on a foundation of gravel, macadam or concrete with a cushion coat of sand and gravel, and with joints filled with sand or gravel or in some cases a cement grout. This type of pavement has given satisfactory service for nine years on automobile roads leading into Berlin. A kleinpflaster pavement on a gravel base costs from $1.06 to $1.48 per sq yd; constructed on a 10-in to 12-in stone layer the cost in one instance was $1.90.

Durax, which is a small stone pavement used in England, is a modification of kleinpflaster. The small stones vary in size, according to the traffic to be taken. They are laid on a mastic cushion ½ in thick of bituminous material mixt with sand on a foundation of broken stone or concrete. The joints between the blocks of the wearing surface are filled with bituminous cement. Finally a coat of shingle gravel is spread over the surface.

18. Brick Pavements

Vitrified Bricks are now almost universally used in the construction of this type of pavement. Both represt and wire-cut lug brick are used. The common size of a paving brick varies from about 2½ by 4 by 8 in to 3½ by 4 by 8½ in.

The dimensions of brick given in the 1918 Specifications of the Am. Soc. of Mun. Imps. are as follows: "The standard size of block shall be 3½ inches wide, 4 inches deep, and 8½ inches long. They shall not vary from these dimensions to exceed ⅛ inch in width and depth, and not more than ½ inch in length. If the edges of the brick are rounded, the radius shall not exceed 3/16 in. Only brick with four raised lugs on one side not to exceed ¼ inch nor less than ⅛ inch in height shall be used."

Characteristics. A brick pavement is durable, easy to clean, offers light resistance to traction, is good from the standpoint of non-slipperiness and is

non-productive of dust. This type of pavement, however, when constructed with a cement grout filler is sometimes objectionable from noise.

Laying the Bricks. Broken stone and sand foundations have been used under brick pavements in some cases with good results, but for even medium commercial traffic a 4-in to 6-in concrete foundation should be built (see Art. 11). Generally the cost of brick would not justify its use on any but a concrete foundation. The subgrade should be well compacted with a roller. The concrete foundation should conform to the same crown as the finished surface.

Sand cushion. The concrete is covered with a 2-in sand cushion, which is carefully smoothed off with a board template and rolled with a hand-roller so as to be parallel to the finished surface. For details of construction of template see 1911 Standard Specifications for Brick Pavements of the Am. Soc. Mun. Imps. The bricks are laid on this cushion with their longest dimensions perpendicular to the curbs and so that the end joints are broken by at least 3 in, the work progressing from the sides to the center of the street. At street intersections the bricks are laid in a manner similar to that in stone block pavements. Care should be taken in laying the bricks not to disturb the sand cushion. This may be accomplished by requiring the men to stand on the bricks as they are laid. The bricks are laid to the required crown by the aid of strings, stakes, or board templates. A formula for the amount of crown as given in Art. 9.

Sand-cement Mortar Bed. In this method, the concrete is covered with a mixture of one part cement and four parts sand which is smoothed by a template and rolled to a depth of 1 in. The brick are laid as described above, but this method presupposes the use of a cement grout filler.

Green Concrete Foundation. In this type of construction, the concrete, as soon as deposited, is roughly surfaced. A thin coating of a mixture of one part cement to three parts sand is then spread by means of a double metal template, which first smooths the concrete and then a $^3/_{16}$-in bed of the mortar. Upon this bed the brick are laid in the usual manner.

Expansion Joints. To allow for expansion of the pavement, in cases where cement grout filler is used, it is common practise to insert a board, 1 or more inches thick, between the curb and the pavement during construction, and after the pavement is otherwise completed to remove the board and fill the space left with a bituminous cement. The thickness of this cushion depends upon the width of the street, but it should never be less than 1 in. Some specifications provide for a transverse expansion joint, constructed in a similar manner, at a distance of every 25 to 50 ft along the street. In others, however, this requirement is omitted.

Rolling. After the bricks are laid the surface should be swept and then be rolled with a roller, weighing from 3 to 5 short tons, in a longitudinal direction, starting from the curb at either side and working up to the center of the roadway. After this is accomplished the surface should be rolled transversely in both directions at 45 degrees to the curb lines. It may be necessary to tamp by hand some parts of the pavement which are inaccessible to the roller.

Joint Fillers. The joints between the bricks are filled with sand, with Portland cement grout, or with a bituminous material. In places where the traffic is very light, sand has been successfully used, but as a general rule it will not make a desirable filler.

A Cement Grout Filler is made of 1 : 1 mixture of Portland cement and clean sand, which is mixt in boxes. This mixture is spread on the pavement and brushed into the joints. After a distance of from 45 to 60 ft has been covered in this manner, the same area is regrouted by the same gang in a similar

manner with two parts Portland cement to one of sand. The consistency of the grout is about that of thin cream. The pavement should be sprinkled both before and after the grout is applied. While the initial set is taking place, the surface should be sprinkled and all surplus mixture should be swept into the joints, filling them flush with the top of the brick. After the grout has hardened a ½-in layer of sand is spread over the pavement, and if the weather is very hot, it should be dampened by occasional sprinkling. Traffic should not be allowed on the pavement for at least ten days.

A Bituminous Filler is composed of a tar compound, an asphalt, or a mixture of the two. In applying a bituminous filler, the joints are poured until they are filled to the top of the brick, after which, while the filler is still soft, the surface is covered with a layer of sand. The use of a bituminous filler usually will result in a less noisy pavement than one in which the joints have been filled with a cement grout. A bituminous filler will also take up the expansion of the brick to a great extent. The use of a bituminous filler makes the brick pavement practically impervious.

Cost Data. In the following table are given, for several localities thruout America, 1916 prices of brick pavements constructed with various types of fillers and different thicknesses of sand cushions and cement concrete foundations.

Character and Cost of Brick Pavements in 1916 in Several Cities

From Engineering and Contracting, April 4, 1917

City	Square Yards	Price* per Sq Yd	Guarantee, Yrs	Kind of Filler	Sand Cushion Thickness, Inches	Concrete Foundation	
						Thickness, Inches	Proportions
Elmira, N. Y.	14 400	$2.31	5	Pitch	1½	5	1 : 2½ : 5
Altoona, Pa.	60 350	2.19	5	Cement Grout	1	4	1 : 3 : 6
Baltimore, Md.	15 736	2.25†	5	"	1	6	1 : 3 : 6
Columbus, O.	124 000	1.77†	5	"	1½	6	1 : 3½ : 7
Urbana, O.	8 141	2.01	5	Mastic	1½	6	1 : 3 : 6
Chicago, Ill.	50 000	2.12	5	Cement Grout	2	6	1 : 3 : 6
Ann Arbor, Mich.	11 600	2.06†	5	"	1	6	1 : 2½ : 5
La Crosse, Wis.	3 521	2.20	...	Asphalt	2	5	1 : 2½ : 5
St. Paul, Minn.	10 049	2.68	5	"	1½	5	1 : 3 : 5
Dubuque, Ia.	39 000	2.05	...	"	1½	5	1 : 3 : 5
Topeka, Kans.	34 000	1.77†	...	"	2	5	1 : 2½ : 5
York, Nebr.	26 000	2.02	5	"	1½	4	1 : 2½ : 5
Louisville, Ky.	59 758	1.81†	5	Cement Grout	1½	6	1 : 3 : 6
Vicksburg, Miss.	19 269	2.00†	5	"	1½	4	1 : 3 : 5

* Price covers pavement, foundation, and grading. † Does not include grading.

Maintenance. The maintenance of this type of pavement consists mainly of rectifying any low spots which appear in the surface, due to defective bricks or to insufficient foundation. The maintenance clause of 1918 Specifications of the Am. Soc. of Mun. Imps. is as follows: "The period of guaranty shall be five years. During the period of guaranty, whenever the surface of a vitrified brick pavement becomes uneven, holding water ¼ in or more in depth in a distance of 4 ft or less, or when the pavement has settled over trenches existing previous to the completion of the pavement, then the brick shall be taken up and relaid to proper crown and grade. Any brick which may be found soft, unsound, broken or disintegrated, and all portions of the pavement which may have

become rough by reason of the chipping or breaking of the edges of the brick, so as to produce joints exceeding ½ inch at a point ¼ inch below the surface of the brick, shall be removed, and properly replaced with sound material."

Material Data. The number of bricks required per square yard will vary from about 40 to 65, depending upon the size of brick and the width of joints. The amount of filler will also vary with these factors.

19. Wood Block Pavements

Size and Kind of Blocks. Wood block pavement has been employed with increasing frequency in certain cities of this country. Rectangular blocks treated by some preservative process are universally employed. The commonest sizes of blocks are 5 to 10 inches long, 3 to 4 wide, and 3½ to 4 deep, Variations of $^1/_{16}$ and ⅛ inch in the depth and width of the blocks respectively are allowed after the particular size has been determined upon. Southern yellow pine, Norway pine, black gum, and tamarack are the woods in most common use in this country.

Preservative. The 1918 Specifications of the Am. Soc. of Mun. Imps. state that the amount per cu ft shall not be less than 16 lb of water free oil at the completion of treatment.

Characteristics. A wood block pavement properly built is durable and noiseless. It furnishes a smooth surface which can be easily cleaned, but which, when damp, offers a very poor foothold for horses.

Laying the Blocks. The subgrade should be thoroly compacted with a heavy roller. On this surface a cement concrete foundation should be built as described in Art. 11. The depth of the concrete varies with the traffic conditions, 6 in being common. The surface of the foundation should be finished parallel to the final surface of the pavement. The amount of crown may be determined by the formula as given in Art. 9. A cushion of either sand or cement mortar is spread upon the concrete foundation and struck with a template so as to be parallel to the finished surface. The sand cushion is usually 1 inch thick. A 1 : 3 mixture of Portland cement and sand, spread to a depth of ½ to 1 inch, is specified for the mortar cushion. The concrete foundation should be sprinkled with water before the mortar bed is laid. The blocks are placed upon the cushion with the fiber of the wood vertical and in courses which are straight lines, in most cases at right angles to the curb, but in some instances at an angle of from 45 to 65 degrees with the curb line. The joints are overlapped by at least one-third of the block length, and the blocks are so laid that the joints will not be more than ⅛ inch wide. An expansion joint, about 1 in wide, and filled with bituminous material, should be provided next to the curbs, and it is sometimes specified that one or two rows of blocks, depending upon the width of street, be laid next to the curb with the longest dimensions parallel to the curb. Sometimes transverse joints are required every 100 ft. After the blocks are placed in position, the surface should be rolled with a light steam roller. Care should be taken, if a mortar cushion is used, to finish the rolling before the mortar has set.

Joint Fillers. The joints are filled with either sand, bituminous material, or cement grout. After the joints have been filled, a ½-in layer of fine sand is spread over the whole surface, left for a time, and then removed.

The 1918 Specifications of the Am. Soc. of Mun. Imps. state that: "The joints between the blocks shall be filled with either a pitch or asphalt filler. The filler shall be brought to the proper temperature and poured into the joints, and any filler on the surface of the pavement must be spread as thin as possible by means of squeegees."

Bituminous materials used as fillers are coal-tar pitch, asphalt, or specially prepared

materials. A bituminous filler tends to make the pavement more waterproof, and also provides for expansion to some extent, but it is liable to bleed.

A cement grout filler is composed of equal parts of portland cement and sand. It is of such a consistency that it will readily flow, and should be swept into the joints until they are completely filled. The pavement must be kept closed to traffic for at least seven days in order to give the cement time to set. Very few pavements are laid with grout filler.

Maintenance. The preservative fluid will in some cases ooze from the blocks and be very objectionable. The cause of this "bleeding" has been attributed to the adulteration of the creosote oil used as the preservative with coal-tar pitch; the expansive effect of heat on the blocks; and use of too much preservative fluid per cubic foot. These places in a pavement may be remedied by spreading sand over them or by scraping off from time to time the material that has exuded. The maintenance of wood block pavements, outside of taking care of the "bleeding," consists essentially in periodically sanding to prevent slipperiness, removing any decayed blocks, and in making minor repairs, such as raising low spots or fixing places where the blocks have bulged up. In France and England the experiment is being tried of coating the wood pavements with a bituminous surface. The life of some wood pavements in France has been prolonged by taking up blocks after several years' service and sending them to the municipal plant, where the tops are sawed off so that the blocks are again of uniform depth. These blocks are then laid with the original bottom surface as the wearing surface.

Cost. The average cost of wood block pavements in 1916 in several American cities in the New England and North Atlantic States was about $3.25 per sq. yd. These pavements were laid on a concrete foundation and constructed with bituminous or sand fillers. The average price of the same type of pavement in the South Atlantic, North Central, Northwestern, and Southwestern States was about $2.75 per sq yd.

Character and Cost of Wood Block Pavements in 1916 in Several Cities

From Engineering and Contracting, April 4, 1917

City	Square Yards	Price* per Sq Yd	Guarantee, Years	Kind of Filler	Concrete Foundations	
					Thickness, In	Proportions
Brookline, Mass.	1 307	$3.23	5	Sand	5	1 : 3 : 6
Cambridge, Mass.	970	4.20	.....	Cement Grout	5	1 : 2½ : 5
Middletown, N. Y.	6 200	2.85	.....	Sand	5	1 : 2½ : 5
Dayton, O.	29 227	2.91	5	Bituminous	6	1 : 3 : 6
Fort Wayne, Ind.	7 492	2.00†	5	Sand	5	1 : 3 : 6
West Allis, Wis.	15 800	2.35†	5	"	6	1 : 3 : 6
Rochester, Minn.	19 293	2.24	5	Pitch	5	1 : 3 : 5
St. Paul, Minn.	110 304	2.74	.....	"	5	1 : 3 : 5
Dubuque, Ia.	50 000	2.50	1	Asphalt	5	1 : 3 : 5
Minot, N. D.	25 028	2.89†	5	Pitch	5	1 : 3 : 6
Chattanooga, Tenn.	22 800	2.50†	5	Asphalt	6	1 : 3 : 6
Dallas, Tex.	30 887	3.10	5	Asphalt	6	1 : 8

* Price covers pavement, foundation, and grading. † Does not include grading.

20. Sheet Asphalt and Asphalt Block Pavements

Characteristics. Sheet asphalt is waterproof and ideal from the standpoint of traction, ease of cleaning, and healthfulness. It is non-productive of dust Noiselessness and comfort of use rank fair. Foothold, however, is poor in we

weather, and while asphalt has been used on steep grades, it is not adapted to over 3.5% grades. In cases where illuminating gas has come in contact with it chemical action has taken place, resulting in a spongy surface. It is repaired without difficulty, as an effective bond can be secured between old and new work. Asphalt blocks do not require skilled labor to lay. Many asphalt blocks are chipped off at the edges.

Foundation. It is essential that the foundation for sheet asphalt should be firm and unyielding as the weight of traffic must be carried by it. Usually, therefore, a 1 : 2 : 5 to 1 : 3 : 6 cement concrete is used, being 4 to 6 inches in thickness, depending upon the amount of traffic and subsoil conditions. Concrete must be thoroly set before the binder course is laid. Bituminous concrete has been used in a number of instances, in which cases the binder course is not needed, but this foundation is not advised, since the bond between it and the wearing surface is so firm that the latter can only be removed with difficulty when repairs are necessary and, furthermore, the foundation is not sufficient in most instances where sheet asphalt is economically employed. A 4-in to 6-in macadam coated with asphalt or coal tar has been used, and also ordinary macadam, depressions being filled with a mixture of stone and asphalt. This practise should not be followed.

Binder. Upon this foundation the binder course is laid 1 to 1½ inches deep after rolling, about 40% being allowed for compression. This coat is usually composed of broken stone, sand and asphalt cement, or broken stone and asphalt cement. Material is brought on at 200 degrees to 325 degrees fahr in covered dump carts, deposited, and smoothed down with hot shovels and rakes, taking care not to displace the stone aggregate, and then compacted with a 5-ton to -ton tandem roller.

Specifications for binder stone recommended in 1918 by the Am. Soc. of Mun. Imps. follow: "This shall be clean, hard, broken stone, free from any particles that have been weathered, or are soft. If the stone does not contain the proper amount of material passing the one-half (½) inch screen, the deficiency may be made up by the addition of gravel or sand. Ninety-five (95) percent of the binder aggregate shall pass a screen having circular openings whose diameter shall be three-quarters (¾) the thickness of the binder course to be laid. The remaining five (5) percent shall not exceed in their smallest dimension the thickness of the binder course to be laid. The binder aggregate shall be so graded from coarse to fine as to have the following mesh composition (sieves to be used in the order named):

Passing 10 mesh	15 to 35%	Total passing ½ in, 35 to 85%
Passing ½ in circular opening and retained on 10 mesh	20 to 50%	

The above limits as to mesh composition are intended to provide for such permissible variations as may be rendered necessary by the available sources of supply and the character of the work to be done."

Wearing Surface. The aggregate consists of a finely graded sand and a fine filler, such as stone dust or Portland cement. The wearing surface, consisting of the aggregate and the asphalt cement, is usually laid 1½ to 2 inches deep. The amount of bitumen used varies from 9 to 13% and the filler from 6 to 12%. It is brought on at 280 degrees to 325 degrees fahr, dumped and spread on the binder surface, and then tamped around all manholes, gutters, and curbs, and rolled with a tandem roller weighing 5 to 10 short tons. A continued rolling is very essential, as a constant kneading action is necessary for a well compacted surface. About 200 sq yd per hr should be rolled. Special care must be taken along street-car rails to secure thoro compaction. Usually one to three courses of stone block or brick are laid next the rail. Eighteen to 24 in adjoining the curb are often painted with hot asphalt cement to secure imperviousness to water.

The 1918 Specifications of the Am. Soc. Mun. Imps. covering the manufacture of the surface mixture state that: "The sand and the asphalt cement shall be heated separately to such a temperature as will give, after mixing, a surface mixture of the proper temperature for the materials employed. The sand when used must be at a temperature between 275 degrees and 375 degrees fahr. The asphalt cement when used must be at a temperature between 250 degrees and 400 degrees fahr. The various ingredients shall be brought together and mixt for at least one minute in a suitable apparatus until a homogeneous mixture is produced in which all the particles are thoroly coated with asphalt cement. The weights of all materials entering into the composition of the wearing surface shall be verified in the presence of inspectors."

Asphalt Cement. Satisfactory asphalt cements, as shown by service tests, have been made from California, Mexican and Texas asphaltic oils, gilsonite and asphaltic oil, and "Bermudez" and "Trinidad" refined asphalts.

Sand and Filler. The following description of sand and filler may be considered typical for the materials of a wearing course to be subjected to medium traffic. The sand should be clean, hard grained, and moderately sharp. The product used should show the following mesh composition:

Passing 200 mesh sieve	0 to 5%	25 to 35%
" 100 " and retained on 200 mesh sieve	14 " 25%	
" 80 " " " " 100 " "	6 " 21%	
" 50 " " " " 80 " "	15 " 30%	40 to 50%
" 40 " " " " 50 " "	10 " 25%	
" 30 " " " " 40 " "	8 " 20%	20 to 30%
" 20 " " " " 30 " "	5 " 15%	
" 10 " " " " 20 " "	2 " 15%	

The 1918 Specifications of the Am. Soc. Mun. Imps. covering filler are as follows: "This shall be thoroly dry limestone dust or dust from other equally satisfactory stone or portland cement, the whole of which shall pass a 30-mesh per lin in sieve and at least 66% of which shall pass a 200-mesh per lin in sieve. The surface mixture shall contain from 6 to 20% of this filler, depending upon the kind of sand and asphalt used, and the traffic conditions upon the street or streets to be paved."

Inspection. A. W. Dow recommends that the plant laboratory should be equipped with a set of standard sieves, 10, 20, 30, 40, 50, 80, 100, and 200 mesh; sand scales, a stiff brush for cleaning the sieves, preferably a small one with good stiff bristles; a sand sampler; four screens for testing binder stone, 1-in, ½-in, ¼-in, and ⅛-in screens; thermometers for use with the penetration test and for testing temperature of the paving materials; one penetration machine; a dishpan or other receptacle for holding water at the standard temperature, in which to immerse the asphalt cement samples for testing; two trowels, one 4-inch and the other 6; a 4-in wall brush; a pancake turner for sampling paving mixture while in the mixer; two wooden paddles; pads of unglazed manila paper, 9 by 12 inches is a convenient size; two iron pails and a gas or oil stove, as the conditions may permit.

Maintenance. Failures of sheet asphalt are usually due either to defective foundation, dirt on the binder, too soft asphalt in binder, frost, gas, kerosene or oil, fires, poor construction generally, or lack of traffic to compact. The usual method of repairing sheet asphalt is to cut away the surface for a short distance about the affected spot and then strip it up from the foundation. The edges are cut down square, and after being painted with asphalt cement, a new binder and surface mixture are laid as in new construction.

Asphalt Block Pavements. Compressed asphalt blocks usually have been made of graded trap rock screenings, passing a ¼-in screen, mineral dust and asphalt cement. Good blocks also have been made using copper conglomerate, and for medium traffic, hard limestone has been used as a substitute for trap

rock. The most satisfactory mineral dust, used as a filler, is uniformly pulverized limestone. Portland cement has been used, but it is no better than limestone, and costs more. The blocks usually contain from 6.5 to 8% of bitumen.

Shape and Size of Blocks. Well-made asphalt blocks are uniform in composition, texture and shape; have parallel faces; straight, unchipped and unrounded edges, and unbroken corners; and are not warped or otherwise deformed. The usual dimensions of the face of the block subjected to traffic are 5 by 12 in. The blocks are generally manufactured in three thicknesses, 2, 2½ and 3 in. Blocks should not vary more than ¼ in in length and ⅛ in in width and depth from specified dimensions.

Method of Laying. Asphalt block pavements should be laid on a strong foundation, usually of cement-concrete at least 5 to 6 in in thickness, and, in cases of the heaviest traffic which this type of pavement can carry, 8 or 10 in in thickness. On cement-concrete foundations, the blocks are laid on a ½-in fresh 1 : 4 cement mortar bed. The blocks should be laid with close joints on grades as high as 6%. After being laid, the blocks should be covered with a thin layer of dry, clean, fine, sharp sand, which should

Character and Cost of Sheet Asphalt Pavements in 1916 in Several Cities
From Engineering and Contracting, April 4, 1917

City	Square Yards	Price* per Sq Yd	Guarantee, Years	Wearing Course Thickness, Inches	Binder Course Thickness, Inches	Concrete Foundation	
						Thickness, Inches	Proportions
New Haven, Conn.	5 460	$1.84	5	1½	1½	6	1 : 3 : 6
Providence, R. I.	28 009	1.85	5	1½	1½	6	1 : 3 : 6
Bayonne, N. J.	46 480	1.80	10	1½	1½	5	1 : 3 : 6
Harrisburg, Pa.	10 233	2.00	5	2	1	6	1 : 3 : 5
Baltimore, Md.	398 873	1.45†	5	1½	1½	5	1 : 3 : 6
Columbus, O.	115 964	2.02†	5	2	1	6	1 : 3½ : 7
Logansport, Ind.	44 518	1.95	5	1½	1	5	1 : 3 : 5
Duluth, Minn.	21 507	2.22†	5	2	1	6	1 : 3 : 6
Louisville, Ky.	47 747	1.74†	5	2	1	6	1 : 3 : 6
Denver, Colo.	63 000	1.85	5	2	1½	5	1 : 3 : 5
Oakland, Cal.	52 678	1.50†	.....	1½	2	6	1 : 3 : 6

* Price covers pavement, foundation, and preparation of subgrade.
† Does not include preparation of subgrade.

Character and Cost of Asphalt Block Pavements in 1914 in Several Cities
From Engineering and Contracting, April 7, 1915

City	Square Yards	Price* per Sq Yd	Guarantee, Years	Concrete Foundation	
				Thickness, In	Proportions
Jamestown, N. Y.	9 423	$2.35†	.....	5	1 : 2½ : 5
Toledo, O.	7 794	2.00†	5	5	1 : 3½ : 6
Windsor, Ont.	36 086	2.75†	5	6	1 : 2 : 4
Regina, Sask.	28 525	3.00	15	5	1 : 6

* Price covers pavement, foundation, and shaping subgrade.
† Does not include shaping subgrade.

be thoroly swept into the joints, and the surplus left upon the surface for at least 30 days. On grades above 6%, ⅜- or ½-in joints filled with portland cement have been used. After the grout has partially set, the joints are raked out to a depth of about ½ in. On roadways subjected to heavy motor traffic, anchor blocks should be used.

Cost. The cost of sheet asphalt pavements in 1916, in a number of American cities, averaged $1.90 per sq yd, not including grading. The cost of asphalt block pavements averaged $2.40 per sq yd.

21. Bituminous Concrete Pavements

In 1866 the first bituminous concrete pavement was laid in the United States. Since 1910 there has been a general recognition of the value of the many different types of bituminous concrete pavements for use on roads and streets. The mixing method has not been used as much as the penetration method in the construction of bituminous pavements composed of one product of broken stone on account of the cost of mixing and the fear on the part of public officials and engineers of litigation. The introduction of efficient, low-priced mixing machines and the court decisions of test cases have overcome these drawbacks to such an extent that on account of its inherent advantages the use of this method will rapidly gain favor. The characteristics of bituminous concrete pavements are somewhat similar to those of bituminous macadam pavements. Bituminous concrete pavements are, however, more durable than bituminous macadam pavements.

Classification. Bituminous concrete pavements differ principally in the character of the mineral aggregate and the bituminous material of which the surface is composed. They generally may be grouped in three classes.

Class A. A bituminous concrete pavement having a mineral aggregate composed of one product of a crushing or screening plant.

Class B. A bituminous concrete pavement having a mineral aggregate composed of a certain number of parts by weight or volume of one product of a crushing or screening plant, and a certain number of parts by weight or volume of sand, broken stone screenings or similar material, with or without a filler.

Class C. A bituminous concrete pavement having a predetermined mechanically graded aggregate composed of broken stone, broken slag, gravel or shell, with or without sand, portland cement, fine inert material or combinations thereof.

Foundations. For all classes of bituminous concrete pavements, foundations of gravel, broken stone or slag, old macadam, bituminous macadam and concrete, old brick and stone-block pavements, and cement-concrete have been used. The foundation should be strong enough to carry the traffic to which the pavement is to be subjected. Many failures have occurred due to laying bituminous concrete pavements on weak foundations.

Mixing Plants. A plant for hand mixing of unheated aggregate with hot bituminous cement consists of a mixing platform, three or four heating kettles, long-handled shovels, long-handled dippers, and small tools. Unheated broken stone has been mixed with tars or heavy asphaltic oils in the ordinary type of concrete mixer. Asphalt cements of low penetration at normal temperatures cannot be mixed with unheated aggregates by hand mixing or in this type of plant, as it is generally impracticable to coat the unheated broken stone with the hot asphalt cement.

Cement-concrete mixers with heating devices are used to some extent. There are several different types of this class of mixing plants in current use. In one type, the heat, in the form of hot air, is passed into the mixer by means of a large iron pipe, which runs from the fire-box to the outlet end of the mixer. A second

ype consists of a cylindrical mixer mounted on a four-wheeled truck; heat is btained from a hot-air jacket entirely surrounding the cylinder except on the nds, and by means of a kerosene torch inserted within the drum. In a third ype, hot air obtained by the combustion of oil in air is used to heat the mineral ggregate in the mixing chamber; after the aggregate is heated, the bituminous ement is added. A fourth method of utilizing concrete mixers is to use a rotary rier, as a part of the plant, for drying the aggregate.

In a complete plant for the manufacture of bituminous concrete, the aggregate carried by bucket-elevators to rotary driers, where it is dried and the dust xhausted; from the drier the aggregate is raised by elevators to storage bins r to screens from which the aggregate, in several sizes, falls into storage bins; hen required the aggregate is drawn from the bins to a weighing device, and om there deposited into a mixer. Such plants are also equipped with bitumin-us cement heating tanks and weighing buckets. Plants, especially designed for ne manufacture of sheet-asphalt binder and surface mixtures, have been used a limited extent.

Aggregate of Wearing Surface. The thickness of the wearing surface varies om 1½ to 3 in after rolling in all the classes enumerated. The mineral aggre-ate of class "A" is composed of broken stone, of which a size varying from ⅛ 1¼ inches in the longest dimensions represents average practise. The 1918 pecifications of the Am. Soc. of Mun. Imps. describe an aggregate typical of ass "A" as follows: "Broken stone for the mineral aggregate of the wearing ourse shall consist of one product of a stone crushing and screening plant. It nall conform to the following mechanical analysis, using laboratory screens aving circular openings: All of the broken stone shall pass a one and one-uarter (1¼) inch screen; not more than ten (10) percent nor less than one) percent shall be retained upon a one (1) inch screen; not more than ten o) percent nor less than three (3) percent shall pass a one-quarter (¼) inch reen." In Washington, D. C., bituminous concrete pavements of class B" have been constructed in accordance with the following specifications overing the mineral aggregate. "The paving material shall be composed of usht trap rock screenings, concrete sand, and mineral dust in the following oportions: Trap-rock screenings, two parts; concrete sand, one part, and ineral dust, at least 5 percent of the above aggregate; mixt with asphaltic ement." The trap-rock screenings referred to above varied in size from 1 inch screenings and were devoid of dust. A graded aggregate of class "C," nown as the "Topeka" grading and which nas been used in the construction pavements in many parts of America, is given below. It has been mutually greed that the construction of a pavement with this aggregate as specified is ot an infringement of the Warren Brothers' patent No. 727505.

Bitumen from	7 to 11%
Mineral aggregate passing 200-mesh sieve	5 to 11%
" " " 40 " "	18 to 30%
" " " 10 " "	25 to 55%
" " " 4 " "	8 to 22%
" " " 2 " "	less than 10%

As the Bitulithic pavement has a predetermined graded aggregate, it belongs class "C."

The Bituminous Cements which have been used successfully include asphalt nd tar cements, and cements which are combinations of refined tars and phalts. All types are used in the construction of Class A pavements. Usually phalt cements are used for pavements of Classes B and C. The amount of tumen in bituminous concrete mixtures varies as follows: Class A pave-ents, from 5 to 8% by weight; Class B, 6 to 9%; Class C, 7 to 11%.

Laying. In cases where neither curbs, gutters nor edgings are used, thoroly compacted shoulders are constructed. To serve as definite boundaries and to prevent the bituminous concrete from being forced out over the shoulders during rolling, 2-in planks may be placed on the shoulders at the edges of the roadway and allowed to remain until after the seal coat is applied.

After the bituminous concrete has been deposited on the foundation course, it should be uniformly spread by raking to a depth such that when compacted the wearing course will have the desired thickness, which is usually 2 in. For stiff bituminous concrete mixtures, heated shovels should be used for depositing the mix on the foundation course and hot rakes for spreading the bituminous concrete. Care should be taken to see that shovels and rakes are not overheated, as otherwise the mix may be burnt. Usually the back of the rakes should be used to spread the mix uniformly. If the tines are used for this purpose, the larger stones of the mix may be segregated in the surface of the wearing course.

After being uniformly distributed, the bituminous concrete is rolled while it is still warm and pliable. The rolling should begin at the edges and work towards the center. Rolling should continue, without interruption, until all roller marks disappear and the surface shows no further compressibility. Any places which the roller cannot effectively compress should be compacted with hot iron tampers. Excellent results can be secured by the use of tandem rollers, weighing between 10 and 12 tons and having a compression under the rear roller of from 250 to 350 lb per lin in of roll. Equally good results have been obtained by using a 5 to 8-ton tandem roller to shape up the wearing course, and a 12-ton three-wheeled roller for the thoro compaction of the bituminous concrete. In order to prevent ashes from dropping onto the bituminous concrete, each roller should be provided with an ash pan.

Seal Coat. After the wearing course has been rolled sufficiently so that the wheels of the roller do not make any creases, a coat of bituminous cement is applied to the surface. The seal coat should not be applied unless the surface is clean and absolutely dry. From ⅛ to 1 gal per sq yd of bituminous cement should be used, the requisite amount varying directly with the openness of the compacted bituminous concrete surface and the kind of bituminous cement used. Usually, a seal coat is not used on Topeka pavements. The best method of constructing a uniformly distributed seal coat is to use a hand-drawn gravity distributor or a hand-drawn pressure distributor. As soon after the application of the bituminous cement as possible, a thin layer of dry, clean chips should be uniformly spread over the coat of bituminous cement, the total amount of chips being preferably distributed in two applications.

The Maintenance of bituminous concrete pavements consists in covering with bituminous material all spots where the broken stone is exposed on the surface. Places which disintegrate should be cut out and refilled with either a mixt aggregate or by building the hole up with successive layers of aggregate and bituminous material, the former method giving the best results. At varied intervals it is economy to renew the bituminous surface on the pavement, using from 0.1 to 0.45 gal per sq yd. It will be found that the patrol system of maintenance will do much toward prolonging the life of the pavement at a small expense.

The Cost of Bituminous Concrete Pavements varies with the kind and quantity of bituminous material used, the character of the aggregate, and the method of construction employed. Using an aggregate of stone varying from ⅛ to 1¼ inches in longest dimensions, mixt with 1.5 gal of bituminous material per sq yd and with a flush coat of 0.75 gal per sq yd, the cost would be about 30 to 50 cents in excess of ordinary macadam. The cost of pavements with mixt aggregate varies from $1 to $2.50 per sq yd, including foundation and light grading.

22. Bituminous Macadam Pavements

aterials. The bituminous materials used in the construction of bituminous ments built by penetration methods are: asphalts, heavy asphaltic oils, ed water-gas tars, refined coal tars, combinations of refined tars, and inations of refined tars with certain kinds of asphalts. Broken stone, l, sand, shell, and slag have been used as the road metal. The char-istics of bituminous pavements constructed by penetration methods are ar to those noted for bituminous concrete pavements. The penetration od has come into popular use in constructing country roads and resi-l streets due to low first cost, and the large yardage which can be con-ted per day.

nstruction. In the construction of bituminous macadam pavements it is ed to secure, (1) a stable wearing course consisting of broken stone or similar rial thoroly rolled so that it will be well compacted and keyed together and the several sizes of material uniformly distributed, and (2) a uniform dis-tion and penetration of the bituminous material within the upper 1½ to 3 the crust. Several methods of construction have been devised with a view eeting the above prerequisites. The pavement is generally built in 2 or 3 es, the foundation course or courses being from 4 to 8 in thick after rolling, he top course from 2 to 3 in after rolling.

ethod A. When the metaling in the upper course consists of a naturally ed aggregate with sufficient smaller sized product to practically fill the in the larger, say crusher-run stone from 1¼ to ½ inch in longest dimen-, it is not necessary to further fill the voids by the application of a finer ict before applying the bituminous material. An analysis of the type of er-run stone referred to follows:

Passing	¼-in screen	trace
"	½-in "	18.9%
"	¾-in "	43.1%
"	1-in "	34.4%
"	1¼-in "	3.6%

the upper course is laid the bituminous material is applied, either before or the surface is rolled, some favoring the former method because of the er depth of penetration secured. If the rolling is postponed until after pplication of the bituminous material, the wheels of the roller will have wet or oiled to avoid picking up the surface. When the upper course is l preceding the second application of the bituminous material, a coat of ral matter is spread over the surface before rolling. In certain cases this od is also followed where the upper course is not rolled preceding the cation of the bituminous material. The necessity of a seal coat or a second cation of a bituminous material is determined by the traffic conditions; urface should be given a seal coat if it is to be subjected to a heavy combined -drawn vehicle and motor-car traffic. The material used for the seal coat ne cases has a less penetration and a higher melting point than that used e first application. Certain asphalts and combinations of asphalts and ed tars, penetration 25° C., 55–75, have been used with success for this ose.

ethod B. In case the metaling of the upper course is a uniform product out 1 or 1½ inches in size, it is deposited in a layer of the required thick- on the foundation course and lightly rolled. The bituminous material is applied in an amount from 1½ to 2 gal per sq yd and screened stone chips t ⅜ inch in size are spread on the surface and thoroly rolled. The surface xt broomed with stiff brooms, removing all excess chips, and another coat

of bituminous material, of ½ to 1 gal per sq yd, is applied, covered with a layer of stone chips and rolled. This method is also used with metal of the upper course varying from 1½ to 2½ in, in which case ¾-in stone is used in place of the ⅜-in chips.

Method C. In some forms of construction the voids in the foundation course are filled to a certain extent with sand or small-sized broken stone. After rolling, the excess sand or broken stone is broomed off, and the upper course of metaling spread and lightly rolled. Coarse sand, gravel, or stone chips are then spread and broomed until the voids of the metaling are filled to within about 1 in of the top of the surface. The bituminous material is then applied, using about 1½ gal per sq yd; this coat is covered with a layer of sand, gravel, or screened stone chips and thoroly rolled. This method is often used when the material composing the upper surface is of a large and uniform size. As a rule a second application of bituminous material is not made in this method.

Method D. Another method used, when the metaling of the top course is of a large and uniform size, is to place a layer of sand, ¾ in thick, on the bottom course, the voids of which have been filled. The bituminous material is then distributed on this layer, using about 1 gal per sq yd. The upper course of metaling is immediately placed on the mastic and rolled. Continued rolling forces the material of the upper course down and draws the bituminous mastic up into the voids. Another coat of bituminous material of a lower penetration, using about 1¾ gal to the sq yd, is then applied to the surface of the upper course. A layer of ⅜-in stone, ½ to ¾ inch thick, is spread over this and rolling continued. The work may stop here, or may be carried on a step further by brooming off the excess ⅜-in stone, afterward applying another coat of bituminous material, ½ gal per sq yd, adding a layer of screened stone chips and rolling the same. The Gladwell system, which originated in England, is essentially the same in principle except that a course of screened stone chips mixt with bituminous material is substituted for the sand layer and its coat of bituminous material.

Method E. A bituminous macadam pavement called Pitchmac by the inventor J. A. Brodie, City Engineer of Liverpool, has been used to a considerable extent in England and has been adopted as a standard type by the Road Board of England. It is constructed on a foundation of stone. The wearing course of broken stone varies from 2 to 4½ in in depth, dependent upon traffic conditions. If the wearing course is from 2 to 3 in in thickness, it is constructed in one layer, and if from 4 to 4½ in in two layers. The single layer and, in the case of two layers, the upper layer is composed of broken stone ranging in size from 1¼ to 2½ in. After thoro rolling the bituminous material is applied to the single layer or to each of the layers of the two-layer wearing course. The bituminous compound used in England consists of hot sand mixt with tar pitch. From 1¼ to 2 gal per sq yd are used for the one-layer wearing course and from 3¼ to 3½ gal for two layers. To assist in completely filling the voids, chips varying in size from ⅜ to ¾ in are applied during the rolling of the bituminous grouted layer. This type of pavement has been used to a limited extent in Massachusetts.

Method F. In order to secure uniform sizes of road metal in the course upon which is to be applied the first application of bituminous binder, in some cases recourse has been had to harrowing. After harrowing, the course is shaped and thoroly compacted before the bituminous material is applied. With the limestones commonly used in Illinois, this method has produced satisfactory results when suitable asphalt or tar cements have been properly used in accordance with the specifications of the Illinois State Highway Dept.

Method G. In some cases, when heavy asphaltic oil has been used for the ituminous cement, the following method has been employed. Upon a course f broken stone, prior to rolling, about ½ gal per sq yd of asphaltic oil is distributed. The course is then harrowed, after which the second application of about ½ gal per sq yd is made. The course is next thoroly rolled during which screenings are spread. In some cases, the pavement is finished by a third application of ½ gal per sq yd of oil, screened and rolled. In other cases, the third treatment is followed by a fourth using from ¼ to ⅓ gal per sq yd.

The Maintenance of bituminous macadam pavements consists in covering pots with mineral matter where either an uneven distribution or an uneven enetration has caused an excess of bituminous material to exude on the surface. Otherwise the maintenance is similar to that recommended in Art. 21.

Cost. The cost of bituminous pavements built by penetration methods aries largely with the amount and kind of bituminous material and road metal sed, and the method of construction employed. An average cost, using a otal of 2 to 2¾ gal of bituminous material per sq yd, varies from 25 to 40 ents per sq yd in excess of the cost of ordinary macadam.

Bituminous Macadam Pavement Cost Data by Ill. Highway Commission. ength, 1500 ft; area, 3500 sq yd; thickness, 3 in; labor, 25 cents per hour; teams, o cents per hour; average haul, ½ mile; bituminous binder, 2.54 gal per sq yd; constructed, Sept. 29 to Nov. 25, 1915.

Cost of Labor and Materials

	Per sq yd
Superintendence and inspection	$0.077
Cost of stone and gravel	0.430
Cost of sand and chips	0.085
Cost of bituminous binder	0.217
Hauling stone, gravel and chips	0.210
Spreading stone and screenings	0.058
Rolling	0.039
Heating and applying bituminous binder	0.060
Incidental expense	0.019
Total cost	$1.195

Bituminous Macadam Pavement Constructed with Pouring Cans, Alexandria, a. Cost data compiled by E. C. Dunn, City Engineer. "The cost for an average square rd exclusive of grading was $1.01 in 1915 and $1.135 in 1916, the difference being due increased cost of material in 1916. The details of the average cost per square yard 1916 were as follows:

Concrete Base

	Cents	Cents
Materials:		
Water	0.25	
Sand delivered 165 lb at ½ cent	8.25	
Cinders, on cars, $^1/_7$ cu yd at 16⅔ cents	2.38	
Cement, on cars, $^{11}/_{100}$ bbl at $1.47	16.17	
Tools, etc	1.00	28.05
Labor:		
On concrete and placing 50 lb stone	13.00	
Teams	4.00	
Supervision	0.50	17.50
Total cost of concrete base		45.55

Top Wearing Surface

Material:		
Stone, 350 lb trap on cars at 0.69 cent	24.15	
Asphalt, 2½ gal on cars at 8.4 cents	21.00	
Fuel	0.80	
Tools, etc.	2.00	47.95
Labor:		
Men	12.50	
Teams	6.00	
Supervision	1.50	20.00
Total cost of top wearing surface		67.95

Total cost per square yard $1.135.

Wages: Foreman, 25 cents per hour; laborers, 17 to 18 cents; carts, 30 cents, and double teams, 40 cents per hour; haul, 0.95 mile, average. Work done by day labor, City Engineer's wages not included."

Bituminous Gravel Pavements. The use of gravel in the construction of bituminous pavements by penetration methods has been usually limited to those localities where broken stone costs more than gravel. It is self-evident that it is impracticable to secure the same keying effect with gravel as can generally be obtained with broken stone.

The method of construction and the conclusions relative thereto, as contained in the following abstracts of a statement by P. E. Green, are typical of the practice of many engineers who have used gravel in this class of work. "It was specified that the gravel, as spread on the street, should be of a size that would pass a ring of 2 in diameter and be held on a ring of ½ in diameter. After a small yardage has been spread under these specifications, it was found that there was no lock at all between the stones, and that when filled with the bituminous cement but little strength was secured. This yardage was taken up and the gravel afterwards used was still further screened so as to eliminate practically all pebbles below ¾ in in size. After the surface had been rolled to the satisfaction of the inspector, 1½ gal per sq yd of Texaco asphalt was poured over the pavement, and, immediately behind the asphalt, pea-size gravel was lightly spread over the surface, and the whole rolled again. This was then covered with ½ to ¾ gal per sq yd of asphalt cement, and over this second coat of cement was spread coarse sand, and the whole again rolled. On account of the fact that so much care had to be taken with the gravel used, and that it all had to be screened after being received in order to eliminate small pebbles which prevented the locking together of individual stones, it would probably have been cheaper to have purchased crushed limestone in the first place. Some conclusions may be drawn as to this method of construction. They are as follows: Gravel can be made to lock and bond only after a great deal of labor and trouble are taken with it. It would probably be economical to pay twice as much for crusht stone as for gravel to get equal results."

23. Concrete Pavements

Characteristics. A concrete pavement furnishes a smooth surface, is easy to clean, and is not productive of much dust. It is somewhat noisy, and is more slippery than a broken stone road but less slippery than a sheet-asphalt pavement. Its resistance to traction is low. Cracks may be easily repaired but depressions and disintegrated spots are difficult to repair satisfactorily without cutting away the concrete. A worn out cement-concrete pavement sometimes may be used economically as a foundation for a wearing course of some other kind of paving material.

Foundation. Concrete pavements should only be laid on a well-compacted and well-drained subgrade. If the subgrade is of clay or other heavy soil, it should be replaced with clinker, broken stone, cinders, gravel, or some other suitable material. This surface should be flat or parallel to the finished surface

of the pavement and should be thoroly rolled with a heavy roller. Prior to the deposition of the concrete, the subgrade should be thoroly wet, otherwise the subgrade will absorb water from the concrete.

Ingredients and Proportioning. The materials used for the fine aggregate of a cement-concrete pavement are generally sand or stone screenings, and for the course aggregate either broken stone or gravel. The broken stone employed should be obtained by crushing hard, tough rock. Preferably the stone should be composed of naturally graded sizes and free from dust or dirt. What has been said relative to broken stone applies as well to gravel, since a screened gravel with the fine material eliminated allows a more accurate determination of the correct proportions. The sand used should be clean, sharp, and coarse, free from loam, clay, and vegetable or organic matter. The cement should be a first-class portland cement that will meet the standard specifications of the American Society for Testing Materials. Care should be taken to use clean water, since water which contains any alkalies or acids will be detrimental to the concrete. The proportions which are used in connection with the construction of concrete foundations are not rich enough in either cement or mortar to make satisfactory concrete which is to be subjected to the abrasive and impact forces of traffic. The proportions used for the best class of one-course cement-concrete pavements are about one cement, one and one-half or two fine aggregate, and three coarse aggregate. For two-course pavements, the base may have the proportions 1 : 2½ : 4 and the top, 1 : 1 : 1½.

Construction. In plain cement-concrete pavements constructed by the mixing method, the concrete is deposited in one or two layers. Usually the average transverse slope does not exceed ¼ in per ft. The thickness varies from 5 to 10 in, dependent on traffic requirements and other local conditions. More attention should be paid to obtaining a smooth and regular surface in constructing cement-concrete pavements than is usually accorded concrete foundations. The use of templates to strike the surface of the concrete, and of bridges, which span the concrete surface, thus enabling the laborers to work over the surface without standing on it, should be insisted upon. The surface may be finished by the use of wood floats, belts or light rollers. It is also important to protect the surface from too rapid drying out while the concrete is curing, otherwise shrinkage cracks are liable to occur. This is accomplished sometimes by covering the pavement as soon as it has taken its initial set with a canvas which is kept moist for a few hours. The canvas is then removed and the surface is covered with layer of sand or earth, which is kept thoroly moist for a period of two weeks.

Expansion Joints. Transverse joints always should be provided. There will be more or less contraction and expansion of the concrete due to changes of temperature, variation in the moisture content of concrete, and variation in the condition and character of the subgrade. If expansion joints are not present, when the concrete contracts, the tensile strength of the concrete will be exceeded and the pavement will crack; when it expands it will tend to crush, spall, or bulge. The edges of the joints must be protected from the abrasive action of the traffic, and it is obvious that the joints should be filled with a material that will allow some movement between the joints as the pavement expands and contracts. The width of the transverse joints depends upon the distance between them. It is considered better practice to construct narrow joints at short intervals apart rather than wide ones far apart. Transverse joints are placed from 15 to 50 ft apart, 30 ft being an average distance. If curbs are used, longitudinal joints should be constructed adjacent thereto. The width of the longitudinal joints will depend somewhat upon the width of the pave-

ment. They are usually made from ½ to 1½ in wide and are filled with a bituminous filler or a patented expansion joint is employed.

Reinforced Pavements. Reinforced cement-concrete pavements are usually constructed by the two-course method. The reinforcement usually consists of woven wire or expanded metal, altho a mesh work of small round bars is sometimes used, and generally is placed between the base and the wearing course. At the 1916 National Conference on Concrete Road Building, a committee reported that "it is only recently that reinforcement has been used to any great extent, and the interval is too short for the accumulation of the data necessary to standardized practice."

The Cost of Concrete Pavements varies from $1 to $2 per sq yd, depending upon the location of the work and the amount and character of the materials employed, an approximate average price for a 6-in pavement being $1.35.

24. Sidewalks, Curbs and Gutters

Width and Slope. In business districts sidewalks usually extend from property line to curb, but in residential districts it is common to find a grass plot or row of trees placed between the footway and the curb, while on rural highways the walk is entirely omitted or restricted to a narrow foot-worn path at the side of the road. The widths of sidewalks from property line to curb vary from 4 to 6 ft in residential sections; 8 to 20 ft in business districts. The usual width for each sidewalk is from one-fifth to one-sixth of the total width between property lines. The transverse slope of the sidewalk varies from ⅛ to ½ in per ft, depending upon the kind of material with which the surface is constructed.

Concrete sidewalks usually have a 6-in well-compacted foundation of broken stone, gravel, or cinders. The total thickness of concrete is 3 to 5 in, and it is composed of a 1 : 2 : 4 or a 1 : 2½ : 5 mix. The upper inch, however, is a wearing coat composed of 1 : 1 or 1 : 1.5 mixture of cement and ¼-in sand or stone. The concrete is laid in sections which form slabs 4 to 8 ft long, the wearing course being placed on the lower concrete before the latter has set up. The joints which mark off the slabs in many cases extend thru the entire depth of concrete, thus providing for expansion and contraction. The wearing surface is finished off smooth by troweling and later, if desired, may be marked with a grooving or perforating tool.

For Brick Sidewalks concrete may be used as a foundation, but a good stone, sand, or gravel bed is most common. The bricks are laid either diagonally or perpendicularly to the center line of the sidewalk. The joints are filled with sand, and a light coat of sand is spread over the surface upon completion. The latter, however, is ultimately removed by the action of traffic or otherwise.

Flags of Blue Sandstone or Granite are laid on a 3-in to 4-in foundation of sand or other suitable material, the joints being filled generally with a cement mortar. In Paris the granite slabs are 6 inches in thickness, bedded in the concrete. Flag stones are usually 2 to 3 inches in thickness.

Cinders are laid 6 inches deep and are then flooded and tamped. These sidewalks are apt to be dusty.

Gravel walks are laid 3 to 4 inches thick on a good subsoil. Where the soil is poor it is excavated and the space refilled with crusht stone, clinker, or screened gravel of large size. The total depth may be as much as 8 in, the wearing course being 0.25 to 0.5 inch of fine gravel or torpedo sand.

Tar Concrete sidewalks are built on a coarse gravel foundation, the stones being not more than 2 inches in greatest diameter, which is thoroly coated with

a hot coal tar before it is laid. On top of this course is laid the binding course composed of gravel, not exceeding 1 inch in its greatest diameter, mixt with a hot coal tar in an amount of 1 gal to 1 cu ft of gravel. These two courses are tamped and rolled until no further movement of the stone takes place, and are laid so that the total thickness after rolling shall not be less than 2¼ in. The wearing surface is composed of a mixture of sand and coal tar, 75% sand and 25% tar, and is laid on top of the binding course and rolled in so that the finished depth will be ¾ in at all points.

Asphalt Mastic sidewalks are constructed in France as follows: a mixture of natural rock asphalt, Trinidad asphalt and asphaltic oil is laid to a thickness of about 1 in on a cement concrete base about 4 in thick, the top of which has been treated with a layer of cement mortar ½ to 1 inch in thickness. A sprinkling of gravel is spread over the surface while the asphalt is still warm.

The Costs of various forms of sidewalk construction follow: cement concrete, including excavation and a cinder or gravel foundation, varies from about 80 cents to $1.50 per sq yd; brick on a sand and gravel bed, 70 cents per sq yd, Middle States, and $1.10 per sq yd, New England States, including excavation and sand or gravel bed; flagstone, from $1.75 to $3.00 per sq yd; tar concrete approximately 60 cents per sq yd.

Stone Curbs are usually composed of granite, blue stone, limestone, or sandstone. They are 4 to 12 in wide, 8 to 24 in deep, and 3 to 8 ft long. The top and front faces are drest, the latter being given a slight batter to keep wheels away from the top edge. The ends are square drest, and the curbs are so laid that the ends come about ⅛ in apart. Curb stones are most commonly laid on a well compacted sand base.

Concrete Curbs are common, laid usually "in situ" on a sand base. They should be separated from the sidewalk by an expansion joint. Several types of combination concrete curbs and gutters are on the market, practically all of which are laid "in situ" and some of which are reinforced.

The Cost of Curbing is as follows: granite, $1 per lin ft, including cost of setting; concrete curb 8 in wide by 24 in deep, approximately 60 cents per lin ft, including cost of laying.

Gutters are from 1½ to 6 ft wide, some having the same fall as the roadway, with the lowest point next to the curb, and others having the lowest point at the center, thus forming a trough section. In the latter case the depth at the center will vary from 4 to 9 in. Cobblestones, as commonly used, are laid on a 6-in to 10-in bed of sand or gravel. These stones are 3 to 8 in in their longest dimensions, and are laid on a sand or gravel bed with broken joints. The joints are filled with sand, after which the surface is rammed and covered with a thin sand layer. Gutters are also constructed of brick and stone block. In brick, stone block, and asphalt pavements, the gutters are formed in most cases by extending the pavement to the curb. The cost of cobblestone gutters is approximately 75 cents per sq yd.

25. Street Cleaning and Snow Removal

Formation of Dust. The formation of a large part of street dust is due to the deposition of dirt adhering to the wheels of vehicles coming from adjacent earth, gravel, or macadam streets, the leakage of the contents of loaded vehicles both in transit and while loading and unloading, the excrements of animals, and the abrasive action of traffic. Other sources of street dust are mineral matter applied to the surface of streets, the decay of leaves, bark, twigs, and other vegetable matter, dust from manufacturing concerns, and soot and ashes from chimneys. It is always a public nuisance. Under traffic it forms heavy clouds

to such an extent as to obscure the traveled way; when wet it forms a mud which renders footing dangerous to both man and beast, and may cause skidding of wheels.

Methods which have been found applicable to various types of roads and pavements in urban districts are as follows: Earth, gravel, and broken stone roadways are cleaned by gangs or patrolmen with push brooms or by horse-drawn or motor-driven rotary sweepers. Bituminous surfaces, and good brick, bituminous and wood block pavements are cleaned of coarse dirt by gangs or patrolmen during the day and by hose flushing and squeegeeing or by rotary scrubbers during the night. Brick, in poor condition, and stone block pavements are cleaned during the day by gangs or patrolmen and during the night with rotary brushes and hand or machine flushing methods. In connection with all the methods, the dirt is forced to the gutters and is usually removed by gangs with wagons following the machines.

Hand Cleaning is done either by gangs or patrols. In the first-named method, the pavement is cleaned at frequent intervals, while in the latter case each patrolman has a certain definite area to clean every day, his equipment consisting of a push-scraper, push-broom, shovel, and usually some type of can-carrier to collect the refuse in. The material is collected in cans or bags and placed to one side, or is swept into heaps at the side of the street to await the arrival of wagons which cart the refuse to the public dumping-ground. In almost every city of size in Europe may be found a patrol system well organized and highly efficient, but in this country the poor supervision and general indifference given to the matter are usually responsible for the lax and shiftless systems encountered in so many of our smaller cities. Hand sweeping is entirely confined to the day time.

Horse Sweepers are employed in many cases, the work being done generally at night. Such sweeping should always be preceded by sprinkling to ensure the laying of dust. There are numerous combined sweepers and sprinklers, both horse-drawn and motor-propelled, in use in Europe. This treatment alone is excellent on streets subject to light traffic, but on the heavier-traveled thorofares should be supplemented by hand sweeping during the day. A pavement in good repair is essential for best results with a mechanical sweeper. For cleaning asphalt pavements a sprinkler combined with a rotary scrubber is employed with good results. The scrubber is a cylinder fitted with a series of rubber blades.

Hose Flushing is employed to a considerable extent in Europe, and is used to some extent in this country. The material is carried in suspension to the sewer thru the nearest catch-basin. This treatment is particularly efficient in removing all fine dirt, but it necessitates the preliminary removal of all coarse material, which might in time clog the ordinary sewer-pipe. Flushing is accomplished by applying a stream of water to the surface with a broad, sweeping motion. In addition, the pavement is scrubbed with rubber squeegees.

Motor-Truck Flushing Machines. There are various types of flushing machines used on sheet and block pavements which throw the water in broad fan-shaped sprays over the surface, thus washing the dirt into the gutters.

Snow Removal in Urban Districts. Wm. H. Connell describes and classifies the common methods as follow: (1) Plowing the snow to the side or center of the street by means of automobile or horse-drawn plows, after which it is piled and then loaded into vehicles and hauled to dumps, which may be sewer manholes or rivers and in some cases open lots. This general method of disposal should start when the snow begins to fall and plows should be kept continuously at work during and after the storm until all of the snow is removed from the

streets. (2) Panning (supplemental method), which consists of pushing the snow into the sewer manholes by means of a specially constructed form of scraper, usually constructed of iron, which may be likened to a very large snow shovel. (3) Flushing the snow into sewer inlets or manholes (supplemental method). This is done by means of a hose or by running power flushing machines up and down the streets and forcing the snow to the gutter, following with a gang of men brooming the melted snow into the sewer inlets.

Highways Outside Urban Districts. Conclusions from 1919 Report of Committee, Am. Road Builders' Assn. "On main traveled roads snow must be removed to the road surface and for the full width. Methods: (1) Prevent the excessive accumulation of snow wherever possible by the use of snow fences and the removal of the natural causes of these accumulations. (2) Wherever speed is essential and enough equipment can be provided to keep ahead of the storm, plows attached to the front of motor trucks have proved the most efficient method of removing snow. These must be supplemented by local assistance wherever drifts and excessive accumulations cannot be prevented, and wherever snow must actually be moved away. (3) Wherever speed is not so essential and the depth of snow fall is not too great, road machines and plows drawn by horses or tractors are satisfactory. (4) Wherever the fall is great, the snow may be rolled in the outlying districts, and must be removed from the roadway in thickly settled sections, where hand methods are the only satisfactory solution."

MACHINERY

26. Crushing Plants

A complete crushing plant for road-building purposes consists of a boiler, engine, crusher, elevator, screen, and storage bin.

The Crushers are of two distinct types: namely, the gyratory and the jaw crushers. Both types of crushers have some means of regulating the openings so that, by using the proper opening together with appropriate crushing plates, almost any size of crusht product can be obtained. The size of the crusht product is limited by the smallest opening between the crusher plates at the outlet end of the crusher. The gyratory crusher is a more recent invention than the jaw crusher. That the gyratory crusher produces a more uniform product and is a more durable machine are the main advantages claimed for it over the jaw crusher. With the same horse power this type of crusher will generally produce a larger output.

The type and size of crusher to be used depends upon the nature of the rock to be crusht, the size of the product desired, and whether or not the plant is to be permanent or portable. The output of any crusher will depend to a large extent upon the plant arrangement. This latter consideration should be given serious study. It will be found that the smaller the stone is crusht, the less will be the output of a crusher, since just so much more work must be done in crushing the stone.

The Gyratory Crusher is extensively used for permanent plants. Its great weight and height have not made it generally adaptable to portable plants, altho it is sometimes made for this purpose. A description of portable gyratory crushers is given as follows:

Dimensions of each receiving opening in inches	7 by 32	8 by 35	10 by 40
Capacity in short tons per hour	10 to 20	20 to 40	30 to 60
Diameter of ring in inches	1.5	2	2.5
Horse power required	15 to 20	18 to 25	22 to 30
Weight of mounted crusher in pounds	12 000	16 000	18 000
Approximate price on wheels	$1250	$1600	$1800

In the following are given data of gyratory crushers adaptable for permanent plants:

Dimensions of receiving openings in inches	8 by 30	10 by 38	12 by 44	14 by 52
Capacity in short tons per hour	15 to 40	30 to 70	50 to 90	80 to 120
Diameter of ring in inches	1.5 to 3	1.75 to 3.5	2 to 3.5	2.5 to 4
Horse power required	14 to 21	22 to 30	28 to 45	50 to 75
Weight of crusher in pounds	20 900	31 200	45 500	64 800
Approximate price	$1400	$1700	$2300	$3000

Jaw Crushers are largely used for portable crushers. They are placed on a framework which is supported by four wheels. They are designated by the size of the jaw openings at the top. The size of 8 by 16 is very commonly used. The following shows the weights and capacities of several typical sizes:

Size of openings in inches	8 by 16	9 by 18	10 by 22
Capacity in short tons per hour	9 to 14	12 to 20	16 to 25
Jaws set to, in inches	2	2	2
Horse power required	12	15	25
Weight of crusher in pounds	7500	9600	13 500
Approximate price on wheels	$735	$900	$1400

The crushers may be run by gasoline engines or by steam engines, the latter being perhaps more common. The steam engine is generally mounted upon a horizontal boiler, which is in turn placed on wheels so that it can be easily transported from place to place. This last type of engine and boiler is made in sizes capable of developing from 4 to 50 h p and costs about $30 per h p. Gasoline engines, which develop from 10 to 25 h p, cost from $40 to $50 per h p.

Elevators and Screens. The stone as it comes from the crusher is carried by means of a bucket elevator to the revolving screen, which is fixt over the bin. In portable plants the arrangement of the elevator, the screen, and the bins is such that they can be readily dismantled for transportation purposes. The elevators and screens are made in standard sizes, the lengths depending upon the size of the crusher and bin. Unless the elevator is to be housed in, the buckets should be fixt to a chain drive, rather than fixt to a revolving belt, from the standpoint of durability. The prices of screens and elevators with gears and skids depend upon the length and size. A 10-ft screen with gears and skids will cost from $200 to $300. A 16-ft elevator will cost about $300.

The Portable Bins are generally made with three compartments. These bins are made in sizes varying in capacity from 13 to 50 short tons. In some of the more modern types of portable bins, provision is made so that the bin can be raised to a height which will allow teams to pass beneath the unloading chutes. An average price for this type of portable bin is approximately $10 per short ton capacity.

Small Tools. The prices of small tools are as follows: No 2 hand shovels, $6.00 to $9.00 per dozen; No 2 hoes, 4 by 11¼ in, $4.00 per dozen; railroad picks, $6.00 per dozen; pick handles, $3.25 per dozen; 10-tine stone forks, $17 per dozen; 4-tine stone hooks, $9 per dozen; 2-man 12-tooth rakes, $26.50 per dozen; crowbars, 5 cents per lb; stone hammers and sledges, 8.5 cents per lb.

27. Rollers and Scarifiers

Rollers. Two types are manufactured, 3-wheel and tandem rollers. The common size of the 3-wheel rollers varies in weight from 10 to 18 short tons while the tandem rollers vary from 2.5 to 12 short tons. The 3-wheel roller is used to a great extent in ordinary macadam construction, the heaviest size being used in the construction of roads where the stone is either very large or of an extremely hard nature, such as trap rock. The tandem rollers have been mostly used in the construction of sheet-asphalt, bituminous concrete,

brick and wood block pavements. Both types of rollers in this country are commonly run by steam, altho there are some makes which are run by gasoline engine. Rollers in Europe equipped with this latter power have been found to be particularly advantageous, since the roller can be made more compact; no time is lost in waiting to get up steam; the rollers are less noisy; they are smokeless, and capable of much more rapid movement; and they are found to be particularly useful in repairing patches and on maintenance work. Within the past few years the use of the heaviest types of rollers in constructing bituminous concrete pavements has been found detrimental. A 12-ton roller has sufficient weight to do all the work required. The tandem rollers will in the future without doubt be more largely used on this type of work. Successful construction does not require the heavy weight so much as it does a steady rolling, which gradually brings the different particles of stone together in juxtaposition without actually crushing them. The rear wheels of all 3-wheel rollers are provided with holes in which spikes may be inserted for picking up the road. These spikes, however, do not accomplish the same work as a good type of scarifier.

Cost. Three-wheel rollers varying from 10 to 15 short tons in weight cost from $2500 to $3500 f o b at the factory. Tandem rollers varying from 2.5 to 12 short tons in weight cost from $1300 to $3000.

Scarifiers. In the United States most of these machines consist of a heavy cast-iron block on two or four wheels, which holds a series of steel picks. The blocks weigh about three short tons. These picks can be adjusted in the block or the block itself so arranged that any depth desired up to 5 or 6 in can be picked up. The picks are arranged in either a straight line, or in two lines which together form a V. Most of the scarifiers are so designed that it is not necessary to turn them around. This is accomplished generally by having two sets of picks, one set being used when the machine runs in one direction, and the other when in the opposite direction. Scarifiers of this type are towed by a chain hitched to the roller. The arrangement of the picks and the form of the blocks vary, but all of the machines work on the same principle. The scarifiers of Europe are of two types, one of which is similar to the one just described, while the other is attached to the rear of a 3-wheel roller and is a part of the roller. It consists of a framework supporting a small block to which the picks are attached. This pick block is behind one of the driving wheels and the picks are set at such an angle that they are forced into the road by the tractive force of the roller and the weight of the block. The advantages of this type of scarifier are the ease with which it is handled and the saving in time obtained by having the scarifier so attached that it can instantly be put into operation when the roller is traveling in either direction.

A road surface will be much more easily scarified if it is first thoroly watered. The scarifier will rip up the surface, bringing the stone to the top, and allow the dirt and finer particles to go to the bottom. By using a scarifier it is possible to accomplish as much as one hundred times more work in a day than can be done by hand methods.

The price of many of the American makes of scarifiers is about $500 f o b factory.

28. Mixing Plants

Asphalt-Mixing Plants. A complete mixing plant used in the asphalt-paving industry includes elevators, dryers, storage bins, kettles for melting bituminous material, and a mixer capable of mixing the sand and bituminous material, all suitably arranged so that the operations follow one another in such a manner that none of the ingredients of the mixture have to be handled

by hand from the time they are placed in the receiving end of the machine until they leave the outlet end. The above apparatus is installed as a stationary plant, or is mounted permanently on either one or two flat cars as a portable plant.

Bituminous Concrete Mixing Plants. Mixing plants may be divided into four classes: first, cement-concrete mixers; second, cement-concrete mixers with heating attachments; third, dryers, storage bins, and mixers; fourth, dryers, storage bins, weighing devices, and mixers.

Cement-Concrete Mixers. Unheated broken stone has been mixed with tars or heavy asphaltic oils in the ordinary type of concrete mixer. With this type of plant asphalt cements of low penetration at normal temperatures cannot be mixed with unheated aggregates, as it is generally impracticable to coat the unheated broken stone with the hot asphalt cement. This class of mixers should not be used for the construction of bituminous concrete pavements.

Cement-Concrete Mixers with Heating Attachments. There are several different types of this class of mixing plant in current use. In one type the heat, in the form of hot air, is passed into the mixer by means of a large iron pipe, which runs from the fire-box to the outlet end of the mixer. A second type consists of a cylindrical mixer mounted on a four-wheeled truck. Heat is obtained from a hot-air jacket entirely surrounding the cylinder except on the ends and by means of a kerosene torch inserted within the drum. In a third type, hot air obtained by the combustion of oil in air is led from the combustion chamber into the mixing cube. After the mineral aggregate is heated the bituminous cement is added. A fourth method of utilizing concrete mixers is to use a rotary dryer, as a part of the plant, for drying the aggregate.

Dryers, Storage Bins, and Mixers. In several types of plants the aggregate is heated in rotary dryers from which the dried material is transported by elevators to storage bins. In some cases the aggregate is raised to the storage bins before being dried. As required, the aggregate falls by gravity into the mixer. The heat for the dryers and the mixer is obtained by direct heat from fire-boxes or by oil-burning apparatus.

Dryers, Storage Bins, Weighing Devices, and Mixers. In a complete plant for the manufacture of bituminous concrete, the aggregate is carried by bucket-elevators to rotary dryers, where it is dried and the dust exhausted; from the dryer the aggregate is raised to the storage bins by elevators; when required the aggregate is drawn from the bins to a weighing device and from there deposited into a mixer.

Kettles are made in different shapes, mounted on wheels or otherwise, and vary in size from 50 to 1500 gal. The smallest sizes are generally cylindrical in shape. Portable kettles are usually heated by direct fire. Kettles used in asphalt-paving plants are usually heated by means of steam coils.

Small Tools. The prices of small tools are as follows: asphalt or tar rakes, 18-in iron shank, $12.85 per dozen; asphalt mattocks with crucible steel cutter, $9 per dozen; tar-carrying pails, $2.70 each (see Engr.-Contr., Apr. 5, 1911).

29. Distributors

Distributors may be divided into two classes: namely, pressure distributors and gravity distributors. Under the gravity type may be included the Pouring Pots. There are several different makes of pots, all of which are somewhat similar to the ordinary watering-pots, except that the nozle and spout are replaced with other devices by means of which the material is poured onto the road in a sheet rather than in streams.

Gravity Distributing Machines include hand-drawn gravity distributors from which the material flows in the form of a sheet, tanks from which the bituminous material flows by gravity through a hose and nozle onto the road, machines with distributing apparatus consisting of one or more horizontal pipes pierced with small holes from which the material flows in small vertical streams onto the road surface or onto a flash board and from the board to the surface

of the roadway in the form of a sheet. The general practise in Europe in using machines of this type is to follow the distribution by brushing the material into the road. This is done either by hand brooming or by brooms which are attached directly behind the distributor.

Pressure Distributors. The various types of distributing machines of this class may be grouped in the following subdivisions: hand-drawn distributors, n one type of which the material is pumped from the tank through a length of lexible hose to an iron pipe fitted with one or more nozzles, while in another the heated material is pumped into the distributor and applied to the roadway surface by pressure from a tank of compress air; pressure tanks to which are attached hose and spraying devices or horizontal distributing apparatus; and machines equipped with mechanical power-pumps between the tank and the listributing apparatus. Several machines of the last type have been invented n the United States. The distributing devices of all of these machines are alike in having horizontal pipes fitted with nozzles. The machines differ somewhat in the way the pressure is obtained and applied. Pumps, run by a sprocket-drive attachment on the rear axle, by steam and by gasoline, are utilized. Horses and steam-rollers are used for hauling. Some distributors are mounted on motor-trucks.

The Am. Soc. of Mun. Imps. prescribed in its 1918 specifications the following requirenents to secure uniform distribution and to provide for such details as means of control of proper pressure and temperature and prevention of rutting of the wearing course of bituminous macadam pavements during construction: "The pressure distributor employed shall be so designed and operated as to distribute the bituminous materials specified uniformly under a pressure of not less than 20 lb nor more than 75 lb per sq in n the amount and between the limits of temperature specified. It shall be supplied with an accurate stationary thermometer in the tank containing the bituminous material and with an accurate pressure gage so located as to be easily observed by the engineer while walking beside the distributor. It shall be so operated that, at the termination of each run, the bituminous material will be at once shut off. It shall be so designed that the normal width of application shall be not less than 6 ft and so that it will be possible on either side of the machine to apply widths of not more than 2 ft. The distributor shall be provided with wheels having tires each of which shall not be less than 18 in in width, the allowed maximum pressure per sq in of tire being dependent upon the following relationship between the aforesaid pressure and the diameter of the wheel: for a 2 ft diameter wheel, 250 lb shall be the maximum pressure per linear in of width of tire per wheel, an additional pressure of 20 lb per in being allowed for each additional 3 inch in diameter."

30. Street Cleaning Apparatus

The Ordinary Push-Broom is universally used in street cleaning operations. n this country the brooms are about 16 in wide, and the head is filled with split bamboo, rattan, hickory, or steel wire. One edge of the broom-head is sometimes fitted with a steel scraper. These brooms are used for heavy sweeping. For lighter and more thoro sweeping the head is filled with basswood, which s more pliable than some of the other forms of fillings. In Europe hand brooms made out of birch or other twigs are still used to some extent for light sweeping. Push-brooms, similar to those employed in the United States, are in general use. n one form of push-broom the filling is enclosed by a wooden clamp that can be moved up and down so as to make the filling as stiff as desired. The average price of hand push-brooms is about $8 per dozen. A small rotary sweeper on three wheels, pushed by hand, has been used in Europe. A man with this machine can sweep three times as much area as he can with the ordinary push-broom

Rotary Horse-Drawn Sweepers are made to sweep widths varying from 6 to 9 ft. By means of levers operated by the driver the broom can be made to engage the road surface with either a light or heavy pressure. The brooms are filled with rattan, split bamboo, or hickory. A 2-horse rotary sweeper with steel frame will cost about $500. A 1-horse sweeper with wooden frame will cost about $300.

Motor-Truck Sweepers have been used in Europe for several years. In France it was found that altho they could accomplish four times as much work with this type of machine as with horse-drawn types and with a lower operating cost per square yard, yet the method was too costly on account of the charges for sinking funds and repairs.

Hand Pick-up Sweepers. Several types of pick-up sweepers have been designed and used. One of the most successful is pushed by hand, and consists of a rotary brush geared to the wheels. The brush is covered in with a hood and operates on the carpet-sweeper principle, throwing the sweepings into a pan which is a part of the machine. The sweeper will clean at one passage a strip 30 in wide. The cost of this sweeper is about $90.

Squeegees and Scrubbers. The squeegees are made of wood or metal with a rubber edge in widths varying from 12 to 20 in. They cost about $1 each. The rotary scrubbers are built on somewhat the same principle as the rotary sweeper and are attached to a cylindrical sprinkling wagon. As the water falls upon the surface it is pushed toward the gutter by the scrubber. This machine complete costs about $1300. Hand scrapers for use in cleaning asphalt pavements cost from $2 to $4 each, depending upon the size. They are made about 36 in wide.

Bags and Cans. Sweepings may be collected in galvanized iron cans or in bags which are later removed by wagons. Cans of this sort cost from $2.50 to $4.00 each. Bags cost 5 to 8 cents each. A 2-wheel truck for carrying a bag to hold the sweepings costs about $14. Cans for holding sweepings are fixt to a wheel truck in such a manner that they can be attached and detached from the truck by a very simple operation. A dust pan is also attached into which the sweepings are broomed. The pan can be raised by a lever and dumped into the can. The truck is pushed by hand. This machine complete costs about $30.

Horse-Drawn Sprinklers. In connection with street sweeping, sprinkling is necessary to prevent the dust rising. Sprinkling carts of the ordinary cylindrical-tank type, having capacities varying from 400 to 800 gal, are very common. A 600-gal sprinkling wagon costs about $500.

Motor-Truck Sprinklers, some of which have the sprinkling attachments on the front of the machine instead of in the rear, are used in Europe. The iron tanks of the European sprinklers are rectangular rather than cylindrical in shape as in this country.

SECTION 16

HARBOR AND RIVER WORKS

ASSOCIATE EDITOR

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FORMERLY CHIEF OF THE BUREAU OF YARDS AND DOCKS OF THE NAVY AND GENERAL MANAGER, U. S. SHIPPING BOARD EMERGENCY FLEET CORP.

Wm. M. Torrance, M. Am. Soc. C. E., assisted in compiling data and supervising preparation of drawings.

M. J. McPike and M. Kaufmann prepared drawings.

* Chas. W. Staniford, Chief Engineer, Dept. Docks and Ferries, N. Y., furnished some of the drawings and data in these articles.

Lieut. H. B. Overesch, Jr., C.E.C., USN., and Lieut. H. D. Rouzer, C.E.C., USN., assisted in compiling the data and securing illustrations for the revision of this section. Mr. Joseph Michaelson assisted in the preparation of the discussion of length of sheet piling on page 1730.

HARBORS

1. General Information

A Harbor is a refuge for ships, a port or haven. From the very earliest understanding of the term, it was a sheltered arm of the sea in which vessels could be built and launched. and taken for repair, or seek refuge in time of storm. One of the requisites was a good anchorage or holding ground. Such cargo as was carried by the vessel was transported to the shore by smaller craft or carried to the vessel by hand. Natural harbors are those which, in themselves, are usable without resort to engineering works of improvement. Primitive harbors were almost always of this character. With the increase in size and draft of vessels most of the so-called natural harbors have required works of extension and improvement. As a distinctive term, a natural harbor may be considered as one located on an estuary or an enclosed bay. Artificial harbors, in a more restricted sense, are those requiring the construction of extensive engineering works, such as breakwaters for protection of the shipping from wave action, or where shallow or otherwise unsuitable areas have been excavated and deepened and other improvements made in order to make them available for the purpose intended.

Prevailing usage fails to distinguish clearly between the term " harbor " and " port." When to a harbor is added terminal facilities, the harbor becomes a port. A haven of refuge for shipping from storm and the elements would not be a port unless to the usage made of this shelter there are also added the facilities for loading and unloading the vessels and re-shipping or supplying the necessary cargo by land-drawn and water-borne supply lines and feeders. R. S. MacElwee suggests as a definition, " The harbor may be defined as consisting of the water ways and channels as far as the pier head lines, the port to include everything on the landward side of those lines, that is, the piers, slips, wharves, sheds, tracks, handling equipment, etc.," and suggests that this distinction, while technical and novel, tends to mark off distinctly the lines of the jurisdiction of the Federal Government as against that of the State and municipal authorities. This technical restricted definition would appear to have advantages, but it is believed the term port should be considered the general one and includes the harbor while the harbor does not include the port.

Design. Harbors are usually located by force of circumstance over which the engineer has little or no control. This is especially so with natural harbors where the local geographical advantages and growth of population have resulted in the need for, and location of, harbors irrespective of the existence of better natural harbor facilities at points not far distant. The best natural harbors are probably, by reason of the very facilities they offer, often centers about which important interests and activities grow. In cases where the engineer has some voice in the selection, a thoro survey should be made of the neighborhood, including the foreshore and the depths of water in the vicinity. Borings and soundings should be taken to ascertain the character of the ground, both as to anchorage and as regards the readiness with which it will lend itself to economical dredging or deepening operations, should such be necessary. In an important harbor the depth of water should be sufficient to meet the requirements of the maximum draft of vessel likely to use the harbor, allowance being made for over-depth for the pitching and surging of vessels under wave action and the drag or set of vessels when underway. Borings on land should also be made to indicate probable subsurface conditions with a view to future location of the necessary harbor works. Observations should be made as to the meteorological phenomena, such as prevailing winds, frequency of

storms, height and force of waves, establishment of mean high and low water and range of tide, direction and velocity of prevailing currents, evidence of silting and of littoral drift.

The large ports and harbors of the world, especially the older ones, were undoubtedly located to afford shelter and protection not only from the elements, but from the enemy and to protect the shipping while laid up from destruction and ravages of raids which were frequent in ancient, medieval and comparatively recent times. This condition was common in the Mediterranean and in the North Sea; consequently we find many old ports some distance up rivers, probably located as far up as the vessels of these times could be navigated. The facility with which the river and its tributary branches could be used as a water highway by light draft craft for transporting cargo undoubtedly had an important influence, as water transportation had many advantages over the possible land transportation conveniences available in those times. As a consequence we find the large European ports, such as London, Liverpool, Antwerp, Hamburg, etc., river ports to which, with the change in times and the increase in size of vessels, the sea had to be brought by continuous expensive harbor and river work in order that they might retain their supremacy as ports, or others which have disappeared or are no longer ports, as for example Paris.

On account of the facility with which land transportation, especially rail, can now be handled, the newer ports, especially of the United States, are located on the seacoast or on estuaries, probably explaining the reasons for the location of river and sea ports. This same condition has caused the development and construction in European practice of wet docks together with those reasons given in Art. 24 of this Section.

Tides. The tidal day is always of greater length than the solar day, two tides occurring, generally, in each day, high tide being on an average 50 minutes later every day. While the tides on the coast line are produced by the sun and moon, this is not due to their immediate effect on the waters of the sea adjacent to the coast line, but is due to secondary tidal waves, resulting from primary waves in the wide expanse of the oceans. It is calculated that the primary effect of the moon on rise of tide is 1.34 ft, and of the sun 0.61 ft, or together 1.95 ft, or against each other 0.73 ft, the velocity of the primary tidal wave being from 50 to 60 miles an hour. The momentum of this mass moving around the earth, taken in connection with the shoaling of the ocean and configuration of the coast line, explains the very great variation in range of tide found at different localities. The atmospheric condition, by reason of the pressure on the surface of the water, affects the tidal range inversely as the height of the mercury in the barometer. Sudden variations of the height of mercury in the barometer will give a difference of about 0.35 inches for each foot of tidal range for each inch variation in height of barometer. Tidal range is affected considerably by wind, dependent upon location, and characteristics of the coast line, and the force, direction, and continuation of wind

Tidal Changes. Engineering works of improvement in harbors or rivers may result in quite surprising changes in tidal conditions. In the location and construction of harbors and harbor works, it is of the greatest importance to ascertain definitely the data of high and low water and range of tide, for which purpose automatic tide gages should be used. Elevation of mean high and mean low water at the various important parts of the United States can be readily secured from local authorities, or from the U. S. Coast and Geodetic Survey, but in important new harbor work, it is advisable to establish these independently. Altho the height to which the surface of the sea rises and falls at different points along the coast varies considerably, yet there is a mean level to which,

at a certain stage of the tide, the water returns. This is known as mean sea level. Where possible, it is advisable to refer to this datum.

Tidal Prism. This is the volume of water represented by the area of the bay, river or harbor effected by tidal changes multiplied by the range of tide which would generally be the volume of water entering at the flood and leaving at the ebb. Harbor engineers are loath to permit the construction of any works that will serve to change materially this tidal prism, as a decrease in the tidal prism has a direct effect upon the quantity and velocity of the water of the ebb and flood tide and a consequent effect upon the channel depth and direction by reason of the resulting tidal scour.

2. Waves

Wave Height. In the location of harbors and the design of harbor works, it is necessary to ascertain the probable maximum wave action and forces likely to be encountered. The height of wave to which an exposed entrance may be subjected will depend upon the greatest fetch or reach of open sea from the windward shore; for open exposed locations

$$H = 1.5\sqrt{d}$$

where H is the height of the wave in feet, and d is the length of fetch in miles. For shorter lengths of exposure in bays, or harbors, this height is represented by

$$H = 1.5\sqrt{d} + (2.5 - \sqrt[4]{d})$$

Extensive observations closely check the results obtained by the application of these two formulas, first proposed by T. Stevenson, it being borne in mind that in the open sea the fetch is confined to the distance over which the wind acts continuously in one direction, the rotary character of ocean storms restricting the applicable fetch even in the broadest expanses of the Pacific Ocean to a distance of not over 1000 miles.

Formation of Waves. The direction of the wave crest in deep water is at right angles to that of the wind, but, on approaching the shore, it tends to come parallel to the shore line, due to the lagging effect on the wave of the rising bottom. Most waves are caused by the action of the wind, due to the wind blowing over the surface with varying velocity, acting by friction on the surface and by direct pressure on the rear of the wave crest. A confused sea existing during a heavy storm usually changes to a regular swell afterward, resulting in trochoidal seas. Waves formed in this manner in deep water are generally understood to be oscillatory waves, in which the particles of water forming the wave move thru circular or elliptical orbits. This is not strictly correct as, undoubtedly, all storm waves are to some extent waves of translation. Deep water storm waves on reaching shallow water become waves of translation, the friction of the rising bottom on the lower portion of the wave tending to retard this to the point where the front of the crest of the wave becomes steeper and steeper, and finally breaks and becomes a breaker or comber.

Wave Velocity in deep water is dependent upon the velocity of the wind, taken together with its continuity, and the extent of the surface of the sea over which it acts which, in turn, affects the height of the wave and the wave length, and may be given by

$$V = \sqrt{5.123\,L} \text{ for deep water waves}$$

$$V = C\sqrt{5.123\,L} \text{ for shallow water waves}$$

in which the coefficient C is the square root of the ratio of the axes of surface orbits b/a, it being recalled that the shoaling depth changes the orbits from

circular to elliptical, V being the velocity in feet per second, and L length of wave in feet. It is only necessary to use the coefficient C in cases where the depth of water is less than one-half the wave length. The following are values of C:

d/L =	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
C =	0.552	0.746	0.858	0.922	0.958	0.977	0.988	0.994	0.997

in which d = depth of water, or distance from the center of the surface orbit to the bottom, and L = distance from a wave crest to the next.

The relation between wave height and length is dependent upon wind, sea and other local complex influences; in general the proportion between length and height may be taken as: 33 for moderate wind and light sea, 20 for strong wind and rough sea, 18 for storm and heavy sea.

The wave pressure varies greatly; pressures as high as 7840 lb per sq ft have been recorded. A series of observations at Skerryvore Rocks and at Tyree gave results: 6 ft swell 3041 lb; 10 ft ground swell 3041 lb; 20 ft heavy sea 4562 lb; strong gale, heavy sea 6083 lb per sq ft.

Wave Energy of a wave of one unit width, with length L and height H is

In deep water, $E = 8\,L\,H^2\,(1 - 4.935\,H^2/L^2)$
In shallow water, $E' = 8\,L\,H^2\,(1 - 19.74\,a^2/L^2)$

in which a is the semi-major axis of surface orbits, H and L are in feet, and E is in ft-lb. This energy is for salt-water waves; for fresh-water waves reduce amount by 2½ per cent.

Values of a in Terms of Wave Height H

d/L =	0.10	0.15	0.20	0.25	0.30	0.35	0.40
a =	$0.91H$	$0.68H$	$0.59H$	$0.55H$	$0.52H$	$0.51H$	$0.504H$

When a wave enters a bay, or arm of the sea, between natural or artificial barriers, its height tends to decrease on account of friction of the shoaling bottom and the retardation due to barriers, and on emerging into open, interior water, further decreases in height due to distribution and the tendency of the wave crests to diffuse parallel to the shore line.

Wave Action. The percentage of wave height above water level is important as indicating the height to which harbor structures may be subjected to direct wave action. Observations indicate that, in general, about two-thirds of height of the wave is above mean water level, and about one-third below. Due to the action of high winds, opposing currents and further unknown causes, deep sea waves may break partially in water of sufficient depth for their free propagation, so that a barrier opposed to them in water of great depth may at times be subjected to the direct action of breaking waves. It is known, however, that waves invariably break on reaching water of insufficient depth. Considering the height of the maximum wave as determined by actual observation, or by application of Stevenson's formula, the ratio of depth of water to wave height in which waves may break will vary from 1 to 2.71, or 1.67 being a mean ratio taken from 134 observations. Where wave action is arrested by barriers, a portion, at least, of the wave energy will be exerted against this barrier. It is important in designing harbor works that such artificial barriers be made strong enough to resist the successive attacks of maximum sized storm waves. Wave force is exerted and transmitted against such barriers in divers ways, some of which are as follows: (a) The force may be static pressure due to the height of the column of water. (b) It may result from the effect of rapidly moving particles of the liquid. (c) It may be due to the impact of floating bodies on the surface of the water hurled by the wave against the barrier. (d) It may result from the rapid subsidence of masses of water thrown against the structure, producing a partial vacuum and causing sudden pressure to be exerted from within. (e) Destruction of the structure may result from the fall-

ing on it from above of large and heavy masses of water thrown up by the wave action.

The interior of such barrier structures is also often affected by forces transmitted thru joints or cracks, by hydraulic or pneumatic pressure. The most destructive effects of waves are exerted at or about sea level. The result of wave action may be apparent at considerable depths, altho diminishing considerably in force and extent. It should be considered in the design of structures subjected to wave forces that the material entering into the structure is, at least, partially submerged and that, therefore, the weight of the mass submerged must be considered. Consideration should also be given to the presence and movement of ice where harbor works are located in colder climates.

3. Breakwaters

Uses of Breakwaters. A breakwater is a work or barrier constructed around or in connection with artificially sheltered harbors in order to protect the interior water areas from the effect of heavy seas, and make it possible for this area to be used for the safe mooring, operating, handling, loading, and unloading of shipping. Breakwaters are almost invariably a most prominent and essential feature of artificial harbors, and are employed to convert an open area or roadstead into a harbor or in rendering more secure and usable harbors that are enclosed and protected, excepting against the sweep of the sea with prevailing winds coming from certain directions. Breakwaters on a smaller and less important scale are sometimes constructed in the interior areas of large natural harbors to protect shipping from wave action under conditions of heavy storms from an unfavorable direction. A breakwater, the protected side of which is used as a quay for wharfage, is also known as a Mole.

Design. There are three general types of breakwaters: First, that in which the exposed face is vertical. Second, partially vertical and partially inclined. Third, inclined. The type to be selected is dependent upon local conditions. Claims in favor of vertical type of breakwater are that the action of deep sea waves being purely oscillatory, the wave force encountered is due to water head pressure, dependent upon the height of waves. As has already been stated, nearly all waves are to some degree waves of translation, and as breakwaters are usually located and constructed in comparatively shallow waters, the wave forces to be encountered are more complicated than those due purely to hydrostatic pressure and, undoubtedly, vertical face breakwaters are subjected to severe wave impact. The combination of the inclined and vertical face often results in the throwing up, vertically, of large masses of water which, in turn, fall on the top of the breakwater directly behind the parapet. In crib breakwaters, with heavy timber decks, structures have often been seriously damaged by such action. In inclined breakwaters there is danger, if these are not carried high enough and surmounted by vertical wall, with heavy storm waves, that these will sweep entirely over the breakwater, resulting in a disturbed effect, and quite an appreciable secondary wave action behind the breakwater. When waves impinge obliquely in the direction of the breakwater, they often accumulate size and energy as they travel along its front. In the design of harbor breakwaters, information should first be obtained as to the direction and force of prevailing winds, character of coastal currents, probable maximum height of waves, character of bottom or foundation and cost and availability of materials to be employed in construction.

Rubble-Mound Breakwaters are a more or less heterogeneous assemblage of rubble, riprap, or cobble, deposited without any particular regard to bond or bedding. This is the simplest form of breakwater and is usually constructed by the dumping of material into the sea from barges or cars run out on trestles

or staging. This operation is carried on until the mound or heap emerges from the water and is carried up a distance above the same, the action of sea and waves being depended upon to give the sides a natural stable slope. It is usual in constructing extensive works of this character to make some attempt at grading materials and to control the slopes, the interior being composed of cobble or smaller riprap and the sea-side of the breakwater of heavier and more massive stone. As the force of the wave decreases with the depth in deep water, the slopes are often steeper and smaller stones are used. Such a structure is often supplemented by placing heavy massive stone or concrete blocks on the sea side, while the slopes and faces above the sea level are roughly paved with blocks of stone or concrete.

Fig. 1

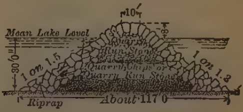


Fig. 2

Rubble-Mound Breakwaters

Figs. 1 and 2 show typical sections of rubble-mound breakwaters, Fig. 1 illustrating the first modification made in the Cleveland, Ohio, breakwater. The earlier breakwater had a complete sand core or hearting; during heavy storms, this hearting was washed out, resulting in the subsidence of the rubble top and eventual disruption. The work was finally built as shown in Fig. 2 with all sand omitted from the core and with rough instead of smooth stone covering. These breakwaters cost about $175.00 a foot. A rubble-mound breakwater of the same general type at Buffalo, N. Y., cost $135.00 a foot.

Timber Breakwaters consist of timber cribs floated out to place and loaded and sunk by being filled with rubble; or are built of close round piles or sheet-piles driven into the bottom, framed, braced and backed up with rock or concrete blocks. Timber cribs of this character are sometimes superimposed on rubble-mound breakwaters. Breakwaters of timber are more or less temporary structures and are principally used in shallow water for unimportant barriers. They have, however, been extensively used on the Great Lakes.

Fig. 3. Timber Crib Breakwater

Fig. 3 shows a typical timber crib breakwater built at Cleveland, Ohio, at a cost of about $175.00 per lin ft. This work was damaged by storms, masses of water projected upward by storm waves fell on the top timber work, breaking the cross members and planking. The work was reinforced by backing the structure with riprap. Somewhat similar timber crib breakwaters at Buffalo, N. Y., cost $151 000 and $165.00 per lin. ft.

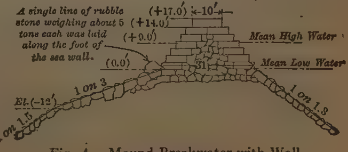


Fig. 4. Mound Breakwater with Wall

Masonry Wall Breakwaters are, in whole, or part, constructed in a regular and systematic manner of coursed stone masonry or concrete with vertical or inclined faces. Such walls are constructed in cofferdam in the dry or under water by divers or diving bells. This character of breakwater is often built from the tide level up surmounting a rubble-mound sub-structure or capping a timber crib.

Fig. 4 shows a low coursed stone wall breakwater surmounting a rubble-mound, at Dog Bar, Gloucester, Mass ; a somewhat similar breakwater at Sandy Bay, Cape Ann, Mass., was considerably strengthened by preventing the large blocks from sliding by fixing steel dowels into the stones on the harbor side against the toe of the course directly above.

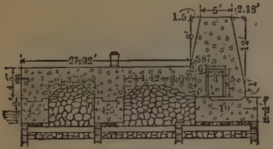


Fig. 5

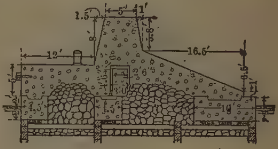


Fig. 6

Timber Breakwaters capped with Concrete

Figs. 5 and 6 show typical sections of the mole breakwater at Harbor Beach, Mich., consisting of a timber crib, capped with concrete blocks. The section shown in Fig. 6 was damaged by storm by having the concrete capping lifted and cracked. The cost of wall and composite wall breakwaters varies widely; the wall breakwater at Dover, England, in 50 ft depth, cost $1733.00 per ft; at Holyhead, a rubble-mound surmounted by a wall in 40 ft of water cost $790.00 per ft.

Reinforced Concrete Breakwaters may be built by means of reinforced caissons built on shore, or in floating structures, launched or lowered into the sea and sunk to place, and settled upon a prepared foundation of rubble or piles, by filling the compartments with stone or sand, or may consist of reinforced concrete piles and sheet-piling banked or filled with sand, riprap or rubble. See Sect. 5, Art. 38, page 565. In the following examples of breakwaters the numbers refer to Fig. 7.

(1) Zeebrügge, Belgium, 1905. Breakwater or mole 6560 ft long, lower course built of monoliths, set in floating caissons of structural steel and plating with concrete lining arches. Lower sections 82 ft long, 24½ to 29½ ft wide, 28 to 36 ft deep. Built in graving dock and floated out. Sills for setting 20 inches deep. Sections sunk at low water by flooding and then filled with concrete. Blocks weigh 1500 to 1600 long tons light and 4000 to 5000 long tons when filled.

(2) Barcelona, Spain, 1906. Section of floating caisson for monoliths for extending the breakwater. Blocks 39 feet long divided into five pockets, which were filled with pre-molded blocks.

(3) Touapsé, Russia, on Black Sea, breakwater 1400 ft long, largest caissons 56 ft long, 21 ft wide, and 21 ft deep divided into 21 divisions, 7 sets longitudinally. Caissons of reinforced concrete. Large boxes weighed 268 long tons when launched and 1600 long tons when filled.

(4) and (5). Talcahuano, Chili, 1908. Breakwater 1750 ft long, section (5), dam (4), each caisson 33 ft long, built of reinforced concrete and filled with sand and stone.

Breakwaters have been constructed employing floating caissons at Bilboa, Bizerta, and Algoma, Wis. See Sect. 5, Art. 38.

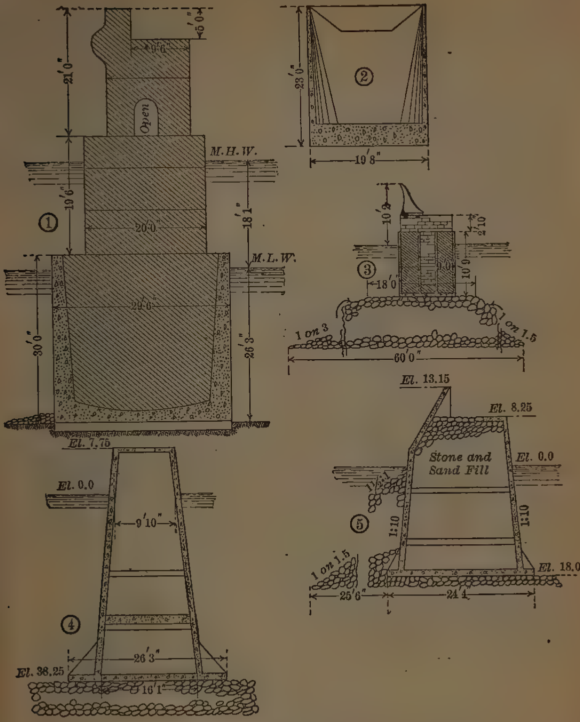


Fig. 7. Reinforced Concrete Floating Caisson Breakwaters

4. Bars. Sea Encroachment

Littoral Drift. The coast line of all exposed foreshores indicates a gradual but continual change, in some locations in the direction of retrogression or washing away, and in other locations in deposition and accretion, the carrying away and depositing of materials being due to wave action in connection with current flow. The breaking of waves on a beach or foreshore serves to stir up loose material and also to break up solid material by direct erosion or by the impact or wearing away, due to the carriage by the water of particles of sand and shingle; the lighter particles of such material remain in suspension long enough for them to be projected some distance along the shore by the resultant of combined wave action and littoral currents. The heavier particles are rolled along

the beach and partake of a zigzag movement, the principal action being generally confined to the reach between high and low water mark. Littoral drift is generally attributable to the prevailing wind direction, altho it is modified by onshore currents. Breaking waves acting on a beach or foreshore by reason of their velocity are carried some distance up the sloping beach, mixing with, and stirring up, the sand and shingle. The wave having spent itself, the water then runs back by gravity, but is met by the next oncoming wave, the returning water carrying in suspension and rolling down the beach some sand and shingle, the oncoming wave by reason of its greatest velocity being at the top or crest is carried over the returning water, the returning water setting up what is known as the "undertow." The material carried down, as the undertow loses its velocity, is in turn carried up again on the beach by returning waves moving backward and forward, as shown in Fig. 8.

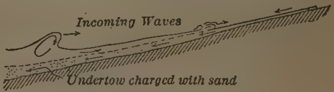


Fig. 8

The general effect of the wind on the foreshore, as shown in Fig. 9, tends to carry this drift of sand and shingle up the beach in a zigzag line. The actual action, as can be readily understood, is far more complex, but the general principle illustrated obtains. The drift varies, being dependent upon the direction and force of the wind, but, in general, the result follows the direction of the prevailing winds of the locality.

Bars. Bars at the entrances of harbors are of four general classes: (a) Natural bars, consisting of hard material not affected by scour or current; (b) Those due to the deposition of alluvial material brought down by river drainage; (c) Casual bars occasionally and irregularly heaped up by storm wave action and afterwards dispersed by similar action, or by one or more currents; (d) Bars consisting of sand or shingle of certain general features and permanent direction, but constantly subjected to alteration by action of winds, waves and varying currents. It is generally concluded that the last given type of bar is that most usually encountered at harbor entrances, and is due, principally, to wave action and the littoral drift engaged in depositing material across the mouth of the outfall channel and the constant tendency of the ebb and flood currents to remove and disperse the same. The result of such drift action and harbor ebb current is shown in Fig. 10, which is typical of the conditions that have affected the entrances to interior bodies of water on sandy coast lines.

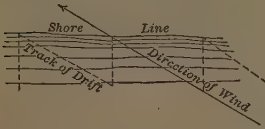


Fig. 9

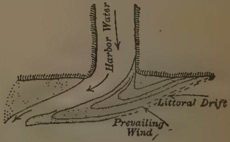


Fig. 10

Influence of Coast Line on Formation of Bars. Where the bed of the sea decreases in depth rapidly, the drift moving along the coast line is less readily carried into the channel by flood tide and is more readily transported by the ebb tide to deep water. Under such conditions bar formation is unlikely. Harbor

entrances on precipitous foreshores of rock materials are not likely to bar formations. Prominent projections.of the coast line on the side from which the flood tide sets in causes the current to run around it with velocity sufficient often to prevent depositing of drift and formation of bars at harbor entrances.

5. Foreshore Protection

Sea Walls, Dikes or Bulkheads are constructed along the shore line to prevent encroachment of the sea by direct wave action, and, as in the case of breakwaters, may consist of loose rubble-mounds or heaps, masonry wall work, usually, however, supplemented with timber, steel, or reinforced concrete sheet-piling driven into the beach and strengthened by wales, guide and brace piles, fascines and mattress work held in place by piles and loaded with rock. The character or massiveness depends on the location and wave forces the work will be subject to. In following notes of sea walls the numbers refer to Fig. 11.

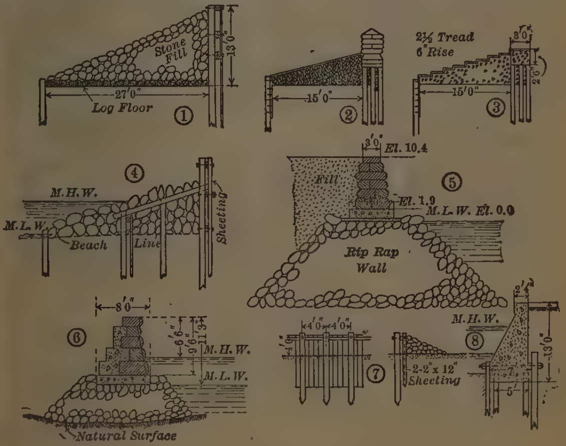


Fig. 11. Sea Walls

(1), (2), (3), (4), Coney Island, N. Y. Section of sea wall.

(5) Governor's Island, N. Y. Sea wall, 1901–1911. Average cost $19.45 per lin ft. Average cost of wall and riprap $47 per lin ft. Average settlement 0.8 ft, maximum settlement 2.4 ft.

(6) New York, Brooklyn Parkway, 1913. Masonry wall. Cost $27.50 per lin ft. Riprap foundation, 40c. per ton.

(7) Atlantic City, N. J.

(8) New Orleans, La., 2650 ft wall, fill behind wall 8 to 12 ft deep, piles 50 ft long, expansion joint every 50 ft, cost of wall, $26 per lin ft, filling 11 ½ c. per cu yd.

Effectiveness of Sea Walls. In many instances large sums of money have been expended on sea wall protection with entirely unsatisfactory results. In exposed locations subjected to high wave action, with a sea current along the beach, on which the sea wall is constructed, the action of the waves, together

with the current, will gradually, but continually, carry the beach material away until the foundation is exposed and undermined and the work destroyed. The most carefully constructed massive works of this character will not endure under such conditions.

Groins are built out from the shore line perpendicular to, or making an angle with, the same. They are generally constructed perpendicular to the direction of the current or drift. Their object is to cut off and prevent the carrying of beach materials along the foreshore. They need not be of massive construction, and usually consist of close piling, or sheet-piling driven between guide wales and piles, or horizontal courses of plank held between vertical timbers or piles driven into the beach. They should be built so as to offer the least resistance to wave action, follow closely at a few feet height the general slope of the beach, and extend from high to low water. When of more massive and stronger construction on a steep beach they should, if built out below low water, extend out into water of depth so that wave action will not seriously disturb bottom conditions, such as would result in scour and undermining and the destruction of the outshore end. When built in this way they are also known as Spur Dikes or Jetties. Groins should cut off the lateral sweep of waves. The distance between groins should equal their length and may be 1¼ times the length. In following notes, the numbers refer to Fig. 12.

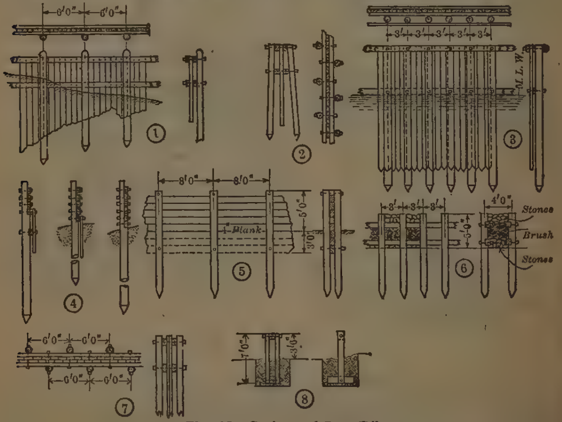


Fig. 12. Groins and Spur Dikes

(1) New York, N. Y., foot of Bay 22nd, Brooklyn. Wales 5 x 10, sheeting 4 in.

(2) Bath Beach, N. Y., Sheeting 3 in, 11 ft long. Top wales, 6 x 6, lower wales, 6 x 8 in.

(3) Coney Island, N. Y. Wales 4 x 10, 4 in sheeting 22 ft long.

(4) Telewana Park, N. Y. Horizontal planking 3 x 10, wales 4 x 8, sheeting 3 in, 6 ft long, piles 15 to 20 ft long.

(5) Atlantic City, N. J. Planking was put in sand as lateral deposit, raised beach, all lumber creosoted.

(6) Atlantic City, N. J.

(7) Asbury Park, N. J. Placed, 1908; successful up to 1914. Beach protected and extended.

(8) Type of low groin adopted by E. Case on English coast with entire success.

Groins of reinforced concrete have been extensively used on the English coast, consisting of grooved reinforced concrete piles and horizontal slabs added as beach rose in height. On the Holland coast spur dikes consisting of brush piles covered by pavement of stone have been successfully used, length about 350 ft.

Principle of Action of Groins. The object of groin construction is to confine the action of the breaking waves on the beach to moving the sand and shingle composing the same up and down locally in the confined space, and at the same time prevent the current along the beach from carrying this sand away to other locations, the beach current being compelled to seek a path outshore of the extremity of the groins at a depth where the wave action will be insufficient to stir up a sandy bottom, so that it may be carried away by the beach current. On the other hand, this current will carry, in suspension and along the bottom, sand or shingle from more distant points not protected by groin construction, and deposit this in the pockets formed by the groins resulting in beach accretion, as indicated in Fig. 13.

Fig. 13

The windward accumulation will be more pronounced, the deposit on the leeward side being materially reduced by eddies caused by the lee side of the outer end of the groins; as the pockets between become filled up, it becomes necessary to raise the height and extend the groin seaward. In most instances it is possible by this method to considerably add to the foreshore where, previous to the work, it was being steadily and continuously carried away. Groins are advantageously employed in connection with sea wall protection.

Dunes. Beside the retrogression or deposition of materials by direct water action of breaking waves, coastal currents and littoral drift, the air currents or wind frequently have a direct effect ot considerable importance, as is evidenced by the erosion, carrying away and deposition of sandy beach materials in the building up and constant movement of dunes. Such action may often become of so important a character in connection with foreshore protection as to require stabilizing.

On the Holland coast this is accomplished by resorting to planting of coarse grass and the placing of brush to protect the sand surface from wind action and also to catch the flying particles of sand and artificially build up dunes or embankments at predetermined places.

6. Channel Regulation

Variation in Channels. In bay or estuary harbors on coast lines subject to appreciable tidal ranges, it is quite frequently the case that the ebb current in finding its way to the sea has a tendency to scour out a channel for itself in the line of least resistance, water from the sea entering the harbor at flood tide, eroding for itself a different channel-way, sometimes across the channel-way of the ebb tide, with a consequent inclination to fill up this channel. The influence of littoral current and drift at the entrance of such harbors, together with that of gales and storms at intervals, often seriously complicate the problem of maintaining navigable channels. However, there is a constant tendency toward an equalization of forces looking to the establishment of more or less fixed channel-way. The navigable channel need not necessarily follow the natural channel eroded by the ebb or the flood tide, but may be established by crossing a series of deep pockets, availing itself, in whole or part, of either or both tidal channels. For methods of marking channels, see Art. 11.

Dredging. Such navigable channels are often established and maintai by excavating shoal reaches between natural channel-ways and deep holes, by straightening out and making more direct the natural channels, by c thru bends, and across bars. A description of the different processes of exca ting under water will be found in Sect. 6, Art. 43. Dredging operations of t character are frequently sufficient in themselves, altho, to some extent, nec sary as a continuous operation for maintenance of channels. In other ca dredging operations are necessary in conjunction with other works of chan control and regulation.

Jetties. In cases where there is considerable littoral drift, resulting in formation of bars across harbor entrances, or formation of bars from depo of sediment, jetties are built out from the harbor entrance into deep water. the construction of jetties, attention must be given to the natural conditi or inclination of the channel to establish itself in a certain direction, with a vi to not unnecessarily disturbing the equilibrium of forces. Where a heavy litto drift is constantly forcing to leeward, the outflow of water should be direct into the sea by a curved channel having its convex side presented to the dir tion from which gales are prevailing, a single jetty of curved section being us on the windward side of the channel. In such construction it will often be nec sary to extend the jetty seaward from time to time, as the foreshore is bu up on the windward side of the jetty; under some conditions such a je would be likely to carry the drift beyond the entrance into deep water, a not require seaward extension.

Converging Jetties run parallel with the general direction of the ebb tide a are carried out from the land and converge, leaving a comparatively narr opening. Jetties of this type tend to guide and restrict the outflowing ti controlling and directing the scouring effect by increasing the velocity. T windward jetty serves to cut off the littoral drift, the leeward jetty assisti the windward jetty in directing the force of the ebb tide to scouring out t drift brought into the channel-way by flood tide. Jetties of this charac often, from time to time, require outshore extension into deep water or the assi ance of dredging. Diverging jetties bring the entrance to a more natural co dition, and afford a better approach for navigation, but have a tendency, ho ever, to scour out a deep hole in the inshore entrance-way.

Design and Construction. Jetties are usually constructed of mounds heaps of large rubble in a somewhat similar manner to the construction of mou or heap breakwaters. The material is usually carried up slightly above t elevation of high tide. In estimating the quantity of material to be used, it is to be borne in mind that considerable is lost thru settlement in soft bottom, besides which, an excess of material must be allowed as the action of the waves will tend to wash down the stone to flatter slopes than would ordinarily obtain. From time to time, after completion, stone will have to be added to insure a high enough elevation to keep cross currents at high tide from sweeping beach drift over the jetty into the

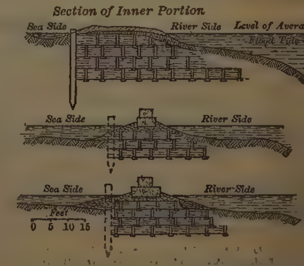


Fig. 14. South Pass, Mississippi River Jetti

channel. Fascine and mattress work with rubble loading is employed in jetty construction. Jetties are also built of wall work and also of reinforced concrete.

A jetty built in 1913, at Shark River Inlet, N. J., consists of two practically parallel walls curved with concave side toward the south, the beach sands drifting toward the north. The jetties consist of a row of 16 inch reinforced concrete sheet-piling, with counterforts at 10 ft intervals, and with a braced reinforced concrete superstructure;

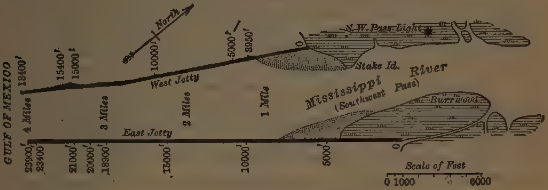


Fig. 15. South West Pass, Mississippi River

the pockets are filled with sand, the toe or apron is protected with riprap. Fig. 14 shows typical sections of jetties at the South Pass of the Mississippi River, and Fig. 15 a general plan of the jetties at South West Pass of this river.

RIVERS

7. Character and Description

Origin. Rivers owe their origin to the natural drainage or flow of water from the land to the sea, the development of their beds and their direction being due to the character of the soil, natural obstructions, inclination to follow the path of least resistance, and to certain modification in these characteristics due to influences, in some cases passed, traceable to the breaking up of the Glacial Period. Their final formation and direction are the results of erosive action of the water, as balanced by soil resistance, resulting, in the cases of most older rivers, of an established equilibrium within certain limitations between the contending forces and the regimen as existing.

Sources of River Water are, in all cases, derived from tidal or rain water, the tidal water entering at the lower end, or mouth, and being directly due to the great primary or secondary ocean tidal waves, which, at high tide, pass up the estuary and the river, raising the level of the water and causing a flow of water up the river, and during the period of low water the process is reversed. The quantity of water passing up the river, due to such tidal influences, is, of course, on the average, the same as the quantity flowing out on the reverse tide. The supply of fresh or rain water coming into the river at its source and constantly augmented by smaller supplies from branches and rivulets and the drainage from the banks along its course, travels solely in one direction from the source to the mouth and is subject to considerable variation in quantity and duration.

Tidal Rivers. Nearly all rivers emptying into a tidal sea are affected by tidal influences for some distance up from the mouth, depending principally upon the range of tide and the slope or fall of the river. At the crest of the tide, the salt water of the sea, by action of gravity, flows up the river; the sea water being denser than the fresh river water, the flow has a tendency to take place

by moving under the less dense fresh water and lifting this up. On the trough of the tidal wave the flow is reversed, causing a continual oscillation in and out, the quantity of the ebb tide being augmented by the fresh water of the river. The volume of the ebb is under normal conditions always in excess of the volume of the flood, this condition being reversed at rare intervals by the occurrence of heavy onshore gales. The method of action of the tide is, that as the flood tide begins to make up the river, the current is at first slowed up, then entirely checked, and then reversed, banking up and preventing the flow and escape of the fresh river water into the sea. On the turn of the tide, the tidal water and the stored accumulated river water flow out into the sea, the greatest velocity occurring at about half flood or half ebb. At the periods of reversal, or of slack water, a short time elapses with no current existing. Contrary to the conditions generally prevailing directly on the coast line, the duration of ebb tide in rivers is longer than that of the flood tide, the difference depending upon the character of the river and the quantity of fresh water flowing down the river as compared with the volume of tidal water entering the stream.

Non-Tidal Rivers are rivers emptying into tideless seas or lakes or the upper reaches of tidal rivers beyond the effect of tidal action. The rise and fall of such rivers are entirely dependent upon the precipitation of rain on the areas drained by such rivers and their tributaries, the melting of snow and ice in such areas or as modified by the characteristics of the surface of the ground, growth of vegetation, and the peculiarities of the stream itself, such as depth, width, and slope.

8. Flow of Water in Rivers

Velocity of Current. The direction of flow and velocity of river water is due solely to the effect of gravity dependent upon the level of the surface of the water in the river. The difference in such level in any given length is termed "slope" or "fall." The particles of water at high level exert a pressure on those below them; these, being free to act in any direction, are pressed downward, forward, and upward toward the lower level, the whole mass being thus set in motion generally in the manner indicated in Fig. 16.

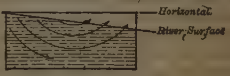


Fig. 16

The particles which come in contact with the bottom and banks of the stream are retarded by friction, not moving with the same freedom as the particles in the center of the stream section. Particles in contact with the bottom and side are also deflected from their true course causing disturbances in the stream action. As the stream moves forward the particles describe orbits, varying in dimension with the section and depth of the stream; in large deep streams, the orbits being larger, the disturbing elements are less potent, the mean velocity being that obtaining in open channels as given in Sect. 9, Art. 15, and may be obtained more directly by

$$V = C\sqrt{2\,R.\,F}$$

in which F equals the fall per mile in feet, R the hydraulic mean depth in feet and C a coefficient of varying value, as follows:

For small streams of about	50 cu ft per sec	$C=0.65$
For larger streams of from 200 to 300 cu ft per sec		$C=0.75$
For tidal rivers of	1000 cu ft per sec	$C=0.85$
For tidal rivers of	10 000 cu ft per sec	$C=0.95$
For tidal rivers of	100 000 cu ft per sec	$C=1.00$
For tidal rivers of	1 000 000 cu ft per sec	$C=1.50$

Transporting Capacity of Water. All rivers are charged with alluvial matter carried by them in suspension, the turbid condition of many rivers during periods of heavy flow illustrating the extent of the work being carried on by them in the transporting of material. In addition to the materials actually carried in suspension, heavier and larger particles are transported by being rolled along the bottom of the stream. The sources of materials carried in suspension are the disintegrating effect of frost, breaking up and loosening of soil materials, the wash and erosion of rain, and the scour or eroding effect of the stream upon its banks. Under normal conditions the sectional area of the river is sufficient to provide for a velocity which will not cause undue erosion, the natural bed remaining in a state of equilibrium. However, any agencies which tend to increase such velocity tend to cause erosion with increase of sectional area and eventual decrease of velocity to the point of equilibrium. With a constant variation of volume of water flowing in rivers, detritus is brought down at one time and deposited, is again taken up and transported further down stream, and so on. See discussion of Experiments by Grove K. Gilbert in Professional Paper 86 of U. S. Geological Survey, 1914; abstract in Eng. News, Sept. 10, 1914.

In a tidal river, the soil material carried by the stream is carried back and forth by the tide; the volume of ebb being greater than the flood, it is eventually slowly carried out to sea, or is deposited on the shores of the river mouth or estuary forming salt marshes. In non-tidal rivers, as the velocity of the current slackens on approaching and emptying into the sea, the material carried in suspension is deposited at the mouth of the river forming deltas. The delta of the Mississippi River is a prominent example.

If a stream is loaded to its full carrying capacity it will tend to flow within the confines of its bank and the bed without further erosion; in some rivers upward of 2 percent in weight of the total volume of water passing along the channel is solid matter. The ability of a stream to carry material is dependent upon the square of the velocity as modified by the depth. Refer to Sect. 9, Art. 17.

Erosion and Scour. A non-tidal river with its flow constant in a down-stream direction has an inclination to flow in a sinuous or meandering course. The particles of water moving in circular orbits or paths tend to impinge against one bank or the other, and on account of the lack of uniformity of the character of soil materials forming the banks, there is a constant tendency to erode or scour on one side or the other. A curved direction having once been assumed, the tendency is to increase the curvature, as illustrated in Fig. 17.

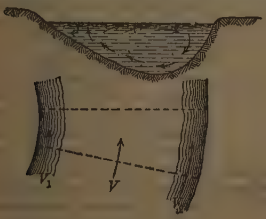


Fig. 17

The stream velocity is V. On account of the curved direction of the stream, the centrifugal force will tend to increase the velocity near the concave bank to V_2, and to decrease the velocity on the convex bank to V_1; the increased velocity on the concave bank, together with the dynamic pressure, due to the centrifugal force, will tend to deepen the stream bed at this side of the stream and erode and scour out the bank, these same forces tending to broaden a river at the bend by decreasing the mean hydraulic depth. The usual condition of a stream is illustrated in Figs. 18 and 19, the first showing a slight reverse bend, and the second a more considerable one.

The current in each case being close to the concave bank, has a tendency to scour out on this side deep channel-ways with consequent steep banks. Where material is carried

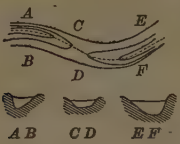


Fig 18

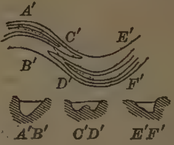


Fig. 19

in suspension the decrease in velocity on the convex side permits the deposition of detritus and the formation of flats, as illustrated in Fig. 20 and, eventually, with the growth of the meandering effect of marsh land.

In some streams the bends become so great that the bights approach each other and a natural cut-off occurs, the bends soon taking the form of crescent lakes. The intersection having once occurred, the new channel is rapidly eroded as the shortest path and the one of least frictional resistance, the meandering effect is more prevalent with less velocity than in torrential rivers in which the increment due to centrifugal force is proportionately less. As the navigable distance is often naturally shortened by such cut-offs, they have been made by dredging; this should only be done after careful investigation as to water height and the consequent velocity thru the cut-off; experience has shown that the shortening of the river course by this means often results in scour and bank caving.

Fig. 20

Channels and Bars. In non-tidal streams the flow being constantly in one direction, the regimen of the stream flow is closely fixt, the flow, quantity and velocity varying with the precipitation of rain so that, within reasonable limitations of time, the location and direction of channels and flats are likely not to vary greatly. In tidal streams the conditions are different and more complex, the variation in fresh water flow being materially complicated by tidal flow as regulated and controlled by the extent and direction of storms. Fig. 21 illustrates the condition often obtaining at or near the mouth of tidal streams, the reversal of current due to the ebb and flood resulting in two or more variable channels with consequent formation of bars, very often shifting in character and extent.

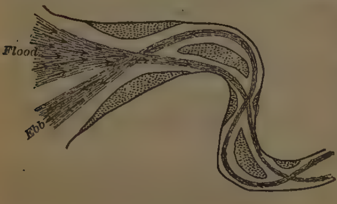


Fig. 21

The width and sectional area of channels in non-tidal rivers and in the upper reaches of tidal rivers bear a certain fixt relation to the drainage area and to the consequent quantity of water discharged. The width and depth of the tidal reaches of a tidal river do not appear to be so fixt, altho the channels once fixt are usually maintained by the tidal flow. Any cause that tends to obstruct the free flow of tidal water and the propagation of the tidal wave is detrimental to the maintenance of the

channel condition and frequently leads to shoaling. It is important to observe this effect in the design and construction of works of proposed improvement.

9. Training Works

Dredging. As with harbor work, the improvement of rivers, while often attempted by dredging, is not always effective as a measure of permanent improvement and usually entails continuous maintenance operations. Channels may be straightened out by cuts across bars and flats and the navigable distances along rivers decreased by excavating between the bends of meandering rivers. Dredging for the improvement of tidal or non-tidal rivers is principally useful in connection with other improvement works for stream regulation and control, and as a regular operation to maintain adequate depth and width of channels, slips, and basins.

Training Walls, also called dikes, are employed to direct the flow of the current in rivers with a view to the establishment of more favorable and fixt channels and often, also, to prevent scour and the erosion and carrying away of river banks. In the navigation of rivers it is important that the curvature of the channel should be of such radius as to permit the ready handling and movement of ships, this depending upon the depth of water and the size of ship to be considered. For large ships, if possible, it should be not less than 2500 feet. Single or double walls may be employed, the selection depending on the object to be accomplished. Two parallel walls may be used in a river flowing thru a broad alluvial valley, in a wide bed and with an inclination to meander, the walls confining and directing the current flow, keeping the space between them open by sluicing and scour. The slack water area inshore of the training walls will tend to shoal and fill up. Such wall work in rivers is similar to and in many respects merely an interior continuation of jetty work at river mouths or harbor entrances.

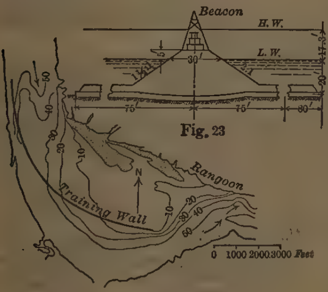


Fig. 22. Training Wall, Rangoon River

At Rangoon, India, 1907, the Rangoon River (Figs. 22 and 23) was cutting out the concave river bank just above the city, causing formation of sand bars and shoaling of water on the opposite side, and on being deflected by a prominent point opposite the city was scouring out and undermining the foundations of wharves and piers. The training wall built was 2 miles in length and consisted of brushwood mattress 3 ft thick, covered with a weighting layer of rubble, a rubble-mound wall surmounted by a concrete wall filled with rubble. Range of tide 20 ft with 5 to 7 mile river current. Wall cost, average, $470 per lin ft. River has scoured out new channel along wall and area behind wall has filled up.

Construction of Training Walls or Dikes, as with jetties, may be of loose rubble-mound construction, with or without a surmounting masonry wall, or may be of timber, timber close piling, timber sheet-piling, steel sheet-piling, or reinforced concrete. In the employment of rubble-mound walls, considerable material will have to be used that will afterward be buried in the bottom, for

as the wall is built and extended, the scour of the stream at its end will tend to deepen the channel at this point, leaving deep holes to be filled with stone as the wall work is carried forward. Training walls should be carried up to the elevation of high tide, and when curved should be given easy curves so as not to cause sudden changes in the direction of channel currents. The area in back of the training wall eventually becomes filled up with sediment deposited by the stream in the slack water. A typical section of a loose rubble-mound dike in use on Arkansas River is shown in Fig. 24, the rubble being deposited on fascines or brushwood mattress. Fig. 25 shows a typical solid, pile brushwood and stone dike in use on Arkansas River.

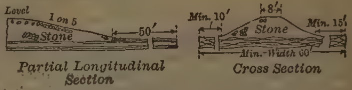


Fig. 24. Stone Dikes, Arkansas River

Spur Dikes, also called Spur Jetties, may be employed to regulate and direct the current of a river by contracting flow area and causing scour and lateral deposition of material behind or in between dikes. The influence of the spur dikes is similar to that of the groins employed in foreshore protection work, as described in Art. 5. Spur dikes are sometimes employed in connection with longitudinal dikes or training walls, or such longitudinal walls or bulkheads may be constructed connecting the end of the spur dikes after the space between them has become filled up with deposit.

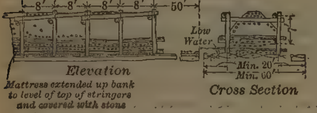


Fig. 25. Solid Pile Dike, Arkansas River

Fig. 26 illustrates the improvement of the Upper Mississippi River by the employment of spur dikes which are built out perpendicular to or making an oblique angle with the direction of the current.

Permeable Dikes, instead of entirely cutting off or diverting the current flow, merely slacken and retard it. They consist of open-work construction, usually of timber piles and cribbing with rubble weighted brushwood mattress work hurdles. The velocity of the current being retarded behind or between such dikes, material in suspension is deposited and the area is filled up, the dikes themselves eventually become buried in a hydraulic fill dam of their own creation. On account of the initial increase in velocity at the contracted area of the dike itself, the fascine or mattress work is built with an apron on each side of the work to prevent scouring and undermining.

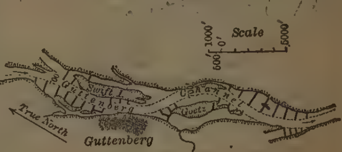


Fig. 26. Spur Dikes, Upper Mississippi

Fig. 27 illustrates the employment by the Chicago and Northwestern Ry. Co. of permeable dikes on the Missouri River at Blair Crossing. The river, as shown in (1), was following a cut-off and threatening the Blair bridge. The work was done in 1913. Permeable dikes were constructed across this cut-off, forcing the river into the old channel. (2) shows a section of the 2-pile dike; the main dike employed a 5-pile section.

Bank Revetment is effected, when practical, by sloping the bank to stable slopes, protecting them from scour by brush mattresses, riprap, or block paving, timber planking, piling and sheet-piling. Such work is necessary, not only on rivers to protect banks from erosion due to the flow and velocity of the current, but at tide level to protect against wave action and wash of passing vessels. See Sect. 11, Art. 16.

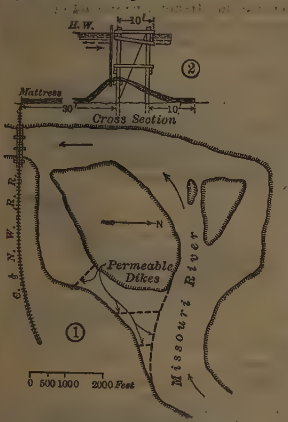


Fig. 27. Permeable Dikes, Missouri River

In Fig. 28 (1), (2), (3), (4), (5), (6), (7), show typical methods of revetment on the Arkansas River. (8) and (9), typical methods Upper Mississippi River. (10), typical methods, the Missouri River.

The standard revetment consists now of grading the bank on a slope of 1 : 3, laying a woven willow mattress 86 ft wide from 3 ft above low water and over the river bed, ballasting and sinking it, and paving the bank from the inshore edge of mattress to the top of bank. If the paving (riprap) fails by erosion, a substantial reinforced concrete revetment is placed extending from the top to almost low water and from there a system of connected concrete flexible blocks about 24 in square, extending beyond low water. This protects the bank and weak point of willow mattress. Cost per lin ft $8 to $10.

10. Flood Control

Overflow. The causes which contribute to a river overflowing its banks are naturally found in exceptionally heavy precipitations of rain and in the melting of snow and ice in the early spring over the area of its watershed, the origin usually being in the more precipitous upper reaches of the stream. Under normal conditions, floods of this character are to be expected periodically and have in certain cases been of considerable assistance in enriching the flat agricultural lands on either side of the stream by the depositing on them of rich alluvial matter brought down and deposited as the river water recedes after the period of flood. This natural regimen is sometimes disturbed by a change in the condition and character of the vegetation and forest growth over the drainage area; the cutting of timber, removal of forest underbrush, and the placing of land under cultivation, serving to increase the rapidity of the run-off, with the possible result that periods of rainfall which would not have ordinarily caused serious floods, under the change in the natural condition of the drainage area may have a tendency to hasten the drainage and outflow of rain water into the river. The conservation of forest growth, especially on the high lands where river tributaries have their source, has been advocated as a measure to reduce rapidity of run off and consequent flood, and at the same time by the retarding or storage effect increase the flow at low stages. Forest growth, undoubtedly, does exert a useful regulating effect on flow and resulting floods, but such effects are materially changed and even reversed by altitude and frost, especially by snow and ice. The effect of forestation on flood would appear to be a local problem, as it can hardly be asserted as a general proposition that changes in such forest areas produce well-marked effects on river floods.

Levees are embankments of earth built up on the sides of rivers to prevent the overflow of the banks. They should be built up two or three feet higher than the highest water recorded in that vicinity. In their design and construction, consideration should be given to the fact that the confining of the stream between the side levees will serve to decrease the stream area during flood flow, and probably entail increase in velocity and in elevation of the highest water over and above that recorded before the construction of the levees. The side

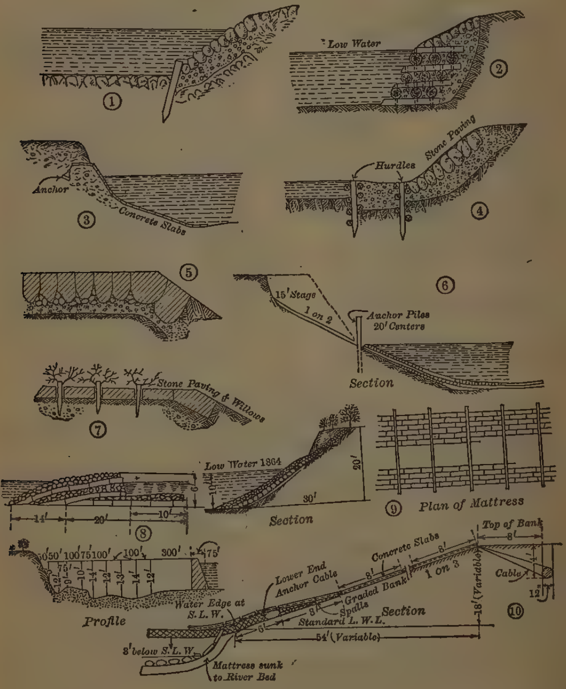


Fig. 28. Revetments

slopes are dependent upon the character of the soil and vary from 2: 1 to 4: 1 The top width of the levee should be sufficient so that in case of settlemen or unusually high water, the crest of the levee may be raised by additiona embankment or by placing of bag extensions. Refer to Sect. 6, Art. 41, Leve Construction, and Sect. 17, Art. 19.

Fig. 29 shows typical sections for various heights of levees employed on the Mississipp

River. Revetment work is employed to protect levee slopes where river conditions are such as to indicate the necessity for such work.

The Dikes of Holland perform a duty somewhat similar to that imposed on river levees, protecting the low lands from inundation by the sea. This system of dikes consists of the natural sand dunes, artificially built up sand dunes, and embankments. The rivers outflowing along the coast line have levees constructed along their banks. These sand dunes are protected against wind erosion by a growth of coarse grass and reed, which

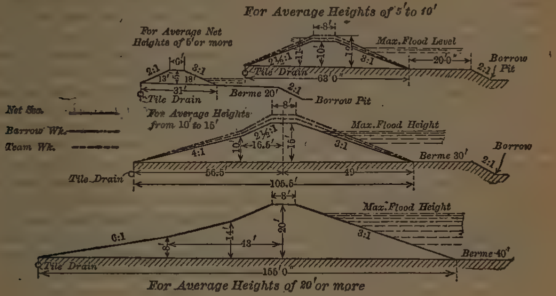


Fig. 29. Mississippi River Levees

also serves to retard and hold the blowing sand and build up the dunes, fresh grass being planted on top of this. Where subject to wave action, the slopes are protected by fascine and mattress work, weighted with basalt or concrete blocks; short timber piles are driven to pin and hold down the work and help to break the force of the waves. In localities where lateral currents occur, spur dikes are built out perpendicular to the main dike.

Flood protection walls may be employed in connection with, or to replace, levee work. Such walls, consisting of two reinforced concrete walls backfilled, were employed at Portsmouth, Ohio. The walls were fitted with flood gates which are closed when the elevation of the river reaches a dangerous height, the sewage discharge from the city is then pumped instead of flowing thru the wall. 6500 ft of wall was built at an average cost of $6 per ft.

Navigation. Most rivers at high stages usually afford the necessary facilities for navigation restricted within the known and expected local requirements. The low stages of rivers often make navigation impossible without the assistance of engineering works. At such low stages with the fall in stream velocity and in the quantity of water discharged, bars extend diagonally across the river, especially at bends, blocking the fairway and making the passage of vessels difficult or impossible. With sudden rises, new channels are eroded thru such bars, varying much in direction and location. Formerly in the Mississippi, when this river carried a large local river traffic, it was customary at New Orleans to hold frequent conferences between pilots of ascending and descending steamboats in order to learn and ascertain the latest changes of channel. Rivers, especially during freshets, bring down large quantities of drift logs, developing snags or sawyers, which tend to obstruct navigation, raising barriers and sand bars and changing the channel conditions; it is important that such snags be removed as soon as possible.

Reserve Basins or Reservoirs. Under favorable conditions, river floods may be somewhat controlled by the construction in the upper reaches of basins

or reservoirs, consisting of large areas with impounding dams, it being contemplated to hold in reserve in these reservoirs the excess flood volume, this to be released during the periods of low stages, and thus to increase artificially such low stage flow with a view to establishing an average flow. The fundamental principle involved is a sound one, but, usually, natural difficulties are encountered that render impractical the obtaining of the desired results. The problem involved, for successful application for such works for flood control, requires special and careful study of local conditions, the areas available for reservoir sites, value of land likely to be submerged, and the existence of suitable sites for dam construction. The plan has been tried on the upper Mississippi River, the region being remarkable for the number of small lakes. A large Indian reservation presented conditions unusually favorable. Five reservoir lakes were immediately available and additional ones possible and in prospect. The watersheds of the five reservoirs constituted 11.8 per cent of the whole area above St. Paul, and 21.5 per cent of that above Minneapolis. The actual result to navigation on the Upper Mississippi system was an average increase in low water depth at St. Paul of about 14 inches. When the release of water was specially regulated, 22 inches increase was obtained; these figures are based on a record of 20 years, during which period the total outlay, including construction and operation, was about $1 500 000. From this it is concluded that, as a general proposition, such reserve reservoir work, while instrumental in controlling flood discharge and assisting navigation, is of use principally in connection with other engineering work of improvement. The difficulty of regulating the discharge from such storage reservoirs can not be over-estimated. The release of reservoir water may be necessary and beneficial to one section, and have quite the opposite effect on other sections served by tributaries or branches which, due to local severe precipitation, are experiencing local floods.

Canalization is a method employed to considerable extent in Europe for improving the navigable condition in rivers. It has been applied to the Ohio River below Pittsburgh for a distance of about 1000 miles. In 1875 this project, by means of 45 movable dams, increased the minimum navigable depth to 6 ft, and in 1905, by 54 movable dams, to 9 ft. The total fall of the river in this distance is 424 ft, of which 26 ft are at the falls near Louisville, around which a canal has been constructed, for use at low water stages. Refer to Sect. **11**, Art. 16, Canals. The employment of dams and canals and canal locks is well suited to such projects where the local conditions are favorable.

11. Signals and Lighthouses

Buoys. Buoys are employed for demarcation of entrances, interior channelways, and for the location of dangerous shoals. They are placed at intervals at either side of the channel-way, the port and starboard boundaries being indicated by the shape and color of the buoy. Buoys must be moored to heavy cast iron mushroom anchors or frequently large blocks of concrete are used with heavy iron eyes cast therein. Dependent upon the range of tide, the buoy is secured to an anchor by a length of cable two or three times the maximum depth in which it is to float. In channel demarcation the distance between buoys varies with the character of the channel, as to its width and straightness. In the interior of harbors mooring buoys are frequently provided at various fixt locations so as to enable vessels to moor without resort to ship's anchors. In harbors of restricted area, at times two buoys, one for bow and one for stern, are provided for mooring. Buoys are used for location of wrecks in outer harbors or in the open sea; also to indicate cable crossings.

Beacons are fixt signals or structures used for means of alignment or for indi-

cating changes of direction. Frequently prominent objects or land marks, natural topographical features, or prominent structures, are used as beacons.

Channel Lighting. The location of channels is often fixt for night use by luminous buoys lighted by means of oil or gas, or the range is fixt by two illuminated points, afloat on buoys or fixt on shore, or one afloat and the other ashore, the change in direction being shown by a similar line or range.

Sound Signals. Light being impractical and useless during heavy mists or fogs, resource is had to sound or warning signals, a bell or whistling buoy being used for this purpose. In deep water where there is always some wave motion, bell buoys depend on this motion to actuate balls or pendent clappers. Whistling buoys depend on the taking in of air and its expulsion as the buoy rises and falls, or the bell may be struck or the whistle sounded by compressed air or gas carried in a tank that is regularly recharged. Sound transmitted thru the air often gives a misleading idea of locality, so that resort is had to submarine sound signals which can be heard a great distance, and from which the direction can be identified.

Lightships. At locations where, on account of the natural conditions, it is impractical to locate and maintain lighthouses, or at locations where luminous buoys would not be striking enough in character, necessary light for the assistance of navigation is placed on light-ships. Signals of this character are more certain and reliable than are light buoys exposed to strong currents in heavy seas. The principal requirements are staunch construction and steadiness under severe storm conditions.

Lighthouses. See Sect. 5, Art. 42. Dangerous promontories, points, and bars are marked by lighthouses, especially at entrance to bays and harbors. The lights are fixt, revolving, or flashing, for purposes of identification. When a navigator has identified a light and knows its height above sea level, the distance at which it first becomes visible at sea is known. Let H be the height of light, h the height of the observer above sea level, both in feet, and D the distance from the vessel to the light in statute miles. When the light is seen exactly on the horizon, then

$$D = 1.32\,(\sqrt{H} + \sqrt{h})$$

If the distance in nautical miles is required, 1.32 is to be replaced by 1.15. For example, if the height of the light is 100 ft, and the observer is 25 ft above the water, he sees the light on the horizon when he is 19.8 statute miles or 17.2 nautical miles from the lighthouse.

The following statements regarding the lights of lighthouses and the structures themselves have been kindly furnished by J. S. Conway, Deputy Commissioner of the U. S. Lighthouse Service.

(1) **Intensity.** Lights are of classes heretofore known as "orders," based on the focal diameter of the lens, as "first-order," "second-order," etc., and miscellaneous smaller lights. Recent practice substitutes the candlepower of the light in place of the now more or less misleading term "order."

(2) **Ranges of visibility,** two classes: the geographical range of the light due to its height above sea level, and the luminous range due to the intensity of the light emitted by the illuminating apparatus.

(3) **Character** of the illuminating apparatus, three classes: catoptric or reflecting, dioptric or refracting, and catadioptric, in which the "lens" is provided with both reflecting and refracting media. The latter type is the most in use.

(4) **Characteristics** of the light, such as Fixt, Flashing, Occulting, Fixt and Flashing, or other combinations for distinguishing purposes. Fixt lights are liable to confusion with neighboring private lights and should be avoided if practicable. Fixt

combined with flashing, and lights with prolonged flashes should be avoided for the reason that the "fixt" and "prolonged" rays emitted by the lens are weaker than those of the "flashes," giving a misleading characteristic at a distance. Multiple lights, except for minor lights, are now considered obsolete, because they largely increase the cost and are less distinctive than flashing lights. Lights of alternating colors should be so designed that the white and colored rays are of equal intensity.

(5) **Arcs of visibility,** two classes: single lights illuminating a large arc of the horizon and two lights in range illuminating a limited arc for purpose of marking narrow channels and passages. Often plates of red glass are introduced in the lantern so that the sector of rays passing through them will cover an outlying danger such as a reef, rock, or shoal, or to indicate the edge of a channel.

(6) **Attendance,** two classes: Watched and Unwatched lights.

(7) **Fog-signals,** four classes: the ordinary bell struck by hand or machinery and submarine bells struck by machinery, also whistles, reed horns, and sirens blown by compressed air. Some of the whistles and sirens are sounded by steam.

(8) **Materials.** Almost all the usual materials of construction have been used in building lighthouses in the United States Lighthouse Service: stone masonry, brickwork, reinforced concrete, framed timber, structural cast iron, structural steel, cast iron plates, steel plates and piping.

(9) **Arrangement.** Light stations situated on land sites usually consist of the light tower, oil house, fog signal building, keepers' dwellings, workshop, water supply and drainage systems, boathouse and ways, barn, and the usual outbuildings, roads and walks, altho, owing to the restricted area of some sites, one or more or all of the buildings may be combined in one. On submarine sites the whole station is practically confined to one structure.

(10) **Foundations.** For land sites, the foundation for a closed tower of masonry, or metal work, is usually a single block of concrete resting upon the foundation soil. Occasionally these blocks are placed upon a timber grillage supported by piles for sites upon low or marshy land. In all cases the block is extended so as to bring the unit pressures within the bearing power of the soil. Occasionally a skeleton structure is placed upon a single foundation block, but usually each column or leg of the tower has its individual block. Where the site is subject to overflow, the buildings are sometimes grouped together, raised upon braced columns and connected by a system of galleries.

For submarine sites, the foundation may consist of a cylindrical cast iron or sheet steel caisson filled with concrete, or a masonry pier. Cast iron caissons are in general use on the Atlantic Coast. When placed upon a ledge of rock, the latter is usually leveled up with concrete bags if below low water, or with tools if exposed, and the ledge or rock is then heavily ragbolted to the concrete filling. For soft bottom, the best method of procedure is to float the caisson out and sink it by the pneumatic process. Both timber and metal working chambers have been used and the depths from the cutting edge to high water have varied from 19 to 85 feet. Other foundations, consisting of timber cribs and concrete blocks, used in fresh water, have been placed either directly on good existing bottom, or upon a layer of small rock usually 3 or 4 ft thick deposited upon the soft or dredged bottom prior to floating the crib out. These timber cribs are usually filled with stone and terminate about 2 ft below low water level, the concrete blocks are then placed to the height of the deck.

(11) **Forces.** Structures upon land sites are exposed to wind pressures, and occasionally to waves in addition. Those upon submarine sites are exposed to wind, wave, currents, and ice. The usual procedure in determining the stability of tower is to locate the common center of effort of all the forces acting upon the structure to overturn it and so proportion the weights of the entire structure (the buoyancy of the water must be taken into account for submarine structures) that the resultant of the active forces and the net weight falls so far within the outer edge of the base that there is compression over its whole surface if the foundation soil is compressible. In seeking for this result it is proper to include the lateral resistance of the soil when the structure penetrates it for some distance for the reason that it is often heavily compressed by a large deposit of riprap and offers good support. The maximum unit pressures both vertical and lateral must not exceed the bearing power of the soil. In case the foundation is rock,

the resultant must fall so far within the outer edge of the base that the maximum unit pressure does not exceed the compressive strength of the materials in contact.

The wind, wave, current, and ice pressures assumed should be the maximum in each instance, as lighthouses are commonly exposed to severe gales and flows of ice. It has been the practise to assume wind pressure for flat surfaces at 60 lb per sq ft, allowing ⅔ of this figure for rotundity. Maximum wave pressures of 6000 lb per sq ft on flat surfaces are assumed, based upon Stevenson's experiments, the force of the wave being greatest at its crest and diminishing to zero at its base. The pressures exerted by currents vary with their rate of speed. The pressures due to ice have been assumed at 30 000 lb per sq ft for a field of melting ice, one ft thick, striking the pier and crushing its way past.

(12) **Superstructures.** The superstructures of all towers, whether separate or combined with other buildings, have certain features in common. There is a main entrance door at the base, a flight of winding stairs, broken by landings in high structures leading to the service room, which in the larger lights is usually separated by an airlock from the watchroom above, the latter supporting the lantern. Occasionally in large lights and usually in small ones the service and watchrooms are combined. The pedestal of the illuminating apparatus usually rests upon the watchroom floor, but in the small lights the lantern floor supports it. The clear glazed opening of the lantern is just sufficient to pass the horizontal rays from the illuminating apparatus, and if the latter is of a revolving type, showing flashes or occultations, the newel post of the tower serves as a weight shaft for the clock. There are railed galleries outside both lantern and watchroom. The tower should be thoroughly fireproof and isolated in this respect from the other buildings. For calculating the strength of closed and skeleton towers the manner prescribed for chimneys and viaduct bents is employed with the exception that great stiffness and rigidity must be provided, as vibrations are detrimental to the proper working of the lamps and clocks of the illuminating apparatus.

QUAYS

12. Design

Definition. Wharves are landing places or platforms built out, into, or on to the water for the berthing of vessels. Wharves parallel to the shore are generally known as quays, and their protection walls as quay walls; wharves built into the stream or fairway perpendicular or oblique to the shore are generally known as piers.

Quay Walls, or bulkheads, are used for wharfage for retaining and protecting embankments or retaining filling. Their proper design and cost are largely dependent upon local conditions and the use to which they are to be put. The character of the foundation and the depth of water to be provided are important factors in the cost and design. Some data on design are given in Sect. 7, Art. 13.

Data on Design. It is important to observe the character of the foundation on which the wall is placed; whether the wall may be built directly on rock or hard-pan, or on a floating foundation on timber piling or grillage work. In the latter case, information must be secured as to the prevalence of marine wood-borers, and, if present and active, precautions must be taken to protect the exposed timber work by creosoting or other preservative or protective methods. Quay walls are designed in a similar manner to retaining walls with the difference that on the water side they are subject to the water pressure varying with the height of the tide, and on the land side to the earth and contained water pressure, with a proper allowance for surcharge. The contained water on the land side of the wall will usually stand higher than the tide water, especially at low tide.

The Combined Pressure of water and earth upon a wall is needed in making a design. Let w_1 be the weight of a cubic foot of water, w_2 the weight per cubic

foot of the submerged earth, ϕ the angle of repose of that earth, and h the heigh of the vertical wall. Then, for one foot in length of the wall,

$$\begin{aligned}
&\text{Water pressure} &&= \tfrac{1}{2} h^2 w_1 \\
&\text{Earth pressure} &&= \tfrac{1}{2} h^2 w_2 \tan^2 (45^\circ - \tfrac{1}{2}\phi) \\
&\text{Combined pressure} &&= \tfrac{1}{2} h^2 (w_1 + w_2 \tan^2 (45^\circ - \tfrac{1}{2}\phi)) \\
&\text{or, Combined pressure} &&= \tfrac{1}{2} h^2 W
\end{aligned}$$

To find W the weight per cubic foot, w_1 of sea water is taken as 64 lb, and w may be found experimentally by taking samples of the earth as near as possib in its natural compacted state, and ascertaining its weight submerged in se water, or may be fixt theoretically in approximate terms by taking the weig of the original material, such as limestone rock, granite, slate, and other materia less the weight per cubic foot of water multiplied by the percentage of soli material per cubic foot, that is, 100 percent less the percentage of voids in th material.

Combined Weight W of Sea Water and Earth, lbs per cu ft
(Equivalent Liquid Pressure)

Slope of repose of earth	Weight w_2 of Submerged Earth, lbs per cu ft							
	40	44	48	52	56	60	64	68
½ : 1	66.2	66.4	66.7	66.9	67.1	67.3	67.6	67.8
¾ : 1	68.4	68.9	69.3	69.8	70.2	70.7	71.1	71.[illegible]
1 : 1	70.9	71.6	72.2	72.9	73.6	74.3	75.0	75.[illegible]
1¼ : 1	73.2	74.2	75.1	76.0	76.9	77.9	78.8	79.[illegible]
1½ : 1	75.4	76.6	77.7	78.9	80.0	81.2	82.3	83.[illegible]
1¾ : 1	77.5	78.8	80.2	81.5	82.9	84.2	85.6	86.[illegible]
2 : 1	79.3	80.8	82.3	83.9	85.4	86.9	88.4	90.[illegible]
2½ : 1	82.3	84.2	86.0	87.8	89.7	91.5	93.3	95.[illegible]
3 : 1	84.8	86.9	88.9	91.0	93.1	95.2	97.2	99.[illegible]
3½ : 1	86.8	89.0	91.3	93.6	95.9	98.2	100	102
4 : 1	88.4	90.8	93.3	95.7	98.1	101	103	105
5 : 1	90.9	93.6	96.3	99.0	102	104	107	109
6 : 1	104	108	112	116	120	124	128	132

This table gives the combined weight W, or the values of $w_1 + w_2 \tan^2 (45^\circ - \tfrac{1}{2}\phi$ the weight w_1 being taken as 64 lb per cu ft for sea water. It should be noted that t submerged weight of the earth will be dependent on both character of original soil ro and on the extent to which the soil is compacted, the angle of repose becoming great as the soil is more compacted. See table A, page 689, for values of angles of repo corresponding to various slopes.

It will be unusual in practise to find conditions where a combined weight W in exce of 90 lb per cu ft should be used. The total pressure in pounds against a linear fo of a vertical wall h feet in height is found by multiplying W by $\tfrac{1}{2} h^2$.

Some observations have been made at various depths on the weight "in situ" of so immersed in sea water, and, as should be expected, they show wide variation.

The Overturning Moment is the product of the total equivalent liqu pressure on the rear side of the wall, and one-third the height, less the produ of the water pressure on the sea side of the wall, by one-third its depth. In d docks, canal locks, masonry quay walls or bulkheads, the height to be tak would be the distance from the top of the fill down, while in walls with timb or other relieving platforms the height would be from the bottom of t platform. When the bulkhead or wall is founded on viscous or plastic so the depth or height to be considered will not necessarily be the height of the wa but may extend some distance below the depth of water at the sea face.

Weight of Soils immersed in Sea Water

Material	Pounds per cu ft		
	Max.	Min.	Average
Gravel and marl	..	42.0	62.9
Gravel and sand......	73	42.0	62.4
Sand................	66	42.0	58.3
Gravel, sand and clay	80.9	51.2	70.0
Stiff clay............	64.8	38.4	47.8
Stiff clay and gravel...	70.3	44.8	52.6

Practise and Custom. It is too frequently the case in the design and construction of wharfage work to be guided entirely by the accepted custom of practical wharfbuilders. It should be remembered that the same fundamental principles apply here as to other engineering structures. If the wall is a gravity structure, the safe bearing capacity of the foundation soil must be considered, it being remembered that the submerged part of the structure loses dead weight to the extent of the buoyancy effect of the water displaced by it.

Where piles are employed, their safe bearing capacity should be obtained by driving test piles and either applying a test load or making observations of weight and drop of hammer, penetration when bringing up, and use these data in connection with one of the well-known pile driving formulas, the pile structure being examined as a column, and, if of timber, due allowance made for character of material used, the creosoting of timber reducing the strength and safe carrying capacity of the pile. If capped with timber, consideration should be given to the safe bearing power across the grain of the timber used. In wharf work under conditions generally obtaining, timber piles 14 inches in diameter at butts, with the usual framing, should not be counted on to give more than 18 tons carrying capacity per pile. If creosoted timber, not more than 15 tons. The safe bearing capacity of reinforced concrete piles is frequently taken at twice this, but is, of course, dependent on the pile section and the driving of the pile. In reinforced concrete piling the bearing capacity at the top of the pile is reduced by the weight of the pile. In a reinforced concrete pier built in Havana, Cuba, piles 20 inches square, and up to 85 ft long, were used, and gross bearing capacity of 40 tons per pile allowed, or net capacity of 25 tons, 15 tons being allowed as the weight of pile. When employing batter, brace, or spur piles, this bearing capacity should be somewhat reduced, as the driving effect of the pile driver hammer will be less effective, on account of the inclination and consequent friction on the leaders or guides.

Expansion Joints. In long quay or sea walls or bulkheads, expansion joints at intervals of about 50 feet, are necessary in order to avoid unsightly shrinkage and settlement cracks. It is desirable to so design these as to permit irregular vertical movement in adjoining sections, but to lock these sections so as to prevent a horizontal break. A simple and effective joint of this character is made by casting one or more "V" or grooves in the end of one section, and coating this with pitch or tar, or

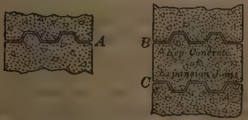


Fig. 30. Expansion Joints

the end of two adjoining sections may be built in this manner a few feet apart and the space filled up. The two methods are shown in Fig. 30.

For joint *A* tar is applied with brush to side first concreted. For joints *B* and *C* tar is applied with brush to body concrete after forms are removed and before key concrete is placed.

13. Foundations.

Sheet-piling. When sheet-piling is employed in quay walls or bulkheads in connection with or without relieving platforms, the sheet-piling will act as a simple or partially fixt beam supported where driven into the bottom and at its anchorage or fastening at the platform. Due precautions should be taken to see that the sheet-piling is driven deep enough into the bottom to secure a proper hold and to guard against being pushed out, for which purpose it must be driven to a depth so that the passive resistance of earth will take the bottom reaction. In reinforced concrete sheet-piling, steel can be placed accordingly, but in any case sufficient reinforcing steel must be placed on both faces so that the pile will be strong enough to be lifted and handled while being placed in the work. It is usually advisable to lift the piling at two points so that the negativ and positiv bending moments are equal, and to fix these points by casting gas pipe section or eyes in the piling.

A too frequent cause of failure of walls or bulkheads is the pushing out of the bottom or toe of the sheet-piling, or, in very soft material where sheet-piling is not long enough, the running out or mudwave effect underneath the bottom of the sheet-piling.

Method of Computing Depth to which Sheet Piling should be Driven, size of piles, struts or tension of tie backs.

In Fig. 30*a* the problem is shown in its simplest form.

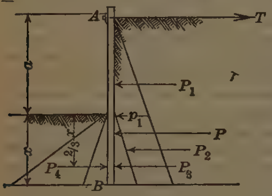


Fig. 30*a*

AB is a sheet pile driven to depth x into the soil and a the known height above the soil to which it is retaining earth, assume it is held at *A* by a strut or tie *T*. P_1 is the known active pressure above the outside soil. p_1 the intensity at the level of the soil, p the increment of intensity per unit of depth for active pressure above or below outside soil. kp the increment of intensity per unit of depth for the passive pressure below the outside soil. $P_2 = p_1x$; $P_3 = \frac{1}{2}px^2$; $P_4 = \frac{1}{2}kpx^2$; $P_1 = \frac{1}{2}pa^2$ or $P = P_1 + P_2 + P_3 = \frac{1}{2}p(a+x)^2$.

Take moments around *A* and $P \times \frac{2}{3}(a+x) = P_4(a + \frac{2}{3}x)$.

Which is an equation with one unknown quantity x. Also $P_4 + T = P$ or T tension in tie or compression in strut $= P - P_4$.

Fig. 30*b* shows the usual trench sheeting, the two struts at *D* and *C* take the place of the tie, *T* in Fig. 30*a* and under these conditions $P_4 + S_1 + S_2 = P$, and from this the size of sheeting and depth to which sheet piling is to be driven can be determined, and also the dimensions of struts and wales. Sheet pile *AB* being considered as a beam loaded as shown in *ABF*. The same analysis as above is applied to the general type of sheet pile bulkhead, Fig. 30*c*, where *AB* is the sheet pile *MN* the elevation of water. The triangle *ALM* indicates the earth pressure above tide level while *MLIB* indicates the total active pressure

of submerged earth and water below tide level on the land side. *MHB* indicates the total water pressure on the water side, and *EGH* the total active soil pressure on the water side and *EHF* the total passive pressure. Superimposing the active

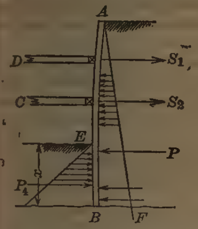


Fig. 30*b*. Length of Sheeting and Dimensions of Braces

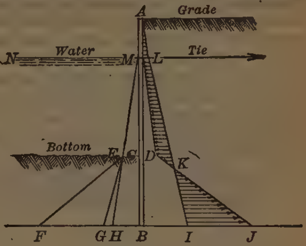


Fig. 30*c*. Bending Moment and Shear on Sheet Pile

and passive pressure on the water side on the active pressure on the land side *AMDKL* represents the intensity of the forces outward. *KIJ* represents the intensity of the active and passive forces inward which with *T* equals the outward forces. From this may be constructed the moment diagram and the size and details of the sheet piling actually desired.

Fig. 30*d* is the general case of a platform wall with surcharge, the length and size of the sheet piling may be determined as follows:

Find the known active force P_1 extending from the relieving platform to the level of the outside soil. Find the intensity p_1 at the level of the outside soil. Find p the increment of intensity per unit of depth for the active pressure below the outside soil. Find kp the increment of intensity per unit of depth for the passive pressure below the outside soil. Then $P_2 = p_1x$, $P_3 = \frac{1}{2}px^2$ and $P_4 = \frac{1}{2}kpx^2$. Take the moment of all the forces below the platform about the point O, giving

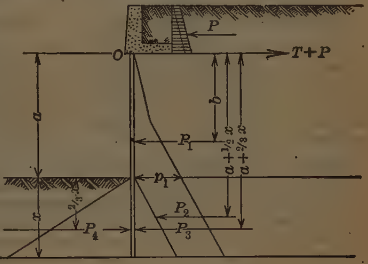


Fig. 30*d*

$$P_4(a+\tfrac{2}{3}x) - P_3(a+\tfrac{2}{3}x) - P_2(a+\tfrac{1}{2}x) - P_1b = O;$$

$$\tfrac{1}{2}px^2(a+\tfrac{2}{3}x)(k-1) - p_1x(a+\tfrac{1}{2}x) - P_1b = O;$$

$$\tfrac{1}{3}p(k-1)x^3 + \tfrac{1}{2}ap(k-1)x^2 - \tfrac{1}{2}p_1x^2 - ap_1x - P_1b = O.$$

An equation with one unknown x. Solve for x. Then determine P_2, P_3 and P_4 and $T = P_1 + P_2 + P_3 - P_4$.

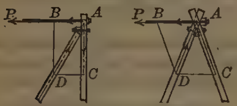
In applying this method the table of equivalent liquid pressure in article 12 page 1728 may be employed. kp the increment of intensity of passive pressure

is dependent upon the character of soil bottom and must be obtained by experiment or experienced inspector.

Brace Piles, when driven with opposite inclinations, are subjected to a pull in one case and a push in the other, and connection should be made accordingly. It is too frequently the case that brace piles, tho provided for in ample number and of sufficient bearing capacity, have weakened and improper connections, so that the full lateral effect is not secured from them.

A quay wall or bulkhead can be anchored against the thrust or horizontal pressure by dead-men or anchors buried in the embankment some distance back and below the plane of rupture made by the angle of repose, or by anchors of brace piles. Having determined the amount of this thrust, the anchorage can be readily designed.

Such anchorages are shown in Figs. 31, 32, 33. In Figs. 31 and 32, the amount of horizontal thrust is represented by A B, A D would be the push or bearing



Figs. 31, 32. Pile Anchorages

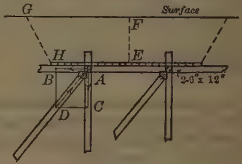


Fig. 33. Pile Anchorage with Platform

on the brace pile, and A C the uplifting effect or pull on the vertical pile in Fig. 31, and inclined pile in Fig. 32; Fig. 33 is an anchorage in connection with a relieving platform, or might be used separately as anchorage for a bulkhead or wall. A C is the uplifting effect, but the actual pull or uplift on the vertical pile shown is decreased by the weight of the soil or backfill surcharge on the platform, represented by $E\,F\,G\,H$.

Effective and Ineffective Anchorage details of brace or anchorage connections are shown in Figs. 34 and 35. Fig. 34 shows five typical brace pile anchors for a quay wall or bulkhead. Fig. 35 also shows five typical cases of improper brace or spur, pile details or connections. Some of these have been taken from actual work, Fig. 35 (1) and (2) having been discovered and sketched only after the failure of the work by the outward movement and overturning of the wall. It will be noted that in these two cases the bearing or thrust in the brace pile is

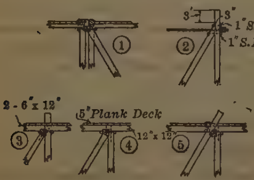


Fig. 34. Effective Pile Anchorage

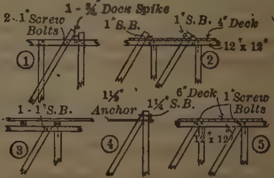


Fig. 35. Ineffective Pile Anchorage

entirely taken by the bending in two 1-inch screw bolts and one ¾-inch dock spike. These, apparently, held for some time, but, being submerged, even this

upport was greatly weakened by the corrosion of fastenings. In Fig. 35 (3) nd (4) depend entirely upon the strength in bending of one screw bolt, as does 5), except that the deck planking directly above the piles furnishes an additional bearing. In designing work that will be submerged, or kept partially o, if the work is to be considered permanent, it is important, in so far as posible, to avoid entire dependence upon iron or steel fastenings. Galvanizing as been attempted with indifferent success, and, in one or two instances, exruded brass has been used for fastenings. As far as possible, work of this charcter should be detailed for dependence upon the fitting of timber joints and the ise of wedges, dowels, and treenails. The general practise in engineering design hould be applied to obtain dimensions of caps and stringers, thickness of plankng or decking, bearing surface in contact, and other details. Liberal allownce must be made, however, for the weakening of aged water-logged timber r the deterioration of timber by rot, when exposed to successive wetting and lrying out; and in reinforced concrete construction to the possible disintegrating ffect of concrete when submerged in salt water, especially when subject to the ction of frost.

Safe Bearing Capacity of Groups of Piles. As stated in Sect. 6, Art. 23, he bearing capacity of piles is dependent upon the support at the bottom of he pile; if the pile penetrates a short distance to hardpan or rock, the strength f the pile as a column must be considered; when the pile penetrates some disance into soft materials, the bearing capacity depends upon the skin frictional esistance. It would be well, however, to note that the skin frictional resistance s the means by which the load on the pile is transmitted thru the pile and thru vertical cone of earth, of which the pile is the center, to the subsoil at the plane f extreme penetration of the pile, the bearing capacity of the soil also being ncreased by being compacted by the driving of the pile. For this reason the safe earing capacity of one pile is no criterion for fixing the safe bearing capacity of number of piles driven closely together, as in the latter case the cones of resisance overlap and the bases of these cones at the bottom of the piles will, in cases f a group of piles driven closely together, each be required to carry the load, at east in part, of several piles. While the bearing capacity of the subsoil at some listance beneath the surface, as at the foot of long piles compacted by pile drivng, will safely carry considerably higher loads than the same character of subsoil t or near the surface not compacted, there is, nevertheless, a limitation which an be readily exceeded by placing piles too close together, expecting them to arry considerable loads. It is inadvisable in pile foundations, for harbor work, o place piles closer together than 2½ ft to 3 ft. It will be seen that at 2½ ft enters, in carrying loads of 18 tons each, the interior piles of a large group vill subject the subsoil to a load of approximately 3 tons per sq ft.

Anchorage or Pull on Piles. Where brace piles are used for anchorage of uay walls or bulkheads, or in connection with pier work, as previously stated, he bringing into play of these brace piles to resist lateral movements frequently esults in an upward pull on some inclined or vertical piles. As a general rule, iles driven some distance into the soil and dependent upon skin friction for heir carrying or bearing capacity can be safely counted on as giving an anhorage effect or resistance to pulling equal to the estimated bearing or carryng capacity. Experience in pulling up or attempting to pull up piles driven n harbor work fully confirms this statement. Observations made on piles 55 to 5 ft in length, driven in 23 to 35 ft of water, indicate that a pull of from 20 to 5 tons was necessary, in many instances tops of piles pulling off before the piles vere moved. This resistance to up-pull is dependent upon the skin friction of he pile or actual mechanical anchorage in the case of types of concrete piles

with enlarged bases, which, in turn, is dependent upon the weight submerged in water, of the cones or pyramids of soil engaged by the pile and having their small ends or apex at the bottom or foot of the pile and the enlarged bases at the river or sea bed. Hence, the anchorage effect in a group of piles driven closely together, will be less than the sum of the anchorage effect of a like number of single piles, because the anchorage is entirely dependent upon the weight of the mass of earth engaged by the group of piling and in no case should be expected to exceed the weight of the affected inverted prism of soil to the bottom of the piles of the group. The effective weight is, when below tide level, of course, the submerged weight of the soil, for which refer to Art. 12.

14. Timber Quays or Bulkheads

Inexpensive and Temporary Character. When work is to be of a more or less temporary character and of inexpensive type, timber is largely employed, often, where necessary, treated by one of the preservative processes. In harbors where marine wood-borers do not exist or are not active, timber work below mean tide level, where always kept wet, will last indefinitely and can be employed in the foundation and other parts of permanent harbor work.

Creosoted Timber. Available information shows that the life of creosoted timber, submerged in water where marine wood-borers are prevalent, varies a great deal and this is due to differences in quantity and quality of the oil used and to the method of treatment. A variation in quantity of 10 to 24 lb per cu ft is frequently employed. Short leaf pine, on account of its structure, readily takes more oil per cubic foot than pitch pine, and, while not possessing the same strength, is, in many instances, a better material to use where creosoted lumber is required.

Yellow pine piles treated with 16 lb of creosote per cu ft, after being in place in Charleston harbor 12 years, show no evidences of having been attacked by marine wood-borers. In some of the other South Atlantic and Gulf ports, where the marine wood-borer is very active, there are examples of creosoted timber piles that have not been seriously damaged by the marine wood-borers, and have been in place over 20 years.

Examples of various timber quays are shown in Fig. 36, the numbers referring to the brief descriptions given below.

(1) Garrison Ave., Bronx River, New York City. Front close piling filled with riprap.

(2) Harlem River, New York City. Three inch sheetpiling backed with riprap. Anchor bents 10 ft centers.

(3) Echo Bay, New Rochelle, New York. Close piling backed with riprap. Bents 6 ft 6 in centers.

(4) Oak Point, New York.

(5) Hudson River, New York. Riprap embankment. Bents of platform 10 ft on centers.

(6) Astoria, L. I., New York, Astoria Light and Power Co. Crib filled with rubble, pockets 8 ft square formed by logs.

(7) Hudson River, New York, Interboro Rapid Transit Co. Slope paved with riprap. Bents 5 ft centers.

(8) Harlem River, 171st Street, New York. Three inch sheetpiling, backfilled with riprap. Guide piles 5 ft centers, two anchor piles every 10 ft.

(9) Rockway Inlet, New York, Barren Island. Bents 10 ft centers, one brace pile to each.

(10) San Pedro, Port of Los Angeles, California. Live load allowed 500 lb per sq ft. Piles creosoted, bents 15 ft centers.

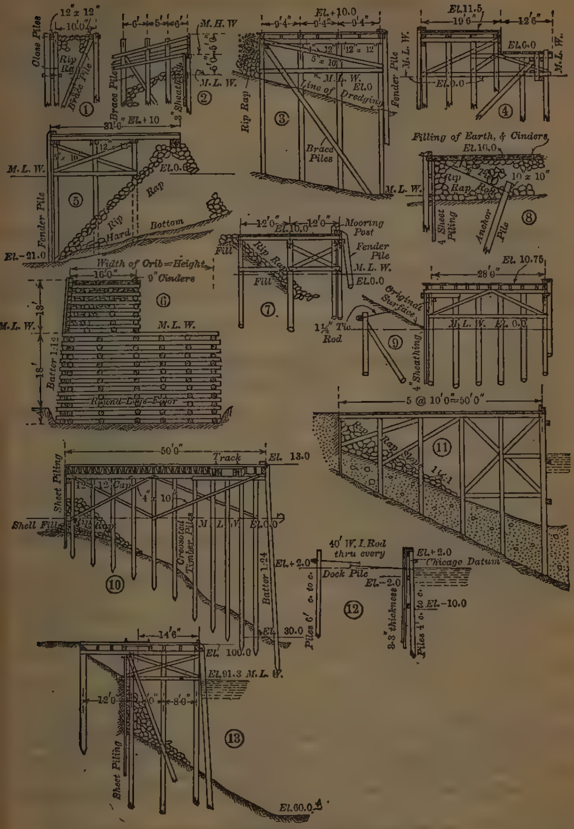


Fig. 36. Timber Quays

(11) Missouri River, Kansas City, Mo. Timber and piles creosoted. 526 ft long. Cost $43 per lin ft.

(12) Chicago, Ill. Standard Sheetpile bulkhead. Sanitary District.

(13) Navy Yard. New York, 1912. 200 ft long. Cost $38 per lin ft. Platform with 6 in sheetpiling. Bents 10 ft centers.

15. Masonry, Concrete, and Timber Quays

Foundations. In the construction of quays or bulkheads of more permanen character where rock or hard unyielding foundation can not be readily secured timber piling and grillage work are often employed for the foundations or ma be carried up to mean tide level, the work being surmounted by a masonry o reinforced concrete structure.

Riprap in locations where it can be secured in quantity and at a low cost i extremely valuable as a Quay wall material, for the foundations of the wal the protection of slopes, filling behind the wall, and for the underwater part c the wall itself. On account of the voids the material in mass submerged doe not weigh more than earth fill and stands at a much steeper slope, less than on 1.

Essential Details of various precedents are given in the following state ments, the numbers referring to Fig. 37.

(1) Savannah, Ga. Standard Fuel Supply Co. 1912. Twelve inch reinforced concret wall on timber piles, 15 ft 6 in centers reinforced concrete deck slab. Live load 600 lb Cost $50 per lin ft.

(2) Brooklyn, N. Y. Gowanus Canal. Timber platform, with concrete wall on pile and riprap.

(3) Chicago, Ill. Proposed plan for bulkhead wall alongside Long Pier.

(4) San Diego, California. 1912. 2675 ft long. Cost $124 per lin ft. Concrete in cased piles 7 ft centers.

(5) Iloilo, P. I. 1912. Cylinders 12 ft centers. Reinforced concrete on timber piles Cost $70.35 per lin ft.

(6) Chicago, Ill. Long Pier bulkhead.

(7) Charleston, S. C. 1911. 4000 ft long. Untreated timber piles, sheetpiling an concrete wall. Reinforced concrete sheetpiling, 3 ft wide, to protect timber work fror marine borers. Cost $35 per lin ft. The cost of an alternative design, having a rein forced concrete wall, was $36 per lin ft.

(9) New York (South Brooklyn), N. Y. One of several types of wall used by De partment of Docks and Ferries.

(10) Boston, Mass. Northern Avenue. 1911. At Commonwealth Pier. Lengt 645 ft. Cost $106.70 per lin ft.

(11) Schenectady, N. Y. N. Y. State Barge Canal. (12) Amsterdam, N. Y. N. Y State Barge Canal. (13) Utica, N. Y. N. Y. State Barge Canal.

(14) Providence, R. I. Fields Point. Bents 4 ft centers, two brace piles to each bent Sheetpiling 6 in and 8 in.

(15) New York, N. Y. Central R. R. of New Jersey, Bronx Terminal. Bents 8 f centers, two brace piles to a bent, sheeting 6 in.

(16) Boston, Mass. U. S. Navy Yard. (17) Norfolk, Va. U. S. Navy Yard. 191c Timber platform, surmounted by concrete wall on reinforced concrete sheetpiling, 5 ft long. Similar wall faced with granite ashlar, built at New York Navy Yard, 1912 Cost $180 per lin ft. Bents, 5 ft centers.

(18) Berlin, Germany. Spree Canal. (19) Neufahrwasser, Commercial Railroad Germany.

16. Masonry and Reinforced Concrete Quays

Foundations. Where the foundation is of rock or unyielding material, qua walls built up of large stone blocks, concrete blocks, or concrete monoliths, ar permanent and effective. In localities where granite is readily procurable a a reasonable price, this type of wall can not be improved upon both as to per manency and cost. With less satisfactory foundation conditions, especiall where marine wood-borers are prevalent, reinforced concrete piles or column are effectively used. In masonry walls the underwater work, if of blocks, is usu

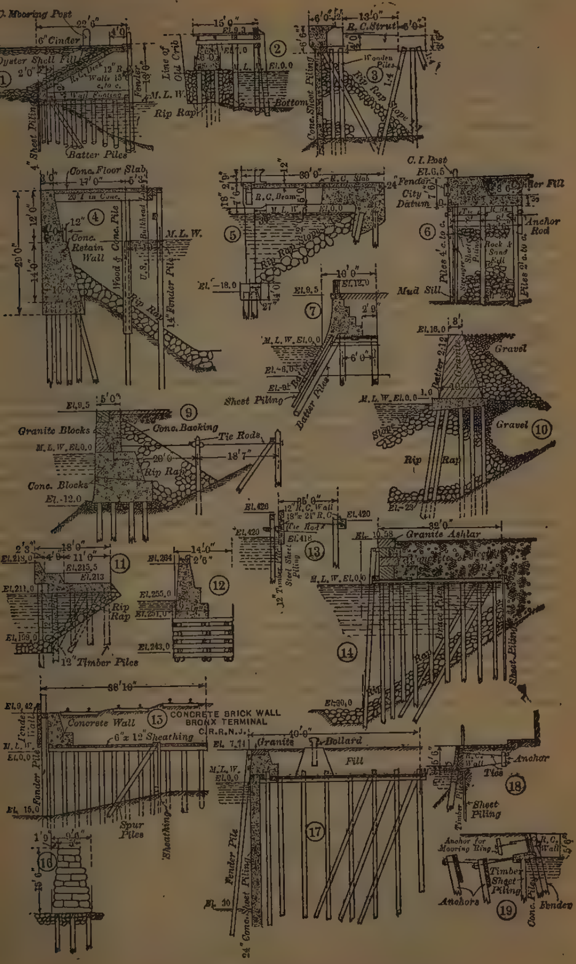


Fig. 37. Masonry, Concrete, and Timber Quays

ally laid by divers to elevation of low water. Where monolithic concrete block are used these may be premolded and lowered to place by heavy derricks, th foundation being prepared by the use of riprap and broken stone or concret bag work. Concrete or stone masonry walls are also built in the dry in coffe dams, or use is made of reinforced concrete or timber floating caissons or boxe built on shore, launched, and sunk in place by filling.

Essential Details of various precedents are given in the following state ment, the numbers referring to Fig. 38.

(1) Key West, Fla. U. S. Navy Yard. Reinforced concrete piles and deck. Ben 10 ft centers. (2) Albany, N. Y. State Barge Canal. (3) Los Angeles, California, 191 Reinforced concrete piles and deck. Cost $170 per ft. Bents 20 ft centers; anchors, vertical, 4 brace timber piles. (4) Nantes, on Loire, France. Reinforced concrete. (Spandau, Germany. Municipal Quay. (6) Nantes, France. (7) New York, N. Y. Po Morris, Bronx, N. Y. C. & H. R. R. (8) Oakland, California. 2005 ft long. (9) Bo ton, Mass. Fish Pier. Wall around Pier 6. Cost, about $260 per ft. (10) Baltimor Md. Proposed design. Est. cost, $50 per ft. (11) Portsmouth, N. H. Navy Yar Loose coursed granite laid by diver. A type of wall used along New England Coas 1907–1912. 320 ft. $308 per ft. (12) New York, N. Y. 116th Street, Department Docks and Ferries. (13) New York, N. Y. Cedar Street. Department of Docks ar Ferries.

The Department of Docks and Ferries of New York City has for many years em ployed large premolded concrete blocks for Quay wall construction. With rock found tion within 40 ft of surface, if material over rock is soft, it is dredged off and rock cleane and the wall based on bag concrete brought up to level by divers. Upon this successiv tiers of concrete blocks are laid up to low tide level; above this, granite ashlar, backe with mass concrete, is used. The blocks weigh from 25 to 95 tons. Where the foundatio is such that piles can be driven to clay, sand or other firm bottom, soft material is exc vated to 30-ft depth, piles driven and cut off 15 ft below low water and given lateral resis ance by an embankment of riprap and gravel brought up to tops of piles. Upon the piles a single tier of concrete blocks is laid, each weighing 80 tons. A granite ashlar wa facing with concrete backing surmounts the blocks. Where the bottom is soft, the wa is provided with a relieving platform. In some walls in soft bottom, settlements high as 4 ft have occurred, which were remedied by adding courses of granite. Th blocks are handled by scows and set in place by a 100-ton or a 40-ton crane. Cost h varied greatly from $100 to $750 per lin ft.

Quay walls, in which monoliths are employed consisting of floating caissons, in a sim ilar manner to their use in Breakwaters, see Art. 3, have been constructed at Zeebrugg Belgium; Rotterdam, Holland; Norresundby, Denmark; Talcahuano, Chili, and hollo reinforced concrete boxes at Passau on the Danube. At Antwerp, Belgium, a quay wa was constructed using caissons launched and floated to place and then sunk by fixin over their upper part a pneumatic caisson or diving-bell handled by two floats, the se tion being then filled with concrete.

17. Failures of Quay Walls

Movement and Settlement of quay walls and their eventual failure in who or part are not uncommon, even in well-designed walls or bulkheads where amp provision has, apparently, been made for the outward thrust; it is not unusu to find after work has been started evidences of settlement at the toe and irre ular outward movement at the top. This may, in some cases, be accounted fo by the slight settlement of the foundation under the front of the wall on accoun of the concentration of load and the outward thrust at that place and also i walls where timber platforms or anchors, employing brace piles, are used, t the slight movement allowed in the bringing up of bearings and connection In most instances, this will not continue to any great extent, and may be co rected in so far as appearance goes, by resetting the coping. On walls or bul heads having ornamental copings or balustrades, it is often advisable to post

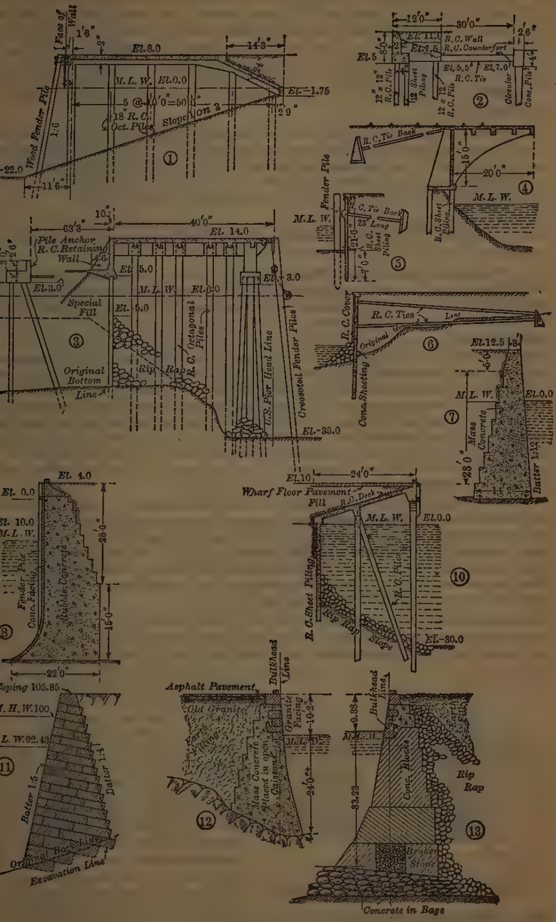


Fig. 38. Masonry and Reinforced Concrete Quays

pone the placing of these until a short time after the wall or bulkhead has been built and has had an opportunity to take its set. In gravity masonry walls, a not infrequent form of failure, or partial failure, is the moving or sliding out, in mass, of the wall on its base. It is customary, where possible, to lay the foundation and upper courses of masonry at an inclination to overcome this.

Examples of Failures and the corrective measures successfully applied in some instances:

(a) An outward movement was observed during the construction of The Fish Pier wall in Boston Harbor, the wall resting on an inclined riprap bed, laid on clay overlaying hardpan and rock. The movement was arrested by driving piles directly in front of and at the toe of the wall and constructing in the rear of the wall a relieving platform. 400 ft of wall were involved; the cost of repair was about $90.00 per lin ft.

(b) A section about 300 feet long of a masonry and timber platform wall built at the New York Navy Yard, failed by outward movement and overturning, the cause being, undoubtedly, improper brace pile connections, and as referred to in Art. 12.

(c) A somewhat similar wall in Washington, D. C., built on a very narrow platform without brace pile ties, moved outward 10 ft when the backfilling had only been carried to mean tide elevation. When repaired, by constructing a timber relieving platform with adequate brace piles, backfilling was carried to grade and no further movement of importance found.

(d) A wall in Charlestown, S. C., harbor, 4000 ft long, was located on a salt marsh; the river silt of increasing compactness and density extended to a depth in excess of 90 ft below tide. The depth of water at face of wall varied from 0 to approximately 8 ft. The support for the wall consisted of a timber platform and brace piles with sheet-piling driven at an inclination. Filling of river silt was placed behind the wall by hydraulic dredging. In two locations the filling material blew out underneath the sheet-piling, pushing out the bottom of the sheet-piling. This was corrected by loading the toe of the wall with riprap and, in some instances, by driving heavy vertical sheet-piling inside and back of the wall platform.

(e) A platform bulkhead built in New York Harbor, consisting of a timber platform with sheet-piling on the inside with brace piles at 5 ft intervals, with 15 ft depth of water at the sheet-piling, and 30 ft depth along the face, gave evidences of outward movement when the depth of water was increased by dredging to 34 ft. This was corrected by placing 2 in round steel anchors every 20 ft and carrying these back 40 ft inshore of the sheet-piling to anchors of timbers, dead-men, and piles, and placing riprap along the slope decreasing the depth of water at sheet-piling to 12 ft.

(f) A wall built at the Navy Yard in New York, about 1885, of gravity type consisted of heavy granite blocks, with massive counterforts, laid by divers on a timber pile foundation and of granite ashlar work above tide level. It was designed at a time when the draft of the vessels did not exceed 20 ft. With the increase in draft of vessel, the basin directly in front of this wall was dredged out and, finally, to a depth slightly in excess of 30 ft. A somewhat similar wall in another locality failed by outward movement and overturning. The wall in question moved outward and gave every indication of early failure. It was repaired by the method indicated in Fig. 39, the ashlar work being stripped off to low water, sheet-piling driven directly in front of the wall, and this tied back in turn by timber ties to a platform in back of the wall provided with vertical and brace piles, the space between the face of the wall and the sheet-piling filled with riprap, and a new wall built directly atop of the sheet-piling, and the work backfilled.

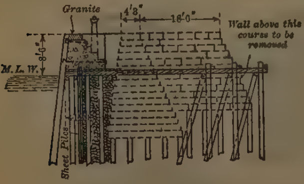


Fig. 39. Quay Wall Repair, New York

(g) A crib wall in D., L. & W. R. R. yard, Hoboken, N. J., the top of which had given way and needed renewal, was repaired at a cost of $13.21 per ft, as shown in Fig. 40.

(h) A seawall built on a timber platform, the protective face of which under water consisted of 3-lap, 2-in yellow pine treated with creosote, began to fail on account of the destruction of the wood by the teredo. Eight-inch grooved and tongued reinforced concrete sheet piles were driven and a reinforced concrete face put on top of this and the structure tied back by tie rods. The work will cost about $85.00 a linear foot in 1919 as shown in Fig. 40a.

Fig. 40. Quay Crib Repair, Hoboken, N. J.

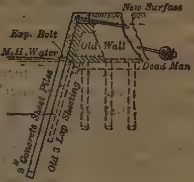


Fig. 40a. Wall Repair, Hampton Roads, Va.

There are numerous other examples, especially of walls or bulkheads constructed to retain filling that have failed by the outward movement of sheet-piling at the toe or bottom, or by outward movement at the top, and overturning on account of insecure and insufficient anchorage.

The increase in the depth of harbors, due to the increased draft of vessels, has, in instances, rendered insecure wall structures designed with sufficient strength and stability for the conditions existing when they were built.

LANDING PIERS

18. General Information

Pier and Bulkhead Line. In the interior of harbors and on navigable rivers, two lines are usually established in the United States by the National government, or, outside of this jurisdiction, by local port authorities. The bulkhead line is the line to which solid, or solid filled structures may be built. The pier-head line is the line fixing the boundaries of the fairway to which wharf or pier structures may be built, but when so built must be of open construction, permitting the flow through them of the full sweep of the current or tide, such open construction being in the nature of pile or column bents. It is usually permissible, if desired, to make the structure from mean-low-tide up continuous or solid.

Length and Width of Piers. Piers should be of at least sufficient length to berth the longest ship likely to use them, and hence should project out into the stream a short distance beyond the bow or stern of the ship to protect the hull of the vessel from collision by passing navigation or floating ice. Piers are often built long enough for berthing two or more vessels on either side. Allowance should be made in determining the length of piers for the growth in dimension of shipping. The width of piers is largely dependent upon the use to which they are to be put, altho a safe minimum width, in order to secure lateral stability by bracing, would be fixt by the depth of water and the length of the pier and the character and size of the vessels to which it will be required to afford wharfage. Piers to be used for the berthing of vessels, but not to any extent for the discharge, handling, or storing of cargoes, should be not less than 40 to 50 ft wide. For piers in shipbuilding, and repair yards, carrying standard gage rail-

road track, and for heavy crane track service, requiring the handling of bulky material, 75 to 100 ft which is frequently used. For commercial pier work and in an important port where the largest ships are likely to be berthed, and cargo handled and re-handled, widths of 100 to 250 ft are used, while in the same character of piers, in connection with railroad terminals carrying one or more lines of standard gage railroad tracks and provided with sheds for handling and storing of cargo, widths of piers in excess of 200 ft are often used. No fixt rule can be given for the length of piers. Local conditions and the value of property must control. If piers are designed long enough for two or more vessels to lay alongside, the traffic congestion incident to handling and rehandling cargo at the entrance to the pier must be considered. As the width of piers, or transit shed erected on the pier, should afford storage space for at least one cargo outgoing or incoming, that is, considering whether a one- or two-story shed is used, the cargo capacity of the vessel at the pier and the necessity for aisles or passage ways thru the storage space, and for road ways and railroad tracks to take in and bring out cargo, must be considered. There has been a tendency to increase the width of piers at cargo handling terminals.

The width of piers for handling incoming and outgoing cargo is directly related to the quantity of cargo to be accumulated on the pier for loading or the quantity to be landed in unloading while awaiting transfer and distribution. At a properly laid-out terminal vessels should be promptly loaded or unloaded. In determining the proper width the cargo carrying capacity and length of the largest vessel likely to use the pier should be taken. For man handling 5 ft high would appear to be the highest economical height of stowage; while 40 cu ft can be allowed for a cargo ton, or with passageway, etc., allow 12 sq ft of pier or wharf per cargo ton. With mechanical stacking and handling this could be decreased or the width of piers can be decreased by using two decked transit sheds, altho in that case one deck is generally employed for incoming and the other for accumulating outgoing. On account of the increase in sizes of vessels and the importance of space for accumulating cargo in advance and discharging without delaying vessels, the tendency is to construct wider piers. The New York City pier and waterfront congestion is undoubtedly due largely to too narrow piers for the size of vessels using these.

Width of Slips. The berthing space or slip between two piers should be wide enough to readily accommodate two vessels, one lying at each pier, and also give room for the entry or departure of either. Sufficient space should also be provided for loading and unloading by barges or lighters on the off pier side of the vessel. The greatest beam of the largest ships is now about 100 ft, so that in slips where the largest vessels are to be accommodated the width should not be less than 250 ft, and it should preferably be more than this.

Depth of Water at Wharves. In former years the amidship, hull section of wooden ships was such that the maximum draft required was not necessary directly in front of quays or piers, but with the advent of steel hulls of practically rectangular amidship section, with bilge keels, the maximum draft is required directly in front of the wharf to be occupied, unless the vessel is kept off by spur shores, or floats, in which case there is a loss of space in the slip and inconvenience in the placing of gangways and the handling of cargo.

At the first-class piers on the Atlantic Coast 40 ft appears to have been recently fixt as the desired depth for port approach channels. One or two notable examples exist of transatlantic liners which have a depth of approximately this at full load. It is doubtful that the tendency to increase full load draft design of ocean-going vessels will continue beyond this 40-ft limitation.

On the Atlantic Coast, in the important harbors, where possible, a usable depth of not less than 35 ft should be provided, as some of the larger vessels in commercial use, when fully loaded, have a draft in excess of this. Provision should be made for this usable depth at extreme low water by allowing additional depth for silting up, and to provide for movement on account of wave action.

Naval vessels seldom have a maximum displacement draft in excess of 30 ft. An allowance, however, must be made for over-draft, especially in case of injury, such as the flooding of compartments.

Direction of Piers. In the prevailing American practice, piers are built out perpendicular to the shore line and to the direction of the channel. Some advocate placing piers obliquely, urging that the task of bringing vessels into and out of slips is thereby rendered easier. This is, of course, dependent upon tidal and current conditions. In a tidal harbor with the ebb and flood of the tides, this would be so under one of the two conditions of the tide, but it would be more difficult, or impossible, to bring vessels in or out with the reverse condition of current.

Laying Out of Wharf Work. The location of piles or columns in bents are usually fixt by batten ranges placed on shore, the spacing being laid off on two bases, a short distance apart, parallel with the bents, and the distance out to bents determined by similar ranges on adjoining structures, or by steel or cloth tape lines measuring out from a fixt base, the piles being held in place by stay-lathing or temporary braces below the cut-off or finished grade. The elevation of cut-off is given by level and carried from the given point by straight edge and checked by instrument. Piles are cut off below water, by divers using a hand saw, or by using a horizontal circular saw operated from the surface.

Piers in Deep Water. When piers are laid out so that for the entire length or at the outer extremity they are in water of considerable depth, resort is sometimes had to raising the bottom, into which the piles are to be driven by making embankments of riprap. In pile piers, if placed after the piles are driven, there is likely to be a settlement in the structure due to the weight of the riprap or cobble. This method of construction has been employed along the New York City waterfront, and in the case of the New York Central pier, at Watt Street, resulted in a settlement of nearly 2 ft. Riprap embankments, under water, are also used to give the pier lateral strength, especially where the piles can not be driven deep enough into the bottom to insure good holding. The raising of the bottom by embankments of riprap or cobble is also resorted to to strengthen the pier laterally and especially to support the piling, which would otherwise involve consideration of the carrying capacity of such piles as an extremely long unsupported column.

19. Design

The Live Load carrying capacity of piers and quays depends largely on the use to which they are to be put, and ranges from 250 to 1200 lb per sq ft. The minimum should be used for piers intended solely for berthing light traffic and the support of oil and water mains, and the maximum for especially heavy work, such as the handling and storage of ordnance, armor plate, and heavy machinery. In commercial wharves, 600 lb per sq ft should be used for piers, and 750 lb for quays. The foundations are to provide for this live load and dead load of the structure itself. The foundation for standard gage railroad track, or heavy crane tracks, shed or building foundations, require special treatment as in other engineering structures.

Prevailing Practise. In pier design it will be found that details frequently have to conform with established rules fixt by local authorities. Where this is

not the case, the deck or slab should be designed to carry full live load and dead load, the stringers, rangers or beams to carry full live load and dead load. Caps or girders, 80 percent of live load, and full dead load. Piles or columns 75 percent of live load and full dead load. For other structures such as sheds, railroad tracks, and special work, design in conformity with general building practise.

Timber Pier Framing is largely done by the methods used in ship framing. Pile caps, if not in one piece, are usually scarfed at such locations as to break joints in adjoining bents. In substantial work piles are usually tenoned and caps mortised. Frequently clamps are substituted for caps, these consisting of two timbers, the pile being shouldered out on each side to receive them. String pieces or rangers are either butted over caps and drifted, or spiked, to the cap, or the joint occurs in the middle of the bay, the timbers being fish-plated. In heavy and important timber piers, the deck system usually consists of heavy planking with somewhat lighter wearing sheathing laid diagonally to the direction of traffic over it. Out-shore corners on piers are usually rounded and heavily reinforced. In harbors where floating ice is prevalent, especially in tidal rivers, the bays at the outer end of the pier are made of double width, and the pile or column bents at either side protected between tides with steel or iron plates. The fastenings used are, in general, similar to those employed in timber, ship, and bridge work, and are frequently galvanized. Piers are often built with a slight crown at the center to facilitate drainage.

Fender System. Wharves, when used for berthing, are protected by fenders fitted so as to be readily removed for repair and replacement. The fender system is made up of vertical fenders or fender spring piles and horizontal wearing ribbons. The outer ends or corners of piers are usually protected by heavy clusters of fender piles, or by separate pile clusters or dolphins. In reinforced concrete and masonry piers the fender system is often arranged on an elastic system with sufficient spring to save the main structure from the shock of vessels coming alongside. For this purpose the spring of the vertical and horizontal timbers is employed, or groups of heavy car springs used. Frequently such fender systems are assisted or replaced by floating fenders, rolling logs or spars of single or made-up timbers sometimes served with old hemp line and fastened at the ends with lines or chains fixt to the side of the piers or quays.

Fender System. In addition to the fenders employed it is sometimes customary to construct dolphins or clusters of piles along the side of a pier to breast off vessels with. Such dolphins are also used without the pier for mooring purposes while vessels are awaiting for pier or berthing space or being loaded or unloaded by lighters.

Moorings. Piers are provided with mooring posts or bollards, bitts, and cleats, one for each 50 ft length of berth, the inshore and outshore moorings for bow and stern lines to be heavy bollards or posts fitted with horns or pins. Typical details of mooring fittings are shown in Fig. 41. When vessels are berthed alongside of quays, it often becomes necessary to spur them off so that at low tide the bilge keel will not rest on bottom slopes. Spur shores of heavy spars are used for this purpose, one end resting against the vessel's side, and being supported from it by lines or chains, the other end resting on wheels or rollers on the wharf platform, and made fast by lines or chains to bitts or mooring posts. The same object is accomplished by placing floats in between the vessel and the wharf.

Accessibility of Wharf Platforms. In harbors where the range of tide is only moderate in extent, the platform elevation of main piers and of second floors of pier sheds are usually so fixt that access may be had to cargo openings in sides of ships, for placing and transfer of cargo, by portable gangways. Where

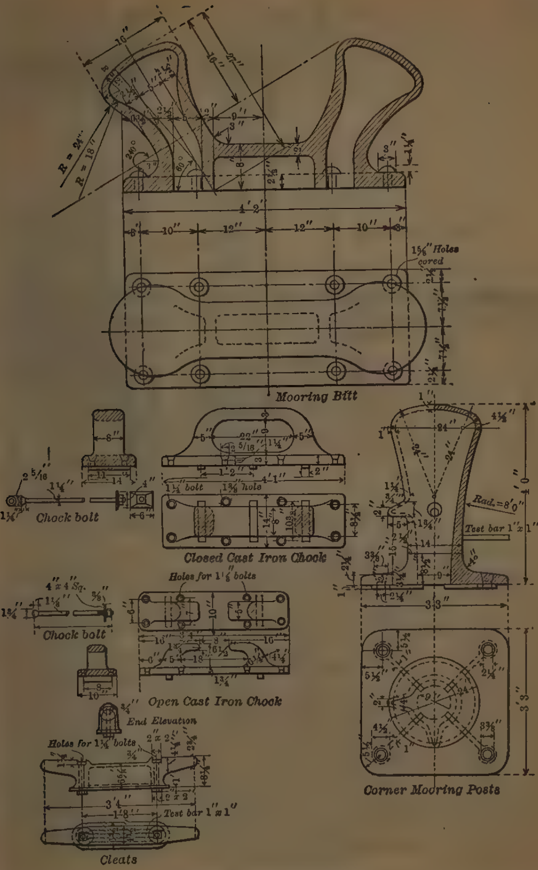


Fig. 41. Mooring Fittings

there is considerable range in tide, piers are sometimes built on two levels, the low one used at low tide, and the upper one at high tide, or inclines are built on the sides of the wharf, and sometimes these are provided with drop or adjustable gangways. A detail of such a gangway, in use at Philadelphia, Pa., is shown in Fig. 42.

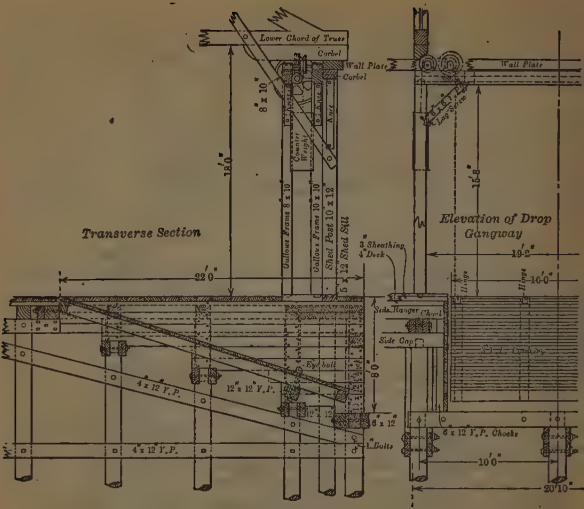


Fig. 42. Drop Gangway, Philadelphia

Ocean Piers are generally built for passenger service, landings or recreation purposes. Ocean piers have been constructed for commercial purposes in locations where natural harbors were not available or convenient. Piers of the recreation type are found at resorts near populous cities, and are very often constructed of steel or iron piles or columns with steel and timber superstructure. The piles or columns are made up of pipe, or structural shapes, iron or steel disks or sections of screws being fixt to the lower end. The piles are put into position, with the aid of water jets, an attempt being made to place them sufficiently deep so that the sand surrounding them will not be disturbed and washed away by wave action. These iron or steel piles or columns, when made of pipe, offer a minimum resistance to wave action; the requisite lateral stiffness is usually secured by cross bracing of steel rods. Piers of this character have also been built employing reinforced concrete piles surmounted by a reinforced concrete deck system. Ocean piers of the commercial type have been constructed on the southern coast of Cuba and in South America; in some cases, instead of being continuous piers built out into the sea, they consist of a detached platform or pier in deep water, the cargo being handled to and from the shore by means of overhead cableways.

20. Life and Cost of Piers

The Life of Piers is entirely dependent upon the location of the work and character of construction employed. Timber piers built in harbors where marine wood-borers are not active will last indefinitely below mean tide, but above this are subject to rapid deterioration and decay. Under such circum-

stances the life of a timber pier, in its entirety, could be probably fixt at 20 years. There are instances of work of this character requiring extensive repair, amounting to replacement, in 15 years, while in other cases piers after a life of 25 to 30 years are still in use. C. W. Staniford, Ch. Eng., Dept. of Docks and Ferries, New York Harbor, gives the following facts in Trans. Am. Soc. C. E., Vol. 77, 1914:

Description	Percentage of total original cost	Renewal required
Sheathing	12	Every 6 yrs
Backing log	1.8	" 8 "
Fender chocks, including vertical cheating	4	" 10 "
Fender piles	4.7	" 12 "
Decking	11.3	" 15 "
Bracing	7.1	50% in every 20 yrs
Rangers and caps	24.4	50% " " 20 "
Piles, above mean low water	34.7	33⅓%" " 20 "

From the above it is derived that the average life of the entire pier is 21.6 years. The timber piles, however, below mean tide elevation, which are still in good condition, can be made use of by building a new superstructure above and resting on them. Climatic conditions, the care with which the construction work is performed, and the character of timber employed will materially modify length of life. Timber piers, with creosoted piles, in ports where marine wood-borers are active, may have a length of life between 15 and 25 years. In timber piers with reinforced concrete decks, the length of life is dependent on the length of life of rangers and caps, and, when of substantial construction in a locality such as New York Harbor, can be taken as 30 years. Masonry, reinforced concrete, masonry and reinforced concrete on timber foundations or platforms where marine wood-borers are not prevalent or active, have practically an indefinite length of life, altho under some conditions there is a possibility of deterioration of concrete work immersed in sea water requiring extensive repair and renewal. It seems safe, however, to fix the length of life of this character of structures at 60 years. Untreated, unprotected timber in harbors where marine wood-borers are prevalent and active, are destroyed rapidly, in some instances the piles being entirely eaten away and destroyed in two or three years.

Costs of Piers. The data available as to the cost of piers are likely to prove extremely misleading. These costs are, of course, dependent upon details of construction, character of foundations, live load carrying capacity for which designed, the character and cost in the particular location of the material employed, availability and cost of labor and appliances. For general estimating purposes, for the cost per superficial foot of different types of pier work, in American ports, the following may be taken, the range between the maximum and minimum being dependent on location, foundations, and detail of design. These costs do not include auxiliary structures, shed foundations, railroad track foundations, subways, galleries, or other structures:

	Cost per sq ft
Untreated pine timber piers	\$0.75 to \$1.20
Creosoted pile untreated timber superstructure piers	\$1.00 to \$1.50
Untreated timber piers with reinforced concrete deck	\$0.90 to \$1.45
Creosoted timber with reinforced concrete deck	\$1.15 to \$1.75
Untreated timber platform with concrete cross-walls or reinforced concrete columns and deck	\$1.40 to \$2.00

	Cost per sq ft
Creosoted timber piles with reinforced concrete columns and deck	\$1.70 to \$2.35
Reinforced concrete piles and timber system and deck	\$1.70 to \$2.55
Reinforced concrete piles and deck and deck system	\$1.90 to \$2.75
Timber platform to low water and solid fill above	\$1.50 to \$3.00

On reinforced concrete deck piers there should be added to the above figures, if asphalt wearing surface is to be placed, from 10 to 16 cents per sq ft, and if creosoted wood block wearing surface is to be placed, 20 to 28 cents per sq ft. On filled piers, in addition to this, add 8 to 12 cents per sq ft for concrete base. It is to be noted that on the last type of filled platform pier, the enclosing wall structure representing the principal cost of the structure, the width of the pier would be a controlling feature in the cost. No cost can be given for solid filled piers, that is, piers enclosd by one of the various types of quay walls, as this cost will depend entirely on the cost of the wall and the width of the pier.

Cost of Piers. The cost herein given as is the case with cost generally is that obtaining previous to the years 1914 and 1915. These are probably no criterion, even, for the cost obtaining at present, and likely to obtain. Increase in the wages of labor, cost of materials, and various other economic reasons have disturbed cost so that during the year 1918 on the Atlantic coast the cost of wharf work has increased, two, three and in cases four times. In consideration of this condition the tabulation to be given hereinafter as to annual charges and capital cost has value only as a comparative measure of estimate for different types, wherein it undoubtedly still holds.

Annual Charges or Capitalized Cost. The first cost of construction of pier work is not the sole or true criterion of cost, as this should be considered with the probable length of life, cost of repair, and insurance risk.

Undoubtedly the proper method to make comparison is to consider the annual charge representing the interest on first cost, cost of sinking fund to replace, cost of repair and insurance, and, if desired, capitalize this.

While the insurance rate would apply to the first cost of the pier itself, the character of construction will also affect the rate on cargo handled and stored on the pier, and the type of construction would be of importance to the pier user, emphasizing the need of adopting a fire-resisting type of construction for piers on which valuable cargo is to be handled or stored. In planning wharf work, the considerations affecting first cost may often outweigh the apparent advantages of a lower annual charge. Irrespective of the financial problems, it is difficult or impossible to fully foresee future requirements, and it is frequently the experience that port improvements become obsolete and require extensive changes or must be removed, in which case the advantage is apparent of having selected a design of low cost and one readily torn down.

In preparing the table, p.1749, the following governing conditions have been taken: Live load, 600 lb per sq ft. Pier platforms 10 ft above mean low tide level, 5 ft range of tide, 30 ft depth of water, length of piles required, 70 to 75 ft to top; 65 to 70 ft, low water platforms. Yellow pine piles 15 tons bearing, creosoted, 12 tons bearing, concrete piles, 25 tons bearing and dead weight of pile. Uniform unit prices applied to quantities in each case and are prices prevailing in the Port of New York.

Types 1, 2, 3, 4, 5, and 9 .... 10 ft bays.
Types 7, 8, 10, 11, and 12 .... 5 ft bays.
Types 6 .... 15 ft bays.
Types 1 to 9, piers 70 ft wide, 700 ft long.
Types 10 to 12, piers 200 ft wide, 1000 ft long.
Types 1, 3, 5, 6, 7, 8, 10, and 11, for harbors in which marine wood-borers are not active.
Types 2, 4, 9, and 12, in harbors where marine wood-borers are active.

Data on Costs and Annual Charges for Piers

Type of piers	Cost per sq ft Dollars	Cost per sq ft with asphalt top Dollars	Cost per sq ft with asphalt and concrete base Dollars	Average length of life Years	Annual charge per cent Int. 5% Sinking Fund 5% Repair, Insurance	Annual charge Cents per sq ft
Piers 70′ wide, 700′ long						
(1) Timber piles and top......	1.633	.....	.....	20	12.02	12.42
(2) Timber top, creosoted piles	1.42	.....	.....	20	12.02	17.10
(3) Reinforced concrete deck and timber piles........	1.177	1.317	.....	30	8.51	11.21
(4) Reinforced concrete deck and creosoted piles......	1.596	1.737	.....	20	10.02	17.39
(5) Timber piles and platform concrete column and deck	1.421	1.561	.....	60	6.53	10.19
(6) Timber piles and platform, concrete cross wall and deck............	1.545	1.685	.....	60	6.53	11.00
(7) Timber piles and platform, concrete ret. wall filled..	2.109	.....	2.35	60	6.1	14.34
(8) Timber piles and platform, concrete ret. wall, granite faced..............	2.452	.....	2.692	60	6.1	16.42
(9) Reinforced concrete piles and deck...............	1.812	1.952	.....	60	6.53	12.75
Piers 200′ wide, 1000′ long						
(10) Timber piles and platform, conc. ret. wall filled......	1.653	.....	1.893	60	6.1	11.55
(11) Timber walls and platform, conc. ret. wall, granite faced................	1.793	.....	2.043	60	6.1	12.40
(12) Solid filled surrounded by timber platform, conc. wall, reinf. conc. sheet piles..................	1.643	.....	1.883	60	6.1	11.49

Types 10, 11, and 12, show the decrease in cost per square foot, of these three types of piers (the first two being similar to Types 7 and 8, 70 ft piers), when piers are made wider.

The tabulation is subject to material change in the order of costs and annual charges, dependent upon location, cost of material and labor, purpose for which pier is intended, and various other factors.

21. Timber and Timber Composite Piers

The Temporary Character of timber piers which, apparently, serves as a reason against their extensive use, in many instances loses much of its force in view of their low first cost, simplicity in design, and short length of time in which they can be built. Furthermore, in localities or harbors where a comprehensive and well-planned port improvement project has not been developed, the wharf work is often laid out and constructed to serve the immediate purpose, to be later torn down and rebuilt on new lines. The growth in

importance of many American ports has been so rapid, and unforeseen, that no attempt at port planning in the past could have adequately provided for the future. The unprecedented and unforeseen growth in the size of vessels has also made necessary far-reaching and sweeping changes in port facilities. For these reasons timber piers of temporary character were as serviceable as more permanent piers, it being easier and less expensive to remove them to make way for other more extensive work. The timber pier has played a not unimportant part in the rapid growth and expansion of American harbors. The engineer in planning pier work when considering cost, is hardly justified in employing a permanent structure of a probable life of 50 or 60 years, when it is apparent that the wharf work will long before that time require extensions and improvements amounting to rebuilding.

While timber wharf construction in American ports has played the part indicated in the development of such ports, and has made it possible to materially re-arrange and re-build without excessive cost, this very condition has brought about a more haphazard construction with very little serious and comprehensive planning for port problems and construction of port facilities. It is to be regretted that pier or transit sheds are frequently used in American practice for long storage periods and cargo is not expeditiously brought to and taken away from vessels. Vessels frequently lie at and monopolize wharfage space when they might as well be at anchor in the stream or be laid up at dolphins and thus make available wharves for other vessels awaiting for loading or unloading cargo. As an illustration the area devoted to wharves at the port of New York is several times that devoted to similar purposes at ports such as Hamburg, Liverpool and London, all of which ports have in recent years handled about the same quantity and value of incoming and outgoing cargo.

Composite Piers, in localities where the under-water timber work will not be attacked by marine wood-borers, are practically permanent structures and, as a rule, are economical in first cost.

Crib Work may be employed for pier construction on a yielding or rock foundation. The crib consisting of round or square timbers is built in courses with pockets filled with riprap or rubble, in soft ground the crib being sunk to place on a prepared foundation of piles cut off to grade, on a level bed of broken stone or gravel, or directly on the bottom. On rock the crib is built up to rest and fit on the ledge, the contours of which have been previously determined by soundings.

Essential Details of various precedents are given in the following statements, where the numbers refer to Fig. 43.

(1) Philadelphia, Pa. 1912. Pier 5. 657 ft long; bents 10 ft centers. Live load 500 lb, inner 400 ft, 300 lb outer. Supports 40 ton crane track and standard gage track. R. con. top on timber piles. Cost $2.78 per sq ft inner end; $2.34 per sq ft outer end.

(2) New York, N. Y. Navy Yard, Pier 3. 1905. 520 ft long. Timber piles below water; 35 ft depth of water. Cost $2.80 per sq ft.

(3) New York, N. Y. Dept. Docks and Ferries, West 77th St. Timber piles and framing to mean tide elevation. Reinforced concrete walls, 12 in thick; 10 in reinforced concrete deck.

(4) Norfolk, Va. Proposed Old Dominion Terminal. Untreated timber piles. Surrounding wall with 16 in 45 ft long reinforced concrete sheet-piling. Wall bents, 4 ft centers; pier bents, 8 ft centers. This type of construction to bulkhead line; open creosoted timber pile pier to pier head line. Estimated cost $2.10 per sq ft.

(5) Norfolk, Va. Proposed alternate plan, Old Dominion Terminal. Cost $2.05 per sq ft.

(6) New York, N. Y. Navy Yard, Pier 2. 1901. Timber low water platform, filled

and banked with riprap, supports coaling plant. Length of pier 500 ft. Cost $2.85 per sq ft.

(7) New York, N. Y. Navy Yard, Pier D. 1912. Timber pile platform to low water; 34 ft of water. Piles 55 to 60 ft long. Granite ashlar faced surrounding wall. Paved creos. W. B. pavement. 2 standard gage and one 50 ton crane track entire length of pier. 440 ft long. Live load, 600 lb per sq ft. Cost, $3.25 per sq ft.

(8) Hoboken, N. J. D., L. & W. R.R. Terminal, Pier 1. 1909. 958 ft long. Bents 10 ft centers; 12 in caps; 6 in deck. Cost, $2.76 per sq ft.

(9) and (10) New York, N. Y. Navy Yard. Alternate designs, (10) adopted. Two

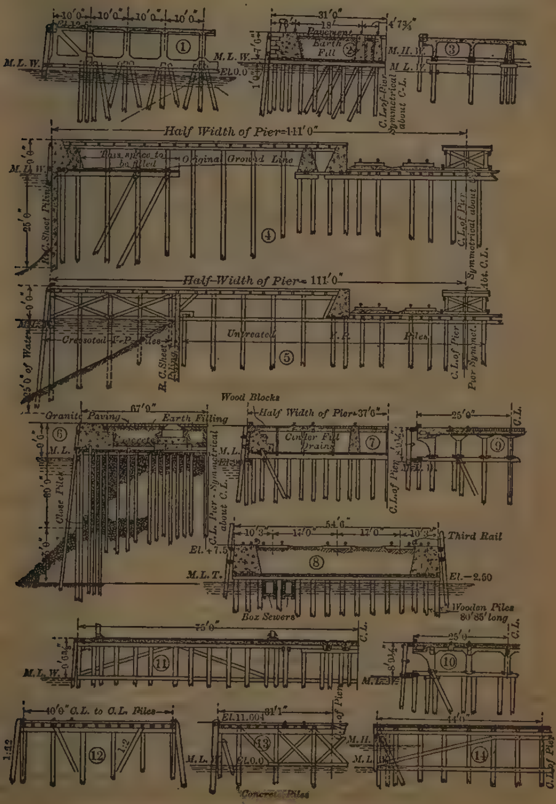


Fig. 43. Timber and Timber Composite Piers

piers, each 500 ft long. 1913. Timber piles and grillage below mean tide level. Piles 85 ft long, depth of water 35 ft. Reinforced concrete columns and deck system, columns premolded and set. Piers carry 2 lines of standard gage track, subways, and piping. Paved in center, creos. W. B. pavement. Cost, $2.08 per sq ft.—(9) Mushroom system, and (10) Beam system.

(11) New York, N. Y. Dept. Docks and Ferries. Typical construction at present in use. Untreated timber structure with 10½ in reinforced concrete deck with 2¼ in asphalt pavement.

(12) Puget Sound, Wash. Navy Yard. Timber approach wharf. Cost, $0.92½ per sq ft.

(13) Charleston, S. C. Navy Yard, Pier 4. 1905. 650 ft long. Reinforced concrete piles, 16 in square. Timber braces, clamps and deck system, 8 x 16 top clamp; 6 x 10 low water clamps; 3 in deck; bents 10 ft centers. Live load, 500 lb per sq ft. Creosoted timber pile fenders. Cost $2.10 per sq ft. Similar approach wharf 45 x 920. 18-inch reinforced concrete piles 40 to 50 ft long. 10-inch reinforced concrete deck. Cost, $2.00 per sq ft. Similar construction employed in Railroad terminal pier, Brunswick, Ga.

(14) Philadelphia, Pa. Clyde Line. 1900. Pier 556 ft long; bents 10 ft. Timber piling, bracing, deck system.

22. Masonry and Reinforced Concrete Piers

Filled-in Piers consist of walls similar to Quay walls of masonry, or of the platform type, the interior being filled with earth; when properly built they are permanent and unyielding and require little or no repair work, other than that due to the settlement of the filling usually occurring only during the first few years of use. Such settlement prevents the pier from being paved or kept level during the period of settlement. They can only be employed inshore of the "Bulkhead Line." On account of the cost of the surrounding wall work, narrow filled piers are not economical, except possibly at times when pier work is located on flats, in shallow water, or on high land, and the approach channels, slips or berths alongside require excavation or dredging, so that this dredged material can be used for filling.

Reinforced Concrete Piles and Cylinders are employed in pier construction where marine wood-borers are active, for economy in locations where the cost of timber piles is too high, or where the character of the pier foundation is such as to require the use of cylinders.

Essential Details of various precedents are given in the following statement. The numbers refer to Fig. 44.

(1) Balboa, Canal Zone. 1911. Lumber pier; live load, 400 lb per sq ft; length 706.6 ft; bents 30 ft centers. Hollow concrete cylinders built in 6 ft sections of 1 : 2 : 4 concrete, provided with steel cutting edge shoe; sunk to rock by jetting and interior excavation, filled with 1 : 3 : 5 concrete.

(2) Olongapo, P. I. U. S. Navy Yard, Pier B, 332 ft long; bents 12 ft centers; reinforced concrete cylinders, 6 ft; reinforced concrete deck. Cost $2.60 sq ft.

(3) Boston, Mass. Pier 6. 1912. 1200 ft long; 300 ft wide. Surrounding wall of courses of split granite 2 to 3 ft high, laid by diver. Cost $1.90 sq ft.

(4) Puget Sound, Wash. U. S. Navy Yard, Pier 8. 397 ft long; bents 16 ft centers. Live load 400 lb sq ft. Cylinders 3 ft diameter at top, 6 ft at bottom, shell 4 in thick, 23 to 52 ft long; sunk to hardpan by interior excavation. 2 standard gage tracks full length of pier. Cost, $3.25 sq ft. In the same location a similar designed pier No. 4, 490 ft long; cylinders 42 in diameter, 11 ft diameter at bottom, 43 ft long. Cost, $3.53 sq ft. This was constructed by placing wooden shells, sinking these by interior excavation, sealing bottom with tremie concrete and filling with concrete.

(5) Havana, Cuba. Pier No. 1, 660 ft long by 162 ft wide, bents 20 ft centers. A similar pier No. 2, 680 ft long, was built, and a quay 800 ft x 80 ft. Reinforced concrete piles 16 in, 18 in, and 20 in square. Water 20 to 40 ft deep. Soft bottom. 12 in rein-

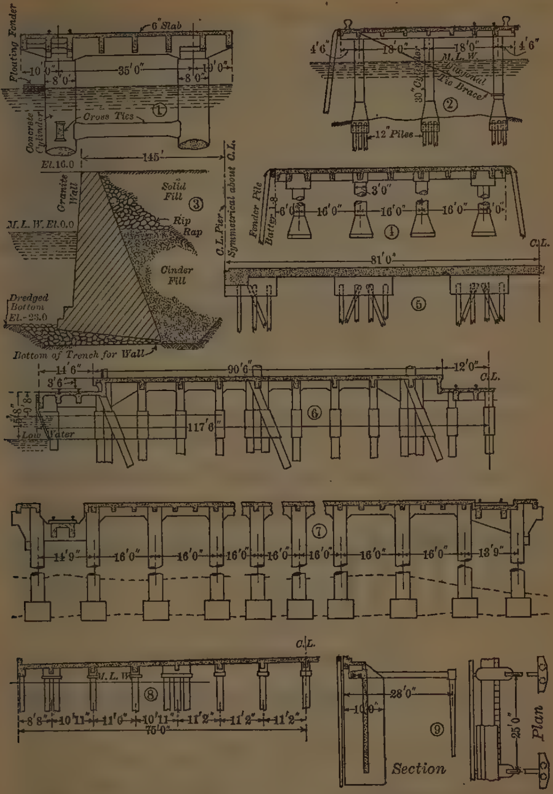


Fig. 44. Masonry and Reinforced Concrete Piers

forced concrete deck, deck live load 250 lb per sq ft and shed load. Cost, $3.35 per sq ft.

(6) Halifax, N. S. Pier No. 2. 1913. 686 ft long. 24 in sq reinforced concrete piles, 45 to 77 ft long; reinforced concrete brace piles cambered 2 to 5½ in. Special cement employed with minimum of aluminum to resist salt-water action. Bents 18 ft centers; rock foundation 68 to 52 ft below surface; area under pier filled by dredging to increase lateral stability. Piles loaded 80 to 90 tons each, test load 120. 1 : 2 : 4 concrete. Piles protected against ice between tides by timber sheathing. Live load 1000 lb sq ft.

(7) San Francisco, Cal. Pier 28, 677 ft long; bents 15 ft centers. Creosote W. B.

pavement, 31 ft wide down center, remainder of pier asphalt. Car springs fender employed. Type of construction general in this harbor, which is best given by a brief abstract of specification: "The excavation for the cylinders shall be made inside of steel caissons which shall be driven into the hard bottom and sealed. The steel caisson shall be of such strength and rigidity as to resist external pressure, and of such size that they may be withdrawn without injuring cylinder bases. The forms shall be wood stave forms of the diameter indicated. After the steel caissons have been set in place and the excavation made, as hereinbefore specified, the wood stave forms and reinforcement shall be set in place in the exact location shown on the plans. The concrete shall then be deposited thru a tube or bottom dump bucket. Under no circumstances will the placing of any concrete be allowed in or under water." Cylinders are sunk 12 to 14 ft below dredge line, and where bottom is soft, timber piles are driven to support the concrete. Cost of this pier was $3.20 per sq ft. Cost of Pier 26, $3.16 per sq ft. Piers 30 and 32, $2.52 per sq. ft. Pier 39, $3.57 per sq ft.

(8) Baltimore, Md. Broadway pier. Reinforced concrete piles, 16 in sq; 8 in reinforced concrete deck. Bents 10 ft centers.

(9) Baltimore, Md. Piers 4, 5 and 6, filled piers. Surrounding wall consists of flattened steel cylinders 25 ft on centers, sunk, excavated and filled with concrete. Reinforced concrete sheet piles between, anchored by reinforced concrete anchors and piles. Cylinders 27 ft in length, built up in sections of 4 ft. Cylinders driven, excavation made by jet and pumping. Cost of surrounding wall $97 00 per lin ft. Cost of piers, $1.57 per sq ft.

Two examples of piers of widely different types constructed during the period of the war in 1918 and 1919 are herewith shown.

Fig. 44*a* shows Pier 4 at Norfolk, Va, 1919, 1000 ft long. Interior untreated pine piling, 4-ft centers live load 900 to 1200 lb per sq ft. 350 long ton fixed tower revolving

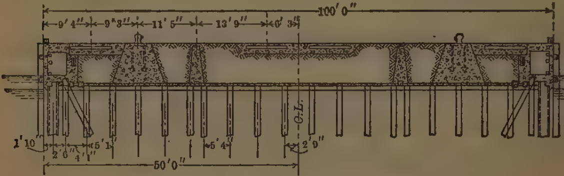


Fig. 44*a*. Army Base Piers, Norfolk, Va.

hammer head crane, see Fig. 91*f*, inclosed space by 60 ft concrete reinforced sheet piles 40 ft depth of water at mean low water, range of tide, 3 ft. Cost $7.47 per sq ft.

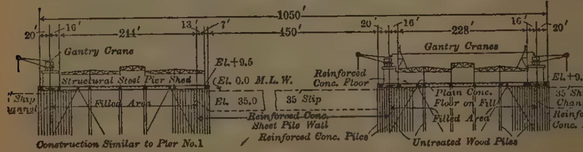


Fig. 44*b*. Shipbuilding. Filling Out. Navy Yard, Norfolk, Va.

Fig. 44*b* shows two piers for general loading and unloading of cargo of miscellaneous character built 1919, 1328 ft long. Filled center surrounded with reinforced concrete sheet-piles, interior untreated piles, exterior concrete piles.

23. Pier or Transit Sheds and Warehouses

Shelter from the Weather is provided on piers used for the loading and unloading of cargoes, by the construction on the piers of sheds, one or two stories in height, the floor area covered by the sheds being devoted to the handling and distribution of incoming and outgoing cargo and used for the storage of this cargo for a short time. Warehouses are usually provided on shore for goods to be stored for periods of more or less lengthy duration. When such warehouses are for the purpose of storing dutiable articles, remaining under custom seal until called for, and the custom imposed paid, they are known as Bonded Warehouses. Pier sheds are constructed of timber framing and siding with roofing covering of appropriate roofing material; timber framing with corrugated sheet metal siding and roofing; structural steel framing with corrugated sheet metal siding and roofing; of reinforced concrete and other combinations of materials. On open pier construction, lightness of construction within safe limits is essential, and the tendency is to make such structures fire retarding or resisting, especially where quantities of valuable cargo are handled. The width of the shed is, in most cases, made nearly the same as the width of the pier, a narrow space being provided on either side for string pieces, location of mooring posts and bitts, and for handling lines. The outshore end of piers is usually left open so as to permit of ready handling of the bow and stern lines of incoming and outgoing ships.

Pier or Transit Sheds are of one or two story, dependent upon local conditions and practice and the availability of water front and value of the same. On account of the conditions, pier or transit sheds should be capable of affording floor space for the storage of the cargo of incoming vessel or the accumulation of cargo ready to be placed in an outgoing vessel. Two-story sheds permit of a narrow pier width, or the second story is utilized for passenger traffic. That the two-story pier sheds apparently are preferred in the American practice is witnessed by the newer piers in New York, Philadelphia, and Boston. One-story sheds are preferred in European practice.

Railroad Tracks, when carried out on piers, are as a rule run either directly thru the center of the pier, or at the sides of the pier. Tracks are laid in a depression, so that the level of the car platform will be at or about the level of the pier deck, or are laid on the deck level. When railroad tracks are run on the side, they are not usually covered by the main pier shed. When run in the center, the pier shed is designed so as to afford proper head room and ventilation for the locomotive gases.

Details of Construction of Sheds are generally the same as those followed in building construction it being often customary to give the sides of the shed a slight inclination inwards or "tumble-home." The second floor in two-story sheds is sometimes given a slight crown and the lower chords of roof trusses a camber. Openings should be provided at frequent intervals in the sides used for berthing, or should be made continuous so as to afford ready access to the various ship hatchways and side openings. Doors may be either swung from the top and counterweights employed to facilitate ready and rapid opening and closing, or slide horizontally on overhead trackways. (Various folding cargo doors and metal rolling shutters are also used.) When continuous openings are provided, several lines of overhead tracks are placed, so that the door leaves may be rolled by one another. When the siding of the shed is of light sheet metal, on either structural steel or timber framing, it is advisable to protect this on the inside to a height of about 6 ft by cargo battens placed at a slight distance from the inside of the siding.

Lighting and Ventilation. In wide sheds the lighting arrangements are of considerable importance; natural light is provided by a continuous or series of skylights, or by monitors constructed in the roof, or by both methods. The employment of monitors also affords opportunity for ventilation, but is likely to permit leakage in heavy driving rain storms. Circular sheet-metal ventilators of various types may be employed on the roof ridge spaced 50 to 100 ft apart. As occasions frequently arise where the loading and unloading of vessels and handling of cargo must be done at night, proper artificial lighting is important, for which the practise is similar to that obtaining in general building lighting.

Miscellaneous Appliances. There have been many disastrous pier fires, so that adequate fire protection and service systems are important adjuncts to properly constructed pier sheds. Piers, if of timber or timber composite, being highly inflammable, the fire insurance rate on the structure itself and on cargo stored is an item that can properly be fixt in the annual charge and capitalized cost. Important commercial structures should have installed modern fire alarm and indicator systems, automatic sprinkler systems, preferably of the dry pipe type, and fire service mains, laterals, risers, connections and fire hose on reels or racks readily accessible. If local fresh or salt water pressure lines are not available, elevated storage tanks, with pumps and overboard suction connections, must be included in the equipment of the pier shed. If water mains are carried out on piers, precautions should be taken to prevent freezing by means of proper insulation, or by draining the lines when not in use.

Essential Details of various precedents are given in the following statements, where the numbers refer to Fig. 45.

(1) Havana, Cuba. Pier No. 1. 1913. Reinforced concrete pier shed. Live load, second floor, 400 lb per sq ft. Cost, $3.00 per sq ft. One of two piers and an 80 ft wide quay structure composing waterfront terminal, all similar in construction with ornamental street frontage. Freight entering is divided into three classes; first class consisting of soap and food-stuffs; second class, light hardware and machinery—both handled on first floor; third class, crates, bales, and merchandise, handled on second floor. Elevators and escalators provided from first to second floor and above. In another structure, third floor provided with refrigerating plant of 30 000 cu ft capacity.

(2) New York, N. Y. 33d St., South Brooklyn, Dept. Docks and Ferries. Bays, 20 ft.

(3) Halifax, N S. 677 ft long. Reinforced concrete structure, with expansion joints transversely every 90 ft. Live load, second floor, 500 lb per sq ft.

(4) San Pedro, Cal. Port of Los Angeles, Pier A, Shed No. 1. Timber construction.

(5) San Francisco, Cal. Pier 28. Bays 30 ft. Structural steel frame with reinforced concrete roof 2¾ in thick, covered with 5-ply roofing. Siding of reinforced concrete. Wire glass employed. Cost, $1.12 per sq ft. Pier 26, cost, $0.97 per sq ft. Piers 30 and 32 include steel girder runways and rails for inside cranes and outside ship towers. Cost, $0.87 per sq ft. Steel rolling doors employed in these sheds, cost per sq ft of shed being: Pier 26, $0.27; Piers 30–32, $0.22 each per sq ft. Piers 26, 30, 32, timber sheds with steel columns.

(6) San Pedro, Cal. Port of Los Angeles, Municipal Pier No. 1. Bays 20 ft. Equipped with side uprights for cargo handling.

(7) New York, N. Y. Typical pier, Chelsea section, Dept. Docks and Ferries. Swinging lift doors, provided with counter-weights. Uprights at sides of shed equipped for cargo handling.

(8) Jersey City, N. J. Pier No. 9, D., L. & W. R. R. Steel frame. Reinforced concrete roof, siding and floor. Cost of shed, $2.00 per sq ft, including column footings. Cost of quay and platform, $2.25 per sq ft.

(9) New York, N. Y. Dept. Docks and Ferries, Recreation Pier, E. 24th Street. Bays 20 ft. Typical example of recreation piers constructed in this harbor.

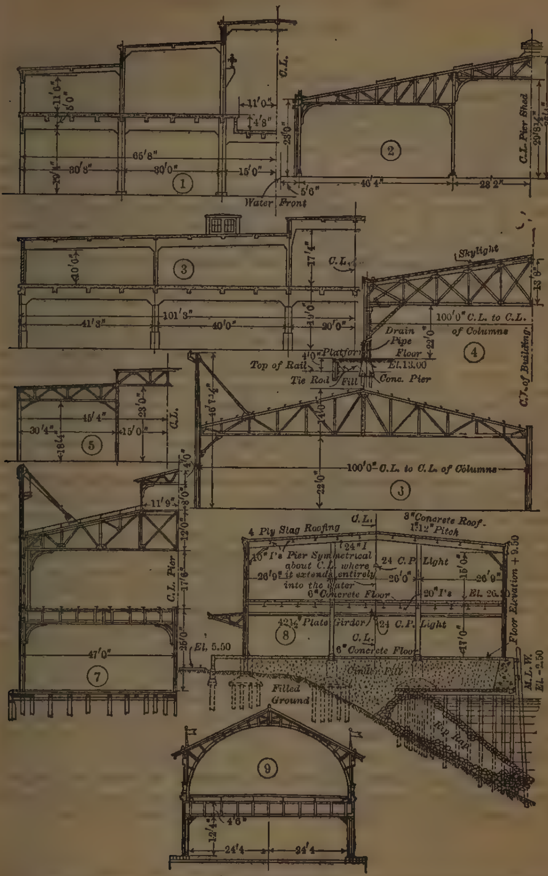


Fig. 45. Pier Sheds

WET AND DRY DOCKS

24. Wet Docks

Definition. Docks may be divided into two general classes—harbor docks and repair docks. The former, known as Wet Docks, are enclosed, or partially enclosed basins, usually provided with locks and entrance gates, similar to canal locks. They are sheltered basins for the receiving of shipping, so arranged that the water in the dock may be kept at more or less constant level to facilitate the loading and unloading of cargo.

Use of Wet Docks. Their usefulness is apparent in harbors where there exists a considerable range of tide, and especially wherein under low tide conditions the approach to the harbor itself is not navigable for deep-draft vessels. They also have a special field of usefulness in harbors or rivers where considerable silting up takes place, the dock area being kept free of such silting by keeping out turbid river or harbor water and often by supplying water to the dock basin from other sources. This type of structure, in its restricted sense, is rarely found in American practise, altho it is employed in some South American ports.

The reasons given for the use of wet docks are undoubtedly not the only ones that have occasioned their appearance in European practise, as otherwise they would undoubtedly have before this been employed in American ports where a considerable range of tide is involved. They have undoubtedly been resorted to in order to retain the maritime commerce of river ports, where the increase in draft of vessels has become greater than the normal navigable depth of the river. The wet dock has been developed as the instrument by means of which Old World ports have retained maritime supremacy.

Design of Wet Docks. It is apparent that in wet docks, where entirely enclosed and provided with entrance gates, as the surface of the water is to be kept at approximately a constant elevation, the sides and bottom of such a basin must be practically impervious, or arrangements must be made to replenish the water lost by leakage. This is especially so in all seas having a considerable variation in tide, and a consequent appreciable difference at times in the height of water in and outside of the wet dock. Where the basin is located in a practically impervious soil, such as clay or clay soils, and many kinds of rock, these conditions are generally provided for. For this reason, a thoro exploration of the subsoils in locations where wet docks are projected is of the greatest importance. In locations where the subsoil is readily water bearing, the bottom must be blanketed with clay puddle or a masonry lining placed. The dock walls can be treated in a manner similar to that described under Quay Walls, in Art. 12; the walls, however, of dock basins being at times subjected to external hydrostatic pressure in excess of that to which they would be subjected if located on open basins. This is, usually, not a serious matter, as the combined earth and contained water pressure, together with the passive resistance of the earth backfill, will be found sufficient to provide for this condition.

Locks and Lock Gates. The condition governing the design and construction of these is, in all respects, similar to that found in canal work. Information on this subject will be found under Canals, in Arts. 17 and 19, of Sect. 11.

25. Marine Railways

Definition and Description. Repair docks may be marine railways, lift docks, graving docks, or floating docks. Marine railways are inclined slips or ways, extending out some distance into and under the water, and running up on the foreshore a sufficient distance so that the vessel when hauled out will be entirely clear of the water. A platform or cradle moves on these ways, usually

on nests of rollers, and is hauled out of the water by means of chain or wire rope cable. Racking and pawls are usually provided, so that if the hauling device should break, the cradle and vessel will not slip back into the water. It is apparent that even without considering the feature of range of tide, the application of this type of repair dock is limited to vessels of comparatively small size, as in long vessels of deep draft the ways would have to be carried out a great distance under water, approximately twice the length of the vessel to be docked, making the entire length of the ways over 3 or 4 times the length of the vessel to be docked. This condition is sometimes materially improved by use of a collapsible cradle. While there is no reason why sufficient capacity of hauling device can not be designed, nevertheless important considerations would tend to limit this. This type of repair dock is, undoubtedly, valuable and economical for the docking of smaller vessels. Its application, however, appears to be limited. No marine railways have been built in excess of 5000 tons lifting capacity. Fig. 46 shows a marine railway of 2000 tons capacity. Estimated cost, $100-000.00.

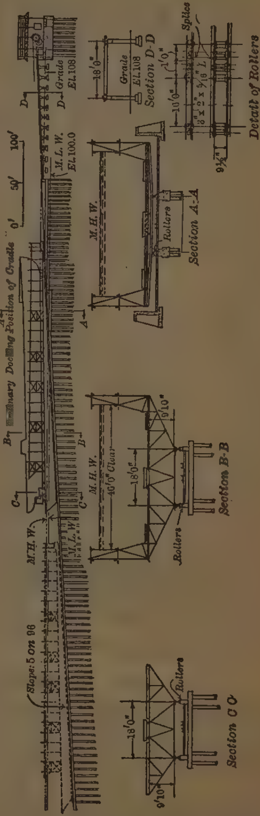


Fig. 46. Marine Railway

Marine Railways were designed in 1918–1919 of 3200 tons capacity; the estimated cost has been $300 000 or nearly twice the cost per ton capacity as that given hereinbefore.

The Foundation should, if possible, be incompressible. In many larger and more important marine railways the ways are built on piling. The intensity of pressure, while not great, is, nevertheless, in the form of a moving load, becoming greater as the vessel hauled out loses its buoyancy. The ways are, in many instances, similar to the ways employed in shipbuilding work, which, however, do not go out under water to the same extent, and besides this, the weight of a vessel when launched is usually considerably less than would be the docking weight of the same vessel.

Ways. The ways are laid at a gradient of between 1 on 13 and 1 on 25, 1 on 20 being the usual slope for large marine railways. The ways consist of

from 2 to 4 rails of iron or steel securely fastened to longitudinal bearers which, in turn, rest on cross-ties bearing on the foundation. The racking to receive the cradle safety pawls is usually placed in the center of the ways between the two central rails, if 4 rails are used. The under-water work is either performed in the dry by the construction of a coffer-dam around the site, or by divers. If no foundation piles are used, the grading must be carefully done, working from over-water battens. If piles are used the underwater portion of the ways is built in sections, floated to place, and lowered onto the foundation by weighting and fastened securely in place.

The Cradle or Platform is usually constructed of timber or structural steel, and is provided with keel and bilge blocks. The cradle is constructed on longitudinal sleepers, placed directly over the longitudinal ways, and provided on the underside with an iron track. The cradle is moved on nests of rollers placed in between the upper and lower lines of track. The rollers are usually of cast iron, double flanged, and held in place by flat bars or links; sometimes the rollers are fixt on the cradle and run on tracks on the longitudinal way-sleepers. In large marine railways, the cradle is built with side frame work, which is used for working platforms. Frequently, cradles, instead of having the same inclination as the ways, are built with the platform horizontal, or nearly so, so that the ship being docked will immediately come to a bearing and not be lifted first by the bow.

Broadside Marine Railways. With a view to overcoming the difficulty as to length of marine railway, they are sometimes built so as to withdraw vessels from the water broadside on.

Power Absorbed in Friction. The power required to raise a vessel is dependent upon the inclination of the ways and the force required to overcome friction. Experiments made on the power absorbed in friction indicates a variation from 3.3 percent to 7½ percent of the weight lifted. On larger and well-designed marine railways, the percentage is in the neighborhood of 4 percent, to which should be added 5 percent for initial friction in starting from rest. Thus in a marine railway having an inclination of 1 to 20, the hauling machinery would have to exert a pull of 14 percent of the weight of the vessel lifted.

Lift Docks are platforms lowered into and raised from the water. This type of structure is of limited capacity and is not now used to any great extent. It is very often employed in connection with pontoons or floating dry docks, hydraulic power being generally employed, applied thru a series of hydraulically operated cylinders placed on both sides of the platform and connected together transversely by girders. A small dock of this type has recently been built in the Island of Curaçao.

26. Graving or Basin Dry Docks

Definition. Basin dry docks, often called graving docks, are basins generally made by excavation in the foreshore of the harbor, having entrance-ways closed with gates or portable caissons and are usually of such dimensions as to be capable of receiving, with sufficient clearance, the maximum sized ship to be docked therein. After the ship is floated into the dry dock, the opening is closed by means of a gate or caisson and the water in the interior is removed by pumping, the ships settling on blocks provided for that purpose. In some graving docks advantage is taken of the fall of tide to reduce the amount of pumping necessary. In nearly all cases the graving dock must be pumped out twice for the docking of the vessel. The dock is usually left unwatered so that when word is received that a certain vessel is to use the dock, the blocking may be arranged to conform with the profile of the keel and the character of the vessel. The

dock is then flooded and the vessel floated in and centered and held in the proper position, the opening closed and the water removed. On completion of the work on the ship, the dock is flooded and the ship removed, the opening again closed, and the water removed from the dock.

Foundations. In large graving docks, water at considerable head must be excluded, and for that reason the character of the subsoil and foundations are of considerable importance in locating structures of this type. A graving dock can, altho at times at a prohibitive cost, be constructed in almost any location, but, where possible, it should be located at a site where sound bed rock will be encountered at some elevation not below the subgrade of the structure or in locations where marl, stiff clay, or mixtures of marl or stiff clay, sand and gravel are encountered. Suitable foundation conditions, other than rock, sometimes result in less expensive and more expeditious construction Fig. 47 shows a half transverse section at the entrance sill of a graving dock on pile foundations.

Graving docks have been constructed in unstable water-bearing soils, but at high cost. With a view to securing a suitable site, or to enable proper precautions to be taken in design and in method of constructions, if selection of a site

Half Section through inner sill, Looking Inboard

Fig. 47. Section of Graving Dock with Pile Foundation

is restricted, an important preliminary step is the taking of exploration borings, operations for which are described in Sect. 11, Arts. 29 and 30. It is advisable, where possible to do so, to secure core borings showing the condition of the soil " in situ " and, in special cases, to supplement these with test pits. See Sect. 11, Art. 31. In water-bearing soils such test pits may have to be made by employing the pneumatic process.

Masonry Graving Docks. In the design and construction of such docks, provision must be made for counterbalancing the hydrostatic pressure by sufficient weight of masonry or other anchorage effect. In porous water-bearing soils, the hydrostatic pressure to be considered will be at least the full hydrostatic pressure due to maximum tide elevation, and under certain conditions, on account of the presence of ground, surface or artesian waters, a pressure in excess of this. Where the dock is located on a solid rock ledge without water-

bearing fissures, this condition will not obtain, altho ideal foundation conditions of this character are seldom, or never, encountered in practise, and it is, therefore, customary on this character of foundation to provide for relieving the water pressure by under-draining the floor of the structure. In some soils, such as tenacious marls or clays, or hardpan, the same precautions may be taken, the walls of the graving dock being designed as retaining or quay walls. In graving docks built in water-bearing soils, full hydrostatic pressure should be expected and provided for. A number of graving docks have been constructed in which the weight of the masonry mass is somewhat less than the uplifting force due to full hydrostatic pressure, their integrity depending upon the friction of the backfill on the back of the side-walls, and, in some cases, to anchorage or pull on foundation piles. In stable waterbearing soils where the uplifting forces are clearly defined, dependence can be placed on the anchorage effect of the foundation piles, where such exist. This anchorage effect, however, should not exceed the submerged weight of the mass of subsoil engaged by piling, as stated in this section, Art. 12. The friction of the backfill on the rear of side-walls being indeterminate at best, should not be given a definite value, and should be considered merely as a factor of safety. In a dry dock completed at the New York Navy Yard in 1912, the anchorage effect against hydrostatic uplift was secured by deep piers with flared-out basis, secured into the floor by a system of anchor rods, the anchorage provided in each being in the neighborhood of 1 000 000 lb, these piers being sunk to position by the pneumatic process.

A masonry graving dock is subjected to three following conditions:

(a) When empty: The bottom or floor is subjected to hydrostatic pressure, usually considerably in excess of the weight of the floor itself. This excess is transmitted to the heavy side-walls by actual or virtual inverted arch action, the abutments of the arch being furnished by the weight of the side-walls taken together with the thrust of the earth and water on the rear of the wall and the passive resistance of the earth itself. When this abutment action is insufficient, reinforcing steel should be provided in the floor.

(b) The dry dock empty with a ship of maximum tonnage in the dock: The conditions are the same as (a), except that there is concentrated at the center line on keel blocks, and on the side blocking, the weight of the ship; for which concentration, where the foundation is somewhat yielding, special provision must be made by distribution of this load by reinforcing the masonry with steel, or the foundation directly under the blocking designed accordingly.

The loading per running foot of dry dock is dependent on the length, depth, and width of the structure and the character of ships to be docked. For war vessels of the largest size weights as high as 90 long tons per running foot should be allowed, as the concentration under machinery and gun turret locations are liable to run this high. For commercial steamers the loads will be less. Under ordinary docking conditions $\frac{5}{8}$ of this load will come on the keel blocks and $\frac{3}{16}$ on each side, docking keel or bilge block. In the work of bottom repair, as blocking is removed, the concentrations are shifted, this, however, only occurring locally over small areas.

(c) Dry dock full of water: Under this condition, the foundation, considered in its entirety, is subjected to the greatest load, consisting of the submerged weight of masonry to tide level, plus the weight of masonry above tide level, the conditions being somewhat altered with the presence of surface or artesian ground water in the soil. Under this condition the thrust on the rear of the wall is due to submerged earth pressure to ground or tide water level, tide or ground water pressure and surcharge of earth pressure above tide or ground water level, and surcharge on wall, reduced by water pressure on the interior of the dry dock. It is often customary to use combined liquid pressure to the coping of the structure and neglect surcharge (Art. 12). Under this condition, very often little or no virtual or actual arch action can be counted on, and the foundations under the side-walls should provide for the weight directly above them, or provisions for distribution made by employing reinforced steel. In yielding foundations, if this is not done, settlement at the side-walls will be in excess of the settlement under the center

f the floor, causing cracks at the toe of the walls, and along the center line of the dry ock. It should be noted, as between Conditions (a) and (c), there is likely to be a comlete reversal of stresses in the floor.

To these three conditions a fourth may be added, that is, of the dry dock while under onstruction. This should be given careful consideration and analysis, the omission of vhich has, in some instances, resulted in damage to the uncompleted structure. Many lder masonry graving dry docks have been constructed of rubble or ashlar stone work, iced with ashlar work. Nearly all of the more modern graving docks are constructed f concrete, unlined, or lined with granite ashlar work, or, as in the case of the graving ock built at the New York Navy Yard in 1912, vitrified brick, in either case caisson or ate seats consisting of granite stone work with radial joints for the bottom and lower arts of the side steps, the contact faces of which are fine cut work, or, at times, rubbed. 'he beds and joints of caisson-seat stone work should not exceed ¼ inch, and, in some ases, are limited to ⅛ inch.

Examples of recent graving docks are shown as follows:

Figs. 47a and 47b show a plan and a transverse section of a group of four dry docks t the Norfolk, Va., Navy Yard. Dry Dock No. 3 was built between 1903 and 1911

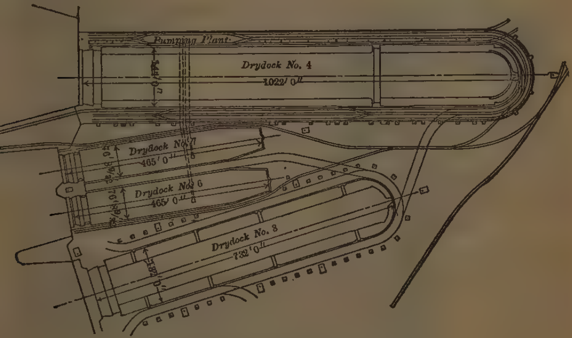


Fig. 47a. Group of Four Dry Docks for Norfolk, Va.

vhile Dry Docks Nos. 4, 6 and 7 are in process of completion; the larger for the Navy nd the two smaller ones for the United States Shipping Board Emergency Fleet Cor-

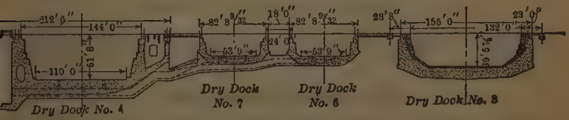


Fig. 47b. Transverse Section of Docks, Norfolk, Va.

poration. Details of all four docks as to dimensions and cost can be found in the tabuation on page 1476, Art. 30. One pumping plant consisting of three 54-in vertical volute umps serves all three of these docks. No. 4 Dry Dock is provided with an intermediate aisson gate seat so that the inner end of the dry dock can be used for smaller vessels on ong time repair work, and the outer portion continue available for docking larger vessels. This arrangement practically provides two docks in one. A similar arrangement is

provided in the Philadelphia Navy Yard Dry Dock under construction and in the Commonwealth Dry Dock at Boston, Mass. These arrangements entail the additional cost of the interior seat, two dry dock caisson gates and independent pumping for the two sections of the dock. The older dry dock of the group, No. 3, as will be noted in Fig. 47*b*, is lined with granite ashlar masonry, while the three new docks are of concrete thruout without other facing or lining. A section of Dock No. 4 shown in Fig. 47*b* is at the pump well. The half-section to the right of the center line is so designed that the dock could be constructed on the natural stable slope of the excavation which was made by drag line in the dry. The floor invert is 20 ft in thickness. The floor of the two smaller docks are 10′ 6″ in thickness. Excavation for these docks is also being made in the dry. Water is kept out of the excavation site by a coffer dam of steel sheet-piling, wooden sheet-piling and earth embankment. Dry Dock No. 4, the large dock, has been completed in about 2 years, while Dry Docks Nos. 6 and 7 will be completed within one year, indicating that this type of dry dock can, under favorable conditions, be constructed as expeditiously as floating docks of similar size.

Fig. 47*c* is a half typical section of a graving dock under construction at the Philadelphia Navy Yard, and is similar in dimensions to that just completed at the Norfolk

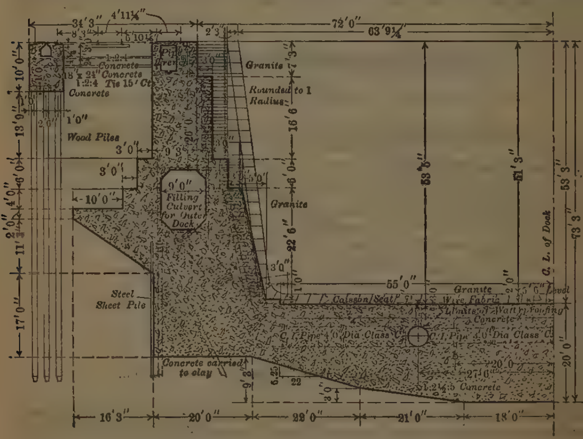


Fig. 47*c*. Half Section Graving Dock, Philadelphia, Pa.

Navy Yard. The steel sheet-piling shown surrounds the entire site at the bottom and was placed to cut off the flow of water in the sand and gravel above the clay sub-soil.

27. Design of Graving Docks

Data on Design. A graving dock should be designed having in mind the three conditions enumerated in the paragraph above, and consideration should also be given to the possible fourth condition. The necessary information having been obtained as to the character of the soil, its probable submerged weight and angle of repose, allowance made for the safe weight of masonry per cubic foot, stress diagrams should be drawn for all conditions. Fig. 48 is such a diagram for Condition (a). In this case the hydrostatic pressure is just counterbalanced by the weight of masonry. The line of pressure does not pass thru the middle-

third at the base of the side-wall or in the center. If on analysis the maximum unit pressure in the virtual arch is not too high, the masonry at the center above the virtual arch may be considered inert, altho it may be desirable to employ reinforcing metal to avoid unsightly longitudinal center line cracks.

Under these conditions the vertical components of the downward forces would be in excess of the vertical components of the upward forces of hydrostatic pressure, this excess being counterbalanced by the soil pressure or passive resistance and would appear as a vertical closure in the polygon of forces (Fig. 48). In analyzing the floor arch thrust in the same way it may be necessary to not only consider the active earth and water forces on the rear of the retaining wall, but the soil pressure or passive resistance that would be afforded to this wall acting as an abutment for the inverted floor arch thrust, from which it will be seen that it is entirely probable in the preparation of the design and in the analysis of the structure and in the making of the stress diagram to insert exterior soil resistance or passive pressure forces.

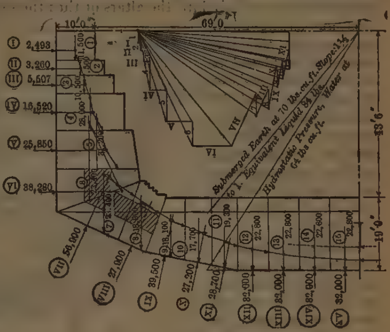


Fig. 48. Stress Diagram

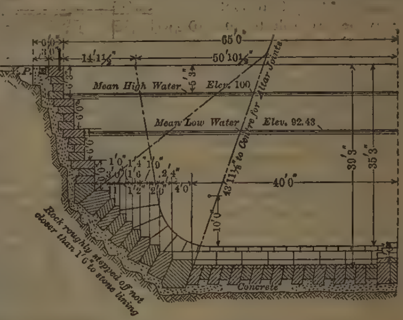
When the weight of the section is in excess of the hydrostatic pressure, the diagram would have to take into consideration the bearing of the structure on the foundation, or soil pressure, which could be considered as equally distributed over the entire area of the foundation. However, in a case such as that shown where the line of pressure was outside of the middle-third at the center, in order to develop longitudinal cracks on the tension side, the floor at the center would have to lift slightly, relieving the soil directly beneath it of pressure and transferring the entire soil pressure to the sides, resulting in raising the line of pressure to within the middle-third as indicated by the dotted line.

Masonry Graving Docks on Rock Foundations. The same considerations govern the design, excepting that provision can probably be made for relieving, in whole or part, hydrostatic pressure; under the most ideal conditions the rock can be excavated out to the rough dimensions of the graving dock, and then lined with masonry, with or without relieving drains. A graving dock built at Hunter's Point, San Francisco, Cal., was built in this way, the excavation being made in impervious serpentine rock, and lined with a comparatively thin layer of concrete. No provision was made for draining. In graving docks founded on ledge rock, the necessary stress diagram should be made for all conditions. Fig. 49 shows a half transverse section of a graving dock on rock foundation.

Altars are provided in the side-walls at convenient heights, the first at elevation of about mean high tide and additional altars at intervals below this. They consist of steps or platforms 2 to 3 ft wide, and are used for the placing of shores to hold upright the vessel being docked, and as a means of access from and to plank stages or working platforms placed alongside the hull.

Stairways and Timber Slides. Stairways are provided for going into and

coming out of the graving dock, two sets on either side of the entrance and at or on either side of the head with intermediate stairways to suit convenience. These are built directly into the altars or thru the side-walls. Timber slides are used for letting down and hauling up material and are placed at convenient intervals in the side-walls on both sides of the structure. In order to simplify design and construction, portable stairways may be provided, leaving the altars clear and continuous.



Half Section thru Body of Dock

Fig. 49. Graving Dock, Rock Foundation

Drainage. The floor is drained by means of either open gutters at the sides or in the center of the floor, or by drainage culverts with drain connections, located at intervals. Where gutters or drains are used the floor must be graded transversely to these, the gutters or drains being given gradients to insure self-cleaning.

Head. The inshore end of the graving dock is known as the Head, as distinguished from the outshore end, or Entrance. Heads are built rectangular, trapezoidal or circular or elliptical, in which case the walls are designed and figured as retaining walls, or are built as simple or compound, circular or elliptical arches.

Timber Graving Docks have, in the past, been extensively employed in American practise, when vessels of smaller size with less draft and of different hull section were constructed for the Merchant and Naval Service. They had the advantage of lower first cost and more expeditious construction and were especially suited for location in stable waterbearing subsoils. The principle of design and construction is different from that obtaining with respect to masonry graving docks and is based upon the partial, or entire, reduction of hydrostatic pressure on the bottom by drainage and the building of the side-walls at a slope approximating the natural slope of the soil, relieving this portion of the structure from the earth thrust. The flow of ground or tide water thru these walls is cut off by a continuous and enclosing wall of sheet-piling driven at the coping or some distance back from it, the timber altars of the side-wall being laid on inclined stringers supported on piles, a sloped wall of puddled clay being laid between the soil and the timber altars. The flow of water onto the floor is cut off or reduced by a continuous enclosure of sheet-piling driven at the foot or toe of the wall or slope and 1 to 3 rows of timber sheet-piling are driven across the entrance, the floor itself consisting of heavy timber planking, fastened to dimensioned caps bolted to the foundation piling, with which the bottom is studded at 3 to 4 ft intervals, additional piles being driven in the center, where necessary, under the keel blocks, and under the probable position of bilge blocks. Three to 5 ft of concrete is placed under the timber floor in between timber caps and piling. In this type of structure a certain amount of leakage or self-draining is expected and allowed for. Among the advantages

claimed are that the sloped sides give better light and ventilation. While this argument, undoubtedly, is of importance as applied to the older types of vessels, it loses its force when steel vessels of rectangular amidship sections, with practically flat bottoms, are considered, as, in both masonry and timber graving docks, artificial light must be employed, under such hulls. The disadvantage of this character of construction is that in order to secure necessary width of graving dock at the height of the blocks, the width at the coping line becomes excessive. Thus, with a vessel 100 ft beam, and 30 ft draft, with allowance for clearance, the width of the timber graving dock at the coping line would have to be nearly 200 ft as against 140 ft for a masonry graving dock, entailing the requirements of a considerably larger area of land, and difficulty in handling materials for building or repair by side-wall cranes. Timber graving docks have a further disadvantage of not being a permanent structure and require continuous and expensive repair and replacement. The surface of the ground around the structure is in a continuous process of settlement due to the percolation of water thru it, and to the washing out by this water of the finer particles of soil. On account of the difficulty of obtaining complete and intact sheet-pile water cut-off walls, frequently extensive washouts and undermining occur in this character of structure.

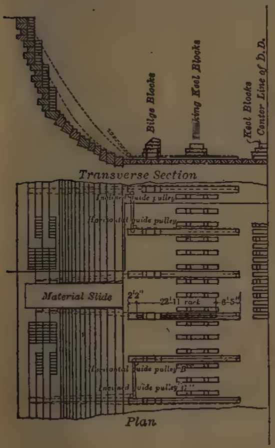


Fig. 50. Arrangement of Blocking

Blocking, Shores, and Fittings. Keel blocks are provided on the center line of the floor and consist of heavy dimensioned hardwood timbers. They should be so laid out as to afford sufficient bearing to avoid undue crushing under the maximum weight likely to be brought upon them. In American practise they usually consist of three or more tiers, placed transversely of the dock on 4 to 2 ft centers longitudinally. Heights of blocks above floor should not be less than 3½ to 4½ ft to give clearance for working under a ship.

In European practise the lower blocks are often of cast iron, sometimes wedge-shape, to permit of ready removal. The lower course is fastened to the floor by anchor bolts, and the upper course, if of timber, by iron dogs. In addition to these blocks, bilge blocks are provided on both sides of the keel blocks, but placed at longer intervals apart. Some of the larger modern ships, especially in the Naval service, are provided with four docking keels, two on either side of the center line, so that in being docked the ship will come to a horizontal bearing on central keel and side docking keel blocks avoiding the expense and delay of holding the graving dock while blocking is being rearranged to fit the hull of the particular ship to be docked. When docking keel blocks are not employed, bilge blocks are used, consisting of several tiers of blocks, the upper one slightly inclined or hinged to the block below and provided with a wedge. These blocks slide on bilge block bearers placed transversely of the dock. They can be drawn in under the ship and against the bilge by means of hauling chains,

cast iron racking being placed on the bearers and adjustable automatic pawls attached to the lower bilge block, holding the block in place when it has been drawn under the ship. These pawls are lifted and the block released when the hauling-back chain is employed. The docking keel and bilge-block bearers are sometimes made of reinforcing concrete, or entirely done away with, the cast-iron racking being fastened directly to the floor of the structure. Fig. 50 shows a half cross-section and floor plan giving position and arrangement of blocking.

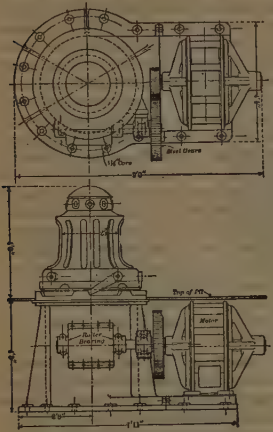


Fig. 51.—Electric Capstan

Shores are used with, or to replace, bilge blocks, to hold a vessel upright when the graving dock is unwatered. They bear against the vessel's side at frames and are wedged against the walls, rest on the altars, or are suspended from the coping by lines, for which purpose small cleats are placed at convenient distances. In order to assist in bringing and holding the vessel to alignment in position over the blocking, brest lines are employed, final adjustment often being made by block and tackle, for which small hand winches are installed at convenient intervals along the top of the dock wall.

Capstans and Bollards. As it is impracticable to place ships in graving docks under their main engine power by tugs, power capstans are very often provided for this purpose. In smaller docks hand capstans are employed, assisted by windlasses on the ship being docked. On large graving docks, power capstans should be provided at the head end of the dock on either side of the entrance and in long structures midway of the length. These should have two speeds, the maximum for slow hauling from 15 000 to 35 000 lb at the capstan barrel, the minimum is used for overhauling lines so as not to permit too much slack and possible snarling. Bollards or mooring posts are

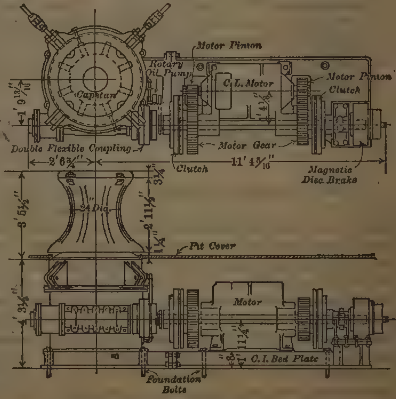


Fig. 51a. Electric Capstan.

placed on both sides at intervals of about 50 ft. Figs. 51 and 51*a* show electric motor driven capstan of the latest type. Fig. 51*a* is designed with barrel height so as to permit the passage over it of the heavy dry dock traveling cranes.

28. Construction of Graving Docks

As graving docks are always located on the foreshore of harbors, the problem of construction has an important influence on design. With the increase in draft of vessels and the necessity for providing for overdraft in case of accident, the clearance depth over entrance sills of graving docks has increased greatly and with it the head of ground or sea water to be taken care of during the construction period.

In certain locations it becomes impossible or impractical, within the range of ready execution, to construct graving docks by any method involving the unwatering of the site. When such cases occur, recourse must be had to a method of construction that will make the unwatering of the site unnecessary. As the method of construction is an important item in graving dock design, it is well worth while illustrating various methods that have from time to time been employed.

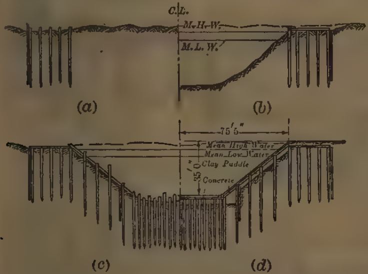


Fig. 52. Timber Graving Dock

The usual course followed in the construction of timber graving docks is illustrated in Fig. 52 where (a) indicates the original conditions of the site. The entire site is surrounded by a wall of sheet-piling, a portion of the round piling being driven previous to excavation, (b) shows the excavation, in part completed, (c) illustrates the driving of the round piling on the side slopes and in the bottom, and of the bottom enclosing sheet-piling, and (d) shows the work completed. This method was generally followed in the construction of two timber dry docks built in the New York Navy Yard, and two similar structures built in South Brooklyn, N. Y.

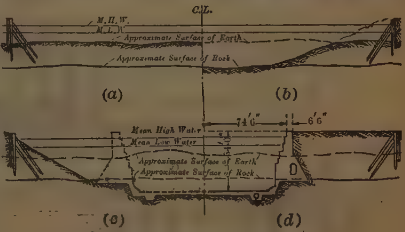


Fig. 53. Graving Dock, Boston, Mass.

Dry dock proposed for the Port of Boston: The contemplated method of construction for this dock is illustrated in Fig. 53. The projected structure is located on ledge rock, covered with a blanket of soil under water. On either side and around the head of the proposed dock double timber bulkheads were placed, filled in with excavated material. The proposed steps of construction (a), (b), (c), (d), are shown, (b) with the space between bulkheads filled and excavation made to ledge rock; (c) excavation in ledge rock; (d) completed structure backfilled. In this design the side-walls are retaining walls, the

floor or bottom is shallow, and is to be under-drained, so that the underside of the floor will not be subjected to hydrostatic pressure.

Naval Dry Dock at Charleston, S. C., was located on a salt marsh, with underlying,

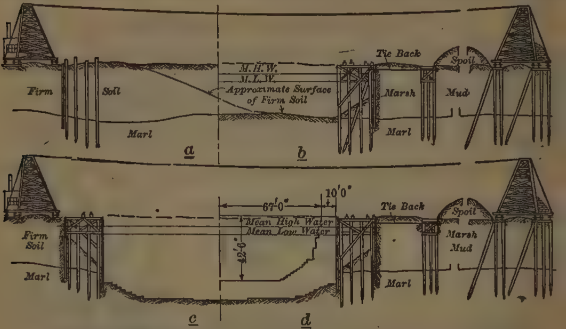


Fig. 54. Graving Dock, Charleston, S. C.

marl foundation. The entire site was surrounded with heavy sheet-piling, held in place by timber trestle work, brace piles, and shoring, as illustrated in Fig. 54. The final excavation into the marl is shown in (c), and completed structure in (d). The method employed in making the excavation for the structure was,—the first 10 ft were taken out by hydraulic dredging, the remainder of the excavation in the marl being taken out by clam shell buckets operated by cable-ways and by pick and shovel and blasting. Cofferdam around the entrance to the work is illustrated in (e).

The method of constructing the Naval Dry Dock at Mare Island, Calif., is as illustrated in Fig. 55. The excavation was made in water. Foundation piling was then driven in the bottom and the t mber crib of the general dimensions of the structure, built

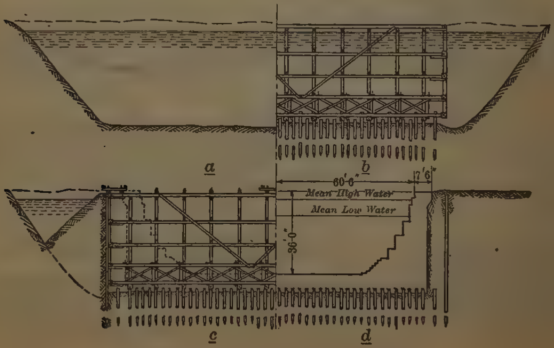


Fig. 55. Graving Dock, Mare Island, Calif.

up on the surface of the water, loaded, and lowered until it rested on certain of the foundation piling, as shown in (b). Timber sheet-piling was then driven around the crib work and then backed up with earth, as shown in (c). The interior of the crib was then pumped dry and the structure completed as illustrated in (d).

The side-wall, or English plan, is illustrated in Fig. 56. This method has been used, with success, in the construction of some European graving docks, (a) illustrating the preliminary step of driving sheathing and excavating a trench for the side-walls; b) completion of this trench and driving of the side-wall foundation piles; (c) the completion of the side-wall and the excavation of the interior or "dumpling," and the driving of the interior foundation piles; (d) completion of the work. In this method of construction there is a tendency for the side-walls to be thrust or move inward when the interior of the "dumpling" is removed. The floor work should be constructed in sections, or heavy bracing placed between the wall work pending completion of the bottom. A modification of this method was employed in the preliminary attempts at construction of Dry Dock No. 4, Navy Yard, New York, but failed, the uncompleted side-walls being thrust in.

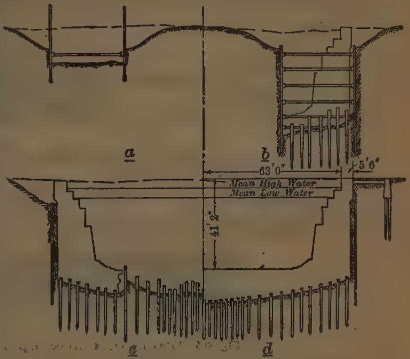


Fig. 56. Graving Dock Construction, English Trench Method

The dry dock at the New York Navy Yard was constructed by the method illustrated in Fig. 57. This dry dock was located in waterbearing quicksand soils. Open excavation

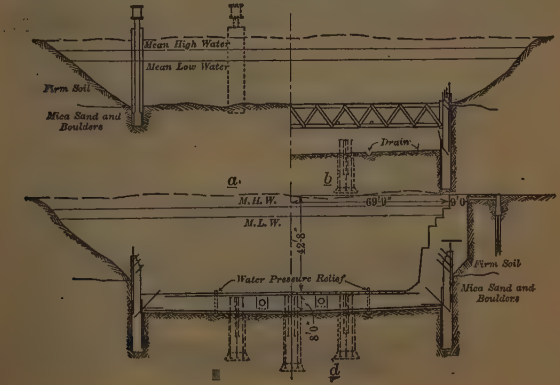


Fig. 57. Graving Dock, New York

was made by steam shovel to the quicksand. The entire site was then surrounded by a reinforced concrete cut-off wall, placed by pneumatic process. The excavation of the interior was then made, bracing being placed between cut-off walls, as illustrated in (b). Interior rectangular piers or anchors were placed by pneumatic process. The

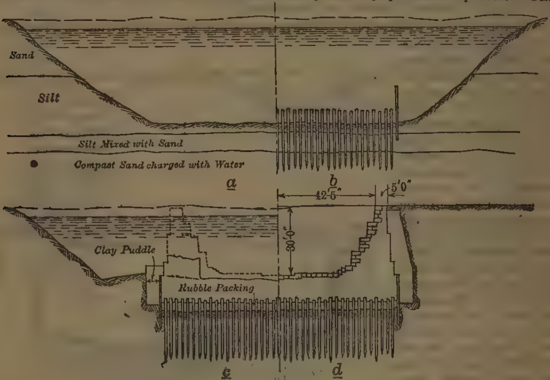


Fig. 58. Graving Dock, Kobe, Japan

floor was then built in 20-ft sections, entirely across the structure, as shown in (c). Side-walls were then built up and the structure backfilled.

At Kobe, Japan, the character of the site was such that after an unsuccessful attempt to construct in cofferdam in the dry, it was decided to employ a method not requiring the unwatering of the site. The plan followed is illustrated in Fig. 58. Excavation was made in water, as shown in (a), and piling then driven as shown in (b). Concrete was then deposited by skips and buckets under water, as illustrated in (c). Upon completion of placing of the underwater concrete up and above elevation of high water, the structure was pumped out and interior lining placed, as shown in (d).

A somewhat different method of construction proposed in connection with another dry dock is illustrated in Fig. 59. Excavation made under water, foundation piling

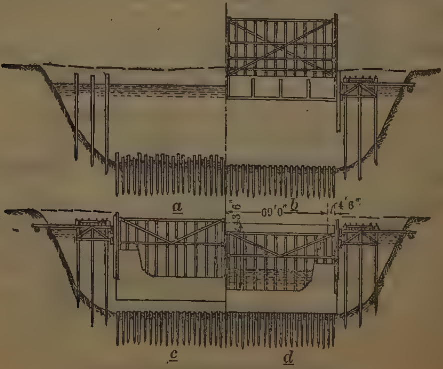


Fig. 59. Construction Employing Floating Caissons

driven as shown in (a), the foundation piling then cut off under water to grade, and the graving dock structure built in sections by means of caissons lowered to place, as illustrated in (c) and (d), in a somewhat similar manner to the method employed for Breakwater and Quay Wall Construction. See Arts. 3 and 15.

The above method was proposed for the construction of the Pearl Harbor, Hawaii, Navy Dry Dock. In the construction of this dock the general principle was used but the 16 sections were constructed on a floating dock, and then by lowering the floating dock a steel tank coffer dam was floated over a section and bolted to it. The floating dock was then lowered and the whole section floated to place, the tank coffer dam flooded and the section lowered, the side-walls and bottom of the section built complete in the coffer dam, the tank-coffer dam then floated by removing water from the tank, and then floated away and placed on the next section and so on. Space between sections was closed by concrete deposited in Tremie.

29. Gates and Pumping Plant

Caissons or Gates. Graving dock gates are, in many respects, similar to lock gates, description of which will be found in Sect. 11, Art. 19. In American practise caissons are more often employed for closing entrance to graving docks, and are usually box- or ship-shaped structures which are flooded and sunk in place at the seat, the removal of water from the interior of the dock drawing them up against the seat. When to be removed after the graving dock has been flooded, they are pumped out and hauled or towed to one side. The sliding type of caisson for which, in connection with the dry dock structure, a recess is provided at the entrance, is moved across the entrance by a hauling device, and returned to its recess in a similar manner, an illustration of which is shown in Fig. 60. This is the favorite type in English graving dock practise and it is employed for the entrance of the Quebec graving dock. The removable type of caisson is rectangular or ship-shaped, as shown in Fig. 61, or hydrometer-shaped, as shown in Fig. 62, the second type having a contracted section for a depth of 6 to 12 ft at the water line, to admit of more rapid raising and lowering when being pumped out or flooded. Or the caisson may be of the rectangular or fin keel type. Where the caisson bears on the sill, water is excluded by either depending on the close fit of a timber meeting piece, or on rubber or hemp gaskets. To resist water pressure the caissons are built and designed with a series of interior horizontal girders. When one of the types of ship-shaped caissons is in place and the graving dock pumped out, the lifting effect of the exterior water is only active on the surface of the caisson in contact with water, so that the buoyancy at such time is somewhat over half of that of the caisson when floating. However, in nearly all cases the dead weight without the water ballast is not sufficient to hold the caisson in place under these conditions. There are usually installed on the caisson, besides the pumping plant for raising the same, hand or power driven capstans at both ends and a series of large valves, power or hand actuated, so that the graving dock may be flooded thru the caisson by opening these valves Caissons of this type being floating structures, care must be taken in the design to assure proper stability under all conditions, and for that purpose it is necessary to make the calculations and curves transverse and longitudinal showing displacement, center of gravity, center of buoyancy, contained water, and metacentric heights under various conditions, especial attention being given to the effect of the contained free water during flooding and lowering. On completion of the caisson these calculations should be checked by inclining experiments.

Pumping Plant. A graving dock is freed of water by a power pumping plant, electricity or steam being generally used, the selection depending upon the special conditions obtaining in the locality. The plant should be divided into

two or more units, so that in case of a breakdown it would be still possible to operate the graving dock. The units employed are usually vertical or horizontal centrifugal pumps and should be of such combined capacity and design that the dock will be unwatered by the entire pumping plant in from 1½ to 2½ hours The water head at the beginning of the pumping will be o, the power being used up in friction and in velocity head, so that the pumping plant must be designed

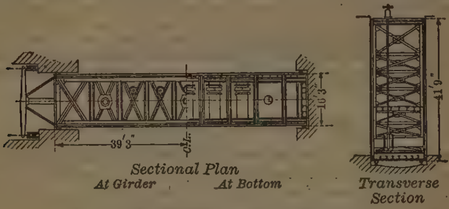


Fig. 60. Entrance Caisson, Sliding Type

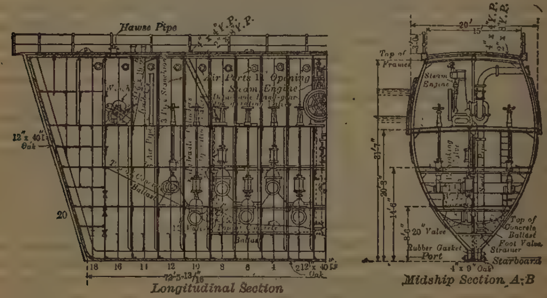


Fig. 61. Ship-Shape Caisson Gate

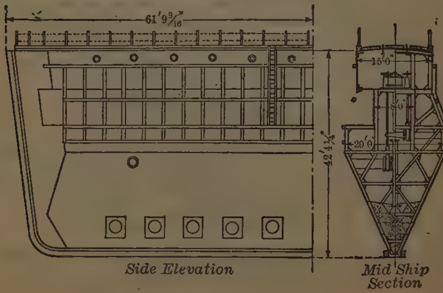


Fig. 62. Hydrometer-Shape Caisson Gate

for the especial conditions in order to obtain overall efficiencies, or over 50 percent. It is usually impossible to remove the last 2 ft of water over the floor

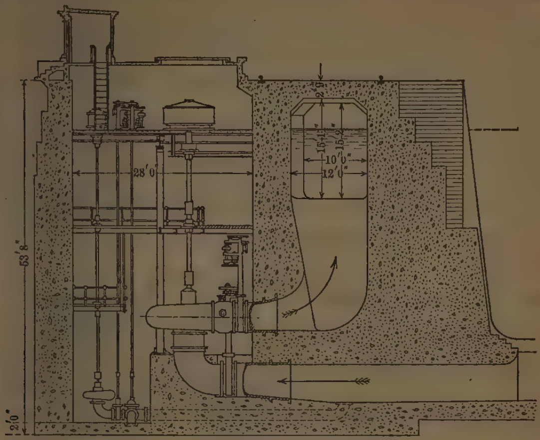


Fig. 63. Pumping Plant

by these main pumps, the blocking and block bearers holding back the water so as to result in a surging effect in the drainage forebay. Where possible, main pumps should be placed low enough and with intakes so arranged as to postpone as long as possible this surging effect. For finally freeing the graving dock of water and for regular drainage purposes, smaller drainage pumps are provided. Fig. 63 shows a sectional elevation of the graving dock pumping plant for Dry Dock No. 4, at the New York Navy Yard; there are 3 main pumps and 2 drainage pumps.

Fig. 63a shows a sectional view of the graving dock pumping plant for the new dry dock at the Navy Yard, Philadelphia, Pa. There are three electric-motor-driven pumps of 54 in size. The same general type of pumping plant is employed at the new Navy Dry Dock in the Navy Yard, Norfolk, at the State Dry Dock at Boston, Mass., and the Balboa Dry Dock at Panama.

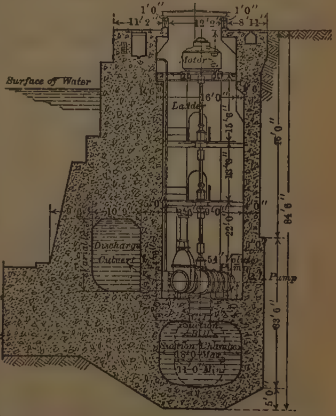


Fig. 63a Cross-section of Pumping Plant, Phila., Pa., and Norfolk, Va.

In the design of the main pumping

machinery, the work to be done by the entire plant is obtained by the total volume and weight of water contained in the graving dock at high tide; the product of this weight by the distance between its center of gravity, and the elevation of water surface at high tide, will give the work done. The required capacity of the pumping plant may then be obtained by dividing the product by the number of minutes length of time allowed for pumping out the dock, and in the same way the required motor horse-power obtained by assuming a 50% efficiency. It is customary to employ constant speed induction motors.

Flooding of the structure is usually accomplished by flooding valves installed in the entrance caisson or by sluice-ways and gates making connection with the sea or by both. These should be designed so that the structure can be flooded within 1 hour. On account of the location of the flooding gates in the lower part of gate caissons, frequently under certain conditions considerable sediment is carried into the dock by this method of flooding, so that sluice-ways and gates, connecting to tide water at a higher elevation, are often more desirable. When of large area, they also make more rapid the final leveling of water in the interior and exterior of the graving dock, and make possible a more expeditious removal of the entrance caissons.

30. Cost of Graving Docks

The Cost of Graving Docks varies between wide limits, depending, as it does, upon the character of foundation, dimensions of the graving dock, particularly the usable depth and the kind of material employed in its construction. In masonry dry docks the lining of the structure with stone work, particularly granite, will add considerably to the cost. Excluding land value, timber graving docks cost between $700 and $1200 per running foot of dock, masonry graving docks between $1500 and $3500 per running foot of dock, and may, in instances, exceed this figure.

Masonry Graving Docks

No.	Location	Built		Lgth.	Cost		
		Begun	Finished	Feet	Total	Per running foot	Per ton, max. ship docked
1	Boston, Mass., N. Y. #2	1899	1905	748	$1 100 000	$1470	$28.80
2	Charleston, S. C., N. Y. #1	1902	1908	575	1 250 000	2170	36.80
3	Mare Island, Cal., N.Y., #2	1899	1910	752	1 680 000	2210	44.20
4	New York, N. Y., N. Y. #4	1905	1912	703	2 450 000	3470	49.70
5	Norfolk, Va., N. Y. #3	1903 1910	1908 1911	732	1 730 000	2360	38.50
6	Phila., Pa., N. Y. #2	1899	1908	754	1 470 000	1950	38.50
7	Portsmouth, N. H. #2	1899	1906	751	1 125 000	1500	29.70
8	Puget Sound, #2	1908	1913	838	2 100 000	2500	32.70
9	Boston, Mass., State	1914	1919	1188	3 231 000	2500	28.09
10	Quebec, P. Q., Canada	1914	....	1190	3 000 000	2500	30.30
11	Norfolk, Va., N. Y. #4	1917	1919	1022	4 356 000	4250	46.85
12	Norfolk, Va., N. Y. #6	1918	1919	471	765 000	1625	63.75
13	Norfolk, Va., N. Y. #7	1918	1919	471	714 000	1515	70.00
14	Balboa, Panama	1911	1915	1100	2 795 000	2540	34.10
	Timber Graving Docks						
15	New York, N. Y. #3	1873	1897	668	555 000	835	23.30
16	Norfolk, Va., N. Y. #2	1887	1889	500	505 000	1010	38.30
17	Phila., Pa., N. Y. #1	1889	1891	500	549 000	1100	45.00
18	Puget Sound, W. N. Y. #1	1892	1896	650	633 000	970	27.20

The cost per ton of maximum size ship that can be docked was obtained by taking the product in feet of the usable length, allowing 5 feet clearance; the width of entrance at 1 foot above blocks, less 2 ft clearance; and the depth to top of blocks or 6 inches over sill at high water, dividing this by 35 to obtain long tons and taking 0.7 of this as a probable block coefficient. The following numbers refer to the table on page 1776.

(1) Founded on blue clay and gravel and is built of concrete lined with granite.

(2) Founded on stiff marl, is of concrete lined with granite.

(3) Founded on piles driven in soft clay. Built of concrete with granite sills and entrance.

(4) Founded on reinforced concrete surrounding walls and interior pedestals placed by pneumatic caissons in fine running mica sand and built of reinforced concrete lined with vitrified brick, granite sills and entrance.

(5) Founded on piles in clay and marl. Built of concrete, granite lined. This dock was, after completion, extended in length.

(6) Founded on piles driven in sand and gravel. Built of concrete, granite sills and entrance.

(7) Founded on ledge rock, concrete lined with granite.

(8) Founded on sand, gravel, clay; concrete and granite.

(9) Work nearing completion, founded on ledge rock, concrete, granite, entrance sills, coping and steps, intermediate seat dividing dock into two sections when desirable, side-walls lined with granite. Actual cost will be in excess of that given by perhaps $500 000, on account of war conditions.

(10) Completed in 1918. Founded on rock, concrete with granite altars and entrance, sills, sliding caissons at entrance.

(11, 12, 13) These docks founded on so-called marl containing much shell detritus at entrance and blue clay. Concrete laid directly on marl and clay except at entrance to No. 6 and No. 7, founded on timber piling.

The cost of graving dock appurtenances, while independent of foundation conditions, bears some relation to the character and size of the structure. Electric power capstans, including foundations, depending upon size, will vary from $7000 to $9000 each, and the pumping plant from $40 to $70 per motor or engine horse power. Caisson gate costs will be a function of the width and depth of the entrance and, in general, will vary as the product of the square of the width and the square of the depth.

The upkeep and maintenance cost of the body of masonry graving docks is small and should not exceed ¼ of 1 percent per annum. The upkeep cost of graving dock appurtenances varies with the character of the same, the cost of repair and replacing of blocking being high and, to some extent, dependent upon the frequency of use, and may be taken at from 10 to 15 percent of first cost, per annum. The upkeep cost of caisson gates is, in general, similar to that of the hulls of steel vessels and may be taken at from 3 to 5 percent per annum.

31. Floating Dry Docks

Definition. A floating dry dock is a structure of wood, wood and steel, steel, or reinforced concrete, capable of being submerged by the admission of water to its interior compartments, at which stage, if desired, a ship is floated into position; the structure is then raised by the removal of water from its interior compartments by pumping, or other means.

Relative Advantages. Lower first cost, less time to complete, less operating expense in pumping, and the fact that it is a movable property, are claimed as advantages for floating docks, as compared to graving docks.

The comparative first cost is entirely dependent upon the type of the floating dock and the location of the site available for the graving dock. Under the most favorable conditions for graving dock construction, the time consumed in the construction of a floating dock is somewhat less than would be the case with a graving dock. A graving dock has the advantage of permanence, more rigidity, less danger of accident, less expensive maintenance and repair, and less annual charges on account of its greater length of life. The selection of the type of structure is entirely dependent upon local conditions and requirements.

Types of Floating Docks are shown in Fig. 64: (1) The simplest type of floating dock is the solid trough type of timber or steel. It has the advantage of simplicity, of longitudinal rigidity, with the disadvantage that it must be built in its entirety intact, and can not be self-docked for repairing, cleaning, and painting.

(2) Sectional docks consist of a number of solid trough sections or slices of Type (1), and are constructed of such length that any one section can be docked, as shown, by one or two other sections. The sections are connected together by locking logs which hold them in alinement, but are not intended to furnish longitudinal rigidity for the entire structure. The operation of the dock usually requires skill and care in order that each section may take the load directly above it, the buoyancy of sections being adjusted by the amount of water pumped from them. This adjustment of weights, however, is not always necessary. With ocean-going steel vessels, the vessel itself is designed with sufficient longitudinal girder strength to distribute inequalities of load, unless the hull structure has been seriously injured, in which case the vessel should not be docked on a sectional type of floating dock. This type of structure is a favorite type in timber, especially for commercial use; on account of its low first cost and its flexibility, additional sections can be added, increasing the length of the dock.

(3) The Rennie type consists of continuous side-walls forming girders for longitudinal rigidity. These side-walls rest on, and are fastened to, a series of pontoons of such dimensions that one or more pontoons may be removed from under the side-walls and lifted for repair by the remaining structure. Docks of this character are built with steel walls and timber pontoons, or entirely of steel.

(4) In the Clark and Stanfield type the side-walls are continuous and extend to practically the full

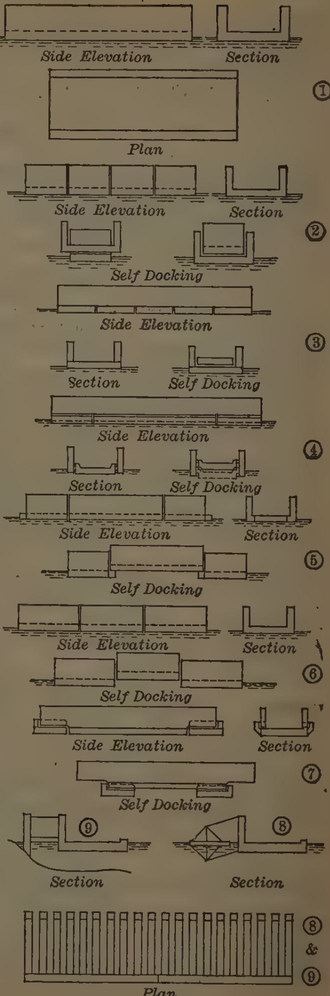


Fig. 64. Types of Floating Docks

depth of the structure, the pontoons being fastened between the side-walls, the fastening consisting of fish-plates and bolts. Pontoons can be self-docked; repair, cleaning and painting of side-walls being performed by careening of the entire structure. The side-walls being deeper than in the Rennie type, more rigidity is secured. Disconnection of pontoons for self-docking and repair is complicated. This type of dry dock is usually constructed of steel. The Havana dock now in commercial use in New York harbor, and the Naval dock at Algiers, La., are constructed on this principle.

(5) The Pola type is a sectional dock consisting of three sections, each a solid trough. The central section is of about the same size as the two end sections together. The sections are connected together for longitudinal rigidity, which connection, however, is complicated. The central section is locked by the two end sections by means of a projection on each. The end sections are docked by the center section.

(6) The Cunningham Sectional type of dock is, in many respects, similar to Type (1), Sectional dock, but consists of larger sections, provision being made for joining the sections together by fish-plates and bolts. Self-docking is accomplished by raising one section with two other sections.

(7) The Maryland Steel Company type consists of a main pontoon of solid trough section with side-walls extending beyond its length and carried on two shorter end pontoons having low independent side-walls for stability in self-docking operations. In self-docking the two end pontoons raise the central section and the two end pontoons are in turn raised by the center section. This type is constructed in steel. The Naval dock "Dewey" at Olongapo, P. I., is of this type.

(8) The Single Side Wall type consists of one rectangular side-wall and pontoon or a series of sectional pontoons. On the other side of the wall is provided a shallow pontoon with vertical members and series of two parallel members pin-connected to the vertical members and to the side-wall of the dock. The purpose of this pontoon is to afford stability and hold the structure upright when raising or lowering.

(9) The Off-Shore type is similar to the Single Side Wall dock, but the vertical member is fixed directly on shore, the pontoon being done away with.

(10) Types (8) and (9) can both be constructed with openings between pontoons so that the vessel after being lifted can be deposited on a separate floating structure or fixed platform, so that while the ship is undergoing repair the dock is ready to lift another vessel and is then known as a Depositing Dock. Neither of these types is in extensive use.

In the Burgess sectional dock the side-walls are replaced by an open framework in which slide ballast pontoons, which always remain on the surface and give stability to the structure. A structure of this character was built of timber in New York Harbor, but is not now in use.

Location. Floating docks require considerable depth of water for their operation, this depth being fixt by the draft of the maximum ship to be docked, height of blocks, clearance of blocks to keel of ship, depth of floating dock pontoons, and the allowance under the dock for clearance and for filling up. The requisite depth must often be secured by dredging and the expense of dredging operations at such great depth is considerable. In many harbors, the excavation for deep floating docks will rapidly fill up. Provision must also be made for mooring the structure, which is usually done by placing it alongside a pier built for that purpose. When this is not done, approach piers to the floating dock must be constructed. A pier or quay is also necessary outside of the entrance to the floating dock to moor vessels while awaiting docking, or to assist in guiding the vessel into the floating dock. Such a structure is also necessary in connection with the use of graving docks. Sheltered locations are desirable for floating docks; this being more necessary on account of the difficulty of handling the dock in a high wind.

Data on Design. A floating dock must have sufficient transverse strength so that the excess buoyancy of the side-walls and the pontoons at the sides will be enabled to carry the concentrated weight of that portion of the ship carried on the central keel and side bilge-blocking, and also when floated light carry

the excess weight at the sides of the side-walls and the pumping machinery. The floating dry dock, when fully sunk, ready to receive a ship, is usually flooded with water to within a small distance from the position of the outside water. This is especially so on timber floating docks. To guard against accidental over-flooding in the sinking of the dock, safety decks are often provided in the side-walls, above which the dock can not be flooded. When a ship has been placed in the dock and the operation of raising is commenced, the contained water is, as a rule, first removed from the side-walls. As the displacement of these side-walls per foot of lift is from necessity considerably less than the weight per foot lift of the maximum sized ship to be docked, the water is entirely removed from the side-walls when the ship has only been lifted a few feet, and the maximum water pressure due to difference between external and internal water head, occurs at the period when the side-walls have been emptied of contained water. The exterior sheathing or plating of the structure, beams, frames, and trussing must be analyzed for this condition of external pressure, which is illustrated in Fig. 65. In most floating dry docks this will not exceed

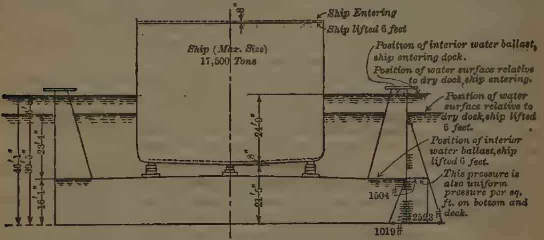


Fig. 65. Maximum Water Pressure on Dock

20 ft of water head. This condition may be avoided by pumping from compartments of the pontoon directly under the ship, before removing all the water contained in the wing walls. It is to be borne in mind, however, that with a floating dry dock it is possible to partially raise out of the water a ship of greater displacement than could be lifted clear of the water, and for this reason it is often advisable to design the structure to withstand water pressure equivalent to the maximum submersion of the pontoon deck. In timber structures, as the timber is more or less constantly immersed, a reduction should be made in the allowable strength in tension, compression, and bearing. In steel floating docks, the practise of structural steel and steel ship design should be followed, the plating being considered as a continuous beam and not a catenary.

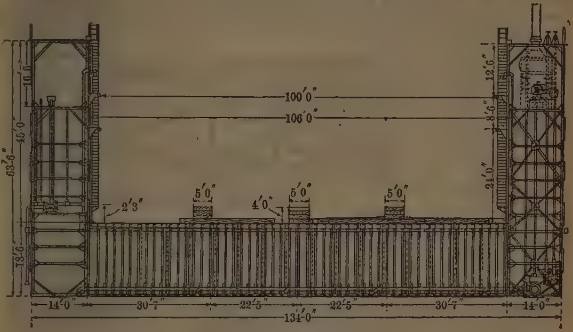
Fig. 66 is a typical cross-section of a self-docking steel floating dock, Maryland Steel Company type, having a gross lifting capacity of 18 000 long tons.

Fig. 66a is a plan longitudinal section and cross-section, side and end elevation of a 45 000-ton steel floating dock of the bolted, three-section trough type (Cunningham) designed but not built.

Fig. 67 is a half cross-section thru a section of a sectional timber dry dock, a type favored in commercial use on account of its flexibility, that is, the possibility of increasing its length and sizes by adding additional sections. The floating dock in question consists of 6 sections, each 80 ft long, having a combined maximum lifting capacity of 18 000 long tons.

Fig. 67a is a plan longitudinal and cross-section and half end elevation of a 12 000-ton timber sectional dry dock.

Loading. The loading conditions to which a floating dry dock is subjected are dependent upon the method employed in removing the contained water from the structure. It is desirable, to some extent, to pump water from the various compartments so that the buoyancy or lifting effect will bear some relation to the load or weight of that part of the vessel directly above this section of the floating dry dock. In order to obtain the maximum lifting effect of a floating dry dock, it is necessary to remove all of the contained water from the structure,



Transverse Section

Fig. 66. Steel Floating Dock

with the result that the bending strain must be taken by the dock structure or the ship being docked. Consequently the manner of regulating the pumping

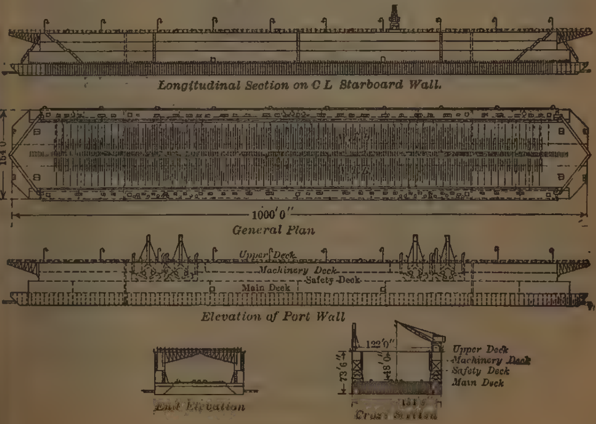


Fig. 66a. Cunningham 45 000-ton Dock.

and lifting dock and ship is of greatest importance, as it might readily be so operated as to seriously strain both ship and dock.

The Bending Moment produced in the ship and dock is shown graphically by Fig. 68: (a) showing the effect when lifting a long ship; (b) when lifting a short heavy ship, and (c) the method employed to decrease the bending moment by only removing a portion of the water from the end compartments. Fig. 69 illustrates the transverse loads to which a section of a floating timber dry dock is subjected.

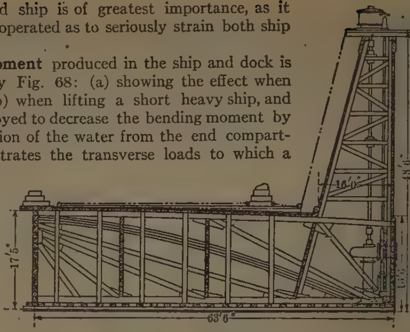


Fig. 67. Timber Floating Dock

Effective Capacity. The effective lifting capacity of a properly designed floating dry dock, whether of timber or steel, or a combination of both, will vary between 60 and 70 percent of the displacement of the pontoons In general, it may be taken that one-third of this displacement will represen the dead-weight of the structure and the contained water that can not b removed, and two-thirds will represent the effective maximum lifting capacity In timber floating dry docks the buoyancy effect of the timber must b counteracted by ballast of rock or iron, in which case proper allowance mus

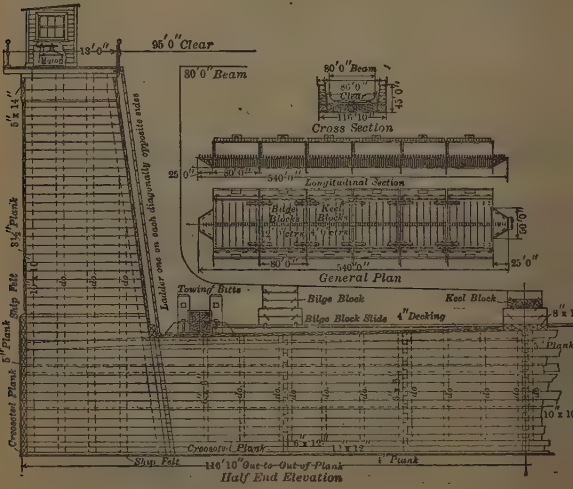


Fig. 67a. 12 000-ton Timber Dock

be made for the reduction in weight of this ballast by reason of its submersion. Ballast is not needed in steel floating dry docks and usually is not necessary in floating dry docks of steel and timber.

Costs of Floating Dry Docks of different types can be given only approximately, as these are influenced by the varying costs of materials, size of structure, and location in which it is to be built. Such approximate relative costs may be taken as follows:

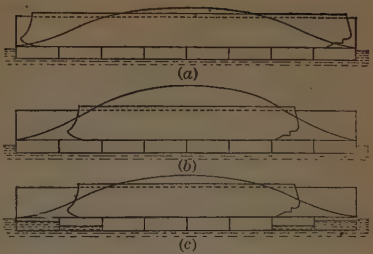
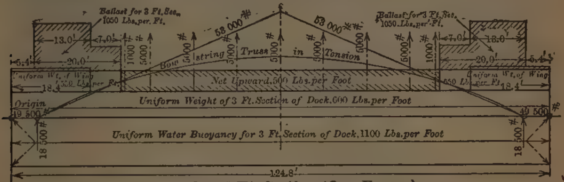
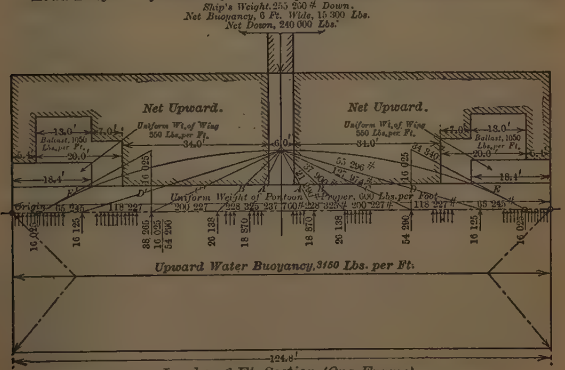


Fig. 68. Longitudinal Bending Moment



Load on 3 Ft. Section. (One Frame)

Load Diagram of Floating Dock, Light Condition, No Water in Dock.



Load on 3 Ft. Section. (One Frame)

Load Diagram of Floating Dock, Full Load, Ship on Dock.

Fig. 69. Transverse Loads

Timber solid trough type	\$28 to \$35 per lift long ton
Timber sectional type	\$25 to \$32 " " " "
Timber pontoons, steel wing walls	\$42 to \$48 " " " "
Steel, sectional or solid trough	\$52 to \$65 " " " "

These costs are for the floating dry dock complete, but take no account of the cost of the necessary work required in connection with the operation of the structure, such as piers, approaches, dredging, and other work. The variation given allows for differences in size and details and for location at which the floating dry dock is built, the higher figures for the steel structures being based on costs on the Pacific Coast of the United States.

The above given costs were those obtaining 1914-1915, the estimated cost in 1918-1919 taken from the bids and estimates on structures then designed are timber solid trough and sectional type, \$85 per lift ton; timber pontoon and steel wing walls \$95 per lift ton; steel solid trough or sectional, \$120 per lift ton.

The upkeep and maintenance costs of floating dry docks depend on the location and the materials employed. All timber floating dry docks have been in continuous use for 60 years, with only slight repair to them. In waters in which marine borers are prevalent, the exterior of timber pontoons must be protected by yellow metal, copper, tar felt covered with creosoted sheathing plank, or all planking creosoted. The interior of compartments will not require protection. The annual charge for upkeep of nine steel docks, as given by L. E. Clark, was 1.12 percent of first cost. Experience with such docks in warm climates indicates that annual upkeep cost will, in instances, exceed 3 percent of first cost. Steel structures in sea water require periodic painting to prevent undue corrosion. The most effective interior covering is by coating with bitumastic compound, costing from \$4 to \$7 per ton weight of structural steel in the hull.

32. Stability of Floating Docks

Stability. In order to secure the requisite stability against overturning when light or when lifting a ship, provision must be made for longitudinal and transverse watertight compartments. In a rectangular floating structure the general stability varies directly as the square of the number of watertight divisions. In timber floating dry docks, in most cases, the structure is divided into two transverse watertight compartments, additional longitudinal timber bulkheads being provided on either side of the central watertight bulkhead, these bulkheads merely acting as strengthening and swash bulkheads, and do not materially affect the transverse stability. In steel floating dry docks four to six transverse watertight divisions are usually provided. The metacentric height is represented by $GM = (I - GB \times V - \Sigma i)/V$

in which I is the moment of inertia of the water-plane of flotation, GB the distance between center of buoyancy and center of gravity, V the volume of displacement in cubic feet, and Σi is the summation of moment of inertias of interior contained water surfaces. Or the stability moment is

$$M_1 = \sin\theta/35\ (I - GB \times V - \Sigma i)$$

This calculation should be made for both transverse and longitudinal conditions, it being remembered that with a ship on the dry dock the stability of the ship must be taken into consideration; I, GB, V and Σi of the formulas being the combined expressions for ship and floating dock.

In order to apply the necessary formula, diagrams or curves should be available showing the characteristics of the floating dry dock under various conditions.

Fig. 70 gives an example which shows the curves of displacement, center of buoyancy, center of gravity, weight of contained water, and moment of inertia of exterior and interior water planes. Employing this and information as to similar characteristics of the maximum sized ship to be dry docked, curves can

be drawn showing the metacentric heights at various drafts of ship and dock as is shown in Fig. 71, which is a typical case for a 12 000 long ton capacity Timber Sectional Floating Dock.

Imprisoned Air. When sinking a floating dry dock it is necessary to afford means for the escape of the imprisoned air. This is done by providing piping

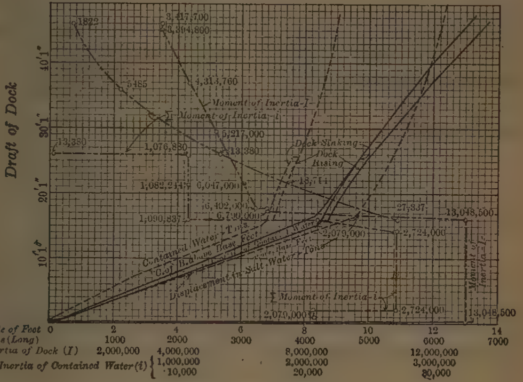


Fig. 70. Displacement and other Curves

for air escape with connections at the upper parts of compartments, or in timber dry docks, by providing air escapes directly thru the deck.

Wind Effect. Sufficient stability should be provided so that with a maxi-

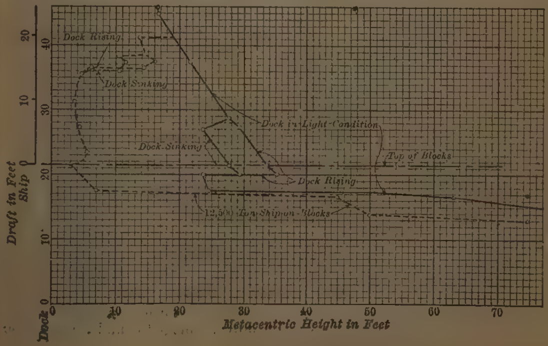


Fig. 71. Transverse Stability Curves

mum sized ship on the dry dock there will be no undue heeling of the structure due to wind action. The angle or heel of inclination θ is obtained from

$$\operatorname{Cot} \theta = D \times GM / A \times CL$$

in which A is total pressure of wind in pounds, D is the displacement in pounds, GM is the metacentric height, and CL is the distance between center of pressure and center of lateral resistance.

Pumping and Flooding Plant is so arranged that each watertight compartment can be controlled in pumping or flooding, independently of other compartments. In a solid trough dock, one or two main pumping plants are usually provided, on one or both sides of the dock connected to a pumping main, having a lateral connection to each compartment. Fig. 72 shows the general arrangement of such a pumping plant. Each section connection is controlled by direct-acting wedge-shaped valves, the valve rod being brought to one station by a system of bell crank levers, the dockmaster located at this station controlling the entire pumping of the dock. In sectional floating dry docks, it is necessary to provide a separate pump plant in each section. These may be driven individually or by means of continuous sectional shafting connected between section by universal couplings, the operating power being either steam or electric. The most satisfactory system at present in use is electrically operated vertical shaft centrifugal pumps. In sectional docks, with two pumps in each section, one on each side of the structure, a cross-connection is often provided so that in case one pump should break down, the section could still be operated by the other pump. This involves a longer time for pumping out and also serves to decrease the stability of the entire structure by decreasing the stability of one of its sections. The sinking by flooding is carried out by flooding valves or gates, leading to the various compartments, the dockmaster controlling this entire operation to insure even settlement without undue straining of dock or ship. Floating docks are often fitted with an indicator system to show in a central control station the depth of water in all compartments. An entirely satisfactory system has not as yet been successfully applied.

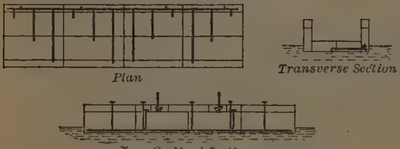


Fig. 72. Arrangement of Pumping Plant

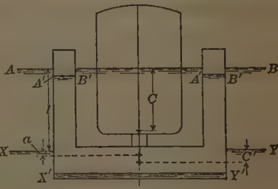


Fig. 73

It has been proposed to raise floating docks by compressed air, by removing the water from compartments by forcing air into them, and in a somewhat similar manner sinking by allowing this air to escape. While this has been tried experimentally on a small scale, it has not been successfully used in practise. The general idea has had some success applied to raising sunken vessels by pumping air into the hull or by forcing it into submerged cylindrical pontoons made fast to the hull by chains or wire cables.

Power Required in Operating a Floating Dry Dock. The work done in raising a floating dry dock with a ship is directly proportional to the weight of the dock light and weight or displacement of the ship.

$$E = D\,(l - c) + D'\,c' \pm (D - D')\,a$$

in which, as shown in Fig. 73, D is the displacement in pounds of ship and floating dock ready to lift, c is depth in feet below surface of water of center of buoyancy of D, D' is displacement in lbs of dock or ship and dock wholly or partially raised, c' is depth in feet below surface of water of center buoyancy of D', l is height in feet raised, and a is distance above or below surface of water of center of gravity of water removed from dock when dock is in its raised position.

The necessary data for application of the formulæ can be obtained from curves showing displacement and position of center of buoyancy of the ship and displacement of dock, weight of contained water and positions of center of gravity and center buoyancy. The characteristics of the pumping plant can then be obtained by fixing on the time in which it is desired to raise the floating dry dock with a ship of maximum weight, usually taken at from 1 to 1½ hours. The overall efficiency of an electrically driven centrifugal pump of the type herein described should be between 50 and 60 per cent.

Fig. 74. Blocking

Blocking. The arrangement of blocking on a floating dock is in many respects similar to that employed in graving docks, and is shown in Fig. 74.

FERRIES AND SPECIAL WORKS

33. Ferry Racks

Ferry Racks. The landings or terminals for ferries consist or a slip with piles and fenders on each side called racks, transfer bridge, platform, and ferry shed. The racks are usually built diverging outshore, the inshore ends closely fitting the guard rails of ferry-boats. The racks consist of one or more rows or banks of piles with upper, lower, and one or more intermediate string pieces or wales on which are fastened vertical fender planking of oak, yellow pine, or some other hard wood. Where the ferry-boats are small or light, and the traffic infrequent, a single row of piles is used, the piles being spaced about 3 ft centers. The rack is stiffened at the inshore end by additional piles or clusters of piles, as this portion of the rack receives the principal wear and shock. The height of the rack is dependent upon the range of tide, it being evident that the fender system must be of sufficient height, or depth, so that the guard rail of the ferry-boat, when loaded or unloaded, will never be below or above the rack. The wales are made up of several thicknesses of heavy planking or dimensioned timber, so as to be readily fitted to the required curve and are bolted to the round piles. The vertical fender planking is fastened to the wales by drift bolts or dock spike at the upper and lower extremity. Ribbon pieces are fastened to the wales and thru the vertical fenders by countersunk head screw bolts. The upper ends of fender planking are sawn off at a bevel to shed water, and the tops of piles rounded for the same purpose and often covered with canvas or zinc for protection against the entry of water and consequent rot. Frequently the fenders at the inner end of ferry racks are coated with heavy oil, or grease.

In heavy ferry rack construction, two, three, or four rows of piles are employed, the piles in the rear rows being staggered, and the tops of rear rows are cut off at a lower elevation. In two bank racks two or more wales are fastened to the back of the front high row and one or two heavy wales to the front of second pile row, so as to transmit the shock of the incoming ferry-boat to the second row of piles, and if three rows of piles are used, the third row is also provided with wales of this type. The outshore end or entrances of racks are usually constructed with a cluster of piles, driven a slight distance apart, and drawn together at the top, and lashed with wire cable fastened with staples, these dolphins or clusters fre-

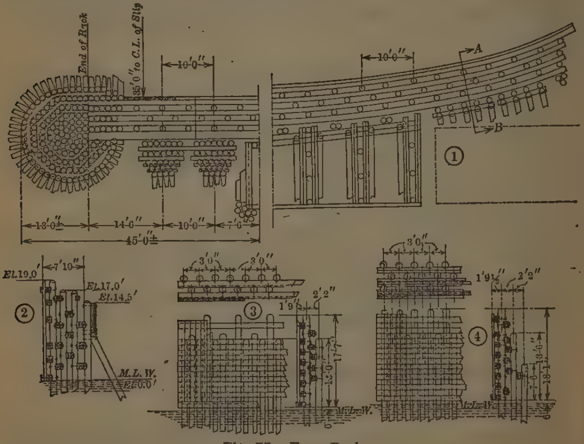


Fig. 75. Ferry Racks

quently receiving the first impact of the entering ferry-boat, guiding the ferry-boat into the slip. Fig. 75 shows general arrangement of various types.

(1) Plan of three and four bank rack for Municipal Ferry, New York City, 39th Street, Brooklyn.

(2) Typical section of (1). Front wales, two 6 x 12 in. Rear wales, three 4 x 12 in. Fenders, 5 x 12 in white oak.

(3) Typical elevation, section, and plan of two bank rack, D. L. & W. R.R. Passenger service.

(4) Elevation, section, and plan of three bank rack, D. L. & W. R.R.

The work can be laid out by securing plans of the guard rails of various ferry-boats that are to use the slip, and from these make a plan showing maximum dimensions. Lay out the slip with racks fitting this plan and measure ordinates from and perpendicular to the center line to pile locations. Construct a light temporary platform along the slip and on this locate points on which batten ranges can be fastened, giving the transverse range of piles, the piles being located and driven by measuring out the proper distance on this range. All piles need not be so located, but piles every 10 or 12 ft having been accurately located, intermediate piles can be driven in general conformity with the desired curve The work can be laid out from a temporary platform or existing adjoining pier or structure alongside of the site of the ferry slip. A ferry-boat can be moored in place and governing dimensions obtained by driving piles at intervals against the guard rail of the template used.

34. Transfer Bridges

Ferry Transfer Bridges should be of such length that at extreme tides, with maximum and minimum loading of ferry-boat, the gradient will be possible for traffic, the range being from low tide with heavily loaded boat having little freeboard to high tide with light load and greater freeboard. The outshore end of the bridge, when light, is usually carried by a pontoon, so that it will, at all times, float to about the elevation of the ferry-boat deck. For this adjustment of height the buoyancy of the pontoon is employed sometimes with the assistance of counterweights fastened to the bridge by wire or chain cables, and carried up over sheaves mounted on an overhead frame. The final adjustment of height is made by hand or power operated windlass fastened to the end of the bridge. The outshore end of the bridge is usually curved to fit the bow or stern of the ferry boat, and is provided with heavy hardwood or iron toggles which can be moved out to rest on the deck of the ferry-boat, so that when the weight of traffic leaves the ferry-boat and moves on to the bridge, this weight, in excess of the buoyancy effect of the pontoon, is carried by the ferry-boat and not by the overhead gear. The inshore end of the bridge is supported on rollers, in roller chocks, supported on the platform, or is carried on heavy hinges on a rolling or sliding bearing fitted with car springs or rubber buffers to assist in

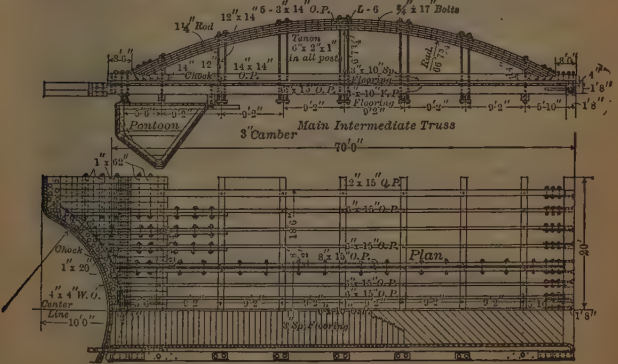


Fig. 76. General Traffic Transfer Bridge

taking up the shock of the incoming boat. The ferry-boat is secured to the bridge by means of two lines made fast to eyes or chocks on the dock of the ferry-boat, the slack in the lines being taken up by windlasses. The bridge itself usually consists of a platform suspended from two or four structural steel or timber trusses, a timber bow-string truss being frequently used. The overhead gear with counterweight should be of sufficient strength to support the outshore end of the bridge when unloaded, so that the pontoon may be removed when necessary for calking and repair. Fig. 76 shows a ferry bridge and pontoon and is typical of this character of structure in general use. Where double-deck ferry-boats are employed, elevated adjustable gangways or bridges are provided for the upper-deck traffic.

Ferry Houses or terminal sheds are, as regards materials employed and details, similar to pier sheds, structural steel framing with metal siding and sheathing being largely employed. The shed is carried out over the transfer bridge and often projects out a distance over the slip to afford protection against the weather.

Railroad Car Transfer slips are more rectangular than ferry slips for general traffic, and the racks are consequently built straighter or no rack used. Fig. 77 is a section of a typical three-bank rack; the fender planking is horizontal, and this appears to be standard practise for car transfer racks. The transfer bridges must be of sufficient length to insure a reasonably flat slope for variations in tide and freeboard of the car float. Where possible grades should not exceed 5 percent. It is not desirable to run locomotives on the float, and the transfer of cars is effected by using two or more pushers or empties to couple up with. When operating on a steep grade the locomotive remains on the level platform. The outer end of the bridge is supported on a pontoon or by means of a counter-weight.

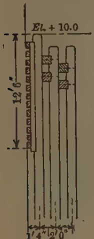


Fig. 77.

Fig. 78 shows a transfer bridge with pontoon and overhead gear. The adjustment of the height of track is made by removing water from the compartments of the pontoon by means of a small power pump or by filling it, the final adjustment being made by the overhead gear with hand-operated winches.

Fig. 79 is a two-track transfer bridge, employed by the N. Y. Central R.R. in Weehawken, N. J. In this the supporting pontoon is replaced by two counterweights each, weighing 46 short tons, the adjustment of height being made by a power-operated vertical

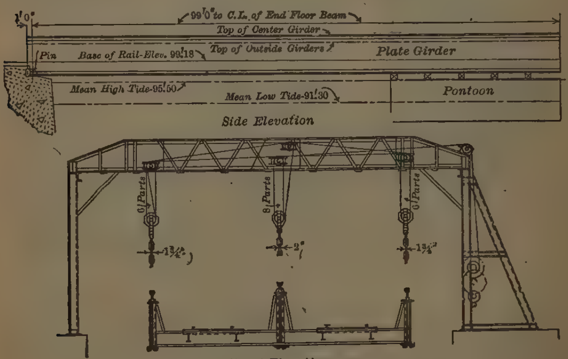


Fig. 78. Railroad Car Transfer Bridge

screw shaft. In this type the weight of cars is almost entirely carried by the overhead gear. The estimated weight of structural steel for one transfer is 740 000 lb.

The general type of railroad and car transfer in use often consists of a bridge made up of two or more timber Howe trusses hinged, or supported, on a rolling log inshore,

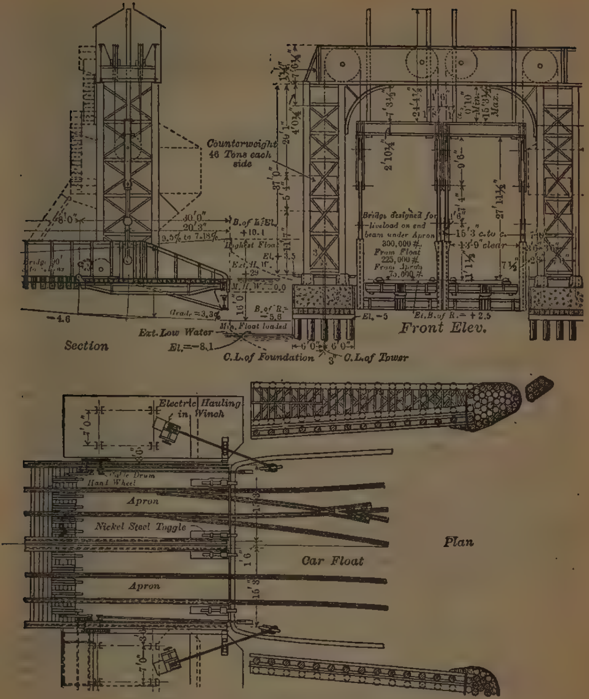


Fig. 79. Transfer Bridge, N. Y. C. & H. R. R.R.

with the outshore extremity resting on a float or pontoon. Directly over the outer end, and supported on pile foundations, on either side, an overhead or gallows frame is provided; the bridge is connected to this by chain or wire cable tackle, and the final adjustment of height made by this means.

River Car Transfers are employed in connection with ferrying railroad cars across rivers. When the range of tide is not very great, such transfers are similar to those used in harbors. However, when the elevation of tide fluctuates to a considerable extent, as in the great interior rivers of the United States, it is customary to use a method of transfer similar to that shown in Fig. 80.

This system has been in successful use for many years for transferring railroad cars from or to tracks from car floats. The arrangement is known as a "cradle." Standard gage railroad track is laid on an incline, usually diagonally, along the bank or river levee slope. The cradle running on this track is hauled up and down the incline accord-

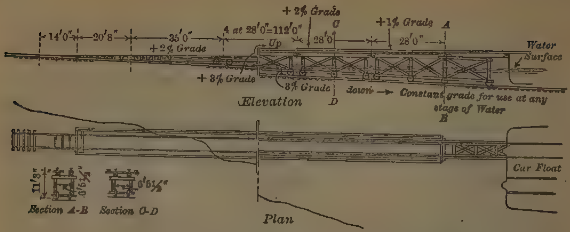


Fig. 80. Cradle, Ill. Cent. R.R., Mississippi River

ing to the stage of tide. Fig. 80 shows a cradle in use at Harahan and Baton Rouge, by the Ill. Cent. R.R. for transferring cars across the Mississippi River.

35. Coal, Ore, and Grain Plants

Coal Handling Plants are usually of two general types: (a) The wharf storage type, in which the coal is received by rail or otherwise from shore, by ship or barge from the water, and stored on the wharf itself for distribution and use. (b) The type where the coal is received by ship or barge cargo, and transferred to, and stored on shore, for shore consumption, or to be again distributed for ships' use; or delivered to the pier from the shore by rail or otherwise, and distributed directly to ships or barges. The last-mentioned type is often provided with some pier storage as an auxiliary feature.

Type (a) is, manifestly, somewhat limited in capacity, depending on the dimensions of the pier and the consequent storage-bin area, as it is inadvisable to store coal in bins to a greater depth than 20 ft, on account of the possibility of spontaneous combustion, and the consequent necessity for re-handling coal. The bins are usually given a bottom slope to permit of ready self-discharging, with a minimum of man-handling, the discharging being effected by portable chutes, collapsible or fixt, for discharge into ships or barges' holds, or for discharge on shore under the bins into carts or railroad cars, the loading of hoppers being effected by fixed hoists with collapsible cantilever arms, on which the bucket or unloading device travels transversely on trolleys or carriages, the distribution in the length of the bin being performed by bucket or belt distribution, or the entire loading device may travel longitudinally of bins for the purpose of distribution. Fig. 81 shows such a plant built in 1901, at the Navy Yard, New York.

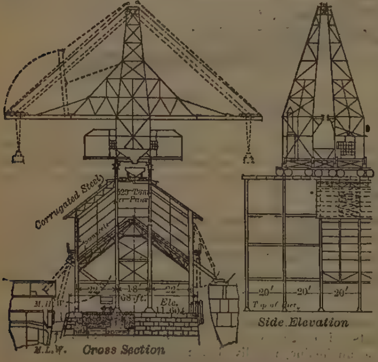


Fig. 81. Coaling Plant, New York

The arrangement of coal handling plants, of Type (b), is multitudinous in character, and no at-

tempt will be made to cover this feature in this section, other than to state that for small plants the stationary or movable coal tipple is employed with great economy. In larger plants, a very satisfactory method appears to be a loading apparatus, cable operated railroad dump cars. discharging to shore storage bins or piles. When coal is required for ship or barge cargo, cars are run out on the pier and discharged into smaller bins or hopper chutes convenient to ship or barge hatchways, cars being discharged directly over the bins by bottom or side dumping car, or, in some cases, by tipping the car.

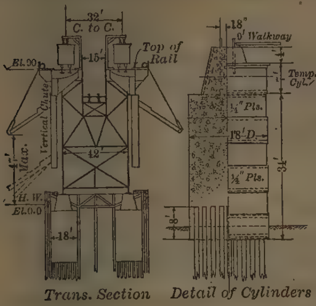


Fig. 82 shows the latest coal pier and plant of Norfolk & Western R.R., Lambert's Point, Va., the method of operation being: Coal from mine arrives at yard and travels by gravity over automatic weighers, and is hauled up incline to pier by power hauling cable to tipping platform, where the car is inverted and contents of car dumped, the capacity being 30 cars per hour. Empty car, on being righted, returns to yard by gravity. Special electric trolley pier cars are raised to the top of the pier by elevators, discharging coal thru hoppers and chutes. Empty pier cars are returned for new load. Capacity of this coal pier ranges between 3000 and 5400 tons per hour.

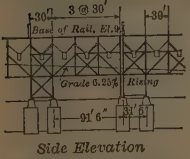


Fig. 82. Coaling Plant, Lambert's Point, Va.

Fig. 83 shows the layout of a somewhat similar plant in use by the D. L. & W. R.R., Jersey City, N. J.

Where a considerable amount of coal is to be stored, it may be placed in piles on shore in close proximity to the coaling pier by being dumped by cars from overhead tracks or trestles, or directly from grabs or buckets operating on travelers. When required, coal is re-loaded by means of grabs or buckets operated by travelers or locomotive cranes and placed in cars. Vessels are loaded with coal from coal barges placed alongside by manhandling in bags, and hoisting by ship's gear and winches. This method of loading is often found most economical, as the ship's crew is employed in the operation, and has other advantages

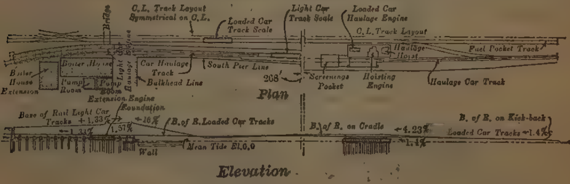


Fig. 83. Coaling Plant, Jersey City, N. J.

where coal bunkers are small, widely distributed, and access to them is difficult, such conditions especially applying to Naval vessels. Floating mechanical coal conveyors are also employed for loading coal on steam vessels.

Coal is sometimes stored under water in compartments or enclosures for that purpose, as there is no deterioration due to atmospheric action, no losses due to dust blowing away,

and the possibility of spontaneous combustion and consequent expensive re-handling is avoided. Coal stored under water 14 years shows a deterioration of only 1 ½ percent.

Ore Handling Devices for loading and unloading are generally similar in layout to Type (b), Coal Handling Plants, except that the material is seldom moved back and forth, but is usually moved in one direction either from the source to be loaded or from the ship to be unloaded. An important feature in the design is the considerably greater weight per cu ft of the ore.

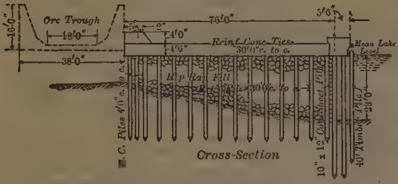


Fig. 84. Ore Pier, Cleveland, O.

Fig. 84 is a section showing an ore trough and pier at Cleveland, O. Ore can be delivered from the vessels directly to freight cars, to stock piles, or placed in the ore trough to be picked up for distribution by conveyors. The ore pier is provided with loading and unloading traveler operating longitudinally of the pier. Fig. 85 shows an ore pier at Two Harbors, Minn. The cost of this plant ranged between $3800 and $4500 per pocket, excluding dredging.

Grain Handling Plants. When cargoes of grain are to be handled in bulk, it is usually loaded and unloaded by means of grain elevators, the material

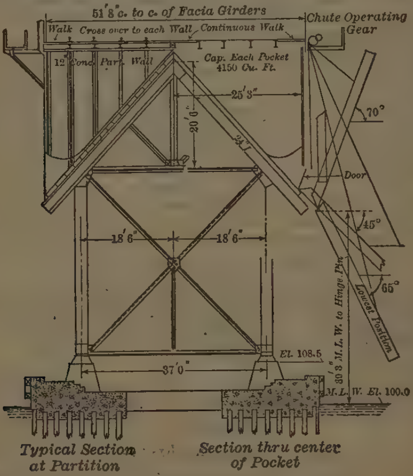


Fig. 85. Ore Pier, Two Harbors, Minn.

being stored in bins. Grain elevators may be portable, mounted on barges, or constructed on piers or quays. The material is handled by endless belts, a succession of buckets on a continuous chain, by pneumatic power, or in lesser

bulk by buckets or grabs. The pneumatic system of unloading has the advantage of extreme flexibility and can be applied in any position, saving the cost of hand trimming. Grain unloading plants for handling large cargoes are found on the Great Lakes, but not in other American ports where the plants are solely for loading into vessels. For unloading, floating equipment is most often employed, unloading the cargo of the vessel and loading barges for distribution and trans-shipment in the same operation.

Fig. 86 shows the latest type of grain elevator constructed at Girard Point, Philadelphia, Pa., Penn. R.R., having a total capacity of over 1 000 000 bushels. Grain comes in on cars on a descending grade, and is hauled to and from the unloading shed by cable.

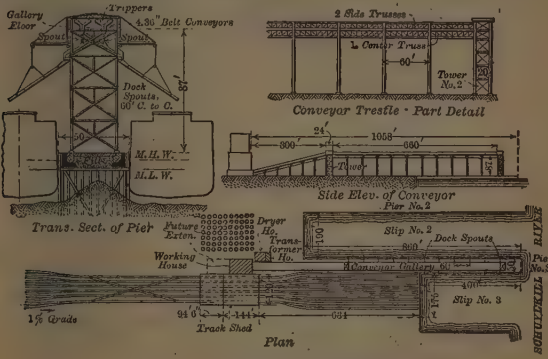


Fig. 86. Grain Handling Pier, Philadelphia

Unloading is performed by power shovels into receiving hoppers, discharging therefrom on receiving belts, and is raised to receiving garners and weighed, and thence by means of spouts distributed to the storage bins or to the pier shipping conveyor consisting of four 36 inch belts, from whence it is distributed by trippers thru side pier spouts.

36. Dumps and Landing Stages

Dumps are constructed and set aside for the handling of refuse material. On account of the importance of securing and retaining ample depth for the purpose of navigation, in most ports it is against the law to dump material within the navigable water area, whether the material is of such character as to sink to the bottom or float on the surface of the water and interfere with navigation or be unsanitary or obnoxious. In some

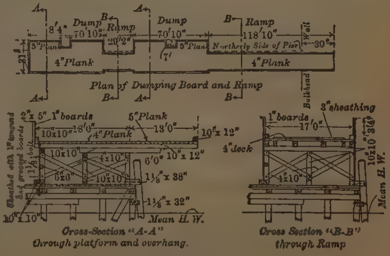


Fig. 87. City Dump

of the larger cities public dumps are provided where excavated material or refuse may be dumped into barges for trans-shipment for local filling, or disposal at sea. Fig. 87 shows a dumping board at Pier 33, New York City. Fig. 88 is a typical example of a dumping board for distribution of excavated material.

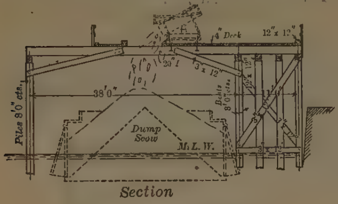


Fig. 88. Railroad Car Dump

Floating Landing Stages are provided where there is considerable range of tide and it is not desirable to incur the expense for the construction of a wet dock, or where, for a similar reason, it is inadvisable to construct a fixt pier or quay with two or more platforms to be used at various stages of tide. Floating landing stages on a small scale are employed for small boat landings and consist of a float or pontoon with a gangway from the pontoon to the pier or quay platform. When constructed, the deck is supported by its own buoyancy or on hollow pontoons, the buoyancy of which must be sufficient to carry the weight of passengers or cargo. The pontoon must be divided into two or more longitudinal and two or more transverse watertight compartments in order to insure stability against overturning when the compartments are partially full of water. The formula to be applied is given in Art. 32 under Stability of Floating Dry Docks.

Fig. 89. Landing, Memphis, Tenn.

The floating landing stage at Liverpool, Eng., consists of a deck supported on pontoons, and is 2500 ft long by 80 ft wide, and has eight gangways or bridges connecting it with the shore. The maximum range of tide at this port is 30 ft.

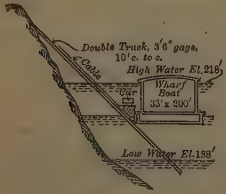


Fig. 90. Car Handling

On many of the interior rivers of the United States, there is considerable freight and passenger traffic, loading and unloading taking place at various places along the bank where no regular landings are provided, the light draft, stern wheel steamboats employed running close to the bank and transferring traffic by platforms or gangways. At many of the more important ports where the elevation of the river varies greatly, and ordinary wharves are not available or advantageous, floats are employed. Fig. 89 shows the usual arrangement, illustrating particularly the levee landing at Memphis, Tenn. The landing stages or floats are moved up and down the bank or levee incline as the river falls or rises.

Where the river bank is steep, and bulky cargo is to be handled, landing stages or floats are employed in connection with inclines, as shown in Fig. 90. This type of incline is usually double-tracked with a platform on each track, the two platforms being connected

together by a cable running over a drum of a hoisting engine which operates the mechanism. This particular type is in use on the Tennessee, Mississippi, and Ohio Rivers used in connection with the handling of iron and steel products, coal, and cotton. Telpherage transfer system is also extensively employed for river landings; typical example of which is shown in Fig. 91, employed by the Ill. Cent. R. R., South Memphis, Tenn.

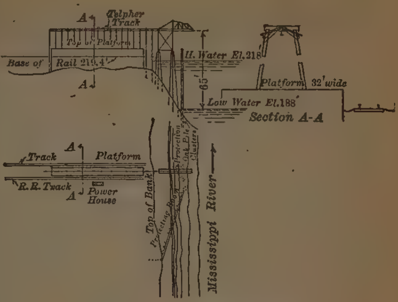


Fig. 91. Telpherage Landing

37. Shipbuilding Ways

While the construction and launching of ships come within the province of the naval architect, the design and construction of the platform or ways on which the vessel is built is a question of foundation design and one that will be dealt with briefly herewith.

Shipbuilding Ways are in many respects similar to marine railways, and are primarily inclined slips on which a vessel is built, and which project far enough out into and under the water so that when the vessel is ready to be launched it may pass down from the land to the water. As with marine railways, building ways may be of the lengthwise or broadside type. The broadside type is resorted to where the basin or stream into which the vessel is launched is narrow and does not afford the requisite width for lengthwise launching. Broadside ways are common on the Clyde River. Building ways consist of the fixt ways which is the incline or platform fastened to the foundation and the cradle or sliding ways. The vessel itself is built on the fixt ways or on blocking and the fixt ways are frequently not placed until shortly before the v ssel is ready for launching. Then the weight of the vessel is transferred to the cradle or sliding ways and in turn to the fixt ways by means of wedges. A lubricant is placed between the sliding and the fixt ways, usually a mixture of tallow and soap, and at the proper time the vessels released by sawing or a trigger device. The gradient or inclination of the ways is fixt dependent on the type and launching weight of the vessel so that with the release, by whatever method secured, the sliding ways and vessel will move down to the water and be launched into it. It is customary as a matter of precaution to place horizontal jacks to give the sliding ways and vessel a push or "kick."

The fixt ways consist of two longitudinal streaks, one on each side of the center line of keel, and of sufficient width to carry the maximum moving load of the vessel and sliding ways, the safe bearing capacity of the timber used across the grain being considered.

Inclination of Ways. Inclination or the gradient of the ways is dependent upon the length, type and probable launching weight of the ship to be built and is seldom greater than ⅞ in to the foot run. Ways for moderate sized-steel vessels with a launching weight of from 2000 to 4000 tons have a gradient of ⅝ to ¾ in; on larger heavier vessels ⅝ to ½ in is used while on very large vessels of great length such as the largest transatlantic liners and battle cruisers ½ or $^7/_{16}$ in is employed. For the last two classes of ships the ways are often built with a varying gradient; that is, with a very flat circular profile, so that the

ship when launched and moving down the ways moves on the arc of a great circle. This arrangement has the advantage of bringing less strain on the trigger or launching device, and the vessel once released and moving down the ways receives an increasing acceleration. With the vessel of considerable length this has the further advantage of decreasing the required length of the out-board or under-water portion of the ways; that is, the requisite depth of water at the end can be obtained in a shorter run.

Foundations. The foundations of the blocking, or fixt ways, must, of course, be designed to carry the maximum weight of that section of the vessel built directly above it, and the lower, or out-board portion, must be designed with a view to the fact that as the vessel is launched the maximum weight section will move down, superimposing its load over the entire lower ways as it passes over it.

As the vessel reaches the water and becomes submerged therein she is immediately in part water borne, and the foundation directly under the stern becomes relieved in increments of the superimposed weight, while the load under the bow or upper end of the vessel is increased by the upward force of the outboard vertical buoyancy effect. This change in condition continues until the stern or outboard end of the vessel is entirely water borne, and about to be lifted from the ways when a pivoting action about the upper end of the bow occurs, and that portion of the weight of the vessel representing the upper end reaction is concentrated at the bow or upper end. This pivoting pressure is the maximum pressure brought to bear upon the ways foundation and has a decreasing effect continuing down the ways from the initial point of pivot until the vessel leaves the ways entirely. With larger vessels a pivot cradle is constructed under the bows detached from the sliding ways or cradle, the hull of the vessel at the bow end resting in a trunnion saddle (a) which sets in the pivoting cradle (b), all shown in Fig. 91*a*. In designing ways it is imperative that all of the load and maximum weight pressure conditions be obtained. Fig. 91*b* shows the buoyancy pivoting hogging curves and maximum way pressure curves for a 3500-ton dead weight cargo coal barge. The ways must be extended outboard sufficiently so that the depth of water at the end will be approximately the draft of the vessel at the bow, where the sliding ways leave the fixed ways when the vessel is launched; otherwise the vessel would drop through a vertical distance represented by the excess in draft at this point. In practice, it is quite usual to launch expecting and providing for some drop, the opening between the fixt ways at the outboard end being cleared sufficiently so that the hull of the vessel at the bow will not strike any obstruction and cause damage to the hull or the ways.

Fig. 91*a*

The type of foundation required is dependent entirely upon the location and can be treated similarly to other foundation conditions. However, due to the necessary restrictions of being located on some foreshore, timber or concrete piling is the general rule. When timber piling is employed it is, of course, subject to rot and decay above the wet line. Sometimes to avoid the necessity of constantly repairing this portion of the ways, the piling is cut off at mean tide or wet line and concrete or masonry cross walls or concrete piles built on the timber piles, or the necessity of such construction is avoided by the use of reinforced concrete piles of the pre-cast or cast in place type, the relative advantage of the two types of construction being, as in other foundations, a question of the particular conditions and restrictions involved. It is impossible to lay down a rule as to the relative desirability or permanency in ways foundations as it so frequently

happens that for reasons of development and change in ship dimensions and design or for other reasons it becomes necessary to materially change the carrying capacity of a ways if it is utilized during the life of that particular shipbuilding

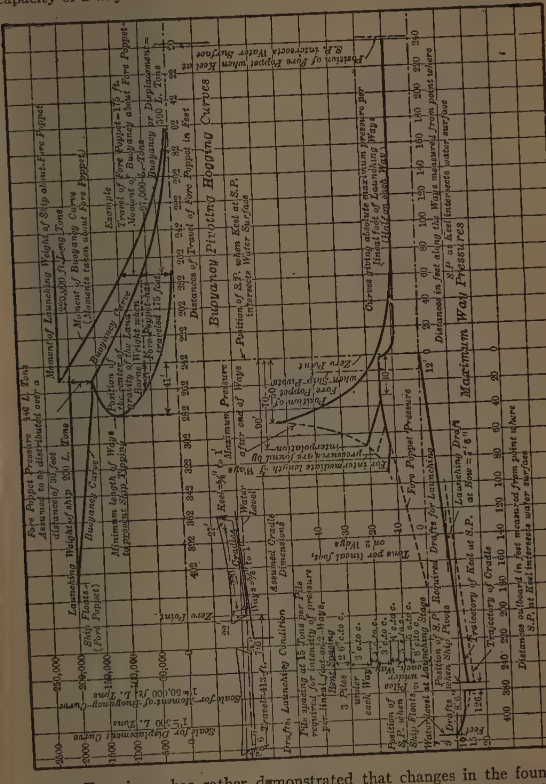


Fig. 91b. Hogging Curves and Way Pressusre

plant. Experience has rather demonstrated that changes in the foundation, usually with a view to increasing its carrying capacity, are called for whenever a new ship is to be laid down.

The outboard end of the ways, as with the marine railway, must be constructed below water level, and must be heavily braced longitudinally and transversely. A portion of the ways can be built by sawing off the foundation piles by an underwater saw arbor, or by employing divers, or the entire site can be enclosed by a coffer-dam and the water then removed and the ways constructed in the dry. The last method is frequently resorted to in new ways,

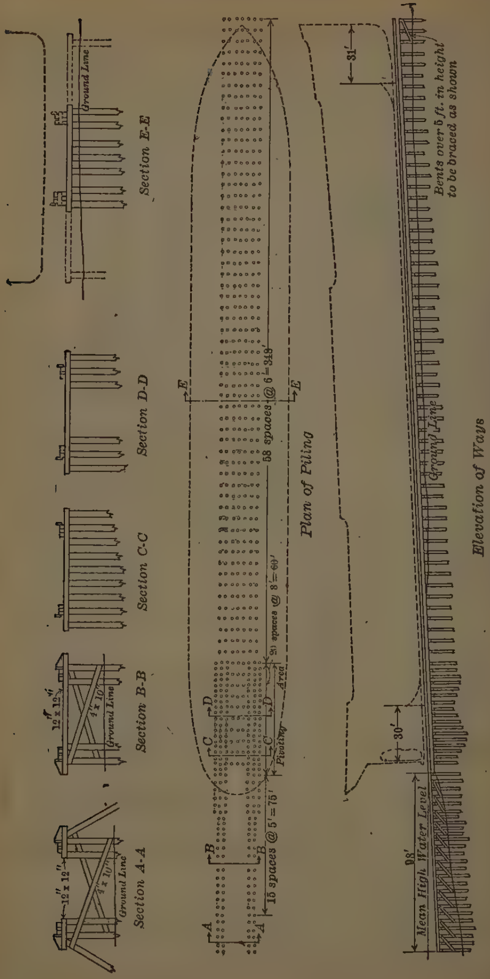


Fig. 91c. Cross-section Showing Transverse Bracing

as the under water part of the ways is so subject to damage in the course launching or damage by passing vessels, or floating objects, it is customary ore launching to carefully inspect this portion of the ways by divers and to uild it also when needed by this method. Fig. 91*c* shows the plan and eleva- 1 of a typical building and nching ways for a 9000-ton d weight cargo ship, the nching weight of which is umed at 2850 tons. Atten- 1 is invited to the piling n necessary at the out- rd end where the founda- 1 at *CC* and *DD* is greatly nforced. This is to take e of the pivotal pressure einbefore referred to.

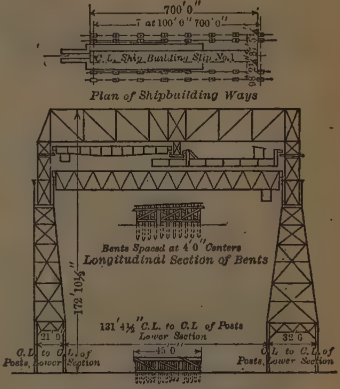


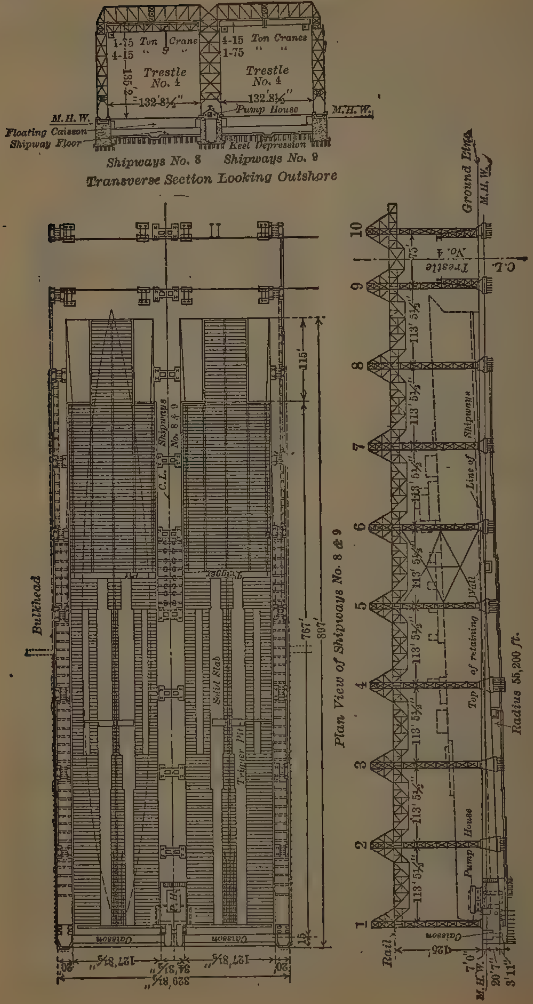
Fig. 91*d*. Timber Building Ways

n construction of merchant sels frequently the launch- weight is always the weight displacement of the bare light ship, the boilers, ma- nery and most of the acces- ies having been placed ile the vessel is on the ways ler construction. It is en desirable either for the pose of rapidly clearing ways and getting it ready another ship, or by reason of the fact that the particular plant does not struct and install the machinery and other ship's gear, to launch the hull l place the machinery, etc., afterwards, in which case the launching weight o that extent so much less. In the construction of naval vessels, especially tleships and cruisers having heavy armor and armament, the exterior armor, ret armor and often turrets, together with much of the machinery and all of guns, are placed after the vessel is launched, so that the launching weight is ally from 30% to 40% of the full load displacement.

Types of Building Ways. The ways shown in Fig. 91*c* is of the simplest e, and that most generally used, and has the advantage of economy of t cost. As previously stated a more permanent foundation is obtained by use of concrete cross walls or pillars. Ways of this character have the dis- antage that the outboard end is under water, not subject to visual inspection, l when preparing to launch there is, of course, some difficulty in providing proper lubricants under water. They must always be extended some tance out to insure a depth for clearance.

A twin set of ways have been constructed by the Newport News Shipbuilding ., a plan, section and side elevation of which are shown in Fig. 91*e*. In these ys the water is excluded from the lower end by means of floating caisson es. The ways have the advantage of permanency, shorter length and less ght required for overhead crane run way structure. They undoubtedly olve the element of a much higher first cost and entail drainage of the lower tion of the site during the process of building.

The type of ways above described naturally leads to the consideration of the of a shallow graving dock for shipbuilding purposes. Ships have in the past



Longitudinal Section Looking North Showing Battle Cruiser on W...

Fig. 91e. Timber Ways

en built in a basin or graving dock which results, of course, in tying up this portant part of the :epair plant and preventing its use for ship docking and air work; such use would not be resorted to in ordinary circumstances unless ere was assurance that the dock could not be profitably used for ordinary cking purposes. A shallow dock has been built at the Puget Sound Navy rd especially for shipbuilding purposes. A shipbuilding ways of this type particularly adaptable to this location on account of the extreme range of tide, arly 20 feet, which would have made the length and cost of an ordinary build- ; ways and the maintenance of the under water portion very expensive as mpared to other locations.

Shipbuilding Cranes. As this matter begins to come within the scope of e shipbuilding plant layout, and design, it will be very briefly dealt with as a

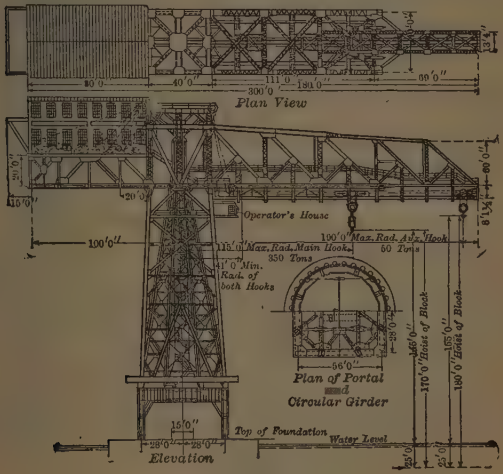


Fig. 91f. 350-Ton Fitting-out Crane

tter of general information. A simple and very effective method of handling frames, shapes, plates, built-up bulkheads, etc., for a ship is the employ- nt of two or more aerial cable ways, usually arranged longitudinally, picking material at the head of the slip and moving it longitudinally and trans- sely to the desired location. It has been effectually employed in merchant pbuilding, and is especially adaptable to certain types of work.

An effective type of crane is the light, fixt hammer-head tower crane. This ally has a lifting capacity of from 5 to 15 tons, and must be so arranged as cover the entire area of the vessel and be able to handle material from storage l assembly spaces alongside of the ways. They are usually grouped on either e of the ways, three, five or seven being employed. The arms must be put varying elevations so as to avoid interference and give the requisite clearance the depth of hull, etc.

A similar type crane is sometimes employed on one or both sides of the ways, but instead of being fixt travels on a standard gage or special gage track parallel with the ways and to one side, or cranes of this character may be of the jib or luffing type.

When a large complicated passenger vessel such as the transatlantic liners or a battleship or battle cruiser is to be built it has often been considered desirable to resort to more complete and extensive weight handling devices than can be afforded by the fixed or movable tower crane service. The building ways of several of the important shipbuilding plants are served by longitudinally traveling cranes; it will be noted in Fig. 91*e* that the twin ways of the Newport News Shipbuilding Co. are each served with one 75-ton and four 15-ton longitudinal cranes, which pick up material from the assembly and storage space at the head of the ways. Fig. 91*d* is a plan and cross-section of typical building ways and overhead crane structure employed by the Navy for the shipbuilding ways at the New York, Philadelphia and Norfolk yards The upper cranes each span one-half of the width of the ways, are of 15 tons capacity and the lower, larger crane is of 40 tons capacity. They handle material from the head of the ways and also are so arranged as to pick up material from cars running parallel with the ways. Section 91*f* is a side elevation and plan of a 350-ton hammer head fixt crane which is to be employed for placing of heavy weights on vessels after they are launched. It is designed to be placed on a special fitting out pier and vessels are brought to the pier and traversed longitudinally along its side so as to come within the reach of this appliance.

38. Cargo Handling Cranes

Cargo Handling devices and appliances for miscellaneous merchandise do not appear to any extent in successful use in American port practise, altho in more favor in European harbors. In American practise the cargo is unloaded and loaded largely by using ship gear, the hoists being made by ship boom or from gallows frames and uprights along the side of the pier shed; mixed cargo when handled on the pier is sorted by hand and distributed by man-handling on push trucks. In two-story pier sheds freight elevators, chutes, and continuous belt inclines are sometimes used.

In European practise, which is in favor in some larger South American ports, shore cranes are often employed for loading and unloading, the cranes being generally of the portable type. Numbers in the following examples refer to Fig. 92.

(1) and (2) Antwerp, Belgium, on the Scheldt. Quay wall runs entire length of the city. Principal wharfage capacity is derived from interior wet docks. Range of tide is 16 ft. Open sheds constructed some distance from quay wall. Portable crane spans outside railroad track; cranes handle cargo from the ship to cars, or to the face of the shed, in which it is handled by hand trucks for storage or transfer to cars in the rear for reshipment. Shed is for transfer or temporary storage. Warehouses for permanent storage are usually found with railroad connections close by.

(3) Hamburg, Germany, on the River Elbe. A greater amount of wharfage room is secured by interior basins. The plant of the Hamburg American Line is the one illustrated. The sheds are about 2500 ft long and 175 ft wide, and are from 30 to 35 ft from the face of the quay wall. In this space are operated electric cranes which span an outside railroad track which handles the cargo from steamer to railroad track or to side of ship. In interior of shed cargo is handled by hand trucks thru the shed for local storage or to carts or railroad tracks in the rear for direct delivery, or delivery to warehouses.

(4) Bremerhaven, mouth of the Weser on the North Sea. Tide has range of 8 ft, but on account of insufficient depth of water, wet docks are employed. Shows typical section of the North German Lloyd terminal. Sheds are about 200 ft wide, of corru-

gated iron siding wooden roof trusses, tar and gravel roofs. Electric cranes are provided for loading and unloading; cranes operate by picking up cargo from the holds of steamers by circular movement, swinging on to railroad cars on three lines of track, or to the sides of the shed. Loading and unloading are accomplished by this system on the shore side, and by barges and ship's boom and tackle on the water side.

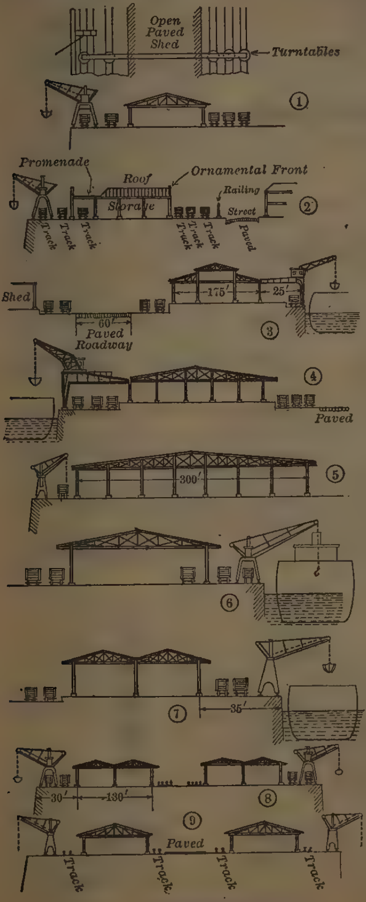


Fig. 92. Cargo Handling Methods

(5) Havre, France, at the mouth of the Seine. Range of tide 25 ft. Wet docks employed. Coffee and cotton landings. Sheds are 300 ft wide and 2600 ft long. The sides are continuous rolling doors, roof of tar and gravel, with glass skylights. Cargo is handled by crane from ship to side of shed or to railroad track.

(6) Southampton, Eng., American Line terminal.

(7) London, Eng., shows system of handling at Albert dock. Sheds are of light corrugated iron construction and run entire length of quays.

(8) London, Eng., Tilbury docks. (9) Glasgow, Scotland. Typical arrangement of cargo handling.

Movable Shore Cranes for miscellaneous cargo handling, while not generally employed in American practise, are usually employed in connection with handling of homogeneous cargo in bulk, various types of standard gage locomotive cranes being used for that purpose. Special cranes or traveling transporters are extensively used, the following types being shown in Fig. 93.

(1) One of four loading and unloading bridges at Rotterdam, Holland, 8 long tons capacity. (2) One of two transporters, 10 long tons capacity. (3) Coal loading and unloading bridge. (4) Loading appliances.

Fig. 94 shows type of heavy locomotive crane employed in connection with

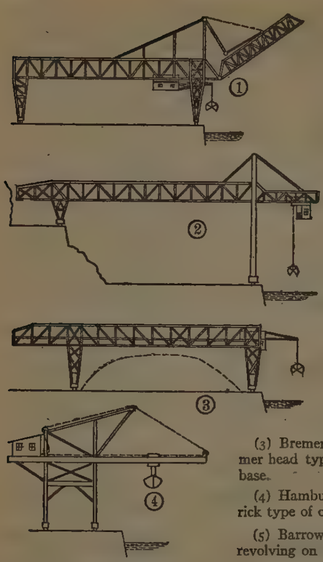


Fig. 93. Cargo Handling Travelers

ship repair and dry dock work. The crane is of 50 long tons capacity at 68 ft radius, of lesser capacities at greater reach and of 15 long tons capacity at 123 ft reach, and travels 120 ft per minute, with a maximum load. Cost, about $50,000. Cranes of this type are in extensive use in various U. S. Navy Yards.

Stationary Cranes. Fig. 95 shows types of fixed shore cranes for heavy lifting and for building or repair purposes.

(1) 150 ton hammer head crane at Bremen, Germany. Interior spindle is supported on base step bearing, the crane revolving in the 4 leg tower.

(2) 150 ton revolving crane at Kiel, Germany. Crane of the hammer head revolving type. Interior spindle, stepped at the base, operating within the 3 leg support.

(3) Bremen, Germany, capacity 100 tons, hammer head type, revolving on circular track at the base.

(4) Hamburg, Germany, 100 tons, stiff leg derrick type of crane.

(5) Barrow-in-Furness, Eng., 120 ton jib crane, revolving on pedestal base.

Cranes of the hammer head type have been constructed of 250 long tons lifting capacity. Fig. 96 illustrates English practise, and shows a crane at Portsmouth Dock Yard.

Fig. 97 illustrating German practise, and is a 250 long ton capacity crane constructed for Blohm and Voss at Hamburg, Germany, in 1912. The crane is supported at a bear-

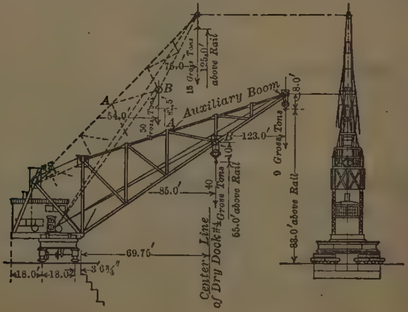


Fig. 94. Traveling Dry Dock Crane

ing on the top of the fixed spindle or tower, the enclosing structure revolves with the crane and bears against a track at the base.

Sheer legs are often used for heavy lifting. Fig. 98 showing a 100 long ton sheer leg with luffing jib device, the reach of the sheer leg being regulated by power operated horizontal screw, connected to the foot of the rear inclined member. This horizontal screw is sometimes replaced by a vertical quadrant with worm screw drive or pinion and racking. Sheer legs are also used for floating [redacted]

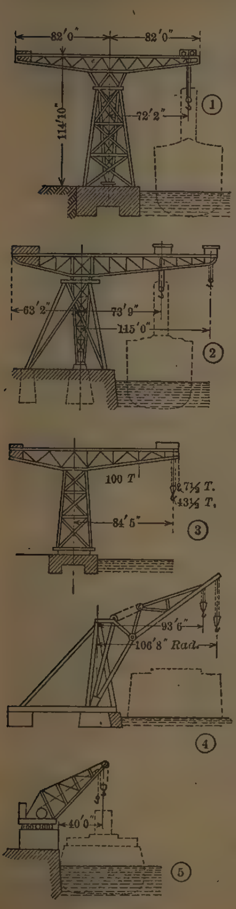


Fig. 95. Heavy Fixed Cranes

Floating Cranes are employed in connection with shipbuilding and repair, for loading and unloading heavy weights, handling lock gates, and for wrecking purposes. Fig. 99 shows the "A" frame boom type in extensive use in American practise. The "A" frame may be held in position by back stays of structural material or by wire cables, or in smaller derricks the "A" frame may be replaced by a mast. Cranes of this kind are in use from 10 tons to 250 tons capacity, the larger capacity being obtained in one crane by a lift directly over the bow, employing water ballast at the other end.

Fig. 100 shows a revolving tower floating crane of 75 long tons capacity. A crane of this type is employed in New York by the Dept. of Docks and Ferries, for placing quay wall concrete blocks. A similar crane is in use at Navy Yard, Boston, Mass. The load moment is taken care of by fixed ballast, water ballast, and the stability of the structure.

The Bridge type of floating crane is shown in Fig. 101, the bridge or trolley running thru the uprights so that the load may be transported from shore or vessel to the deck of the crane, or thru the crane to be handled on barge or shore at the other end. This crane was constructed in 1901, and rebuilt in 1910, and is at the Navy Yard, New York. Cost, $205 000. A similar crane is at the Navy Yard at Bremerton, Wash. Two bridge cranes of 150 tons capacity at the Navy Yards, Boston, Mass., and Pearl Harbor, H. T., cost slightly over $300 000 each. The load moment is counterbalanced by power, automatically operated counterweights or by water ballast.

An Incline Bridge Crane is illustrated in Fig. 102, and is in use by Schneider & Company, Chalons, France, built in 1911, of 25 tons capacity. The revolving jib type crane is shown in Fig. 103. One of 250 long tons capacity is self-propelling and is in use at Wilhelmshaven, Germany. Two similar cranes of same capacity, with less reach, are in use in the Isthmian Canal Zone, where they are to be employed for removing and placing lock gates, and for other purposes. These cranes cost about $450 000 each, and can lift 672 000 pounds. The pontoons are 150 ft long, 88 ft wide, and 15 ft 9 in deep.

39. Design of Cranes

The Design of cranes involves the field of mechanical, electrical, and civi engineering, and for floating cranes, also Naval architecture. For stationary o movable shore cranes, the structural features are analyzed and designed in accordance with prevailing structural steel practise, the operating machinery in accordance with electrical and mechanical machine design. Having decided on the required lifting capacity and the speed of lift, in order to obtain the horse power of the power unit, allowance must be made for the loss in friction in hoisting blocks, gearing, bearings, and operating mechanism. Three to five percent loss in friction should be allowed for each block sheave. The diameter of sheaves should preferably be 50 times the diameter of the wire cable and should not be less than 30 diameters. Steam power is generally used, altho on larger stationary or floating cranes electric motor operation has come into general use, generated by central plants or by isolated plants installed on the crane.

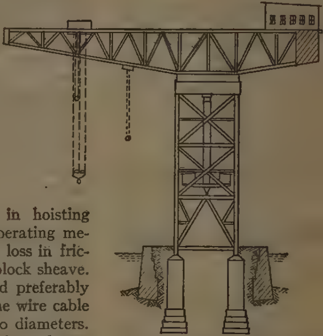


Fig. 96. 250 Ton Crane, Ports mouth, Eng.

With the application of the maximum load moment, structural steel work must be designed to carry this and also the wind load, taken at 10 lb per sq ft of exposed surface or without load, 40 lb per sq ft. For floating cranes the longitudinal and transverse stability is obtained, applying the formulas

$$GM = (I - B\,G.V - \Sigma i)/V = \text{Metacentric Height}$$

$$RM = \sin\theta/35\ \ (I - B\,G.V - \Sigma i) = \text{Righting Moment}$$

for which purpose the position of the center of gravity for the entire structure is secured. This calculation is made for the crane when light and when lifting the maximum load at maximum reach. The point of application of the lifted load will be at the center of the sheave pin on the jib or boom of the main structure. The overturning moment due to the lifting of this

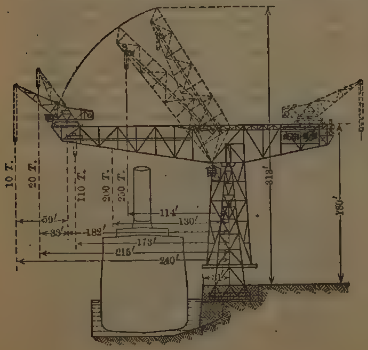


Fig. 97. 250 Ton Crane, Hamburg, Germany

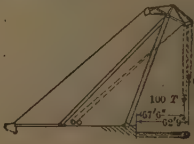


Fig. 98. Sheer Legs

load is the product of the load by the horizontal distance from the center of the load to the center of the float or pontoon, and the change of trim due to the application of

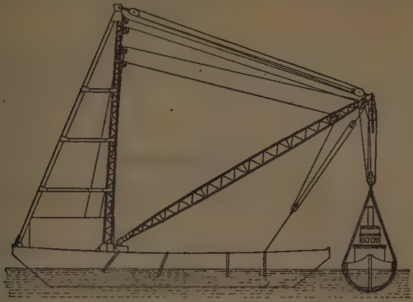


Fig. 99. "A" Frame Boom Crane

such load may be obtained in approximate terms by dividing this overturning moment by the moment due to inch change of trim.

Moment due to change of trim of 1 in $= D \times GM / Wl$

In these formulas GM = Metacentric height in feet; I = Moment of inertia in feet of water plane of flotation; BG = Vertical distance in feet from Center of Buoyancy to center of gravity; V = Volume of displacement in cu ft. D = Displacement in tons or lbs; Σi = Summation of moment of inertia in ft of interior water planes; Wl = length of waterline in inches. Half of the total change of trim will represent the reduction in

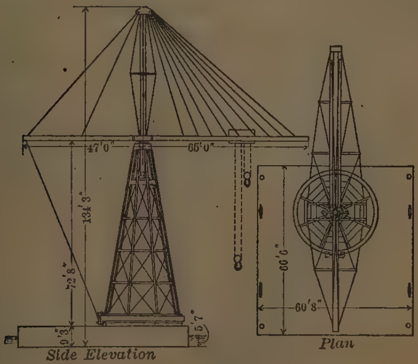


Fig. 100. Revolving Tower Crane

freeboard at one end of a rectangular pontoon and the increase in freeboard at the other end. The heel or inclination due to wind pressure is given by

Cot $\theta = (D \times GM) / (A \times C\ L)$

in which A = Total wind pressure in tons or lbs, and

$C\ L$ = distance in feet from center of pressure to center of lateral resistance.

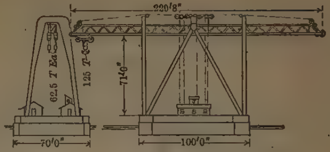


Fig. 101. Bridge Type Floating Crane

40. Port Development

The Growth and Development of a port involve many problems other than those of improving its strictly natural advantages by engineering works. The popular idea of the essential attributes of a port is its foreign water-borne commerce, with the necessary approaches, channels, quays, piers, docks, and structures to adequately receive, protect, and afford berths to vessels while loading or unloading. Imports and exports are generally considered as principally involved, little attention being given to domestic cargo, land-hauled freight, with the requirement of rail terminals and interchange facilities, which all go to complicate the intricate problem. Proper provision for water-front handling as it affects ocean-borne trade is undoubtedly necessary, as is also similar provision for domestic or coastwise shipping. However, incoming cargo is not consumed at the water-front of a port, or even within the port itself, nor does outgoing cargo originate on the water-front or even entirely come into being in the port. The water carriage in nearly all instances is only a part of the transportation involved. In view of this condition, port development at a large commercial center includes a proper provision for transfer between water carriers, land carriers, water and land carriers, and land and water carriers. Short land hauls from the

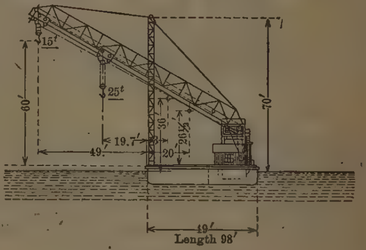


Fig. 102. Incline Bridge Crane

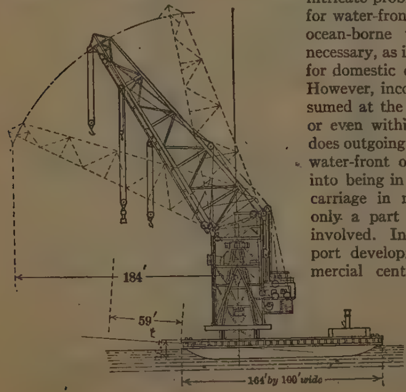


Fig. 103. Revolving Jib Type

water-front terminal may be made by cartage, but the bulk of land transportation would, of course, be by railway, so that any comprehensive port plan must take account of and provide for economical co-ordination of water and land carriers, with expeditions and complete interchange of freight at reasonable rates. In ports with a large and populous hinterland two or more railroad trunk lines with their terminal will be involved.

It is obvious that for outgoing and incoming cargoes these must be delivered or taken from all of the rail or other inland carriers and in turn delivered to, or taken from the vessel. A vessel, can of course be moved from pier to pier accumulating her cargo in that manner. Considering the usual high charter rate or per diem cost of large seagoing vessels, this involves loss of time and expense and must be avoided where possible so that it is customary to, in as far as possible, accumulate cargoes in advance.

Belt Lines, connecting main railroad feeders, are essential to proper port development, and should comprise in general an outer belt, which, while perhaps not a part of the port itself, serves to divert heavy traffic and afford an interchange between railroads without the freight tonnage having to go through the congested terminals of the port, and in that way relieves freight congestion in the port terminals; an intermediate belt, which affords means of transfer between port railroad terminals and upon which will often be located industrial works and factories of the port community itself, so that these may have ready access, by means of this belt, to interior shipping points or to water-borne carriers; a water-front belt, which serves as the direct interchange between water-borne and land carriers, and this belt may be an actual railroad belt or its equivalent in whole or part,—a lighterage belt, that is, the transfer is effected by lighter in the waters of the harbor and its tributaries, the lighterage taking place in loose bulk or packages or by lighterage of railroad cars or car floats. New York City is a conspicuous example of a large port with such a lighterage belt. The facility and economy with which such interchange and co-ordination are made by lighterage undoubtedly have contributed largely to the growth and prosperity of the port of New York.

Development of the port of New York thru lighterage has undoubtedly been an artificial one forced by certain economic competition between rail trunk lines who have absorbed in the freight rate all or a greater part of the lighterage cost.

Imports and Exports, while in the popular mind the most important business feature of a port, are in reality, in nearly all cases, the least important as regards tonnage, altho the providing of proper facilities for the export and import trade is essential to the continuation of the growth of the port. The very conditions involved have complicated the problems further, as it is found that in nearly all instances large commercial ports are thickly populated and the centers of populous districts, seldom or never entirely self-sustaining as regards food, clothing, implements, machinery, building materials, etc., so that a very large portion of the incoming freight, land- or water-borne, principally the former, is for local consumption or for redistribution from the port, inland or coastwise, as a manufacturing or sales center. This condition makes it necessary to give attention to features of port development other than those involved directly in export and import facilities.

Port Planning involves intricate economic studies based upon statistical research of present and probable future growth, character of local manufactured products, consumption of materials, rail and other transportation rates, which in all need the careful attention of experienced specialists. Haphazardly placed piers, wharves, warehouses, transit sheds, sometimes placed in too exposed locations and without due correlation to other economic requirements, have undoubtedly resulted in considerable loss of time and ill-advised expenditure of funds.

The Free Port has not been resorted to in American port policy. The nearest approach is in the bonded warehouse referred to in Art. 23. A free port is an area on the seacoast or on navigable water within the boundaries of which imports or foreign cargoes may be brought, unloaded and stored without being required to pay the legal tax impost of the country in which a port is located. The advantages of a free port are that importers and foreign merchants can store and retain dutiable goods within the area without the additional investment of the duty involved, and can retain such goods within the area of the free port until it is brought inland, when duty is collected at the boundaries of the port, or it may be trans-shipped without being subjected to the collection of duty or expense other than that of handling and storage. Raw materials may be worked into manufactured articles within the confines of a free port, and such articles trans-shipped without payment of custom fees or if taken over the boundary of a free port are subject to the impost on these manufactured articles. The various advantages of the free port have greatly stimulated the growth of such ports, as is witnessed in the case of Hamburg, and this method of port legislation and regulation might with profit be followed in American port economics.

41. Shipping Terms

Tonnage. When displacement of a vessel or docking capacity of a dry dock is referred to the tons are always long tons of 2240 pounds, and in all ship and dock computations 35 cu ft of sea water or 36 cu ft of fresh water are taken as weighing one ship ton.

The gross tonnage of a vessel is the measure of her cubic capacity in terms of 100 cu ft and is the volume in cubic feet of all enclosed spaces inside of frames divided by 100 cubic feet.

The net registered tonnage is in the same way the gross tonnage less the volume in 100 cu ft of certain machinery and other spaces not accessible to cargo. Such as crew space, steering-gear house, chain lockers, stores, etc.

The term "registered," as applied to net tonnage, refers to the measurements for that particular vessel as measured and registered by one of the national registry agencies, such as British Lloyds, Bureau of Veritas of France, or the American Bureau of Shipping. The displacement of a vessel is the weight of the vessel or water displaced by the vessel under any particular conditions and varies with the draft of the vessel at any particular time.

The dead weight tonnage of a vessel is the gross carrying capacity in tons and is the displacement of the vessel fully loaded, full draft to the Plimsoll mark, less the displacement of the vessel light and bare without coal, crew, and ship or crews' supplies. The cargo carrying capacity of a vessel in tons is something less than her dead weight tonnage; being this dead weight tonnage less the weight of bunker coal, crew, ship and crew supplies, etc.

The actual cargo carrying capacity of any vessel is dependent on the spaces or cubage available for cargo loading and dead-weight tonnage exclusive of fuel, etc., carrying capacity. The ideal arrangements for which stevedores contend is to take advantage of full weight carrying capacity and completely fill volume of cargo carrying holds. In general and for approximation purposes cargo carrying steamers in the ocean trade will carry about two-thirds of the tons of freight represented by dead-weight tonnage or about the number of tons weight represented by the numerals in the term "gross tonnage." Forty cubic feet are generally considered as representing a cargo ton.

In the design and construction of dry docks the weight to be lifted or the draft considered is that of the vessel light with no cargo and little coal. Dockage and other tolls are usually based upon the registered gross tonnage; an extra charge being made for cargo or excess fuel, etc. The Plimsoll mark is the mark made on the sides of the hull amidship to indicate the depth to which the vessel may be safely loaded.

SECTION 17

IRRIGATION AND DRAINAGE

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IRRIGATION

1. Fundamental Considerations

Definition. The practice of supplying water to crops by artificial means is called irrigation. Irrigation is essential in arid districts and may be profitable in semi-humid regions where the rainfall is occasionally insufficient to mature crops and for certain purposes in humid regions as an assurance against droughts. Irrigation water may be obtained from gravity flow by the diversion of natural streams or by pumping from surface or sub-surface supplies.

Localities where irrigation is essential to successful agriculture may be found in western United States and Canada, the western countries of South America, southern Europe, Egypt, Persia, India, Australia and other parts of the globe. The census of 1910 showed that in the United States there were approximately 32 000 000 acres of land under irrigation projects either wholly completed or in process of construction. Of this nearly 15 000 000 acres were being actually irrigated. The total expenditure for irrigation works in the United States up to 1910 was $308 000 000 and the total estimated cost of all irrigation works, including the cost of completing those under construction, was $424 000 000. The estimated cost of preparing 32 000 000 acres of land for irrigation was $443 000 000.

The Development of the Irrigation Resources of a country usually begins with the bottom lands lying close to the stream which provides the water supply. In the early days of irrigation, ditches were built to serve a single farm or a small group of farms. Later, community ditches were built and irrigation companies were formed which irrigated larger areas under a single system. In the United States and other countries whose irrigation resources have been extensively developed, the more accessible lands are under irrigation and the flow of streams during the irrigation period has been largely appropriated. There thus remains for development only the more inaccessible lands and those which must be supplied by the storage of flood flows and water not available during the irrigation season. In general, irrigation may be divided into four operations: the storage of water, the diversion of water, the conveyance of water, and the application of water to the land.

The Soils of arid districts extend to a greater depth and are commonly more fertile than those of humid localities. In the irrigated sections of the United States the soils are generally deep and of uniform texture. Hardpans are found near the surface of the ground in some localities, but they are usually dissolved by irrigation. Care should be taken to determine the crops to which an irrigated area is best adapted. This may depend upon the length of growing season as well as the soil formation. A strong growth of sage brush, bunch grass or other vegetation indicates fertile soil. Salt grass and grease wood are indicative of alkali. In general, the soils of arid districts are well supplied with lime, potash, and nitrogen, but are low in humus content.

2. Disposal of Irrigation Water

Soil Moisture occurs in three forms, usually termed hygroscopic, capillary, and gravity water. Hygroscopic water is the moisture which perfectly dry soil will absorb from the water vapor in the air. It either may form an extremely thin coating of the soil particles or be held in loose chemical combination. It can not be removed without artificial heat and has no value in sustaining plant life. Capillary water exists as a thin coating of the soil particles. It moves from a wet soil to a drier soil in any direction. Capillary water is of chief value in supplying moisture for vegetation. Gravity water is the water contained by soil beyond its capacity to contain capillary water. It moves downward by

gravity and is not retained by the soil, as such, if there is satisfactory drainage. After rains or if water is applied by irrigation, gravity water near the surface of the ground in passing downward may become capillary water in the lower strata.

The amount of hygroscopic water in the soil depends upon the temperature, humidity of the air, texture of the soil, and other less important factors. It varies from a minimum of about 0.5 percent of the weight of the soil for coarse sand to maximums of 8 and 17 percent respectively for silt and clay. The limit of soil to hold capillary water is usually reached when it contains from 15 to 20 percent of its weight of moisture. Water will rise from 0.5 to 2 ft in one day and from 0.7 to 4 ft in one week by capillary action. The rate of capillary movement increases with the fineness of the soil particles. The distance which moisture may be drawn by capillarity varies from about 2 ft for sand to 5 ft for fine silt or clay. Cultivated soils contain from 30 to 50 percent of voids which marks the limit of their capacity to hold water.

General Methods of Disposal. Irrigation water applied to land may be disposed of by (1) plant transpiration, (2) evaporation from the ground, (3) percolation to strata beyond the reach of plant roots and (4) surface waste. Only the water used in plant transpiration serves a useful purpose, and in successful irrigation disposal by the other three methods should be reduced to a minimum.

Plant Transpiration is the process by which growing plants obtain nutriment from the soil. Carbon, which constitutes about one-half of the dry plant, is taken by leaf action from the carbon-dioxide in the air. Plants contain from 2 to 10 percent of mineral matter obtained from the soil moisture in which it is dissolved. The remainder of the plant is composed of the elements of water combined with the mineral matter and carbon. There is a continual upward current of water through the plant. The soil moisture, containing mineral matter in solution, enters the hair roots of the plant, the water being dispelled by the leaves into the air in the form of water vapor. The rate of movement of the water usually varies from 1 to 6 ft per hour, but may be much greater than this. The rate of transpiration from the ground is greater for moist soils. It may be equivalent to a depth of ½ in over the ground surface per day when the ground is wet, but reduces as the capillary water in the soil becomes depleted.

Evaporation from Soil. The rate of evaporation from the surface of saturated soil may be nearly three times as great as the rate from free water surface in the same locality. Under ordinary field conditions, however, the evaporation is usually less than from free water surface. In general, the amount of evaporation decreases with the soil moisture. The evaporation is greatest from uncultivated ground where capillary movement of the moisture is continually bringing water to the surface of the ground. Cultivation of ground forms a mulch which greatly retards capillary activity and consequently reduces evaporation. A mulch of 6 in will save 80 percent of the water which would be evaporated from uncultivated ground.

Percolation. When irrigation water is applied to land the surface becomes saturated and the water gradually percolates to the lower strata. In the passage capillary water is retained. If the amount of water applied to the land is small enough, the percolation will not extend beyond the depth where it may be reached by the roots, but if a larger amount is applied, some of the water will pass beyond the reach of the roots and be wasted.

Sandy loam soils will retain about 2 in of water within a depth of 6 ft, but when more than this is applied, the greater part of it will pass to a greater depth. Coarse soils will retain less moisture than fine soils and therefore require more frequent irrigation.

Surface Waste. There is always a certain portion of irrigation water which is lost before it soaks into the ground This may occur in two ways, by evapora-

tion and by running off the surface of the ground. The first of these is ever present and can not be controlled to any extent. The latter loss, however, may be prevented or materially reduced by care in irrigating. Where proper care in using water is not exercised, this loss is frequently more than 20 percent of the water applied.

3. Water Requirements

Units. The unit of volume commonly used in irrigated districts to express storage or the quantity of water required to irrigate a given area of land is the acre-ft. It is the amount of water required to cover an acre of land 1 ft in depth or 43,560 cu ft. An acre inch is 1/12 of an acre ft. The unit used to express flowing water is cu ft per sec. One cu ft per sec flowing for 24 hrs is equal to 1.9835 acre-ft. A unit of measure for flowing water that is becoming obsolete in the miners inch (see Sect. 13, Art. 28).

The Factors Affecting the Quantity of Water required to mature crops are: the character of soil, the climate, the time of irrigating, the number of irrigations, and the method of irrigating. The first two of these conditions are fixed, but careful study should be given to each locality to determine the system of irrigating which will provide the most economical use of water. This matter has been carefully investigated by the United States Department of Agriculture and by many of the western states. It has been found that while the use of water up to a certain amount is beneficial, beyond this amount little if any apparent increase in the productivity of the land results. It has also been found that where the supply of water is insufficient to irrigate all of the available land lying under it, there is an economical limit to the amount of water which should be applied. In other words, a greater profit from a given quantity of water may, in some cases, be obtained by spreading it over a larger area. This point is illustrated in the following example by Widtsoe (Principles of Irrigation Practice, page 337), assuming that sugar beets are worth $5 per ton and the cost of producing them, including interest on investment, is $60 per acre:

30 acre-inches spread over	Inches of water on each acre	Yield of beets per acre (tons)	Total yield of beets (tons)	Price paid for ton of beets	Gross income from beets	Cost per acre	Total cost.	Net income from beets
1 acre........	30.0	21.0	21	$5	$105	$60	$60	$45
2 acres......	15.0	19.5	39	5	195	60	120	75
3 acres......	10.0	18.6	56	5	280	60	180	100
4 acres......	7.5	16.3	65	5	325	60	240	85

The above table shows that the greatest yield per acre is obtained by applying the 30 acre-inches of water to 1 acre of land, but the greatest net income for this quantity of water is obtained by spreading it over 3 acres.

The Character of Soil greatly affects the amount of water used for irrigation. From two to four times as much water is used on coarse sandy soils as on silt and clay soils. On the Umatilla project in Oregon, where a coarse sandy soil predominates, an average of nearly 16 acre-feet per acre was used in 1912. By proper methods of irrigating the amount of water required may be greatly reduced. On coarse soils much water is usually lost by percolating to the lower strata beyond the reach of roots. This can, in a large measure, be avoided by irrigating only a small area at one time and using a large head of water so that the area can be irrigated quickly. Since coarse soils have less capacity for holding capillary water than fine soils, they require more frequent irrigations, but a

smaller quantity of water at each application. In general, the more fertile the soil the less water is required. Cultivating the soil forms a mulch over the surface and thus reduces the evaporation and the water requirements.

The Kind of Crop has an important bearing on the amount of water required. Excepting fruits the water required will vary with the length of the growing period. Alfalfa, which grows thruout the entire season, requires more water than plants of quicker growth.

The Quantity of Water Used for different crops in the United States as determined from extensive experimental data is given approximately in the following table. Frequently much larger quantities than the maximum given are used, but in most cases these could be reduced by better methods of irrigating. Results of investigations in southern Europe, Egypt, India and Australia indicate requirements practically the same as the United States.

Water Requirements for Different Crops

Crop	Soil	Water requirements in acre-feet per acre		
		Average maximum used	Average minimum used	Average for most economic use of water
Alfalfa (three cuttings)	Clay and sandy loam	4.0	1.5	2.5
	Porous sandy soil	10.0	4.0	6.0
Grain	Clay and sandy loam	3.0	1.0	1.5
	Porous sandy soil	5.0	1.5	2.5
Potatoes	Clay and sandy loam	3.0	1.0	1.5
	Porous sandy soil	5.0	2.0	2.5
Sugar beets	Clay and sandy loam	3.0	0.5	1.2
	Porous sandy soil	5.0	1.2	2.0
Deciduous fruits	Clay and sandy loam	2.0	0.7	1.0
	Porous sandy soil	4.0	1.5	2.0
Citrus fruits	Clay and sandy loam	3.0	1.2	1.7
	Porous sandy soil	5.0	2.0	3.0

The Times to Irrigate to secure the best results from a given water supply will be such that come nearest to fulfilling the ideal condition that the soil shall have an adequate and uniform moisture content thruout the growing period. Since transpiration from large plants is greater than from smaller ones, and also because maximum growth is obtained late in the summer when evaporation is greatest, more water should be applied during the latter part of the growing season. After vegetation has attained its full growth and becomes well matured, but little water is required. When water is obtained directly from streams this ideal condition is not usually attainable, since the water supply which is plentiful during the early part of the growing season usually decreases rapidly as the summer advances. If the source of supply is from storage reservoirs water is available when needed. An important consideration is that the ground should be thoroly moist at seeding time or when plant growth begins. To accomplish this, fall, winter or early spring irrigation has been practiced with excellent results. If the fall and winter rains are heavy enough to supply adequate spring moisture such irrigation is unnecessary. In localities where water is not available during the growing period, short-season crops such as grains and early fruits have been grown successfully with fall and winter irrigation only.

If the soil is well filled with moisture at the time plant growth begins in the spring, the first irrigation of the crop should be deferred until the need for more water is apparent. This permits the roots to obtain a more vigorous growth. After the first irrigation successive irrigations should follow as required. As a rule frequent irrigations using comparatively small quantities of water are advisable. The usual practice, for fine soils, is to apply from 3 to 5 in of water at each irrigation. A smaller quantity of water should be used for coarse soils, if practicable, but the irrigations should be more frequent.

The Number of Irrigations varies with the crop and the character of the soil. For fine soils, alfalfa should be given one or two irrigations for each cutting; potatoes, sugar beets, corn and similar crops are usually irrigated from 3 to 5 times, the bulk of the water being applied from July 1st to August 15th in the northwestern states; grains are given from 1 to 3 waterings near the time of seed formation; deciduous fruits require a fairly uniform distribution of moisture, with rather more at the time fruit is forming; citrus fruits require irrigation thruout the year, with the maximum requirement in the fall. Coarse soils require more frequent irrigations than the finer soils.

Duty of Water is a coined term which has been used variously to express the water used and the water required for irrigation. The meaning of the term is ambiguous and its use could well be dispensed with. As more commonly applied the duty of water for a given crop means the quantity of water used for irrigating the crop, as for example, 2 acre-feet per acre or a continuous flow of 1 cu ft per sec to 80 acres. The net duty of water for an irrigation project may mean the average rate of use per acre for the project, water being measured at the farmers' intakes. The gross duty of water will then be the average rate of use per acre measured at the diversion, and thus includes all conveyance losses.

4. Conveyance Losses

Seepage and Evaporation. All canals and other conduits used for conveying water are subject to losses from seepage and evaporation. Earth canals form fully 90 percent of the water-carrying systems of irrigation projects and from these, seepage losses usually amount to a large percentage of the water diverted. Evaporation losses in earth canals are always much less than seepage losses and the two sources of loss are usually considered together. When water is first turned into a canal, the banks and sides of the canal absorb water rapidly until the ground has been filled to its capacity with capillary water, after which there is a steady and fairly uniform loss from gravity water. The coarser the soil, the greater the seepage. If canal waters contain silt this will tend to deposit in the pores of the soil and reduce seepage.

Less important factors that are believed to influence the amount of seepage are the depth of water in the canal, velocity of water, and temperature of soil and water. When canals are located below other irrigation systems there may be an inflow of water which, in a measure, will counteract the effects of seepage and evaporation. Conveyance losses in canals may be expressed conveniently as a function of the wetted area. Extensive experiments in this and other countries have been made to determine the losses for different soils. The following table represents approximately the mean of these investigations for canals that have been in service five or more years, but it should be considered as subject to a possible error of at least 25 percent. The main difficulty will be found in proper classification of the soil.

Canal Linings have been used extensively to reduce seepage losses. The most satisfactory linings that have been developed are concrete or cement mortar, crude oil, and clay puddle. Which of these should be used depends upon the materials that are most readily available and other local conditions. Concrete linings are also effective in reducing friction losses and in the case of

canals excavated in rock this consideration may be more important than the prevention of seepage. Canal linings are always expensive and for this reason they have not been used extensively except where water is very valuable. In southern California many entire canal systems are lined thruout, while in other localities only those portions of canals are lined where seepage losses are greatest.

Seepage and Evaporation Losses in Canals in Second Feet per Mile for Average Conditions

Wetted perimeter of canal section in feet	Kind of material							
	Clay	Clay loam	Sandy clay loam	Sandy loam	Fine sand	Coarse sandy soil	Sandy soil with some gravel	Very gravelly soil
5	.08	.12	.27	.38	.49	.61	.92	1.53
10	.15	.24	.55	.76	.98	1.22	1.83	3.06
15	.23	.37	.82	1.15	1.47	1.83	2.75	4.58
20	.31	.49	1.10	1.53	1.96	2.44	3.67	6.11
25	.38	.61	1.37	1.91	2.44	3.06	4.58	7.64
30	.46	.73	1.65	2.29	2.93	3.67	5.50	9.17
35	.53	.86	1.92	2.67	3.42	4.28	6.41	10.69
40	.61	.98	2.20	3.06	3.91	4.89	7.33	12.22
45	.69	1.10	2.47	3.44	4.40	5.50	8.25	13.75
50	.76	1.22	2.75	3.82	4.89	6.11	9.17	15.28
55	.84	1.34	3.02	4.20	5.38	6.72	10.08	16.81
60	.92	1.47	3.30	4.58	5.87	7.33	11.00	18.33
65	.99	1.59	3.57	4.97	6.36	7.94	11.92	19.86
70	1.07	1.71	3.85	5.35	6.84	8.56	12.83	21.39
75	1.15	1.83	4.12	5.73	7.33	9.17	13.75	22.92
80	1.22	1.96	4.40	6.11	7.82	9.78	14.67	24.44
85	1.30	2.08	4.67	6.49	8.31	10.39	15.58	25.97
90	1.37	2.20	4.95	6.87	8.80	11.00	16.50	27.50
95	1.45	2.32	5.22	7.26	9.29	11.61	17.42	29.03
100	1.53	2.44	5.50	7.64	9.78	12.22	18.33	30.56

Concrete Linings are made of various thicknesses ranging from ½ to 5 in. For the thinner linings only very fine aggregate should be used, and for linings 1 in or less in thickness a cement mortar is preferable. The proportions of cement to aggregate should be about 1 to 8 for the thicker linings and 1 to 4 for linings less than 1 in thick. When properly made, concrete linings will reduce the amount of seepage from 75 to 95 percent, the thicker linings being more effective. There is also a tendency for silt to deposit in the pores of the concrete which renders it more impervious with age. For a lined canal of a given area, the cross-section should be such as will give the maximum hydraulic radius practicable, but the slopes of the banks should not be less than the angle of repose of the material thru which the canal is constructed. Canal linings of cement mortar from ½ to 1 in thick which have been constructed in western United States have given excellent service. By using a comparatively dry mixture, concrete may be placed without forms on canal banks having a slope of 1 to 1. For steeper slopes forms will be necessary. The very thin linings should be used only for small canals. The following is the approximate range of costs of canal linings in western United States. In a few cases the costs have been much greater than are here given:

Thickness of lining in inches	Cost in cents per sq ft	Thickness of lining in inches	Cost in cents per sq ft	Thickness of lining in inches	Cost in cents per sq ft
½	2.5 to 3.5	1½	4.2 to 6.0	3	6.6 to 9.7
¾	3.0 to 4.1	2	5.0 to 7.2	4	8.4 to 12.3
1	3.4 to 4.8	2½	5.8 to 8.6	5	10.1 to 14.8

Oil Linings are made by sprinkling from 1 to 3 gal per sq yd of crude oil on the sides and bottom of the canal. Several light applications are used, and in some cases the surface is worked with a rake or harrow after each application. Good results have been obtained, however, by leaving the surface undisturbed after applying the oil. The oiled surface is generally rolled or otherwise compacted. Experiments indicate that oil linings when new will reduce seepage losses in canals from 50 to 75 percent. They deteriorate, however, and their effectiveness diminishes with age. The average cost of preparing oil linings in the United States, not including the cost of the oil, has been about 1.5 cents per gal of oil used.

Clay Puddle Linings have been used in localities where suitable material is available within a reasonable distance. The clay is deposited in a layer from 3 to 6 in thick along the bottom and sides of the canal, after which water is turned into the canal and the material is puddled by harrows or the tramping of animals. A good clay puddle lining will reduce the seepage loss by from 60 to 80 percent. It does not deteriorate, but becomes more effective with age. Fine surface soil thrown into a canal containing water and thoroly stirred up by teams dragging heavy chains or otherwise agitated has in some cases been quite effective in reducing seepage.

5. Water Supply

Total Water Supply. One of the first considerations in the investigation of an irrigation project is the determination of the amount of water available for the project. The discharge of the stream or streams forming the source of supply should be carefully investigated. A knowledge of the distribution of the flow as well as the total annual discharge for a period of years is essential. The best data on which to base an estimate of future discharge are continuous discharge records covering a period of years. Such records, for many streams in the United States, are published in the Water Supply and Irrigation Papers of the U. S. Geological Survey. They may also in some instances be obtained from State Engineers' offices or from private sources. On many streams, however, available discharge records cover only a comparatively short period or they may be entirely lacking. It also may be found that discharge measurements on a stream have been made at a place quite remote from the proposed point of diversion. Precipitation records are valuable for supplementing incomplete stream discharge data. Such records, published by the U. S. Weather Bureau, are available at one or more points for practically every drainage basin in the United States.

The safest data for estimating future discharges are actual discharge measurements, near the place of diversion, covering a period of not less than ten years. A record for this period will probably include ordinary high and low water stages of the stream, and show the general distribution of flow thruout the year. If a project is under investigation and discharge records near the place of diversion are not available, a gaging station should be established at the first opportunity. If there is any doubt as to the adequacy of the water supply for a proposed project, construction work should be delayed until supplementary data sufficient to provide a satisfactory estimate of discharge have been obtained. In general, at least 2 years of continuous discharge records should be available near the place of diversion and these should be supplemented with such additional data that a reasonably reliable record for a period of at least 10 years may be estimated

Prior Rights to Water. In estimating the water available for irrigation from a given stream, all prior rights should be investigated. In western United States the streams are controlled by the states and the laws regulating their use are not uniform and are continually being changed by new statutes and court decisions. The state laws governing the stream should therefore be understood. The waters of some streams have been adjudicated by the states and in such cases records of prior rights are obtainable at State Engineers' offices. In many instances it will be found that the records of water rights are in such a condition that an examination of each canal diverting from the stream must be made to determine the amount of water actually used and the area of land under it that is entitled to water.

Water Supply Available. The difference between the total water supply and the quantity of water represented by prior appropriations gives the water supply available. The estimate of available water supply is usually prepared by months by deducting from the discharges for each month of the period covered by measured or estimated discharges the water represented by prior rights for corresponding months. These differences give the estimated amount of water that would have been available for each month during the period considered and if the period is long enough (preferably at least 10 years) they may be taken to represent the available future supply.

The Water Supply Required for a given area will be that necessary to provide for the irrigation of crops and in addition all seepage and evaporation losses in canals and reservoirs. (Arts. 3 and 4.) An estimate of the water supply required should be prepared for each month by adding the estimated seepage and evaporation losses. By comparing the water available with the water required, the deficiency for any month of the period covered by records may be determined. If storage is to be provided, such deficiencies will give the amount of storage required. About 20 percent of an irrigable area will be occupied by roads and buildings or for other reasons will not be irrigated. This area should be deducted for purposes of estimating water requirements for an irrigation project.

6. Canal Systems

The System of Conduits and structures necessary to convey water from the place of diversion to the farmers' laterals is commonly called the canal system. It may consist of any or all of the following: Canals, flumes, pressure pipes, tunnels, drops, wasteways, and various small structures. These should be so designed and constructed as to provide for the economic delivery and distribution of the water used by the project.

Canal Sections. (See Sec. 11, Art. 24.) The canal section of a given area which will provide the greatest carrying capacity, other things being equal, is the one having the greatest hydraulic radius. With the side slopes ordinarily

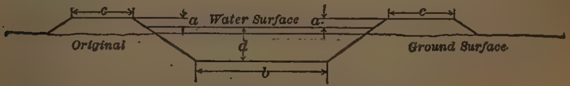

Fig. 1. Canal Section for Level Ground

necessary in earth canals the section of maximum hydraulic radius has a bottom width less than the depth. There are practical objections to the use of such sections for earth canals. Figs. 1 and 2 represent respectively cross-sections

of earth canals for level and sloping ground. As commonly constructed $b = 3d$ to $4d$, $a = 0.4d$ to $0.6d$, $c = 1d$ to $2d$. The side slopes of canals in level ground and of the embankment side in sloping ground are usually 1 on 2 or 1 on 1½. On steep hillsides the uphill side of the canal may have a slope of 1 on 1, all slopes being kept as steep as possible in order to reduce excavation. In designing canals the cross-sectional area of the excavation is usually made about 10 percent greater than that of the embankments to allow for waste and shrinkage.

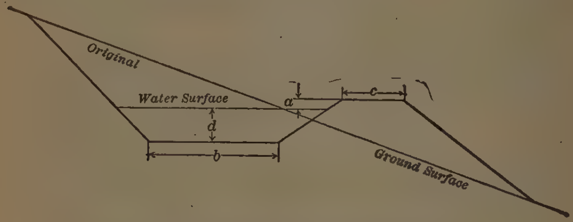

Fig. 2. Canal Section for Sloping Ground.

The Most Economical Velocity of water for earth canals is usually the maximum velocity which may be maintained without erosion. (See Sect. 9, Art 17.) This permits the smallest cross-sectional area of the canal prism and results in a minimum construction cost. The lower velocities are also objectionable in that they may permit the deposition of silt and the growth of aquatic weeds and grasses. In exceptional cases velocities lower than would otherwise be advisable may be used to conserve head. The velocity at which erosion begins varies with the character of material, the following being approximately the maximum allowable mean velocities in ft per sec for different kinds of earth:

Very light find sand	1.0 to 1.5	Clay	3.0 to 4.0
Coarse sand	1.5 to 2.0	Fine gravel	3.5 to 4.0
Sandy soil	1.8 to 2.2	Coarse gravel	4.0 to 5.0
Light alluvial soil	2.0 to 2.5	Cemented gravel or hardpan	6.0 to 7.0
Clayey loam	2.5 to 3.5	Rock	8.0 to 15.0

In a canal designed to carry water at a given velocity care should be taken to maintain this velocity at all points. Where a drop is constructed in a canal the opening in the drop should be just sufficient to discharge the capacity of the canal while maintaining the normal depth of water. Similar precautions should be taken above any structure which modifies the conditions of flow. A disturbance usually exists for some distance above and below any structure which requires a change in velocity and riprap or some other form of protection for the bottom and sides of the canal is usually necessary. The velocity of the water will accelerate at the concave side of a bend in a canal and on sharp curves some protective lining may be necessary.

Canals in Rock are excavated with steep side slopes, commonly 1 on ¼, and with bottom widths not more than two times the depth. They are also sometimes built with a semi-circular cross-section. Canals excavated in rock are usually lined with concrete. For canals in hardpan or other indurated material the conditions are intermediate between those for earth and rock.

Flumes (see Sect. 11, Art. 15) are used to convey water across streams, ravines or other depressions of the ground, and also around hillsides to avoid difficult

excavation. They are built of wood, steel and concrete, or of combinations of these materials. The substructure may consist of piers or posts of concrete or of wooden or steel posts or trestles. Trestles should be braced longitudinally and posts should be well tied together with both transverse and longitudinal bracing. Concrete bases or mud sills may be used for supports. Flume channels usually have rectangular cross-sections when constructed of wood

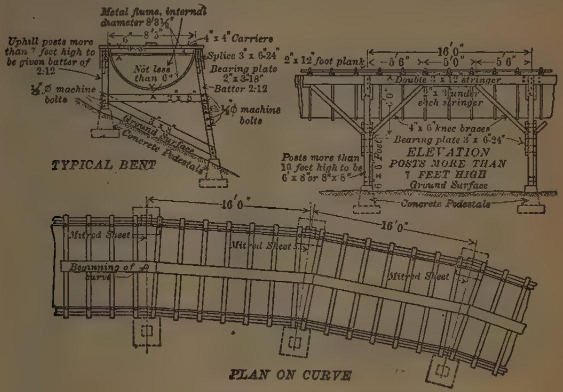

Fig. 3. Steel Flume with Wooden Substructure.

or concrete and semi-circular cross-sections when constructed of steel. Fig. 3 shows a steel flume with wooden substructure and concrete pedestals. Short flumes have been built over 100 ft high, but as a rule flumes are better adapted to conditions where the height is not much over 20 ft. For greater heights especially for long depressions pressure pipes will usually be more economical.

Pressure Pipes (see Sect. 10, Art. 21 and Sect. 11, Arts. 13 and 14) for irrigation purposes are usually built of wood, steel, or reinforced concrete. For small quantities of water, if the pressure head does not exceed 20 ft, vitrified sewer pipe and plain concrete pipe have proved satisfactory. Pressure pipes are used to carry water across depressions or around steep hillsides; small pressure pipes may be used as laterals for distribution systems.

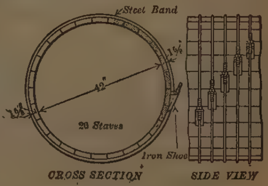

Fig. 4. Wooden Stave Pipe

Reinforced concrete pipes are best adapted to pressure heads of less than 100 ft. Wooden stave pipes have been used for heads of 300 ft or more, tho 250 ft is probably very near to the economical limit of such construction. For the higher heads, steel is used exclusively. The average life of steel pipe is greater than that of wooden pipe, but the life of each is dependent upon the care with which it is maintained. (Art. 32.) Under ordinary conditions of use, concrete pipe should last indefinitely. Wooden pipe being smoother than concrete or steel has a greater capacity for a given diameter. De-

posits will form on the inner surface of a steel pipe, which may reduce its capacity from 25 to 30 percent after a few years of service. In determining the most economical pipe for a given locality, where continuous service is required, the first cost, life, and carrying capacity of each kind of pipe should be considered. Fig. 4 is an example of a wooden stave pipe, as manufactured by the Washington Pipe and Foundry Company.

Tunnels (see Sect. 11, Arts. 32 to 41 incl.) may be constructed across points to shorten canal lines, or thru divides to divert water to a different drainage basin. They are also built in the sides of rocky canyons where canals, flumes, or pressure pipes would be expensive to maintain. In difficult side hill location, the water may be carried successively in canals, flumes, tunnels, or pipes, following each other in any order.

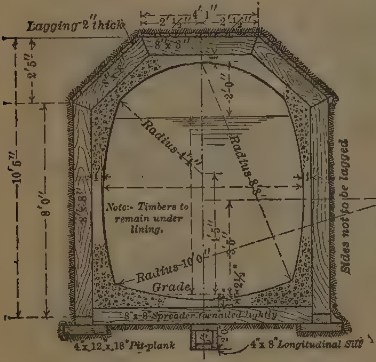

Fig. 5. Strawberry Tunnel, Shale Section.

Tunnels for irrigation purposes are usually lined with concrete, tho short tunnels in firm rock are sometimes left unlined. Concrete linings serve the double purpose of preventing caving of tunnels and of reducing resistance to the movement of water, thus increasing their capacities. The minimum dimension of the cross-section of a tunnel, except for very short tunnels, should not be less than 6 ft and preferably 7 ft in order to allow free movement of the workmen. Beyond this limit they should be kept as small as practicable in order to reduce the cost. Velocities of 10 to 12 ft per second in concrete lined tunnels are permissible. Fig. 5 shows a cross-section of the Strawberry Tunnel, Utah. The following table gives data relative to some of the more important tunnels of the U. S. Reclamation Service. All tunnels are concrete lined.

Data for Tunnels of U. S. Reclamation Service

Name of tunnel or project	Shape	Maximum height, ft	Maximum width, ft	Length, ft	Capacity cu ft per sec	Area, sq ft	Cost per lin ft
Gunnison......	Arched...	11.4	10.5	30 580	1300	100	95
Strawberry.....	Arched...	8.5	7.0	19 900	500	56	60
Tieton........	Circular..	6.1	6.1	10 860*	336	29	30
Truckee Carson.	Arched...	14.3	12.0	2 725†	1200	160	44
Belle Fourche..	Horseshoe	8.0	8.0	1 306	320	54	
Huntley.......	Arched...	9.0	9.2	2,654*	400	76	45

* Total length of four tunnels. † Total length of three tunnels.

The Grade and elevation of water surface at the controlling points of a canal system should be thoroly investigated. Special consideration is required wherever a change in the velocity of the water occurs. When water from a canal

enters a pipe or culvert, changing to a higher velocity, allowance should be made for velocity head and head lost at entrance. A similar allowance should be made where a canal is changed to a smaller section. In this case the difference in elevation of water surfaces above and below the change in section represents head allowance for entrance conditions and change of velocity. If change from a higher to a lower velocity is made gradually some of the velocity head may be converted into static head.

In designing a canal or other structure for the passage of water the engineer should make liberal estimates of capacities. Uncertainties as to the proper values of coefficients necessarily exist and in general it will be safer to design a structure with a capacity slightly greater rather than less than that required.

7. Drops and Chutes

The Grade in Earth Canals should be ordinarily that which will give the greatest velocity that may be carried without eroding the sides or bottom of the channel. When, therefore, it is desired to drop an earth canal to a lower elevation, the excess grade must be taken up by a structure designed for the purpose. The function of such a structure is to maintain the normal velocity of the water in the upper canal and deliver it at the normal velocity to the canal at the lower elevation. Two types of structures, commonly called Drops and Chutes, have been used for this purpose.

Drops are structures which concentrate the fall at one or more points. They usually are built of concrete or wood, the latter being satisfactory for temporary construction. The essential parts of a drop are the brest-wall, which extends across the channel and connects the upper and lower canals, the water cushion or platform on which the water falls, the various wing walls which connect the

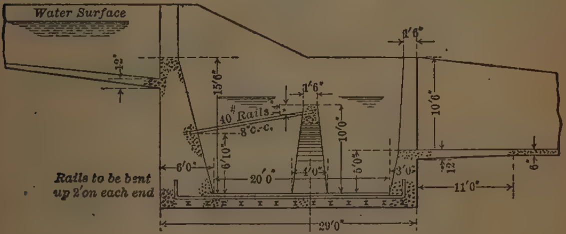

Fig. 6. Drop with Grating and Water Cushion

structure to the sides of the canal, and the canal linings, which prevent erosion immediately above and below the structure. The brest-wall extends from the elevation of the highest water surface in the upper canal to the bottom of the lowest part of the structure. Sufficient opening should be left in the portion of the brest-wall which extends above the bottom of the upper canal to discharge just the required volume of water while maintaining the normal depth of water in the canal. The opening in the brest-wall may be either a weir or one or more notches. Trapezoidal notches, having the narrower of the two parallel sides flush with the bottom of the canal, are frequently used. Such notches may have semi-circular lips projecting from the base which causes the water to spread out and reduces the impact of the fall.

The notched form of fall crest with water cushion below has been used with excellent results. It appears to reduce the difficulties from erosion above and below the fall

Steel rails placed under the fall have not proved very satisfactory. Cut-off walls should be built into the backfilling, which should be moistened and thoroly tamped to prevent the water from breaking thru. Reinforced concrete has largely replaced other materials for the construction of drops. Fig. 6 shows a section of a 10-ft drop with grating and water cushion at Uncompahgre, Colorado. Ordinarily, drops are not built more than about 15 ft high. For greater heights two or more drops in a series may be used. The amount of excavation for the canal leading from the drop increases as the slope of the ground decreases. In gently sloping ground a series of low drops may, therefore, be more economical than one higher structure.

Chutes are either lined canals or flumes, constructed on steep grades which connect two canals of different elevations. In addition to the conduit a chute must have a structure at the entrance to connect it to the upper canal and a structure at the exit to retard the velocity of the water before discharging it into the lower canal. The opening at the entrance to the chute should be just large enough to discharge the required volume of water while maintaining the normal depth of water in the canal. The conduit usually discharges into a water cushion in which baffles may be built to help break up the high velocity of the water. Canals should be paved for a short distance above and below the chute. The conduit may be of any length, and chutes several hundred feet long are not uncommon. Chutes with short conduits built on steep inclines are used in place of drops, Fig. 7 shows a reinforced concrete chute with a short conduit, of the Okanogan project, Washington.

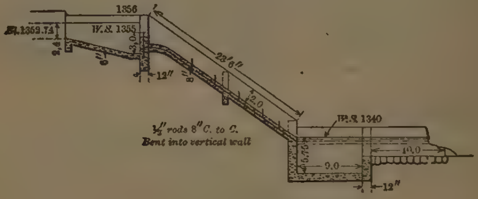

Fig. 7. Reinforced Concrete Chute

The velocity of water in a chute will accelerate until it is just great enough to overcome the frictional resistance. To determine the required dimensions of the conduit it should be divided into short reaches and the hydraulic computations made for each reach. The rate of reduction in area of the cross-section of the conduit will be more rapid at the upper end.

8. Minor Structures

Canal Crossings for Natural Waterways. At every place where a canal crosses a natural drainage channel, whether such channel carries water continuously or at infrequent intervals, provision must be made for conveying the natural drainage to the lower side of the canal. This may be done in either of the following ways: (1) by carrying the water under the canal; (2) by carrying the water over the canal; (3) in the case of small flows which may be expected only at rare intervals, by allowing the water to flow into the canal with provisions for it to escape over the opposite embankments; (4) by intercepting ditches above the canal which collect the drainage from small channels and convey it to some other channel for which a crossing is provided. Before designing a structure for this purpose a careful estimate of the maximum flow that the

channel may be expected to carry should be made, and a liberal waterway should be provided in all cases.

If the bottom of the canal is high enough natural drainage waters in small quantities may be carried under the canal in culverts or pipes. For crossing streams where large flows may be expected, it will generally be safer to convey the canal water over the stream in a flume, preferably supported by a truss, or under it in a pressure pipe. Short flumes or overchutes are sometimes used to carry small quantities of water over canals. Where drainage water is allowed to enter a canal a depression should be left in the upper embankment which, together with a short section of the canal, is usually lined with concrete. The objection to this method is that sediment is washed into the canal.

Wasteways are structures which may, in an emergency, divert the entire flow of a canal into a natural drainage channel. They are intended primarily as a safeguard to the canal and should preferably be located above the weaker portions where breaks are most likely to occur. In case of a break or other trouble, if there is a wasteway above, the water may be turned out of the canal more promptly than if the headgate alone must be relied upon for this purpose and the resulting damage will be less.

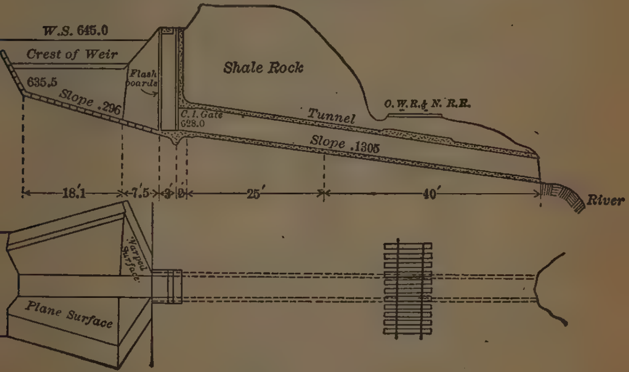

Fig. 8. Wasteway from Canal

The essential features of a wasteway are a system of gates in the lower embankment of the canal and provision for diverting the water thru them. Many different types of wasteways have been built. Usually the gate sills are placed at an elevation lower than the bottom of the canal in order to increase the head on the gates. A low submerged weir is sometimes placed across the canal below the gates to help divert the water thru them. A similar weir upstream from the gates will retard the velocity of the water in the canal when the gates are open and reduce the erosion above the structure. If built in earth the canal for a short distance above the wasteway and the channel leading from it should be concrete lined or paved. Fig. 8 shows a wasteway discharging thru a tunnel of the Umatilla project, Oregon.

Spillways are constructed by depressing the lower embankment of a canal to the elevation of the highest water surface which the canal is designed to carry. A spillway in an earth canal should have its crest and both slopes of the embankment lined with concrete. Spillways protect a canal against overflow, when drainage water enters it, when the water is backed up by an obstruction in the canal, or when a sudden rise in the stream from which the diversion is made allows too great a volume of water to enter the headgates.

Bridges must be provided where a canal is crossed by road or railway. Highway bridges may be built of wood or steel, or concrete arches may be used. Wooden bridges resting on concrete piers and abutments are in very common use. For earth canals, where the velocity of the water is low, piers or posts in the channel are not objectionable except that they create a slight tendency to erosion in their vicinity.

At Railway Crossings it may be found that the road bed is too low to allow clearance for the water under a bridge. In such cases short inverted siphons or pressure conduits are used. The intake and outlet of short pressure conduits should be carefully designed to provide for the entrance and exit of water with the least possible disturbance, in order

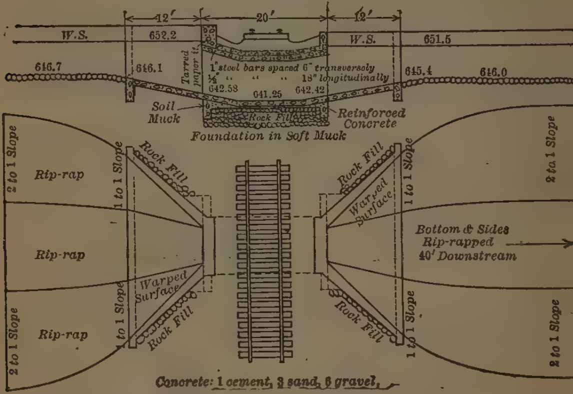

FIG. 9. Canal Crossing under Railroad

to make the crossing with a minimum loss of head. Fig. 9 shows a crossing of the Feed canal of the Umatilla project under the O. W. R. and N. railway.

Sand Gates and Sand Traps. Streams used for irrigation frequently carry large quantities of silt, especially during flood stages. Diversion works on such streams may be designed to exclude a portion of the silt, but some of it will usually enter the canal. The finest sediment will be carried in suspension by the minimum velocities that usually obtain in a canal system while the coarser material will be rolled along until it is finally deposited in the bottom of the canal. The fine material that is carried thru a canal system to the irrigated land is a valuable fertilizer but the coarser material which is deposited in canals reduces their capacity and is objectionable. The water in canals acting as feeders to reservoirs should be made as free from silt as practicable in order to reduce to a minimum the deposition of silt in the reservoir.

Different types of structures have been designed to remove the coarser material carried by canals. In general such structures consist of an enlarged canal section which terminates in a low weir placed across the channel or a basin having its bottom depressed below the subgrade of the canal, with sluice gates at the lower end. Owing to the retarded velocity of the water at this place the coarser sediment will move closer to the bottom or be deposited and a portion of it may be scoured out or allowed to pass out thru the sluice

gates. Frequently sand traps are built a short distance below headgates to remove the sediment as soon as possible after it enters the canal. In such cases, when there is more water in the river than is required for the canal, the sluice gates may be left open allowing a continuous discharge thru them. If there is no surplus water, the gates are opened only occasionally for sluicing out the sediment which has been deposited. Sand traps built on this plan may also serve as wasteways, or regulating works for the purpose of regulating the flow in the canal.

Telephone Systems are essential for the proper operation of large irrigation projects. Telephone booths should be placed at intervals along main canals and at other critical points and connected to the operator's house at the headgate. This enables the water to be shut off promptly in case of a break in the canal or other trouble and also assists in the economical distribution and delivery of water.

9. Diversion Works

Purpose. Wherever water is diverted from a natural stream into an artificial conduit some form of diversion works is necessary. This usually consists of a diversion dam or weir, which controls the elevation of the water surface in the stream, and a headgate, just above the diversion dam, which regulates the amount of water allowed to enter the canal. Sluice gates thru the dam and adjacent to the headgate or other devices to prevent the entrance of silt into the canal may be provided. Structures, called *regulating works*, are sometimes constructed a short distance below headgates for the purpose of diverting surplus water back into the stream. With such a structure the amount of water entering the canal may be more effectively regulated than when the headgate alone is relied upon for the purpose. A sand gate is frequently combined with the regulating works. The laws of most states require that fish ladders be constructed in dams across natural streams. In some cases logways must also be provided.

Low Diversion Dams. In general, when the diversion is made at a place where the stream lies in a flat sedimentary bottom, the natural regimen of the stream should be changed as little as possible. Under such conditions a very low dam or weir is preferable. If the stream has a flood plain subject to inundation, the diversion dam should extend across the natural channel of the stream and earth embankments several feet higher than the highest flood stage should connect the structure to the higher land on either side of the stream.

Low diversion dams are commonly built of concrete or wood or a combination of the two. Concrete dams are usually constructed on rock formations. For earth or gravel foundations, a substructure consisting of a rock and timber grillage or bearing piles tied together with walings and caps covered with a deck of heavy planking, will be satisfactory provided it is built low enough to be continually submerged. On this substructure either a wooden or concrete weir may be built. Fig. 10 shows a plan of the diversion works and a cross-section of diversion weir of the Umatilla project, Oregon.

High Diversion Dams may be built where a suitable site is available. Any type of dam may be used for this purpose, the problem being similar to that encountered in constructing a dam for other purposes. High diversion dams may be built in rock canyons to avoid expensive canal construction, the proper height being that which will result in the lowest combined cost of canal and dam.

Between the highest and lowest types of diversion weirs any intermediate height may be used. Each diversion presents special features and the height and type of dam must be chosen to conform to them. Collapsable weirs may be used when it is required to

reduce the flood elevation of the water above the dam. Temporary brush and rock dams are sometimes used for diverting water into small canals.

Headgates control the entrance of water into the canal. They are preferably built on the line of the bank of the stream at right angles to the diversion dam. They should be joined to the diversion weir so as to form part of the same structure with it. Headgates usually consist of a series of gates separated from each other by piers. The piers rest upon a floor, of the same elevation as the bottom

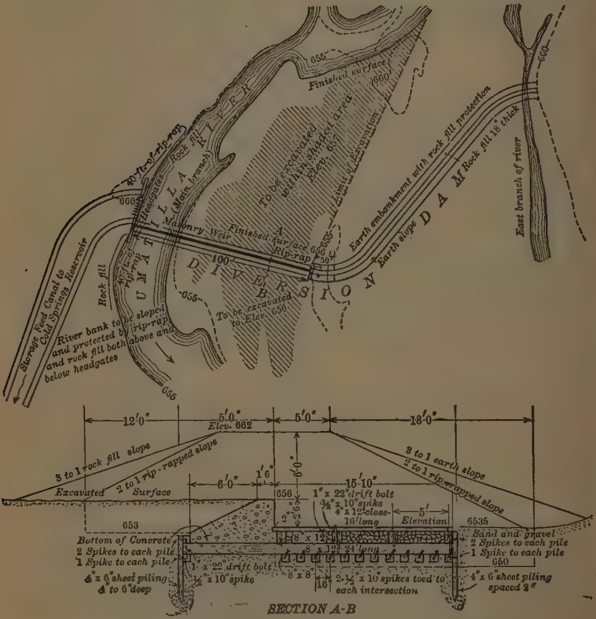

Fig. 10. Low Diversion Weir with Earth and Gravel Foundation

of the canal, which should be of sufficient thickness to give the required strength and stability. Cut-off walls should be provided to prevent erosion and excessive seepage beneath and around the structure. The width of gate opening will be the distance between piers and the height of opening is regulated by curtain walls between the piers. A walk is usually built across the top of the structure and hoisting devices to operate the gates rest upon the walk. Gates should preferably be of such a size that they can be operated by one man. The total gate opening should be sufficient to discharge the maximum capacity of the canal in the contingency that one gate cannot be operated.

Headgates are commonly built of concrete, but wood may be used for temporary construction. Gates may be made of cast iron, steel, or wood. They are usually set in steel or iron guides imbedded in the concrete. The bottom and sides of an earth canal should be protected by riprap or concrete lining for some distance below the headgate to prevent erosion. Headgates diverting from streams which carry drift during flood stages should have the noses of piers protected by fenders to prevent clogging of the gate openings. Fig. 11 is a cross-section of the headgate shown in Fig. 10.

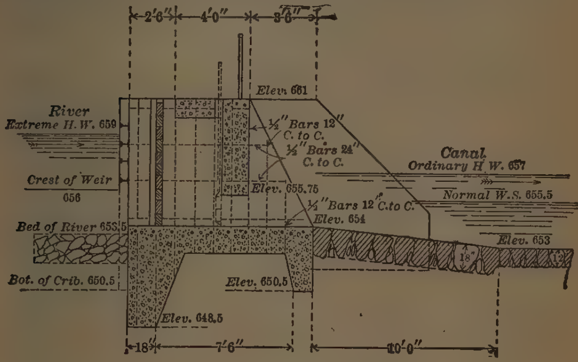

Fig. 11. Headgate for Low Diversion Weir

Sluiceways are openings in a diversion weir, usually adjacent to the headgate. They may consist of open panels in the weir or gates near the base of the weir. The latter are especially adapted to the higher dams. Sluiceways are necessary only on streams that carry considerable sediment. They serve to keep an open channel immediately in front of the headgates and to prevent the heavier sediment from entering the canal. They may also be used to regulate the elevation of the water surface above the dam. Sluiceways must be kept closed when it is desired to divert all of the water in the stream, but with a surplus of water they may be left partially or entirely open.

10. Storage

Necessity for Storage. Storage is essential to the complete utilization of a stream for irrigation purposes. Only those waters are directly available which flow during the irrigation period and are not in excess of the immediate requirements of the area which they serve. Without storage all water in excess of that which is directly available must be wasted. On many streams, further development for irrigation purposes is restricted to the use of stored waters.

Storage Reservoirs. The possibility of economical storage is dependent upon the availability of suitable reservoir sites. Natural lakes or basins may be used for storage when a suitable dam site exists at the outlet. A reservoir may be located on a stream from which water is diverted, or on a tributary of this stream. The water supply from a tributary may not be sufficient to fill a reservoir located upon it, in which case the flow may be supplemented by a feeder canal

diverting from some other stream. If conditions are favorable water may be taken thru a canal or tunnel from an adjoining watershed.

An investigation of the feasibility of storage for a stream should include a complete reconnoissance of the drainage basin. Each possible site should be first roughly investigated to determine approximate data relative to capacity, character of dam site, geological formation of the area to be flooded, availability of construction materials, value of rights of way and other information relative to the cost and value of storage. From these data the sites appearing to possess the greatest merit may be selected and for them a more thoro examination may be made. Topographic maps of reservoir sites are indispensable in computing capacities.

Seepage and Evaporation from Reservoirs. One of the first considerations in investigating the feasibility of a proposed reservoir should be to make a careful study of the geological formation of the area to be flooded with a view to estimating possible seepage losses. Many instances can be cited where reservoirs have proved worthless after construction because of their inability to hold water. Such a condition results in great financial loss and humiliation to the responsible parties. The problem requires the mature judgment of the most experienced engineer. No rules can be laid down which will be safe guides in each case. In general, it may be stated that seepage losses from basins lying in rock (not volcanic rock) or clay will not be excessive. Reservoirs built in formations of fine loam or volcanic soil have generally given satisfaction, tho seepage losses from such reservoirs have been greater than from those lying in rock or clay. If any portion of the banks or bottom of the reservoir is formed of sand or gravel, even tho it may be overlain with a blanket of fine soil, and such sand or gravel might form an underground channel leading around or under the impounding dam or to some other drainage basin, there is grave danger of excessive seepage loss. On account of the large areas involved, efforts to reduce seepage in reservoirs by clay puddle or other methods have usually been unsatisfactory. If the water flowing into the reservoir carries much sediment, there will be a tendency for the reservoir to silt up and reduce seepage losses. Seepage losses from reservoirs in rock or clay formations should not be more than 5 percent of the inflow, and in fine loam or other soil probably not more than 10 or 20 percent. In coarse sand or gravel they may be anything up to the entire inflow. In determining the net supply of storage water evaporation losses should be deducted. These may be obtained from tables of evaporation losses from free water surfaces. (See Sect. 13, Art. 19.) The apparent capacity of a reservoir may be considerably increased by BANK STORAGE. (See Sect 9, Art. 43.)

The following rules of caution are given by A. P. Davis (Engineering News-Record, April 4, 1918; p. 665): 1. Avoid reservoirs adjacent to gypsum deposits and to limestone deposits which show evidence of caves. 2. Examine critically reservoirs in volcanic rock, as a few have failed in such locations. Coarse-grained sandstone seems to be an object of suspicion and should be critically examined. 3. Natural depressions are treacherous and should be examined with care, and if they are near deep canyons or underlain with coarse material where water might readily escape, no superficial tightness will avail to make them effective.

Storage Problems commonly encountered in irrigation work are the determination of reservoir capacity to supply a given quantity of water at specified rates of use, and determination of the amount of water available for irrigation from a reservoir of a given capacity. Where a reservoir receives all or a part of its water thru a feeder canal it will also be necessary to determine the capacity of canal required to supply the storage. The solution of such problems may be readily accomplished by applying the principles of the mass diagram. (See Sect. 9, Art. 43.) In all cases irrigation water will be drawn from the reservoir at a variable rate and the use line will be curved.

11. Distribution Systems

The Function of a distribution system is to convey water from the main canal to each parcel of land to be irrigated. Water is usually diverted first into main distributing canals, then into main laterals and sub-laterals, and finally into the farmers' ditches, from which it is applied directly to the land. It is desirable to have as few openings as possible in the banks of main canals in order to reduce the danger from washouts and hence farm ditches should divert from laterals and sub-laterals rather than from main canals. The distribution is usually made thru a system of earth canals, but where water has a high intrinsic value, or where seepage losses would otherwise be excessive, water may be delivered in flumes, pipes, or lined canals.

Location. Water should be drawn at proper intervals from main canals into moderate sized branches so located as to command the greatest area and serve the land in the most direct manner practicable. The distribution is most economically affected when the main distributing canals are located along the tops of ridges so that they can supply water to laterals on either side. The laterals likewise should conform to the dividing lines between watercourses. It may not be found practicable, however, to conform rigidly to this plan. On flat areas distributing canals may be run along or parallel to property lines. Fig. 12 illustrates distributing canals and laterals diverting from a main canal.

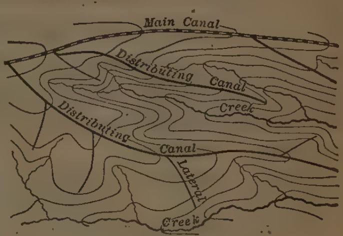

Fig. 12. Distributing System

A Topographical Map of the area to be irrigated, with a contour interval of not more than 5 ft, should be prepared before attempting the design of a distribution system. In addition to topographical features, this map should show non-irrigable lands and other areas to be excluded from the project. Special features, such as vegetation, property lines, improvements and soil formation, are in some instances required.

Design of Distribution System. In general, the entire distribution system should be planned, and the scheme for irrigating each part of the project should be worked out before any construction work is begun. This should be done even tho the construction for a portion of the system is to be deferred for some time. As the plan is developed the position of all canals, flumes, pipes, and other structures should be projected on the topographic map, and the area to be irrigated under each should be determined.

Capacities of Canals should be based upon the water requirements for the period when the greatest amount of irrigating will be done. It is customary to determine the number of acres that will be served by 1 cu ft per sec, during the period of maximum requirements. The required capacity of main canals and distributaries may vary from 1 cu ft per sec for each 120 to 160 acres irrigated, for fruit lands where the water supply is fairly uniform and maximum economy in the use of water is practiced, to 1 cu ft per sec for each 50 acres irrigated, where the supply of water is available for comparatively short periods and conditions are otherwise unfavorable. For diversified crops where the water supply is regular a capacity of 1 cu ft per sec for each 70 to 90 acres irrigated is commonly used. The capacity of farm laterals should not be less than

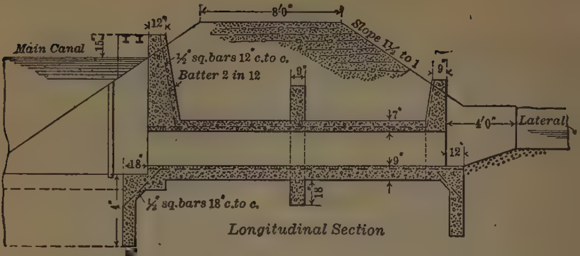

Fig. 13. Reinforced Concrete Turnout to Lateral

the maximum head to be used in irrigating. This will vary from 1 to 2 cu ft per sec where furrow irrigation is practiced to 5 to 10 cu ft per sec for flooding methods. If rotation irrigation is to be practiced the capacity per unit area served should be greater for laterals than for main canals in order to provide flexibility in operation. In general, an allowance for seepage and evaporation should be added to the above figures.

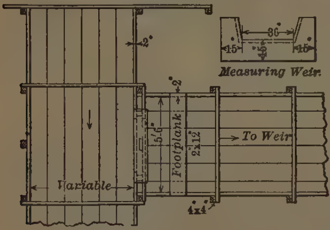

Fig. 14. Wooden Head to Lateral

Structures for Distribution Systems are similar to those for main canals. Turnouts must be provided, where water is diverted from main canals to distributaries and at the heads of all laterals. Flumes or pressure pipes are constructed for carrying water across depressions. Drops are required in earth canals where the slope of the ground is greater than the allowable grade. These structures may be built of wood or concrete. Wood is more commonly used for the small turnouts and lateral heads. Measuring devices, generally weirs, are placed at the heads of farm laterals. Figs. 13, 14, and 15 are examples of structures for distribution systems.

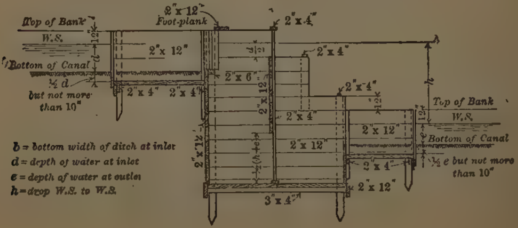

Fig. 15. Wooden Drop for Lateral

12. Irrigation by Pumping

The Source of Water Supply may be either a natural stream, canal, or well. The pumping plant may be of any capacity from a single pump capable of serving a small farm to a number of large units supplying several thousands of acres. Large pumping projects are usually supplied with water from some surface source, while small areas may be irrigated from wells or surface water. Limited areas of bench land lying above the main canals of gravity projects may in some instances be economically irrigated from these canals by utilizing water power generated at drops. Any kind of power that is cheap enough may be used for pumping. Steam and crude oil engines have been used successfully for large plants in the United States, while gasoline engines and electric energy transmitted from central stations have proved satisfactory for irrigating small areas.

The Feasibility of irrigation by pumping depends upon the cost of water and its value to the land. The first cost of a pumping project may be low as compared to the cost of many gravity projects, but the operating expense and resulting cost of water will generally be higher. Pumping against heads of over 100 ft is not profitable except where the crop is valuable or the water requirements small. Irrigation of fruit lands by pumping from deep wells has been practiced for many years.

Cost of Pumping for irrigation depends upon the unit cost of power, and the head or vertical lift. The power required will be the theoretical power necessary to raise the water at the required rate divided by the efficiency of the pumping plant. In general the efficiency of pumps increases with their size, and will vary from 30 to 80 percent, depending upon the size, make, and care with which installations are made.

The Unit Cost of Power is generally less for large than for small plants. This is because greater efficiency and smaller cost of attendance is possible for the larger installations. The cheapest power is that obtained directly from water power. Power may be produced from coal or oil in large plants at from 1 to 2 cents per horse-power hour, while in small plants of low efficiency the cost may be as much as 5 cents per horse-power hour.

Cost of Raising Water per Acre-foot with Power at 1 Cent per Horse-power Hour

Pump efficiency	Head in ft									
	10	20	30	40	50	60	70	80	90	100
100	$0.14	$0.27	$0.41	$0.55	$0.69	$0.82	$0.96	$1.10	$1.24	$1.37
80	.17	.34	.51	.69	.86	1.03	1.20	1.37	1.54	1.72
70	.20	.39	.59	.78	.98	1.18	1.37	1.57	1.77	1.96
60	.23	.46	.69	.92	1.14	1.37	1.60	1.83	2.06	2.29
50	.27	.54	.82	1.10	1.37	1.65	1.92	2.20	2.47	2.75
40	.34	.69	1.03	1.37	1.72	2.06	2.40	2.75	3.09	3.43
30	.46	.91	1.37	1.83	2.29	2.75	3.20	3.66	4.12	4.58
20	.69	1.37	2.06	2.75	3.43	4.12	4.80	5.49	6.18	6.86

13. Preparation of Land

Clearing. Arid lands are usually covered to some extent with native vegetation. This may consist of grasses and small brush which can be plowed under, or heavy brush or scrub trees. All surface vegetation and roots that will interfere with the cultivation of the land should be removed and burned.

Sage brush is the most widely distributed of all of the desert plants. It commonly attains a height of from 2 to 5 ft. A substantial growth of sage brush usually indicates a fertile soil. Sage brush may be removed by grubbing, or broken off by a team dragging a steel rail over it, or by other special devices. The cost of clearing land of sage brush usually varies from \$3 to \$6 per acre. The cost of removing large brush and trees may vary from \$25 to \$75 per acre, depending upon the size and thickness of growth.

Grading. Unimproved land must usually be graded before water can be distributed over it uniformly. Proper grading is very essential to the economic use of water. It reduces the expense of irrigating and insures maximum crop returns with a minimum consumption of water. Many areas are comparatively smooth and the cost of preparing the land for irrigation is inconsequential. The cost of grading rough or rolling land may be from \$15 to \$40 per acre. The method to be employed in irrigating (see Art. 14) may greatly affect the amount of grading required. Before beginning excavation, a detailed plan of the grading to be done should be worked out on an accurate topographic map.

In many irrigated districts the soil is of uniform texture for some distance below the surface. Other localities have a comparatively thin surface soil which is underlain by a less fertile sub-soil. Where the latter condition obtains, too extensive excavation may seriously affect the productivity of the soil, and it may be advisable to modify the plan of grading and system of irrigation from that which would otherwise be desirable. Where land is sufficiently valuable, it may be practicable to move to one side the surface soil from the areas to be excavated or filled, and after grading to recover them with the surface soil. Grading is usually done with scrapers. Buck-scrapers and Fresno scrapers may be used to advantage if the haul is short, while wheeled scrapers are preferable for long hauls. Road graders and various types of home-made levelers are used for smoothing the surface of the ground.

The First Cultivation of raw land is frequently difficult and expensive and the first crop returns will usually be less than later ones. Light sandy soils are especially difficult to subjugate, as they are apt to drift and allow the seeds to be removed by wind action. Such soils must usually be cleared and planted in small areas, and protection from wind provided as soon as possible. Straw or manure placed over the exposed surface may provide a satisfactory protection. Sometimes rows of sage brush, about 50 ft apart, at right angles to the direction of the wind, are left temporarily on the ground to act as wind breaks. A hardy grain, such as rye, is frequently used for the first crop.

14. Application of Water to Land

Classification of Methods. The different methods of applying water to land may be classified as surface and sub-surface methods. Sub-surface irrigation, or sub-irrigation, is usually more expensive than surface irrigation and is not commonly practiced except where particularly favorable conditions exist. Surface irrigation may be accomplished by lateral percolation or by flooding.

Farm Ditches. In any plan of irrigation farm ditches should be laid out with a view to providing the maximum efficiency and economy in the distribution and use of water. The water must be delivered to the highest point of the area to be irrigated, from which point it is conveyed by a field lateral to distributing ditches.* Fig. 16 shows a system of ditches that may be used for several methods

of applying water where the topography of the ground is suitable. Preferably farm ditches should run along fences and property lines or be parallel to them, but often they must be curved to better conform to topographic features. The best practice is to put farm ditches from 330 to 660 ft apart. Their capacity should be the maximum irrigating head that is to be used.

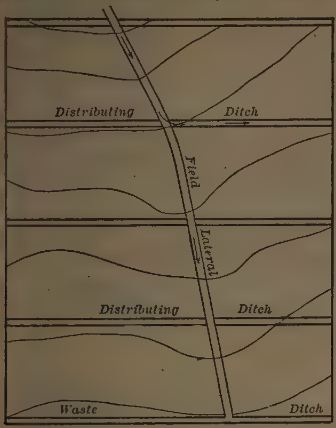

Fig. 16. System of Farm Ditches

Uncontrolled Flooding requires less grading than other methods of irrigating, but it is more wasteful and does not provide a uniform distribution of water. This method is illustrated in Fig. 17. A ditch is constructed along the upper end of the field and at intervals of preferably not more than 20 rods other ditches roughly parallel to the upper ditch are constructed. The area between any two ditches may be irrigated by opening the upper ditch at several points and allowing the water to flow over the land to the lower ditch. Usually many small dikes must be thrown up while irrigating to force the water over the higher areas.

Fig. 17. Flooding of Meadows.

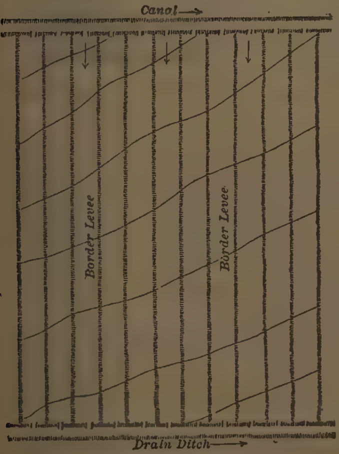

Fig. 18. Flooding by Border Method

A modification of the above method consists in making small temporary ditches or furrows which lead from the supply ditch along the ridges and higher places in the field. Water is turned into these temporary ditches which overflow their banks and cause water to be distributed over the field. This method is used for irrigating grain and alfalfa and is an improvement over the uncontrolled flooding method.

The Border Method of Flooding is shown in Fig. 18.

Parallel ditches should extend across the field at intervals of from 20 to 40 rods. Small levees called borders are built between ditches from 2 to 4 rods apart. The areas between each set of borders should be graded so that the profile of the ground on any cross-section will be approximately level. Water turned on the land between two borders is confined to the area between them and if the grading has been done properly the water is distributed uniformly. The objection to this method of irrigating is that the upper portion of the area receives more water than the lower portion.

This method is best adapted to fields having a gentle uniform slope. A slope of about 1 ft in 500 ft is preferable. Where the natural slope is greater than this the grade may sometimes be reduced by running the borders diagonally across the field. The cost of preparing land for irrigation by the border method, including ditches, will vary from $10 to $30 per acre.

The Contour Check Method of Flooding requires a supply canal at the upper end of the field with distributing ditches leading from it which pass directly down the slope (Fig. 19). Check levees are constructed along contours between

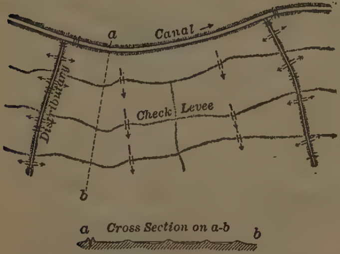

Fig. 19. System of Check Levees

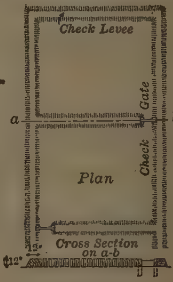

Fig. 20. Flooding by System of Squares

the distributing ditches, and cross levees divide the basins thus formed into two approximately equal parts, which are called checks. Each check may then be filled with water from the ditch adjacent to it. Levees should be about 9 in high and at least 6 ft wide at the base. Material for construction of levees is taken from the high places and the surface within each check should be graded to a fairly level surface. The cost of grading, where the slope of the ground does not exceed 50 ft per mile, is from $10 to $30 per acre. The size of checks which has proved most satisfactory is from 1 to 3 acres. A large head, preferably 5 to 10 cu ft per sec, should be available for irrigation by this method, the full head being used to fill one check at a time.

A modification of the above method of irrigating has been used on rough land and hillsides where the slope exceeds 1 in 20. The cost of grading is proportionally higher as the slope or degree of roughness of the land increases. The ground is graded to a series of level benches and terraces with a check levee at the top of each terrace. The levees follow roughly the direction of the contours, but the checks are usually irregular in shape and vary considerably in size.

The Square Check Method of Flooding is best adapted to gently sloping ground. This method is similar to the contour check method, except that the

field is laid out in square checks without regard to contours. The size of checks varies from less than 1 acre to 5 acres or more, but for best results checks of not more than 3 acres are preferable. Sandy soils require smaller checks than the more impervious ones. Water may be turned into checks from distributing ditches or if the checks are small it may be allowed to flow from one check to another (Fig. 20). The latter plan has the disadvantage of giving the upper checks more water than the lower ones. The cost of grading for irrigation by this method, including the cost of ditches, will usually range from $10 to $30 per acre.

The Basin Method is a modification of the check method employed for irrigating orchards. Levees are built around each tree and water is supplied from ditches placed between alternate rows of basins or allowed to flow diagonally across basins as indicated in Fig. 21. Mounds of earth are usually placed around the trees to prevent the areas immediately adjacent to them from becoming submerged. Jointed pipes extending from a supply canal to the lowest basin coming under it are sometimes used for filling the basins. After the lowest basin is filled, joints of pipe are removed and the next basin above is filled. This process is continued until all of the basins in the row have been irrigated, when the operation may be repeated for the other rows. This method is economical of water and is well adapted to the use of a small irrigating head.

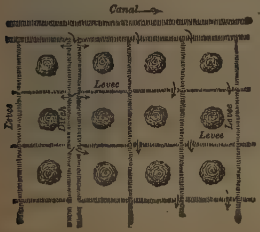

Fig. 21. Basin System of Irrigation

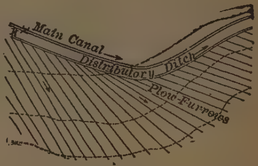

Fig. 22. Furrow Irrigation of Grain

The Furrow Method of Lateral Percolation may be used in place of flooding methods for irrigating grain or meadows (Fig. 22) and is used almost exclusively for irrigating crops planted in rows. The land is prepared by constructing supply ditches across the field at suitable distances apart and running furrows down the slopes between them. Furrows are made with plows or with special machines called markers. Very little grading is necessary for irrigating by this method and the cost of preparing the land for irrigation is less than for any of the flooding methods. Furrows are frequently made on slopes as great as 5 or 10 ft per 100 ft, but light soils may be eroded if placed on too steep slopes. The grade of furrows may be reduced by running them diagonally down a hill.

For irrigating grains and alfalfa the furrows are made from 3 to 6 in deep and spaced from 2 to 4 ft apart. Crops planted in rows may have a furrow in each space between rows, or in alternate spaces between rows. For young orchards a furrow is usually placed on either side of the trees and after the trees mature several furrows are used for each row. Deep furrows are believed to give more satisfactory results than shallow ones. The irrigation is accomplished by turning water into several furrows at one time and allowing it to flow to the end of each furrow, the moisture being distributed thru the soil

by lateral percolation. This method has the advantage of reducing evaporation and providing a uniform distribution of moisture. It is objectionable in some cases in that large heads cannot be handled to advantage, and in general it requires more labor to apply the water to the land than other methods of irrigation.

Sub-Surface Irrigation or sub-irrigation is the name applied to all those methods in which the water is applied below the surface of the ground. The water is conveyed in underground channels containing openings, beneath the area to be irrigated, and thru these openings the water percolates and is distributed thru the soil by capillary action. The underground channels may consist of ordinary tile drains or perforated pipes. The cost of installation of such systems is high and in general the advantage obtained by them has not warranted the additional expense. In most localities surface irrigation is to be preferred to sub-irrigation.

Some lands are naturally sub-irrigated. Generally these are bottom lands underlain by gravel where the surface of the ground is but a few feet above the normal water surface of the stream. Water is drawn from this sub-surface supply by capillary attraction to or nearly to the surface of the ground. Deep rooted crops such as alfalfa are especially adapted to naturally sub-irrigated lands. Porous soils underlain by an impervious stratum quite near the surface may sometimes be sub-irrigated by simply digging ditches at intervals of from ¼ to ½ mile. Water flows down the slope between the two strata, and if near enough to the surface, is available for plant growth.

Spray Irrigation has long been used in cities for watering lawns and small gardens. In certain humid districts it is used for supplementing the rainfall on larger areas devoted to the production of truck and small fruits. The essentials for this method of irrigating are a pumping plant and distributing pipe system. Hydrants may be located at suitable intervals over the field from which the water is applied to the land through hose, or parallel pipe lines from 50 to 60 ft apart, elevated a few feet above the ground and provided with nozzles at suitable intervals may be used to distribute the water over the field. An irrigation system of this kind is expensive, but the increased returns will often more than justify the expense.

15. Distribution and Use of Water

Distribution of Water among Users. At certain periods of the irrigation season the water requirements of crops are greatest and the demand for water may be excessive. The use of water cannot exceed the capacity of the irrigation system, and some adjustment in the use of water is usually necessary during certain portions of the irrigation season. On many large projects the farmer is required to give written notice from 1 to 6 days in advance, stating when he will want water and the head that he desires. When it is not possible to deliver water to all users on the date requested without exceeding the supply of water, or the capacity of the irrigation system, the distribution is adjusted so as to accord as closely as possible with the requests of the various consumers.

Rotation in Irrigation. The consumer's right to the use of water is frequently expressed as a certain continuous flow, usually in cubic feet per second, thruout the irrigation season. As it is not economical to irrigate with a small head, it is customary for farmers whose rights to water are thus expressed, to rotate in its use, each taking a suitable head for such a time that the total water used will not exceed the allotment for the season. In many irrigated districts the right to water is expressed in some unit of volume, such as acre-feet per acre, or inches depth for the area to be irrigated. Such units are preferable for districts with an assured water supply.

Measurement of Irrigation Water. In order that irrigation water may be equitably distributed some plan for measuring the water turned into each farm lateral is necessary. The ideal method of measuring such water is with a meter that records the flow in some unit of volume such as cubic feet or acre-feet. Tho numerous devices for this purpose have been suggested none of them has been developed to a practicable working state. The present practice is to measure the rate of flow, and to multiply this rate by the time to get the volume of discharge. Weirs have been generally used for measuring the flow of irrigation ditches where sufficient head is available. Headgates may be used as submerged orifices by rating them with a current meter and determining the coefficient of discharge for different gate openings and heads. Similarly a discharge curve may be obtained from current meter measurements for a flume or a permanent section of a canal. Cone's Venturi flume may be used for measuring discharges where sufficient head is not available for installing a weir.

Economy in Use of Water. In most arid districts the area of fertile land is much in excess of that which can ever be irrigated by the most effective use of all available waters. The limit of irrigation is therefore fixed by the water supply and not by the land area. Any water that is wasted or used in excess of that which may be applied to the land beneficially reduces the irrigable area. On the other hand, any improvement in methods of irrigation which reduces the water requirements, and all waste prevention, increase the area of land which may be reclaimed. Investigation of the use of water that will yield the greatest net return is therefore an important branch of irrigation engineering.

The Method of Payment for water has much to do with the economy of its use. Commonly the farmer's contract calls for a maximum quantity of water and there is no incentive for him to use a smaller amount. A better method of payment is for the farmer to pay for just what water he uses, or, if his contract calls for a fixed quantity of water, to allow a bonus for any smaller amount that he may use. Under such conditions, the farmers will readily become interested in practicing economy in the use of water and in developing more efficient methods of irrigation.

SWAMP AND OVERFLOWED LANDS

16. General Considerations

Classification. Those areas which are covered with water or whose soils contain an excess of water either continually or to such an extent as to render them unfit for cultivation are commonly spoken of as swamp or overflowed lands. There are three general divisions: inland marshes, river-bottom lands, and tide lands. Most of these areas are susceptible of reclamation, and in many instances their development is of great economic importance.

Location and Distribution. Swamp and overflowed lands are distributed quite generally thruout the world and many such areas have been successfully reclaimed. The largest enterprises of this kind have been in Europe, where land values are high. Notable examples are the English Fens, where 700 000 acres of tide lands have been reclaimed, and Haarlem Lake, Holland, where a shallow lake 43 000 acres in extent has been converted into a fertile farming district. Large drainage enterprises have also been successfully completed in France, Italy, and other European countries. Of the large areas of swamp and overflowed lands in the United States only a small fraction have been reclaimed.

The following is the reclaimable swamp area by states, according to the statistical abstract of the United States for 1910, issued by the Department of Commerce and Labor:

	Acres		Acres
Alabama	1 120 000	New Jersey	601 900
Arkansas	5 760 000	New York	576 000
California	1 850 000	North Carolina	2 400 000
Connecticut	37 700	North Dakota	226 000
Delaware	200 000	Ohio	200 000
Florida	18 500 000	Oklahoma	35 000
Georgia	2 400 000	Oregon	500 000
Illinois	2 688 000	Pennsylvania	96 000
Indiana	1 000 000	Rhode Island	17 900
Iowa	800 000	South Carolina	1 760 000
Kansas	160 000	South Dakota	226 000
Kentucky	224 000	Tennessee	800 000
Louisiana	9 600 000	Texas	1 620 000
Maine	240 000	Vermont	70 000
Maryland	356 000	Virginia	384 000
Massachusetts	138,700	Washington	75 000
Michigan	4 400 000	West Virginia	2 500
Minnesota	4 500 000	Wisconsin	2 500 000
Mississippi	6 173 000	Wyoming	25 000
Missouri	1 920 000		
Nebraska	256 000	Total	74 481 700
New Hampshire	43 000		

Soil. Marsh lands are commonly covered with decayed vegetable matter and are usually very fertile. In many cases, however, some element of plant food is deficient and certain crops will not grow satisfactorily without artificial fertilization. This is especially true of peat and muck lands, which are usually lacking in potash. The subsoil of inland marshes is ordinarily similar to that of the surrounding country. Overflowed lands along rivers are composed largely of sedimentary deposits, silt and fine sand usually predominating. Tidal marshes may be formed of clay, sand and silt in any proportions. The soil formation of swamp and overflowed lands should always be carefully investigated with a view to determining its fertility and its influence on construction methods and plan of drainage.

Natural Vegetation. Inland marshes frequently contain heavy growths of timber, or where the timber has been cleared they may be covered with dense underbrush, stumps, and fallen trees. Other marshes contain growths of coarse grasses, reeds, or moss. In all cases the expense of clearing must be included in the cost of reclamation.

Topographic Map. The general characteristics of a drainage project can best be studied from a topographic map, which should be prepared early in the investigation. Contours of the area to be reclaimed should ordinarily be shown for 1 ft intervals, and natural drainage channels should be carefully indicated. Information as to soil formation, property lines, and source and amount of water to be disposed of may be recorded on the map. Existing maps may sometimes be of value in determining tributary areas which naturally drain into the area to be reclaimed.

17. Run-off

Surface Drainage. The channels and outlets of a drainage system should be of adequate capacity to provide for the removal of water at such a rate that the ground will be ready for early cultivation in the spring, and that the growth and harvesting of crops will not be seriously retarded. It is therefore the maximum or flood flows which must be considered in designing a surface drainage system, and the same principles apply regardless of the scheme of reclamation. Excessive run-off may result either from heavy rains, or the sudden melting of snows, but the most severe floods are caused by the former.

The Maximum Run-off per unit area that may be expected from any drainage basin decreases as the area increases. In general, the law of maximum discharge from flat areas, similar to those encountered in large drainage projects, may be represented by the equation

$$Q = KM^{0.75}$$

in which Q is the maximum discharge in cubic feet per second, M the drainage area above the point at which Q is required, in square miles, and K an empirical coefficient. If A is the area in acres, the formula may be written

$$Q = 0.00786KA^{0.75}.$$

Values of K obtained from measurements of maximum run-off of a number of drainage projects in the Mississippi valley range from 20 to 60. They correspond to rainfalls of from 3 to 7 in in 24 hours. The measurements were made on areas varying from approximately 2 to 200 square miles. The value of K depends primarily upon the maximum rainfall, but is influenced to a large extent by several other factors. It increases with the slope of the ground and the degree of imperviousness of the soil. The condition of the ground also influences the value of K. A heavy rain falling on wet ground will cause a greater run-off than the same rain falling on dry ground. If the ground is frozen or covered with ice, conditions will be favorable for a large surface run-off. Precipitation records covering as long a period as possible should be consulted to determine the maximum rainfall to be provided for. This need not be necessarily the heaviest recorded rainfall, as it may be considered a better business proposition to allow the crops to suffer damage once in several years than to go to the expense of providing drainage adequate for the immediate removal of flood waters resulting from the severest storms.

The Total Annual Run-off from a given area is the difference between the annual precipitation and the water returned to the atmosphere thru evaporation and plant transpiration, subject to modifications from ground water conditions and possible deep seepage losses. The effect of the two latter considerations is commonly neglected in approximate calculations. Many factors affect the amounts of evaporation and transpiration, the more important being the mean annual temperature, and the amount and distribution of rainfall. The humidity of the air, velocity of the wind, character of vegetation, degree of cultivation, and soil formation are also important influences.

An accurate estimate of transpiration and evaporation losses for a given area involves a thoro investigation of the various factors involved. Rough estimates may be made by considering only the temperature and precipitation. Mean values of combined transpiration and evaporation losses, based upon observations in the United States, are given in the following table. The yearly run-off in inches is obtained by subtracting the proper tabulated value from the annual precipitation. Observed values may be expected to differ by as much as 5 in from those given in the table.

Average Annual Transpiration and Evaporation Losses in Inches

Annual precipitation, inches	Mean annual temperature (Fahr.)								
	35°	40°	45°	50°	55°	60°	65°	70°	75°
25	16	17	18	19	20	21	22	23	24
30	17	18	19	20	21	22	23	24	25
35	18	19	20	21	22	23	24	25	26
40	19	20	21	22	23	24	25	26	27
45	20	21	22	23	24	25	26	27	28
50	21	22	23	24	25	26	27	28	29
55	22	23	24	25	26	27	28	29	30
60	23	24	25	26	27	28	29	30	31
65	24	25	26	27	28	29	30	31	32
70	25	26	27	28	29	30	31	32	33

18. Pumping Plants

Capacity. The removal of water may be accomplished by gravity or pumping or by a combination of the two methods. A pumping plant should be of adequate capacity to remove water at a rate that will prevent injury to crops up to the point where additional capacity will cost more than the damage to crops that will result if the additional capacity is not provided. The time which land may be flooded without serious damage differs for different crops, truck being more susceptible of damage than corn, sugar cane, grains and similar crops, but in general the land should not be submerged for more than 24 hours. The capacity of pumping plant must be based upon a consideration of maximum run-off (Art. 17) and also upon the storage provided by canals, the maximum length of time that crops may be submerged without damage, and the slope of the reclaimed area. Areas with considerable slope require greater pump capacity than more level areas, because of the tendency of water to collect and flood crops at the lower elevations.

In southern Louisiana pumping capacities sufficient to remove from 1 to 1.5 in per 24 hours have proved sufficient for areas of from 5000 to 10 000 acres. One inch of run-off in 24 hours is equivalent to approximately 27 cu ft per sec per sq mile. In the upper Mississippi valley a pump capacity of from 0.25 to 0.50 in for similar areas has been found satisfactory. The records of the U. S. Weather Bureau at New Orleans for the 22 years preceding 1914 show that for this period there were 43 storms during which the precipitation in 24 hours exceeded 3 in. These storms are classified as to their intensity as follows:

43 rains exceeding 3 in in 24 hours.
19 rains exceeding 4 in in 24 hours.
7 rains exceeding 5 in in 24 hours
3 rains exceeding 6 in in 24 hours.
2 rains exceeding 7 in in 24 hours.
2 rains exceeding 8 in in 24 hours.
0 rains exceeding 9 in in 24 hours.

The Period of Operation. Ordinarily pumps will be operated early in the spring to get the ground ready for cultivation, and after heavy rains during the

growing period. In the lower Mississippi valley, where plant growth continues thruout the year, the pumping plant must be ready to operate at all times. In northern United States pumping usually is not required for more than 20 days, while in the extreme south it may be required from 40 to 50 days in the year.

Location. Pumping plants should be located near the natural drainage outlet if the area is approximately level, so that the water will travel the minimum length of canal. Unless electric power is supplied from a central station the facilities for transporting fuel must be considered in selecting a location for the plant. The ground should also be explored with a view to obtaining a favorable foundation for the plant.

Buildings. The footings under machinery and buildings should be of concrete, and if built on earth they should be supported by piling. The pump should be mounted on the same block of concrete as the engine that operates it so that subsequent settlement will not throw them out of line. Buildings should be durable, of fireproof construction, and of sufficient stability to withstand the heaviest winds.

Power. The kind of power to be used in a particular case must depend upon local conditions, but each possible source of power should be investigated. The power finally selected should be that which requires the smallest operating expense, based upon interest charges, attendance, fuel, maintenance and depreciation. Pumping by electricity has in many cases proved satisfactory and where several drainage projects are in the same vicinity the generation of electric energy at a common central station might be advantageous. Windmills have been used in Holland, but the uncertain character of this power renders it unsuitable for drainage purposes, except possibly as an auxiliary power. Steam, oil, gas-producer, and gasoline have been used for pumping in the United States. Gasoline engines have been used chiefly for small plants. Gas-producer plants have generally proved unsuccessful for intermittent service. Oil engines using crude oil have proved very satisfactory both as regards oil consumption and the cost of attendance. Steam is more generally used for large plants than other sources of power, but the large initial cost in foundations, settings, and appurtenances, and the high grade of mechanical ability required to operate steam plants, and the expense of keeping them continually ready for service are serious objections.

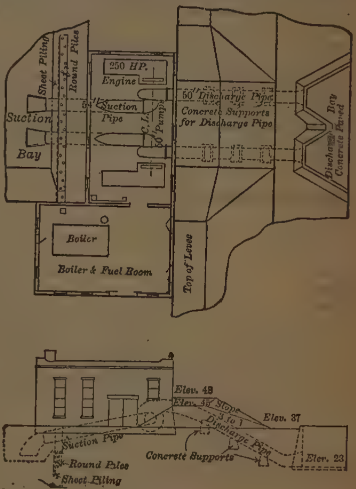

Fig. 23. Pumping Plant for Drainage System

Pumps should be selected and operated so as to secure the maximum efficiency practicable. The maximum lift will not usually exceed 10 ft and the bulk of the water will not be lifted more than 3 or 4 ft. It is at the lower lifts also that the greatest pumping capacity is required. It is desirable, therefore, that the pumps should operate at very nearly their maximum efficiency for the lower lifts and operate at a satisfactory efficiency thru the entire range of lift. Centrifugal pumps have been quite generally used for drainage reclamation work. Fig. 23 shows an outline of the plan and elevation of the pumping plant for Louisa-Des Moines Drainage District No. 4, Iowa.

The rated capacity of ordinary low head centrifugal pumps is based upon a velocity thru the discharge opening of from 10 to 12 ft per sec. The loss of head due to friction may therefore equal or exceed the static lift. For this reason the suction pipe and discharge pipe should be made as short as possible and sharp bends should be avoided. The outlet should be submerged so as to induce siphon action in the pipes, and the discharge pipe should be flared at the lower end so that the water will discharge at a velocity of not over 5 ft per second. The suction pipe must be air-tight and the water should be drawn from a concrete lined intake several feet deeper than the minimum elevation of water surface to be maintained. Centrifugal pumps if properly designed and installed should give efficiencies of 60 to 70 percent thruout the range of lift ordinarily encountered in drainage work, but improperly selected pumps may give very much lower efficiencies. These figures refer only to the pumps, and friction losses in pipes and velocity head at exit must be added to the static lift to obtain the actual head against which the pumps must operate.

Average Loss of Head in Feet per 100-ft Length of Suction or Discharge Pipe of Pumps

Velocity in pipe ft per sec	Diameter of pipe in inches													Add for each 90° bend
	6	9	12	15	18	21	24	30	36	42	48	54	60	
8	4.5	2.7	1.9	1.4	1.1	0.9	0.8	0.6	0.5	0.4	0.3	0.3	0.3	0.2
9	5.7	3.4	2.4	1.8	1.4	1.2	1.0	0.8	0.6	0.5	0.4	0.4	0.3	0.3
10	6.9	4.2	2.9	2.2	1.7	1.4	1.2	0.9	0.7	0.6	0.5	0.4	0.4	0.4
11	8.3	5.0	3.5	2.6	2.1	1.7	1.5	1.1	0.9	0.7	0.6	0.5	0.5	0.5
12	9.8	5.9	4.1	3.1	2.5	2.0	1.7	1.3	1.0	0.9	0.7	0.6	0.5	0.6
13	11.5	6.9	4.8	3.6	2.9	2.4	2.0	1.5	1.2	1.0	0.8	0.7	0.6	0.8
14	13.2	7.9	5.5	4.2	3.3	2.7	2.3	1.7	1.4	1.1	1.0	0.8	0.7	0.9
15	15.1	9.0	6.3	4.8	3.8	3.1	2.6	2.0	1.6	1.3	1.1	1.0	0.8	1.0

Cost. The cost of drainage pumping plants varies widely according to the type of machinery, expense of transportation, character of foundation and difficulties of erection. The following tables give costs of power plants and single unit pumping plants and fuel costs for different types of installation in Texas and Louisiana from data obtained in Bulletin 71 of the U. S. Department of Agriculture. The costs of pumps include foundations and piping, and the costs of power plants include engines, boilers, and their foundations and auxiliaries. The costs of intakes, buildings, and discharge canals or flumes are not included.

Cost of Power Plants and Single Unit Pumping Plants

Indicated horsepower	Type of engine: Simple slide valve, non-condensing, From	To	Compound condensing slide valve, From	To	Compound condensing Corliss, From	To	Single unit centrifugal pumping plant, From	Total	Capacity of pump Gallons per min.
20	$1 100	$1 500	....	....	....	....	$2 000	$3 500	5 000
50	2 500	3 500	....	....	....	....	2 000	4 000	10 000
100	4 000	6 500	$2 000	$3 500	$12 000	$14 000	2 500	6 000	20 000
200	....	....	8 000	10 000	13 000	18 000	3 500	7 500	30 000
300	....	....	10 000	12 000	15 000	21 000	4 500	8 500	40 000
400	....	....	11 000	15 000	19 000	22 000	6 000	9 500	60 000
500	....	....	....	....	22 000	24 000	7 500	10 500	80 000
600	....	....	....	....	24 000	26 000	9 500	12 000	100 000
700	....	....	....	....	26 000	28 000	12 000	15 000	120 000
800	....	....	....	....	28 000	30 000	14 000	17 000	140 000

Relative Fuel Costs of Several Types of Pumping Plants

For 1000 acres, 6 ft difference in levels, 8 ft total head. Amount pumped = 1 acre-foot per acre per season, at rate of approximately 1 acre-inch per acre in 24 hours, or 42 cu ft per sec.

Water horse power	Type of plant	I.H.P. of steam engine B.H.P. of gas engine	Total cost of fuel oil per hr at $1.25 per barrel	Total cost of gasol'n per hr at 12 cents per gallon	Total fuel cost per year of removing total 12 in at pump efficiencies named: 100%	70%	60%	50%	40%	30%
41	Simple slide-valve engine, belted	H.P. 50.5	$0.88	......	$238	$340	$397	$476	$595	$795
41	Simple slide-valve engine, direct connected	45.5	.80	......	218	311	364	436	545	727
41	Gasoline engine, belted	45.5	......	$0.68	184	263	307	368	460	613
41	Gasoline engine, direct connected	41.0	......	.61	165	236	275	330	412	550
41	Simple Corliss engine, belted	49.5	.50	......	135	193	225	270	338	450
41	Simple Corliss engine, direct connected	44.5	.45	......	122	174	203	244	305	407
41	Compound condensed slide valve, direct con-connected	45.5	.36	......	98	140	163	196	245	326
41	Compound condensed Corliss, direct connected	44.5	.31	......	83	119	138	166	208	276

19. Levees and Ditches

General. Excepting the pumping plant, where pumping is necessary, the bulk of the labor of land reclamation by drainage consists of the construction of levees and ditches. Drainage ditches are required to remove surplus water, and levees must be built to prevent the entrance of outside water upon lands subjected to overflow. The study, therefore, is largely one of designing and locating these levees and ditches so as effectively to protect and drain the tract at a minimum cost, and in such a manner as to require the smallest future outlay for maintenance and repairs.

Levees Subject to Current and Wave Action should be designed on liberal lines and carefully constructed (Fig. 24). They should preferably be located on the higher ground and between ridges to reduce excavation and secure a firm foundation to build on. The alignment should be as regular as practicable, however, and sharp turns should be avoided.

The slope of the face of a levee on the exposed side should not be steeper than 1 on 3 and if built of light material a slope of at least 1 on 4 should be used. The inner slope should be at least 1 on 2 and preferably 1 on 3. The borrow pit, which is always on the outer side of the embankment, should be separated from it by a berm of 30 to 50 ft. The width of the top of the embankment, should be at least 6 ft and a freeboard of 3 ft or more should be allowed between the highest water planned for and the elevation of the top of the levee. A tile drain is usually placed along the inner slope of the levee to intercept seepage water and make the base of the levee more firm. All vegetable matter should be removed from the strip of land which the levee is to occupy and usually this area is corrugated with longitudinal plow furrows before the embankment is placed. A cut-off trench, filled with selected material, may be built near the center of the base of the levee. Levees subject to wave action should be protected by rock riprap or other construction to prevent erosion. (See Sect. 6, Art. 41.)

Levees in Still Water may have a somewhat smaller cross-section than those subjected to current or wave action. They are commonly located on property lines without regard to the topography. This results in regularly shaped districts and minimum length of levee, but frequently the cost of construction and maintenance may be reduced by following the higher and firmer land and deviating somewhat from the more direct line.

Common dimensions for this type of levee (Fig. 24) are 1 on 2 or 1 on 3 for embankment slopes, a top width of embankment of 6 ft, a berm of 10 ft and a freeboard above the maximum high water elevation of 3 ft. To reduce seepage, the borrow pit should

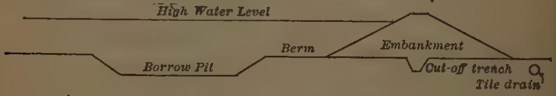

Fig. 24. Cross-section of Levee

be on the outer side of the levee, but it is sometimes placed within the tract and thus serves as a drainage canal. All vegetable matter and humus should be removed from the strip of land on which the levee is to be built and as far as practicable only the better grade of material should be placed in the embankment. Carelessness in these regards may result in excessive seepage thru the levee. Levees built of the mud commonly found in swamps will shrink from 20 to 50 percent of its original volume, and allowance for this shrinkage should be made when the material is placed. The shrinkage will be less for sandy soils than for those in which clay predominates.

Ditches of various sizes must be constructed and so located as thoroly to drain the area to be reclaimed. On pumping projects the larger ditches also

serve for storing water after heavy rains and thus give greater flexibility to the pumping plant. In many instances the latter function is fully as important as the former, and quite commonly some of the main ditches are made much larger and deeper than would be necessary for drainage alone in order to provide more storage.

The size of each drainage ditch must be carefully determined from the duty which it has to perform. The general features are shown in Fig. 25. The side slopes are usually made about 1 on 1½. When excavated with dredges it may be more convenient to leave

Fig. 25. Cross-section of Drainage Ditch

steeper banks and allow additional width with the idea that the banks will cave in and eventually conform approximately to 1 on 1½ slopes. Excavated material should be deposited in spoil banks on either side of the ditch, leaving suitable berms between the ditch and banks.

Excavating Machinery. Levees on firm dry land may be built either with teams and scrapers or with machinery. In marshy land the main canals and levees can usually be built better with floating dredges than by any other means. If the ground is soft and there is nothing to interfere with its use, an orange peel dredge may be used. If the tract contains stumps and fallen timber a dipper dredge will be more serviceable. On account of its longer reach the orange peel type of dredge is preferable where the conditions are favorable for its use. Small open field ditches are frequently excavated by hand after the main body of the land has been drained.

Maintenance of Levees and Ditches. Bushes and other vegetation of rank growth should not be allowed on levee banks since water will follow the roots and increase seepage. After the soft material in a levee has dried sufficiently it should be smoothed off and preferably seeded to Bermuda grass. It may then be mowed with a mowing machine, or the grass may be kept down by allowing stock to graze upon it. A heavy sod will assist in preventing erosion and in keeping out weeds and other objectionable vegetation. Grass and weeds should be cut out of ditches from 1 to 3 times each year. At least once in 2 years the ditches should be cleaned of the accumulated mud and silt.

The Cost of dredging drainage ditches in the United States prior to 1916 varied approximately from 7 to 11 cents per cubic yard. At the same time the cost of building levees with dredges varied from 10 to 19 cents per cubic yard. The cost of clearing right of way, where clearing is necessary, should be added to the above figures. The cost per acre of reclamation by drainage where both levees and interior ditches are required (exclusive of the cost of pumping plants) varies from $15 to $45 per acre. The average cost of excavation on over 50 drainage projects in the Mississippi valley has been approximately $23 per acre. Of this amount the ditches cost about one-third and the levees two-thirds.

20. Inland Marshes

Source of Water. The first step in the investigation, preliminary to deciding upon the plan of reclamation, is to determine the source of the water or the reason that the land is too wet. The water may come from direct rainfall, from visible streams, from the run-off from neighboring hills, or from springs.

The condition may be due largely to the fact that the natural outlet channel is obstructed or of insufficient depth or capacity to properly drain the area.

Main Drainage Channel and Outlet. In order that a swamp area may be properly drained it is necessary that the outlet and main drainage channel should be deep enough and of sufficient capacity to carry the drainage of the swamp and all tributary areas while maintaining an elevation of water surface low enough to provide for effectively draining the area to be reclaimed. The natural drainage channel thru an inland marsh may not be clearly marked or it may be very winding and obstructed with fallen timber and debris. One of the first requirements in such cases is to excavate or straighten and enlarge and deepen such channels and carry them to a natural outlet. If no satisfactory outlet exists, pumping may be resorted to, but inland marshes are usually drained entirely by gravity.

Auxiliary Drainage. The amount of drainage required in addition to that provided by the main channel will depend largely upon the size of the marsh, the natural slope of the ground, the soil formation, and the source of under-

Fig. 26. Reclamation of Inland Marsh

ground water. If the subsoil is sandy and especially if the ground has a fair slope toward the main channel no additional drainage may be necessary. Also if the marsh is small no additional ditches may be required. If the subsoil is of impervious formation auxiliary drainage ditches across the tract may be required at ¼- or ½-mile intervals. When springs occur on the tract they should be cut off by intercepting drains, that is drains which tap the source of supply before it reaches the surface of the ground. This result may usually be accomplished by means of an underground tile drain. If there is much flow on to the tract from surrounding hills a catchwater open ditch at the junction of the high and low land may be advisable. After the main drainage channel and principal auxiliary ditches have been completed, if further drainage is required it may be accomplished by open farm ditches or tile drains. Fig. 26 indicates the different methods of drainage that may be used in reclaiming inland marshes.

The Water Table thruout the tract should not be closer than 3 or preferably 4 ft from the surface of the ground at any point. Marsh lands commonly

settle from 1 to 2 ft after cultivation, the settlement being greater in proportion to the depth of decayed vegetable matter with which the tract is covered. Proper allowance should be made in designing ditches and drains, so that the land will be drained to a sufficient depth after settlement has taken place.

21. Overflowed Lands

Classification. There are two general classes of overflowed lands: tidal marshes which are periodically submerged and uncovered by the rise and fall of tides, and river bottom lands which are temporarily inundated during high stages of the streams. The lower lying marshes in either case may be permanently submerged. Some lands, like those in southern Louisiana, may be subject to overflow from both causes. As regards topography, the area to be reclaimed may either adjoin high land on one side, or be entirely surrounded by overflowed land. In general, a similar plan may be adopted for reclaiming all overflowed lands, the details being changed to suit the particular conditions.

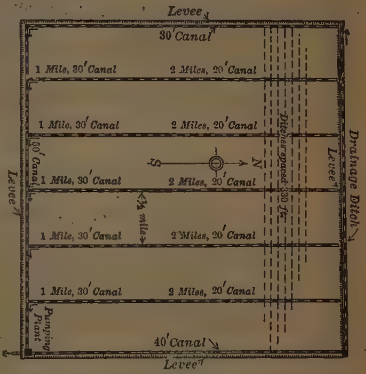

Fig. 27. Reclamation of Overflowed Land

Protection from Overflow is secured by constructing levees around the area to be reclaimed, (Art. 19.) If the tract joins high land on one side, levees will be required on but 3 sides. In general the area of such lands that may be reclaimed by one levee system is limited to that which is included between the main stream or body of water and two of its tributaries. A levee runs along the bank of the main water and return levees follow the tributaries to the high land. Diversion ditches may be constructed near the foot of the slope where the hills join the flat land to deflect the hill waters and prevent their entering the tract. If the area to be reclaimed is surrounded by overflowed lands, levees must be built on all sides. Fig. 27 shows the system of levees and ditches of Area No. 7, Gueydan, Louisiana. Fig. 28 shows the method of reclaiming a tidal marsh near Dorchester, New Jersey.

Drainage. The drainage system consists of one or more main ditches with auxiliary ditches spaced as required, but usually from ¼ to ½ mile apart. Open farm ditches or drain tile may then be used to provide whatever additional drainage may be necessary. Open ditches should preferably be located on property lines or parallel to them, in order that they may interfere as little as possible with farming operations.

The Depth of Ditches should be such that the entire tract may be satisfactorily drained to a depth of 3 to 4 ft. The practice in the lower Mississippi valley, Fig. 27, is to dig farm ditches 4 ft deep. The main ditches must have dimensions and slopes which will provide satisfactory outlets for the smaller ditches.

Marsh lands will frequently settle 1 to 4 ft after reclamation, and this settlement may continue for several years. The probable settlement of the ground should be considered in designing the drainage system.

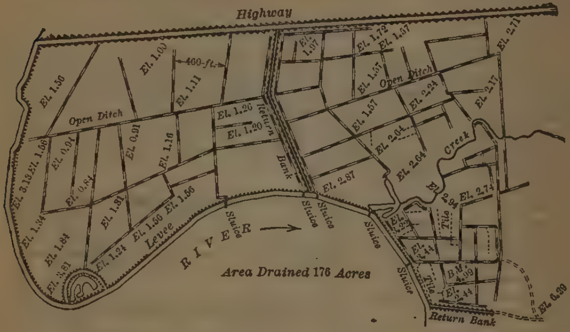

Fig. 28. Reclamation of Tidal Marsh

The Outlet for the drainage system may be provided by gravity or pumping or by a combination of the two. If a natural gravity outlet is relied upon a sluice gate must be provided where the water passes thru the levee. Sluice gates should act automatically and allow the water to pass out when the water surface is higher within than without the tract and close when the opposite condition obtains. A flap valve, hinged at the top, is ordinarily used for this purpose. A hand-operated gate should be installed in conjunction with the automatic gate to be used in case the latter gets out of order. River bottom

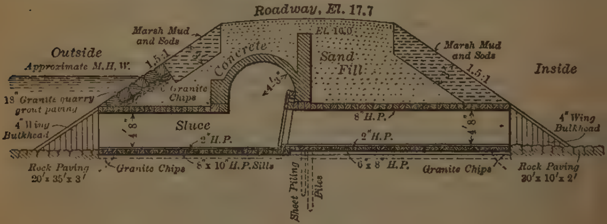

Fig. 29. Longitudinal Section of Levee through Sluice

lands when subject to overflow for only short periods may be drained by gravity outlets if the water surface in the river is normally low enough, otherwise auxiliary pumping will be necessary satisfactorily to drain the tract. Tidal lands may be drained through automatic sluice gates, the gates being opened during low tides and closed during high tides. In localities where the tidal range is small and the elevation of the ground is low, reclamation can be accom-

plished only by pumping. In many instances where tidal lands are drained by gravity the expense of auxiliary pumping would be more than justified by increased returns from the land.

In general, tide lands drained only by gravity lying 1 ft or less above mean low water have no value for agricultural purposes, those between 1 and 2 ft above mean low water will produce hay, and those between 2 and 3 ft above mean low water will produce rye, potatoes, corn and other crops. The best results are obtained from land 3 ft or more above mean low water elevation.

Sluices should be carefully designed and constructed with a view to obtaining both safety and effective service. Cut-off walls and thoroly tamped backfulling will reduce the danger of leakage along the structure. Sluices may be made of either wood or concrete, but the latter is preferable. The gates should be well balanced and fitted so that they will move easily and allow a minimum of leakage and all hardware should be of bronze or some other non-corrosive metal. The tops of sluices should be below ordinary low tides, as they thus afford better drainage and the gates being submerged will operate more easily. The capacity of a sluice should be sufficient to discharge the maximum run-off to be provided for in addition to back leakage. Gates may be set either at the outlet of the sluice or in a chamber within the sluice. Fig. 29 shows a cross-section of a levee and sluice at Wellfleet, Massachusetts.

FARM DRAINAGE

22. General Considerations

Soil Moisture is one of the essentials of plant life, but all water in excess of that required by vegetation is harmful. The only water that is needed by plants is contained in the capillary water (Art. 2) which exists as a thin coating to the soil particles. Gravity water, that is water in excess of the soil's capacity to contain capillary water, is not beneficial. Air within the soil is also essential to healthy plant growth, and this is excluded when the ground contains water in excess of plant requirements. In a soil completely saturated with stagnant water, air is completely excluded and the ground becomes cold and sour and entirely unproductive. The conditions most favorable to plant growth obtain when the gravity water passes freely downward, leaving in the soil only as much water as is beneficial to plants, and allowing a free circulation of air thru the soil.

The Ground-water Level, for ordinary soils and crops, need seldom be at a depth of more than 4 ft, and in some cases less. When too near the surface, water will be continually drawn up by capillary action, the percolation of precipitation will be retarded, and the soil will become cold and improperly aërated.

Lands Requiring Drainage. Three classes of land may be considered: (*a*) That which is entirely unproductive or which cannot be profitably farmed without drainage, (*b*) that which may be profitably farmed without drainage but which with drainage will produce increased returns sufficient to justify the expense of drainage, (*c*) that which cannot be sufficiently improved to justify the expense of drainage. Low lying marshy and springy lands belong to the first class. It is probable that all lands not having a porous subsoil, and especially clay lands, can be benefited by drainage. Whether it will pay to drain such lands depends on the value of the land before drainage as compared to its value after drainage. A fair criterion to apply is whether the increased returns after drainage will exceed the interest on the investment required to drain the land.

Benefits of Drainage. Drainage benefits farm lands in the following ways: (*a*) By removing the surplus water, which is harmful to plants, and aërating the soil; (*b*) by warming the soil and thus decreasing danger from frosts, causing the seeds to germinate earlier, and grow more promptly, permitting earlier cultivation of the land, and increasing the length of the growing season; (*c*) by making the soil more granulated, porous, and friable, and thus increasing its ability to absorb rainfall. In general, the results of farm drainage are to make certain the production of crops each year and to increase the yields and profits.

An Objectionable Feature of drainage is that the ground water in passing down and away carries with it in solution many elements that are valuable as plant food. There appears to be no remedy for this condition in humid regions, since adequate drainage is the first requisite. In irrigated districts the difficulty may be in a large measure overcome by restricting the use of water to the actual requirements of crops.

Preliminary Investigation. Before laying out a plan of drainage for a given area a careful preliminary study should be made and a topographic map should be prepared showing contours and intermediate elevations at controlling points. It is important that natural drainage lines should be distinctly shown. A soil survey of the area should be made and the character of the soil and subsoil at different points may be conveniently marked on the map, supplemented by additional field notes if necessary. The ground water also should be studied, especially if there is evidence that it comes from some source other than precipitation upon the area to be drained. The position of the outlet should be determined from the field investigation and marked upon the map. The map and other data may then be used as a basis for the plan of drainage.

23. Drains and Drain Tile

Principles of Drainage. There are two kinds of drainage: Surface drainage, by which water is conveyed over the surface of the ground; and sub-surface drainage, by which water is conveyed beneath the surface of the ground. Surface drainage removes the water that cannot percolate into the ground as fast as it is supplied by precipitation or melting snow. Sub-surface drainage removes the water which cannot be retained as soil moisture.

Farm lands require surface drainage to remove the excess water resulting from heavy rains, but, in general, it is better to have as much water percolate thru the soil into the sub-surface drainage as possible. If ground slopes are too steep heavy showers will run off the surface before the ground has been wet as deep down as is desirable. Better results will usually be obtained from gently sloping land, where the bulk of the water soaks into the ground and that which is in excess of plant requirements is promptly removed by sub-surface drainage. It is frequently desirable to retard the surface drainage on sloping ground which may be accomplished in a measure by cultivating the land across the slopes. Farm drainage is primarily a problem of supplementing natural sub-surface drainage.

Open Ditches receive water from the surface of the ground and also water that passes thru the soil. They therefore assist in surface drainage, which may be an advantage on very flat areas. In the lower Mississippi, where the rainfall is heavy and the land practically level, open farm ditches are used almost exclusively (Fig. 27). Farm ditches occupy land which might otherwise be cultivated, and make it difficult to move farm machinery from one part of the farm to another. They are expensive to maintain and in clay soils they are apt to become puddled and prevent the free entrance of ground water.

Sub-surface Drains are conduits below the surface of the ground with openings of a size that will permit the ground water to enter freely and exclude the

surrounding earth. Box drains have been used to a limited extent. They are made of four boards nailed together to form a rectangular cross-section with cleats between the top and sides to leave narrow openings for the entrance of water. Blind ditches partially filled with stones or poles were formerly used, but they were not permanent and have been abandoned. Tile drains have been found to provide the most satisfactory drainage, and they are used almost exclusively.

Kinds of Tile. There are two kinds of drain tile in common use: Clay tile and cement tile. Clay tile are sometimes made hexagonal or octagonal, but more commonly they are round. They range from 2 to 36 in in diameter and from 12 in in length for the smaller sizes to 30 in for the largest. Cement tile under 12 in in diameter are more expensive than clay tile, but over this size the cost of production may be less. The following comments on clay and cement tile are contained in Farmers' Bulletin 524 of the U. S. Department of Agriculture:

Clay Tile. Soft, medium, and hard-burned or vitrified clay tile are made. It costs less to make the soft-burned than the hard-burned tile and the selling price is lower, but the quality is not so good. Soft-burned tile have done good service, however, and when put under ground below the frost line have lasted indefinitely. The best tile are burned to a cherry red and when struck by a piece of steel give a sharp metallic ring. In the north, where they are laid above the frost line, only the hard-burned tile should be used. Hard-burned, or vitrified, tile are practically nonporous; thus, they absorb little moisture and unless water stands in them they are not injured by freezing temperatures. A tile that cracks and shatters in winter when lying in the yards unprotected from the weather is not fit to use above the frost line and is not the best under any conditions. Thick tile make the best joints. Those with thin sides, especially in the smaller sizes, are likely to slip out of place and leave openings. Some manufacturers who ship tile make them with thin sides to reduce the weight and the consequent cost of transportation. A 4-in tile should weigh at least 6 lb, a 5-in 8 lb, and a 6-in 11 lb. Some factories make them heavier than this, which is better so far as utility is concerned.

Cement Tile. There are a number of machines on the market for making cement (or concrete) tile that cost from $50 to $100. One part of cement to about 3 parts of aggregate should be used. The largest pieces in the aggregate should not be more than one-half the thickness of the wall. The cement and aggregate should be well mixed and sufficient water should be added to make the mixture moist but not wet. After the tile are taken from the machine they should be placed in the shade. The longer the mold is allowed to remain around the tile the better. After the mold is removed the tile should be sprinkled twice a day for 3 or 4 days, or until thoroly cured. A well-made and well-cured cement tile, like a good clay tile, when struck with a piece of iron will have a sharp metallic ring. Large cement factories are equipped with steam driers, with which the curing can be done to the best advantage.

24. Location of Drains

The General Plan of draining a given area should be determined before any drains are constructed, in order that all parts of the system when completed will work together effectively even tho the construction is to extend over a long period. Details not pertaining to the general scheme of drainage may be worked out as required. When more than one farm will be benefited by the construction of main drains, each party interested should contribute in the investigation in so far as the work affects his property. State laws usually cover the legal points involved

The Outlet is usually the first consideration. On rolling or hilly lands a natural outlet ordinarily exists. On low level land an artificial outlet is usually necessary. The elevation of the water surface in the outlet should be low enough to provide for the effective drainage of the area which it serves. It should be of

sufficient capacity to take care of the requirements of surface and sub-surface drainage. The expense should be assessed against all of the farms using the outlet in proportion to the benefits accruing to each. Outlets may be either open ditches or covered pipes. Tile up to 36 in in diameter have been used for this purpose.

Drainage Systems. There are two general systems of drainage: Partial and complete. Partial drainage systems are used on rolling land when it is desired to drain only isolated portions of a tract, as illustrated in Fig. 30. The complete system is used when all of the land of a comparatively large area is to be drained. Examples of complete drainage systems are shown in Fig. 31 and 32. It will be noticed that certain areas adjacent to the main laterals are double drained, and for maximum economy this double drained area should be as small as practicable. For this reason long parallel field drains are preferable. The system indicated in Fig. 32 requires a smaller amount of tile than that indicated in Fig. 31.

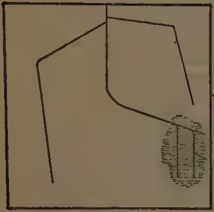

Fig. 30. System of Partial Drainage

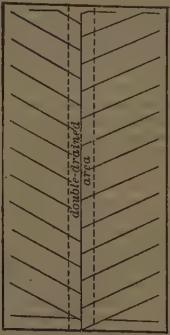

Fig. 31. System of Complete Drainage

Fig. 32. System of Complete Drainage

Field drains should be laid in the direction of the greatest slope. This insures the greatest velocity and capacity. High velocities of water assist in scouring out the drains and in keeping them clean. It is very important that drains in sandy land should have velocities great enough to scour out the sand which filters into the drains between the tile. A 4-in tile drain in sandy land should not be laid on a grade of less than 0.3 ft to 100 ft. The laterals, submains and mains should perferably be located approximately in the lines of natural drainage, since the slope of the ground leads the water in this direction. The drains should be laid as straight as practicable and bends should be made by smooth curves. It usually will be found necessary to deviate somewhat from the natural drainage channel in order to improve the alignment.

The Depth and Frequency of Drains that will produce the most satisfactory results depends largely upon the character of soil. Where deep drainage can be used the drains may be spaced farther apart. The more porous the soil the deeper the drains can be placed. In heavy clay soils of close textures, drains placed to a depth of from 2 to 3 ft have produced good results when deeper drains have failed. In other cases clay soils have been satisfactorily drained at depths of from 3 to 4 ft. A depth of about 4 ft appears to be the best for sandy soils.

Practice also varies widely regarding the proper spacing of drains. In dense clay soils they should probably be placed from 30 to 40 ft apart, and in sand a spacing of 200 ft may be used. For intermediate grades of material distances of from 50 to 150 ft may be selected to suit the conditions.

Before deciding definitely as to the proper depth and frequency of drains for a given area a careful study should be made of all factors likely to affect the problem. Conditions vary widely and results obtained in one locality may not apply to another locality where conditions appear quite similar. Local conditions especially should be studied, and if time is available and other conditions seem to warrant it, experiments with different depths and spacings of drains should be made before the completion of the work as a whole is undertaken.

25. Capacity and Size of Drains

The Required Capacity of a sub-surface drain depends upon the rate at which water will pass from the surface of the ground to the drain. The two principal factors affecting the rate at which water will enter a drain are the porosity of soil and the amount and rate of precipitation.

Drainage Coefficient. The inches depth of water which must be removed in 24 hours to satisfactorily drain any area is called the drainage coefficient of that area. Its value for different soils must be obtained from experiments. An investigation by the U. S. Department of Agriculture of a number of underground drains in Illinois and Iowa that were doing satisfactory service showed maximum discharges in 24 hours of from 0.11 to 0.27 in, most of the results varying from 0.15 to 0.20 in. These results were obtained on areas having fairly porous subsoils. Heavy clay soils and coarse sand soils will have respectively smaller and larger drainage coefficients than the above. It also appears that the drainage coefficient of rolling land is about 20 percent greater than for flat land. The following may be considered average values of drainage coefficients for different soils:

Compact clay soil, flat, 0.12; rolling, 0.14.
Medium porous soil, flat, 0.18; rolling, 0.22.
Porous sandy soil, flat, 0.30; rolling, 0.36.

With the drainage coefficient decided upon, the capacity of drain in cu ft per sec required to drain a given area can be obtained by multiplying the area in square miles or acres by the proper value in the accompanying tables:

Decimals of an Inch of Run-off per Twenty-four Hours Expressed as cu ft per sec per square mile

Inches	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	......	.27	.54	.81	1.08	1.34	1.61	1.88	2.15	2.42
.1	2.69	2.96	3.23	3.50	3.76	4.03	4.30	4.57	4.84	5.11
.2	5.38	5.65	5.92	6.18	6.45	6.72	6.99	7.26	7.53	7.80
.3	8.07	8.34	8.60	8.87	9.14	9.41	9.68	9.95	10.22	10.49
.4	10.76	11.02	11.29	11.56	11.83	12.10	12.37	12.64	12.91	13.18
.5	13.44	13.71	13.98	14.25	14.52	14.79	15.06	15.33	15.60	15.86
.6	16.13	16.40	16.67	16.94	17.21	17.48	17.75	18.02	18.28	18.55
.7	18.82	19.09	19.36	19.63	19.90	20.17	20.44	20.70	20.97	21.24
.[illegible]	21.51	21.78	22.05	22.32	22.59	22.86	23.12	23.39	23.66	23.93
.[illegible]	24.20	24.47	24.74	25.01	25.28	25.54	25.81	26.08	26.[illegible]5	26.62
1.0	2[illegible]									

Decimals of an Inch of Run-off per Twenty-four Hours Expressed as cu ft per sec per Acre

Inches	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	......	.00042	.00084	.00126	.00168	.00210	.00252	.00294	.00336	.00378
.1	.00420	.00462	.00504	.00546	00588	.00630	.00672	.00714	.00756	.00798
.2	.00840	.00882	.00924	.00966	.01008	.01050	.01092	.01134	.01176	.01218
.3	.01260	.01302	.01344	01386	.01428	.01470	.01512	.01555	.01597	.01639
.4	.01681	.01723	.01765	.01807	.01849	.01891	.01933	.01975	.02017	.02059
.5	.02101	.02143	.02185	.02227	.02269	.02311	.02353	.02395	.02437	.02479
.6	.02521	.02563	.02605	.02647	.02689	.02731	.02773	.02815	.02857	.02899
.7	.02941	.02983	.03025	03067	.03109	.03151	.03193	.03235	.03277	.03319
.8	.03361	.03403	.03445	.03487	.03529	.03571	.03613	.03655	.03697	.03739
.9	.03781	.03823	.03865	.03907	.03949	.03991	.04033	.04075	.04117	.04159
1.0	.04201									

Size of Drain. The size of drain required to discharge a given quantity of water can be computed from one of the open channel formulas. The Manning formula for the smallest diameter of circular conduit that will discharge a given quantity of water may be written,

$$d = 12\left(\frac{2Qn}{s^{1/2}}\right)^{3/8},$$

in which d is the diameter of the conduit in inches, Q is the discharge in cu ft per sec, s is the slope or fall per foot, and n is the coefficient of roughness, which is to be given the same value as Kutter's n. Values of n depend upon the degree of roughness of the pipe and the care that is taken in placing the tile to get smooth joints, the extreme range of variation being from about .011 to .017. A value of .014 should be attained from a good grade of tile if proper care is taken in laying them. Values of d from the above formula with $n = 0.14$ given in the table cover the ordinary ranges of Q and s. For other values of n multiply d as given in the table by the following factors:

For n equal to.............	.011	.012	.013	.015	.016	.017
Multiply d in table by......	0.91	0.94	0.97	1.03	1.05	1.08

Formerly 2-in tile were used for farm drains, but modern practice is to use no tile smaller than 3 in in diameter. The size of tile selected for a given case should ordinarily be the commercial size nearest to and larger than the computed size, excepting that no tile smaller than 3 in should be used. The following are diameters in inches of common commercial tile: 2, 2½, 3, 4, 5, 6, 7, 8, 10, 12, 15, 18, 20, 21, 22, 24, 27, 30, 33, and 36. For main drains above 15 in in diameter vitrified sewer pipe is commonly used.

Diameters of Tile Drains in Inches and Decimals of an Inch Required to Discharge Various Quantities of Water at Different Slopes

Discharge in cu ft per sec	s = fall per foot											
	.0005	.001	.002	.004	.006	.008	.01	.02	.03	.04	.05	.1
.01	2.3	2.0	1.8	1.6	1.5	1.4	1.3	1.2	1.1	1.0	1.0	0.9
.02	3.0	2.6	2.3	2.0	1.9	1.8	1.7	1.5	1.4	1.3	1.3	1.1
.04	3.9	3.4	3.0	2.6	2.5	2.3	2.2	2.0	1.8	1.7	1.6	1.4
.06	4.5	4.0	3.5	3.1	2.9	2.7	2.6	2.3	2.1	2.0	1.9	1.7
.08	5.1	4.4	3.9	3.4	3.2	3.0	2.9	2.5	2.3	2.2	2.1	1.9
.1	5.5	4.8	4.2	3.7	3.5	3.3	3.1	2.8	2.6	2.4	2.3	2.0
.2	7.1	6.3	5.5	4.8	4.5	4.2	4.1	3.6	3.3	3.1	3.0	2.6
.3	8.3	7.3	6.4	5.6	5.2	4.9	4.7	4.2	3.9	3.7	3.5	3.1
.4	9.3	8.1	7.1	6.3	5.8	5.5	5.3	4.6	4.3	4.1	3.9	3.4
.5	10.0	8.8	7.7	6.8	6.3	6.0	5.7	5.0	4.7	4.4	4.2	3.7
.6	10.8	9.5	8.3	7.3	6.8	6.4	6.1	5.4	5.0	4.7	4.5	4.0
.8	12.0	10.5	9.3	8.1	7.5	7.1	6.8	6.0	5.6	5.3	5.1	4.4
1.0	13.1	11.5	10.1	8.8	8.2	7.8	7.4	6.5	6.1	5.7	5.5	4.8
1.2	14.0	12.3	10.8	9.5	8.8	8.3	8.0	7.0	6.5	6.1	5.9	5.2
1.4	14.8	13.0	11.4	10.0	9.3	8.8	8.4	7.4	6.9	6.5	6.2	5.5
1.5	15.2	13.3	11.7	10.3	9.5	9.0	8.7	7.6	7.1	6.7	6.4	5.6
2.0	16.9	14.9	13.1	11.5	10.6	10.1	9.7	8.5	7.9	7.4	7.1	6.3
2.5	18.4	16.2	14.2	12.5	11.6	10.9	10.5	9.2	8.5	8.1	7.8	6.8
3.0	19.7	17.3	15.2	13.3	12.4	11.7	11.2	9.9	9.1	8.7	8.3	7.3
3.5	20.9	18.3	16.1	14.1	13.1	12.4	11.9	10.5	9.7	9.2	8.8	7.7
4	22.0	19.3	16.9	14.9	13.8	13.1	12.5	11.0	10.2	9.7	9.3	8.1
5	23.9	21.0	18.4	16.2	15.0	14.2	13.6	12.0	11.1	10.5	10.1	8.8
6	25.6	22.4	19.7	17.3	16.0	15.2	14.6	12.8	11.9	11.2	10.8	9.5
7	27.1	23.8	20.9	18.3	17.0	16.1	15.4	13.6	12.6	11.9	11.4	10.0
8	28.5	25.0	22.0	19.3	17.9	16.9	16.2	14.3	13.2	12.5	12.0	10.5
10	31.0	27.2	23.9	21.0	19.4	18.4	17.7	15.5	14.4	13.6	13.1	11.5
12	33.2	29.1	25.6	22.4	20.8	19.7	18.9	16.6	15.4	14.6	14.0	12.3
14	35.1	30.8	27.1	23.8	22.0	20.9	20.0	17.6	16.3	15.4	14.8	13.0
16	36.9	32.4	28.5	25.0	23.2	22.0	21.1	18.5	17.1	16.2	15.6	13.7
18	38.6	33.9	29.8	26.1	24.2	22.9	22.0	19.3	17.9	17.0	16.3	14.3
20	40.2	35.3	31.0	27.2	25.2	23.9	22.9	20.1	18.6	17.7	16.9	14.9
25	43.7	38.3	33.7	29.6	27.4	26.0	24.9	21.9	20.3	19.2	18.4	16.2
30	46.7	41.0	36.0	31.7	29.3	27.8	26.7	23.4	21.7	20.6	19.7	17.3
35	49.5	43.5	38.2	33.5	31.1	29.4	28.2	24.8	23.0	21.8	20.9	18.3
40	52.0	45.7	40.1	35.3	32.7	31.0	29.7	26.1	24.2	22.9	22.0	19.3
45	54.4	47.8	42.0	36.8	34.2	32.4	31.0	27.3	25.3	23.9	22.9	20.2
50	56.6	49.7	43.7	38.3	35.5	33.7	32.3	28.3	26.3	24.9	23.9	21.0
55	58.7	51.5	45.2	39.7	36.8	34.9	33.5	29.4	27.2	25.8	24.7	21.7
60	60.6	53.2	46.7	41.0	38.0	36.0	34.6	30.4	28.1	26.7	25.6	22.4
65	62.5	54.9	48.2	42.3	39.2	37.1	35.6	31.3	29.0	27.5	26.3	23.1
70	64.2	56.4	49.5	43.5	40.3	38.2	36.6	32.2	29.8	28.2	27.1	23.8
80	67.5	59.3	52.1	45.7	42.4	40.2	38.5	33.8	31.3	29.7	28.5	25.0
90	70.6	62.0	54.4	47.8	44.3	42.0	40.2	35.3	32.8	31.0	29.8	26.1
100	73.4	64.5	56.6	49.7	46.1	43.7	41.9	36.8	34.1	32.3	31.0	27.2
110	76.1	66.8	58.7	51.5	47.8	45.2	43.4	38.1	35.3	33.5	32.1	28.2

26. Construction of Tile Drains

Staking Out. Tile drains should be set true to line and accurately to grade. When curves are necessary they should be smooth and regular. Before beginning excavation stakes should be set 50 ft apart parallel to and about 2 or 3 ft from the center line of the drain on the side opposite to that on which the earth will be thrown. Elevations of tops of stakes should be taken with a level and the cut marked on each. These stakes will serve as guides for the rough excavation. After the excavation is approximately to grade, batter boards should be placed across the trench opposite each stake at the same distance, preferably about 6.5 or 7 ft above grade. The center line is then marked on the batter boards and a string connecting these points will be directly above and parallel to the grade line. The center line at any point may then be obtained by dropping a plumb bob from the line, and the grade by measuring down with a pole of proper length.

Fig. 33
Tile Hook

Excavation should be begun at the outlet. The trench may be dug either by hand or with a trenching machine. If much work is to be done the trenching machine will prove more economical. For 6-in. tile or smaller the trench may be made 12 in wide at the top and 7 in at the bottom. The ditch should be finished at the bottom with a drain scoop. It is especially important that the bottom of the ditch be excavated accurately to grade and line before the tile are placed.

Trenching Tools and Machines. The special tools commonly used in digging trenches or drain tile are tile spades, a shovel, pick, drain scoop, and tile hook. There are two kinds of tile spades, solid and open, the latter being used for wet sticky soils. Tile spades are from 16 to 22 in long and from 5 to 6 in wide. A drain scoop is used for finishing the bottom of the trench preparatory to laying the tile. It should be curved to a radius approximately equal to the outer radius of the tile. A tile hook, Fig. 33, is used for placing tile in the trench. It consists of a bent ½-in rod attached to a wooden handle. There are a number of types of trenching machines on the market. They vary from different styles of plows, which loosen the earth so that it may be more readily shoveled, to large machines of the wheel-bucket and endless chain type, which do all the work of excavation. The larger trenching machines vary in cost from $1800 to $6500. Trenching plows may be obtained at $20 to $300.

Laying of Tile, like the excavation of trenches, should begin at the outlet. If the bottom of the trench has been properly prepared the laying of the tile should proceed rapidly. The smaller sizes of tile are usually laid with a tile hook by a man standing at the top of the trench, Care should be taken to see that the tile joints fit tightly, especially at the top. Crooked and imperfect tile should be thrown out and if used at all they should be placed together near the upper end of the drain. Curves are usually made by using crooked tile or by chipping off one edge of the tile. It is easier and better, however, to have special curved shapes, either drain or sewer tile, available for the purpose. Y-junctions should be placed where needed, the open end being plugged up until the connection is made. Before the tile are covered the work should be inspected to see that all joints are properly made and that the tile are laid true to line and grade.

Backfilling. Before the trench is filled the tile may be surrounded by coarse hay, twigs, burlap, small stones, or pieces of brick. This provides for a freer entrance of water and helps to exclude fine sand from the drain. The first earth should be put into the trench carefully so as not to move the tile. The

remainder of the backfilling may be done with shovels, scrapers, or a turn plow.

Outlets. Where a tile drain discharges into an open waterway the outlet should be protected by building a substantial retaining wall around it. The foundation should extend well below the bottom of the channel so that the wall will not be undermined by the current in the waterway or the water discharging from the drain. Only vitrified clay or metal pipe should be used near the outlet, as the softer clay tile which will be satisfactory under ordinary conditions may disintegrate under the alternate conditions of freezing and thawing to which they will be subjected at the outlet.

Surface-Inlets are sometimes constructed at low places to supplement surface drainage. They are commonly made of wood, concrete, or sewer pipe. They consist essentially of a small basin that extends to or below the bottom of the drain to which they are connected, with an opening at the top for the entrance of surface water. The surface opening should be covered with an iron grate which also should be protected by a covering of loose rock to prevent the entrance of dirt and drift.

Silt-Basins or Sand-Traps are basins placed in drains or at the junctions of drains for the collection of silt and sand. They also afford an opportunity for the inspection of drains. The bottoms of the basins should be 2 or 3 ft below the tile to provide a receptacle for the silt, which may be cleaned out as often as is necessary. These structures are not required for clay or loam soils, but are useful where drains are laid in sandy soils.

27. Costs and Profits

Cost of Round Drain Tile f o b Factory

Prices quoted by manufacturers, subject to discount of from 35 to 55%.

Size, in	Length of one tile, ft	Weight per linear ft, lb	Price per 1000 ft	Price of Y's, T's, ells and curves, each	Linear ft per car load of 30 000 lb
2	1	3	$ 12	$0.10	9000
2½	1	4	15	.10	7500
3	1	5	20	.20	6000
4	1	7½	30	.20	4000
5	1	10	40	.30	3000
6	1	13	55	.40	2500
8	2	20	90	.60	1500
10	2	30	135	1.00	1000
12	2	40	180	1.50	750

Amount of Tile per Acre. Where, over a considerable area, field drains are placed parallel and an equal distance apart, the number of linear feet of tile required per acre may be obtained by dividing 43560 by the distance between drains in feet. The following are linear feet for distances commonly used. An allowance of 5 percent should be made for breakage and imperfect tile.

Distance apart....	25	30	33	40	50	66	80	100	150	200
Lin ft per acre....	1742	1452	1320	1089	872	660	545	436	291	218

Cost of Constructing Drains. Under favorable conditions, for 6 in tile or smaller, the cost of digging trenches 3 ft by hand varies from 2 to 3 cts per ft.

On large contracts, trenching machines have done similar work for from 1½ to 2½ cts per ft. About 50 percent should be added to these figures for trenches 4 ft deep. Laying tile and backfilling costs from ¼ to ½ ct per ft. If the ground is rocky or contains many roots the expense of excavation will be greatly increased. Very wet soil and especially quicksand may add to the cost of the work. The cost of laying a 12-in drain will be about twice as great as laying a small drain to the same depth.

The Cost per Acre of drainage will depend upon the distance between drains. If drains are 100 ft apart, 436 lin ft of tile will be required. With the cost of 4-in tile at $20 per thousand delivered to the ground, the cost of tile per acre will be $8.72. If trenching costs 2½ cts per ft and laying tile and backfilling ⅜ cts per ft the total cost of drainage, exclusive of mains and laterals and other accessories, will be $20.25 per acre. Where field drains are spaced from 50 to 120 ft apart the corresponding costs of drainage may range from $45 to $15 per acre. A common cost of tile drainage is about $25 per acre.

Profits from Drainage. The profits to be derived from farm drainage depends largely on the character of the land. The greatest profits usually result from the drainage of lands too wet for cultivation. The following examples are given in the Yearbook of the U. S. Department of Agriculture for 1914 as typical of results obtained from properly draining farm lands in the humid region of the United States:

In the coastal plain of North Carolina about 25 acres that were producing nothing were tile drained for perhaps $250, probably not including costs of teaming and of supervision, and since then have produced a bale of cotton per acre. A field of 6 acres was drained for about $160, and the owner makes good crops on soil worthless without drainage. In the black prairie belt of Alabama, a field that had not been cultivated in years because too wet was drained with tile; then it produced one bale of cotton per acre and repaid the entire cost of drainage the first year. The following year the field yielded 50 bush of corn per acre, twice the rate from other parts of the farm. Another drained field produced one bale of cotton per acre, while the undrained land produced only half a bale. A 10-acre field that yielded practically nothing in 1912 was tile drained, and in 1913 produced 60 bush of oats per acre; in 1914 the rate was again 60 bush of oats, in contrast to 10 bush per acre from the adjoining 15-acre field planted to the same grain. The cost of most of the tile drainage in Alabama has been about $25 per acre, some of it as high as $30 to $35, but increases of 50 to 200 percent in yields and the assurance of good crops every year instead of only in very favorable seasons are very satisfactory returns. The cost of drainage there has usually been repaid in 2 to 3 years by improved crops. In Iowa, a field of 40 acres too wet for planting was tile drained at a cost of $24 per acre, after which it produced 60 bush of corn per acre. Another field was drained for $23 per acre, thereby increasing the yield from 15 bush to 40 and 50 bush of corn per acre. In Arkansas, on one of the State farms, 1 bale of cotton per acre was secured in favorable years, and nothing at all when the early part of the season was wet; the year following the installation of tile the yield was 1½ bales per acre. In Nebraska a tract of more than 700 acres was tile drained at $24.25 per acre, the pumping plant cost $2 per acre, and as part of a larger district the cost of levees to protect from overflow was $9 per acre. The improvement, at a total cost of $35 per acre, immediately increased the crop on about 80 acres of corn 22 bush, and on another part the increase in 2 years was from nothing to more than 30 bush of wheat per acre.

28. Drainage of Irrigated Land

Necessity. More than 10 percent of the area in United States, which has been under irrigation for a considerable period, has become partially or entirely unproductive thru waterlogging or becoming impregnated with harmful mineral salts. These conditions are brought about by over-irrigation and inadequate natural drainage, and the remedy lies in supplementary artificial drainage.

he necessity for artificial drainage or the system of drainage to be employed s not apparent usually until after several seasons of irrigation, but from a study f the topography and soil strata it should be possible to foretell approximately he future drainage requirements of a newly irrigated tract. Some irrigation rojects do not require artificial drainage and others require it only to a limited xtent, but in all cases the matter should be thoroly studied in connection with he investigation and design of an irrigation system.

Alkali is the popular name applied to a number of mineral salts, harmful to vegetation, hich are found in the soils of arid districts. The more common of these are sodium hloride, sodium sulphate, magnesium sulphate, calcium chloride, calcium sulphate and odium carbonate. All of these except the last appear on the ground as a white crust nd are called white alkali. Sodium carbonate, or black alkali, is the most injurious to lants. It is indicated by dark spots on the surface of the ground due to the dissolution f humus and vegetable matter which it causes. These salts are readily soluble in water nd irrigation waters after passing thru the soil become more or less impregnated with hem. When the ground water reaches a sufficiently high elevation, moisture is drawn om it by capillary attraction to the surface of the ground, where the water is evapoated and the alkali is left as a thin deposit. After this action continues long enough ufficient alkali collects on the ground to injure or destroy vegetation. The height to hich water will rise by capillarity varies from about 2 ft for coarse sand to 5 ft for fine lay.

Removal of Alkali. Lands which contain an excess of alkali either from atural causes or from irrigation usually may be cleansed by irrigation after dequate drainage has been provided. The water must be lowered below he limit of capillary action, and then by applying water the salts are dissolved nd carried thru the soil to the ground water. Usually two good irrigations ill reduce the alkali on the surface to a harmless quantity, tho in rare instances wo years or more may be required for reclamation.

Objects of Drainage. The specific results to be accomplished by the drainge of irrigated land may be one or more of the following: (*a*) The lowering f the ground water to a level where it will not interfere with the penetration f plant roots to the required depth; (*b*) the removal of alkali from the surface f the ground; (*c*) the removal of excess soil moisture and the aëration of the oil. The problem differs from that encountered in humid regions in that kaline soils are peculiar to arid districts and the amount of water which the nd receives under irrigation may in a large measure be controlled.

Source of Water. A drainage system cannot be designed successfully until e source of the damaging water is known and its movement understood. The urce of the water may be (*a*) irrigation water applied to the tract itself, (*b*) rigation water applied to an area at a higher elevation, (*c*) a canal or reservoir a higher elevation. In the first case the movement of the water will be downard thru the soil. In the last two cases the water will move laterally either in orous strata near the surface or under pressure beneath an impervious stratum hardpan or clay.

Depth of Drains. The drainage should be such as to keep the ground water vel below the depth which will allow the movement of water to the surface of e ground by capillary action. The depth must be more than 4 ft for fine soils. xperience has shown that the depth of drains should be never less than 5 ft d a depth of 6 to 8 ft will produce more efficient results. Intercepting drains ould be placed at the depth that will most effectively cut off the flow of underound water.

Location of Drains. The location of drains will always depend upon the source d movement of the damaging water, and no general rules can be laid down. arallel drains may be used on flat areas where the water does not come from

some clearly defined outside source. Such drains are usually spaced 300 ft or more apart. Intercepting drains may be used to cut off the ground water flowing from higher areas. They are usually placed near the foot of a hill or where there is an abrupt change of slope. A secondary intercepting drain is sometimes placed below and approximately parallel to the main drain to collect water that passes the upper drain.

The area enclosed by the shaded line, Fig. 34, shows a tract of land near Hyde Park, Utah, which had become waterlogged and alkaline from the irrigation of lands at higher elevations. This land was successfully reclaimed by the system of intercepting drains indicated in the figure. Fig. 35 shows a tract of land near Garland, Utah, that was reclaimed by a combined system of intercepting drains and field drains. Where water is carried from a higher area thru a porous stratum the intercepting drain should extend down to the impervious material, as indicated in Fig. 36, otherwise some of the water will pass below the drain. Fig. 37 shows a method of intercepting water under pressure beneath an impervious stratum. This method may be used when the water is at a great depth.

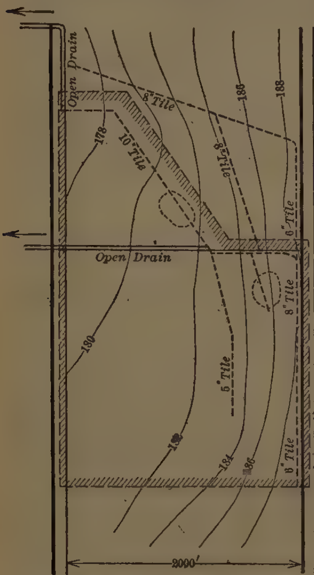

Fig. 34. Drainage of Irrigated Land by Intercepting Drains

Capacity of Drains. The amount of drainage to be provided for will depend upon the source of the water. Where the water comes entirely from the irrigation of the tract being drained it will usually be sufficient to provide for the removal of 0.05 to 0.1 in of water per 24 hours. This is equivalent to 9 to 18 in for a growing period of 6 months. Where the damaging water comes from an outside source, the determination of the proper drain capacity may be very difficult and definite rules cannot be given. If the water comes from other irrigated lands their areas and the amount of water percolating from them should be determined as accurately as possible. If it comes from irrigation canals, the seepage losses in such canals should be investigated. The careful consideration of these matters may assist in a satisfactory solution of the problem.

Drains. Either open ditches or closed drains may be used. The relative advantages and disadvantages of the two types are practically the same as with drains in humid regions. Nothing smaller than 4-in drain tile should be used on irrigated lands. The outlet may be either a natural drainage channel or an irrigation ditch. In the latter case, the drainage water will be available for irrigating areas lower than the one drained. The cost of trenching for tile

drains will be relatively higher than in humid regions owing to the greater depth of drains. If the banks tend to cave, timbering must be resorted to, which will add materially to the cost of the work.

Over-irrigation and Drainage. The necessity for the drainage of irrigated lands is brought about largely thru over-irrigation. If water is applied to the

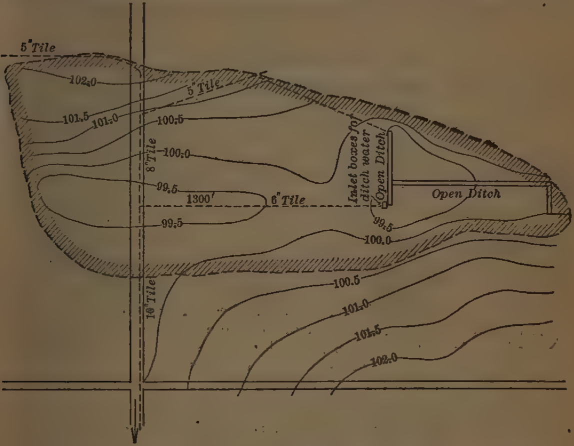

Fig. 35. Drainage of Irrigated Land by Intercepting and Field Drains

land only within its capacity to contain capillary water the needs of vegetation may be provided for and no water will be lost thru deep seepage This ideal condition cannot always be attained, but in general the use of less water will be beneficial in many ways. The injurious effects of waterlogging and alkali

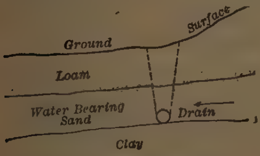

Fig. 36. Intercepting Drain

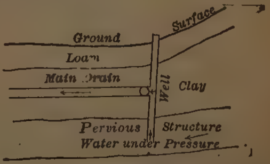

Fig. 37. Intercepting Drain for Water under Pressure

usually can be overcome by drainage, but in the same manner that alkali is removed, various mineral elements of the soil, and humus and nitrogen which are valuable plant foods are washed thru the soil and out of reach of plant roots. In all districts where over-irrigation is practiced, this process of leaching out the soil is in progress and the ground is gradually losing its valuable

elements of plant food. By exercising care in the use of water newly irrigated districts may, while providing adequate moisture for crops, avoid this waste and possibly prevent the necessity of constructing expensive drainage systems in the future.

LEGAL AND ECONOMIC FEATURES

29. Administration of Streams

Early History. During the early colonial period of the United States it was found that domestic legislation could not be enacted immediately to cover all subjects and provide for all contingencies. In order that some supplementary law might be available to govern unusual cases, the states generally accepted the Common Law of England. This was presumed to be a temporary matter and it was believed that statutory law would soon replace this legislation borrowed en bloc during a time of emergency. The states thus introduced, as a part of the Common Law of England, the doctrine of riparian rights.

The Doctrine of Riparian Rights was given its first concrete definition by Lord Ellenborough, in the English House of Lords in 1805. A final definition of the doctrine was given in the House of Lords in 1833 in th ecase of Mason *v.* Hill as follows: " A riparian proprietor can have no larger right than he has by nature against those above and below him. Hence the right to have a stream flow in its natural state without diminution or alteration is an incident to the property in the land thru which it passes." The doctrine as commonly understood and interpreted is: " All persons owing land abutting on a natural stream have the right to demand that the waters of that stream shall pass their lands undefiled in quality and undiminished in quantity."

The doctrine of riparian rights is negative and cannot be supported on arguments dealing with the beneficial use of water or public interests. It came into being before streams were of much importance in England save for navigation, and at a period when public interests were not considered. The English colonies have abrogated the doctrine wherever streams are of importance to the welfare of the people, especially where irrigation is an essential to agriculture. The defects of the doctrine were not soon discovered in the United States. The eastern part of the United States does not differ greatly from England in so far as the use of streams is concerned. Water resources were not appreciated during the early history of the United States. As the waters of streams and lakes were diverted for beneficial use conflicts arose between users, and the courts were appealed to for a definition of the rights concerned. The courts have had an impossible task and the record of their inability to administer a resource which must rest on technical judgment may be found among the archives of nearly every tribunal of justice.

Principle of Prior Appropriation. When gold was discovered in California the miners established rules of their own making. The first man to stake out a claim had the right to the ground. He posted a notice which defined the limiting boundaries of his claim. Since most of the mining industry of California in the early days depended upon the use of water, some rule was necessary to define their rights to the streams. They naturally accepted the same rule that applied to their claims, that is the first to use water had the first right to the stream. This was the birth of the doctrine of priority. This doctrine is now accepted as one of the fundamental principles in the arid districts of the United States.

California Legislation. In 1873 the legislature of California passed an act which recognized the doctrine of prior appropriation and also provided for the posting of a notice at some conspicuous place on the river bank setting forth

the claims of the prospective water user. In 1878 California created the office of State Engineer, who was presumed to have general supervision of the streams. Some time later a contest was waged between the doctrine of appropriation and the doctrine of riparian rights. The leading case, Lux v. Haggin (69 Cal. 255, at 398 et seq., 10 Pac. 674) was decided by the Supreme Court of California in 1884, upholding the common law doctrine. This decision has retarded development in California and prevented the enactment of legislation which might have brought about an effective administration of water resources.

Colorado Legislation. Colorado adopted the best features of the California legislation and was the first state to abrogate the doctrine of riparian rights. The office of State Engineer was created in 1881. Claims to water are made by filing in the office of the county clerk, the courts determining the rights accruing under the claims. This system has resulted in expensive litigation, and unnecessary delay in the settlement of claims. The main features of the Colorado law were adopted by other states and in many cases an attempt was made to enforce its provisions and at the same time accept the doctrine of riparian rights.

The Wyoming Water Laws. Laws very similar to those of Wyoming have been adopted by practically every western state with the exceptions of California, Colorado, Montana and Washington. The Province of Alberta, Canada, has also adopted the principal features of these laws. The Wyoming water laws represent the first attempt to place the control of water in the state. They were drafted by Dr. Elwood Mead, Territorial and later State Engineer. The following clauses were placed in the constitution of Wyoming:

ARTICLE I

"Sec. 31. Water being essential to industrial prosperity, of limited amount, and easy of diversion from its natural channels, its control must be in the state, which, in providing for its use, shall equally guard all the various interests involved."

ARTICLE VIII

"Sec. 1. The water of all natural streams, springs, lakes or other collections of still water, within the boundaries of the state, are hereby declared to be the property of the state."

"Sec. 2. There shall be constituted a board of control, to be composed of the state engineer and the superintendent of the water divisions; which shall, under such regulations as may be prescribed by law, have the supervision of the waters of the state, and of their appropriation, distribution and diversion, and of the various officers connected therewith. Its decisions to be subject to review by the courts of the state."

"Sec 3. Priority of appropriation for beneficial uses shall give the better right. No appropriation shall be denied except when such denial is demanded by the public interests."

"Sec. 4. The legislature shall by law divide the state into four (4) water divisions, and provide for the appointment of superintendents therefor."

"Sec. 5. There shall be a state engineer who shall be appointed by the governor of the state and confirmed by the senate; he shall hold his office for a term of six (6) years or until his successor shall have been appointed and shall have qualified. He shall be president of the board of control, and shall have general supervision of the waters of the state and of the officers connected with its distribution. No person shall be appointed to this position who has not such theoretical knowledge and practical experience and skill as shall fit him for the position."

Under the law which was enacted by the first legislature of Wyoming, the board of control determines all rights to the use of water. Litigation has practically disappeared. Every water user knows the limits of his rights and the state takes the initiative in their determination and protection. But few appeals have been taken from the board and it has been upheld in practically every case. To initiate a right, an application for permit is filed with the state engineer. If approved, this becomes a permit and the date of the

priority is fixed by the time of filing the application in the office of the state engineer. When the irrigation or other works are completed the proper division superintendent takes proof of the water user and a certificate of appropriation is issued. This is the final guarantee of the right by the state. The state requires complete plans to be filed showing the details of all construction that is contemplated. Where dams over 5 ft in height are to be built across the channels of running streams or over 10 ft in height elsewhere, the state may appoint an engineer to supervise construction—the builder paying the cost of this work. Some of the fundamental principles embraced in the law are contained in the sections which follow:

"Sec. 724. A water right is the right to use the water of the state, when such use has been acquired by the beneficial application of water under the laws of the state relating thereto, and in conformity with the rules and regulations dependent thereon. Beneficial use shall be the basis, the measure and the limit of the right to use water at all times, not exceeding in any case, the statutory limit of volume. Water being always the property of the state, rights to use shall attach to the lands for irrigation, or to such other purpose or object for which acquired in accordance with the beneficial use made and for which the right receives public recognition, under the law and the administration provided thereby. Water rights cannot be detached from the lands, place or purpose for which they are acquired, without loss of priority."

"Sec. 725. Water rights are hereby defined as follows according to use: Preferred uses shall include rights for domestic and transportation purposes; existing rights not preferred, may be condemned to supply water for such preferred uses in accordance with the provisions of the law relating to the condemnation of property for public and semi-public purposes. Such domestic and transportation purposes shall include the following: First—water for drinking purposes for both man and beast. Second—water for municipal purposes. Third—water for the use of steam engines and for general railway use. Fourth—water for culinary, laundry, bathing, refrigerating (including the manufacture of ice), and for steam and hot water heating plants. The use of water for irrigation shall be superior and preferred to any use except where turbine or impulse water wheels are installed for power purposes."

The four water divisions are further subdivided into water districts. In these smaller areas water commissioners have charge of the diversion and division of water, under the supervision of the division superintendent. The water commissioners have police powers.

30. Irrigation and Drainage Development Acts

State and National Laws have been enacted for the purpose of encouraging or assisting irrigation development in the United States. There has been no national drainage legislation, but drainage statutes have been enacted by most of the states. Under certain conditions more effective results could doubtless be obtained thru government legislation, especially for the reclamation of river bottom lands along interstate streams where drainage is incidental to flood prevention and the problem becomes a matter of national as well as local importance.

The Desert Land Act, approved in 1877, represents the first legislation enacted for the purpose of stimulating irrigation development. Like all development acts of this class, it provides a method of securing title to public lands, making their reclamation thru irrigation, one of the requisites for patent. The entryman, under this act, must comply with the water laws of the state and proof of irrigation in compliance with such laws must be submitted. The desert land act originally permitted an entryman to secure 640 acres by irrigating at least 5 acres in each legal subdivision of 40 acres. Since lands have become more valuable, Congress has limited the entry to 160 acres and the entryman is required to reclaim all of the irrigible lands of the entry.

The Irrigation District Law originated in California in 1887. A certain district or area is defined by the people interested in irrigation development and under the law an organization is effected which has some of the power and

authority of a municipality. Bonds are issued for the purpose of financing the construction of irrigation works. The lands of the district are held as security for the bonds. The officers of the district comply with the laws of the state with regard to water rights. The state exercises no supervision beyond this point.

The Carey Act was authorized by Congress in 1894. Under its provisions lands are donated to the states under the condition that the states provide for their reclamation. In the first measure passed by Congress, one million acres of land were donated to each arid state which elected to accept the imposed responsibility by legislative enactment. The states are authorized to contract with companies which perform the construction that is required, and then dispose of interests in the same (commonly referred to as water rights) to settlers in accordance with the area irrigated by each freeholder. At a certain stage in development, usually when the irrigation works are completed, the lands are patented to the state and as settlement takes place, and freeholders pay their installments to the constructing company and water is beneficially applied, the state patents the land to the settler.

The United States Reclamation Act was authorized by Congress in 1902. It provides that all revenues derived from the sale and rental of public lands shall be employed in the construction of irrigation works designed for the reclamation of arid lands. The United States Reclamation Service was organized gradually to take charge of the work. Projects are carefully studied and the irrigation works are then designed and laid out. When the works are finished settlement is invited. Settlers acquire lands in areas that are estimated to support a family comfortably. Under some projects the farm areas are as small as forty acres. These are entered under the homestead act and the settler is given 20 years to complete his payments toward a proportionate interest in the irrigation works. Water users' associations are formed under each project and as the settlers make their payments they become active participants in the management of the irrigation works. Finally, the government withdraws and leaves all responsibility in the hands of the local organizations.

The earliest drainage law enacted in the United States was by the colony of New Jersey in 1772. Statute books of all the states have subsequently been filled with similar legislation. In 1850 the Swamp Land Act of the Federal Government patented to the Public Land States swamp land now aggregating over 63 000 000 acres. The Act contemplated the reclamation of this land by the states as an incident to the perfection of good title. In the interval the states have parted with most of this land to railways, corporations, and individuals, and little or nothing has been undertaken toward its reclamation.

District Drainage Laws enacted in various states are generally based on the following fundamental principles of self-government, namely, (1) the consent of the majority of the interested landowners is necessary before a drainage district can be organized; (2) the drainage district cannot be organized unless it is clearly shown that the benefits to be derived will exceed the cost; (3) no drainage district can be organized unless it is shown that the work will be conducive to public health or general welfare. In general, the state laws require presentation of a petition by interested land owners setting forth the necessity and nature of the improvement accompanied by a bond. This petition is in some states presented to the county commissioner; in others to county supervisors or judges of the county court.

The drainage laws of some states provide for the appointment of county drainage commissioners and surveyors, in others for the appointment of a competent engineer. In some states the laws are based primarily on the idea of drainage for sanitation and the appointment of a sanitary or drainage commissioner for the counties. In others, drainage districts are created from time to time by statute, and provide for drainage commissions and for boards of assessors.

31. Cost and Value of Project

Cost Estimates. A preliminary estimate is usually prepared for the purpose of studying the feasibility of an undertaking. The final estimate which is used as a basis for letting construction contracts and other negotiations should be based upon a thoro investigation of the project and should consider all costs and expenses from the inception of the project to the time when it becomes a going concern. Cost estimates for land reclamation projects may be divided into four divisions: preliminary costs, the cost of construction, interest on investment, and the cost of colonization.

Estimates of Preliminary Costs include preliminary surveys and investigation, financing, rights of way, water rights, damages, legal fees, and all expenses incurred in perfecting rights and titles so that nothing will arise later to interfere with the progress of the work or the rights of settlers. Bond discounts may properly be considered a part of the expense of financing. Legal complication or expenses likely to be incurred from the diversion of waters, deflecting the course or modifying the flow of natural channels, should be investigated with particular care.

Estimates of Cost of Construction include the costs of all labor, supplies and equipment required for building the project. They should be liberal and make proper allowance for unforeseen contingencies such as unusual floods, and possible rise in the price of labor and cost of materials. Estimates of the cost of structures and excavation should be based upon a reasonable knowledge of foundation conditions and the character of material to be excavated. Unexpected difficulties are usually encountered in excavating under water, and these should be allowed for in making estimates. The cost of engineering, which includes final surveys, plans and supervision will usually range from 5 to 10 percent of the construction cost.

Interest on Investment, which begins with the first expenditures, increases in amount until the project is completed, and remains a charge against the cost of construction until it can be carried by revenues received from the project. As each tract of land is settled it carries its proportionate share of the interest charge. In order to reduce this expense to a minimum the construction work should be carried forward without undue delay and an active campaign for colonizing the land should be undertaken at as early a date as practicable.

The Colonization of a reclaimed area is an important feature of its development and an independent advertising and selling organization is usually advisable. Frequently the colonizing of a project is given out by contract, at a stipulated price per acre, to a colonization company, the commission being deducted from the revenues received from sales. No matter how attractive a newly reclaimed project may be, its advantages must be advertised and opportunities for showing the land must be provided. Frequently free railroad transportation from long distances is furnished by colonizing agencies to prospective settlers. It is important that a project should be settled and put under cultivation with as little delay as possible, in order to reduce interest charges and secure prompt returns on the investment. For this reason a vigorous colonizing campaign, with a view to settling the greater part of the project in a comparatively short time is generally advisable even tho it may require a larger expenditure than would be necessary if the settlement were extended over a longer period of time. The cost of colonization may vary from \$5 to \$25 per acre, depending upon the desirability, location, accessibility, and price of the land as well as the efficiency of the organization and general business conditions.

The above comments refer more particularly to projects developed by private capital, but in general the same principles apply to government reclamation work. Under

the terms of the U. S Reclamation Act, the reclamation fund is available without interest and the expenses resulting from delays in construction and colonization, tho just as real, are not so readily apparent. The government has provided no method of effectively advertising and colonizing its projects, and much of the available reclaimed area has not been settled.

The Value of Raw Land after reclamation will depend upon the fertility of soil, the kind of crops to which it is adapted, market conditions, transportation facilities, the efficiency of reclamation works, temperature, precipitation, length of growing season, and general business conditions. The safest basis for estimating the value of land is the market price of similar lands in the same or neighboring localities, making proper allowance if necessary for the expense of subjugating wild land.

The Cost of Reducing Raw Land from its wild state to the condition where it can be profitably cultivated may be considered as a part of the total cost of reclamation. If, as is usually the case, the farmer bears the expense of preparing his land for cultivation, the cost of this work in addition to the difference between the value of crops that might be expected from the land if thoroly subjugated, and those actually obtained while the land is being brought to a condition of profitable cultivation, should be deducted from the market price of similar cultivated lands in the locality to obtain the approximate value of the raw land. The three principal items of cost are: Clearing the land of native vegetation, grading, and putting in and successfully growing the first crops.

The Value of a project before reclamation may be estimated by deducting the estimated investment required for development from the estimated value of the land after reclamation. This difference may also be considered as the estimated profit from the investment.

Reclamation Projects as Security for Investment. After a reclamation project is completed and colonized and in successful operation it furnishes ideal security for investment. It is permanent and continually increases in value as improvements are made and the land is brought to a higher state of cultivation. Such security is comparable to a farm mortgage. A proposed or partially completed project, however, has a market value equal only to the value of the raw land which it embraces, and it is only within this limit that it provides safe security for an investment. The securities usually issued for the development of land reclamation projects are common and preferred stocks and bonds.

Projects may be financed with stock alone, with bonds alone, or with stock and bonds. If only stock is issued, the stock represents the actual investment, and the risk of the venture is shared by all of the stockholders. If only bonds are used for obtaining funds it is usual to issue common stock to the amount of the bond issue, the former representing the speculative feature of the investment. A large part of this stock is then distributed with the bond buyers as a bonus, the remainder being used in promoting the enterprise. Financing with stock and bonds requires an actual investment in stock which places the bonds on a sounder basis.

Failure of Reclamation Projects may result from improper engineering or poor business management. Engineering failure may result from under estimating construction costs, improper designs, or, in the case of irrigation projects, over-estimating the available water supply. From the business standpoint, failure may be caused by extravagance in the use of funds, inefficient management, inadequate arrangements for financing, colonization difficulties, or the short-sighted policy of putting an exorbitant price on land. The revenue from every project is dependent upon its successful colonization, and this not only requires that the project be settled with a good class of people, but that they be maintained in a prosperous and contented condition. Reclaimed lands are usually sold on the partial payment plan, the initial payment being frequently 25 percent of the selling price, with equal annual payments

and interest on deferred payments until the total amount is paid. It is a common experience on newly reclaimed areas that the settlers do not appreciate the expense or time required to subjugate raw land and having invested too much of their capital, either because of paying an exorbitant price for their land or buying too large an acreage, they do not have enough money left to develop their land, and as a result they are not able to make deferred payments and the land goes back to the original owners.

32. Operation and Maintenance

Organization. The right to participate in the benefits of an irrigation or drainage system is inherent with every tract of land which it embraces. This implies that every land owner within a project is entitled to a voice in its management. In some instances reclamation works are owned and operated by companies who contract with land owners to provide the benefits of the system at a fixed annual rate. More commonly the ownership of reclamation works ultimately passes to an organization composed of the land owners each of whom participates in the operation of the system proportionately to the area of land which he owns. This may be accomplished by forming a stock company in which 1 share of stock is issued for each acre of land. Such an organization has general direction of the operation of the system.

Water Users' Associations are formed by the water users of the projects of the U. S. Reclamation Service as soon as it appears probable that a project is to be constructed. The object of these associations is to facilitate dealings with the Department of the Interior and, after the construction work is completed, to participate in the operation of the project. The ownership, and operation of storage works remains in the hands of the Government while the distribution of water is left to the management of the Water Users' Associations.

The Operation of a reclamation system may be divided into three general divisions: (*a*) The operation proper, which includes the operation of pumps and other machinery, and for irrigation projects the delivery and distribution of water; (*b*) maintenance; and (*c*) clerical work. The force necessary will depend upon the size of the project. The organization should be such as to provide for effective and efficient service from the system, to keep the project maintained in a satisfactory operating condition, and to keep all records and accounts in a proper manner. General supervision will rest with the board of directors, or its equivalent, who determine the general policies of the system. Direct supervision is under a general manager, who is the responsible head.

The Head of the Operating Department of a drainage project may be the chief engineer of the pumping plant. On irrigation projects he is the water superintendent, and has charge of the distribution of the irrigation water. Under the superintendent are ditch riders who apportion the water among the various users. One ditch rider can oversee the distribution of water for 2000 to 3000 acres, where water is controlled at the farmers' headgates. If delivered to lateral headgates he serves from 5000 to 10 000 acres. The water superintendent should be thoroly versed in the principles of water measurement and the ditch riders should be sufficiently skilled to be able to make an equitable distribution of the water.

Maintenance as applied to reclamation projects includes the repairs, and labor necessary to keep the system in successful operation. Repairs to canals or structures damaged by floods, cleaning canals, replacement of broken parts of machines and similar work belong properly to maintenance, and it may also include minor extensions and betterments and the replacement of inexpensive structures. A regular maintenance force should be employed to keep the system

in satisfactory operating condition, except in case of unusual accidents, in order that the expense of maintenence for different years may, as nearly as practicable, be uniformly distributed.

Depreciation. There is a natural limit to the life of many of the parts of a reclamation system. Canals, levees, earth and rock dams and similar structures if properly maintained, and not accidentally destroyed should last indefinitely. Also it cannot be said that there is any limit to the life of many properly constructed concrete or masonry structures. Wooden and steel structures, however, will eventually deteriorate to such a state that they can no longer be maintained in a satisfactory working condition. The depreciation of the different parts of a reclamation system should be estimated in advance and provisions for replacements should be made by an amortization fund created by equal annual contributions, which are included as a part of the operating expense.

The Average Life, in years, of various parts of reclamation systems is approximately as follows:

Wooden flumes, redwood	15 to 20
Wooden flumes, fir	12 to 15
Wooden flumes, pine	8 to 10
Wooden stave pipe, fir, uncoated	12 to 20
Wooden stave pipe, fir, wellcoated	20 to 25
Wooden stave pipe, redwood, uncoated	20 to 25
Wooden stave pipe, redwood, wellcoated	25 to 30
Miscellaneous small wooden structures	10 to 20
Steel riveted pipes, uncoated	20 to 30
Steel riveted pipes, wellcoated	30 to 40
Centrifugal pumps	12 to 20
Steam engines	15 to 25
Boilers	12 to 20
Crude-oil engines	10 to 15
Pole lines for telephones or transmission	10 to 15
Electric motors	15 to 20
Buildings and improvements	25 to 75

The life of a structure depends largely upon the care that is taken to preserve it and the conditions of use to which it is subjected. Wood that is alternately wet and dry will deteriorate more rapidly than if kept continually wet. Structures above water may be preserved by keeping them well painted. Wood or steel buried in the ground or submerged or in contact with water for considerable periods will last longer if well coated with asphaltum or tar. The following, relative to the life of wood stave pipe (by D. C. Henny, Reclamation Record, August, 1915), was collected from a large number of installations:

(*a*) Under favorable conditions of complete saturation, fir well coated may have the same life as redwood uncoated.

(*b*) Either kind of pipe will have a longer life if well buried in tight soil than if exposed to the atmosphere. Such life may be very long, 30 years or over, if a steady pressure is maintained.

(*c*) Either kind of pipe will have a longer life if exposed to the atmosphere than if buried in open soil, such as sand and gravel and volcanic ash, provided in a hot or dry climate it be shaded from the sun.

(*d*) Under questionable conditions, such as light pressure or partially filled pipe, fir, even if well coated, may have only one-third to one-half the life of redwood.

(*e*) Under light pressure the use of bastard staves should be avoided.

(*f*) The use of wooden sleeves in connection with wire wound pipe is objectionable and has caused endless trouble and expense.

(*g*) If wooden sleeves are employed they should be provided at least for sizes from 10 in up with individual bands to permit taking up leaks.

Annual Cost of Operation includes all expenditures required to secure the benefits from the reclamation works and maintain them in working condition.

There is a wide variation in costs for different projects. The following represent the annual range of costs on a large number of projects:

Cost per acre of distributing water	$0.40 to $1.00
Cost per acre of maintenance	.50 to 1.20
Cost per mile of canal of maintenance	50.00 to 125.00
Cost per acre of general expense	.40 to .80
Cost of pumping water. (See Art. 18.)	
Cost of depreciation; each part of system must be considered separately.	
Total cost per acre of operation	2.00 to 4.00

Other costs are taxes, insurance, interest on investment and sinking fund, any or all of which may be included as a part of the annual expense.

The following is an estimate of the annual cost of irrigating 40 000 acres of land in southern Texas by pumping. It is based upon a total yearly water consumption of 120 000 acre-feet pumped against an average head of 20 ft. It does not include taxes, insurance, interest on investment or sinking fund:

Pumping Plant			
Depreciation, 7½ percent of $145 000		$10 875	
Labor			
3 enginemen	$4 200		
10 firemen	3 650		
Day labor	2 400		
		10 250	
Fuel, 40 000 bbls oil at $1.50		60 000	
Incidental supplies and repairs		3 000	
			$84 125
Canal System			
Depreciation of flumes, 7½ percent of $40 000		$3 000	
10 ditch riders at $900		9 000	
1 superintendent (civil engineer)		2 400	
1 assistant		1 500	
Cleaning canal system		25 000	
Renewals and maintenance, structures		8 000	
Miscellaneous		3 000	
			51 900
General			
Office rent		$1 200	
General manager		6 000	
Assistant		1 800	
2 clerks		2 400	
2 automobiles, operation and depreciation		2 000	
Miscellaneous expense		5 000	
			18 400
Total annual expense			$154 425

33. Engineering Reports

Purpose. A report on a land reclamation project should set forth, in clear and concise form, the essential characteristics of the enterprise. All data on which conclusions are based should be given, in order that the report will withstand a critical inspection by other engineers, but the controlling features, on which the feasibility of the project depends, should be intelligible to any good business man. Voluminous details, such as run-off, temperature, and pre-

cipitation records may be included in an appendix. Reports are always required in connection with the financing of reclamation projects, and their value in this regard may depend, in a large measure, on the standing and reputation of the engineer who makes them. In order to bring a project properly before investors, the promoters of the enterprise should have the report of a reputable engineer. Financial concerns, before investing, ordinarily require an independent report from their own engineer. Frequently several reports are made before a project is financed. Reports may be made at intervals during construction to determine whether work is progressing satisfactorily and in case of failure the creditors may require a report to determine the status of their securities.

An Investigation must precede the preparation of a report. This includes a field examination and a search for all data bearing on the development or success of the project. Ordinarily all surveys and other data of the field engineers will be available and these should be scrutinized with sufficient care to justify conclusions as to their reliability. Information relative to land values, crops, market conditions and similar matters may frequently be obtained from interviews with reliable persons, but data obtained from such sources should be accepted with caution. Government reports should be used if possible for securing temperature, precipitation, and stream flow data. Information obtained from investigating other projects in the locality is always valuable. Frequently additional surveys, test pits or other work will be required, but these should not be of extensive character. If data sufficient for an intelligent report are not available, this fact may be stated in the report, or the investigation may be postponed until full and complete data are secured. The field investigation ordinarily should not require more than one or at the most two weeks.

The Contents of a Report should be such as will set forth clearly the feasibility of the project from both engineering and business standpoints. In general, the main points to be shown are: (*a*) the efficiency of the reclamation system, (*b*) the cost of reclamation, and (*c*) the value of the project after reclamation or the net profit to result from the development of the project. The material should be presented in logical order and each subject should be discussed under an appropriate heading. The following are headings, with outlines of subject matter to be treated under each. They will have to be modified to suit the particular conditions of different projects:

(1) **Introduction.** There should be a few introductory statements leading up to the main body of the report. These may include the authority for making the report, the data on which the report is based, the engineer's opinion of the reliability of such data, the period covered by the investigation, and other general information of a similar nature.

(2) **Location.** It is important that the general location of the project should be described. Principal boundaries, railway and water transportation facilities, and distances to the more important markets should be given. A small scale general map showing this and other information will be valuable.

(3) **General Description.** A general description of the project should be given before entering into details. This may include important topographical features, the area of the project, and a comprehensive outline of the scheme of reclamation. A map of the project showing the general plan of reclamation should be included with the report and referred to in the discussion.

(4) **Climate.** Since the growth and time of marketing crops depends in a large measure on climate, this subject may be discussed in considerable detail. Tables may be prepared from government records showing maximum, minimum, and mean temperatures and amounts of precipitation for the different months of the year. Dates of last killing frosts in the spring and first killing frosts in fall should be given.

(5) **Soil.** The soil formation of the project from the standpoints of texture, fertility, adaptability to particular crops, and drainage conditions should be discussed. If the

soil formation varies greatly a soil map may be included. Government and state publications give valuable data relative to soils for many localities.

(6) **Crops.** Information as to the kinds, yields, and values of crops to which the lands of the project are particularly adapted should be included in the report. The costs of growing, the times of marketing, and the net returns to be expected from producing different crops may also be given. Information of this kind may frequently be shown to advantage in tables.

(7) **Markets.** The market for crops is usually an important consideration, and this should be discussed. Transportation facilities and freight rates should be given consideration in this connection.

(8) **Water Supply.** The success of an irrigation project depends in a large measure upon the adequacy of its water supply and this subject should be treated with considerable detail. The two things to be determined are the water supply available and the water supply required, and the analysis of the problem should show the data used and the steps taken in arriving at conclusions in regard to these matters. The discussion may be incorporated under the following headings.

(8*a*) **Period of Irrigation** showing the time when different crops will ordinarily be irrigated and the number of irrigations for different crops, with conclusions as to the approximate distribution in the use of water.

(8*b*) **Water Requirements** for crops or duty of water based upon a study of the needs for irrigation of the crops likely to be grown on the project.

(8*c*) **Water Supply Necessary** for the project determined from considerations of plant requirements, actual area to be irrigated, and seepage and evaporation losses in canals and reservoirs:

(8*d*) **Water Supply Available,** determined from a study of stream flow and other hydrological data, with proper deductions for prior rights to water. This phase of the problem will require careful investigation and all conclusions should be supported by as many records as practicable. If such records are too voluminous to be incorporated in the body of the report they should be put in the appendix.

(9) **Run-off.** The run-off from an area to be reclaimed by drainage should be discussed in detail and all precipitation records and other data necessary to support conclusions should be given. If pumps are to be used at the outlet the reasons for selecting a pumping plant of a certain capacity should be fully explained.

(10) **Cost of Project.** An estimate of the cost of developing a project, up to the point where it will become a going concern, should be included in the report and the main elements entering into the cost should be discussed. Estimates of this class should be made on a liberal basis so that ample allowance will be made for all contingencies.

(11) **Annual Cost.** An estimate of the annual cost of operating the project should be submitted. This, like the estimate of cost, should be on a liberal basis.

(12) **Value of Land.** The various conditions affecting the value of lands after reclamation should be discussed and an estimate of values should be submitted. It is important to distinguish between the value of raw lands after reclamation and the value of cultivated lands.

(13) **Net Returns.** An estimate of net returns on the investment may be submitted. This will be obtained by deducting the total expense of reclamation from the estimated value of land.

(14) **General Conclusions.** A few short, clear-cut paragraphs summarizing the main points brought out in the investigation should be given at the beginning or at the end of the report. The conclusions should contain definite statements of opinion regarding the value of crops, the adequacy of water supply for irrigation projects, the estimated cost of reclamation, the cost of operation, the value of land, estimated returns from the investment, and other important features. In some instances the report may be concluded with the engineer's recommendations.

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